Facilitating Instructional Change: A Case Study on Diffusion of Curriculum Innovation

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While much research has been conducted on train-the-trainer models for spreading curriculum innovations at the K-12 level, not much is known about how such models play out at the undergraduate level, especially with newer curriculum innovations using student-centered instruction. I present findings from one such project: a case study on the second-generation facilitation of a professional development group focused on supporting instructors teaching with the Inquiry-Oriented Abstract Algebra (Larsen et al., 2013) curriculum materials. These are student-centered instructional materials designed to engage students with developing and formalizing mathematical ideas as members of a classroom community, as opposed to passive lecture-based teaching. I investigate the relationship between the intent of the instructional support model and the facilitator's beliefs and goals for the professional development, using video data collected from a series of online meetings and from the facilitator's classroom in the year prior to his facilitation. Results indicate that the facilitator's orientations and goals around sharing authority and creating supportive learning environments, especially for women participants, both modify and stabilize the intentions of the TIMES project (NSF Awards: #1431595, #1431641, #1431393) as a whole, and the train-the-trainer model as a subsidiary.

Contents

List of Abbreviations

IO Inquiry-Oriented

IOAA Inquiry-Oriented Abstract Algebra

IOI Inquiry-Oriented Instruction

OWG Online Working Group

PD Professional Development

PDF Professional Development Facilitator

TIMES Teaching Inquiry-oriented Mathematics: Establishing Supports

Chapter 1

Introduction

The last few decades have seen increasing calls for reform in undergraduate mathematics instruction due to growing concern among policy makers that outdated and ineffective teaching practices are hindering science and technology progress in the United States (e.g, Center for Science, Mathematics, and Engineering Education, 1999; Udall, 2006). To many educators and researchers, this necessitates a fundamental shift in the undergraduate mathematics education paradigm. In response, many have called for post-secondary mathematics teaching to "provide students with learning experiences that are rich and meaningful: centered on students' ideas, requiring their mental engagement in and out of class, and accountable to their prior understandings" (Laursen & Rasmussen, 2019, p. 129). Advocates for student-centered instruction like inquiry-based learning (IBL; Ernst et al., 2017) and inquiry-oriented instruction (IOI; Kuster et al., 2017) have added to this chorus. Research has shown that such student-centered instruction supports lasting conceptual learning (e.g., Larsen et al., 2013; Kwon et al., 2005;) and improves STEM retention rates (Rasmussen et al., 2013; Hutcheson et al., 2011).

In spite of this expanding body of research demonstrating the widespread benefits of these kinds of pedagogical reforms, student-centered curricula and instructional practices are not widely taken up (Johnson, 2019). Because these reform curricula have failed to spread at scale, there is a growing focus on researching supports for such radical changes in instructional practices (Henderson et al., 2011). The challenges instructors face as they implement curricular innovations are one difficulty that researchers have identified in taking such innovations to scale; for instance, developing an understanding of student thinking, planning for and leading discussions, and leveraging students' contributions to meet mathematical goals are just some difficulties instructors new to student-centered instruction may face (Johnson & Larsen, 2012; Speer & Wagner, 2009). As argued by Andrews-Larson et al. (2019), finding ways to support research-based instructional change may be one of the most critical questions facing education researchers today. In particular, fostering the development of undergraduate mathematics instructors' pedagogical reasoning is of the utmost importance (p. 3). One hopeful solution to these dissemination challenges is Henderson et al.'s (2011) finding that ongoing professional development supports effective instructional change.

One project aimed at supporting instructional change was Teaching Inquiry-Oriented Mathematics: Establishing Supports (TIMES; NSF Awards: #1431595, #1431641, #1431393). The TIMES project's central goal is to investigate what structures are needed to support

undergraduate mathematics instructors as they implement inquiry-oriented curricula, specifically in the context of undergraduate abstract algebra, linear algebra, and differential equations. These inquiry-oriented curricula have been tested and shown to create positive learning outcomes for students (Kwon et al., 2005; Larsen et al., 2013), but the innovations alone are not guaranteed to generate widespread instructional change. Thus, by providing curricular support materials, summer workshops, and ongoing online working groups (OWGs), the TIMES project investigated an instructor support model grounded in research on instructional change and professional development.

In the OWG meetings throughout the semester, participating instructors, called TIMES Fellows, planned and reflected on their instruction in small groups with the support of a facilitator trained in IOI. In the first year of the TIMES project, principal investigators (PIs) served as facilitators. In the second year, TIMES Fellows from the year prior were chosen and trained to facilitate OWGs with new instructors. This was a critical component of the instructional support model; by implementing a train-the-trainer structure for spreading IOI, the project investigated a promising way to bring student-centered curriculum innovations to scale.

In this study, I focused analysis on one former TIMES Fellow serving as an OWG facilitator for instructors implementing Inquiry-Oriented Abstract Algebra (IOAA) in the Fall of 2016. I sought to gain insight into the relationship between the facilitator's beliefs about IOI, his goals for professional development, and his experiences with instruction in the previous year. Broadly, I investigated one instance of the train-the-trainer model for diffusion of innovation in the case of an instructional support model for teachers new to implementing inquiry-oriented instruction. In particular, I aimed to capture the influence of the facilitator's orientations and goals on his facilitating actions within the OWG and ultimately clarify the ways in which those actions modified or stabilized the intent of the professional development.

Chapter 2

Review of Literature

Teachers' implementation of inquiry-oriented instruction (IOI) has been a significant focus of reform-based curriculum research in recent years. The structure and supports provided by professional development programs for undergraduate instructors are of particular interest to those looking to develop curricula with the capacity to spread at scale on the undergraduate level. This study explores diffusion of innovation through the train-the-trainer model in the context of an inquiry-oriented instructional support system. To contextualize this study at the intersection of the principals of IOI and how researchers can design effective supports for instructors new to implementing reform curricula, I explicate the goals of the TIMES project and the particular definition and principals of IOI in regard to equity, authority, and classroom norms. I also review the literature on professional development, especially as it pertains to train-the-trainer models.

This investigation takes place within the context of a broader project, Teaching Inquiry-Oriented Mathematics: Establishing Supports (TIMES; NSF Awards: #1431595, #1431641, #1431393), a research and development project focused on supporting large-scale implementation of several inquiry-oriented mathematics curricula at the undergraduate level, in part through a train-the-trainer model of professional development. These PDs are essential because instructors face many challenges when they begin to teach in student-centered ways. For instance, it is challenging to develop an understanding of student thinking (Johnson & Larsen, 2012) and to build on students' contributions and solutions (Speer & Wagner, 2009; Wagner et al., 2007), which are often communicated in informal and idiosyncratic ways. Instructors also have difficulty planning for and leading discussions about instructional tasks utilizing unfamiliar presentations of mathematical ideas (Rasmussen & Marrongelle, 2006). To investigate best practices in supporting new instructors as they bring IOI to scale, I focus on one of the three instructional supports TIMES provided: an online working group (OWG) supporting instructors' first-time implementation of IOI.

The OWG aimed to provide targeted feedback on specific instructional practices, tools for adapting materials to specific instructional contexts, opportunities to discuss student learning and instruction, as well support instructors in developing pedagogical reasoning practices. The instructors in this OWG were engaged in implementing the inquiry-oriented abstract algebra (IOAA) curriculum materials (Larsen, 2013). These curriculum materials were designed to leverage students' reasoning to develop formal concepts from abstract algebra through cycles of inquiry and formalization using inquiry-oriented instruction.

Inquiry-oriented instruction, or IOI, is a type of student-centered instruction aimed at supporting students as they work individually and collaboratively on carefully designed mathematical tasks (Rasmussen & Kwon, 2007), grounded in the instructional design heuristics of Realistic Mathematics Education (RME; Freudenthal, 1991; Gravemeijer, 1999). Although IOI shares much of its philosophy and goals with other forms of inquiry-based learning, it should be noted that the definition used in TIMES stipulates a strict characterization of IOI, centered on four instructional principles (Kuster et al., 2017): *generating student ways of reasoning, building on student contributions, developing a shared understanding* and *connecting to standard mathematical language and notation* (p. 2). These four instructional components can be elaborated into six local practices of IOI, which are the specific instructional actions that make up the components (Kuster et al., 2019). I focus on the sixth local instructional practice, where teachers guide the development of the mathematical agenda of the class (p.189). The mathematical agenda is not just the collection of mathematical knowledge comprising the goal of an instructional sequence, but includes providing opportunities for students to engage in meaningful mathematical practices like conjecturing, proving, defining, creating and using algorithms, and modeling (Moschkovich, 2002; Rasmussen et al., 2005, Laursen & Rasmussen, 2019), so that the mathematical knowledge developed in the classroom is regarded by students as "their own private knowledge, knowledge for which they themselves are responsible (Gravemeijer & Doorman, 1999, p. 116). To encourage this kind of student ownership of the mathematics, teachers guide students through targeted task sequences aimed at reinventing a mathematical idea over the course of weeks. These guided reinventions require much from instructors, who need not only a deep understanding of the mathematics and curriculum task sequences, but also a deep understanding of students' knowledge and reasoning, as well as adaptive and responsive facilitation skills (Laursen & Rasmussen, 2019). Because these skills are unlikely to develop consistently on their own, active support for instructors new to IOI is needed.

Professional development (PD) can be a successful tactic for supporting these drastic instructional changes. It can grow faculty's knowledge for teaching, which is critical to IOI (Johnson, 2013; Johnson & Larsen, 2012, Speer & Wagner, 2009; Wagner et al., 2007) and aid in sustaining instructional change by creating continual opportunities for development (Henderson et al., 2012). In the research literature, high impact professional development is long-term and ongoing, as opposed to only lasting a few days (e.g., workshops; see Hayward et al., 2015; Henderson et al., 2011, 2012). It is also subject-matter focused; in terms of student-centered mathematics instruction, impactful PD attends to students' mathematical thinking and to faculty's broader instructional goals (Hill, 2007; Kazemi & Hubbard, 2008). Finally, PDs are more effective when they are integrated into instructors' daily routine (Franke, Kazemi, & Battey, 2007; Putnam & Borko, 2000) and provide relevant and specific feedback (Elmore, 2002). A train-the-trainer model is one proposed mode of professional development that could meet these recommendations and be implemented at greater scale.

Train-the-trainer is a promising method for scaling up curriculum innovations. Although little is known about train-the-trainer models at the undergraduate level, applicable lessons

have been learned from research at the K-12 level. Train-the-trainer models can be integral in forming supportive learning communities (Pancucci, 2007), they can develop the pedagogical knowledge essential for both teaching and facilitation (Perry & Boylan, 2018), and are broadly perceived as effective ways to scale up professional development (Taylor et al., 2010; Bolondi et al., 2017; Perry & Boylan, 2018; Hart, 2023). The US Bureau of Education and Research (BER, 2023) regularly conducts National Train-the-Trainer Institutes where staff become skilled facilitators who can guide teachers toward successful practices using video clips, one kind of faculty collaboration-based PD that has been more deeply studied (van Es, 2012; van Es et al., 2014; Barnett, 2002; Andrews-Larson et al., 2019). In studying a train-the-trainer model of professional development, it is critical to attend to the facilitator's role.

The work of a facilitator is demanding; they must be committed to the agenda and goals of the broader program they represent and remain responsive to participating teachers' diverse needs and contexts (Borko et al., 2014), all while setting agendas and norms for the group, supporting teachers' sharing experiences and insights, moderating discussions, providing feedback, and enabling teachers to work towards the group's goals (Schwarts et al., 2022). Little work has been done, however, on the role facilitators play in supporting undergraduate instructors as they implement reform curricula. Existing research suggests that facilitators should attend carefully to how instructors engage with mathematics content as they coordinate the goals of the lesson, students' reasoning, and instructors' own pedagogical decision-making to theorize about possible instructional moves (Andrews-Larson et al. 2019). The literature also emphasizes facilitation that attends to how instructors' beliefs about mathematics may align with or diverge from the intent of the curriculum (Fortune & Keene, 2021). Similarly, Kelley and Johnson (2022) suggest that certain roles taken up by members of OWGs align with the tenants of IOI more than others. In parallel, this study aims to document the ways in which the facilitation of a similar OWG reflected or modified the tenants of inquiry-oriented instruction, a pivotal goal driving the instructional support model.

The instructional support model presented in the TIMES project can be viewed on two levels. On the first, it is an instructional change model aimed at disseminating curricula and pedagogy and developing reflective faculty (Henderson et al., 2011). On the second, due to its status as a train-the-trainer PD, it is a diffusion innovation, where instruction is changed by altering many individual instructors' practices (Borrego & Henderson, 2014, p. 228). Due to the multi-step diffusion process involved in train-the-trainer models, it is necessary to investigate whether the on-the-ground implementation of the innovation abides by the values and goals behind its design, especially because adaptations due to contextual pressures and individual interpretations are likely. That is, researchers must be wary of the broken-telephone effect, where the intent behind the innovation becomes distorted across repeated enactments. Distortions of particular concern are *lethal mutations*, modifications that so radically depart from the values and intentions behind an innovation that the benefits of it can be entirely lost. When implementing new curriculum innovations, mutations

of this kind have been identified in flipped-classroom undergraduate courses in calculus and in engineering (Bagley, 2019; Espino et al., 2023). Examples of mutations – though perhaps less than lethal – to IOI as a curriculum innovation have also arisen in research. Johnson et al. (2013) investigated mathematician's experiences with implementing an IOAA curriculum, finding one instructor in particular whose implementation carried noticeable mutations. This instructor prioritized students' opportunities to engage in mathematical activities like conjecturing and proving over his responsibility to cover curriculum topics, taking the position that moving students toward certain curriculum goals was actually at odds with the process of reinventing concepts (p. 752). Because a local instructional practice key to the principles of IOI includes advancing the mathematical agenda of the class, this can be considered a mutation of the innovation that occurred when taking IOAA curriculum to scale. Researchers, as well as facilitators, must be aware of the potential for departures like these, especially in disseminating a model of professional development that relies on generational transfer between trained-trainers and new faculty. In the case of TIMES's instructional support model, facilitators were responsible for upholding the instructional design principles of RME and the tenants of IOI while attending to the the treatment of productive classroom norms and authority dynamics. The extent to which their facilitation modified or stabilized this intent is of particular concern.

Because IOI emphasizes classroom social interactions as grounds for the reinvention of mathematical concepts, it is crucial for both facilitators and teachers to attend to the negotiation of norms that foster meaningful learning opportunities (Gravemeijer, 2020). Classroom norms were conceptualized by Cobb and Yackel (1996) as the behavioral expectations that "characterize regularities in communal or collective classroom activity" (p. 178). These norms are developed collaboratively between the instructor and students as they interact in the classroom; for example, it is often normative for students to raise their hands and wait to be called on to speak. Sociomathematical norms are of particular interest to researchers studying IOI; these are the norms specific to mathematical activities, and may include expectations for what counts as an acceptable solution or explanation, as well as criteria for different or more sophisticated solutions (Hershkowitz & Schwarz, 1999). Recent investigations in IO classrooms have shown that social and sociomathematical norms can empower students to create, explain, and justify mathematical ideas and can provide opportunities for students to develop productive beliefs about the nature of mathematics and their mathematical identities (Laursen & Rasmussen, 2019). Similarly, normative behaviors for participation within teacher professional development groups often arise. In addition to sociomathematical norms (Dean, 2005; Tatsis & Koleza, 2008; Elliott et al., 2009), teachers also develop norms specific to discussions about teaching mathematics, called *sociopedagogical* norms (Dick et al., 2018). Thus, a facilitator's task is two-fold: they must support the development of productive sociopedagogical norms for discussions within PD and lead discussions which support instructors as they foster productive sociomathematical norms in their own classrooms.

Because fostering students' mathematical autonomy is one central goal of inquiry-oriented

instruction, facilitators must attend to supporting teachers as they adjust their behaviors and roles in the classroom to share authority with students. To create more opportunities for students to autonomously build and justify their own mathematical ideas, as well as develop more productive attitudes and mathematical identities, instructors must move away from a classroom authority tradition based on their own power and mathematical expertise (Langer-Osuna, 2017; Wilson & Lloyd, 2000; Amit & Fried, 2005). Sharing authority among teachers and students is a mutual goal of inquiry pedagogies (Gerson & Bateman, 2010; Hicks et al., 2021), but perspectives differ on what precisely constitutes shared authority. In the most hierarchical sense, it requires a relinquishing of authority from teacher to students (e.g., Bleiler-Baxter et al., 2023; Fried, 2020), or a minimizing of the teacher's authority to create and assess mathematical ideas (e.g., Gerson & Bateman, 2010). More progressive conceptions focus not on the teacher's abdication of authority but rather on its effective wielding to promote students' authority (e.g., Bishop et al., 2022; Oyler, 1996). In IOI, teachers are tasked with authoritative roles like guiding discussion and leveraging students' ideas to advance the mathematical agenda of the class while simultaneously bolstering students' authority to create and justify mathematical ideas. Facilitators are key agents in supporting instructors as they adjust their teaching practices to share authority with students in this way.

Further, this shift in authority relations creates a possibility for peer interactions to foster gender inequities in group work and whole class discussions (Serbin et al., 2020). While it has been suggested that inquiry pedagogies like IOI could make math classes more equitable across genders (Laursen et al., 2014), more recent studies demonstrate both inequitable participation and inequitable learning outcomes for women and men students (Smith et al., 2019; Johnson et al., 2020, Reinholtz et al., 2022). This is a pressing issue for those designing professional development and instructional support due to the unique opportunities such settings provide to aid teachers in explicitly addressing gender inequity in their classrooms. Further, equitable instructional practices are considered to be a pillar of inquiry-based mathematics education (IBME), of which IOI is characterized as a subset (Laursen & Rasmussen, 2019). Few studies have explicitly identified instructional practices that support gender equity in inquiry pedagogies, but the results are promising. Instructors can create more opportunities for women students to contribute in mathematically sophisticated ways by explicitly attending to the frequency and quality of contributions (Hicks et al., 2021), and they can foster norms that encourage productive beliefs about students' mathematical abilities and the nature of mathematics (Serbin et al., 2020). These cases are compelling evidence for the influence that instructors can have on the success of women students when they focus on creating equitable opportunities for participation. PD facilitation that supports instructors in developing these practices is then in line with the intentions of the instructional support model.

The existing literature contextualizes this study at the intersection of the principles of IOI and the design of effective supports for instructors implementing reform curricula. The facilitator's treatment of authority dynamics as they relate to community norms and gender

equity is essential for investigating this professional development as a diffusion innovation. To capture how the facilitation modified or stabilized the intent of the TIMES instructional support model, I explicate the precise lens through which I analyzed the facilitator's actions in the next chapter.

Chapter 3

Theoretical Framework

Because both facilitators and teachers are tasked with guiding others' learning, there exist commonalities in their practices that can be leveraged to study PD facilitators. To conceptualize professional development facilitator's motivations and practices, I lift frameworks for understanding teachers' decision-making practices and for analyzing classroom authority dynamics to the professional development context. First, I adapt Schoenfeld's (2010) resources, orientations, and goals (ROGs) framework. I employ three constructs:

1. **resources** (R): the intellectual, material, contextual, and social means upon which a person draws;
2. **orientations** (O): a person's beliefs, values, preferences, and attitudes;
3. **goals** (G): purposes a person consciously or unconsciously sets out to achieve.

This creates the foundation for my analysis of the facilitator's ROGs in context of the pillars of inquiry pedagogies (see Laursen & Rasmussen, 2019). Because a broader aim of the pillars of IOI is fostering students' mathematical autonomy, there is an expected shift away from a classroom authority tradition based on the expertise and power of the instructor and toward a new authority structure where students are empowered to create and justify their own mathematical ideas. That is, authority in IO classrooms must be shared between teachers and students to fulfill the goals of IOI. While these shifting authority relations have finally begun to be studied in the undergraduate mathematics settings (e.g., Gerson & Bateman, 2010; Hicks et al., 2021), they remain unexplored in the case of undergraduate instructor professional development. Due to the specific definition of IOI in this study, I adjust a framework used to capture authority relations in an inquiry-based calculus classroom to better describe the dynamics of inquiry-oriented instruction, lifting key constructs to the facilitator level to capture authority relations in professional development.

Authority can be defined more generally as the ability to influence the beliefs or behaviors of others. It is commonly held in the literature that different types of authority are resultant from different claims to legitimacy made by the authority bearer (Amit & Fried, 2005). In a typical classroom, a mathematics instructor may bear authority based on their position within the university or on their knowledge as an expert in mathematics. Taking more specific direction from Gerson and Bateman (2010) and Bishop et al., (2021), I view authority as an enacted relationship between individuals where "a receiver recognizes the merit of a

Table 3.1: Authority types and subtypes with brief definitions

Authority type	Sub-type	Legitimized by . . .
Hierarchical	Institutional	position as instructor or facilitator
	Granted	permission from instructor or facilitator
Expertise	Ownership expertise	one's creation of an idea or solution
	Mathematics expertise	the bearer's mathematical expertise
	Pedagogical expertise	the bearer's pedagogical expertise
Mathematical	Mathematics community	shared norms, ideas, and practices
	Justification	mathematical reasoning

claim given by a bearer and is influenced to change in some way" (p. 198), noting that the roles of bearer and receiver fluctuate across contexts and activities. Thus, authority is shared when facilitators and participants, as well as instructors and students, engage in activities like conjecturing, justifying, representing, evaluating, and qualifying pedagogical and mathematical ideas. An authority relation is indicated by changes in behavior regarding what is said, what is written, and how people are thinking about the subject, be it mathematics or pedagogy. I adapt constructs from Gerson and Bateman's (2010) hierarchical, expertise, and mathematical authorities in my analysis, summarized in Table 3.1.

Hierarchical authority is legitimized by a person's position in the community, and is most often borne by instructors when they direct the behavior of the class through verbal or written instructions. In parallel, a facilitator directing the behavior of participants in professional development would also bear hierarchical authority. This authority may be received by participants when they follow directions, and by students when they request permission to address the class verbally or by writing on the board. Two sub-types of hierarchical authority are institutional authority, which is most often held by the instructor of the class or the facilitator of the PD based on their position within an institution, and granted authority, which is held by participants who have been invited to bear authority by either the instructor or the facilitator. The decision to grant participants and students authority is crucial to a dynamic shift toward shared authority, as is desirable in IOI.

Expertise authority can be held by both students and instructors (Amit & Fried, 2005; Gerson & Bateman, 2010), and its legitimacy is based on the perception that the bearer has expertise, due to their education or credentials (Oliveira et al., 2007; Benne, 1970), or due to their exhibited ability or knowledge. Two sub-types of expertise authority were identified by Gerson and Bateman (2010): ownership expertise authority and mathematics expertise authority. A person bears ownership expertise authority over a solution or idea because they

created or helped to create it. A person bears mathematics expertise authority when their claims are legitimized due to their demonstrated mathematics expertise (p. 204). Similar to mathematics expertise authority, I employ the construct of pedagogical expertise authority, where a person's claim to authority when they talk about instruction is legitimized by their expertise as a teaching professional.

Specific to mathematics contexts, mathematical authority has been identified as the authority to "determine appropriate procedures and methods to solve and verify problems and to make sense of situations, issues, and questions" (Wilson & Lloyd, 2000, p. 148). Mathematical authority differs subtly from mathematics expertise authority. For instance, students might accept claims made by their instructor without understanding the mathematical reasoning behind them, hence receiving the authority on the basis of the instructor's perceived expertise, rather than on the basis of the mathematical argument made. Additionally, the authority of the mathematics community, sometimes called the authority of the discipline, has been identified as a relation legitimized by the norms and shared practices of the mathematical community (Amit & Fried, 2005; Boaler, 2003). For example, the norm that a mathematical claim demands a mathematical proof could legitimize a person's mathematics community authority when they request a proof for a claim. By attending to areas of expertise in this way, I appeal to Benne's (1970) notion of *anthropogogical authority*, where instructors carefully leverage authority in order to enculturate students in mathematics, so that students are empowered to bear mathematical authorities themselves.

These more democratic conceptions where students and teachers share types of authority across contexts dove-tail with the goals of inquiry-oriented instruction. That is, implied within IOI is the notion that students must bear mathematical authority to create and justify their ideas, as well as the notion that instructors must carefully leverage authority to create rigorous mathematical learning and identity building. Because instructors bear the novel responsibility of fostering shared authority dynamics in the interest of such collective learning, it is vital to attend to authority in the professional development context. My analysis of one PD facilitator's orientations and goals around authority aims to explicate the relationship between the intent of the professional development within TIMES and its enactment within the OWG.

Chapter 4

Context and Methods

To investigate best practices in bringing IOI to scale, TIMES utilized three forms of instructional supports: curricular support materials, summer workshops, and online working groups. The curricular support materials provided additional online resources to teachers. The summer workshops provided a three-day intensive session where participants became acquainted with the instructional tasks and curricular resources. Finally, the online working groups met for one hour each week during the semester following the summer workshop to work on instructional practices and discuss implementation issues. My primary focus is the facilitation of an online working group (OWG) of three instructors all new to IOI that met for 14 weeks in the fall semester of 2016.

The OWG consisted of two instructors currently implementing the IOAA curriculum materials, Elena and Laura, one instructor preparing to teach in the following academic quarter, Roger, and one facilitator with previous experience teaching IOAA as a TIMES fellow in the previous year, Mickey. Elena, Laura, and Roger had experience teaching abstract algebra, but were new to the IOAA curriculum materials. While Roger was unable to teach the course synchronously with Elena and Laura – his course was held during the winter quarter after the end of the fall semester – the OWG continued to meet with Roger after their classes concluded to support his instruction during the winter quarter. This was beyond the official 14-week commitment they made to the TIMES project, so no data was collected from these meetings.

The instructors were introduced to the curriculum materials in the summer workshop and met synchronously online via Google Hangouts once per week to reflect on the curriculum and implementation with the facilitator. Members of the OWG were regularly asked to share reflections on their instruction and investigate the curriculum tasks from a mathematical perspective. These meetings were recorded for retrospective analysis. Additionally, the facilitator met with one of the TIMES project's principal investigators (PI) each week in a series of Debrief meetings, wherein the primary topics of discussion were goals for facilitation, beliefs about teaching IOAA, and updates on the OWG's proceedings. These Debrief meetings were also recorded online via Google Hangouts.

In order to identify Mickey's orientations and goals for facilitation, three sets of video data were analyzed: the OWG meetings, the Debrief meetings, and Mickey's teaching with the IOAA curriculum materials in the previous year. This analysis had four main phases. In the first phase, I began distilling the data by creating analytic notes from meetings $2 - 13$

of the OWG. An error in data collection caused meeting 1 to be recorded without audio. Similarly, meeting 14 was only partially recorded due to technical difficulties. The analytic notes log was organized into five columns: Video File Name, Timestamp, Quote, Speaker(s), and Notes. A new row was added to the log when I identified an instance where Mickey was talking about or facilitating discussions around implementing IOI. Of particular interest were conversations focused on or aligned with the instructional principles and practices of IOI, as well as the equitable instructional practices documented in the literature (see Chapter 2). In general, these analytic memos focused on how Mickey was talking about and facilitating discussions around implementing IOI.

In addition, instances indicating norms that encourage productive beliefs about mathematical abilities and the nature of doing mathematics previously documented in the literature, as well as discussions around creating opportunities for meaningful participation, were transcribed with analytic notes. Additional notes were made describing interactions and discussions among the whole group that suggested the development of these norms. Taking direction from Clark et al.'s (2008) conditions for documenting the development of norms, it was inferred that a behavior was becoming normative to the OWG if it was commonly enacted without prompting, participants did not challenge someone enacting that behavior, and/or participants challenged someone when they did not comply with that behavior. This led to 107 instances transcribed in the first round of analytic note-taking.

In the second phase, I drew on data from the analysis of the OWG meetings, creating an analytic notes log of the Debrief meetings with identical structure to the log in phase one, but more strongly focused on the subject-matter of instances that had been noted in the OWG. For example, Mickey's habit of asking open-ended questions to prompt discussion arose repeatedly in the OWG analytic note log, so instances where he discussed this facilitating strategy in the Debrief meetings were of particular interest. Instances of Mickey attending to gender equity in facilitation were also emergent in this part of analysis. In total, this process led to 32 instances transcribed with analytic notes from the Debrief meetings.

In the third phase of analysis, I drew on data from the analytic notes logs in phases one and two to build descriptive themes of Mickey's prevailing orientations and goals. This iterative thematic analysis (Braun & Clarke, 2021) attended to commonalities between the content of the analytic notes logs. Early commonalities arising from this included facilitating moves like encouraging reflective teaching practices, asking open-ended questions, discussing equitable practices for both teaching and facilitating, and avoiding reliance on Mickey's expertise. In addition, establishing productive norms for both the OWG and the participants' classrooms emerged as a focus in both data sets. Table 4.1 provides a summary of prevailing themes and the associated facilitating habits documented in the OWG and Debrief meetings.

Table 4.1: Summary of prevailing themes.

Prevailing Orientation/Goal	Mickey's Habits	Prototypical quotes	No. of instances
Orientation toward Sharing Authority	Avoids positioning himself as the expert	"I don't consider myself an expert by any means" "If you're looking to me to be the expert in the room, I don't have a clue."	13
	Discusses authority in the classroom	"What we're seeing is authority - or power- shift. Where is the authority lying?" "If you're trying to do [inquiry-oriented instruction], you know, there's a power shift toward the student."	11
	Asks group to discuss, rather than giving advice	"I have no idea how to handle this . . . I want to open it up to everyone in the room." "Your input counts more than mine."	9

Prevailing Orientation/Goal	Mickey's Habits	Prototypical quotes	No. of instances
Creating a Supportive Environment	Asks open-ended questions/avoids "right answer" thinking	"Tell me more about your thought-process here." "I'll leave it to you all to discuss. Remember: there's no right answer here."	19
	Encourages reflection on pedagogy & mathematics	"Now that you can reflect on it, what went through your mind at that moment when you were handling that situation?" "Roger, can you expand on that [idea about cancellation]?" "Okay, suppose that comes up, how would you all handle that? What teaching moves – what pedagogical moves – would you use for that?"	14
	Co-creates norms around sharing difficulties & showing support	"Well thank you, Laura, for sharing that [challenge]." LAURA: "I'm just struggling with the pacing . . . I feel like I'm just way behind where I want them to be." MICKEY: "You'll get 'em there!"	11

Prevailing Orientation/Goal	Mickey's Habits	Prototypical quotes	No. of instances
Supporting Women's Participation	Leverages Women's Contributions	"Can you say that again? We're going to write it up here on the board." ELENA: How do you get the students to correct their own errors?" MICKEY: "Great question! What do you all say to that?"	18
	Discusses Women's experiences	"So, Elena, what about you? How do you deal with assessment in this class?" "So I think Laura's trying to figure out her place in this whole thing."	6
	Explicitly attends to gender equity	"I want it to be organic, like, I don't want to go 'You're female, please participate', you know?"	3

The following themes arose during this analysis which can be viewed either as orientations or goals: sharing authority and creating a supportive learning environment, especially for women to participate. I took two components as evidence of a supportive learning environment: (1) explicit descriptions from participants about their experiences and (2) norms (both social, sociomathematical, and sociopedagogical) that have been previously shown to support learning (Serbin et al., 2020; Mitchell, 2022, in press). In documenting Mickey's orientation toward sharing authority, I applied my theoretical authority framing to each instance, attending to a) who was bearing authority to participate and b) what their claim to legitimacy was (hierarchical, expertise, mathematical, and sub-types). Of the 139 instances documented in phase 1 of analysis, 35 were not included in the final phase of analysis due to their lack of cohesion with both the prevailing themes and each other. These instances neither formed a recognizable pattern with, nor contradicted, the instances in the three prevailing themes.

In the fourth phase of analysis, I looked to recordings of Mickey's classroom implementation of IOI [1]. The classroom data came from an upper-division undergraduate abstract algebra course at a large doctoral-granting institution in the Midwest US, taught the year prior to Mickey's facilitation. Specifically, videos of whole-class activities from the units on quotient groups and group isomorphism were analyzed, looking for confirming or conflicting evidence of Mickey's orientations and goals for IOI from earlier analysis. These three classes were

[1]The phenomena identified in the classroom are not the central focus of this study; they are a secondary data set used to provide context and evidence for Mickey's instructional experiences in order to investigate their relationship to Mickey's interactions with the instructional supports as a trained-trainer facilitator, which is consistent with the aim of TIMES.

Table 4.2: Summary of confirming classroom instances.

Prevailing Orientation/Goal	Mickey's Classroom Habits	No. of instances
Sharing Authority	Invites students to evaluate contributions, rather than Mickey	4
	Minimizes authority of correct answers	6
Creating a Supportive Environment	Publicly honors student contributions	8
	Fosters norms around productive failure	5
	Elicits in-progress thinking over finished solutions	3
Supporting Women's Participation	Publicly values women students' contributions	4
	Intentionally solicits women students' participation	5

chosen due to their proportion of whole-class discussion to small group work (other classes largely consisted of the latter, which did not focus on Mickey's teaching). In addition, they all occurred in the middle or late term of the semester, so I assumed that classroom norms were firmly established by then.

In identifying confirming and conflicting instances of Mickey's orientations and goals for IOI, I kept in mind that many habits were context-dependent and would likely arise differently due to differences in experience level and age between Mickey and his students. For instance, the OWG norm of sharing difficulties and showing support was dependent on a shared experience among participants as equal colleagues, and any habit that arose analogously in the classroom would be grounded in a context dependent on a teacher-student relationship. In the case of the above example, Mickey co-created and reinforced a norm of honoring productive failures, which was dependent on his position as the instructor, but clearly habituated his goal of creating a supportive learning environment. Similarly, I transcribed instances when Mickey avoided positioning himself as the expert and asked open-ended questions in the classroom, and looked for conflicting instances where Mickey might leverage his own expertise or ask closed-ended questions. A summary of confirming instances is provided in Table 4.2.

Chapter 5

Results

Within the online working group, the debrief meetings, and the classroom, Mickey's actions suggest an over-arching goal of supporting participants and students as they build subject-specific autonomy. In the OWG, participants are supported in building pedagogical autonomy, and in the classroom, students are supported in building mathematical autonomy. Mickey's supportive facilitating and teaching practices are mediated by his orientations toward sharing authority and his goal of creating a supportive learning environment, especially for women.

5.1 Orientations toward Authority

Mickey's decision-making practices appear to be influenced by an aim to shift traditional authority relationships in both the online working group and the classroom context. Mickey first demonstrates this intention in an explicit discussion of classroom authority in the early weeks of the OWG meetings. In the following OWG episode, Elena had been reflecting on her pedagogical choice to give her students the definition of bijection when they could not recall it independently.

> ELENA: I didn't feel like that was something they could just "inquiry out" – it wouldn't just happen.
>
> MICKEY: Sure, but the thing is is that, holistically, are you doing that more often than not? Because . . . What you're seeing is authority- or power-shift.
>
> ELENA: Right.
>
> MICKEY: Right. Where is the authority lying? Is it lying with you? Well then, they're only going to do things that satisfy you, instead of doing things that satisfy themselves, you know, for them to learn.

Online Working Group, Excerpt O-1

This episode suggests a belief that shifting authority to be shared between teacher and students promotes learning. Mickey's last statement implies that if authority lies with the

instructor, then students' engagement will not result in learning. For Mickey, students must take up authority in order to learn. By explicitly motivating participants to attend to these authority dynamics within their own classrooms, Mickey demonstrates his orientation toward authority. This is best characterized as a belief or value in shifting authority structures so that participants, be they students in a classroom or instructors in professional development, rely on their own mathematical or pedagogical expertise to reason about their quandaries, rather than turning to Mickey for his. This practice of shifting authority arises in both the online working group and the classroom when Mickey shares expertise authority with participants while carefully leveraging his authority to advance the agendas in both contexts.

5.1.1 Expertise Authority

Throughout his teaching and his facilitation, Mickey rejects the notion that he is an expert. This refusal to bear expertise authority was displayed in the OWG when Mickey directly discussed his lack of expertise and consistently elicited participants' contributions before making his own, thus sharing pedagogical expertise and ownership expertise authorities among participants. This practice is reflective of an orientation that shared authority supports the development of pedagogical reasoning, a key goal of the instructional support model (see Chapter 2, page 3). This attitude toward expertise authority is made more apparent in discussions from the debrief meetings and episodes from Mickey's teaching.

In the online working group, Mickey displayed his orientation toward sharing authority by refusing to position himself as the expert in the room, consistently eliciting participant thinking *first* before sharing his own ideas, and sometimes refusing to give advice at all. Often, when a participant asked Mickey a direct question, he would redirect it to the entire group with phrases like, "So that [question] is for everybody, I can answer later," and "I think this [topic] merits at least a three-to-five-minute discussion. I'll add a disclaimer that I have no idea how to handle that situation." In a quintessential episode, Elena has asked Mickey how he would handle the situation of having an exceptionally bright student eager to share all of the answers:

> MICKEY: If you're looking at me to be the expert in this situation, I don't have a clue. I have conjectures and thoughts about this, but I want to open it up to everybody in the room.

Online Working Group, Excerpt O-2

Mickey directly states that the participants should not view him as an expert. Furthermore, by choosing to give the opportunity to answer Elena's question to the whole group, Mickey receives the pedagogical expertise authority of the remaining participants. This is a clear invitation for participants to bear pedagogical expertise authority to reason about

Elena's quandary, as well as ownership expertise authority to create and discuss possible solutions. The belief that this practice of sharing expertise authority is essential for the OWG participants' learning motivates Mickey's decision-making as a facilitator.

Mickey's orientation toward avoiding expertise authority is also apparent in discussions from the Debrief meetings, where he overtly describes his relationship to his own expertise. An instance of dialogue that captures this is:

> MICKEY: It's hard even with them sometimes, because when I ask a question, they're seeking "Is this the answer you wanted?" and like, a colleague of mine introduced me the other day as a pedagogical expert and . . . I am so not that.

Debrief Meetings, Excerpt D-1

Even more explicitly, he says,

> MICKEY: I don't consider myself an expert in inquiry-oriented [sic] by any means.

Debrief Meetings, Excerpt D-2

Rather, than viewing himself as an expert in IOI, Mickey is aware that his position as facilitator garners him institutional authority, but continually avoids bearing that authority, often through sharing pedagogical and ownership expertise authorities. Mickey indicates that he prefers for his instructors to lead discussions and generate ideas. This attitude is reflected in his debrief conversations when he says, "I know I usurp the conversation every once in a while, but I want others to give their approaches, because they must be different than mine." Mickey's usage of the word "usurp" in particular suggests that, in taking long turns of talk to share his approach to a pedagogical challenge, he is somehow encroaching on the participant's opportunities to make contributions of their own. This is indicative of Mickey's orientation toward the importance of sharing pedagogical and mathematical expertise authorities in professional development.

Likewise, in discussing his own views on assessment during a debrief meeting in Week 8, Mickey mentioned that his OWG seems to struggle consistently with student learning assessments, trying to lean on his experience for guidance. He said, "This keeps coming up over and over and over again, and I keep asking them, 'What are the goals of your course?' For me, your assessment should match your goals." Mickey shared his personal teaching philosophy but rejected any positioning of himself as the authority on what kinds of assessments were best. He encouraged his instructors to create their own standards for assessment, rather than to look for the "correct way" to assess their students based on his expertise. This clearly demonstrates Mickey's goal of sharing expertise authorities. His comments about answering

participants' questions based on his expertise authority emphasize his intention to shift traditional authority structures within his facilitation to create dynamics of shared authority. In general, Mickey's orientations toward authority are evidenced by his facilitating actions, which can be characterized by his explicit discussion of classroom authority dynamics and his repeated refusal to bear expertise authorities, instead sharing among the instructors in the OWG.

Mickey's teaching actions provide context for these facilitation practices and the authority beliefs that inform them. Because of the contextual differences between the classroom and the online working group, it is no surprise that this orientation arose differently during Mickey's teaching in the year prior. In the classroom, Mickey's orientation toward traditional authority dynamics can be characterized by his intentional and particular distribution of opportunities to bear expertise authorities among his students through a process-over-product value selection of contributions and an invitation for students to share in the mathematical authority to evaluate each others' contributions.

In mathematics classrooms, it is often the case that students with correct solutions most consistently bear granted authority to contribute to discussion. Mickey subverts this authority tradition by minimizing the contributions of students with correct solutions and granting authority to students with partial or in-progress solutions to contribute their thinking. In the classroom, this careful distribution of opportunities to bear authority arises when Mickey leverages his institutional authority to prompt students to share their thinking, especially when he asks students to share what he calls "productive failures". One explanation of productive failures that Mickey gives in class is:

> MICKEY: Try stuff. In Algebra, you're going to cook a lot of equations up. Some of them may be fruitful, some of them may not be. The biggest thing, though, is to explore. Because, some help, some don't. Learning from your mistakes, having that productivity when you're failing, is a huge part of this.
>
> Mickey's IOAA Class, Excerpt C-1

This explanation also serves as evidence that Mickey is publicly honoring productive failure, which will be discussed further in Section 5.2.2. In class, Mickey regularly grants students the authority to share these productive failures, and it should be noted that students are expected to discuss these processes as part of their course grade.

By granting students authority to contribute their in-progress mathematical ideas, Mickey subverts the tradition that only correct solutions can legitimize a students' mathematics expertise authority, shifting toward a dynamic where students are enculturated into the mathematical community through opportunities to share their informal, in-progress reasoning. In an exemplifying episode, the class is attempting to prove a conjecture about a generating set for a symmetry group. The student who made the conjecture begins to share his completed

proof, and Mickey interrupts him to call on a student who is still working on the conjecture, saying, "Hold on, hold on, let's see, Madison, can you help me out with this?" Mickey has not received the mathematical authority the student with the correct solution is attempting to bear, and instead grants authority to Madison to share her reasoning about the conjecture so far. In inviting her to bear granted authority, Mickey creates the opportunity for Madison to bear both ownership expertise and mathematics expertise authorities. This is supportive of Madison's, and arguably many other students', mathematical autonomy because it values the process of creating a proof over the final product, which helps students' to develop productive beliefs about their position in the mathematical community (see Serbin et al., 2020, p. 4). This support is also created by centering Madison's in-progress thinking, which aids other students in gaining insight into their classmate's reasoning.

In another episode, a student has been granted authority to share their solution to the third isomorphism task (Larsen et al., 2013) with the class on the document camera. Mickey has realized that the student has a correct solution, and motions for the student to sit down before they're finished sharing. He says, "Before we get to the boards, because some people aren't there yet, let's go ahead and do proofs of task 1 and task 2, who would like to volunteer ?" By stopping the student who has the correct answer from sharing first, Mickey is not receiving the mathematical authority of students who have the correct solution. This de-emphasis also allows students who have not solved the task to continue working. After a discussion on tasks 1 and 2, Mickey prompts the whole class to present their work on task 3 by saying, "Alright, task 3, if you'll put those on the boards." By granting every student in the class the authority to share their work on task 3, Mickey publicly honors all students' mathematics, not just the student who arrived at a correct solution first. By subverting the expectation that students with correct solutions should be granted authority, Mickey supports his students' mathematical autonomy, thus leveraging his authority to encourage students to bear authority themselves.

Mickey also subverts the tradition that the instructor bears the institutional and mathematics expertise authorities to evaluate students' contributions by inviting students to bear authority to evaluate each others' ideas. Following the episode above where all students have been prompted to share work for task 3 on the whiteboard, Mickey asks, "So, what do we think? And remember, we're in a non-judgemental phase in our lives right now, so keep your comments very productive, okay?" By empowering his students' to evaluate each others' contributions through constructive feedback, Mickey minimizes his mathematical and expertise authorities, creating opportunities for students to bear mathematical authority to evaluate each others' work and ownership expertise authority to respond to evaluations of their own ideas. This supports their mathematical autonomy by engaging students in reasoning about the contributed solutions and giving them ownership of mathematical ideas.

Mickey's orientations toward subverting expected authority dynamics to create opportunities to share authority have parallels across contexts. His refusal to bear pedagogical expertise authority in the online working group parallels his refusal to bear mathematical expertise authority to evaluate students' contributions in the classroom, instead inviting students to

evaluate each others' contributions. Similarly, inviting OWG participants to share their thinking before sharing his own subverts authority in much the same way as his choice to honor students' partial solutions over completed ones; in both instances, Mickey invites participants to share expertise authorities specific to the learning environment. The practice of sharing authority in both settings demonstrates Mickey's intent to subvert traditional authority dynamics. This practice arises as an artifact of Mickey's belief that sharing authority supports the development of subject-specific autonomy in participants.

5.1.2 Authority to Advance the Agenda

Although Mickey refused to bear the expertise authority his participants often requested of him, there was tension between Mickey's authority values and his responsibility to advance the agenda of the OWG. Advancing the agenda required Mickey to bear institutional authority both as the facilitator in the OWG and as the instructor in the classroom.

Mickey often prompted participants and guided group discussion to meet the goals of the OWG, bearing institutional authority in order to advance the agenda of the OWG (see Chapter 2, p.3). This is evidenced on the individual level when Mickey directly asks participants for their thoughts on the mathematics, as seen in the following exchange between Mickey and Roger during a group discussion of Task 4.

> MICKEY: Turning toward the content in Lesson 4, why is it, mathematically, that in every group Cayley table, each symmetry appears exactly once in each row and each column?
>
> ROGER: It has to do with cancellation?
>
> MICKEY: Can you expand on that? Pardon the pun.
>
> Online Working Group, Excerpt O-3

When Mickey bears his authority as the facilitator to prompt Roger to share his thinking, he advances the professional development agenda by guiding participants to engage with the curriculum materials from a mathematical perspective.

He also directly prompts the participants to reflect aloud on their teaching experiences. After Laura had discussed a difficulty she was having with using one of her student's contributions to formalize a mathematical idea, Mickey prompted her to dig deeper, saying "Now that you can reflect on it, what went through your mind at that moment when you were handling the situation?" and later directing the remaining participants to discuss potential pedagogical moves to apply to Laura's quandary. By exercising his institutional authority as the facilitator in this way, Mickey advances the agenda of the OWG by inviting participants to engage in pedagogical reasoning.

So, while Mickey shares authority often by refusing to be the bearer of expertise authority in the Online Working Group, he is willing to bear the kind of institutional authority associated with the facilitator's responsibility to advance the agenda to meet the OWG's goals. Although these actions do not depart drastically from hierarchical authority relations, they are consistent with a motivation to support participants' pedagogical reasoning and autonomy, which is in line with both Mickey's goals and the intent of the instructional support model.

Similarly, Mickey must bear institutional authority in the classroom to direct students' activity and advance the mathematical agenda of the course. He does this through providing tasks and granting students authority to share their solutions, as is typical of instructors who bear hierarchical authority. In the following episode, the class is investigating a conjecture made by another student that a particular cycle generates a subgroup of S_3, the symmetric group on a set of three elements. Here, the elements are permutations written in cycle notation, and the identity element is represented with the symbol e. A previous student has conjectured that the permutation (123) generates a subgroup of S_3, and Mason is explaining why his classmate is correct.

> MICKEY: So (123) squared is equal to (132).
>
> MASON: And then (123) cubed goes to e because the inverse of (123), and so you just have . . . yeah.
>
> MICKEY: Sorry, can you say that again?
>
> MASON: So (132) is the inverse of (123) in this notation, and so if we have this (123) squared we know it goes to that, so if we add another one, it's gonna go to the identity.
>
> MICKEY: Aaaahhh, okay. By the way, can you generalize that to make a conjecture about the inverse of any cycle?
>
> MASON: Uuhh, well, its the same as - wait, are you asking me to do it, or just someone?
>
> MICKEY: You! Go ahead.

Mickey's IOAA Class, Excerpt C-2

This leads to the class creating and adding a conjecture about general subgroups of S_n to their list of things to prove, hence advancing the mathematical agenda of the class. Mickey's choice to leverage student contributions in this episode was an authoritative one; he leveraged his own institutional authority to encourage Mason's mathematical authority. Mickey receives Mason's mathematical authority by prompting him to repeat his statement, and Mason receives Mickey's institutional authority by responding to Mickey's questions. Additionally, Mickey is bearing institutional authority to advance the mathematical agenda

of the class by prompting Mason's contribution and asking him to make a conjecture, hence granting Mason authority.

Similar to the OWG, Mickey's orientation toward his own institutional authority is somewhat unsurprising in the classroom context, but by using it to grant authority to students to contribute, Mickey acts with the goal of supporting the development of his students' mathematical autonomy and advancing the mathematical agenda. What is most striking about Mickey's orientation toward authority in the classroom regarding the mathematical agenda is the way this agenda informs Mickey's choice of which students are granted authority to contribute. By de-emphasizing the mathematical authority of students with correct answers, Mickey creates more opportunities for students to bear ownership and mathematics expertise authorities, thereby promoting their mathematical autonomy.

To fulfill his roles as instructor and facilitator, Mickey had to take on the responsibility of advancing the agenda in both contexts, be it the agenda of the online working group to engage participants with the curriculum's mathematical ideas and to create reflective teaching practices, or the agenda of the classroom to create and formalize a shared understanding of abstract algebra and its requisite skills and practices. Both of these required Mickey to bear institutional authority, but his decision-making as a teacher and a facilitator was informed by a goal of supporting the development of autonomous reasoning in participants and, as will be discussed further in section 5.2, creating a supportive learning environment.

Additionally, Mickey often brought his pedagogical and mathematical experience to bear on these situations. Hence, authority to advance the agenda can be seen as a context-specific authority that requires the bearer to have both institutional authority and subject expertise. Note that this is not expertise authority, because Mickey's authority is not being legitimized on the basis of his expertise, but on the basis of his role in a given context. Expertise is, however, a necessary component of Mickey's choices. Without mathematical expertise, advancing the agenda of the classroom would be impossible. Without pedagogical expertise with the curriculum, the agenda of the OWG would seem nebulous and unfocused.

5.2 Creating a Supportive Environment

Supportive learning environments are known to promote participant engagement, interaction, and self-confidence, and are often characterized by relationships of trust and mutual respect between members (Niu et al., 2022). Indeed, these characteristics may play a role in supporting many of the desirable heuristics found in IO curriculum materials, like promoting student engagement in discussions of mathematical reasoning (Solomon, 2005) and encouraging student ownership of mathematical ideas (Solomon, 2007). It is no surprise that Mickey's facilitating and teaching actions suggest a goal of creating such environments. Across contexts, Mickey demonstrated his goal of creating a supportive learning environment by publicly honoring productive struggle and supporting the development of non-judgemental

practices of inquiry and reflection.

5.2.1 Supporting Inquiry and Reflection in the OWG

In the online working group, analysis revealed Mickey's intent to create a supportive learning environment for instructors. To this end, results showed that Mickey asked open-ended questions, prompted instructors to share their reasoning about pedagogical and mathematical issues, and co-created a norm of showing support when a member of the group shared a struggle. For instance, when a participant asked for advice about leveraging an unexpected student contribution, Mickey posed the question to the whole group, saying, "Okay, suppose that comes up, how would you all handle that? What teaching moves – what pedagogical moves would you use for that?" By asking open-ended questions, Mickey creates an environment where participants are engaged and their thoughts are valued outside of their correctness.

Mickey reflected on this practice in the debrief meeting with the PI, describing how participants were often worried about the correct mathematical answer expected of them . Mickey was aware that this caused tension with his goal of creating an environment where the participants are encouraged to share their thinking without worrying about how it will be evaluated. In the following episode from the debrief four weeks into the semester, Mickey described an experience with this tension while the OWG was investigating the Sudoku task (Larsen et al., 2013).

> MICKEY: It was really interesting because Elena kept saying, "Is this what you want me to say?" and I kept going, "I don't even know what I want you to say!" Like, they're still in a mode of -
>
> PI: "Right answer".
>
> MICKEY: Yeah like, what is right and what is not, and I think that's underlying the discussion as well, when it comes to the mathematics, since there is a right answer.

Debrief Meetings, Excerpt D-3

This demonstrates a nuance of Mickey's orientation toward creating a supportive learning environment. By focusing on generating discussion around mathematical or pedagogical quandaries rather than on generating the "correct" answer, Mickey's facilitating actions were aimed at supporting the participants' development of a non-judgemental practice of inquiry into their own mathematics and pedagogy. Further, by avoiding assessment of the instructors' mathematical contributions, Mickey aimed to create an environment where the participants were encouraged to share their reasoning about the curriculum tasks.

Mickey began most meetings by prompting the instructors to share how they thought their classes were going. This often motivated participants to share difficulties they were having and seek advice from other members of the OWG. It was normative for others, including Mickey, to respond to these kinds of contributions with encouragement and support (see Mitchell, 2022, in press).

Further, participants' reflections on their experience suggest Mickey's success in creating a supportive learning environment in the OWG. Consider the following exchange between two participants toward the end of the semester:

> ROGER: I'm really grateful . . . knowing that I have this community of support.
>
> ELENA: Yeah! It's just like, in every stage, everyone is there to help everyone, and that's really beautiful.

Online Working Group, Excerpt O-4

This episode makes it evident that participants viewed the OWG as a supportive environment. Additionally, it should be noted that the members of the OWG agreed to continue their weekly meetings after the semester concluded in order to support Roger's implementation of the IOAA curriculum tasks. Mickey's actions that encouraged this kind of camaraderie within the online working group can be viewed in line with his intention to create a supportive environment where participants can develop practices of inquiry and reflection. The following episodes from Mickey's classroom the previous year provide insight into how his teaching experiences inform these goals for facilitation.

5.2.2 Supporting Productive Failure and Non-judgemental Feedback in the Classroom

In the classroom, Mickey created an atmosphere where students' mathematical contributions were treated with enthusiasm and respect, and where failure was valued as a necessary and welcome part of learning. By expecting students to give productive, non-judgemental feedback, encouraging productive failures, treating students' reasoning as valuable to the classroom community, and leading explicit discussions about the value of creativity and productive failure in mathematics, Mickey demonstrated his goal of creating a supportive learning environment. It is likely that encouraging students to make mathematical contributions and treat each other with mutual respect and encouragement aided in their development of a strong sense of self-efficacy and mathematical autonomy (Serbin et al., 2020).

In the following episode, the class has been working toward a more formal definition of isomorphism and is discussing one definition proposed previously by another student. Cara shares the following thought without being called on.

cara: I think those [homomorphisms] would have to be commutative for that to work.

mickey: [*excitedly*] Ooooooh!

cara: Can I write what I wrote?

mickey: Yeah absolutely! Here, come on up. You've got this!

"Cara's Conjecture, part I"
Mickey's IOAA Class, Excerpt C-3.1

Mickey immediately expresses enthusiasm for Cara's thinking, which prompts her to volunteer to present her argument on the document camera, which he also encourages by saying, "You've got this!". By inviting Cara's contribution and responding with enthusiasm, Mickey publicly honors her thinking. This demonstrates Mickey's goal of creating a learning environment where students feel that their thinking is valued beyond evaluations of correctness.

Notably, Mickey still has the responsibility of guiding the class toward the correct formal mathematical ideas. This means that when Cara's proposed proof in the next episode contains an error, Mickey gently steps in.

mickey: Okay, I'm going to have to intervene here, unfortunately.

cara: Do it!

mickey: So, there's a misconception here. You're trading a property of the function for a property of the element, okay, so . . . can I take this over?

cara: Go for it.

"Cara's Conjecture, part II"
Mickey's IOAA Class, Excerpt C-3.2

Mickey treats his own intervention as unfortunate, and asks Cara for permission to discuss the remainder of the proof. Cara receives this correction with calm enthusiasm, saying things like "Do it!" and "Go for it." After they've worked through the issue as a class, Cara and Mickey have the following exchange.

mickey: Okay, but this is awesome, and I'll tell you why: some others were probably confused about pre-image and inverse like this –

cara: Yeah [*laughs*] I do that like every time I work with pre-image and inverse!

mickey: And so this failure was very productive!

"Cara's Conjecture, part III"
Mickey's IOAA Class, Excerpt C-3.3

Mickey leverages Cara's mistake to advance the mathematical agenda of the class. Further, he publicly honors her productive failure, describing explicitly how Cara's error was a learning opportunity for the class. Cara does not treat her mistake as shameful; rather, she laughs when she realizes that this error is common for her. This enthusiasm for her own productive failure can also be taken as evidence that an environment has been created where Cara feels that her contributions have value even if they are somewhat incomplete or incorrect. This evinces that Mickey's teaching actions have supported the development of Cara's mathematical autonomy.

As discussed in previous sections, Mickey expects students to provide constructive feedback on each others' contributions, saying things like "And remember, we're in a non-judgemental part of our lives right now, so keep your comments very productive, okay?" This expectation is met with enthusiasm by students, as shown in the following episode. Mickey's habit of publicly honoring students' in-progress reasoning about partial solutions over correct and complete solutions, as discussed earlier in Section 5.1.1, also appears to be related to Mickey's valuing of nonjudgemental feedback and productive failure. In the following episode, Mollie has just presented a proof to the class and asked for feedback. After a round of student questions and comments, the following interaction occurred.

> MICKEY: [*very gently*] We have to prove that [the identity] is in there - you've assumed it.
>
> MOLLIE: Ooooh [*smiling*] oh I did, yeah, okay.
>
> MASON: You've shown it's unique, which is nontrivial!
>
> MICKEY: That's true, that's true. So, there's the issue, but thank you. There's some really good things there. There's some stuff to improve on, but there's some really good things here.

"Mollie's Proof"
Mickey's IOAA Class, Excerpt C-4

Not only is Mason's interjection a form of productive feedback for Mollie, but by publicly honoring her contribution in the face of an error, Mason is simultaneously upholding the norm that partial solutions are valuable contributions and the norm that productive failure is a necessary mathematical activity. This is significant because Mickey's enthusiastic affirmation of Mason's comment demonstrates a co-creation of these norms by instructor and students. Because norms are most often co-created in this way, it is unlikely that this norm would have been established independently of Mickey's actions.

To support the development of students' mathematical autonomy, Mickey leads explicit discussions about the value of creativity and productive failure in class. Not only did he make sharing productive failures part of students' course grade, he also assigned reading about fostering mathematical curiosity and shared his own experiences with developing mathematical resilience. In the following excerpt, Mickey refers to one such reading he assigned.

> MICKEY: Finally, I want to get to the fostering mathematical curiosity paper for two seconds. And the two seconds is this . . I think the thing I got most out of it was this – and hopefully the way I'm structuring the tasks makes this clear – [*reads from the paper on the projector*] "Students are rarely asked to view a solution to a problem as a starting point in problem-solving." As a starting point! What got me interested in mathematics was that it was expansive. I once thought of mathematics like, I could never create a new mathematical theorem, right? All these geniuses did this, I couldn't do this. But, if you think of it this way [*gestures to the quote he just read*], you could always create something. All you need to do is just push the boundaries.

Mickey's IOAA Class, Excerpt C-5

In this aside, Mickey discusses his own experiences with developing mathematical reasoning and problem-solving skills, relating them to the reading he assigned. His statement about viewing the solution as a starting point is indicative of the relationship he sees between developing mathematical autonomy and valuing the process of problem-solving over the final product. Mickey describes how he started to believe he was able to create mathematical ideas when he began to value the problem-solving process over the final solution.

Due to contextual differences between his roles in the OWG and the classroom, Mickey's goal of facilitating the development of a supportive learning environment arises differently, but parallels can be drawn between his facilitating actions and his teaching actions. By asking open-ended questions and prompting the OWG members to discuss their pedagogical quandaries, Mickey creates an environment where participants are welcome to contribute to discussion free of evaluations of worthiness. By publicly honoring productive failure, Mickey creates a similar environment in the classroom. Fostering mutual respect and support that encourages students to take risks and make mistakes, ultimately bolstering their mathematical autonomy, can be viewed in parallel with Mickey facilitating the development of camaraderie within the OWG. Notably, when Mickey fosters a supportive learning environment, his actions reflect a more specific goal of supporting women participants' pedagogical and mathematical autonomy.

5.3 Supporting Women Participants

Throughout the OWG and Debrief meetings, Mickey attends explicitly to women's experiences, demonstrating a goal of supporting women by creating the kinds of supportive learning environments described in Section 5.2. He prioritizes the establishment of norms where participants support and encourage each other, centers women participants' experiences while reflecting on his facilitation, and leverages their contributions to generate discussion around pedagogical difficulties. There is a clear influence here between Mickey's teaching experiences and his facilitating actions; in the classroom, Mickey intentionally solicits women students' mathematical ideas, leverages their partial solutions to advance the mathematical agenda of the class, and co-creates norms around valuing productive failures and providing constructive feedback.

Within the OWG, Mickey's goal of facilitating equitable participation for women arose as he revoiced and leveraged women participants' contributions to discussion and facilitated the establishment of the same norms that lead to better learning outcomes for women students in the classroom, such as sharing one's struggles (Hassi & Laursen, 2015; Serbin et al., 2020). While discussing the curriculum tasks from a mathematical perspective, Mickey often prompts women participants to repeat their contributions, saying things like, "Can you say that again? We're going to write it up here on the board", in order to present their ideas to the whole group for discussion. He also revoices contributions made by women participants in order to clarify their meaning and reiterate the idea to the rest of the participants. These prompts often take the form, "So, Laura, you're saying that [repeats idea as he understands it]? What do you all think about that?"

It is important to note that Mickey also attends to and leverages the contributions of the male participant, Roger, and that "promoting equitable participation" is not conflated with "only allowing women participants to speak" within the OWG. The idea here is that by publicly honoring women participants' experiences and contributions *alongside the male participant's*, Mickey is encouraging women participants to engage in challenging pedagogical quandaries in much the same ways he encouraged his women students to engage with mathematical difficulties, ways that have been documented as promoting productive beliefs about women's mathematical efficacy (Serbin et al,. 2020; Gutierrez, 2002; Hassi & Laursen, 2015).

In the debrief meetings, Mickey's intention to promote equity often takes the form of wrestling with his facilitation procedures, where he expresses frustration with implementing this goal. He says things like, "I want it to be organic, like, I don't want to go 'You're female, please participate', you know?" This shows that not only does Mickey explicitly attend to creating equitable opportunities for women to contribute to discussions, but he also has what Schoenfeld (2011) might call a subgoal of eliciting women participants' contributions in a way that feels natural. Additionally, he focuses on Elena and Laura's individual experiences as participants and instructors and intends to establish the norms within the OWG that have been documented in the literature (Serbin et al, 2020) as being conducive to women's

equitable participation.

In a discussion with the PI during week 4 regarding some discomfort the participants expressed with eliciting students' thinking, Mickey says, "so I think [Laura]'s trying to figure out herself in this whole thing, and I think that that's, I had the same moment of sorts." Mickey often relates his own experiences to Laura's in this way, which shows that he deliberately attends to her thinking about teaching IOAA.Similarly, when the PI asked how he thinks Laura feels about teaching with the IOAA curriculum, he gave the following statement.

> MICKEY: I think she's hot and cold . . .To me, two things happen: I think Elena's very warm to all this, and she may be reinforcing or helping, and then also Laura still giving out these reports indicates she has some attention and awareness to what's going on.

Debrief Meetings, Excerpt D-4

Not only does he attend to how Elena and Laura experience IOI, but how their interactions in the OWG influence each other. Notably, he also sees support and encouragement as an obvious goal of his facilitation.

> MICKEY: We've set up this like, support system, so that when [Laura] does feel down, we're there to help her slash pick her up . . . and I think that as a facilitator, I've got to set up those norms, at least try to set up those norms, right?

Debrief Meetings, Excerpt D-5

By prioritizing the establishment of norms where participants encourage and support each other, and by centering the women instructors' experiences while reflecting on his facilitation, Mickey demonstrates his goal of creating a supportive learning environment for women participants. This goal is clearly influenced by Mickey's teaching experiences; Mickey created a supportive learning environment for women students by enthusiastically leveraging women's contributions to advance the mathematical agenda of the class, publicly valuing productive failures as part of the learning process, and expecting students to provide constructive feedback.

In "Cara's Conjecture" (Excerpt C-3, Section 5.2), Mickey publicly values Cara's thinking by inviting her contribution with enthusiasm. Further, he leverages Cara's struggle with preimage and inverse to advance the mathematical agenda of the class, noting that her error was valuable to the learning of the entire classroom community. These actions contribute to

a learning environment where women students can believe that their mathematical ideas are valuable and worth sharing. Further, in "Mollie's Proof" (Excerpt C-4, Section 5.2), Mickey affirms Mason's choice to publicly honor Mollie's contribution in spite of an error. That is, Mickey supports students in their practice of honoring each others' productive failures and responding with constructive feedback. His teaching actions directly supported the development of this environment when he said things like, "And remember, we're in a non-judgemental phase of our lives right now, so keep your comments very productive," when inviting students to assess each other's work on the boards.

In addition to creating a learning environment where women students' feel supported in developing their mathematical autonomy, Mickey's teaching actions reflected an intention to create more equitable participation opportunities for women and men students by soliciting women students' participation more often than the comparison sample of instructors teaching under the TIMES project. Using equity ratios, Smith et al. (2019) found that within all TIMES IOAA classes, women were called on to participate relatively proportionally to their representation. Since Mickey had a class composed of 29% women, we would expect him to call on women students 29% of the time (at minimum) to be consistent with other TIMES instructors. In contrast, Mickey actually called on women 53% of the time. Although this gives the appearance that Mickey's solicitation is skewed toward women and away from men students, it should be noted that men students made contributions to discussion without being called on 87% of the time. Mickey's over-solicitation of women students' contributions can be viewed as an attempt to remediate this difference in proportion (men students should be expected to make contributions without being called on only 71% of the time). This pattern of teaching actions is indicative of Mickey's goal of supporting women.

Further, it has been shown that Mickey's IOI had positive achievement outcomes for women students; their average scores on the Group Theory Content Assessment (Melhuish, 2015) were higher than the comparison sample of women, an outlier for TIMES fellows (Johnson et al., 2019). Serbin et al. (2020) documented norms established in Mickey's class that could, they argue, promote gender-based equity in instruction. In this section, I have highlighted Mickey's role in the creation of these documented norms, as well as emergent teaching practices that reflect Mickey's goal of creating a supportive environment for women. I have also made inferences about the influence these teaching experiences had on Mickey's OWG facilitation.

As in the previous section, contextual differences between the online working group and the classroom must be taken into account when discussing Mickey's actions and goals. Mickey's abstract algebra classroom consisted of 6 women and 15 men college-aged students, while the online working group consisted of 2 women instructors and 1 man instructor. Because all 4 members of the OWG were experienced university instructors, their peer dynamics more closely reflected their status as equal colleagues. This is in contrast to the discernibly different dynamics occurring within the classroom, where students do not typically treat the instructor as a peer or colleague and vice versa. Hence, Mickey's goal of facilitating equitable participation, especially for women, arose differently across contexts. In the classroom,

Mickey's teaching actions in line with this goal often took the form of intentionally prompting women students to share their contributions more often and demonstrating enthusiasm for their thinking. Additionally, Mickey tended to honor partial solutions over correct solutions, giving students' in-process contributions attention over students with completed work, expecting non-judgemental and constructive feedback from others, and valuing productive failures as a necessary part of mathematical activity. In the online working group, his facilitating actions toward this goal could be characterized by attending to women participants' experiences, soliciting and leveraging their reasoning about instructional and mathematical quandaries, and supporting the establishment of norms where participants encourage and support each other.

Finally, it is worth noting that throughout the semester the online working group met, Mickey indicated both implicitly and explicitly that he was trying to model inquiry-oriented instruction within his facilitation. He made comments like "I hope I'm modeling what I would like to do in the classroom," and even described the activities of the OWG in parallel to the four instructional principles of IOI in Kuster et al. (2017): "We're generating teacher thinking, building on teacher thinking, developing a shared understanding, and then bringing it into the formal world." This description makes it clear that Mickey viewed his facilitation as an opportunity to model the kind of instruction he had in mind for his participants' implementation of IOI. This is direct evidence of the influence that Mickey's teaching experiences with IOI had on his interaction with the OWG as an instructional support. Mickey's facilitating actions are influenced by a general goal of supporting participants as they develop autonomy, either as teachers or students, and this support is mediated by this set of orientations and goals around authority, support, and equity.

Chapter 6

Conclusions

This study serves as a proof-of-concept for the train-the-trainer model of professional development in taking the IOAA curriculum innovation to scale, while maintaining the goals and values behind the innovation's design. It serves as an auspicious step toward establishing effective ways to support research-based instructional change, one of the primary concerns facing education researchers today (Andrews-Larson et al., 2019). Additionally, my analysis supports the conclusion that this particular PD meets many research-based recommendations for establishing supports, as seen in Henderson and Dancy (2008). First, it satisfies the recommendation that instructors be provided with easily modifiable materials (p. 88). While Mickey treated the curriculum tasks as intended for use with their original phrasing and particular order, he also encouraged and guided instructors in adapting assessments like homework, quizzes, and tests to meet their unique instructional goals (Section 5.1.1). A second recommendation is that providers understand and disseminate both the details and principles behind curriculum design choices (Henderson & Dancy, 2008, p. 88) which is satisfied in the OWG when Mickey attends to mathematical and pedagogical quandaries in discussions and supports instructors' attempts to shift toward shared authorities that encourage students' mathematical autonomy. Mickey's refusal to take on pedagogical expertise authority indicates his view of the OWG participants as equal partners, as suggested by Henderson and Dancy (2008, p. 89). Finally, Mickey acknowledges that change is difficult and supports instructors in sharing their pedagogical and mathematical difficulties, also consistent with these recommendations.

In addition to making a promising step toward a broader goal of the education research community, this study also characterized the interactions between a facilitator's instructional experience and further instructional supports. The excerpts analyzed serve to illustrate the ways in which Mickey brought his orientations and goals for inquiry-oriented instruction to bear as he facilitated an online working group of instructors practicing IOI for the first time. Because I was able to identify specific instances where Mickey's facilitating actions demonstrated a set of orientations and goals that also arose in the classroom, this case study points to the importance of understanding the pedagogical values and goals of candidates for facilitator selection. Further, because Mickey's orientations and goals mediated an overarching motivation to support students and participants in developing their autonomy as learners and teachers of mathematics, this PD evoked Benne's (1970) notion of *anthropogogical authority*, an authority dynamic in line with IOI's heuristic of enculturation into the community of practitioners of mathematics through student engagement in meaningful

mathematical practices (Kuster et al., 2019, p. 185).

While these orientations and goals often worked harmoniously toward this end, Mickey's orientations were sometimes in conflict with his goals. Fittingly, Mickey's orientation toward authority in Section 5.1.2 supports his goal of developing a supportive learning environment for Laura when he prompts her to share her pedagogical reasoning. This careful leveraging of institutional authority allowed Mickey to publicly value Laura's thinking while also advancing the agenda of the OWG. However, there were instances where Mickey's orientation toward authority was in conflict with his broader goal of supporting participants' autonomy. Excerpt C-5 (Section 5.2.2) can be viewed as Mickey momentarily bearing expertise authority to deliver a mini-lecture on problem-solving. Although this is not in line with his orientation toward sharing authority, it is consistent with his goal of creating a supportive learning environment where students' productive failures are valued as part of the learning process.

This analysis of Mickey's authority practices not only provides insight into ways that authority can be leveraged to create learning opportunities in IOI, but how such practices translate to facilitating IOI professional development training. While Gerson and Bateman (2010) take the position that an ideal instructional environment to promote shared authority would limit the instructor's institutional and mathematics expertise authorities (p. 206), IOI demands that the instructor bear pedagogical expertise authority and mathematics expertise authority to advance the mathematical agenda of the class. In the case of Mickey, I identified instances where his careful leveraging of expertise and institutional authority in the OWG paralleled his classroom experience, in both cases advancing the mathematical or pedagogical agenda. Balancing authority and the responsibility to advance the agenda in this way, as an instructor or facilitator, is no simple task. While Gerson and Bateman's (2010) work suggests the importance of teaching practices that allow students many opportunities to bear granted, justification, ownership expertise, and mathematics community authorities, this study demonstrates ways that PD can support the development of these teaching practices for instructors new to IOI. Not only do these practices have great potential to promote students' mathematical autonomy, I have shown that they can also inform powerful facilitating practices within a train-the-trainer model of professional development. I conjecture, then, that opportunities to share authority with students during inquiry-oriented instruction provide a rich and productive background from which to conduct professional development, a clear strength of this train-the-trainer model.

In spite of its strengths, this study also has limitations. The analyses presented here were based on existing data I acquired after it was collected; hence, methods of video collection could not be adjusted to target the specific phenomena I observed. For this same reason, subsequent interviews with the facilitator providing more detailed insight into his orientations and goals for the PD could not be collected. The analyses were also grounded in research on professional development at the K−12-level due to a lack of research on collegiate PDs. Future studies on train-the-trainer modes of professional development looking to document potential mutations to the model should attend carefully to methods of data collection, including interviews, and look to new research on collegiate professional development

to inform theory. Additionally, analysis of both OWG and classroom video data did not explicitly account for potential effects of racialized social dynamics. Specifically, the OWG participants were two white women and one black man, so nuances in authority relations within the OWG may be better explained through more intersectional analysis pertaining to identity positioning in discourse.

Viewing both instruction and PD facilitation through the lens of shifting authority dynamics can be useful in future investigations of both teachers' implementation of IOI and their participation in online professional development programs. This work contributes to the small but growing bodies of literature on authority in IOI (e.g., Hicks et al., 2021; Dawkins, 2014) and on effective professional development (Fortune & Keene, 2021), especially as it applies to train-the-trainer models for university educators. As undergraduate mathematics faculty seek more convenient and ongoing support for implementing ambitious teaching practices like IOI, research on virtual professional development like this is vital. Further, the mathematics education research community needs more data on how to effectively support teachers implementing IOI. Future research should investigate how the instructors in this study implemented IOI, with an eye toward authority and gender equity, and similar work on other trained-trainer facilitators should be conducted and comparatively analyzed to enrich our community's knowledge about best practices for facilitating professional development.

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