# Elevation Based Classification of Streams and Establishment of Regime Equations for Predicting Bankfull Channel Geometry

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Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science

In

Civil Engineering

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July 1, 2013

Blacksburg, VA

Keywords: Stream classification, elevation, joint probability distribution, hydraulic geometry, aspect ratio, sinuosity, channel gradient, bankfull channel dimensions, nondimensionalization, multiple regression analysis, universal regime relations, regional regime models, residual errors

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## Abstract

Since past more than hundred years, fluvial geomorphologists all across the globe have been trying to understand the basic phenomena and processes that control the behavioral patterns of streams.  A large number of stream classification systems have been proposed till date, but none of them have been accepted universally. Lately, a large amount of efforts have been made to develop bankfull relations for estimating channel geometry that can be employed for stream restoration practices. Focusing on these two objectives, in this study a new stream classification system based on elevation above mean sea level has been developed and later using elevation as one of the independent and nondimensionalising parameters, universal and regional regime equations in dimensionless forms have been developed for predicting channel geometry at bankfull conditions.

To accomplish the first objective, 873 field measurement values describing the hydraulic geometry and morphology of streams mainly from Canada, UK and USA were compiled and statistically analyzed. Based on similar mode values of three dimensionless channel variables (aspect ratio, sinuosity and channel slope), several fine elevations ranges were merged to produce the final five elevation ranges. These final five zones formed the basis of the new elevation based classification system and were identified with their unique modal values of dimensionless variables. Performing joint probability distributions on each of these zones, trends in the behavior of channel variables while moving from lowland to upland were observed. For the completion of second objective, 405 data points out of initial 873 points were selected and employed for the development of bankfull relations by using bankfull discharge and watershed variables as the input variables. Regression equations developed for width and depth established bankfull discharge as the only required input variable whereas all other watershed variables were proved out to be relatively insignificant. Channel slope equation did not show any dependence on bankfull discharge and was observed to be influenced only by drainage area and valley slope factors. Later when bankfull discharge was replaced by annual average rainfall as the new input variable, watershed parameters (drainage area, forest cover, urban cover etc.) became significant in bankfull width and depth regression equations. This suggested that bankfull discharge in itself encompasses the effects of all the watershed variables and associated processes and thus is sufficient for estimating channel dimensions. Indeed, bankfull discharge based regression equation demonstrated its strong dependence on watershed and rainfall variables.

## Dedicated

This thesis is dedicated to those who lost their lives in the tragic incident of "Uttarakhand flash floods" India, June 2013. May their souls always rest in peace. Apart from them, I would also like to dedicate this thesis to few of my family members who always abided by me and supported me during my entire academic stay at Virginia Tech. My parents "Mr & Mrs Jha" for their unconditional love and care; my sisters in law "Maya Agnihotri and Shraddha Puranik" for always believing in me; my brothers "Amit Jha, Anand Murthy and Abhishek Jha" for their continuous motivation and feedbacks and my school teacher since 5$^{th}$ grade "Mrs Sreekala Madhavan" who has always been a constant source of inspiration to me. This thesis is also dedicated to two more people "Professor Mohd. Afaq Alam" and "Mr. R.K Malhotra". Both with their consistence guidance played a pivotal role in encouraging me to pursue masters in civil engineering.

## Acknowledgments

I would like to express my deepest gratitude to my advisor and committee chair, Dr. Panayiotis Diplas. He has always believed me and encouraged me during the entire period of this research work. Without his support and motivation, this thesis would have been a distant dream. I am indebted to him and feel blessed to have worked with him and learned under him. He has left an everlasting impression on me and my mind and he will always keep on inspiring me in the every phase of life ahead.

I would like to thank my committee members, Dr. Glenn E. Moglen and Dr. Jennifer L. Irish at Virginia Tech for their unending support provided during my research. I would also like to thank Dr. Paolo. R Scardina for his immense help and guidance provided during my academic stay at Virginia Tech.

I would like to thank Dr. Shrey K. Shahi for his incredible assistance and knowledge provided during the statistical analysis of my research work. Without his inputs and help, the analysis presented in this thesis would not have been successfully completed. This thesis also deserves a special acknowledgment to Mr. Sudhir K. Pathak whose continuous feedbacks helped in building the results of my research works.

I thank all my friends at Virginia Tech and India for being there and always supporting me in the hard times: Anisha Nijhawan, Ankur Rathor, Abhinav Gupta, Chaintanya Hedau, Garima Sharma, Karthik, Manisha Rai, Pankaj Agrawal, Ripan Singh, Saloni sood and Shreya Dubey. In the end, I would also thank my lovable brothers and sisters for always making me feel special " Sanjeev, Priyanka, Rahul, Aprajita, Pragya, Sakshi and Samiksha".

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# Chapter 1: Introduction

Rivers show considerable variability in their hydraulic properties and behavioral patterns while moving upstream to downstream. They also keep changing naturally with time due to the effects of several climatic, geological and hydrological variables (Schumm, 2005). In any fluvial system, the morphology of a reach is defined by the combined effects of various complex watershed processes continuously acting on it (Schumm & Litchy, 1965). The dynamic interplay between them and the river makes the stream even more variable in its behavior.

Streams may be classified on the basis of their age into young, mature or old (Davis 1899) or on the basis of their pattern into straight, meandering or braided (Leopold and Wolman, 1957). Culbertson et al. (1967) proposed a classification system which was based on braiding patterns, sinuosity, bank heights, flood plains etc. Later, Rosgen (1994) divided streams into 7 major types on the basis of entrenchment and aspect ratio, sinuosity, gradient and channel material. However, even with the existence of so many classification systems, none of them have been accepted universally till date and there still lies a need to develop a stream classification system that can provide a consistent framework for communicating stream behavior and its properties (Ward & D'Ambrosio, 2008). In this paper, efforts have been made to develop such a framework for addressing streams by identifying them with their elevation property above mean sea level. The guiding principle behind the use of this parameter is the physical property it represents, potential energy, which is the driving mechanism for river flows.

In addition to the new classification system, this work also aims on developing bankfull regime equations for estimating channel properties. The first breakthrough work in this regard was done by Leopold and Maddock (1953) who developed power regression equations for estimating channel width and depth on the basis of its mean discharge value. Significant work in this field was also performed by Parker [1979], Andrews [1984], Parker and Toro-Escobar [2002], Parker et al. [2003] and Millar [2005]. In all these previous approaches only bankfull discharge was used for developing the bankfull equations. Contrary, in this study along with bankfull discharge, several watershed variables and climatic variables have also been quantitatively included in the development of regime equations for predicting bankfull channel properties. These regime equations can serve out to be of significant help in developing natural channel design for various stream restoration purposes, numerical and physical modeling in laboratory.

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# Chapter 2: Classifying streams on the basis of elevation above mean sea level- a statistical approach

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## 2.1 Abstract

In this study, an alternate way of classifying streams, based on elevation rather than median grain size has been examined. To accomplish this goal, 873 field measurements covering stable channel reaches of UK, USA and Canada were compiled and statistically analyzed. The complete dataset was divided into final five elevation zones (0-250ft, 250-1500ft, 1500-3500ft, 3500-5000ft and 5000ft-above) and most probable values of aspect ratio (Ar), channel slope (Sc) and sinuosity (P) occurring together in nature were calculated for each of these zones. Values confirmed that aspect ratio initially increases while moving from lower to higher elevation ranges and then reduces above 5000 ft of elevation. Channel gradient always showed an increasing trend while moving upstream. Sinuosity was found to be high only at the lowest elevation range of 0-250 ft and for all other zones it was observed to be fairly constant. In the later section, dataset of sandy, gravel and cobble streams were divided using these elevation zones. The most probable values of channel variables for each of these channel types showed behavior patterns similar to the one described above when no distinction was made on the basis of grain size. Based on these results it was concluded that elevation based classification is a more suitable universal classification system where hydraulic properties of streams within each elevation zone follow a consistent trend.

**Keywords:** Aspect ratio, Channel gradient, Sinuosity, Elevation, Joint probability distribution, Stream classification, Most probable values

## 2.2 Introduction

During the past 100 years, more than 20 different stream classification systems have been proposed. Streams have been classified on the basis of their patterns, orders, hydraulic geometry, bed material, sediment inputs etc. Davis in 1899 proposed the first recognized classification system where he divided streams in terms of their age (youthful, mature and old age) [Ward & D'Ambrosio, 2008]. Strahler in 1952 introduced the concept of stream order where smallest headwater tributaries were called the first order streams and when two first order streams met a second order stream as formed. Similarly, when two second order streams met a third order stream was formed and so on [Ward & D'Ambrosio, 2008]. The first morphology based classification of stream channels was proposed by Leopold and Wolman (1957) where the streams were distinguished on the basis of their patterns as braided meandering and straight. Later Schumm (1977) came up with another morphology based classification system where he divided streams on their basis of its sediment transport behavior as erosion, deposition or transport streams. Rosgen (1994, 1996) developed a new approach to channel classification system where he divided the streams into four hierarchical levels. He identified these levels with the stream's conditions, morphological descriptions, geomorphic characterization, etc.

Even with the existence of so many available classification systems, none of them have been accepted universally till date and thus there lies a need to develop a classifying technique which could provide a better understanding of the stream's behavior and morphology. In this work, a new parameter "elevation above mean sea level" has been introduced for stream classification purposes and patterns have been identified in the stream behavior as one move from lower to higher elevation reaches. Additionally in this study (using joint probability distribution), an attempt has also been made to understand the existing combinations among stream geometric variables and how occurrence of one affects the other. Most probable values of hydraulic geometric variables existing together have been calculated which can be very helpful to engineers or geo-morphologists in producing stable channel dimensions [Schumm, 1977]. Analysis such as these can lead to a profound understanding of the stream morphology, flow hydraulics and its response to various external watershed activities.

## 2.3 Field data and study sites

The analysis presented in this study was based on a cumulative dataset of 873 field measurement values, compiled by many researchers and reported in various resources published in the past. All these 873 points satisfied the basic criteria of having the corresponding values of at least four major channel variables: Aspect ratio (Ar), channel gradient (Sc), sinuosity (P) and median grain size ($D_{50}$). By using the means of Google earth, the author located the value of "elevation above mean sea level" for each of these 873 field locations and included them in this study as the fifth major channel variable of the dataset. In terms of regions, these field data points covered stable channel reaches from three different countries: Canada, UK and USA. Table 2-1 provides a detailed description of regions covered within each country and the total number of data points belonging to each country.

Table 2-1 : Region based grouping of 873 field data points

| Country | Regions covered | Total no of data points | Type of streams covered based on grain size. | Data source |
|---|---|---|---|---|
| Canada | Yukon, British Columbia, Alberta, Manitoba, Saskatchewan | 92 | Sandy, Gravel & Cobble | Church & Rood (1983) |
| UK | Wales, Scotland, Staffordshire, Lancashire, Herefordshire, Durham county | 74 | Gravel & Cobble | Charlton et all (1978), Hey and Thorne (1986), Church & Rood (1983) |
| USA | Arizona, New Mexico, Oklahoma, Navajo, Missouri, Virginia, Maryland, West Virginia, New York, Montana, Washington state, Florida, Georgia, Alabama, Tennessee, Colorado, Michigan, Kentucky, | 707 | Sandy, Gravel & Cobble | Mccandless (2003), Metcalf (2005), Wirtanen & yard (2003), Elliott et al (1984), Metcalf et all (2009), Brockman et al. (2012), Keaton et al (2005), Horton (2003), Krstolic & Chaplin (2007), White (2001), Lawlor (2004), Mulvihill et al (2009), Dutnell (2010), Sutherland (2003), Cinotto (2003), Moody (2003), Morse (2009), Lotspeich (2009), |

The number of data sources for Canadian and UK streams were limited to three in number whereas the number of sources available for US streams were as large as 18. This explains why the number of data points for US streams is as large as 707 which are approximately 5 times the combined field values of UK and Canada streams. Additionally based on $D_{50}$, the data points from Canada and USA covered streams from all the three types: sandy, gravel and cobble whereas UK streams were either gravel or cobble in nature.

## 2.4 Methods

In the first step, thirteen fine elevation zones were created (0-250ft, 250-500ft, 500-1000ft, 1000-1500ft, 1500-2000ft, 2000-2500ft, 2500-300ft, 3000-3500ft, 3500-4000ft, 4000-5000ft, 5000-6000ft, 6000-7000ft and 7000ft & above ). Based on its respective elevation value, the combined 873 field points were grouped in these elevation zones (Table2-2). For each of these zones, the modal values of the aspect ratio, channel gradient and sinuosity were calculated respectively. Zones with similar modal values of the three channel variables were merged to generate the final five elevation zones. Further, most probable values (MPV) of [Ar, Sc, and P] occurring together in nature for each of these five zones were calculated using joint distribution estimation.

**Table 2 -2 : Distribution of combined 873 field values into 13 finer elevation zones**

| **Elevation range (ft)** | **Notation** | **Number of data points** |
|---|---|---|
| 0-250 | A | 79 |
| 250-500 | B | 89 |
| 500-1000 | C | 158 |
| 1000-1500 | D | 113 |
| 1500-2000 | E | 65 |
| 2000-2500 | F | 58 |
| 2500-3000 | G | 44 |
| 3000-3500 | H | 46 |
| 3500-4000 | I | 31 |
| 4000-5000 | J | 38 |
| 5000-6000 | K | 46 |
| 6000-7000 | L | 51 |
| 7000 + | M | 55 |

The complete statistical analysis in this study was performed using the statistical software R. Using a nonparametric method of “Kernel density estimation (Gaussian kernel)”, probability density functions of the three dimensionless variables (Ar, Sc, and P) were estimated and plotted for each of the final five elevation zones. The peak of each of these probability distribution plots corresponded to the modal values of the variables for the respective elevation zone. Using similar technique of kernel density estimating and smoothing on a fine grid in R, joint probability plots in 3 dimensional forms were also obtained for each of the established five zones. The peak in the plots represented the MPVs of the three variables [Ar, Sc, P] occurring together in the nature.

In the later section, a comparison was made between the grain size based classification system and the elevation based classification which has proposed in this study. The cumulative field measurements of 873 points was divided on the basis of its $D_{50}$ values into groups of sandy, gravel and cobble streams(Table2-3) and central tendency values were calculated for each of these groups. Additionally, the MPVs of [Ar, Sc, P] occurring together were also calculated separately for sandy, gravel and cobble streams. Later, the dataset of each stream type was further subdivided into the established five elevation zones as described above. Median, mode, standard deviations, and MPVs were also calculated for each of these fifteen refined zones and variations observed among the five zones of each stream type were investigated and discussed further.

**Table 2 - 3: Distribution of 873 field values into sandy, gravel and cobble stream regions**

| Channel type | $D_{50}$ range (mm) | Number of data points |
|---|---|---|
| Sandy | 0-2 | 209 |
| Gravel | 2-64 | 450 |
| Cobble | 64 & above | 214 |

## 2.5 Results

### 2.5.1 Formation of final five elevation groups

Table 2-4 highlights the modal values of the three channel variables "Ar, Sc and P" calculated for each of the thirteen elevation zones.

**Table 2 - 4 : Modal values of Ar, Sc and P calculated for each thirteen elevation group**

| Elevation range (ft) | Notation | Mode | | |
|---|---|---|---|---|
| | | Aspect ratio (Ar) | Channel slope (sc, %) | Sinuosity (P) |
| 0-250 | A | 10.5 | .075 | 1.33 |
| 250-500 | B | 16 | .17 | 1.15 |
| 500-1000 | C | 15 | .15 | 1.14 |
| 1000-1500 | D | 17 | .16 | 1.08 |
| 1500-2000 | E | 25 | .22 | 1.13 |
| 2000-2500 | F | 24 | .21 | 1.15 |
| 2500-3000 | G | 23 | .24 | 1.15 |
| 3000-3500 | H | 24 | .25 | 1.12 |
| 3500-4000 | I | 30 | .35 | 1.07 |
| 4000-5000 | J | 31 | .37 | 1.08 |
| 5000-6000 | K | 22 | .50 | 1.13 |
| 6000-7000 | L | 23 | .53 | 1.12 |
| 7000 + | M | 21 | .59 | 1.11 |

The modal value of the aspect ratio was least for the elevation group A. Aspect ratio then significantly increased to an average value of 16 and remained fairly constant for the B, C, D elevation ranges. From E to H, the aspect ratio again increased to an average value of 24 and remained almost same for all the groups between E and H. Aspect ratio again increased for I and J groups and acquired an average value of 30.5. Later for the higher elevation ranges group "K, L & M", the aspect ratio fell down to an average value of 22. Thus one can summarize the complete behavior of aspect ratio as a variable whose value generally increases with the rise in elevation, but finally falls down at higher mountainous elevation regions. At mountainous or upland regions, the river banks are predominantly bedrock and resistant to erosion [Elliott et al, 1984]. Any adjustment to increased discharge compensates mainly by increase in depth and thereby making the aspect ratio comparatively smaller than erosional stream bank channels.

Similar pattern was observed in the modal values of channel gradient for 13 different elevation regions. The first elevation group A followed a unique minimum gradient value of .075%. Channel gradient then increased and remained constant between B and D elevation ranges with an average value of .16%. Elevation groups between E and H were observed to have similar gradient values with an average of .23%. I and J groups also had very close gradient values and the average was .36%. Finally, K, L, M groups were considered to be equivalent in their channel gradient values with an average of .54%. Thus overall, the complete behavior of channel slope can be summoned as increasing with the increasing elevation with the rate of steepness increasing at mountainous region elevations.

Sinuosity showed highest modal value of 1.33 for the first elevation group and for all other higher elevation groups, it remained approximately equal with values ranging between 1.15 and 1.07. At lowland regions, the streams are usually identified with perennial flows, low gradient and fine grained firm cohesive banks that provide necessary conditions for streams to meander [Leopold & Langbein, 1966; Dijk et al 2013].

Based on the merging of elevation groups that displayed similar values of channel properties, the elevation zones were redefined and finally divided into five major zones (0-250ft, 250-1500ft, 1500-3500ft, 3500ft-5000ft and finally 5000ft & above). These five zones (table 2-5) form the basis of the new stream classification system proposed in this study where each zone represents similar channel characteristics.

**Table 2 - 5: Division of cumulative field dataset into 5 distinct elevation zones**

| Elevation range (ft) | Number of data points | Mode | | |
|---|---|---|---|---|
| | | Aspect ratio (Ar) | Channel slope (sc, %) | Sinuosity (P) |
| 0-250 | 79 | 10 | .075 | 1.33 |
| 250-1500 | 361 | 14 | .16 | 1.11 |
| 1500-3500 | 212 | 24 | .23 | 1.14 |
| 3500-5000 | 69 | 31 | .37 | 1.09 |
| 5000-above | 152 | 21 | .54 | 1.13 |

### 2.5.2 Joint probability distribution of channel variables for final five elevation zones

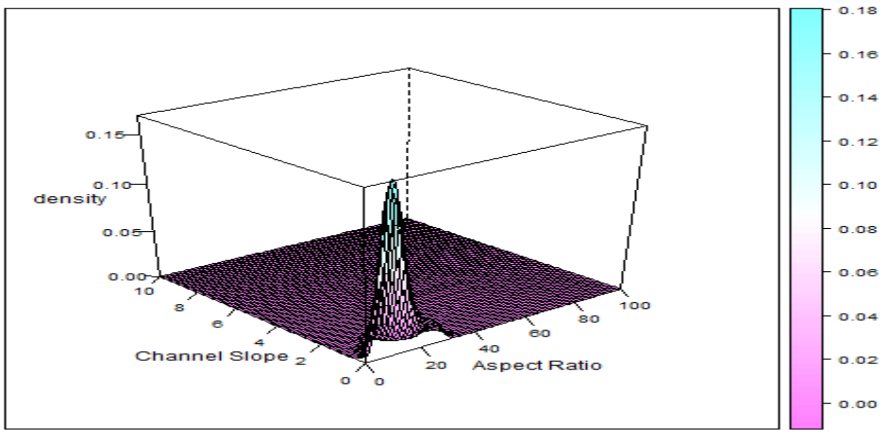
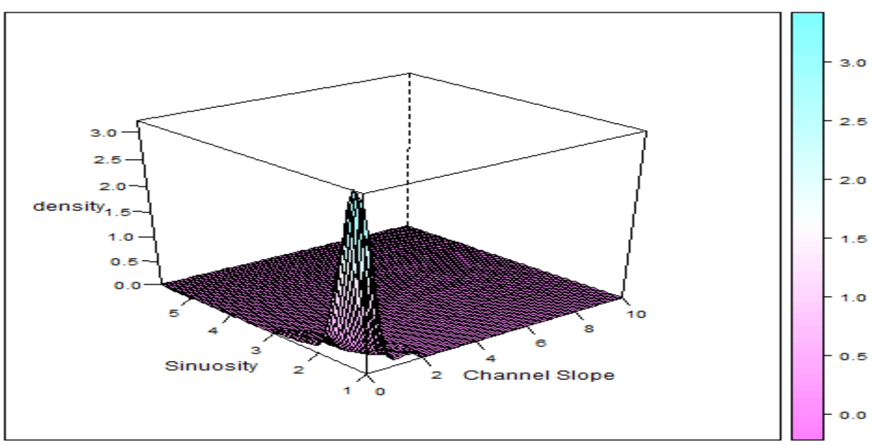
After the establishment of elevation based classification system, the second objective of this paper was to evaluate the interdependency existing among the stream variables and how occurrence of one affected the value of the other. In this section, joint probability distribution was performed among the Aspect ratio, channel slope and sinuosity and the most probable values (MPVs) of these dimensionless variables occurring together in stream world were calculated. These MPVs were calculated for each of the five elevation zones and distinct differences were observed between the MPVs of each group (Table 2.6). Additionally, as described in the method section, three dimensional density plots were also obtained using kernel estimation technique in software R. Visualizing these plots, one could easily identify the peaks where the probability of occurrence of Ar, Sc and P was maximum and thereby determine the combination of the dimensionless variables that is most likely to exist in nature for each elevation range.

**Table 2 - 6: MPVs of Ar, Sc and P for each five elevation zones.**

| Elevation range (ft) | MPVs of [Ar, Sc, P] |
|---|---|
| 0-250 | [10.5, .15, 1.24] |
| 250-1500 | [16.0, .25, 1.14] |
| 1500-3500 | [26.0, .45, 1.16] |
| 3500-5000 | [38.0, .45, 1.14] |
| 5000-above | [22.0,.61, 1.16] |

Table 2.6 clearly suggests that the most probable values of only aspect ratio significantly changed from one elevation range to another. The value of sinuosity was highest for the lowest elevation range and for all other ranges the most probable value of sinuosity was almost constant and centered on the average value of 1.15. As far as channel gradient was concerned, it's most probable value increased from lower elevation ranges to highest mountainous ranges. One would have expected this phenomenon as the steepness of topography usually increases with increase in altitude.

For each elevation range, the three dimensional plots were obtained by taking two variables at a time. The two axes on x-y plane represented the two variables considered under study and the vertical plane represented the joint probability values of these variables. At this point, one must realize that since mathematically it was not possible to draw four variables (three variables and their joint density values) on a three dimensional plots, only two variables at one time were considered. Figure 2.1, 2.2, 2.3, 2.4 and 2.5 represent the respective three dimensional plots of all the five elevation ranges. For all the plots, uni-modal behavior was observed which suggested that for each of the five elevation zones, there occurs a unique set of variable values whose probability of concurrent occurrence is highest for stable channel reaches lying within the respective elevation zone. The peaks in each of these plots represent the most probable value of the variables occurring together. At lower elevation ranges (0-250 ft and 250-1500 ft), one can observe sharp peaks whereas at higher elevation ranges the plots (with aspect ratio as one of the variables) shows broad shaped distributions. This suggests that at higher elevation the variations observed in modal values of aspect ratio would be greater than those observed at lower elevation regions. In some cases for the same elevation zones, there lies considerable difference between modal values of a hydraulic parameter and its corresponding value as a part of MPV calculations. For example, the individual modal value of channel slope in the lowest elevation range of 0-250ft was as low as .075% whereas its value in combination with Ar and P appeared to be twice and equal to .15%. In fact, in all of the five elevation zones, the individual modal values of the channel slope always came out to be less than its corresponding part in MPV. Similarly for the elevation range of 3500-500ft, the individual modal values of aspect ratio was found to be 31 and when calculated in combination with Sc and P, joint modal increased to 38. For all other regions, individual modal values of aspect ratio did not differ much with the corresponding joint modal values. Similarly for sinuosity, it was only for the lowest elevation range that the joint modal value differed from the corresponding individual modal value.

| Combination of variables | 3D plots for elevation range 0-250ft |
|---|---|
| Aspect ratio+ Channel slope |  |
| Channel slope + Sinuosity |  |

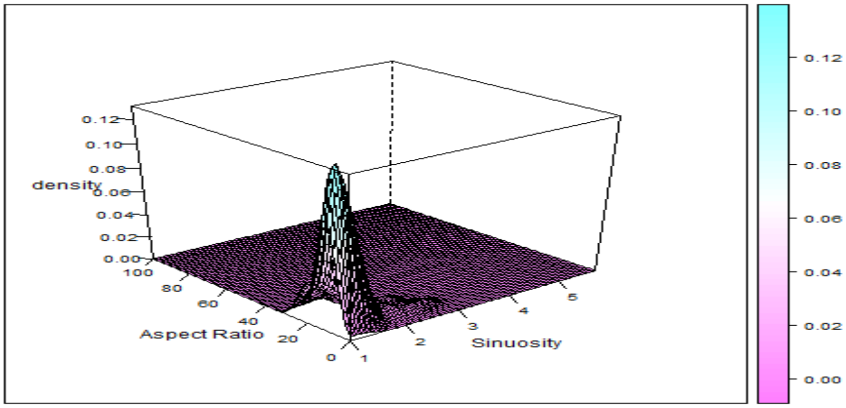
| | |
|---|---|
| Sinuosity + Aspect ratio |   |

**Figure 2 - 1: Three dimensional probability distribution plots for elevation range 0-250 ft.**

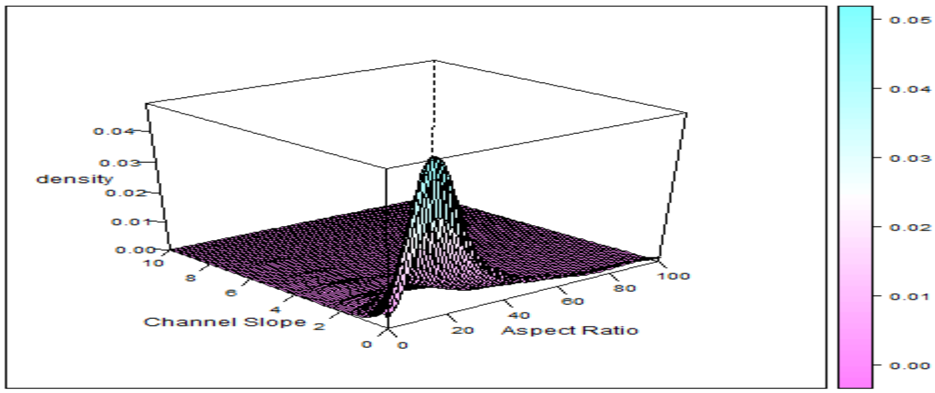
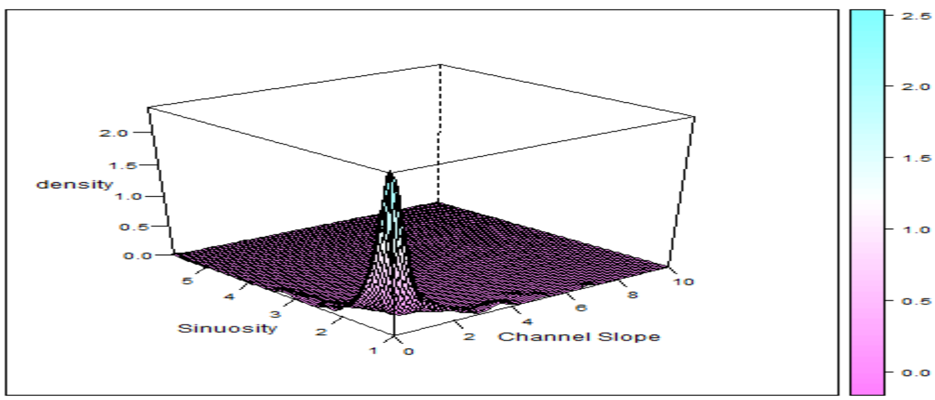
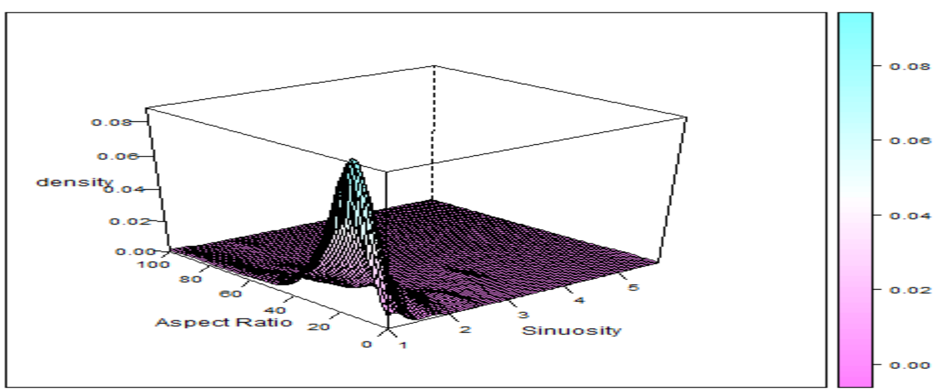
| Combination of variables | 3D plots for elevation range 250-1500ft |
|---|---|
| Aspect ratio+ Channel slope |   |
| Channel slope + Sinuosity |   |
| Sinuosity + Aspect ratio |   |

**Figure 2- 2: Three dimensional probability distribution plots for elevation range 250-1500 ft.**

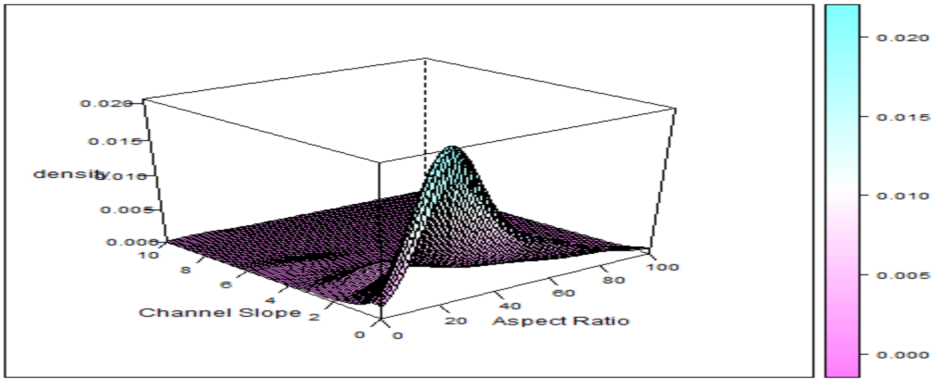
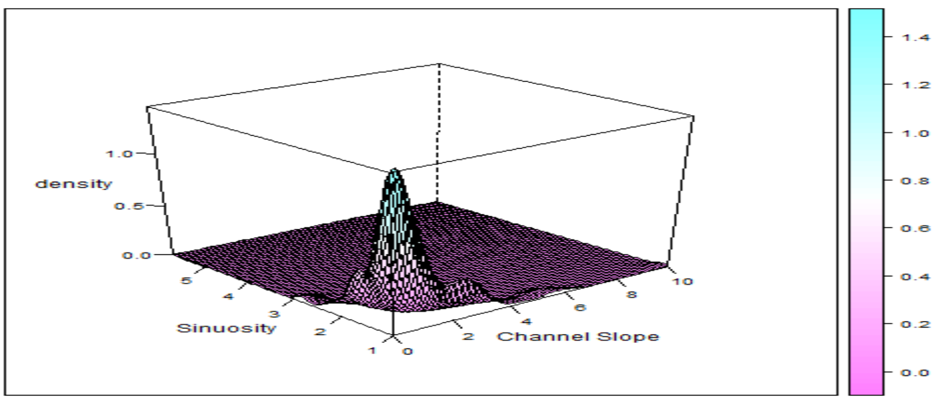
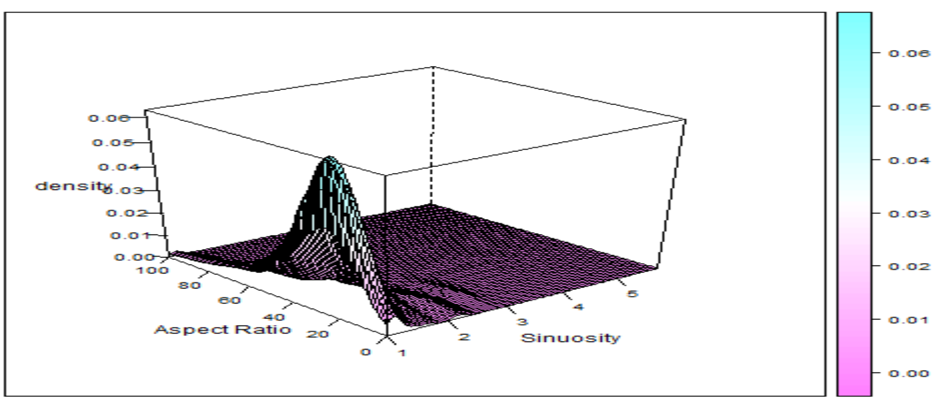
<table>
<tr><td>Combination of variables</td><td>3D plots for elevation range 1500- 3500ft</td></tr>
<tr><td>Aspect ratio+ Channel slope</td><td>
</td></tr>
<tr><td>Channel slope + Sinuosity</td><td>
</td></tr>
<tr><td>Sinuosity + Aspect ratio</td><td>
</td></tr>
</table>

**Figure 2- 3 : Three dimensional probability distribution plots for elevation range 1500-3500 ft.**

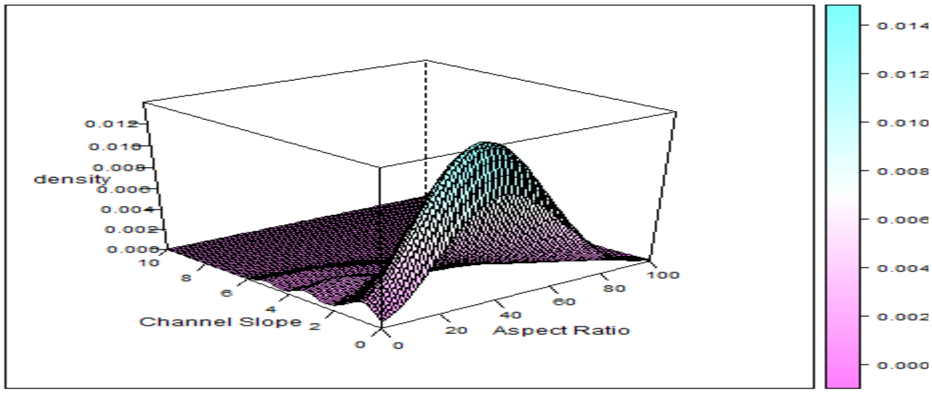
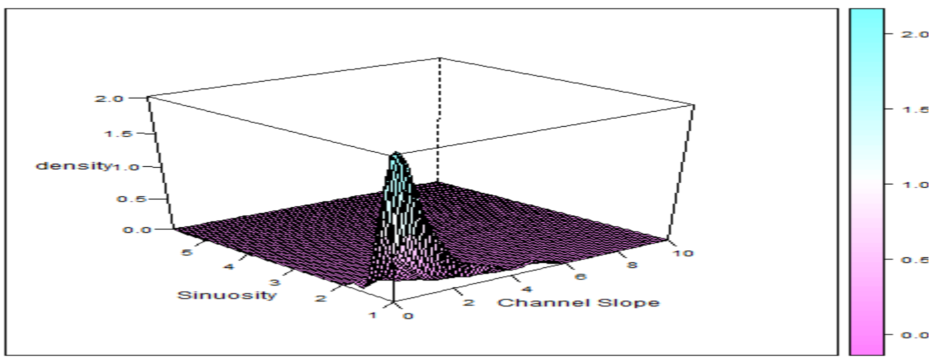
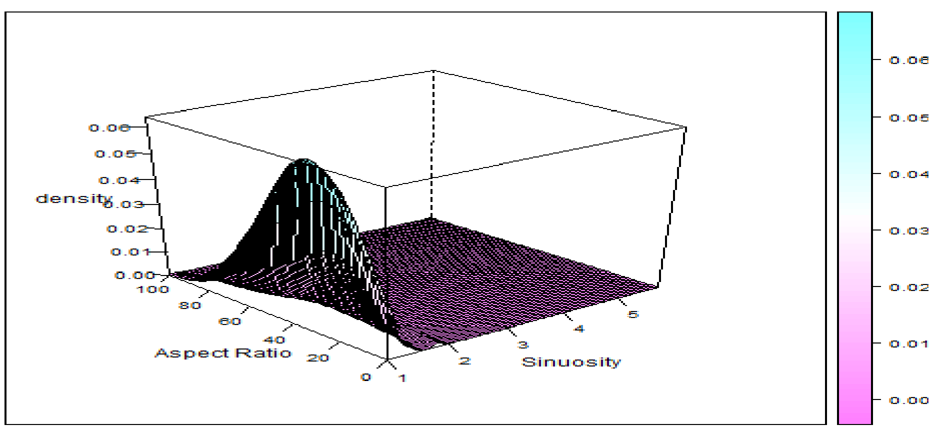
<table>
<tr><td>Combination of variables</td><td>3D plots for elevation range 3500- 5000ft</td></tr>
<tr><td>Aspect ratio+ Channel slope</td><td>
</td></tr>
<tr><td>Channel slope + Sinuosity</td><td>
</td></tr>
<tr><td>Sinuosity + Aspect ratio</td><td>
</td></tr>
</table>

**Figure 2- 4 : Three dimensional probability distribution plots for elevation range 3500 - 5000 ft.**

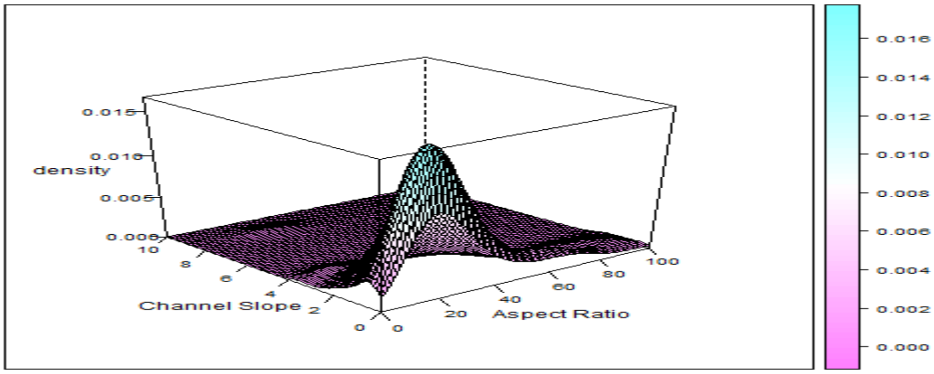
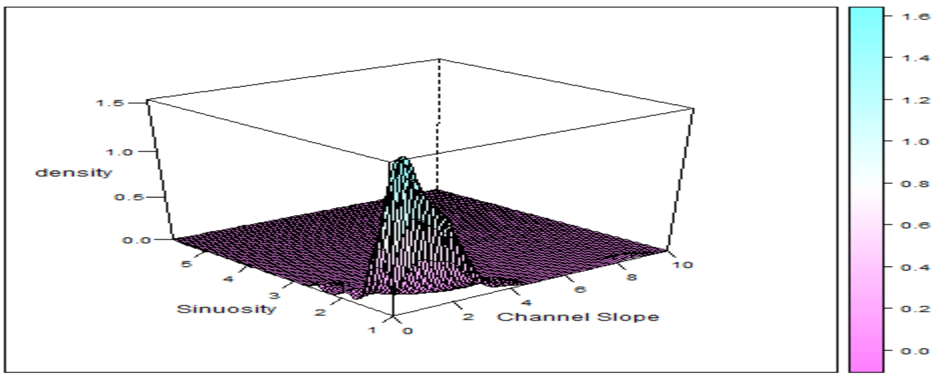
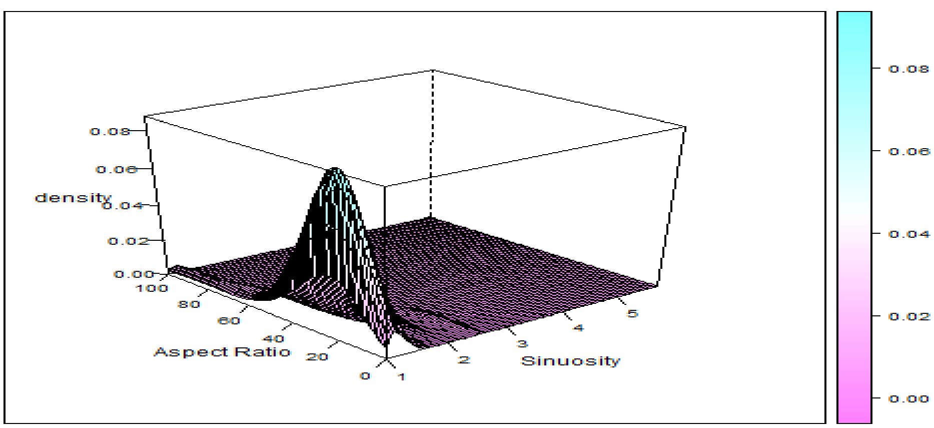
<table>
<tr><th>Combination of variables</th><th>3D plots for elevation range 3500- 5000ft</th></tr>
<tr><td>Aspect ratio+ Channel slope</td><td>
</td></tr>
<tr><td>Channel slope + Sinuosity</td><td>
</td></tr>
<tr><td>Sinuosity + Aspect ratio</td><td>
</td></tr>
</table>

**Figure 2- 5 : Three dimensional probability distribution plots for elevation range 5000 ft. and above.**

### 2.5.3 Elevation based refinement of sandy, gravel and cobble streams

As described in the introduction, the most traditional method of classifying streams has been on the basis of median grain size. Streams with $D_{50}$ less than 2 mm are broadly classified as sandy, between 2-64 mm are classified as gravel, between 64-256 mm are termed as cobble and finally 256 mm and coarser are called as boulder streams. Since in this study only few field points belonged to the boulder streams criteria, these data values were merged with the cobble stream data.

In this section of results, the complete dataset of 873 field values were divided on the basis of median grain size into sandy, gravel and cobble streams. The individual modal values of the three dimensionless parameters were calculated separately for these three stream types. In case of Ar, the modal values increased while moving from sandy streams (16) to gravel streams (25) and finally to cobble streams (33). Similar trend was observed for Sc where the individual modal values of sandy, gravel and cobble streams were found to be .09, .38 and .47 respectively. However for P, reverse trend was observed where sandy streams showed maximum modal value of 1.35, gravel stream showed a modal value of 1.15 and cobble streams displayed a value equal to 1.10. Utilizing the similar concept of kernel density estimation as employed in previous section, joint probability distributions were calculated for each of the three stream types. Table 2.7 highlights the MPVs of sandy, gravel and cobble streams. These values were approximately equal to the individual modal values and once again clearly suggested that sandy streams can be characterized by low aspect ratios, flat channel slopes and high sinuosity. In comparison, gravel streams can be characterized as less meandering, having higher aspect ratios and relatively steeper channel beds. Cobble streams can be characterized with highest aspect ratios and channel slope values and almost straight channels.

**Table 2- 7 : MPVs of channel variable occurring together for sandy gravel and cobble streams**

| Channel type | MPVs of [Ar, Sc, P] |
|---|---|
| Sandy | [15, .1, 1.25] |
| Gravel | [25.0, .40, 1.16] |
| Cobble | [32, .45, 1.12] |

At this juncture of research, the major question which arises is whether the MPVs calculated for the sandy, gravel and cobble streams are the true representative of each channel type. In an attempt to find an answer to this question, sandy, gravel and cobble streams were further subdivided into five established elevation zones and joint probability estimation were exercised on each zone. Tables 2.8, 2.9 and 2.10 highlight the MPVs calculated for each of the 5 elevation zones.

**Table 2- 8 : MPVs of Ar, Sc and P for each five elevation zones of sandy streams**

| Elevation range (ft) | MPVs of [Ar, Sc, P] |
|---|---|
| 0-250 | [10.0, .05, 1.36] |
| 250-1500 | [16.5, .10, 1.16] |
| 1500-3500 | [37, .25, 1.19] |
| 3500-5000 | [31, .25, 1.14] |
| 5000-above | [24.0,.4, 1.19] |

Table 2- 9 : MPVs of Ar, Sc and P for each five elevation zones of Gravel streams

| Elevation range (ft) | MPVs of [Ar, Sc, P] |
|---|---|
| 0-250 | [11, .20, 1.34] |
| 250-1500 | [16.0, .35, 1.19] |
| 1500-3500 | [23.0, .45, 1.19] |
| 3500-5000 | [38.0, .50, 1.14] |
| 5000-above | [19, .8, 1.16] |

Table 2- 10 : MPVs of Ar, Sc and P for each five elevation zones of Cobble streams

| Elevation range (ft) | MPVs of [Ar,Sc, P] |
|---|---|
| 0-250 | [11, .35, 1.27] |
| 250-1500 | [16.0, .35, 1.14] |
| 1500-3500 | [28, .60, 1.16] |
| 3500-5000 | [41 .58, 1.16] |
| 5000-above | [26, 1.15, 1.12] |

From the above tables, it is clearly indicated that the most probable values of the channel variables significantly change with the change in elevation ranges. For sandy streams, till 5000 ft the Ar increased three times from value of 10 to 31 and then dropped down to the value of 19. Exactly similar trend was observed for Ar in case of gravel and cobble streams too. In fact, this pattern of aspect ratio was also seen in the previous result section (2.5.2) when no distinction was made on the basis of stream types.

Also in case of channel gradient, one can observe the values of sandy streams to rise continuously from as low as .05% to as steep as .4%. Even for the gravel streams, the channel slope value increased with the rise in elevation from a value of .2% to .8%. Cobble streams can be observed to have fairly steep channel slopes even at the low elevation ranges. But it follows the identical trend of increasing with the rise in altitude as seen in sandy and gravel streams. In fact at the highest elevation range cobble streams displayed a significantly high gradient value of 1.15%.

For sandy streams, the sinuosity value was observed to be high only at lowest elevation range of 0-250ft and for other ranges the value varied between [1.19-1.14]. Even for gravel streams, the sinuosity was found to be as large as 1.34 at the lowest elevation zone and for other higher zones it showed a value varying between 1.14-1.19. Cobble streams also showed a fairly large sinuosity value (1.27) at the lowest elevation range and for other elevation ranges displayed values located between 1.12-1.16. These observations clearly suggested that even sinuosity followed a similar behavior pattern for all the three channel types.

Based on these above refined MPVs, it can also be observed that at higher elevation ranges such as 3500-5000 ft; the joint modal value of Ar for sandy streams was found to be much greater than the Ar values of gravel and cobble streams at lowest elevation range. Even for Sc, sandy streams at 3500-5000ft showed value greater than the gravel streams Sc value at the lowest elevation range. Both cobble and gravel streams at lowest elevation range, showed sinuosity values much larger than the corresponding sandy streams value at 3500-5000ft region. All these observation clearly rule out the initial understanding that sandy streams are more sinuous, have gentle slopes and low aspect ratio. These hydraulic properties are not dependent on grain size parameter and can be defined better with the help of elevation.

Thus from all the above results, it can be clearly suggested that refining any channel type on the basis of elevation significantly improves the consistency of channel properties. Additionally, it also provides an understanding of how stream properties behave while moving upstream to downstream. The trends observed in aspect ratio, channel slope and sinuosity were not only similar for the each channel type but also for the case when grain size was not employed in dividing the channel type (2.5.2). Based on all these understandings, elevation can be concluded as a pivotal classifying parameter which when employed does have the capability of producing a more universal and consistent classification system.

## 2.6 Summary and Conclusions

Eight seventy three field measurements describing the hydraulic geometry of stable channel reaches from three different geographic regions (UK, USA & Canada) were compiled from various published resources and further divided into 13 fine elevation ranges. The modal values of aspect ratio (Ar), channel slope (Sc) and sinuosity (P) were calculated for each of these 13 ranges. Based on similar modal values observed for these three channel variables, several fine ranges were merged to produce the final five elevation zones : "0-250ft, 250-1500ft, 1500-3500ft, 3500-500ft, 500ft-above". These zones were identified with their unique modal values of dimensionless channel variables and were concluded as the new elevation based classification of stable stream channels into groups of five.

One of the other major objectives of this study was to calculate the most probable values (MPVs) of the channel variables (Ar, Sc and P) that occur together in nature. Joint probability distribution was performed on the five elevation zones and most probable values (MPVs) of Ar, Sc and P occurring together in nature was calculated. These values can prove out to be very useful for designing of irrigation canals or channels. The present work suggests that at equilibrium these channels would tend to follow MPVs of their respective elevation zones. Even while choosing representative channel characteristics for pursuing numerical modeling or physical modeling in the laboratory, these MPVS can be very helpful. Additionally, 3-dimensional plots (representing the combined density values of the three channel variables) were also obtained for each elevation zone. The unimodal behavior of these plots confirmed that for each of the elevation regions only one set of MPVs exist in the stream world. This implies that elevation leads to a single valued function where it produces a unique set of values (Ar, Sc and P).

Based on these MPVs, the behavior pattern of the channel variables were visualized while moving from lower elevation regions to higher ones. Aspect ratio increased from a small value of 10.5 to 38 while moving from 0 to 5000 ft and above 5000 ft, it decreased to a value of 22. Channel gradient always showed a rising trend while moving downstream to upstream with MPV ranging between.15% to .61%. Sinuosity was found to be high only at the lowest elevation range (1.24) and for other ranges it remained below 1.16.

In the final section, the same dataset of 873 field values was divided on the basis of D50 into sandy, gravel and cobble streams. Each of these stream types was further subdivided into the five elevation zones. MPVs were calculated in the similar manner as described above for all the five zones of each channel types. In all the stream types, the aspect ratio initially increased till 5000ft elevation range and then above 5000 ft experienced a dip. For sandy streams, its value of aspect ratio at 3500-500 ft elevation range was found to be 31 which was found to higher than aspect ratio values of cobble and gravel streams at 1500-3500ft elevation range. Channel gradient for all the three stream types always showed a progressive increment in its value while moving downstream to upstream. Even sandy streams at higher elevation ranges displayed channel gradient values equal to .4% which is higher than the channel gradient value of cobble and gravel streams located at 1500 ft. Even sinuosity values followed a trend which was similar to all the three channel types. At lowest elevation ranges, sinuosity was observed to be high for all sandy, gravel as well as cobble streams. Above all the most interesting feature of this refinement was that the trends observed for all the channel variables were not only similar to each other but also to the previous analysis of data of five zones where no differentiation was made on the grounds of grain size.

These results clearly suggest that elevation does have the capability of dictating the channel properties for any stable stream and thus should be utilized in classifying the streams universally.

## Notation

Ar = Aspect ratio

P= Sinuosity

Sc = Channel slope

$D_{50}$ = Median grain size

MPVs = Most probable values

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# Chapter 3: Dimensionless regime equations for predicting stream Morphology from watershed variables

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## 3.1 Abstract

Universal and regional regime equations were developed for predicting bankfull channel geometry (width, depth and channel gradient) of stable channel reaches. Using elevation as one of the independent and repeating variables, dimensional analysis was employed to convert all pertinent fluvial parameters into dimensionless forms. Then, multiple regressions were implemented to derive regime equations. The work in this study was divided into two different sections. In phase I, universal equations were developed using bankfull discharge, drainage area, and channel median grain size ($D_{50}$) and valley slope as the independent/input variables. Three forty nine (349) field data points describing the hydraulic geometry of 13 different states of USA and fifty six (56)data points describing the gravel and cobble streams of UK were employed for this analysis. The equation for channel gradient showed its complete dependency only on valley slope and drainage area and discharge came out to be an insignificant variable. However, in the expressions developed for width and depth, bankfull discharge emerged as the only significant and required variable. All other watershed variables were found to be statistically insignificant. The main reason behind their insignificancy can be understood by realizing that bankfull discharge in itself encompasses the effects of several other watershed variables and thus is more of a dependent output variable than being considered as an independent one. In addition to these universal equations, regional regime equations were also developed using the similar regression approach and they showed behavior similar to the universal equations.

In phase II of this study, new universal and regional regime equations were developed using watershed variables and annual average rainfall as the inputs. Regime equations were

developed not only for width, depth and slope but also for bankfull discharge. For analysis in phase II, a combined dataset of 234 field values were used covering 5 different states of US and several different gravel and cobble streams of UK. Channel gradient equation did not show any difference with the phase I expression as in both phases it was found to be dependent only on valley slope and drainage area. Unlike phase I, all the watershed variables in phase II such as drainage area, valley slope and $D_{50}$ emerged statistically significant in determining width and depth. The bankfull discharge equation also showed its major dependency on rainfall and drainage area, but was found to be independent of valley slope. In phase II, regional regime equations were also developed separately for 6 different regions of UK and USA and all of them showed similar behavior to the universal equations.

In both of these models, the validity of the universal equations was checked against two independent datasets covering stream morphology values of Ohio and Wyoming. Equations from both models delivered satisfactory performance in predicting the width and depth, with model I giving better results. Additionally, the universal and regional regime equations in both models were also statistically verified with the help of residual error scatter plots and the frequency distribution plots.

**Keywords**: Bankfull geometry, nondimensionalising parameter, multiple regression, residual errors, dimensionless regime equations, stream power

## 3.2 Introduction

Stream properties   reflect the outcome of coupled watershed processes subject to precipitation forcing.  Initial relief, geology, climate, vegetation density, drainage area, runoff and sediment supply are few variables which influence these processes and dictate the outcome(s), the morphology of any stable channel reach [Schumm and Litchy, 1965].  Stream responds to perturbations released into the fluvial system (such as climatic change, tectonic activities, etc.) by changing or adjusting the value of its properties such as bed armoring, aspect ratio, sinuosity, channel gradient etc. Extend of these changes vary according to the degree of freedom that a stream has (or can employ) to adjust to a new equilibrium condition. Additionally these changes also depends on the threshold values of the stream properties where disturbance will typically alter the grain size first (sandy streams), bedforms second, aspect ratio third and finally channel gradient in the end [Buffington, 2012]. In this way the stream not only helps itself in achieving the dynamic equilibrium but maintains a state of balance between the input and outputs of the fluvial system.

The main objective of this study is to quantify the effects of all the above watershed variables and the threshold limits of stream properties and thus develop empirical relations that can be used universally for any stable channel reach. Lately due to severe manmade disturbances (such as urbanization, straightening of the stream, etc.) and climatic changes, various stream channels all across the globe have degraded, which in effect has deteriorated the water quality and the aquatic life in the streams. It is in these situations that the empirical or regime equations developed can play instrumental role in restoring the degraded streams back to their stable forms [Johnson, 2008].

The first primal work in the development of regime equations can be traced to early 20th century, when Lindley (1919) proposed the regime theory for understanding alluvial channel behavior and designing canals in Indian subcontinent. Later, Lacey (1929-30, 1946, 1957-58) worked further on the regime concept and developed equations for calculating mean depth, channel slope, wetted perimeter, etc in terms of mean discharge and lacey's factor [Singh, 2003]. One of the major drawbacks associated with Lacey's equations was the lack of dimensional homogeneity and inconsistency in its performance. It was only in the year 1953 that Leopold and Maddock established following relationships as power functions of mean discharge:

$$W = aQ^{b}, \quad d = cQ^{f}, \quad V = kQ^{m}$$

$$n = NQ^{p}, \quad S = sQ^{y}, \quad L = pQ^{j}$$

where W is the channel width, d is the flow depth, V is the velocity, Q is the flow discharge, n is the manning's roughness factor, S is the slope, L is the rate if sediment transport ; and a, b, c, f, k, m ,N, p, s, j and y are parameters. These empirical relations were based on 63 different stream reaches from the state of Wyoming, Montana, Kansas, Nebraska and few others. Average value of the exponents computed over these cross sections came out to be: b=.26, f=.4 and m=.34 and for downstream geometry came out to be b=.5, f=.33-.4 and m=.1-.17. The major problem with Leopold's model was again its dimensional non homogeneity and thus inability to reveal the physics underlying the relations. A significant improvement over the development of empirical equations appeared during the era of 1979 to 2005, when various researchers such as Parker [1979], Andrews [1984], Parker and Toro-Escobar [2002], Parker et al. [2003] and Millar [2005] developed dimensionless forms of equations for predicting bankfull geometry of single thread gravel bed streams [Parker, 2007]. Considerable work based on theoretical formulation of the problem has been undertaken as well (e.g. Diplas and Vigilar 1992, Vigilar and Diplas 1997 & 1998).

Even though, several approaches and equations have been proposed in the past, there still lies missing a universal model that could be employed in predicting channel properties of any stable reach. Parker et al. [2007] did come up with quasi universal dimensionless relations for gravel streams, but the equations were confined only to single thread gravel bed streams. Later Wilkerson and Parker [2011] developed quasi universal relationships for sandy streams, but again these equations were confined for single thread sand bed rivers. In this study an attempt has been made to develop such universal dimensionless regime equations that won't be bounded by any channel type or regional limitations. Based on a cumulative dataset of four hundred and five field measurement values, universal regime equations for predicting bankfull channel geometry of any stable stream type (using bankfull discharge and watershed variables as the inputs) were developed. In this paper new form of bankfull relations have also been developed using precipitation as the new input variable in place of bankfull discharge. The study also introduces elevation as the new key parameter which could be used in place of median grain size for non dimensional purposes.

## 3.3 Field data and study sites

Channel geometry and watershed morphology of stable stream channels from various regions of UK and USA were considered for this study. The amount of work accomplished by the USGS in collecting channel morphology data for USA streams/creeks is so enormous that around 85%

of the sites considered in this study belong to 13 different states of USA and rest 15% of the data belong to streams located in United Kingdom. The overall geographic area of USA is divided into 8 different physiographic regions and 25 provinces where all regions and provinces show significant differences amongst themselves in terms of precipitation, runoff, climate, topography, tectonic activities etc. For example, the Great Plains province in south eastern Wyoming is characterized by a high elevation range of 1100ft to 7500ft, coarse gravel and cobble streams, semi arid climate and 10-20 inches of average rainfall occurring annually. In comparison coastal plain physiographic province in north-west Florida is characterized by low elevation range of 75 to 405 ft, sandy streams, humid sub-tropical climate and 52-65 inches of average annual rainfall. Picking up data points belonging to all of these 8 physiographic provinces assure that even if most of the stream channels in this study are located in USA, they can be considered representative of streams all across the world.

## 3.4 Methods applied

In this study, we aim to develop dimensionless regional and universal regime equations that can be used to estimate bankfull channel geometry from watershed variables. The complete work of this study can be divided into two phases. For each phase the method applied can be summarized as a three step process where multiple regressions are performed on the non dimensional variables of a large stream morphology dataset.

### 3.4.1 Compilation of channel morphology data

Channel morphology data of four hundred and five (405) stable reaches from UK and USA were compiled from various published sources. Number of data points belonging to USA was three forty nine (349) and number of data points belonging to UK was fifty six (56). Three major criteria's were considered while short listing these field measurement values. First, the streams should be stable and in quasi equilibrium state. Second, each field point should have values of at least the following stream and watershed variables: bankfull discharge, drainage area, bankfull width and bankfull depth, channel gradient, median grain size, elevation and valley slope. Third, bankfull discharge should have been measured in a direct way rather than assuming a flood of a certain return period. Considering these criterions more than fifteen hundred (1500) procured field data points were neglected and a final cumulative set of 405 points was prepared. It was on the basis of these 405 field points that dimensionless regime equations were formulated. Additionally, a separate dataset of 72 field points belonging to Ohio & Wyoming State were also prepared which had values of all the stream and watershed variables as mentioned above except the valley slope. The prime reason of including these 72 points in our analysis was to evaluate the validity of the dimensionless relationship developed between bankfull discharge and channel geometry in our study.

For establishment of different dimensionless regime equations, the complete dataset of 405 points was divided or classified into two different ways. In the first way, all the data points were divided region wise into 11 different groups. In the second way, the 405 points were divided on the basis of median grain size into groups of sandy, gravel and cobble. The complete break up of each of these classification systems have been represented in Table 3.1.

**Table 3- 1 : Region wise and grain size based classification of field data**

| 1. Region wise data classification | | |
|---|---|---|
| **Region/State** | **Number of Data Points** | **variables covered** |
| Arizona | 27 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR |
| Colorado | 18 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR |
| Florida, Georgia, Alabama, Tennessee** | 31 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR |
| Kentucky | 29 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv |
| Maryland | 57 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR |
| Missouri | 35 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR,U,F,G |
| Montana | 50 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR |
| New Mexico | 27 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv |
| Virginia | 17 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR,F, U |
| Washington | 58 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR |
| UK Gravel Rivers | 56 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, Sv, AAR,Qs, Vegetation |

| 2. Grain size based classification | | |
|---|---|---|
| **Stream type** | **D50 range (mm)** | **Number of Data Points** |
| Silt/Sandy | 0.0-2.0 | 90 |
| Gravel | 2.0-64.0 | 214 |
| Cobble & coarser | 64.0 and above | 101 |

** Since there was a paucity of data for the individual regions of FL, AL, Tenn. and GA, these states were combined in the regional analysis. Moreover stream locations covered from all these states were found to be located commonly in the coastal plain region which is a major division of Atlantic plain physiographic region.

Apart from the above 405 data points, additional 72 field values covering stable reaches of Wyoming and Ohio were also utilized for verification purposes (Table. 3. 2)

**Table 3-2 : Region wise classification of independent dataset**

| Region/State | Number of Data Points | variables covered |
|---|---|---|
| Ohio | 37 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, AAR |
| Wyoming | 36 | $Q_{bf}$, W, d, Sc, DA, $D_{50}$, Elev, AAR |

### 3.4.2 Conversion of dimensional variables into non-dimensional forms

**Differentiating between input and output variables**

Before proceeding with this conversion step, it was necessary to divide the variables into groups of input and output variables.

During the first phase following group was made:

| | |
|---|---|
| *Input Variables* | *$Q_{bf}$, Qs, Sv, F, U, G, $D_{50}$, DA, Elev, Vegetation type* |
| *Output Variables* | *W, d and Sc* |

In the second phase, bankfull discharge was considered as an output rather than an input and regime equations were developed not only for channel geometry but also for the bankfull discharge. In this case, a new variable, annual average rainfall (AAR) was introduced as the input. Grouping of variables for this phase can be shown as:

| | |
|---|---|
| *Input Variables* | *AAR, Qs, Sv, F, U, G, $D_{50}$, DA, Elev, Vegetation type* |
| *Output Variables* | *$Q_{bf}$, W, d and Sc* |

**Application of Buckingham Pi theorem**

Using the concept of Buckingham Pi theorem, all the input and output variables mentioned above were converted into dimensionless forms. Unlike Parker et al. [2007] who used D50, ρ and g as the repeating variables, in this study Elev, ρ and g were as the repeating ones. Table 3.3 shows the non dimensional terms formed using both of these sets of repeating variables**:

**Table 3-3: Dimensionless forms of watershed and stream variables**

| Variable | Dimensionless form using Elev, rho and g | Dimensionless form using D50, rho and g |
|---|---|---|
| Bankfull Discharge, $Q_{bf}$ | $\frac{Qbf}{(\sqrt{gD50}) \times D50^2}$ | $\frac{Qbf}{(\sqrt{gElev}) \times Elev^2})$ |
| Drainage Area, DA | $\frac{DA}{D50^2}$ | $\frac{DA}{Elev^2}$ |
| Annual Average Rainfall, AAR | $(\frac{AAR}{D50})$ | $(\frac{AAR}{Elev})$ |
| Sediment supply, Qs | $\frac{Qs}{(\sqrt{gD50}) \times \rho \times D50^2})$ | $\frac{Qs}{(\sqrt{gD50}) \times \rho \times Elev^2)}$ |
| Bankfull width, W | $\frac{W}{D50}$ | $\frac{W}{Elev}$ |
| Bankfull depth, d | $\frac{d}{D50}$ | $\frac{d}{Elev}$ |
| Elevation above mean sea level, Elev | $\frac{Elev}{D50}$ | ---- |
| Median grain size, D50 | ----- | $\frac{D50}{Elev}$ |

** Variables (Sv, Sc,F,U,G) are unaffected by these two different methods.

### 3.4.3 Multiple regression analysis on the dimensionless variables

In the final step, best fit relationship was determined between the output and input variables using multiple regression technique. All the dimensionless input and output variables were converted into logarithmic forms and linear regression was performed between them. Regressions were kept on repeating till only statistically significant terms remained in the final equation. This whole method of formulating final regime regression equations can be termed as

"Backward stepwise regression on log transformed values" For the ease of utilization; these linearly regressed relationships were converted and presented in their respective power forms. Additionally, to indicate the strength, the $r^2$ values of the linearly regressed logarithmic relationships were also provided alongside the power equations.

Phase 1: The universal equations developed for the first phase of the study (when inputs used were Qbf, DA, D50, Sv, and Elev) were expected to have the following forms:

$$\frac{\mathrm{W}}{\mathrm{Elev}} = a\,\left(\frac{\mathrm{Qbf}}{(\sqrt{\mathrm{gElev}})\times \mathrm{Elev}^2}\right)^{p1} \left(\frac{\mathrm{DA}}{\mathrm{Elev}^2}\right)^{q1} \left(\frac{\mathrm{D50}}{\mathrm{Elev}}\right)^{r1} (\mathrm{Sv})^{s1}$$

$$\frac{\mathrm{d}}{\mathrm{Elev}} = b\,\left(\frac{\mathrm{Qbf}}{(\sqrt{\mathrm{gElev}})\times \mathrm{Elev}^2}\right)^{p2} \left(\frac{\mathrm{DA}}{\mathrm{Elev}^2}\right)^{q2} \left(\frac{\mathrm{D50}}{\mathrm{Elev}}\right)^{r2} (\mathrm{Sv})^{s2}$$

$$\mathrm{Sc} = c\,\left(\frac{\mathrm{Qbf}}{(\sqrt{\mathrm{gElev}})\times \mathrm{Elev}^2}\right)^{p3} \left(\frac{\mathrm{DA}}{\mathrm{Elev}^2}\right)^{q3} \left(\frac{\mathrm{D50}}{\mathrm{Elev}}\right)^{r3} (\mathrm{Sv})^{s3}$$

The regional equations developed for this phase of study followed similar power format and transformation technique as adopted during universal regression analysis. However for some regions which had more input variables available in their dataset (such as forest cover, urban cover, sediment supply, grass cover and vegetation), the equation had more number of terms in the right hand side.

Phase 2: For the second phase of our study (when inputs were AAR, DA, D50, Sv and elevation), the universal power equations developed for bankfull discharge, width, depth and channel gradient had the following forms:

$$\frac{\mathrm{Qbf}}{(\sqrt{\mathrm{gElev}})\times \mathrm{Elev}^2} = d\,\left(\frac{\mathrm{AAR}}{\mathrm{Elev}}\right)^{p4} \left(\frac{\mathrm{DA}}{\mathrm{Elev}^2}\right)^{q4} \left(\frac{\mathrm{D50}}{\mathrm{Elev}}\right)^{r4} (\mathrm{Sv})^{s4}$$

$$\frac{W}{Elev} = a\left(\frac{AAR}{Elev}\right)^{p1}\left(\frac{DA}{Elev^2}\right)^{q1}\left(\frac{D50}{Elev}\right)^{r1}(Sv)^{s1}$$

$$\frac{d}{Elev} = b\left(\frac{AAR}{Elev}\right)^{p2}\left(\frac{DA}{Elev^2}\right)^{q2}\left(\frac{D50}{Elev}\right)^{r2}(Sv)^{s2}$$

$$Sc = c\left(\frac{AAR}{Elev}\right)^{p3}\left(\frac{DA}{Elev^2}\right)^{q3}\left(\frac{D50}{Elev}\right)^{r3}(Sv)^{s3}$$

Again, in case of deviation from standard normal distribution of residual frequency curves, box cox transformation technique was employed. Also, the regional equations developed here, depending on the additional input variables available had few more extra terms on the right hand side.

In both these models, the statistical significance of the input exponents was checked using the concept of hypothesis testing. Null hypothesis referred to exponents being significant and alternate hypothesis referred to exponents being statistically insignificant. During the regression analysis, p value of each input exponent was determined and compared with the value of significance level α of .05. If the p value was found to be less than α, null hypothesis was rejected and the respective exponent and thus the input variable was considered to be statistically insignificant [Rice, 2007].

The validity and quality of the regressions performed were checked by the behavior of its residual plots. Residual errors produced by regression were plotted against the value of each significant input variable. If the residual plots showed a discernible pattern and was not evenly distributed about the horizontal axis, the log transformed linear regression was considered to be invalid and a new regression model was adopted. Additionally, the frequency distribution of residuals was also plotted. Ideally these probability distributions should be normally distributed with skewness equal to zero and kurtosis value equal to 3. In case the distributions came out to be significantly skewed, box cox transformation technique was employed to make the residual data normal. In box-cox transformation, the output variable was raised to an appropriate exponent value (.5, 2,-.5,-2,-1, etc.) following which regression was performed between the output and input variables. The new residuals were now seen to follow a normally distributed behavior [Cox, 1964].

## 3.5 Results

Since the complete work involved in this study was divided into two phases, it would be more appropriate to discuss results phase wise and thus divide it into two sections. In the first section, bankfull regression equations were developed for bankfull width, depth and channel slope using bankfull discharge and watershed variables as the input variables. In the second section, bankfull regression equations were developed for bankfull discharge, width, depth and channel slope using annual average rainfall and watershed variables as the inputs. Additionally in a separate section (III) section, a comparison was made between elevation, median grain size and drainage area based regression equations to determine which of these three serves better and physically more meaningful nondimensionalising parameter.

### 3.5.1 Section 1: Results obtained from phase 1 of the study

Equations developed in this phase can be thought of two different kinds: universal equations developed on the basis of cumulative 405 points dataset and regional equations developed on the basis of their respective individual datasets. For each of these kinds, the input and the output variables were always nondimensionlized using average reach elevation, g and ρ as the three repeating variables. Therefore, it won't be wrong and indeed would be more pertinent to address this approach as "elevation based approach of formulating regression regime equations".

### Establishment of universal regime equations:

Restating again, the universal equations presented in this subsection have been developed on the basis of the cumulative dataset comprising of 405 field points. These 405 field points correspond to all the eight physiographic regions of USA and major counties/states of UK and thus confirm the tag of universality to these equations. As mentioned in the method section, the output variables for which these equations have been developed were bankfull width, depth and channel slope and the input variables considered for formulating these equations were bankfull discharge, drainage area, median grain size and valley slope.

i) **Universal Width equation**: Performing linear regression (using Microsoft excel)on the logarithmic values of width and the inputs, following universal equation in power form was obtained:

$$\frac{W}{Elev}=11.7\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.43}\left(\frac{DA}{Elev^2}\right)^{-.02}\left(\frac{D50}{Elev}\right)^{.015}(Sv)^{-.048},\ R^2 = .93$$

In the above regression, the p value of only discharge exponent came out to be less than the significance level of .05. Rest all other showed p values greater than .05 and thus were suggested to be relatively insignificant in predicting bankfull channel width.

Neglecting the insignificant variables, the regression analysis was once again performed and following final equation in power form was developed.

$$\frac{W}{Elev}=7.7\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.40} \quad , R^2 = .96$$

The residual errors scatter graph plotted against the corresponding bankfull discharge values (Figure: 3.1) was found to be widely scattered about the horizontal axis. Even the histogram frequency plot of the residuals (Figure: 3.1) were observed to be normally distributed with a skweness value of .03 and kurtosis equal to 2.9. Both these plots clearly suggested that the regression approach adopted in developing the width equation were statistically acceptable and valid. Additionally, the high $R^2$ confirmed that the regression model developed had good prediction strength.

ii) **Universal depth equation:** Formulation of universal bankfull depth equation followed the exact same set of procedure as implemented during development of width equation. For the linear logarithmic regression analysis considered here, the output variable was the logarithmic values of bankfull depth and input variables were the logarithmic values of bankfull discharge, drainage area, median grain size and valley slope (all in dimensionless forms). The regression resulted in formulation of equation having the following power form:

$$\frac{d}{Elev}=.38\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.42}\left(\frac{DA}{Elev^2}\right)^{-.000082}\left(\frac{D50}{Elev}\right)^{.015}(Sv)^{-.08}, R^2 = .97$$

Once again the p values of all the watershed variables were found and to be greater than .05 and thus were concluded to be statistically trivial. Performing regression analysis again with only discharge as the input variable, following bankfull depth relation in its final form was obtained:

$$\frac{d}{Elev} = .70\left(\frac{Qbf}{(\sqrt{gElev}) \times Elev^2}\right)^{.43} \quad , R^2 = .97$$

With an $R^2$ value as high as .97, the above depth equation can be suggested to have a strong predicting capability. Also, the residual error scatter plot for the above regression (Figure: 3.1) did not show any trend with the bankfull discharge and were found to be equally distributed about the x axis. Even the histogram frequency plots of the residuals (Figure 3.1) followed a standard normal distribution.

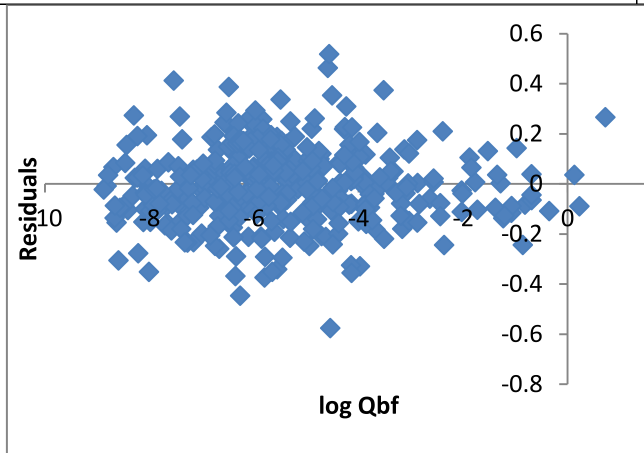
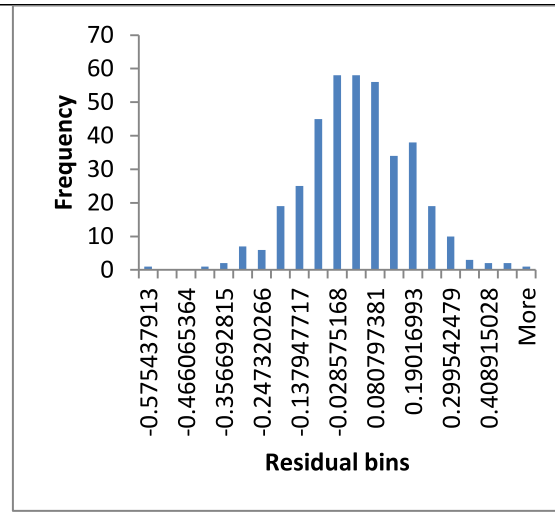
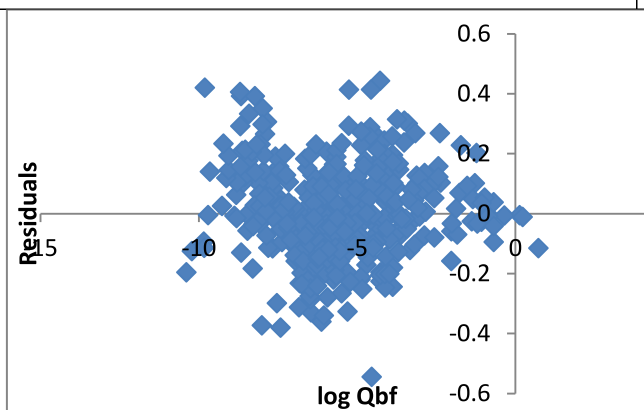
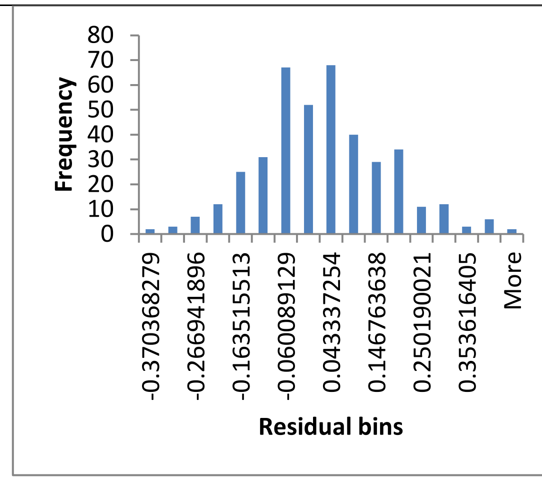
<table>
<tr><th>Channel Variable</th><th>Residual Plot</th><th>Frequency distribution plot</th></tr>
<tr><td>Bankfull width</td><td>
</td><td>
</td></tr>
<tr><td>Bankfull depth</td><td>
</td><td>
</td></tr>
</table>

**Figure 3 - 1: Residual error and frequency distribution plots for width and depth regression**

**iii)** **<u>Universal channel gradient equation:</u>** Developing the equation for predicting channel gradient can be considered as one of the most challenging works of this study. In the initial attempt, same approach of performing linear regression analysis on logarithmic values was followed. The regression equation using this model came out to be :

$$Sc=.89\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.00005}\left(\frac{DA}{Elev^2}\right)^{-.023}\left(\frac{D50}{Elev}\right)^{.008}(Sv)^{.97655}$$

The p values of the exponents of two variables (Qbf, D50) were found to be greater than significance level value α of .05. This clearly suggested exclusion of these terms from the above regression and developing a new equation having the following reduced form:

$$Sc=.0001\ \left(\frac{DA}{Elev^2}\right)^{-.02}\ (Sv)^{.98}$$

Even though the r-square value of this reduced equation was calculated to be as high as .98, the relationship was rejected on the basis of its residual plots (Figure 3.2 and 3.3) and the skewness of its probability distribution plot (Figure 3.4). The plots conveyed existence of a non random pattern in the residuals and were found to be skewed to the right.

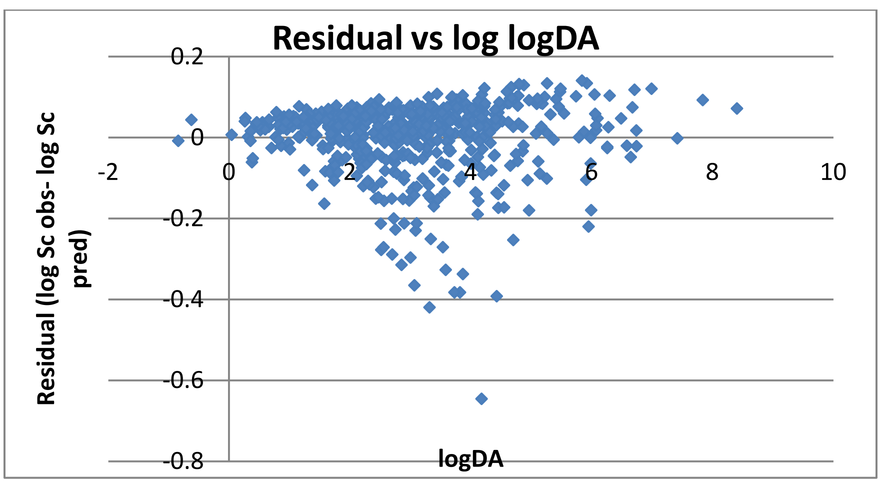



**Figure 3-2 : Residue in logarithm value of channel slope versus logarithmic value of drainage area**

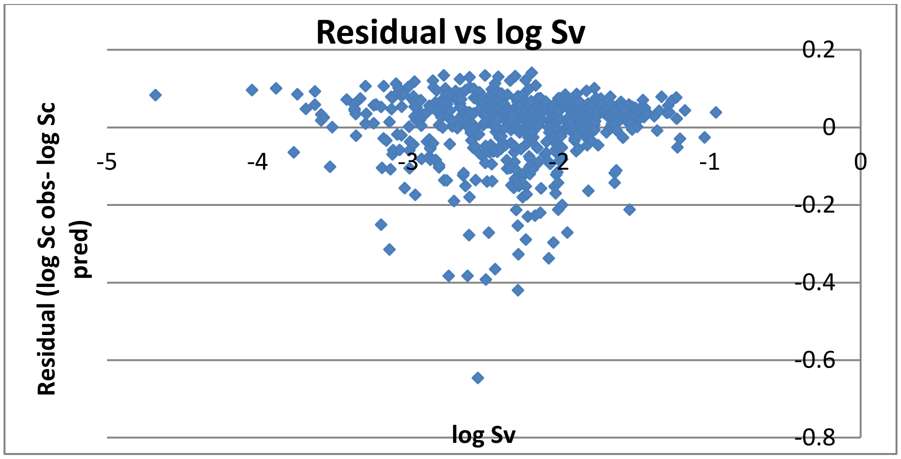



Figure 3- 3 : Residue in logarithm value of channel slope versus logarithmic value of valley slope

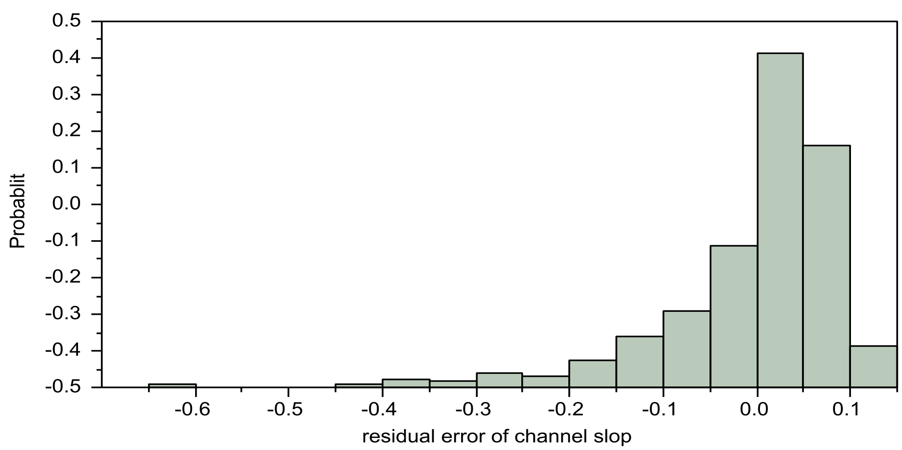



Figure 3- 4 : Probability distribution of channel slope residuals

Application of Box-Cox power transformation:

Since the above regression model of channel gradient was found to be statistically unacceptable, a new approach for determining the correct model was adopted. Starting with the basic relationship of calculating channel slope ($Sc=\frac{Sv}{P}$), this approach was directed towards finding the expression for sinuosity (P). Once the regression equation of sinuosity was developed it could be easily used to estimate channel slope. Based on the inputs of bankfull discharge, median grain size and drainage area, following equation was obtained for predicting sinuosity.

$$P=1.06\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{-.0014}\left(\frac{DA}{Elev^2}\right)^{.0176}\left(\frac{D50}{Elev}\right)^{-.0023}$$

Based on the concept of p value of the coefficients, discharge and grain size were concluded to be insignificant terms and were neglected. The new reduced equation for sinuosity estimation came to be:

$$P=1.12\left(\frac{DA}{Elev^2}\right)^{.016}$$

However, the probability plots of the above equation once again proved it to be rightly skewed. To overcome this skewness, Box-Cox transformation approach was applied on the regression model [Cox, 1964]. Using Box-Cox, an appropriate exponent was identified for the output variable (sinuosity) which would transform the residual data into a normal distribution. Using this technique the equation came out to be:

$$\sqrt{\log(P)} = .21 + .024\log\left(\frac{DA}{Elev^2}\right)$$

The residual and the probability plot (Figure 3.5 and 3.6 respectively) for above model confirmed that the expression was acceptable and the data had a standard normal distribution of its residual.

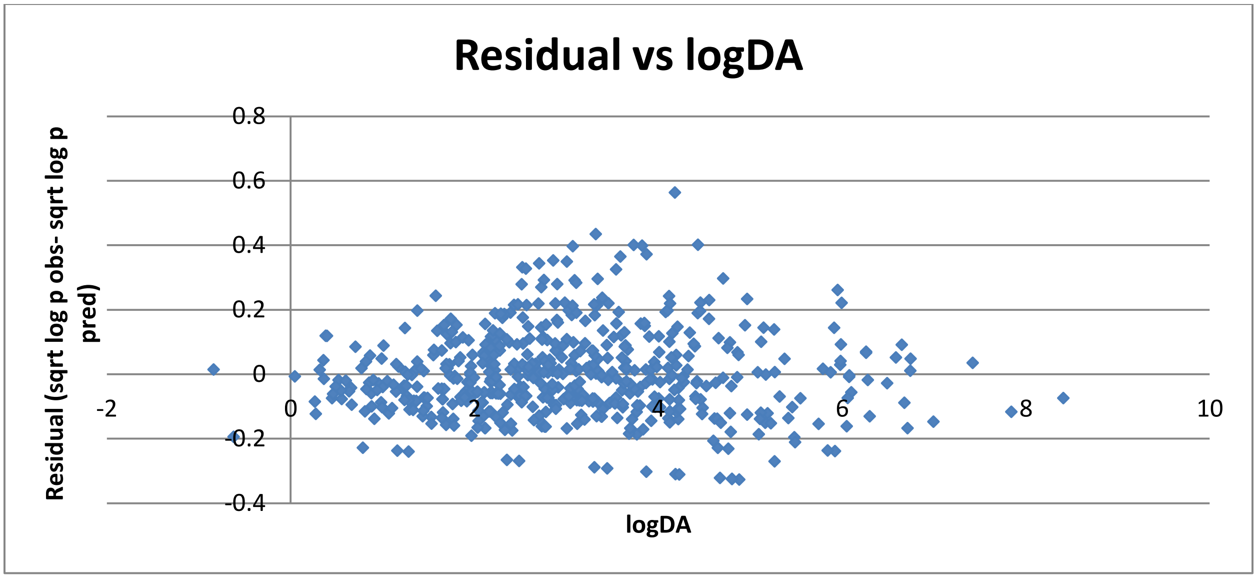



**Figure 3- 5 : Residual error scatter plot versus log DA after box cox transformation**

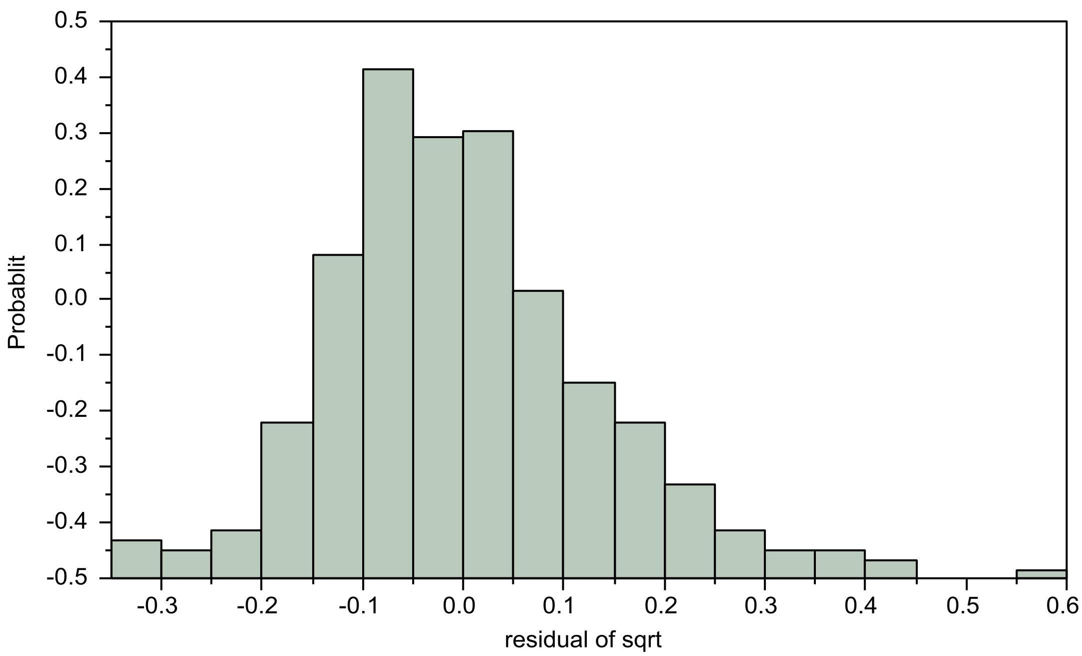



**Figure 3- 6 : Probability plot of residual error after box cox transformation**

Hence, the final acceptable expression for predicting channel slope can be written as:

$$Sc= \frac{Sv}{10^{(.21 + .024 \log \frac{DA}{Elev^2})^2}}$$

Looking at the small exponent values of drainage area in the above equation, one may argue that drainage area does not have much influence over the prediction of channel gradient and thus may be removed from the analysis. Countering it, the author would like to say that drainage in the above equation helped in actually predicting the sinuosity first which in turn was later utilized in making the channel slope prediction. Since sinuosity usually differs by only a small margin of values, even the minimal contribution of drainage area in predicting it cannot be ignored.

**iv) <u>Summarizing the universal equations in their final forms :</u>**

The final results obtained in this subsection have been summarized in table 3.4.

**Table 3-4: Universal regression models of phase I**

| **Stream Variable** | **Dimensionless Regime Equation** | **$R^2$** |
|---|---|---|
| Bankfull width | $\frac{W}{Elev}=7.7 \left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.40}$ | .96 |
| Bankfull depth | $\frac{d}{Elev}=.70 \left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.43}$ | .97 |
| Channel gradient | $Sc= \frac{Sv}{10^{(.21 + .024 \log \frac{DA}{Elev^2})^2}}$ | .98 |

Looking at the above equations, one can clearly conclude that bankfull is the only required factor in determining bankfull width and depth. The regime equation for channel gradient shows independency from discharge and is totally dependent on drainage area and valley slope. This result seems reasonable as discharge mainly dictates only those dynamic stream variables which have a tendency to keep changing over human time scale and show a low threshold value. Channel gradient nearly remains constant over the human time scale and thus is dependent on only those factors (DA,Sv) which also remain constant over this time scale. Physically, channel gradient also depends on several geological events such as tectonic activity and extreme flood events [Buffington, 2012].

### Establishment of regional regime equations:

Dimensionless regime equations were developed separately for 11 different states/regions of USA & UK and have been presented in tabular manner as shown below in table 3.5, 3.6 and 3.7. The method followed here remains the same as was used in formulation of universal equations. Backward stepwise linear regression was performed between the logarithmic transformed values of dimensionless input and output. For consistency purposes, the value of significance level α for this regional regression analysis was kept the same as during universal regression analysis, equal to .05. The significance and the reliability of the developed relationships were verified by inspecting the residual error scatter and histogram plots. The prime motive of developing these regional equations was to quantitatively and qualitatively capture the effects of extended watershed variables (forest cover, urbanization, sediment supply) on the channel dimensions. Since not all the regions covered values of these extended variables, the universal equations formed above lacked including them and thus their effects too. Relationships derived for the state of Missouri, Piedmont region, Non urban valley and UK gravel Bed Rivers do suffice this motive and thus indicates the effects of including them in predicting channel morphology. For simplification purpose this subsection has been divided into 3 parts. Each part discusses the three channel variable (width, depth and channel gradient) separately and how much do they differ from their respective universal counterpart. Ideally the regional regressions should not be much different from the universal forms and indeed should have a better predictive ability. However the key result not lies in finding that but in realizing if universal equations can be used as a surrogate for the regional regime equations.

**i) Dimensionless regional equation for width :**

For each of the 11 state and regions, the regression equation for estimating bankfull width was kept in the following form:

$$\frac{W}{Elev} = p \left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^a \left(\frac{DA}{Elev^2}\right)^b \left(\frac{D50}{Elev}\right)^c (Sv)^d (U)^e (G)^f (F)^g \left(\frac{Qs}{(\sqrt{gElev})\times\rho\times Elev^2)}\right)^h$$

Not all the 11 regions mentioned in this study covered all the dimensionless input variables as has been shown in the above equation. For example only three out of thirteen regions dataset comprised of urban cover and forest cover values. Sediment supply data was available only for UK gravel streams and thus was seen to be included only into UK regression equations. Therefore depending on the number of input variables dataset of each region had, different regression relationships were developed for all regions. Table 3.5 shows a summary of the coefficients, exponents and r2 values that were obtained for each region using backward stepwise log transformed linear regression.

**Table 3-5: Exponents and coefficients of regional width equations**

| Region/ State | Coefficient (p) | Bankfull Discharge exp (a) | Drainage area exp (b) | D50 exp (c) | Valley slope exp (d) | Urban cover exp (e) | Grass cover exp (f) | Forest cover exp (g) | Sediment supply (h) | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Arizona | 4.0 | .37 | .08 | 0 | 0 | NA | NA | NA | NA | .91 |
| Colorado | .02 | .14 | .12 | 0 | 0 | NA | NA | NA | NA | .51 |
| Florida, Georgia, Tennessee & Alabama | .50 | .44 | 0 | -.28 | 0 | NA | NA | NA | NA | .83 |
| Kentucky | .15 | .39 | 0 | 0 | 0 | NA | NA | NA | NA | .83 |
| Maryland | .19 | .29 | .13 | 0 | 0 | NA | NA | NA | NA | .98 |
| Missouri | .06 | .13 | .22 | 0 | 0 | 0 | 0 | 0 | NA | .82 |
| Montana | 3.9 | .39 | .11 | 0 | 0 | NA | NA | NA | NA | .93 |
| New Mexico | 1.22 | .31 | .07 | 0 | 0 | NA | NA | NA | NA | .91 |
| Virginia (Piedmont Province) | 1.32 | .29 | .13 | .05 | 0 | -.07 | NA | 0 | NA | .95 |
| Washington | 9.5 | .37 | .05 | .08 | 0 | NA | NA | NA | NA | .94 |
| UK Gravel rivers | .636 | .28 | .15 | 0 | 0 | NA | NA | NA | 0 | .97 |

Except for Colorado State, all other regions do show a very high correlation value. The low $r^2$ for Colorado can be mainly attributed to the banks at the study sections which were vertical and erosion resistant and thus width varied very little with discharge [Elliot et all, 1984]. In the above table one can find several entries as NA or 0. NA refers to the non availability of that particular input in the regions dataset and thus has not been included in the regression analysis. 0 denotes that even though the particular input variable was available for that region, statistically it came out to be insignificant and thus was ignored from the regression analysis. The criteria for significance testing remained the same where if the input variable had p value of its hypothesis testing greater than .05 then statistically it could be neglected. Based on this concept, urban cover, sediment supply, forest cover and grass cover in the all the applicable regions came out to be statistically insignificant and thus were awarded with an exponent value of 0. Probable reason behind such observation can be attributed to the bankfull discharge variable. Bankfull discharge information may duplicate the need for having the information of these watershed variables and thus delivers them as relatively insignificant in the regressions. In 8 out of 11 regions the median grain size was also credited with an exponent value of 0 and in rest 3 had a value close to 0 (except for region FL, AL, GA, TN). This clearly suggested that median grain size cannot be considered as a principle input parameter in determining channel width. In 5 states, drainage area acquired exponent value equal to or close to 0 and in almost all other states showed less relevancy than the respective discharge exponents. Investigating further, one can easily figure out that in almost all the regions the sum of discharge and drainage area exponent was approximately between .40 and .45 which is similar to the exponent value of bankfull discharge in universal width equations. Based on all these observation, it can be once again suggested that bankfull discharge serves as the only significant and pivotal variable in predicting bankfull width of any stable channel. Thus, new regressions were once again developed for each of 10 regions (Colorado excluded) by using discharge as the only input variable. Table 3.6 highlights the new exponents and coefficient values of the 10 regions.

**Table 3-6: Exponents and coefficients of revised regional width equations**

| Region/ State | Coefficient (p) | Bankfull Discharge exp (a) | $R^2$ |
|---|---|---|---|
| Arizona | 29 | .47 | .90 |
| Florida, Georgia, Tennessee & Alabama | 6.6 | .38 | .83 |
| Kentucky | 10 | .42 | .83 |
| Maryland | 8.1 | .41 | .97 |
| Missouri | 5.0 | .39 | .70 |
| Montana | 38 | .49 | .88 |
| New Mexico | 9.2 | .41 | .90 |
| Virginia (Piedmont Province) | 8.5 | .42 | .97 |
| Washington | 16 | .44 | .94 |
| UK Gravel rivers | 6.9 | .40 | .97 |

Results of the revised regression for width, clearly indicates that almost all the regions have an average exponent value of discharge lying between .40-.45 which was similar to the exponent of discharge in universal width equation. Even after removing the watershed variables from the analysis, all the regions showed high r2 values and thus had strong prediction ability. Thus, bankfull discharge can once again be concluded to be an adequate required variable for making width predictions.

**ii) Dimensionless regional equation for depth:**

The dimensionless regional regime equations for depth were developed separately for each of the 11 different UK and USA states and the mathematical form of the equation was kept similar to the one employed during formulation of regional width equations.

$$\frac{d}{Elev} = p\left(\frac{Qbf}{(\sqrt{gElev}) \times Elev^2}\right)^{a} \left(\frac{DA}{Elev^2}\right)^{b} \left(\frac{D50}{Elev}\right)^{c} (Sv)^{d} (U)^{e} (G)^{f} (F)^{g} \left(\frac{Qs}{(\sqrt{gElev}) \times \rho \times Elev^2)}\right)^{h}$$

As described in the regional width section, not all the regions covered data points for all the possible input variables. In such cases the exponents of the unavailable input variables were assigned with a NA and thus were not included as a part of their respective regression analysis. Table 3.7 summarizes the coefficients and the exponent vales obtained for each of the 11 regions after regression analysis was performed on log transformed data of each region.

Table 3-7: Exponents and coefficients of regional channel depth equations

| Region/ State | Coefficient (p) | Bankfull Discharge exp (a) | Drainage area exp (b) | D50 exp (c) | Valley slope exp (d) | Urban cover exp (e) | Grass cover exp (f) | Forest cover exp (g) | Sediment supply (h) | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Arizona | .16 | .36 | 0 | 0 | 0 | NA | NA | NA | NA | .82 |
| Colorado | 13.5 | .56 | 0 | 0 | 0 | NA | NA | NA | NA | .85 |
| Florida, Georgia, Tennessee & Alabama | 1.01 | .43 | 0 | 0 | 0 | NA | NA | NA | NA | .96 |
| Kentucky | .19 | .33 | 0 | 0 | 0 | NA | NA | NA | NA | .80 |
| Maryland | .30 | .43 | 0 | -.09 | 0 | NA | NA | NA | NA | .99 |
| Missouri | .90 | .40 | 0 | .10 | 0 | 0 | 0 | 0 | NA | .92 |
| Montana | .057 | .28 | 0 | 0 | 0 | NA | NA | NA | NA | .70 |
| New Mexico | .24 | .38 | 0 | 0 | 0 | NA | NA | NA | NA | .87 |
| Virginia (Piedmont Province) | .35 | .39 | 0 | 0 | 0 | 0 | NA | 0 | NA | .95 |
| Washington | .086 | .38 | 0 | 0 | -.22 | NA | NA | NA | NA | .94 |
| UK Gravel rivers | .64 | .39 | 0 | 0 | 0 | NA | NA | NA | 0 | .98 |

Compared to all the input variables, only bankfull discharge emerged as the most significant variable on bankfull depth of 11 regions depends. Rest all watershed variables (similar to width case) came out to be relatively insignificant and had exponent values either equal to or very much close to 0. All these observations once again suggested that in predicting bankfull depth, bankfull discharge is the only relevant and determining parameter. When regressions were once performed (including only discharge as the input), almost similar coefficients and exponent values of discharge were obtained. Even the $r^2$ values for all the 11 regions did not show any significant change. Therefore, no further revised table has been provided for the regional depth regressions of 11 regions.

**iii) Dimensionless regional equation for channel gradient:**

The regime equation derived for estimating channel slope for each region was kept in the following form:

$$Sc = \frac{Sv}{10^{( p + q\, og \frac{DA}{Elev^2})^2}}$$

Where p, q (Table 3.8) are the coefficients obtained after the regression analysis was performed on each regional dataset. On observing carefully, one can clearly see that the structure of this equation is completely identical to the one derived for predicting universal channel gradient. Working on exactly similar concept, log transformed regression equation of sinuosity was initially formed using drainage area as the only input parameter as other variables (Qbf, D50, Qs, F, U, and G) were concluded to be statistically insignificant. In the later step, box-cox transformation technique was implemented for achieving a standard normal non residual probability plot.

### Table 3-8: Exponents and coefficients of regional channel slope equations

| Region/ State | Coefficient (p) | Drainage area exp (q) | $R^2$ |
|---|---|---|---|
| Arizona | .21 | -.0012 | .99 |
| Colorado | .12 | .088 | .99 |
| Florida, Georgia, Tennessee & Alabama | .19 | .037 | .97 |
| Kentucky | .10 | .11 | .98 |
| Maryland | .29 | .0007* | .96 |
| Missouri | .14 | .017 | .99 |
| Montana | .025 | .11 | .98 |
| New Mexico | .26 | -.013 | .97 |
| Virginia (Piedmont Province) | .49 | -.05 | .99 |
| Washington | .3 | .025 | .98 |
| UK Gravel rivers | .16 | .027 | .99 |

### Evaluating the prediction accuracy of regional and universal equation

Till now, the complete discussion in section 1 dealt with the determination of statistically valid dimensionless regression regime equations. We successfully formulated universal and regional relationships that can be implemented in predicting bankfull width, depth and channel gradient for stable channel reaches belonging to various physiographic regions. However, one major task which still needed to be accomplished was evaluating the prediction accuracy of these regional and universal regime equations and making a comparison between the two. Results displayed in this subsection is not only an attempt to furnish this impending task and but also in realizing if regional equations can be replaced by universal ones.  Table 3.9 lists the average error with standard deviations calculated for each of the three hydraulic variables using universal as well as the corresponding regional equations.

Table 3-9: Comparison between regional and universal equations

| Region/state | Average error in predicting width (%) | | Average error in predicting depth (%) | | Average error in predicting channel gradient (%) | |
|---|---|---|---|---|---|---|
| | From regional equation (μ ± σ) | From universal equation (μ ± σ) | From regional equation (μ ± σ) | From universal equation (μ ± σ) | From regional equation (μ ± σ) | From universal equation (μ ± σ) |
| Arizona | 13 ± 12 | 18 ± 16 | 15 ± 14 | 35 ± 40 | 4±8 | 9±5 |
| Colorado | 49 ± 29 | 75 ± 58 | 23 ± 17 | 36 ± 29 | 5±5 | 7±6 |
| Florida, Georgia, Tennessee & Alabama | 31 ± 40 | 45 ± 38 | 14 ± 11 | 44 ± 18 | 18±23 | 19±26 |
| Kentucky | 22 ± 45 | 20 ± 40 | 20± 32 | 31±37 | 24±24 | 42±47 |
| Maryland | 17 ± 13 | 17 ± 13 | 13± 14 | 22±16 | 11±10 | 12±10 |
| Missouri | 21 ± 17 | 21 ± 18 | 12± 11 | 26 ± 14 | 4±4 | 7±3 |
| Montana | 19 ± 16 | 31 ± 54 | 19± 15 | 28 ± 27 | 10±16 | 14±21 |
| New Mexico | 15 ± 15 | 23 ± 33 | 18±22 | 31± 40 | 6±6 | 7±5 |
| Virginia (Piedmont Province) | 12 ± 11 | 13 ± 12 | 13±8 | 22±10 | 10±6 | 15±16 |
| Washington | 13 ± 11 | 24 ± 13 | 11± 10 | 19±15 | 16±18 | 21±27 |
| UK Gravel | 17 ± 15 | 18 ± 15 | 10±11 | 36±15 | 20±26 | 20±23 |

As expected, regional equations in almost all the regions give better prediction of channel variables than the universal equations. Using the respective regional equations, more than 80% of the 11 different regions had their average error in width and depth less than 25% and average error in channel gradient less than 15%. Using the universal equations on the same regions, the average errors encountered in majority of states while predicting the bankfull width, depth and slope was less than 30%, 35% and 20%, respectively. All these numerical average error values clearly suggest that universal equations predictions are not much different from the regional ones and thus can be successfully utilized in estimating the hydraulic geometry of any stable stream reach.

Moving ahead, one cannot ignore but notice the high standard deviation values shown by few states such as Colorado, Kentucky, Georgia, Alabama, Florida and Tennessee in predicting channel width and depth. Of all the plausible reasons, error in measurement of bankfull discharge or approximating it with 1.5-2.0 year flood value can be considered as the most dominant reason behind such high standard deviation values. In fact when observed carefully, the channel gradient regression equations (both regional and universal) do not involve discharge as an input and thus can be understood as to why they have significantly low standard deviation values for almost all regions. Instead the only inputs on which channel slope has been shown to be dependent are valley slope and drainage area and apparently both these watershed variables are much less sensitive to measurement errors than other variables. Another compelling reason behind such greater variations observed in width and depth but not in slope can be attributed to the remarkable differences observed in the threshold values of these channel variables. Buffington [2012] in his paper has described how various channel responses (width, depth, bed material, channel gradient) vary over spatial and temporal scales and can be grouped into small scale adjustments to large scale changes.

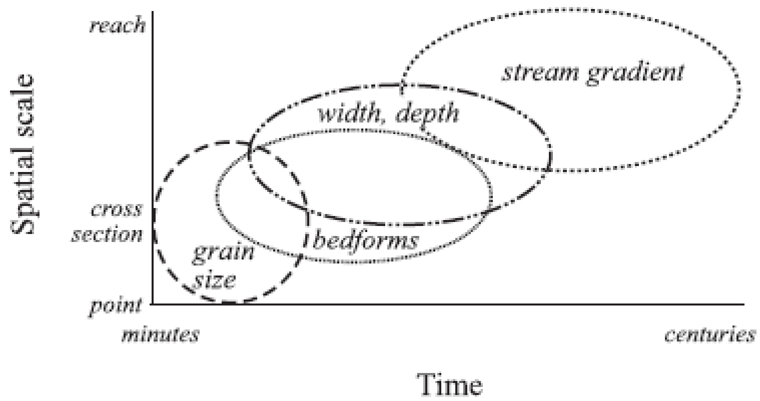



**Figure 3- 7 : Channel responses over spatial and temporal scales**

Figure 3.7 proposed by Buffington [2012] clearly suggests that any given disturbance will alter the grain size first (generally valid for sandy streams), width and depth second and finally will impact the channel gradient in the end. In fact over human time scales or also referred to as graded time scales, the channel gradient remains relatively constant with only slight variations about the mean [Schumm & Lichty, 1965]. Looking at all the above reasons one can now clearly understand as to why width and depth show more variations in the predicted values of almost all the regions as compared to channel gradient.

## Variation of stream power in 11 different regions of UK and USA

In the above section, the various reasons that we discussed for larger standard deviation values observed in predicted width and depth for some regions as compared to others were all qualitative in nature. In this section, we try to bring a quantitative aspect to our understanding and analyze the behavior of specific stream power for each of different 14 regions. Specific stream power is a simple function of discharge, slope and channel geometry which when combined together represents the rate of potential energy dissipated in a stream moving downstream (Rhoads 1987). Mathematically, it can be expressed as $? = ?_0U/g$, where $?_0$ is bottom shear stress, U is average water velocity, and g is gravitational acceleration. However, for study purposes this equation was not used as it did not account for the sediment composition on the channel bed and thus instead following expression in dimensionless form was considered.

$$\omega^{*} = \omega/(\rho_s \sqrt{RgD_{s50}^{3}})$$ ,[ Almedeij & Diplas, 2005].

Ideally, for regions having low prediction accuracy of their regional equations, the dimensionless stream power should vary over a large range of values and vice versa. Using the above equation, various central tendency values of $\omega^{*}$ were calculated for the 11 regions. Table 3.10 and box plot figure 3.8 summarizes the summary statistics of $\omega^{*}$ (mean, median, std dev, $75^{th}$ & $25^{th}$ percent quartile, data points, outliers) for the 11 regions.

**Table 3- 10 : Statistical summary of dimensionless stream power for 11 regions**

| Region | 75th % quartile -25th % quartile | Mean± Std dev μ±σ | Median (M) |
|---|---|---|---|
| Arizona | .05 - .012 | .09±.17 | .02 |
| Colorado | 2.02 - .34 | 1.9±3.5 | .65 |
| Florida, Georgia, Tennessee & Alabama | 7.3 - .63 | 5.3±8.8 | 1.8 |
| Kentucky | 8.5 - .066 | 9.0±18 | .17 |
| Maryland | 1.06 - .07 | 1.2±2.0 | .18 |
| Missouri | .27 - .05 | .40±1.0 | .010 |
| Montana | .22 - .04 | .72±2.5 | .08 |
| New Mexico | .20 - .05 | .75±1.4 | .08 |
| Virginia (Piedmont Province) | .90 - .065 | 1.2±2.4 | .09 |
| Washington | .18 - .06 | .13±.14 | .07 |
| UK Gravel rivers | .08 - .04 | .078±.066 | .055 |

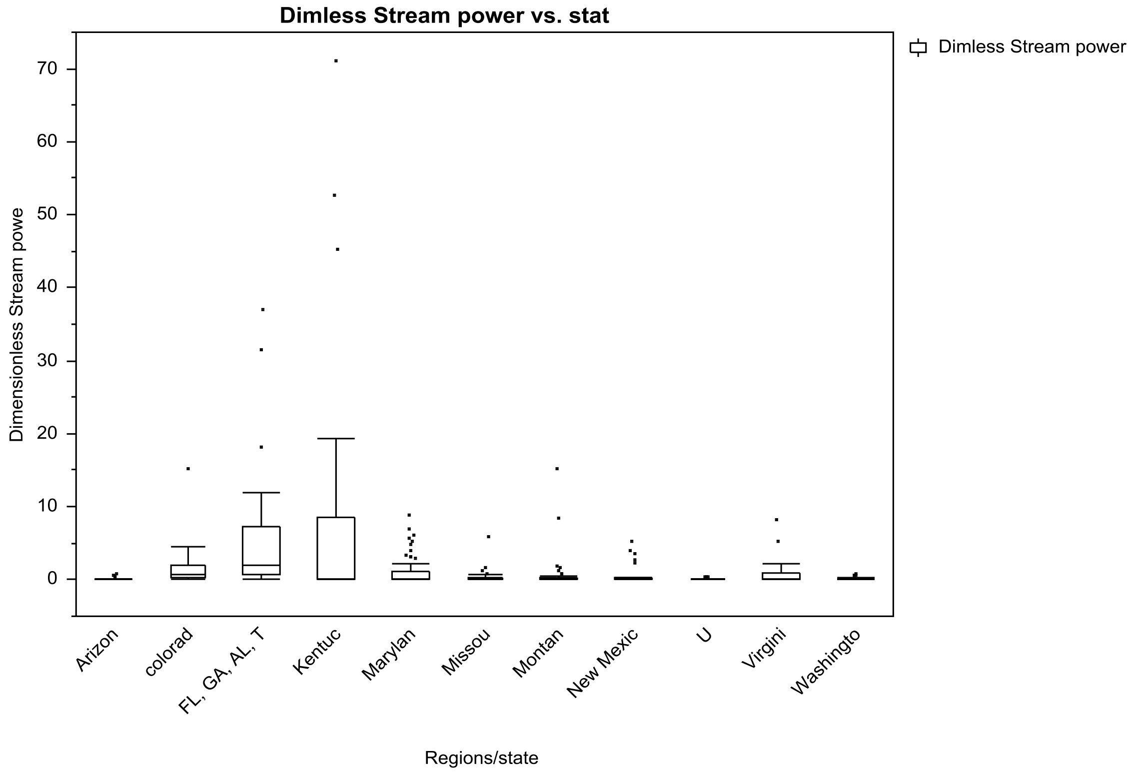



**Figure 3- 8: Dimensionless stream power variation for 11 different states of UK and USA.**

The summary table (3.10) and the box plot (figure 3.8) clearly shows that out of all the regions "Florida, Georgia, Alabama, Tennessee, Kentucky" are the ones that have the maximum difference between their 75$^{th}$ percentile and 25$^{th}$ percentile quartile values of dimensionless stream power. Larger range of stream power values indicate greater tendency for degradation and aggradation and thus greater deviation from state of quasi-equilibrium [Ferencevic& Ashmore, 2011]. Additionally, these states also have a higher standard deviation value of their means in comparison to others. This is in complete agreement to what was observed in previous section where above mentioned states displayed the maximum variability and low prediction accuracy of their respective regional regime equations.

### Verification of the width and depth universal equations using two independent dataset

For successful validation of any regression model, it is essential to test the relations against independent datasets. In this section, we predict the values of bankfull width and depth of two different independent regions which were not included during the regression analysis. Predicted values of 36 data points of Wyoming State and 37 data points of Ohio State were calculated using the developed universal equations and plotted against their respective actual values (figure 3.9 and 3.10).

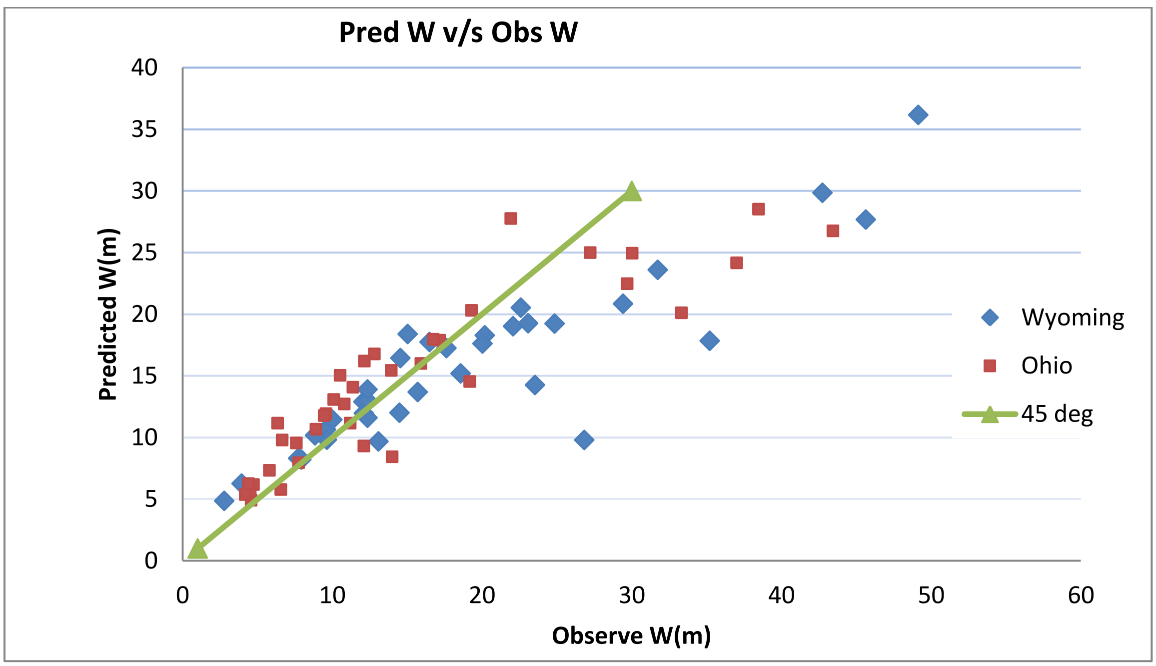



**Figure 3- 9: Comparison of universal width equation with independent datasets**

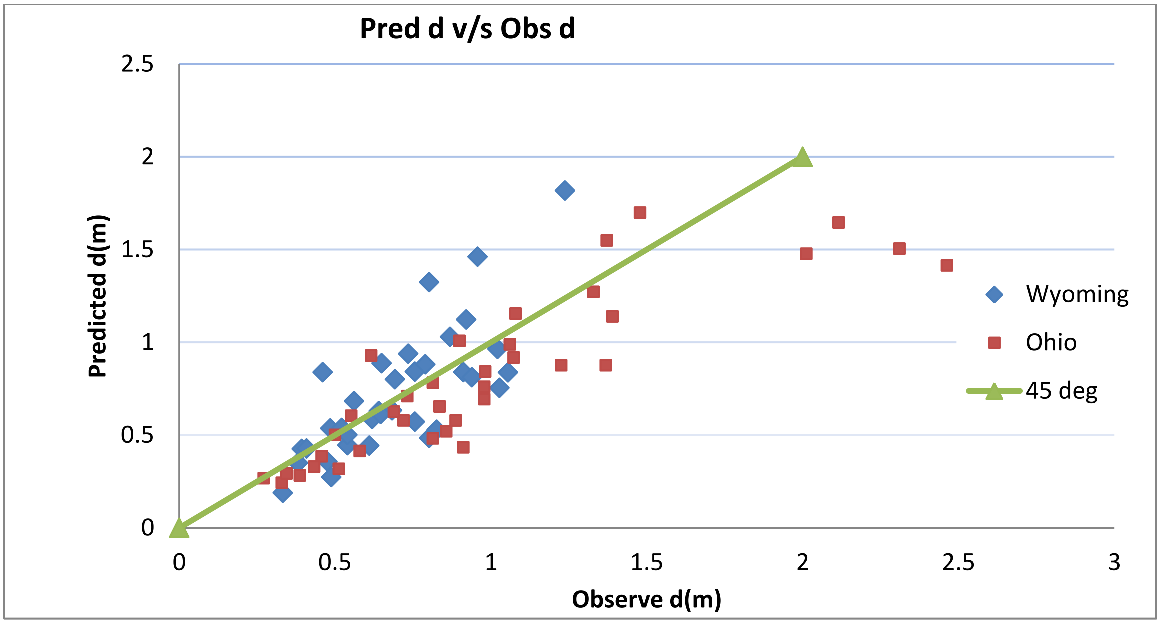



**Figure 3- 10 : Comparison of universal width equation with independent datasets**

Except for the higher values of width and depth, almost all the data points (in case of the both the plots) can be seen to be centered about the 45 degree line. This observation clearly indicates that the universal equations developed in this paper are not only valid equations but also do have a universal appeal of estimating the dimensions of stable stream channels. A summary table (3.11) was also prepared which comprised of various central tendencies calculated for the predicted versus observed ratio of width and depth for the two states.

Table 3- 11 : Statistical summary of predicted versus observed values of width and depth for two independent datasets

| Central tendencies | Ohio | | Wyoming | |
|---|---|---|---|---|
| | Pred W : Obs W | Pred d : Obs d | Pred W : Obs W | Pred d : Obs d |
| Mean ± std dev $\mu \pm \sigma$ | 1.05±.29 | .99±.26 | .94±.27 | 1.2±.34 |
| 1st Quartile | .78 | .81 | .74 | .94 |
| Median | 1.07 | .94 | .91 | 1.13 |
| 3rd Quartile | 1.26 | 1.13 | 1.07 | 1.38 |

The average error in predicting width for Ohio state is as low as 5 % and for Wyoming state it is even less by 3%. The average error for predicting depth in case of Ohio is almost close to null and in case of Wyoming is approximately equal to 20%. With all these low error percentages, it gets even more evident that the equations developed in this study are acceptable can be conferred with the title of "Universal".

## Comparing the above developed Universal models to the prior established Leopold and Maddock's model

Before concluding all the results derived in phase 1, it would be interesting to see how these equations behave in comparison to the models developed in the past. Leopold and Maddock in their study (1953) proposed that the width and depth of stream channels at any given cross section or along the length of channel varies with mean discharge as simple power functions. They can be described as having following mathematical forms:

$W = aQ^b$ ,   $d = cQ^f$

To calculate the values of exponent's b and f, Leopold and Maddock in their study included 63 different stream reaches spread across the states of Nebraska, Wyoming, Montana Kansas and Missouri. Based on regression applied to these data points, the average value of b and f for downstream variations were found to be equal to .5 and .4 respectively. Both these values are comparable to the ones obtained in this paper's universal regression models (b=.40 and f=.44). This small difference can be mainly accounted for three major reasoning. First, the vast difference in the number of data points considered in this study to the limited field values (63) employed by Leopold and Maddock in their works. Second, unlike bankfull discharge being used in this study, Leopold and Maddock used average discharge values. Third, all the equations developed in this paper were in dimensionless forms whereas this was not the case with Leopold and Maddock's. In fact by making the equations dimensionless with the help of elevation, two variables are being utilized than one in predicting the channel geometry. Also, elevation in itself probably includes the effects of various other factors such as relief, tectonic activities etc whose effects otherwise may not have been possibly captured in the models established in this study.

Amidst all the differences that have been discussed so far, there still lies a strong similarity between the two models that precisely connects them so well. Initially, this study was motivated by making use of all the possible stream and watershed variables which could possibly define the prediction of stream's width, depth and channel gradient. But what appeared in the beginning was totally transformed after the analysis. Except for bankfull discharge all other watershed variables (drainage area, forest cover, grass cover, bank vegetation, sediment supply etc) followed to be statistically insignificant variables and were eventually dropped from the regression analysis. This is exactly what Leopold and Maddock emphasized in their works, if not directly. Their works was indicative of suggesting mean discharge as the only regulating input variable which seemingly controlled the width and depth of a stream channel. This leads one to believe that the dimensionless regression models developed in this paper are perhaps refined version of the Leopold and Maddock model. However, at this point one must realize that the in this study all the watershed parameters affecting stream flow were taken into consideration while formulating the bankfull relationships which otherwise have remained absent in Leopold and Maddock approach and other relationships developed later.

## Comparison of the universal width and depth equations developed in this study to Parker et al.'s gravel model developed in 2007

Parker et al in 2007, developed quasi-universal relations in dimensionless forms for calculating bankfull hydraulic geometry of alluvial gravel bed streams. Standard linear regression method adopted by them yielded fowling equations in power form:

$$\frac{W}{D50}=4.63\ \left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{.467}$$

$$\frac{d}{D50}=.382\ \left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{.40}$$

$$Sc=.1\ \left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{-.344}$$

The baseline dataset used by Parker in developing these relationships comprised of a total of 72 field points. He in his study included 16 stream reaches from Alberta, 23 from Britain I, 23 from state of Idaho and 10 reaches of Colorado River from western Colorado and eastern Utah. Additionally, he also used three independent sets of data consisting of streams from Maryland, Britain II and Colorado. Parker showed that almost all the predicted values of the independent dataset found using his power equations were well within ½ and 2 times the reported values. It was only the streams from UK that showed substantial deviation from universality. The average value of the ratio between predicted width and reported width for these Britain streams was found to be as high as 1.34. Similar was the case with its depth, where the ratio between predicted and observed was found to be equal to.91. Parker justified the deviation observed in width by making use of classification proposed by Hey and Thorne [1986]. Hey and Thorn in their works, classified UK stream data on a scale of 1 to 4 in terms of the density of its bank vegetation, where 1 denoted least density and 4 denoted the highest. Using this information, Parker et al. calculated the ratio of predicted versus observed width for each four classes of streams and found that the ratios increased progressively from class 1 to class 4 (class1: .93, class2: 1.21, class3: 1.45 and class4: 1.66). Thus, it was concluded that density of bank vegetation controls the deviation of width from universality, with class 1 being closest to universality and class 4 farthest.

Unlike Parker's model which used surface median grain size, model developed in this study used elevation as the independent variable for nondimensionalization purposes. Using this approach, universal equations for estimating width, depth and channel gradient were developed. These universal equations are not limited to any specific stream type (sandy, gravel or cobble) and may be employed independently for any stable reach type. On the contrary, Parker emphasized on building bankfull relations based on channel types and thus developed equations separately for sandy and gravel bed streams. At this point, one must understand that this subsection of comparison is not intended towards proving whether classifying streams is a relevant criterion in developing equations or elevation is a better nondimensionalising parameter than median grain size. These questions have been dealt in the section later titled "Power of nondimensionalization". Currently, in this subsection an attempt has been made to see how the universal width and depth equations developed in this study perform when applied to the gravel datasets used by Parker et al. If the prediction came out to be within acceptable limits and the error was comparable to the ones produced by Parker et al., then probably we can say that the approach proposed in this paper is universally a valid one and may be used as an alternative to Parker's gravel stream model.

Out of 7 datasets originally used by Parker, 5 datasets were used for comparison purposes. Two datasets were left out as the author was unable to locate the precise elevation values of the field locations used in these two datasets. The five datasets can be described as follows: i) 25 field values from Alberta compiled by Kellerhals et al [1972]; ii) 23 field values from Britain gathered by Charlton et al [1978] ; iii) 24 stream reaches from Colorado by Andrews [1984]; iv) 11 Maryland and Pennsylvania gravel reaches [McCandless, 2003]; v) 62 British reaches by Hey and Thorne [1986]. Datasets numbered i) and ii) were two of the four datasets which Parker et all had used for forming equations. Rest three was employed to test the developed equations. In this study, datasets numbered iii), iv) and v) were a part of the cumulative dataset from which universal models were developed. Rest two datasets had not been included in the regression and thus would serve even better in giving a true accuracy of our universal relations.

Using both the approaches (Parker's and Author's) average ratio of predicted and observed values of width and depth for each of the 5 regions were calculated. In addition to the mean, median, standard deviations, 25$^{th}$ percent quartile and 75$^{th}$ percent quartile were also reported for each region. Table 3.12 and 3.13 summarizes all these central tendency values calculated separately for the 5 regions.

**Table 3- 12 : Statistical summary of predicted versus observed width ratio for 5 different datasets.**

| **Statistical Parameter** | **Kellerhals data [1972]** | | **Charlton data [1978]** | | **Andrews data [1984]** | | **McCandless data [2003]** | | **Hey & Thorne data [1986]** | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) |
| $\mu \pm \sigma$ | .93±.50 | .83±.50 | 1.16±.70 | 1.37±.94 | 1.0±.30 | 1.07±.24 | 1.0±.24 | 1.09±.40 | 1.13±.30 | 1.33±.34 |
| 1$^{st}$ Quartile | .62 | .71 | .68 | .71 | .84 | .92 | .84 | .83 | .91 | 1.05 |
| Median | .92 | .97 | 1.05 | 1.26 | .95 | 1.03 | .94 | .95 | 1.1 | 1.31 |
| 3$^{rd}$ Quartile | 1.17 | 1.20 | 1.48 | 1.7 | 1.15 | 1.20 | 1.21 | 1.24 | 1.27 | 1.53 |

**Table 3 - 13 : Statistical summary of predicted versus observed depth ratio for 5 different datasets.**

| Statistical Parameter | Kellerhals data [1972] | | Charlton data [1978] | | Andrews data [1984] | | McCandless data [2003] | | Hey & Thorne data [1986] | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) | Pred/obs (our model) | Pred/obs (Parker model) |
| μ±σ | 1.12±.80 | 1.27±.67 | 1.30±.83 | .90 ±.55 | 1.17±.22 | 1.1±.18 | 1.2±.29 | .97±.20 | .86±.19 | .58±.11 |
| 1st Quartile | .60 | .50 | .55 | .39 | 1.0 | .97 | 1.03 | .86 | .70 | .51 |
| Median | 1.12 | .90 | 1.25 | .91 | 1.17 | 1.06 | 1.27 | .97 | .87 | .58 |
| 3rd Quartile | 2.2 | 1.8 | 1.73 | 1.22 | 1.35 | 1.22 | 1.36 | 1.1 | 1.0 | .65 |

From the summary tables, it is evident that both the models display comparable performances in estimating channel width and depth. In some cases such as Hey and Thorn's and McCandless's, model developed in this study gave even better prediction results than Parker's. Using this study models, the average error encountered in estimating width for Hey and Thorne dataset was around 13% where as using Parker's model, it came out to be approximately 33%. As mentioned earlier, Parker in his paper deals with this large deviation by relating it with the variation observed in the density of bank vegetation, with lowest bank vegetation density (Type I) streams being closest to universality and vice versa. Interestingly when this study model was used in quantifying the effect of vegetation on width, an altogether different result was seen. In terms of universality, Type II streams became closest to it and rest other show comparatively less deviation from universality.  Table 3.14 summarizes the mean, median and std dev values of predicted and observed width calculated using both the models for all the four vegetation types.

**Table 3- 14 : Statistical summary of predicted & observed width ratio for Hey and Thorn data segregated on the basis of vegetation types.**

| Statistical Parameter | Type I vegetation | | Type II vegetation | | Type III vegetation | | Type IV vegetation | |
|---|---|---|---|---|---|---|---|---|
| | Our model | Parker model | Our model | Parker model | Our model | Parker model | Our model | Parker model |
| Mean ± std dev $\mu \pm \sigma$ | .79±.09 | .91±.09 | 1.02±.12 | 1.20±.15 | 1.18±.19 | 1.45±.26 | 1.41±.29 | 1.64±.28 |
| Median (M) | .78 | .92 | .97 | 1.15 | 1.22 | 1.39 | 1.33 | 1.62 |

Based on these values, it may seem legit to suggest that the approach adopted in this study does have the capability of capturing to some extend the effects of bank vegetation on channel width. Also, this analysis leads us to question the previous established conclusion of whether bank vegetation density varies inversely with the universality.

## Discussion for Section I

Results obtained in phase I strongly indicate that bankfull discharge as the single most important parameter dictating the bankfull width and depth for any stable stream channel. Other parameters such as drainage basin, urbanization, forest and grass cover, grain size distribution and even the sediment supply proved out to be statistically insignificant parameters and thus did not play a significant role in determining the channel dimensions. It was only in the case of formulating channel slope equation, that other parameters (valley slope and drainage basin) emerged as the decisive ones and discharge became an insignificant variable. The reason that discharge dominates over the channel dimensions and watershed parameters appear trivial can be understood in terms of the flow chart as described in the figure 3.11:

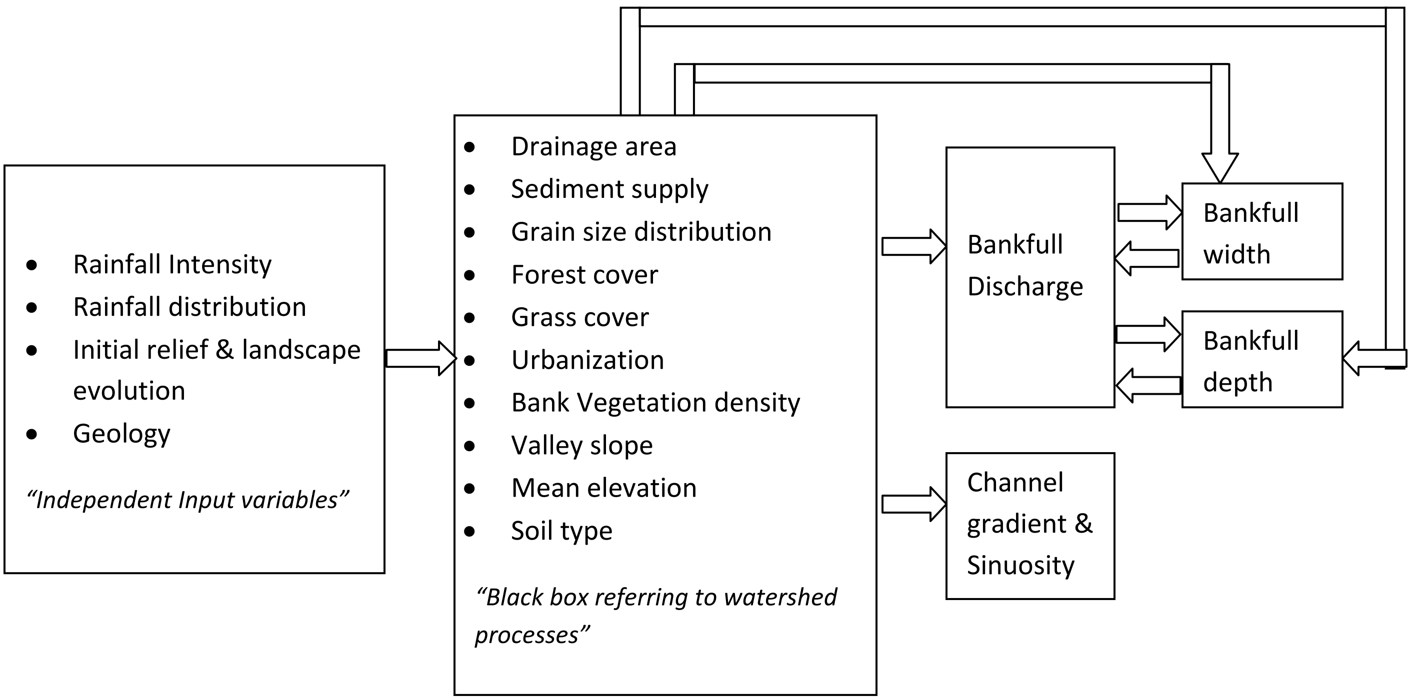



**Figure 3 - 11 : Flowchart depicting relationship between independent input variables, watershed processes and output variables.**

The flow chart is self-explanatory in realizing that bankfull discharge is not an independent input variable but a culmination of various watershed processes acting on the actual independent variables namely: climate, relief, geological factors, forest cover, bank vegetation and drainage area. Till now, the analysis presented in phase I of our study was only based on half of the above picture and rest all other watershed processes and inputs were not being considered. This possibly explains as to why all the watershed variables turned out to be statistically insignificant during our regression analysis for width and depth. The watershed processes can be considered equivalent to a black box where various processes combine together to perform a series of complex operations on the precipitation falling on the landscapes. These operations ultimately deliver output in the form of discharge and which in turn determines the stream dimensions.

Physically and quantitatively, precipitation functions as a non relevant factor to channel gradient and thus gradient is found to be statistically disconnected from discharge effects. Factors which mainly govern the channel gradient are geology, relief and landscape evolution and the combined effects of these factors can be very well captured by the values of valley slope, mean elevation and drainage area. Interestingly, these are the only three variables formulating the universal and regional regime equation for channel gradient.

### 3.5.2 Section 2: Results obtained from Phase 2 of the study

In phase I of the study, we saw how bankfull discharge is more of an output variable as far as watershed processes are concerned and input variable as far as stream properties are concerned. Bankfull discharge is a result of several watershed forces and variables acting on the precipitation being received by the basin area. Based on this notion, it seemed very much legitimate to consider a different approach for developing these regime equations which should include all the steps of the flowchart rather than just following a part of it. In phase II of this study, bankfull discharge was replaced by the annual average rainfall in the regression and equations were formulated not only for bankfull width, depth and slope but also for bankfull discharge. The complete methodology of developing these new set of regression equations remained the same as was used during Phase I. All the input variables were initially nondimensionlized using elevation as the key variable. These nondimensionlized variables were then transformed into their respective logarithmic forms and backward stepwise regression was performed to develop the regime equations. For the equations to be labeled valid, the residual error plots were graphed against each input variable and checked for randomness about the horizontal axis. Also, the frequency histogram of the residuals were plotted to verify its resemblance with standard normal distribution with skewness and excess kurtosis close to the value of 0. In case the histogram appeared to be skewed left or right, box-cox transformation was performed on the output variable to form a different valid form of regression equation.

Not all the 405 data points which were used during Phase I could be included in phase II analysis. Simple reason being, not every unique field point had its corresponding annual average rainfall value too. Bounded by this limitation, the total number of data points contributed towards the formulation of universal regime equations for phase II studies were 234 in number. These data points belonged to following 6 different regions: UK, Virginia, Maryland, Missouri, Montana and Washington State. The original datasets of only UK, Virginia and Montana contained within themselves the required annual average rainfall values. For rest of the three regions, the annual mean precipitation values were determined by the author from the annual average rainfall records for each stream location. Based on these 234 field values, universal regime equations for bankfull discharge, width, depth and channel gradient were developed. Later, the individual datasets of each 6 regions were operated to compose the respective regional regime equations. In the subsections that follow, universal and regional equations derived using "backward stepwise logarithmic regression" have been presented and discussed.

### Establishment of universal regime equations

Restating again, the equations described in this subsection were developed based on following independent input variables in non-dimensional forms: annual average rainfall, drainage area, median grain size and valley gradient. Since these were the only four variables that occurred common to all the individual regional datasets, the universal equations were developed using these as the inputs. Exercising regression between the logarithmic values of nondimensional inputs and output variables, power equations were obtained in its final form as shown in the table 3.15:

**Table 3 - 15 : Universal equations developed using rainfall and watershed variables as inputs.**

| Stream Variable | Dimensionless Regime Equation | $R^2$ |
|---|---|---|
| Bankfull Discharge | $\frac{Qbf}{(\sqrt{gElev})\times Elev^2} = .037\,(\frac{AAR}{Elev})^{1.8}\,(\frac{DA}{Elev^2})^{.49}\,(\frac{D50}{Elev})^{.22}$ | .95 |
| Bankfull width | $\frac{W}{Elev} = .52(\frac{AAR}{Elev})^{.67}\,(\frac{DA}{Elev^2})^{.18}\,(\frac{D50}{Elev})^{.11}\,(S_v)^{-.22}$ | .95 |
| Bankfull depth | $\frac{d}{Elev} = .05\,(\frac{AAR}{Elev})^{.68}\,(\frac{DA}{Elev^2})^{.19}\,(\frac{D50}{Elev})^{.14}\,(S_v)^{-.19}$ | .93 |
| Channel gradient | $Sc = \frac{Sv}{10^{(.20+.03\log\frac{DA}{Elev^2})^2}}$ | .97 |

All the four regime equations presented above displayed a very high correlation value confirming strong prediction ability for each. In the discharge equation, one can clearly see valley slope not being included as one of the input variables. The reason behind it can be explained on the grounds of p value concept. After the first initial regression was performed, the p value of the valley gradient came out to be .34 which is greater than the significance level $\alpha$=.05. Rest all other inputs displayed p value much less than .05 and thus were included to be a part of the discharge relation. When interpreted physically this result may seem valid, as based on continuity principle discharge should not change from one point to another on a steady reach. It should mostly get affected by the amount of runoff being added to the stream flow from the watershed. Precipitation and drainage area are the two major components that determine this runoff and this explains why annual average rainfall and drainage area are raised to such high exponent values. Apart from these two variables, runoff generated also depends on several other vital factors such as the soil type, forest cover and the bank vegetation density. All these factors are greatly influenced by the geology and relief of the watershed system and elevation may be considered as an indirect measure of these parameters. Thus by using elevation as the nondimensionalising parameter the effects of these factors are being captured in the bankfull equation. The significance of using elevation as the key variable has been discussed much more in detail in the section III of this paper.

Now moving on to the dimensionless width and the depth equation, all the four input variables had their respective p value much less than the value of $\alpha$ and thus were included in the final equations. Surprisingly, in both these equations the exponents of precipitation and drainage area was calculated to be almost equal. This may be expected as precipitation and drainage area both majorly combine to form the runoff value and runoff may be considered to have approximately equal influences on both width and the depth. In fact the exponents in the regression equations of phase 1 obtained as a function of $Q_{bf}$, indicative of surface runoff, are very similar. Additionally, valley gradient in the two equations was observed to be inversely proportional to these channel dimensions. Williams [1970] observed similar relationship in one of his laboratory flume experiments. In 177 flume experiments that he performed he observed that at constant discharge and depth as the flume width became wider the channel slope became flatter. Similarly when width and discharge was kept constant, rise in depth made the channel flatter too. In fact when Williams performed multiple regression analysis on his depth data, following relationship with channel slope was obtained:

$$d \;\alpha\; S^{-.28}$$

This is very much comparable to what was achieved in the above regression analysis, where depth was shown to vary with slope as "$d\alpha\ S^{-.19}$ ". The variation in exponent values may be explained on the grounds of difference in the channel bed particles of the two analyses. Williams in his experiments used only sand particles having a median grain size of 1.35mm whereas in this analysis the median grain size varied from being cobble to gravel to sandy. Moreover, one must realize that all the data points in this analysis were the actual field values whereas Williams' analysis was based on his experimental flume data.

Moving on to the channel gradient equation, one can easily observe that the relation formulated in the phase II of the study is almost identical to the equation we had derived in phase I. One should have expected this as channel gradient in both phase studies proved to be dependent only on two common input variables, i.e. drainage area and valley slope.

In the three universal equations that were developed for discharge, width and depth; median grain size was another input variable that emerged statistically significant. Qualitatively, this may be understood with the help of following two reasons. First, bed material of any stream reach is capable of carrying the upstream information and thus may be considered as an indirect measure of discharge coming from the upstream. Second, grain size distribution may be indicative of the sediment load and the type of soil of the watershed system.

### Validity check of the above regressed universal equations

Figures 3.12, 3.13 and 3.14 summarize all the residual error scatter plots and frequency plots obtained for the four output variables.

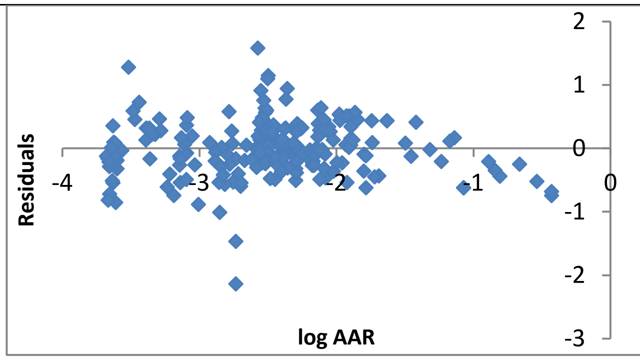
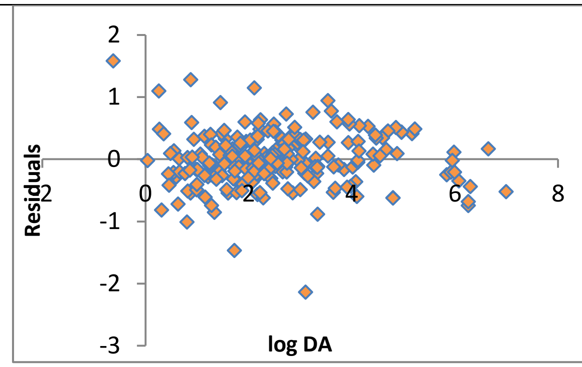
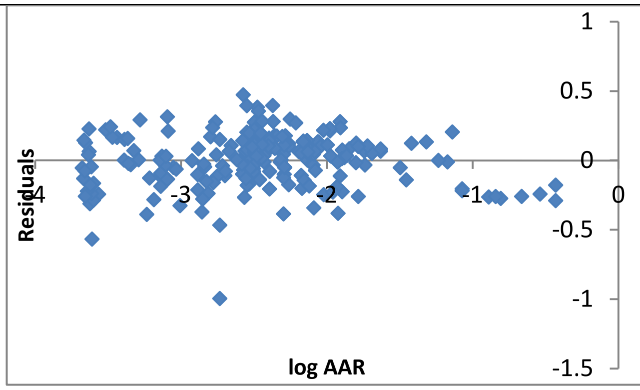
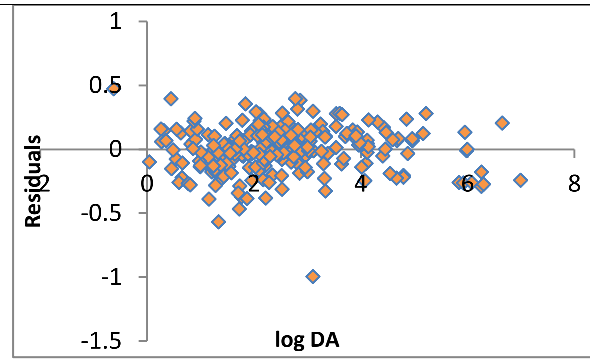
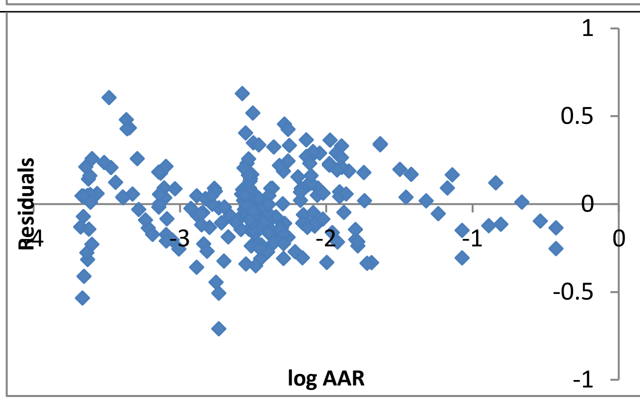
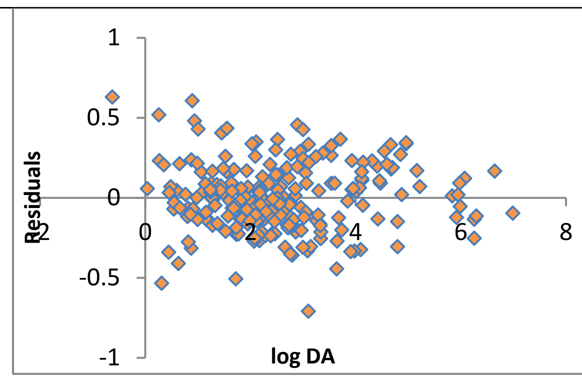
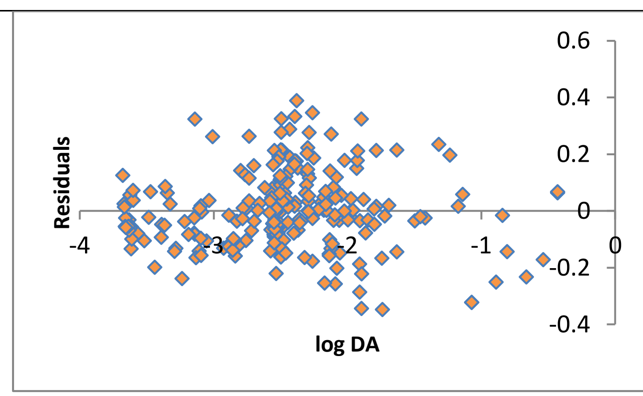
| Output Variable | Residual error against logarithmic annual average rainfall | Residual error against logarithmic drainage area |
| --- | --- | --- |
| Bankfull Discharge |  |  |
| Bankfull width |  |  |
| Bankfull depth |  |  |
| Sinuosity | Sinuosity does not depend on annual average rainfall. |  |

**Figure 3 - 12 : Residual error scatter plots against each independent input variable for phase II**

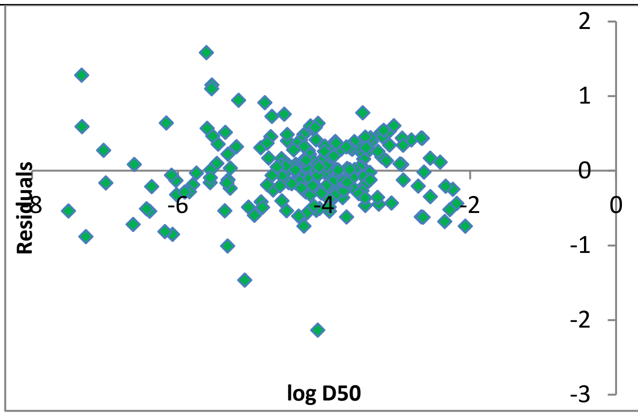
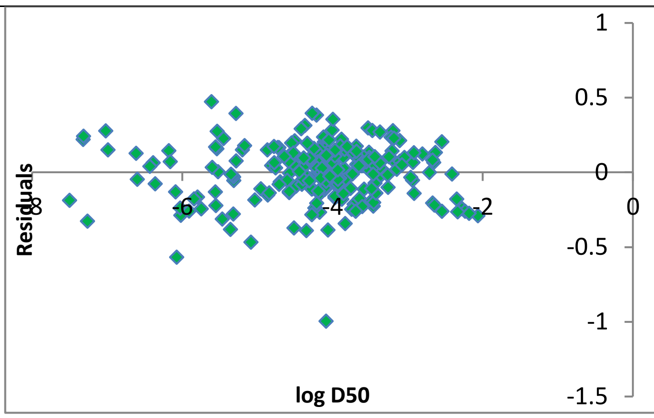
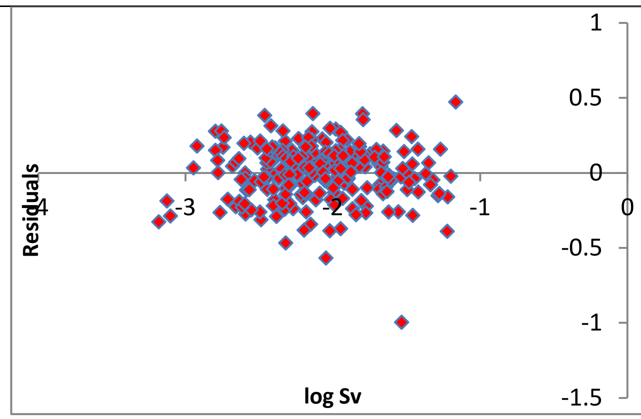
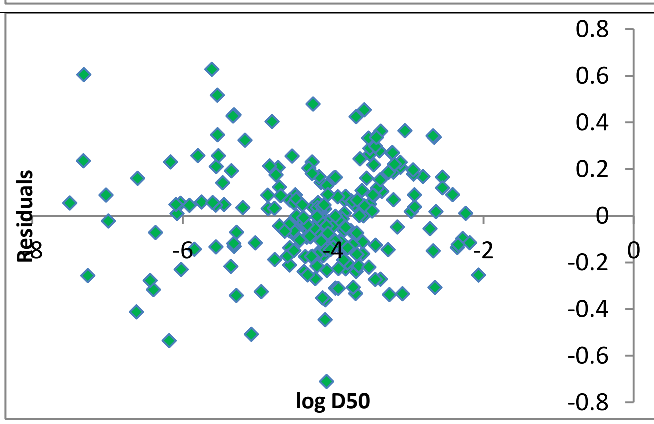
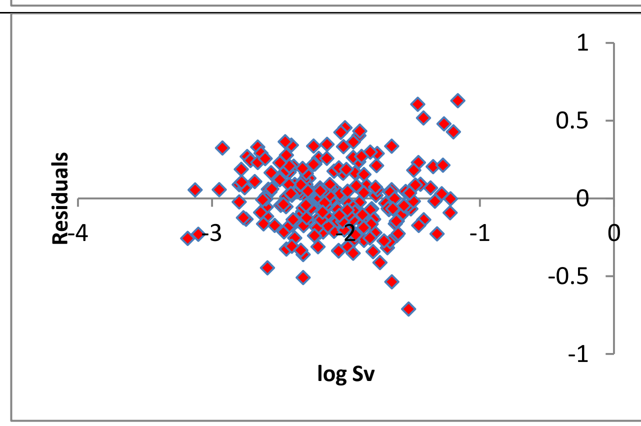
<table>
<tr><td>Output Variable</td><td>Residual error against logarithmic median grain size</td><td>Residual error against logarithmic valley slope</td></tr>
<tr><td>Bankfull Discharge</td><td>
</td><td>Bankfull discharge is independent of valley slope</td></tr>
<tr><td>Bankfull width</td><td>
</td><td>
</td></tr>
<tr><td>Bankfull depth</td><td>
</td><td>
</td></tr>
<tr><td>Sinuosity</td><td>Sinuosity does not depend on median grain size.</td><td>Valley gradient was not considered as one of the input variable during regression as sinuosity is defined as ration of valley slope and channel slope.</td></tr>
</table>

**Figure 3 - 13 : Residual error scatter plots against each independent input variable for phase II**

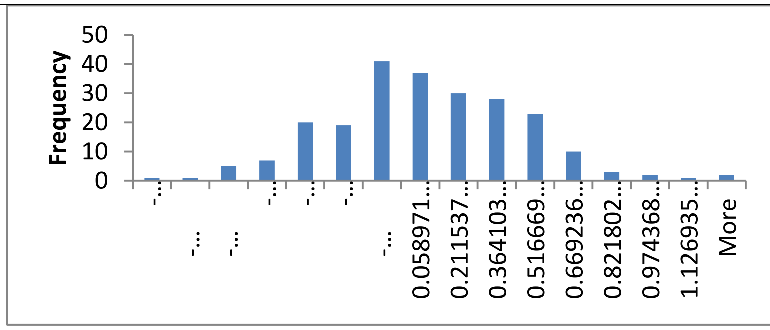
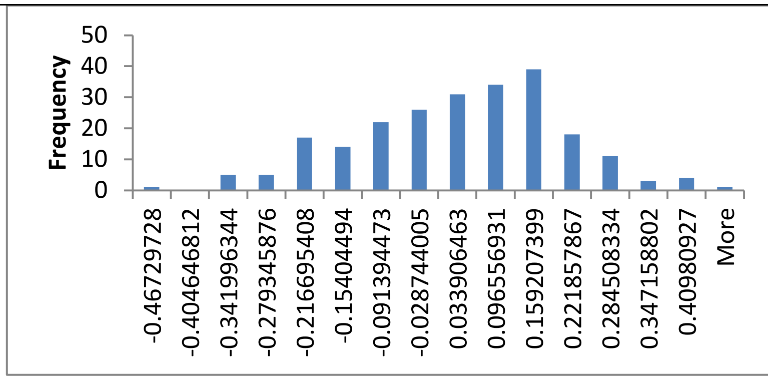
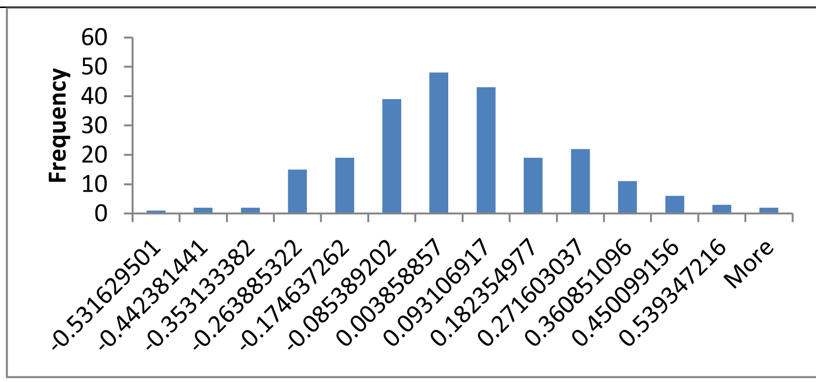
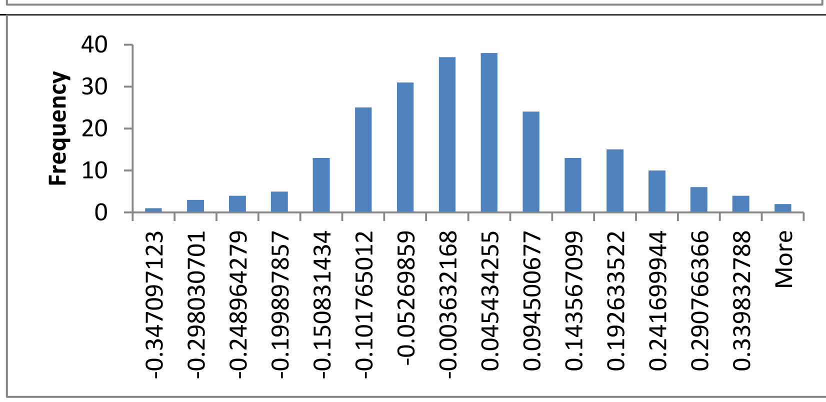
<table>
<tr><th>Output</th><th>Histogram frequency plots of residual errors</th><th>Skewness</th><th>Kurtosis</th></tr>
<tr><td>Bankfull discharge</td><td>
</td><td>.21</td><td>3.3</td></tr>
<tr><td>Bankfull width</td><td>
</td><td>-.19</td><td>2.85</td></tr>
<tr><td>Bankfull depth</td><td>
</td><td>.14</td><td>3.08</td></tr>
<tr><td>Sinuosity</td><td>
</td><td>.27</td><td>3.18</td></tr>
</table>

**Figure 3- 14 : Histogram frequency plots of the residual errors for all the four output of phase II**

From the above scatter plots it is clearly evident that residual errors for all four outputs do not follow any trends with the input variables and were randomly distributed about the horizontal axis. All the four histogram frequency plots further confirm that residual errors (for each output variable) was normally distributed with skewness and excess kurtosis close to 0.

The skewness and kurtosis values presented in these figures, served as yet another proof that residual error histogram may be considered equivalent to a standard normal distribution. These values may not be exactly equal to the Ideal values of skewness equal to 0 and kurtosis equal to 3 but still may be considered under acceptable limits. Bulmer [1979] suggested a rule of thumb that if skewness is between ±.5, the distribution is approximately symmetric. The range of skewness that we calculated in analysis occurred between -.19 & +.27 and thus based on his rule may be considered valid. Regarding kurtosis, Pearson [1905] advocated that if the excess kurtosis for any distribution appeared close to 0, the distribution is termed mesokurtic and can be considered equivalent to a normal distribution. Again, the excess kurtosis in all our 4 cases of residuals came out to be within decimal values and thus the distributions were approximately normal.

### Verifying the universal models using independent datasets of Ohio and Wyoming

Based on 73 independent data points from the state of Ohio and Wyoming, the prediction accuracy of the phase II universal model was checked. The accuracy of channel gradient equation could not be checked using these dataset as they lacked the valley slope measurements. Also, since the universal equations developed above for width and depth included the valley slope variable in their equations, the valley slope was approximated by channel gradient values. This may be acceptable for verification purposes as valley gradient and channel slope values differ only marginally and won't produce much difference in the prediction performances of equations. Figures 3.15, 3.16 and 3.17 compare the predicted values of width, depth and discharge of the two states with their respective reported values.

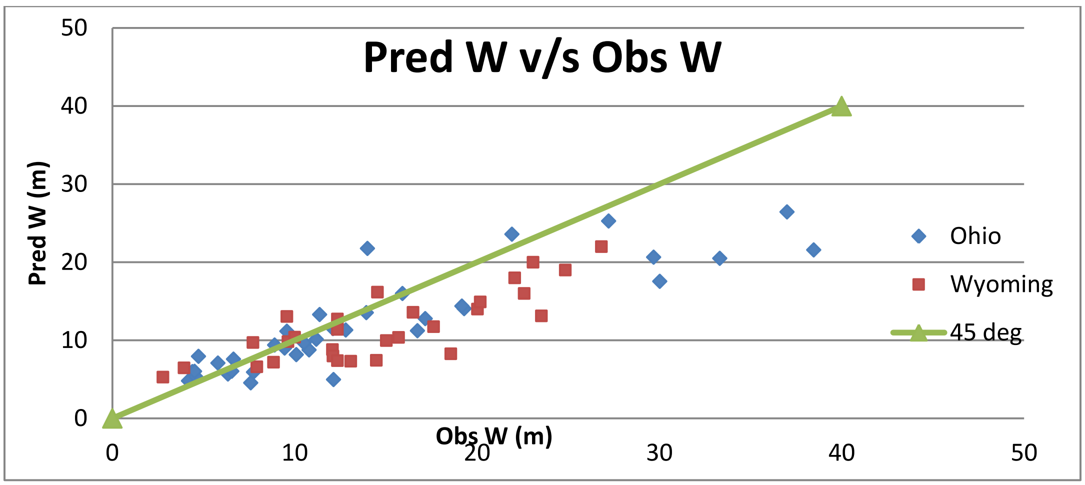



**Figure 3 - 15 : Predicted versus reported values of width for Wyoming and Ohio**

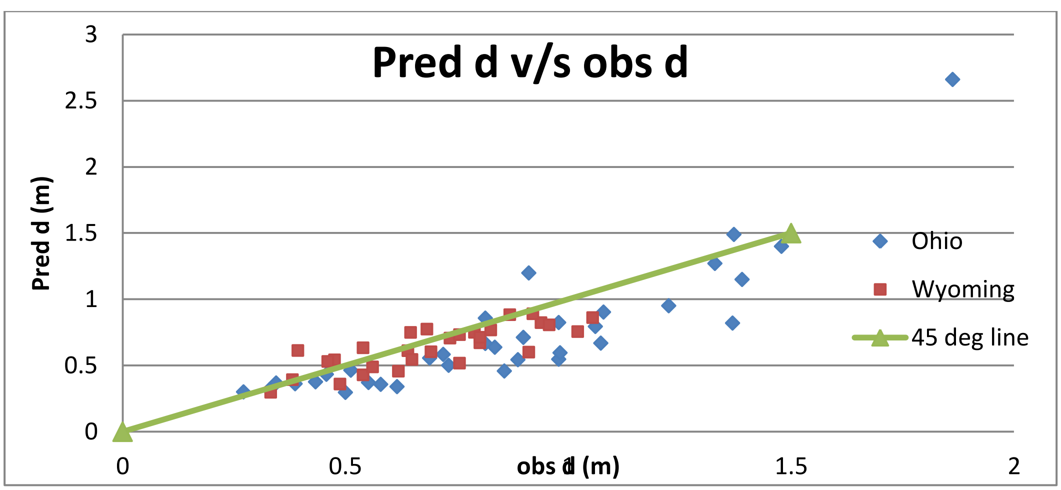



**Figure 3- 16 : Predicted versus reported values of depth for Wyoming and Ohio**

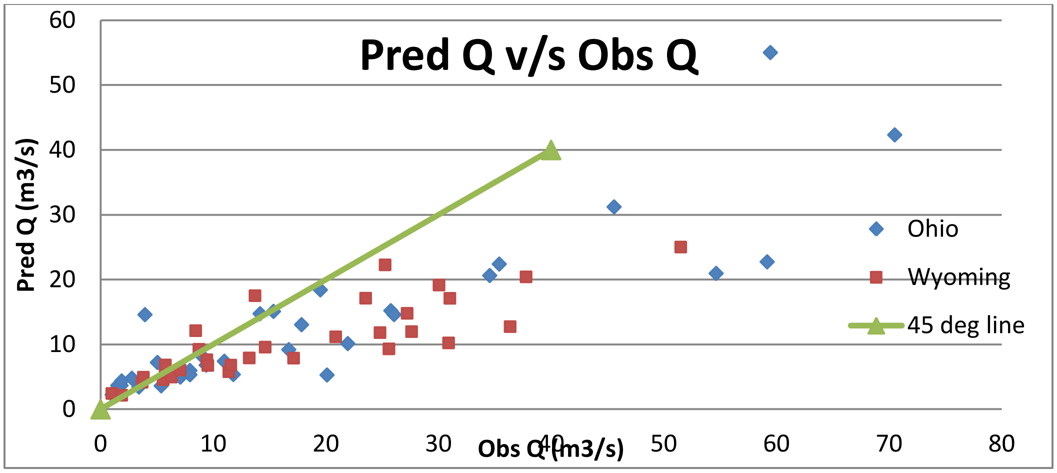



Figure 3 - 17 : Predicted versus reported values of discharge for Wyoming and Ohio

Looking at the above plots, one can clearly say that universal equations developed for width and depth have acceptable prediction ability but for most of the cases under predict the values. The average error produced while predicting width and depth (for both states) came out to be 23% and 19% respectively. It was only in the case of bankfull discharge equation that the error produced was fairly high with an average value of 38%. The interesting aspect about bankfull discharge equation was that it under predicted the discharge values for 95% of all the Wyoming and Ohio locations. It may have arisen due to the inclusion of annual average rainfall in the analysis without considering its intensity and distribution over the entire annual year. Also, in this approach the whole of discharge was considered to being produced by only precipitation falling on the watershed area whereas the discharge coming to a reach from upstream was neglected. Even though the median grain size in the universal equations can be considered indicative of upstream discharge, upstream discharge may not have been captured completely.

## Establishment of regional regime equations

Implementing similar approach as used during phase II formulation of universal equations, regional regime equations were developed separately for six different regions of UK and US. The structure of these equations for all the four outputs were kept similar to the universal ones except with the addition of few extra terms on the input side. Dataset of few regions such as UK and Montana comprised the values of additional input variables such as forest cover, sediment supply etc. and thus it would be interesting to note the effects of these variables on our outputs which otherwise remained unknown during universal regression analysis.

**i)** <u>Equations developed for bankfull discharge</u>

The regional bankfull discharge equations developed in this phase II of our study had following power form:

$$\frac{Qbf}{(\sqrt{gElev})\times Elev^2} = P\left(\frac{AAR}{Elev}\right)^{a}\left(\frac{DA}{Elev^2}\right)^{b}\left(\frac{D50}{Elev}\right)^{c}(Sv)^{d}(U)^{e}(G)^{f}(F)^{g}\left(\frac{Qs}{(\sqrt{gElev})\times\rho\times Elev^2)}\right)^{h}$$

Table 3.16 summarizes the value of the all coefficients and exponents calculated for the six different regions. The input terms that proved out to be statistically insignificant for a region were given the value of 0 in the table. Also, the input terms whose values were missing from a region's dataset and thus could be a part of that region's equation were assigned with the characters "NA" in the table. These concepts of "0" and "NA" were followed during the development of regional equations for width, depth and slope too.

**Table 3 - 16 : Coefficient and exponents values of bankfull discharge equation calculated for all six regions.**

| Region/ State | Coeff (p) | Ann Avg Rainfall exp (a) | Drainage area exp (b) | D50 exp (c) | Valley slope exp (d) | Urban cover exp (e) | Grass cover exp (f) | Forest cover exp (g) | Sediment supply (h) | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Missouri | 3.18E-07 | .80 | .67 | .27 | 0 | .15 | 0 | 0 | NA | .71 |
| Maryland | .00006 | .81 | .79 | .30 | 0 | NA | NA | -.26 | NA | .91 |
| Montana | 1.8E-05 | 1.01 | .83 | .06 | 0 | NA | NA | -.40 | NA | .83 |
| Virginia (Piedmont Province) | 3.15E-07 | .60 | .94 | .05 | 0 | .17 | NA | 0 | NA | .95 |
| Washington | .002 | 1.8 | .49 | .04 | 0 | NA | NA | NA | NA | .93 |
| UK Gravel rivers | .04 | .39 | .56 | .90 | 0 | NA | NA | NA | .10 | .98 |

In all the above six regions, it is clearly visible that bankfull discharge was mainly dependent on the rainfall and drainage area factors. The exponent values for both these inputs were highest for almost all the six regions. This is very much in agreement with what was observed during the formulation of universal regression equations. In the above table, urban cover variable in Missouri and Virginia State is seen to have direct proportionality with the bankfull discharge and forest cover variable in Maryland and Montana is seen to have inverse relationship with the bankfull discharge. Urbanization increases the runoff coefficient and thus has a direct proportionality relationship where forest cover for any regions accounts for rainfall losses and thus is seen to have an inversely relation with discharge.

**ii)** Equations developed for bankfull width

The bankfull width equation formulated for all 6 regions may be represented by the following equation mentioned below:

$$\frac{W}{Elev} = P\left(\frac{AAR}{Elev}\right)^{a}\left(\frac{DA}{Elev^2}\right)^{b}\left(\frac{D50}{Elev}\right)^{c}(Sv)^{d}(U)^{e}(G)^{f}(F)^{g}\left(\frac{Qs}{(\sqrt{gElev})\times\rho\times Elev^2}\right)^{h}$$

The different values of coefficients and exponents for all the regions have been summarized in the table 3.17.

**Table 3 - 17 : Coefficients and exponent values of regional bankfull width equations**

| **Region/ State** | **Coeff (p)** | **Ann Avg Rainfall exp (a)** | **Drainage area exp (b)** | **D50 exp (c)** | **Valley slope exp (d)** | **Urban cover exp (e)** | **Grass cover exp (f)** | **Forest cover exp (g)** | **Sediment supply (h)** | **$R^2$** |
|---|---|---|---|---|---|---|---|---|---|---|
| Missouri | 2.85 | 1.14 | .30 | 0 | -.08 | .03 | 0 | 0 | NA | .80 |
| Maryland | .0018 | -.12 | .50 | .04 | -.09 | NA | NA | 0 | NA | .92 |
| Montana | .07 | .42 | .29 | .07 | -.23 | NA | NA | -.15 | NA | .83 |
| Virginia (Piedmont Province) | .02 | .11 | .40 | .08 | -.07 | 0 | NA | -.16 | NA | .93 |
| Washington | .84 | .61 | .23 | .13 | -.11 | NA | NA | NA | NA | .89 |
| UK Gravel rivers | .13 | .32 | .20 | .20 | -.05 | NA | NA | NA | 0 | .96 |

Once again, rainfall and drainage area emerge as the most significant input variables in estimating the width for all the six regions. Valley slope may also be concluded to be an influential input character having an inverse relationship with the width. Alike the bankfull discharge, forest cover in the states of Montana and Virginia is found to be inversely proportional to the width variable.

**iii)** Equations developed for bankfull depth

Continuing the same approach, bankfull depth equation had the following power form:

$$\frac{d}{\mathrm{Elev}} = P\left(\frac{\mathrm{AAR}}{\mathrm{Elev}}\right)^{a}\left(\frac{\mathrm{DA}}{\mathrm{Elev}^2}\right)^{b}\left(\frac{\mathrm{D50}}{\mathrm{Elev}}\right)^{c}(Sv)^{d}(U)^{e}(G)^{f}(F)^{g}\left(\frac{\mathrm{Qs}}{(\sqrt{g\mathrm{Elev}})\times\rho\times\mathrm{Elev}^2)}\right)^{h}$$

Table 3.18 summarizes the exponents and coefficient values derived for all 6 regions:

**Table 3 - 18 : Coefficients and exponent values of regional bankfull depth equations**

| Region/ State | Coeff (p) | Ann Avg Rainfall exp (a) | Drainage area exp (b) | D50 exp (c) | Valley slope exp (d) | Urban cover exp (e) | Grass cover exp (f) | Forest cover exp (g) | Sediment supply (h) | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Missouri | 1.8E-08 | 1.8 | .47 | .04 | -.26 | .08 | 0 | 0 | NA | .71 |
| Maryland | .017 | .60 | .23 | .09 | -.09 | NA | NA | 0 | NA | .87 |
| Montana | .001 | .30 | .15 | .04 | -.10 | NA | NA | .06 | NA | .83 |
| Virginia (Piedmont Province) | .001 | .26 | .33 | .04 | -.18 | 0 | NA | -.13 | NA | .93 |
| Washington | .016 | .72 | .13 | .08 | -.30 | NA | NA | NA | NA | .91 |
| UK Gravel | .03 | .25 | .24 | .23 | -.06 | NA | NA | NA | 0.03 | .96 |

The exponent values in this table is yet another proof our understanding that rainfall and drainage area for any fluvial system carry the maximum information necessary for determining the properties of any stable channel reach. All other factors mostly influence these two variables and thus are seen to be playing moderate roles in making channel predictions (D50). In fact ,when D50 was removed from the universal regression of the phase II, not much difference was observed in the r2 values of each equation. Its significance may have partially arisen due to some statistical anomaly.However, D50 is still included in these equations as results suggest them to be statistically significant variable. This may serve as a scope to further study these universal models and employ more than the 234 data points for developing them.

**iv)** Equations developed for channel gradient

Since channel gradient was found to be dependent only on valley slope and drainage area, one would expect the regional regime equation developed for estimating channel slope in this phase to be identical to the ones developed in phase I. In both these phases, the form of the equation for channel gradient was:

$$Sc = \frac{\mathrm{Sv}}{10^{(\mathrm{p} + \mathrm{q} \log \frac{\mathrm{DA}}{\mathrm{Elev}^2})^2}}$$

Table 3.19 comprises the values of coefficients and drainage area exponents for all the 6 regions:

Table 3 - 19 : Coefficients and exponent values of regional channel slope equations

| Region/ State | Coefficient (p) | Drainage area exp (q) | $R^2$ |
|---|---|---|---|
| Missouri | .15 | .017 | .99 |
| Maryland | .12 | .3 | .98 |
| Montana | .025 | .11 | .97 |
| Virginia (Piedmont Province) | .50 | -.05 | .99 |
| Washington | .3 | .025 | .98 |
| UK Gravel rivers | .16 | .027 | .99 |

## Validity check of the above regional regressions

The residual error scatter plots for all the above regions were observed to be randomly scattered about the x axis confirming that the residual errors do not follow any identifiable pattern with any of the input variables. Even the frequency plots of the residual errors for all the six regions were found to be approximately normally distributed with skewness and excess kurtosis being close to 0.

## Application of Manning's equation in verifying the above universal regression model

As mentioned earlier in this study, all the field data points that we have used in our analysis so far belong to only stable channel reaches. For all practical purposes and especially for this subsection let us assume the flow in these reaches to be uniform too. Manning [1890] developed the following empirical formula for estimating velocity in an open uniform channel flow:

$$V = \frac{k}{n} R_h^{2/3} . S^{1/2}$$

Since for stream channels the depth is usually small in comparison to width, the hydraulic radius in the above equation may be approximated by the channel depth. Typically for aspect ratios greater than 20, the hydraulic radius is very much equivalent to the depth. Velocity in the above equation can be written in terms of discharge. Implementing these two changes and rearranging the terms, the Manning's equation can be written as:

$$Q = \frac{k}{n} . W . d^{1.67} . S^{.5}$$

Substituting the expressions for width and depth in the right hand side of the Manning's equation and combining the same input variables together, following equation was obtained:

$$Q = \frac{K\prime}{n} \left(\frac{AAR}{Elev}\right)^{(.67+1.67*.68)} \left(\frac{DA}{Elev^2}\right)^{(.18+1.67*.19)} \left(\frac{D50}{Elev}\right)^{(.11+1.67*.14)} (Sv)^{(.5-.22-1.67*.19)} (Elev)^{2}$$

$$Q = \frac{K\prime}{n} \left(\frac{AAR}{Elev}\right)^{(1.80)} \left(\frac{DA}{Elev^2}\right)^{(.49)} \left(\frac{D50}{Elev}\right)^{(.34)} (Sv)^{(.03)} (Elev)^{(2.67)}$$

*K'* in the above expression represents the combined multiplied value of the coefficients of all the input variables. Also, at this point one must realize that in the above expression channel gradient has been considered equal to valley slope variable. This is justified for the above analysis as the channel gradient equation derived earlier had its major dependency on only valley slope and the only other input variable that appeared in the equation "drainage area "had an exponent value of as low as .03.The original universal discharge equation which had been derived previously in the phase II had following form:

$$\frac{Qbf}{(\sqrt{gElev}) \times Elev^2} = .017 \left(\frac{AAR}{Elev}\right)^{1.8} \left(\frac{DA}{Elev^2}\right)^{.49} \left(\frac{D50}{Elev}\right)^{.22}$$

Comparing the above equation with the one derived using manning's, it is clearly visible that the exponent values for rainfall and drainage area in both the expressions are exactly the same. Valley slope in universal discharge equation was statistically found to be an insignificant input variable and in case of manning's derived equation was found to be raised to a negligible exponent value of .03. It was only in the case of grain size variable that the exponent values for the two equations were observed to be slightly different. This slight variation may have occurred due to inclusion of roughness coefficient factor only in the manning's equation but not in the regressions that we had performed. In fact when roughness coefficient in manning's equation was replaced by the Strickler's [1923] formula "$n=.047d50^{1/6}$ ", the difference between the exponents of D50 reduced to as low as .04. Based on all these observations, it can be concluded that the universal equations developed in the phase II of our study do satisfy the manning's formula and thus is a valid regression model.

## Discussion for Phase II

All those watershed variables which had proved out to be trivial during our phase I analysis, emerged statistically significant in the phase II. This once gain reassures our understanding that bankfull discharge in itself includes the effects of all other watershed variables and thus is alone sufficient in determining the width and depth of any stable channel. In phase II, when bankfull discharge was replaced by the new variable "annual average rainfall" all other variables (drainage area, grain size, valley slope and forest cover) became significant and started showing noticeable contribution in estimating the channel variables.

Even though the equations developed in this phase II have appreciable r-square values and do satisfy the manning's formula and the residual error tests, there are still few limitations regarding the applicability of these expressions.  The phase II analysis does not takes into account the distribution pattern of the annual average rainfall. Mean value of precipitation may not be true representative of the rainfall variable and thus the universality of these expressions can be significantly improved by inclusion of distribution factor. However in our study of phase II, ignoring the distribution factor did not affect the results much as the dataset used in our analysis mostly covered only those regions where rainfall occurred almost whole of the year round. Apart from the distribution factor, sediment discharge can be considered as another substantial input variable absent from our universal model. But surprisingly in the regional regression analysis of UK gravel streams, sediment supply came up with very low exponent values and thus seemed to have minimal effect over the channel width, depth and discharge. Verhoog [1987] in one of his works mentioned that low precipitation results in reduced bank vegetation density which in turn increases the peak discharge value and that greatly increases the sediment load in the channels. In short sediment supply can be considered to have an indirect co-relation with the precipitation intensity and distribution and thus may be exempt from the analysis. It is mainly the distribution factor that needs to be incorporated in our equations which would probably be watching over several other unknown and essential factors.

As is evident from the limitations above, there still lies scope of improvement over the universal and regional models that we have developed in our phase II study. Nonetheless, this kind of work is first of its kind and thus may be prone to various challenges in future.

### 3.5.3 : The power of nondimensionalization: Effect of changing repeating variables on universal regressions

The complete results of phase 1 and phase 2 were based upon using elevation as the key nondimensionalising parameter. Why were other variables such $D_{50}$ and DA not considered for the same? Does the accuracy and consistency of regression relationship vary shifting from one variable to another variable? Or else whatever variable one selects, it doesn't makes a difference. This section is completely based on finding answers to these questions and understanding the effects of choosing one variable over the other for nondimensionalization purpose.

Initially, cumulative dataset of 405 field points covering streams of UK and USA were considered for this analysis. Regression equations were developed between channel dimensions and bankfull discharge using all the three possible groups of repeating variables: (Elev, ρ, g), ($D_{50}$, ρ, g) and (DA, ρ, g). The relationships obtained from this analysis have been summarized as shown in the table 3.20.

**Table 3 - 20 : Comparison of regression equations developed using three different repeating variables for complete dataset**

| Bankfull Variable | Non Dimensional Relationship (Elevation based) | Non Dimensional Relationship (D50 based) | Non Dimensional Relationship (Drainage area based) |
|---|---|---|---|
| Width | $\frac{W}{Elev}$=7.8$\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.40}$<br><br>$R^2$=.95, average error= 16%<br>Median error=14%,<br>Std dev=30% | $\frac{W}{D50}$= 6.5$\left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{.41}$<br><br>$R^2$=.93, average error= 20%<br>Median error=19%,<br>Std dev=28% | $\frac{W}{\sqrt{DA}}$= 9.4$\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.42}$<br><br>$R^2$=.92, average error= 25%<br>Median error=14%,<br>Std dev=21% |
| Depth | $\frac{d}{Elev}$=.7 $\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.43}$<br><br>$R^2$=.97, average error= 24%<br>Median error=20%,<br>Std dev=40 | $\frac{d}{D50}$=.35$\left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{.42}$<br><br>$R^2$=.65, average error= 27%<br>Median error=23%,<br>Std dev=37 | $\frac{d}{\sqrt{DA}}$=.8$\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.42}$<br><br>$R^2$=.65, average error= 20%<br>Median error=18%,<br>Std dev=41 |
| Channel Gradient | Sc=.0006$\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{-.15}$<br><br>$R^2$=.26, average error= 150%<br>Median error=59%,<br>Std dev=356% | Sc =.03$\left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{-.16}$<br><br>$R^2$=.37, average error= 113%<br>Median error=57%,<br>Std dev=290% | Sc =.36$\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.21}$<br><br>$R^2$=.18, average error= 150%<br>Median error=66%,<br>Std dev=295% |

The value of the correlation coefficient along with the average error, median error and standard deviation calculated for all the three channel variables, clearly suggest that none of the repeating variable builds a more superior equation in predicting the channel dimensions than the other. The level of accuracy displayed by all three repeating variables is nearly the same and thus selecting one variable over the other does not make a difference. To further strengthen this conclusion, similar regression analysis was made using separate datasets of sandy, gravel and cobble streams. The results once again proved that no difference exists in choosing one repeating variable over the other. The values of the average error, median error and other statistical parameters were found to be the same and thereby making it evident that power of nondimensionalization remains unaffected by changing the repeating variable. Results derived separately for sandy, gravel and cobble streams have been presented in table 3.21, 3.22 and 3.23 respectively.

**Table 3 - 21 : Comparison of regression equations developed using three different repeating variables for sandy stream dataset**

| Bankfull Variable | Non Dimensional Relationship (Elevation based) | Non Dimensional Relationship (D50 based) | Non Dimensional Relationship (Drainage area based) |
|---|---|---|---|
| Width | $\frac{W}{Elev}=7.6\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.39}$<br>$R^2$=.97, average error= 25%<br>Median error=22%,<br>Std dev=33% | $\frac{W}{D50}= 5.2\left(\frac{Qf}{(\sqrt{gD50})\times D50^2}\right)^{.43}$<br>$R^2$=.75, average error= 29%<br>Median error=25%,<br>Std dev=26% | $\frac{W}{DA}= 11\left(\frac{Qb}{(\sqrt{g})\times DA^{5/4}}\right)^{.45}$<br>$R^2$=.79, average error= 21%<br>Median error=20%,<br>Std dev=28% |
| Depth | $\frac{d}{Elev}=.98\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.44}$<br>$R^2$=.96, average error= 24%<br>Median error=30%,<br>Std dev=34% | $\frac{d}{D50}=.20\left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{.45}$<br>$R^2$=.92, average error= 30%<br>Median error=36%,<br>Std dev=32% | $\frac{d}{\sqrt{DA}}=.608\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.41}$<br>$R^2$=.93, average error= 31%<br>Median error=36%,<br>Std dev=33% |
| Channel Gradient | $Sc=.00015\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{-.18}$<br>$R^2$=.26, average error= 140%<br>Median error=63%,<br>Std dev=390% | $Sc=.30\left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{-.29}$<br>$R^2$=.37, average error= 186%<br>Median error=75%,<br>Std dev=480% | $Sc=.06\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.16}$<br>$R^2$=.18, average error= 185%<br>Median error=81%,<br>Std dev=600% |

**Table 3 - 22 : Comparison of regression equations developed using three different repeating variables for gravel stream dataset**

| Bankfull Variable | Non Dimensional Relationship (Elevation based) | Non Dimensional Relationship (D50 based) | Non Dimensional Relationship (Drainage area based) |
|---|---|---|---|
| Width | $\frac{W}{Elev}$=7.8$\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.40}$<br>$R^2$=.93, average error= 22%<br>Median error=14%,<br>Std dev=33% | $\frac{W}{D50}$= 5.8 $\left(\frac{Qf}{(\sqrt{gD50})\times D50^2}\right)^{.42}$<br>$R^2$=.93, average error= 23%<br>Median error=22%,<br>Std dev=45% | $\frac{W}{DA}$= 7.0$\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.40}$<br>$R^2$=.97, average error= 20%<br>Median error=13%,<br>Std dev=26% |
| Depth | $\frac{d}{Elev}$=.62 $\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{.43}$<br>$R^2$=.94, average error= 30%<br>Median error=24%,<br>Std dev=34% | $\frac{d}{D50}$=.42$\left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{.40}$<br>$R^2$=.90, average error= 23%<br>Median error=18%,<br>Std dev=34% | $\frac{d}{DA}$=.21$\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.39}$<br>$R^2$=.96, average error= 18%<br>Median error=27%,<br>Std dev=30% |
| Channel Gradient | Sc=.001$\left(\frac{Qbf}{(\sqrt{gElev})\times Elev^2}\right)^{-.13}$<br>$R^2$=.20, average error= 119%<br>Median error=52%,<br>Std dev=340% | Sc =.09$\left(\frac{Qbf}{(\sqrt{gD50})\times D50^2}\right)^{-.27}$<br>$R^2$=.33, average error= 102%<br>Median error=51%,<br>Std dev=301% | Sc =.035$\left(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}}\right)^{.09}$<br>$R^2$=.10, average error= 118%<br>Median error=58%,<br>Std dev=349% |

**Table 3 - 23 : Comparison of regression equations developed using three different repeating variables for cobble stream dataset**

| Bankfull Variable | Non Dimensional Relationship (Elevation based) | Non Dimensional Relationship (D50 based) | Non Dimensional Relationship (Drainage area based) |
|---|---|---|---|
| Width | $\frac{W}{Elev}=7.9(\frac{Qbf}{(\sqrt{gElev})\times Elev^{2}})^{.40}$<br>R2=.95, average error= 20%<br>Median error=16%,<br>Std dev=26% | $\frac{W}{D50}= 5.8\ (\frac{Qbf}{(\sqrt{gD50})\times D50^{2}})^{43}$<br>R2=.96, average error= 21%<br>Median error=17%,<br>Std dev=27% | $\frac{W}{\sqrt{DA}}=8.8(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}})^{.41}$<br>R2=.96, average error= 14%<br>Median error=13%,<br>Std dev=14% |
| Depth | $\frac{d}{Elev}=.6\ (\frac{Qbf}{(\sqrt{gElev})\times Elev^{2}})^{.43}$<br>$R^2$=.90, average error= 20%<br>Median error=20%,<br>Std dev=41% | $\frac{d}{D50}=.42(\frac{Qbf}{(\sqrt{gD50})\times D50^{2}})^{.40}$<br>$R^2$=.95, average error= 25%<br>Median error=23%,<br>Std dev=53% | $\frac{d}{\sqrt{DA}}=.36(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}})^{.42}$<br>$R^2$=.47, average error= 21%<br>Median error=18%,<br>Std dev=39% |
| Channel Gradient | $Sc=.001(\frac{Qbf}{(\sqrt{gElev})\times Elev^{2}})^{-.14}$<br>$R^2$=.31, average error= 70%<br>Median error=45%,<br>Std dev=106% | $Sc=.06(\frac{Qbf}{(\sqrt{gD50})\times D50^{2}})^{-.25}$<br>$R^2$=.39, average error= 78%<br>Median error=50%,<br>Std dev=100% | $Sc=.005(\frac{Qbf}{(\sqrt{g})\times DA^{5/4}})^{.016}$<br>$R^2$=.29, average error= 87%<br>Median error=67%,<br>Std dev=94% |

Figures 3.18 and 3.19 show the comparison between the predicted and observed values of each variable using all the three repeating variables employed for complete dataset.

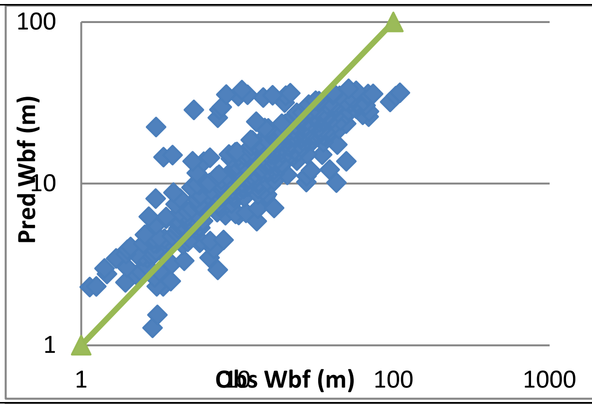
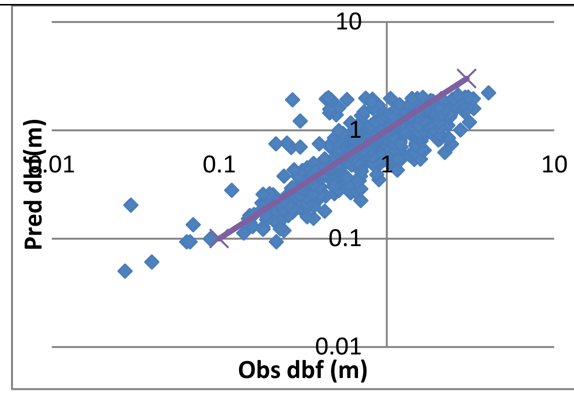
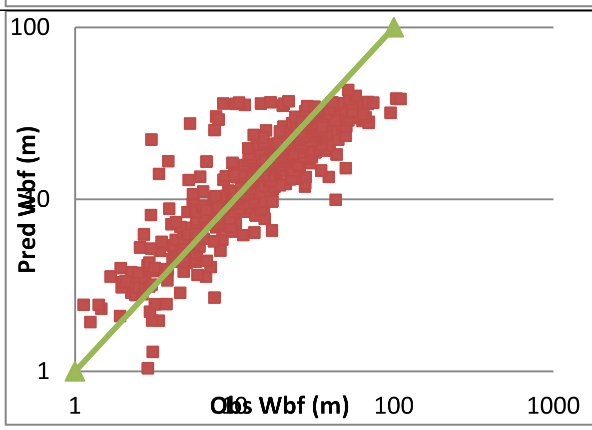
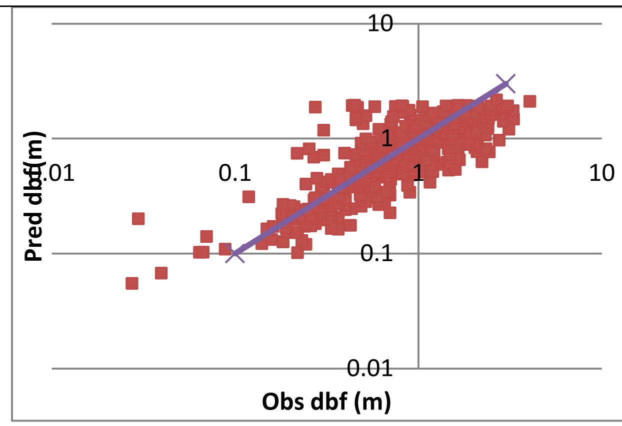
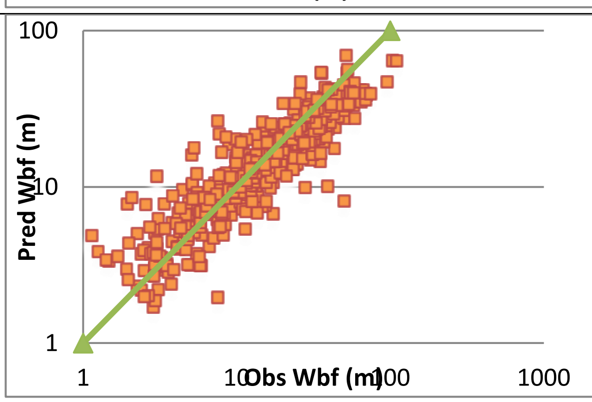
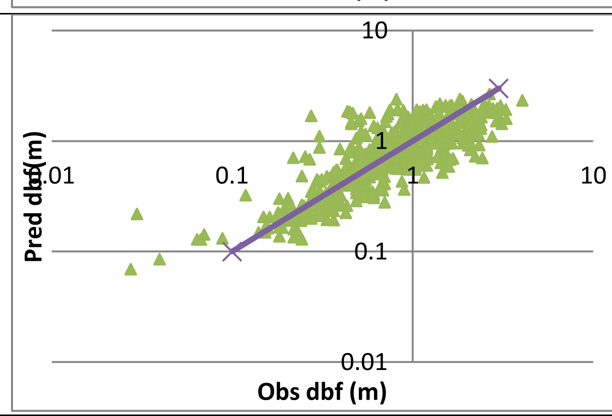
| Repeating Variable | predicted width versus Observed width | Predicted depth versus observed depth |
|---|---|---|
| Elevation based approach |  |  |
| Median grain size based approach |  |  |
| Drainage Area based approach |  |  |

**Figure 3 - 18 : Comparison between predicted and observed width and depth values using three nondimensional techniques.**

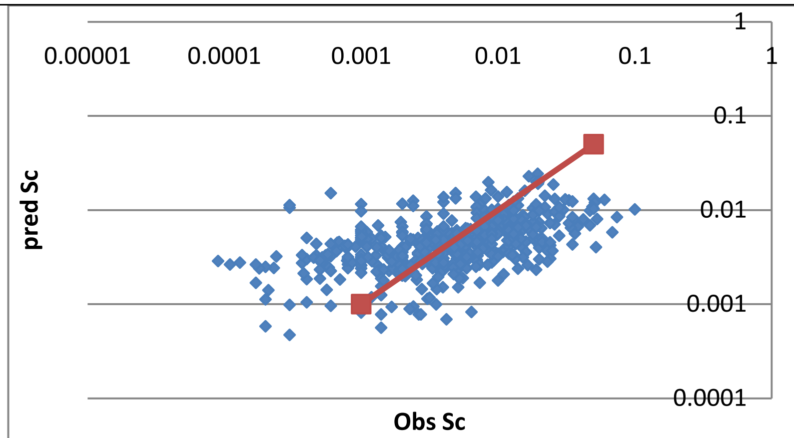
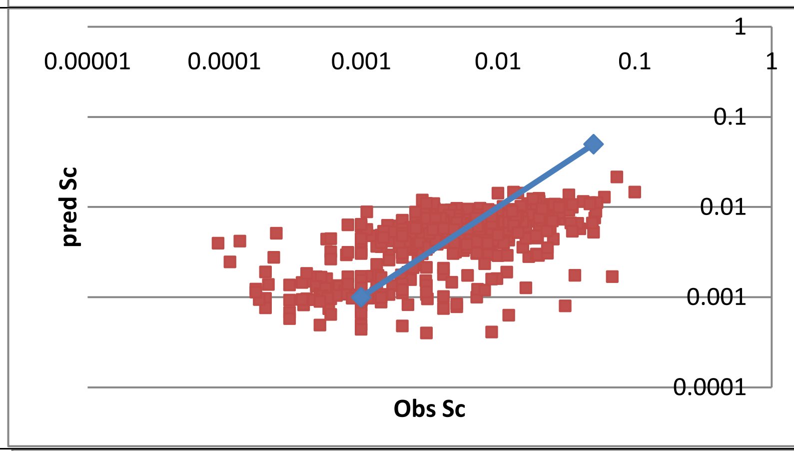
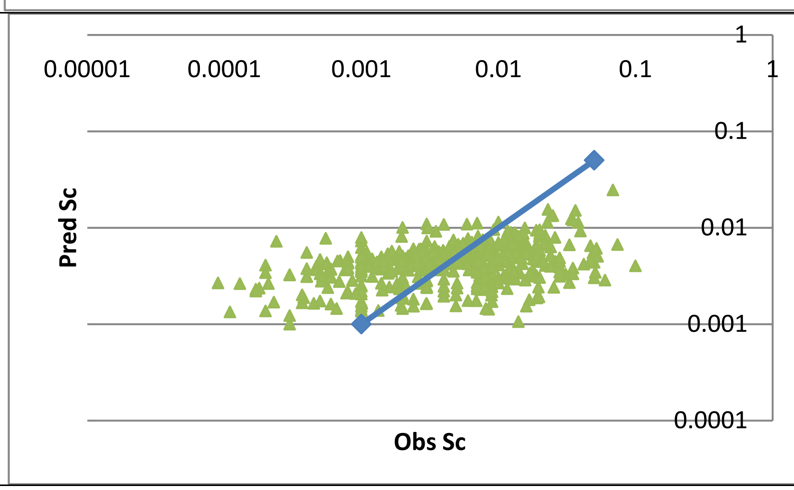
<table>
<tr><th>Repeating Variable</th><th>predicted width versus Observed width</th></tr>
<tr><td>Elevation based approach</td><td>
</td></tr>
<tr><td>Median grain size based approach</td><td>
</td></tr>
<tr><td>Drainage Area based approach</td><td>
</td></tr>
</table>

**Figure 3 - 19 : Comparison between predicted and observed channel slope values using three different nondimensional techniques.**

## Discussion for section III

Even though all the nondimensionalising variables (in the analysis above) were found to be equivalent in their performances, this paper recommends using elevation over the other repeating variables. Favoring elevation over grain size and drainage area can be explained on the basis of following reasons.

- With the advancement in current technologies, determining elevation for any location does not require any field wok and thus is a much easier task than determining drainage area and $D_{50}$. This makes elevation based analysis an "office desk based approach" requiring less time and effort.

- Unlike $D_{50}$, elevation is an independent input variable which probably changes only once in millions of years. $D_{50}$ is more susceptible to measurement errors as the values are comparatively small whereas in case of elevation, errors in the magnitude ± 50 feet would also not affect its performance much.

- Moreover, elevation is an indirect measure of several other fluvial system variables such as relief, geology, temperature, precipitation [Duckstein et al, 1972] and even bank vegetation density. Thus by using elevation as the key parameter in the regime equations, one is assured of including several other influential parameters which otherwise would not have been considered.

## 3.6 Conclusions of this study

The complete work in this study was divided into two different phases. In phase I, dimensionless universal equations in power form were developed for estimating channel width, depth and slope. In the equations developed for width and depth, bankfull discharge emerged as the most significant parameters. Channel gradient was found to be independent of bankfull discharge and majorly dependent on valley slope and drainage area variables. This is in contrast to the results derived by Parker et al. [2007] where channel gradient equation was based on discharge as the only input parameter. Later in phase I, regional regime equations were also developed for 11 different regions of US and UK. In the regional equations developed for width, the exponent values of bankfull discharge once again acquired the most significant value and rest watershed variables (Drainage area, $D_{50}$, valley slope, urban cover, forest cover, etc.) appeared relatively insignificant.

The universal equations for width and depth established bankfull discharge as the only required variable for predicting channel dimensions. All other watershed variables (drainage area, $D_{50}$ etc) were found to be statistically insignificant. Two independent datasets from Ohio and Wyoming were used to test the prediction accuracy of these universal width and depth equations. The values estimated using equations were in close agreement with the reported field values. The universal models were also in good agreement with the Leopold & Maddock [1953] model and Parker [2007] gravel model. In some datasets which Parker had used in his regression analysis such as Hey and Thorne data [1986] and McCandelles data [2003], the universal equations (developed in this paper) gave better prediction results and this even more confirmed the universal behavior of the equations. Also, the universality of the universal models comes from the fact that a large number of 405 field measurement values were used in the analysis, covering 11 different regions of UK and US.

Bankfull discharge in itself includes the effects of all other watershed variables and this possibly explains why other variables appeared insignificant in the regression. This result also led to the realization that bankfull discharge is more of an output variable than being considered as an independent input parameter. The precipitation received by a basin area is acted upon by various watershed processes to produce bankfull discharge thereby confirming discharge as a dependent output variable.

In phase two of the study, annual average rainfall was used as the primary input variable to formulate the universal and regional regime equations for four different output variables: Bankfull discharge, width, depth and channel gradient. In both the universal and regional expressions, along with precipitation all the watershed variables (drainage area, grain size, valley slope, urbanization and forest cover) were found to be statistically significant. The universal and regional equations developed for channel slope in phase II were identical to the one developed in phase I. This is expected as channel slope in both the phases were found to be dependent only on valley slope and drainage area variables. In the equations developed for width, depth and discharge, annual average rainfall and drainage area were found to be the major contributors. Valley slope proved out to be an insignificant variable for the universal discharge equation but followed substantial inverse relationship with the width and depth equations. The universal equations for discharge, width and depth satisfied the manning's open channel formula and thereby established the robustness of these universal equations. The validity of these universal equations developed in phase II was checked using two independent datasets from Ohio and Wyoming. In both these states, the performance of width and depth equations was found to be satisfactory but slightly less accurate than the values predicted using the phase I model. For all the data points of Ohio and Wyoming, the bankfull discharge equation under predicted the discharge values by an average of 38%. This error could have been significantly reduced by taking into account the time distribution factor of annual average rainfall which otherwise has been not included in this analysis. Ideally phase II equations should be favored for estimating the channel dimensions but since it is not always possible to capture all the watershed variables, therefore from engineering point of view revised equations developed in phase I should be used. Also, in cases where it is easier to measure the bankfull width value, a reverse approach may be utilized in finding the bankfull discharge value of the stable channel reaches.

In an additional section, regression equations were separately developed for width, depth and channel gradient by using three different repeating variables: elevation, $D_{50}$ and drainage area. The predicting performance of all the three repeating variables in building nondimensionlized equations was found to be similar. This suggested that choosing one variable over the other does not make a difference in the regressions. However in this paper, elevation is recommended over the other repeating variables as it is easier to determine and also elevation in itself encompasses the affects of several other fluvial system variables such as climate, geology, relief etc.

## Notation

AAR = Annual average rainfall

d= Bankfull depth

$D_{50}$ = Median grain size

DA = Drainage area

Elev = Elevation

F = Forest cover (%)

G = Grass cover (%)

g = Gravitational acceleration

n = Manning's roughness

P = Sinuosity

$Q_{bf}$ = Bankfull discharge

Qs = Sediment supply

Sc= Channel slope

Sv = Valley slope

U = Urban cover (%)

v = Mean flow velocity at bankfull flow

w = Bankfull width

ρ = Density of water

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