A Fair Division Approach to Performance-based Cross-Asset Resource Allocation

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Word Count:

Text	-	3792
Figures	1 x 250	250
Tables	4 x 250	1000
Total	-	5042

Submitted for Presentation at the 9th International Conference on Managing Pavement Assets, Alexandria, Virginia, May 18-22, 2015

December 8, 2014

ABSTRACT

Resource allocation mechanisms have become a major issue for transportation agencies in the United States and around the world. In order to meet budgetary restrictions resulting from reductions in funding, transportation agencies have explored alternatives to modify the traditional approaches to funding allocation. Most of the alternative methods for funding allocations focus on maximizing infrastructure performance, obviating the consideration of equity. Equity considerations often influence allocation decisions; therefore, the impact of equity should be considered in funding allocation analyses. This paper presents a methodological framework for performance-based cross-asset resource allocation using the fair division method. The fair division method allocates resources in such a way that participants believe they are receiving a fair share based on utility functions. Collective utility functions are used to conduct tradeoff analyses of different allocations in terms of total utility and total envy which are compared to the predicted asset performance. A case study using performance data maintained by the Texas Department of Transportation was conducted to demonstrate the applicability of the proposed framework. Results from the study suggested that the proposed framework for cross-program resource allocation could be an effective and reliable tool for transportation agencies to allocate resources in an objective manner. Additionally, this framework provides the necessary means to incorporate equity factors in the allocation processes, addressing a major shortcoming associated with most traditional approaches to resource allocation.

Keywords: Transportation asset management; resource allocation; fair division; equity consideration;

INTRODUCTION

Allocating resources to finance transportation projects is one of the major concerns of the state Departments of Transportation (DOTs) in the U.S. and other transportation agencies around the world. The way resources are allocated will greatly impact the performance of transportation agencies in terms of achieving their goals and objectives. Increasing levels of transportation demand with limited capacity and constrained resources have forced transportation agencies to do more with less. As cited in the 2013 Critical Issues in Transportation (*1*), all modes of transportation systems must contend with aging infrastructure and capacity problems for which revenues are no longer adequate.

NCHRP Report 736 reported the general mechanism used to allocate resources by state DOTs in the U.S. (*2*). The overall allocation procedure is driven by DOT policies, performance goals, and priorities, which ultimately define the goals of the organization. The budget of DOTs includes tax revenue, user fees, federal funding, credits, and funding from other sources. DOTs allocate the available funds to each of the existing programs, such as preservation, safety, operations, based on high-level strategies, policies, and performance objectives. The methodology, rationale, and analytical support for these decision-making processes vary significantly in practice, ranging from negotiation and adjustment of historical shares for various programs to data-driven decision models based on program performance and need (*2,*3*).* Resource allocation approaches found in the literature can be categorized in four groups: historical/formulas allocation, performance appropriation, optimization schemes, and cross-asset optimization tools (*4,5,6*). Recent studies have shown that DOTs across the nation have a genuine interest in developing methods to deploy cross-asset optimization tools in their resource allocations (7, *8, 9*)

Currently, transportation agencies focus on efficiency rather than equity in their resource allocation mechanisms. However, a combination of efficiency and equity has the potential to create more defensible funding allocation mechanism. The fair division approach is a contemporaneous and active area within the management science field, in which algorithms are developed to divide up limited resources among competing interests and satisfy a suitable equity criterion. The fair division method was first introduced by Steve Brams and Alan Taylor in their book: Fair Division: From Cake-cutting to Dispute Resolution (*10*).

The "fair division" approach has been widely adopted in the computer science field, where algorithms are developed for computer programs to deal with allocations of CPU time, memory and bandwidth (*13*). In transportation field, the fairness concept was first introduced in the late 1990s, when studies were conducted to analyze the impact of road pricing on users. Later, Litman reported potential mechanisms to incorporate equity impacts into transportation planning (*14*). Additionally, types of equity, ways to evaluate equity, and practical ways of incorporating equity into the decision-making process were presented (*14*). With regards to infrastructure asset management, a Texas Department of Transportation (TxDOT) project investigated fair division algorithms as a mechanism for allocating funds and resources among competing interests. Finally, Gurolla proposed the integration of fair division concepts along with a local search optimization method to determine a sequential allocation of funds that minimizes envy (*13*).

OBJECTIVE

The objective of this paper is to develop a methodological framework for performance-based cross-asset resource allocation using the fair division method, aiming at providing new alternatives for transportation agencies and creating a more defensible resource allocation mechanism. Utility functions are used to allocate resources fairly among multiple players. Moreover, social welfare and collective utility functions are proposed to conduct tradeoff analyses among potential allocations scenarios.

KEY CONCEPTS

Funding Allocation Considering Equity

Equity (also fairness) refers to the distribution of benefits and whether that distribution is considered appropriate (*14*). Transportation funding allocation decisions have significant and diverse equity impacts for the following reasons:

- The quality of transportation service available affects people's opportunities and quality of life;
- Transportation allocation decisions affect the location and type of development that occurs in an area, and therefore accessibility, land values and economic development;
- Transportation facilities, activities and services impose various indirect and external costs, such as congestion delay and accident risk imposed on other road users, infrastructure costs not funded through user fees, pollution, and undesirable land use impacts.

Considering equity in funding allocation process can be difficult because there are different types of equity, numerous impacts to consider, and various ways of measuring these impacts (*14*). A particular decision may seem equitable when evaluated one way, but inequitable when evaluated another. In general, there are three major categories of equities that should be considered in transportation funding allocation as illustrated in Table 1.

Table 1 Equity in Transportation Funding Allocation

Equity	Features
With Regard to Rate of Return	This equity is concerned with the allocation of resources among competing programs considered equal in terms of rate of return of generated revenues. According to this definition, programs should receive the same percentage of resource as they contribute. Consequently, funding allocation policies should avoid favoring one program over others by using rate of return as a measure.
With Regard to Performance	This equity is concerned with the allocation of resources between programs or districts that differ in terms of performance or condition. By this definition, funding allocation policies are considered equitable if they favor conditionally disadvantaged programs, therefore compensating for overall inequities. Policies favoring programs with greater need are called progressive, while those that restrict funding allocation to disadvantaged programs or programs are called regressive. This definition is used to support more funding allocation to programs with greater need.
With Regard to Need	This definition is concerned with the allocation of funding between programs or districts that differ in transportation needs, and therefore the degree to which the transportation system meets the needs of travelers. This definition is used to support allocation based on demand, which means that transportation resources should be allocated according to the actual needs of different programs or districts.

Adopted from source (*14*).

Fair Division Approach

The fair division is a new and active area within management science, in which algorithms are developed to divide up limited resources among competing interests and satisfy a suitable fairness criterion. Since its initiation by Brams and Taylor, this approach has been employed to solve a variety of allocation problems such as divorce settlements, company merges, shore divisions, and computer memory allocations (*10,11,12,13*).

A fair division problem is defined as follows: assuming there is a set of N players ($P_1, P_2, \ldots, P_N$) and a set of goods $\boldsymbol{S}$, the objective is to divide $\boldsymbol{S}$ into N shares ($S_1, S_2, \ldots, S_N$) so that each player gets a fair share of $\boldsymbol{S}$. A fair share is a share that, in the opinion of the player receiving it, is worth $\frac{1}{N}$ of the total value of $\boldsymbol{S}$. It is assumed that any player is capable of deciding whether his/her share is fair; in other words, it is assumed that any player is capable of assigning unambiguous values to $\boldsymbol{S}$ and to various parts of $\boldsymbol{S}$ (*15*).

As such, the fair division scheme is a systematic procedure for solving a fair division problem, possessing the following properties:

- The procedure is considered decisive, meaning that if the rules are followed, a fair division of the goods $\boldsymbol{S}$ is guaranteed;
- The procedure is internal to the players with no outside intervention required to carry out the procedure;
- The fair division method assumes that the players have no useful knowledge of each other's value system;
- The players are assumed to be rational, meaning that they base their actions on logic, not emotion.

The last assumption is imperative because a fair division scheme does not guarantee that each player will get a fair share; it only guarantees that each player can get a fair share if he or she plays rationally (*10,15,16,17*)

Fair Division Scheme Requirements

The fair division schemes attempt to satisfy four requirements: proportionality, envy-free, equitability, and efficiency. Proportionality implies that each of the P_N participants receives what he or she considers being at least $1/S$ of the total value of the object or objects divided. Envy is experienced by a player if he or she would prefer to trade his or her portion of the division with other players. Consequently, an allocation is considered envy-free if no player strictly prefers the portion assigned to player. Envy cannot be entirely eliminated in many allocation protocols; however, the degree of envy can be measured. A similar concept closely related to proportionality is equitable. A fair division allocation will be equitable if and only if each participant believes he or she has received the same fraction of the total value of the object or objects divided. The most fundamental efficiency criterion is the Pareto condition. An allocation is called Pareto efficient (or Pareto optimal) if there is no other feasible allocation that would make at least one player strictly better off while not making any of the others worse off (*15,16,18*).

Utility Function in the Fair Division Method

Allocating resources fairly among multiple programs requires the concept of utility function. In the case of transportation agencies, all players (assets) have an equal attractiveness to the available funds and thus it is important to quantify the satisfaction of players. In this sense, utility can be defined as a measure or relative satisfaction. A common assumption is that the utility of a player, or a program in a transportation agency, depends only on the goods that it receives, rather than on goods received by the other players (or programs) outside the allocation process. The utility can be defined as a function of the ratio of needs and allocated funds as shown in Equation 1. The utility of each player (or program) i can be defined by using Equation 1 as shown in Equation 2.

$$Utility=\frac{Allocated\ Funds}{Needs} \quad (1)$$

$$U_i=\frac{F_i}{N_i},\ \forall\ i \in I \quad (2)$$

where,

i = the ith player (or program) competing for resource;
U_i = utility value of the ith player (or program);
F_i = funding received by the ith player (or program);
N_i = resource needed by the ith player (or program) .

Equations 1 and 2 ensure that the value of utility is always between 0 and 1, since the highest amount of funding received by a player (or program) will never exceed the budget requested. For example, a player (or program) that receives no funding will have a utility value of 0 representing the lowest satisfaction, whereas a player (or program) receiving funding equal to the requested needs has a utility of 1, corresponding to the maximum satisfaction.

Social Welfare and Collective Utility Functions

Utility is a measure of the relative satisfaction only, rather than an indication of the fairness of a potential allocation. While fairness is clearly a major consideration in the division of goods, another important consideration is the social welfare resulting from the division. Apparently, a division may be envy-free but very inefficient, e.g., in the total welfare it provides to the players. In principle, all sorts of indicators are taken into account when judging fairness.

One particular method for incorporating fairness criteria is to obtain the individual utility level of the players, which is known as the welfarist approach (*12*). Technically, this means that rather than looking at allocations and assessing their relative fairness, the utility value after the allocation process is the only criterion that needs to be considered and compared. A whole range of fairness and efficiency criteria can be defined in terms of so-called social welfare orderings and collective utility functions (CUFs). Some of the most important CUFs are utilitarian, egalitarian, elitist, and Nashwhich are shown in Table 2 with mathematical formulation and features (*12*).

TABLE 2 Collective Utility Functions

CUFs	Features	Formulation
Utilitarian	-Objective is to maximize the sum of individual utilities; -Completely ignores fairness considerations; -Some players may not receive utility; -Easy to implement.	$SW_{util}(u) = \mathbf{F} = \arg\max_{\mathbf{F}} \sum_{i \in I} U_i$ (3)
Egalitarian	-Objective is to maximize the minimum of individual utilities; -All players are equally satisfied in terms of their utility; -Reduces efficiency and requires optimization.	$SW_{egal}(u) = \mathbf{F} = \arg\max_{\mathbf{F}} \left(\min_i U_i\right)$ (4)
Elitist	-Objective is to maximize the maximum individual utility; -Some players will be fully satisfied while others may not receive any funding at all; -The advantage of this method is that some players get very high funding.	$SW_{elit}(u) = \mathbf{F} = \arg\max_{\mathbf{F}} \left(\max_i U_i\right)$ (5)
K-rank	- Objective is to maximize the kth ranked utility; -It is "blind" with respect to agents that are either extremely well or extremely badly off. -Intervention is allowed; -k=1, egalitarian; k = n, elitist.	$SW_{krank}(u) = \mathbf{F} = \arg\max_{\mathbf{F}} U_{(k)}$ (6)
Nash	-Combine efficiency and fairness considerations; -Like the utilitarian CUF, it favors high total utility, but it also encourages inequality-reducing transfers of utility at the same time. For example, the utilitarian CUF cannot distinguish between ⟨4,4⟩ and ⟨2,6⟩, while the Nash CUF will favour the former.	$SW_{elit}(u) = \mathbf{F} = \arg\max_{\mathbf{F}} \prod_i U_i$ (7)

where:

$\mathbf{F}$= resource allocation results and $\mathbf{F} = \{F_i\}_{i \in I}$
F_i = resource received by the ith player
I = set of players and $I = \{1, 2, \ldots\}$
U_i = utility value of the ith player (or program)
$U_{(k)}$ = the utility of the kth ranked player after allocation.

METHODOLOGY

After carefully analyzing challenges, opportunities, and gaps associated with resource allocation problems, a framework for performance-based cross-asset resource allocation using the fair division method is proposed. The standardized conceptual framework guides the resource allocation process and can be customized to accommodate the needs of various state DOTs and other transportation agencies. The overall framework is shown in Figure 1.

FIGURE 1 Performance-based Cross-Asset Resource Allocation Framework

Identify Goals and Objectives

The procedure begins with the strategic planning, comprising the goals, the allocation philosophy, and objectives that govern the way in which the agency would be operated and measured. At this stage, transportation agencies need to clearly identify goals and objectives for the resource allocation procedure. Moreover, agencies must define which assets would be considered as part of the proposed methodology.

System Condition

In this step, the objective is to evaluate the condition of the infrastructure system to generate an overall score for each of the assets being analyzed, which starts by defining the assets that will be used to achieve the goals set by the transportation agency. Some examples of assets that can be established to receive funds are pavements, bridges, culverts and signs.

The condition of each asset is calculated based on performance measurements. Since each asset may have different performance measures characterizing its condition, different approaches could be used to develop an overall score for each asset. How to measure and compare benefits is one of the challenges in the cross-asset funding allocation. It is difficult to quantify the benefits received from maintenance actions across various types of assets in a standardized way, such as in the case of comparing agency savings, cost effectiveness of bridge improvement, reduction of traffic delays, and sign replacement. Various methods allow decision makers to compare measurement, units, attributes, and factors. Some of these methods are the Analytical Hierarchy Process (AHP), the scaling-scoring-weight, and the multi-attribute utility (*20*).

Finally, performance-funding relationships are used to measure the effects of funding levels on overall condition scores for each asset, which follow an exponential form (*20*). Transportation agencies can use historical funding and performance data to calibrate this model. The general form of this function is:

$$Performance = A \cdot (allocated\ funds)^{B} \qquad (8)$$

where,

A and B = calibration parameters.

Allocation Protocol

The allocation protocol has the objective of allocating funds using the fair division approach to incorporate equity in the allocation procedures. Allocating resources fairly among multiple programs requires the concept of utility function. Additionally, time horizon should be defined by decision makers to plan allocations that better capture their goals and objectives. CUFs shown in Table 2 provide the proposed methods to allocate the funds using different functions.

Moreover, in order to compare the social welfare of the different CUFs, parameters such as total utility and total envy are suggested. Their mathematical formulations are presented as follows:

$$Total\ Utility = \sum_{i=1}^{N} U_i = \frac{F_i}{N_i} \tag{9}$$

$$Envy = \epsilon_{ij} = \begin{cases} |U_i - U_j|, & if\ (U_i - U_j) > 0 \\ 0, & otherwise \end{cases} \tag{10}$$

$$E = \sum \epsilon_{ij} \tag{11}$$

where,

i = the ith player in the competition for resource;
U_i = utility value of the ith player;
F_i = funding received by the ith player;
N_i = resource needed by the ith players;
ϵ_{ij} = envy experienced by i from j;
E = the total allocation envy.

Allocation fairness can be measured in terms of envy experienced by each program based on the total utility and total envy obtained from each CUF. The comparison of these parameters could provide transportation agencies with the ability to introduce equity parameters in the decision-making. Additionally, the CUFs would enable the agency to conduct trade-offs in terms of the fairness of the funding allocation.

The next step is to determine the predicted performance for each asset group. The predicted performance will be determined using allocated funds for each scenario and the performance-funding relationship defined as part of "System Condition" discussed earlier. Predicted performance together with allocated funds would provide decision makers with various scenarios for resource allocation, which can track equity and efficiency parameters.

Trade-off Analysis

The last step is to evaluate the various funding alternatives obtained from the proposed methodology. A funding allocation alternative represents a possible strategy of allocating funds on the basis of various considerations. Based on the proposed cross-asset resource allocation framework, all potential alternatives should be evaluated in terms of fairness and optimality.

Fairness in the allocation is measured using envy while the optimality is quantified using total utility and predicted performance.

CASE STUDY

The roadway network of Travis County located in Texas was used to demonstrate the applicability of the proposed framework. This roadway network is managed by TxDOT. For simplicity in the result analysis, only two asset groups were used to conduct the case study: pavements and bridges. Additionally, the time horizon was defined as three years. The available funds were assumed to be 75 percent of the total estimated needs for both asset groups. Table 3 shows the input data used in the case study.

TABLE 3 Case Study Profiles

Parameter	Pavements	Bridges
Condition Measurement	Condition Score (CS)	Sufficiency Rating (SR)
Database	PMIS	PonTex
Average CS 2012 [1]	90.14	-
Average SR 2012 [2]	-	90.00
Estimated Needs ($million) [2,3]	-	-
2013	83	28
2014	139	33
2015	139	35
Performance-Funding[4]	-	-
2013	A = 46.55 ; B = 0.15	A = 56.63; B= 0.15
2014	A = 19.81 ; B = 0.31	A = 56.63; B= 0.15
2015	A = 19.81; B = 0.31	A = 56.63; B= 0.15

[1] Information from TxDOT Pavement Management Information System (PMIS) database
[2] Information from TxDOT PonTex database
[3] Performance Analysis Tools for Highway Pavement (PATH-P) (*21*)
[4] Values obtained from source (*20*)

RESULTS

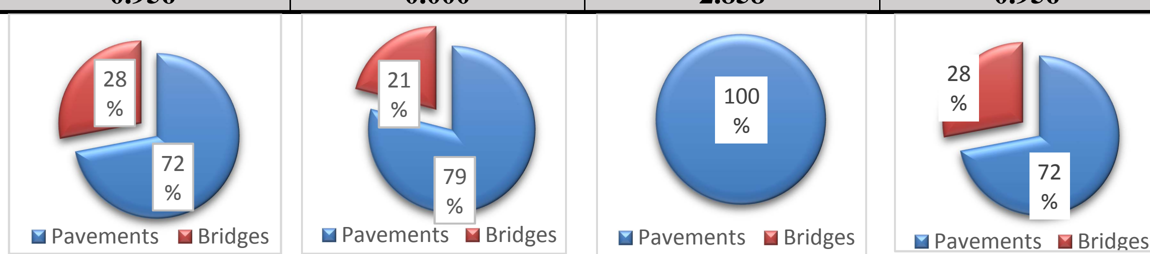
Table 4 shows the summary of the analysis results for the case study. Funds were allocated in accordance with the proposed methodological framework. Moreover, as part of the allocation protocol, four CUFs were used: utilitarian, egalitarian, elitist, and Nash. In order to compare efficiency and fairness, the total utility, total envy, and performance were computed. By examining the results shown in Table 4, the following observations can be obtained:

- All CUFs generated different allocation scenarios, which is expected since each approach response to a different objective. On one hand, the utilitarian approach favors bridges rather than pavements because bridges have fewer needs than pavements. On the other hand, the egalitarian approach allocated partial funds to all the programs in such a way that all program utilities are viewed as a fair share. Moreover, the proposed approach parameters, such as total utility, total envy and performance to conduct trade-off analyses, follow the goals and objectives of transportation agencies;

- With regards to total utility, the utilitarian and the Nash approaches represent the highest value; while the egalitarian and the elitist showed the lowest values. The total utility can be taken as a measure of efficiency, indicating which allocation is the most attractive among potential alternatives;
- As expected, the egalitarian approach represented the lowest envy; while the elitist CUF showed the highest value (i.e., envy for the egalitarian is null compare to the 2.858 resulted in by the elitist approach). Usually, the highest value of envy would represent the lowest value of total utility.
- The CUFs played an important role in the proposed methodology because they provide the necessary means to conduct trade-off analyses. Instead of allocating funds following fixed formulas, the agency has the option of adopting a different CUF to allocate funds in a more data-oriented manner. For example, if the pavement condition shows a high utility value and the bridge condition has a low rating, an approach favoring bridge would benefit the resource allocation strategy. The proposed methodological framework suggests potential improvements in fund allocations.

TABLE 4 Summary Results

		Utilitarian		Egalitarian		Elitists		Nash	
		Pavements	*Bridges*	*Pavements*	*Bridges*	*Pavements*	*Bridges*	*Pavements*	*Bridges*
Allocated Funds ($million)	2013	55	28	62	21	83	0	55	28
	2014	96	33	104	25	129	0	96	33
	2015	96	35	104	26	131	0	96	35
	Total	247	96	271	72	343	0	247	96
Performance	2013	81.91	90.00	85.30	87.78	91.25	78.41	81.91	90.00
	2014	75.69	90.95	81.89	82.51	89.45	62.73	75.69	90.95
	2015	73.21	91.52	76.98	74.26	87.63	56.45	73.21	91.52
Utility	2013	0.666	1.000	0.750	0.750	1.000	0.001	0.666	1.000
	2014	0.691	1.000	0.750	0.750	0.928	0.000	0.691	1.000
	2015	0.687	1.000	0.750	0.750	0.939	0.000	0.687	1.000
	Total	2.043	3.000	2.250	2.250	2.867	0.001	2.043	3.000
	Sum	**5.043**		**4.500**		**2.868**		**5.043**	
Envy	2013	0.334		0.000		0.991		0.334	
	2014	0.310		0.000		0.928		0.310	
	2015	0.312		0.000		0.939		0.312	
	Total	**0.956**		**0.000**		**2.858**		**0.956**	



CONCLUSIONS

The overall objective of this paper is to present a framework for performance-based cross-asset resource allocation using the fair division method, aiming at providing new alternatives for transportation agencies in their quest of allocating limited resources in a more defensible manner. The applicability of the developed methodology was successfully demonstrated with the case study. The major conclusions drawn from this study include:

- Various studies in asset management have identified resource allocation across assets as a significant gap. The method proposed in this paper addresses this deficiency by adopting the fair division approach. However, the fair division approach need to be further customized to transportation infrastructure management to facilitate its acceptance. This study presents a first step by incorporating the fair division approach in the cross-asset resource allocation;
- New resource allocation alternatives for transportation agencies are needed. Methodologies, such as fair division, can serve as a viable alternative to existing allocation methods, including historical appropriations and consensus formulas;
- Currently, transportation agencies focus on efficiency rather than equity in their resource allocation mechanisms. However, a combination of efficiency and equity have the potential to yield more defensible funding allocation mechanism. The proposed methodological framework provides the means to conduct trade-off analysis by simultaneously taking fairness and efficiency into consideration;
- The proposed framework has the potential to become a decision-support tool for the allocation of funds at the program level by introducing equity parameters, which could provide the agency with the means to intervene more in the allocation process. Moreover, parameters, such as total utility and total envy, could be used as important inputs to the allocation procedure to achieve agency goals and objectives.

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