APPENDIX ONE

THE KRONECKER PRODUCT OF TWO MATRICES

Let **A** be a $p \times m$ matrix and **B** a $q \times n$ matrix, let a_{ij} denote the element in the i^{th} row and j^{th} column of **A** and let b_{rs} denote the element in the i^{th} row and j^{th} column of **B**. The $pq \times mn$ matrix with $a_{ij}b_{rs}$ as the element in the $(iq + r)^{th}$ row and the $(jn + s)^{th}$ column is called the Kronecker or direct product of **A** and **B** is denoted by $\mathbf{A} \otimes \mathbf{B}$; that is,

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \cdots & a_{1m}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \cdots & a_{2m}\mathbf{B} \\ \vdots & \vdots & & \vdots \\ a_{p1}\mathbf{B} & a_{p2}\mathbf{B} & \cdots & a_{pm}\mathbf{B} \end{bmatrix}.$$