COMPARISON BETWEEN MATHEMATICAL AND MODEL ANALYSIS OF STRESSES IN BUILDING FRAMES

BY

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Acknowledgment is made to Professor D. H. Pletta of the Applied Mechanics Department whose instruction and assistance made this thesis a possibility.

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INTRODUCTION

Statement of Problem

Object. The object of this thesis is to show as clearly as possible a comparison between stresses in beams and frames as calculated by mathematics and that obtained from models by the deformeter method. It also illustrates the possibilities of using models in the design of structures.

Scope: It is not the intention to take a number of different problems and show a solution to them, but to take a few simple cases and show the application of the principles of model testing.

Summary

The first experiment consisted of the making and testing of a cardboard model of a beam fixed at both ends. The results obtained from the model compared very favorably with the mathematical solution as there was a variation of only 1.8 percent.

The second experiment was performed with a cardboard model of a bent fixed at both supports. The results from the model as compared to the mathematical solution could not be made to check very closely. There was a variation up to 10 percent in the influence lines for moment for horisontal loads, and a difference as high as 50 percent for vertical loads.

It was found after the experiment was performed that the modulus of elasticity of the columns was about four times as great as the modulus in the girder. Then after making the correction in the mathematical solution for the difference in the modulus of elasticity there were only three points that showed a variation of over six percent, and most of the points were under $3\frac{1}{2}$ percent.

REVIEW OF LITERATURE

History

The problem of Structural Engineering is to create a structure that will fulfill safely, rationally, and economically the particular purpose for which it is intended. To solve such problems a number of different methods have been developed and the structural model is one of them, known for a long time but until recently not very often used.

Professor Beggs*of Princeton University has developed the deformeter method of model analysis to a very high degree, but the high cost of apparatus used in his laboratory prevents most engineers from being able to experiment in model testing. A modification of the Beggs apparatus has been developed by T. Werner and Fred L. Plummer* and it is from their article in Civil Engineering that the ideas were obtained for the design of the apparatus used in testing models for this thesis.

Probably the most widely used engineering models have been models of dams, but with the introduction of low cost apparatus the deformeter method of model analysis is an ever increasing field of study.

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* See Bibliography

The Use of Engineering Models*

In considering the necessity of using models to solve engineering problems properly, the limitations of our present engineering knowledge have to be clearly realized as well as the complexity of some of the engineering problems that are often assumed to be simple.

As in every exact science, the engineer deals with the phenomena occurring in nature, but a pure scientist is interested in building up a theory that would explain exactly these phenomena and the laws that govern them. The engineer takes a different point of view and feels satisfied if a theory can be developed covering a particular angle of a problem in a way that would make possible a solution, perhaps only approximate, but sufficiently accurate for practical purposes.

Present mathematical knowledge is limited and no general solution is available for the differential equations set up by introducing the theory of stress. Consequently the solving of each structural problem resolves itself into the finding of the partial or approximate solutions of the general equations which will, with a greater or less degree of accuracy, solve the particular stress problem. In order to make even such solutions possible a number of simplifying

* Taken from A. V. Karpov's artical on the Use of Engineering Models in Military Engineer. V26 N150, Nov. - Dec. 1934

assumption have to be made. They have to be such as to make possible the solving of the equations and to assure that, for the particular problem and within certain limits, there is a reasonably close agreement between the evaluated and actual stress.

Many such assumptions pertain to the physical properties of the material used in the structure. The actual physical properties of materials are governed by such complicated relationships that they very seldom can be set forth in the form of simple and useful mathematical expressions. The most common method is to introduce a relatively simple approximate relationship covering some of the physical properties of the material. Such a simplified expression is applicable, as a general rule, only within a comparatively narrow range. The best known is the assumption of direct proportionality between strains and stresses of the material usually expressed by stating that the material follows Hooke's law. This assumption is the basis of the present theory of elasticity and covers with a fair degree of accuracy many materials, provided the stress is kept sufficiently low or within the so-called elastic limits. Consequently the differential equations that were solved by introducing this assumption are applicable only within the limits in which the assumption of a linear stress-strain relationship represents the actual behavior of the material. If applied beyond these limits, the solved equations will give results that may be very different from actual conditions.

There are large numbers of cases where the partial solutions of the general equations may be obtained or indicated by the introduction of a number of auxiliary equations. The postulate that the external and internal work have to be equal, the method of least work or deformations, the method of virtual displacements, the elastic energy theory may be cited as a few of the methods that are most extensively used.

Every structure and every element of a structure consist of physical bodies that are three-dimensional. Therefore only a three-dimensional state of stress is possible. The existence of the three-dimensional state of stress is clearly recognized, but the solving of the involved mathematical expressions may become very difficult and even impossible.

Since no theory is available covering the physical properties of materials and no general solution of the differential equations of the theory of stress is possible, every one of the available methods for determining stresses is approximate. All such methods are based on a number of assumptions and may be applied only within certain limits. The correctness of these assumptions, the practical limits of applicability of each method, and the divergence between the evaluated and actual conditions of a structure must be found by test. From the theoretical point of view, the most satisfactory method of testing would be to build in the natural surroundings, a full sized structure and test it. Such procedure in most cases would require a large amount of time and money. Therefore, a demand has been created for some more speedy and less expensive method of testing.

The use of models is the engineer's response to that demand.

Any advanced model study is a research problem and is to be approached with an open mind. The danger of the use of improper models must be realized; by neglecting the similarity conditions it is easy to build a model the behavior of which will have no proper relation to the behavior of the prototype and which will prove a predetermined but erroneous theory or idea. The behavior of models that fulfill the major similarity conditions will have a natural relationship to the behavior of the prototype and the results of studies made on such models are the ones that will advance the art.

In general a paper design does not add greatly to engineering knowledge. A properly conducted model study, however, not only is a tool permitting the best possible solution of a particular problem, but in nearly all cases it will advance understanding and knowledge; it is inspiration for future improvements and more advanced engineering work.

If the present great engineering achievements are to advance still further, much time and effort must be devoted to the theoretical and research work which will enable the engineer to base future structural designs on sound engineering knowledge. The use of scientifically designed models will contribute largely to this result.

Similitude Requirements In Model Design

In its broadest sense, similitude exists when there is some systematic relation between the behavior of a model and that of its prototype. Roy W. Carlson* states that meeting the requirements of two simple but exacting rules comprises the basis of model design in the field of both structures and hydraulics.

These two rules are as follows:

Rule 1. - The model shall be geometrically similar to its prototype, except as to dimensions which do not effect the behavior of the model.

Rule 2. - The force scale-reduction factor shall be the same for forces arising from each of the various influences.

While these rules insure strict similarity they also allow free choice to be made of a certain number of scale-reduction factors such as lengths, times, widths, and elastic modulii.

Rule 1 merely signifies that the ideal model is proportioned exactly like the prototype, all dimensions in the prototype (including thickness of the individual members) being reduced by exactly the same factor to obtain the model.

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* Roy W. Carlson. Similitude Requirements in Model Design. Engr. News Rec., Aug. 23, 1934 P 235

In other words the first rule fixes the relative dimensions of the model.

Rule 2 fixes the properties of the material from which the model can be built and also interprets the measurements on the model in terms of the prototype. It is apparent that since all influences must be reduced in the same proportion, the more influences there are the more difficult it becomes to fulfill exactly the requirements of Rule 2. Sometimes the designer must either sacrifice exactness or resort to an indirect use of the model. In a number of simple problems, practically the only similarity condition to be met is the identity of the material of model and prototype. In others, the proper scalar representation of the prototype and the external force is to be added.

For the fixed ended beam tested in this experiment, plate III, the only similitude requirement that has to be met is that the scale ratio between the model and prototype be known. Most any material that will spring back to its original position after being deflected the small amount required is satisfactory for use.

The bent, plate IV and V, presents a more difficult problem, as there are several requirements to be met. The lengths of the members must be in proportion to those of the prototype, the modulus of elasticity* of the different mem-

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* See bibliography of George Erle Beggs works.

bers must be in the same ratio as in the full size structure, and the widths of the members must be in proportion to the cube roots of the moments of inertia of the full scale cross sections because the members are of a constant thickness b.

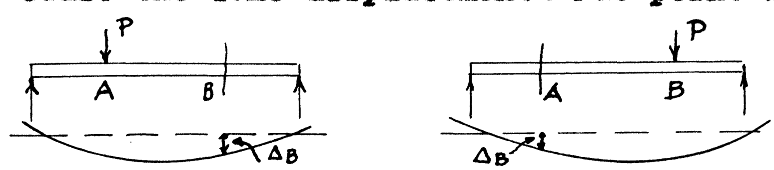
$$\frac{Im}{Im'} = \frac{\frac{bd^3}{12}}{\frac{bd_1^3}{12}} = \frac{I}{I_1}$$

$$\frac{d}{d_1} = \sqrt[3]{\frac{I}{I_1}}$$

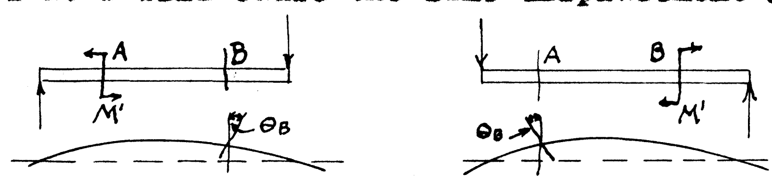
THEORY OF MODEL TESTING

Maxwell's Law* of the reciprocity of displacement is the basis for model testing by the deformeter method. Three cases of the law as applied to beams are as follows:

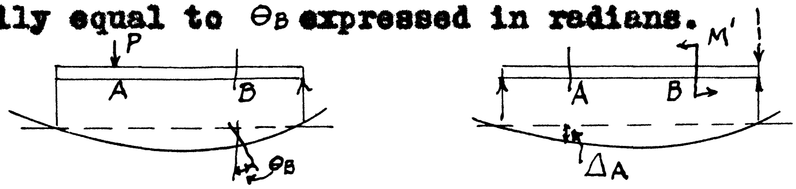
If a force P applied at a point A in a beam causes a displacement Δ_B at B, then the same force P applied at B will cause the same displacement Δ_B at point A.



If a couple M' applied at point A in a beam causes a displacement θ_B at point B, then the same couple M' applied at B will cause the same displacement θ_B at A.



If a force P applied at point A in a beam, causes an angular displacement θ_B at point B, then a couple M' expressed in inch-pounds and numerically equal to the force P, applied at point B, will cause a linear displacement Δ_A at point A. Δ_A expressed in inches is numerically equal to θ_B expressed in radians.



* Elastic Energy Theory - Van Den Broek

Influence Lines

An influence line is a graph or chart showing, for successive positions of a unit load, the value of some direct linear function of the load on the structure. In other words, an influence line shows the value of the moment, shear, or reaction at one point in a structure due to a unit load applied in succession at any number of other points in the structure.

In the deformeter method of model analysis a unit twist is applied at one support and the deflections measured at several points in the structure to obtain the deflection curve which is used as the influence line for moment at the twisted support. In like manner the support is moved a unit distance in the direction of the reaction to obtain the deflection curve which is used as the influence line for reaction at that support. The influence line for shear is obtained by moving the support in the direction of the shear a unit distance and measuring the deflection.

The following paragraphs contain the proof of the fact that the deflection curves of deformed structures are the influence lines of the superfluous elements at the points where the unit movements are applied.

Deflection Curves as Influence Lines*

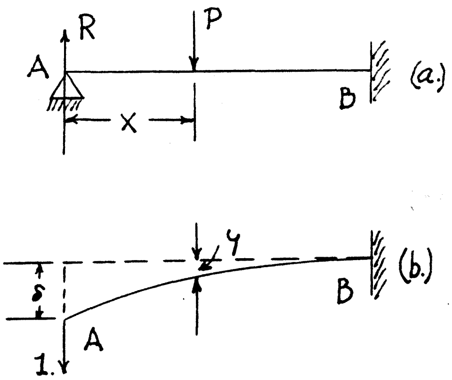


Fig. 1.

Let us consider the real condition of the beam, shown in fig. 1(a), as the first state of stress. The reaction R at the support A is taken as the superfluous element. For the second state of stress, the fictitious state shown in fig. 1(b) is considered. The superfluous support is removed and instead of the unknown reaction R, a force equal to unity acting in a downward direction is taken. This second state of stress is a very simple one and its deflection curve can be found without any difficulty. Let δ denote the deflection at the end A, and y denote the deflection at the point of application of the force P. The work done by the forces of state (a) on the corresponding displacements for the state (b) is $Py - R\delta$. The work of the force unity, of the second state on the corresponding displacement of the first state is equal to zero because the displacement of the point A in the first state is equal to zero. Due to the fact that the displacements at the built in end are equal to zero, it is not necessary to consider the reactions at this end, for the corresponding work is always

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* Applied Elasticity - Timoshenko & Lessells

equal to zero. The reciprocal theorem gives $Py - R\delta = 0$ from which $R = Py/\delta$. It is seen that in the case of a moving load P, the reaction R changes in proportion to y. The deflection curve of the state (b) gives a complete picture of the manner in which R varies due to the motion of P. This curve is called the line of influence of the reaction R.

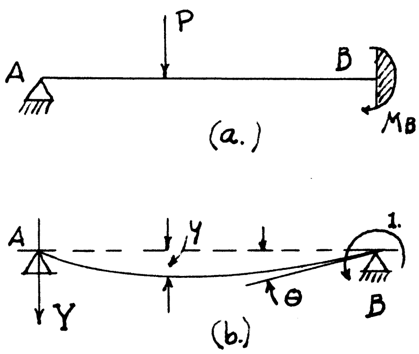


Fig. 2.

If instead of the reaction at the support A, the reactive couple M_B at B be considered as the superfluous reactive element, the two states of stress shown in fig. 2 must be compared. The case 2(a) represents the real condition of the beam. The case 2(b) is the fictitious state of stress in which the superfluous fastening at B is removed, and, instead of the unknown reactive couple M_B a couple equal to unity is applied. This second state of stress is a simple one. The deflection y and the rotation θ can easily be obtained. Applying the reciprocal theorem we have $Py - M_B\theta = 0$; $M_B = \frac{Py}{\theta}$ Again, the deflection curve for the fictitious state of stress represents the line of influence for the superfluous reactive element M_B.

THE INVESTIGATION

Method of Procedure

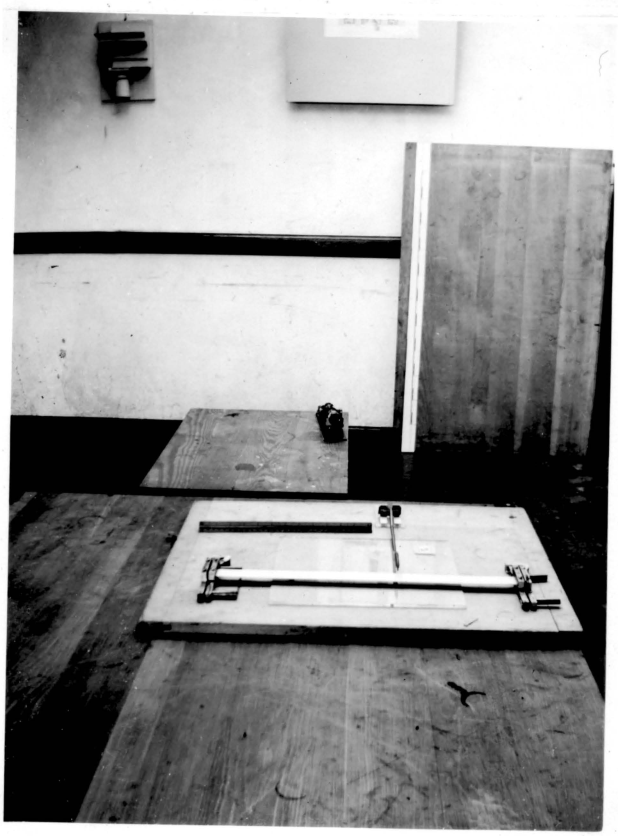


Plate I. Set Up of Apparatus

Description of Apparatus Used

The apparatus required for finding deflections in the models by the deformeter method is shown in plate I.

The deformeters used, as shown in fig. I, are made from mild steel bars one-half by three-eights inch in cross section. Springs are used to hold the rocker bar in its position on the fixed bar which was fastened to the drawing board by wood screws D. The supports of the model are clamped between the rocker bar and the bar H. For rotation, (moment), a filler is inserted between C and the glass plate. For movement in an axial direction, (reaction), fillers of equal thickness are inserted between B and C and the glass plate. For lateral movement, (shear), a filler is inserted between A and the glass plate.

The fillers used in the experiments were razor blades, and a thickness gage.

The mirror used was a piece of plate glass mirror about $\frac{3}{4}$ x $1\frac{3}{4}$ inches on the back of which a razor blade was glued. When in use one edge of the razor rested on a mirror plate which was composed of a glass plate 2" x 4" on which two strips of paper were glued to keep the razor blade from slipping. The top edge of the mirror was moved by a mirror bar which was in turn moved by the model. This mirror bar was made from a piece of copper tubing about 12" long with a needle in one end to stick in the model, and a piece of paper glued to the flattened end which rested on the mirror razor blade.

The scale, plate II, was made on a long strip of

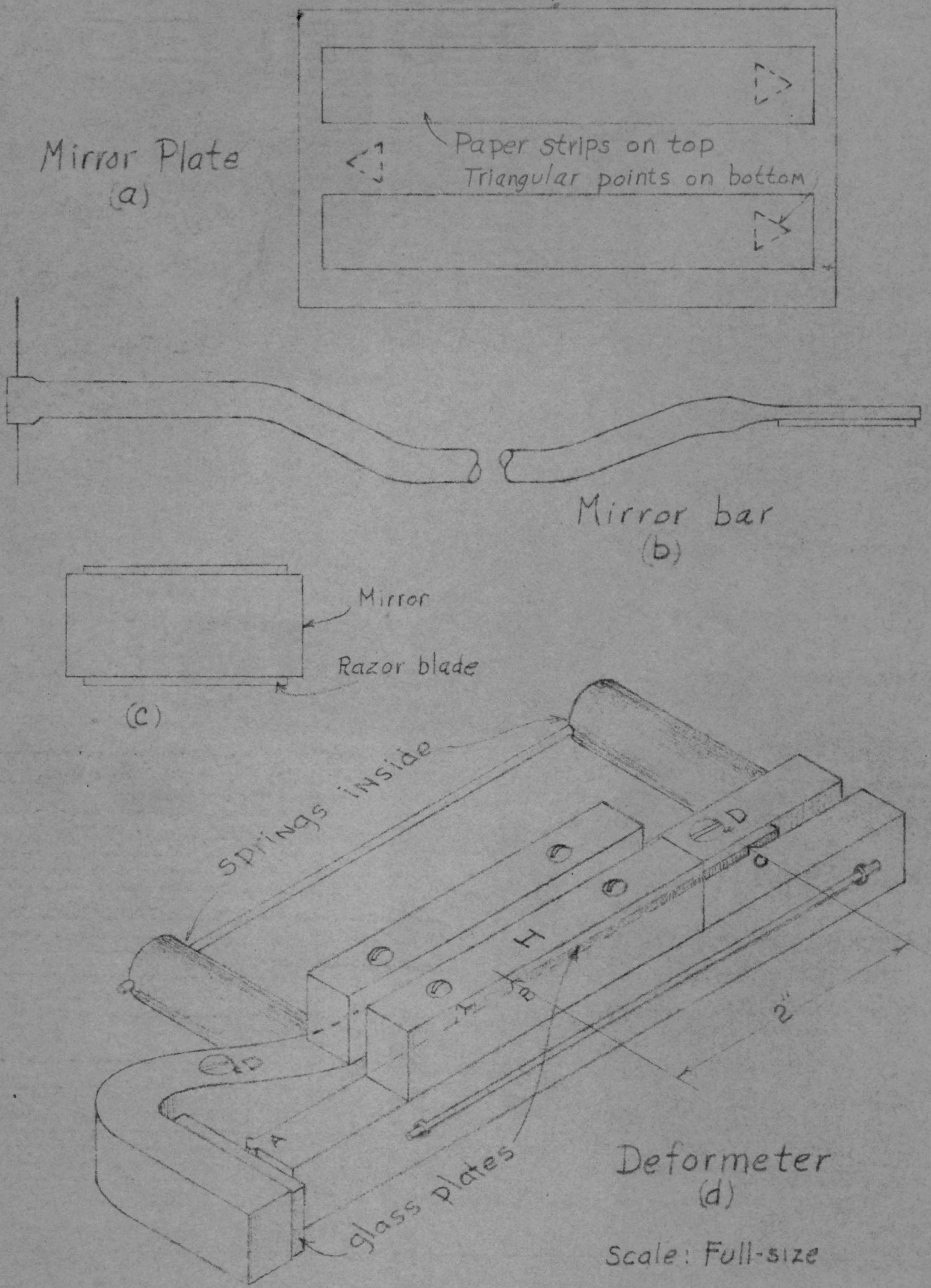
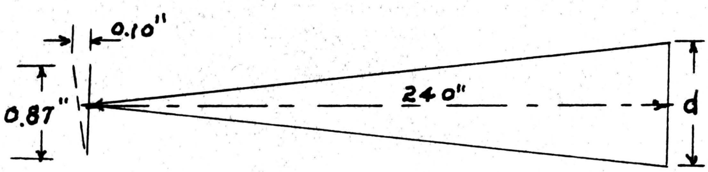

Mirror Plate
(a)
Paper strips on top
Triangular points on bottom
Mirror bar
(b)
Mirror
Razor blade
(c)
Springs inside
glass plates
Deformeter
(d)
Scale: Full-size
2"

Fig. I.

Plate II. The Scale

paper mounted on a large drawing board. The units for the scale were determined as follows:



by similar triangles:

$$0.10 : \frac{d}{2} = 0.87 : 240$$

$$\frac{.87\,d}{2} = 240$$

$$d = 55.175''$$

The scale was made 57.175 inches long and than divided into a thousand equal parts so that a deflection could be read directly to one ten thousandth of an inch.

The plane table alidade, plate I, was borrowed from the Civil Engineering Department. It was used to read the scale which was reflected through the mirror. The deflections were read directly in inches as known deformations were applied to the model.

To reduce friction the model rested on quarter inch steel ball bearings and these in turn were free to roll on a ¼ inch thick glass plate. At several points along the model small weights were placed above the bearings to keep the material from buckling.

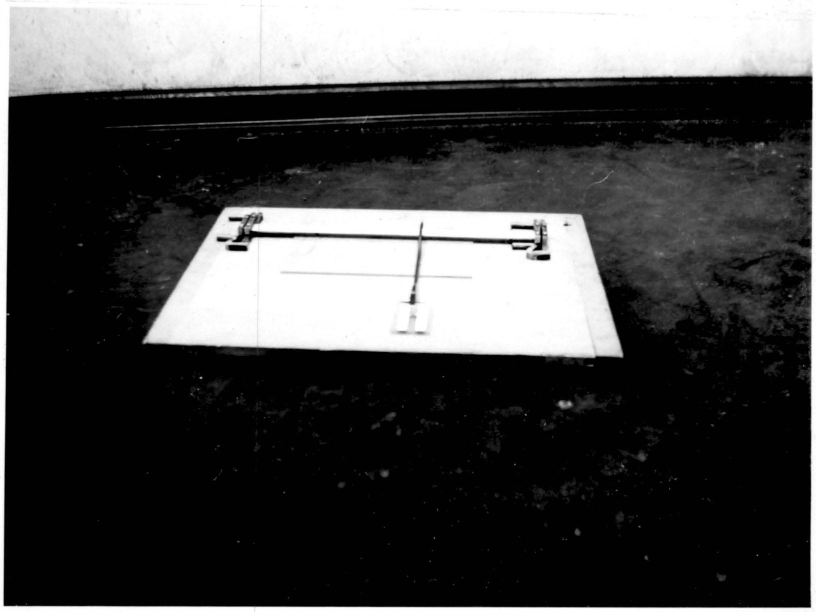


Plate III. Fixed Ended Beam

RESULTS

Fixed Ended Beam

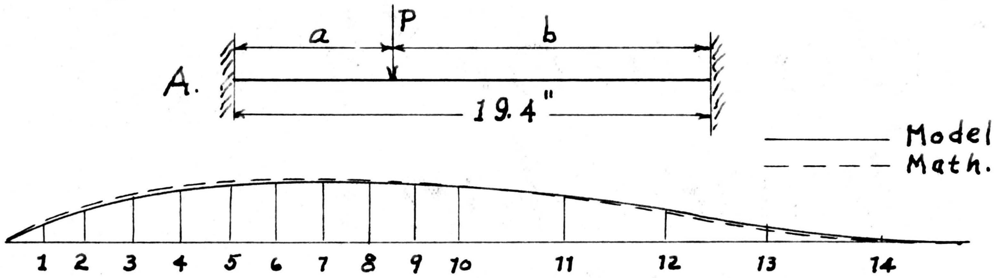


Fig. 2. Influence Line for Moment at A.

Point	Model $\frac{\Delta}{\theta}$	Math. $\frac{Pab^2}{L^2}$
1	0.80	0.817
2	1.52	1.545
3	2.08	2.095
4	2.48	2.492
5	2.72	2.740
6	2.82	2.860
7	2.88	2.868
8	2.76	2.780
9	2.56	2.610
10	2.38	2.375
11	1.88	1.780
12	1.13	1.120
13	0.54	0.518
14	0.112	0.1105

Table I. Data for Fig. 2

Point	Model	Math.
1	0.360	0.3675
2	1.160	1.1760
3	1.880	1.8200
4	2.280	2.2935
5	2.600	2.6160
6	2.770	2.8000
7	2.850	2.8640
8	2.820	2.8240
9	2.660	2.6950
10	2.470	2.4975
11	4.260	4.1550
12	3.010	2.9000
13	1.670	1.6380
14	0.652	0.6285
15	0.840	0.8300
Total	32.002	32.1050

Table II. Moment in Inch Pounds.

(Area Under Curve)

The results from table II show a variation of only 0.3% between the total moments of the mathematical and model solutions. The fixed ended moment is equal to $wL^2/12$ which gives 31.4 inch-pounds. This shows the variation of 1.8%. Fig. 3 is the influence line for reaction at A.

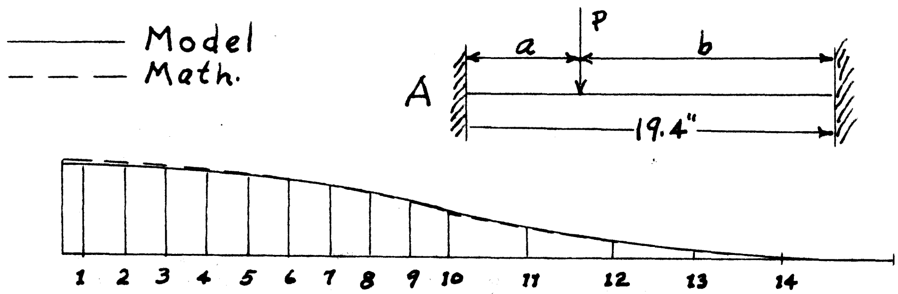


Fig. 3. Influence Line for Reaction at A.

Point	Model $\frac{\Delta}{\text{Filler}}$	Math. (Mom. about B)
1	.991	.993
2	.970	.972
3	.943	.945
4	.893	.894
5	.840	.841
6	.779	.778
7	.711	.710
8	.638	.637
9	.560	.561
10	.482	.484
11	.297	.299
12	.195	.196
13	.084	.086
14	.015	.017

Table III. Data for Fig. 3

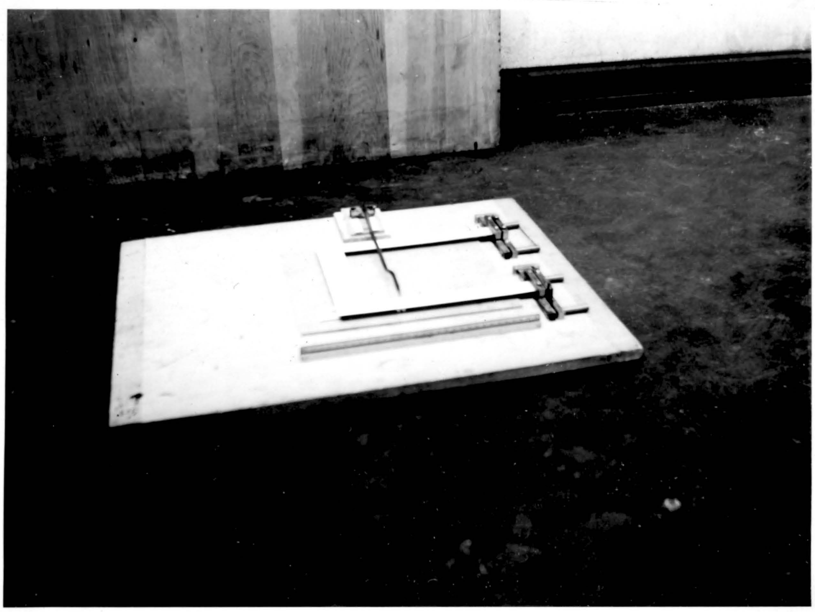


Plate IV. The Bent.
Showing position for horizontal loads.

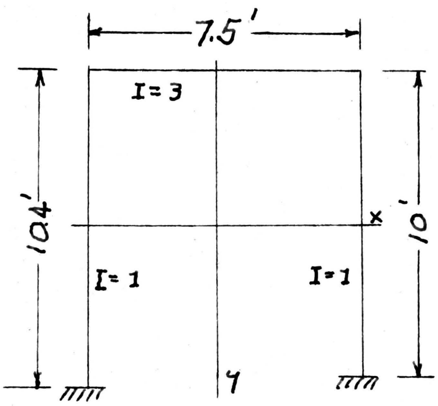


Fig. 4.

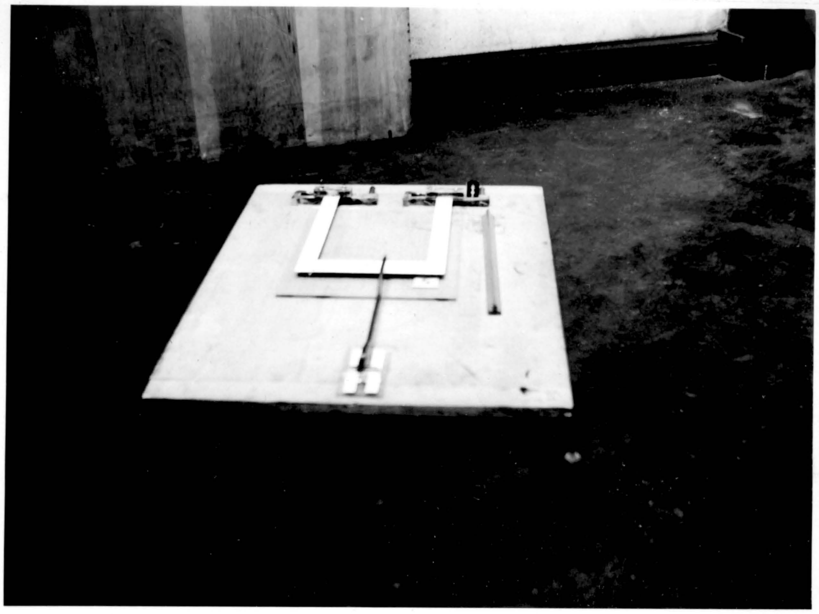


Plate V. The Bent.
Showing position for vertical loads.

The Bent

The bent, as shown in fig. 4, was first solved mathematically without knowledge of the fact that there was a difference in the modulus of elasticity of the different members.

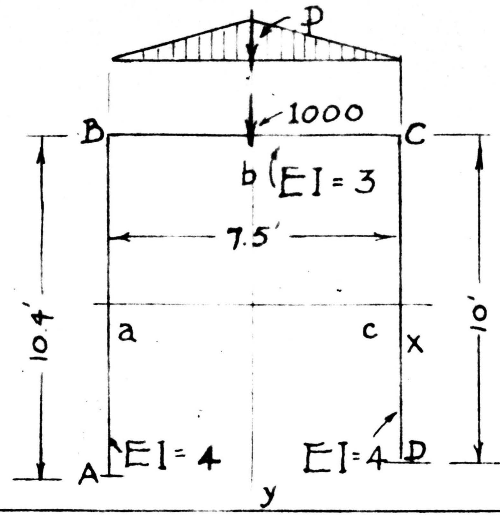
After finding that too great a difference occured between the mathematical and model solutions the modulus of elasticity of the members was checked and found to be in a ratio of about 4 to 1, (the columns being of the stiffer cardboard). The bent was then solved taking into account this difference in the modulus of elasticity, table IV.

Table IV shows the column analogy solution for a load at point 14.

Figures 5, 6, and 7 show the comparison between the mathematical and model influence lines for moment, shear and reaction respectively.

Tables V, VI, and VII show the values of the influence lines in the same order as figures 5, 6, and 7.

Note: See Cross & Morgan on "Continuous Frames of Reinforced Concrete" for complete details.



Properties of Section											Elastic Load				
Given					Computed						Given			Computed	
Member	Length	EI	x_1	y_1	El. Area a	Statical Moments ax	ay	Products of Inertia $ax^2 + I_x$	$ay^2 + I_y$	$axy + I_{xy}$	P	x	y	Mx	My
a	10.4	4	-3.75	-0.2	2.6	-9.75	-.52	36.58 0	104 23.44	1.95 0					
b	7.5	3	0	5	2.5	0	12.5	0 11.72	62.50 0	0 0					
c	10.0	4	3.75	0	2.5	9.375	0	35.15 0	0 20.82	0 0					
Corrections to Centroid			-.0494	1.576	7.6 A	-.375	11.98	83.45 .02	106.864 18.88	1.95 -.592	2344	0	5	0 -115.9	11,720 3,690
		Corrections for Dissymmetry						83.43 .07	87.984 .07	2.542				+115.9 +232.1	8030 3.5
								83.36	87.91					-116.2	8026.5

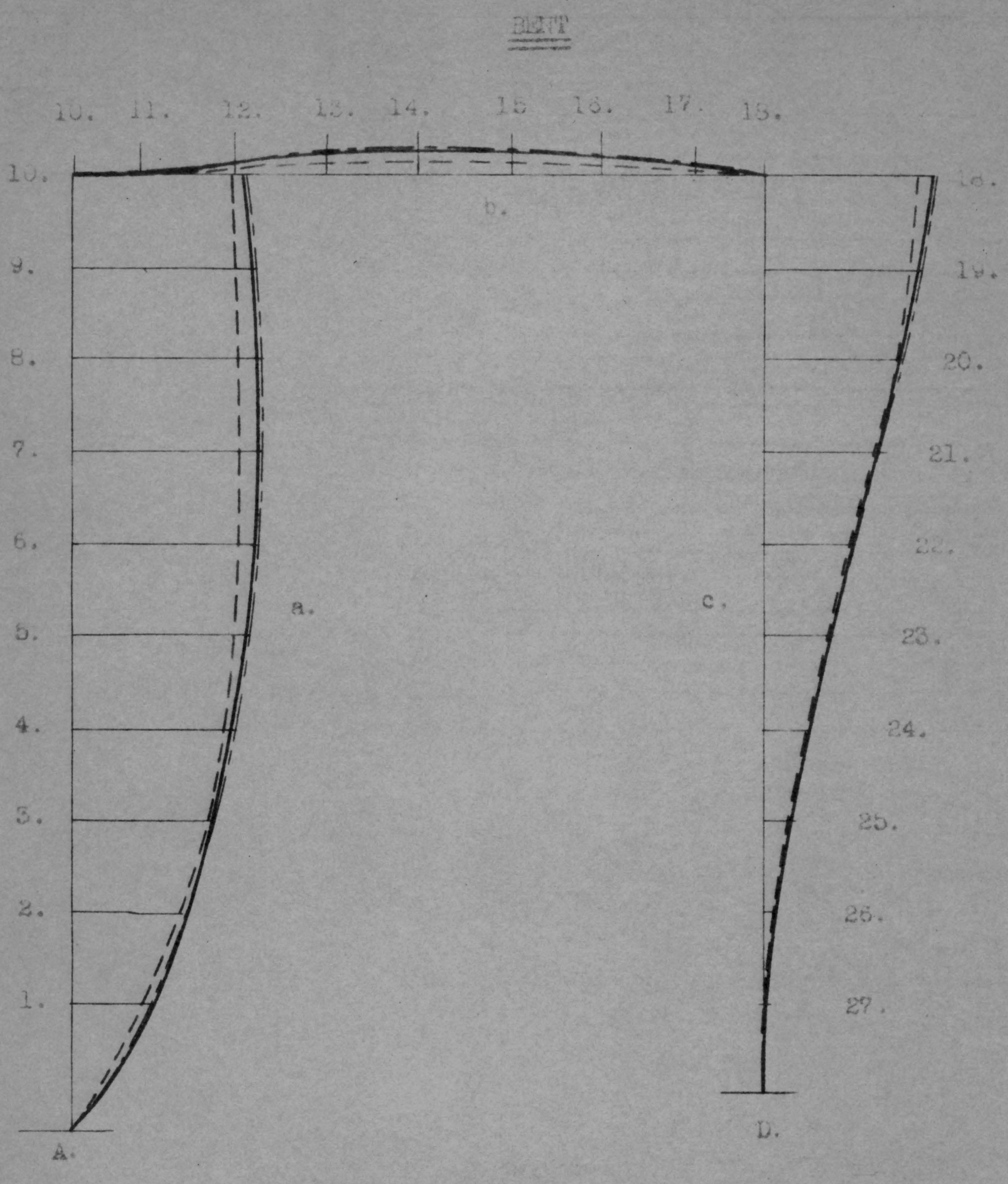
$$M_i = \frac{P}{A} + \frac{M_x'}{I_x}x + \frac{M_y'}{I_y}y \qquad M = M_s - M_i$$

Table IV. Column Analogy (Load at point 14)

Point	Model	Math. Constant E	% Variation from model Constant E	Math. E=4:1	% Variation from model E=4:1
1a	-1280	-1186	7.3	-1205	5.8
2a	-1860	-1794	3.5	-1846	0.7
3a	-2280	-2232	2.1	-2326	2.0
4a	-2610	-2525	3.2	-2662	1.9
5a	-2820	-2690	4.6	-2879	2.0
6a	-2960	-2755	6.9	-2995	1.1
7a	-2980	-2755	7.5	-3022	1.4
8a	-2980	-2688	9.8	-2994	0.5
9a	-2900	-2632	9.2	-2904	0.1
10a	-2780	-2558-	8.0	-2810	0.1
10b	+ 50	0	0	0	0
11b	+ 80	+ 48	40.0	+ 82	1.6
12b	+ 180	+ 101	43.0	+ 183	1.4
13b	+ 270	+ 140	51.0	+ 269	0.4
14b	+ 320	+ 160	50.0	+ 323	1.0
15b	+ 330	+ 157	52.0	+ 331	0.15
16b	+ 280	+ 127	54.0	+ 276	1.3
17b	+ 120	+ 67	44.0	+ 152	21.0
18b	+ 20	+ 0	0	0	0
18c	-2740	-2556	6.7	-2820	2.8
19c	-2580	-2395	7.1	-2534	1.8
20c	-2240	-2024	9.6	-2220	0.9
21c	-1890	-1850	2.1	-1857	1.7
22c	-1540	-1510	1.9	-1485	3.5
23c	-1140	-1105	3.0	-1113	2.3
24c	- 810	- 801	1.1	- 764	5.6
25c	- 520	- 485	6.8	- 458	12.0
26c	- 260	- 233	10.0	- 218	16.0
27c	- 70	- 61	13.0	- 57	18.0

Table V. Values for Fig. 5.

(Moments in foot-pounds.)



Model

Math. constant E.

Math. E ratio 4:1

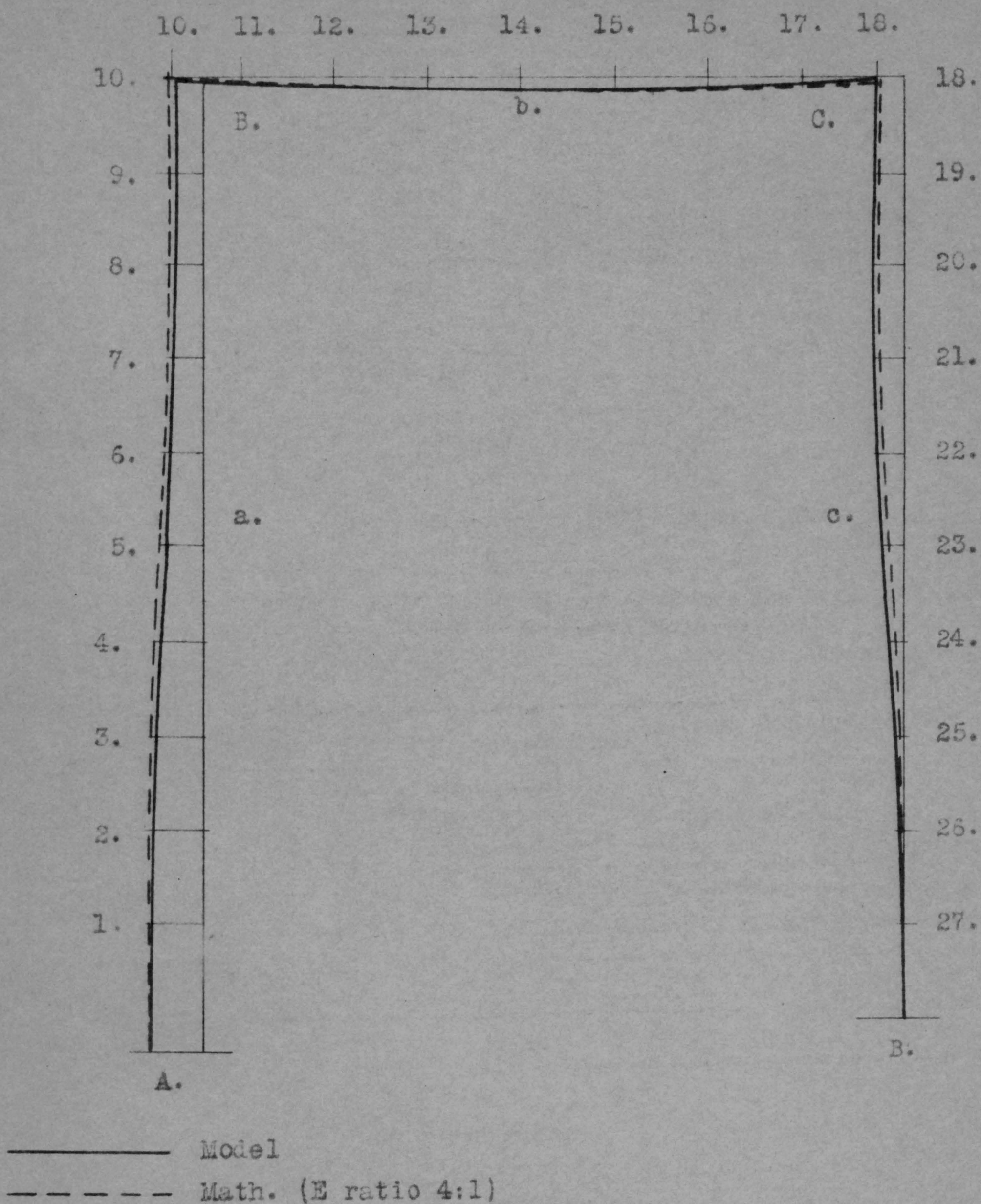
Influence Lines For Moment at A.

Fig. 5.

Point	Model	Math.
1a	.990	.990
2a	.946	.950
3a	.855	.895
4a	.794	.850
5a	.770	.790
6a	.702	.724
7a	.641	.656
8a	.614	.640
9a	.481	.530
10a	.469	.477
10b	.015	0
11b	.046	.033
12b	.068	.065
13b	.085	.085
14b	.091	.091
15b	.084	.084
16b	.061	.064
17b	.030	.032
18b	0	0
18c	.496	.478
19c	.427	.425
20c	.374	.367
21c	.305	.300
22c	.229	.240
23c	.183	.179
24c	.145	.122
25c	.076	.073
26c	.045	.034
27c	.015	.009

Table VI. Values for Fig. 6.

BENT



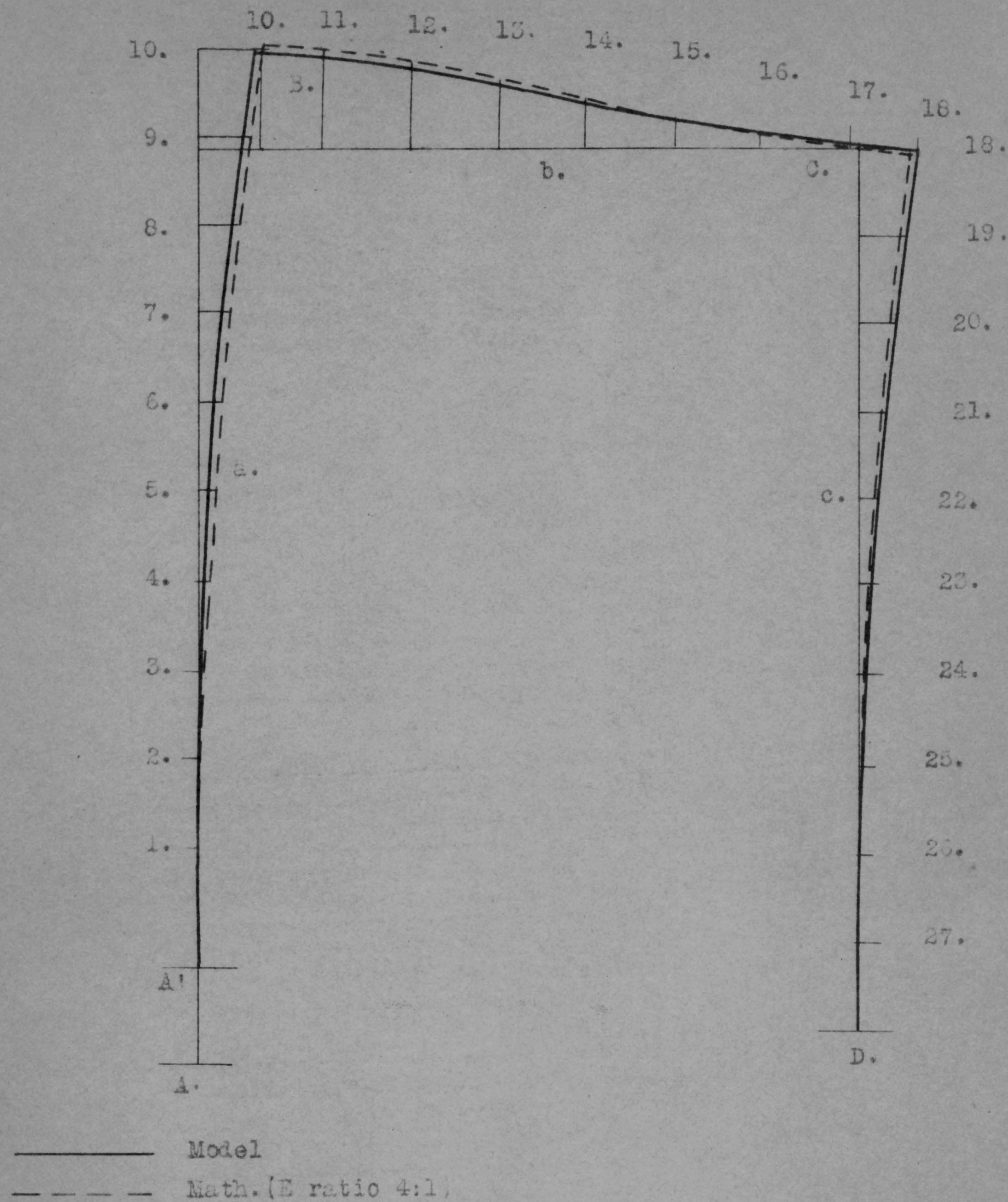
Influence Lines For Shear at A.

Fig. 6.

Point	Model	Math.
1a	0	.001
2a	.023	.030
3a	.048	.060
4a	.107	.104
5a	.137	.155
6a	.214	.219
7a	.290	.306
8a	.405	.378
9a	.443	.476
10a	.580	.584
10b	.992	1.000
11b	.886	.907
12b	.748	.780
13b	.634	.644
14b	.496	.501
15b	.359	.359
16b	.221	.221
17b	.091	.090
18b	0	0
18c	.587	.585
19c	.480	.475
20c	.397	.375
21c	.305	.289
22c	.224	.212
23c	.153	.148
24c	.069	.095
25c	.053	.053
26c	.015	.025
27c	0	.006

Table VII. Values for Fig. 7.

BENT



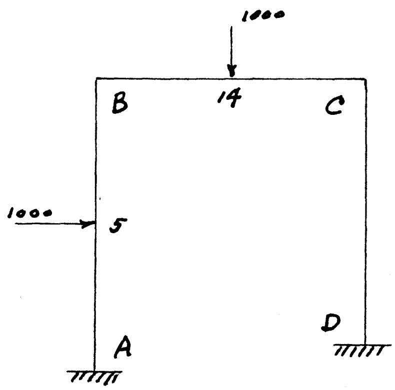
Influence Lines For Reaction at A.

Fig. 7.

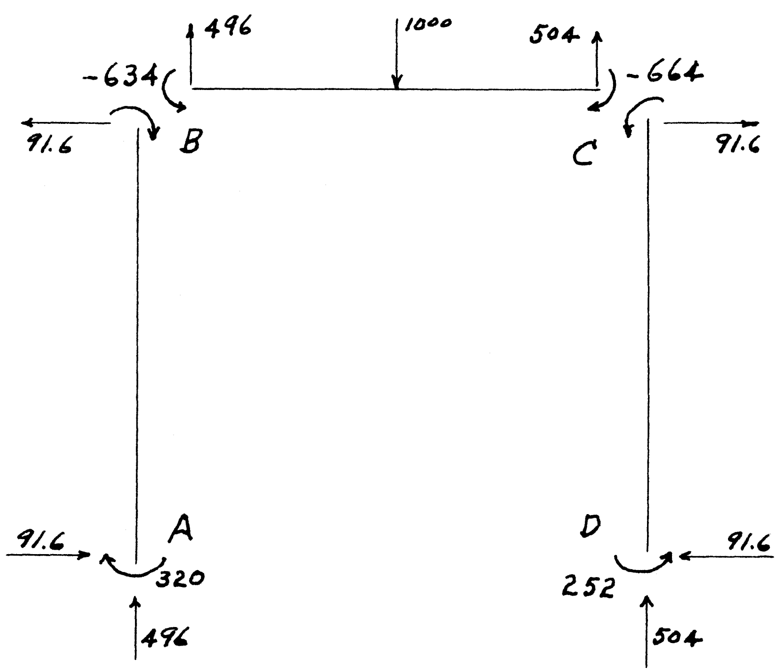
Application of Model

When a known set of loads are in evidence it is not necessary to construct the whole influence line, but just find the moments shears and reactions at one support for each load. With these values the moment diagram can be worked out and the cross sections checked to see if they are properly stressed.

Illustration 1.
1000 pounds
at points 5 & 14



For the load at point 14



Moment at B (mom. about B)

+ 320 - 91.6 x 10.4 = -634 ft. lbs.

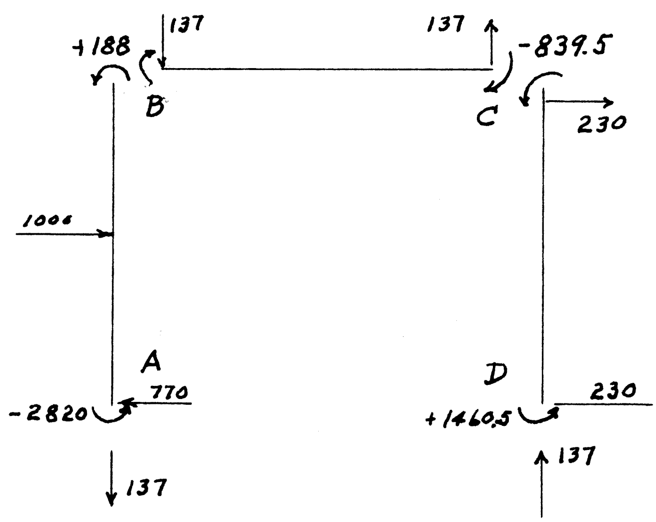
Moment at C (mom. about C)

- 634 - 3750 + 496 x 7.5 = -664 ft. lbs.

Moment at D (mom. about D)

+91.6 x 10 - 664 = +252 ft. lbs.

For the load at point 5



Moment at B (mom. about B)

-1000 x 5 -2820 + 10.4 x 770 = +188 ft. lbs.

Moment at C (mom. about C)

+188 - 137 x 7.5 = - 839.5 ft. lbs.

Moment at D (mom. about D)

+230 x 10 - 839.5 = + 1460.5 ft. lbs.

Total Moments

A = - 2820 + 320 = -2500 ft. lbs.

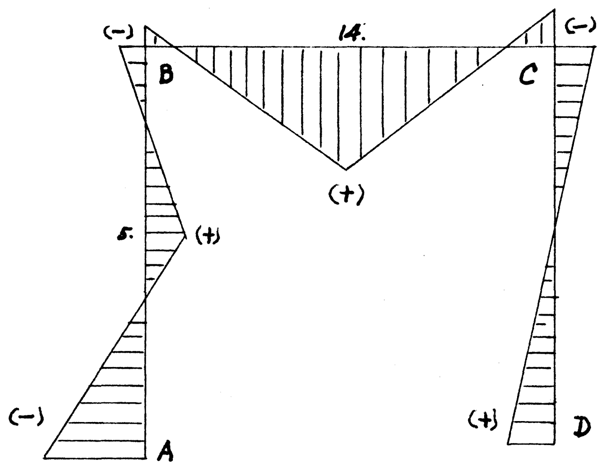
B = - 634 + 188 = - 446 ft. lbs.

C = - 839.5 - 664 = -1503.5 ft. lbs.

D = 1460.5 + 252 = + 1712.5 ft. lbs.

14 = + 5902

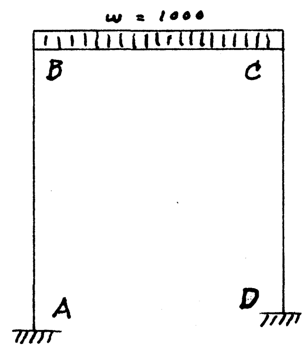
5 = + 1260



Final Moment Diagram

Illustration 2

Uniform load on girder of 1000 lbs. per ft.

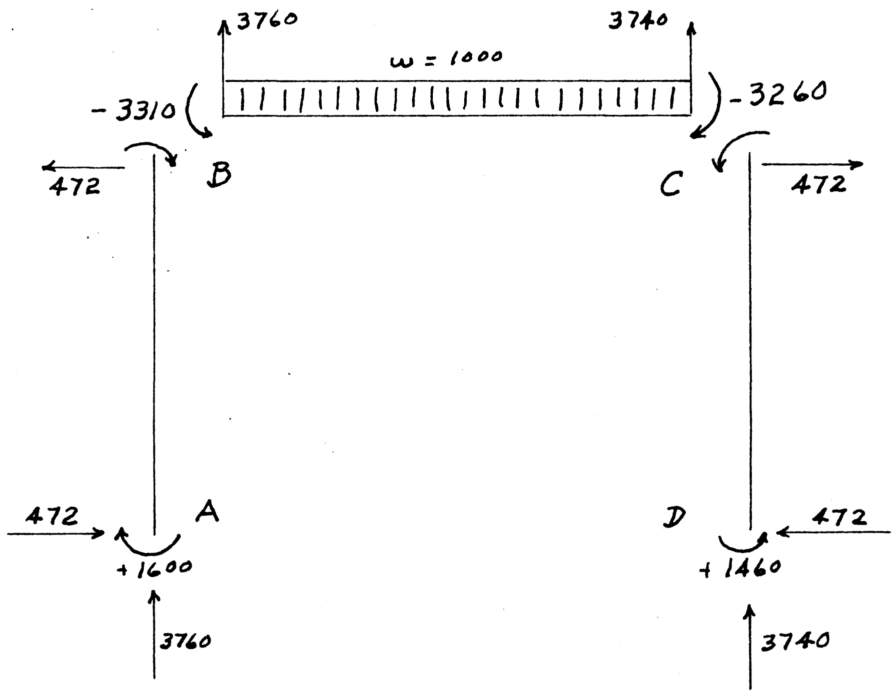


From the area under the curves:

Moment, 1600 ft. lbs. (A)

Shear, 472 lbs. (A)

Reaction, 3760 lbs. (A)



Moment at B (mom. about B)

$$- 472 \times 10.4 + 1600 = - 3310 \text{ ft. lbs.}$$

Moment at C (mom. about C)

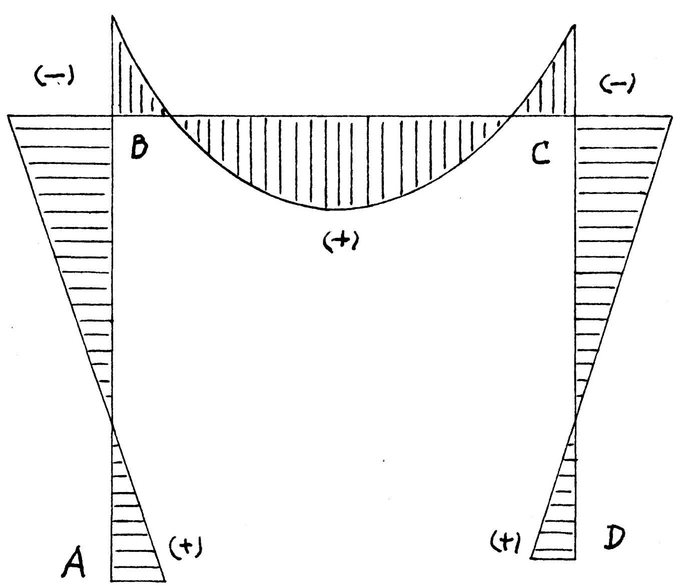
$$- 3310 - 7500 \times 3.75 + 3760 \times 7.5 = - 3260 \text{ ft. lbs.}$$

Moment at D (mom. about D)

$$- 3260 + 472 \times 10 = + 1460 \text{ ft. lbs.}$$

Moment at center of girder

$$+ 1600 - 472 \times 10.4 + 3760 \times 3.75 - 3750 \times 1.875 = + 3755 \text{ ft. lbs.}$$



Bending Moment Diagram

DISCUSSION OF RESULTS

As a whole the results obtained compared very favorably with the mathematical theory. The variation from the mathematical solution was due to three major causes: namely, the under-developed technique of the experimenter, the limited amount of materials and apparatus, and the inaccuracies occuring by not carrying the mathematical solution far enough.

Even though the writer did not have such a finely developed technique he has every reason to believe that the model is just as accurate as the mathematics. Take, for example, the results on the fixed ended beam. W hen the influence line for moment was constructed mathematically and the moment found by taking the area under the curve, an error was introduced by not carrying the solution to the last decimal place.

Then there is a chance for inaccuracies in cutting out the model. It is not a difficult problem to make a model of a beam, but extra precaution must be taken in forming a model of a structure where the cross-sectional area of the different members are not the same.

The apparatus used was reasonably accurate, but if one wanted to experiment further in model testing, the deformeter should be redesigned. That is, rotation should occur about a point on the center line where the model comes out of the deformeter instead of about a point which is four tenths of an inch from the outside face of the clamp.

CONCLUSIONS

1. The results from the deformeter method of model testing can be made as accurate as the model and the apparatus used.

(a) The model can be made as true a representation of the full size structure as is desirable.

2. The model is believed to be more accurate in its results when designing such structures as bents because there has always been a question as to where the column ends and the girder begins.

3. The chances of making mistakes when designing by models are greatly reduced due to the fact that the steps used in reaching the solution are few in number as compared to the mathematical analysis.

4. Where the structure contains members of variable cross-section such as a haunched beam or a tapered column, it is the opinion of the writer that less time would be consumed in designing by the use of a model than by the use of mathematics.

5. The use of models enables one to see just how the structure will act when deflected, therefore making it easier for the designer to show why reinforcement is placed at certain points in the structure.

6. The use of models in design is an ever increas-

ing field, and it is to be expected that model testing will attain a very definite and important position as soon as engineers can be made to realize the possibilities of this method. Already certain types of models are being used in the design of the largest engineering projects as a check on the mathematical solution. In closing it might be added that the future of the deformeter method of model testing depends upon the accuracy of the results obtained by the experimenters.

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