THEORETICAL INVESTIGATION OF THE SECOND SHOCK IN THE BLAST WAVE

by

Melvin Lowell Collier, Jr.

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LIST OF SYMBOLS

r	Radius to a point	L
t	Time	T
$u(r,t)$	Particle velocity	$L\ T^{-1}$
$p(r,t)$	Pressure	$F\ L^{-2}$
(r,t)	Density	$F\ T^2 L^{-4}$
$\overline{p}$	Initial outside pressure	$F\ L^{-2}$
$\overline{\rho}$	Initial outside density	$F\ T^2 L^{-4}$
γ	Ratio of specific heat at constant pressure to specific heat at constant volume	
$\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial r}$	Co-moving derivetive	T^{-1}
E_{tot}	Total energy released upon explosion	$F\ L$
E	Constant $\times E_{tot}$ - energy term	$F\ L^{\alpha-2}$
R_o	Initial radius of sphere	L
$R(t)$	Radius to the first shock wave	L
$\overline{R}(t)$	Radius to the second shock wave	L
$U(t)$	Velocity of first shock wave	$L\ T^{-1}$
$\overline{U}(t)$	Velocity of second shock wave	$L\ T^{-1}$
$K = (E/\overline{\rho})^{1/5}$	Constant	$L\ T^{-2/5}$
$\pi_1 = \frac{r}{R(t')}$	Non-dimensional independent variable	

$$\alpha = \begin{cases} 1 \text{ - Plane explosion} \\ 2 \text{ - Cylindrical explosion} \\ 3 \text{ - Spherical explosion} \end{cases}$$

$\Pi_2 = \frac{\bar{p}\, r^{\alpha}}{E}$ - Non-dimensional independent variable

$\bar{\Pi}_1 = \frac{r}{R(t')}$ - Non-dimensional independent variable

t_o Constant time T

$t'/t_o = \frac{t + t_o}{t_o}$ - Non-dimensional time

g_o Zeroth order pressure function - function of Π_1 or $\bar{\Pi}_1$

h_o Zeroth order density function - function of Π_1 or $\bar{\Pi}_1$

f_o Zeroth order velocity function - function of Π_1 or $\bar{\Pi}_1$

$\theta_o = f_o/\Pi_1$ - Another velocity function

g_1 First order pressure function - function of Π_1 or $\bar{\Pi}_1$

h_1 First order density function - function of Π_1 or $\bar{\Pi}_1$

f_1 First order velocity function - function of Π_1 or $\bar{\Pi}_1$

A_1, A_2, A_3 - Non-dimensional functions of g_o, h_o, θ_o, g_1, h_1, and f_1

$Z_o = \gamma \frac{g_o}{h_o}$

Subscript "1" refers to the front or outside of a moving wave.
Subscript "2" refers to the back or inside of a moving wave.
Subscript "i" refers to conditions inside the sphere at time zero.

I. INTRODUCTION

In the study of blast waves, it has been observed that in addition to the primary strong shock wave ahead of the driving gas, a second shock also forms inside of this driving gas. This formation of the second shock will invalidate all the existing theories of blast waves in that an additional non-isentropic region actually exists inside of the interface between the driven and the driving gas. This thesis investigates the actual behavior of the second shock and the resultant flow pattern.

Let us consider a finite spherical container under a high pressure or containing a large potential energy surrounded by a gas. In this thesis, air is considered to be the gas inside and outside the container. However, the theory developed is valid regardless of the gas considered, provided the explosion can be considered a blast wave and not a flame front. That is, dissociation and combination of the two gases must be considered in that case. When the energy is released from the container, a strong spherical shock wave, the primary shock, is formed and is propagated outward. The initial velocity of the wave is very large, but its velocity decreases with time until it finally comes to rest. This thesis only considers the relatively short period of time when the pressure ratio across the primary shock is sufficiently large compared to unity such that the approximate Rankine-Hugoniot boundary conditions apply.

The particles on the surface of the sphere move outward following behind this primary shock wave. This surface is called the interface. After a short period of time the interface starts to implode on the origin causing the pressure inside to increase to such an extent that a second shock wave forms inside the driving gas. The fact that the interface implodes on the origin was brought out by Oswatitsh[8], and Oswatitsh also predicted that a second shock wave might occur. The gas between the interface and the primary shock, the driven gas, is being compressed between these two surfaces. Although the pressure increases inside the driving gas, the pressure, as well as the velocity, is continuous across the interface. The density, however, is discontinous across this interface.

There are, thus, three regions of flow to be considered: the region between the center and the second shock where the flow may be assumed isentropic, the region between the second whock and the interface where the flow is definitely not isentropic and the region between the interface and the primary shock where again the flow is non-isentropic. The regions discussed are shown in Figure (7).

Although, the flow is non-isentropic, it may be considered homentropic or particle isentropic. That is, the entropy along each particle line is constant, but varies from particle to particle with a sudden jump in entropy across the shock waves. The homentropic flow equation plus the equation of continuity and the equation of motion constitute the three non-linear partial differential equations in the dependent variables velocity, pressure and density and in the independent variables displacement and time.

The boundary conditions to be considered are conservation of mass, momentum and energy across both shock waves and the interface. The approximate Rankine-Hugoniot equations apply across the primary shock, but the exact equations must be applied across the second shock. Also, the primary shock can be considered to be of constant strength to the zeroth order, whereas the second shock strength will vary with time. This is to be expected since at first, the second shock is an outward facing wave but implodes on the orgin after only a short period of time. The boundary conditions across the interface are identically satisfied regardless of the density increase. Pressure and velocity are continuous across the interface. The conditions inside the driving gas will determine the density ratio. This type of surface is called a singular discontinuity.

Since the partial differential equations are highly non-linear they must be reduced to a set of ordinary differential equations in order to be solved. Pressure, density and velocity are thus expanded in a variable series. This thesis will consider only the zeroth and first order terms of this series. The zeroth order equations will be the non-linear ordinary differential equations of the point source blast wave problem and the first order equations, which are linear, will make corrections to the blast wave solution.

In the solution to this problem, we shall refer to the motion as being "self-similar to the first or second shock wave." This means that the velocity is expanded in a series in terms of the velocity of the shock wave and that the motion at any time is similar to the motion at any other time. The term "almost self-similar" means the motion is self-similar to the zeroth order but not to the first order.

II. REVIEW OF THE LITERATURE

Sedov's[1] book is one of the leading works on similarity methods in mechanics, and he devotes a considerable number of pages to the problems of one-dimensional unsteady motion of a gas. He considers only those problems where the solutions are self-similar and the dependent variables of the processes can be expressed in terms of a single non-dimensional independent variable containing only one-dimensional constant. He considers a number of problems all of which are governed by the three partial differential equations used in this thesis. By transforming the equations to the one non-dimensional independent variable, he reduces the three partial differential equations to three ordinary differential equations. He also makes a change of variable transformation which permits him to get an exact solution to certain problems. In getting these exact solutions, he makes use of a field of integral curves plotted in a phase plane; a standard technique of non-linear mechanics but not often tried in the field of fluid mechanics. Sedov does consider the problem of the spherical detonation but considers that the explosion originates from a point. Thus, he has only an expanding weak interface inside of which the gas is at rest. He does obtain an exact solution for the propagation of the spherical shock wave up until the time when the shock can no longer be considered strong. For the problems which can not be solved exactly, he mentions only that a numerical integration is required.

Since this thesis uses the blast wave solution as the zeroth order approximation to the finite explosion, it seems appropriate to devote a few pages to a more detailed summary of the work done by Sedov.

The non-dimensional variables which are pertinent to the strong explosion problem can be considered to be

$$\gamma, \qquad \lambda = \frac{\rho_1^{1/5}\, r}{E_o^{\ 1/5} t^{2/5}}, \quad \tau = \frac{p_1^{\ 5/6}\, t}{E_0^{\ 1/3} \rho_1^{1/2}} \ .$$

If the initial pressure is considered negligible in comparison to the pressure inside ($p_1 \approx o$), then the variable τ drops out and there remains only two parameters to consider, γ and λ . Thus the disturbed gas motion can be considered self-similar. The variable λ can be written in a more general way as

$$\lambda = \frac{r}{B^{\ 1/m} \quad t^{\delta}},$$

and in the case under consideration

$$B^{1/m} = \left[\frac{E_o}{\rho_1}\right]^{1/5} \text{ and } \quad \delta = 2/5.$$

Since p and ρ contain the mass, another constant which also contains the mass must be specified in order to obtain non-dimensional variables. Sedov assumes this constant in the form

$$[a] = M\, L^{k}\, T^{s}.$$

For the blast wave a becomes simply the energy released and thus $k = 2$ and $s = -2$.

The velocity, density and pressure can then be written

$$v = \frac{r}{t} V(\lambda), \qquad \rho = \frac{a}{r^{k+3} t^{s}} R(\lambda),$$

$$p = \frac{a}{r^{k+1} t^{s+2}} P(\lambda),$$ where V, R, and P are functions of the one independent variable λ.

These relationships are only true if the motion is self-similar. A non-self-similar expression for V might be

$$v = \frac{r}{t} \left(V(\lambda) + \gamma V_1(\lambda) + \gamma^2 V_2(\lambda) + \cdots \gamma^n V_n(\lambda) + \cdots \right)$$

and similar expressions for p and ρ. These expressions reduce to the self-similar expressions for $\gamma = 0$.

If the expressions for v, p, and ρ are substituted into the equations of continuity, motion and energy and a new variable

$$Z = \gamma \frac{P}{R}$$

is defined, three non-linear ordinary differential equations are obtained. These equations must be solved consistent with the Rankin-Hugoniot boundary conditions. It can be shown that points of the parabola

$$Z(\lambda) = [V(\lambda) - \delta]^2$$

transform into themselves. The parabola will then be points of weak discontinuities. Points Z_1, V_1 on one side of the parabola will transform into points Z_2, V_2 on the other side of the parabola, and if the transformation is from supersonic relative velocity to subsonic relative velocity, the transformation indicates a shock. Pressure and velocity are zero outside the strong shock and thus

$$Z_1 = 0, \quad V_1 = 0.$$

The transformation then gives

$$V_2 = \frac{4}{5(\gamma + 1)}, \quad R_2 = \frac{\gamma + 1}{\gamma - 1}, \quad Z_2 = \frac{8\gamma}{25}\frac{(\gamma - 1)}{(\gamma + 1)^2}$$

Z_2, V_2, and R_2 then give values to start the integration of the differential equations.

It is interesting to note that by considering an energy integral an exact solution for Z as a function of V can be obtained. The curve is similar to the one shown in figure (8). In order to obtain V. P, and R as functions of λ , however, numerical integration must be carried out. The curves for V, P, and R are similar to the ones shown in figures (1), (2), and (3).

For self-similar gas motion, the motion of the shock can be easily determined from the single non-dimensional variable parameter λ . For the spherical blast wave.

$$\lambda = \frac{r}{(E/\rho_1)^{1/5} t^{2/5}} .$$

For λ = constant, r = R the radius of the shock which is a function of time only. Thus

$$R = \left(E/\rho_1\right)^{1/5} t^{2/5} \lambda ,$$

where λ is any constant other than zero. This is the only possible solution since it is impossible to combine t, ρ_1, and E in a non-dimensional form.

For convenience λ is taken equal to unity and the term E is then not necessarily the energy released. E must, however, be proportional to the energy released and the value of E is then determined from the equations of motion. The shock wave would have the same form regardless of the equations of motion used, provided no new dimensional constants were introduced in the equations. It is interesting to note here, that if the motion is not self-similar, λ in the above equation for R may be a function of time.

Several other papers have been written on this subject of strong plane, cylindrical and spherical shock waves. Among the most important are those written by Taylor(2), Sakurai(3) (4) and Lin(5). Taylor(2) transforms the partial differential equations to ordinary differential equations in much the same manner as Sedov(1).. He uses a numerical integration procedure in order to find his dependent variables in terms of the one non-dimensional independent variable. He gets a similar solution as Sedov[1] for the propagation of the shock and compares his results with date obtained from the New Mexico atomic bomb explosion in 1945. Sedov[1] also uses Taylor's[2] experimental data to illustrate the tremendous amount of energy released on explosion.

Lin[5] follows Taylor's[2] procedure to obtain the shock propagation velocity for a cylindrical wave. He also uses a numerical integration process. He mentions that his results can apply to the exploding thin wire or to obtain the fluid properties in the field of travel of a meteor or a hypersonic traveling missile.

Sakurai[3] [4] considers the case of plane and cylindrical shock waves again issuing from a point. His solutions are constructed in power series of $(c/u)^2$, where c is the sonic velocity of the undisturbed atmosphere and u is the propogation velocity of the shock. In his first paper (I), he obtains a numerical solution using only the zeroth term of the power series. This is a first approximation to the actual solution. His second paper (II) is an extension of the first using the first term in the power series. His method of getting a solution differs from that of Sedov[1], Taylor[2] and Lin[5] in that he uses two non-dimensional independent variables.

It appears that Burt[12] and Sedov[1] developed similar theories for the similarity solution to the point source blast wave at approximately the same time. Burt's[12] method differs from Sedov's[1] only in the fact that he considered an n-dimensional space. It is significant that Burt[12] also obtained an exact solution to his equations.

The six papers discussed above are all similar in several respects. First, the explosion is assumed to originate from a point. Under this assumption, no interface and consequently no second shock wave can be formed. Secondly, each author makes some assumptions as to the nature of the dependence of velocity, pressure and density with respect to the displacement and time. This is necessary in order to reduce the partial differential equations to ordinary differential equations. Finally, all four authors use a numerical integration procedure in order to solve the resulting equations. Sedov[1] and Burt[12] are the only authors who mention that an exact solution is possible in certain cases.

Guderley[6] is another author who is able to obtain an exact solution to a problem involving the same governing equations as the explosion problem. His problem is that of the reflection of a shock wave from the center. A weak shock wave is involved here, rather than the strong shock as in the other problems. He transforms the fluid mechanics equations to ordinary differential equations, makes another transformation and makes use of the phase plane to obtain an exact solution to his problem. In so far as this thesis is concerned, this problem is only of interest because Guderley[6] was able to obtain an exact solution.

The method of characteristics is a powerful tool in obtaining graphically some approximate solutions to problems in fluid mechanics. Several authors discuss the method of characteristics as applied to strong shock waves. Sauer[7] considers the method of characteristics as applied to strong spherical and cylindrical shock waves in an isentropic flow case. He defines a potential function which satisfies the equation of motion automatically. By transforming to a new set of coordinates, he obtains the characteristic equation. He mentions the fact that this potential equation is non-linear and can only be linearized for the case of plane shock waves. However, Sauer[7] does show that an approximate graphical construction can still be used. He does not solve any particular problem using this method.

Oswatitsh[8] and Döring[9] use the method of characteristics to solve the explosion problem. Oswatitsh[8] derives the characteristic equations for a non-isentropic process and then goes on to solve the explosion problem. In order to apply the method of characteristics, he must assume that the energy was released from a finite container. Thus an interface exists in his problem. His problem deals primarily with plane shock waves and he only mentions briefly the spherical shock wave. Oswatitsh[8] mentions that the two gases across the interface could be different, and consequently could have different specific heat ratios. However, in solving his problem he assumes the specific heat ratios across the interface to be the same. After obtaining the characteristic (Mach) lines in the displacement-time plane, Oswatitsh[8] goes on to plot pressure and velocity verses displacement for different values of time. For the spherical shock wave, he shows the sharp pressure rise on one time value and mentions that a second shock wave might exist in the gas to the left of the interface. However, he does not assume this shock wave to exist in solving his problem. Oswatitsh's[8] paper differs from the other explosion problems mentioned in that he assumes a large pressure ratio at time zero and does not mention the energy released on explosion.

III. THE INVESTIGATION

1. Basic Equations and Assumptions

The problem to be considered here is that of determining the fluid state at any point at any time arising from the sudden release of energy from a spherical container. It is assumed that the gas inside and outside the container initially is air and that both gases are at approximately the same temperature. The gas inside, however, is initially at a high pressure and density, or in other words there is a large potential energy stored in the container. Upon explosion or release of this energy, a strong shock wave of assumed spherical symmetry is propagated outward. It is assumed that the specific heat at constant pressure and constant density is a pure constant. That is, there is no heat transfer and no dissociation of the gas. Due to the fact that high temperatures are generated upon the release of this energy, this last assumption is not a particularly good one. However, the assumption is generally the one assumed for a blast wave. The basic equations are, of course, the continuity equations, the momentum equation and the energy or homentropic equation. The continuity equation is

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + (\alpha - 1) \rho \frac{u}{r} = 0 \qquad 1.1$$

where $\alpha = 3$ for spherical symmetry, 2 for cylindrical symmetry and 1 for plane symmetry. The momentum equation is

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial r} + \frac{\partial p}{\partial r} = 0. \qquad 1.2$$

The homentropic equation, which says the entropy is constant along a particle path is

$$\frac{d}{dt} \left(p/\rho^{\gamma} \right) = 0. \qquad 1.3$$

or

$$\frac{\partial}{\partial t} \left(p/\rho^{\gamma} \right) + u \frac{\partial}{\partial r} \left(p/\rho^{\gamma} \right) = 0. \qquad 1.4$$

where γ is the ratio of specific heat of constant pressure to the specific heat of constant volume. γ is assumed constant and equal to approximately 1.4 for air.

2. Dimensional Analysis and Expansion of the Variables

Consider that the dimensional parameters important to the solution of the problem are as follows:

$$E, \ \bar{p}, \ \bar{\rho}, \ r, \ t,$$

where E is a quantity having the dimensions of energy but not necessarily equal to the energy released. The dimensionless numbers which can be formed using these parameters are as follows:

$$\pi_1 = \left(\frac{\bar{\rho}}{E}\right)^{1/5} \frac{r}{t^{2/5}} \qquad 2.1$$

$$\pi_2 = \frac{\bar{p}^{5/6}}{E^{1/3}} \frac{t}{\bar{\rho}^{1/2}} \qquad 2.2$$

However, if the radius of the first shock is assumed to be approximately the same as the radius of a point source explosion, then these two dimensionless numbers can be expressed in a slightly simpler manner. That is, taking the radius of the first shock as

$$R = \frac{E}{\bar{\rho}}^{1/5} t^{2/5}, \qquad 2.3$$

the first dimensionless number can now be written

$$\pi_1 = \frac{r}{R(t)},$$

where π_1 now has a value of one along the shock. Since the density and energy are contained in the first π number, the pressure is the only additional parameter which needs to be considered in the second π number. A convenient combination is obtained by considering r, E, and $\bar{p}$. Thus,

$$\pi_2 = \frac{\bar{p}\, r^{\alpha}}{E} \qquad 2.5$$

However, according to equation (2.3) the radius of the shock wave is zero at time zero. This does not satisfy the boundary conditions which says that at t = o, the radius of the sphere is finite and not zero. Therefore, if we replace t by $t' = t + t_o$ and write

$$R = \left(\frac{E}{\rho}\right)^{1/5} t'^{\,2/5} \qquad 2.6$$

then when t = o

$$R_o = \left(\frac{E}{\rho}\right)^{1/5} t_o^{\,2/5} \qquad 2.7$$

where R_o is the initial radius of the sphere, and if R_o is specified t_o is defined. This is a simple translation of the axes r and t. However, the initial boundary conditions are now satisfied.

Possibly a better expansion of R(t') would be of the form

$$R = (E/\bar{\rho})^{1/5}\ t'^{2/5}\ (1 + L(t')\ \pi_2), \qquad 2.8$$

to the first order terms. However, $L(t')$ is another parameter introduced into the problem which cannot be solved using only the equations of motion. This expansion will be discussed later on.

The variable π_1 will vary from zero to one, and $\pi_1 = 1$ is the value of this variable everywhere along the first shock wave. Since $\bar{p}$ will be assumed small and E is large, the variable π_2 will always be a small number. Since π_2 is small the pressure, density and velocity can be expanded in terms of this variable. The expansions would be as follows

$$u = U\left[f_0(\pi_1) + \pi_2 f_1(\pi_1) + \pi_2^2 f_2(\pi_1) + \cdots\right. \quad 2.9$$

$$p = E/r^3\left[g_0(\pi_1) + \pi_2 g_1(\pi_1) + \pi_2^2 g_2(\pi_1) + \cdots\right. \quad 2.10$$

$$\rho = \frac{E t'^2}{r^5}\left[h_0(\pi_1) + \pi_2 h_1(\pi_1) + \pi_2^2 h_2(\pi_1) + \cdots\right. \quad 2.11$$

If only the terms to first order are considered, then the terms from π_2^2 on are dropped. In this thesis, only the zeroth and first order terms will be considered. It is assumed that since π_2 is to be considered very small the series will converge rapidly and the terms past the first order will be negiligible as compared to the zeroth and first order terms. The dependent variables f_0, g_0, h_0, f_1, g_1, and h_1 are now non-dimensional. The term U is the velocity of the shock front and is

$$U = \frac{dR}{dt'} = n\,K\,t'^{\,n-1} = n\,\frac{R\,(t')}{t'}, \qquad 2.12$$

where $K = (E/\bar{\rho})^{1/5}$ and $n = 2/5$ for a spherical blast wave. The terms E/r^3 and Et^2/r^5 are assumed expressions which have the dimensions of pressure and density respectively.

The governing equations in terms of π_1 are now obtained by substitution equations (2.9), (2.10), (2.11), into equations (1.1), (1.2), (1.4), and using

$$\frac{\partial}{\partial t'} = -\frac{\pi_1^{\,2}\,U}{r}\,\frac{\partial}{\partial \pi_1}, \qquad 2.13$$

$$\frac{\partial}{\partial r} = -\frac{\pi_1}{r}\,\frac{\partial}{\partial \pi_1} + \frac{\alpha\,\pi_2}{r}\,\frac{\partial}{\partial \pi_2}, \qquad 2.14$$

Also, if we let

$$p_o = E/r^3\,, \qquad 2.15$$

$$\rho_o = \frac{E\,t'^{2}}{r^5}\,, \qquad 2.16$$

then

$$p_o/\rho_o U^2 = \frac{\pi_1^{\,2}}{n^2}\,, \qquad 2.17$$

$$\frac{r}{\rho_o U} \frac{\partial \rho_o}{\partial t} = \frac{2 \pi_1}{n}, \qquad 2.18$$

$$r/\rho_o \frac{\partial \rho_o}{\partial r} = -5, \qquad 2.19$$

$$\frac{r}{p_o} \frac{\partial p_o}{\partial r} = -3, \qquad 2.20$$

$$\frac{r}{\rho_o U^2} \frac{\partial p_o}{\partial r} = -3 \left(\frac{\pi_1}{n}\right)^2, \qquad 2.21$$

$$\frac{r}{U^2} \frac{\partial U}{\partial t} = \frac{n-1}{n} \pi_1, \qquad 2.22$$

The zeroth order governing equations of motion, continuity and entropy are, after some simplification,

$$\frac{d g_o}{d \pi_1} = \frac{g_o h_o \theta_o \left[(3-2\gamma)\theta_o + \frac{3n(\gamma-1)-\gamma}{n}\right] - \frac{3\gamma}{n^2} g_o^2}{\pi_1 \left[h_o (\theta_o - 1)^2 - \frac{\gamma}{n^2} g_o\right]}, \qquad 2.23$$

$$\frac{dh_o}{d\pi_1} = \frac{h_o(\theta_o - 1)\left[h_o\left(3\theta_o^2 - \frac{7n+1}{n}\theta_o + 5\right) - \frac{3+5\gamma}{n^2} g_o\right] + \frac{3 g_o h_o \theta_o}{n^2}}{\pi_1 (\theta_o - 1)\left[h_o (\theta_o - 1)^2 - \frac{\gamma}{n^2} g_o\right]}, \qquad 2.24$$

$$\frac{d\theta_o}{d\pi_1} = \frac{\frac{3 g_o}{n^2}(\gamma\theta_o - 1) - h_o \theta_o (\theta_o - 1)(\theta_o - 1/n)}{\pi_1 \left[h_o (\theta_o - 1)^2 - \frac{\gamma}{n^2} g_o\right]}, \qquad 2.25$$

where the substitution $\pi_1 \theta_o = f_o$ was made in order to simplify the results. The first order equations are, after simplification

$$\frac{d g_1}{d \pi_1} = \frac{\gamma \pi_1 g_o(\theta_o-1) A_1 - \gamma g_o A_2 + (\theta_o-1) A_3}{\pi_1^4 h_o(\theta_o-1) \left[h_o(\theta_o-1)^2 - \frac{\gamma g_o}{n^2} \right]}, \text{ --} \tag{2.26}$$

$$\frac{d h_1}{d \pi_1} = \frac{h_o(\theta_o-1)^2 A_1 + h_o(\theta_o-1) A_2 - 1/n^2 A_3}{\pi_1^4 h_o(\theta_o-1)^2 \left[h_o (\theta_o-1)^2 - \frac{\gamma g_o}{n^2} \right]}, \tag{2.27}$$

$$\frac{d f_1}{d \pi_1} = \frac{\frac{\gamma g_o \pi_1}{n^2} A_1 + h_o(\theta_o-1) A_2 - 1/n^2 A_3}{\pi_1^3 h_o^2 (\theta_o-1) \left[h_o(\theta_o-1)^2 - \frac{\gamma g_o}{n^2} \right]}, \tag{2.28}$$

where

$$A_1 = - \frac{d h_o}{d \pi_1} f_1 - \left(5 + \pi_1 \frac{d \theta_o}{d \pi_1} + \theta_o\right) h_1, \tag{2.29}$$

$$A_2 = - \left(4 \theta_o + \pi_1 \frac{d \theta_o}{d \pi_1} + \frac{n-1}{n}\right) h_o f_1 - \left(3 g_o - \pi_1 \frac{d g_o}{d \pi_1}\right) \times \frac{h_1}{h_o h^2}, \tag{2.30}$$

$$A_3 = - \gamma \pi_1 (\theta_o-1) \left(5 h_o - \pi_1 \frac{d h_o}{d \pi_1}\right) g_1 - \frac{3 g_o h_o}{\theta_o-1} f_1 - \left[\pi_1^2(\theta_o -1) \frac{d g_o}{d \pi_1} + \pi_1 g_o \left((2 \gamma -3)\theta_o - 5 \gamma \right) \right] h_1 \tag{2.31}$$

Thus it is necessary to solve six ordinary differential equations in order to find g_o, f_o, θ_o, g_1, h_1, and f_1 as functions of π_1. The zeroth order equations can be solved independently of the first order equations. However, for convenience, the six equations were solved simultaneously. The numerical procedure of Runge-Kutta was applied to fourth order accuracy.

3. Boundary Conditions Across a Shock Front

The assumptions are made that the discontinuity is infinitesimally thin and that the Rankine-Hugonoit equations of conservation of energy, mass and momentum apply. The equations are:

conservation of mass

$$\rho_1 (u_1 - U) = \rho_2 (u_2 - U), \qquad 3.1$$

conservation of momentum

$$\rho_1 (u_1 - U)^2 + p_1 = \rho_2 (u_2 - U)^2 + p_2, \qquad 3.2$$

and conservation of energy

$$\rho_1 (u_1 - U) \left[(u_1 - U)^2 + \frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \right] = \rho_2 (u_2 - U)$$

$$\left[(u_2 - U)^2 \quad \frac{2\gamma}{\gamma - 1} \frac{p_2}{\rho_2} \right] \qquad 3.3$$

where U is the velocity of the front. This thesis will be concerned with two types of discontinuity, the shock front and the contact front. For the interface $u_1 = u_2 = U$ and the first and third equations are satisfied identically. The second equation gives the relationship that $p_1 = p_2$. This means that the density ratio across a contact discontinuity is arbitrary and must be determined by other conditions. This type of discontinuity is called a singular discontinuity. The boundary conditions across the interface are given in equations (3.2), (3.3), and (3.4).

For a shock front

$$\rho_1(u_1-U) = \rho_2 (u_2-U) \neq 0$$

and the conservation of energy becomes

$$(u_1-U)^2 + \frac{2\gamma}{\gamma -1} \frac{p_1}{\rho_1} = (u_2-U)^2 + \frac{2\gamma}{\gamma -1} \frac{p_2}{\rho_2} . \qquad 3.4$$

Substituting equations (2.9), (2.10), and (2.11) into the above expressions, we obtain three zeroth order equations

$$h_{o1} (f_{o1}-1) = h_{o2} (f_{o2}-1), \qquad 3.5$$

$$h_{o1} (f_{o1}-1)^2 + \frac{\pi 1^2}{n^2} g_{o1} = h_{o2} (f_{o2}-1)^2 + \frac{\pi 1^2}{n^2} g_{o2}, \qquad 3.6$$

$$(f_{o1}-1)^2 + \frac{2\gamma}{\gamma -1} \frac{\pi 1^2}{n^2} \frac{g_{o1}}{h_{o1}} = (f_{o2}-1)^2 + \frac{2\gamma}{\gamma -1} \frac{\pi 1^2}{n^2} \frac{g_{o2}}{h_{o2}}, \qquad 3.7$$

where the subscript 1 refers to the front side of the wave and the subscript 2 to the back side of the wave. Making the substitution

$$z_{o1} = \gamma \frac{g_{o1}}{h_{o1}} , \qquad 3.8$$

the above equations can be solved for the "2" variables in terms of the "1" variable. The equations are

$$(f_{o2}-1) = (f_{o1}-1)\left[1 + \frac{2}{\gamma+1}\cdot\frac{\frac{\pi l^2}{n^2} z_{o1} - (f_{o1}-1)^2}{(f_{o1}-1)^2}\right], \quad 3.9$$

$$\frac{\pi l^2}{n^2} z_{o2} = \frac{\gamma-1}{\gamma+1}\,\frac{1}{(f_{o1}-1)^2}\left[(f_{o1}-1)^2 + \frac{2}{\gamma-1}\,\frac{\pi l^2}{n^2}\right] z_{o1}$$

$$\left[\frac{2\gamma}{\gamma-1}(f_{o1}-1)^2 - \frac{\pi l^2}{n^2} z_{o1}\right], \quad 3.10$$

and

$$h_{o2} = \frac{h_{o1}(f_{o1}-1)}{(f_{o2}-1)}. \quad 3.11$$

The three first order equations which are obtained are as follows:

$$h_{11}(f_{o1}-1) + h_{o1} f_{11} = h_{12}(f_{o2}-1) + h_{o2} f_{12}, \quad 3.12$$

$$h_{11}(f_{o1}-1)^2 + 2 h_{o1}(f_{o1}-1) f_{11} + \frac{\pi l^2}{n^2} g_{11} =$$

$$h_{12}(f_{o2}-1)^2 + 2 h_{o2}(f_{o2}-1) f_{12} + \frac{\pi l^2}{n^2} g_{12} \quad 3.13$$

$$2 f_{11}(f_{o1}-1) + \frac{2\gamma}{\gamma-1}\,{\frac{\pi l}{n}}^{2}\left(\frac{g_{11}}{h_{o1}} - \frac{g_{o1}}{h_{o1}^2} h_{11}\right) =$$

$$2\, f_{12}\ (f_{o2}-1) + \frac{2\,\gamma}{\gamma-1}\ \frac{\pi\, 1^2}{n^2} \left(\frac{g_{12}}{h_{o2}} - \frac{g_{o2}}{h_{o2}^2}\, h_{12} \right). \qquad 3.14$$

These equations can be solved for the "2" variables in terms of the "1" variables by inverting the matrix form of the equations

$$\begin{bmatrix} f_{o2} & o & f_{o2}-1 \\ 2\, h_{o2}\ (f_{o2}-1) & (\pi\, 1/n)^2 & (f_{o2}-1)^2 \\ 2\ (f_{o2}-1) & \frac{2\,\gamma}{\gamma-1}\ \frac{\pi\, 1^2}{n^2\, h_{o2}} & \frac{-2\,\gamma\ g_{o2}\ \pi\, 1^2}{(\gamma-1)\, n^2\, h_{o2}^2} \end{bmatrix} \begin{bmatrix} f_{12} \\ g_{12} \\ h_{12} \end{bmatrix} =$$

$$\begin{bmatrix} h_{11}\ (f_{o1}-1) + h_{o1}\ f_{11} \\ h_{11}\ (f_{o1}-1)^2 + 2\, h_{o1}\ (f_{o1}-1)\ f_{11} + \frac{\pi\, 1^2}{n^2}\, g_{11} \\ 2\ (f_{o1}-1)\ f_{11} + \frac{2\,\gamma}{\gamma-1}\ \frac{\pi\, 1^2}{n^2\, h_{o1}}\ (g_{11} - \frac{g_{o1}}{h_{o1}}\, h_{11}) \end{bmatrix}. \qquad 3.15$$

For the second shock wave, this calculation is most easily done on a computer. However, for the first shock wave

$$p_1 = \frac{E}{r^3} \left[g_{o1} + \frac{\bar{p}\, r^3}{E}\ g_{11} \right] = \bar{p} \qquad 3.16$$

$$u_1 = U \left[f_{o1} + \frac{\bar{p}\, r^3}{E}\ f_{11} \right] = o \qquad 3.17$$

$$\rho_1 = \frac{E\, t'^2}{r^5} \left[h_{o1} + \frac{\bar{p}\, r^3}{E}\ h_{11} \right] = \bar{\rho} \qquad 3.18$$

where $\bar{p}$ and $\bar{\rho}$ are the initial outside pressure and density respectively. Thus $f_{o1} = f_{11} = g_{o1} = h_{11} = 0$ and $h_{o1} = g_{11} = 1$ and $\pi_1 = 1$. Solving the equations with the above initial values gives the conditions inside the first shock, which are as follows:

$$z_{o2} = \frac{2\gamma(\gamma-1)}{n^2(\gamma+1)^2}, \qquad 3.19$$

$$f_{o2} = \frac{2}{\gamma+1}, \qquad 3.20$$

$$h_{o2} = \frac{\gamma+1}{\gamma-1}, \qquad 3.21$$

$$g_{o2} = \frac{2}{n^2(\gamma+1)}, \qquad 3.22$$

$$f_{12} = \frac{-2\gamma}{n^2(\gamma+1)}, \qquad 3.23$$

$$g_{12} = -\frac{\gamma-1}{\gamma+1}, \qquad 3.24$$

$$h_{12} = \frac{-2\gamma(\gamma+1)}{n^2(\gamma-1)^2}. \qquad 3.25$$

4. The Solution Inside the First Shock Waves

For the first shock wave $\eta = 2/5$, $\pi_1 = 1$ and the initial value of the other variables are determined by the boundary conditions across the shock front. The boundary conditions will be discussed later. The numerical solutions to the equations are shown in figures (1), (2), (3), (4), (5), and (6). The increment on π_1 was taken as .001.

Under the above assumptions the zeroth order solutions are self-similar to the first shock wave since the solutions depend on only the one non-dimensional variable π_1. The first order solutions are of course not self-similar because of the π_2 variable and the resultant pressure, density and velocity are then non-self-similar. The solutions to the equations can only apply between the first shock wave and the original particle line of the sphere; the contact discontinuity or interface. In order to find this contact discontinuity, it is only necessary to solve the ordinary differential equation

$$U = \frac{d\left(\frac{R_c}{R_o}\right)}{d\,(t'/t_o)} = \frac{U}{K_o^{-3/5}}\left[f_o\,(\pi_1) + \pi_2\, f_1\,(\pi_1)\right] \qquad 4.1$$

for $\frac{R_c}{R_o}$ as a function of t'/t_o with the initial conditions of $\frac{R_c}{R_o} = 1$ when $t'/t_o = 1$.

These non-dimensional variables correspond to r= initial radius of the sphere when t = 0. The $f_o(\pi_1)$ and $f_1(\pi_1)$ are the solutions shown in figures (3) and (6). The solution to this equation was obtained using the IBM 1620 Computer, and the solutions for R(t), (first shock wave), and $R_c(t)$ (interface) are shown in Table I. The plot of these curves is shown along with the plot of the second shock wave in figure (7).

In order to find a solution inside the contact discontinuity, we must solve equations (2.23) thru (2.28) with initial conditions starting inside the contact surface. The Rankine-Hugoniot boundary conditions across this surface are

$$\frac{u_2}{u_1} = 1, \qquad 4.2$$

$$\frac{p_2}{p_1} = 1, \qquad 4.3$$

$$\frac{\rho_2}{\rho_1} = \delta \qquad 4.4$$

where δ is arbitrary. In order to get a numerical solution δ must be specified. The value of δ will be used then as a constant but a variable parameter and a mass integral will be written to check the assumption. That is, at any time the mass inside the contact surface must be the same as the original mass. The original pressure and density ratio will depend on the assumption on δ . From previous experimental work on blast waves, where the original temperature ratio was one, a value of δ of 2.65 was measured.[13][14]* Therefore, δ will be taken arbitrarily as 2.65 in this example. Thus, the non-dimensional variables for velocity, pressure and density become

$$f_{02} = f_{01},\ f_{12} = f_{11},\ g_{02} = g_{01},\ g_{12} = g_{11}$$

$$h_{02} = 2.65\ h_{01},\ h_{12} = 2.65\ h_{11} \qquad 4.5$$

where

$$f_{01},\ f_{11},\ g_{01},\ g_{11},\ h_{01},\ h_{11}$$

are conditions on the front side of the interface. The equations will have to be solved using the above initial values and the value of π_1 given along the contact surface at a particular time. Thus, the solutions can no longer be considered self-similar between the interface and the second shock since time must be specified for each solution.

Now consider how the equations behave inside the contact surface. Define a new variable

$$z_o = \frac{\gamma\ g_o}{h_o} \qquad 4.6$$

and solve equations (2.23), (2.24) and(2.25) for

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* numbers in parentheses refer to bibliography

$$\frac{d\theta_o}{d Z_o} = \gamma \, (\theta_o - 1) \Bigg[\frac{3 Z_o}{\gamma n^2} (\gamma \theta_o - 1) - \theta_o (\theta_o - 1) (\theta_o - 1/n)$$

$$- \frac{3 Z_o \theta_o}{\gamma n^2} - \gamma (\theta_o - 1) \left(3 \theta_o^2 - \frac{7n+1}{n} \theta_o + 5 \right.$$

$$\left. \left. - \frac{3+5\gamma}{n^2} \frac{Z_\theta}{\gamma} \right) \right] \Bigg/ Z_o \left[\gamma \theta_o \left((3-2\gamma) \theta_o \right. \right.$$

$$\left. \left. + \frac{3n(\gamma - 1) - \gamma}{n} \right) - \frac{3 \gamma Z_o}{n^2} \right] (\theta_o - 1) \qquad 4.7$$

This equation has a singular point at $Z_o = 0$ and $\theta_o = 1$. There are other singular points, but this is the only one of interest in this thesis. The graph of the curves going through this singular point is shown in figure (8); the arrows indicate the direction of increasing π_1. The curves shown are for $\eta = 2/5$. The point A of

$$Z_o(1) = \frac{2 \gamma (\gamma - 1)}{n^2 (\gamma + 1)^2} \qquad 4.8$$

$$\theta_o(1) = \frac{2}{\gamma + 1} \qquad 4.9$$

is the initial point inside the first shock wave and the solution of the zeroth order equations in this region is given by the curve AB. After the jump conditions across the contact surface, the solution follows a curve such as CD. It is evident from the zeroth and first order equations that as points along the curve

$$\frac{Z_o}{n^2} = (\theta_o - 1)^2 \qquad 4.10$$

or

$$\frac{\gamma g_o}{n^2} = h_o \ (\theta_o - 1)^2 \qquad 4.11$$

are approached, the gradients of g_o, h_o, θ_o, g_1, h_1, and f_1 become very large. Points close to this parabola will then indicate points where a second shock wave must occur. The second shock wave can then be located by choosing points where the gradients of the functions first become large. The location of this shock is only approximate and the solution obtained between the contact surface and this shock can not be correct since the boundary conditions across the shock involve the velocity of the second shock and the equations were solved for $n = 2/5$ which means the velocity of the first shock is involved in the equations. The approximate location of this second shock is given in Table I and shown in figure (7). In order to solve the equations inside the contact surface, the velocity of the second shock must be found and it is assumed that the motion is almost self-similar between the interface and the second shock. That is, it is assumed that the motion is almost self-similar except on the boundaries. A plot of log ($\bar{R}/R_o$) versus log (t'/t_o) is shown in figure (9). $\bar{R}$ (t'/t_o) is the radius to the second shock. For

$$1.0 < t'/t_o < 2.0, \quad \bar{R}/R_o$$

can be expressed as

$$\frac{\bar{R}}{R_o} = (t'/t_o)^n \qquad 4.12$$

where $n = .285$. For $t'/t_o > 2.0$, the equation of the curve can be expressed as

$$\frac{\bar{R}}{R_o} = (t'/t_o)^n + \text{constant}. \qquad 4.13$$

However, since only the velocity is involved in the equation, n is the only important parameter and the constant is immaterial. As the velocity of the second shock is now known, the equations can be solved inside the contact surface. Since the motion is to be assumed self-similar from the interface to the second shock, a new variable $\bar{\pi}_1$ is defined as

$$\bar{\pi}_1 = \frac{r}{\bar{R}(t')} \qquad 4.14$$

and thus $\bar{\pi}_1 = 1$ along the second shock wave. The velocity of this wave is given by

$$\bar{U} = n \frac{\bar{R}}{t'} \qquad 4.15$$

and the dependent variables are now expanded as

$$u = \bar{U} \left[f_o + \pi_2 f_1 \right] \qquad 4.16$$

$$p = \frac{E}{r^3} \left[g_o + \pi_2 g_1 \right] \qquad 4.17$$

$$\rho = \frac{E\, t'^2}{r^5} \left[h_o + \pi_2 h_1 \right] \qquad 4.18$$

where f_o, f_1, g_o, g_1, h_o, and h_1 are now functions of $\bar{\pi}_1$. The velocity of the second shock $\bar{U}$ will now appear in the differential equations. The differential equations on g_o, h_o, θ_o, g_1, h_1, f_1 are the same as equations (2.23) thru (2.28) with the exception that π_1 is changed to $\bar{\pi}_1$ and n is no longer 2/5 but now depends on (t'/t_o).

At the interface π_1 is given and $\bar{\pi}_1$ can then be calculated; $\bar{\pi}_1$ is of course greater than one since the interface is outside the second shock.

$$\bar{\pi}_1 = \frac{r}{\bar{R}} = \frac{r}{R} \frac{R}{\bar{R}} = \pi_1 \frac{R}{\bar{R}} = \pi_1 (t'/t_o)^{2/5-n} \qquad 4.19$$

At the contact surface or interface

$u_1 = u_2$ and $U \left[f_o (\pi_1) + \pi_2 f_1 (\pi_1) \right] =$

$$\bar{U} \left[f_o (\overline{\pi}_1) + \pi_2 f_1 (\overline{\pi}_1) \right] \qquad 4.20$$

and

$$f_o (\pi_1) = \pi_1 \theta_o (\pi_1), \; f_o (\overline{\pi}_1) = \overline{\pi}_1 \theta_o (\overline{\pi}_1).$$

Thus

$$\theta_o (\overline{\pi}_1) = \frac{2/5}{n} \theta_o (\pi_1) \qquad 4.21$$

and

$$f_1 (\overline{\pi}_1) = \frac{2/5}{n} \frac{R}{\bar{R}} f_1 (\pi_1) \qquad 4.22$$

For $p_2 = p_1$ and $\rho_2 = \delta \rho_1$ we get respectively

$$g_o (\overline{\pi}_1) = g_o (\pi_1) \qquad 4.23$$

$$g_1 (\overline{\pi}_1) = g_1 (\pi_1) \qquad 4.24$$

$$h_o (\overline{\pi}_1) = \delta h_o (\pi_1) \qquad 4.25$$

$$h_1 (\overline{\pi}_1) = \delta h_1 (\pi_1) \qquad 4.26$$

where f_o, f_1, g_o, g_1, h_o, h_1 are known values for π_1. Using these values as initial values equations (2.23) thru (2.28) can be solved to give $f_o (\overline{\pi}_1)$, $f_1 (\overline{\pi}_1)$, $g_o (\overline{\pi}_1)$, $g_1 (\overline{\pi}_1)$, $h_o (\overline{\pi}_1)$, $h_1 (\overline{\pi}_1)$. The solution to the equations was obtained for $t'/t_o < 2.0$ where $n = .285$. The solution for $t'/t_o > 2.0$ would be obtained in a similar manner, the only difference being in the value of n. According to this theory then the second shock wave should occur at $\overline{\pi}_1 = 1.0$. However, due to the fact that n can only be approximated and the fact that this is a numerical solution $\overline{\pi}_1$ is not exactly 1.0. The greatest deviation from 1.0 occurs at $t'/t_o = 2$ where $\overline{\pi}_1 = .985$. For $t'/to < 2$ $\overline{\pi}_1$ is about .995. This does not change the location of the second shock appreciably and therefore these values will be taken as the location of the second shock wave.

There are actually several points within a range of π_1 of .001 which satisfy the boundary conditions across the second shock. A change in π_1 of .001, however, does not essentially alter the location of the second shock wave. The pressure, density, and velocity do change considerably because of the steep gradients at these points. However, the pressure, density and velocity ratios must increase or decrease continuously with time. There is then only one point at each time which satisfies this condition and the curves of pressure, density and velocity ratios vs t'/t_o are shown in figure (10). The interval on $\overline{\pi}_1$ in the region close to the second shock was taken as .0001 but there is still the possibility that an error can exist in the location of the second shock wave.

The pressure, density and velocity can now be calculated from the first shock wave to the second shock wave, by using the original assumptions on the dependent variables. These equations can be more conveniently expressed in a non-dimensional form as follows:

$$\frac{pR^3}{E} = \left[\frac{g_o(\overline{\pi}_1)}{\pi_1^{\,3}} + \frac{\bar{p}R_o^{\,3}}{E}\left(\frac{R}{R_o}\right)^3 g_1(\overline{\pi}_1) \right] \qquad 4.27$$

$$\frac{\rho R^5}{E\, t_o^{\,2}} = \left(\frac{t'}{t_o}\right)^2 \left[\frac{h_o(\bar{\pi}_1)}{\pi_1^{\,5}} + \frac{\bar{p} R_o^{\,3}}{E}\left(\frac{R}{R_o}\right)^3 \frac{h_1(\bar{\pi}_1)}{\pi_1^{\,2}} \right. \qquad 4.28$$

$$\frac{u}{U} = \frac{n}{2/5}\left(\frac{t'}{t_o}\right)^{n-2/5} \left[\overline{\pi}_1\, \theta_o(\overline{\pi}_1) + \frac{\bar{p} R_o^{\,3}}{E}\, \pi_1^{\,3} \right.$$

$$\left. \left(\frac{R}{R_o}\right)^3 f_1(\overline{\pi}_1) \right] \qquad 4.29$$

For the region between the first shock wave and the interface $n = 2/5$, $\overline{\pi}_1 = \pi_1$ and the equations are somewhat simplified. These values of pressure, density and velocity are shown in figures (11), (12), and (13).

5. The Region Inside the Second Shock Wave

Inside the second shock wave, a rarefaction wave must emanate from the point of the initial radius of the sphere. That is, a wave must be propagated inward after the explosion and a particle inside the initial radius would not being to move until the wave reached the initial location of this particle. Under this assumption, the solution inside the second shock wave will be an isentropic centered wave and the characteristics will be straight lines originating at the initial radius.

From the theory of characteristics, we have two equations

$$\frac{dr}{dt} = u - c \qquad 5.1$$

and

$$u + \frac{2}{\gamma - 1} c = \beta = \text{constant} \qquad 5.2$$

where $c = \sqrt{\gamma p/\rho}$ and is the isentropic speed of sound. The first equation gives the fact that u and c are constant along the straight characteristics and the second equation gives how u and c vary from one characteristic to another. Once u and c are known inside the region the pressure and density can be calculated from

$$c^2 = \frac{\gamma p}{\rho} \qquad 5.3$$

and

$$p/\rho^{\gamma} = \text{constant}. \qquad 5.4$$

Putting these equations into the non-dimensional form used in this thesis if follows that

$$\frac{d\left(\frac{r}{K\, t_o^{2/5}}\right)}{d\,(t'/t_o)} = \frac{u}{U} - \frac{c}{U} = \frac{R}{R_o}\left[\frac{.4}{t'/t_o}\,\frac{u}{U} - \sqrt{\frac{\frac{\gamma p R^3}{E}}{\frac{\rho R^5}{E\, t_o^2}}}\right] \qquad 5.5$$

where

$$\frac{c}{U} = \frac{(t'/t_o)}{.4}\sqrt{\frac{\frac{\gamma p R^3}{E}}{\frac{\rho R^5}{E\, t_o^2}}} \qquad 5.6$$

and

$$\frac{R}{R_o}\left[\frac{.4}{(t't_o)}\,\frac{u}{U} + \frac{2}{\gamma - 1}\sqrt{\frac{\frac{\gamma p R^3}{E}}{\frac{\rho R^5}{E\, t_o^2}}}\right] = \beta. \qquad 5.7$$

Let

$$\frac{d\,\frac{r}{R_o}}{d\,(t't_o)} = \sigma$$

and u/U and c/U are obtained as follows:

$$\frac{.4}{(t'/t_o)}\,\frac{u}{U} = \frac{\beta - \sigma}{6\, R/R_o} \qquad 5.8$$

$$\sqrt{\frac{\frac{\gamma p R^3}{E}}{\frac{\rho R^5}{E\, t_o^2}}} = \frac{\beta + \frac{2\sigma}{\gamma - 1}}{6\, R/R_o} \qquad 5.9$$

Since β is a known constant and σ is a known slope, the velocity function and the speed of sound function are known. This gives one equation on the pressure and density function. The isentropic equation for pressure and density in non-dimensional form becomes

$$\frac{\frac{pR^3}{E}}{\left(\frac{\rho R^5}{E\, t_o^2}\right)^\gamma} = \frac{p/\rho^\gamma \quad \frac{\bar{\rho}^{\,1/5}\, t_o^{6/5}}{K^3}}{(R/R_o)^4} = \text{constant} \qquad 5.10$$

or

$$\left(\frac{R}{R_o}\right)^4 \frac{p\, R^3/E}{(\rho\, R^5/E\, t_o^2)^\gamma} = \text{constant} = B$$

The constants in these equations at points just inside the second shock are now checked; these values for various times are shown in Table II. Also, shown in the table is the point of intersection of the rarefaction waves on the non-dimensional radial axis. These numbers are of course supposed to be 1.0. Thus, there is about 2.5% error in the location of the rarefaction wave. This means that there can be as much as 2.5% error in the location of a point where pressure, density and velocity functions are determined. This error can be tolerated. The constants B and β are somewhat deceptive, since a small error in the pressure and density functions can result in a large error in these constants. For example, consider a function of three variables $F = F(x,y,z)$. The error in F due to an error in x, y, and z is given by

$$\frac{dF}{F} = \frac{\partial F}{\partial x}\frac{dx}{F} + \frac{\partial F}{\partial y}\frac{dy}{F} + \frac{\partial F}{\partial z}\frac{dz}{F} \qquad 5.11$$

and the maximum error in F can be written

$$\left|\frac{\Delta F}{F}\right| = \left|\frac{\partial F}{\partial x}\frac{\Delta x}{F}\right| + \left|\frac{\partial F}{\partial y}\frac{\Delta y}{F}\right| + \left|\frac{\partial F}{\partial z}\frac{\Delta z}{F}\right|$$

If we consider $B = p/\rho^{\gamma}$ and consider that p and ρ are within engineering accuracy of 5% then the error in B is

$$\left|\frac{\Delta B}{B}\right| = \left|\frac{\Delta p}{p}\right| + \left|\frac{\gamma \Delta \rho}{\rho}\right| .$$

For $\gamma = 1.4$, $\frac{\Delta B}{B} \times 100 = 12\%$, or a 12% error in B is due to a 5% error in p and ρ. Thus the values of B and β listed in Table II are justified as being within engineering accuracy.

The computer program could be rewritten, taking small increments on π_1 and the errors could be minimized. This does not seem justified at this time, since we can take an average value of B and β and interpolate for the new values of the pressure and density function. It must be noted that an average value of B and β must be used since an error in these numbers at the second shock are greatly magnified on moving into the region behind the shock.

The non-dimensional times of 1.8 and 2.0 are not included in Table II because the errors in B and β are too large for these numbers to be of significience. Several factors could account for this. First, the primary shock wave has decreased in strength and possibly can no longer be assumed infinitely strong. Secondly, and probably most important, the value of n of .285 is not correct at these larger values of time.

Using equations (5.8), (5.9), and (5.10) we can calculate the pressure, density and velocity function inside the second shock wave. The solution to these equations are shown in figures (14), (15), (16). Table III also shows the value of σ , $\frac{p\ R^5}{E}$ and $\frac{\rho\ R^5}{E\ t_o^2}$ where the velocity is zero. Of course, the pressure and density inside this region are constant since the fluid is at rest. Dividing the density function by $(R/R_o)^5$ a constant at each time of approximately 254 is obtained and, dividing the pressure function by $(R/R_o)^3$ a constant of approximately 2.935 is obtained. The problem is thus solved, and the pressure, density and velocity are determined as functions of position and time for all regions inside the explosion. Figure (23) shows the physical plane at a specific time. It remains to check the original assumption on the density increase across the interface.

6. Mass Integral

In order to verify the assumption on the density increase across the interface, consider the conservation of mass integral. That is, the original mass contained in the sphere must be the same as the mass contained between the interface and the origin at any time.

$$\text{Mass} = \int_0^c \rho \ \text{dvol.} \qquad 6.1$$

and substituting the variables used in this thesis and the fact that dr = Rd π_1,

$$\rho_i \frac{4}{3} \pi R_o^3 = \int_0^{\pi_{1c}} \frac{\rho R^5}{E t_o^2} \frac{E t_o^2}{R^5} 4 \pi r^2 R d \pi_1 \qquad 6.2$$

which become upon simplification

$$\frac{\rho_i}{\rho} = \frac{3}{(t'/t_o)^{4/5}} \int_0^{\pi_{1c}} \frac{\rho R^5}{E t_o^2} \pi_1^2 d \pi_1 \qquad 6.3$$

The curves of $\left(\rho R^5/Et_o^2\right)\pi_1^2$ vs π_1 are shown in figures (17), (18) and (19). The term $\rho R^5 \pi_1^2/Et_o^2$ is called a non-dimensional mass. The curves beyond the value of π_1 where the velocity equals zero are not plotted since the density is constant in this region and the curves are simple parabolas and the area easily calculated. The values of $\rho_i / \overline{\rho}$ as calculated from the mass integral for the indicated times are shown in Table IV. Also shown in this table is the value of $\rho_i / \overline{\rho}$ as calculated from the governing differential equations. That is, consider $\frac{\rho R^5}{E t_o^2}$ where $R = \left(\frac{E}{\overline{\rho}}\right)^{1/5} t'^{2/5}$

$$\frac{\rho R^5}{E t_o^2} = \frac{\rho E t'^2}{E t_o^2 \overline{\rho}} = \frac{\rho}{\overline{\rho}} (t'/t_o)^2$$

or

$$\frac{\rho_i}{\overline{\rho}} = \frac{1}{(t'/t_o)^2} \frac{\rho_i R^5}{E t_o^2} \qquad 6.4$$

Notice in Table IV that the density ratio inside the rarefaction wave is approximately constant and that the percentage differences between the two calculations are small. This density ratio of approximately 255 is, of course, the original density ratio, since the gas is at rest inside the rarefaction wave. That is, the gas inside the rarefaction wave has not been disturbed and therefore the conditions must be the same as the initial conditions, zero velocity, high pressure, and high density. This density ratio depend on the original assumption on δ . It is now necessary to check the initial pressure ratio by considering an energy integral.

7. Energy Integral

As we have said before, there are two energy terms in the equations; E_{tot} which is the energy released upon explosion and E which has the dimensions of energy but is not necessarily the energy released. Now calculate E_{tot} in terms of E.

$$E_{tot} = \int_0^{s1} \left[\frac{u^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p}{\rho} \right] \rho \, d \, vol \qquad 7.1$$

Putting u, p and ρ in the non-dimensional form used in this thesis and using

$$d \, vol = 4 \pi r^2 dr = 4 \pi r^2 R \, d \pi_1,$$

$$E_{tot} = 4 \pi E \int_0^1 \left[\frac{.08\left(\frac{u}{U}\right)^2 \frac{\rho R^5}{E t_o^2}}{(t'/t_o)^2} + \frac{\gamma}{\gamma - 1} \frac{pR^3}{E} \right] \pi_1^2 \, d \pi_1 \qquad 7.2$$

is obtained after simplification.

The integrand is called a non-dimensional energy. The curves of the non-dimensional energy vs. π_1 for various times up to the point of zero velocity are shown in figures (20), (21), and (22). Since the velocity is zero and the pressure constant inside the rarefaction wave the non-dimensional energy vs. π_1 is a parabola and the area is easily calculated. Finding the area under these curves, (integrating the above equation for different times), we find that $E_{tot} \cong 40$ E. Translating these energy terms into an initial pressure ratio, the results from the two methods are tabulated in Table V with the per cent error for each time. In order to obtain the pressure ratio from the energy ratio, consider the following:

$$\frac{p_i \quad R^3}{E \ (R/R_o)^3} = \frac{p_i \ R_o^3}{E} \cong 2.935 \qquad 7.3$$

The 2.935 was approximately the value of $\frac{p_i \ R_o^3}{E}$ calculated inside the rarefaction wave. This value was constant for the three times considered. Thus

$$\frac{p_i \ R_o^3}{E \ \left(\frac{\bar{p} \ R_o^3}{E}\right)} = \frac{p_i}{\bar{p}} = 200 \ \frac{p_i \ R_o^3}{E} \qquad 7.4$$

since $\bar{p} \ R_o^3/E$ was chosen as .005. Consider now $\frac{\bar{p} \ R_o^3}{E}$ where E_{tot} for an ideal gas is given by

$$E_{tot} = \frac{\gamma}{\gamma - 1} \ \frac{p_i}{\rho_i} \left[\rho_i \ \text{Initial volume} \right]$$

$$E_{tot} = \frac{\gamma}{\gamma - 1} \frac{4}{3} \pi \ R_o^3 \ p_i \qquad 7.5$$

and

$$\frac{E}{\bar{p} \ R_o^3} = \frac{\gamma}{\gamma - 1} \ \frac{4 \pi}{3} \ \frac{E}{E_{tot}} \ \frac{p_i}{\bar{p}} = 200 \qquad 7.6$$

From the energy integral for E_{tot}/E and equations (7.4) and (7.6) above, we can calculate the initial pressure ratio as obtained from the solution to the equations of motion, and the initial pressure ratio as obtained from the energy integral. The results are tabulated in Table V. The errors in these calculations are not to be considered large since these errors are assumulated from two sources. First a small error in pressure, density and velocity at the rarefaction wave would show up in these calculations. Secondly, a small error in pressure, density and velocity would be magnified in calculating E_{tot}/E. Notice that the error in the density ratio is small. This is to be expected since the calculations are much simpler. Thus, from the pressure ratio and density ratio calculations, it can be readily stated that the error in pressure, density and velocity is less than 5%.

IV. SUMMARY

Consider how any other spherical blast problem with different initial conditions might be solved using this method. First, an assumption on n for the first shock wave must be made. This is more fully discussed in the discussion section. In this thesis the point source blast wave value of 2/5 was used. The pressure, density and velocity are expanded in non-dimensional functions where the zeroth order terms assume an outside pressure of zero. These functions are shown in equations (2.9), (2.10) and (2.11). That is,

$$u = u\ (E, \bar{p}, \bar{\rho}, r, t, n)$$
$$p = p\ (E, \bar{p}, \bar{\rho}, r, t, n)$$
$$\rho = \ (E, \bar{p}, \bar{\rho}, r, t, n)$$

where E, $\bar{p}$, and $\bar{\rho}$ are constants; r and t are independent variables and n is a variable parameter.

Substituting the above values into the equation of motion, the continuity equation and the homentropic equation, equations similar to (2.23) thru (2.28) are obtained. That is,

$$g_o' = g_o'\ (g_o, h_o, f_o, \pi_1, \gamma, n)$$
$$h_o' = h_o'\ (g_o, h_o, f_o, \pi_1, \gamma, n)$$
$$f_o' = f_o'\ (g_o, h_o, f_o, \pi_1, \gamma, n)$$

$$g_1' = g_1' (g_o, h_o, f_o, g_o', h_o', f_o', g_1, h_1, f_1, \pi_1, \gamma, n)$$
$$h_1' = h_1' (g_o, h_o, f_o, g_o', h_o', f_o', g_1, h_1, f_1, \pi_1, \gamma, n)$$
$$f_1' = f_1' (g_o, h_o, f_o, g_o', h_o', f_o', g_1, h_1, f_1, \pi_1, \gamma, n)$$

where the prime symbol denotes the derivative with respect to π_1, the independent variable, and g_o, h_o, f_o, g_1, h_1, and f_1 are the non-dimensional functions; γ is the ratio of specific heats and n is the variable parameter.

The boundary conditions are applied across the first shock wave, in order to get the initial values for the non-dimensional functions. Equations such as (3.5) thru (3.15) are obtained where the conditions inside the first shock wave are expressed in terms of the conditions on the outside of this shock. Since the initial conditions are now known, a numerical procedure can be applied in order to integrate the governing equations. The interface can now be found by solving the equation for a particle originally on the surface of the sphere.

The boundary conditions across the interface $u_2 = u_1$, $p_2 = p_1$, $\rho_2 = \delta \rho_1$, where δ is another variable parameter, are applied and the equations are again integrated until the solution becomes discontinuous. This is then the approximate location to the second shock wave. The approximate velocity of the second shock wave can now be obtained graphically and the parameter n changed in the equations. A new variable

$$\overline{\pi}_1 = \frac{r}{\bar{R}(t')}$$

is defined and the new conditions at the interface obtained. That is, the motion between the interface and the second shock wave is assumed almost self-similar. The governing equations are, of course, of the same form as equations (2.23) thru (2.28). The equations are again integrated from the interface to the second shock wave. The boundary conditions, equations (3.5) thru (3.15) are applied across the second shock to obtain the conditions inside this region. The pressure, density and velocity can now be found from the original assumptions on these variables by assuming a value for π_2, the small pertubation independent variable.

Since the region inside the second shock wave is an isentropic rare-faction wave, the solution can be obtained by applying the characteristic equations

$$\frac{dr}{dt} = u - c,$$

$$u + \frac{2}{\gamma - 1} c = \text{constant},$$

$$c^2 = \frac{\gamma p}{\rho} \text{ and}$$

$$p/\rho^{\gamma} = \text{constant}.$$

Once a solution has been obtained in this region, a mass integral such as equation (6.3) can be calculated to check the assumption on the parameter δ . This integral can be used to obtain the original density ratio if a portion of the gas is still as rest. An energy integral such as equation (7.2) can also be calculated to give the relationship between E and E_{tot}. This integral can also be used to calculate the initial pressure ratio.

Tables IV and V give the final results for the initial pressure and density ratios. Figures (11) thru (16) show how the pressure, density and velocity vary from the first shock wave to the last rareaction wave where the gas is still at rest.

V. LIST OF TABLES

Table I

t'/t_o	R/R_o	$\frac{R_c}{R_o}$	R/R_o
1.0	1.0	1.0	1.0
1.2	1.07565	1.06375	1.05096
1.4	1.14407	1.11843	1.10280
1.6	1.20684	1.16606	1.14650
1.8	1.26505	1.20794	1.18020
2.0	1.31951	1.24493	1.20866
2.2	1.37078	1.27780	- -
2.4	1.41933	1.30711	1.23620
2.6	1.46551	1.33324	1.22809
2.8	1.50960	1.35658	- -
5.8	2.02009	1.49524	- -
6.6	2.12725	1.48853	- -
7.2	2.20259	1.47505	- -
8.6	2.36483	1.41942	

Table II

t'/t_o	B	β	R_o/R_o
1.2	1.218×10^{-3}	.6660	.975
1.4	1.370×10^{-3}	.6481	.976
1.6	1.178×10^{-3}	.5942	.974
Avg.	1.25×10^{-3}	.6358	

Table III

t'/t_o	σ	$\frac{\rho R^5}{Et_o^2}$	$\frac{pR^3}{E}$	u/U
1.2	-.127	367	3.66	0
1.4	-.127	498	4.39	0
1.6	-.127	655	5.188	0

Table IV

t'/t_o	$\rho_i/\bar{\rho}$ Mass Integral	$\rho_i/\bar{\rho}$ Equations	% Difference
1.2	240.8	254.8	5.49
1.4	245.0	254.1	3.58
1.6	246.3	255.8	3.71

Table V

t'/t_o	$\frac{E_{tot}}{E}$	$p_i/\bar{p}$ Energy Integral	$p_i/\bar{p}$ Equations	%Difference
1.2	40.79	556.38	588.24	5.42
1.4	39.72	541.78	586.32	7.60
1.6	40.32	549.96	590.38	6.85

VI. LIST OF FIGURES

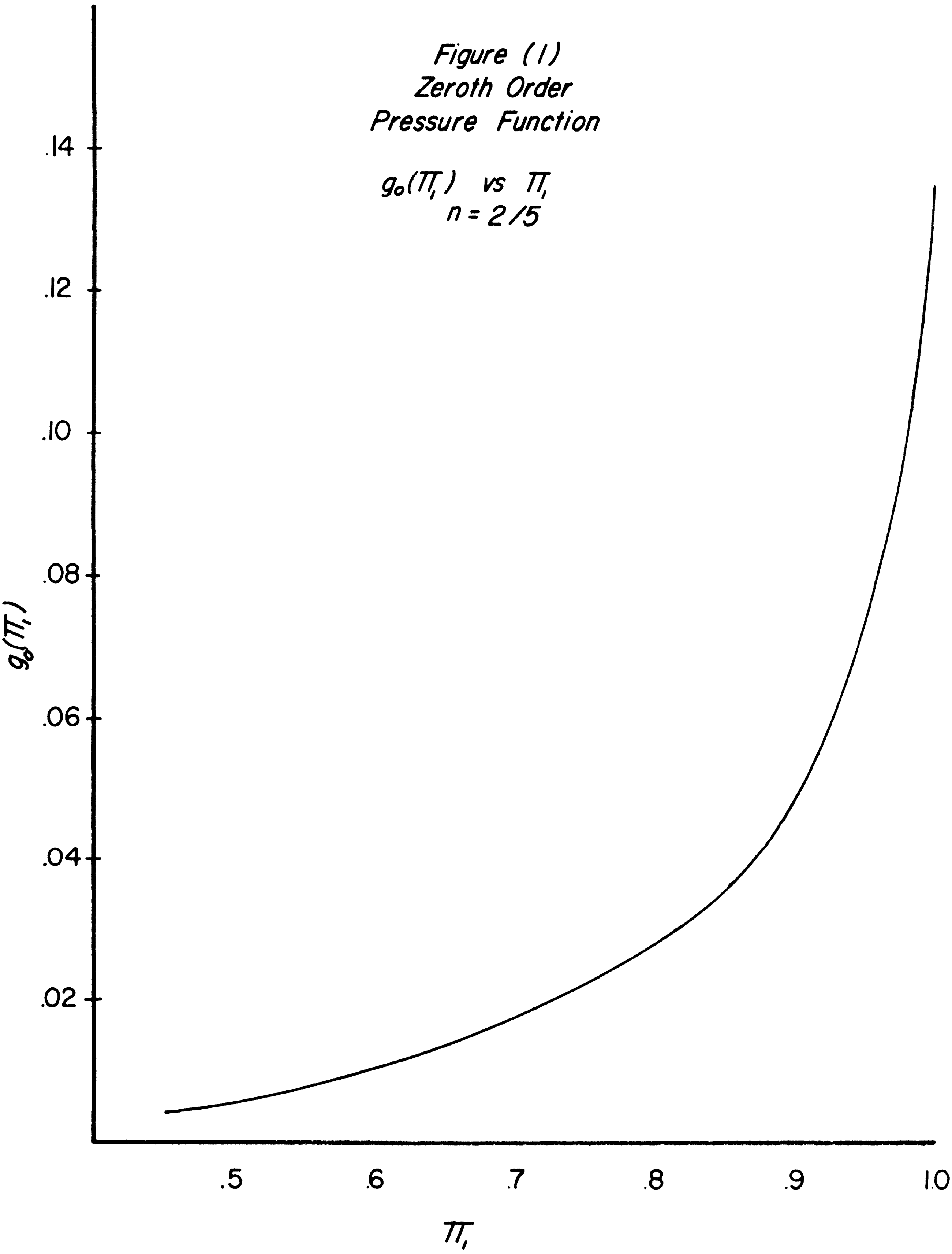


Figure (1)
Zeroth Order
Pressure Function
$g_o(\Pi_1)$ vs Π_1
n = 2/5
.14
.12
.10
.08
.06
.04
.02
$g_o(\Pi_1)$
.5
.6
.7
.8
.9
1.0
Π_1

Figure (2)
Zeroth Order
Density Function

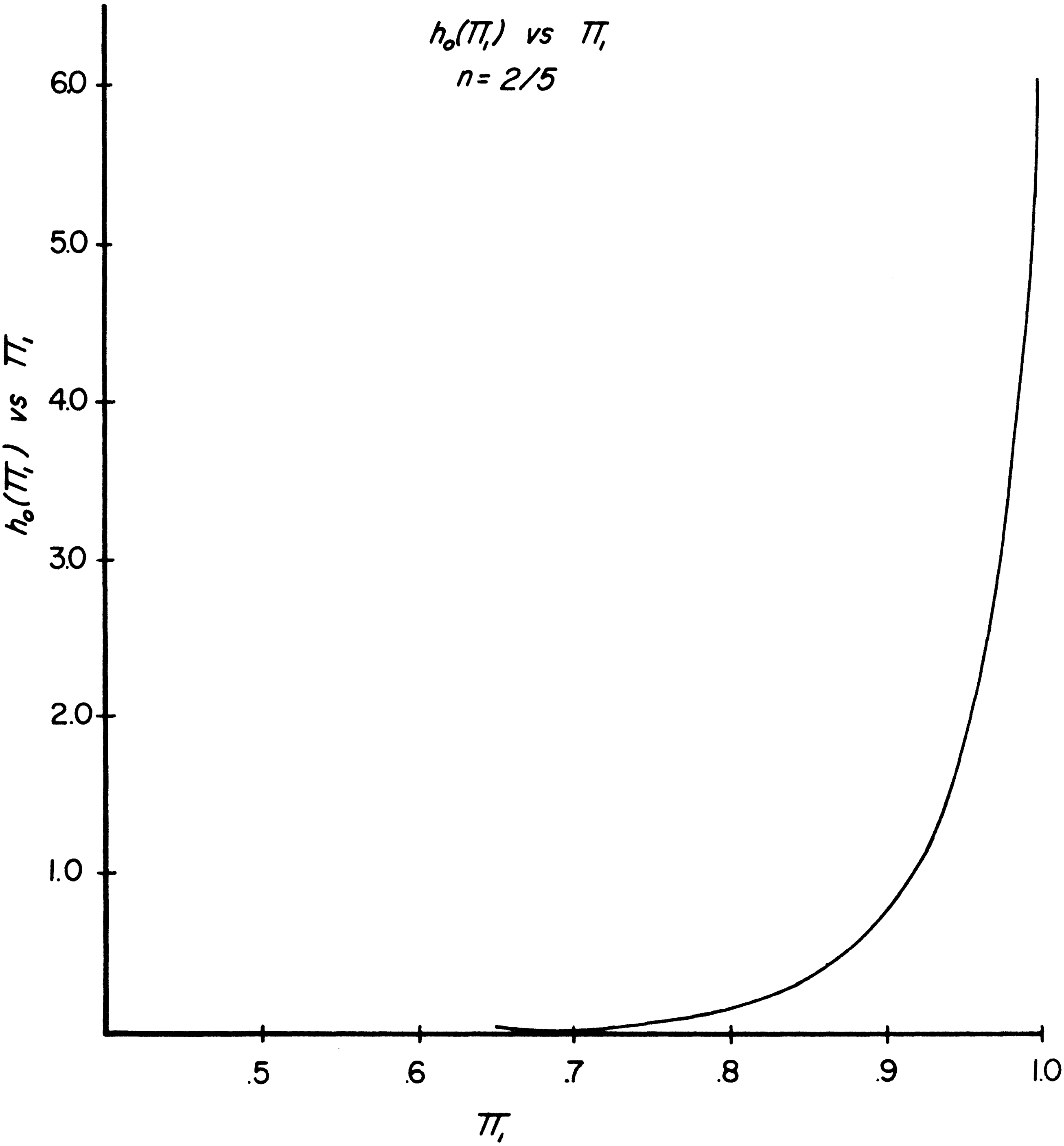


Figure (3)

Zeroth Order

Velocity Function

$$\theta_o(\pi_1) = \frac{f_o(\pi_1)}{\pi_1} \quad \text{vs} \quad \pi_1$$

$n = 2/5$

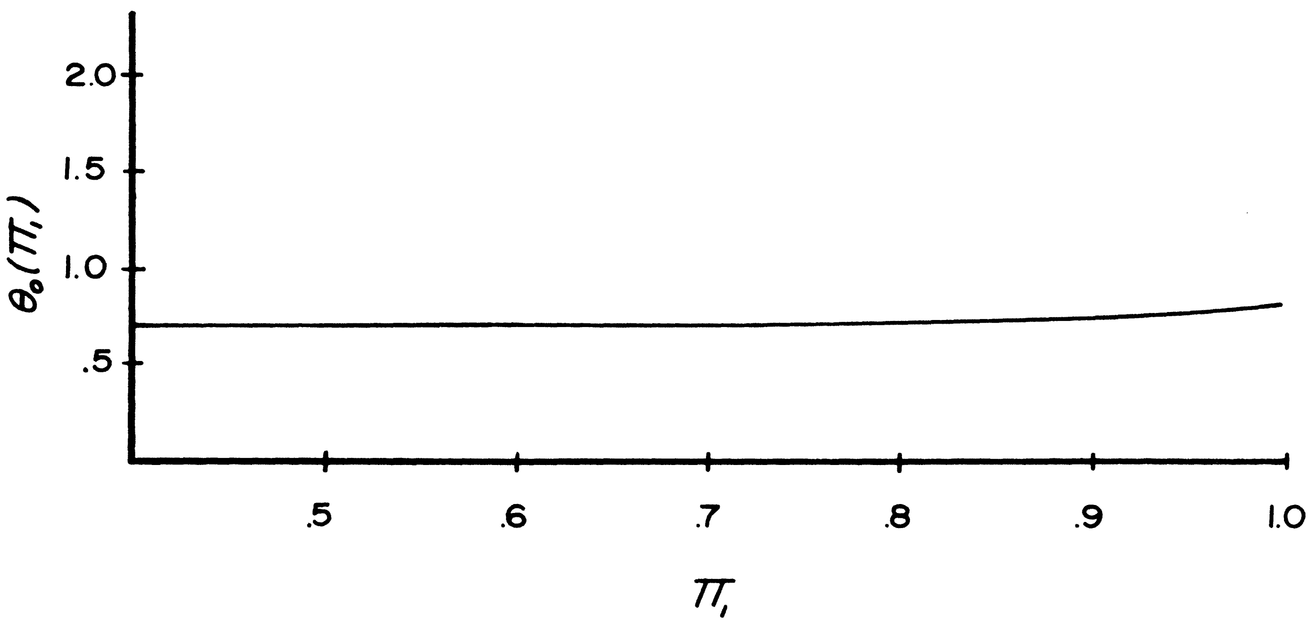
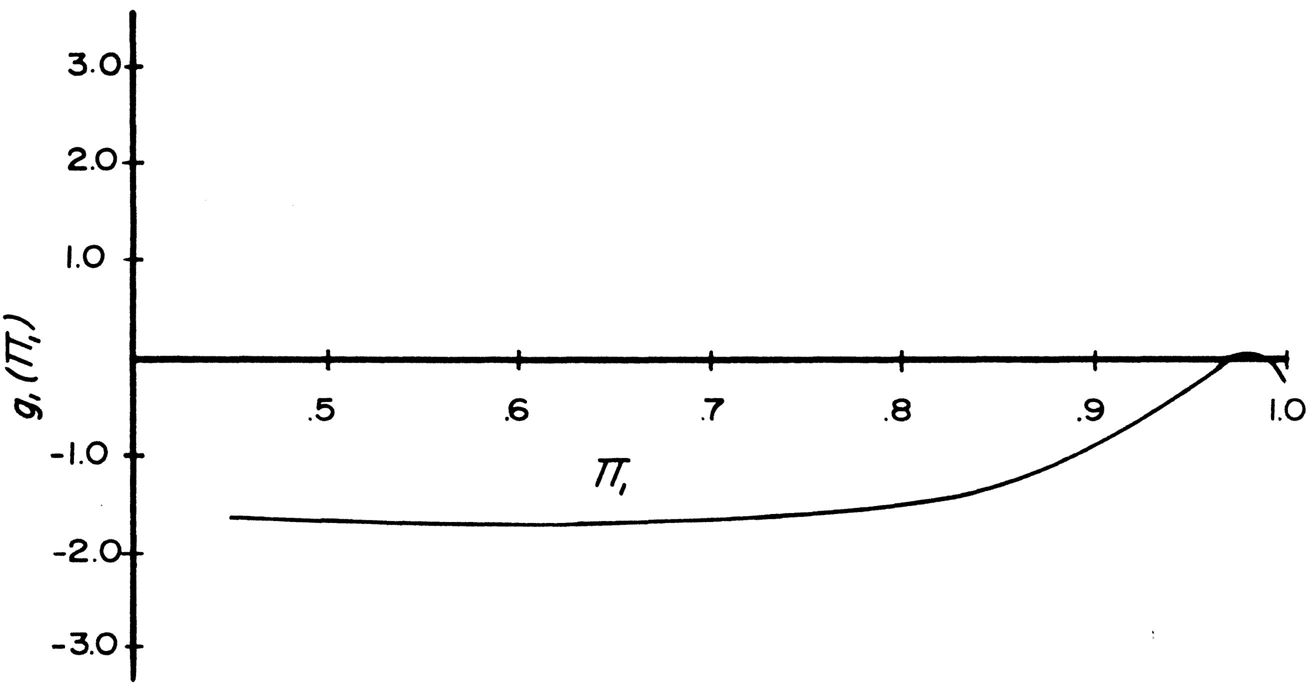


Figure (4)
First Order
Pressure Function

$g_1(\Pi_1)$ vs Π_1

$n = 2/5$



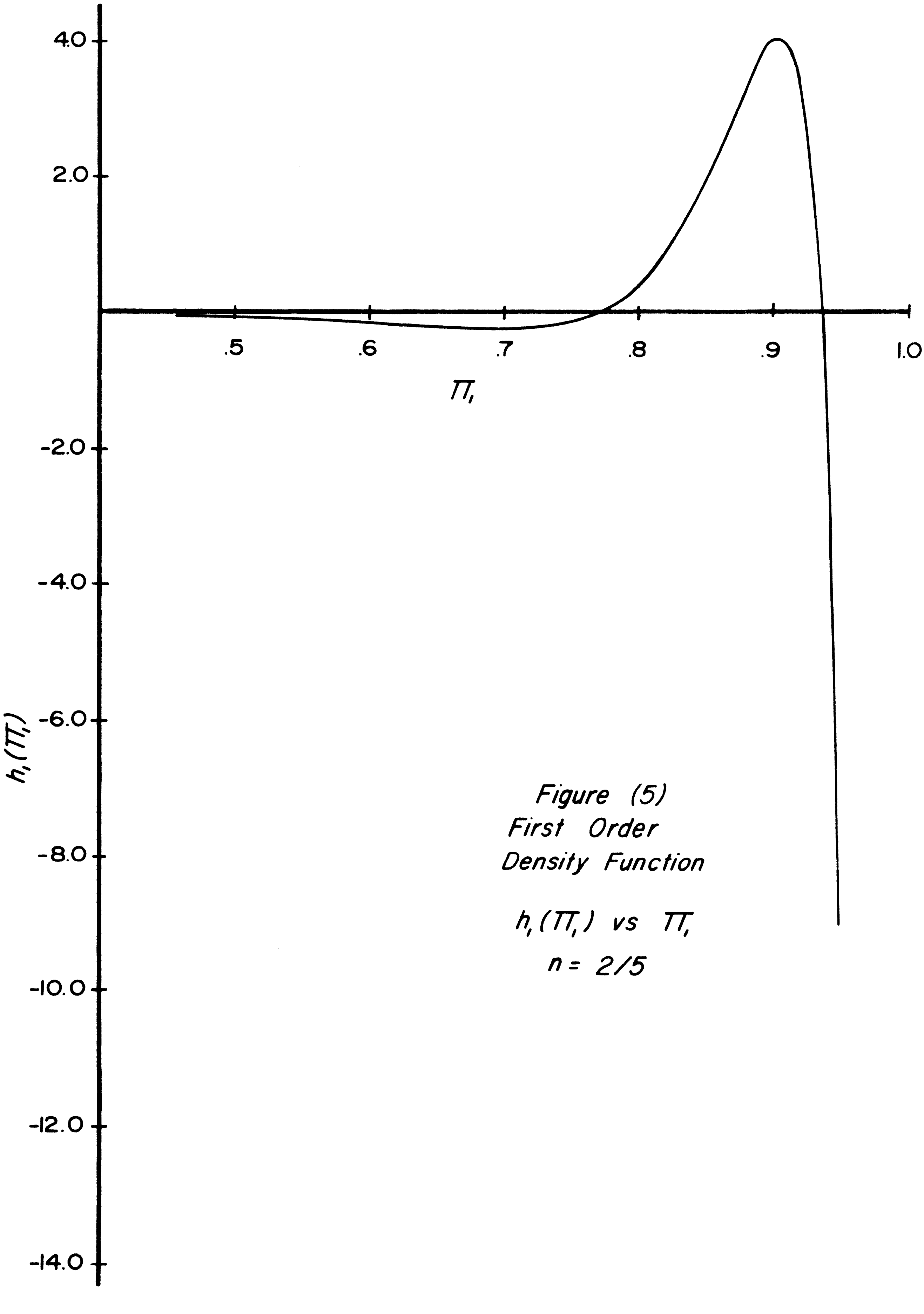


Figure (5)
First Order
Density Function

$h_1(\pi_1)$ vs π_1

$n = 2/5$

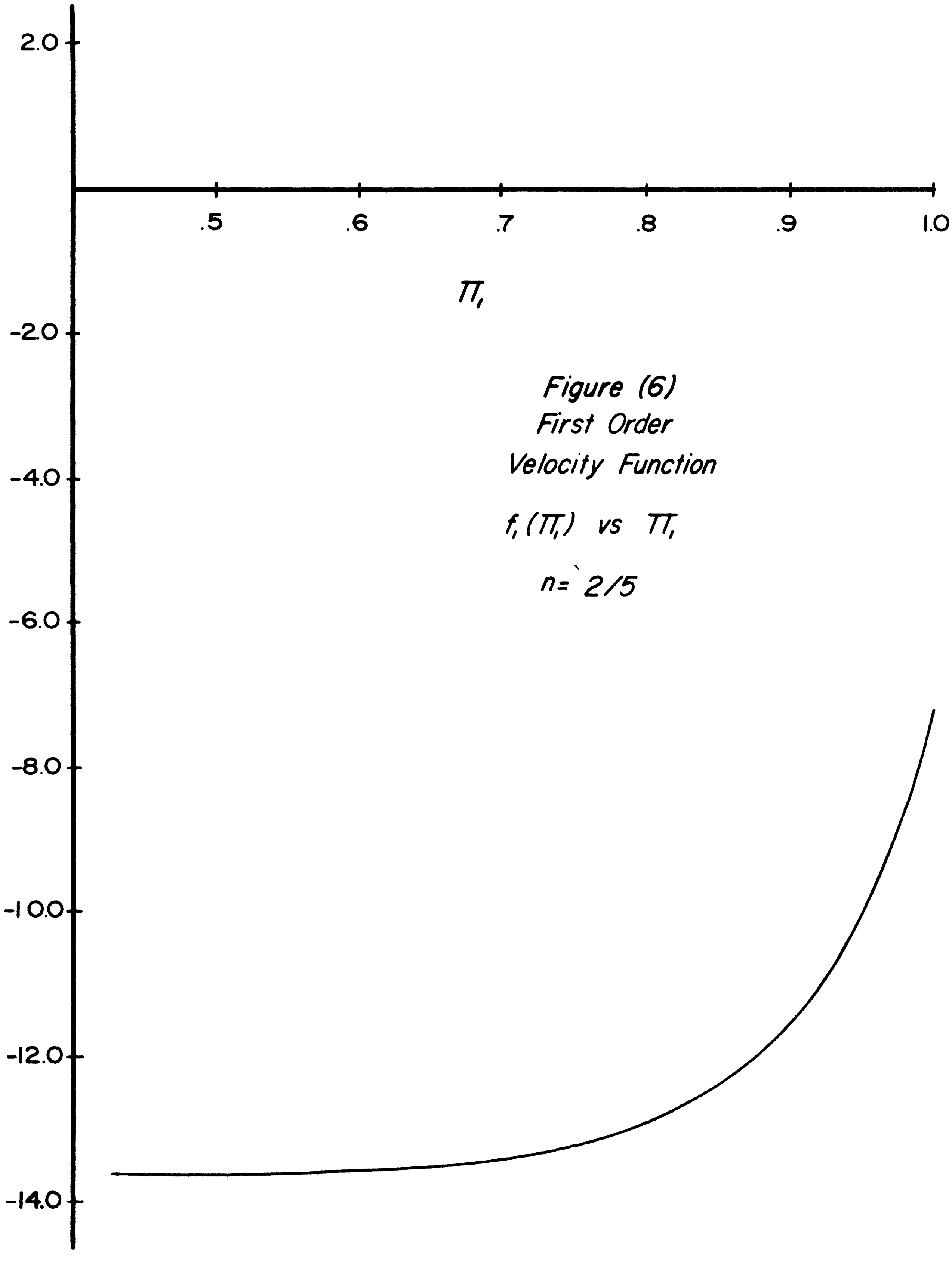


Figure (6)
First Order
Velocity Function

$f_1(\pi_1)$ vs π_1

$n = 2/5$

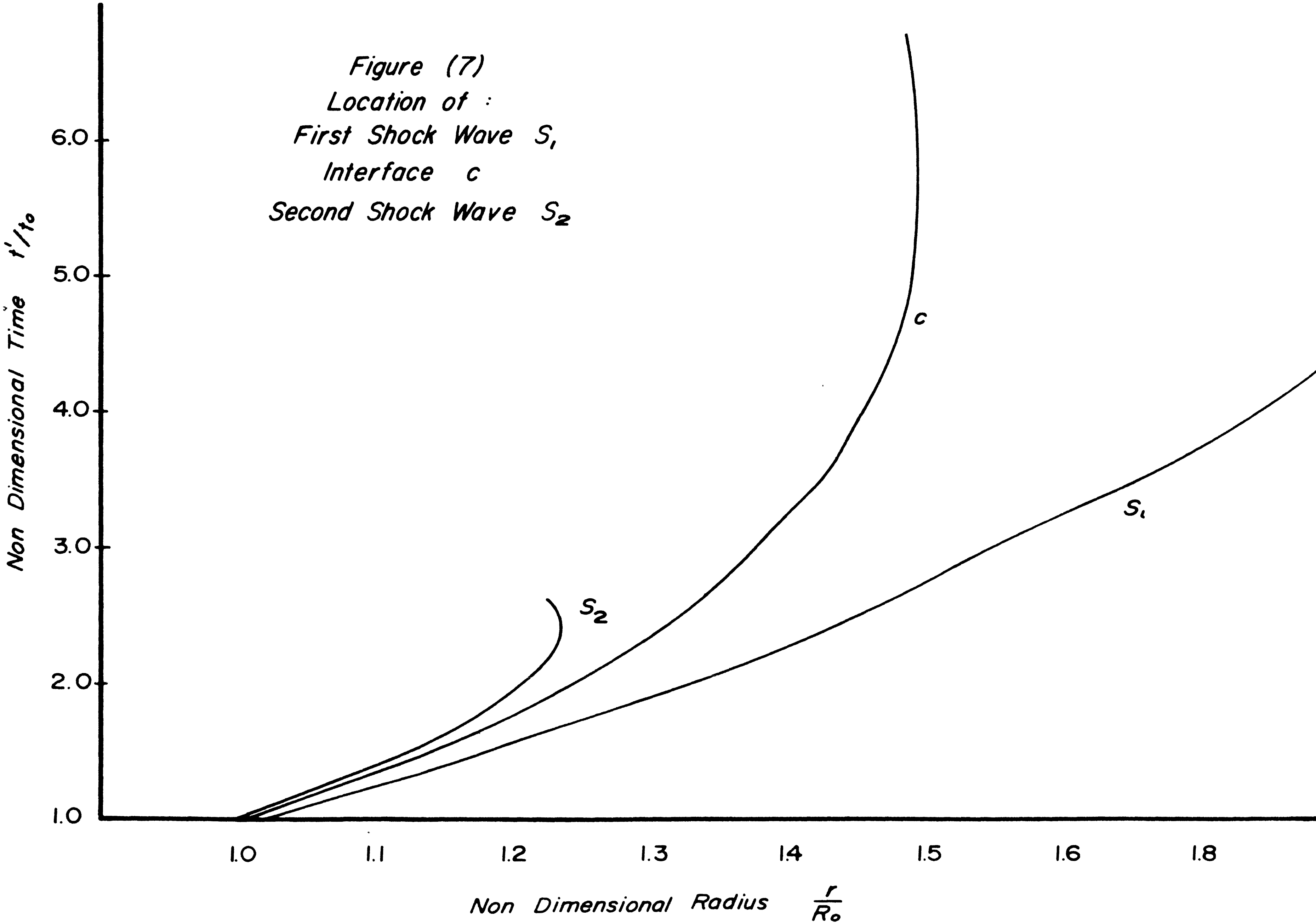


Figure (7)
Location of :
First Shock Wave S_1
Interface c
Second Shock Wave S_2

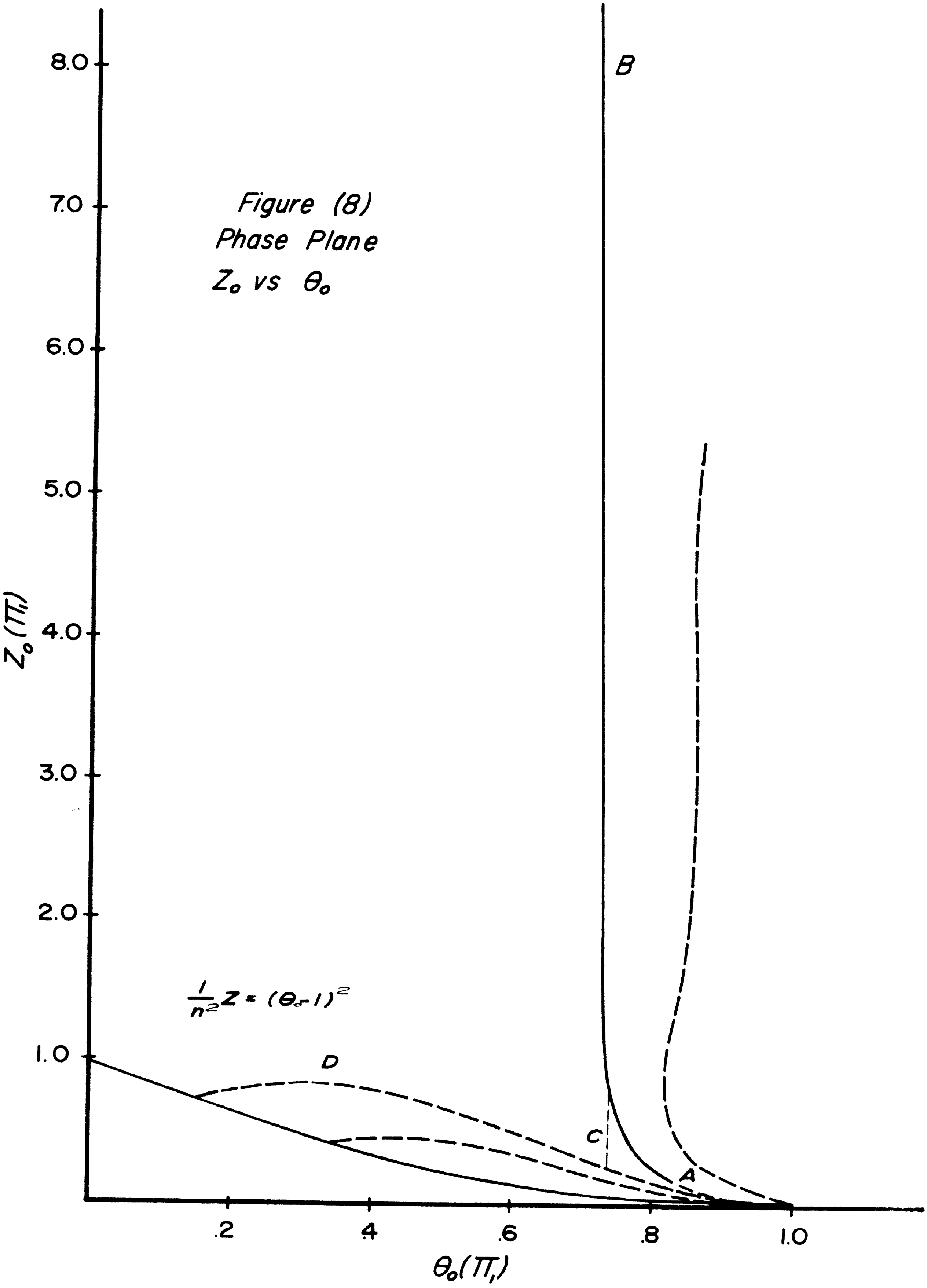
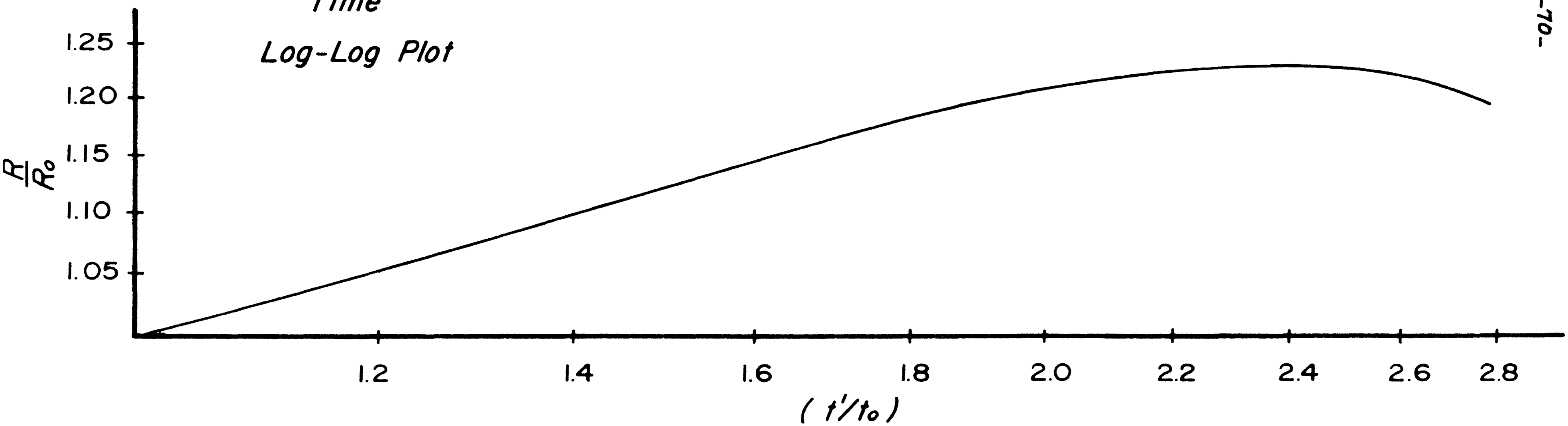


Figure (8)
Phase Plane
Z₀ vs θ₀
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
.2
.4
.6
.8
1.0
Z₀(Π₁)
θ₀(Π₁)
B
A
C
D
1/n² Z = (θ₀-1)²

Figure (9)
Position of:
Second Shock
vs
Time

Log-Log Plot



$1.0 \leq t'/t_o \leq 2.0 \qquad \frac{\bar{R}}{R_o} = t'/t_o^{\,n} \qquad n = .285$

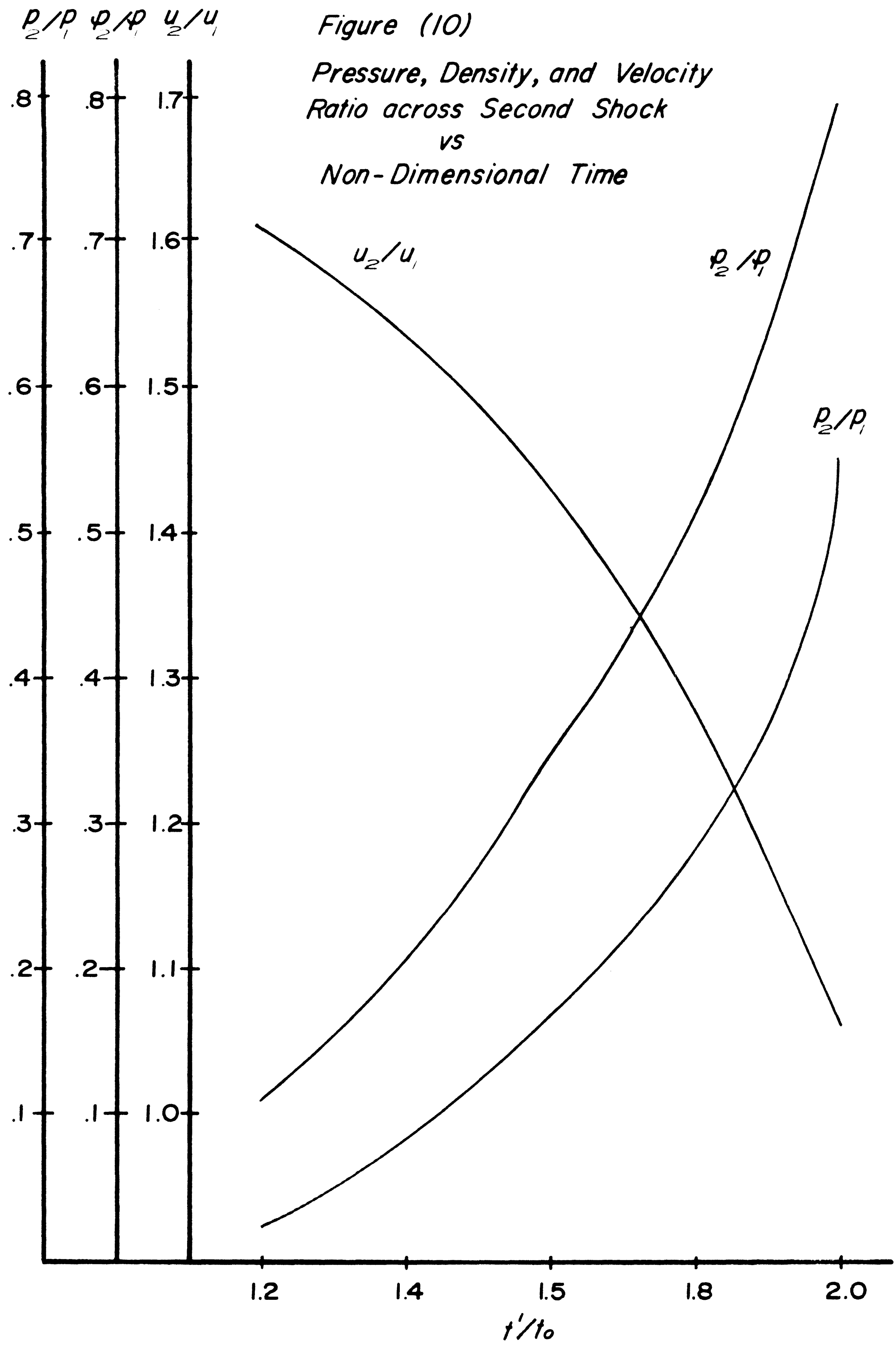


Figure (10)

Pressure, Density, and Velocity Ratio across Second Shock vs Non-Dimensional Time

Figure (11)

$\frac{pR^3}{E}$ vs π_1

From First Shock

To Second Shock

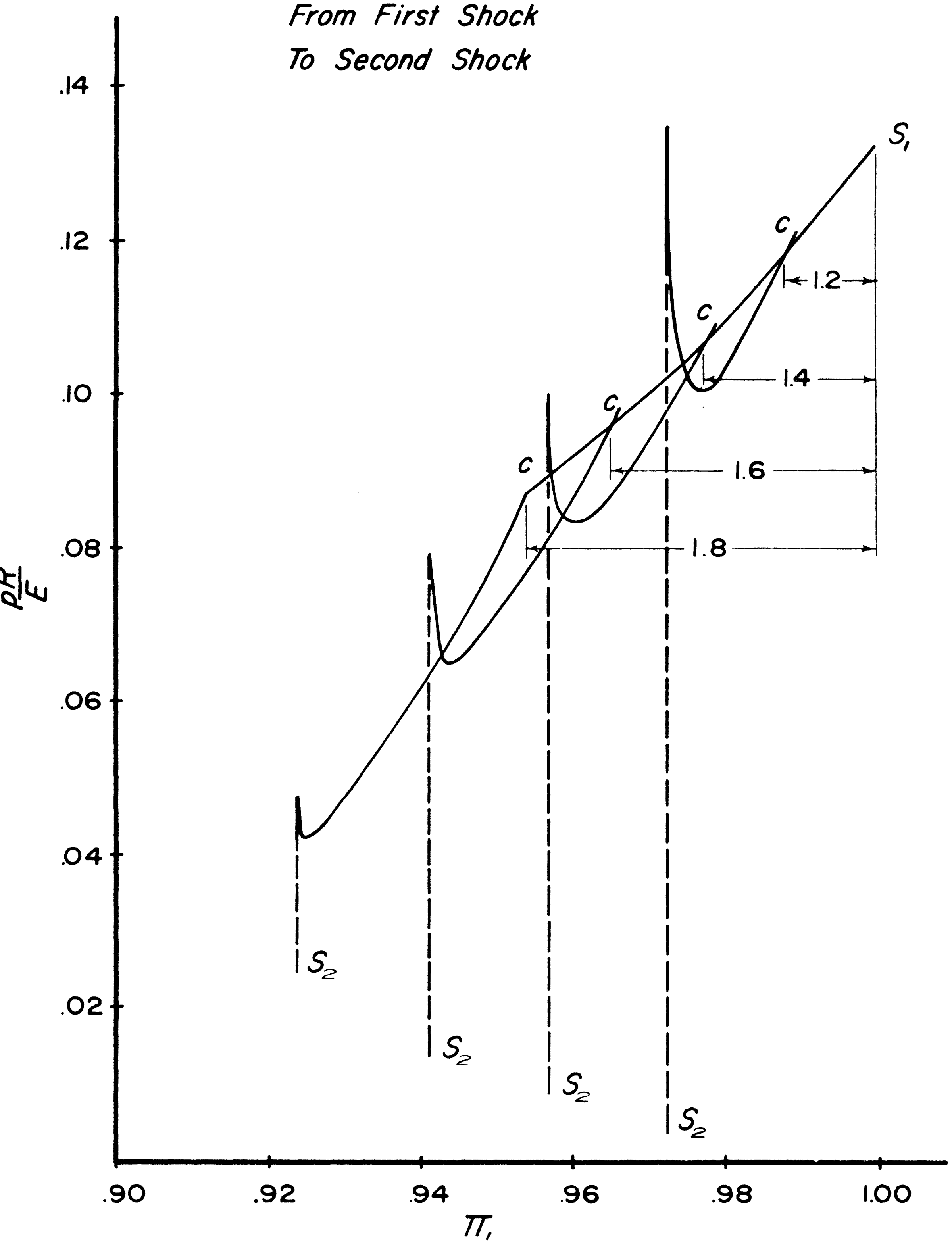
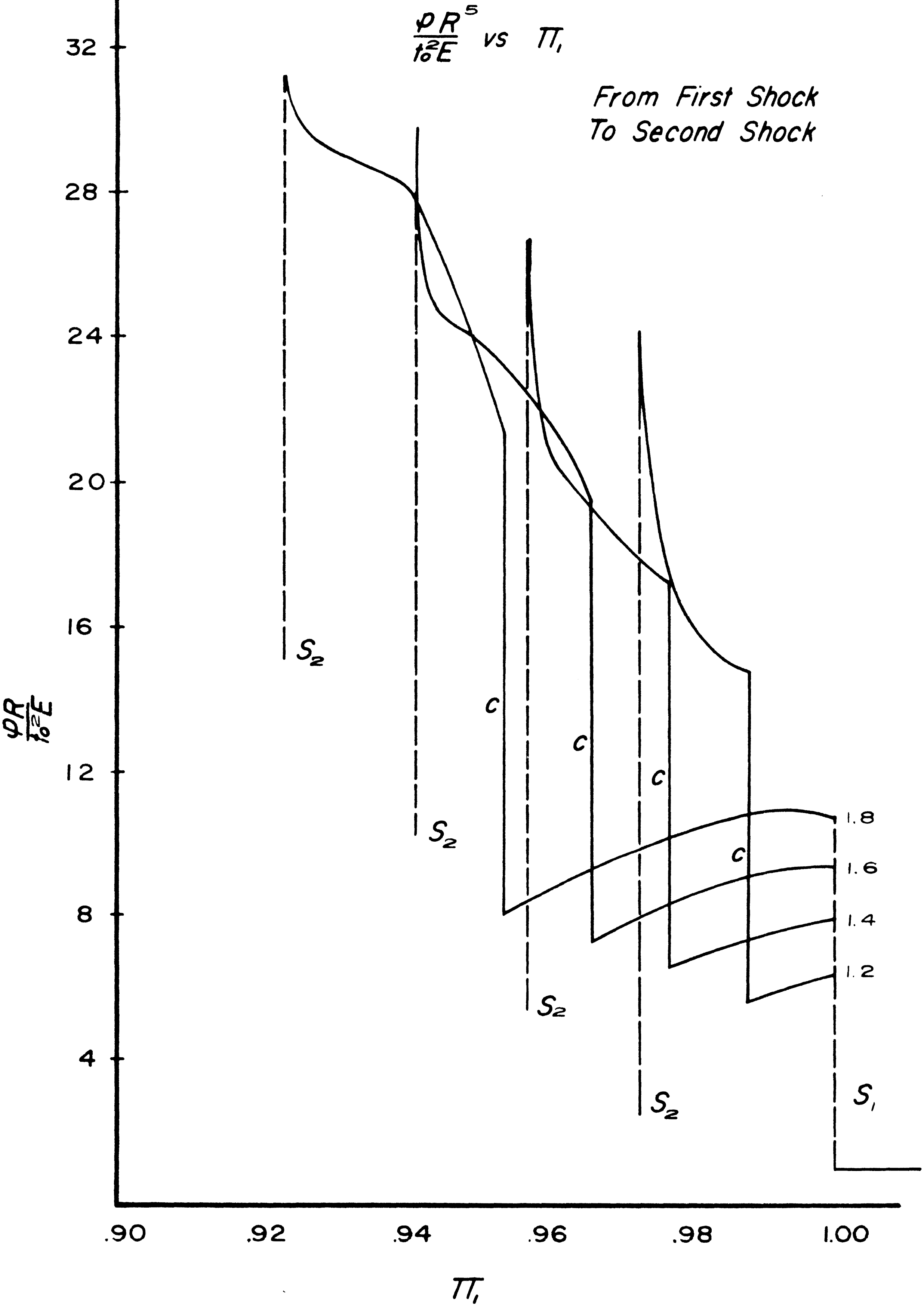


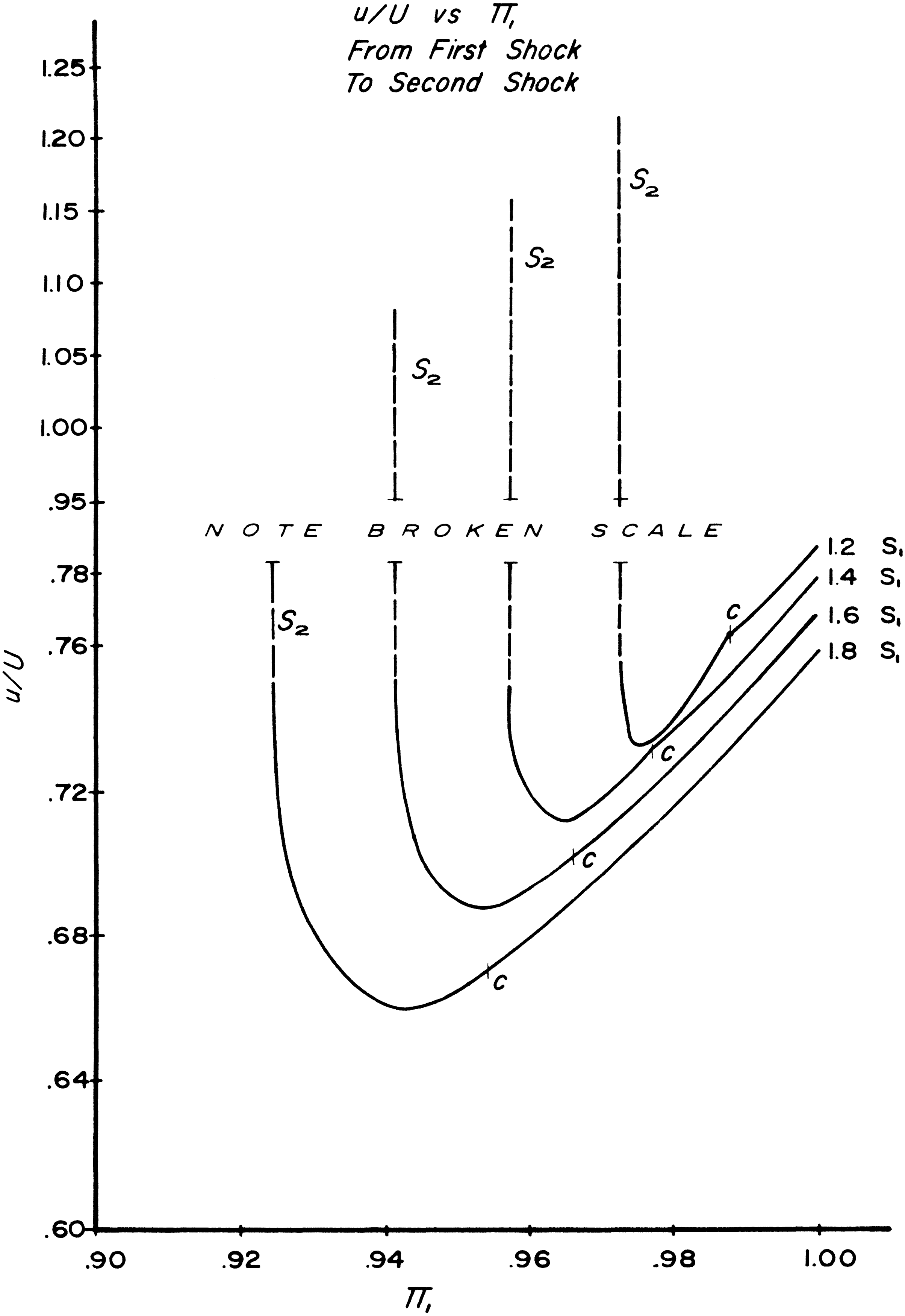
Figure (12)



$\frac{\varphi R^5}{t_0^2 E}$ vs Π_1
From First Shock
To Second Shock
32
28
24
20
16
12
8
4
$\frac{\varphi R}{t_0^2 E}$
S_2
S_2
S_2
S_2
S_1
c
c
c
c
1.8
1.6
1.4
1.2
.90
.92
.94
.96
.98
1.00
Π_1

Figure (13)

u/U vs Π_1
From First Shock
To Second Shock



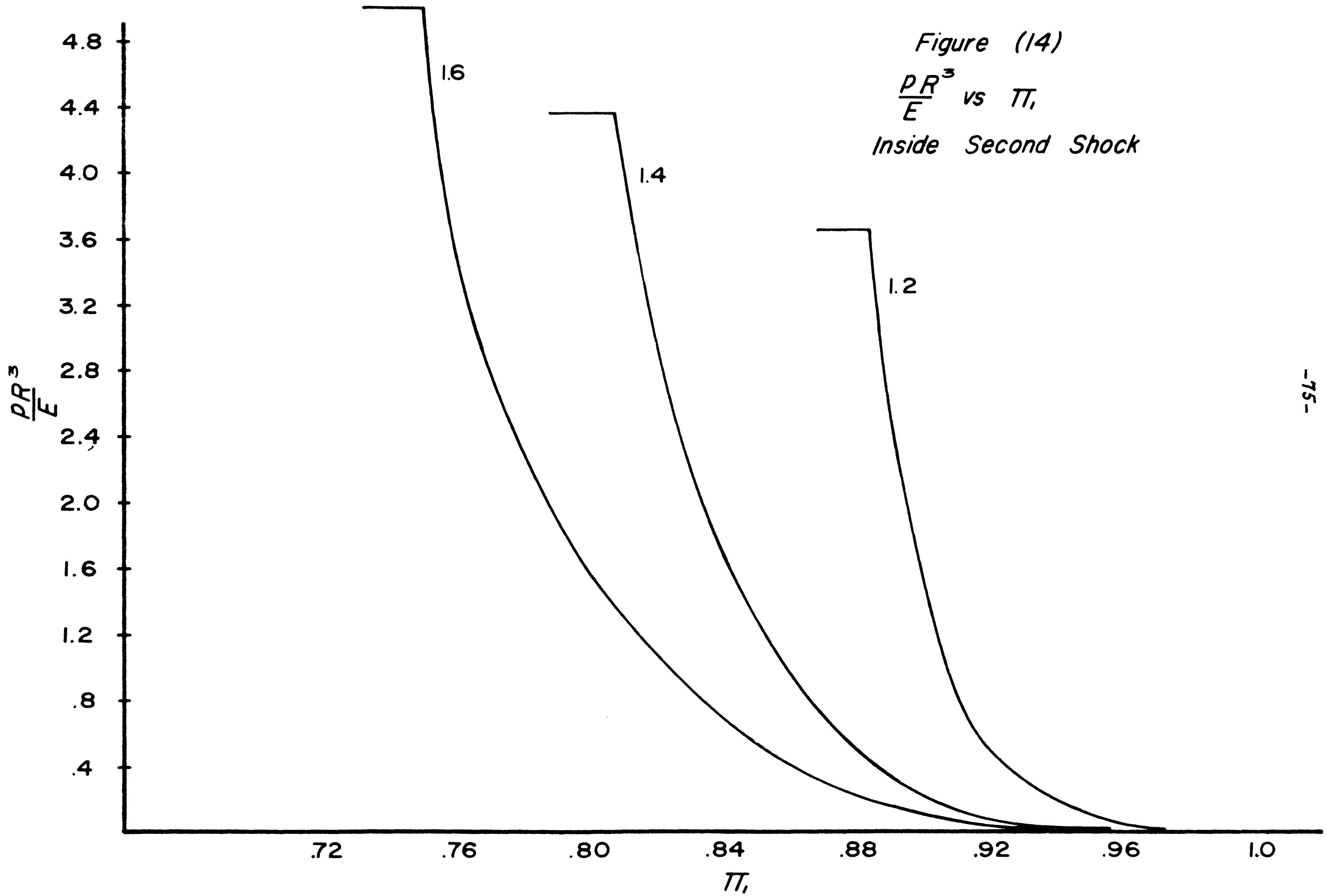


Figure (14)

$\frac{pR^3}{E}$ vs π_1

Inside Second Shock

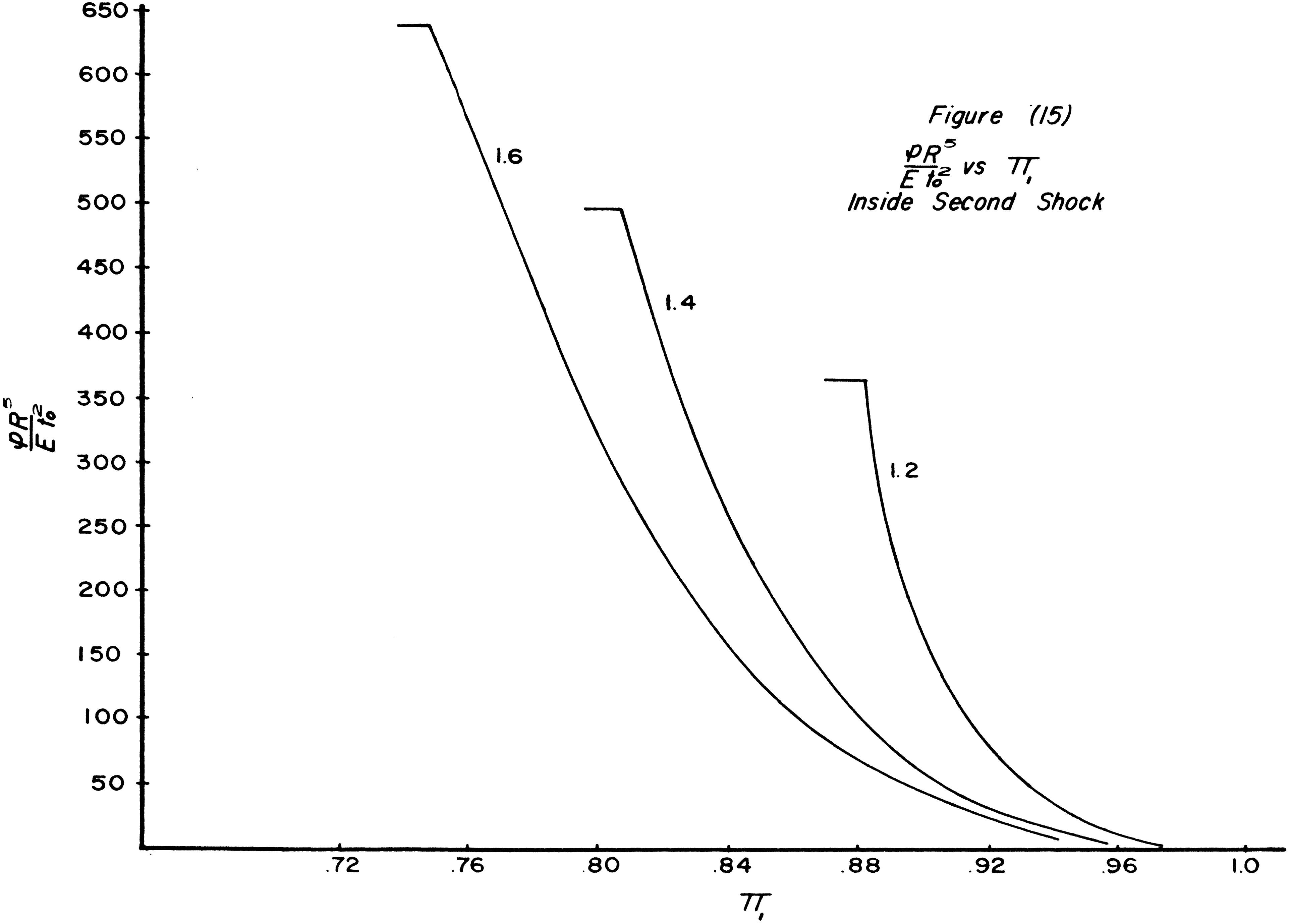


Figure (15)

$\frac{\varphi R^5}{E t_0^2}$ vs Π_1

Inside Second Shock

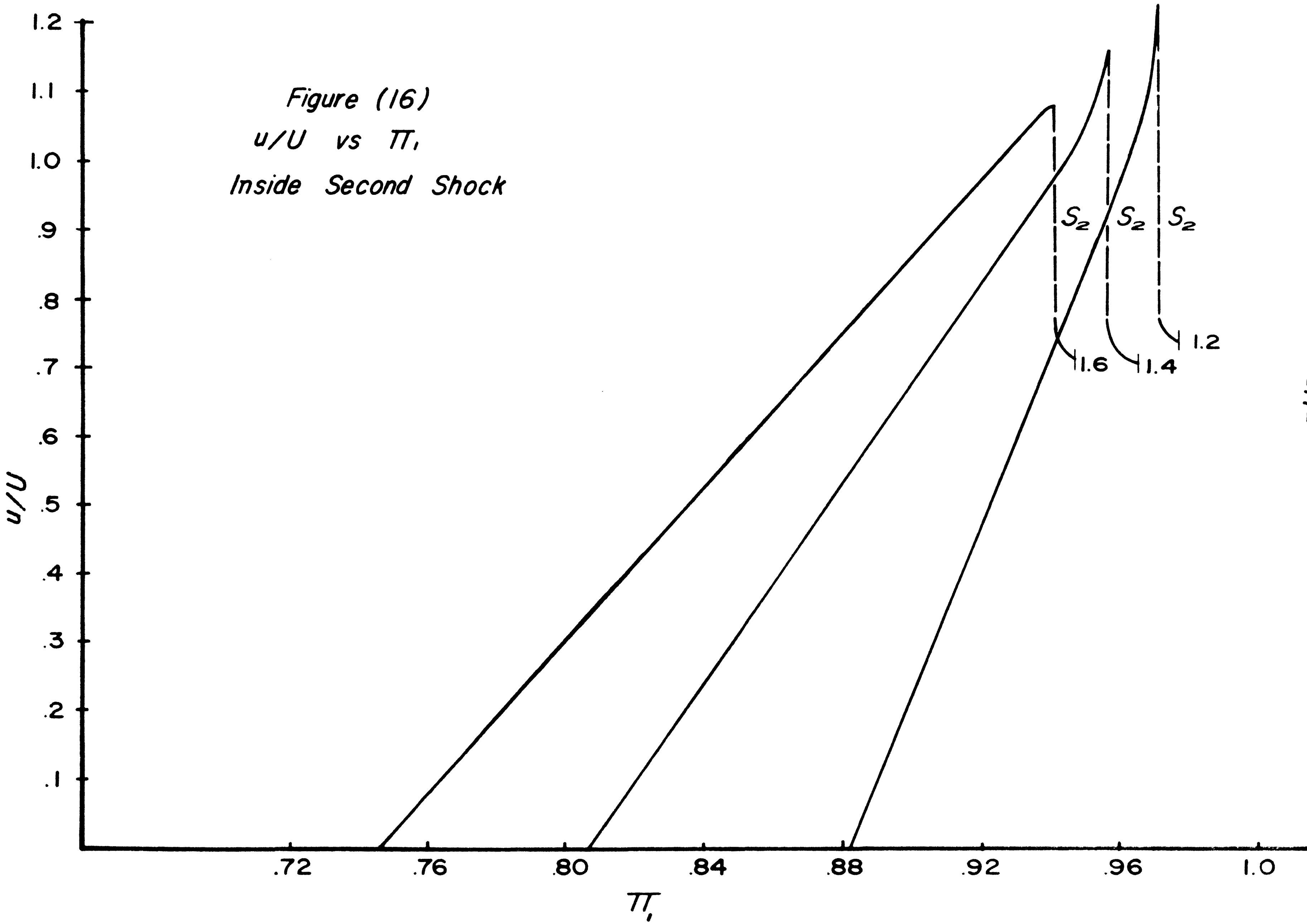
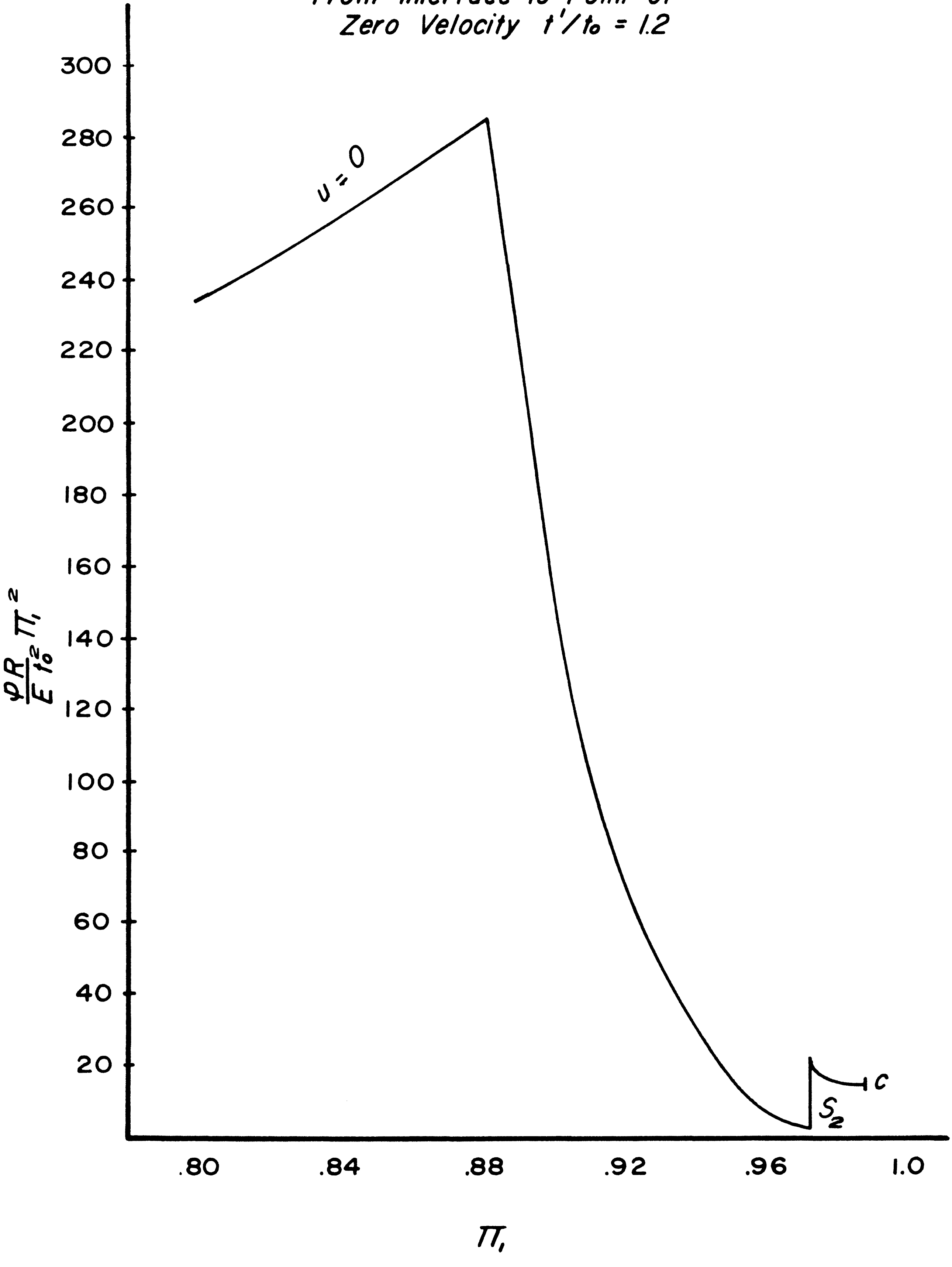


Figure (16)
u/U vs π_1
Inside Second Shock

Figure (17)

Non-Dimensional Mass vs π_1
From Interface to Point of
Zero Velocity $t'/t_0 = 1.2$



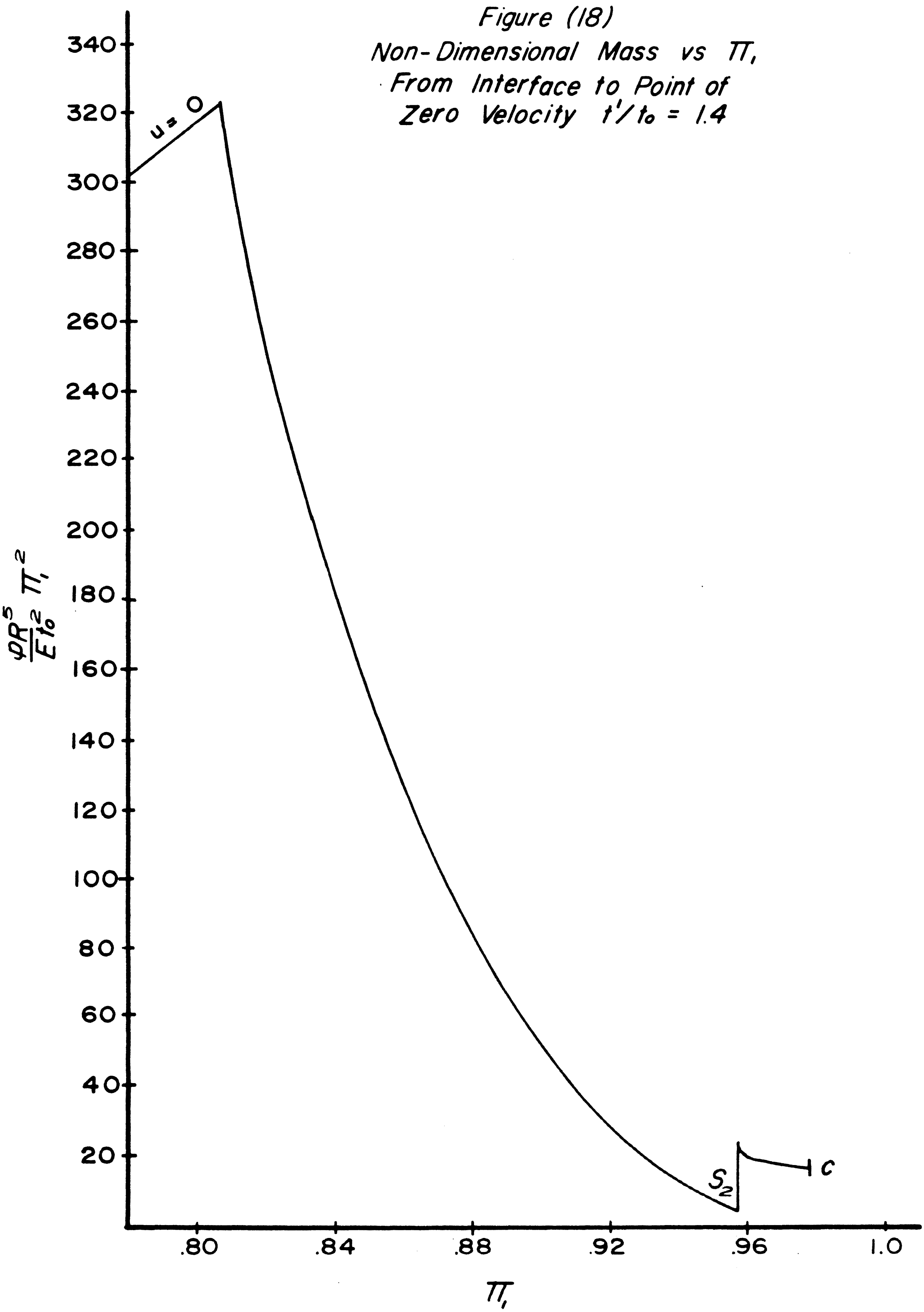
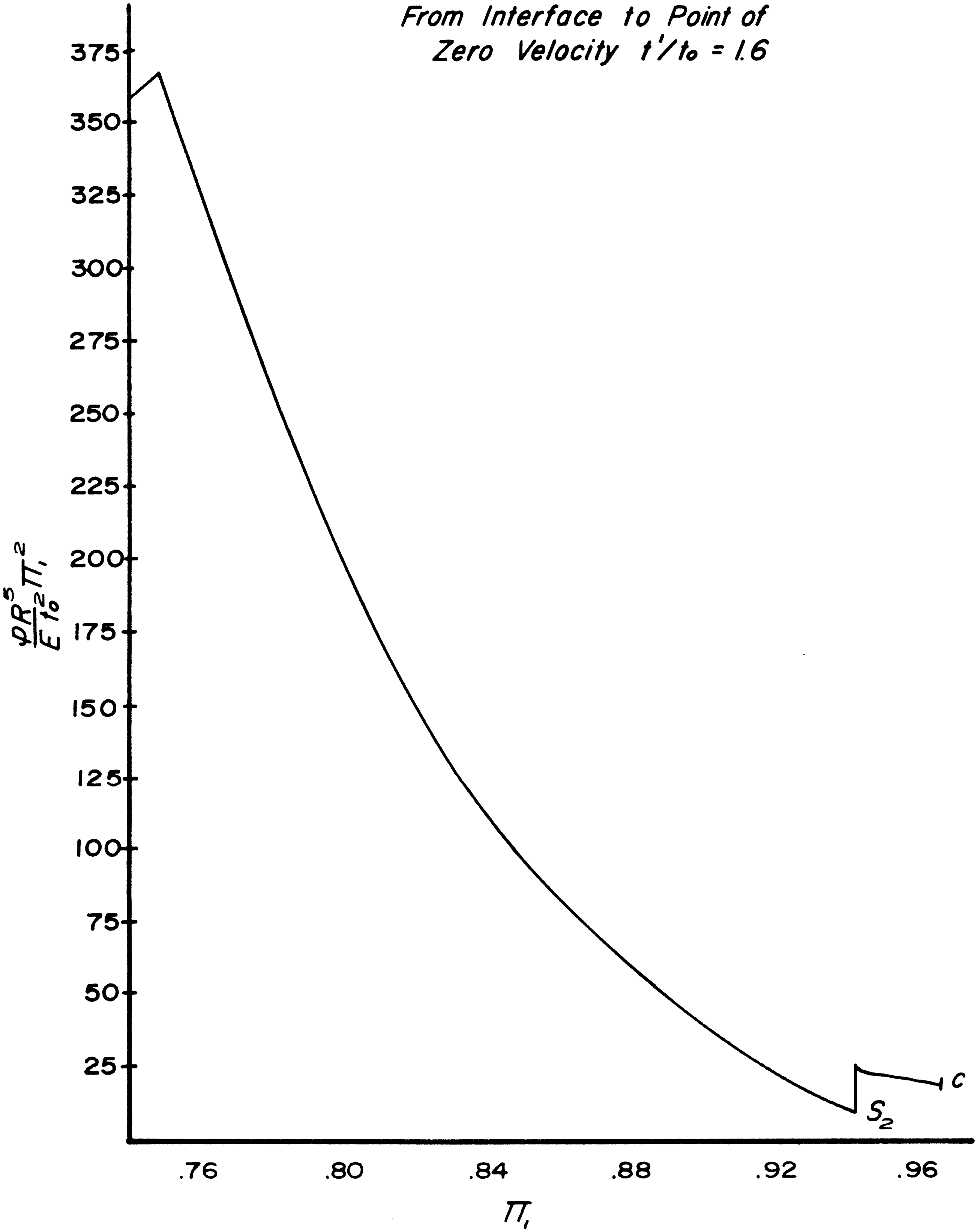


Figure (18)
Non-Dimensional Mass vs Π_1
From Interface to Point of
Zero Velocity $t'/t_0 = 1.4$
u = 0
S_2
c
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
$\frac{\rho R^5}{E t_0^2} \Pi_1^2$
.80
.84
.88
.92
.96
1.0
Π_1

Figure (19)
Non-Dimensional Mass vs π_1
From Interface to Point of
Zero Velocity $t'/t_0 = 1.6$



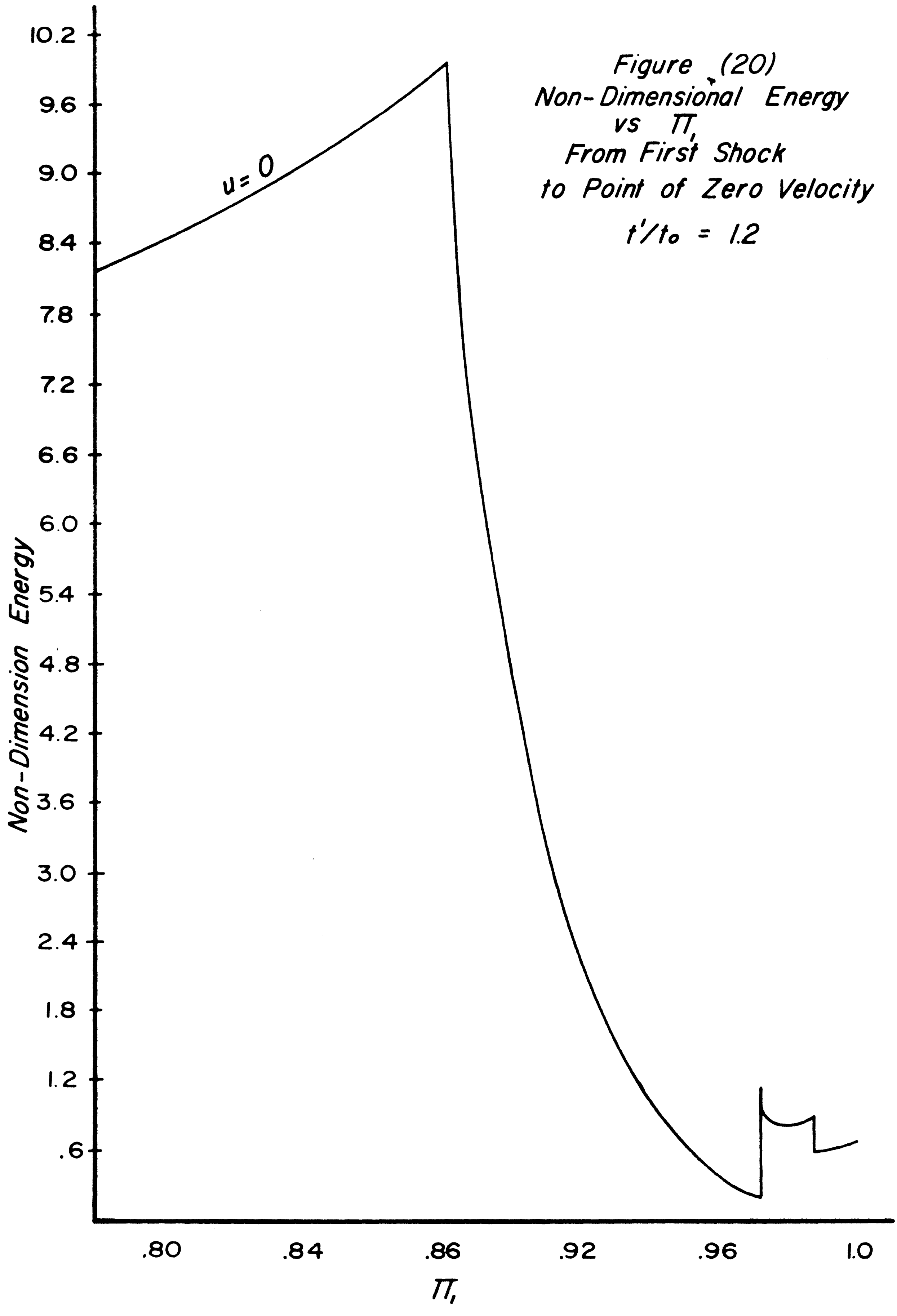


Figure (20)
Non-Dimensional Energy
vs π_1
From First Shock
to Point of Zero Velocity
$t'/t_o = 1.2$

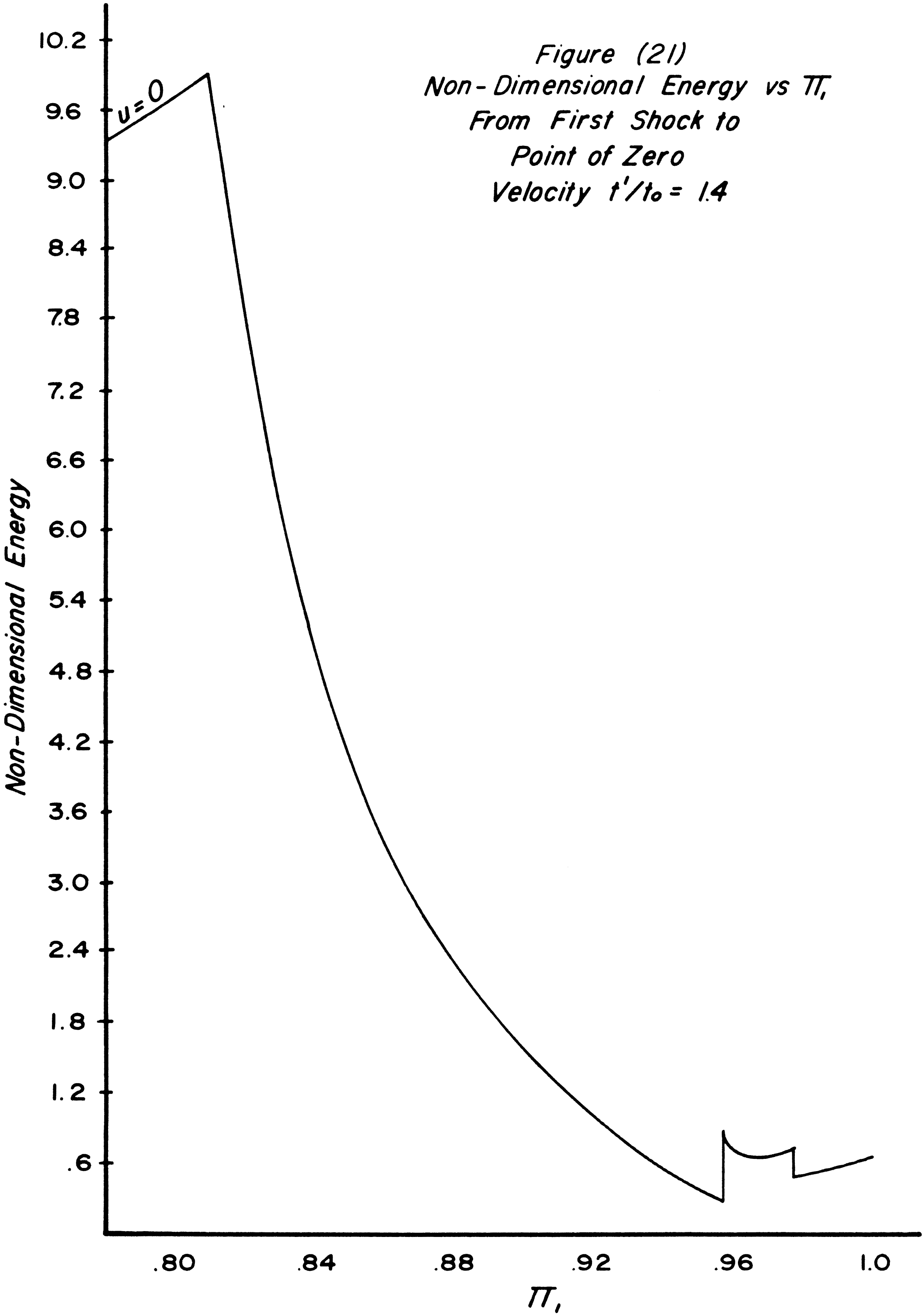
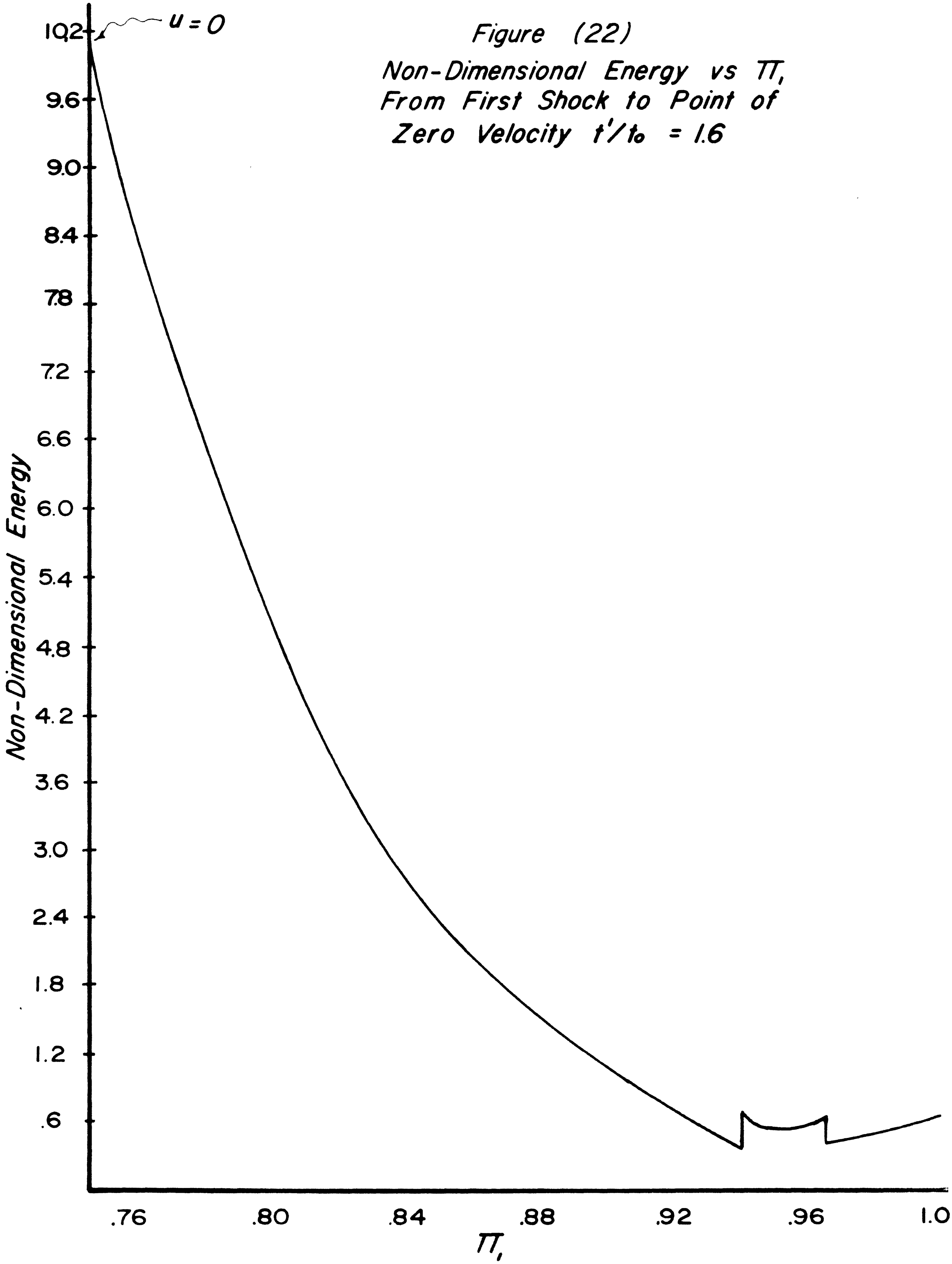


Figure (21)
Non-Dimensional Energy vs Π₁
From First Shock to
Point of Zero
Velocity t'/t₀ = 1.4
v=0
10.2
9.6
9.0
8.4
7.8
7.2
6.6
6.0
5.4
4.8
4.2
3.6
3.0
2.4
1.8
1.2
.6
Non-Dimensional Energy
.80
.84
.88
.92
.96
1.0
Π₁



u = 0
Figure (22)
Non-Dimensional Energy vs TT,
From First Shock to Point of
Zero Velocity t'/to = 1.6
10.2
9.6
9.0
8.4
7.8
7.2
6.6
6.0
5.4
4.8
4.2
3.6
3.0
2.4
1.8
1.2
.6
Non-Dimensional Energy
.76
.80
.84
.88
.92
.96
1.0
TT,

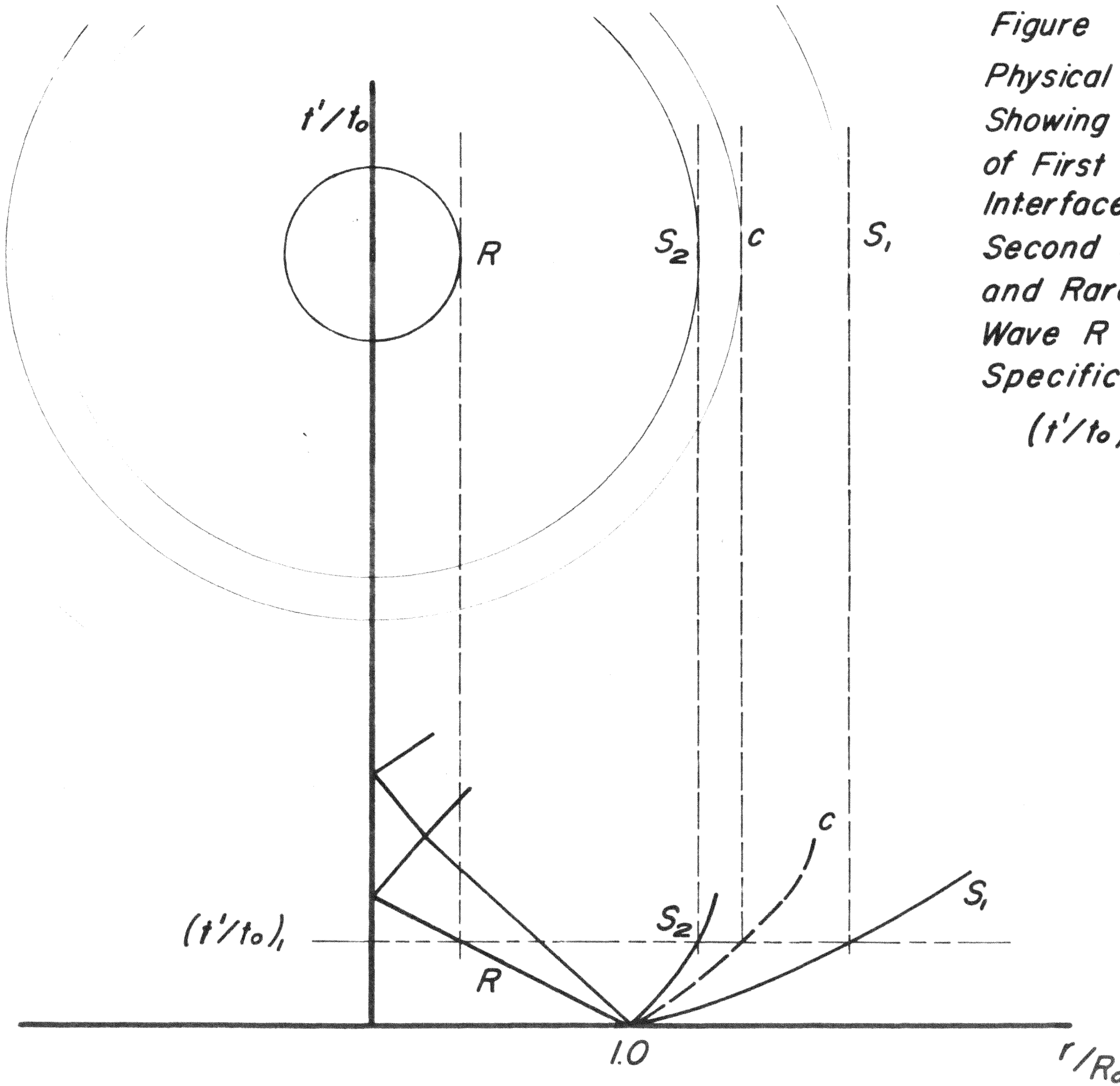


Figure (23)
Physical Plane Showing Position of First Shock S_1, Interface C, Second Shock S_2 and Rare Faction Wave R at a Specific Time $(t'/t_o)_1$

VII. DISCUSSION OF RESULTS

The problem set forth to be solved was the investigation of the second shock wave and the resultant flow properties after the explosion from a finite spherical container. The formation of a second shock wave from a finite explosion had been predicted analytically[8] and observed experimentally,[13] but this was the first attempt to find the location of this shock front and to determine the flow properties resulting from the formation of this shock. It was stated that the spherical container was to be initially at a high pressure and density and that the temperature of the inside air was approximately at the temperature of the outside air.

An approximate self-similar type solution was used and the dependent and independent variables were non-dimensionalized. The primary shock wave was assumed to have, to the zeroth order, the same form as the shock formed from a point source blast; that is of the form time to the 2/5 power.[1] [2] From dimensional considerations, it is necessary, to the zeroth order, that the wave have this form. However, the problem set forth in this thesis has additional dimensional parameters, the pressure directly ahead of the first shock and the finite radius, and the 2/5 power assumption need not be exactly correct. Actually, any power or polynominal series in time, can be assumed for the radius to the first shock. Since the problem investigated is approximately a blast wave, it seems appropriate to start with the 2/5 power assumption.

Since a sonic wave is a straight line in the r - t plane and a blast wave is a 2/5 power function, then an intermediate first shock wave (stronger than a sonic wave but not infinitely strong) can be expressed, to the zeroth order, as a power function of time, where the power is between one and 2/5. It is necessary to start with an assumption of shape on the first shock wave. The question is, of course, what power function is assumed for any intermediate shock wave? Experimental results would seem to be the best guide. However, if the wave is weak then a power of one may be used to the zeroth order and first order terms added to alter the straight line shape. Conversely, a power of 2/5 may be used if the wave is strong and again first order terms added. In either case, convergence should be fairly rapid. The intermediate shock wave is the hardest to solve since the power function is unknown. The second shock wave in this thesis is an intermediate shock, and thus it is difficult to find the equation of this shock.

The outside pressure and density and an energy term were the dimensional constants, along with r and t, which were assumed to be important to the solution. It is very easy to make the assumed outside pressure and density the actual outside pressure and density because of the boundary conditions across the first shock wave. However, since our non-dimensional independent variables were arbitrarily choosen, there are no conditions which say that the dimensional energy term must be the energy released. As a matter of fact, it is proven that the total energy released is approximately forty times the energy term considered. The inside pressure and density however, depend on the total energy released. For this reason, the initial pressure or density ratios are not known, but this initial pressure and density ratio must come from the solution to the problem. The initial conditions on the problem have thus been satisfied by the blast wave assumptions. The initial pressure and density are very high, thus assuring that approximately a blast wave problem has been solved. That is, the initial pressure ratio was found to be approximately 588, the initial density ratio approximately 254 and the initial temperature ratio only 2.3.

It can be seen that the pressure, density and velocity decrease from the first shock wave to the discontinuity. This compares with the point source blast wave solution where pressure and density decreases, but the velocity reaches a limiting value. Because of the contact surface or interface of the finite source explosion, the velocity does not reach a limiting value and the velocity can actually become negative. This, of course, happens at a time after the interface starts to implode on the origin. The interface is the big difference between the point source explosion and the finite source explosion and must exist in the finite source explosion. In this region between the first shock and the interface, the flow was assumed to be almost self-similar to the first shock and the velocity was expanded in terms of the first shock velocity. This is necessary in order to satisfy the strong shock wave boundary conditions.

At the interface, the velocity and pressure are continuous but there is a jump in density. This jump in density can be arbitrary since the boundary conditions of conservation of mass, momentum and energy are identically satisfied across the front. This type of front is called a singular discontinuity. It is necessary then to assume a value for this density jump in order to obtain a numerical solution. However, after the solution is obtained a mass integral can be obtained to verify the assumption on the density jump. That is, the mass contained between the origin and the interface at any time must be the original mass contained in the sphere. In this thesis a density increase of 2.65 was assumed. This was approximately the density increase observed in experiments on both spherical blast waves and plane blast waves. After the solution was obtained, a mass integral was written to verify this assumption on the density increase across the interface. It can be seen that the discrepancy in initial density ratios between the solution to the equations and the mass integral solutions is only about 4%. This seems to verify the fact that the density increase across the interface is not in error by more than 5%.

Inside the interface the pressure and velocity decrease, but there is a sharp increase in density indicating that the gas is being compressed between another wave and the interface. Also, as the solution is carried further into this region there is a very sharp rise in pressure and velocity. As a matter of fact, the solution blows up at certain points, indicating that a continuous solution across a certain curve in the r - t plane is impossible. Thus this curve in the r - t plane must be a second shock wave formed due to the fact that the original gas in the sphere is being compressed inside the interface. Using the same perturbation solution as in the first region, it is found that the boundary conditions cannot be satisfied for this curve is to be a shock. The variable π_1 varies along this curve while for the first shock wave π_1 is constant, actually made equal to one. It must then be assumed that the flow between the interface and the second shock is almost similar to the second shock.

However, in order to make this assumption, the velocity of the second shock must be known. Thus, this curve in the r - t plane is only the approximate location of the second shock. Using this approximate location, the velocity of the second shock is known at every time and a new variable $\overline{\pi}_1$ is defined, which is constant along the second shock. Notice that from dimensional considerations, it is impossible for a second shock to form from a point source explosion. Making the solution to the equations almost self-similar between the interface and the second shock wave, the equations can be integrated to again determine the approximate location of this second shock front. Upon integration, the solution for pressure, density and velocity between the interface and the second shock is obtained. It is interesting to note that the pressure, density and velocity from this solution are considerably different from the solution obtained using a perturbation from the first shock, but the location of the second shock using both methods is approximately the same. Notice that it is impossible to locate the second shock wave exactly since the shock is assumed to be infinitisimal in thickness and this, of course, is not true for a real shock wave. However the error in the location does not exceed .1%. The boundary conditions across this shock front and the fact that the entropy is approximately constant inside this shock front then give the other conditions necessary to determine the location of this shock and the pressure, density and velocity at this shock. The fact that the entropy checks at each time within 9 to 11% indicates that the pressure and density and thus the density ratio across the interface is in error by no more than 5%. This is borne out by the fact that a 5% error in pressure and density can result in as much as a 12% error in entropy.

The important point to make here is that two almost similar solutions have been patched together across the interface. The solutions are not similar to each other but one solution is almost similar to the first shock wave and the other solution is almost similar to the second shock wave. Similarity laws have been used for the zeroth order expansions while actually the motion is not exactly self-similar. This is due to the fact that another dimensional constant, the dimensional initial radius has been introduced into the solution. The outside pressure is not taken into account in the zeroth order solution and in order to consider this pressure, a first order term must be added to the solution. The final solution for pressure, density and velocity is, of course, now not self-similar.

Notice that this method could be used to find the solution after two spherical shock waves interact. After two shocks interact, there are two regions separated by an interface with a shock wave on the outside of each region. If the conditions are known outside each shock wave, approximate similarity assumptions can be used to find the solution inside the wave and patch the two solutions across the interface. Since the position of the shock waves are probably not known after interaction, this problem would be rather complex. However, the general method used in this thesis would apply. That is, assume the region between one shock and the interface almost self-similar to that shock and integrate from the shock towards the interface. The boundary conditions could then be checked at the interface.

At this point, the problem has been solved from the outside of the first shock wave to the inside of the second shock wave. The two regions considered here are the most important since this is the first attempt at combining two approximate similarity type solutions.

The region inside the second shock wave is quite easy to solve, since this region is a simple rarefaction wave. From the solution already obtained there is about 2.5% error in the location of the rarefaction wave. This error is within engineering accuracy and since the location of the wave depends on pressure, density and velocity, this would indicate that an error in pressure, density and velocity is within an allowable 5%. Applying the equations of characteristics and assuming the region inside the second shock is isentropic, a solution inside the second shock wave is obtained. Notice that the velocity decreases to zero as the solution is carried out inside the region and that there is an increase in pressure and density. This is to be expected since that portion of the gas at rest at any time has the same pressure and density as the initial gas. The solution was carried out for only three specific times in this region, since at large time values, the rarefaction waves starts to interact with each other and the solution becomes more complicated. This region is important to the solution only in the fact that it serves as a check on the solution obtained between the two shocks. That is, after the solution is obtained in this region, a mass integral can be written to check continuity, and an energy integral can be written to find the proportional constant between E and E_{tot}. As these integrals check with the equations within a few percent, it can be concluded that the solutions between the two shock waves is valid.

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THEORETICAL INVESTIGATION OF THE SECOND SHOCK

IN THE BLAST WAVE

by

Melvin Lowell Collier, Jr.

Abstract

In the study of blast waves from a finite sphere, it has been observed that in addition to the primary strong shock wave ahead of the driving gas, a second shock also forms inside of this driving gas. This formation of the second shock will invalidate all the existing theories of blast waves in that an additional non-isentropic region actually exists inside of the interface between the driven and the driving gas. This thesis investigates the behavior of this second shock and the resultant flow patterns.

The governing equations are the conservation of mass, conservation of momentum and conservation of entropy along a particle line. The boundary conditions are the Rankine-Hugoniot conditions of conservation of mass, momentum and energy across a shock front. The problem is solved by dimensional analysis and similarity assumptions. The radii to any point and time are the independent variables and an energy term, the outside pressure and the outside density are dimensional constants considered important to the solution. Two new non-dimensional variables are defined,

one (π_1) which is independent of the outside pressure and the other (π_2) which contains the outside pressure. Since the pressure outside is considered small this second variable is very small and can thus be considered a perturbation variable. The dependent variables of pressure, density and velocity are expanded in a non-dimensional series such as $a_0(\pi_1) + a_1(\pi_1)\pi_2 + a_2(\pi_1)\pi_2^2 + ----$ The zeroth order term considers that the outside pressure is zero and the other terms make corrections for a finite outside pressure. Only the zeroth and first order terms were considered in this thesis. The motion between the first shock wave and the interface has thus been assumed self-similar to the zeroth order. Since this is approximately a blast wave, the zeroth order approximation of the first shock wave is assumed to have the same form as the blast from a point source. The boundary conditions are applied in order to get the initial value of the functions and the equations are numerically integrated towards the interface up to the first order of π_2 .

The interface is obtained by solving the equation for the velocity of a particle originally on the surface of the sphere. The pressure and velocity are continuous across the interface, but there is an increase in density. This is a singular discontinuity and the boundary conditions are identically satisfied. Therefore, the density jump across the interface is assumed and then later checked by writing a mass integral. The equations are again integrated from the interface inward until the solutions become discontinuous. This is then the approximate location to the second shock. The boundary conditions for a second shock to exist are, however, not satisfied. Since an approximate location to the second shock has been obtained, the approximate propagation velocity of this shock can be estimated. A new independent variables $\overline{\pi}_1$ and π_2 are defined and the motion between the interface and the second shock is assumed self-similar to the zeroth order of π_2 . The equations are again integrated from the interface to the second shock using the new assumptions up to the first order of π_2 . The position of the second shock is not appreciably altered under the new assumptions, but the pressure, density and velocity are changed such that they now satisfy the boundary conditions for a shock to exist.

The boundary conditions across the second shock are applied and the region inside this shock should be approximately isentropic. Since this region is a rarefraction wave, the solution can be obtained by the method of characteristics.

Finally a mass integral may be written to check the assumption on the density jump across the interface and to obtain the initial density ratio. An energy integral is also written to find the relationship between the energy term used in the thesis and the total energy released. The initial pressure ratio can now be found. The assumptions of a blast wave are good for the sample calculation given in this thesis since the initial pressure ratio was found to be 588 and the initial density ratio 254. The region inside of the second shock was found to be approximately isentropic, and the mass and energy integrals also checked within reasonable errors.