TIME DEPENDENT REDUNDANCY OPTIMIZATION

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(ABSTRACT)

Three different time-dependent optimal reliability design models with a series-parallel system are formulated. An efficient heuristic procedure for each problem is elucidated, and the results are described. The redundancy optimization problem using the time-dependent reliability function is solved so that the system reliability exceeds the target reliability over a time period at a minimum cost. The number of the redundancies in the system is obtained to ensure that the target reliability can be matched with the system reliability, which is time-dependent, as closely as possible without violating a cost constraints. The system configuration is optimized so that the system reliability matches the target reliability as much as possible while maintaining the system reliability at certain level under a cost constraint. Various applications of simple and efficient heuristic algorithms for solving time-dependent redundancy optimization problems are also provided. These techniques are used and tested in systems containing a substantial number of subsystems. The heuristic approaches appear more successful than other optimization techniques in solving these problems, and can be applied to any constrained redundancy optimization problems without any tedious formulation and computation.

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I. INTRODUCTION

1.1 Reliability in our everyday life - The need for reliability

In today's modern society, we do many things with machinery, automatic equipment, robots, computers, and other devices. The failure of these equipment can often cause effects from inconvenience and customer dissatisfaction to a severe impact on our society. People's demand for highly reliable and more complex products is increasing continuously. Nevertheless there is a dilemma between complexity and reliability. As the system has grown more complex, the consequences of its unreliable behavior have become more severe in terms of cost, effort, lives, etc.. Therefore, the importance of obtaining highly reliable systems and components has been recognized in recent years.

1.2 Definition of reliability

The term "reliability" can be applied to various types of human activities as well as to the performance of physical systems or functional objects. Reliability is a measure of the ability of a system to operate without failure when put into service. Reliability has been defined in various ways. One of the best is provided by the Radio Electronics and Television Manufacturers Association (1955), now known as the Electronics Industries Association, as " the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered ".

1.3 Methods of improving reliability

Reliability can be improved in the following ways [1]:

1. Use large safety factors

2. Reduce the complexity of the system to the minimum essential for the required operation. (Unnecessarily complex configurations only increase the probability of system failure.)

3. Increase the reliability of components through a product improvement program

4. Practice a planned maintenance and repair schedule

5. Use component redundancy

Combinations of these methods are possible. For example, a high degree of reliability might be achieved by practicing a planned maintenance and repair schedule on the system and using higher reliability components.

All these methods should be included in a full reliability program; however, redundancy is the most common and effective method during the system planning stage. Therefore, redundancy optimization will be discussed next.

1.4 Redundancy optimization - Why optimization

Redundant configurations can be broadly classified into the following three types:

(1) System-level redundancy (parallel-series configuration) (shown in Figure 1.1)

(2) Component-level redundancy (series-parallel configuration) (shown in Figure 1.2)

(3) Redundancies at different levels (combinations of the above two) (shown in Figure 1.3)

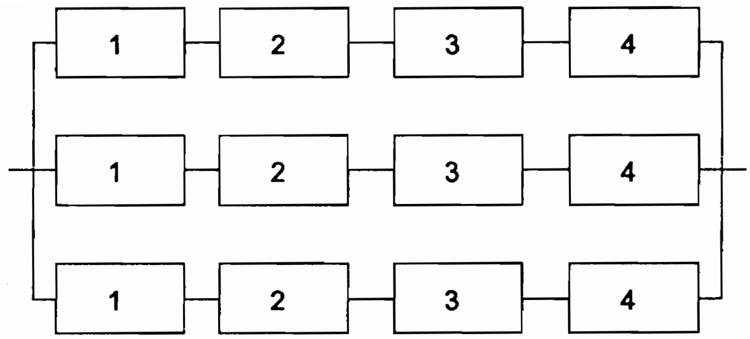

Fig 1.1 System-level redundancy

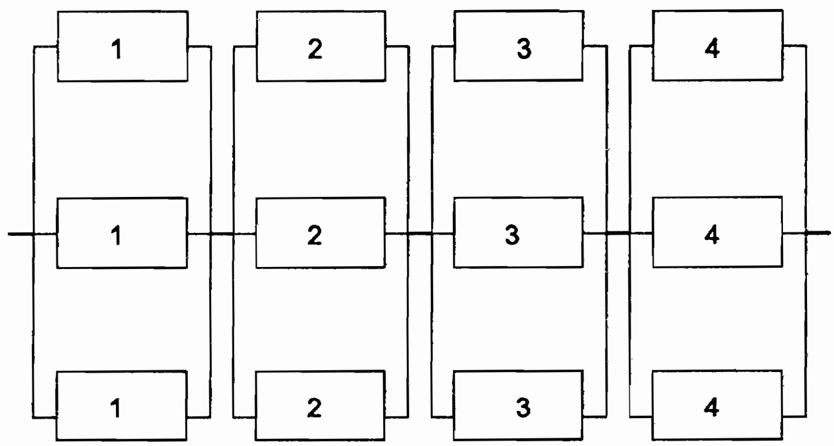

Fig 1.2 Component-level redundancy

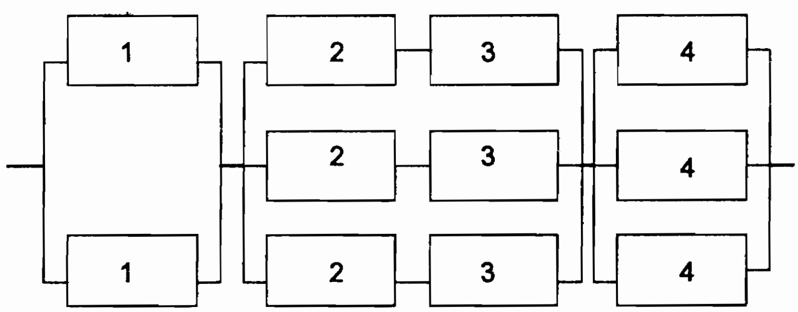

Fig 1.3 Redundancies at different levels

From an economic point of view, the system-level redundancy is usually expensive and wasteful as compared to the component-level redundancy, but it may be necessary because of safety. In many cases, an increase in redundancy at the detail part level, where cost, space or weight increases are minor, yields a greater reliability increase than redundancy at the system-level. However, the choice depends on the nature of the system, time, its cost, and its mission. When redundancy is required for improving system reliability, a number of design trade-offs must be examined to determine what kinds of redundancy are to be employed - "system-level", "component-level" redundancy, or combinations there of.

It is common to improve system performance by using component redundancy. At the beginning of a system design effort, the number of redundant components of the system is decided and built in by using various optimization techniques for the redundancy allocation and applying well known reliability theory for the reliability evaluation. Increasing redundancy components can improve system reliability, but there are always some resource constraints on the system such as cost, weight, and volume of the system that put a limitation on the amount of redundancy to be used. Therefore, it raises an optimization problem.

1.5 Time-dependent redundancy optimization - statement of the problem

Various papers have presented different approaches to obtaining redundancy optimization by assuming time-independent reliability. In contrast, very little research has been done on optimal system design with time-dependent reliability. Time is one of the most important elements in reliability definition because it represents a measure of the period during which a certain degree of system performance can be expected. Therefore, it is necessary that time must be considered as part of the reliability specification and measurement. Assuming time-independence, the definition of reliability would be relatively meaningless.

Therefore, the objective of this study is to extend the traditional redundancy optimization using time-dependent reliability. The purpose of this research is to methodically develop the procedures which will minimize the total system cost while maintaining the system reliability at certain level, and match the target reliability as much as possible without violating the resource constraints.

1.6 Thesis outline - Overview of remaining chapters

The following chapter provides an overview of previous research in the area of redundancy optimization problems. In chapter 3 the proposed mathematical models for time-dependent redundancy optimization problems are derived. The heuristic algorithms and results are presented in chapter 4. Finally, in chapter 5 conclusions are drawn and recommendations for further work are also included.

II. LITERATURE REVIEW

2.1 Introduction

In the past 30 years, many authors [2 - 21] have presented various optimization techniques to approach optimal reliability design. However, only a few optimization techniques have been proven effective when applied to large-scale problems. Most of the redundancy optimization involves finding the optimal number of redundant components subject to the resource constraints. Various optimization techniques have been employed to deal with the redundancy optimization with different system structures. The accuracy of the solutions is dependent on the specific technique used. Therefore, the previous work on redundancy optimization can be classified as follows:

(1) on the basis of assumed system structure

a. Series

b. Parallel

c. Series-parallel, Parallel-series, and combined model

d. K-out-of-n

c. General (bridge, non-series-non-parallel)

(2) on the basis of optimization techniques

a. Dynamic programming method

b. Heuristic algorithm

c. Integer programming method

d. Miscellaneous method

(3) on the basis of the accuracy of the solution

a. Approximate approach

b. Exact approach

c. Heuristic approach

2.2 Optimal system reliability models with redundancy

In this section, a brief review each of the models considered in the survey are discussed.

1. Series structure (1-out-of n : F) (e.g. Ghare and Taylor [15]) (shown in Fig 2.1):

In series systems, the function operation depends upon the proper operation of all system components. If one component fails, the system fails.

2. Parallel structure (1-out-of n : G) (e.g. Tillman [14]) (shown in Fig 2.2):

In parallel systems, there are n paths connecting the input to the output, and the system fails only when all components fail.

3. Series-parallel, parallel-series, and combined model (e.g. Aggarwal et al. [7] , Misra [19]) (shown in Fig 1.1 and Fig 1.2).

Fig 2.1 An n-stage Series system

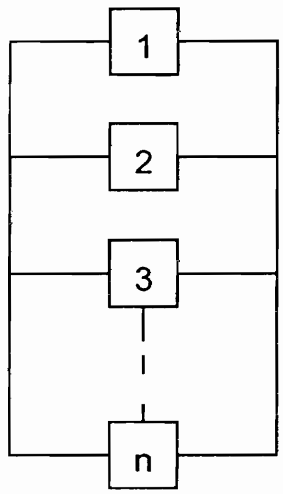

Fig 2.2 An n-stage Parallel system

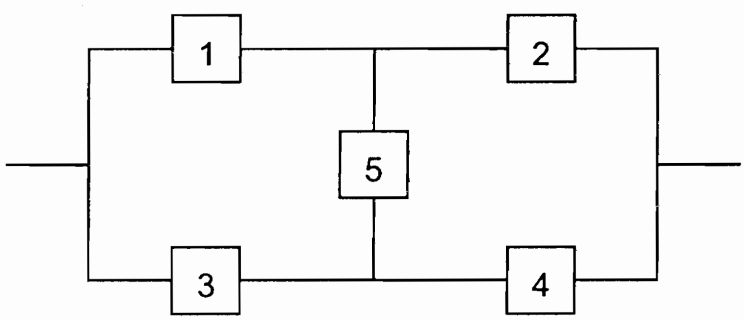

Fig 2.3 A bridge system

4. K-out-of -n (e.g. Bai et al [20]):

A k-out-of-n system consists of n identical independent components of which at least k ($1 \leq k \leq n$) of the components should succeed in order for the system to succeed. For k = 1, it becomes a parallel system, and for k = n, it becomes a series system.

5. General (bridge, non-series-non-parallel) (e.g. Shi [9] , Aggarwal [8]) (shown in Fig 2.3):

The reliability of general (complex) systems can be evaluated by using either conditional probability or other approaches.

2.3 Optimization techniques for obtaining optimal system configuration

a. Dynamic programming method [2,3,4]

Dynamic programming is the collection of mathematical tools used to analyze multi-stage decision processes and problems. In general applications of dynamic programming, several computational difficulties do arise, but, if successful, it will result in a less restricted optimal solution. A concise review of the dynamic programming approach to reliability optimization with redundancy is given by Kettle [2], who presented a direct dynamic programming approach to the one-constraint problem and Proschan & Bray [3] and Barlow and Proschan [4],who extended this problem to the two-constraint case.

b. Heuristic algorithm [5,6,7,8,9,10]

Generally, heuristic algorithms are quite efficient and simple compared to dynamic programming, integer programming or other methods; however, they do not guarantee a global optimal solution.

Tillman et al. [5] proposed a method which combined Hooke and Jeeves's pattern search [6] and the heuristic approach developed by Aggarwal et al. [7] . Aggarwal et al. [7] developed a heuristic algorithm by choosing the greatest ratio of " decrement in unreliability " to " product of decrements in slacks" to determine which component is added. Aggarwal [8] presented another algorithm for optimum allocation of redundancies in general systems. Shi [9] presented an algorithm for complex systems using the ratio of the minimal path set reliability to the percentage of consumed resources to select an added component. Nakagawa and Nakashima [10] dealt with an n-stage series system by repeatedly using a more reliable candidate at the stage that has the greatest value of the " weighted sensitivity function " without violating any of the constraints for solving the optimal redundancy allocation problem.

c. Integer programming method [11,12,13,14]

Although integer programming secures integer solutions, it does not guarantee that an optimal solution can be obtained in a reasonable time. A short review of integer programming approaches to reliability optimization with redundancy problem is as follows.

Misra [11] formulated the redundancy optimization problem as a zero-one integer programming and solved the problem using Lawler and Bell's [12] algorithm. Tillman [13], [14] used integer programming for minimizing cost or maximizing system reliability subject to multiple constraints.

d. Miscellaneous method [15,16,17,18,19]

Ghare & Taylor [15], and Mcleavey [16] applied branch-and-bound to approach the parallel redundancy. Nakagawa [17] also used branch-and-bound to solve reliability allocation problems with multiple nonlinear constraints. Li & Haimes [18] used a decomposition approach for optimization of a large system with a general network structure. Misra [19] provided two optimization techniques, Lagrange multipliers and Maximum principle, for optimizing the reliability of a series-parallel system subject to linear constraints.

2.4 Approximate, exact, heuristic approaches for reliability optimization with redundancy

In most optimization techniques used for the system reliability with redundancy, there is an inevitable battle of computational cost versus the quest for higher levels of optimality. This raises the question of measuring the computational efficiency.

1. Approximate approach [19]: (discrete maximum principle, linear programming, lagrange multipliers)

The advantage of the approximate techniques is a saving in computation time as compared with exact methods which are computationally tedious, time-consuming and costly. They treat integer variables as real numbers then round them off to the nearest integer to obtain a solution.

2. Exact approach [2,3,4,11-17]: (branch-and-bound, dynamic programming and integer programming)

The exact techniques, such as branch and bound, dynamic programming, and integer programming, require more complicated formulation and long computation time, but they provide an exact solution for reliability optimization problems.

3. Heuristic approach [5,6,7,8,9,10]

Heuristic approach is a simple and efficient intuitive procedure to generate solutions which are near optimal or suboptimal in a reasonably short time. Integer solution is another advantage of heuristic approach.

2.5 Summary

The time-independent redundancy optimization problems have been studied extensively. However, the reliability literature on redundancy optimization with time-dependence is relatively sparse. There appears to be only one paper specifically dealing with the redundancy optimization with time-dependent reliability. Nakashima and Yamato [21] formulated an optimal reliability design problem of a system with time-dependent reliability and then obtained the solution using dynamic programming. Although optimization techniques have been widely used in all kinds of redundancy optimization as indicated in the literature cited above, only a few of optimization techniques have been proven effective when applied to large-dimension problems. Moreover, most of the redundancy optimization problems are nonlinear integer programming problems. The problem is further compounded by the introduction of time factor; presenting a greater challenge to the study.

III. PROBLEM STATEMENT

3.1 Introduction

A brief review of the time-dependent reliability, problem statement, and mathematical formulation are presented in this chapter.

3.2 Review of time-dependent reliability

The time-dependent reliability can be obtained from a known hazard function, $\lambda(t)$,or life distribution function , $F(t)$, using

$$R(t) = 1 - F(t) = \exp\left[- \int_0^t \lambda(x)dx \right] \tag{3-1}$$

The nature of $\lambda(t)$ depends on the distribution of failure time. In this section, the effect of time-dependent reliabilities (based on exponential distribution) on the evaluation of system reliability for some of the commonly occurring system configurations is described.

3.2.1 Series configuration

For n components in series, as shown in Figure 2.1, let λ_i (t) be the hazard rate for the i-th component. Then

$$R_i(t) = \exp\left[- \int_0^t \lambda_i(x)dx \right] \tag{3-2}$$

and the system reliability is

$$R_s\ (t) = \prod_{i=1}^{n} \exp\left[- \int_0^t \lambda_i(x)dx \right] \tag{3-3}$$

If all the components have exponential failure distributions, then

$$R_i\ (t) = \exp\left(- \lambda_i t \right) \tag{3-4}$$

where λ_i is the constant failure rate for the i-th component. The series system reliability can be expressed as

$$R_s\ (t) = \prod_{i=1}^{n} \exp\left(-\lambda_i t \right) \tag{3-5}$$

or

$$R_s\ (t) = \exp\left[\left(-\sum_{i=1}^{n} \lambda_i \right) t \right) \tag{3-6}$$

3.2.2 Parallel configuration

For a parallel configuration of n components, as illustrated in Figure 2.2, the system reliability is given by

$$R_s\ (t) = 1 - \left[\prod_{i=1}^{n} \left\{ 1 - \exp\left(- \int_0^t \lambda_i(x)dx \right) \right\} \right] \tag{3-7}$$

Assuming all the components have constant hazards, let λ_i be the constant hazard rate for the i-th component. Then

$$R_s\ (t) = 1 - \prod_{i=1}^{n} \left[1 - \exp\left(-\lambda_i t \right) \right] \tag{3-8}$$

3.3 Problem statement and objectives

People always ask " Is this system reliable ? ". From an economic point of view and given today's industrial trends, " Is this system reliable enough ? " should be asked instead. Setting up the target reliability can solve the above question. The target reliability for a system is normally determined by the system designer and the manager. The selected target value is based on the customer need and mission. Figure 3.1 shows the relationship between system reliability and total system cost from producer's and customer's points of view. Finding an "economic balance" between producer cost and user cost can help people to determine to what level of reliability one should aspire and what is the optimum target reliability. If a system has to operate for long period under a no maintenance and no repair condition, the system must be designed to ensure a sufficient reliability throughout its mission. Therefore, it is evident that reliability and economics play a major integrated role in the decision-making process. When the component redundancy is used to increase the system reliability, several design trade-offs among the allocated reliability, cost, space, and weight must be examined until the system's target reliability is obtained.

The target reliability can be pre-determined according to the considerations stated above. However, the time factor, which is directly related to the durability of the products over the service period, has never been considered in the overall process. Therefore, the goals of the present research consist of (1) formulating a redundancy optimization problem using the time-dependent reliability function to ensure that the system reliability shall exceed the target reliability over a time period at a minimum cost, (2) finding the number of the redundancies in the system so that the target reliability can be matched with the system reliability, which is time-dependent, as closely as possible without violating a cost constraints, and (3) finding the optimal system configuration so that the system reliability matches the target reliability as much as possible while maintaining the system at certain level under a cost constraint. It should be pointed out that the target reliability in these cases is also time-dependent.

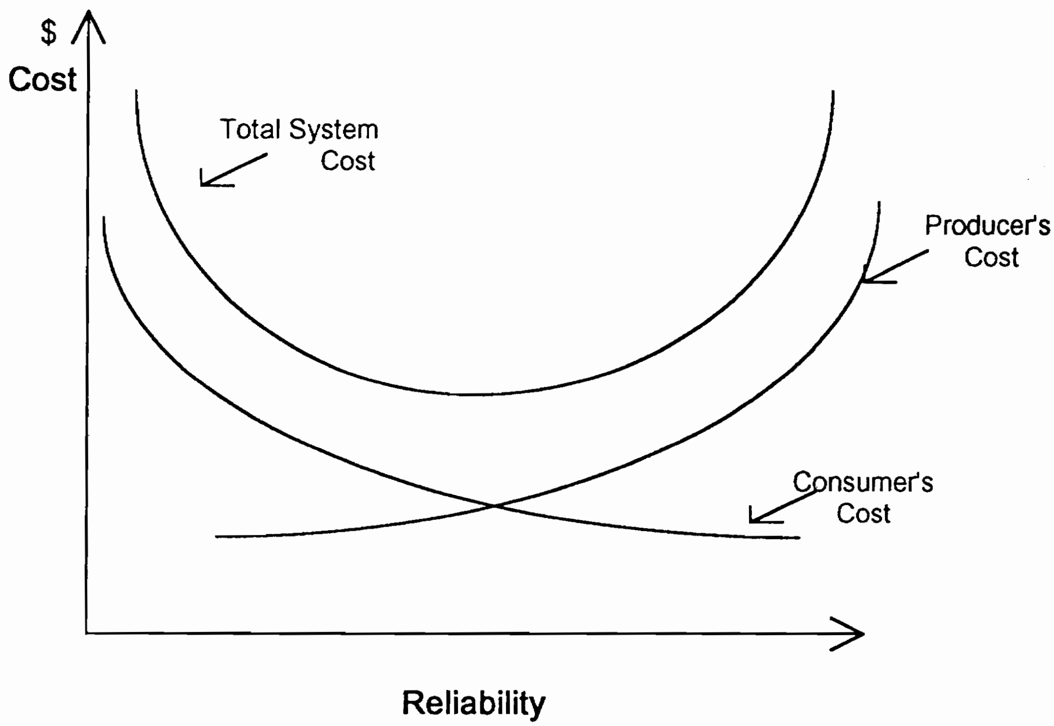

Fig 3.1 The relationship between system reliability and total system cost.

3.4 Mathematical formulation

In this section, the redundancy optimization problems which are stated above are discussed with more detail mathematically and graphically.

A system, shown in Figure 3.2, with a series-parallel structure is considered in this research. The system consists of n stages (serial) and each stage is composed of x_i components (parallel). For the system to operate, at least one component in each stage needs to function.

The system reliability, $R_s(t,x)$, over a given period of time is the product of the reliabilities of the stages (subsystems):

$$R_s(t,x) = \prod_{i=1}^{n} R_i(t,x_i) \tag{3-9}$$

Stage i has x_i identical components with a reliability P_i

Therefore,

$$R_i(t,x_i) = 1-(1-P_i)^{x_i} \tag{3-10}$$

In the time dependent case, each P_i will be given by

$$P_i = e^{-\int_0^t \lambda_i(u)du} \tag{3-11}$$

where $\lambda_i(t)$ is the hazard rate of component in the i-th subsystem and t is the mission time.

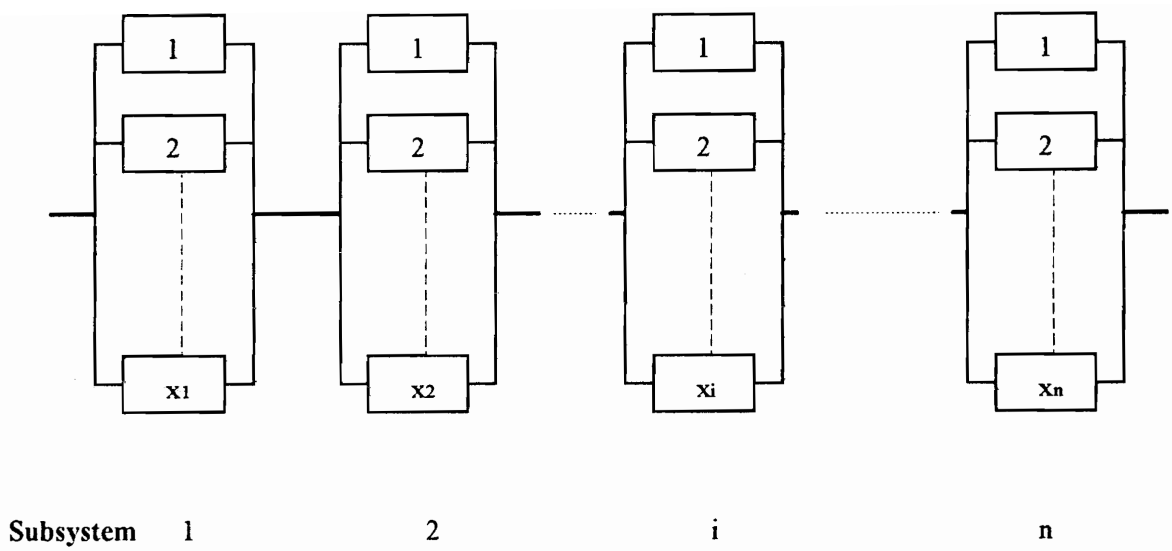

Fig 3.2 A Mixed Series-Parallel System

Problem 1: Finding an optimum redundant component allocation, $X = (x_1, x_2, \ldots, x_n)$, for a system which ensures that the system reliability shall exceed the target reliability over a time period at a minimum cost.

Mathematically,

Minimize Cost: $$C(c_i, x_i) = \sum_{i=1}^{n} c_i x_i$$

subject to the time-dependent system reliability constraint

$$R_s(t, x) \geq R_{Target}(t)$$
$$\forall t \leq T_f \tag{3-12}$$

$R_s(t, \underline{x})$:	system reliability;
$R_{Target}(t)$:	time-dependent target reliability;
$C(c_i, x_i)$:	total cost of the system;
C_i:	cost of each component in subsystem i;
T_f:	given specified time;
t:	mission time;
x_i:	number of components in stage (subsystem) i.

For illustration, the hypothetical target and system reliabilities are shown in Figure 3.3. The system reliability shall exceed the target reliability by selecting the optimum number of redundancies at a specified time period.

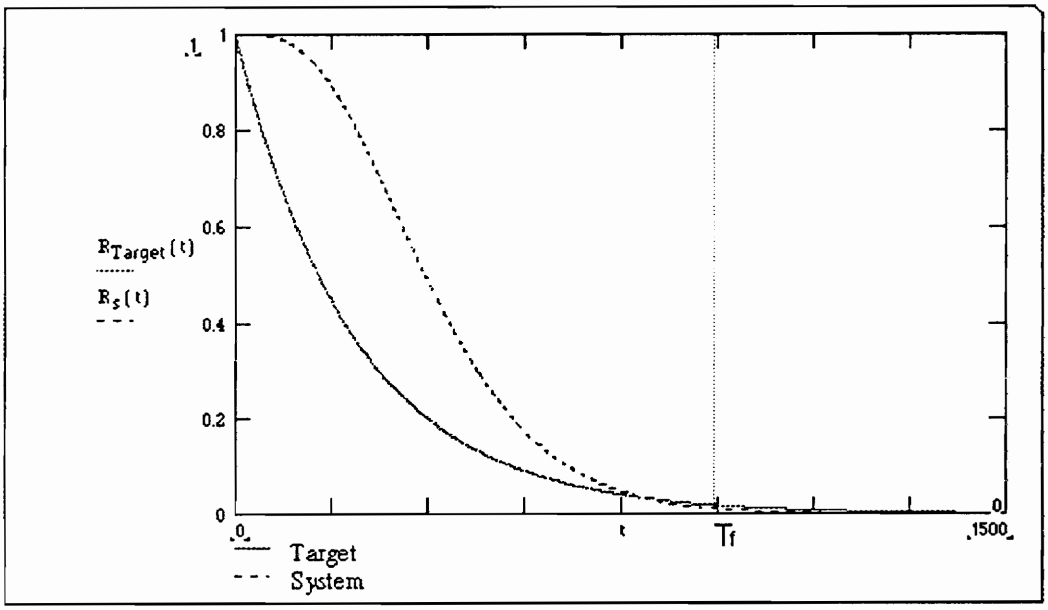

Fig 3.3 System reliability shall exceed target reliability at minimum cost

$$R_{Target}(t) := e^{(-0.004 \cdot t)}$$

$$R_s(t) := \left[1 - \left[1 - e^{(-0.002 \cdot t)}\right]^{\alpha}\right] \cdot \left[1 - \left[1 - e^{(-0.003 \cdot t)}\right]^{\beta}\right] \cdot \left[1 - \left[1 - e^{(-0.0035 \cdot t)}\right]^{\gamma}\right]$$

Problem 2: Finding the number of redundancies in the system so that the target reliability can be matched with the system reliability as closely as possible without violating a cost constraint.

Mathematically, the problem is equivalent to minimizing $G[R_s(t,\underline{x}), R_{Target}(t)]$

where

$$G[R_s(t,\underline{x}), R_{Target}(t)] \equiv \int_0^\infty [R_s(t,\underline{x}) - R_{Target}(t)]^2 \, dt$$

or

$$G[R_s(t,\underline{x}), R_{Target}(t)] \equiv \int_0^\infty \left| R_s(t,\underline{x}) - R_{Target}(t) \right| dt$$

subject to cost constraint

$$\sum_{i=1}^{n} c_i x_i \leq C \tag{3-13}$$

For example, the hypothetical target and system reliabilities are depicted in Figure 3.4. The objective is to find the system reliability which approaches the target reliability as much as possible by choosing the optimum number of redundancies. However, the system reliability is not necessarily greater than the target reliability over a period of time.

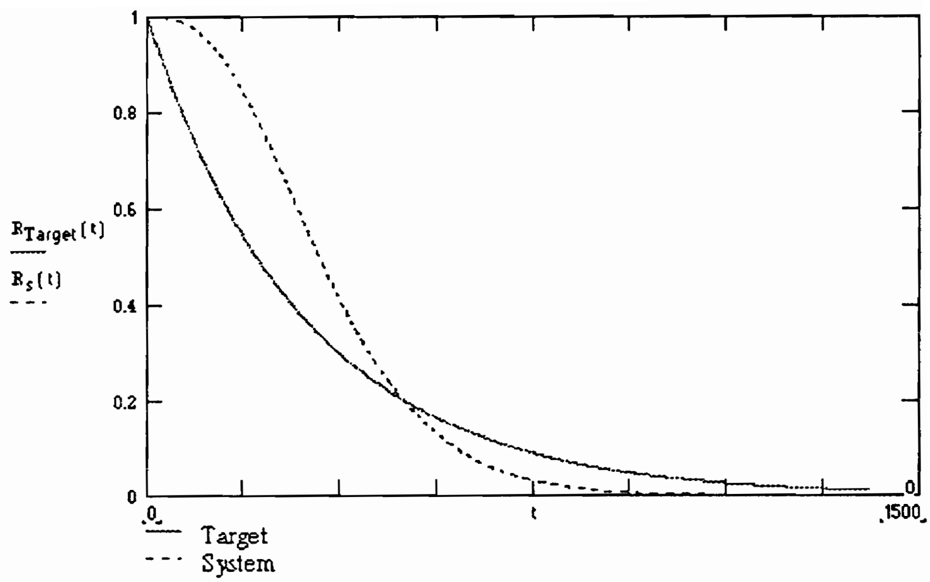

Fig 3.4 System reliability match the target reliability as closely as possible

$$R_{Target}(t) := e^{(-0.003 \cdot t)}$$

$$R_S(t) := \left[1-\left[1-e^{(-0.002 \cdot t)}\right]^{\alpha}\right] \cdot \left[1-\left[1-e^{(-0.003 \cdot t)}\right]^{\beta}\right] \cdot \left[1-\left[1-e^{(-0.0035 \cdot t)}\right]^{\gamma}\right]$$

Problem 3: Finding an optimum redundant component allocation, $X = (x_1, x_2, \ldots, x_n)$, for a system which matches the target as much as possible while maintaining the system at certain level under the cost constraint.

Mathematically, the problem is equivalent to minimizing $G[R_s(t,\underline{x}), R_{Target}(t)]$

where

$$G[R_s(t,\underline{x}), R_{Target}(t)] \equiv \int_0^\infty [R_s(t,\underline{x}) - R_{Target}(t)]^2 dt$$

or

$$G[R_s(t,\underline{x}), R_{Target}(t)] \equiv \int_0^\infty \left| R_s(t,\underline{x}) - R_{Target}(t) \right| dt$$

subject to cost and reliability constraints

$$\sum_{i=1}^{n} c_i x_i \le C$$

$$R_s(t,x) \ge R_{Target}(t) \tag{3-14}$$

$$\forall t \le T_f$$

The objective is to find the optimal system configuration so that the system reliability matches the target reliability as much as possible. Further, the system reliability should exceed the target reliability over a time period under a cost constraint.

IV. SOLUTION PROCEDURES AND NUMERICAL EXAMPLES

4.1 Introduction

In this chapter, solution procedures for three different redundancy optimization problems are presented and numerical results for several examples are obtained by heuristic approaches.

4.2 Notations & Assumptions

4.2.1 Notations

n	number of series subsystems
i	subsystem number, i=1,2,.....,n
$R_s(t,\underline{x})$	reliability of system at time t
$R_i(t)$	reliability of subsystem i at time t
$R_{Target}(t)$	target reliability at time t
$\underline{X}$	$(x_1, x_{2,}\cdots\cdots\cdots, x_n)$
x_i	number of parallel units in subsystem i
T_f	given specific time
C	available cost limitation
C_i	cost of one element at subsystem i
λ_i	component failure rate at subsystem i
t	mission time
T	system lifetime

4.2.2 Model assumptions

This research concentrates on to a series-parallel system with statistically independent components and the following assumptions are made in this section for the sake of simplicity of the model.

(1). The system contains n subsystems in series (1-out-of-n: F) with x_i i.i.d. units in parallel ($x_i \geq 1$).

(2). Each subsystem is 1-out-of-x_i: G, i.e., a subsystem is good until all x_i components fail.

(3) All the units as well as all the stages are s-independent.

(4). All components in parallel in the same stage have the same life distribution.

(5). All the components have exponential lifetimes with constant failure rates λ_i.

(6). No repair and maintenance.

(7). Units, subsystem, and the system are either good or failed

(8).Total system cost is a linear combination of individual costs by the elements in each stage.

4.3 Solution procedure and numerical examples for problem 1

[Problem 1]: Find a minimum cost redundant component allocation, $\underline{X} = (x_1, x_2, \ldots, x_n)$, for a system which ensures that the system reliability shall exceed the target reliability over a time period.

[Mathematical model]:

Minimize Cost: $C(c_i, x_i) = \sum_{i=1}^{n} c_i x_i$ (4-1a)

subject to the time-dependent system reliability constraint

$$R_s(t,x) = \prod_{i=1}^{n} R_i(t,x_i) = \prod_{i=1}^{n} [1-(1-e^{-\lambda_i * t})^{x_i}]$$

$$= [1-(1-e^{-\lambda_1 t})^{x_1}] * [1-(1-e^{-\lambda_2 t})^{x_2}] \cdots [1-(1-e^{-\lambda_n t})^{x_n}]$$

$$\geq R_{Target}(t)$$

$$\forall t \leq T_f$$

x_i : integer i=1,2,.......,n (4-1b)

[Solution procedure]:

This heuristic approach is based on the concept that redundant components are added one at a time which derives from the incremental reliability per pound (IREPP) method by Barlow & Proschan [4]. The new policy is generated from the previous policy by adding a single component to the subsystem that gives the best increment in reliability per unit cost.

The computational procedure is presented in the following steps:

Step 1. Assign $x_i = 1$ for i = 1,2,.....,n. Because this is a series system, there must be at least one component in each stage and the system should not violate any resource constraints.

Step 2. Compute the increment in the reliability per unit cost for one added element (redundant)

$$\int_{\infty}^{0} \left[R(x_1, x_2, \ldots, x_i + 1, \ldots, x_n, t) - R(x_1, x_2, \ldots, x_i, \ldots, x_n, t)\right] \Big/ C_i \; dt$$ [1]

Step 3. Find

$$i^* \equiv \underset{i}{Max}\{\int_{\infty}^{0} \left[R(x_1, x_2, \ldots, x_i + 1, \ldots, x_n, t) - R(x_1, x_2, \ldots, x_i, \ldots, x_n, t)\right] \Big/ C_i \; dt\}$$

Let $x_{i^*} = x_{i^*} + 1$

Step 4. Check the reliability constraint

$$R_s(t, \underline{x}) \geq R_{Target}(t)$$

If yes, go to step 5.

If no, repeat step 2 and step 3.

Step 5. Remove the last redundant component added in step 3, and check all feasible solutions in the last application of step 2. Find i^* such that the system reliability exceeds the target reliability over a time period at a minimum cost, i.e.,

---

[1]This integral is approximated by Simpson's rule.

$$\underset{i}{Minimize} \quad C_i \text{ and}$$

$$R_s(t,\underline{x}) \geq R_{Target}(t,\underline{x}), \forall t \leq T_f$$

Let $x_{i^*} = x_{i^*} + 1$

$\Rightarrow$ $\underline{x} = (x_1, x_2, \ldots, x_n)$ is an optimal solution.

[Numerical examples]

Example 4.3-1 An eight-stage series system is considered to illustrate the application of solution procedure to time-dependent redundancy optimization problems. The component reliability and cost data are given in Table 4-1.

Table 4-1 Data associated with example 4.3-1

Stage i	1	2	3	4	5	6	7	8
Component reliability $R_i(t), e^{-\lambda_i t}, \lambda_i =$	.0010	.0011	.0012	.0013	.0014	.0015	.0016	.0017
Cost , C_i	1.5	1	2	1.5	1	1.5	1	1

$R_{Target}(t) = e^{-0.008t}$ $\quad T_f = 600 \quad t = 0,\ldots,900$ [2]

Then, the problem becomes

Minimize cost: $1.5x_1 + x_2 + 2x_3 + 1.5x_4 + x_5 + 1.5x_6 + x_7 + x_8$ (4-2a)

[2]Both the system and the target reliabilities approach zero at t=900.

subject to the time-dependent system reliability constraint

$$R_s(t,\underline{X}) = \prod_{i=1}^{8} R_i(t,x_i) = \prod_{i=1}^{8}[1-(1-e^{-\lambda_i t})^{x_i}]$$

$$=[1-(1-e^{-\lambda_1 t})^{x_1}]*[1-(1-e^{-\lambda_2 t})^{x_2}]\cdots[1-(1-e^{-\lambda_n t})^{x_n}]$$

$$\geq R_{Target}(t) = e^{-0.008t}$$

$$\forall t \leq T_f = 600$$

$$t = 0,..,900$$

(4-2b)

Starting with $\underline{X}$=(1,1,1,1,1,1,1,1), one redundant component is added to a stage at a time as shown in Table 4-2. After completing the computational procedures, the optimal system configuration is $\underline{x} = (x_1, x_2, \ldots, x_8)$= (1,2,1,1,2,1,2,2) with the optimal system cost of 14.5.

Table 4-2 Results of Example 4.3-1

Number of components in stage								Objective
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Min $\sum_{i=1}^{n} c_i x_i$
1	1	1	1	1	1	1	1*	10.5
1	1	1	1	1	1	1*	2	11.5
1	1	1	1	1*	1	2	2	12.5
1	1*	1	1	2	1	2	2	13.5
1	2	1	1	2	1	2	2	14.5

Example 4.3-2 This example is a system composed of 20 subsystems operating in series. The component reliability and cost data are shown in Table 4-3.

Table 4-3 Data associated with example 4.3-2

Stage i	1	2	3	4	5	6	7	8	9	10
λ_i	0.0001	.00011	.00012	.00013	.00014	.00015	.00016	.00017	.00018	.00019
C_i	3	4	3	3	4	2	2.5	1.5	1	2
Stage i	11	12	13	14	15	16	17	18	19	20
λ_i	.00020	.00021	.00022	.00023	.00024	.00025	.00026	.00027	.00028	.00029
C_i	2.5	2	1	1.5	1.5	0.5	1.5	1	0.5	1

$R_{Target}(t) = e^{-0.003t}$ $\quad T_f = 1000 \quad t = 0,..,1500$ [3]

The initial configuration of each stage is $\underline{X}$= (1,1,......,1). Following the steps of the computational procedures, the optimum configuration is $\underline{x} = (x_1, x_2, \ldots, x_{20})$ = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,1,2,3,2). The optimum system cost is 42.5. The results are presented and are shown in Table 4-4.

[3]Both the system and the target reliabilities approach zero at t=1500.

Table 4-4 Results of Example 4.3-2

Number of components in stage																				Objective
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	Minimize $\sum_{i=1}^{n} c_i x_i$
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1*	1	39
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1*	1	1	2	1	39.5
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	2	1*	40
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1*	2	2	41
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2	2*	2	42
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2	3	2	42.5

*This indicates that a redundant component is to be added to the stage.

4.4 Solution procedure and numerical examples for problem 2

[Problem 2]: Find the number of the redundancies in the system so that the system reliability matches the target reliability as closely as possible without violating a cost constraint.

[Mathematical model]:

Minimizing $G[R_s(t,\underline{X}), R_{Target}(t)] \equiv \int_0^{\infty}[R_s(t,\underline{X}) - R_{Target}(t)]^2\,dt$ (4-3a)

subject to the cost constraint

$$\sum_{i=1}^{n} c_i x_i \le C \tag{4-3b}$$

where C is a fixed constant.

[Solution procedure]:

The objective is to minimize $\int_0^{\infty}[R_s(t,\underline{X}) - R_{Target}(t)]^2\,dt$ in successive steps. The procedure at each step is to add one redundant component to the stage with the smallest value in equation 4-3a, and to stop when certain criteria as shown in step 6 are met or the cost constraint is violated. This solution procedure utilizes the same concept as the IREPP algorithm.

The computational procedure for solving problem 2 is stated as follows:

Step 1. Initialize $x_i = 1$ for all i = 1,2,...,n

And set $\int a(t)dt = \int [R_s(\underline{x},t) - R_{Target}(t)]^2\, dt = L$

Step 2. Add one component each at a time, and compute

$$\int a_i(t)dt = \int [R_s(x_1, x_2, .., x_i + 1, .., x_n, t) - R_{Target}(t)]^2\, dt \quad [4]$$

Step 3. Find $i^* \equiv \underset{i}{Min} \ \{ \int a_i(t)\ dt \}$

If $L \geq \int a^*_{new}(t)dt$, go to step 4.

Otherwise, the current value of $\underline{X}$ is an optimal solution.

Step 4. Let $x^*_i = x^*_i + 1$

$$L \equiv \int a^*_{new}(t)dt = \text{(New lower bound)}$$

Step 5. Check to see if the constraint, $\sum_{i=1}^{n} c_i x_i \leq C$, is violated.

(I). If the solution is still feasible, go to step 6.

(II). If the cost constraint is violated, cancel the proposed addition of redundant component, remove that stage from further consideration and repeat step 2.

(III). If the cost constraint is exactly satisfied, the current value of $\underline{x}$ is an optimal solution.

[4]This integral is approximated by Simpson's rule.

Step 6. (Stop criterion)

If (1) $\Delta L = L_{old} - L_{new} \leq \varepsilon$ (given)

or (2) redundant component is continuously added to the same subsystem twice, then stop.

Otherwise, go to step 2.

[Numerical examples]:

Example 4.4-1 Consider the system composed of 8 stages operating in series. The problem is to minimize the objective function, $\int_0^\infty [R_s(t, \underline{X}) - R_{Target}(t)]^2 dt$, under cost constraint. The data associated with this example are in Table 4-5:

Table 4-5 Parameter data for example 4.4-1

Stage i	1	2	3	4	5	6	7	8
λ_i	0.001	0.0011	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017
C_i	1.5	1	2	1.5	1	1.5	1	1

Target reliability $\equiv e^{-0.008t}$ t $\equiv 0,....,900$ $C \equiv 15$

Following the steps of the algorithm, the results presented in Table 6 are obtained. The optimal allocation with respect to the cost constraint is $\underline{x} = (x_1, x_2, \ldots, x_8) = (1,1,1,1,1,1,2,4)$ with the system cost of 14.5.

Table 4-6 Results of Example 4.4-1

Number of components in stage								Objective	Constraint
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	$\int [R_s(t,\underline{X}) - R_{Target}(t)]^2 dt$	$\sum_{i=1}^{n} c_i x_i \le C$
1	1	1	1	1	1	1	1*	2.413	10.5
1	1	1	1	1	1	1*	2	0.806	11.5
1	1	1	1	1	1	2	2*	0.063	12.5
1	1	1	1	1	1	2	3*	0.028	13.5
1	1	1	1	1	1	2	4	0.025	14.5

*This is the subsystem to which a redundant component is to be added.

Example 4.4-2 Consider the system composed of 20 subsystems operating in series and subject to a linear cost constraint with the following data:

Table 4-7 Data associated with Example 4.4-2

Stage i	1	2	3	4	5	6	7	8	9	10
λ_i	0.0001	.00011	.00012	.00013	.00014	.00015	.00016	.00017	.00018	.00019
C_i	3	4	3	3	4	2	2.5	1.5	1	2
Stage i	11	12	13	14	15	16	17	18	19	20
λ_i	.00020	.00021	.00022	.00023	.00024	.00025	.00026	.00027	.00028	.00029
C_i	2.5	2	1	1.5	1.5	0.5	1.5	1	0.5	1

$R_{Target}(t) = e^{-0.0035t}$ $\quad t = 0,\ldots,1500 \quad C = 50$

The steps of solution for this system are summarized in Table 4-8.

Table 4-8 Results of Example 4.4-2

Number of components in stage																				Objective	Constraint
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	$\int [R_s(t,\underline{X}) - R_{target}(t)]^2 dt$	$\sum_{i=1}^{n} c_i x_i \le C$
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1*	0.7911	39
1*	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	0.1090	40
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2*	0.0131	43
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3*	0.0022	44
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	0.0013	45

*This is the subsystem to which a redundant component is to be added.

4.5 Solution procedure and numerical examples for problem 3

[Problem 3]: Find an optimum redundant component allocation for a system which matches the target as much as possible while maintaining the system reliability at certain level under the cost constraint.

[Mathematical model]:

Minimize $G[R_s(t,\underline{X}), R_{Target}(t)] \equiv \int_0^{\infty}[R_s(t,\underline{X}) - R_{Target}(t)]^2\,dt$ (4-4a)

subject to cost and reliability constraints

$$\sum_{i=1}^{n} c_i x_i \le C$$

$$R_s(t,\underline{X}) = \prod_{i=1}^{n} R_i(t,x_i) = \prod_{i=1}^{n}[1-(1-e^{-\lambda_i * t})^{x_i}]$$

$$= [1-(1-e^{-\lambda_1 t})^{x_1}]*[1-(1-e^{-\lambda_2 t})^{x_2}]\cdots[1-(1-e^{-\lambda_n t})^{x_n}]$$

$$\ge R_{Target}(t)$$

$$\forall t \le T_f$$

x_i : integer i=1,2,.......,n (4-4b)

[Solution procedure]:

A two-phase algorithm is developed for solving this nonlinear integer programming with cost and reliability constraints. The phase I serves as a search procedure for initial feasible solution set which satisfies the system reliability's requirement under the cost constraint. The phase I procedure is based on the concept that a component is added to the stage where its addition produces the greatest ratio of "increment increases in reliability" to the "increment increases in resource (cost) usage". Following the phase I procedure , a lower bound for the objective value is generated, and then continue the phase II procedure to search for better solutions.

The computational procedure is presented in the following steps:

Phase I: Initial solution (Search for initial feasible solution)

Step 1. Assign $x_i = 1$ for all i = 1,2,...,n

Step 2. Compute the increment in the reliability per unit cost for one added element (redundant)

$$\int \left[R(x_1, x_2, \ldots, x_i + 1, \ldots, x_n, t) - R(x_1, x_2, \ldots, x_i, \ldots, x_n, t)\right] \Big/ C_i \quad dt$$

Step 3. Find

$$i^* \equiv \underset{i}{Max}\{\int \left[R(x_1, x_2, \ldots, x_i + 1, \ldots, x_n, t) - R(x_1, x_2, \ldots, x_i, \ldots, x_n, t)\right] \Big/ C_i \; dt\}$$

Let $x_{i^*} = x_{i^*} + 1$

Step 4. Check the cost constraint

$$\sum_{i=1}^{n} c_i x_i \le C$$

If the solution is feasible, go to step 5.

Otherwise, cancel the proposed addition of redundant component, remove that stage form further consideration and repeat step 2.

When all the stages are excluded from further consideration, stop, indicating no feasible solution.

Step 5. Check the system reliability constraint

$$R_s(t,\underline{X}) \geq R_{Target}(t) \quad \forall\, t \leq T_f$$

If yes, go to step 6.

Otherwise, go to step 2.

Step 6. Remove the proposed redundant component added in step 3. Check all feasible solutions in the last application of step 2. To find i^* from all feasible solutions in last step 2,

Minimize $\int_0^\infty [R_s(t,\underline{X}) - R_{Target}(t)]^2 dt$

Let $x_i^* = x_i^* + 1$

And set $\int a(t)dt = \int [R_s(\underline{x},t) - R_{Target}(t)]^2 dt = L$

Phase II. Search for a better solution (Apply the same procedure as for problem 2)

Step 1. Add one component each at a time. And compute

$$\int a_i(t)dt = \int [R_s(x_1, x_2, .., x_i + 1, .., x_n, t) - R_{Target}(t)]^2 \, dt$$

Step 2. Find $i^* \equiv \underset{i}{Min} \; \{ \int a_i(t)dt \}$

If $L \geq \int a^*_{new}(t)dt$, go to step 3.

Otherwise, the current value of $\underline{X}$ is an optimal solution.

Step 3. Let $x^*_i = x^*_i + 1$

$L \equiv \int a^*_{new}(t)dt$ = (New lower bound)

Step 4. Check to see if the cost constraint, $\sum_{i=1}^{n} c_i x_i \leq C$, is violated.

(I). If the solution is still feasible, go to step 5.

(II). If the cost constraint is violated, cancel the proposed addition of redundant component, remove that stage from further consideration and repeat step 1.

(III). If the cost constraint is exactly satisfied, the current value of $\bar{x}$ is an optimal solution.

Step 5. (Stop criterion)

If (1) $\Delta L = L_{old} - L_{new} \leq \varepsilon$ (given)

or (2) redundant component is continuously added to the same subsystem twice, then stop.

Otherwise, go to step 1.

[Numerical examples]:

The following examples are presented to show the capabilities of the solution procedure.

Example 4.5-1 Consider the system composed of 8 subsystems operating in series and subject to cost and reliability constraints with the following data:

Table 4-9 Parameter data for example 4.5-1

Stage i	1	2	3	4	5	6	7	8
λ_i	0.001	0.0011	0.0012	0.0013	0.0014	0.0015	0.0016	0.0017
C_i	1.5	1	2	1.5	1	1.5	1	1

$R_{Target}(t) = e^{-0.008t}$ $\quad C = 15$

$t = 0, \ldots\ldots, 900$ $\quad T_f = 600$

Then, the problem is

$$\text{Minimize } \int_0^{\infty} [R_s(t, \underline{X}) - R_{Target}(t)]^2 dt \equiv \int_0^{\infty} [R_s(t, \underline{X}) - e^{-0.008t}]^2 dt \tag{4-5a}$$

subject to cost and reliability constraints

$$1.5x_1 + x_2 + 2x_3 + 1.5x_4 + x_5 + 1.5x_6 + x_7 + x_8 \le C = 15$$

$$R_s(t,\underline{x}) = \prod_{i=1}^{8} R_i(t,x_i) = \prod_{i=1}^{8} [1-(1-e^{-\lambda_i t})^{x_i}]$$

$$= [1-(1-e^{-\lambda_1 t})^{x_1}]*[1-(1-e^{-\lambda_2 t})^{x_3}]\ldots[1-(1-e^{-\lambda_8 t})^{x_8}]$$

$$\geq R_{Target}(t) = e^{-0.008t}$$

$$\forall t \leq T_f = 600$$

$$t = 0,\ldots,900$$

(4-5b)

The steps of solution for this system are summarized in Table 4-10

Table 4-10 Results of Example 4.5-1

Number of components in stage								Objective	Constraint
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	$\int_0^{T_f}[R_s(t,\underline{X}) - R_{Target}(t)]^2 dt$	$\sum_{i=1}^{n} c_i x_i \leq C$
1	1	1	1	1	1	1	1*	#2.4133	10.5
1	1	1	1	1	1	1*	2	#0.8064	11.5
1	1	1	1	1*	1	2	2	#0.0633	12.5
1	1*	1	1	2	1	2	2	#0.5368	13.5
1	2	1	1	2	1	2	2	2.2217	14.5
2	1	1	1	2	1	2	2	2.0322	15

*This is the subsystem to which a redundant component is to be added.

#This indicates that the reliability constraint is still violated.

Example 4.5-2 Consider an example of a system with 20 stages for which the component reliability, target reliability and cost data are as follows:

Table 4-11 Data associated with example 4.5-2

Stage i	1	2	3	4	5	6	7	8	9	10
λ_i	0.0001	.00011	.00012	.00013	.00014	.00015	.00016	.00017	.00018	.00019
C_i	3	4	3	3	4	2	2.5	1.5	1	2
Stage i	11	12	13	14	15	16	17	18	19	20
λ_i	.00020	.00021	.00022	.00023	.00024	.00025	.00026	.00027	.00028	.00029
C_i	2.5	2	1	1.5	1.5	0.5	1.5	1	0.5	1

$R_{Target}(t) = e^{-0.003t}$ $\quad T_f = 1000 \quad t = 0,..,1500 \quad \mathrm{C}=50$

Following the steps of the computational procedures, the optimum configuration is

$\underline{x} = (x_1, x_2, \ldots, x_{20})$ = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,1,2,3,2). The optimal objective value is 0.1045.

Table 4.-12 Results of Example 4.5-2

Number of components in stage																				Objective	Constraint
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	$\int [R_s(t,\underline{X}) - R_{Target}(t)]^2 dt$	$\sum_{i=1}^{n} c_i x_i \le C$
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1*	1	#5.0044	39
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1*	1	1	2	1	#2.8864	39.5
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	2	1*	#1.3654	40
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1*	2	2	#0.2625	41
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2	2*	2	#0.0739	42
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2	3	2	0.1045	42.5

*This indicates that a redundant component is to be added to the stage.

#This indicates that the reliability constraint is still violated.

4.6 Comparison between time dependent and time independent cases

4.6.1 Time dependent cases

This section provides a comparison of problems 1, 2, and 3 for an 8-stage system. Each example has the same cost and reliability data, and the target reliability. As shown in Figure 4.1 and Table 4-13, the optimal system configuration of example 4.3-1 for problem 1 gives the highest system reliability with the lowest cost. However, it does not match the target reliability well. The objective of problem 2 is to find the optimal number of redundancies in the system so that the target reliability can be matched with the system reliability as closely as possible without violating the cost constraint. Therefore, the optimal system reliability of example 4.4-1 for problem 2 approaches the target reliability very well under the cost constraint, but the optimal redundant component allocation of example 4.4-1 gives the lowest system reliability. One can also see from the Table 4-13 and the Figure 4.1 that the optimal allocation of example 4.5-1 for problem 3 has the intermediate performance.

Table 4-13 Optimal solutions of examples 4.3-1,4.4-1,4.5-1

Example	Optimal system configuration								$\int_0 [R_s(t,\underline{X}) - R_{Target}(t)]^2 dt$	$\sum_{i=1}^{n} c_i x_i$
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8		
4.3-1	1	2	1	1	2	1	2	2	2.222	14.5
4.4-1	1	1	1	1	1	1	2	4	0.025	14.5
4.5-1	2	1	1	1	2	1	2	2	2.0322	15

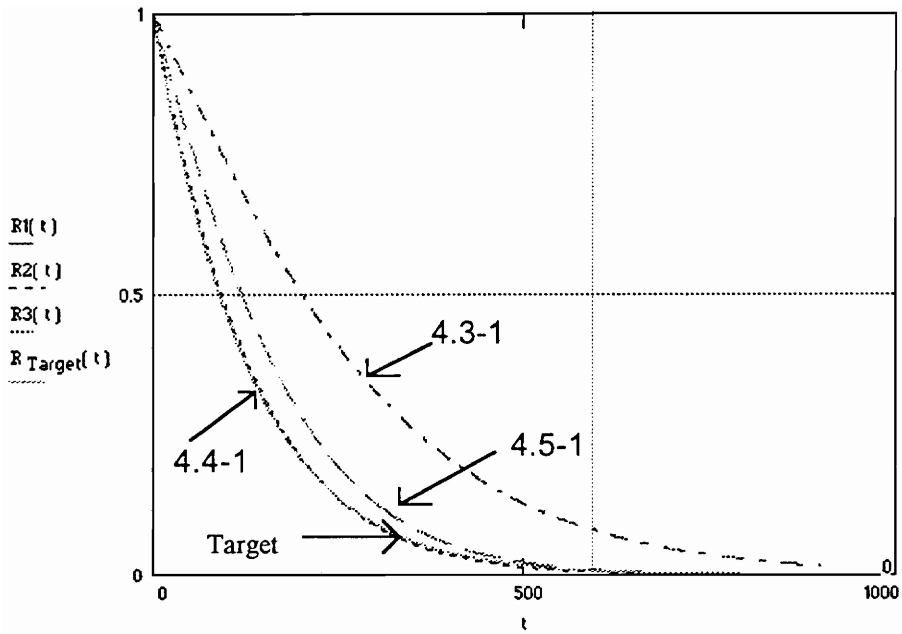

Fig 4.1 System reliabilities of examples 4.3-1,4.4-1,and 4.5-1

and target reliability

4.6.2 Time dependent vs. Time independent

In this section, a comparison of problem 1, which is using time dependent reliability, with the time independent case is provided. For comparison and computational purposes, the example of the time independent case has the same cost and reliability data, and the same target reliability as those in example 4.3-1 for problem 1. The time dependent data in the example 4.3-1 are computed at one particular point in time, say t = 10, and the system parameters are given in Table 4-14.

Table 4-14 Data associated with time independent case

Stage i	1	2	3	4	5	6	7	8
Component reliability $R_i(t=10), e^{-\lambda_i t} =$	.9900	.9891	.9881	.9871	.9861	.9851	.9841	.9831
Cost, C_i	1.5	1	2	1.5	1	1.5	1	1

$$R_{Target}(t=10) = e^{-0.008t} = 0.9231$$

A detailed computation procedure for time independent case leading to an optimal allocation of (1,1,1,1,1,1,2,2) by the IREPP algorithm is outlined in Table 4-15. As is clear from this table, in three steps, the reliability constraint is met with the optimal system cost 12.5 . A comparison of Tables 4-2 and 4-15 shows that the optimal redundancy allocation by the time independent approach , $\underline{x} = (x_1, x_2, \ldots, x_8) = (1,1,1,1,1,1,2,2)$, is not equal to the optimal allocation of (1,2,1,1,2,1,2,2) using the time dependent approach.

Table 4-15 Results of time independent case by IREPP method

Number of components in stage								System reliability	Objective
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	$R_s = \prod_i R_i$	$\text{Min} \sum_{i=1}^{n} c_i x_i$
1	1	1	1	1	1	1	1*	#0.8976	10.5
1	1	1	1	1	1	1*	2	#0.9128	11.5
1	1	1	1	1	1	2	2	0.9273	12.5

*This indicates that a redundant component is to be added to the stage.

#This indicates that the reliability constraint is still violated.

As shown in Figure 4.2, the optimal redundancy allocation according to the time independent approach can meet the system requirement at the beginning of the stage. However, if the system has to be operated for a long period of time under no maintenance and no repair conditions , the optimal system configuration obtained by the time independent approach dose not always meet the mission reliability criterion. When $t \geq 110$, the system reliability of time independent approach is less than the target reliability. In contrast, the time dependent approach provides more comprehensive solutions for the system designer during the redundancy optimization process.

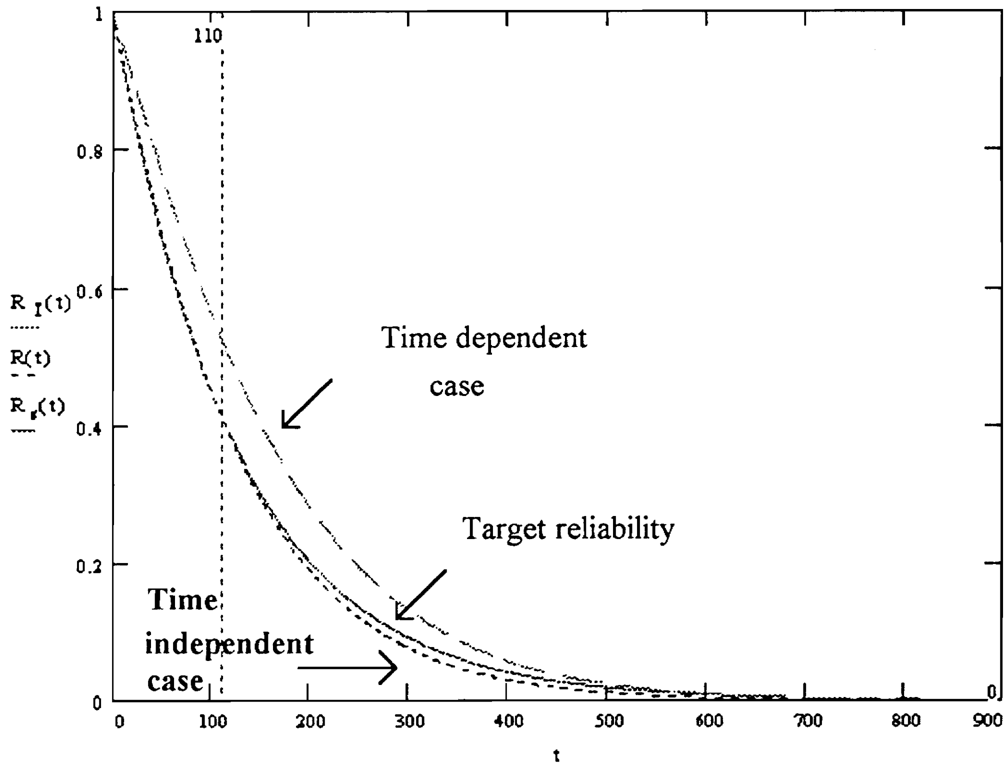

Fig 4.2 Time dependent vs. Time independent

V. SUMMARY, CONCLUSIONS, & RECOMMENDATIONS

5.1 Introduction

The previous chapter was devoted to the development of solution procedures for three different time-dependent redundancy optimization problems using heuristic approaches. This chapter serves to highlight the important efforts undertaken in this research to methodically develop the procedures for solving redundancy optimization problems. A summary and conclusions are presented according to the results obtained. Certain recommendations and suggestions for further research are also included in this chapter.

5.2 Summary and Conclusions

In this research, some optimal reliability design problems of a series-parallel system with time-dependent reliability are formulated, and an efficient procedure for each problem is elucidated. Various applications of simple and efficient heuristic algorithms for solving time-dependent redundancy optimization problems are provided. As evidenced by the examples, these techniques have been used and tested in systems containing a substantial number of subsystems. The heuristic approaches appear to be more successful than other optimization techniques in solving these problems. The "IREPP" algorithm, derived from Barlow and Proschan [4] in 1965, is widely adopted by many heuristic approaches to efficiently solve redundancy optimization problems. This research further extends the concept of IREPP algorithm to solve time-dependent redundancy optimization problems.

In resource allocation processes, it is often beneficial to develop a solution that yields an optimal value of the measure of desirability of the system. However, a problem may arise when one attempts to evaluate reliability of the redundant system where failure rates vary with time. The Simpson's rule is used to solve these difficult computational problems. As a result, there is some inevitable small error because of the computer limitation on numerical accuracy. A major asset of the heuristic algorithms presented in this study is that they can be applied to any constrained redundancy optimization problems without any tedious formulation and computation.

Several time-dependent redundancy optimization models are presented in this study. The optimization techniques employed for solving these problems are briefly outlined. The objective of the problem 1 is to minimize the system cost subject to a nonlinear time-dependent reliability constraint. A fast, practical and simple heuristic algorithm which is based on the concept of increment reliability per unit cost is developed. From the results of the examples, this heuristic approach seems to be very efficient in solving these problems. In problem 2, when the system reliability matched the target reliability at certain level, the only way to improve the objective value is to add redundant component at some particular subsystem which has the smallest hazard rate. Therefore, introducing the proper stop criterion can help prevent the redundancy when searching for the optimal solutions. The phase II for problem 3 seems to be not very effective to search for better solution. The optimal solution for most problems can be obtained using phase I; therefore, the phase II serves more to verify the optimal solution than to search for better solutions.

In a series type of reliability relationship, every subsystem must be much more reliable than the goal for the mission, and every component must be more reliable than its system. Therefore, given the system's target reliability, the system designer needs to know what reliability each subsystem should meet. Moreover, the redundancy optimization process focuses attention on the relationship between reliability and resources allocation, thus leading to a more complete understanding of the basic reliability problem inherent to the design.

In summary, it can be said that the efforts of this research have contributed to the extension of traditional redundancy optimization for time-dependent cases and the development of general heuristic procedures for time-dependent redundancy optimization problems.

5.3 Recommendations for further research

Since little work has been done on time-dependent redundancy optimization, the field is open for further developments in all aspects. In order to fully understand and utilize optimal redundancy allocation, the following areas can be targeted for further research to make the reliability model more realistic and complete:

(1) The exponential distribution plays an important role in reliability and life testing because of its mathematical tractability. However, it only serves well on certain portions of the life data. The incorporation of other life distributions on component's hazard rate should help derive more general algorithms for redundancy optimization and make the present study more complete. One particular interest would be the Weibull distribution because, through the appropriate choice of its parameters, a variety of failure rate behaviors can be modeled.

(2) As mentioned in the literature review of this study, there are different system structures in reliability model and the redundant components are allowed to be added in parallel, standby, or k-out-of-n, etc.. Therefore, it would be interesting to investigate the systems arranged in ways other than series-parallel. Using other optimization techniques for obtaining optimal system configurations also provides another potential area for study.

(3) Another direction for further research is to extend the current nonrepairable system to repairable redundant system. Further, consider the maintenance policy where a unit is maintained preventively when it works for a specific time without failure. Finally, the effects of preventive maintenance on redundant systems may be worth examining.

(4) For the sake of mathematical simplicity, most of the reliability models assume that all the units as well as all the stages are statistical independent; however, such an assumption is often questionable. There are some systems in which the failure rates of the surviving components depend on the number of failed components. Therefore, there is a great potential for further investigation on optimization model for state-dependent systems using time-dependent reliability.

All these above areas are beyond the scope of the present work, but they could serve as excellent sources for further investigation to shed light on this unexplored research area where the time factor is considered in the optimization process.

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