A STUDY OF THE EFFECT OF NORMAL STIFFNESS ON KINETIC FRICTION FORCES BETWEEN TWO BODIES IN SLIDING CONTACT

by

John Alton Elder, Jr.

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APPROVED:

Chairman, Dr. N. S. Eiss, Jr.

Dr. H. H. Mabie

Dr. C. J. Hurst

Dr. J. B. Jones, Head of Department

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LIST OF SYMBOLS

ΔF_{avg}	Average stick-slip force fluctuation for a particular normal stiffness
$F_{(t)}$	Friction force as it varies with time
K_H	Stiffness of rider support in direction tangential to sliding direction
K_N	Stiffness of rider support normal to the plane of the contacting surfaces
K_{N1}	Normal support stiffness of 5 lb/in
K_{N2}	Normal support stiffness of 32 lb/in
K_{N3}	Normal support stiffness of 71 lb/in
N	Normal load
$N_{(t)}$	Normal load as it varies with time
m	Mass of rider
p	Penetration hardness
s	Shear strength
S.R.	Surface roughness
V	Velocity of sliding surface
V_c	Critical velocity or sliding velocity above which stick-slip will not occur
W	Weight of rider
x	Displacement of the rider in the tangential direction to sliding
$\ddot{x}$	Acceleration of the rider in the tangential direction to sliding
y	Displacement of the rider in the normal direction to sliding
$\ddot{y}$	Acceleration of the rider in the normal direction to sliding

σ	Standard deviation
μ	Coefficient of friction
μ_{int}	Coefficient of friction for the initial pass over a surface
μ_{fin}	Coefficient of friction for the final pass over a surface
μ_s	Static coefficient of friction
μ_{sl}	Sliding coefficient of friction

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I. INTRODUCTION

Friction-excited vibration or discontinuous kinetic friction force and its effects have been noted in many different sliding systems. Some of the familiar occurrences of friction-excited vibration are: clutch jerking, brake squeaking, machining chatter, jerky motion of rubbing elements in instruments, squeaky door hinge, and brush vibration on a slip ring. The phenomenon of discontinuous kinetic friction force was studied by F. P. Bowden and L. Leben (3) as early as 1940 and many researchers since have attempted to analyze the phenomenon in order to understand and control it because of its wide and usually undesirable occurrence. However, these works do not yet give a complete theory of frictional vibration as is evidenced by the several conflicting theories.

This thesis will be concerned with the continued investigation of friction-excited vibration, commonly called stick-slip because of the periods of zero relative motion and then relative motion between sliding and stationary surfaces in contact as shown in Figure 1. The approach of this work is different from most other investigators in that it studied the effect of stiffness, contact surface displacement (roughness), natural frequency, and similar variables in the direction normal to the plane of contact between the sliding surfaces.

The principal justification of this study is simply that it is obvious in the systems previously named that normal roughness,

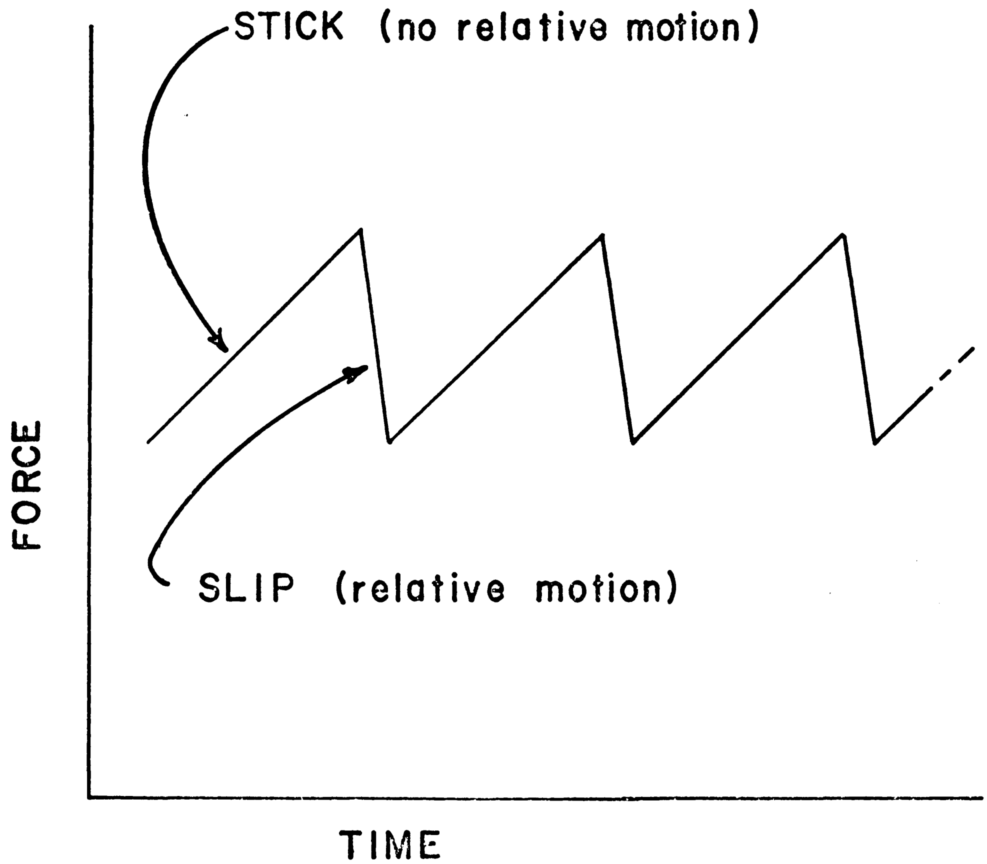

FIGURE I - GRAPHIC ILLUSTRATION OF STICK-SLIP BETWEEN BODIES IN SLIDING CONTACT normal stiffness, and normal natural frequency do exist and in most cases are important parts of the system, (e.g. the normal stiffness of a clutch, a slip-ring brush, and a tool post in machining). Other researchers generally neglected the effects of these variables even though they were present in their experimental systems and were often varied inadvertently with other system parameters being studied. Only one other work (34) has been found in which normal motion and normal damping were studied for their effect on friction-excited vibration. This work gave positive results for the effect of the mentioned normal functions on frictional vibration and thus gave further justification for a study to determine if normal stiffness has an influence on stick-slip.

II. REVIEW OF LITERATURE

The terms friction-excited vibration, stick-slip, and discontinuous kinetic friction force have been used to describe the same phenomenon of sliding friction as illustrated and explained in the introduction. There are also several theories which attempt to explain why and how the phenomenon occurs. Some of the various theories concerning friction-excited vibration are presented in this section to give the background for this study and to illustrate that no one theory exists which explains friction-excited vibrations. Those characteristics, assumptions, and conclusions made by previous researchers that aid in the justification of a study of normal properties, which exist and could influence the friction force of a sliding system, are stated.

One of the first theories for discontinuous friction force was presented by Bowden and Leben (3). They designed a very sensitive system which would indicate any slight movement in the horizontal direction of a rider, the member of a two body sliding system that remains in the same relative position with respect to ground during sliding, by reflecting light from the rider through a set of lenses onto oscillograph paper. The apparatus had a high stiffness in the vertical direction to insure that if contact was broken it would be regained immediately, but no means to record the vertical movement or determine its effect was incorporated into the equipment. By the use of this apparatus they observed the phenomenon of stick-slip. From this Bowden and Leben theorized that kinetic friction force was a vibrational phenomenon which was due to a welding and breaking of the weld between sliding surfaces. During weld the rider would move with the sliding surface, thus causing deflection of the rider support proportional to the sliding velocity. Due to the deflection there would be an increasing force in the support which opposed the motion of the rider with the sliding surface. This increasing force in the support would be balanced by an increasing friction force until the force in the support became great enough to overcome the maximum strength of the weld. Then, with the weld broken, the rider would move in the direction opposite that of the sliding body due to the force in the support attempting to return it to zero deflection. The motion would continue until sufficient time of contact or increase in temperature occurred to allow welding to again take place and repeat the cycle of discontinuous kinetic friction force.

H. Blok disagreed with Bowden and Leben on the bases of studies by P. J. Papenhuyzen and himself (1). Papenhuyzen's research showed that frictional vibrations occurred during sliding between materials that could not weld and that some slip was present at all times. Blok observed stick at the beginning of sliding before pressure or sliding temperatures were great enough to create a welded joint. Thus, Blok believed that the welding theory could not be the primary cause of frictional vibration. His theory stated that vibration would occur if the frictional force decreased with an increase in sliding speed. The explanation for this is that the decrease in force would allow relative motion between the slider and base, thus causing a continually varying force on the slider as it tried to match the base speed. Blok pointed out that the decrease in friction force could be due to a decrease in load as well as a decrease of the coefficient of friction but he did not discuss the details of this possibility. Since the calculations to consider a system with a decreasing friction force with increasing velocity, as proposed, would be very difficult, Blok assumed a definite (constant) static friction force and a lower constant kinetic friction force with change in velocity to derive dynamic property relationships and to show that vibrations will occur if a varying friction force is present. No experimental evidence was presented to establish the accuracy of his equations and he admitted that his study was not completely representative of actual sliding because of his assumptions.

Iu. I. Kosterin and I. V. Kragel'skii (15) agreed that Blok's decreasing friction theory was good, but they did not feel it was complete or that any other theory defined the vibrations accurately. Therefore, they developed a theory which they said was not the ultimate but was probably closer to the actual case than any other. They began by defining the varying friction force as relaxation oscillations which consist of two stages, one of uniform motion with zero relative displacement and one with non-uniform relative displacement between stationary and moving surfaces, similar to Bowden (3). The elasticity of the system was identified as the fundamental parameter of mechanical relaxation vibration. The theory they developed is as follows:

"Mechanical relaxation oscillations appear in elastic friction systems when the static friction coefficient depends on the duration of stationary contact and is larger than the sliding friction coefficient and also when the relation between the sliding friction characteristic and velocity has a decreasing characteristic." (15)

They also concluded that changing parameters of the elastic system such as stiffness, damping, and mass could only reduce the vibration but not eliminate it because the oscillations are caused by the frictional characteristic of the surfaces in contact. No consideration was given to any normal properties even though their model and experimental apparatus (a clutch) had the ability to move and stiffness in the normal direction.

E. Rabinowicz (23), (24) tried to present all possible causes of frictional vibration in his study of stick-slip. He stated that, in general, the vibration is due to the variation of friction force as a function of some other variables. He explained how the vibrations could occur as a function of position (displacement), time, or velocity. He made no effort to relate force fluctuation to variance of normal properties even though normal vibrations were not restricted on his experimental apparatus.

Another school of theory states that there are properties of the sliding system, none of them being in the normal direction, which dictate the occurrence of friction-excited vibrations as much as properties of the coefficient of friction. In these theories, properties as damping, mass, and stiffness are used to predict a sliding velocity (critical velocity) above which frictional vibration cannot occur.

C. A. Brockley, R. Cameron, and A. F. Potter (6) developed an analytical model which predicted the critical velocity. This model indicated the following relationships between system parameters to decrease or eliminate friction-excited vibrations.

"1) A small load and a large stiffness of the elastic suspension.

2) A small difference in the value of maximum static coefficient of friction and minimum coefficient of friction.

3) A large damping ratio.

4) A small value of the static-friction growth constant" (6) (increase in static coefficient of friction with time of contact).

An experimental model was also developed to test the validity of the previous predictions. Data from the experimental model correlated with theoretical predictions. Thus, this theory identified a larger number of parameters which could affect friction-excited vibrations.

Another supporter of the theory that sliding contact systems have a critical velocity dictated by system parameters is B. R. Singh and his co-workers. Singh (31) attempted to verify his theoretical work with experimentation on a milling machine and with a simulated system on an analog computer. The theoretical and analog computer-predicted values for critical velocity agreed; but even though a critical velocity was obtained experimentally on the milling machine, its value was consistently 10% to 25% lower than the predicted values.

Work by Singh and H. B. Mohanti (27) indicated that if a fluctuating drive force was applied to a slider the value of the critical velocity increased above the initially predicted critical velocity if the frequency of the force was the same as the natural frequency of the horizontal system. They also made other variations in the physical nature of the sliding system, but still found that a resonance condition altered the predicted critical velocity. Thus the theory expanded the number of parameters presented by Brockley and also illustrated that a resonance condition can influence friction-excited vibrations. No experimental data supported these new parameters in this article or in similar articles by Singh and V. Push (20),(29),(30). Futhermore, as in previous works, no normal properties were studied in works in which Singh was involved.

R. P. Jarvis and B. Mills (14) have published a theory stating that friction-excited vibrations do not depend on the relationship of velocity to coefficient of friction. In fact they allowed the coefficient of friction to be constant in part of their analytical work. The theory they presented indicated that system instability or capacity to vibrate was due to the manner in which the motions of the components were coupled (the configuration of the supports of the sliding members). Equations were derived to predict instable configurations and test equipment consisting of a disk and rider was used to confirm the results of the equations. The limitation of the equations, however, was their inability to cope with complex support configuration.

F. Morgan, M. Muskat, and D. W. Reed (17) attempted to verify the behavior of stick-slip as indicated by Bowden and Leben. They obtained similar results, but did not agree on the welding theory. They did comment that a small variation of flatness in the tangential plane of the moving surface would create a large change in normal load; however, they did not evaluate the magnitude of the effect.

D. Godfrey (11) studied the effect of normal vibration on friction force and determined that the vibration reduced the average value of the friction force. He carried this further and showed that the reason for the force decrease was that the normal load decreased during vibration due to lifting of the rider.

The majority of the works already described did not recognize any effect of normal properties on friction force, and the investigators that mentioned the possible effect did not try to evaluate it. The following papers are primarily concerned with properties in the normal direction (stiffness, displacement, and damping) and how they affect friction force.

D. M. Tolstoi's (33),(34) work has been to establish the existence of normal displacement during sliding and to show that this displacement is important in friction-excited vibrations. He states that he has proved the existence of normal microvibrations during sliding. In addition he gave a relationship which indicated that decreasing friction force with increasing velocity, which others used for the bases of their theories about frictional vibration, cannot exist unless there is some normal displacement. Tolstoi states that the normal movement may be caused to some degree by the microgeometry. He experimentally endeavored to confirm his theory by retarding the normal displacement of the experimental system to show the effect on friction-excited vibrations. The results of his work show that he was able to prevent frictional vibration with added vertical damping. Consequently, he supported his theory of the effect of normal displacement on stick-slip and showed that normal damping could eliminate it.

Tolstoi and S. R. Grigorava (12) made a check of the effect of normal vibration on friction force similar to D. Godfrey (11). Unlike Godfrey, they tried to establish a frequency for maximum effect on friction force. They obtained a frequency for maximum decrease in friction force with vibration and tried to relate the corresponding frequency to the natural frequency of the rider. Poor correlation was obtained with this attempt to relate the two frequencies and it was credited to the difficulty in obtaining the contact stiffness of the rider which was used in the natural frequency calculations. No mention, however, was made of an endeavor to compare applied frequencies with a natural frequency determined from the stiffness and mass of the rider support.

I. R. Bogue (2) developed an analytical model for stick-slip of a cantilever friction contact in which stiffness of the support of the cantilever in the normal direction was considered. The contacting surfaces were likewise assumed to be elastic. Hence, he was unable to solve his equation because of the difficulty in defining the elasticity of the surfaces. However, he did acknowledge the presence of normal stiffness in sliding friction systems in his theory.

The references mentioned previously give some of the major theories in the field of kinetic friction force vibration. These theories are varied in the properties of a system they choose to consider and are in some cases contradictory to each other. Some mention and evidences are given by these researchers of possible influences on discontinuous friction force of variations in the properties in the normal direction. This investigation will attempt to further the knowledge of any effects on friction of normal stiffness and properties related to this stiffness.

III. INVESTIGATION

A. Object

The object of this work was to determine the influence of normal stiffness on stick-slip or friction-excited vibrations.

B. Development of the Experimental System

In the development of the system it was necessary to insure the following:

(1) that the tangential and normal stiffnesses and motion were not coupled in the rider support and the varying of the vertical stiffness did not affect properties of the system not directly related to the stiffness.

(2) that the conditions of sliding were such that stick-slip would be experienced.

(3) that the recording equipment was sensitive enough and of the type and location to record the friction-excited vibration phenomenon.

To allow for normal and tangential independence (see Figures 2 and 3) of the rider support a cantilever beam to hold the rider was used such that it had stiffness in the tangential direction, but bearings at its ground connection gave it complete freedom in the normal direction. This beam remained constant in size and shape for all tests and accordingly gave a

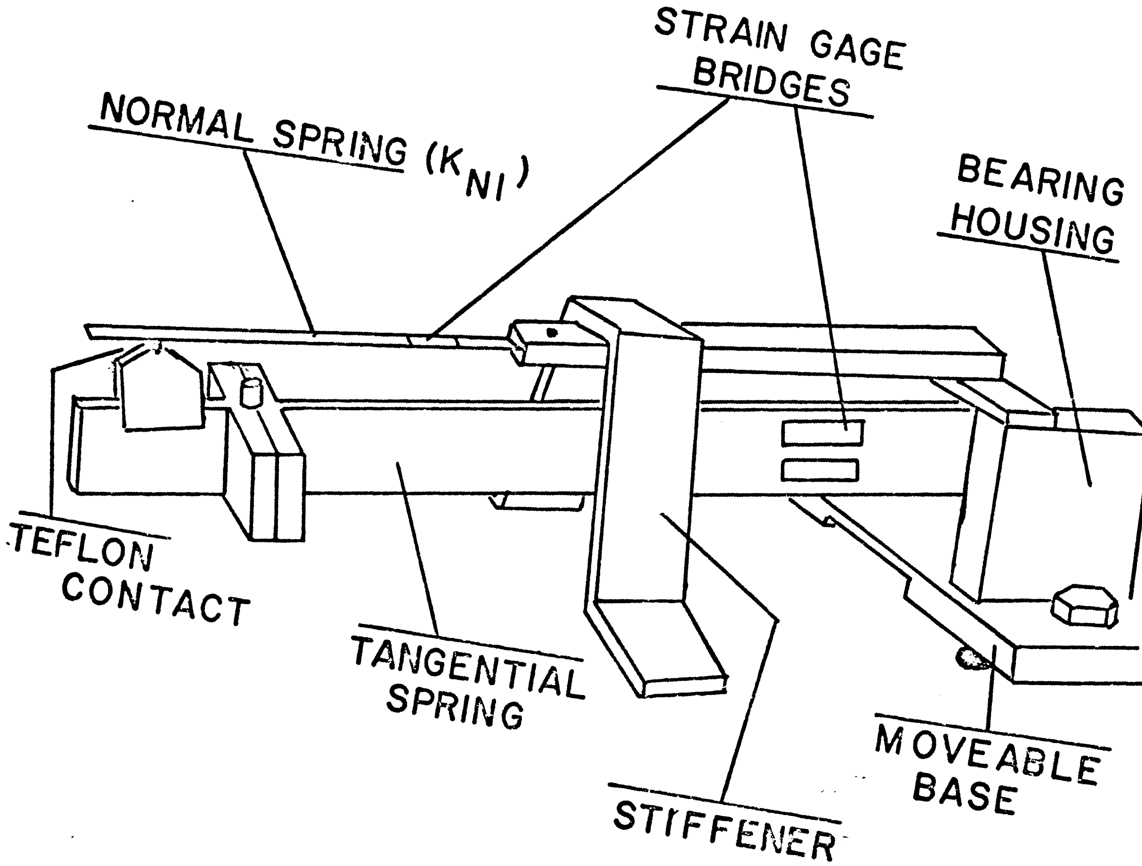

FIGURE 2 - RIDER SUPPORT SYSTEM

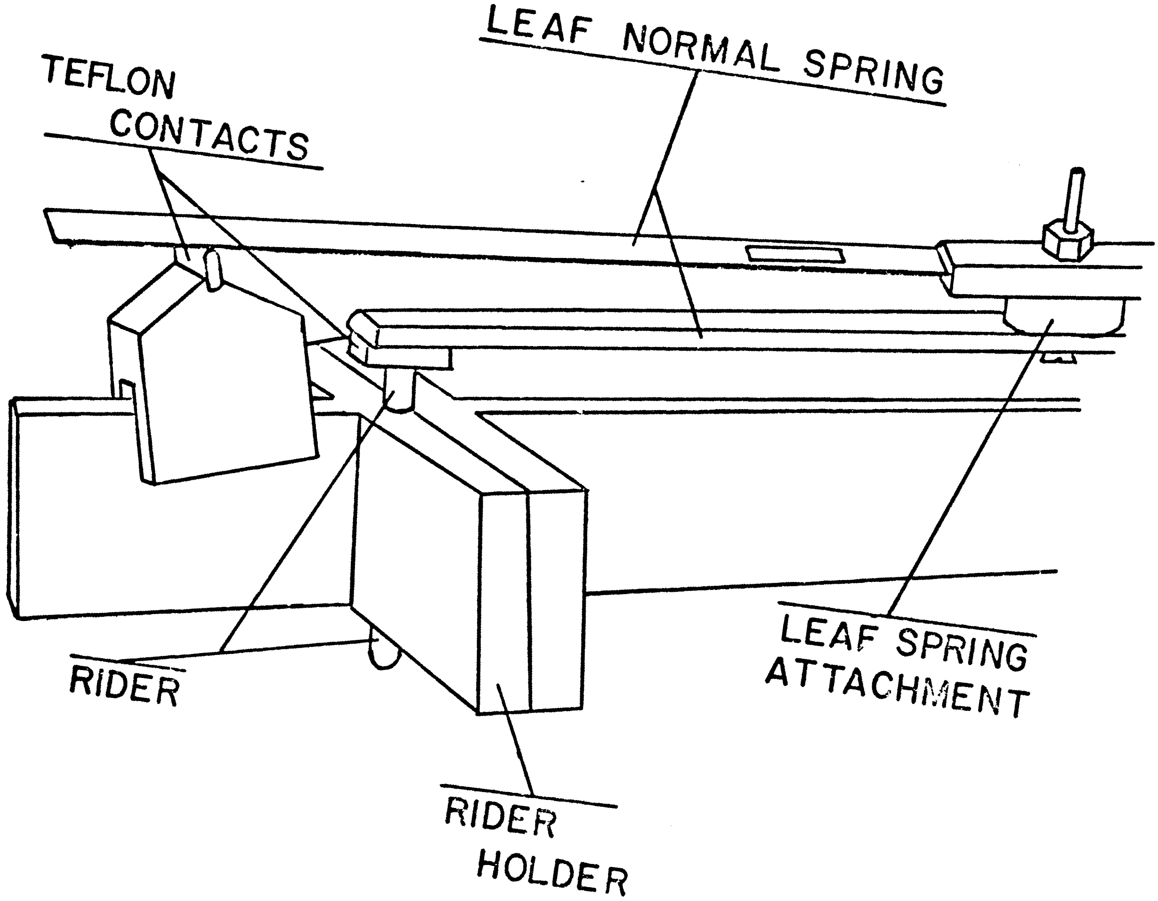

FIGURE 3 - RIDER SUPPORT SYSTEM WITH A LEAF NORMAL SPRING

constant tangential stiffness to the support for all tests. The normal stiffness required a different type of mechanism since it had to be variable in order to conduct the desired test. An arrangement similar to a leaf spring was used in which one low stiffness (K_{N1} = 5 lb/in) cantilever spring was in contact with the tangential cantilever spring for all tests (except zero stiffness test) and other springs with several different stiffnesses were brought into contact for different tests to obtain several normal stiffnesses. The normal cantilever springs were not fastened to ground by means of bearings as was the tangential beam, however the points of contact between the normal and tangential springs had teflon contacting surfaces. Therefore, a system with independent normal and tangential motion was obtained without affecting properties of the system not dependent on normal stiffness. (Calibration of this will be discussed later).

To develop a system in which stick-slip would be experienced, it was first decided from the literature (21),(26) that the contacting surfaces should be mild steel, one with a flat surface and the other with a rounded end (1/4" diameter) rider. (See Figure 4.) However obtaining the proper drive and isolation for the test specimen to insure that the results recorded were due to frictional vibration and not created by the vibration or natural frequencies of the system, required some testing. The first attempt was with a rotational system similar to a cam and follower which proved unsatisfactory

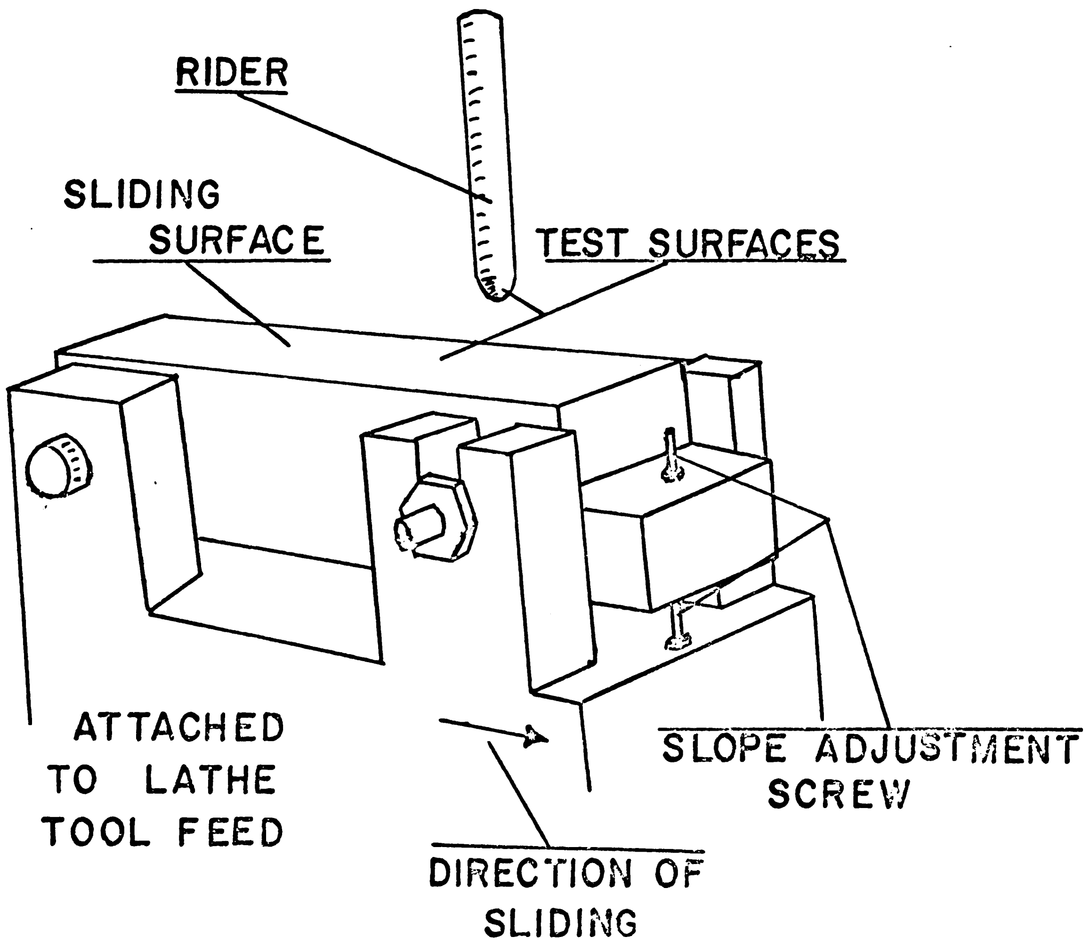

FIGURE 4 -TEST SPECIMENS WITH MOUNTING MEMBER AND SLOPE ADJUSTMENT

because the minimum speed obtainable with available equipment was 1 in/sec. This speed set up natural frequency vibrations in both the tangential and the normal springs. Although the vibrations in the tangential direction could be related to the results of D. Sinclair (26) and A. Watari and T. Sugimoto (35), this was not the classic form (Figure 1) of stick-slip motion and the normal vibration could not be justified. In fact the occurrence of the normal vibration was believed to indicate a vibrational effect in the system other than the one caused by friction. Also, the fact that the edge of the cam was to simulate a completely flat surface passing under a rider dictated that the cam rotate within 0.00006 of an inch (normal sensitivity) of round. This was very difficult to achieve with the bearings being used and, as was, extended the time to mount specimens for one run into hours. After unsuccessful attempts to modify the original system, the system described next was developed and used for the experimental work.

In order to obtain the low velocity of sliding needed to obtain classic stick-slip and to avoid forced natural frequency vibrations, the tool feed of a lathe was used to drive the sliding member. Speeds as low as 0.01 in/sec could be obtained. With the ways of the lathe bed being sufficiently flat, the sliding test specimen was ground flat and mounted in a device which allowed precision adjustment of the slope (see Figure 4) of the specimen. This combination eliminated error in the measurements due to gross variations in surface flatness of the sliding specimen. Moreover, with the lathe drive motor completely isolated from the test system with vibration damping material no difficulties were experienced as the result of the vibration of the drive system.

To acquire the normal and tangential force and deflection data strain gages were used. A four arm bridge of SR-4 gages (type C-1,500 ohms, gage factor of 3.56) was used on the tangential beam, and a two arm bridge of SR-4 gages (type C-5,342 ohms, gage factor of 3.49) was used on the normal beam. The tangential bridge signal went directly to a Sanborn Carrier Preamplifier Model 150-1100 and was recorded by the Sanborn recorder. The normal bridge operated with an Ellis Bridge Amplifier Meter and this signal was again amplified in the Sanborn AC-DC Preamplifier Model 150-1000 and recorded on two-channel recorder paper with the horizontal signal. Maximum sensitivities were required of both bridges in order to study the frictional vibration phenomenon. The sensitivities of these systems as calibrated are given in Table I.

Also included in the system was a microswitch which made a record on the recorder paper for every 0.125 inch of travel of the sliding specimen. This allowed the sliding speed to be calculated and indicated its consistency. Another feature of the experimental system was the mobility of the ground connection (see Figure 2) of the tangential and normal springs

TABLE I

Sensitivities

	Tangential	Normal
Force	0.025 lb/mm*	---
Deflection	0.0001 in/mm*	0.00006 in/mm*

*(per millimeter of recorder paper with a maximum width of 50 millimeters)

which allowed runs to be made on tracks parallel to one another by moving the beams relative to the sliding specimen.

Dial indicators and a calibrated spring scale were also mounted and used during calibration. These will be discussed later.

C. Procedure

Normal stiffness and surface roughness were varied to determine what effect these normal properties had on friction-excited vibration. However, a basic procedure was followed for all tests with appropriate changes at various steps to alter the tests as desired. The basic procedure is presented in the following discussion.

First a method to insure the specimens were positioned uniformly for all tests was established. A dial indicator with the sensitivity to indicate 0.0001 of an inch was used to insure the flatness of the sliding specimen. The vertical spring with strain gages in combination with the Ellis Bridge was used to set the preload at approximately 0.3 of an ounce for each test. The preload was accomplished by the positioning of the rider specimen since the deflection of the normal spring, and therefore the load added by this spring was dictated by the length of the rider which extended below the horizontal spring rider holder. The surface of the rider was then polished with No. 1 grit emery paper and cleaned with acetone as recommended in the literature (26). The sliding surface had various surface finishes and therefore was not polished for every run but it was cleaned with acetone before each run. The surfaces finishes studied were completely opposite in nature in order to evaluate vertical stiffness for the extreme cases. One surface was intentionally roughed with tool marks perpendicular to the direction of movement to a roughness of 300 microinches. The other was ground with tool marks in the direction of movement to a roughness of 10 microinches.

After the specimens were ready, the desired normal stiffness was obtained by either removing the spring K_{N1} for a stiffness of zero or adding other springs to obtain stiffness of 32 lb/in (K_{N2}) or 71 lb/in (K_{N3}). K_{N1} was also used with no alterations, thus giving four normal stiffnesses.

To obtain stiffnesses of K_{N2} and K_{N3} it was necessary to insure that the end of the spring that was clamped to ground was tightened with the same and sufficient force for each test of a particular stiffness. This had to be done without a gross increase in normal load due to the deflection of the added spring but with enough preload to insure the added spring would remain in contact with the rider. The error presented by the variation of tightness of the fixed end of the spring was not realized initially and much work was done until a better system of calibration pointed out the error and thus rendered portions of the initial work of no value.

With the system ready for test (specimens in position and desired stiffness), calibration of the normal load, normal stiffness, and tangential stiffness were made to insure the tests were uniform and that only the desired variables had been changed. Two dial indicators and a spring scale were used for these calibrations. For normal calibration an increasing force would be applied to the cantilever beam at the location of the rider by means of the spring scale. This would be done until the rider had been raised to the same height it would be if in contact with the sliding surface. This was indicated by the Ellis Bridge Recorder in combination with the strain gages mounted on spring K_{N1}. The force thus determined would be the normal load. Further increases in the force applied by the spring scale were made with the deflection of the beam being indicated by a dial indicator or in the case of K_{N1} read from a scale. Thus the normal stiffness could be calculated. A similar procedure of increasing force with the spring scale and measuring deflection with a dial indicator was used to determine horizontal stiffness. Sensitivities were determined by applying a known load or deflection to the beam and noting the corresponding value on the Ellis Recorder or the Sanborn Recorder. Values noted on the Ellis Recorder were converted to the Sanborn Recorder. Several readings were made at each data point to insure the accuracy of the readings.

After calibrating, the surfaces were again cleaned with acetone and positioned for the beginning of the test. The drive motor was started but the drive of sliding surface was not engaged. The Sanborn chart paper was then started at a speed of 2.5 mm/sec and the drive speed and zero signal were recorded. The sliding surface drive was then engaged and a run made. The distance of the run was 3/8 of an inch. A pass was made over the same track with the same rider for at least eleven times or until stick-slip became apparent at a chart speed of 2.5 mm/sec. The last run of a test was made with a recorder chart speed of 50 mm/sec to obtain an accurate record of the stick-slip motion. Values and trends were then read from the traces and compared.

IV. DATA AND RESULTS

The data is presented in the form of comparisons of typical friction force traces and a table of values indicating the conditions for each run and the results taken from the traces. Typical traces were used because the actual data for one run was recorded on Sanborn recorder paper in lengths of five to six feet and because a method to describe the vibrational variation of force with numbers only could not be done accurately or clearly.

The effect of normal stiffness on stick-slip is shown in Table III and Figure 11.

Each force fluctuation average in Table III was calculated from the data obtained in four runs. From each run the maximum and minimum fluctuation would be taken, thus giving eight data points to be averaged in order to determine the fluctuation for each stiffness.

Overall average results noted from the experimental data are:

1) $\mu_{fin} = 0.48$

   $\sigma = 0.030$

2) μ_{int} (S.R. = 10 microinches) = 0.152

   $\sigma = 0.029$

3) μ_{int} (S.R. = 300 microinches) = 0.234

   $\sigma = 0.027$

The value of μ_{fin} was calculated by averaging the values of the coefficient of friction for the final pass of each run. Values of

μ_{int} (S.R. = 10 microinches) and μ_{int} (S.R. = 300 microinches) were calculated in a manner similar to μ_{fin} but only eight values were included in each average since the runs were separated according to the surface roughness.

TABLE II

DATA

K_H = 220 lb/in   V = 0.01 in/sec   Pass Length = 3/8 in

Run	K_N (lb/in)	S.R. (microinches)	No. of Passes	N (lb)	FORCE FLUC (lb) min	FORCE FLUC (lb) max	μ_{int}	μ_{fin}
1	0	10	17	1.13	0.075	0.100	0.177	0.50
2	0	10	14	1.13	0.075	0.075	0.177	0.49
3	0	300	19	1.13	0.075	0.100	0.243	0.49
4	0	300	13	1.13	0.050	0.100	0.265	0.49
5	5	10	12	1.15	0.075	0.100	0.174	0.50
6	5	10	21	1.15	0.050	0.075	0.174	0.49
7	5	300	12	1.15	0.050	0.075	0.261	0.41
8	5	300	12	1.15	0.075	0.100	0.239	0.43
9	32	10	23	1.34	0.025	0.050	0.131	0.50
10	32	10	21	1.38	0.025	0.050	0.145	0.53
11	32	300	14	1.44	0.025	0.050	0.227	0.45
12	32	300	12	1.41	0.025	0.050	0.230	0.48
13	70	10	14	1.31	0.025	0.050	0.115	0.48
14	70	10	21	1.31	0.000	0.013	0.115	0.48
15	70	300	21	1.44	0.025	0.038	0.187	0.49
16	70	300	20	1.38	0.025	0.038	0.217	0.45

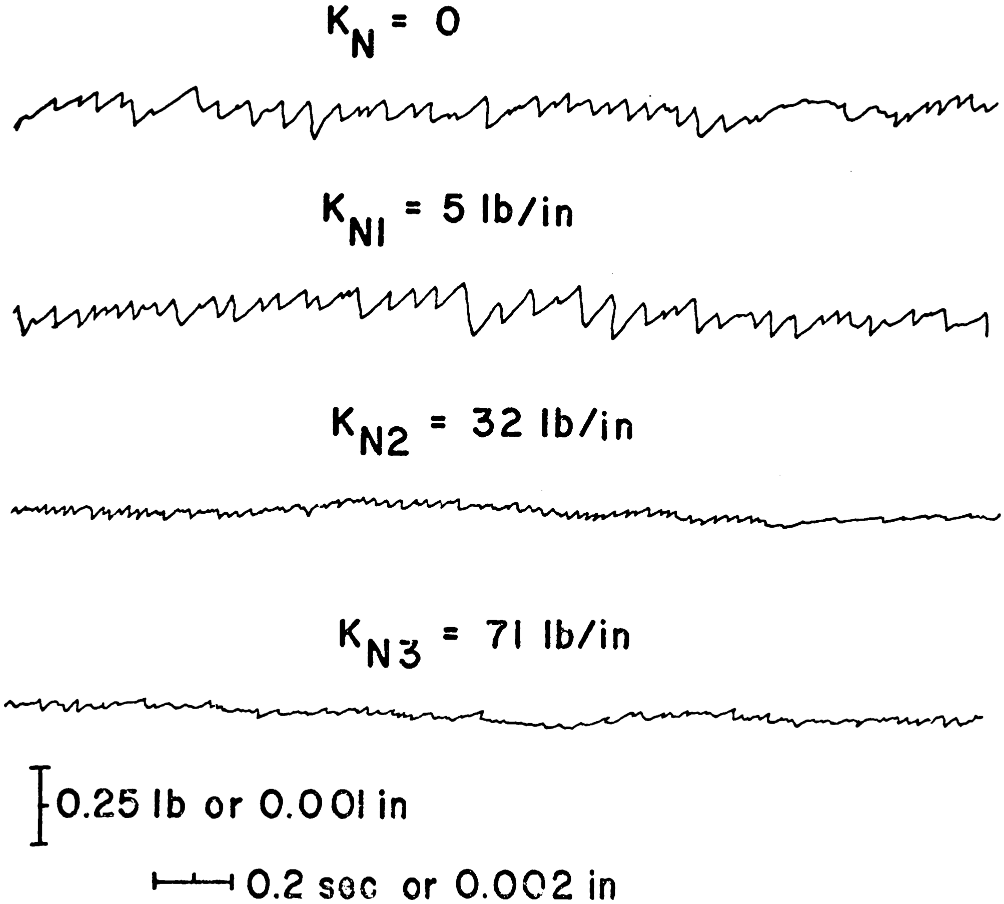

FIGURE 5 - VARIATION OF STICK-SLIP AMPLITUDE WITH NORMAL STIFFNESS FOR A ROUGH CONTACT SURFACE (300 μin)

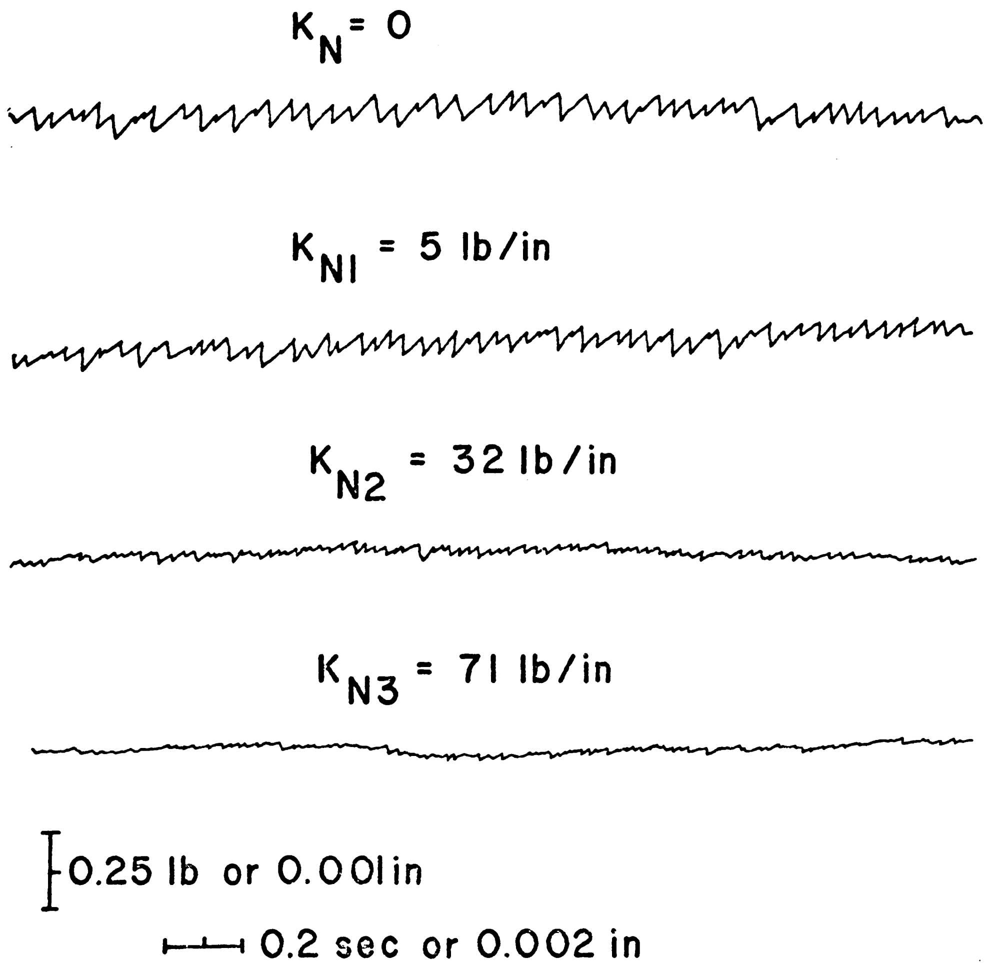

FIGURE 6 - VARIATION OF STICK-SLIP AMPLITUDE WITH NORMAL STIFFNESS FOR A SMOOTH CONTACT SURFACE (10 μin)

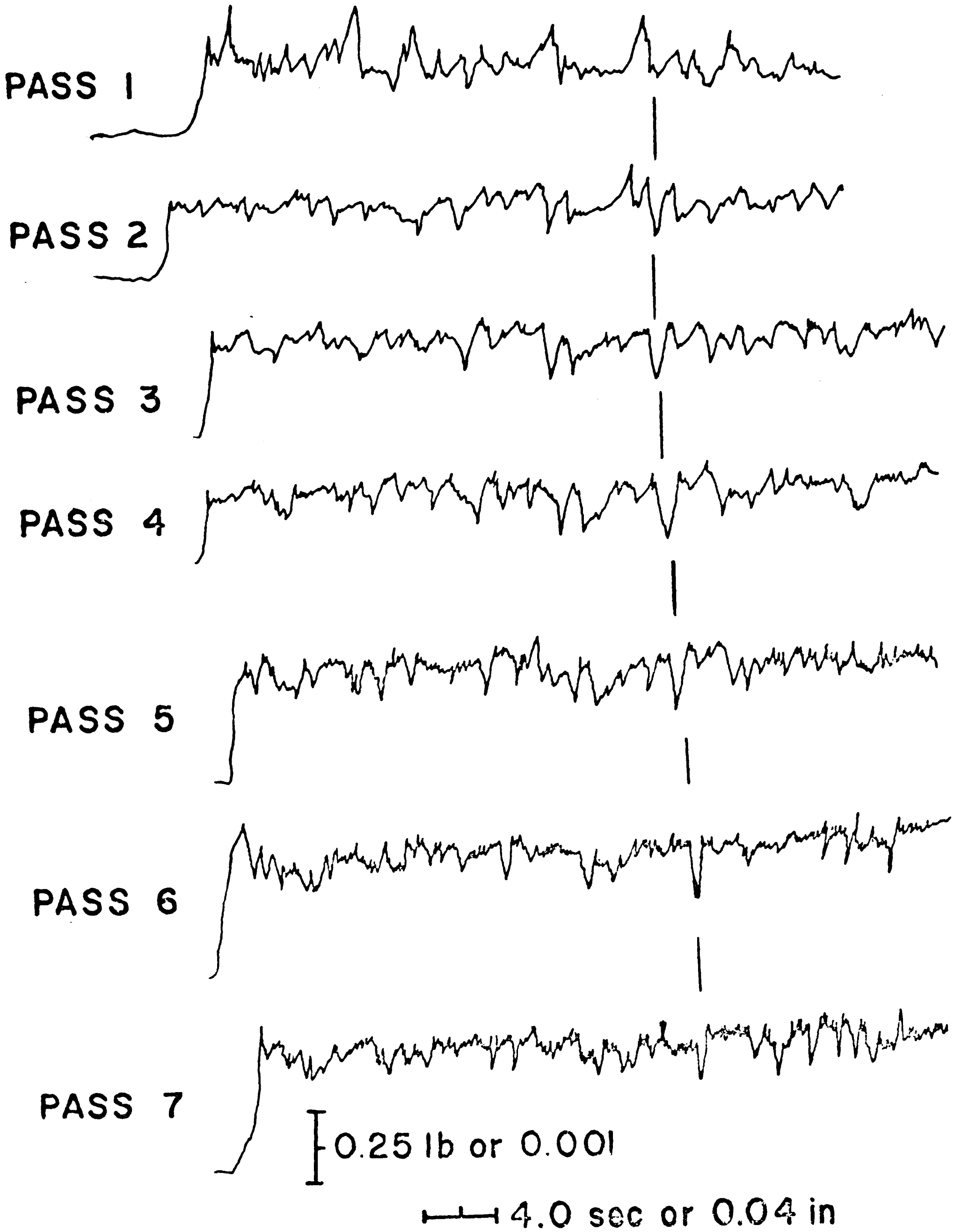

FIGURE 7 - VARIATION OF FRICTION FORCE WITH THE NUMBER OF PASSES OVER THE SAME ROUGH (300 μin) SURFACE (K_{NI} = 5 lb/in)

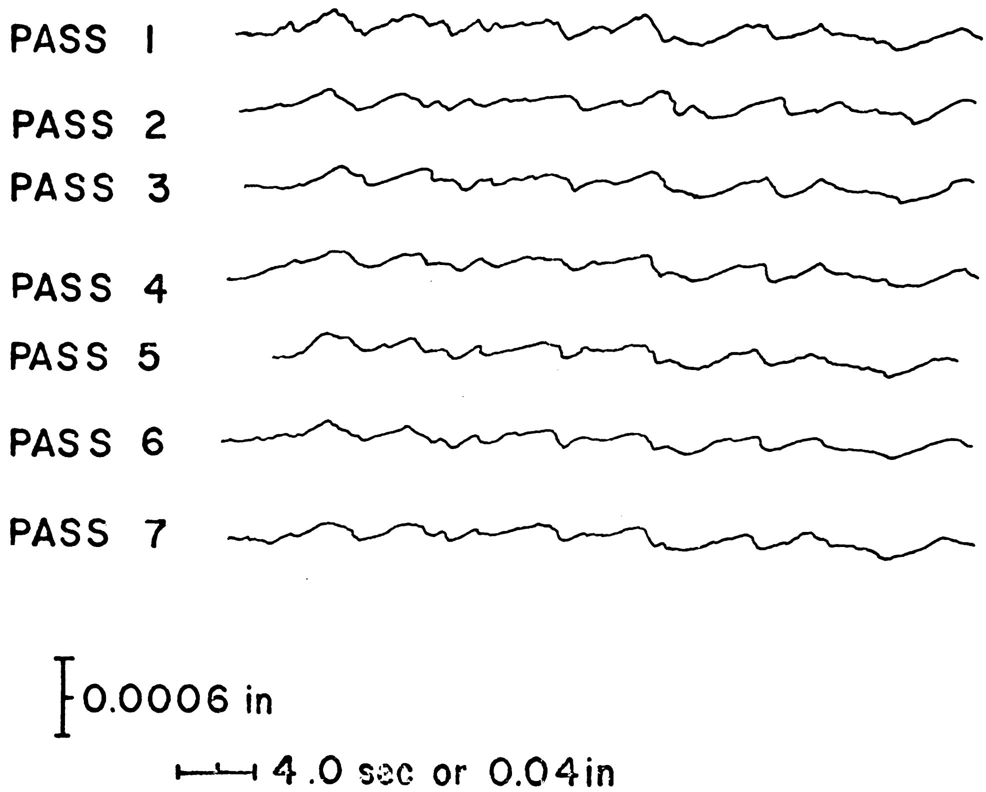

FIGURE 8 - SURFACE PROFILES CORRESPONDING TO PASSES OF FIGURE 7 (S. R. = 300 μin)

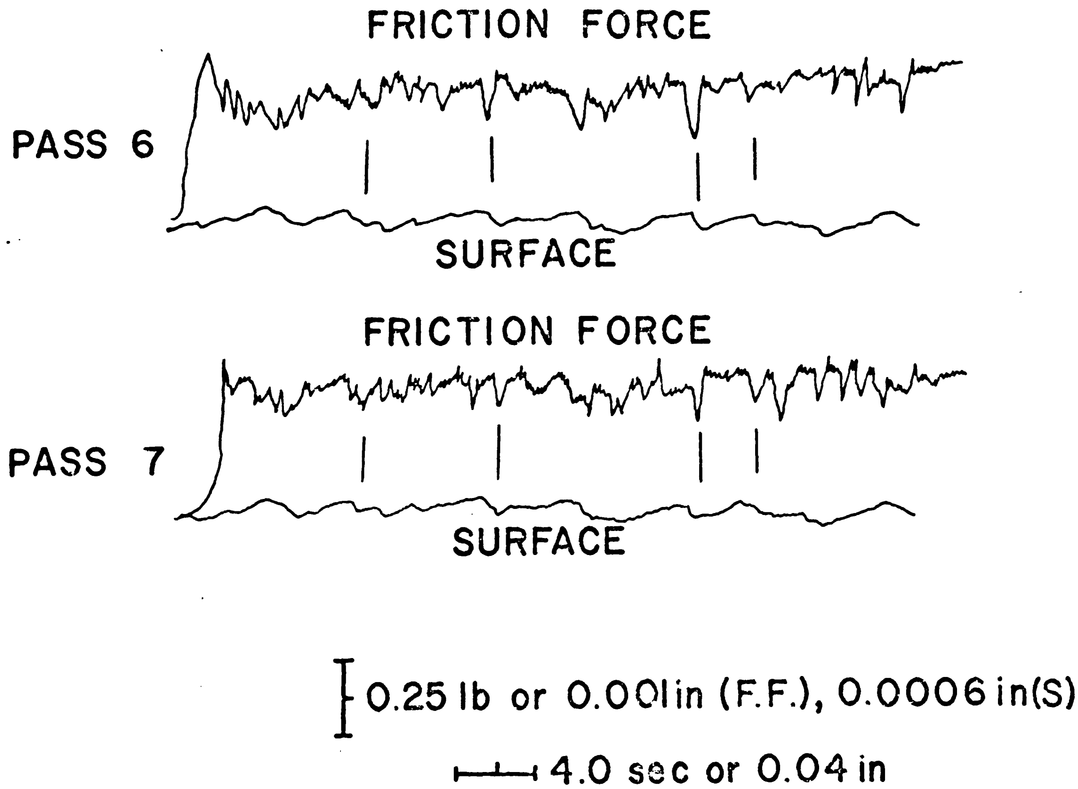

FIGURE 9 - INFLUENCE OF THE SURFACE PROFILE ON FRICTION FORCE (S. R. = 300 μin) (K_{NI} = 5 lb/in)

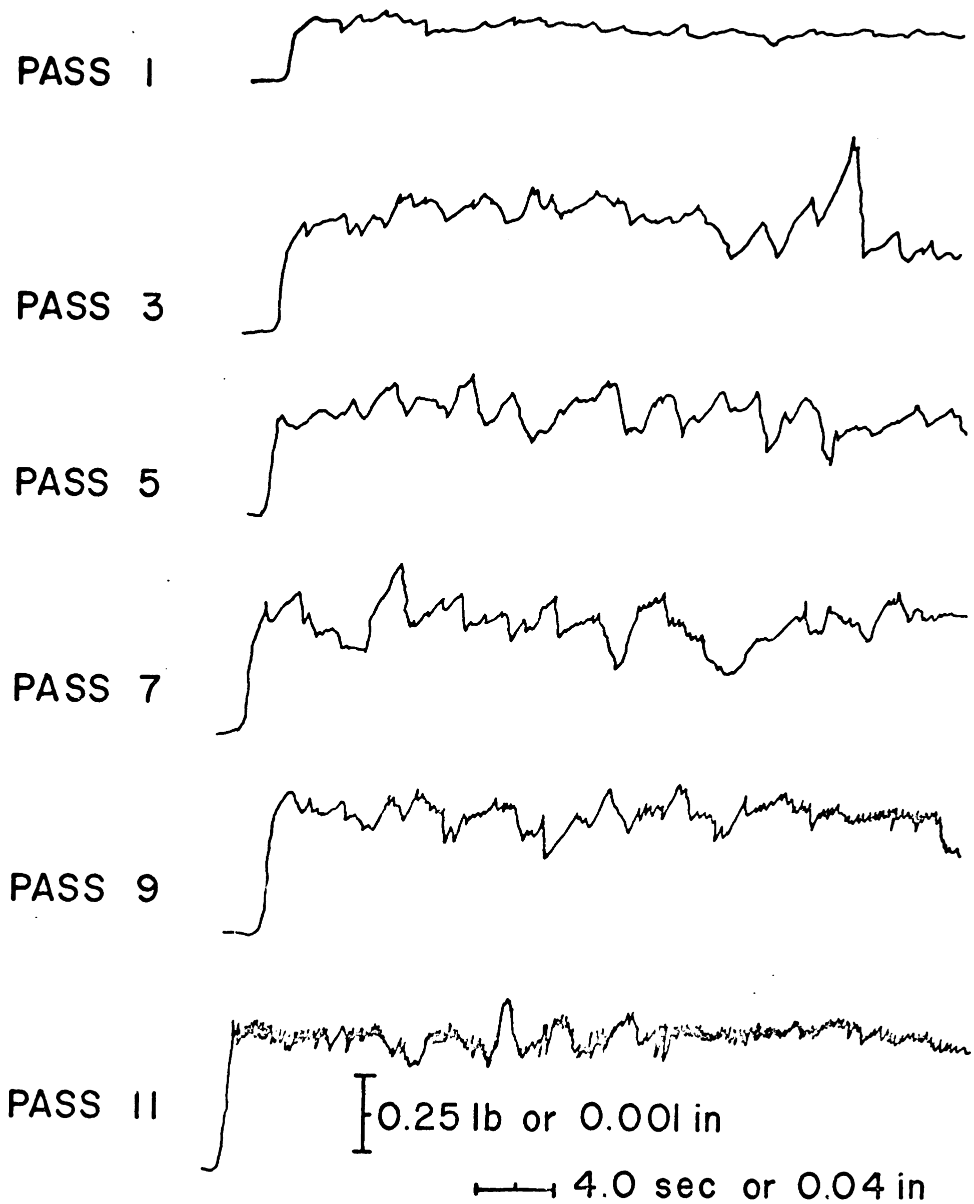

FIGURE 10 - VARIATION OF FRICTION FORCE WITH THE NUMBER OF PASSES OVER THE SAME SMOOTH (10 μin) SURFACE (K_{NI} = 5 lb/in)

TABLE III

Variation of Stick-Slip Amplitude with Normal Stiffness

K_N (lb/in)	ΔF_{avg} (lb)	σ (lb)
0	0.081	0.018
5	0.075	0.019
32	0.038	0.013
71	0.027	0.012

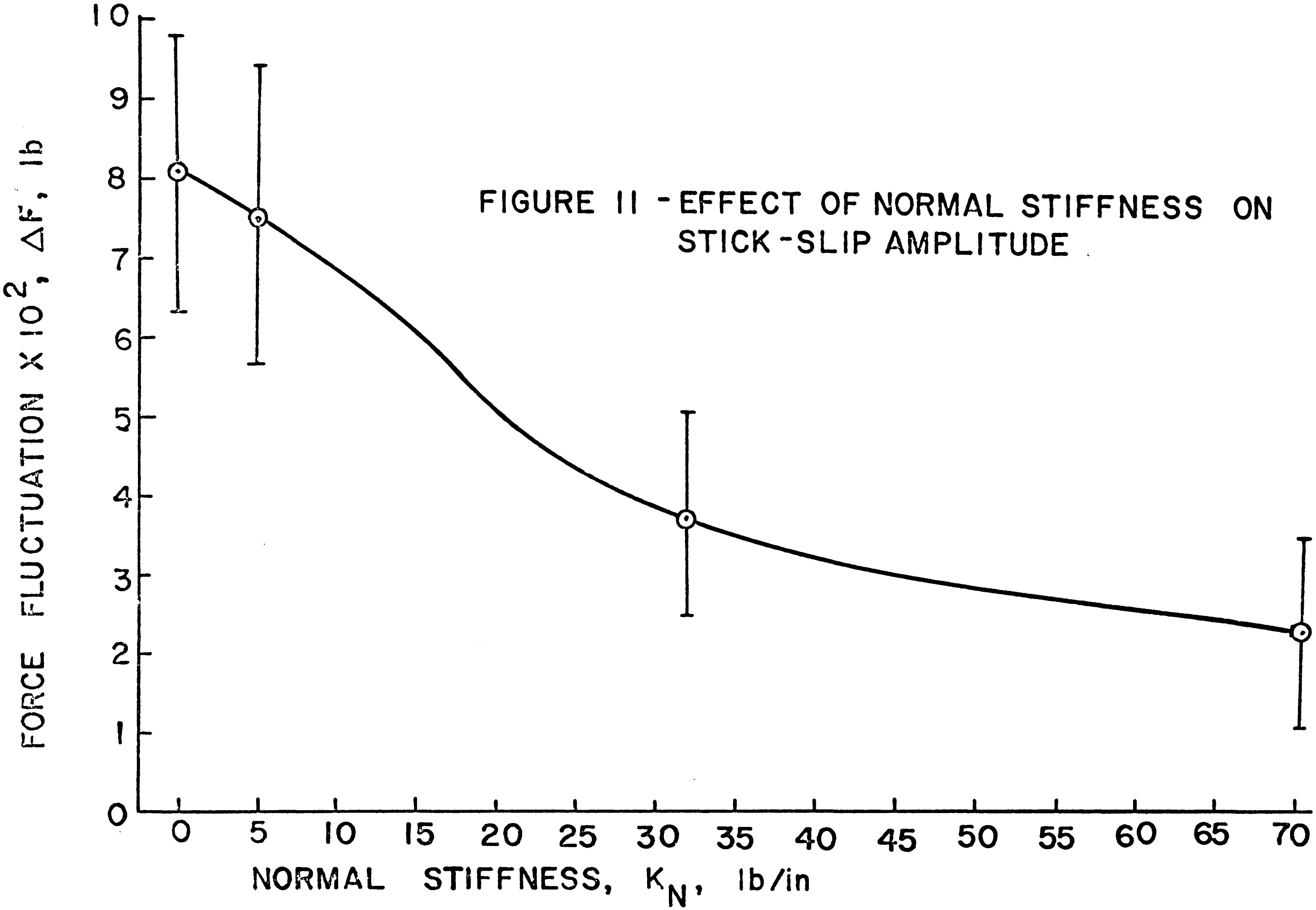

FIGURE 11 - EFFECT OF NORMAL STIFFNESS ON STICK-SLIP AMPLITUDE

V. DISCUSSION OF RESULTS

A. Idealized Model

In studying stick-slip the idealized model generally used for a sliding system without damping is similar to the one shown in Figure 12a. A more realistic model is shown in Figure 12b but the effect on stick-slip of the spring in the normal direction has not been determined. Possible equations to describe the motion of the system of Figure 12b would be:

Normal Motion

$$m\ddot{y} + K_N y + W = N_{(t)} \tag{1}$$

Coupling Equation

$$F_{(t)} = \mu N_{(t)} \tag{2}$$

Tangential Motion

$$m\ddot{x} + K_H x = F_{(t)} \tag{3}$$

In order for these equations to be derived several assumptions had to be made about the system. These were:

1) normal stiffness only affects stick-slip by changing the normal load due to normal movement [This possibility was mentioned by both H. Blok (1) and F. Morgan (17)]
2) the coefficient of friction was constant

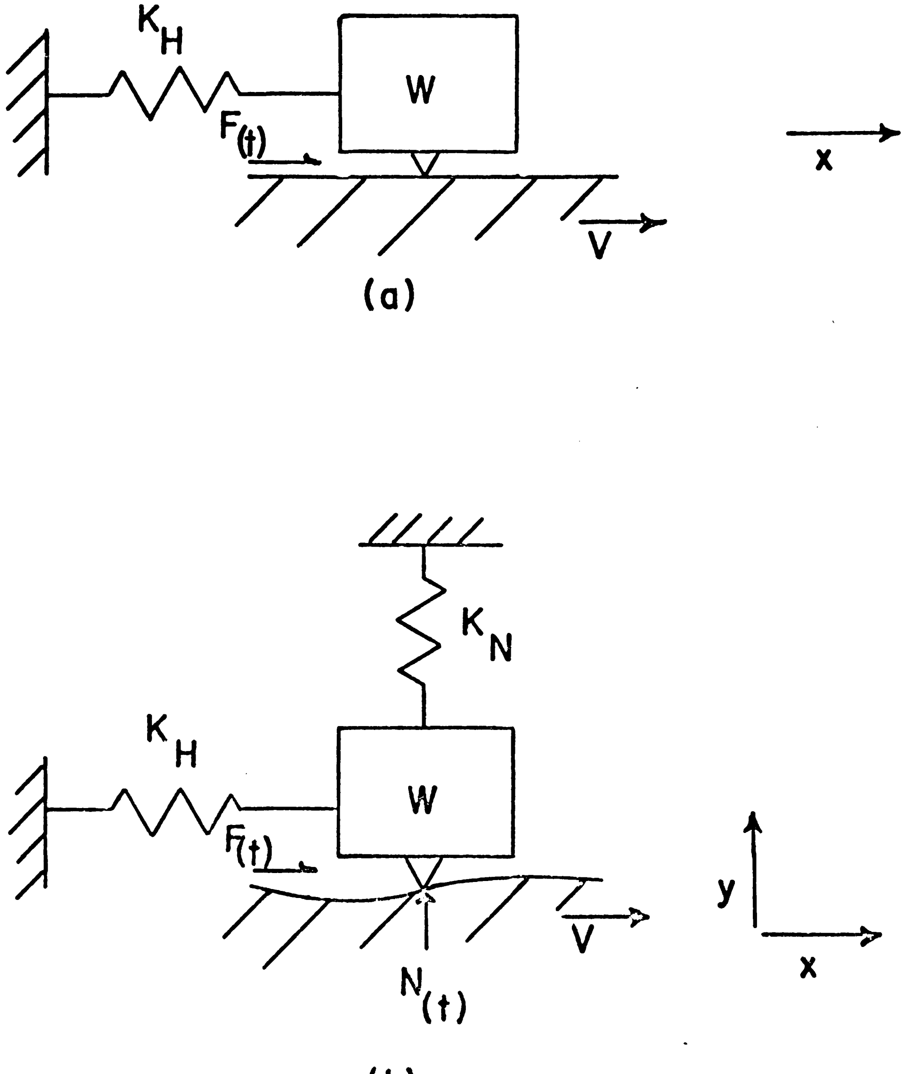

FIGURE 12 - IDEALIZED MODELS OF STICK-SLIP SYSTEMS

3) contacting surfaces were completely inelastic
4) horizontal and normal springs were only coupled by the friction force
5) the rider (W) was always in contact with the sliding surface

After some study of the assumptions made in obtaining equations (1),(2), and (3), it was decided that assumptions (2) and (3) were not in accordance with the literature concerning stick-slip and friction force. Therefore unless assumption (1) could be shown experimentally to be accurate any analytical or analog computer solution of equations (1),(2), and (3) would be of no value in this study.

The validity of assumption(1) was determined by making sliding tests with different surface roughness (different normal movement). These tests showed that friction force would vary with elevation of the surface (Figure 9). Also, with each pass over the rough surface there would be some similarity in the friction force trace due to the roughness (Figures 7, 8, and 9) but no such similarity for the smooth surface (Figure 10). However, no effect on stick-slip due to the variation of normal movement could be detected. F. P. Bowden and L. Leben (3) indicated that they found no effect of surface roughness on stick-slip, but since they were not concerned with normal stiffness it was necessary to re-examine the dependence for this study.

Since this experimental work showed that gross normal movement did not affect stick-slip, assumption (1) and thus assumptions (2) and (3) were found to be inaccurate and no analytical method was available to correct them. Therefore further analytical work with equations (1), (2), and (3) could not be justified and the major emphasis of this work was switched to an experimental determination of the effect of normal stiffness on stick-slip.

B. Effect of Normal Stiffness

The main result of this work was the observation of the effect of normal stiffness on stick-slip. Figure 11 shows a significant decrease in amplitude of stick-slip with an increase in normal stiffness. This change can also be observed in the typical data traces shown in Figures 5 and 6.

Before this change in amplitude can be credited to the change in normal stiffness alone it must be noted that the normal load also increased as much as 20% due to the unavoidable change in preload in going from minimum to maximum normal stiffness. Therefore, if the effect of normal stiffness is to be stated, the effect of increasing normal load must be known.

The literature indicates than an increasing normal load would be expected to cause an increasing amplitude of stick-slip. E. Rabinowicz (24) derived the following relationship for the amplitude of time-controlled stick-slip:

$$\text{amplitude} \propto \frac{N}{K_H V}$$

Thus if the normal load is increased this relationship shows that the amplitude of stick-slip will be increased.

G. Niemann and K. Ehrlenspiel (18) developed the following equation to predict the critical velocity of stick-slip:

$$V_c = \frac{N(\mu_s - \mu_{s1})}{K_H m}$$

From this equation it can be seen that as the normal load increases the maximum velocity at which stick-slip will occur increases. This implies that as the normal load increases the amplitude of the stick-slip increases for any given velocity.

Further study was made concerning normal load in this work and no significant effect of the load was noted. A normal load of 2.19 lb with a normal stiffness of 70 lb/in (K_{N3}) gave a stick-slip force fluctuation of 0.025 lb to 0.038 lb. Thus with an increase in normal load of approximately 60% (1.36 lb to 2.19 lb) over the load originally obtained for K_{N3} the force fluctuation was unchanged.

Thus, based on references in the literature and observations recorded during this study it will be assumed that an increase in normal load (no mass being added) will not cause the amplitude of stick-slip to decrease. With the role of normal load established further discussion must justify the fact that the observed decreasing amplitude of stick-slip was due to an increasing normal stiffness.

The work of D. M. Tolstoi can be shown in support of this work. The observed decrease of stick-slip amplitude was approximately 70% with an increase from 0 to 71 lb/in in normal stiffness. Tolstoi (34) was able to reduce the amplitude of stick-slip 100% by adding normal damping to reduce the normal microvibrations which he observed during sliding. Normal stiffness would also have the same tendency of preventing sliding surfaces from separating and this restraint would become greater with increasing stiffness. Because the surfaces would be restricted in their ability to separate during sliding Tolstoi hypothesized that the possibility of a decreasing coefficient of friction with increasing velocity would be reduced or eliminated because of the following:

1) a greater real area of contact would exist during sliding
2) an increase in depth of indentation, if any
3) an increase in the resistance to sliding due to plastic deformation
4) a decrease of atomic separation in the direction normal to the slip planes of points in contact, thus increasing molecular forces
5) an increase in the area of contactless adhesion

Since stick-slip is dependent on a decreasing coefficient of friction with increasing velocity according to most theories, stick-slip would be reduced as the magnitude of this relationship was reduced.

The reduction of stick-slip due to the normal stiffness can also be considered from an energy standpoint. The rider would start slipping with a certain stored potential energy due to the deflection of the rider support. This potential energy would increase with normal load but the resistance to sliding would proportionally increase and thus an increase in normal load would not affect stick-slip amplitude. However the quicker this potential energy is dissipated by frictional work the shorter the slip. By increasing the normal stiffness the mechanisms enumerated previously would result in a greater rate of energy dissipation if one body is to slide over another. It can thus be seen that increasing normal stiffness would diminish the potential energy quicker and decrease the length of slip. The stick-slip amplitude would be accordingly reduced.

Basically these two theories are the same in that they predict an increase in kinetic friction force due to reduction of normal motion during sliding. Together they theoretically explain the experimental observation that the amplitude of stick-slip can be reduced by increasing normal stiffness.

C. Other Frictional Observations

In order to observe stick-slip several (12-23) passes of the slider had to be made over the same track. The number of passes needed to obtain stick-slip was found to be independent of surface roughness and normal stiffness. This was apparently due to an oxide layer which had to be worn away. Metal to metal contact would then result and stick-slip was obtained. L. F. Coffin (8) confirmed that repeated passes were necessary to obtain a metal to metal friction contact and D. Sinclair (26) used a similar procedure of several passes to obtain stick-slip in his study. Therefore, the need for repeated passes was expected and does not indicate a preculiarity of this experimental system.

Further confirmation that an oxide was initially present can be obtained from work of E. Rabinowicz (21). He stated that the coefficient of friction for initial passes where oxides were present (all metals except the noble metals such as gold) would be in the range 0.1 to 0.3. Initial values for this investigation fell within this range.

The higher coefficient of friction obtained during the final passes can also be related to the initial presence of an oxide film. The shear strength (s) of the exposed metal would be greater than that of the oxide and the penetration hardness (p) of the metal would be no greater than that of the oxide. Thus using the equation

$$\mu = \frac{s}{p}$$

it is evident that the coefficient of friction would be higher after the oxide had been removed by wear.

Another observation concerned the dependence of the coefficient of friction on surface roughness for the first pass over a track regardless of the normal stiffness. The coefficient of friction for the rough surface (300 microinches) was consistently higher than that for the smooth surface (10 microinches). This difference was due to the interlocking of asperities of the rough surface causing an increase in friction force. A very smooth surface could have an increase in friction force due to adhesion but 10 microinches surface finish is not smooth enough to have an effect which will equal the effect of asperities interlocking for the surface finish of 300 microinches. Results of work by E. Rabinowicz (21) with copper and various surface finishes agrees with this explanation of the effect of surface roughness. Rabinowicz also stated that surface roughness should add only approximately 0.05 to the value of the coefficient of friction. This is the case for this work as μ_{int} (S.R. = 10 microinches) = 0.152 and μ_{int} (S.R. = 300 microinches) = 0.234 (a difference of 0.082).

The effect of surface roughness on the average coefficient of friction, as discussed previously, decreased with the number of passes. This is shown by the fact that the coefficient during the final pass (μ_{fin}) varied little from run to run (See Table II). This can be attributed to the wearing of the sharp edges and asperities which caused the interlocking and thus created the higher friction force for the rough surface during the first pass. However, the gross variations in surface roughness were not affected by the wearing. (See Figure 8.)

VI. CONCLUSIONS

1. The amplitude of stick-slip (friction-excited vibrations) for mild steel sliding on mild steel can be reduced by increasing normal stiffness.

2. Surface wear is necessary to obtain stick-slip for mild steel sliding on mild steel with surfaces cleaned only with acetone and in an atmosphere of air. This wear probably involves the removing of oxides.

VII. RECOMMENDATIONS

From this work with normal stiffness and sliding friction several possibilities for further study were noted. These are:

1) With a similar system of mild steel on mild steel establish the curve presented in this work with a larger number of normal stiffnesses.
2) Determine if normal stiffness has an effect on friction-excited vibrations for contacting materials other than those tested.
3) Determine if wear is required to obtain stick-slip with a noble metal (e.g. gold) which does not have an oxide layer.
4) Examine the tracks made by the rider on the sliding surface with a microscope to determine if there is any correlation between surface topography and the reduction in stick-slip amplitude caused by increasing normal stiffness.

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X. APPENDIX A

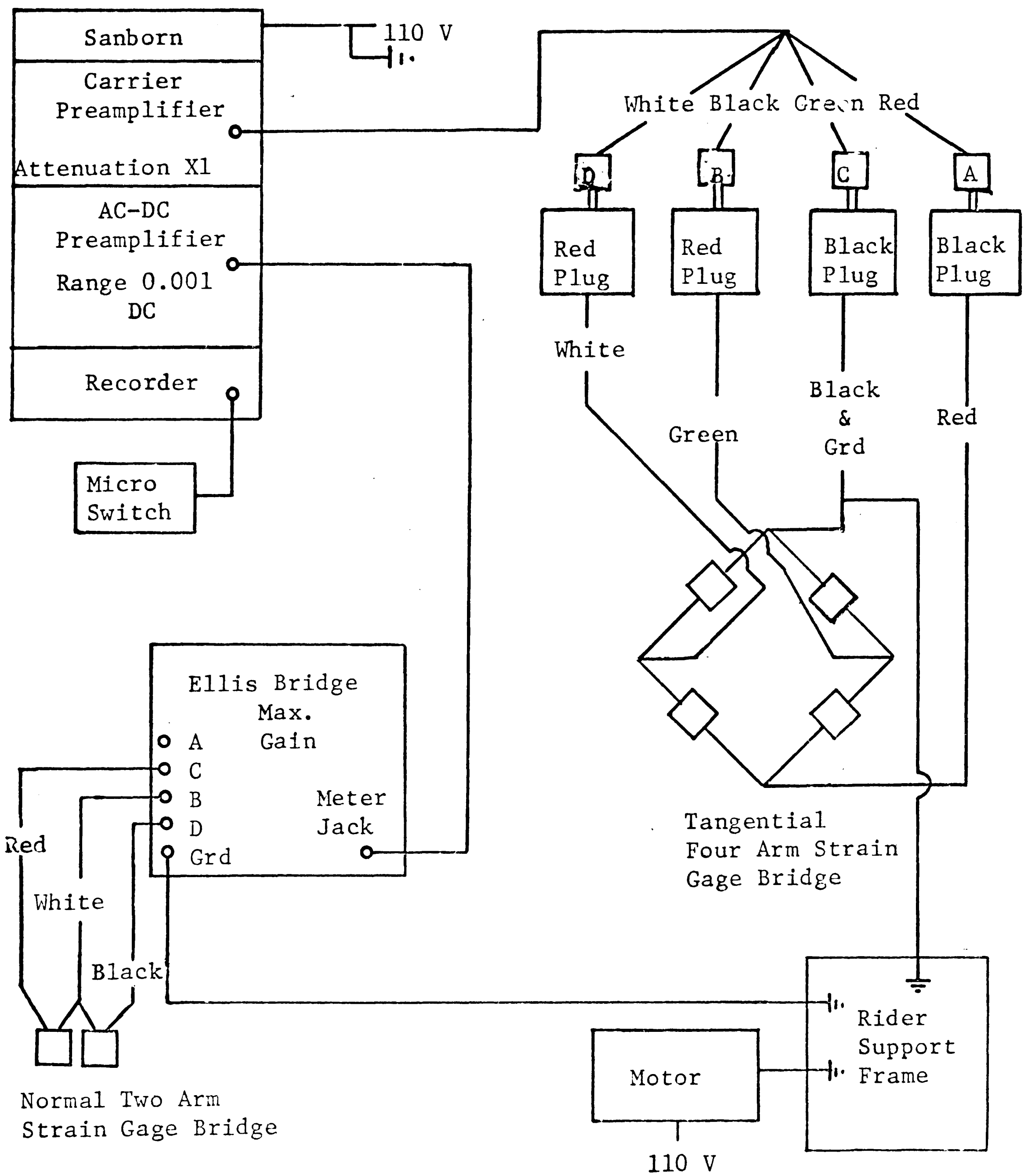
List of Equipment

1. AC-DC Preamplifier, Model 150-1000, Sanborn Co., Waltham, Mass.
2. Bridge Amplifier Meter, Model BAM-1, Ellis Associates, Pelham, N.Y.
3. Carrier Preamplifier, Model 150-1100, Sanborn Co., Waltham, Mass.
4. Dial Indicator, T-2, 0.0001 in/div, range of 0.01 in, Federal Testmaster, Providence, R.I.
5. Dial Indicator, No. 656-441, 0.001 in/div, range of 1.000 in, The L. S. Starrett Co., Athol, Mass.
6. Lathe, Logan Engineering Co., Chicago, Ill.
7. Electrical Switch, BZ-3RW15 type W, Micro Switch, Freeport, R.I.
8. Recorder, Model 154-100B, Sanborn Co., Waltham, Mass.
9. Spring Scale, Model 5605, range 0-72 oz. Ohaus Scale Corporation, Union, N.J.
10. SR-4 Strain Gages, type C-1, 500 ohms, gage factor of 3.56, Baldwin-Lima-Hamilton Corp., Electronics Division, Waltham, Mass.
11. SR-4 Strain Gages, type C-5, 342 ohms, gage factor of 3.49, Baldwin-Lima-Hamilton Corp., Electronics Division, Waltham, Mass.

12. Timer, Model A 211-4, International Register Co., Chicago, Ill.
13. Variable Speed Motor, Model N 30MS2.4 output 0 to 4200 rpm, Trumbull Electric, Plainville, Conn.

XI. APPENDIX B

Wiring Diagram



*Colors describe wires

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A STUDY OF THE EFFECT OF NORMAL STIFFNESS ON KINETIC FRICTION FORCES BETWEEN TWO BODIES IN SLIDING CONTACT

by

John A. Elder, Jr.

ABSTRACT

This work determined what effect stiffness in the direction normal to the plane of contact between surfaces in sliding contact had on the phenomenon of stick-slip.

The results showed that the amplitude of stick-slip vibrations could be reduced by increasing the normal stiffness with contacting surfaces of mild steel. This reduction observed in experimental work could not be predicted analytically.

It was also observed that with the above mentioned surfaces some wear had to take place before stick-slip would occur. This was attributed to the initial presence of oxide on the surfaces.