

**MODELING AND IDENTIFICATION
OF FLEXIBLE JOINTS IN VEHICLE STRUCTURES**

by

Kwangju Lee

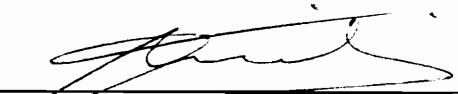
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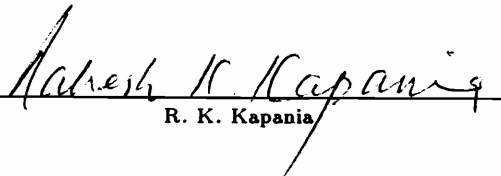
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(ABSTRACT)

A simple, design-oriented model of joints in vehicle structures is developed. This model accounts for the flexibility, the offsets of rotation centers of joint branches, and the coupling between rotations of a joint branch in different planes. The model parameters consist of torsional spring rates, the coordinates of the flexible hinges, and the orientations of planes in which the torsional springs are located. The model parameters are selected to be physically meaningful. In some cases, the behavior of joints can be accurately represented by using simpler models. The conditions under which the joint model can be simplified are discussed. A family of joint models with different levels of complexity are also defined.

A probabilistic system identification is used to estimate the joint parameters by using the measured displacements. The parameters are estimated by minimizing the discrepancies between the measured and predicted displacements. Statistical tests which identify important parameters are also presented. These tests can be used to simplify the joint models without significantly reducing the accuracy in predicting structural responses.

The identification methodology is applied to automotive structures with joints and also to isolated subassemblies consisting of joints and attached branches.

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TABLE OF CONTENTS

1. Introduction	1
1.1 Background	1
✓ 1.1.1 Joints in Structures	1
1.1.2 System Identification	5
1.1.3 Definition of a Joint	8
1.2 Objectives	9
1.3 Outline	11
2. Joint Models	13
✗ 2.1 Behavior of Joints	13
2.2 Joint Parameterization – Full Model	17
2.3 Stiffness Matrix – Full Model	20
2.4 Joint Parameterization – Reduced Models	29
2.4.1 Reduced Model I	29
2.4.2 Reduced Model II	31
2.4.3 Reduced Model III	32
2.4.4 2-D Bracketed Model	32

3. Probabilistic System Identification	34
3.1 Nonlinear Weighted Estimation (Classical Estimation)	35
3.2 Bayesian Estimation (Recursive Estimation)	38
3.3 Sources of Measurements	40
4. Identification of Important Parameters	43
4.1 Overparameterization and Underparameterization	43
4.2 Covariance Matrix of Parameter Estimates	45
4.3 Confidence Contours	46
4.4 Principal Direction Analysis	48
4.5 Lack of Fit Test	49
5. Reduced Model I	51
5.1 Classical Estimation	52
5.2 Bayesian Estimation	55
5.3 Principal Direction Analysis	57
5.4 Effect of Number of Measurements	59
5.5 Effect of Errors in Modeling the Overall Structure	60
6. 2-D Bracketed Model	63
6.1 Importance of Offsets of Rotation Centers	63
6.2 Bias in Parameter Estimates	65
6.3 Consideration of the Worst Loads	67
6.4 Effect of the Ratio of k_i/k_{ij}	68

7. 3–D Isolated Joints Consisting of Simple Box Beams	70
7.1 Description of the Subassemblies	71
7.2 Importance of Offsets of Rotation Centers	73
7.3 Effect of Panel Thickness	74
7.4 Effect of Member Length	76
8. 3–D Isolated Joint of a Real Automotive Structure	79
8.1 Description of the Subassembly	80
8.2 Identification of the Subassembly without Considering Coupling	82
8.3 Identification of the Subassembly Considering Coupling	84
8.4 Comparison of Joint Models under the Worst Loads	87
8.5 Effect of Rocker Length	90
8.6 Effect of Measurement Errors	94
9. Summary and Conclusions	98
References	102
Tables	106
Figures	134
Vita	176

LIST OF TABLES

5.1	Material and geometric properties of a car structure.	107
5.2	Classical estimation considering 4 joints, k^1, k^2, k^3 , and k^4	108
5.3	Classical estimation considering 3 joints, k^1, k^3 , and $k^4 (k^2 = \infty)$	109
5.4	Classical estimation using 6 measurements under load F_1	110
5.5	Bayesian estimation using the prior estimates in Table 5.4 and 6 more measurements under load F_2	111
5.6	Bayesian estimation using the prior estimates in Table 5.5 and 6 more measurements under loads F_2 and F_3 applied at the same time. . . .	112
5.7	Classical estimation using 18 measurements at the same time. Among 18 measurements, each 6 measurements were obtained under each of loads F_1, F_2 , and F_2 and F_3	113
5.8	Classical estimation of 3 parameters, k^1, k^2 , and k^3 at a 3-branch joint. . . .	114
6.1	Actual values of joint parameters for a car model in Figure 6.1.	115
6.2	The results of estimation for different actual values of offset length.	116
7.1	The results of estimation of a T-shaped subassembly of a joint and attached branches.	117
7.2	The changes in on-branch offsets and WSOS with change in the rocker panel thickness.	118
7.3	The changes in the joint model parameters and WSOS with change in the B-pillar length.	119
7.4	The changes in on-branch offsets with change in the rocker length.	120

8.1	Material and geometric properties of the subassembly of the B-pillar to rocker joint and the attached branches.	121
8.2	The results of estimation for different values of the rocker length – Reduced model I.	122
8.3	The results of estimation for different values of the rocker length – Reduced model II.	123
8.4	The results of estimation for different values of the rocker length – Reduced model III.	124
8.5	The results of estimation for different values of the rocker length – Full model with fixed on-branch offsets.	125
8.6	Comparison of joint models under the worst loads that maximize the difference in strain energies when the rocker length was 474.8 (mm). . . .	126
8.7	Comparison of joint models under the worst loads that maximize the ratio of strain energies when the rocker length was 474.8 (mm). – from 6×6 flexibility matrices.	128
8.8	Comparison of joint models under the worst loads that maximize the ratio of strain energies when the rocker length was 474.8 (mm). – from 5×5 flexibility matrices.	129
8.9	The results of estimation for different magnitudes of measurement errors when the rocker length was 1138.3 (mm) – Reduced model II.	130
8.10	The results of estimation for different magnitudes of measurement errors when the rocker length was 1138.3 (mm) – Full model with fixed on-branch offsets.	131
8.11	The coefficients of variation of the parameter estimates when the rocker length was 1138.3 (mm) – Reduced model II.	132
8.12	The coefficients of variation of the parameter estimates when the rocker length was 1138.3 (mm) – Full model with fixed on-branch offsets.	133

LIST OF FIGURES

1.1	A joint in a car structure.	135
2.1	Finite element joint models.	136
2.2	The behavior of a joint branch which is described using a concept model. . . .	137
2.3	Coupling between rotations of a branch in different planes.	138
2.4	Coupling between twist rotation and linear displacements.	139
2.5	A joint element – Full model.	140
2.6	A joint with 3 branches in a plane – Reduced model I.	142
2.7	A joint with 3 branches in a plane – Reduced model II.	143
2.8	A joint with 3 branches in a plane – 2-D bracketed model.	144
3.1	Bayesian estimation.	145
5.1	A 2 dimensional car model with 4 joints.	146
5.2	Measurements used to estimate parameters at 4 joints.	147
5.3	Sectional drawing of confidence contour in $k^1 k^2$ -plane.	148
5.4	Predicted displacements with a simplified model (k^1, k^3 , and k^4).	149
5.5	A 2 dimensional car model with a 3-branch joint.	150
5.6	Effect of number of measurements on the estimation results.	151
5.7	Normalized parameter estimates and WSOS (weighted error sum of squares) with error in the stiffness of roof elements.	152

5.8	Normalized parameter estimates and WSOS with error in the stiffness of B-pillar elements.	153
5.9	Normalized parameter estimates and WSOS with error in the stiffness of the whole car structure.	154
5.10	Displacements predicted with error in the stiffness of the whole car structure. .	155
6.1	A 2 dimensional car model with 4 joints of 2-D bracketed model.	156
6.2	Normalized WSOS for different models.	157
6.3	Variation of bias in parameter estimates as a function of the offset length. .	158
6.4	Displacements predicted with different models when offsets at the second joint, J_2 , are 50.8 (mm).	159
6.5	Normalized WSOS for different ratio of k_i/k_{ij} and for different reduced models.	160
7.1	A T-shaped subassembly made of two box beams.	161
7.2	Concept model of the subassembly in Figure 7.1.	162
7.3	Detailed finite element model of the subassembly in Figure 7.1.	163
7.4	Predicted displacements in F/A direction.	164
7.5	Predicted displacements in I/O direction.	165
7.6	Predicted twist rotations.	166
7.7	Variation of stiffness in F/A direction with change in rocker panel thickness. .	167
7.8	Variation of stiffness in I/O direction with change in rocker panel thickness. .	168
7.9	Variation of twist stiffness with change in rocker panel thickness.	169
7.10	Variation of stiffnesses of three rotational modes with change in rocker length.	170

8.1 Detailed finite element model of a subassembly which consists of
a B-pillar to rocker joint and attached branches. 171

8.2 Predicted displacements in F/A direction under a force applied
in the same direction. 172

8.3 Predicted displacements in I/O direction under a force applied
in F/A direction 173

8.4 Portions of total strain energies due to force components
under the worst loads, which are normalized using
total strain energies of the detailed model. 174

8.5 Predicted displacements in I/O direction under a force applied
in the same direction. The parameters were estimated from subassemblies
with different rocker length. 175

LIST OF SYMBOLS

$\mathbf{b}$	=	vector of the parameter estimates
$\mathbf{C}$	=	covariance matrix
$\mathbf{f}$	=	force vector
h	=	off-branch offset for rotation center of a joint branch
$\mathbf{K}$	=	global stiffness matrix
$\mathbf{K}_J$	=	stiffness matrix of a joint
$\mathbf{K}_R$	=	stiffness matrix of the remaining vehicle structure without joints
k_i	=	joint stiffness of due to the flexibility of each branch
k_{ij}	=	joint stiffness due to coupling between rotations of different branches
n	=	number of measurements
p	=	number of parameters
p_i	=	i-th spring axis
q	=	probability level used in statistic test
r	=	on-branch offset for rotation center of a joint branch
$\mathbf{u}$	=	vector of displacements
$\mathbf{u}_m$	=	vector of measured displacements
$\hat{\mathbf{u}}$	=	vector of estimated displacements
WSOS	=	weighted error sum of squares or weighted sum of squares of errors
$\mathbf{x}$	=	vector of regressor variables

R	=	correlation matrix
Z	=	matrix of sensitivity derivatives of displacements with respect to parameters
β	=	vector of the parameters
γ	=	vector of transformed parameters in principal direction analysis
ϵ	=	vector of measurement errors
ρ	=	coefficient of variation
σ	=	standard deviation
ϕ_{ij}	=	angle between i-th spring axis and j-th global axis

subscripts and other notations :

b	=	parameter estimates
F/A	=	fore/aft (inplane) direction
I/O	=	inboard/outboard (out-of-plane) direction
T	=	twist
ϵ	=	measurement errors
$\hat{\cdot}$	=	estimates

1. INTRODUCTION

Very detailed finite element models of vehicle structures are used to predict their static and dynamic structural response. However, for preliminary design, it is more reasonable to employ simple, design-oriented models. These models should be inexpensive to analyze so that many design iterations can be executed and they should be defined in terms of a small set of design variables to further alleviate the designer's job. These models are called "concept models". One of the major difficulties in developing and employing a concept model of a vehicle structure is to have a simple and accurate model of joints at body connections. The objectives of this dissertation are to develop a simple, design-oriented model of joints and to develop a method for identifying the model parameters.

1.1 Background

The modeling and identification of flexible joints has been studied by various researchers in several engineering fields. However, only a few articles are released for publication due to the practical importance and competition in the related industry.

1.1.1 Joints in Structures

Joints in vehicle structures are formed by overlapping metal sheets fastened by spot or other types of welds. An example of a joint in a car structure is shown in Figure 1.1. The structural behavior of joints is complicated and, therefore, it is difficult to describe

it by concept models. It is generally accepted that the flexibility of joints may dominate the structural response. Chang [1974] used torsional springs to model joints in automotive structures, and he demonstrated that the joint flexibility is important. Chon [1986] proved the existence of upper and lower sensitivity thresholds for the parameters of a joint which is modeled by torsional springs. More specifically, he demonstrated that if a joint is too stiff or too flexible in rotation in a particular plane compared to the remaining structure, then it can be considered as rigid or free to rotate, respectively. In this case, a joint model can be simplified by neglecting those parameters. Borowski et al. [1973] used translational and torsional springs to represent the flexible joints in car frames. He determined the unknown values of the spring stiffness parameters by tuning them so that theoretical predictions match dynamic test results. Garstecki [1988] studied optimal design of joints in linear elastic beams and frames. Ioannidis and Kounadis [1988] studied the effect of joint flexibility on the nonlinear buckling load of metal portal frames.

Joints have been also studied experimentally as well as analytically by the analysis of detailed finite element models. The observations made in these studies are important in the development of concept models. Garro and Vullo [1986] demonstrated that when joints in automotive structures deform, the plate edges, which are facing each other along the weld spots, tend to detach from each other. Sunami et al. [1987,1990] studied the behavior of isolated T- and L- shaped joints made of two simple box beams under in-plane and out-of-plane bending, respectively. They demonstrated that for in-plane bending the deformation mechanism consists of three components that are called release, shear deformation, and elongation. Release is the increase of the turning angle of a joint branch, which is caused by the nonuniform normal stress distribution along a line where two surfaces of the connecting

branches meet. According to Sunami et al., rotation of a joint branch is also caused by shear deformation of the core of the joint and the elongation of the sides of the core that are induced by the applied moment. The flexibility of T-shaped joints subjected to out-of-plane bending is also due to the shear deformation of the cross section of the horizontal member. Sunami et al. also found that bulkheads, which are plates inserted inside the cross sections, are effective in increasing the joint rigidity because they prevent release and cross sectional shear deformation. Hughes [1988] also found that the flexibility of joints in ship structures is due to shear deformation of their core. Sakurai and Kamada [1988] studied a T-shaped joint consisting of box beams. More specifically, they investigated the effects of the panel thickness, bulkheads, and service holes on the joint stiffness. They found that it is more effective to increase the joint stiffness by increasing the thickness of the outer panel of the horizontal member than that of its inner panel because the vertical member is connected with the outer panel in their joint. Rao et al. [1983] identified a range of joint stiffness values which provide an automotive structure with an acceptable NVH (Noise, Vibration, and Harshness) response. These ranges of values can be used as design targets in the early design stages of a new car model.

Previous studies on concept models of joints have usually assumed that the joint branches rotate about the center of the joint, which is defined as the intersection of the shear axes of the connecting branches when all these shear axes are located in the same plane [Lee and Nikolaidis 1991 A]. However, it has been experimentally observed that the rotation center of a joint branch does not coincide with the center of the joint. Rao et al. [1983] set up a test procedure to find the instantaneous rotation center by measuring displacements at different locations along a branch of a joint.

In some types of vehicle structures, joints are stiffened by brackets. Hughes [1988] developed models of joints in transverse frames of ships. His models are two dimensional, and besides the joint flexibility, they also account for the rigid behavior of the bracketed branches in the vicinity of joints. The joint branches rotate about the reference point of the joint and they are connected by torsional springs. Hughes demonstrated that standard frame analysis, which treats joints as rigid, leads to errors in the deflections, which might reach 50 % of the corresponding actual values. Furthermore, he also demonstrated that it is important to account for both the joint flexibility and for the rigid portions of its brackets.

Another characteristic of the joint behavior is the coupling between rotations of a branch in different planes. More specifically, a moment applied to a joint branch about a particular axis induces rotations about the other two axes which are normal to the direction of the applied moment as well as rotation in the direction of the applied moment. Gangadharan [1990] developed a joint model that considers coupling between rotations in different planes. The coupling was taken into account by treating some entries of the stiffness matrix of the joint as unknown parameters. Yamazaki and Inoue [1989] developed a joint model which also accounts for the coupling. In their model, for each joint branch, there are three springs which are located in mutually orthogonal planes. The coupling is accounted for by assuming that, for a joint branch, the orientations of the planes of the springs are arbitrary, and that they coincide with the principal planes corresponding to the three orthogonal mode shapes.

There is another kind of coupling which is between rotations of different connecting branches of a joint [Lee and Nikolaidis 1991 B]. This type of coupling is different from the coupling between rotations of a branch in different planes, which was described in the previous

paragraph. If a force is applied to a branch while clamping the center of a joint, the other branches may also deflect due to the coupling between rotations of different connecting branches.

There has been also work on joints in other types of structures. Bouwkamp et al. [1980] studied the effects of joint flexibility on the structural behavior of offshore towers. They found that the effects can be significant, especially in areas where the joint member lengths are relatively short. McGowan et al. [1987] found the importance of joint flexibility in spacecraft trusses. They used springs to account for the flexibility and they observed that the ordering of the vibration modes can switch as the spring stiffnesses are changed, which indicates the importance of correctly characterizing the structural properties at joints.

1.1.2 System Identification

Since this study employs system identification to identify and simplify the concept models of joints, some recent works in this area are reviewed. This review is limited to works in the identification of structural system and to the identification methodology which is generally applicable to different engineering areas. The objective of system identification is to identify a model that represents the behavior of a given system with acceptable accuracy. The basic idea is to find the values of model parameters so that some measure of the discrepancy between measurements and predictions is minimized.

System identification has many applications in dynamic models of structures. Baruch [1984], and Berman and Nagy [1983] developed system identification procedures to determine the parameters of a structural system from dynamic test results. They determined the parameters by identifying a set of minimum changes in the original stiffness or mass matrices

of the model that represents the system so that the eigensolution exactly agrees with test measurements. However, only in Baruch's method the rigid body modes of the identified system remain the same as those of the original system. Usually, it is difficult to measure the mode shapes of a vibrating system. Methods have been recently introduced for identifying the mode shapes from dynamic response measurements [Baruch and Khatri 1987]. Baruch and Itzhack [1978] corrected nonorthogonal modes of experimental vibration tests so that they satisfied the orthogonality requirement.

System identification has been also applied in other areas of structural analysis. Butkunas et al. [1986] successfully identified regions in models of automotive structures that should be changed so that finite element predictions exactly agree with experimental results. In this approach, the overall car model was calibrated so that theoretically predicted natural frequencies and mode shapes exactly match measurements. Hajela and Soeiro [1989] found that the response of a damaged structure was different from predictions obtained from an analytical model of the original structure. They used both eigenmodes and static displacements in the identification procedure. The identification of nonlinear models of bolted joints was studied by Gaul and Bohlen [1987]. They used measured hysteresis to find model parameters.

The errors in measuring responses were not considered in the aforementioned studies. By considering the measurement errors, the parameter estimates can be more reliable. The above two approaches are called deterministic and probabilistic system identification, respectively [Jazwinski 1970]. Collins et al. [1974] developed a probabilistic identification method that uses measurements of mode shapes and natural frequencies to estimate the parameters of a linear dynamic system. This method is essentially a sequential linearization

technique [Draper and Smith 1981, Bard 1974, Myers 1986]. A probabilistic approach for system identification of nonlinear systems, which employs a recursive filtering algorithm, was proposed by Yun and Shinozuka [1980]. In this approach, the model is identified by using measurements of the time history of the dynamic response of the system.

Recursive estimation [Mendel 1987], which falls in the category of probabilistic system identification, has been used and found to be efficient and effective in determining the parameters of a model. This technique, which is also called Bayesian estimation, can be used to identify the parameters of a system in a recursive way. In each step of this recursive process, the parameter estimates are updated by taking new measurements of the response of a system and by using the results from the previous cycle. Isenberg [1979] reviewed this type of estimation. Allen and Martinez [1989] integrated a commercial software package for finite element analysis (MSC/NASTRAN) and a computer code for Bayesian estimation of linear systems. Their computer code was used to successfully identify models of a truss structure and of an electronics package.

A comparison between deterministic and probabilistic approaches leads to a conclusion that the latter is more reliable than the former [Jazwinski 1970]. Probabilistic estimation considers the statistics of the measurement errors. More specifically, each measurement is weighted according to its accuracy, by scaling it with the inverse of the standard deviation of the measurement error. Moreover, probabilistic estimation provides measures of the errors in the parameter estimates that are derived from the statistics of the measurement errors. In the deterministic estimation, the statistics of the measurement errors are neglected. Thus, no information on the reliability of the parameter estimates can be provided.

1.1.3 Definition of a Joint

A joint is defined to be a region of a structure, which contains the intersection or junction of two or more load-carrying members [Sin et al. 1991]. However, this definition is not complete because the boundaries between a joint and attached members are not specified. Some attempts have been made to standardize the specification of these boundaries for joints in automotive structures. More specifically, values of the distance between the geometric center and the boundary which separates a joint from each attached branch have been recommended by Koterba and Rao [1981]. However, these values have not been widely accepted and there have been some concerns about the effect of the length of the attached branches on the behavior of a joint. Lee et al. [1991] demonstrated that this effect might be significant. They concluded that the joint behavior should be studied and its parameters should be identified by considering a subassembly whose branches are long enough. The subassembly is defined as a part of overall structure which consists of a joint and portions of the attached branches. It was found that there is an upper threshold for the length of the attached branches. The joint behavior and the values of the estimated joint parameters are not sensitive to the length of the attached branches if the length exceeds the aforementioned threshold.

In this study, the locations of the boundaries between a joint and the attached branches are assumed to be unknown, and their coordinates are treated as unknown parameters. The values of these parameters are specified by minimizing the discrepancy between theoretically predicted and measured displacements. This is discussed in Section 2.2.

1.2 Objectives

The objectives of this study are as follows ;

1. To develop a concept model of joints in vehicle structures, which is simple and reasonably accurate. This joint model accounts for the joint flexibility, the offsets of rotation centers of the joint branches, and the coupling between rotations of a branch in different planes and also between rotations of different branches. The model is defined in terms of physically meaningful parameters that make sense to designers, and can be easily extracted by experiments.
2. To investigate the importance of the effects which are mentioned in item 1.
3. To develop a method to estimate the joint model parameters and to identify the most important parameters by using the displacement measurements of the whole structure or of a subassembly. The subassembly is defined as a part of overall structure which consists of a joint and portions of the attached branches.
4. To investigate the effect of the length of the attached branches on the joint behavior.

The parameter estimates, which are obtained by minimizing the measured and predicted displacements, may not represent the actual behavior of joints when they are considered as parts of the whole structure or of a subassembly with longer branches. The subassembly is defined as a part of the whole structure which consists of a joint and portions of the attached branches. More specifically, the behavior of a joint included in the whole vehicle structure may be considerably different from that of an isolated joint. The following are the advantages of considering the whole vehicle structure or a subassembly whose branches are long enough in the estimation of parameters :

1. The difference between important and unimportant parameters is that only the former

significantly affect the response of the overall vehicle body or the subassembly. Thus, the most appropriate way to exploit this fact and to identify the important parameters is to examine the sensitivity of the response of the overall vehicle structure or the subassembly consisting of a joint and attached branches with respect to all parameters. The proposed method accounts for this sensitivity and thus it is expected to be more effective than methods that deal with measured response of isolated joints.

2. The behavior of a joint connected to the other structural members of a vehicle may be considerably different from that of an isolated joint whose ends are either constrained or artificially reinforced by attaching plates. Due to the coupling between the axial and torsional responses of the branches, and also due to the fact that the joint flexibility is caused by the shear deformation of the cross sections of the attached members, the joint has effective torsional stiffnesses that may be considerably different from those of a joint isolated from the rest of the vehicle [Sharman 1982]. The proposed method estimates these effective stiffnesses rather than non-representative values of the stiffnesses which are estimated from isolated joints.
3. The joint parameters are estimated by considering a very large number of response measurements, some of which are obtained at the positions located away from the center of a joint. Thus, the estimates of the joint parameters are based on a large amount of information. This cancels out the effect of errors in the response measurements. As a result, these estimates are more accurate than those obtained by considering a small number of measurements, which are usually obtained from an isolated joint.

1.3 Outline

In Chapter 2, a concept model of joints is described. It is assumed that the joint behavior is linear elastic. The torsional spring rates, the coordinates of flexible hinges, and the angles which define the orientations of the axes of the springs are the joint model parameters. A family of models with different levels of complexity are also introduced. The most complete model is called the full model while others are called the reduced models. The reduced models are derived from the full model by fixing the values of some parameters.

In Chapter 3, the estimation problem is formulated by using the relation between the response of a vehicle and the joint model parameters. Probabilistic nonlinear estimation [Draper and Smith 1981, Bard 1974] is used to estimate the model parameters and to identify the important parameters by using the measured responses of a vehicle structure. More specifically, the joint models are identified by minimizing the discrepancy between theoretically predicted and measured responses.

In this study, the responses are the displacements under static loads. The displacements can be measured from the overall vehicle structure which contains several joints or from a subassembly consisting of a joint and attached branches. In both cases, the measurements are taken at positions away from a localized area around a joint. This is useful to find important parameters. Indeed, the difference between important and unimportant parameters is that only the former significantly affect the displacements of the attached branches or of the whole vehicle body.

The displacements can be measured by the following ways :

1. Perform experiments.

2. Generate a very detailed finite element model of a subassembly which contains a joint.

The displacements calculated in the detailed model are used to simulate the measured displacements.

3. Use random number generators to simulate measurements assuming that a joint model completely describes the behavior of a joint. This procedure can only be used to test the identification procedure.

In this study, the last two procedures are used.

In Chapter 4, some tools which identify important parameters of a joint model are presented. These tools are used to simplify a joint model and to find the most appropriate model of a given joint.

The above methodology is applied to the identification of joints in two dimensional frame models of automotive structures (Chapters 5 and 6) and three dimensional subassemblies (Chapters 7 and 8). In the examples of these chapters, both the full and the reduced models are considered. The predicted displacements that are obtained by using the different modeling options are compared with measurements as well as the displacements calculated under the assumption that joints are rigid. Some conditions are determined under which the joint model can be simplified without losing accuracy in predicting the static response of a vehicle.

2. JOINT MODELS

A family of concept models representing the behavior of joints are presented in this chapter. The concept models are the models which are not only simple and design-orientated but also reasonably accurate compared to the detailed models. This family consists of alternative modeling options with different levels of complexity (Figure 2.1). The most complex and accurate model is called the full model. The remaining models, called the reduced, are simplified versions of the full model. The parameters in the full model are torsional springs, the coordinates of the flexible hinges of each branch, and the angles which define the orientations of axes of the torsional springs. In Section 2.1, the behavior of joints under static loads is studied. In Section 2.2, the full model and its parameters are described. The stiffness matrix of the full model is derived in Section 2.3. In Section 2.4, the reduced models are defined. It is also discussed how to derive the stiffness matrices of the reduced models from that of the full model.

2.1 Behavior of Joints

In this section the physical behavior of a typical joint is studied. The most important characteristics that will be accounted for by the joint model are described. The assumptions in modeling joints are also discussed.

Consider an n -branch joint whose one branch is free while the remaining $n - 1$ branches are clamped. For example, a beam representation of a 3-branch joint is shown in Figure 2.2.

This joint has one branch whose end is free (the free branch) and two other branches whose ends are clamped (the clamped branches). All three branches are straight before loads are applied. The axes of the local coordinate system, $x_L y_L z_L$, of the free branch are defined as follows ; x_L -axis coincides with the shear axis of the branch while the other two axes are normal to the shear axis. The response of the free branch under a static moment is examined. The response can be decomposed into three components. The first two components are due to the deformation of the clamped branches and the free branch, respectively. The third component is the rigid body motion of the free branch. It is the third component that is due to the joint flexibility.

The rigid body motion of the free branch, which is due to the joint flexibility, can be broken down into three rotations and three translations. The rotation about x_L -axis corresponds to twist and the rotations about y_L - and z_L -axes correspond to bending. It is assumed that the translations are negligible in the rigid body motion of the free branch. When a bending moment is applied about negative z_L -axis at the end of the free branch, the behavior of the free branch is characterized as follows (Figure 2.2) :

1. The displacement in the negative y_L -axis is larger than that calculated under the assumption of a rigid joint. This is due to the flexibility of a joint.
2. Under the assumption of a rigid joint, the free branch rotates about the intersection of three branches. However, it is experimentally observed that the free branch rotates about another point, which is called the center of rotation. The distance between the center of rotation and the intersection of the three branches is called the offset of rotation center.
3. When a moment is applied only about z_L -axis, the free branch rotates not only about

z_L -axis but also about x_L - and y_L -axes. This is called the coupling between rotations in three local planes that are defined by the local coordinate system, $x_L y_L z_L$.

The above characteristics of the joint behavior can be described by considering flexible hinges, H_1 , H_2 , and H_3 , along the free branch (Figure 2.2). They are called flexible hinges because a torsional spring is located at each of them. The torsional springs account for the flexibility of a joint. The hinges are located at different positions along the free branch in order to account for the offset of the rotation center. The torsional springs lie in planes, P_1 , P_2 , and P_3 (Figure 2.2). These planes do not necessarily coincide with the local planes, $x_L y_L$, $y_L z_L$, and $z_L x_L$. By changing the orientations of these planes, the coupling between rotations in different local planes can be accounted for.

Another example is given in Figure 2.3 which is illustrative to understand how the coupling can be accounted for in the concept joint model. In this figure, there is only one flexible hinge for simplicity. This hinge H_i lies in a plane P_i which in general is not parallel to any of three local planes, $x_L y_L$, $y_L z_L$, and $z_L x_L$. If a moment is applied about y_L -axis at the free branch, the branch rotates not only about y_L -axis but also about x_L - and z_L -axes. Only when P_i is parallel to $x_L z_L$ -plane, the branch rotates only about y_L -axis under a moment about y_L -axis.

There is also coupling between twist rotation and translations in the plane that is normal to the shear axis of a branch (Figure 2.4). The shear axis is defined as a locus of shear centers at all cross sections. Ideally, if a twist moment is applied about shear axis, translational displacements are zero. However, this is not always true at a joint whose geometry is complicated. For example, when there are service holes or when the shear axes of all connecting

branches do not intersect at the same point, a twist moment can induce translational displacements. The above coupling effect can be accounted for by considering that one hinge may not lie on the shear axis. This hinge is the one that mostly accounts for the flexibility in twist rotation. Thus, the coordinates of this hinge in y_L - and z_L -axes are considered as unknown parameters.

Based on the above discussion, a concept joint model that accounts for the following effects is developed ;

1. the joint flexibility,
2. the fact that the rotation centers of each joint branch do not coincide with each other and that their locations are unknown,
3. the coupling between rotations, between translations, and between rotations and translations.

It is widely accepted that the joint flexibility should be accounted for [Chang 1974]. Lee and Nikolaidis [1991 B] studied the effect of the offsets of rotation centers of joint branches with respect to the reference point for 2 dimensional joints. The reference point was defined as the intersection of the shear axes of the joint branches. They found that this effect is significant if the offset length is large. Finally, Yamazaki and Inoue [1989] demonstrated that the accuracy of a concept joint model can be significantly improved by accounting for the coupling between rotations in different planes.

The following assumptions are made in modeling joints ;

1. The behavior of a joint is linear elastic.
2. The joint branches are rigidly connected in translation.

The second assumption is realistic because a joint is reinforced in an area close to its center, which restrains the relative translational displacements of the joint branches. However, the flexibility in rotation still needs to be considered because a small rotation significantly affects the displacements at positions which are located away from the center of a joint.

2.2 Joint Parameterization – Full Model

A concept joint model is constructed by modeling each joint branch separately with a joint element (Figure 2.5). A joint element consists of the following components ;

1. Three flexible hinges.
2. One flexible section which is a portion of the attached beam.
3. Three rigid sections which are connected using the hinges.
4. Three torsional springs each of which is located at the corresponding hinge. Each spring restrains the relative rotation between rigid sections or between a rigid and a flexible sections.

For an n -branch joint, there are n joint elements. All joint elements at the same joint are assembled together to form a concept joint model.

A joint element without springs is shown in Figures 2.5 (a) to (c) ; 3-D, front, and top view, respectively. The local coordinate system, $x_L y_L z_L$, is defined in a way that x_L -axis coincides with the shear axis of the flexible section of the corresponding joint element. In Figures 2.5 (a) to (c), node i is called the “reference point” of the joint element. The reference point is defined as the intersection when the shear axes of all other connecting branches are projected on the shear axis of the branch corresponding to the particular joint element. When the intersections from several branches are different, the reference point is

considered as the intersection which is the farthest from the flexible section. Each branch of a joint has a joint element and a reference point. All the reference points of the joint are rigidly connected to each other.

Each of the three springs is located in a plane, which is normal to each of the three axes, z_1, z_2 , and z_3 . In this study, these axes are called "spring axes". For each spring axis, two other axes which are orthogonal to each other and also to the corresponding spring axis can be found. These three axes form an orthogonal coordinate system which is called the spring coordinate system. Because there are three spring axes, there are also three spring coordinate systems for each joint element. The coupling between rotations in different planes is accounted for by assuming that the orientations of the spring axes are arbitrary and unknown. The orientation of each spring axis is defined with three angles between the spring axis and three local axes. Among three angles, only two are independent. The remaining angle can be found from the other two independent ones. Thus, in general, 6 independent angles are needed to define the orientations of three spring axes. In some cases, the spring axes are mutually orthogonal or they may even coincide with the local axes. In the former case, the spring axes are called the principal axes by Yamazaki and Inoue [1989]. In the latter case, the rotations in different planes are uncoupled. If a joint is geometrically symmetric with respect to the $x_L y_L$ and $x_L z_L$ planes, then the spring axes coincide with the local axes.

In Figures 2.5 (b) and (c), a joint element is shown with all nodes. At each hinge, there are two nodes because rotations may be different across the hinge. For example, r and c are two nodes at the first hinge, nodes q and b at the second one, and nodes p and a at the third one. Nodes r and c define a boundary between a joint and an attached branch

in the proposed model. Thus, by adopting the proposed model, and by considering the distance from the reference point to the farthest hinge, r_1 , as unknown parameter, the boundary between a joint and an attached branch is to be found. The boundary is found in the parameter identification procedure (Chapter 3). In this procedure, the discrepancy between the measured response and that predicted by the concept joint model is minimized. In Figure 2.5 (b), node j must be selected at a position so that the length of a joint element be longer than the distance from the reference point to the boundary between a joint and an attached branch. If the estimated length of r_1 is close to the length of a joint element, it is an indication that the joint element is not long enough. In this case, the position of node j must be adjusted in a way that the length of a joint element is increased.

The above joint element is defined in terms of 18 independent parameters in general ; the Cartesian coordinates of the three hinges at nodes p, q, r (9), the spring rates (3), and two independent angles to define the orientation of each of three spring axes, z_1, z_2 , and z_3 (6). The numbers in the parentheses correspond to the numbers of parameters. This model of a joint element is simplified as follows ; The two hinges that mostly account for the bending flexibility are assumed to be located on the shear axis of the branch. For example, nodes c (or r) and b (or q) are located on the shear axis of the corresponding branch (Figure 2.5 b). Thus, only 5 independent parameters are needed to determine the location of three hinges. These are the distances of the hinges from the reference point (node i) along the shear axis of the branch, r_1, r_2 , and r_3 , respectively, and the coordinates of node a (or p) along the y_L - and z_L -directions, h_y and h_z , respectively. The distances r_1, r_2 , and r_3 are called “on-branch offsets” while h_y and h_z are called “off-branch offsets”. Under the above assumption, the number of independent parameters of a joint element reduces to 14.

A joint model is an assembly of all joint elements at the same joint. In this model, the reference points of all joint elements are rigidly connected to each other. Thus, the total number of independent parameters of an n -branch joint is $14 \times n$.

2.3 Stiffness Matrix – Full Model

The stiffness matrix of a joint element is derived in this section. The following procedure is applied to find the stiffness matrix :

1. The stiffness matrix of a flexible section, which is a part of the joint element, can be found. This 12×12 matrix relates 12 equations for generalized forces with 12 generalized displacements at nodes j and r (Figure 2.5 b).
2. Using the assumption that translational displacements are the same at 2 nodes of each hinge (nodes r and c , q and b , and p and a of Figure 2.5 b), and using the relations between generalized displacements at 2 ends of each rigid section (nodes c and q , b and p , and a and i of Figure 2.5 b), 3 translations at node r can be expressed with 12 generalized displacements (6 generalized displacements at node i and 3 rotations at each of nodes p and q). Thus, there are 21 unknowns and 12 equations for generalized forces at nodes r and j . In this process it is temporarily assumed that the three hinges are pinned. In other words, any moment is not transmitted through these hinges. This assumption will be relaxed in step 4. Using the equilibrium of generalized forces, 9 additional equations can be found, which ends up with the stiffness matrix of size 21×21 . The entries of this matrix correspond to the following displacements ; 6 degrees of freedom (d.o.f.'s) at the reference point (node i), 6 d.o.f.'s at the other end of the joint element (node j), and 3 rotational d.o.f.'s at each of 3 hinges (nodes p , q ,

and r).

3. The effect of 3 torsional springs is added to this matrix.
4. It is considered that each of the three hinges are pinned only in a plane which is normal to the corresponding spring axis. Then, the rotations in the other two planes which are normal to the plane defined by the corresponding spring axis are constrained to be the same at two nodes of each hinge (nodes a and p, b and q, and c and r). Thus, for each hinge, the total d.o.f.'s are reduced by 2, which makes the number of independent unknowns equal to 15. Finally, the stiffness matrix of size 15×15 is obtained for a joint element. These include 6 d.o.f.'s at the reference point (node i), another 6 d.o.f.'s at node j, and 1 rotational d.o.f. at each of 3 nodes, p, q, and r. There is only one independent d.o.f. at each of nodes p, q, and r because each of these nodes are constrained to rotate about the spring axes, z_1 , z_2 , and z_3 , respectively.

In the following, the derivation of the stiffness matrix of a joint element is described in detail.

The derivation of the stiffness matrix of a joint element starts with the stiffness matrix of a flexible section in the local coordinates, $[\mathbf{K}_{12 \times 12}]_L$, which relates the generalized forces and displacements at nodes r and j in the local coordinates ;

$$\{f_{r1}, \dots, f_{r6}, f_{j1}, \dots, f_{j6}\}_L^T = [\mathbf{K}_{12 \times 12}]_L \{u_{r1}, \dots, u_{r6}, u_{j1}, \dots, u_{j6}\}_L^T. \quad (2.1)$$

In Eq.(2.1), f_{ri} and u_{ri} are the generalized forces and displacements at node r, respectively. For example, f_{r1} is a force in x_L -direction at node r and u_{r5} is a rotation about y_L -axis at node r. The same notation applies to the generalized forces and displacements at the other nodes. The 3 displacements at node r, u_{r1} , u_{r2} , and u_{r3} , are equal to the 3 displacements

at node c, u_{c1}, u_{c2} , and u_{c3} , because nodes r and c are rigidly connected in translation ; $u_{rk} = u_{ck}$ ($k=1,2,3$). The 3 displacements at node c, u_{c1}, u_{c2} , and u_{c3} , can be expressed in terms of 3 displacements and 3 rotations at node q, $u_{q1}, u_{q2}, u_{q3}, u_{q4}, u_{q5}$, and u_{q6} because nodes c and q are located in the same rigid section. All the above relations are written in the local coordinates. There are similar relations between displacements and rotations for the other rigid sections. Thus, there are the following relations between displacements and rotations in the local coordinates :

$$\begin{aligned} \{u_{r1}, u_{r2}, u_{r3}\}_L^T &= [C_{36}]_1 \{u_{q1}, \dots, u_{q6}\}_L^T, \\ \{u_{q1}, u_{q2}, u_{q3}\}_L^T &= [C_{36}]_2 \{u_{p1}, \dots, u_{p6}\}_L^T, \\ \{u_{p1}, u_{p2}, u_{p3}\}_L^T &= [C_{36}]_3 \{u_{i1}, \dots, u_{i6}\}_L^T. \end{aligned} \quad (2.2)$$

In the above equations, $[C_{36}]_k$ can be written as follows ;

$$\begin{aligned} [C_{36}]_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & +d_1 \\ 0 & 0 & 1 & 0 & -d_1 & 0 \end{bmatrix}, \\ [C_{36}]_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 & -h_z & +h_y \\ 0 & 1 & 0 & +h_z & 0 & +d_2 \\ 0 & 0 & 1 & -h_y & -d_2 & 0 \end{bmatrix}, \\ [C_{36}]_3 &= \begin{bmatrix} 1 & 0 & 0 & 0 & +h_z & -h_y \\ 0 & 1 & 0 & -h_z & 0 & +d_3 \\ 0 & 0 & 1 & +h_y & -d_3 & 0 \end{bmatrix}, \end{aligned} \quad (2.3)$$

where $d_1 = r_1 - r_2$, $d_2 = r_2 - r_3$, and $d_3 = r_3$ (Figure 2.5 b). Using the relations between displacements and rotations in Eq.(2.2) and considering that $f_{rk} = f_{ik}$ ($k=1,2,3$) due to force equilibrium, Eq.(2.1) can be written as follows :

$$\begin{aligned} &\{f_{i1}, f_{i2}, f_{i3}, f_{r4}, f_{r5}, f_{r6}, f_{j1}, \dots, f_{j6}\}_L^T \\ &= [K_{12 \times 21}]_L \{u_{i1}, \dots, u_{i6}, u_{p4}, u_{p5}, u_{p6}, u_{q4}, u_{q5}, u_{q6}, u_{r4}, u_{r5}, u_{r6}, u_{j1}, \dots, u_{j6}\}_L^T. \end{aligned} \quad (2.4)$$

In Eq.(2.4), there are 12 equations with 21 unknowns. Thus, 9 more equations for f_{i4}, f_{i5}, f_{i6} , $f_{p4}, f_{p5}, f_{p6}, f_{q4}, f_{q5}$, and f_{q6} are needed for completion. In order to make the derivation of

stiffness matrix simple, it is temporarily assumed that the three flexible hinges at nodes a and p, b and q, and c and r are pinned. This assumption will be relaxed later by considering the spring axes which constrain the rotation planes of flexible hinges. Under the above assumption, the moments about any local axis at nodes a, b, and c are zero ; $f_{ak} = f_{bk} = f_{ck} = 0$ ($k=4,5,6$). There are also relations between forces due to equilibrium; $f_{ik} = f_{pk} = f_{qk} = f_{rk}$ ($k=1,2,3$). Using the moment equilibrium inside each rigid section and the above relations, the following 9 equations can be obtained :

$$\begin{aligned}\{f_{q4}, f_{q5}, f_{q6}\}_L^T &= [C_{33}]_1 \{f_{i1}, f_{i2}, f_{i3}\}_L^T, \\ \{f_{p4}, f_{p5}, f_{p6}\}_L^T &= [C_{33}]_2 \{f_{i1}, f_{i2}, f_{i3}\}_L^T, \\ \{f_{i4}, f_{i5}, f_{i6}\}_L^T &= [C_{33}]_3 \{f_{i1}, f_{i2}, f_{i3}\}_L^T.\end{aligned}\tag{2.5}$$

In the above equations, $[C_{33}]_k$ is :

$$\begin{aligned}[C_{33}]_1 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -d_1 \\ 0 & +d_1 & 0 \end{bmatrix}, \\ [C_{33}]_2 &= \begin{bmatrix} 0 & +h_z & -h_y \\ -h_z & 0 & -d_2 \\ +h_y & +d_2 & 0 \end{bmatrix}, \\ [C_{33}]_3 &= \begin{bmatrix} 0 & -h_z & +h_y \\ +h_z & 0 & -d_3 \\ -h_y & +d_3 & 0 \end{bmatrix}.\end{aligned}\tag{2.6}$$

By augmenting the 12 equations in Eq.(2.4) with the new 9 equations in Eq.(2.5), a 21×21 stiffness matrix of a joint element, $[K_{21 \times 21}]_L$, is obtained. This matrix relates the generalized forces with the generalized displacements by the following :

$$\begin{aligned}&\{f_{i1}, \dots, f_{i6}, f_{p4}, f_{p5}, f_{p6}, f_{q4}, f_{q5}, f_{q6}, f_{r4}, f_{r5}, f_{r6}, f_{j1}, \dots, f_{j6}\}_L^T \\ &= [K_{21 \times 21}]_L \{u_{i1}, \dots, u_{i6}, u_{p4}, u_{p5}, u_{p6}, u_{q4}, u_{q5}, u_{q6}, u_{r4}, u_{r5}, u_{r6}, u_{j1}, \dots, u_{j6}\}_L^T.\end{aligned}\tag{2.7}$$

The effect of 3 torsional springs, k_1, k_2 , and k_3 , is going to be added to the stiffness matrix of a joint element ; k_1 between nodes r and c, k_2 between nodes q and b, and k_3 between nodes

p and a. It can be considered that k_1 is located between nodes r and q, k_2 between nodes q and p, and k_3 between nodes p and i because the rotations of two nodes in the same rigid section are the same. Three angles, $\phi_{z_1 x_L}$, $\phi_{z_1 y_L}$, and $\phi_{z_1 z_L}$, are defined as angles between z_1 and x_L axes, between z_1 and y_L axes, and between z_1 and z_L axes, respectively. Then the stiffness matrix due to spring k_1 in the local coordinates can be written as follows ;

$$[\mathbf{K}_{k1}]_L = \begin{bmatrix} +\mathbf{K}_{11} & -\mathbf{K}_{11} \\ -\mathbf{K}_{11} & +\mathbf{K}_{11} \end{bmatrix}, \quad \text{where} \quad [\mathbf{K}_{11}] = k_1 \begin{bmatrix} C_1^2 & C_1 C_2 & C_1 C_3 \\ C_1 C_2 & C_2^2 & C_2 C_3 \\ C_1 C_3 & C_2 C_3 & C_3^2 \end{bmatrix}, \quad (2.8)$$

where $C_1 = \cos \phi_{z_1 x_L}$, $C_2 = \cos \phi_{z_1 y_L}$, and $C_3 = \cos \phi_{z_1 z_L}$. The 6 d.o.f.'s of $[\mathbf{K}_{k1}]_L$ correspond to $u_{q4}, u_{q5}, u_{q6}, u_{r4}, u_{r5}$, and u_{r6} . The stiffness matrices $[\mathbf{K}_{k2}]_L$ and $[\mathbf{K}_{k3}]_L$ due to springs k_2 and k_3 , respectively, can be obtained similarly. The stiffness matrix of a joint element in local coordinates, $[\bar{\mathbf{K}}_{21 \times 21}]_L$, can be obtained by adding $[\mathbf{K}_{k1}]_L, [\mathbf{K}_{k2}]_L$, and $[\mathbf{K}_{k3}]_L$ to $[\mathbf{K}_{21 \times 21}]_L$ in Eq.(2.7) :

$$[\bar{\mathbf{K}}_{21 \times 21}]_L = [\mathbf{K}_{k1}]_L + [\mathbf{K}_{k2}]_L + [\mathbf{K}_{k3}]_L + [\mathbf{K}_{21 \times 21}]_L. \quad (2.9)$$

The stiffness matrix of a joint element in the global coordinates system, $x_G y_G z_G$, can be obtained by the following transformation ;

$$[\mathbf{K}_{21 \times 21}]_G = [\mathbf{T}_{21 \times 21}]^T [\bar{\mathbf{K}}_{21 \times 21}]_L [\mathbf{T}_{21 \times 21}], \quad (2.10)$$

where $[\mathbf{T}_{21 \times 21}]$ is the transformation matrix relating the local and global coordinate systems.

Now the assumption that the three flexible hinges at nodes a and p, b and q, and c and r are pinned will be relaxed. Due to this assumption, there are 21 independent d.o.f.'s for the stiffness matrix of a joint element, $[\mathbf{K}_{21 \times 21}]_G$ in Eq.(2.10) ; 6 d.o.f.'s at the reference point (node i), 6 d.o.f.'s at the end of the flexible section (node j), and 3 d.o.f.'s at each

of 3 nodes, p, q, and r. However, the 3 nodes, p, q, and r, are actually constrained to rotate about spring axes, z_1, z_2 , and z_3 , respectively. This means that there is only one independent d.o.f. at each of the nodes p, q, and r. A coordinate system $x_1 y_1 z_1$ is defined so that the z_1 axis coincides with the spring axis of a hinge at node r (Figure 2.5 b). This coordinate system is called the spring coordinate system for the first torsional spring. The orientation of the other two axes, x_1 and y_1 can be arbitrary as long as $x_1 y_1 z_1$ forms an orthogonal coordinates. Due to the constraint that the hinge at node r (or c) is flexible about z_1 -axis and rigid about x_1 - and y_1 -axes and due to the fact that rotations are the same at 2 nodes (c and q) of the same rigid section, there are the following relations in the first spring coordinates, x_1, y_1 , and z_1 ;

$$\{u_{q4}, u_{q5}\}_{P_1}^T = \{u_{r4}, u_{r5}\}_{P_1}^T, \quad (2.11)$$

where P_1 denotes the first spring coordinates, x_1, y_1 , and z_1 , and u_{q4} and u_{q5} are rotations about x_1 and y_1 axes at node q, respectively. The same notation applies to u_{r4} and u_{r5} . Using the transformation from spring coordinates to global coordinates, the following relations are obtained ;

$$\begin{aligned} \{u_{q4}, u_{q5}, u_{q6}\}_{P_1}^T &= [\mathbf{T}_{3 \times 3}] \{u_{q4}, u_{q5}, u_{q6}\}_G^T \\ \{u_{r4}, u_{r5}, u_{r6}\}_{P_1}^T &= [\mathbf{T}_{3 \times 3}] \{u_{r4}, u_{r5}, u_{r6}\}_G^T, \end{aligned} \quad (2.12)$$

where

$$[\mathbf{T}_{3 \times 3}] = \begin{bmatrix} \cos \phi_{x_1 x_G} & \cos \phi_{x_1 y_G} & \cos \phi_{x_1 z_G} \\ \cos \phi_{y_1 x_G} & \cos \phi_{y_1 y_G} & \cos \phi_{y_1 z_G} \\ \cos \phi_{z_1 x_G} & \cos \phi_{z_1 y_G} & \cos \phi_{z_1 z_G} \end{bmatrix}. \quad (2.13)$$

Using Eqs. (2.12) and (2.13), Eq.(2.11) can be written as follows ;

$$[\mathbf{T}_{2 \times 3}] \{u_{q4}, u_{q5}, u_{q6}\}_G^T = [\mathbf{T}_{2 \times 3}] \{u_{r4}, u_{r5}, u_{r6}\}_G^T, \quad (2.14)$$

where $[\mathbf{T}_{2 \times 3}]$ is a matrix of size 2×3 that is equal to the first two rows of $[\mathbf{T}_{3 \times 3}]$. Eq.(2.14) has 2 relations between rotations at nodes q and r in the global coordinates. The plane where a corresponding torsional spring is located is defined using the orientation of z_1 -axis only (independent of the orientations of x_1 - and y_1 -axes). The following procedure is used to express the relations in Eq.(2.14) in a way that they are independent of the orientations of x_1 and y_1 axes.

Eq.(2.14) can be written as follows ;

$$[\bar{\mathbf{T}}_{3 \times 3}] \{u_{q4}, u_{q5}, u_{q6}\}_G^T = [\bar{\mathbf{T}}_{3 \times 3}] \{u_{r4}, u_{r5}, u_{r6}\}_G^T, \quad (2.15)$$

where

$$[\bar{\mathbf{T}}_{3 \times 3}] = [\mathbf{T}_{3 \times 3}] - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \cos \phi_{z_1 x_G} & \cos \phi_{z_1 y_G} & \cos \phi_{z_1 z_G} \end{bmatrix}. \quad (2.16)$$

In Eq.(2.16), there are 3 equations. However, the third equation is trivial and thus redundant. By premultiplying Eq.(2.15) by $[\bar{\mathbf{T}}_{3 \times 3}]^{-1}$ and by discarding the third equation, the following can be obtained ;

$$[\mathbf{L}_{2 \times 3}] \{u_{q4}, u_{q5}, u_{q6}\}_G^T = [\mathbf{L}_{2 \times 3}] \{u_{r4}, u_{r5}, u_{r6}\}_G^T, \quad (2.17)$$

where

$$[\mathbf{L}_{2 \times 3}] = \begin{bmatrix} (1 - \cos^2 \phi_{z_1 x_G}) & -\cos \phi_{z_1 x_G} \cos \phi_{z_1 y_G} & -\cos \phi_{z_1 x_G} \cos \phi_{z_1 z_G} \\ -\cos \phi_{z_1 x_G} \cos \phi_{z_1 y_G} & (1 - \cos^2 \phi_{z_1 y_G}) & -\cos \phi_{z_1 y_G} \cos \phi_{z_1 z_G} \end{bmatrix}. \quad (2.18)$$

Eq.(2.17) consists of 2 equations relating the rotations at nodes q and r. Note that these equations are independent of the orientations of x_1 and y_1 axes because all terms are expressed in terms of the three angles between z_1 axis and x_G, y_G , and z_G axes, respectively.

Depending on the values of $\phi_{z_1 x_G}, \phi_{z_1 y_G}$, and $\phi_{z_1 z_G}$, two d.o.f.'s among u_{r4}, u_{r5} , and u_{r6} can

be expressed in terms of the remaining d.o.f. at node r and 3 d.o.f.'s at node q, u_{q4}, u_{q5} , and u_{q6} . The d.o.f. whose corresponding directional angle with respect to the first spring axis, z_1 , is the smallest, is considered as independent one. Similarly, the six d.o.f.'s at nodes p and q can be reduced into two independent ones. Thus, 21 generalized displacements can be reduced into 15 independent ones.

Using the above relations between displacements, the stiffness matrix of a joint element is going to be reduced. The equilibrium equations for a joint element are written as follows ;

$$\mathbf{f} = [\mathbf{K}_{21 \times 21}]_G \mathbf{u}, \quad (2.19)$$

where $[\mathbf{K}_{21 \times 21}]_G$ is the stiffness matrix of size 21×21 , which is shown in Eq.(2.10). The $\mathbf{f}$ and $\mathbf{u}$ are a force and a displacement vectors, respectively, and have 21 elements ; 6 at the reference point, 6 at the end of the flexible section, and 3 at each of 3 nodes, p, q, and r. Using the relations between displacements, $\mathbf{u}$ can be written as follows ;

$$\mathbf{u} = [\mathbf{T}_{21 \times 15}] \mathbf{u}_I, \quad (2.20)$$

where $\mathbf{u}_I$ is a vector of independent displacements, which has 15 elements ; 6 at the reference point, 6 at the end of the flexible section, and 1 at each of 3 nodes, p, q, and r. The $[\mathbf{T}_{21 \times 15}]$ is the transformation matrix, which is found by Eq.(2.17) for the first hinge and by similar relations for the other two hinges. Using Eq.(2.20), Eq.(2.19) can be written as follows :

$$\mathbf{f} = [\mathbf{K}_{21 \times 21}]_G [\mathbf{T}_{21 \times 15}] \mathbf{u}_I. \quad (2.21)$$

Premultiplying the above equation by $[\mathbf{T}_{21 \times 15}]^T$, it is finally found that

$$\mathbf{f}_I = [\mathbf{K}_{15 \times 15}]_G \mathbf{u}_I, \quad (2.22)$$

where $\mathbf{f}_I = [\mathbf{T}_{21 \times 15}]^T \mathbf{f}$ and $[\mathbf{K}_{15 \times 15}]_G = [\mathbf{T}_{21 \times 15}]^T [\mathbf{K}_{21 \times 21}]_G [\mathbf{T}_{21 \times 15}]$. The $[\mathbf{K}_{15 \times 15}]_G$ is the stiffness matrix of a joint element of size 15×15 in the global coordinates. The above equilibrium equations in Eq.(2.22) can be written in detail as follows ;

$$\begin{aligned} & \{f_{i1}, \dots, f_{i6}, f_p, f_q, f_r, f_{j1}, \dots, f_{j6}\}_G^T \\ &= [\mathbf{K}_{15 \times 15}]_G \{u_{i1}, \dots, u_{i6}, u_p, u_q, u_r, u_{j1}, \dots, u_{j6}\}_G^T, \end{aligned} \quad (2.23)$$

where each of p, q, and r in f_p, f_q, f_r, u_p, u_q , and u_r corresponds to one independent d.o.f. among the 3 d.o.f.'s at the corresponding node.

The global stiffness matrix, $\mathbf{K}$, of a vehicle structure with m flexible joints can be found by assembling the stiffness matrices of the joint elements at all joints, $\mathbf{K}_{J_{ij}}$ ($i=1, \dots, m$, $j=1, \dots, n_i$) with that of the remaining structure without joints, $\mathbf{K}_R$;

$$\mathbf{K} = \sum_{i=1}^m \sum_{j=1}^{n_i} \mathbf{K}_{J_{ij}} + \mathbf{K}_R \quad (2.24)$$

where n_i is the number of branches in the i-th joint ($i=1, \dots, m$).

Finally, the total degrees of freedom (d.o.f.'s) of $\mathbf{K}$ are reduced because all the reference points of a certain joint are rigidly connected. For example, let nodes A and B be the reference points of two branches, respectively, at the same joint. There are the following relations between the generalized displacements at nodes A and B, $\mathbf{u}_A$ and $\mathbf{u}_B$, respectively ;

$$\begin{aligned} \{u_{B1}, u_{B2}, u_{B3}\}_G^T &= [\mathbf{D}_{36}] \{u_{A1}, \dots, u_{A6}\}_G^T, \\ \{u_{B4}, u_{B5}, u_{B6}\}_G^T &= \{u_{A4}, u_{A5}, u_{A6}\}_G^T, \end{aligned} \quad (2.25)$$

where $[\mathbf{D}_{36}]$ is found to be similar to $[\mathbf{C}_{36}]_3$ in Eq.(2.3) ;

$$[\mathbf{D}_{36}] = \begin{bmatrix} 1 & 0 & 0 & 0 & +l_z & -l_y \\ 0 & 1 & 0 & -l_z & 0 & +l_x \\ 0 & 0 & 1 & +l_y & -l_x & 0 \end{bmatrix}. \quad (2.26)$$

In the above equation, l_x, l_y , and l_z are the directional components of the length from node A to node B, in x -, y -, and z -axes, respectively. Thus, using the above relations between $\mathbf{u}_A$ and $\mathbf{u}_B$, 6 dependent d.o.f.'s at node B are removed from the global stiffness matrix. For a joint with n branches, the d.o.f.'s corresponding to the reference points of $(n - 1)$ branches are removed, which reduces the total d.o.f.'s of the global stiffness matrix by $6(n - 1)$.

2.4 Joint Parameterization – Reduced Models

In some cases, the behavior of joints can be adequately represented by using simpler models. In this section, a family of the reduced models are defined and their stiffness matrices are derived from that of the full model. The conditions under which the joint model can be simplified will be discussed in Chapter 4. All the concept joint models which are defined in this dissertation are summarized in Figure 2.1.

2.4.1 Reduced Model I

Most studies on joints have employed a model in which the flexibility of joints is accounted for by using torsional springs to constrain the relative rotations between branches at the same joint [Chang 1974, Chon 1986, Lee and Nikolaidis 1991 A]. This is the simplest model in the family of joint models that are analyzed in this study. This model is called “reduced model I”. The following are the characteristics of the reduced model I :

1. The flexibility of joints is accounted for by using torsional springs.
2. The coupling between rotations in different planes is neglected.
3. A joint branch rotates about the same point in all three independent rotational modes, which correspond to 2 bending and 1 torsional modes. The common rotation center is the reference point of the branch. Furthermore, all the branches rotate about the same

point. In other words, the on-branch and off-branch offsets of the rotation centers are assumed to be zero.

The reduced model I of an n-branch joint consists of $3 \times \binom{n}{2}$ torsional springs for a joint with n branches in space. For joints which have 2 or 3 branches, the stiffness matrix of the reduced model I can be found from that of the full model by fixing the values of the following parameters of the full model ; $r_i = h_y = h_z = 0$ (i=1,2,3), $\phi_{z_1x_L} = \phi_{z_1y_L} = 90^\circ$ and $\phi_{z_1z_L} = 0^\circ$, $\phi_{z_2x_L} = \phi_{z_2z_L} = 90^\circ$ and $\phi_{z_2y_L} = 0^\circ$, and $\phi_{z_3y_L} = \phi_{z_3z_L} = 90^\circ$ and $\phi_{z_3x_L} = 0^\circ$ (Figure 2.5).

For joints which have more than 3 branches, the stiffness matrix can be found as follows. A finite element model can be developed for an n-branch joint by considering n nodes located at the reference points. Thus, there are 6n degrees of freedom (d.o.f.'s), which correspond to three translations and three rotations for each node. According to the assumption that the joint branches are rigidly connected in translation, the translational d.o.f.'s of all connecting nodes are the same. Therefore, the total d.o.f.'s for an n-branch joint in space are reduced to $(3n+3)$. The stiffness matrix corresponding to this joint model is a $(3n+3) \times (3n+3)$ matrix, whose first three rows and columns correspond to translational displacements and contain zeros. The remaining 3n rows and columns correspond to the rotations of the n nodes which are located at the reference point. The above procedure can be also used to find the stiffness matrix of a joint with 2 or 3 branches. For example, the stiffness matrix, $[K_J]$, of a 3-branch joint in a plane (Figure 2.6) is as follows ;

$$[K_J] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{ij} + k_{ik} & -k_{ij} & -k_{ik} \\ 0 & 0 & -k_{ij} & k_{ij} + k_{jk} & -k_{jk} \\ 0 & 0 & -k_{ik} & -k_{jk} & k_{ik} + k_{jk} \end{bmatrix}. \quad (2.27)$$

2.4.2 Reduced Model II

The “reduced model II” is a special case of the full joint model. This model is identical to the one developed by Rao et al. [1983], which accounts for the flexibility and the on-branch offsets of the rotation centers. It is assumed that the rotation centers lie on the shear axis of the corresponding branch. The reduced model II neglects the coupling between rotations of the branches in different planes. It also neglects the coupling between rotations and translations. Thus, the full model reduces to the reduced model II by assuming that the hinges of each branch rotate in the three planes, $x_L y_L$, $y_L z_L$, and $z_L x_L$, and that all off-branch offsets are zero. For each branch, there are 6 independent parameters ; 3 torsional springs and 3 on-branch offsets of the rotation centers.

Usually, the on-branch offset of the twist center is small compared to the on-branch offsets of the bending centers. Moreover, the effect of changes in this offset on the response of the joint is negligible. Therefore, it may be assumed that the twist offset is zero without losing any accuracy in predicting the response of the joint. This reduces the number of independent parameters of the reduced model II into 5. In conclusion, the reduced model II of an n-branch joint is described in terms of 5n parameters ; 3 torsional springs and 2 on-branch offsets of rotation centers (bending only) for each branch.

In Figure 2.7, a 3-branch joint in a plane is shown, which is modeled using the reduced model II. Each branch is represented by a joint element. The stiffness matrix of the reduced model II can be found from that of the full model by fixing $h_y = h_z = 0$, $\phi_{z_1 x_L} = \phi_{z_1 y_L} = 90^\circ$ and $\phi_{z_1 z_L} = 0^\circ$, $\phi_{z_2 x_L} = \phi_{z_2 z_L} = 90^\circ$ and $\phi_{z_2 y_L} = 0^\circ$, and $\phi_{z_3 y_L} = \phi_{z_3 z_L} = 90^\circ$ and $\phi_{z_3 x_L} = 0^\circ$ (Figure 2.5).

2.4.3 Reduced Model III

The “reduced model III” is also a special case of the full model but it is more accurate than the reduced model II. It accounts for the joint flexibility, both the on-branch and off-branch offsets of the rotation centers, and the coupling between rotations of a branch in different planes. However, the spring axes that define the orientations of the rotation planes of the 3 hinges of a branch, z_1, z_2 , and z_3 , are mutually orthogonal. Note that although axes z_1, z_2 , and z_3 are orthogonal with respect to each other, their orientations are arbitrary with respect to the local axes. Since z_1, z_2 , and z_3 are orthogonal, three angles are sufficient to define their orientations. Therefore, the reduced model III is defined in terms of 11 independent parameters ; 3 spring rates, 3 on-branch offsets, 2 off-branch offsets, and 3 angles defining the orientations of the spring axes. The reduced model III is similar to the one developed by Yamazaki and Inoue [1989] except that both the on-branch and off-branch offsets are not considered in the latter.

2.4.4 2-D Bracketed Model

Coupling between rotations of different branches of a joint might be important. This type of coupling should be distinguished from the coupling between rotations in different planes which is accounted for by considering that the orientations of spring axes are arbitrary. The former type of coupling can be demonstrated as follows : If a force is applied to one branch while fixing a reference point of a joint element, the other branches might deflect. This deflection should be attributed to the coupling between rotations of different connecting branches. This model is called bracketed because the springs that account for the coupling look like brackets that support joint branches (Figure 2.8).

The “2-D bracketed model” for a joint in a plane is defined as follows :

1. The flexibility of a branch is accounted for by using each of torsional springs (k_i, k_j , and k_k).
2. The branches of joints rotate about positions which are different from the reference point, which is accounted for by considering on-branch offsets (r_i, r_j , and r_k).
3. The coupling between rotations of different branches is also accounted for by using torsional springs (k_{ij}, k_{jk} , and k_{ki}).

The k 's and r 's in the above parentheses are shown in Figure 2.8. In this figure, springs with two indices, k_{ij}, k_{jk} , and k_{ki} , account for coupling between rotations of different connecting branches. Springs with one index, k_i, k_j , and k_k , account for the flexibility of each branch. The r 's are the on-branch offsets of flexible hinges.

The 2-D bracketed model is not a special case of the full model because the coupling between rotations of different connecting branches is not considered in the full model. There are $2n + \binom{n}{2}$ parameters in order to describe the 2-D bracketed model for a joint with n branches ; n torsional springs to account for the flexibility of branches, n on-branch offsets of flexible hinges, and $\binom{n}{2}$ springs to account for the coupling between rotations of different connecting branches. The stiffness matrix of the 2-D bracketed model can be constructed by adding the stiffness of 3 springs, k_{ij}, k_{jk} , and k_{ki} , which constrain rotations of any two connecting branches, to the stiffness matrix of the reduced model II.

3. PROBABILISTIC SYSTEM IDENTIFICATION

Probabilistic system identification is used to estimate the values of the joint model parameters. The values of parameters are estimated by minimizing a certain measure of the discrepancy between the measured and predicted responses. In this study, the responses are the static displacements and rotations of structures. The estimation technique in this study is called probabilistic because it accounts for the uncertainties in the measurements and thus it provides information on the reliability of the parameter estimates. It is also characterized as nonlinear and weighted. It is nonlinear because the structural responses are nonlinear functions of the model parameters and weighted because the measured responses are weighted according to their accuracy.

A recursive system identification method, which is called Bayesian estimation, is also presented. This method can be used to identify the joint parameters in a recursive way. In each step of the recursive process, the parameter estimates are updated by using new measurements and the estimation results from the previous cycle.

In Section 3.1, the nonlinear weighted estimation technique is described. The Bayesian estimation technique is introduced in Section 3.2. In both sections, the formulae for the parameter estimates and their covariance matrix are provided. In Section 3.3, it is discussed how to obtain the measured response of a structure.

3.1 Nonlinear Weighted Estimation (Classical Estimation)

The estimation model in this study is nonlinear. More specifically, the analytically predicted displacements of a structure that contains a joint are related to the joint model parameters according to the following nonlinear equation ;

$$\mathbf{u} = \mathbf{K}(\boldsymbol{\beta}, \mathbf{x})^{-1} \mathbf{f}, \quad (3.1)$$

where $\mathbf{u}$ is a displacement vector, $\mathbf{f}$ is a force vector, and $\mathbf{K}$ is a global stiffness matrix whose elements are functions of the vector of the joint parameters, $\boldsymbol{\beta}$, and the vector of the independent (or regressor) variables, $\mathbf{x}$. The joint model parameters consist of the torsional springs, the coordinates of flexible hinges, and the angles which define the orientations of the torsional spring planes (Chapter 2). The independent variables are also called the regressor variables because estimation is called regression in statistics. The regressor variables are all other effects except parameters, which affect the response.

There are discrepancies between analytically predicted and experimentally measured displacements, which are due to the following :

1. Errors in measuring displacements.
2. Errors in modeling joints.
3. Errors in modeling the remaining structure without joints.

In this study, only the first type of error is considered in the estimation of parameters. However, the effect of the second type of error is studied in Chapters 5 to 8 by using joint models with different levels of complexity. The effect of the third type of error is also discussed in Section 5.5.

Considering the first type of error, the measured displacements are written as ;

$$\mathbf{u}_m = \mathbf{K}(\boldsymbol{\beta}, \mathbf{x})^{-1} \mathbf{f} + \boldsymbol{\epsilon}, \quad (3.2)$$

where $\mathbf{u}_m$ is the vector of the measured displacements and $\boldsymbol{\epsilon}$ is the vector of the measurement errors. The elements of $\boldsymbol{\epsilon}$ are assumed to be zero mean Gaussian random variables. However, they are not identically distributed because the errors are not the same for all measurements. The statistics of $\boldsymbol{\epsilon}$ are defined by its covariance matrix, $\mathbf{C}_\epsilon$. The diagonal entries of $\mathbf{C}_\epsilon$ are the variances of the measurement errors. Each nondiagonal entry is the covariance of the measurement errors between the displacements corresponding to the position of the entry.

In nonlinear weighted estimation [Draper and Smith 1981, Bard 1974, Myers 1986, Isenberg 1979], the vector of the parameter estimates, $\mathbf{b}$, is determined by minimizing the weighted sum of squares of errors (WSOS) between the predicted and measured displacements ;

$$(\mathbf{u}_m - \hat{\mathbf{u}})^T \mathbf{C}_\epsilon^{-1} (\mathbf{u}_m - \hat{\mathbf{u}}), \quad (3.3)$$

where $\mathbf{u}_m$ is the vector of the measured displacements and $\hat{\mathbf{u}}$ is the vector of the predicted (estimated) displacements corresponding to $\mathbf{b}$. In the above equations, $\boldsymbol{\beta}$ is different from $\mathbf{b}$; $\boldsymbol{\beta}$ is the vector of the actual values of parameters and $\mathbf{b}$ is the vector of the estimated values of parameters. The inverse of the covariance matrix of the measurement errors, $\mathbf{C}_\epsilon^{-1}$, is introduced in Eq.(3.3) in order to account for the level of accuracy associated with each measurement and weight measurements accordingly.

The linearization method is used to solve the nonlinear estimation problem [Draper and Smith 1981, Bard 1974]. In this method, the vector of displacements, $\mathbf{u}$, in Eq.(3.1) is

linearized about some initial value of the vector of the parameter estimates, $\mathbf{b}_i$, by taking the first order Taylor series expansion. Then the weighted error sum of squares (WSOS) in Eq.(3.3) can be expressed using the vector of the linearized displacements. The vector of the improved parameter estimates, $\mathbf{b}_{i+1}$, is found by setting the derivative of the WSOS with respect to $\mathbf{b}$ equal to zero ;

$$\mathbf{b}_{i+1} = \mathbf{b}_i + (\mathbf{Z}^T \mathbf{C}_\epsilon^{-1} \mathbf{Z})^{-1} \mathbf{Z}^T \mathbf{C}_\epsilon^{-1} (\mathbf{u}_m - \hat{\mathbf{u}}). \quad (3.4)$$

In the above equation, both $\hat{\mathbf{u}}$ and $\mathbf{Z}$ are calculated for $\boldsymbol{\beta}$ equal to $\mathbf{b}_i$. In Eq.(3.4), $\mathbf{Z}$ is the matrix of sensitivity derivatives whose j-th column, $\frac{\partial \mathbf{u}}{\partial b_j}$, is the derivative of the displacement vector, $\mathbf{u}$, with respect to the j-th parameter, b_j . Each column of $\mathbf{Z}$ is found by the Direct method [Haftka et al. 1990] ;

$$\mathbf{K} \frac{\partial \mathbf{u}}{\partial b_j} = \frac{\partial \mathbf{f}}{\partial b_j} - \frac{\partial \mathbf{K}}{\partial b_j} \mathbf{u}. \quad (3.5)$$

Eq.(3.4) can be used to define an iterative procedure for estimating $\mathbf{b}$. Starting with an initial guess for the vector of the parameter estimates, $\mathbf{b}_o$, Eq.(3.4) is used to obtain a sequence of vectors of the parameter estimates, $\mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_n$. In most cases this sequence converges to the vector of the actual values of parameters, $\boldsymbol{\beta}$. The above iterative procedure is stopped according to some convergence criterion.

The convergence of the linearization method can be improved by using a step factor [Myers 1986] to restrain the change in the vector of the parameter estimates at every iteration. This idea is implemented by using the following equation for the updated vector of the parameter estimates, $\mathbf{b}_{i+1}$;

$$\mathbf{b}_{i+1} = \mathbf{b}_i + \lambda \Delta \mathbf{b}_i, \quad (3.6)$$

where λ is a step factor and $\Delta \mathbf{b}_i$ is a vector of increments in the parameter estimates determined at the i -th iteration stage without using a step factor. The $\Delta \mathbf{b}_i$ is obtained from Eq.(3.4) as the difference between $\mathbf{b}_{i+1}$ and $\mathbf{b}_i$. At the beginning of each iteration cycle, λ is set to be equal to 1.0. If the value of the objective function in Eq.(3.3) at $\mathbf{b}_{i+1}$ is larger than the value at $\mathbf{b}_i$, λ is reduced by half. The above procedure is repeated until the value of the objective function at $\mathbf{b}_{i+1}$ becomes smaller than the value at $\mathbf{b}_i$.

The covariance matrix of the parameter estimates is calculated from

$$\mathbf{C}_b = (\mathbf{Z}^T \mathbf{C}_\epsilon^{-1} \mathbf{Z})^{-1}, \quad (3.7)$$

where $\mathbf{Z}$ is calculated at the converged value of the vector of the parameter estimates, $\mathbf{b}$. The diagonal elements of $\mathbf{C}_b$ are the variances of the parameter estimates. The nondiagonal elements are the covariances of the parameter estimates. By comparing the standard deviations of the parameter estimates with those of the measurement errors, the effect of the latter on the former can be assessed.

3.2 Bayesian Estimation (Recursive Estimation)

In Bayesian estimation, a large set of measurements is divided into smaller subsets and parameters are estimated using each data subset successively. Every time a new data subset is considered, the prior parameter estimates and their covariance matrix are used in order to update the parameter estimates and their covariance matrix [Isenberg 1979, Allen and Martinez 1989]. The procedure of Bayesian estimation is shown in Figure 3.1. For example, consider the $(j+1)$ -th data subset with the vector of the prior parameter estimates, $\mathbf{b}^j$, and their covariance matrix, $\mathbf{C}_b^j$, which have been found by using the j -th data subset.

The vector of the updated parameter estimates, $\mathbf{b}^{j+1}$, is found by minimizing the weighted error sum of squares (WSOS) plus the difference between the vector of the prior parameter estimates, $\mathbf{b}^j$, and that of the updated ones, $\mathbf{b}^{j+1}$, weighted by the inverse of the covariance matrix of the prior parameter estimates, $[\mathbf{C}_b^j]^{-1}$, [Isenberg 1979, Allen and Martinez 1989] ;

$$(\mathbf{u}_m - \hat{\mathbf{u}})^T [\mathbf{C}_\epsilon]^{-1} (\mathbf{u}_m - \hat{\mathbf{u}}) + (\mathbf{b}^j - \mathbf{b}^{j+1})^T [\mathbf{C}_b^j]^{-1} (\mathbf{b}^j - \mathbf{b}^{j+1}). \quad (3.8)$$

In the above equation, $\mathbf{u}_m$ is the vector of the measured displacements in the (j+1)-th data subset and $\hat{\mathbf{u}}$ is the corresponding vector of the estimated (predicted) displacements. In Bayesian estimation, the iterative procedure for the vector of the updated parameter estimates is obtained from the following equation ;

$$\mathbf{b}_{i+1}^{j+1} = \mathbf{b}^j + \{[\mathbf{C}_b^j]^{-1} + \mathbf{Z}^T \mathbf{C}_\epsilon^{-1} \mathbf{Z}\}^{-1} \mathbf{Z}^T \mathbf{C}_\epsilon^{-1} \{\mathbf{u}_m - \mathbf{u} - \mathbf{Z}(\mathbf{b}^j - \mathbf{b}_i^{j+1})\}. \quad (3.9).$$

Eq.(3.9) is used to update the estimates of parameters by using the (j+1)-th data subset, the vector of the prior estimates, $\mathbf{b}^j$, and their covariance matrix, $\mathbf{C}_b^j$. The superscripts j and (j+1) in Eqs. (3.8) and (3.9) denote the data subset number. The subscripts i and (i+1) indicate the iteration number in the linearization method (Figure 3.1). Both $\mathbf{u}$ and $\mathbf{Z}$ are calculated for $\boldsymbol{\beta}$ equal to $\mathbf{b}_i^{j+1}$. The covariance matrix of the updated parameter estimates is calculated from the following equation ;

$$\mathbf{C}_b^{j+1} = ([\mathbf{C}_b^j]^{-1} + \mathbf{Z}^T [\mathbf{C}_\epsilon]^{-1} \mathbf{Z})^{-1}. \quad (3.10)$$

If the parameters are estimated using the first data subset, very large values are used for the diagonal elements and zeros for the off-diagonal elements of $\mathbf{C}_b^j$ in Eqs. (3.9) and (3.10). This means that there is no prior information on the reliability of the parameter estimates.

When the estimation model is linear with respect to the parameters, Bayesian estimation gives the same results as a procedure that employs a classical technique (Section 3.1) where the parameters are estimated by considering all the data subsets simultaneously. When the estimation model is nonlinear with respect to the parameters, Bayesian estimation gives results which are only approximately equal to those from classical estimation.

Bayesian estimation provides an efficient way to improve the reliability of the parameter estimates. For example, consider a case that some parameters are poorly estimated because all measurements that have been used in the estimation are not sensitive to these parameters. The whole results do not have to be discarded because the estimates of other parameters might be acceptably accurate. In this case, additional measurements, which are believed to be sensitive to the parameters whose estimates are poor, can be taken in order to update their estimates. If classical estimation technique is used, all the previous and new measurements must be considered at the same time, which is not efficient in terms of computational work. Instead, only the new measurements are considered in order to update the estimates by applying Bayesian estimation. This procedure will be also discussed in Section 4.1 and will be demonstrated in Section 5.2.

3.3 Sources of Measurements

In this study, the measurements of static response of a structure are used in the system identification techniques which were described in Sections 3.1 and 3.2. In general, the measured responses can be obtained by the following ways :

1. Perform experiments.
2. Simulate measurements from the analysis of very detailed finite element models.

3. Simulate measurements by using random number generators.

Only in the first case, actual measurements are obtained. However, it is not always possible to perform experiments. Moreover, it might be impractical to study the effect of design changes on the parameter estimates by performing experiments.

In the second case, the measured displacements are simulated from the analysis of very detailed finite element models whose results are expected to be accurate and to correlate well with experimental results [Sunami et al. 1987]. It is cheaper to simulate the measured displacements by analyzing the detailed finite element models than by performing experiments. This is also helpful in the investigation of the effects of design changes on the behavior of joints in the early design stage of a new vehicle structure. However, in this case, the statistical tools and tests that will be introduced in Chapter 4 may not be directly applicable because the displacements obtained by detailed finite element analysis do not involve random errors in the measurements.

The third case is useful in testing the estimation method. The following procedure is applied in this case : It is assumed that the developed joint model accurately represents the behavior of a joint and thus that there is no error in modeling joints. The displacements are calculated by analyzing a joint model whose parameters are known, and they are contaminated by adding random errors to them. The errors are obtained by assuming that they follow a given probability distribution, and by using a random number generator in the computer. The contaminated displacements simulate the measured ones. The errors are assumed to be independent, zero mean Gaussian random variables in this study.

In this study, the second method is used in Chapters 7 and 8 and the third method is

used in Chapters 5 and 6. In Section 8.4, both the second and third methods are used. In other words, the displacements which are calculated from the analysis of detailed finite element models of joints are contaminated using random numbers in order to simulate the measurements from an experiment.

4. IDENTIFICATION OF IMPORTANT PARAMETERS

Under some conditions, the estimation procedure does not converge or yields poor parameter estimates. These conditions are described in Section 4.1. The procedures to detect these conditions and to improve the accuracy of the parameter estimates and the predicted responses are discussed in Sections 4.2 through 4.5.

4.1 Overparameterization and Underparameterization

There are two conditions under which the parameter estimates are poor or even the estimation procedure does not converge : “overparameterization” and “underparameterization”.

A model is called overparameterized if it contains redundant parameters. In this case, the estimates of some parameters and the fit to the measurements might be poor. Overparameterization occurs in two cases. In the first case, the response of a structure is insensitive to the redundant parameters. This usually occurs when a joint is either too stiff or too flexible in some directions compared to the rest of the structure [Chon 1986]. For each spring rate of a joint model, there are two values called lower and upper sensitivity thresholds. If the value of the spring rate is out of the range defined by the lower and upper sensitivity thresholds, then it does not affect the response of the structure. Depending on the specific case, the joint model can be simplified by considering that the joint is infinitely flexible or stiff in some particular direction. In the second case, some of the parameters are strongly correlated. It is called “multicollinearity” [Draper and Smith 1981]. In this case,

all dependent parameters, which can be expressed in terms of a set of independent ones, must be discarded. Otherwise, the estimation process may not converge.

When there are not enough parameters to accurately represent the behavior of a joint, the model is called underparameterized. Due to underparameterization, the fit to the measurements may be poor and the parameter estimates may be biased. Bias in the parameter estimates is a systematic error in the estimates, which can not be eliminated or reduced by increasing the number of measurements or by decreasing the magnitudes of the measurement errors. With bias in the parameter estimates, the error between measurements and predictions is also systematic.

The estimation procedure may also fail because of “poor design of experiment”. This occurs when all displacement measurements are insensitive with respect to some parameters. Sometimes the poor design of experiment is confused with the overparameterization. In both cases, the parameter estimates are poor. However, in the former case, there are some measurements which are sensitive to the poorly estimated parameters but have not been considered in the estimation. In the latter case, all measurements are insensitive to the poorly estimated parameters. In order to find out if the poor estimation results are due to the overparameterization or the poor design of experiment, additional measurements, which are believed to be sensitive to these parameters, can be taken. Then, Bayesian estimation (Section 3.2) is applied to new measurements. If the parameter estimates improve significantly, then these parameters are important and they should be retained in the model. In this case, it is concluded that the poor parameter estimates are not due to the overparameterization but due to the poor design of experiment. On the other hand, if the estimates do not improve, the corresponding parameters should be neglected because they are redundant.

Some tools are introduced in Sections 4.2 to 4.4, which can be used to identify redundant parameters. Lack of fit test is also introduced in Section 4.5, which can be used to check if a model includes all the important parameters.

The statistical tools that are presented in Sections 4.2 through 4.5 are exact for linear estimation problems. In nonlinear estimation, which is the case of this study, these tools yield good approximations. The methods described in this chapter are applicable to cases in which there are random measurement errors. Thus, these methods are directly applicable to cases that the displacement measurements are obtained either by performing experiments or by using computer simulation with random number generators (cases 1 and 3 in Section 3.3, respectively). Even when the displacement measurements are obtained from the analysis of detailed finite element models (case 2 in Section 3.3), these methods are useful because they use the sensitivity derivatives of responses with respect to parameters and they provide a measure of the error in the parameter estimates due to the error in measurements.

4.2 Covariance Matrix of Parameter Estimates

The covariance matrix of the parameter estimates, C_b , is obtained from Eq. (3.7) or (3.10). The diagonal elements of this matrix are the variances of the corresponding parameter estimates. The variances are measures of the reliability of the corresponding estimates ; the larger the variance, the poorer the estimate. Practically, the reliability of a parameter estimate is assessed by using the coefficient of variation (COV), ρ_i , which is given by ;

$$\rho_i = \frac{\sigma_{b_i}}{b_i}, \quad (4.1)$$

where σ_{b_i} is the standard deviation of the i -th parameter, which is a square root of the variance of the i -th parameter, and b_i is the estimate of the i -th parameter. The COV,

ρ_i , is more appropriate than the variances, $\sigma_{b_i}^2$, because the former is nondimensional and independent of the order of magnitudes of parameters. A large COV of a particular parameter implies that either this parameter is unimportant or that the measurements used in the estimation are insensitive to this parameter (overparameterization or poor design of experiment, respectively).

The correlation matrix of the parameter estimates can be obtained by normalizing the corresponding covariance matrix. The elements of the correlation matrix are called correlation coefficients. The correlation coefficients between the i -th and j -th parameters, R_{ij} , is obtained from

$$R_{ij} = \frac{C_{ij}}{\sigma_{b_i}\sigma_{b_j}}, \quad (4.2)$$

where C_{ij} is the covariance between the i -th and j -th parameters, and σ_{b_i} and σ_{b_j} are the standard deviations of the i -th and j -th parameter estimates, respectively. The correlation coefficient has a value between -1.0 and $+1.0$. If the values of some correlation coefficients are very close to -1.0 or $+1.0$, then either one of two corresponding parameters is not independent. Therefore, the joint model may be simplified by replacing the corresponding parameters with a smaller set of independent parameters.

4.3 Confidence Contours

A confidence contour is a region in the parameter space, centered around the parameter estimates, such that the actual values of the parameters fall within this region with a prescribed probability [Draper and Smith 1981]. The smaller the size of a confidence contour in a particular direction, the more reliable the corresponding parameter estimate.

If the measurement errors are normally distributed, then the probability distribution of the parameter estimates is approximately normal [Draper and Smith 1981]. Therefore, $L(\mathbf{b}) = (\boldsymbol{\beta} - \mathbf{b})^T \mathbf{Z}^T \mathbf{C}_\epsilon^{-1} \mathbf{Z} (\boldsymbol{\beta} - \mathbf{b})$ follows the χ^2 -distribution with p degrees of freedom (d.o.f.'s), where p is the number of parameters. The weighted error sum of squares, $S(\mathbf{b}) = (\mathbf{u}_m - \hat{\mathbf{u}})^T \mathbf{C}_\epsilon^{-1} (\mathbf{u}_m - \hat{\mathbf{u}})$, follows the χ^2 -distribution with $(n-p)$ d.o.f.'s, where n is the number of measurements. Then, the ratio $\frac{L(\mathbf{b})/p}{S(\mathbf{b})/(n-p)}$ follows the F-distribution with p and $(n-p)$ d.o.f.'s. The vector of parameters, $\boldsymbol{\beta}$, which satisfies the following inequality,

$$(\boldsymbol{\beta} - \mathbf{b})^T \mathbf{Z}^T \mathbf{C}_\epsilon^{-1} \mathbf{Z} (\boldsymbol{\beta} - \mathbf{b}) \leq \frac{p}{n-p} (\mathbf{u}_m - \hat{\mathbf{u}})^T \mathbf{C}_\epsilon^{-1} (\mathbf{u}_m - \hat{\mathbf{u}}) F(p, n-p, 1-q) \quad (4.3)$$

gives an approximate confidence contour, where $F(p, n-p, 1-q)$ denotes the F-distribution with p and $(n-p)$ d.o.f.'s at the $(1-q)$ probability level. The probability that the actual parameter values lie within the contour is $1-q$.

In practice, sectional drawings of the confidence contour are plotted in planes which correspond to each pair of parameters β_i and β_j . The values of all other parameters are set to be equal to their estimated values $\beta_k = b_k (k \neq i, j)$ in each of these planes. If all sectional drawings in different planes are very long in the direction that corresponds to a particular parameter, then either this parameter is unimportant, which implies that the model is overparameterized, or the experimental measurements used in the estimation are insensitive to this parameter (overparameterization or poor design of experiment, respectively).

Sometimes, the confidence interval for each parameter is used to assess the reliability of the parameter estimate. The confidence interval for a parameter is obtained by fixing the values of all other parameters equal to their estimates.

4.4 Principal Direction Analysis

Principal direction analysis identifies the important parameters. In this procedure, a vector of the transformed parameters is found. This new vector is the projection of the vector of the original parameters on the coordinate system defined by the eigenvectors of the covariance matrix of the regressor variables of the linearized model [Chatfield and Collins 1980, Harris 1975]. The response of the linearized model, u , is expressed as $u = \sum_{i=1}^p \beta_i x_i$, where p is the number of parameters, β_i is the i -th parameter, and x_i is the i -th regressor variable of the linearized model. In general, the regressor variables are all effects except the parameters, which affect the response. However, since the principal direction analysis is performed on the linearized model, the regressor variables are the derivatives of the responses with respect to parameters [Draper and Smith 1981].

The covariance matrix of the regressor variables is the inverse of the covariance matrix of the parameter estimates. The transformed parameters, which are statistically independent of each other, are linear combinations of the original parameters ;

$$\gamma = E^T \beta, \quad (4.4)$$

where γ is a vector of transformed parameters, E is a matrix whose i -th column corresponds to the i -th eigenvector of the covariance matrix of regressor variables, and β is the vector of the original parameters. It can be shown that each eigenvalue of the covariance matrix of regressor variables is equal to the change in the weighted error sum of squares due to a unit change in the value of the corresponding transformed parameter [Draper and Smith 1981]. Thus, the magnitude of an eigenvalue is a measure of the relative importance of the corresponding transformed parameter; the larger the eigenvalue, the more important the

corresponding transformed parameter. The magnitudes of elements in the eigenvector indicate the relative contributions of the original parameters to the corresponding transformed parameter.

Principal direction analysis can be used to identify unimportant parameters. An original parameter is important if it mostly participates in eigenvectors whose corresponding eigenvalues are large compared to the smallest one. On the other hand, an original parameter is unimportant if it mostly participates in eigenvectors whose corresponding eigenvalues are small compared to the largest one.

Principal direction analysis can also be used to reduce the number of parameters by replacing them with a smaller number of new parameters which are linear combinations of the original ones. These linear combinations are defined by the eigenvectors which correspond to the largest eigenvalues. The procedure is demonstrated, in Section 5.3.

4.5 Lack of Fit Test

The lack of fit test [Draper and Smith 1981] can be used to check if a joint model fits the measured data, which is equivalent to checking if all the important parameters have been included in the model (i.e. to verify that the model is not underparameterized).

The discrepancy between the measured and predicted responses can be decomposed in two components ; the pure error and the error due to the lack of fit. The former is due to the random noise in the measurements. The latter is a systematic error due to an incomplete model. In the lack of fit test, the former is compared to the latter. If the error due to the lack of fit is significantly larger than the pure error, then the discrepancy between the

measured and predicted responses can not be attributed to the randomness involved in the experiment. In this case, the lack of fit test is failed. If a model passes the lack of fit test, there is no reason to suspect that the model is incomplete. It can be tested whether a simplified model, which is obtained after discarding unimportant parameters, fits the data by performing the lack of fit test. This is useful to avoid discarding important parameters.

The lack of fit test requires repeated measurements which are taken by repeating the experiment while keeping the values of all regressor variables unchanged. Both the pure error and the error due to the lack of fit, weighted by the inverse of the covariance matrix of measurement errors, follow the χ^2 -distributions [Draper and Smith 1981]. Then, the ratio of the pure and the lack of fit errors divided by the corresponding degrees of freedom follows the F-distribution ;

$$\frac{(\hat{\mathbf{u}} - \bar{\mathbf{u}})^T \mathbf{C}_\epsilon^{-1} (\hat{\mathbf{u}} - \bar{\mathbf{u}}) / (n - p - n_e)}{(\mathbf{u}_m - \bar{\mathbf{u}})^T \mathbf{C}_\epsilon^{-1} (\mathbf{u}_m - \bar{\mathbf{u}}) / n_e} \sim F(n - p - n_e, n_e, 1 - q), \quad (4.5)$$

where $\bar{\mathbf{u}}$ is a vector whose elements are average displacements among each set of repeated measurements, n is the number of total measurements, p is the number of parameters, m is the number of different sets of values of regressor variables, $n_e = n - m = \sum_{j=1}^m m_j - m$ is the number of degrees of freedom of the pure error, $(1-q)$ is the probability level, and m_j is the number of repeated measurements at the j -th set of values of regressor variables. If the ratio in Eq.(4.5) is smaller than $F(n - p - n_e, n_e, 1 - q)$, then the model passes the lack of fit test at a probability level $(1-q)$, which means that the model is not incomplete.

5. REDUCED MODEL I

The methodology, which was introduced in Chapters 3 and 4, was tested by applying it to a two dimensional automotive structure with flexible joints.

The reduced model I, which was defined in Subsection 2.4.1, was assumed to be exact in this chapter. This model accounts for the joint flexibility only. A random number generator was used to simulate the measurement errors (Section 3.3). More specifically, random numbers were generated from a Gaussian distribution and added to the displacements which were calculated with known values of the joint parameters. The resulting sums simulated the measured displacements. Then, it was assumed that the values of the joint parameters were unknown and the estimation methodology was applied to the simulated measurements in order to estimate the parameters.

Since only static displacements were considered, it was assumed that the standard deviations of the measurement errors were only one percent of the actual displacements (Sections 5.1 to 5.3). The estimation methodology was also tested by considering measurement errors whose standard deviations were five percent of the actual displacements (Sections 5.4 and 5.5). The car model that was used in this chapter is shown in Figure 5.1. The material and geometrical properties of the car structure are given in Table 5.1.

In Section 5.1, the application of nonlinear weighted estimation method to the identification of joints in a car structure is demonstrated. In Section 5.2, Bayesian estimation is tested

in order to improve the parameter estimates which are inaccurate due to poor design of experiment. In Section 5.3, it is discussed how to identify important parameters by using principal direction analysis. In Section 5.4, the effect of the number of measurements is investigated. In Section 5.5, the effect of errors in the model of the overall structure on the reliability of the parameter estimates and the fit to the measurements is investigated.

5.1 Classical Estimation

In this section, the nonlinear weighted estimation methodology was applied to the identification of four flexible joints in a two dimensional automotive structure. The methodology studied in Chapter 4 was also used to identify unimportant parameters.

Figure 5.2 shows a car model with 4 flexible joints, $J_i (i = 1, \dots, 4)$. All other joints were assumed to be rigid. Since it was assumed that the reduced model I was exact, the joint model parameters were the spring stiffnesses at these joints. In Figure 2.6 and Subsection 2.4.1, k_{ij} was defined as the spring which constrains the relative rotation between the i -th and j -th branches. For the car model in Figure 5.2, a different notation was used for the stiffness parameters ; k^1 stands for k_{12} of joint J_1 , k^2 for k_{12} of joint J_2 , k^3 for k_{13} of joint J_3 , and k^4 for k_{13} of joint J_4 , where J_i is the i -th flexible joint of a car model in Figure 5.2. The corresponding number of branches, i and j , of k_{ij} , are denoted in Figures 5.1 and 5.2. Branches 1 and 3 at joint J_1 were rigidly connected, while branches 2 and 3 were free to rotate ($k_{13} = \infty$ and $k_{23} = 0$). Branches 1 and 2 at joints J_3 and J_4 were rigidly connected, while branches 2 and 3 were free to rotate ($k_{12} = \infty$ and $k_{23} = 0$). The actual values of the stiffness parameters were $k^1 = 1.13 \times 10^8$, $k^2 = 5.65 \times 10^9$, $k^3 = 1.13 \times 10^8$, and $k^4 = 1.13 \times 10^7$ (N-mm/rad). These actual values are in the ranges of stiffnesses of

typical car structures. However, the actual value of k^2 was intentionally taken larger than those of typical car structures, in order to test the procedure of model simplification.

The displacements $u_1, v_1, u_2, v_2, u_3, u_5$ were measured with load F_1 applied. The magnitude of F_1 was 4448.0 (N). Each displacement was measured 5 times under which the structure was subjected to the same load. Thus, the total number of measurements was 30. The errors in the measurements were assumed to be Gaussian. The parameters were estimated by applying nonlinear weighted estimation (classical estimation) to the above measurements. The estimation model was also tested for underparameterization by performing the lack of fit test.

The results of classical estimation and principal direction analysis are summarized in Table 5.2. In Figure 5.3, the sectional drawing of the confidence contour in k^1k^2 -plane is plotted. This sectional confidence contour is the intersection of the hyper ellipsoid representing the confidence contour of the parameter estimates with the plane of $k^3 = \hat{k}^3$ and $k^4 = \hat{k}^4$. This confidence contour corresponds to a probability level of 0.95. In the above, $\hat{k}^i$ denotes the estimate of the i -th parameter, k^i .

The following are observed from Table 5.2 and Figure 5.3 :

1. The estimate of k^2 and its coefficient of variation are large compared to those of the other parameters.
2. In principal direction analysis, the smallest eigenvalue is almost zero. Only k^2 contributes to the corresponding eigenvector. Furthermore, k^2 does not contribute significantly to the first three eigenvectors which correspond to the three largest eigenvalues.
3. The sectional drawing of the confidence contour in k^1k^2 -plane is a thin ellipse whose

long axis is almost parallel in the direction of k^2 . The other sectional drawings of the confidence contour in $k^2 k^3$ - and $k^2 k^4$ - planes are also thin ellipses in the direction of k^2 . Actually the hyper ellipsoidal confidence contour in a multidimensional space is very long in the direction of k^2 .

4. The estimate of k^2 is large compared to the estimates of other parameters, which indicates that the actual value of k^2 may exceed the upper sensitivity threshold (Section 4.1). In order to check the possibility that the actual value of k^2 may be smaller than the lower sensitivity threshold, the weighted error sums of squares (WSOS's) were calculated with either $k^2 = 0$ or $k^2 = \infty$. The WSOS was smaller when $k^2 = \infty$, which means that k_2 is unimportant because this joint is rigid.

Based on the above observations and the fact that the model in Table 5.2 passed the lack of fit test, it appears that k^2 can be discarded ($k^2 = \infty$) without losing significant accuracy.

Table 5.3 summarizes the results of classical estimation and principal direction analysis considering only three parameters k^1, k^3 , and k^4 ($k^2 = \infty$). By comparing Tables 5.2 and 5.3, it is observed that the car model which considers 4 flexible joints fits the measurements equally well as the car model which considers only 3 flexible joints ($k^2 = \infty$). Indeed, the weighted error sums of squares (WSOS's) are almost equal in both cases. Furthermore, the estimated values of k^1, k^3 , and k^4 in Table 5.3 are closer to the actual values than those in Table 5.2. Therefore, the car model which considers only 3 flexible joints is sufficiently accurate for predicting the response of the car body.

Finally, the lack of fit test in Eq.(4.5) was performed to verify that the car model with k^1, k^3 , and k^4 includes all the important parameters. The probability level used in this test was 0.95. The left hand side of Eq.(4.5) was found to be equal to 0.11, which is less than the

right hand side, $F(3,24,0.95)=3.01$. Therefore, there is no reason to reject the hypothesis that the car model which considers only 3 flexible joints fits the measurements.

In Figure 5.4, the predicted displacements considering 3 flexible joints, the displacements calculated by a rigid joint model, and the measured displacements are plotted. For illustrative purposes, the displacements were magnified by 200 times. It was observed that there were large discrepancies between the displacements calculated with a rigid joint model and the measurements, which demonstrates the importance of considering the flexibility of joints. The predicted displacements considering 3 flexible joints were in very good agreement with the measurements.

5.2 Bayesian Estimation

In this section, the use of Bayesian estimation to improve the reliability of the parameter estimates was demonstrated.

The joint parameters of the same car body which was used in the previous section (Figures 5.1 and 5.2) were estimated by measuring the displacements $u_1, v_1, u_2, v_2, u_3, u_5$ once with load $F_1 = 4448.0$ (N) applied. Thus, the total number of measurements was 6. The results of estimation and principal direction analysis are summarized in Table 5.4. As in the previous example, it was observed from Table 5.4 that the measured displacements were not sensitive to k^2 . However, it is difficult to draw any definite conclusion regarding k^4 because, although the coefficient of variation of k^4 is large compared to those of k^1 and k^3 , principal direction analysis indicates that k^4 is important.

In order to decide whether k^4 is important, some additional measurements, which were

expected to be sensitive to k^4 , were taken. Bayesian estimation was applied to these new measurements in order to update the prior parameter estimates which are shown in Table 5.4. For this purpose, a set of displacements $u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4$ were measured once, with load F_2 applied. The number of measurements was also 6 and the magnitude of F_2 was 1334.4(N). Bayesian estimation was applied to the new measurements using the prior parameter estimates in Table 5.4. In Table 5.5, the results of principal direction analysis as well as the results of Bayesian estimation are summarized. Then another set of displacements $u_1, v_1, u_2, v_2, u_4, v_4$ were measured once, with loads F_2 and F_3 applied at the same time. The number of measurements was 6 and the magnitude of F_2 and F_3 were 1334.4 and 3336.0 (N), respectively. Bayesian estimation was applied to the new measurements using the prior parameter estimates in Table 5.5. In Table 5.6, the results of Bayesian estimation and principal direction analysis are summarized. By comparing Tables 5.4, 5.5, and 5.6, the following were observed :

1. The reliability of the estimates of all parameters improved by using Bayesian estimation.
2. Principal direction analyses in all the tables indicate that k^2 is not important.
3. The estimate of k^2 was dramatically changed. However, its coefficient of variation remained significantly larger than those of other parameters after applying Bayesian estimation. This indicates that the weighted error sum of squares and the displacements are not sensitive to k^2 .
4. The large value of the estimate of k^2 indicates that it may exceed the upper sensitivity threshold.

Considering all these observations, it is concluded that k^2 can be discarded.

On the other hand, the improvement in the reliability of the estimate of k^4 was significant.

From the value of its coefficient of variation and the results of principal direction analysis, k^4 appears to be the most important parameter. Therefore, k^4 should be considered in the car model.

The parameters were also estimated considering all the 18 measurements at the same time ; each 6 measurements from each of 3 loading conditions F_1 , F_2 , and F_2 and F_3 . In Table 5.7, the results of classical estimation and principal direction analysis are summarized. It is found that the parameter estimates and their coefficients of variations in Tables 5.6 and 5.7 are approximately equal except the estimate of the unimportant parameter k^2 , which can be discarded. The results of principal direction analysis are also approximately equal in both tables. In conclusion, it is found that Bayesian estimation yields results which are approximately equal to those from classical estimation.

5.3 Principal Direction Analysis

In this section, principal direction analysis was used to reduce the number of parameters by combining them into a smaller set of parameters.

Figure 5.5 shows a 3-branch joint of a car model in a plane, where 3 parameters k^1, k^2 , and k^3 of the rocker to B-pillar joint were assumed to be unknown. Note that the notation is different from that in Subsection 2.4.1. All other joints were assumed to be rigid. The values of these parameters were $k^1 = 1.13 \times 10^{12}$, $k^2 = 2.26 \times 10^8$, and $k^3 = 1.13 \times 10^8$ (N-mm/rad). The value of k^1 is larger than the upper sensitivity threshold, which means that the two horizontal branches of the joint are practically rigidly connected. The displacements v_2, v_4, u_5 were measured by applying load F_1 and v_1, v_3, u_6 by applying load F_2 . The magnitudes of

F_1 and F_2 were 3336.0 and 1334.4(N), respectively. Each displacement was measured 4 times. Thus, the total number of measurements was 24.

The results of principal direction analysis are summarized in Table 5.8. For this example, the iteration procedure did not converge because of the strong correlation between parameters. However, principal direction analysis performed after a number of iterations shows how to simplify the model. More specifically, the first eigenvalue is at least 10^5 times larger than the other two. From the smallest eigenvalue and the corresponding eigenvector, it is clear that k^1 is not important. The second eigenvector, which is practically the difference between k^2 and k^3 , is also unimportant because the corresponding eigenvalue is small compared to the largest eigenvalue. The largest eigenvalue and the corresponding eigenvector indicate that k^2 and k^3 can be combined into one new parameter which is their summation. This is the only important parameter. Thus, the joint model can be simplified by considering one parameter, $k^2 + k^3$, which restrains the rotation of the B-pillar with respect to the rocker.

In order to validate the above conclusions, a simplified joint model containing only one parameter $\gamma = k^2 + k^3$ was considered. This new parameter, γ , restrains the rotation between the B-pillar and the rocker. This new parameter is equal to the parallel connection of k^2 and k^3 . By using classical estimation procedure it was found that the estimate of the equivalent spring is $\hat{\gamma} = 3.408 \times 10^8$ (N-mm/rad), which is almost equal to the sum of the actual values of k^2 and k^3 . This model passed the lack of fit test, which indicates that there is no reason to suspect that it is underparameterized.

This example demonstrates that principal direction analysis is useful not only for discarding unimportant parameters but also for combining two or more important parameters into one,

thus, reducing the number of joint model parameters.

5.4 Effect of Number of Measurements

The objective of this section is to study the effect of the number of measurements on the parameter estimates. For this purpose, the displacements $u_1, v_1, u_2, v_2, u_3, u_5$ were measured with load F_1 applied to the same car body that was used in Section 5.1 (Figures 5.1 and 5.2). The values of all four parameters, $k^i (i = 1, \dots, 4)$, were equal to 1.13×10^8 (N-mm/rad). This value is in a region between the lower and upper sensitivity thresholds for each of four parameters. The magnitude of F_1 was 4448.0 (N). The standard deviations of the measurement errors were five percent of the exact displacements. The above set of measurements were taken once and also repeatedly for 10, 20, 30, 40, and 50 times. Thus, the total numbers of measurements were 6, 60, 120, 180, 240, and 300, respectively.

The estimate of k^1 , which is normalized by its exact value, is plotted in Figure 5.6 as a function of the number of measurements. Centered around the normalized estimate of k^1 , is shown a band whose width is equal to two standard deviations of $\hat{k}^1$. It was observed that the normalized estimate approaches the value of one, which implies that the parameter estimate approaches its actual value as the number of measurements increases. It was also observed that the width of the band decreased, which implies that the reliability of the parameter estimate improves as the number of measurements increases. The same trend was observed for the other parameters.

5.5 Effect of Errors in Modeling the Overall Structure

In the estimation, the error in modeling the overall structure was neglected. Only measurement errors were considered. The objective of this section is to investigate the effect of errors in modeling the vehicle structure on the parameter estimates.

The same car body in Figures 5.1 and 5.2 was used as in Section 5.1. The actual values of the stiffness parameters were $k^1 = 1.13 \times 10^8$, $k^2 = 5.65 \times 10^7$, and $k^3 = k^4 = 5.65 \times 10^8$ (N-mm/rad). In order to investigate the effect of errors in modeling the overall vehicle, the parameters were estimated using a different value of Young's modulus than that used for simulating measurements.

A force $F_1 = 4448.0$ (N) was applied to take 6 measurements, $u_1, v_1, u_2, v_2, u_3, u_5$ (Figure 5.2). Each measurement was taken 5 times, thus, the total number of measurements was 30. The measurement errors were assumed to have standard deviations which are 5.0 % of the actual displacements. It was also assumed that there were errors in modeling the finite elements of the roof and the B-pillar. This modeling error was simulated by using the values of the Young's moduli of these finite elements in estimation that were equal to 80.0, 90.0, 100.0, 110.0, 120.0 (%) of the exact value. The exact value was used in simulating displacement measurements. In Figures 5.7 and 5.8, the parameter estimates and the weighted error sums of squares (WSOS's) are plotted. These figures correspond to errors in Young's moduli of the roof and the B-pillar, respectively. The parameter estimates and WSOS's are normalized by the corresponding values in the case that there were no modeling errors. The deviations of the normalized parameter estimates from the value of one indicates the errors in the parameter estimates introduced by modeling error,

which is called bias of the estimates. The values of the normalized WSOS's that are larger than one indicate that there is discrepancy between predictions and measurements that is introduced by modeling error. It was observed that the results of estimation were not significantly affected by modeling errors.

The worst possible case was also investigated in which there were errors in the Young's moduli of all the elements of the whole vehicle structure. To estimate parameters, a force $F_1 = 4448.0$ (N) was applied to take 6 measurements, $u_1, v_1, u_2, v_2, u_3, u_5$, and another force $F_4 = 444.8$ (N) was applied to take another 6 measurements, $u_1, u_2, v_2, u_3, v_3, u_5$ (Figure 5.2). Each measurement was taken 5 times, thus, the total number of measurements was 60 in this case. Then it was assumed that there were modeling errors in the stiffnesses of all the elements of the structure by changing the values of Young's moduli of all the elements to 80.0, 90.0, 100.0, 110.0, 120.0 (%) of the exact value. The parameters were estimated by using these values. In Figure 5.9, the normalized parameter estimates and WSOS's are plotted. In this case, the bias in the parameter estimates was larger and the fit to the measurements was worse compared to the case in which there were modeling errors in the Young's moduli of either the roof or the B-pillar.

Finally, the predicted and measured displacements are compared in order to assess the effect of modeling errors on the prediction of displacements. A force $F_3 = 4448.0$ (N) was applied as in Figure 5.2 and displacements were calculated using the values of parameters estimated from the car model which involved modeling errors. For this purpose, displacements were calculated by using a car model in which the Young's moduli of all the elements of the whole structure were equal to 110.0 (%) of the actual value. The biased parameter estimates were used in calculating displacements.

The plots of the actual displacements, the displacements predicted with the biased parameter estimates, and the displacements calculated under the assumption of rigid joints are shown in Figure 5.10. The actual displacements were calculated using the actual values of parameters without modeling error. In the latter two cases, displacements were calculated with modeling error. For the purpose of illustration, the displacements in Figure 5.10 are magnified by 200 times. It is observed that, even with modeling error, the predicted displacements are in considerably better agreement with the measurements than the displacements calculated under the assumption of rigid joints. This indicates that the proposed joint model and the estimation procedure are likely to be effective even if there are errors in modeling the overall vehicle structure.

6. 2-D BRACKETED MODEL

The estimation methodology was applied to a two dimensional automotive structure with flexible joints. The 2-D bracketed model, which was defined in Subsection 2.4.4, was assumed to be exact in this chapter. A random number generator was used as in Chapter 5 to simulate the measurement errors (Section 3.3). The measurement errors were assumed to be statistically independent Gaussian random variables whose standard deviations were five percent of the exact displacements. The same car structure that was described in Chapter 5 (Figure 5.1) was used. The material and geometric properties of the car structure are given in Table 5.1.

In Section 6.1, the importance of the offsets of rotation centers is studied by comparing a joint model that accounts for the offsets of rotation centers against a model that neglects them. The bias in the parameter estimates is studied in Section 6.2. In Section 6.3, the importance of the springs that account for the coupling between displacements of different branches is studied. In this chapter, some conditions are determined, under which the 2-D bracketed model can be simplified by neglecting the effect of the springs that connect each two branches of a joint or the effect of the rotation center offsets.

6.1 Importance of Offsets of Rotation Centers

The objective of this section is to investigate the importance of on-branch offsets of rotation centers and to determine the conditions under which the joint model can be simplified.

Figure 6.1 shows a car model with 4 flexible joints. According to the notation in Subsection 2.4.4, r_i is the on-branch offset of rotation center of the i -th branch, k_i is the spring that accounts for the flexibility of the i -th branch, and k_{ij} is the spring that accounts for the coupling between rotations of the i -th and j -th branches. The actual values of the joint parameters of the car model are presented in Table 6.1. The displacements in x and y directions, u_i and v_i , respectively, were measured at 5 nodes with each of 4 loads, F_i ($i=1,\dots,4$), applied as they are shown in Figure 6.1. Each displacement was measured twice under which the structure was subjected to the same load. Thus, the total number of measurements was 80. The magnitude of each load was 2224.0 (N).

The parameters at the second joint, J_2 , were estimated assuming that the values of all the other parameters were known. The actual values of the stiffness parameters of this joint were $k_1 = k_2 = 1.13 \times 10^8$ and $k_{12} = 5.65 \times 10^7$ (N-mm/rad). The estimation of these parameters was repeated for four different offset lengths at the same joint in order to investigate the importance of the offset parameters. The actual values of the offsets were 50.8, 76.2, 127.0, and 177.8 (mm), respectively. The values of the offsets of two connecting branches were set to be equal. The parameters were estimated by using the reduced models I and II, which were defined in Subsections 2.4.1 and 2.4.2, respectively. The results of the estimation are summarized in Table 6.2 for different length of offsets and for different modeling options.

In Figure 6.2, the weighted error sums of squares (WSOS's) are plotted as functions of the offset length for different modeling options. The WSOS's are measures of the fit to the measurements which were used in the estimation. The values of the WSOS's in Figure 6.2 were normalized using the value corresponding to the 2-D bracketed model (2-D bracketed

model was assumed to be exact in this chapter). From Table 6.2 and Figure 6.2, the following were observed with respect to the fit to the measurements :

1. Reduced model II yields very good approximations for all cases.
2. Reduced model I gives good approximations when offsets are small.

From the above observations, it appears that parameters k_{ij} representing the coupling between rotations of different branches can always be neglected when offsets are small (less than 80.0 mm in this example). On the other hand, offsets are important if they are large. Finally, it was observed that the assumption of rigid joints leads to significant errors in predicting the response of the structure.

6.2 Bias in Parameter Estimates

In this section, the accuracy of the parameter estimates is studied for different values of the offset length and also for different joint models.

In Table 6.2, the parameter estimates are listed for cases of different offset lengths. These estimates are biased, which means that there is an error whose nature is systematic. In other words, this type of error cannot be eliminated by increasing the number of measurements or even by reducing the measurement errors. The bias is larger in the reduced models I and II than in the 2-D bracketed model. To measure the bias of the estimates of all the parameters, the absolute value of the difference between the estimated and actual values of parameters were calculated and normalized using the corresponding actual values. The above values are averaged for all parameters in each of the 2-D bracketed and the reduced model II and are plotted in Figure 6.3. The bias in the parameter estimates of the 2-D bracketed model is considerably higher for small values of the offsets than those for large

values. This is true because the offsets and the spring rates are almost perfectly correlated when the offsets are small. In other words, change in offsets can be compensated by change in spring rates, which does not affect the structural responses. More specifically, there is an infinite number of combinations of parameter values that correspond to almost the same value of the objective function (WSOS : weighted error sum of squares) in Eq.(3.3). However, this is not true when the length of the offset is large because the displacement shape obtained by increasing offsets will be different from that obtained by increasing spring rates. As a result the bias in the parameter estimates decreases as the offsets increase. In the reduced model II, the estimates of the parameters are more biased than in the 2-D bracketed model. However, this model fitted the measurements well, as it was observed in the previous section.

In order to check if the joint models can accurately describe the behavior of the joints, a load case that was not used in the estimation of parameters was considered in order to compare the predicted displacements for different modeling options. For this purpose, displacements were calculated under a force $L = 4448.0$ (N) for different modeling options (Figure 6.1). The displacements predicted from different joint models are plotted in Figure 6.4 for the offset length equal to 50.8 (mm). The displacement were magnified by 200 times. It was observed that the displacements predicted by the reduced model II and by the 2-D bracketed model were practically identical even though the parameter estimates were biased (Table 6.2). This indicates that the reduced model II is sufficiently accurate in predicting the behavior of the joint. The displacements predicted by the reduced model I were different from those predicted by the 2-D bracketed model. However, they are more accurate than the displacements calculated under the assumption of the rigid joint.

6.3 Consideration of the Worst Loads

The worst possible loads were considered in order to check if the joint behavior can be accurately described with the reduced model II whose parameter estimates are biased.

The loads which are used to compare two models under the worst circumstances can be found by either of the following ;

1. the load that maximizes the difference in the strain energies of two models,

$$\text{maximize} \quad U_{ModelA} - U_{ModelB} = \mathbf{q}^T [\mathbf{F}_{ModelA} - \mathbf{F}_{ModelB}] \mathbf{q}, \quad (6.1)$$

2. the load that maximizes the ratio of the strain energies of two models,

$$\text{maximize} \quad \frac{U_{ModelA}}{U_{ModelB}} = \frac{\mathbf{q}^T \mathbf{F}_{ModelA} \mathbf{q}}{\mathbf{q}^T \mathbf{F}_{ModelB} \mathbf{q}}, \quad (6.2)$$

where U is strain energy, $\mathbf{q}$ is the generalized force vector, and $\mathbf{F}$ is the flexibility matrix.

The load vector which corresponds to Eq.(6.1) can be shown to be an eigenvector which corresponds to the largest eigenvalue of the difference in the flexibility matrices of two models [Strang 1980] ;

$$[(\mathbf{F}_{ModelA} - \mathbf{F}_{ModelB}) - \lambda_{max}] \mathbf{f} = 0, \quad (6.3)$$

where λ_{max} is the largest eigenvalue and $\mathbf{f}$ is an eigenvector which corresponds to λ_{max} .

The load vector which corresponds to Eq.(6.2) can be shown to be an eigenvector which corresponds to the largest eigenvalue of the following general eigenvalue problem ;

$$[\mathbf{F}_{ModelA} - \lambda_{max} \mathbf{F}_{ModelB}] \mathbf{f} = 0, \quad (6.4)$$

where λ_{max} is the largest eigenvalue and $\mathbf{f}$ is an eigenvector which corresponds to λ_{max} .

Because the 2-D bracketed model was assumed to be exact in this chapter, Model A and Model B in Eqs. (6.1) to (6.4) correspond to the 2-D bracketed model and the reduced model II, respectively. The flexibility matrices and strain energies of both models were found at the position where the load L in Figure 6.1 was applied for the offset length equal to 50.8 (mm). The load vector that maximizes the difference in strain energies (Eq.6.1) was considered. From Eq.(6.3), it was found that $\lambda_{max} = 0.2648 \times 10^{-7}$ and $\mathbf{f} = (f_x, f_y, M) = (0.9985, 0.0004, -0.0556)$, where f_x and f_y are force components in x- and y-directions, respectively, and M is a moment in xy-plane. The two joint models were compared under this load vector in terms of strain energies. The strain energy in the reduced model II was 100.02 (%) of that in the 2-D bracketed model, which indicates that the reduced model II is a very good approximation of the 2-D bracketed model.

The load vector that maximizes the ratio of strain energies (Eq.6.2) was also considered. From Eq.(6.4), it was found that $\lambda_{max} = 1.0000$ and $\mathbf{f} = (f_x, f_y, M) = (0.0556, 0.0212, 0.9982)$. Under the above load vector, the strain energy in the reduced model II was 100.00 (%) of that in the 2-D bracketed model. The above results indicate that the reduced model II can be used instead of the 2-D bracketed model without losing accuracy significantly. Actually, all the elements of the two flexibility matrices were equal up to 3 significant digits.

6.4 Effect of the Ratio of k_i/k_{ij}

In this section, the importance of parameter k_{ij} was investigated for different values of the ratio k_i/k_{ij} . The same car model in Figure 6.1 was used. The procedure for measuring displacements and estimating parameters was repeated as in Section 6.1. Only the parameters of the second joint, J_2 , were estimated. The values of the parameters of the other joints

were assumed to be known. The estimation of these parameters was repeated for three different values of the ratio of k_i/k_{ij} and also for four different values of the offsets. The ratio of k_i/k_{ij} was changed by fixing the value of k_{12} equal to 1.13×10^7 and by changing the values of k_1 and k_2 equal to 2.26×10^7 , 1.13×10^8 , and 5.65×10^8 (N-mm/rad), respectively. For each value of k_i/k_{ij} , the actual values of offsets were also changed ; 50.8, 76.2, 127.0, and 177.8 (mm). The values of the offsets of two connecting branches at J_2 were set to be equal. The parameters of J_2 were estimated by using the 2-D bracketed and the reduced models I and II. In Figure 6.5, the weighted error sums of squares (WSOS's) of the reduced models I and II are plotted for different values of the ratio of k_i/k_{ij} as a function of the offset length. First, it was observed that the reduced model II fitted the measurements very well in all cases. It was also observed that both reduced models fit measurements better when offsets were smaller. The fit was also better for higher values of the ratio of k_i/k_{ij} .

In conclusion, for the examples studied in this section, we may always simplify a joint model by neglecting the coupling between rotations of different branches. However, the effect of the offsets of the rotation centers cannot be neglected.

7. 3-D ISOLATED JOINTS CONSISTING OF SIMPLE BOX BEAMS

The estimation methodology was applied to the subassemblies that consist of three dimensional isolated joints and attached branches made of simple box beams (Figure 7.1). The subassemblies used in this chapter are symmetric in xy and xz planes as shown in Figure 7.1. Thus, there is no coupling between rotations in different planes. Therefore, it is expected that the reduced model II, which is defined in Subsection 2.4.2, should accurately describe the behavior of these joints. The reduced model II accounts for both the joint flexibility and the on-branch offsets of rotation centers along the shear axis of the corresponding branch.

To simulate the measured displacements, detailed finite element models of the subassemblies were analyzed using MSC/NASTRAN as discussed in Section 3.3. More specifically, the displacements of the subassemblies were calculated under static loads and they were used as measurements in order to estimate the joint parameters.

In Section 7.1, the geometry of the subassemblies is described. Both the detailed and the concept models of these subassemblies are described. In Section 7.2, the effect of the rotation center offsets of joint models is examined. The effect of the panel thickness of the cross section on the parameter estimates is investigated in Section 7.3. The effect of the member length on the parameter estimates is studied in Section 7.4.

7.1 Description of the Subassemblies

A subassembly that consists of two intersecting box-shaped beams is shown in Figure 7.1. This subassembly is constructed by attaching the vertical member to the top of the horizontal one. This construction does not allow the two horizontal branches to rotate with respect to each other in the rigid body motion. However, the vertical branch rotates relatively to the horizontal one. Thus, this is practically a one-branch joint. In our joint model, only the flexibility due to the vertical branch needs to be accounted for.

The subassembly of a joint and attached branches in Figure 7.1 is symmetric in xy and xz planes. Due to the geometric symmetry, there is no coupling between rotations of the vertical branch in different planes. Moreover, the shear axes of the three branches lie in the same plane. Thus, it can be assumed that the off-branch offsets are zero and that the three spring axes coincide with the local axes. The local axes of a branch are defined as follows ; one axis coincides with the shear axis of the branch while another axis is parallel to the other branch. The last axis is normal to the other two axes. According to the foregoing discussion, only the reduced models I and II (Subsections 2.4.1 and 2.4.2) are studied in this chapter. The reduced model I is described with 3 spring rates. The reduced model II is described with 5 parameters ; 3 spring rates and 2 on-branch offsets of bending rotations. The coupling between the rotations of different branches is not considered because this joint is practically a one-branch joint. A concept model of the subassembly of a joint and the attached branches is shown in Figure 7.2. Only one element in the vertical member is represented by using a joint element, which consists of 2 rigid sections, 1 flexible section, and 3 springs at 3 hinges. It is enough to consider only 2 rigid sections because the offset of twist is zero (Subsection 2.4.2). All the other elements are represented by using beam

elements.

To simulate the measured displacements, a very detailed finite element model for the subassembly (Figure 7.3) was generated which consisted of plate elements, CQUAD4, in MSC/NASTRAN [Schaeffer 1979]. The number of elements depends on the length of branches. For the model with the shortest branch, the number of elements was 3900. According to Sunami et al. [1987], the displacements obtained by analyzing detailed finite element models were almost equal to the corresponding experimental measurements. All the loads were applied at the free end of the vertical member while the ends of the horizontal one were clamped. The displacements were calculated at several positions along the vertical member. Because there were many grids at each cross section of the vertical member in a detailed model, the average of the displacements at all the grids in the same cross section was used as the measured displacement.

The measured displacements, which are simulated from the analysis of detailed finite element models, do not involve random measurement errors. However, it was assumed that all measurement errors were equal when they were expressed as percentages of the measured displacements which were calculated by analyzing a detailed finite element model. The above assumption was used to consider that all the measurements were equally important. In the examples of this chapter, the standard deviation of the measurement error was assumed to be 10.0 % of the measured displacement. Therefore, the effects of all measurements on the results of estimation were all equal. Actually, the estimated parameter values in Eq.(3.4) do not change if all the standard deviations of the measurement errors are scaled by the same number. Only the weighted error sum of squares (WSOS) in Eq.(3.3) and the coefficients of variation of the parameter estimates in Eq.(3.7) will change in that case.

Two subassemblies of different sizes were analyzed. The first subassembly was identical to the joint used by Sunami et al. [1987, 1990]. The second subassembly was larger than the first one and it had almost the same dimensions as B-pillar to rocker joints in typical real car structures. A B-pillar to rocker joint is a joint at the intersection of a B-pillar and a rocker (Figures 5.1 and 6.1). In this case, the horizontal and vertical members correspond to the rocker and the B-pillar, respectively (Figure 7.1). The first subassembly was analyzed in Sections 7.2 to 7.4. The second one was studied in Section 7.4.

7.2 Importance of Offsets of Rotation Centers

The objective of this section is to study the effect of offsets on the prediction of the displacements. For this purpose, the T-shaped subassembly, which was identical to the one used by Sunami et al. [1987, 1990], was identified with different joint models.

The geometric properties of the subassembly are as follows ; the rocker length $L_R = 800.0$, the B-pillar length $L_P = 400.0$, $l_f = l_w = 50.0$ and $t_f = t_w = 1.6$ for the cross section of the B-pillar, and $l_f = l_w = 50.0$ and $t_f = t_w = 1.6$ (mm) for the cross section of the rocker (Figure 7.1). The material properties are as follows ; Young's modulus $E = 2.09 \times 10^5$ and Shear modulus $G = 8.10 \times 10^5$ (N/mm²). Forces in F/A (fore/aft or inplane) and I/O (inboard/outboard or out-of-plane) directions, $f_{F/A}$ and $f_{I/O}$, respectively, and a twist moment, M_T , were applied in order to take measurements. The magnitudes of forces and moment were as follows ; $f_{F/A} = f_{I/O} = 980.0$ (N) and $M_T = 2.45 \times 10^4$ (N-mm). Displacements and rotations in the directions of the corresponding forces and moment were calculated at 18 different positions along the B-pillar. Thus, the number of measurements was 54.

The parameters of the reduced models I and II were estimated by using the estimation methodology, which was described in Chapter 3. In Table 7.1, the results of estimation using each of the reduced models I and II are shown. The estimates of the spring rates of the reduced model I were larger than those of the reduced model II (Table 7.1). This is true because the spring rates of the reduced model I need to be increased in order to account for the effect of the offsets of rotation centers of the reduced model II.

In Figures 7.3 to 7.5, the measured displacements and rotations are compared with the ones which were predicted with different joint models and also which were calculated under the assumption of rigid joint ; The displacements in F/A (fore/aft) and I/O (inboard/outboard) directions and the twist rotations were examined. In Figures 7.3 to 7.5, the effect of axial displacements was not considered. It is observed that the agreement between the predictions of the reduced model II and the measurements is very good. The reduced model I gave better results than those calculated under the assumption of rigid joints. However, the fit was not as good as in the case of the reduced model II. Consequently, the on-branch offsets of rotation centers play an important role in describing the behavior of joints.

7.3 Effect of Panel Thickness

In this section, the effect of the rocker panel thickness on the joint model parameters was investigated. The study in this section is important because it provides information to designers on how to improve the joint behavior in the most efficient way.

The T-shaped subassembly in Figure 7.1 was used. In this figure, the top and bottom panels of the rocker are called flanges while the two side panels are called webs. The thickness of

both flanges and webs ranged between 1.28 and 1.92 (mm). This is equivalent to $\pm 20\%$ change in the thickness from the baseline value of 1.6 (mm), which was used in Section 7.2. When the effect of changes in the flange thickness was studied, the web thickness was fixed equal to 1.6 (mm), and vice versa. All the other dimensions were fixed equal to the values used in Section 7.2. The same material properties were used as in Section 7.2.

The detailed finite element models of the subassembly were analyzed for the different thickness of the rocker flanges and webs. As in Section 7.2, forces in F/A (fore/aft) and I/O (inboard/outboard) directions and a twist moment were applied and the corresponding displacements and rotations were calculated at 18 different locations along the B-pillar. Thus, the number of measurements was 54. The magnitudes of forces and moment were as follows ; $f_{F/A} = f_{I/O} = 980.0$ (N) and $M_T = 2.45 \times 10^4$ (N-mm). The estimates of the parameters of the reduced model II were obtained for all the cases.

The variations of stiffnesses in F/A, I/O, and twist directions due to changes in the rocker panel thickness are shown in Figures 7.6 to 7.8, respectively. According to these figures, the stiffness in the I/O-direction was found to be the most sensitive parameter with respect to the changes in the rocker panel thickness. The stiffness in the F/A-direction seems to be the least sensitive. All three stiffnesses were found to be more sensitive to the changes in the thickness of the rocker web than that of the rocker flange. This indicates that it is more effective to increase the web thickness than the flange thickness in order to increase the joint stiffness. In Table 7.2, the values of the on-branch offsets are shown for all the cases. The values of on-branch offsets are not sensitive to changes in the panel thickness.

7.4 Effect of Member Length

The behavior of a joint in the vehicle structure may be significantly different than that of an isolated subassembly, which consists of the joint and the attached branches. In this section, the effect of the length of the attached branches on the joint model parameters was investigated.

Two subassemblies of different sizes were used in this section. The material properties are the same as the subassembly in Sections 7.2 and 7.3 . The subassembly that was described in Section 7.2 was used in the study of the effect of the B-pillar length. The geometric properties of this subassembly were given in Section 7.2. Another subassembly whose dimensions are same as those of a typical B-pillar to rocker joint in a real car structure was used in the study of the effect of the rocker length. This subassembly is also symmetric in xy and xz planes. Thus, there is no coupling between rotations in different planes. The geometric properties of this subassembly are as follows ; the B-pillar length $L_P = 1200.0$, $l_f = 150.0$, $l_w = 100.0$, and $t_f = t_w = 1.0$ for the cross section of the B-pillar, and $l_f = 150.0$, $l_w = 110.0$, and $t_f = t_w = 1.0$ (mm) for the cross section of the rocker (Figure 7.1).

Three different values for the length of the B-pillar were considered for the small subassembly ; 350.0, 400.0, and 475.0 (mm). This is equivalent to changes of -12.50 and $+18.75$ % in the B-pillar length from the baseline value of 400.0 (mm) which was used in Sections 7.2 and 7.3. As in the previous sections, forces in F/A (fore/aft) and I/O (inboard/outboard) directions and also a twist moment were applied and the corresponding displacements and rotations were calculated at 18 different locations along the B-pillar. Thus, the number of

measurements was 54. The forces and moment were applied at the free end of the B-pillar for all the cases. The magnitudes of forces and moment are as follows ; $f_{F/A} = f_{I/O} = 980.0$ (N) and $M_T = 2.45 \times 10^4$ (N-mm).

The estimation results for all the 3 cases of different B-pillar length are summarized in Table 7.3. In all the cases, the parameter estimates remained almost the same. This can be explained by the fact that the flexibility of a joint branch only depends on the local deformation of the branches in the vicinity of the intersection between the B-pillar and the rocker. This local deformation is not affected by changes of the B-pillar length.

In order to investigate the effect of the rocker length on the joint parameters, seven different values of the rocker length were considered ; 440.0, 450.0, 883.34, 1140.0, 1560.0, 1980.0, and 2400.0 (mm). Forces in F/A and I/O directions and a twist moment were applied at the free end of the B-pillar, and the displacements and rotations in the corresponding directions were calculated at 18 different locations along the B-pillar. Thus, the number of measurements was 54. The magnitudes of forces and moment are as follows ; $f_{F/A} = f_{I/O} = 980.0$ (N) and $M_T = 1.47 \times 10^5$ (N-mm).

The stiffnesses in the F/A, I/O, and twist directions are plotted in Figure 7.9 as functions of the rocker length. The stiffnesses are normalized with respect to the values corresponding to the largest rocker length, $L_R = 2400.0$ (mm). The stiffnesses for bending in F/A direction and twist do not change significantly. However, it is observed that the stiffness for bending in I/O direction changes dramatically. For example, the I/O stiffness corresponding to the rocker length of 2400.0 (mm) is approximately 2.0 % of that for the length equal to 400.0 (mm). The sensitivity of the I/O stiffness to the rocker length is due to the change of the

shear deformation of the rocker. Indeed, for the example studied in this section, the major component of the joint deformation under I/O bending is the shear deformation of the rocker cross section [Sunami et al. 1990]. As the rocker length increases, the shear deformation of the rocker also increases, which increases the joint flexibility. It is also observed that the I/O bending stiffness converges to a limit value for large rocker length. The limit value of the I/O stiffness is 2.754×10^7 (N-mm/rad) in the subassembly considered in this section. Moreover, there is a threshold for the rocker length above which the stiffness is practically equal to the limit value. This sensitivity threshold is approximately 2000.0 (mm) for the example in Figure 7.9.

The estimates of on-branch offsets are shown in Table 7.4. It is observed that they are insensitive to the rocker length.

A joint is usually identified by considering an isolated subassembly, which consists of the joint and portions of the attached branches. However, as it was demonstrated in this section, the behavior of a joint and the values of its parameters depend on the length of the attached branches. In other words, it is impossible to accurately predict the behavior of a joint which is a part of either a subassembly with long branches or a whole car structure if the values of parameters are estimated from a subassembly with short branches. It was also observed that there is an upper sensitivity threshold for the length of branches, above which the joint behavior is insensitive to the length of branches. Based on the results of this section, it is recommended that the length of the attached branches must be larger than the threshold in order to accurately estimate joint parameters. Otherwise, the parameter estimates cannot accurately predict the joint behavior, which results in inaccurate prediction of the behavior of the whole vehicle structure.

8. 3-D ISOLATED JOINT OF A REAL AUTOMOTIVE STRUCTURE

The estimation methodology was applied to a subassembly, which consists of a B-pillar to rocker joint and the attached branches, of a typical car structure. The detailed finite element model of the subassembly is shown in Figure 8.1. This subassembly is not geometrically symmetric. Thus, the coupling between rotations in different planes may need to be considered.

To simulate the measured displacements, a detailed finite element model of the subassembly was analyzed using MSC/NASTRAN as in Chapter 7. More specifically, the displacements of the subassembly were calculated under static loads and they were used as measurements in order to estimate the joint parameters.

In Section 8.1, the geometry of the subassembly is described. In Section 8.2, the subassembly is identified using the reduced models I and II. In Section 8.3, the subassembly is identified using the reduced model III and the full model. The objective of Sections 8.2 and 8.3 is to investigate the effect and the importance of the coupling between rotations in different planes. In Section 8.4, the joint models are compared with detailed finite element model under the worst loads. The effect of the rocker length on the parameter estimates is studied in Section 8.5. The effect of measurement errors on the parameter estimates is studied in Section 8.6, where the displacements from the analysis of detailed finite element models are contaminated using random numbers to simulate the measurement errors.

8.1 Description of the Subassembly

A detailed finite element model of a subassembly that consists of a B-pillar to rocker joint and the attached branches is shown in Figure 8.1. The vertical and horizontal members of the T-shaped joint in Figure 8.1 are called B-pillar and rocker, respectively. This joint can be considered as an one-branch joint because the connection between the two horizontal branches of the rocker is rigid. Thus, only the flexibility at the connection between the B-pillar and the rocker needs to be considered.

As mentioned earlier, this joint is not geometrically symmetric. Therefore, there is coupling between rotations of the B-pillar in different planes. Thus, the full model (Section 2.2) and the reduced model III (Subsection 2.4.3) may need to be considered.

The finite element models of the subassembly were generated for four different values of the rocker length that ranged from 474.8 to 1138.3 (mm). The rocker length of the subassembly in Figure 8.1 is 1353.7 (mm). These models consist of plate elements, CQUAD4 and CTRIA3, in MSC/NASTRAN [Schaeffer 1979]. The model with the shortest rocker (474.8 mm) has 514 elements. The model with the longest rocker (1138.3 mm) has 998 elements. The geometry of the B-pillar of this car structure is complicated. For simplicity, a large portion of the B-pillar was replaced with a straight beam whose length is equal to 500.0 (mm). This beam was placed on the extension of the shear axis of the remaining portion of the B-pillar. The model of this beam consists of 10 elements of the same length which is 50.0 (mm). The crosssectional properties of this beam were chosen to be the same as those of the remaining portion of the B-pillar except the cross product of inertia which was considered to be zero for the attached beam (Table 8.1).

In order to take measurements, loads were applied at the free end of the attached beam while the two ends of the rocker were clamped. The local coordinate systems of the B-pillar and the rocker, $x_P y_P z_P$ and $x_R y_R z_R$, respectively, are defined so that x_P - and x_R -axes coincide with the shear axes of the corresponding members. The y_P - and y_R -axes are found so that the two planes, $x_P y_P$ and $x_R y_R$, are parallel to each other. Thus, z_P - and z_R -axes are also parallel to each other. The origins of the coordinate systems, $x_P y_P z_P$ and $x_R y_R z_R$, coincide with the reference points of the B-pillar and the rocker, respectively. The shear axes of the B-pillar and the rocker do not intersect. The distance from the reference point of the rocker to that of the B-pillar is 32.51 (mm) in the z_P -direction. In the joint model, the reference points of the B-pillar and the rocker were rigidly connected (Chapter 2). The displacements were calculated at 11 positions of the attached beam, which were equally spaced at a distance of 50.0 (mm). The distances between the reference point of the B-pillar and the positions where measurements are taken range from 272.0 (the shortest) to 772.0 (the longest) (mm).

In the examples of Sections 8.2 to 8.5, the displacements which were calculated in the analysis of detailed finite element models were used to simulate the measured displacements. Thus, the measured displacements do not involve random measurement errors. However, it was assumed that there were measurement errors whose standard deviations were 1.0 (%) of the measured displacements. Therefore, all the measurements were practically normalized and thus, they were considered equally important. In Section 8.6, the displacements were also calculated from the analysis of detailed finite element models. However, these displacements were contaminated using random numbers in order to investigate the effect of measurement errors on the parameter estimates.

The material and geometric properties of the B-pillar and the rocker in the corresponding local coordinate systems are shown in Table 8.1. The local coordinate systems are shown in Figure 8.1.

8.2 Identification of the Subassembly without Considering Coupling

The estimation methodology was applied to the subassembly in Figure 8.1 by using the reduced models I and II (Chapter 2). For this purpose, the subassembly whose rocker length, L_R , was equal to 474.8 (mm) was identified.

Forces in F/A (fore/aft or inplane) and I/O (inboard/outboard or out-of-plane) directions, $f_{F/A}$ and $f_{I/O}$, respectively, and a twist moment, M_T , were applied in order to take the measured displacements. The F/A and I/O directions correspond to y_P - and z_P -directions in Figure 8.1, respectively. The magnitudes of forces and moment were $f_{F/A} = f_{I/O} = 980.0$ (N) and $M_T = 2.45 \times 10^4$ (N-mm), respectively. The translations and rotations in the directions of the corresponding forces and moment were calculated at 11 different positions along the beam which was attached to the remaining portion of the B-pillar. Thus, the total number of measurements was 33. All loads and measurements were applied and calculated with respect to the local coordinates of the B-pillar, $x_P y_P z_P$.

The weighted error sums of squares (WSOS's), the mean square errors (RMSE's), and the parameter estimates of the reduced models I and II were compared in the first columns of Tables 8.2 and 8.3 ($L_R = 474.8$ mm). The RMSE is defined as the square root of the sum of squares of the differences between the measured and predicted displacements divided by the number of measurements. In the sum, each difference is normalized using the value of

the corresponding measured displacement. The RMSE is given by the following equation ;

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{y_i - \hat{y}_i}{y_i} \right)^2}, \quad (8.1)$$

where n is the number of measurements, and y_i and $\hat{y}_i$ are the measured and predicted displacements, respectively. The subscript i refers to the measurement number. It is observed that the fit is better in the case of the reduced model II because the WSOS and RMSE of the reduced model I is significantly larger than those of the reduced model II.

It is also investigated how well the identified models predict the static response of the joint under a force which is different from those used to take measurements. More specifically, another beam of length 1000.0 (mm) was attached (Figure 8.1), which is twice as long as the original beam of length 500.0 (mm), and a force, $f_{F/A} = -1960.0$ (N), was applied in F/A direction at the free end of this long beam. The material and geometric properties of the long beam were identical to those of the short beam (Table 8.1). Then the displacements predicted by the identified joint model were compared with those obtained from the analysis of the detailed finite element model. By following the above procedure, the possibility that the identified model cannot accurately predict displacements which were not used in the estimation was avoided.

When $f_{F/A} = -1960.0$ (N) was applied at the free end of the attached beam whose length is equal to 1000.0 (mm), the displacements at the same position were calculated from the analysis of the detailed finite element model. The displacements in F/A and I/O directions were -5.3930 (mm) and -4.6915 (mm), respectively, which are almost in the same order of magnitudes. Thus, for the subassembly of this example, the coupling effect is important. However, under the same force, the axial displacement at the free end was $u_x = 0.1854$ (mm),

which is only 3.4 (%) of the displacement in F/A direction. Thus the axial displacements can be neglected.

The displacements of the attached beam in F/A direction under a force, $f_{F/A} = -1960.0$ (N), are plotted in Figure 8.2. The predicted displacements in F/A direction were almost identical to the corresponding measurements in the case of the reduced model II (Figure 8.2). However, the reduced model I did not predict displacements with good accuracy which indicates that it is important to account for the offsets of the rotation centers.

In Figure 8.3, the displacements of the attached beam in I/O direction are plotted under a force, $f_{F/A} = -1960.0$ (N). The predicted displacements in I/O direction from both models were not accurate (Figure 8.3). This is due to the fact that both models neglect the coupling between displacements in different planes. In other words, the reduced models I and II are not adequate for the subassembly considered in this chapter.

8.3 Identification of the Subassembly Considering Coupling

The estimation methodology was applied to the subassembly in Figure 8.1 by using models which account for the coupling between displacements in different planes. For this purpose, the subassembly whose rocker length, L_R , was equal to 474.8 (mm) was identified by using the reduced model III and the full model (Chapter 2).

Two forces in F/A and I/O directions and three moments in F/A, I/O, and twist directions were applied. For each of five loads, two sets of translations in F/A and I/O directions and three sets of rotations in F/A, I/O, and twist directions were measured. The forces and moments were applied at the free end of the attached beam whose length was equal to 500.0

(mm). The magnitudes of forces and moments were equal to 980.0 (N) and 2.45×10^4 (N-mm), respectively. The above measurements (2 sets of translations and 3 sets of rotations) were taken at 11 different positions along the attached beam for each of 5 loads. Thus, the total number of measurements was 275. Finally, the total number of measurements decreased to 247 after dropping the measurements that are too small to be important.

The iterative estimation procedure did not converge in both the reduced model III and the full model. For several trials with different initial values, and after some iterations, the value of the on-branch offset, r_3 , that corresponds to the third spring axis, z_3 , was found to be practically zero. Moreover, its coefficient of variation (COV) was very large compared to those of other parameters. For example, in one case, it was found that the estimate of r_3 was equal to 0.17 (mm), its COV was equal to 0.66×10^7 , and its 95 % confidence interval was $(-7.12 \times 10^4, +7.13 \times 10^4 \text{ mm})$. The above values for r_3 depend on the initial values of the iterative procedure. However, in all cases, the predicted displacements were insensitive to r_3 . Based on these observations, this parameter was discarded from both the reduced model III and the full model by setting its value equal to zero. The total numbers of parameters became 10 in the reduced model III, and 13 in the full model.

After r_3 was set to be equal to 0.0 (mm), the COV's of all other parameters in the reduced model III were found to be less than 0.01, which indicates that these parameters are important.

In the full model, the iteration procedure did not converge due to a large number of parameters, even after discarding r_3 . Consequently, the values of the two on-branch offsets, r_1 and r_2 , were fixed to be equal to their estimates which were obtained from the reduced

model II (Table 8.3). Thus, 11 parameters were estimated in the full model.

The results of estimation using the reduced model III and the full model are summarized in the first columns of Tables 8.4 and 8.5, respectively ($L_R = 474.8$ mm). Both the reduced model III and the full model account for the coupling between rotations in different planes. However, in the reduced model III, the three spring axes that define the orientations of rotation planes are mutually orthogonal. Therefore, their orientations are defined by using three angles. These three angles are $\phi_{z_1x_p}$, $\phi_{z_1y_p}$, and $\phi_{z_2x_p}$ (Table 8.4), and they are considered as independent parameters. The values of the remaining angles are derived from those of the above independent ones. On the other hand, in the full model, six angles, $\phi_{z_1x_p}$, $\phi_{z_1y_p}$, $\phi_{z_2x_p}$, $\phi_{z_2y_p}$, $\phi_{z_3x_p}$, and $\phi_{z_3y_p}$, are independent parameters (Table 8.5).

In the reduced model III, the three spring axes are found to be close to the three local axes of the B-pillar. The angles between spring axes and the corresponding local axes are as follows ; 8.07° between z_1 and z_p , 15.53° between z_2 and y_p , and 16.99° between z_3 and x_p (Table 8.4). In other words, the springs, k_1 , k_2 , and k_3 , primarily contribute to the flexibility in F/A and I/O bending and twist, respectively. The i -th spring corresponds to the i -th spring axis. In the full model, for the case in which the rocker length is equal to 474.8 (mm), springs k_2 and k_3 primarily contribute to I/O bending and twist, respectively, because the corresponding spring axes z_2 and z_3 almost coincide with y_p and x_p axes, respectively. For example, the angle between z_2 and y_p is 12.78° and the angle between z_3 and x_p is 9.04° (Table 8.5). However, k_1 contributes to the flexibility in both F/A and I/O bending because the corresponding spring axis, z_1 , forms angles that are roughly equal to 45° with respect to both y_p and z_p axes.

The predicted displacements were compared against measurements in Figures 8.2 and 8.3. The displacements were obtained with the attached beam of length equal to 1000.0 (mm) under a force, $f_{F/A} = -1960.0$ (N). It is observed that both the reduced model III and the full model predict the displacements almost equally well.

For the subassembly studied in this section, the displacements in the direction of the applied load can be accurately predicted by any of the reduced models II and III and the full model. However, only the reduced model III and the full model account for the coupling between rotations in different planes. Based on the observations from Figure 8.3 and the fact that the coupling is important, it is recommended that any of the reduced model III and the full model be used to describe the behavior of the B-pillar to rocker joint in Figure 8.1. The reduced models I and II are not accurate.

8.4 Comparison of Joint Models under the Worst Loads

The worst loads were considered to compare the joint models. This procedure is important in order to check if the estimated models can be effective and useful under loads which are not used in the estimation.

For each concept joint model, the worst loads can be found by either of the following :

1. The load that maximizes the difference in the strain energies of the detailed and concept models (Eq.6.1). This load vector can be shown to be an eigenvector which corresponds to the largest eigenvalue of the difference in the flexibility matrices of two models ;

$$[(\mathbf{F}_{Detailed} - \mathbf{F}_{Concept}) - \lambda_{max}]\mathbf{f} = 0, \quad (8.2)$$

where $\mathbf{F}$ is flexibility matrix, λ_{max} is the largest eigenvalue, and $\mathbf{f}$ is an eigenvector which corresponds to λ_{max} .

2. The load that maximizes the ratio of the strain energies of the detailed and concept models (Eq.6.2). This load vector can be shown to be an eigenvector which corresponds to the largest eigenvalue of the following general eigenvalue problem ;

$$[\mathbf{F}_{Detailed} - \lambda_{max}\mathbf{F}_{Concept}]\mathbf{f} = 0, \quad (8.3)$$

where $\mathbf{F}$ is flexibility matrix, λ_{max} is the largest eigenvalue, and $\mathbf{f}$ is an eigenvector which corresponds to λ_{max} .

The worst loads that maximize the difference in strain energies (Eq.8.2) were considered to compare the joint models for the subassembly whose rocker length was 474.8 (mm). The strain energies were found at the end of attached beam whose length was 1000.0 (mm) (Figure 8.1). Table 8.6 shows the worst load vectors, the displacements under the worst load vectors (both from the detailed and concept models), and the ratio of total strain energy of the concept model to that of the detailed model. In terms of total strain energies, both the full model and the reduced model III are very accurate under the worst loads in Eq.(8.2). Figure 8.4 shows the portions of the total strain energies due to each force component for each concept joint model, which are normalized using the total strain energies of the detailed model. The largest portions are due to force components in I/O direction. The portions due to moment components are almost zero and thus, are not included in Figure 8.4.

The worst loads that maximize the ratio of strain energies (Eq.8.3) were also considered to compare the joint models for the subassembly whose rocker length was 474.8 (mm). The strain energies were found at the end of attached beam whose length was 1000.0 (mm)

(Figure 8.1). Table 8.7 shows the worst load vectors, the displacements under the worst load vectors (both from the detailed and concept models), and the ratio of strain energy of the concept model to that of the detailed model. It was found that the moment components about x- and z-axes were the most dominant elements of these load vectors. For example, the worst load vector for the comparison of the full and detailed models was found to be $\mathbf{f} = (f_x, f_y, f_z, M_x, M_y, M_z) = (0.0608, -0.0020, -0.0000, -0.2128, 0.0408, 0.9744)$, where f_x, f_y , and f_z are force components in x-, y-, and z-directions, respectively, and M_x, M_y , and M_z are moment components about x-, y-, and z-axes, respectively. However, portions of the total strain energy due to moment components are almost zero because the elements of flexibility matrices corresponding to moment components are very small in magnitudes. Thus, the largest portion of the total strain energy is contributed from the force component in x-direction because the other two force components are very small.

As a result, the largest eigenvalue, λ_{max} , is 3.4525 and the total strain energy of the full model is only 28.96 (%) of that of the detailed model. From the load vectors for the comparison of the reduced models III, II, and I with the detailed model, the total strain energies of these models were 30.03, 28.02, and 29.62 (%), respectively, of that of the detailed model. This large discrepancy is due to the fact that the axial flexibility is not considered in all the concept joint models. However, the flexibility in the axial direction is much smaller than that in the other two directions. The three diagonal elements that correspond to the flexibility in the axial, F/A, and I/O directions are 0.35174×10^{-4} , 0.27515×10^{-2} , and 0.60900×10^{-1} , respectively, in the detailed model. In the following, the joint models were compared without considering the axial flexibility, which is very small compared to the flexibility in the other two directions and is practically unimportant.

For the comparison of the joint models under the worst loads that maximize the ratio of strain energies (Eq.8.3), the flexibility matrices of size 5×5 were considered. These matrices were the submatrices of the original flexibility matrices where the row and column that correspond to the axial force and displacement were discarded. Table 8.8 shows the worst load vectors, the displacements under the worst load vectors (both from the detailed and concept models), and the ratio of strain energy of the concept model to that of the detailed model. In terms of total strain energies, both the full model and the reduced model III account for 93.12 and 92.98 (%), respectively, of the strain energy of the detailed model.

8.5 Effect of Rocker Length

The behavior of a joint in a vehicle structure may be significantly different from that in an isolated subassembly, which consists of the joint and portions of the attached branches. In this section, the effect of the rocker length on the estimates of the joint model parameters was investigated.

The subassembly in Figure 8.1 was identified. The total length of the rocker of this subassembly was equal to 1353.7 (mm). As it is observed in Figure 8.1, the geometry of the cross section of the rocker significantly changes at the right end. Therefore, the investigation was limited up to 1138.3 (mm) for the rocker length in order to avoid the effect of changes in the geometry of the rocker cross section. Four cases were studied, in which the rocker length was 474.8, 757.6, 930.9, and 1138.3 (mm), respectively.

The subassembly in Figure 8.1 was estimated by using different joint models for four different values of the rocker length. The same measurements that were used in Sections 8.2 and

8.3 were taken : For the reduced models I and II, forces in F/A and I/O directions and a twist moment were applied, and translations and rotations were measured in the directions of the corresponding forces and moment, respectively. The total number of measurements was 33. For the reduced model III and the full model, five loads (2 forces and 3 moments) were applied. For each load, five sets of measurements (2 sets of translations and 3 sets of rotations) were taken. The measurements whose magnitudes were too small were not included. Thus, the total number of measurements decreased to 247. The magnitudes of forces and moments were 980.0 (N) and 2.45×10^4 (N-mm), respectively.

The estimation results are summarized in Tables 8.2 to 8.5. The reduced model I is not accurate for all different values of the rocker length (Table 8.2). The weighted error sums of squares (WSOS's) of the reduced model I are at least 20,000 times larger than those of the reduced model II (Table 8.3) for all different values of the rocker length. The reduced model II is very accurate in predicting translations and rotations in the directions of applied forces and moment, respectively, for all different values of the rocker length. In both models, the joint stiffness decreased in all directions (F/A, I/O, and twist) as the rocker length increased. The joint stiffness in I/O direction, k_2 , was the most sensitive to the change in the rocker length. For example, in the reduced model II, the I/O stiffness decreased to almost one third of the value that corresponds to the rocker length of 474.8 (mm) when the rocker length increased from 474.8 to 1138.3 (mm). The same trend for the stiffness was also observed in the case of the subassembly consisting of simple box beams in Section 7.4. For the subassembly in this chapter, the on-branch offsets decreased as the rocker length increased (Table 8.3) while they remained almost the same for the subassembly consisting of simple box beams (Table 7.4). Finally, it is observed that the sensitivity of both stiffness

and on-branch offset parameters with respect to the rocker length decreases as this length increases.

The parameters of the reduced model III were also estimated for different values of the rocker length (Table 8.4). As the rocker length increased, the WSOS's in Eq.(3.3) and RMSE's in Eq.(8.1) increased significantly. For example, when the rocker length increased from 474.8 to 757.6 (mm), the WSOS's and RMSE's increased by around 30 and 5 times, respectively. When the rocker length increased longer, the WSOS's and RMSE's also increased more. Thus, the fit to the measurements under the assumption of 3 orthogonal spring axes (Subsection 2.4.3) is poor when the rocker is long. This indicates that the reduced model III cannot represent the behavior of this subassembly when the rocker is long.

The same subassembly was identified by using the full model for different values of the rocker length. The fit to the measurements was better in the full model than in the reduced model III. The WSOS's of the full model were 73.4 and 2.1 (%) of the corresponding values of the reduced model III, for the rocker length equal to 474.8 and 757.6 (mm), respectively. The RMSE's were also smaller in the full model than in the reduced model III, which indicates that the former is better than the latter. Indeed, the RMSE's of the full model were 0.0134 and 0.0122 while those of the reduced model III were 0.0157 and 0.0851 for cases in which the rocker length was equal to 474.8 and 757.6 (mm), respectively. Actually, the results from the full model correlated almost perfectly with all measurements for all cases. As the rocker length increased, the orientations of 2 planes, where springs k_1 and k_2 are located, did not change significantly. The spring k_1 contributes to the flexibility in both F/A and I/O bending. The spring k_2 contributes to the flexibility in I/O bending only. As the rocker

length increased, the orientation of the plane, where the spring k_3 is located, changed. More specifically, k_3 contributed to only the flexibility in twist when the rocker length was equal to 474.8 (mm). However, when the rocker length was equal to 1138.3 (mm), k_3 contributed to the flexibility in both I/O bending and twist. As a result, the behavior of the joint in I/O bending was the most sensitive with respect to changes in the rocker length. This is due to the increase in the shear deformation of the rocker as the rocker length increased [Lee et al. 1991].

The behavior of the subassembly studied in this section changed significantly as the length of the attached branches increased. Thus, in the analysis of the whole vehicle structures, it is not reasonable to use the values of parameters, which are estimated from the subassembly whose branches are short, because this leads to a serious underestimation of the joint flexibility or overestimation of the joint stiffness. This is particularly true for the stiffness in the I/O direction which, in the case of the shortest rocker, is almost three times as much as in the case of the longest rocker. For example, using the values of parameters which were estimated from the subassembly whose rocker length is equal to 474.8 and 1138.3 (mm), the displacements of the subassembly whose rocker length is equal to 1138.3 (mm) were predicted. The displacements in I/O direction under a force applied in the same direction are plotted in Figure 8.5. The discrepancy between measured and predicted displacements was large when displacements were predicted using the values of parameters which were estimated from the shortest subassembly ($L_R = 474.8$ mm).

There may be an upper threshold for the rocker length above which the parameters of the B-pillar to rocker joint are insensitive to changes in the rocker length. In the example of this section, the largest rocker length, which was 1138.3 (mm), should be close to the threshold.

It is recommended that a joint should be identified from a subassembly whose branches are long enough (above an upper threshold or as long as possible). The corresponding parameter estimates should be used in all applications where the concept joint model is employed to describe the behavior of the overall car body [Lee et al. 1991].

8.6 Effect of Measurement Errors

The effect of measurement errors on the parameter estimates was investigated for the subassembly whose rocker length was equal to 1138.3 (mm), which is the longest in the examples of this chapter. To simulate the measurement errors, a random number generator was used (Section 3.3). More specifically, random numbers were generated from a Gaussian distribution and then they were added to the displacements which were calculated from the analysis of detailed finite element models. The resulting sums simulated the measured displacements.

Different magnitudes of measurements errors were used to investigate the robustness of the estimation methodology. It was assumed that the coefficients of variation (COV's) of rotation measurements were five times those of translation measurements because it is more difficult to measure rotations. The COV's of measurement errors were 2.0, 5.0, 10.0, and 20.0 (%) and 10.0, 25.0, 50.0, and 100.0 (%) of the measured displacements for translation and rotation measurements, respectively.

The subassembly whose rocker length was 1138.3 (mm) was identified using the reduced model II and the full model. Forces in F/A (fore/aft or inplane) and I/O (inboard/outboard or out-of-plane) directions, $f_{F/A}$ and $f_{I/O}$, respectively, and a twist moment, M_T , were

applied in order to estimate the parameters of the reduced model II. The F/A and I/O directions correspond to y_P - and z_P -directions in Figure 8.1, respectively. The magnitudes of forces and moment were $f_{F/A} = f_{I/O} = 980.0$ (N) and $M_T = 2.45 \times 10^4$ (N-mm), respectively. The translations and rotations in the directions of the corresponding forces and moment were calculated at 11 different positions along the beam which was attached to the remaining portion of the B-pillar. Thus, the total number of measurements was 33.

In order to estimate the parameters of the full model, two forces in F/A and I/O directions and three moments in F/A, I/O, and twist directions were applied. For each load, two sets of translations in F/A and I/O directions and three sets of rotations in F/A, I/O, and twist directions were measured. Forces and moments were applied at the free end of the attached beam whose length was equal to 500.0 (mm). The magnitudes of forces and moments were 980.0 (N) and 2.45×10^4 (N-mm), respectively. The above measurements (2 sets of translations and 3 sets of rotations) were taken at 11 different positions along the attached beam per each of 5 loads. Thus, the total number of measurements was 275. Finally, the total number of measurements decreased to 247 after dropping the measurements that are too small to be important.

In order to avoid the matter of luck, the identification was repeated with three different sets of measurements which have the same magnitudes of measurement errors. Among the results from each of three different sets of measurements, only those from one set are shown in Tables 8.9 to 8.12. The WSOS's (Eq.3.3), RMSE's (Eq.8.1), and the estimates of parameters of the reduced model II and the full model are shown in Tables 8.9 and 8.10, respectively. In Tables 8.11 and 8.12, the COV's of the parameter estimates are shown for the reduced model II and the full model, respectively.

The first columns of Tables 8.9 to 8.12 were obtained with the measurements which were not contaminated using random numbers. However, in this case, the measurement errors whose COV's were 1.0 (%) of the measured displacements were assumed because the parameter estimation procedure in this study is based on the statistics of the measurement errors (Eq.3.4). Thus, the results in the first columns of Tables 8.9 and 8.10 are the same as those in the last columns of Tables 8.3 and 8.5, respectively ($L_R = 1138.3$ mm). In this case, the COV's of the parameter estimates are also available from Eq.(3.7), which are shown in the first columns of Tables 8.11 and 8.12. These COV's are measures of the errors in the parameter estimates due to the errors in the measurements which are all 1.0 (%) in this case.

The estimation procedure seems to be robust with large magnitudes of measurement errors. When the COV's of measurement errors were up to 5.0 and 25.0 (%) for translations and rotations, respectively, the convergence was good for all three different sets of measurements which have the same magnitudes of measurement errors. When the COV's of measurement errors were larger than 5.0 and 25.0 (%) for translations and rotations, respectively, there were cases where the iterative estimation procedure did not converge. This problem of convergence is due to the large magnitudes of measurement errors. For example, when the COV's of measurement errors were larger than 5.0 and 25.0 (%) for translations and rotations, respectively, there were some measurements whose signs were different from those of the displacements from the analysis of detailed finite element models.

From Tables 8.9 and 8.10, it is observed that the estimates of most parameters do not change significantly as the magnitudes of measurement errors increase. The largest change was observed with the estimate of k_1 of the full model. As the magnitudes of the measurement

errors increase, the estimate of k_1 changed as follows ; 2.4465×10^8 , 1.8961×10^8 , 2.8992×10^8 , 0.9033×10^8 , and 0.7739×10^8 (Table 8.10). From Tables 8.11 and 8.12, it is observed that the COV's of most parameters are in the same order of magnitudes of or smaller than the COV's of measurement errors. However, the COV of k_1 of the full model is exceptionally large (Table 8.12). The large sensitivity of k_1 of the full model to the measurement errors is due to the fact that the corresponding spring axis makes angles both with y_p and z_p axes and thus the estimate of the corresponding spring, k_1 , is sensitive to errors in the measurements in the directions of both y_p and z_p axes.

The estimation procedure is effective with relatively large magnitudes of measurement errors. The probabilistic estimation methodology is found to be useful because it provides measure of errors in the parameter estimates due to a certain magnitudes of errors in the measurements.

9. SUMMARY AND CONCLUSIONS

A family of concept joint models were developed. The model parameters are torsional springs, the coordinates of flexible hinges, and the orientations of planes where the torsional springs are located. In the following, joint models with different levels of complexity are summarized. The numbers in the parentheses correspond to the numbers of parameters.

- Full model (14) is the most complete model. It considers the flexibility, the offsets of rotation centers, and the coupling between rotations in different planes. The coupling is accounted for by changing the orientations of the three planes where torsional springs are located (Section 2.2).
- Reduced model III (11) is similar to the full model except that the planes where springs are located are mutually orthogonal in this model (Subsection 2.4.1).
- Reduced model II (5) considers the flexibility and the offsets of rotation centers which are located on the shear axis of a joint branch. The coupling effect is not considered (Subsection 2.4.2).
- Reduced model I (3) considers only the flexibility (Subsection 2.4.3).

A nonlinear weighted estimation methodology was used to identify the joint model parameters. This methodology uses the measured responses of a system, which are the structural displacements in this study. The model parameters are estimated by minimizing a measure of the discrepancy between the measured and predicted displacements. The parameter estimates improve as the number of measurements increases. It is also important to measure

displacements at many different positions rather than at the localized area around joints. Statistical tools which identify important parameters were also presented. They provided information on the accuracy of the parameter estimates and the effect of measurement errors on the estimates, which is useful in designing the experiment.

The estimation methodology was applied to the following examples ;

- Joints in a two dimensional frame model of an automotive structure (Chapters 5 and 6),
- Subassemblies of B-pillar to rocker joints and attached branches (Chapters 7 and 8).

In the examples, the measured displacements were simulated by using a random number generator (Chapters 5 and 6) or by using the displacements which were calculated from the analysis of detailed finite element models (Chapters 7 and 8). However, in an example of the joint subassembly, the displacements which were calculated from the detailed finite element model were contaminated using random numbers (Section 8.6).

Based on the examples considered in this study, the following conclusions are made. In the following, the items 1 to 8 are obtained from the examples of subassemblies consisting of B-pillar to rocker joints and the attached branches. The items 9 to 12 are from the examples of a two dimensional frame model of an automotive body structure.

1. Both the full model and the reduced model III accurately described the behavior of a B-pillar to rocker joint of a real automotive structure when the rocker was short. Thus, in order to describe the joint behavior, it is important to consider the flexibility, the offsets of rotation centers, and the coupling between rotations of the B-pillar in different planes.

2. The full model accurately described the behavior of a B-pillar to rocker joint for all different values of the rocker length. However, the reduced model III was not effective when the rocker was long. Thus, the planes where torsional springs are located are not mutually orthogonal.
3. The behavior of joints was sensitive to the length of the attached branches. In particular, in the case of B-pillar to rocker joints, the stiffness in I/O (inboard/outboard) bending was very sensitive to the rocker length. This is due to the shear deformation of the rocker cross section, which is the most important deformation mechanism in I/O bending [Sunami et al. 1990 and Lee et al. 1991]. Therefore, it is concluded that a joint behaves differently when it is isolated than when it is a part of the overall automotive structure.
4. In a B-pillar to rocker joint consisting of simple box beams, the joint stiffness was increased more effectively by increasing the rocker web thickness than the rocker flange thickness.
5. In a B-pillar to rocker joint consisting of simple box beams, the stiffness in I/O bending was the most sensitive to the change in both flange and web thickness among all the stiffnesses in two bending and twist modes.
6. On-branch offset for twist rotation was not important. When the three spring axes did not coincide with the local axes, the on-branch offset whose corresponding spring axis was the closest to the shear axis of a joint branch was not important. The torsional spring which corresponds to this spring axis mostly accounts for the flexibility in twist.
7. The worst loads were considered in order to compare the concept joint models with a detailed finite element model. The full model and the reduced model III were found

to be accurate under these loads for a subassembly of B-pillar to rocker joint whose rocker is short.

8. The iterative estimation procedure was robust with measurement errors which were simulated using random numbers. The estimate of a spring which accounts for the flexibility in two rotation modes was more sensitive to the measurement errors than those of springs which account for the flexibility in one rotation mode.
9. The estimates of parameters were biased due to modeling errors. However, the fit to the measurements was not significantly deteriorated with the biased estimates.
10. Statistical tools to identify unimportant parameters were tested. These tools were effective in simplifying the joint models.
11. It appears that coupling between rotations of different branches can be neglected.
12. Bayesian estimation provides an efficient way to improve the reliability of the parameter estimates.

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TABLES

Table 5.1 Material and geometric properties of a car structure.

Young's modulus : $E = 2.0638 \times 10^5 (N/mm^2)$

Part	I, Moment of Inertia(mm^4)	A, Area (mm^2)
Shortgun	1.2971×10^7	9.4224×10^3
Hinge	1.2971×10^7	9.4224×10^3
Rocker	1.2051×10^6	1.1778×10^3
Roof	3.6503×10^5	9.0116×10^2
Windshield	3.2662×10^6	6.0077×10^2
B-Pillar	8.6060×10^5	1.0467×10^3
Rear Pillar	4.3030×10^5	5.2335×10^2

Table 5.2 Classical estimation considering 4 joints, k^1, k^2, k^3 , and k^4 .

Classical Estimation :

Weighted Error Sum of Squares = 5.3813×10^{-4}

i	k^i , Actual Value of Parameter (N-mm/rad)	$\hat{k}^i$, Estimate of Parameter (N-mm/rad)	ρ_i , Coefficient of Variation
1	1.13×10^8	1.092×10^8	0.08
2	5.65×10^9	4.226×10^9	2.62
3	1.13×10^8	1.040×10^8	0.07
4	1.13×10^7	1.014×10^7	0.27

Principal Direction Analysis :

j	λ_j , Eigenvalue	e_j , Eigenvector			
1	0.15×10^{-7}	-0.19	0.00	0.18	0.97
2	0.89×10^{-9}	-0.56	0.00	0.79	-0.26
3	0.11×10^{-9}	0.81	0.00	0.59	0.05
4	0.10×10^{-15}	0.00	1.00	0.00	0.00

Table 5.3 Classical estimation considering 3 joints, k^1, k^3 , and k^4 ($k^2 = \infty$).

Classical Estimation :

Weighted Error Sum of Squares = 5.4188×10^{-4}

i	k^i , Actual Value of Parameter (N-mm/rad)	$\hat{k}^i$, Estimate of Parameter (N-mm/rad)	ρ_i , Coefficient of Variation
1	1.13×10^8	1.094×10^8	0.08
3	1.13×10^8	1.053×10^8	0.07
4	1.13×10^7	1.101×10^7	0.13

Principal Direction Analysis :

j	λ_j , Eigenvalue	e_j , Eigenvector		
1	0.15×10^{-7}	-0.19	0.18	0.97
2	0.88×10^{-9}	-0.56	0.79	-0.25
3	0.11×10^{-9}	0.81	0.58	0.05

Table 5.4 Classical estimation using 6 measurements under load F_1 .

Classical Estimation :

Weighted Error Sum of Squares = 4.1555×10^{-5}

i	k^i , Actual Value of Parameter (N-mm/rad)	$\hat{k}^i$, Estimate of Parameter (N-mm/rad)	ρ_i , Coefficient of Variation
1	1.13×10^8	1.142×10^8	0.18
2	5.65×10^9	$1.000 \times 10^{14} \uparrow$	$\ddagger$
3	1.13×10^8	1.216×10^8	0.16
4	1.13×10^7	1.600×10^7	0.40

$\uparrow$ Upper side constraint for the parameter estimate.

$\ddagger$ Very large coefficient of variation.

Principal Direction Analysis :

j	λ_j , Eigenvalue	$\mathbf{e}_j$, Eigenvector			
1	0.27×10^{-8}	-0.19	0.00	0.16	0.97
2	0.15×10^{-9}	-0.58	0.00	0.77	-0.24
3	0.20×10^{-10}	0.79	0.00	0.61	0.05
4	0.32×10^{-26}	0.00	1.00	0.00	0.00

Table 5.5 Bayesian estimation using the prior estimates in Table 5.4 and 6 more measurements under load F_2 .

Bayesian Estimation :

Weighted Error Sum of Squares = 8.1063×10^{-5}

i	k^i , Actual Value of Parameter (N-mm/rad)	$\hat{k}^i$, Estimate of Parameter (N-mm/rad)	ρ_i , Coefficient of Variation
1	1.13×10^8	1.118×10^8	0.15
2	5.65×10^9	1.722×10^9	0.52
3	1.13×10^8	1.205×10^8	0.12
4	1.13×10^7	1.545×10^7	0.16

Principal Direction Analysis :

j	λ_j , Eigenvalue	e_j , Eigenvector			
1	0.79×10^{-8}	-0.07	0.00	0.10	0.99
2	0.24×10^{-9}	0.73	0.01	-0.67	0.12
3	0.51×10^{-10}	0.68	0.01	0.73	-0.02
4	0.21×10^{-13}	-0.01	1.00	0.00	0.00

Table 5.6 Bayesian estimation using the prior estimates in Table 5.5 and 6 more measurements under loads F_2 and F_3 applied at the same time.

Bayesian Estimation :

Weighted Error Sum of Squares = 5.7065×10^{-5}

i	k^i , Actual Value of Parameter (N-mm/rad)	$\hat{k}^i$, Estimate of Parameter (N-mm/rad)	ρ_i , Coefficient of Variation
1	1.13×10^8	1.018×10^8	0.14
2	5.65×10^9	2.310×10^9	0.34
3	1.13×10^8	1.151×10^8	0.11
4	1.13×10^7	1.364×10^7	0.15

Principal Direction Analysis :

j	λ_j , Eigenvalue	e_j , Eigenvector			
1	0.79×10^{-8}	-0.06	0.00	0.11	0.99
2	0.24×10^{-9}	-0.70	0.00	0.71	-0.12
3	0.51×10^{-10}	0.71	0.01	0.70	-0.04
4	0.21×10^{-13}	-0.01	1.00	0.00	0.00

Table 5.7 Classical estimation using 18 measurements at the same time. Among 18 measurements, each 6 measurements were obtained under each of loads F_1 , F_2 , and F_2 and F_3 .

Classsical Estimation :

Weighted Error Sum of Squares = 1.8969×10^{-4}

i	k^i , Actual Value of Parameter (N-mm/rad)	$\hat{k}^i$, Estimate of Parameter (N-mm/rad)	ρ_i , Coefficient of Variation
1	1.13×10^8	1.077×10^8	0.14
2	5.65×10^9	3.139×10^9	0.73
3	1.13×10^8	1.161×10^8	0.11
4	1.13×10^7	1.276×10^7	0.15

Principal Direction Analysis :

j	λ_j , Eigenvalue	e_j , Eigenvector			
1	0.82×10^{-8}	-0.07	0.00	0.12	0.99
2	0.26×10^{-9}	0.72	0.00	-0.68	0.13
3	0.54×10^{-10}	0.70	0.00	0.72	-0.04
4	0.24×10^{-14}	0.00	1.00	0.00	0.00

Table 5.8 Classical estimation of 3 parameters, k^1,k^2 , and k^3 , at a 3-branch joint. †

Principal Direction Analysis :

j	λ_j , Eigenvalue	e_j , Eigenvector		
1	0.14×10^{-7}	0.000	0.709	0.705
2	0.77×10^{-13}	0.002	-0.705	0.709
3	0.53×10^{-18}	1.000	0.002	-0.002

† Estimates are not available because of strong correlation between parameters.

Table 6.1 Actual values of joint parameters for a car model in Figure 6.1.

J_i : the i-th joint (Figure 6.1)
units : k_i, k_{ij} (N-mm/rad) and r_i (mm)

J_i	k_1	k_2	k_3	k_{23}	k_{13}	k_{12}	r_1	r_2	r_3
J_1	∞	7.91×10^7	∞	3.96×10^7	3.96×10^7	3.96×10^7	12.7	50.8	12.7
J_2	1.13×10^8	1.13×10^8	n/a	n/a	n/a	5.65×10^7	50.8	50.8	n/a
J_3	1.13×10^8	1.13×10^8	4.52×10^7	2.26×10^7	2.26×10^7	2.26×10^7	76.2	76.2	25.4
J_4	∞	∞	1.13×10^8	5.65×10^7	5.65×10^7	5.65×10^7	12.7	12.7	25.4

Table 6.2 The results of estimation for different actual values of offset length.

WSOS : Weighted Error Sum of Squares

units : $r_i, \hat{r}_i$ (mm) and $k_i, \hat{k}_i, k_{ij}, \hat{k}_{ij}$ (N-mm/rad)

$k_{1(actual)} = k_{2(actual)} = 1.13 \times 10^8$ (N-mm/rad) and $k_{12(actual)} = 5.65 \times 10^7$ (N-mm/rad)

Model	$\hat{r}_1$	$\hat{r}_2$	$\hat{k}_1$	$\hat{k}_2$	$\hat{k}_{12}$	WSOS
$r_{i(actual)} = 50.80(\text{mm})$						
2-D bracketed	99.07	43.81	9.03×10^7	5.50×10^7	6.10×10^7	0.12040×10^3
Reduced II	182.18	35.87	2.97×10^8	1.19×10^8	n/a	0.12097×10^3
Reduced I	n/a	n/a	n/a	n/a	1.77×10^8	0.13842×10^3
$r_{i(actual)} = 76.20(\text{mm})$						
2-D bracketed	79.40	53.98	8.32×10^7	8.78×10^7	9.05×10^7	0.10438×10^3
Reduced II	41.71	103.56	2.17×10^8	3.58×10^8	n/a	0.10517×10^3
Reduced I	n/a	n/a	n/a	n/a	2.77×10^8	0.11551×10^3
$r_{i(actual)} = 127.00(\text{mm})$						
2-D bracketed	151.52	116.38	7.89×10^7	8.77×10^7	7.17×10^7	0.79724×10^2
Reduced II	65.59	208.45	2.58×10^8	2.37×10^8	n/a	0.88817×10^2
Reduced I	n/a	n/a	n/a	n/a	6.45×10^8	0.13077×10^3
$r_{i(actual)} = 177.80(\text{mm})$						
2-D bracketed	175.74	164.46	9.65×10^7	1.30×10^8	5.81×10^7	0.79823×10^2
Reduced II	128.01	229.06	2.26×10^8	2.39×10^8	n/a	0.89807×10^2
Reduced I	n/a	n/a	n/a	n/a	3.82×10^9	0.14061×10^3

Table 7.1 The results of estimation of T-shaped subassembly of a joint and attached branches. †

WSOS : Weighted Error Sum of Squares
units : $\hat{r}_i$ (mm) and $\hat{k}_i$ (N-mm/rad)

Model	$\hat{r}_{F/A}$	$\hat{r}_{I/O}$	$\hat{k}_{F/A}$	$\hat{k}_{I/O}$	$\hat{k}_T$	WSOS
Reduced II†	27.4	48.9	1.80×10^8	2.16×10^7	4.18×10^7	0.71094×10^{-1}
Reduced I	n/a	n/a	3.65×10^8	4.24×10^7	4.78×10^7	0.20700×10^3

† In this case, the reduced model II is the most complete because there is no coupling between rotations of a joint branch in different planes.

Table 7.2 The changes in on-branch offsets and WSOS with change in the rocker panel thickness.

WSOS : Weighted Error Sum of Squares
 units : $\hat{r}_i$ (mm) and t_f, t_w (mm)
 t_f : the thickness of rocker flange($t_{f_0} = 1.6mm$)
 t_w : the thickness of rocker web ($t_{w_0} = 1.6mm$)

Thickness	$\hat{r}_{F/A}$	$\hat{r}_{I/O}$	WSOS
$t_f = 0.8t_{f_0}, \quad t_w = 1.0t_{w_0}$	27.5	49.3	0.48148×10^{-1}
$t_f = 1.0t_{f_0}, \quad t_w = 0.8t_{w_0}$	26.4	49.2	0.48871×10^{-1}
$t_f = 1.0t_{f_0}, \quad t_w = 1.0t_{w_0}$	27.4	48.9	0.71094×10^{-1}
$t_f = 1.2t_{f_0}, \quad t_w = 1.0t_{w_0}$	27.2	49.0	0.72066×10^{-1}
$t_f = 1.0t_{f_0}, \quad t_w = 1.2t_{w_0}$	28.0	49.0	0.65953×10^{-1}

Table 7.3 The changes in the joint model parameters and WSOS with change in the B-pillar length.

WSOS : Weighted Error Sum of Squares
units : $L_P, \hat{r}_i$ (mm) and $\hat{k}_i$ (N-mm/rad)
 L_P : the length of B-pillar ($L_{P_0} = 400mm$)

L_P	$\hat{r}_{F/A}$	$\hat{r}_{I/O}$	$\hat{k}_{F/A}$	$\hat{k}_{I/O}$	$\hat{k}_T$	WSOS
$0.875 \times L_{P_0}$	26.8	48.8	1.83×10^8	2.19×10^7	4.21×10^7	0.12000×10^0
$1.000 \times L_{P_0}$	27.4	48.9	1.80×10^8	2.16×10^7	4.18×10^7	0.71094×10^{-1}
$1.188 \times L_{P_0}$	26.8	49.1	1.85×10^8	2.20×10^7	4.25×10^7	0.52201×10^{-1}

Table 7.4 The changes in on-branch offsets with change in the rocker length.

units : $L_R, \hat{r}_i(\text{mm})$
 L_R : the length of rocker ($L_{R_0} = 2400\text{mm}$)

L_R (mm)	L_R/L_{R_0}	$\hat{r}_{F/A}$	$\hat{r}_{I/O}$
440.0	0.183	70.13	121.5
450.0	0.188	69.98	121.3
833.34	0.347	68.70	114.6
1140.0	0.475	68.45	112.4
1560.0	0.650	68.44	111.1
1980.0	0.825	68.34	110.4
2400.0	1.000	68.46	110.0

Table 8.1 Material and geometric properties of the subassembly of the B-pillar to rocker joint and the attached branches.

E : Young's modulus (N/mm²)
 G : Shear modulus (N/mm²)
 A : Cross sectional area (mm²)
 I_y : Moment of inertia about y-axis (mm⁴)
 I_z : Moment of inertia about z-axis (mm⁴)
 I_{yz} : Cross product of inertia in yz-plane (mm⁴)
 J : Shear constant (mm⁴)

Material properties :

$$E = 2.07 \times 10^5 \quad G = 7.95 \times 10^4$$

Geometric properties :

	rocker	B-pillar ^a	Beam attached to B-pillar
A	6.4052×10^2	7.2517×10^2	7.2517×10^2
I_y	1.4958×10^6	0.4875×10^6	0.4875×10^6
I_z	1.6971×10^6	3.1363×10^6	3.1363×10^6
I_{yz}	0.6251×10^6	-0.3344×10^6	0.0
J	0.7900×10^6	0.7636×10^6	0.7636×10^6

^a The geometric properties of the B-pillar were found as the averages of those at two positions because the geometry of the B-pillar was not uniform.

Table 8.2 The results of estimation for different values of the rocker length – Reduced model I.

n = 33 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 3 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i-th flexible hinge (N-mm/rad) ^a
 z_i = spring axis of i-th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg)
 r_i = on-branch offset of i-th flexible hinge (mm)
 h_{y_p} = off-branch offset of 3-rd flexible hinge in y_p -direction (mm)

	$L_R = 474.8$	$L_R = 757.6$	$L_R = 930.9$	$L_R = 1138.3$
WSOS	2.6685×10^3	1.9724×10^3	1.7431×10^3	1.6357×10^3
RMSE	8.9924×10^{-2}	7.7311×10^{-2}	7.2678×10^{-2}	7.0404×10^{-2}
$\hat{k}_1$	2.0379×10^9	1.4171×10^9	1.2618×10^9	1.1782×10^9
$\hat{k}_2$	4.1711×10^7	1.9368×10^7	1.4619×10^7	1.3084×10^7
$\hat{k}_3$	6.1913×10^7	4.9971×10^7	4.4269×10^7	4.1771×10^7

^a In the reduced models I and II, k_1 , k_2 , and k_3 correspond to spring stiffnesses in F/A bending, I/O bending, and twist, respectively ($z_1 = z_p$, $z_2 = y_p$, and $z_3 = x_p$).

Table 8.3 The results of estimation for different values of the rocker length – Reduced model II.

n = 33 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 5 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i -th flexible hinge (N-mm/rad) ^a
 z_i = spring axis of i -th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg)
 r_i = on-branch offset of i -th flexible hinge (mm)
 h_{y_p} = off-branch offset of 3-rd flexible hinge in y_p -direction (mm)

	$L_R = 474.8$	$L_R = 757.6$	$L_R = 930.9$	$L_R = 1138.3$
WSOS	2.3239×10^{-3}	6.2886×10^{-2}	5.2534×10^{-2}	5.6223×10^{-2}
RMSE	8.3917×10^{-5}	4.3654×10^{-4}	3.9899×10^{-4}	4.1276×10^{-4}
$\hat{k}_1$	7.0279×10^8	6.0530×10^8	5.7584×10^8	5.5330×10^8
$\hat{k}_2$	2.6596×10^7	1.3386×10^7	1.0259×10^7	0.9203×10^7
$\hat{k}_3$	5.2971×10^7	4.4552×10^7	4.0138×10^7	3.8131×10^7
$\hat{r}_1$	165.63	147.83	141.21	138.82
$\hat{r}_2$	104.11	92.74	90.68	90.63

^a In the reduced models I and II, k_1 , k_2 , and k_3 correspond to spring stiffnesses in F/A bending, I/O bending, and twist, respectively ($z_1 = z_p$, $z_2 = y_p$, and $z_3 = x_p$).

Table 8.4 The results of estimation for different values of the rocker length – Reduced model III.

n = 247 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 10 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i-th flexible hinge (N-mm/rad)
 z_i = spring axis of i-th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg) ^a
 r_i = on-branch offset of i-th flexible hinge (mm)
 h_{y_p} = off-branch offset of 3-rd flexible hinge in y_p -direction (mm)

	$L_R = 474.8$	$L_R = 757.6$	$L_R = 930.9$	$L_R = 1138.3$
WSOS	6.0741×10^2	1.7883×10^4	4.2303×10^4	5.2836×10^4
RMSE	1.5682×10^{-2}	8.5089×10^{-2}	1.3087×10^{-1}	1.4626×10^{-1}
$\hat{k}_1$	9.2350×10^8	7.9816×10^8	8.2401×10^8	8.2425×10^8
$\hat{k}_2$	2.5642×10^7	1.3768×10^7	1.2327×10^7	1.1866×10^7
$\hat{k}_3$	5.6635×10^7	5.0913×10^7	5.2520×10^7	5.3330×10^7
$\hat{\phi}_{z_1 x_p}$	82.99	82.32	81.40	80.94
$\hat{\phi}_{z_1 y_p}$	86.03	86.81	86.31	86.01
$\hat{\phi}_{z_1 z_p}$	8.07	8.32	9.37	9.91
$\hat{\phi}_{z_2 x_p}$	105.40	109.25	112.98	114.31
$\hat{\phi}_{z_2 y_p}$	15.53	19.26	22.98	24.31
$\hat{\phi}_{z_2 z_p}$	91.99	90.49	90.05	89.92
$\hat{\phi}_{z_3 x_p}$	16.99	20.84	24.71	26.15
$\hat{\phi}_{z_3 y_p}$	75.01	71.03	67.35	66.06
$\hat{\phi}_{z_3 z_p}$	97.82	98.31	99.37	99.91
$\hat{r}_1$	145.24	102.81	69.04	53.72
$\hat{r}_2$	92.62	89.83	32.41	5.24
$\hat{h}_{y_p}$	-136.18	-121.72	-121.59	-119.99
$\hat{h}_{z_p}$	50.93	65.13	81.16	88.00

^a Among 9 angles, only 3 are independent ; $\phi_{z_1 x_p}, \phi_{z_1 y_p}, \phi_{z_2 x_p}$.

Table 8.5 The results of estimation for different values of the rocker length – Full model with fixed on-branch offsets. ^a

n = 247 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 11 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i-th flexible hinge (N-mm/rad)
 z_i = spring axis of i-th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg) ^b
 r_i = on-branch offset of i-th flexible hinge (mm)
 h_{y_p} = off-branch offset of 3-rd flexible hinge in y_p -direction (mm)

	$L_R = 474.8$	$L_R = 757.6$	$L_R = 930.9$	$L_R = 1138.3$
WSOS	4.4555×10^2	3.6714×10^2	4.5374×10^2	4.8637×10^2
RMSE	1.3431×10^{-2}	1.2192×10^{-2}	1.3554×10^{-2}	1.4032×10^{-2}
$\hat{k}_1$	3.8010×10^8	2.8333×10^8	2.6346×10^8	2.4465×10^8
$\hat{k}_2$	2.5851×10^7	1.2672×10^7	0.9799×10^7	0.8889×10^7
$\hat{k}_3$	5.8473×10^7	5.5219×10^7	4.7175×10^7	4.2472×10^7
$\hat{\phi}_{z_1 x_p}$	107.32	103.23	100.89	99.16
$\hat{\phi}_{z_1 y_p}$	129.11	134.82	136.56	138.12
$\hat{\phi}_{z_1 z_p}$	44.23	47.83	48.61	49.59
$\hat{\phi}_{z_2 x_p}$	102.76	107.48	109.57	110.16
$\hat{\phi}_{z_2 y_p}$	12.78	17.49	19.59	20.18
$\hat{\phi}_{z_2 z_p}$	90.76	89.53	89.22	89.09
$\hat{\phi}_{z_3 x_p}$	9.04	24.04	37.42	42.40
$\hat{\phi}_{z_3 y_p}$	82.23	66.39	52.88	47.86
$\hat{\phi}_{z_3 z_p}$	94.60	94.28	94.03	93.88
$\hat{h}_{y_p}$	-128.84	-177.59	-215.29	-232.10
$\hat{h}_{z_p}$	41.06	43.30	46.50	47.65

^a On-branch offsets were fixed equal to values in Table 8.2.

^b Among 9 angles, only 6 are independent ; $\phi_{z_1 x_p}, \phi_{z_1 y_p}, \phi_{z_2 x_p}, \phi_{z_2 z_p}, \phi_{z_3 y_p}, \phi_{z_3 z_p}$.

Table 8.6 Comparison of joint models under the worst loads that maximize the difference in strain energies when the rocker length length was 474.8 (mm).

	Full Model	Reduced Model III	Reduced Model II	Reduced Model I
λ_{max}^a	5.5929×10^{-4}	6.3811×10^{-4}	2.4834×10^{-3}	1.1086×10^{-2}
The worst load, $\mathbf{f}_{worst}$				
f_x^b	0.7093	-0.6474	-0.1841	-0.0491
f_y	-0.1354	0.1036	0.6687	0.1870
f_z	-0.6918	0.7551	0.7204	0.9811
M_x^c	-0.0002	0.0002	0.0021	0.0007
M_y	0.0007	-0.0010	-0.0007	-0.0011
M_z	-0.0001	0.0001	0.0006	0.0002
Displacements under $\mathbf{f}_{worst}$ from detailed finite element model				
u_x^d	4.5604×10^{-4}	-4.8908×10^{-4}	-5.0526×10^{-4}	-6.1261×10^{-4}
u_y	-2.0957×10^{-3}	2.1535×10^{-3}	3.5817×10^{-3}	2.8676×10^{-3}
u_z	-4.2885×10^{-2}	4.6622×10^{-2}	4.5582×10^{-2}	6.0228×10^{-2}
θ_x^e	-6.1249×10^{-6}	6.7448×10^{-6}	5.5524×10^{-6}	8.6436×10^{-6}
θ_y	3.7772×10^{-5}	-4.1060×10^{-5}	-4.0203×10^{-5}	-5.3055×10^{-5}
θ_z	-1.9105×10^{-6}	1.9525×10^{-6}	3.3972×10^{-6}	2.6148×10^{-6}
Displacements under $\mathbf{f}_{worst}$ from joint models				
u_x	0.5936×10^{-4}	-0.7593×10^{-4}	-0.4801×10^{-4}	-0.6874×10^{-4}
u_y	-2.0199×10^{-3}	2.0874×10^{-3}	1.9210×10^{-3}	0.7946×10^{-3}
u_z	-4.2499×10^{-2}	4.6140×10^{-2}	4.3793×10^{-2}	4.9351×10^{-2}
θ_x	-6.0291×10^{-6}	6.6289×10^{-6}	0.4261×10^{-6}	0.5413×10^{-6}
θ_y	3.7377×10^{-5}	-4.0404×10^{-5}	-3.8417×10^{-5}	-4.0938×10^{-5}
θ_z	-1.8502×10^{-6}	1.8979×10^{-6}	1.9291×10^{-6}	0.7501×10^{-6}
$U_{ratio}(\%)^f$	98.155	98.214	92.969	81.417

Table 8.6 Comparison of joint models under the worst loads that maximize the difference in strain energies when the rocker length length was 474.8 (mm). – continued

^a The largest eigenvalue (Eq.8.2).

^b f : the force component of $\mathbf{f}_{worst}$.

^c M : the moment component of $\mathbf{f}_{worst}$.

^d u : the translational displacement.

^e θ : the rotational displacement.

^f The ratio of strain energy which is calculated from each of the concept joint models to that of the detailed finite element model (%).

Table 8.7 Comparison of joint models under the worst loads that maximize the ratio of strain energies when the rocker length length was 474.8 (mm). – from 6×6 flexibility matrices.

	Full Model	Reduced Model III	Reduced Model II	Reduced Model I
λ_{max}^a	3.4525	3.3295	3.5687	3.3759
The worst load, $\mathbf{f}_{worst}$				
f_x^b	0.0608	0.0649	0.0733	0.0928
f_y	-0.0020	-0.0019	-0.0023	-0.0034
f_z	0.0000	0.0000	0.0000	0.0001
M_x^c	-0.2128	-0.2459	0.0019	0.0112
M_y	0.0408	0.0544	0.1631	0.5040
M_z	0.9744	0.9656	0.9839	0.8587
Displacements under $\mathbf{f}_{worst}$ from detailed finite element model				
u_x^d	2.2935×10^{-6}	2.4294×10^{-6}	2.8211×10^{-6}	3.7318×10^{-6}
u_y	-8.3529×10^{-6}	-8.4176×10^{-6}	-1.1004×10^{-5}	-1.6661×10^{-5}
u_z	-4.5350×10^{-5}	-4.7780×10^{-5}	-5.8922×10^{-5}	-8.5162×10^{-5}
θ_x^e	-1.0426×10^{-8}	-1.1987×10^{-8}	-3.4036×10^{-9}	-3.2973×10^{-9}
θ_y	3.9169×10^{-8}	4.1250×10^{-8}	5.1972×10^{-8}	7.6445×10^{-8}
θ_z	-7.0907×10^{-9}	-7.1107×10^{-9}	-9.6337×10^{-9}	-1.5134×10^{-8}
Displacements under $\mathbf{f}_{worst}$ from joint models				
u_x	6.6431×10^{-7}	7.2965×10^{-7}	7.9051×10^{-7}	1.1054×10^{-6}
u_y	-2.4194×10^{-6}	-2.5282×10^{-6}	-3.0834×10^{-6}	-4.9353×10^{-6}
u_z	-1.3135×10^{-5}	-1.4350×10^{-5}	-1.6511×10^{-5}	-2.5226×10^{-5}
θ_x	-3.0198×10^{-9}	-3.6002×10^{-9}	-9.5373×10^{-10}	-9.7670×10^{-10}
θ_y	1.1345×10^{-8}	1.2389×10^{-8}	1.4563×10^{-8}	2.2644×10^{-8}
θ_z	-2.0538×10^{-9}	-2.1356×10^{-9}	-2.6995×10^{-9}	-4.4829×10^{-9}
$U_{ratio}(\%)^f$	28.964	30.034	28.021	29.621

^a The largest eigenvalue (Eq.8.3).

^{b c d e f} Described in Table 8.6.

Table 8.8 Comparison of joint models under the worst loads that maximize the ratio of strain energies when the rocker length was 474.8 (mm). – from 5×5 flexibility matrices.

	Full Model	Reduced Model III	Reduced Model II	Reduced Model I
λ_{max}^a	1.0739	1.0755	1.2832	1.5068
The worst load, $\mathbf{f}_{worst}$				
f_x^b	n/a [†]	n/a	n/a	n/a
f_y	-0.0011	-0.0011	-0.0005	-0.0015
f_z	0.0000	-0.0001	-0.0006	0.0004
M_x^c	0.0023	0.0021	-0.2146	0.0768
M_y	-0.0236	-0.0733	-0.5473	0.8743
M_z	0.9997	0.9973	0.8089	-0.4793
Displacements under $\mathbf{f}_{worst}$ from detailed finite element model				
u_x^d	n/a [†]	n/a	n/a	n/a
u_y	-3.1779×10^{-7}	-3.2717×10^{-7}	1.0220×10^{-6}	-6.6783×10^{-6}
u_z	-4.2958×10^{-8}	3.3904×10^{-7}	-5.8171×10^{-6}	-2.9546×10^{-5}
θ_x^e	5.7416×10^{-11}	2.9363×10^{-11}	-1.0447×10^{-8}	3.9376×10^{-9}
θ_y	-7.4083×10^{-11}	-6.1564×10^{-10}	2.3684×10^{-9}	3.0003×10^{-8}
θ_z	2.6990×10^{-10}	2.5888×10^{-10}	1.5881×10^{-9}	-6.9082×10^{-9}
Displacements under $\mathbf{f}_{worst}$ from joint models				
u_x	n/a [†]	n/a	n/a	n/a
u_y	-2.9593×10^{-7}	-3.0420×10^{-7}	7.9645×10^{-7}	-4.4322×10^{-6}
u_z	-4.0003×10^{-8}	3.1523×10^{-7}	-4.5331×10^{-6}	-1.9609×10^{-5}
θ_x	5.3466×10^{-11}	2.7301×10^{-11}	-8.1410×10^{-9}	2.6133×10^{-9}
θ_y	-6.8987×10^{-11}	-5.7240×10^{-10}	1.8456×10^{-9}	1.9912×10^{-8}
θ_z	2.5134×10^{-10}	2.4070×10^{-10}	1.2376×10^{-9}	-4.5848×10^{-9}
$U_{ratio}(\%)^f$	93.121	92.977	77.928	66.368

^a The largest eigenvalue (Eq.8.3).

^{b c d e f} Described in Table 8.6.

Table 8.9 The results of estimation for different magnitudes of measurement errors when the rocker length was 1138.3 (mm) – Reduced model II.

n = 33 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 3 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i-th flexible hinge (N-mm/rad) ^a
 z_i = spring axis of i-th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg)
 r_i = on-branch offset of i-th flexible hinge (mm)
 h_{y_p} = off-branch offset of 3-rd flexible hinge in y_p -direction (mm)
 COV = C_1/C_2 : C_1 and C_2 are coefficients of variation of displacement and rotation measurements, respectively (%).

COV	1.0/1.0 (%) ^b	2.0/10.0 (%)	5.0/25.0 (%)	10.0/50.0 (%)	20.0/100.0 (%)
WSOS	5.6223×10^{-2}	2.0453×10^1	2.4225×10^1	2.7757×10^1	1.6314×10^1
RMSE	4.1276×10^{-4}	2.8897×10^{-2}	1.5118×10^{-1}	3.0386×10^{-1}	1.2528×10^0
$\hat{k}_1$	5.5330×10^8	5.7113×10^8	5.5792×10^8	5.1546×10^8	4.1653×10^8
$\hat{k}_2$	0.9203×10^7	0.9251×10^7	0.9048×10^7	0.8057×10^7	0.6333×10^7
$\hat{k}_3$	3.8131×10^7	3.8273×10^7	3.8071×10^7	2.8051×10^7	2.6030×10^7
$\hat{r}_1$	138.82	133.35	135.82	143.76	177.19
$\hat{r}_2$	90.63	86.82	99.30	124.27	165.34

^a In the reduced models I and II, k_1 , k_2 , and k_3 correspond to spring stiffnesses in F/A bending, I/O bending, and twist, respectively ($z_1 = z_p$, $z_2 = y_p$, and $z_3 = x_p$).

^b From Table 8.3.

Table 8.10 The results of estimation for different magnitudes of measurement errors when the rocker length was 1138.3 (mm) – Full model with fixed on-branch offsets. ^a

n = 247 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 11 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i-th flexible hinge (N-mm/rad)
 z_i = spring axis of i-th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg) ^b
 r_i = on-branch offset of i-th flexible hinge (mm)
 h_{y_p} = off-branch offset of 3-rd flexible hinge in y_p -direction (mm)
COV = C_1/C_2 : C_1 and C_2 are coefficients of variation of displacement and rotation measurements, respectively (%).

COV	1.0/1.0(%) ^c	2.0/10.0 (%)	5.0/25.0 (%)	10.0/50.0 (%) ^d	20.0/100.0 (%) ^e
WSOS	4.8637×10^2	2.8436×10^2	3.0247×10^2	2.2524×10^2	1.8392×10^3
RMSE	1.4032×10^{-2}	8.8969×10^{-2}	3.2072×10^{-1}	7.2916×10^0	6.6906×10^0
$\hat{k}_1$	2.4465×10^8	1.8961×10^8	2.8992×10^8	0.9033×10^8	0.7739×10^8
$\hat{k}_2$	0.8889×10^7	0.8821×10^7	0.8601×10^7	0.8519×10^7	0.9914×10^7
$\hat{k}_3$	4.2472×10^7	4.9236×10^7	3.6851×10^7	3.5771×10^7	4.7390×10^7
$\hat{\phi}_{z_1 x_p}$	99.16	97.08	94.53	79.37	94.25
$\hat{\phi}_{z_1 y_p}$	138.12	144.90	135.02	154.47	148.34
$\hat{\phi}_{z_1 z_p}$	49.59	55.83	45.38	67.07	58.69
$\hat{\phi}_{z_2 x_p}$	110.16	109.43	110.38	109.75	92.09
$\hat{\phi}_{z_2 y_p}$	20.18	19.46	20.40	19.85	3.72
$\hat{\phi}_{z_2 z_p}$	89.09	88.92	89.11	88.06	86.93
$\hat{\phi}_{z_3 x_p}$	42.40	39.17	44.78	30.82	0.60
$\hat{\phi}_{z_3 y_p}$	47.86	51.16	45.51	59.65	89.91
$\hat{\phi}_{z_3 z_p}$	93.88	94.30	94.07	94.85	89.41
$\hat{h}_{y_p}$	-232.10	-209.23	-243.11	-145.19	-186.11
$\hat{h}_{z_p}$	47.65	53.68	45.63	47.09	41.50

^a On-branch offsets were fixed equal to values in Table 8.2.

^b Among 9 angles, only 6 are independent ; $\phi_{z_1 x_p}, \phi_{z_1 y_p}, \phi_{z_2 x_p}, \phi_{z_2 z_p}, \phi_{z_3 y_p}, \phi_{z_3 z_p}$.

^c From Table 8.5.

^d No convergence for 1 set among total of 3 sets of measurements.

^e No convergence for 2 sets among total of 3 sets of measurements.

Table 8.11 The coefficients of variation of the parameter estimates when the rocker length was 1138.3 (mm) – Reduced model II.

n = 33 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 3 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i–th flexible hinge (N-mm/rad) ^a
 z_i = spring axis of i–th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg)
 r_i = on–branch offset of i–th flexible hinge (mm)
 h_{y_p} = off–branch offset of 3–rd flexible hinge in y_p –direction (mm)
COV = C_1/C_2 : C_1 and C_2 are coefficients of variation of displacement and rotation measurements, respectively (%).
 ρ = coefficient of variation of a parameter estimate (%)

COV	1.0/1.0 (%)	2.0/10.0 (%)	5.0/25.0 (%)	10.0/50.0 (%)	20.0/100.0 (%)
$\rho_{\hat{k}_1}$	1.58	3.27	7.98	14.66	23.93
$\rho_{\hat{k}_2}$	1.30	2.59	6.47	12.11	23.78
$\rho_{\hat{k}_3}$	0.32	3.84	9.55	14.06	26.06
$\rho_{\hat{r}_1}$	2.00	4.33	10.34	17.84	22.17
$\rho_{\hat{r}_2}$	3.32	7.00	14.76	20.38	22.79

^a In the reduced models I and II, k_1 , k_2 , and k_3 correspond to spring stiffnesses in F/A bending, I/O bending, and twist, respectively ($z_1 = z_p$, $z_2 = y_p$, and $z_3 = x_p$).

Table 8.12 The coefficients of variation of parameter estimates when the rocker length was 1138.3 (mm) – Full model with fixed on-branch offsets. ^a

n = 247 (Number of measurements) WSOS = Weighted error sum of squares(Eq.3.3)
 p = 11 (Number of parameters) RMSE = Root mean square error(Eq.8.1)
 L_R = the length of the rocker (mm)
 k_i = spring stiffness of i-th flexible hinge (N-mm/rad)
 z_i = spring axis of i-th flexible hinge
 $\phi_{z_i x_p}$ = angle between z_i and local x_p axes (deg) ^b
 r_i = on-branch offset of i-th flexible hinge (mm)
 h_{y_p} = off-branch offset of 3-rd flexible hinge in y_p -direction (mm)
 $COV = C_1/C_2$: C_1 and C_2 are coefficients of variation of displacement and rotation measurements, respectively (%).
 ρ = coefficient of variation of a parameter estimate (%)

COV	1.0/1.0 (%)	2.0/10.0 (%)	5.0/25.0 (%)	10.0/50.0 (%) ^c	20.0/100.0 (%) ^d
$\rho_{\hat{k}_1}$	9.34	24.55	105.51	145.42	369.29
$\rho_{\hat{k}_2}$	0.22	0.85	2.27	12.77	51.50
$\rho_{\hat{k}_3}$	1.38	6.14	15.94	18.59	29.60
$\rho_{\hat{\phi}_{z_1 x_p}}$	0.34	0.70	4.77	7.09	5.97
$\rho_{\hat{\phi}_{z_1 y_p}}$	1.67	3.35	22.53	12.43	48.91
$\rho_{\hat{\phi}_{z_1 z_p}}$	n/a	n/a	n/a	n/a	n/a
$\rho_{\hat{\phi}_{z_2 x_p}}$	0.06	0.22	0.89	1.07	1.87
$\rho_{\hat{\phi}_{z_2 y_p}}$	n/a	n/a	n/a	n/a	n/a
$\rho_{\hat{\phi}_{z_2 z_p}}$	0.06	0.18	0.73	1.80	10.48
$\rho_{\hat{\phi}_{z_3 x_p}}$	n/a	n/a	n/a	n/a	n/a
$\rho_{\hat{\phi}_{z_3 y_p}}$	1.11	3.77	13.28	11.01	11.08
$\rho_{\hat{\phi}_{z_3 z_p}}$	0.15	0.46	1.13	1.85	2.55
$\rho_{\hat{h}_{y_p}}$	1.31	5.52	16.76	22.39	24.94
$\rho_{\hat{h}_{z_p}}$	1.23	4.99	12.90	18.91	34.57

^a On-branch offsets were fixed equal to values in Table 8.2.
^b Among 9 angles, only 6 are independent ; $\phi_{z_1 x_p}, \phi_{z_1 y_p}, \phi_{z_2 x_p}, \phi_{z_2 z_p}, \phi_{z_3 y_p}, \phi_{z_3 z_p}$.
^c No convergence for 1 set among total of 3 sets of measurements.
^d No convergence for 2 sets among total of 3 sets of measurements.

FIGURES

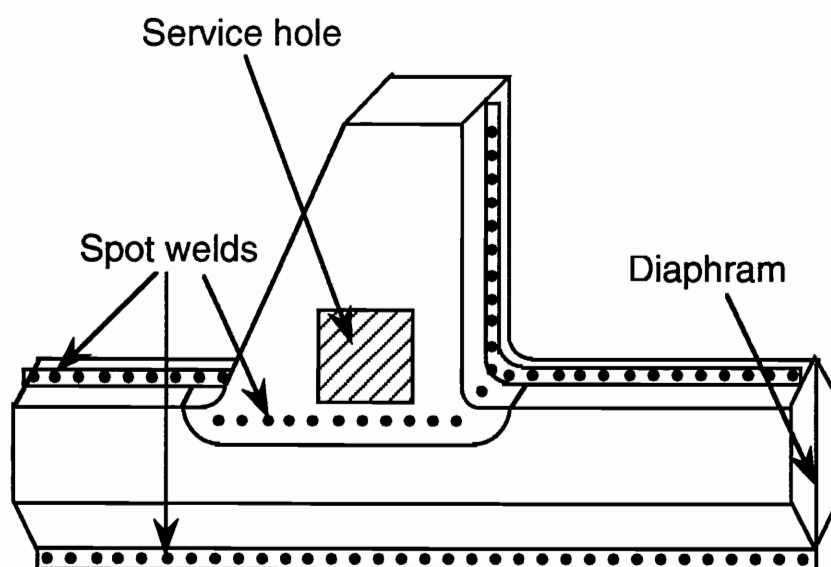


Figure 1.1 A joint in a car structure.

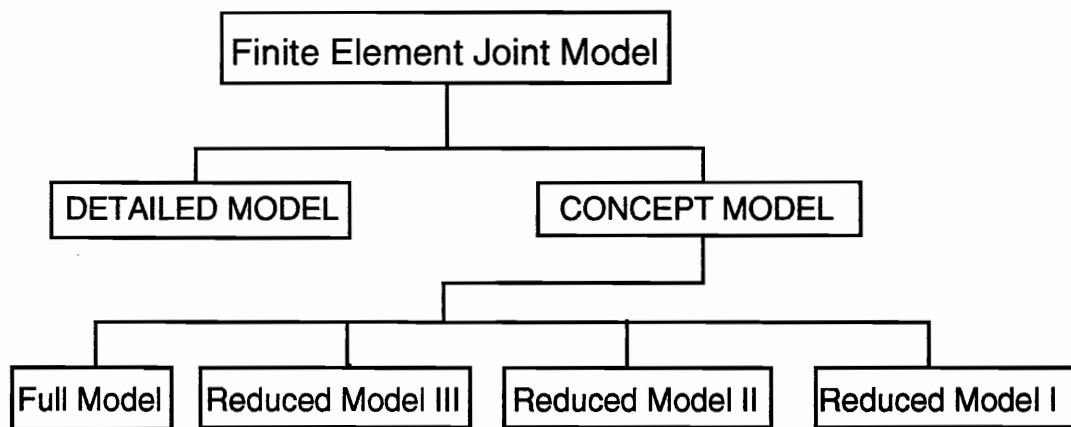


Figure 2.1 Finite element joint models.

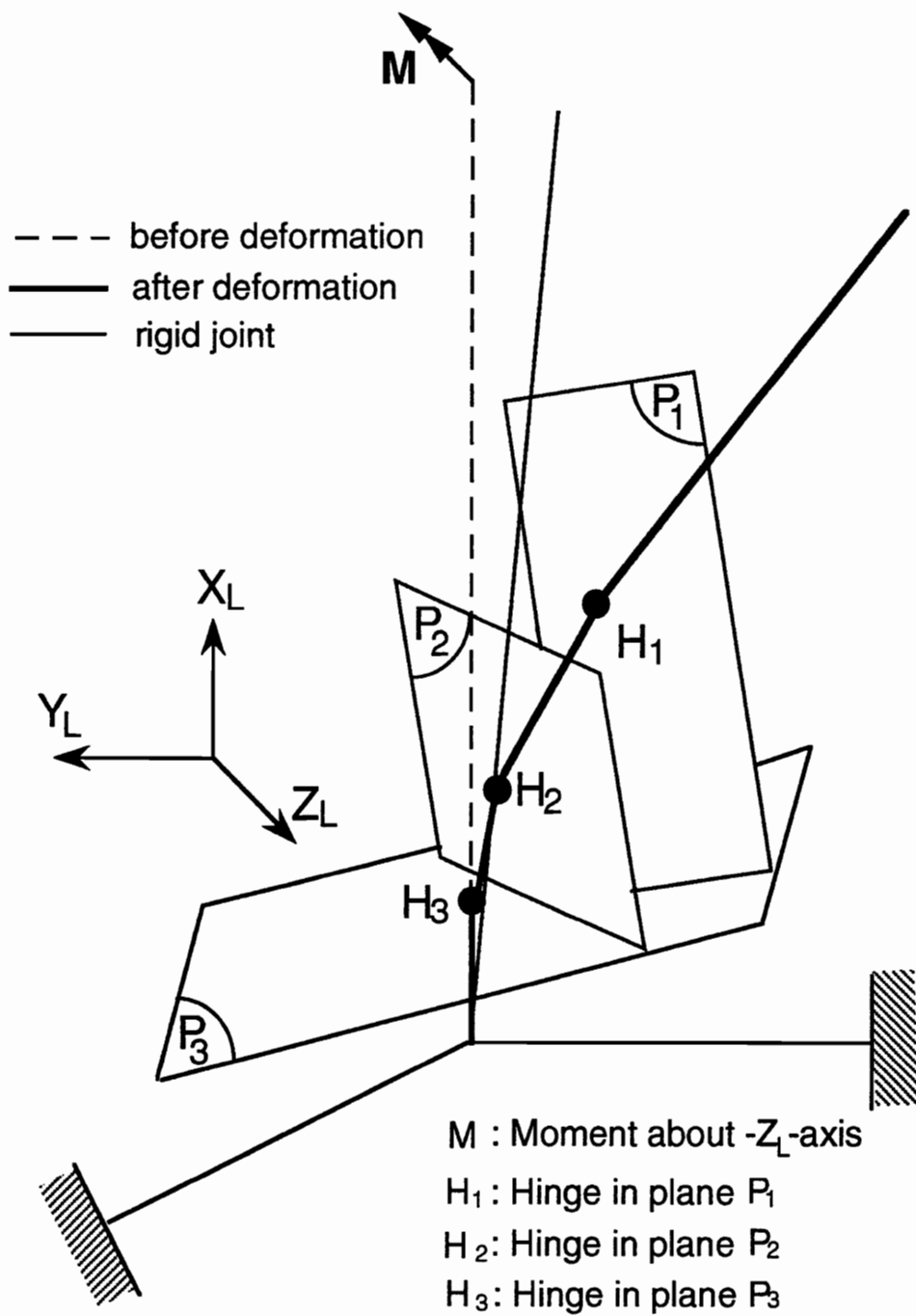


Figure 2.2 The behavior of a joint branch which is described using a concept model. In general, the three planes, P_1 , P_2 , and P_3 , do not coincide with the local planes, $x_L y_L$, $y_L z_L$, and $z_L x_L$.

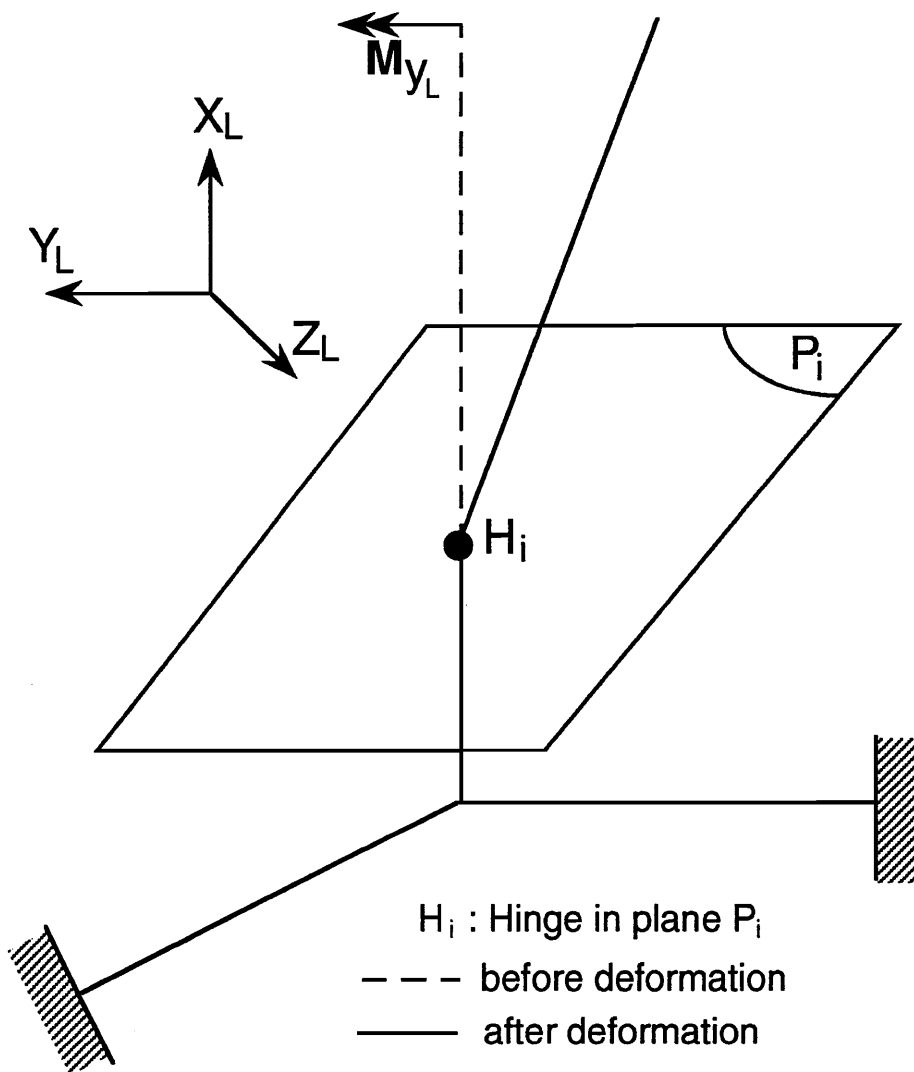
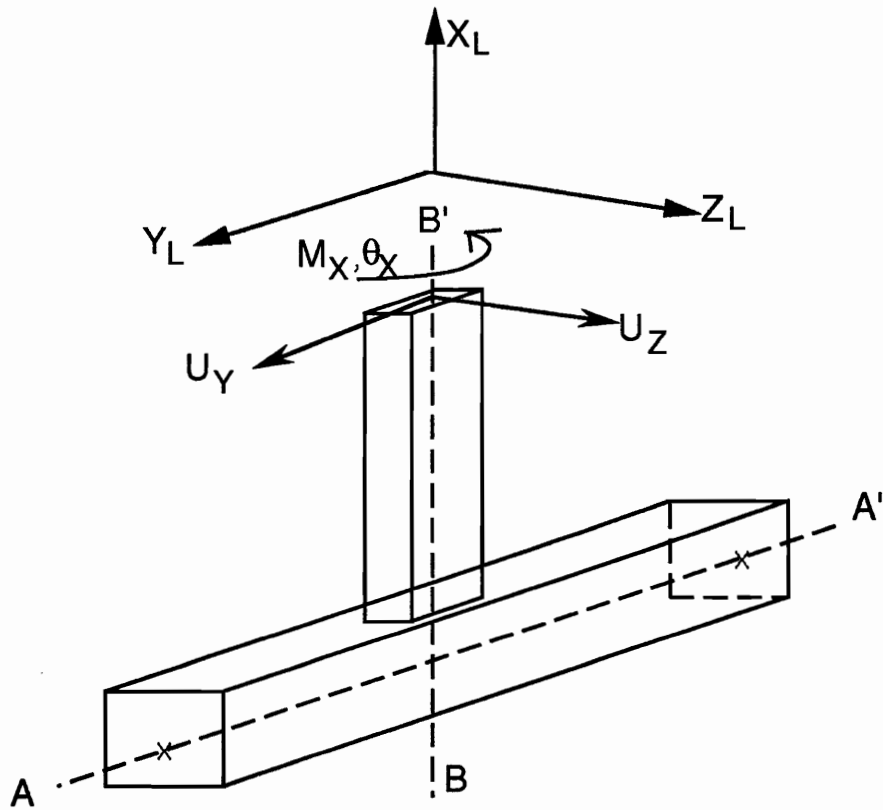
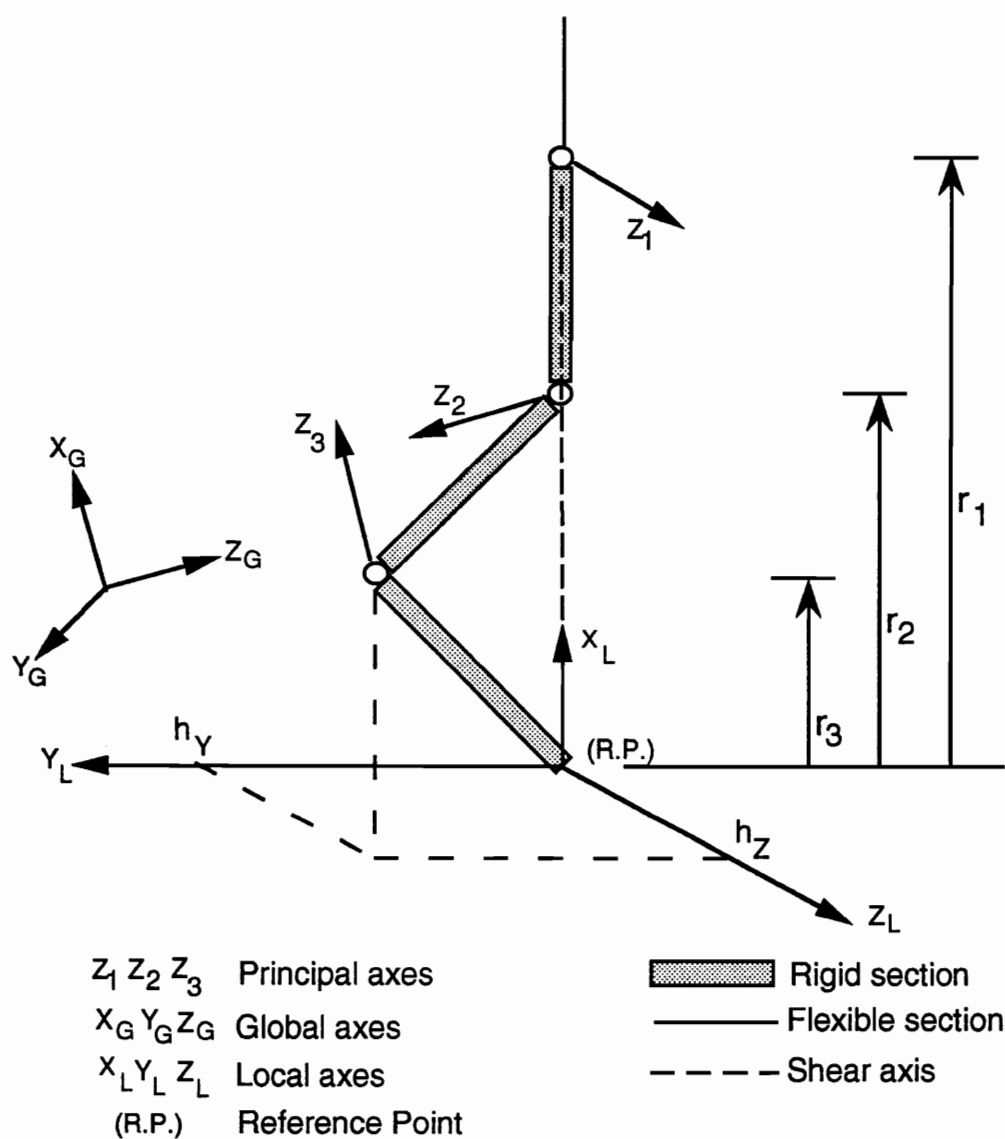


Figure 2.3 Coupling between rotations of a branch in different planes.



M_X : twist moment
 θ_X : twist rotation
 U_Y : in-plane displacement
 U_Z : out-of-plane displacement
 AA', BB' : shear axes

Figure 2.4 Coupling between twist rotation and linear displacements.



(a) 3-D view

Figure 2.5 A joint element – Full model.

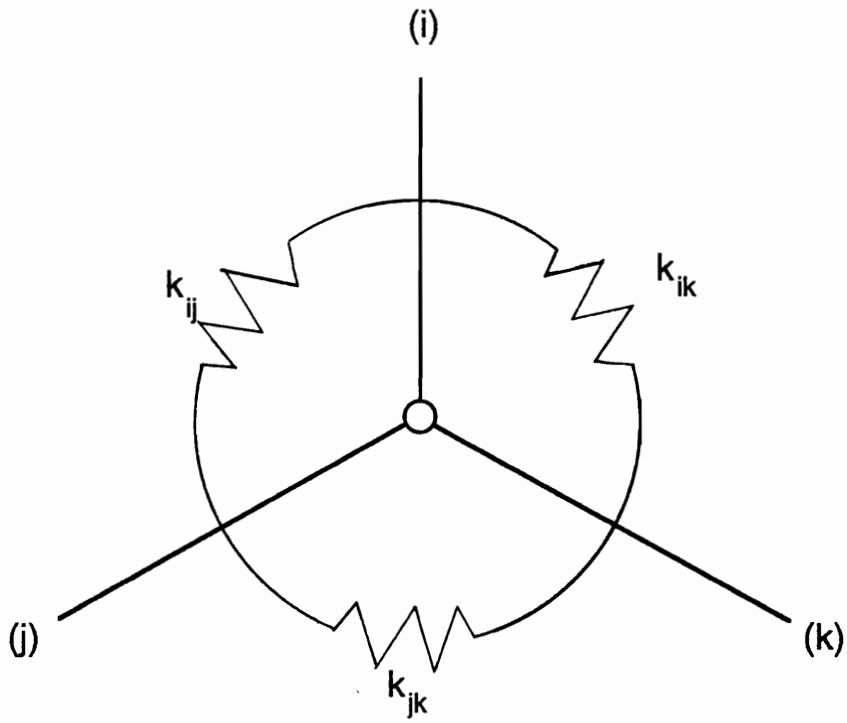


Figure 2.6 A joint with 3 branches in a plane – Reduced model I.

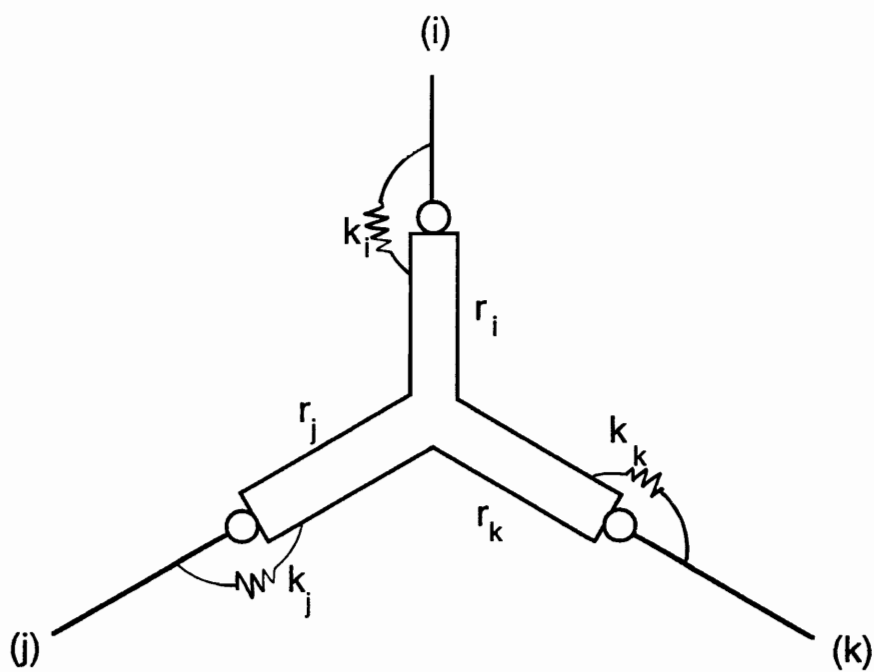


Figure 2.7 A joint with 3 branches in a plane – Reduced model II.

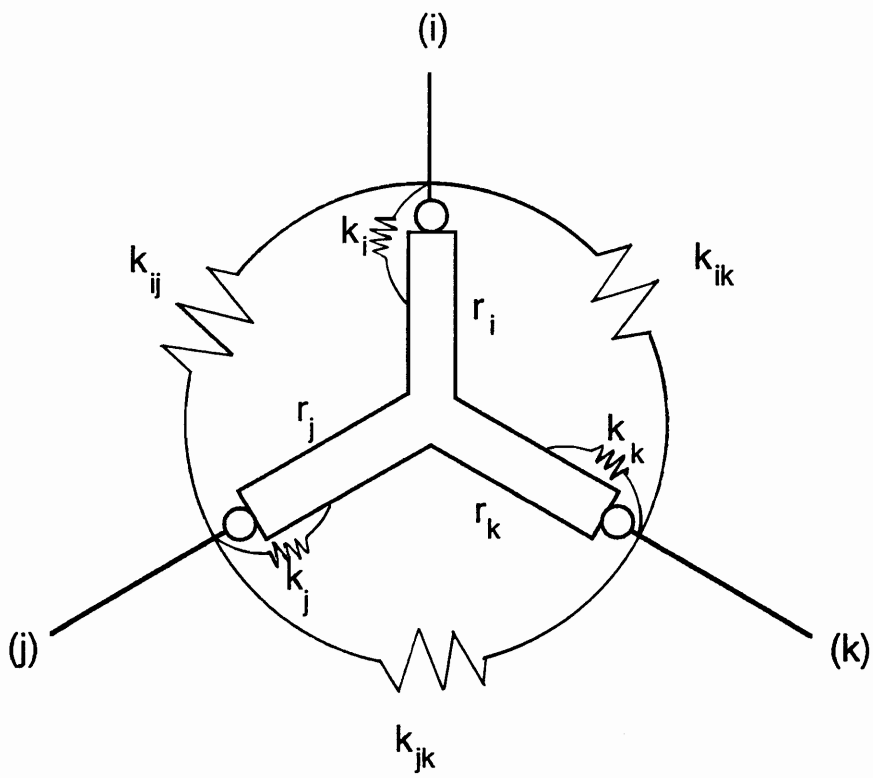


Figure 2.8 A joint with 3 branches in a plane – 2-D bracketed model.

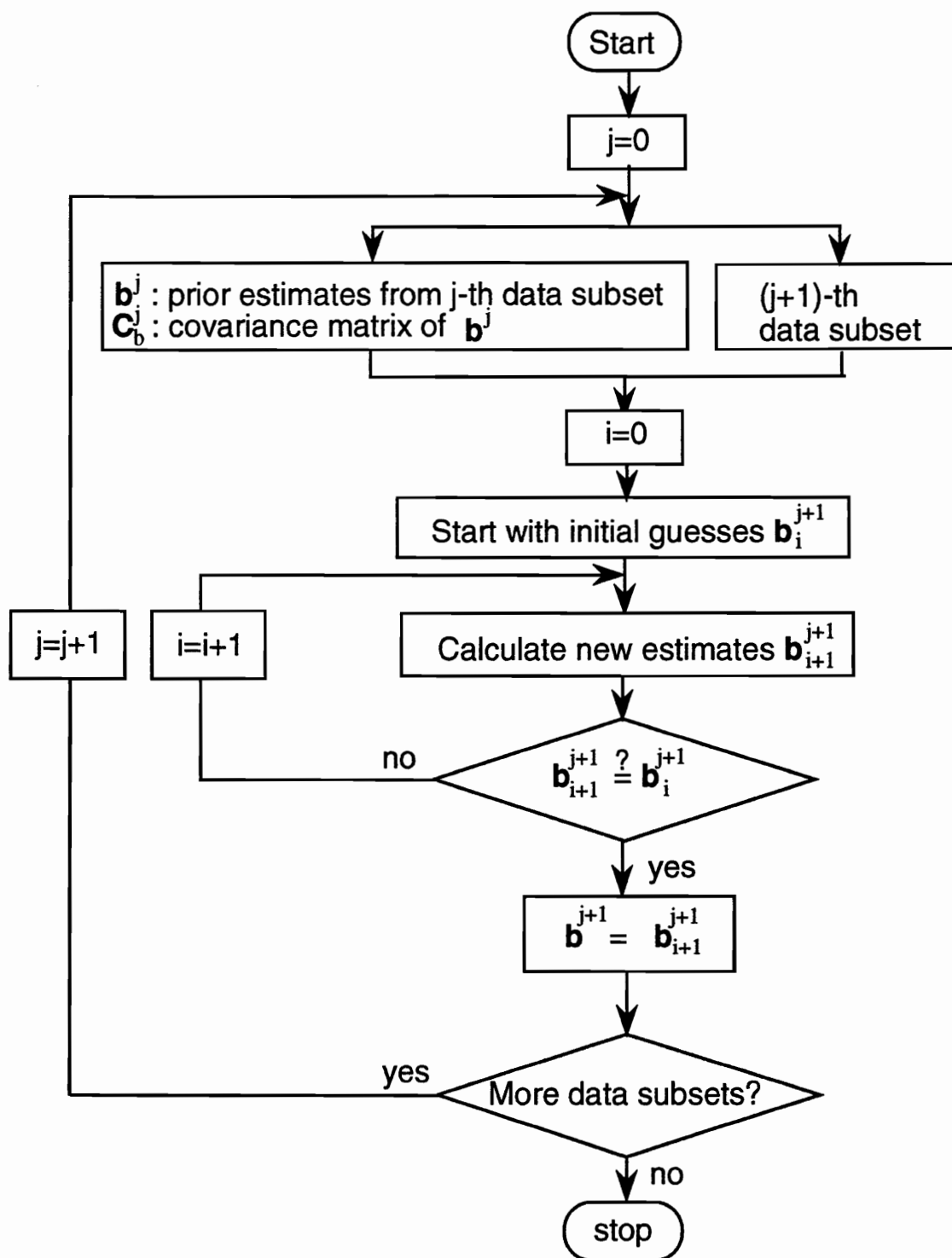


Figure 3.1 Bayesian estimation.

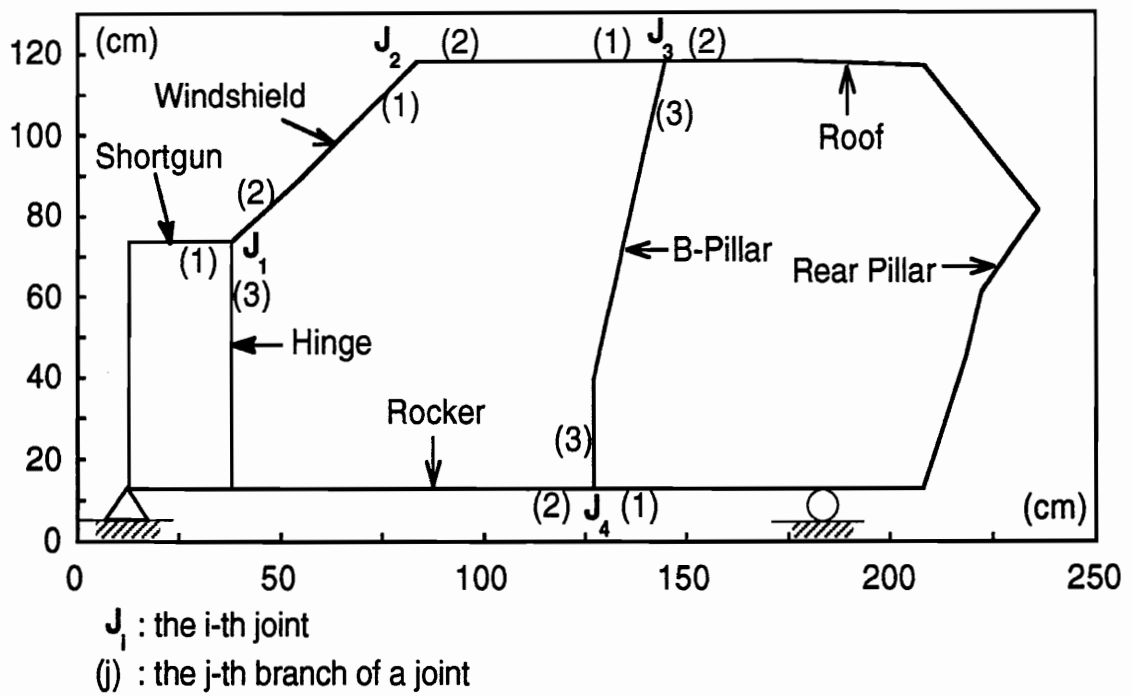


Figure 5.1 A 2 dimensional car model with 4 joints.

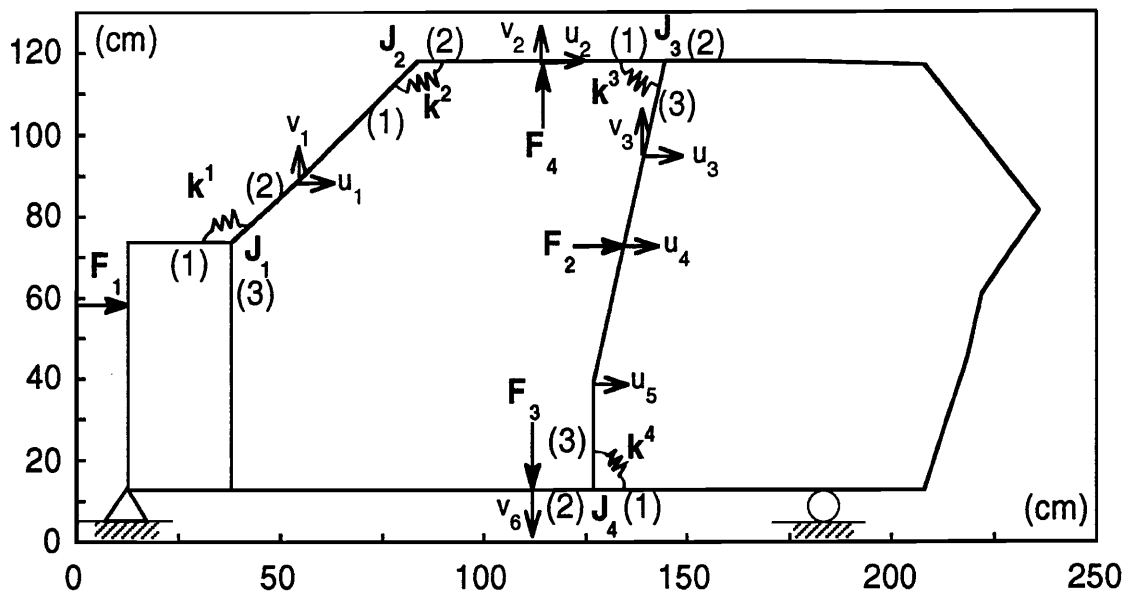


Figure 5.2 Measurements used to estimate parameters at 4 joints.

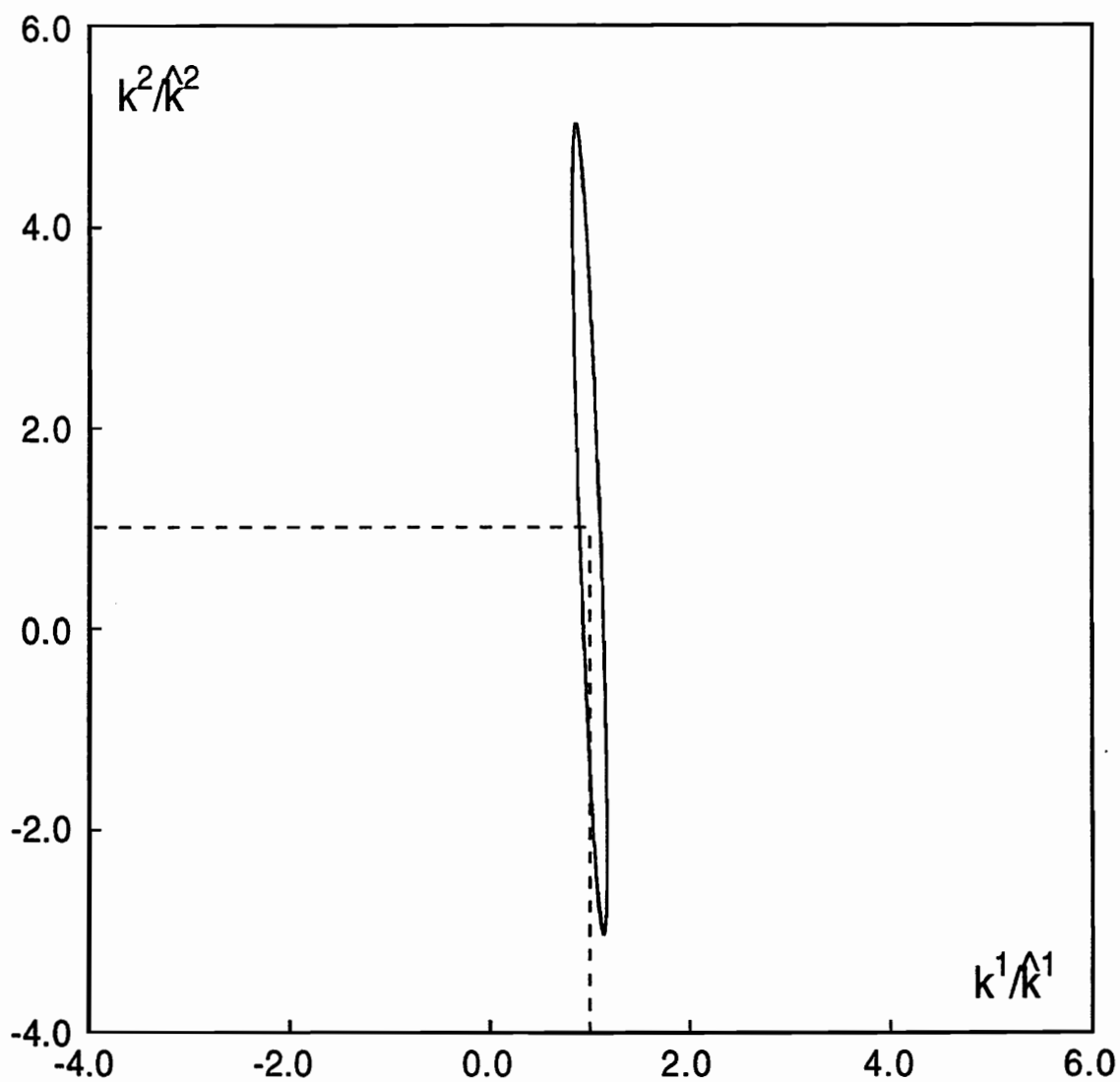


Figure 5.3 Sectional drawing of confidence contour in $k^1 k^2$ -plane.

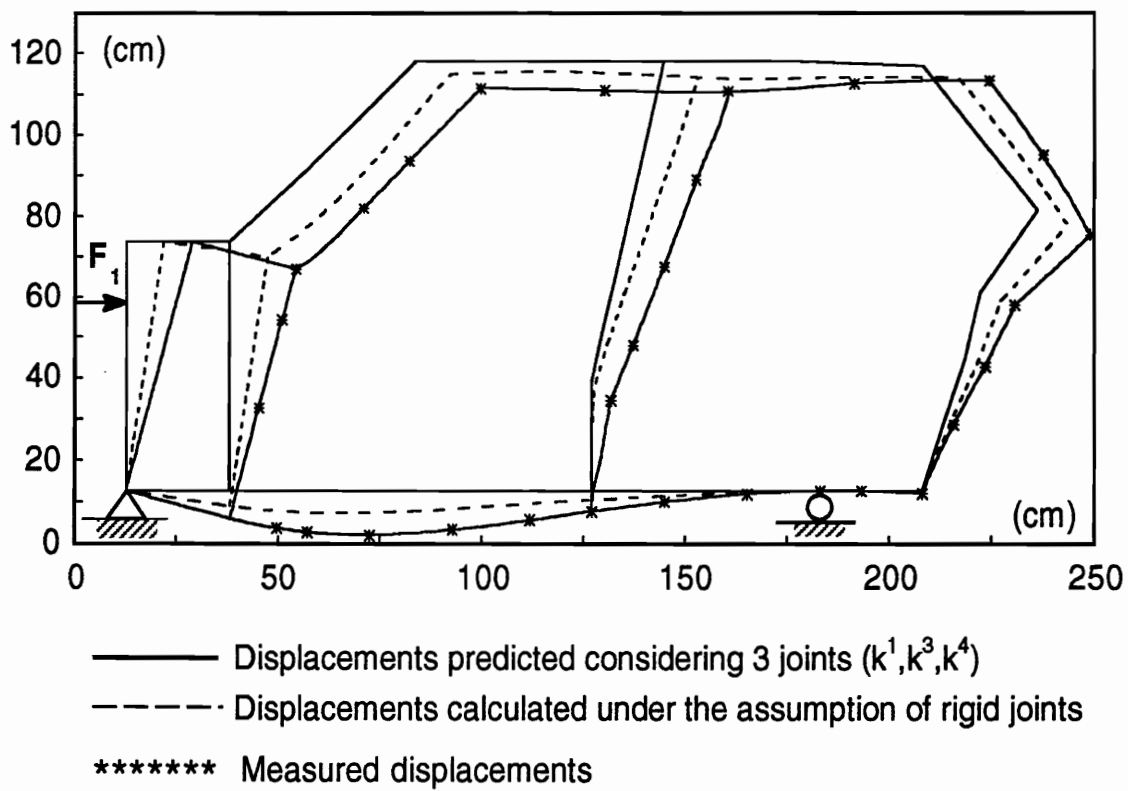


Figure 5.4 Predicted displacements considering 3 joints, k^1, k^3 , and k^4 ($k^2 = \infty$).

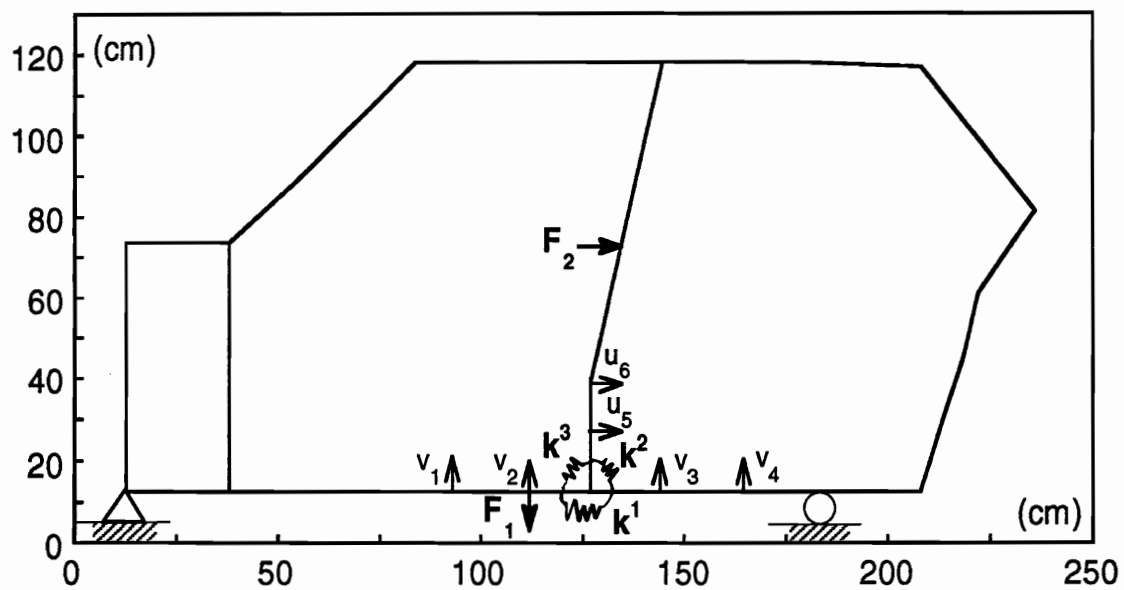


Figure 5.5 A 2 dimensional car model with a 3-branch joint.

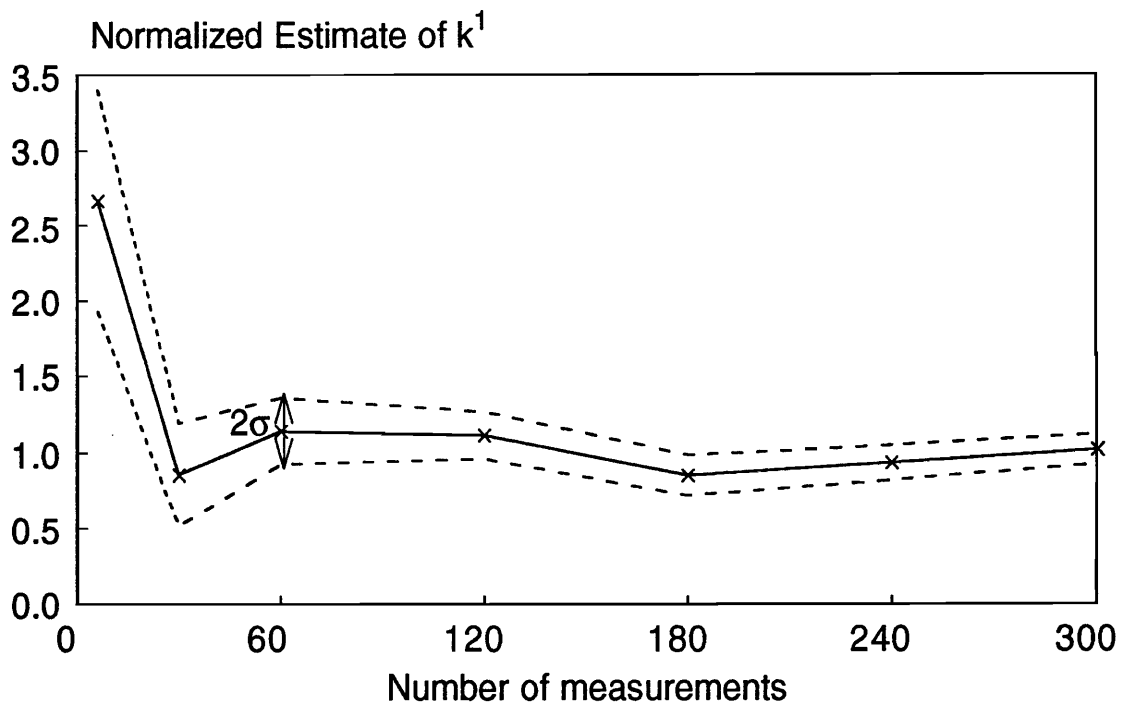


Figure 5.6 Effect of number of measurements on the estimation results.

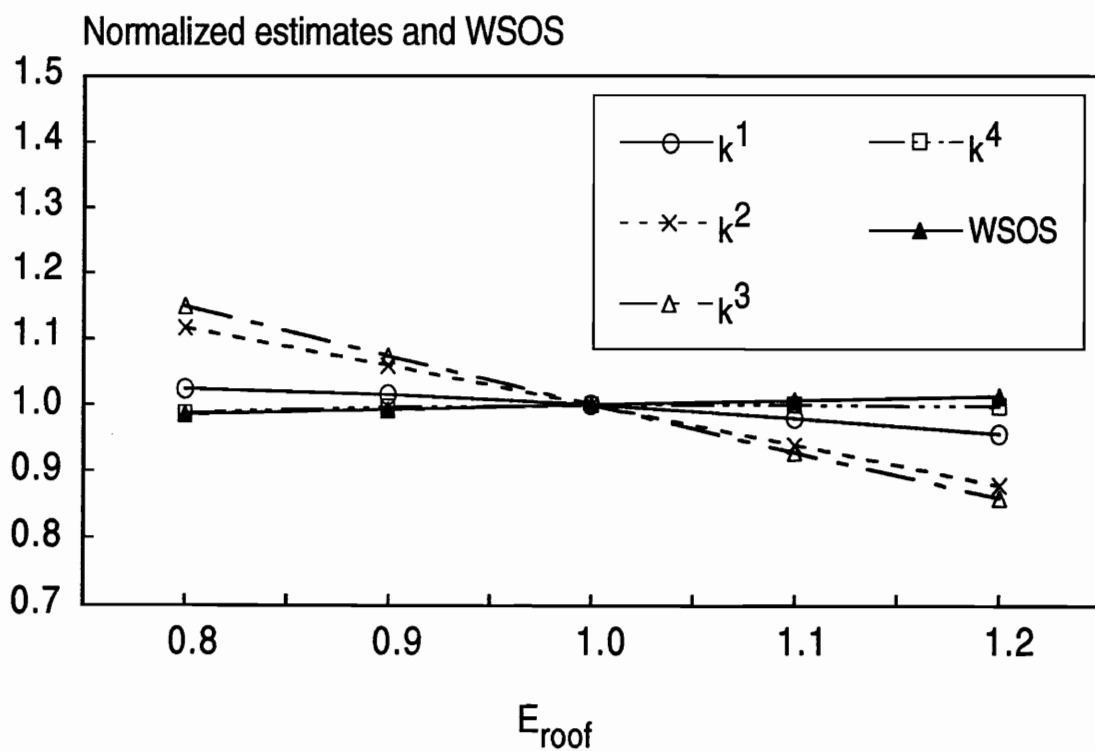


Figure 5.7 Normalized parameter estimates and WSOS (weighted error sum of squares) with error in the stiffness of roof elements.

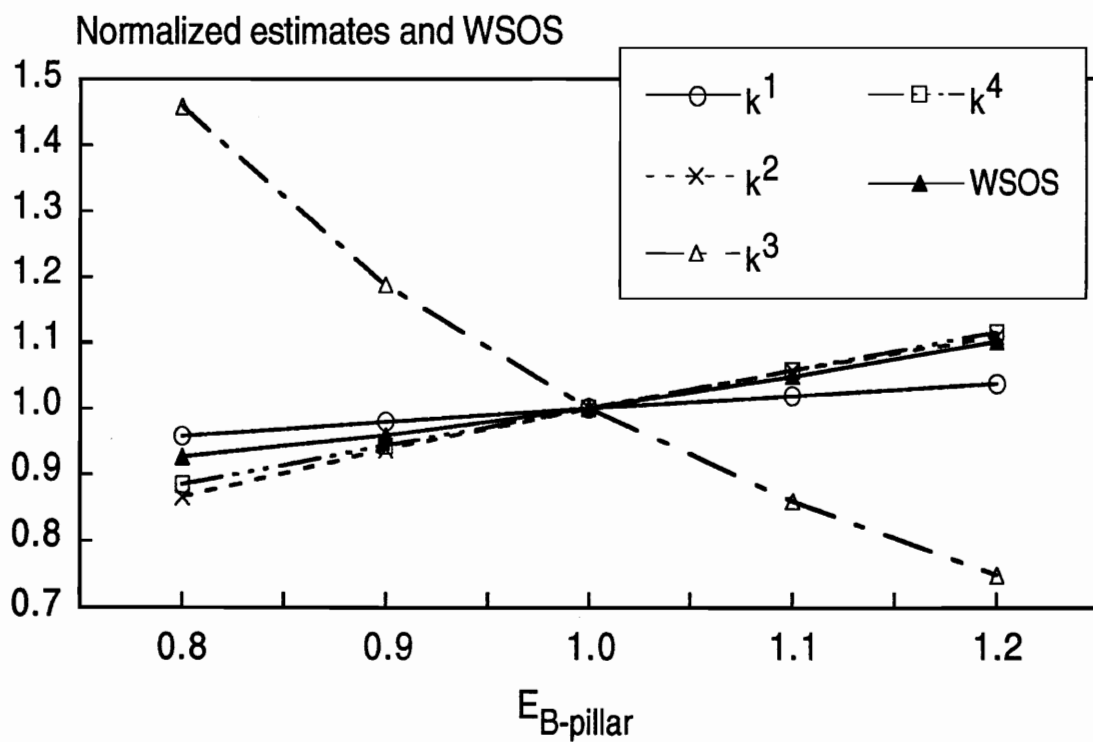


Figure 5.8 Normalized parameter estimates and WSOS with error in the stiffness of B-pillar elements.

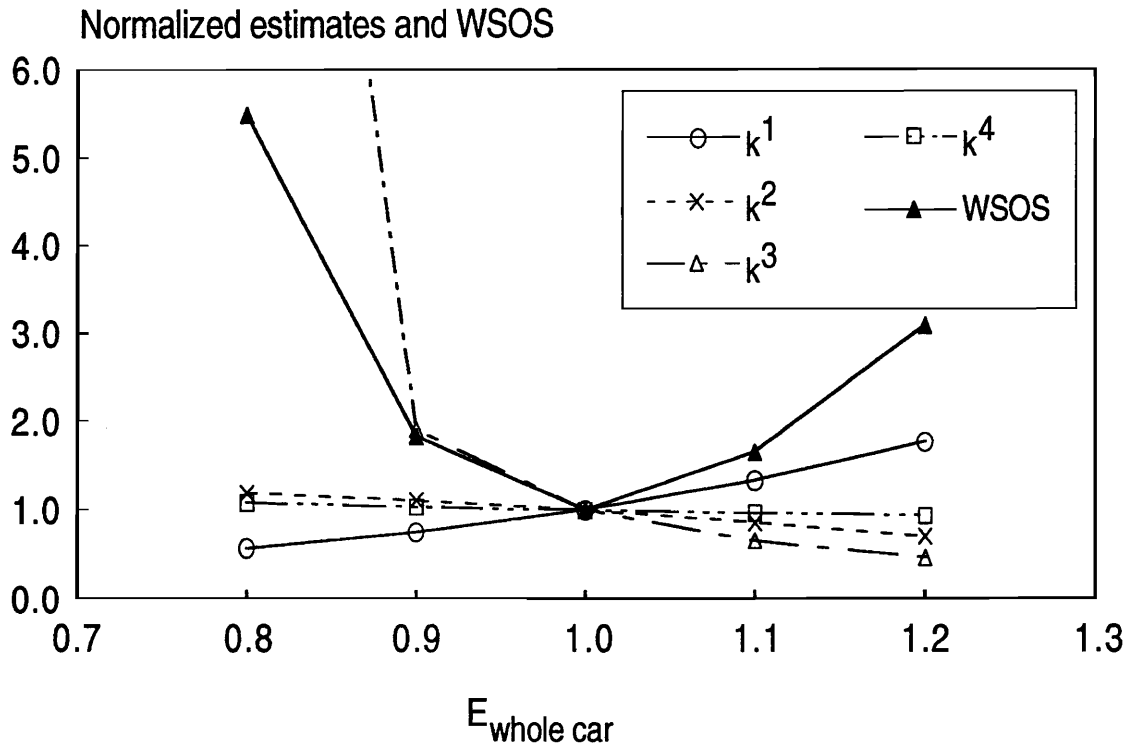


Figure 5.9 Normalized parameter estimates and WSOS with error in the stiffness of the whole car structure.

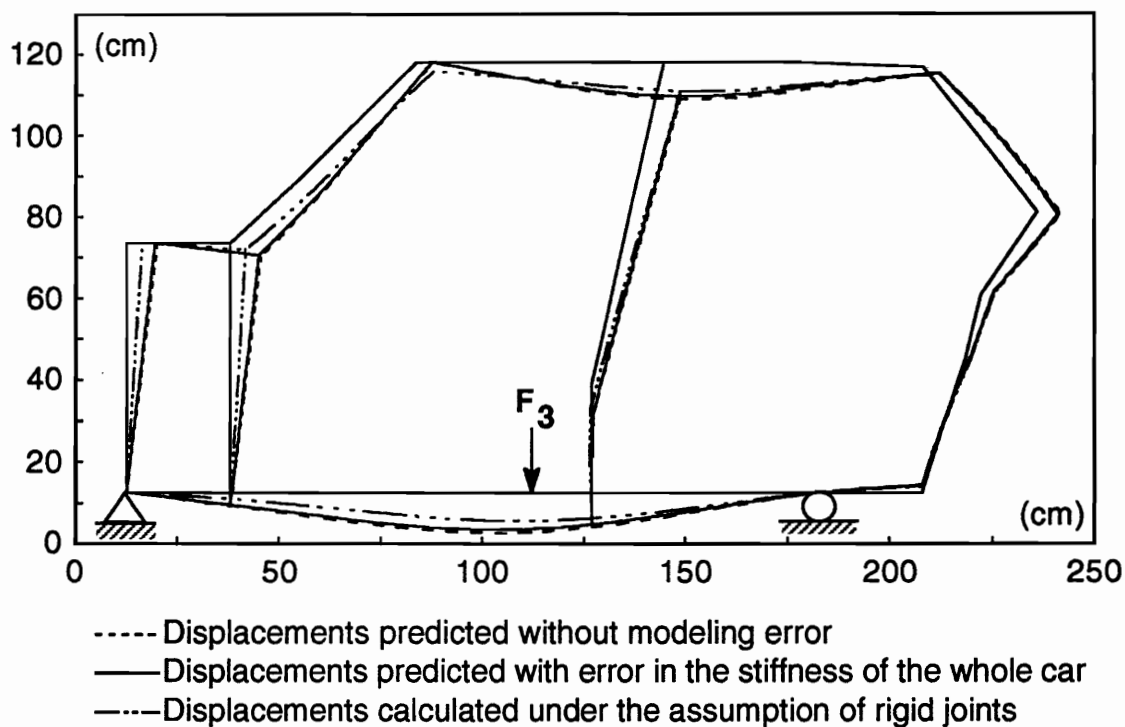


Figure 5.10 Displacements predicted with error in the stiffness of the whole car structure.

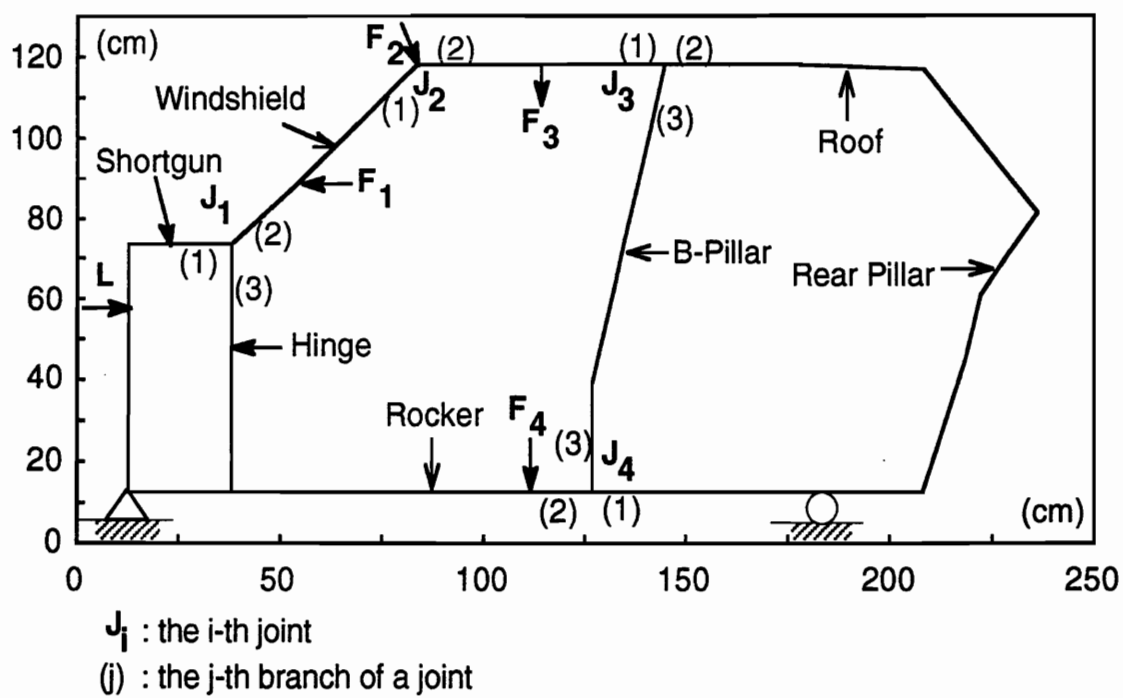


Figure 6.1 A 2 dimensional car model with 4 joints of 2-D bracketed model.

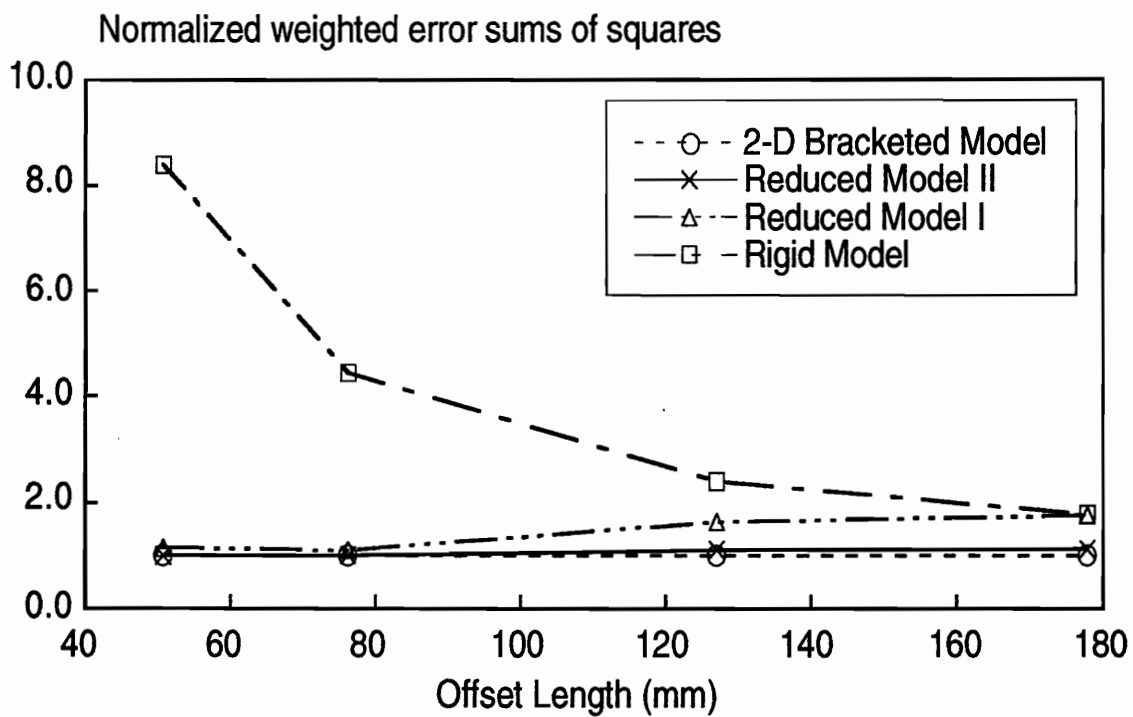


Figure 6.2 Normalized WSOS for different models.

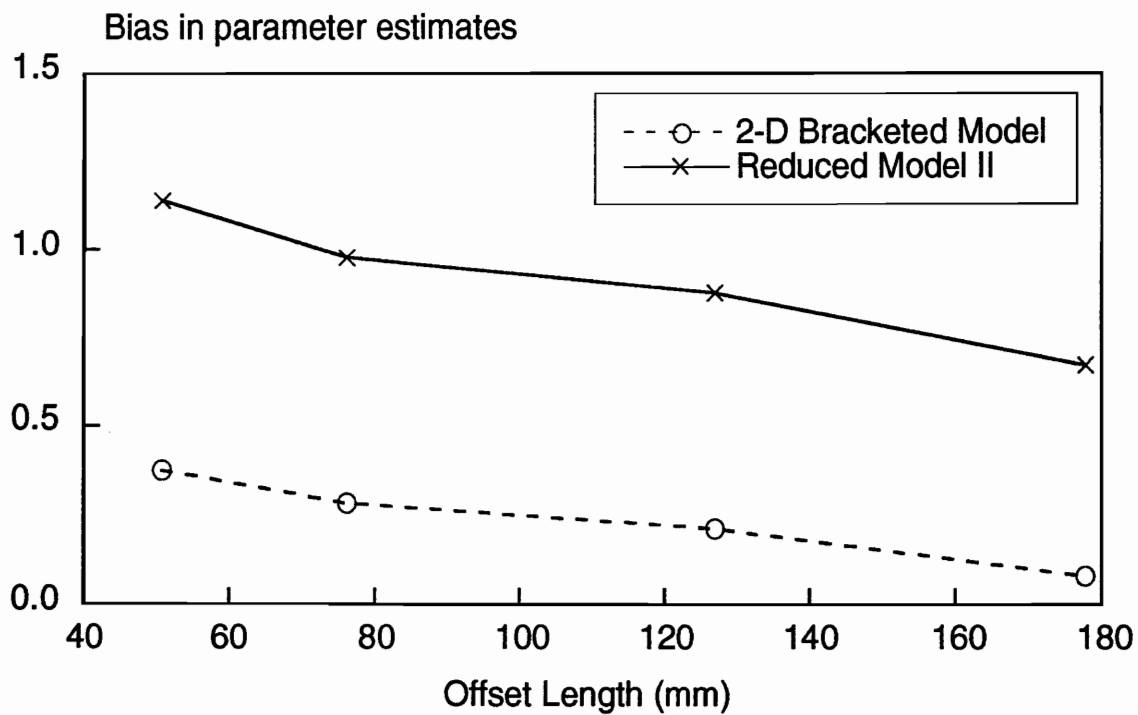


Figure 6.3 Variation of bias in parameter estimates as a function of the offset length.

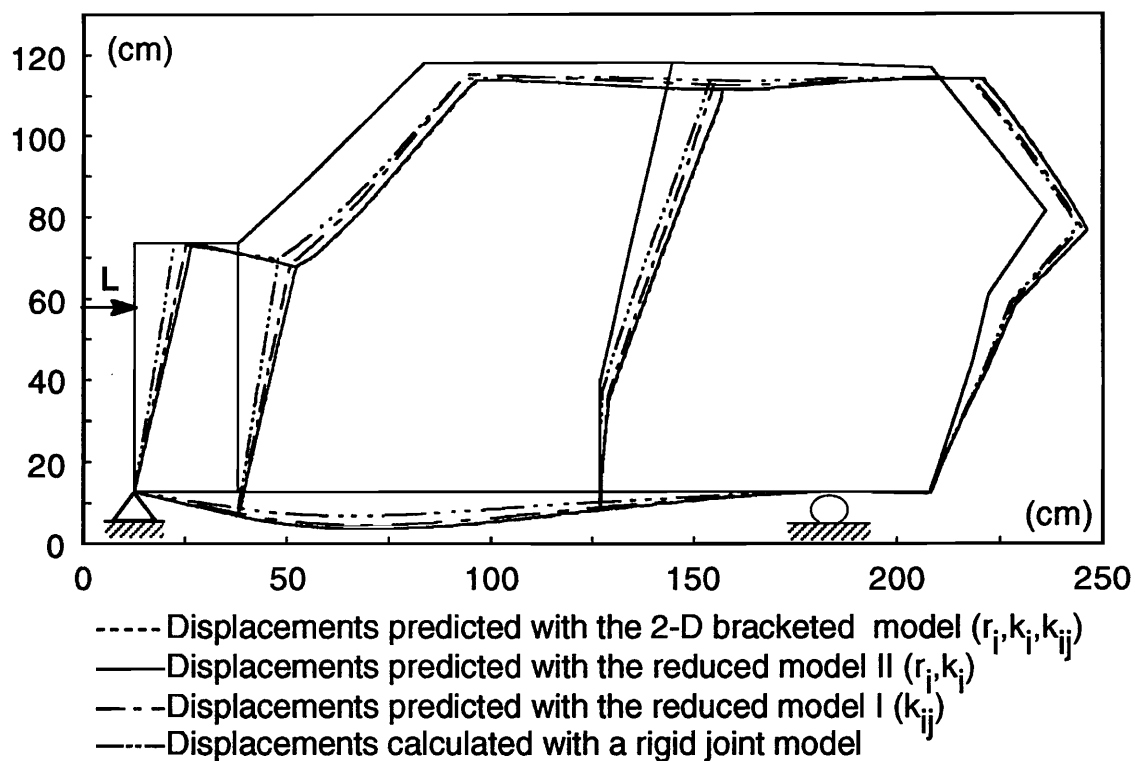


Figure 6.4 Displacements predicted with different models when offsets at the second joint, J_2 , are 50.8 (mm).

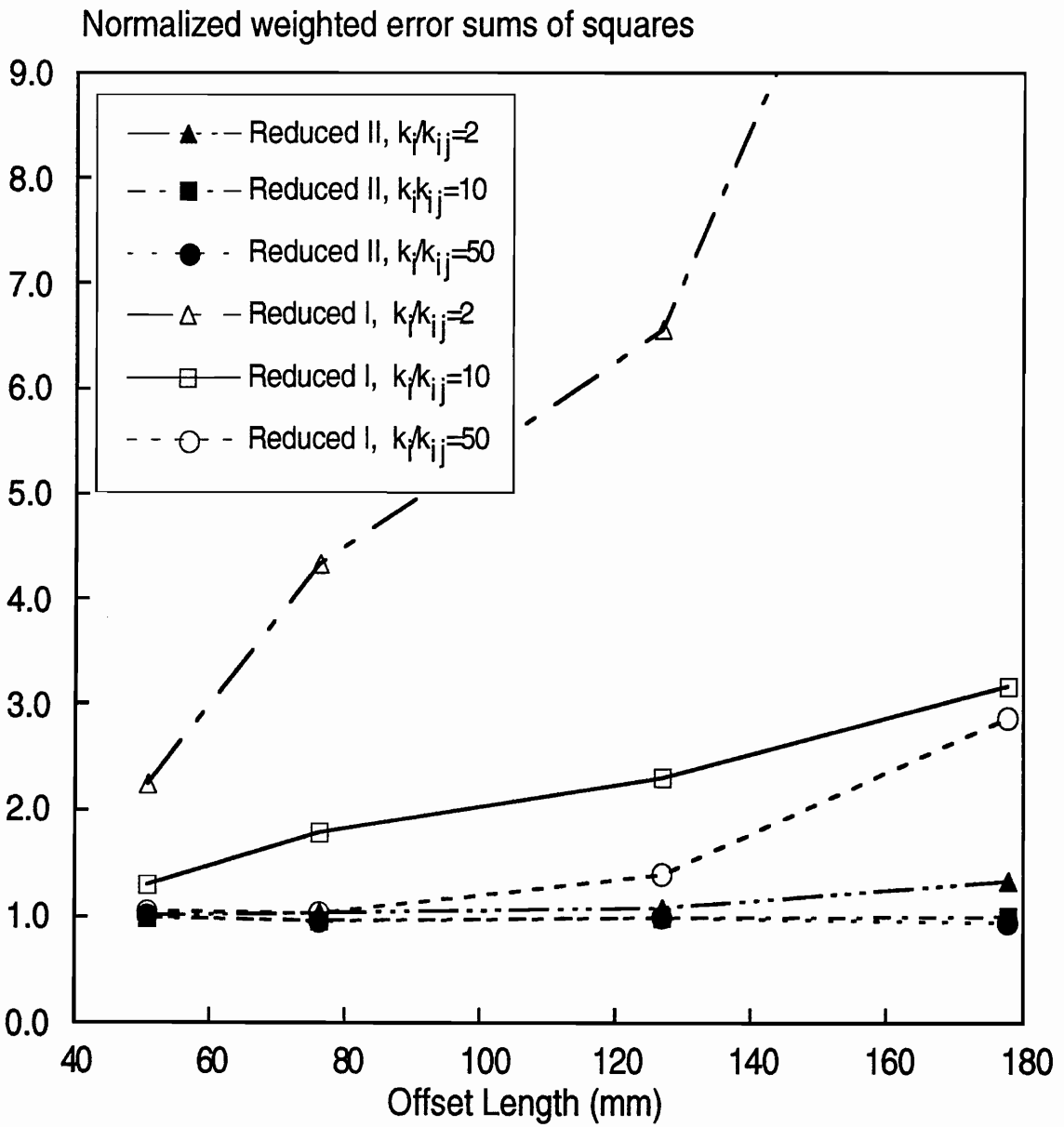


Figure 6.5 Normalized WSOS for different ratio of k_i/k_{ij} and for different reduced models.

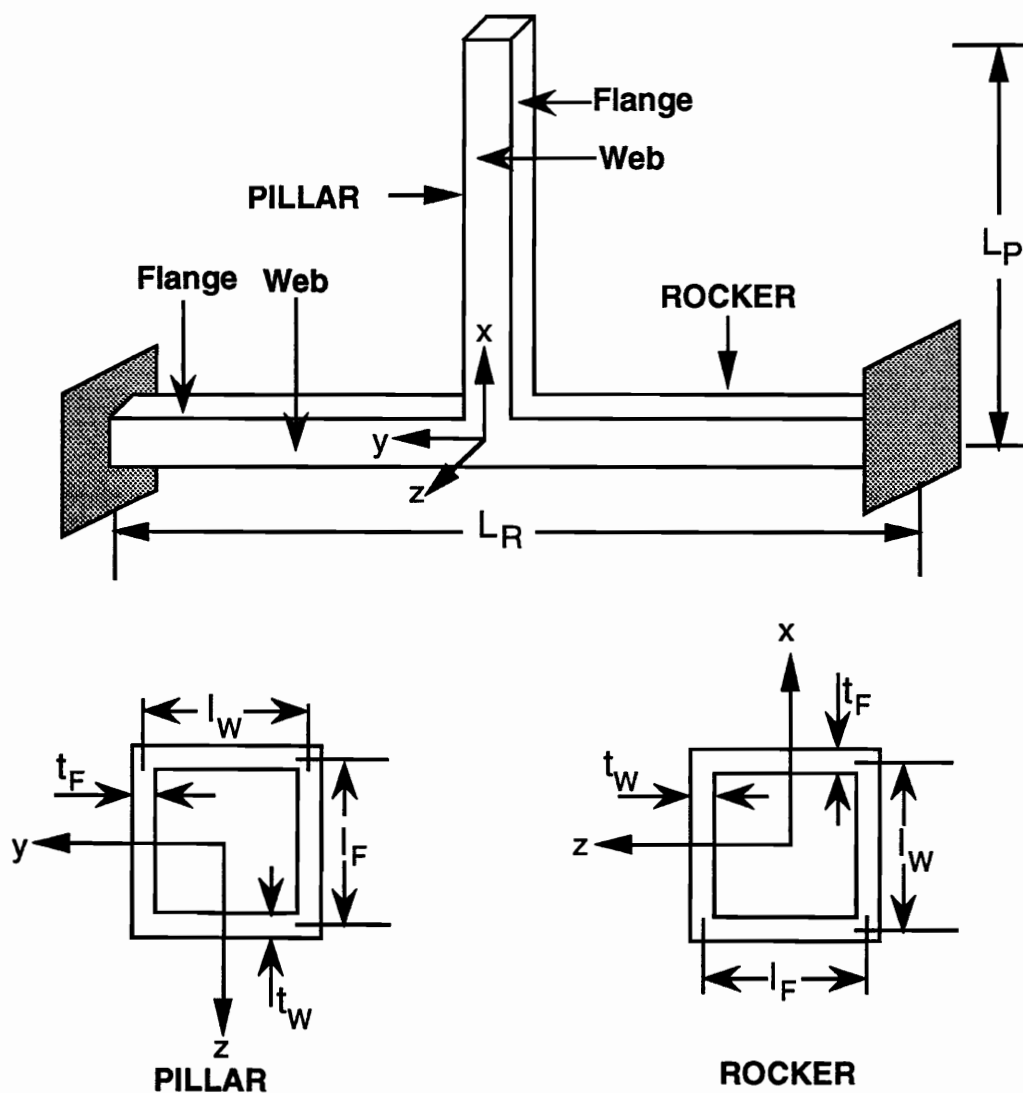


Figure 7.1 A T-shaped subassembly made of two box beams.

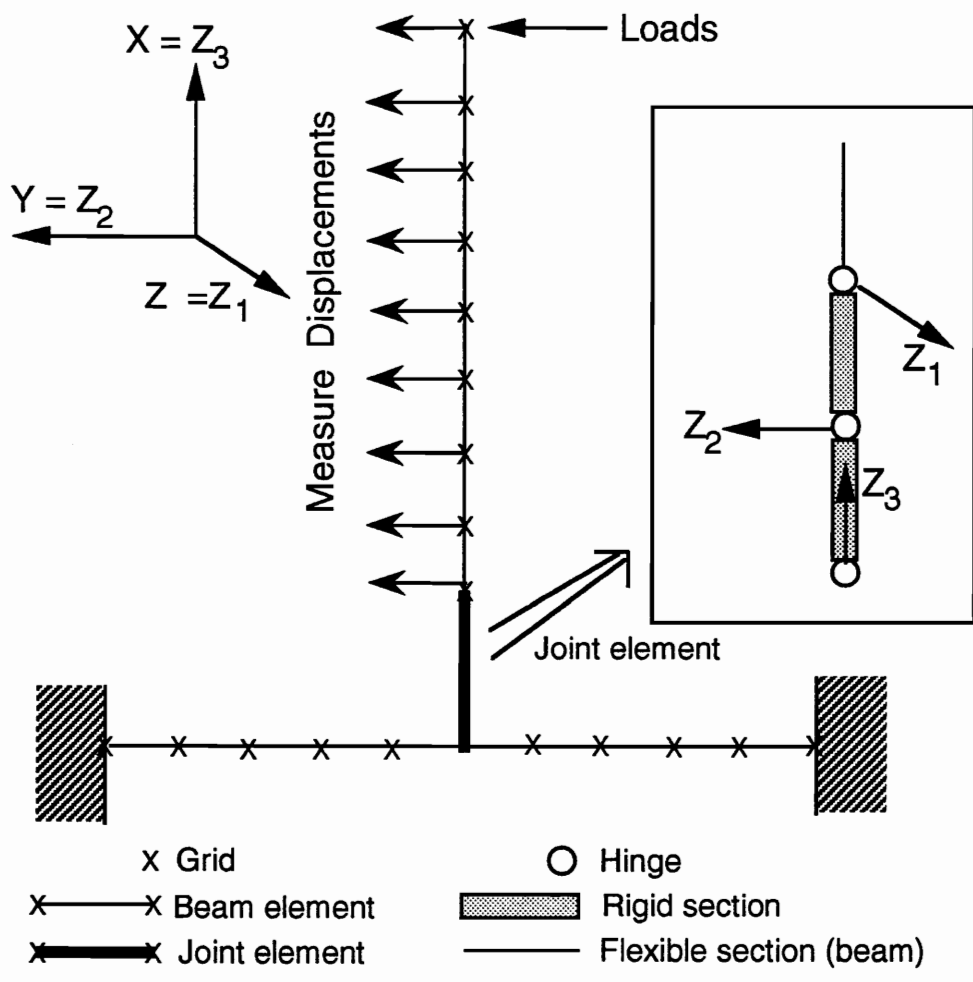


Figure 7.2 Concept model of the subassembly in Figure 7.1.

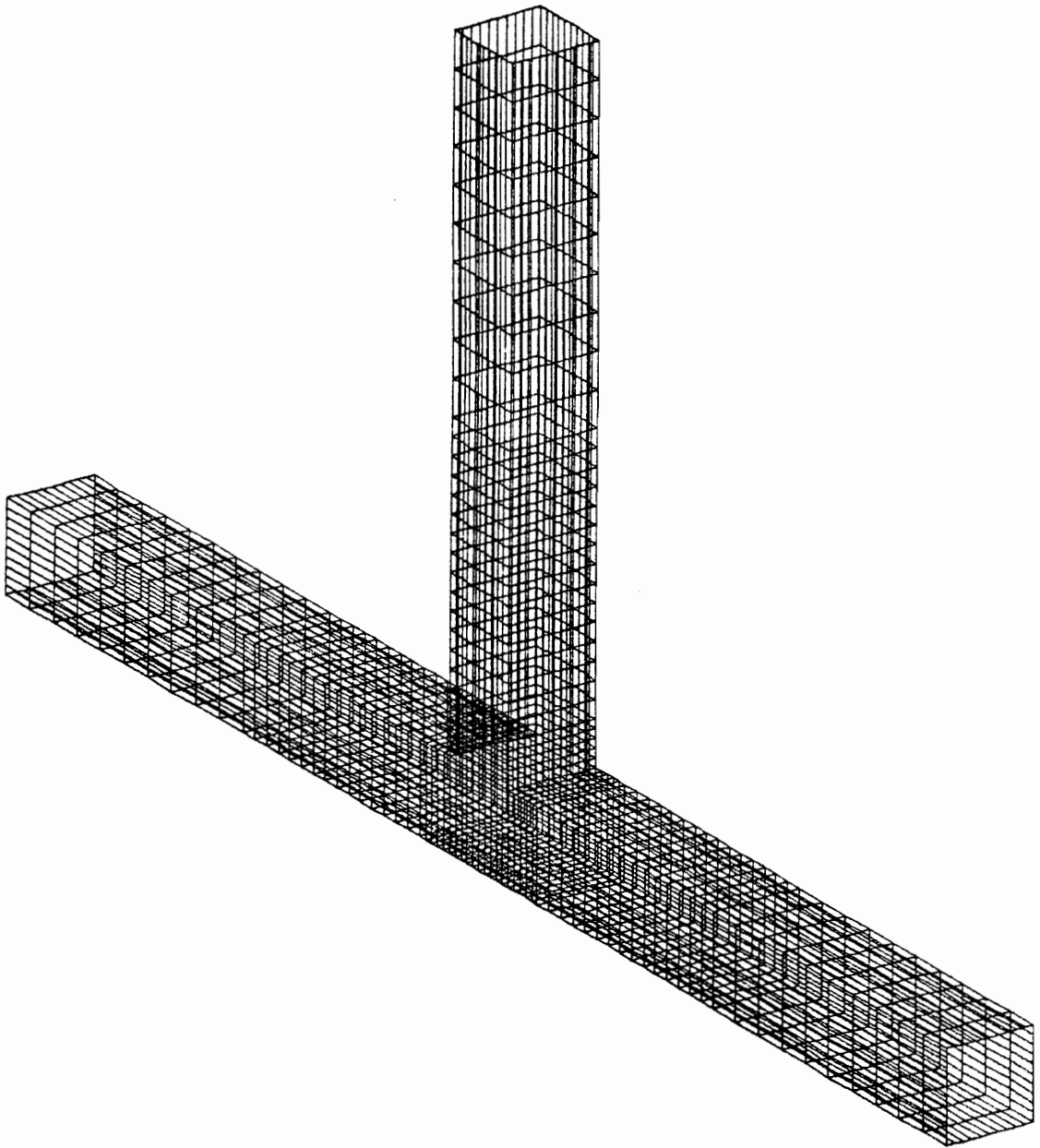


Figure 7.3 Detailed finite element model of the subassembly in Figure 7.1.

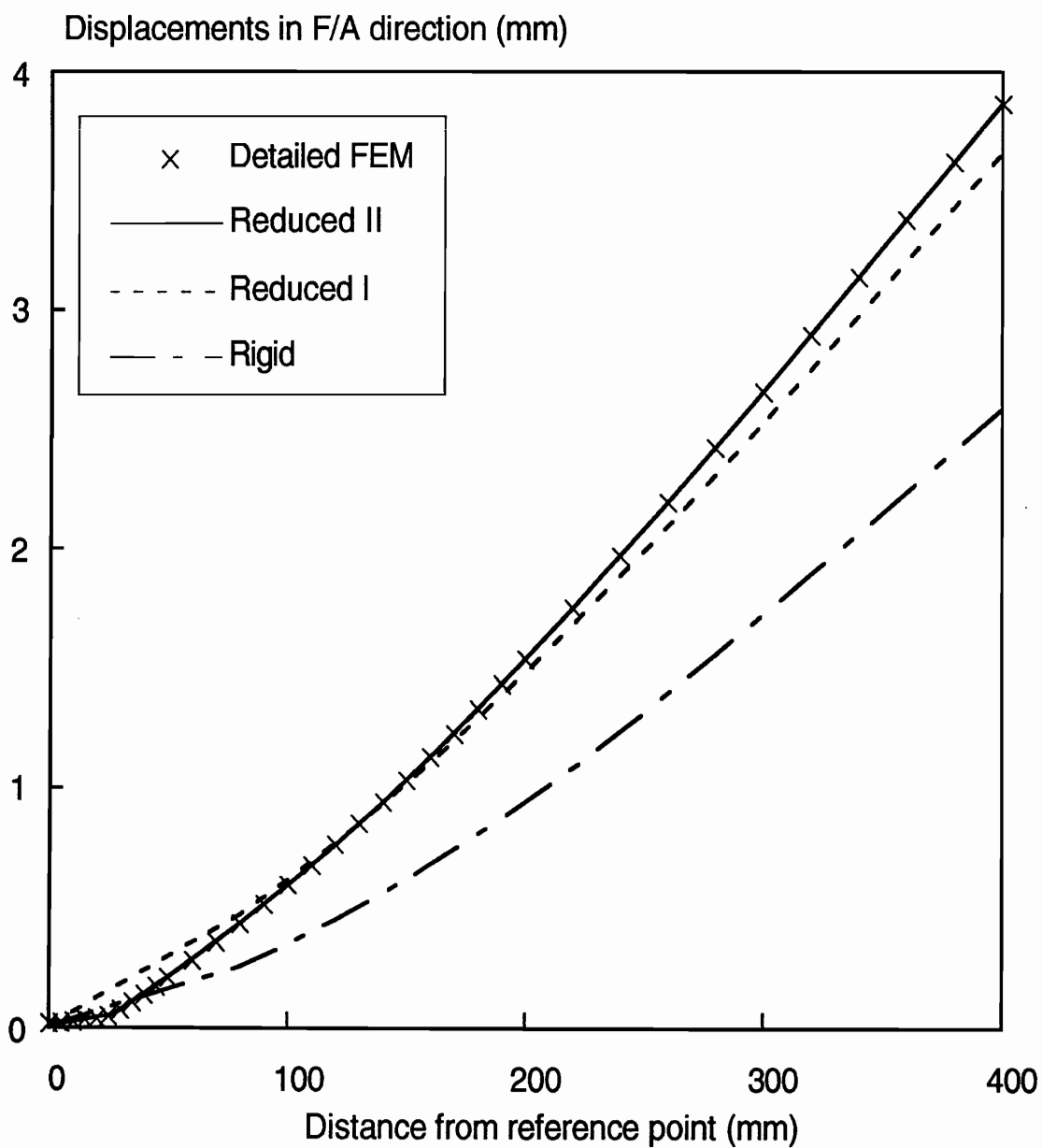


Figure 7.4 Predicted displacements in F/A direction.

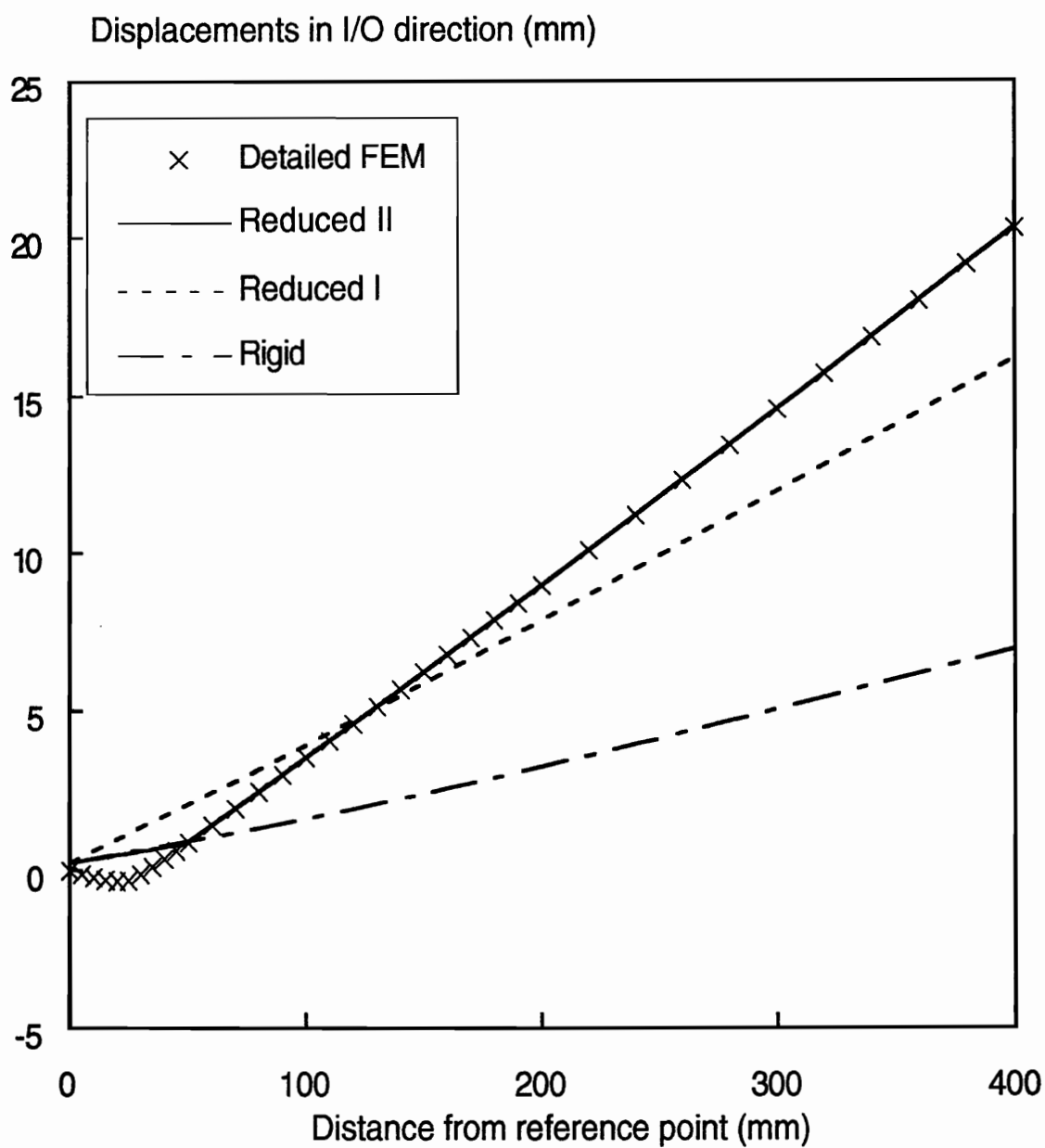


Figure 7.5 Predicted displacements in I/O direction.

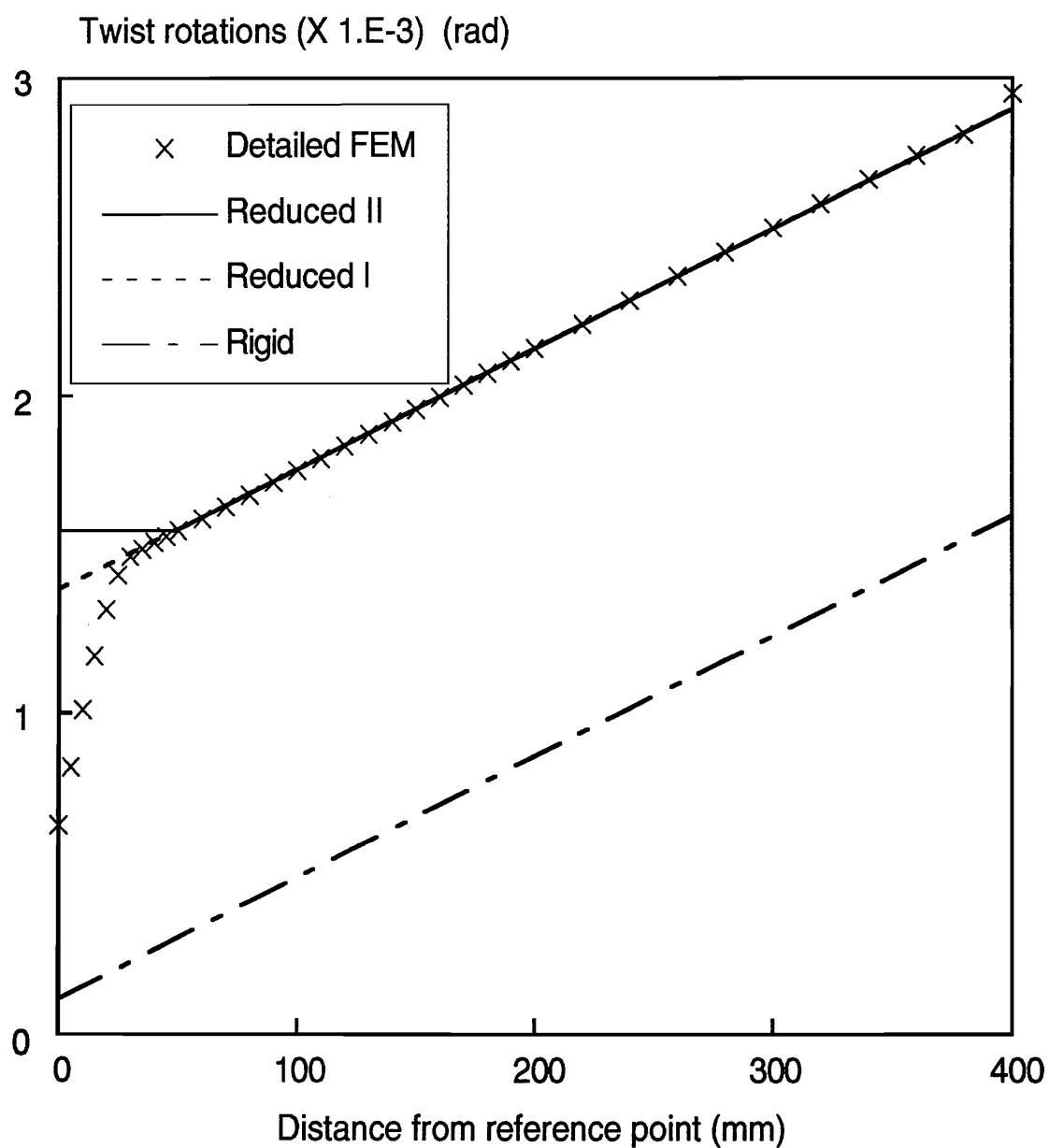


Figure 7.6 Predicted twist rotations.

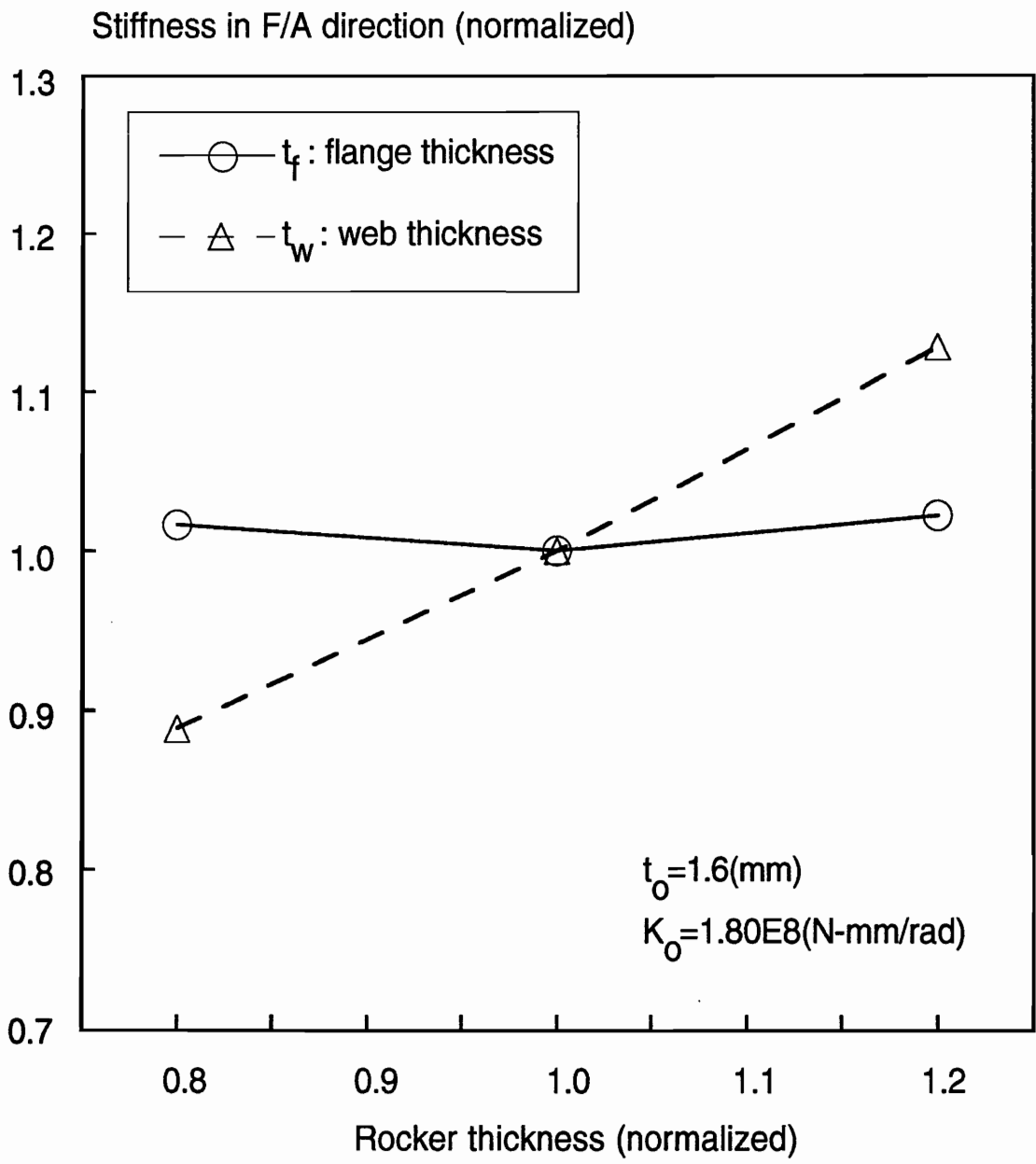


Figure 7.7 Variation of stiffness in F/A direction with change in rocker panel thickness.

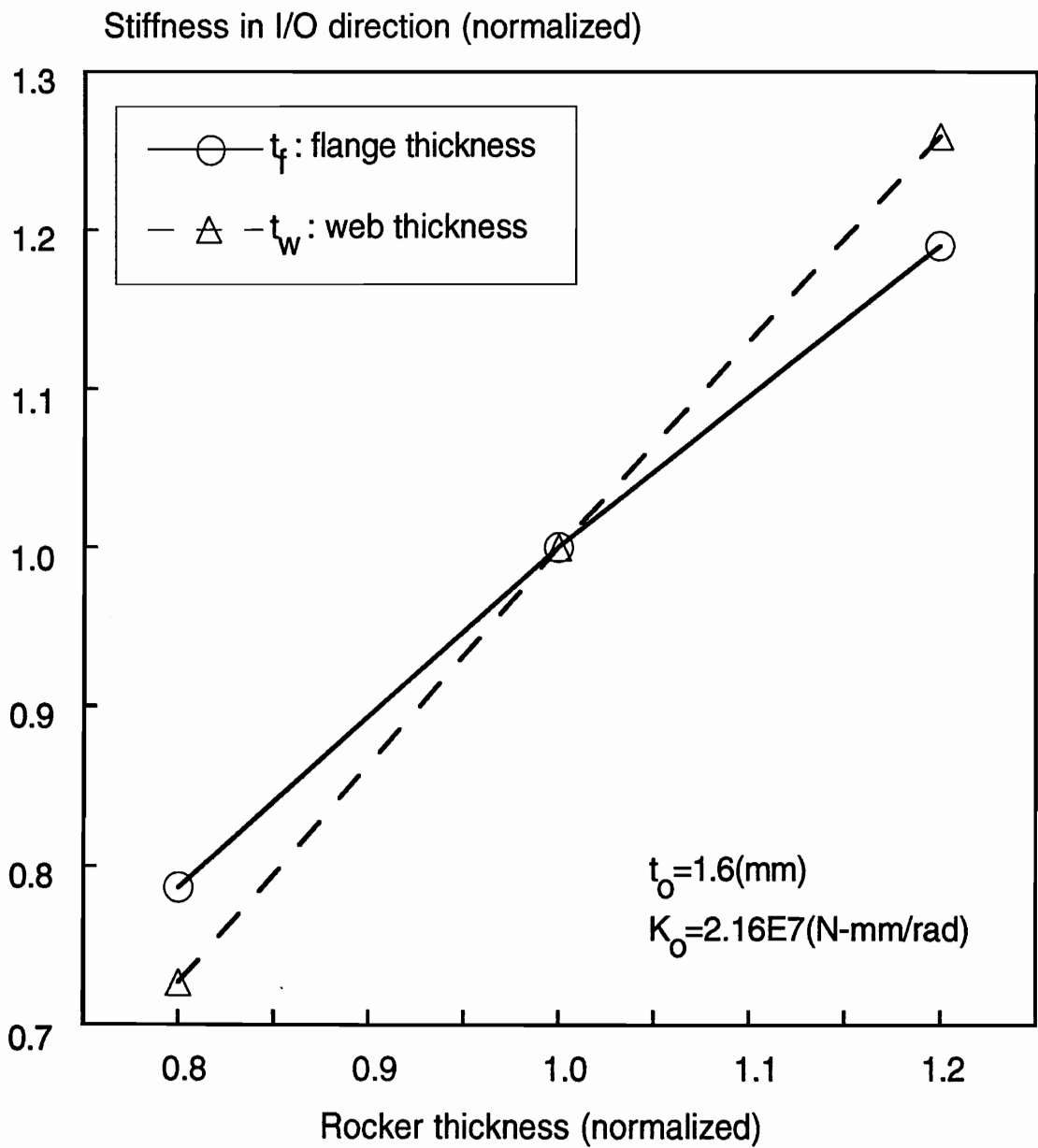


Figure 7.8 Variation of stiffness in I/O direction with change in rocker panel thickness.

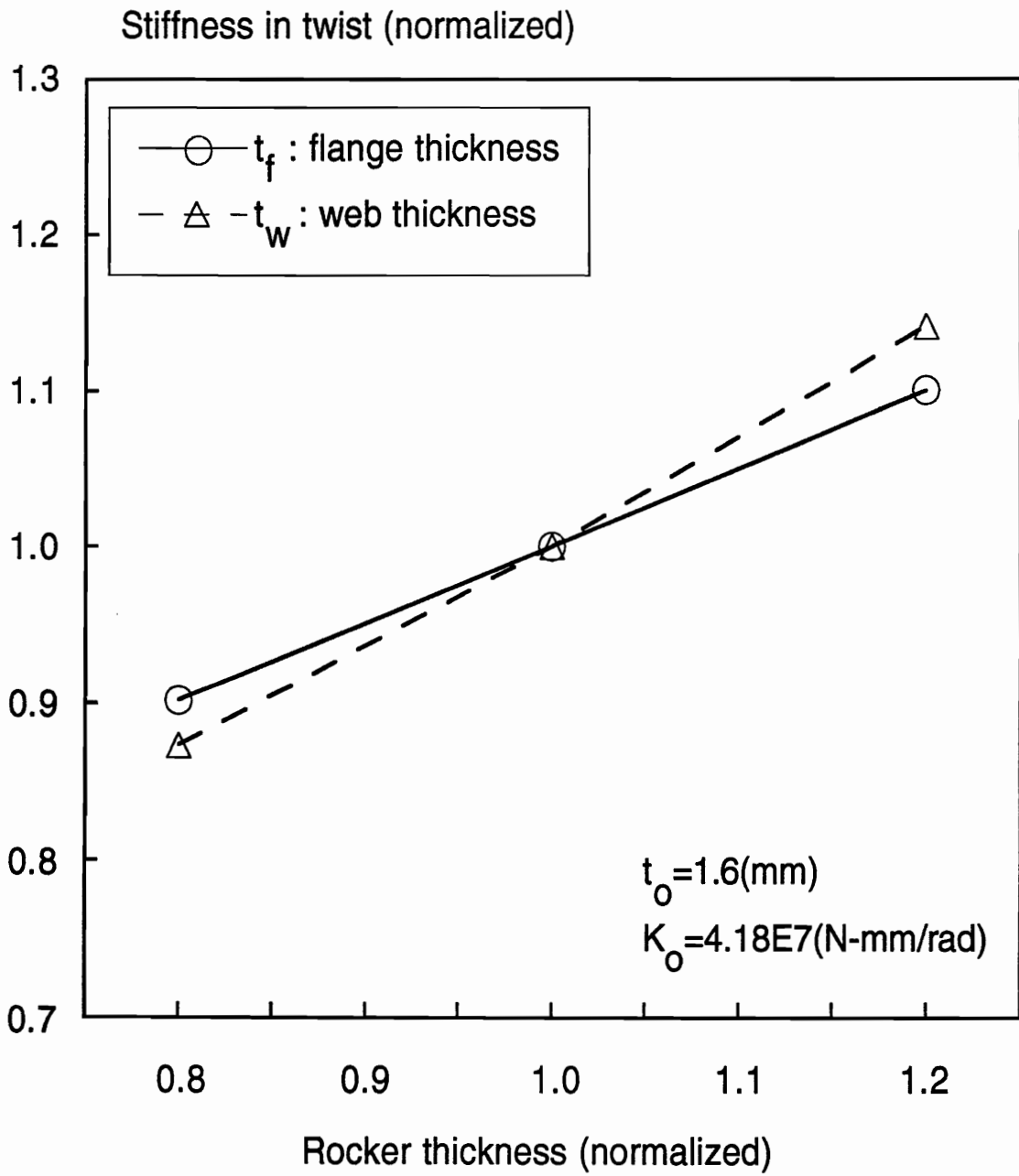


Figure 7.9 Variation of twist stiffness with change in rocker panel thickness.

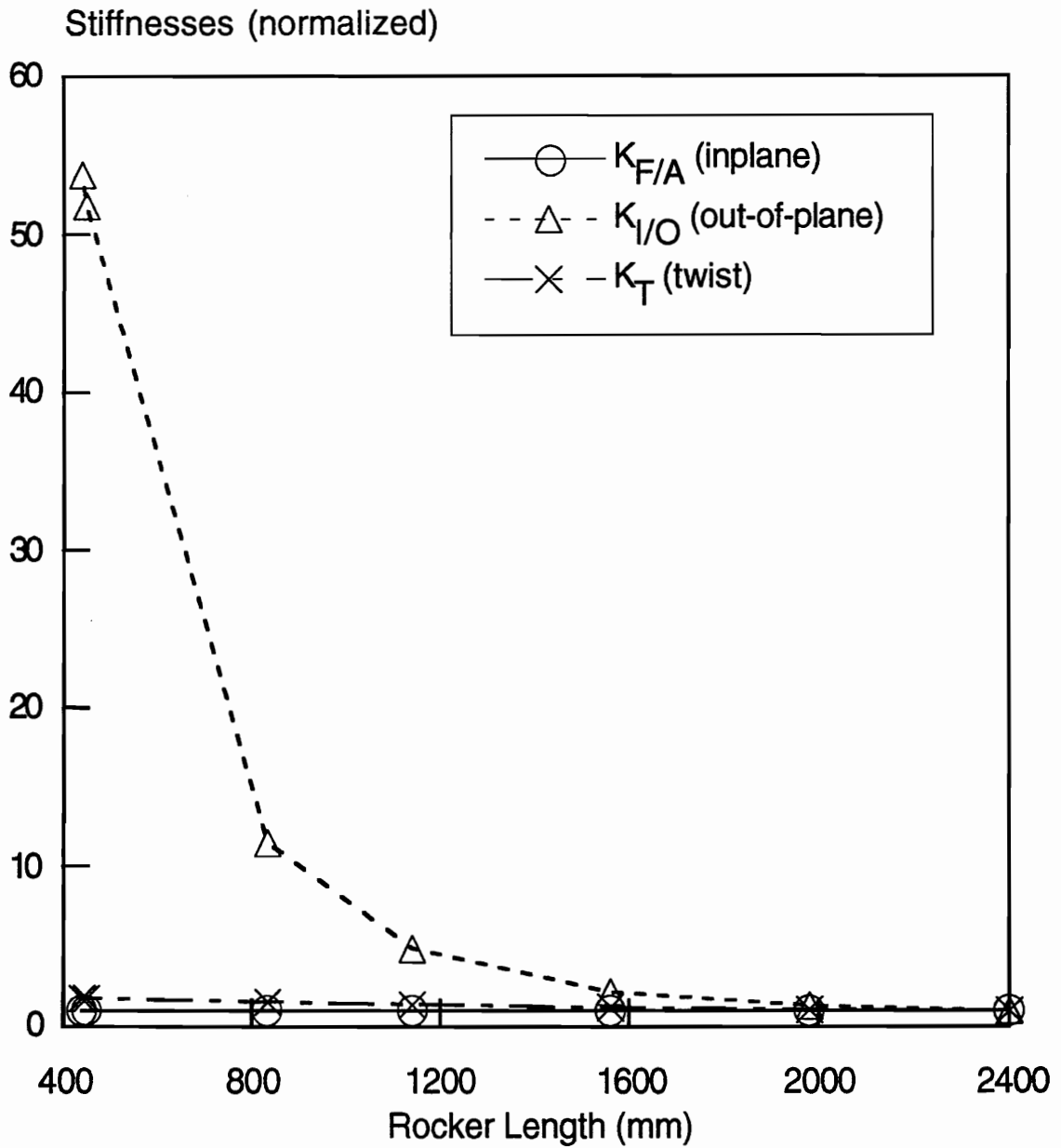


Figure 7.10 Variation of stiffnesses of three rotational modes with change in rocker length.

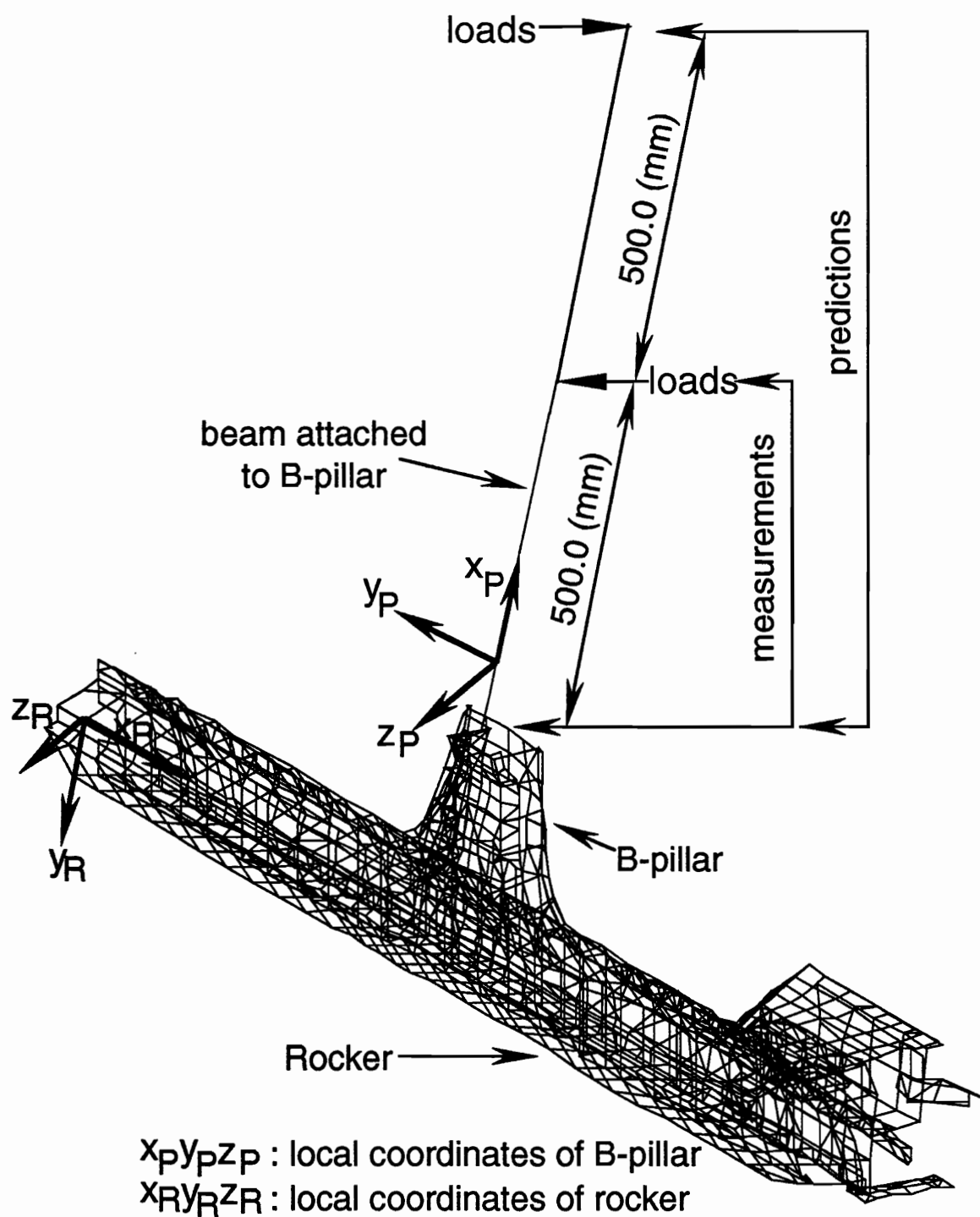


Figure 8.1 Detailed finite element model of a subassembly which consists of a B-pillar to rocker joint and attached branches.

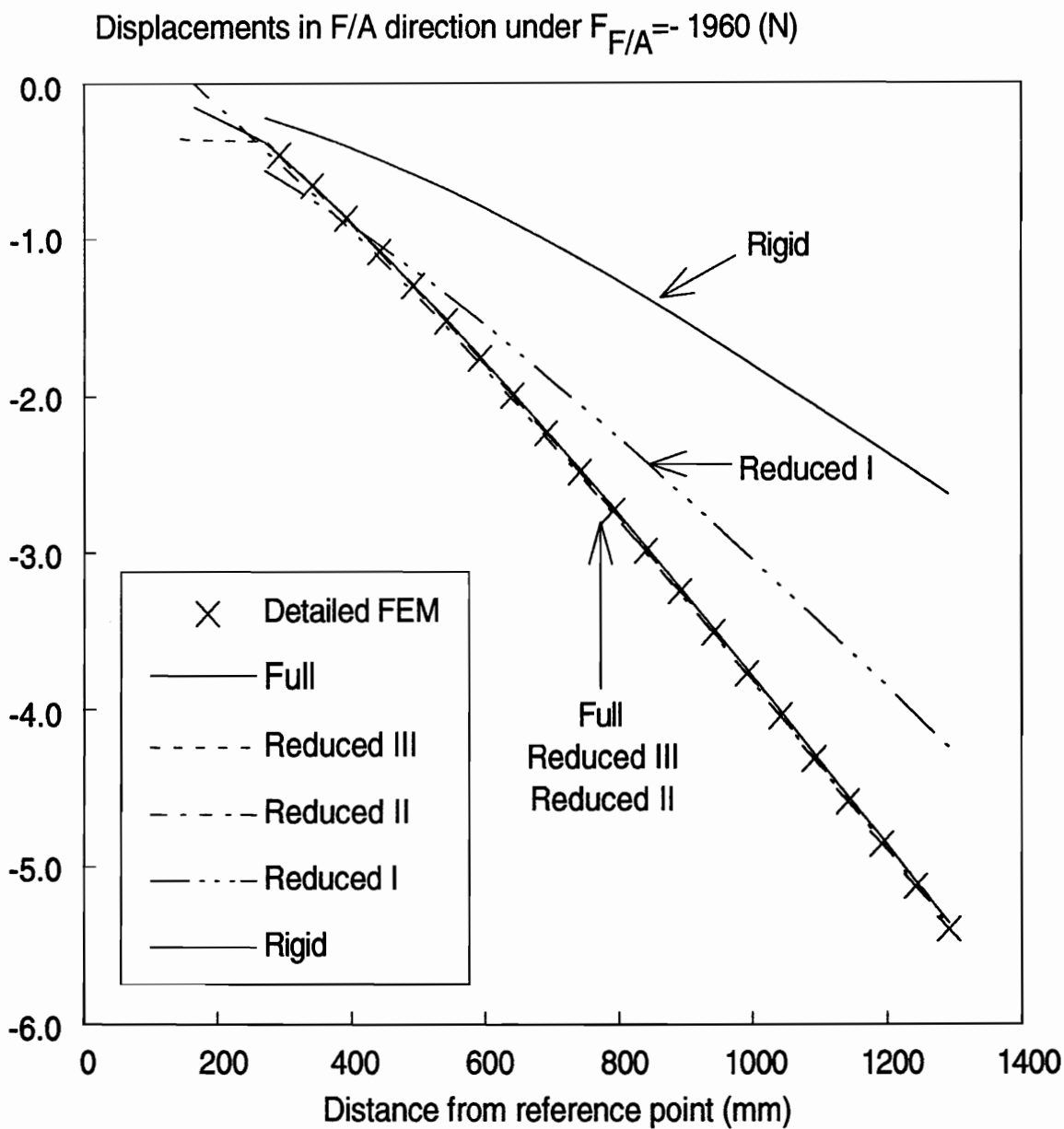


Figure 8.2 Predicted displacements in F/A direction under a force applied in the same direction.

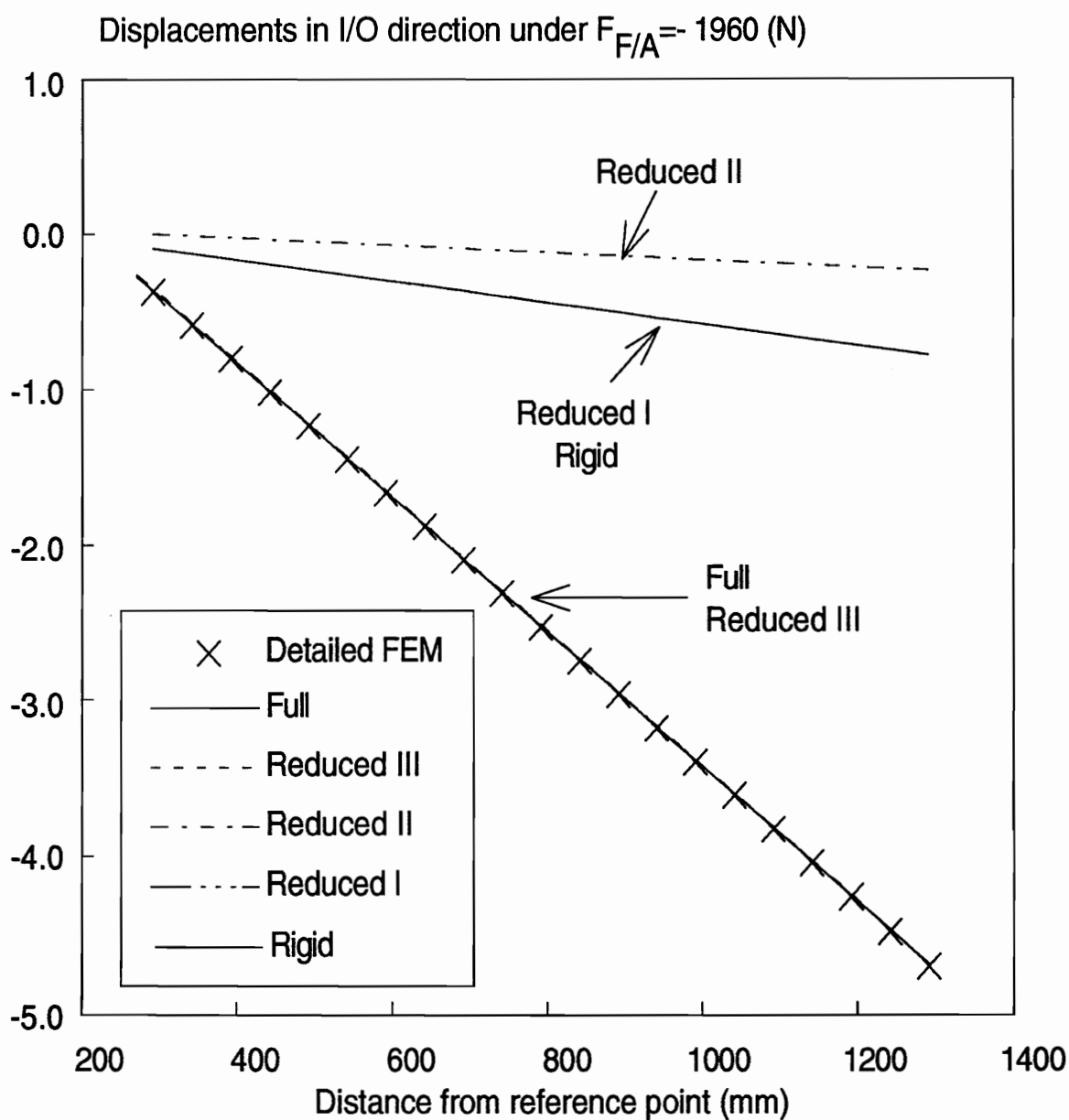


Figure 8.3 Predicted displacements in I/O direction under a force applied in F/A direction.

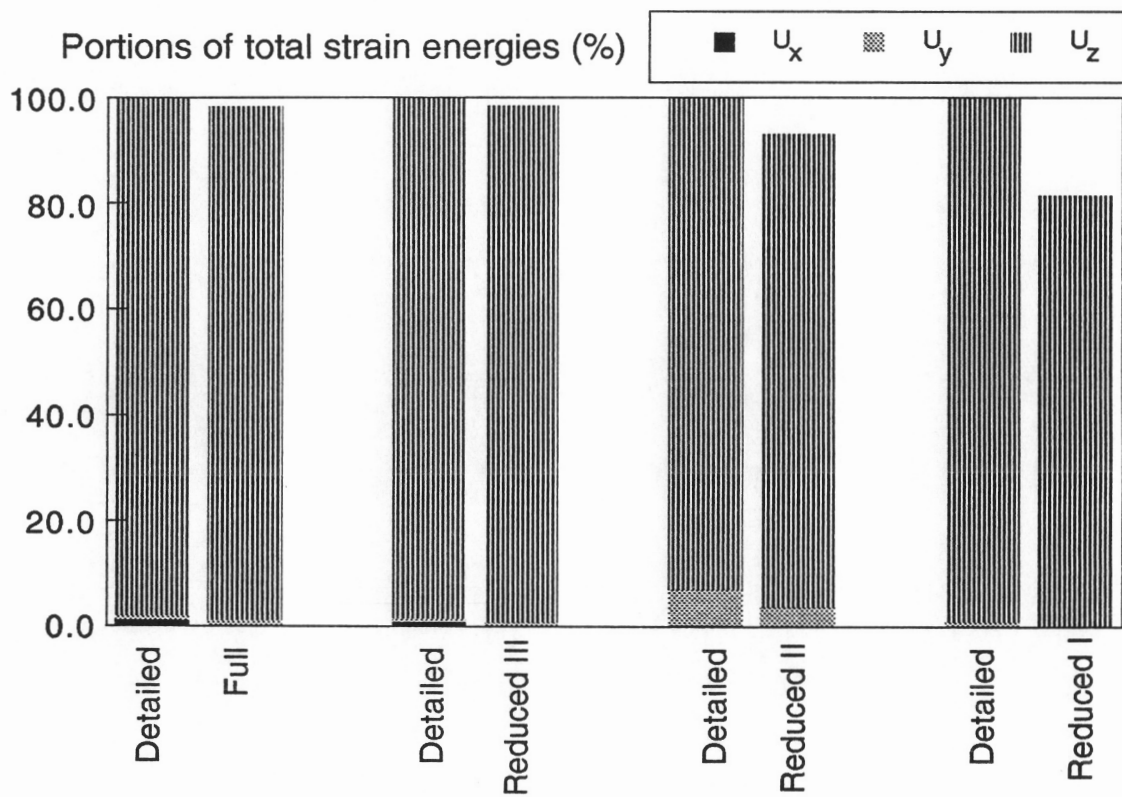


Figure 8.4 Portions of total strain energies due to force components, which are normalized using total strain energies of the detailed model.

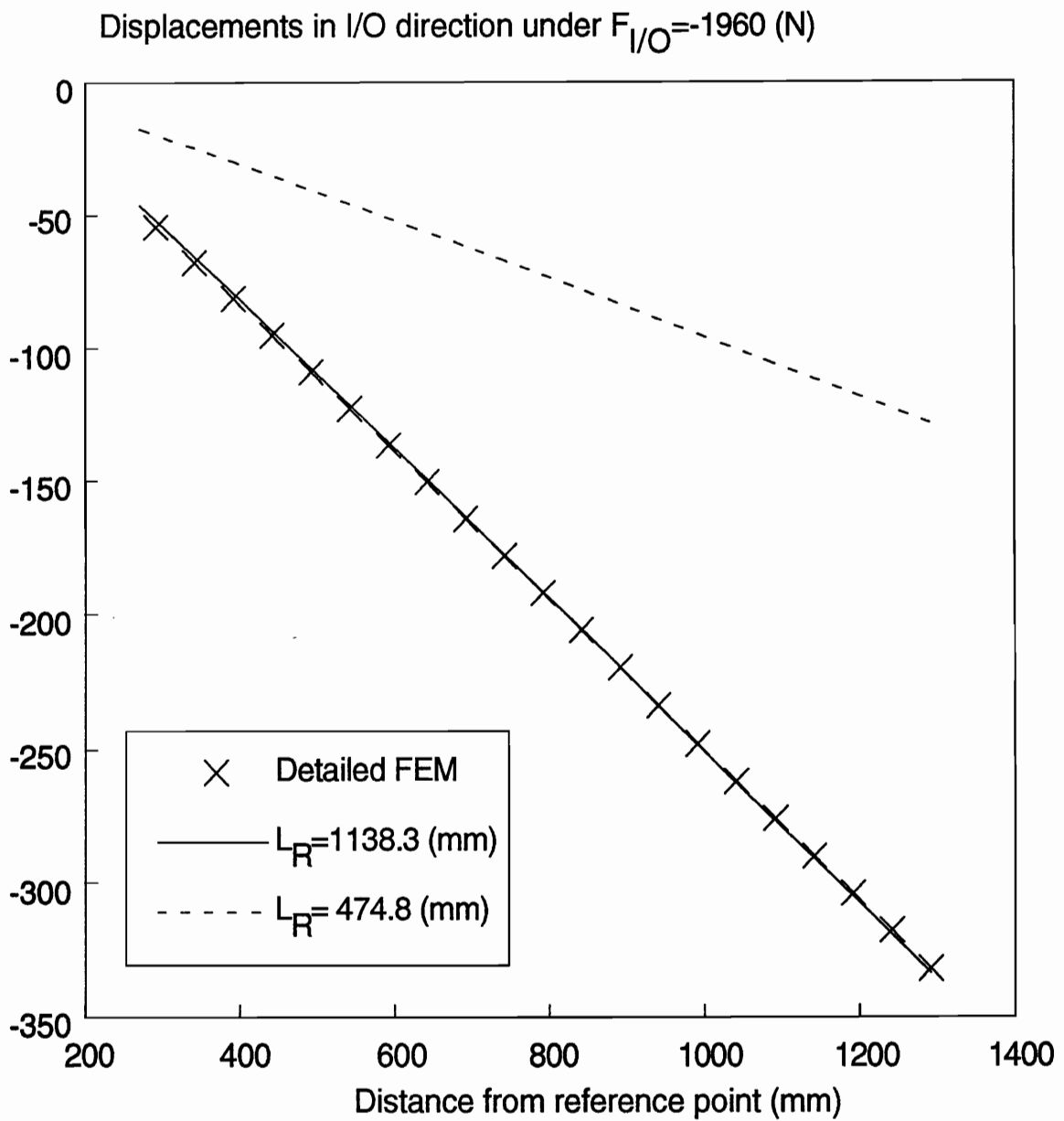


Figure 8.5 Predicted displacements in F/A direction under a force applied in the same direction. The parameters were estimated from sub-assemblies with different rocker length.

VITA

The author was born in Korea in 1961. He graduated from Department of Aeronautical Engineering, Seoul National University in 1983. He received M.S. degree from the same department in 1985. He joined Department of Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University in 1987.

A handwritten signature in black ink, appearing to read 'Kwangju Lee', written over a horizontal line.

Kwangju Lee