# Freeway Travel Time Estimation Based on Spot Speed Measurements 

Wang Zhang<br>Dissertation Submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirement for the degree of<br>Doctor of Philosophy<br>in<br>Civil Engineering

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#### Abstract

As one of the kernel components of ITS technology, Travel Time Estimation (TTE) has been a high-interest topic in highway operation and management for years. Out of numerous vehicle detection technologies being applied in this project, intrusive loop detector, as the representative of spot measurement devices, is the most common. The ultimate goal of this dissertation is to seek a TTE approach based primarily on spot speed measurement and capable of successfully performing in a certain accuracy range under various traffic conditions.

The provision of real-time traffic information could offer significant benefits for commuters looking to make optimum travel decisions. The proposed research effort attempts to characterize typical variability in traffic conditions using traffic volume data obtained from loop detectors on I-66 Virginia during a 3-month period. The detectors logged time-mean speed, volume, and occupancy measurements for each station and lane combination. Using these data, the study examines the spatiotemporal link and path flow variability of weekdays and weekends. The generation of path flows is made through the use of a synthetic maximum likelihood approach. Statistical Analysis of Variance (ANOVA) tests are performed on the data. The results demonstrate that in terms of link flows and total traffic demand, Mondays and Fridays are similar to core weekdays (Tuesdays, Wednesdays, and Thursdays). In terms of path flows, Fridays appear to be different from core weekdays. A common procedure for estimating roadway travel times is to use either queuing theory or shockwave analysis procedures. However, a number of studies have claimed that deterministic queuing theory and shock-wave analysis are fundamentally different, producing different delay estimates for solving bottleneck problems. Chapter 5 demonstrates the consistency in the delay estimates that are derived from both queuing theory and shock-wave analysis and highlights the common errors that are made in the literature with regards to shock-wave analysis delay estimation. Furthermore, Chapter 5 demonstrates that the area between the demand and capacity curves can represent the total delay or the total vehicle-hours of travel if the two curves are spatially offset and queuing theory has its advantages on this because of its simplicity. As the established relationship between time-mean and space-mean speed is suitable for estimating time-mean speeds from space-mean speeds in most cases, it is also desired to estimate the space-mean speeds from time-mean speeds. Consequently, Chapter 6 develops a new formulation that utilizes the variance of the time-mean speed as opposed to the variance of the space-mean speed for the estimation of space-mean speeds. This demonstrates that the space-mean speeds are estimated within a margin of error of 0 to 1 percent. Furthermore, it develops a relationship between the space- and time-mean speed variance and between the space-mean speed and the spatial travel-time variance. In addition, the paper demonstrates that both the Hall and Persaud and the Dailey formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation computes the space-mean speed (harmonic mean) while the Dailey formulation computes the time-mean speed (arithmetic mean).


Chapter 7 focuses on freeway Travel Time Estimation (TTE) algorithms that are based on spot speed measurements. Several TTE approaches are introduced including a traffic dynamics TTE algorithm that is documented in literature. This traffic dynamics algorithm is analyzed, highlighting some of its drawbacks, followed by some proposed corrections to the traffic dynamics formulation. The proposed approach estimates traffic stream density from occupancy measurements, as opposed to flow measurements, at the onset of congestion. Next, the study validates the proposed model using field data from I-880 and simulated data. Comparison of five different TTE algorithms is conducted. The comparison demonstrates that the proposed approach is superior to the TTE traffic dynamics approach. Particularly, a multi-link simulation network is built to test spot-speed-measurement TTE performance on
multi links, as well as the data smoothing technique's effect on TTE accuracy. Findings further prove advantages of utilizing space-mean speed in TTE rather than time-mean speed. In summary, a feasible TTE procedure that is adaptive to various traffic conditions has been established. Since each approach would under-/over-estimate travel time depending on the concrete traffic condition, different models will be selected to ensure TTE's accuracy window. This approach has broad applications because it is based on popular loop detectors.

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## CHAPTER 1. INTRODUCTION

Travel time is not an uncommon concept in transportation engineering. As a simple concept, travel time is defined as "the time necessary to traverse a route between any two points of interest"(1998). Engineers, commuters, consumers, and businesses all understand and have concerns about the every-day travel time needed to get from one point to another, as it can indirectly affect business productivity and efficiency.

Accurately estimating travel time with a certain device or method, or Travel Time Estimation (TTE), is a critical component of Advanced Traveler Information Systems (ATIS) and Advanced Travel Management Systems (ATMS). In addition, TTE plays a key role in Intelligent Transportation System (ITS) applications today. With this in mind, the proposed research is focused on constructing a successful freeway travel time estimation procedure based on spot speed measurements.

### 1.1 Problem Overview

Engineers and planners have used travel time and delay studies since the late 1920s to evaluate transportation facilities and plan improvements. In more modern transportation times, traffic congestion continues as a global annoyance to both engineers and commuters. As TTE has been deemed a critical issue in traffic engineering, engineers have introduced various ITS technologies as a solution to this congestion problem. Because TTE has many significant applications, engineers and researchers continually search for effective ways to provide robust and accurate TTE on both freeways and urban arterial networks. For example, accurate TTE is critical to dynamic travel assignment, as a traffic control center (TCC) can send motorists instantaneous travel information and guidance. According to the real-time TTE outcome on the road, TCC can help decrease congestion delays on a road network system by redirecting traffic to an alternate route. However, incorrect TTE information could make traffic conditions worse by TCC releasing misleading information or travel guidance based on the initial incorrect TTE outcomes. In addition, Partners for Advanced Transit and Highways (PATH) notes that rapid changes in link travel time represent perhaps the most robust and deterministic indicator of an incident. Thus, link travel time could possibly be the most important parameter for ATIS functions like congestion routing.

Various TTE methods that are based on different traffic data collection methods involve different developed mathematic models. TTE is a cross-subject that needs various advanced technologies in computer-aided engineering, electronic engineering, automatic control engineering, and telecommunication engineering, all of which are applied in data collection, information transmission, and signal processing. The major technologies currently being used in TTE include loop detectors, probe vehicle technologies, license plate matching, test vehicle technologies, GPS, and aerial surveys.

Although each technology has some specific advantages, each also has some particular shortcomings, causing the device or technology to work improperly under certain circumstances. For example, loop detectors have trouble measuring low-speed vehicles and only provide point-test time-mean speeds to estimate link travel times as a continuous stream value. Probe vehicle technology cannot provide around-the-clock traffic data collection and it is always related to citizen privacy. License plate matching is time consuming and needs large handling efforts that still cannot avoid human error. Test vehicle technology requires massive labor efforts, with slow data processing. Simultaneously, several mathematic methods are involved in the TTE. For example, statistics models (such as linear regression and Bayesian series), artificial neural network, and Kalman filtering techniques are used popularly.

Although every method has its own advantage, none of methods can provide consistently satisfying outcomes on different common cases. Moreover, no current method has been validated by multiple real-time freeway cases, except with its own test bed. Currently, there is no unanimously satisfying methodology that can estimate travel time with certain accuracy that could be used by ATIS and ATMS. Therefore the method's application accuracy is questioned until it has yielded consistently acceptable outcomes from various normal scenarios. Lastly, most current TTE models perform well under the free-flow traffic conditions, but none of them provide satisfying results
under congested traffic conditions. More specifically, TTE under congestion is exactly the core problem that needs to be solved. Therefore, TTE still is an unsolved problem among transportation engineers and researchers.

With emerging of ITS technologies and a growing demand for accurate TTE and efficient travel, loop detectors as a kind of spot measurement show promising and positive outcomes for successful freeway TTE. Thus, the target of the proposed research focuses on an effective methodology that can provide accurate TTE based on loop detector data, and it also should be effective based on other spot speed measurements. More importantly, the ideal method should be able to pass validation tests on different test beds with acceptable results. In addition, the ideal method should have the capability to work under both free-flow condition and those delays brought on by unexpected events, such as traffic incidents, sudden weather changes, or congestion brought on by construction.

Thus, with this background research and previous knowledge, the proposed research heavily depends on loop detector systems. The test beds for this research effort include three datasets; the first is 1-min. aggregated data on a 15 mile freeway section of Interstate-66 in Northern Virginia that is fully equipped with 31 dual loop detector stations, another is individual vehicle data on a half mile freeway section of Interstate-880 in California that has two dual loop detector stations at both ends; the third dataset is a simulated dataset for the I-66 section of highway. All these stations cover all lanes in both directions at approximately half mile spacing.

The first step involves the construction of a historical traffic information database containing demand, speed, and Origination-Destination, this database from loop detectors also serves as reference of traffic variability and reliability for different locations, time of the day, and day of the week. The next chapter demonstrates the consistency in delay estimation between Shockwave and Queuing theory, this part is inspired by a literature review on traffic Dynamics (Nam D.H. 1998). Here, a conceptual error in Dynamics is highlighted and corrected; the outcome obtained has tremendous help in the travel time estimation section of the dissertation. Subsequently, an approach to estimate space-mean speeds is then introduced and explored; this research effort derives a relationship between the variance of space-mean and time-mean speed and the relationship between the coefficient of variation of space-mean speed and travel time. The last research topic compares 5 different TTEs based on spot speed measurement, on field (I-880) and simulation (INTEGRATION (Assoc. 2002)) test beds. The research proposed two proposed TTE approaches, one is inspired from traffic dynamics and another based on space-mean speed. In addition, data smoothing techniques (Hardle 1991) are applied to enhance travel time estimation. These smoothers act as low-pass filters that remove high frequency noise while maintaining the underlying low frequency signal.

### 1.2 Research Objectives and Contributions

Currently, there is no overall satisfying and accurate methodology that can estimate travel time and be used by ATIS and ATMS successfully. The primary objective of the proposed research is to develop a robust and accurate methodology of TTE based on spot speed measurements. Moreover, the method should also perform well under unusual traffic conditions that are affected by incidents, weather changes, or construction.

The research objectives and contributions can be summarized as follows:
a. Construct travel time datasets for the testing and development of TTEs.
b. Characterize typical day-to-day levels of travel demand variability to identify the need for dynamic roadway travel time estimates.
c. Demonstrate the consistency between Shockwave and Queuing theory for the estimation of roadway travel times and delays.
d. Develop procedures to estimate space-mean speed from time-mean speed, mean travel time from space-mean speed, and travel time confidence limits from spot speed measurements.
e. Develop efficient TTE algorithms based on spot speed measurements.

### 1.3 Dissertation Layout

In terms of the dissertation layout, following the dissertation introduction, a review of the state-of-the-art procedures for estimating roadway travel times from spot speed measurements are presented in Chapter 2. Subsequently, Chapter 3 presents a statistical analysis of daily variations in path flows. This effort demonstrates the need for dynamic roadway travel time algorithms and establishes the variability that is required within simulation software to achieve the desired variability. Chapter 4 demonstrates the consistency in travel time and delay estimates based on Shockwave and Queuing theory. This finding allows modelers to estimate delay using much simpler Queuing theory approaches rather than the more complex Shockwave analysis procedures. In Chapter 5 procedures are developed to estimate space-mean speeds from time-mean speeds, mean travel times from space-mean speeds, and travel time confidence limits from spot speed measurements. In Chapter 6, procedures for estimating dynamic roadway travel times are developed. Finally, Chapter 7 presents the conclusions of the dissertation and recommendations for further research.

## Chapter 2. Literature Review

Given that the layout of the thesis is in the form of papers, the literature review for each topic is provided separately in each paper. Consequently, this chapter only provides a brief overview of the literature in an attempt to provide a broad overview of the topic. More detailed coverage of the literature is provided in each chapter.

The first vehicle detection device was installed at a Baltimore, MD intersection, in the form of a semi-actuated signal; the detector was equipped with a microphone and installed at a pole near the intersection, and it had to be activated by the sound of a car's horn. Simultaneously, a pressure-sensitive pavement detector was introduced, this two-plate system acted with electrical contact to transmit signals and has been widely used for over 30 years (Meddleton, Parker et al. 2004). Inductive loops were first brought to transportation engineering as detection equipment in the 1960s and have become the most-used roadway detection devices since then. Although various problems with loops caused the introduction of numerous non-intrusive detection devices in attempts to replace the failing loops, they still have the most prominence on freeways in the nation.

As the main means utilized by TTE, loop detectors have some specific advantages, such as its low running and maintenance cost, all weather working condition, speedy data processing, and particularly its broad and full-time coverage and accuracy. However, it also has a poor ability to measure low speed traffic. Various TTE methods based on loop detectors have been developed by researchers, which include not only using a simple point-to-stream model but also utilizing detectors as a vehicle identification tool. The traditional practice for estimating speeds and travel times from single loop detectors is based on the assumption of a constant average effective vehicle length. For example, Petty et al. developed a TTE method (Petty and Peter Bickel 1998) based on single loop detectors. The approach used in this method depends on speed estimation from flow/occupancy relationships ( g factor) and statistical analysis. However, this method's effectiveness fails under congested conditions and the flow/occupancy relationship varies in terms of differing traffic conditions. Studies have shown that this assumption provides speed estimates that are sufficiently inaccurate as to severely limit the usefulness of these speed estimates for real-time traffic management and traveler information systems. A similar method (Coifman, Dhoorjaty et al. 2003) was founded by Coifman et al. This method also shows that the simple regression model is effective to short-term travel time prediction. Dailey (Dailey 2004) showed an advanced method that considered vehicle length in speed calculation, which might improve estimation accuracy. Moreover, researchers have investigated the use of median as opposed to mean statistics in order to enhance the robustness of the statistics by ensuring that the measures are not influenced by outlier observations. For example, Lin et al.(Lin, Dahlgren et al. 2004) used the median vehicle passage time as opposed to the mean passage time to estimate speeds from single loop detectors. Similarly, Coifman et al.(Coifman, Dhoorjaty et al. 2003) computed the median speed from the median occupancy in order to reduce speed estimate errors when a wide range of vehicle lengths are present in the traffic stream.

Dual loop detectors have many applications concerning TTE. Compared to single loop detectors, dual loops can successfully measure the speed of each individual vehicle, as well as the vehicle length. Various TTE methods were developed based on dual loops. For example, Dailey (Dailey 2004) developed a Kalman filter on vehicle length estimates while Hellinga (Hellinga 2002) used exponentially smoothed adjacent dual loop detector vehicle length measurements to enhance the speed estimates of single loop detectors. Hellinga demonstrated that the exponential smoothing of 20-s average vehicle length measurements from adjacent dual loop detectors enhanced the accuracy of the speed estimates by approximately 20 percent. Wang and Nihan (Wang and Nihan 2000) using screening procedures to remove intervals with long vehicles and space-mean speed estimates were derived from the intervals with passenger cars only. Above methods mainly concentrate on obtaining accurate spot speed estimates from loops.

Alternatively, Coifman (Coifman 1998; Coifman and Ergueta 2003) developed a TTE method called vehicle re-identification by using vehicle length as the vehicle signature. Coifman proves that it is possible to re-identify the same vehicle at both upstream and downstream loop detectors, while travel time is deduced directly. As known,
the traffic measured by a loop detector on a low speed vehicle is not dependable. Vehicle Re-Identification (VRI) is superiro under congested traffic conditions since it does not need speed (which has obvious measuring error from loop detectors under congested conditions) as the analysis source. Moreover, slower vehicle speeds make vehicle length measurement more precise with lower error and the vehicles tend to travel in comparably fixed sequences with less lane changes during congestion. Therefore, VRI is a feasible method for TTE under congested conditions, while TTE under free-flow conditions depends on the point-test speed method mentioned before because VRI yields more errors under free-flow conditions. When vehicle speed increases and traffic volume decreases, the vehicle length measurement error increases, where more frequent lane changes would increase estimate error as well.


Figure 2.1 Vehicle Length measured by dual loop detectors

A dual loop detector provides two vehicle length measurements for a single vehicle, [1] and [12]. The average is taken as the effective vehicle length:

Length measurement \#1: $\quad L_{1}=20 \cdot 0.3048 \cdot \frac{O T_{1}}{T T_{r}}$
Length measurement \#2: $\quad L_{2}=20 \cdot 0.3048 \cdot \frac{O T_{2}}{T T_{f}}$
Note: 20 feet is the distance between upstream and downstream detectors for one station.
The loop detector is scanned 60 times per second, and depending on vehicle speed, the measured vehicle length error can range between $0.2-0.5 \mathrm{~m}$ for vehicle speeds of $40-120 \mathrm{~km} / \mathrm{h}$, respectively. For a faster traveling speed, there is a higher measurement error. Under congested conditions, vehicles are likely to travel in platoons and less likely to change lanes. By comparing vehicle length estimates at upstream and downstream loop stations, it is possible to capture the identical vehicle travel sequence at both upstream and downstream stations. Once a vehicle
is matched a good estimate of travel times is computed. The link travel time is averaged over all pairs of matched vehicles during a polling interval. Based on a preliminary analysis of I-880 data during congested conditions the algorithm was able to match somewhere between 20 to 30 percent of the traffic stream flow.

A similar technique (Lucas, Mirchandani et al. 2004) developed by Lucas et al. uses platoon recognition instead of vehicle length matching. These two methods share the same characteristic: they do not depend on spot speed measurements but instead use vehicle/traffic signatures to track and measure spatial roadway travel times. However, more often, dual loop detectors are utilized in traditional ways. For example, Coifman (Coifman, Lyddy et al. 2000) developed an algorithm to estimate roadway travel times from speed measurements. The algorithm considers vehicle headways and flow-density relationships and applies a weight to a number of vehicles in order to control estimation error. However, because of inherent deficiencies in the quality of loop detector data, low speed measurement error is still obvious and inevitable. There are many TTE methods developed in terms of this point-to-stream philosophy using dual loop detectors, and it is also considered as one of the main methods in the proposed research.

Regarding accuracy of TTE, it is widely recognized that upstream detectors tend to underestimate travel time, while downstream detectors behave in the opposite way. Smith et al. (Smith, Holt et al. 2004) conducted a sensitivity analysis on methods of TTE based on loop detector data and found that loop detector approaches always tends to overestimate travel time as congestion decreases. Moreover, the locations of detectors impact the system accuracy, and thus detectors should be distributed on the freeway very wisely. Further, the estimation error increases by 50 percent once an adjacent detector station fails.

One more TTE application based on loop detectors is the Dynamics method, which was originally developed by Nam and Drew (Nam D.H. 1998; Nam D.H. 1998; Nam D.H. 1999). The logic behind this approach is to estimate traffic stream density from the difference in cumulative vehicle arrivals and departures within a polling interval, which corresponds to the change in the number of vehicles traveling on the link. Using this approach it is feasible to obtain total travel time during an interval and then use the information further to estimate the average travel time of all vehicles within a polling interval. Furthermore, since travel time is solved with known density, flow, and link length, the main procedures of the Dynamics TTE are to calculate densities from cumulative flows and then calculate travel time from the densities and interval volumes. Dynamics is another approach to estimate travel time that does not use spot speed measurements along a roadway section.


Figure 2.2 Traffic Dynamics, Link TTE from Cumulative Flow

For example, two loop detectors at upstream and downstream placements can represent the two ends of a link, and
the link travel time could be obtained by calculating cumulative flows and interval traffic volumes of both detectors along with link length. The procedure to evaluate travel time is shown below:

$$
\begin{array}{ll}
k\left(t_{n}\right)=\frac{Q\left(x_{1}, t_{n}\right)-Q\left(x_{2}, t_{n}\right)}{\Delta x} \\
m=Q\left(x_{2}, t_{n}\right)-Q\left(x_{1}, t_{n-1}\right) & \\
t_{j}=\frac{\Delta x}{2} \frac{\left[q\left(x_{1}, t_{n}\right) k\left(t_{n-1}\right)+q\left(x_{2}, t_{n}\right) k\left(t_{n}\right)\right]}{q\left(x_{1}, t_{n}\right) q\left(x_{2}, t_{n}\right)}, & \text { when } \mathrm{m}>=0 \\
t=\frac{\Delta x}{2} \frac{k\left(t_{n-1}\right)+k\left(t_{n}\right)}{q\left(x_{2}, t_{n}\right)}, & \text { when } \mathrm{m}<0 \tag{6}
\end{array}
$$

Where:
1: upstream index
2: downstream index
$\mathrm{t}_{\mathrm{n}}$ : time interval n
Q: cumulative flow counts (veh)
q : traffic volume per interval (veh)
$\Delta \mathrm{X}$ : link length (m)
k : density (veh/m)
t : link travel time
m : domain factor, when $\mathrm{m} \geq 0$ it represents normal flow condition (vehicles can enter and exit the link during the same interval), when $m<0$ it represents congested flow condition (vehicles can enter but cannot exit the link during the same interval), according to the $m$ value, different travel time calculations apply.
Vanajakshi and Rilett (Vanajakshi L. 2004) developed an amended method based on Dynamics. The density during free-flow conditions could be obtained from detector's cumulative volume and speed (estimating density from occupancy under free-flow conditions). This is because the original density estimation from cumulative flow is very sensitive and easily yields errors during free-flow conditions. After all, this approach still heavily relies on cumulative flow so any detector malfunction results in serious estimate errors. Therefore, the detector data cleaning and adjustment is very critical to the Dynamics TTE approach. The cleaning procedure includes checks of maximum speed, maximum density, and differentials between upstream cumulative flow volume and downstream cumulative flow volume during an interval. This method, along with the original Dynamics method will be studied in more detail in the proposed research, and a detailed review of them will be shown on Chapter 6.

Ground truth technology, is another means that has gradually increasing applications in TTE. Nanthawichit et al. (Nanthawichit, Nakatsuji et al. 2004) proposed a method that integrates probe technology into fixed detector data within a macroscopic model. It was found that the method could produce better results compared to other methods which only depend on detectors or probe vehicles. However, the proposed method just uses 50 percent of detector data to 50 percent of probe car data with no explanation. Chakroborty et al. presented a method (Chakroborty and Kikuchi 2004) based on bus as Automatic Vehicle Locator (AVL). The method tried to figure out the relationship between bus travel time and travel time of regular traffic flow by a simple linear regression model. However, because of the frequent stop times made by buses and bus routes' limited coverage, the application of AVL is still doubted. Cellular phones have become more popular, causing high percentage of occupancy in motorist. Drane et al. (Drane, Yim et al. 2000) discusses the possibility to calculate TTE by using cell phone signals to locate vehicles. It was found that the usage of cellular phones by motorists is very important to obtain accurate travel time (higher than 5 percent). Moreover, the cell phone signal locating algorithm still has comparatively obvious errors. With the increasing usage of cell phone telecommunication technology, it would be truly practical to obtain accurate TTE by cell phones in the future. Like most of ground truth methods, in order to ensure certain percentage of vehicles traveling, privacy concerns pose a serious problem that cannot be overlooked. Dion and Rakha (Dion and Rakha 2003) published a TTE method by using Automatic Vehicle Identification (AVI) methodology that is able to identify the same vehicle on both upstream and downstream check points. The method also provided a filter
algorithm to screen probe vehicle data in order to obtain precise travel time.
Some literature has stated that the linear regression model is acceptable for short-term travel time prediction, while long-term prediction needs to consider historical data. Zhang et al. (Zhang and Rice 2001) discusses short-term prediction using a time-varying coefficient linear model, but this is an off-line model that does not update real-time traffic data. The model only considers coefficients varying by time-of-the-day, but does not consider traffic distinctness between day-of-the-week. Huisken (Huisken and Berkum 2003) presented a short-term prediction algorithm based on Artificial Neural Network (ANN). It is suggested to use ANN to solve prediction problem since it is fully a multi-input problem. Moreover, Van Lint (Lint and Zuylen 2000) published one ANN algorithm (State Space Neural Network) for travel time prediction. Loop detectors, AVI, and video data were integrated together, with the results showing a 5 minute speed prediction with average error less than 10 percent and 10 minute speed prediction with average error around 16 percent.

The historical database is a necessary component to make long-term prediction Chien et al. (Chien, Liu et al. 2003) developed a prediction model by using Kalman filter technique and historical database by conducting CORSIM simulation validation. The mean absolute relative error, root relative square error, and maximum relative error are 2.8 percent, 3.8 percent, and 9 percent respectively, which is good. Similar research includes Yang (Yang, Yin et al. 2004) with "an on-line recursive short-term traffic prediction algorithm." Schrader et al. discusses a method to predict travel time by using a linear regression historical database. The linear regression model contains 10 parameters, but this model does not consider traffic variation between day-of-the-week. Additionally, there is other literature that discuss travel time prediction on arterial with signal control, such as W. Lin et al., "Arterial travel time estimation for ATIS" (Lin, Kulkarmi et al. 2004).

In this proposed research, data fusion is a key part because the proposed method needs to combine field detector data and simulation data. Xie (Xie, Cheu et al. 2004) proposed three data fusion models for fixed detector data and probe vehicle data, which are sensor data fusion, model data fusion, and hybrid data fusion. Two methods discussed in the paper are multi-layer perception (MLP, with fixed detector estimate, probe vehicle estimate, and probe vehicle sample size) and multi-layer regression (MLR, use a coefficient to control weight of detector data and probe). Sensitivity analysis showed MLP has superiority when probe percentage is low. Furthermore, data fusion was studied deeply in ADVANCE (Rouphail, Tarko et al. 1993; Tarko and Rouphail 1993; Thakuriah and Sen 1993) project in last decade; a very detailed data fusion process was presented by several reports.

Other various literatures discuss travel time estimation when an incident happens, where the main research focus is on how to estimate travel time delay caused by incident. This could be outlined as incident detection time, response time, clearance time, and recovery time.

## Chapter 3. Methodology

Given that the dissertation is composed of four separate publications, the methodology is described in terms of each of these four research efforts.

### 3.1 Traffic Variability and Historical Database

The first research effort involves conducting research to characterize typical daily traffic variability. Specifically, a historical database is created using field data constructed using loop detector data along the I-66 Virginia test bed. As mentioned earlier, the historical database is critical to TTE as it serves as a measure for the desired level of modeling accuracy, especially under typical traffic conditions. The database is composed of 3 to 6 months of traffic data, covering regular week and weekend days as well as some festival days and special events, such as road construction and unusual weather. All traffic data are measured using embedded single loop detectors. The data collected include vehicle time-mean speed, volume, and occupancy, all at 1-minute intervals on a lane-by-lane basis.

The historical database demonstrates traffic variation and basic traffic patterns by time-of-the-day, day-of-the-week, and location. First, the raw data are achieved at 15 -minute and 60 -minute intervals, which are the two major time intervals considered in the proposed research. Volume data is simply the arithmetic average of volume counts per lane [3]. Time-mean speed and occupancy are calculated as volume weighted averages by lane, [8] and [9].

$$
\begin{equation*}
q_{i j}=\frac{\sum_{n=1}^{N_{i}} q_{i j n}}{N_{i}} \tag{7}
\end{equation*}
$$

$q_{i j n}$ : Flow at location i on time j for lane $\mathrm{n}, \mathrm{veh} /$ hour/lane
$q_{i j}$ : Average flow at location i on time j for a direction, veh/hour
$N_{i}$ : Total number of lanes at location i

$$
\begin{equation*}
u_{i j}=\frac{\sum_{n=1}^{N_{i}} u_{i j n} \cdot q_{i j n}}{\sum_{n=1}^{N_{i}} q_{i j n}} \tag{8}
\end{equation*}
$$

$u_{i j n}$ : Time-mean speed at location i on time j for lane $\mathrm{n}, \mathrm{km} / \mathrm{h}$
$u_{i j}$ : Average time-mean speed at location i on time j for a direction, $\mathrm{km} / \mathrm{h}$

$$
\begin{equation*}
o c c_{i j}=\frac{\sum_{n=1}^{N_{i}} o c c_{i j n} \cdot q_{i j n}}{\sum_{n=1}^{N_{i}} q_{i j n}} \tag{9}
\end{equation*}
$$

occ $_{i j n}$ : Occupancy at location i on time j for lane $\mathrm{n}, \mathrm{sec} / \mathrm{min}$
occ $_{i j}$ : Average occupancy at location i on time j for a direction, $\mathrm{sec} / \mathrm{min}$

Two important points to note:

1) There is a HOV-2 (High Occupancy Vehicles) lane during morning peak hours from Monday to Friday in the eastbound direction of travel (5:30 a.m. - 9:30 a.m.) and in the westbound direction (3:00 p.m. -

7:00 p.m.). It is the first left lane of this 4 lane/direction freeway. The speed and occupancy data is calculated separately for HOV lanes.
2) Portion of the traffic data were not measured by loop detectors during the morning peak hour (5:00 a.m. - 10:00 a.m.) in the eastbound direction and afternoon peak hour (3:00 p.m. - 8:00 p.m.) in the westbound direction because the right shoulder was utilized as an extra lane. However, there were no loop detectors in this shoulder lane.

Dr. H. Rakha and Ms. M. Alejandra have spent significant efforts to put this chapter together.

### 3.2 Consistency between Shockwave and Queuing Theory

During the literature review, the Dynamics method developed by Nam and Drew was intensely studied. In computing delay, Nam and Drew claim a discrepancy between shock-wave analysis and queuing theory. This chapter tries to highlight the error and correct the equations that were derived by Nam and Drew. The equation evolvement was re-drawn literally and graphically, two bottleneck-delay examples involving time-varying for constant-arrival rates were demonstrated, with a comparison to calculations with shock-wave analysis and queuing theory, respectively. The results show that queuing theory provides a simple and accurate technique for estimating delay at highway bottlenecks. More importantly, these finds are regarded as the element for one of the TTE approaches discussed in Chapter 7. This chapter was completed by Dr. H. Rakha and the author.

### 3.3 Estimate space-mean speed from loop detectors

Speed measured by point loop detectors is time-mean speed. Link travel time can simply be calculated by knowing the length of the link and representative space-mean speed of the link. Therefore, it is required to estimate space-mean speed from time-mean speed before starting the TTE process.

Time-mean speed and space-mean speed are similar in nature; however, have significant differences in true meaning. Time-mean speed is defined as the arithmetic mean of the speed of vehicles passing a point during a given time interval [10]. Space-mean speed is defined as the arithmetic mean of the speed of those vehicles occupying a given length of road at a given instant [11].

To do this, one will need to follow these steps. Suppose there are N speed samples measured by one loop detector in a certain interval:

$$
\begin{align*}
& \bar{u}_{T M S}=\frac{\sum_{i=1}^{N} u_{i}}{N}  \tag{10}\\
& \bar{u}_{S M S}=\frac{N}{\sum_{i=1}^{N} \frac{1}{u_{i}}} \tag{11}
\end{align*}
$$

Where:
$\bar{u}_{T M S}:$ Time-mean speed ( $\mathrm{m} / \mathrm{s}$ )
$u_{\text {SMS }}$ : Space-mean speed (m/s)
N : Number of speed samples
$u_{i}$ : Speed sample i
It is well known that Wardrop's model (Wardrop 1952) can estimate time-mean speed from space-mean speed, [12]:

$$
\begin{equation*}
\bar{u}_{T M S} \cong \bar{u}_{S M S}+\frac{\sigma_{S M S}^{2}}{\bar{u}_{S M S}} \quad \text { (Wardrop's) } \tag{12}
\end{equation*}
$$

Here the model proposed [9] is:

$$
\begin{equation*}
\bar{u}_{S M S} \cong \bar{u}_{T M S}-\frac{\sigma_{T M S}^{2}}{\bar{u}_{T M S}} \tag{13}
\end{equation*}
$$

The kernel of this formulation is to utilize the variance of the time-mean speed as opposed to the variance of the space-mean speed; it produces an estimate error to within 0 to 1 percent, as is the case for the Wardrop formulation. A series of sensitivity analyses have been done on the proposed model and the results show that the proposed model can produce accurate space-mean speed from time-mean speed if the speed sample's variation is lower than 20 percent. By using this model, all loop detector point-test speed samples (time-mean speed) will be converted to space-mean speed, which can be employed by subsequent analysis (TTE and simulation).

Furthermore, this chapter also derives the relationship between space-mean speed variance and time-mean speed variance, and relationship between space-mean speed and travel time reliability. This observation links Chapters 4 and 7 together and perfectly explains the importance of constructing a historical database to TTE.

Dr. H. Rakha took the leading role in this chapter.

### 3.4 Travel time estimation method and simulation validation

This chapter focuses on TTE algorithms that are based on spot speed measurements. Several TTE approaches are introduced including a traffic dynamics TTE algorithm that is documented in the literature. This traffic dynamics algorithm is analyzed, highlighting some of its drawbacks and impacts on estimation accuracy brought by some key parameters, followed by proposed corrections to the traffic dynamics formulation. The proposed approach estimates traffic stream density from occupancy measurements as opposed to flow measurements at the onset of congestion. Next, the paper validates the proposed model using field data from I-880 and simulated data. Comparison of five different TTE algorithms is conducted. The comparison demonstrates that the proposed approach is superior to the TTE traffic dynamics approach. Particularly, a multi-link simulation network is built to test spot-speed-measurement TTE performance on multi links, as well as the data smoothing technique's effect on TTE accuracy. In summary, the chapter aims to find a feasible TTE procedure that is adaptive to various traffic conditions.

First, Dynamics approach is analyzed; enhancements of the approach and errors and corrections on congestion TTE are presented. Then the sensitivity analysis on interval size and factor " $m$ " is illustrated with validation on both field and simulation test beds. In order to overcome all these shortcomings, a proposed Dynamics model is presented. Main advantages of this approach include: estimating density from occupancy during light traffic condition, using density as threshold to divide traffic regime instead of $m$, and an updated equation for estimating travel time during congested condition.

The next step is to compare five approaches on the same test beds that include field and simulation runs. Approach 1 applies the original Nam and Drew Dynamics theory; Approach 2 is based on original Dynamics but uses occupancy to estimate density; Approach 3 applies Vanajakshi and Rilett's ideas of using occupancy to estimate density and applies a different equation to estimate travel time during the congested condition; Approach 4 uses occupancy for density and divides two traffic regimes by density threshold. Approach 5 is spot speed measurement method that uses the speed measured by a loop detector to assume spot speeds as representative of the link speed and then calculates the link travel time based on the spot speed measurement. Basically, there are three major methods to estimate travel time, including the half distance approach, average speed approach, and minimum speed approach seen below in Equations [14], [15], and [16], respectively.

Half Distance Approach:

$$
\begin{equation*}
t_{i-1, i}=\frac{1}{2}\left(\frac{d_{i}}{v_{i-1}}+\frac{d_{i}}{v_{i}}\right) \tag{14}
\end{equation*}
$$

$d_{i}$ : length of link i between loop i-1 and i
$t_{i-1, i}:$ travel time over link i between detectors i-1 and i
$v_{i-1}$ : speed at loop i-1
$v_{i}$ : speed at loop i

Average Speed Approach:

$$
\begin{equation*}
t_{i-1, i}=\frac{d_{i}}{\left(v_{i-1}+v_{i}\right) / 2} \tag{15}
\end{equation*}
$$

Minimum Speed Approach:

$$
\begin{equation*}
t_{i-1, i}=\frac{d_{i}}{v_{\min }} \tag{16}
\end{equation*}
$$

$v_{\text {min }}$ : minimum speed among $v_{i-1}$ and $v_{i}$
The evaluation of five approaches on I-880 field data demonstrates the advantages and disadvantages of each. I-880 data contains the complete list of traffic conditions on the freeway starting with free-flow, to formation of congestion, to heavily congested, and ending with dissolving of congestion. This is an ideal dataset to test each approach's performance, and under- or over-estimating is frequently found. Such outcomes play a key role in selection of the best approach, as the target is to provide the highest accuracy of TTE under diverse traffic conditions.

Last, a multi-link simulation test bed is introduced to examine the robustness of TTE on a long-length section, which has a more practical application. The travel time on each link is obtained first by spot speed measurement TTE, and then different data smoothing techniques are utilized to reduce the sudden traffic variance's impact on the whole section. Another reason the smoothing should be considered is once the adjacent loop detector fails or provides incorrect speed data, smoothing can help reduce its impact to adjacent link. The simulation scenarios contain the complete periods of free-flow condition, pre-peak condition with traffic increasing, peak-hour condition with maximum traffic, and post-peak condition with traffic decreasing. True travel time is obtained by the simulation probe vehicles and the two kernel smoothers used are Epanechnikov kernel and Triweight kernel.

$$
\begin{align*}
& \text { Epanechikov......... } \frac{3}{4}\left(1-\mu^{2}\right)  \tag{17}\\
& \text { Triwieght............. } \frac{35}{32}\left(1-\mu^{2}\right)^{3}
\end{align*}
$$

Dr. H. Rakha directed this chapter's structural format.

CHAPTER 4. TRAFFIC VARIABILITY AND HISTORICAL DATABASE

# Statistical Analysis of Spatiotemporal Link and Path Flow VARIABILITY 

Wang Zhang ${ }^{1}$ Alejandra Medina ${ }^{3}$, and Hesham Rakha ${ }^{2}$, Submitted to IEEE ITS Conference


#### Abstract

In the absence of advanced traveler information systems, commuters tend to select their routes of travel, within a congested network, primarily based on historical average travel times. Typical traffic conditions can be sufficient if a specific day is similar to these average conditions. However, if traffic conditions vary considerably from the norm, historical information may not be sufficient for commuters to make optimum travel decisions. Under these conditions the provision of real-time traffic information could offer significant benefits. Consequently, the proposed research effort attempts to characterize typical variability in traffic conditions using traffic volume data obtained from 31 dual-loop detector stations along a section of I-66 between Manassas and Vienna, VA during a 3-month period. The detectors logged time-mean speed, volume, and occupancy measurements for each station and lane combination. Using these data, the paper examines the spatiotemporal link and path flow variability on weekdays and weekends. The generation of path flows is made through the use of a synthetic maximum likelihood approach. Statistical Analysis of Variance (ANOVA)(Crow, Davis et al. 1960) tests are performed on the data. The results demonstrate that in terms of link flows and total traffic demand Mondays and Fridays are similar to core weekdays (Tuesdays, Wednesdays, and Thursdays). In terms of path flows, Fridays appear to be different from core weekdays.


### 4.1 INTRODUCTION

According to a Federal Highway Administration (FHWA) report (Institute 2006), not only congestion levels have risen in cities of all sizes, but they extend to more times of the day, more roads, affect more travel, and result in extra travel time when compared to the past. Evidence also suggests that travel-time reliability is valued at a significant "premium" by the traveling public. The accuracy of travel-time prediction is not only important to travelers and freight shippers, but also allows transportation agencies to make significant strides in congestion management, even with the growth of traffic demand (January 2003).
Reliability and variability are two terms closely related, but are slightly different in their focus. These key differences include how they are measured and how they are communicated. In engineering, reliability is defined as the capacity of a component, or a system, to perform as designed while variability occurs when successive observations of a system or phenomenon do not produce the exact same results. In transportation, reliability is related to the level of consistency in transportation service while variability is related to the amount of inconsistency in operating conditions. The two measures can be related due to the fact that the reliability of trip time is directly related to the variability in the performance of the trip route (Institute 2006).

When analyzing a set of system data, generally the higher the variability of the data the lower the system reliability is. In other words, when reliability is high the user can be more confident that the average data will be representative of typical traffic conditions that he/she will encounter in the field. Good field reliability data is crucial to: 1) detect problems; 2) size the magnitude or severity of these problems; 3) make well-informed decisions on proper corrective

[^0]actions; 4) and verify that corrective actions operate as anticipated.(Tobias P. A. and Trindade D. C. 1998)
The paper describes an analysis of flow count data from 31 stations along a section of I-66 between Manassas and Vienna, VA during a 3-month period. The primary purpose of this analysis is to ascertain whether there are significant differences in link flows both temporally and spatially and whether traffic conditions vary temporally from one day to another. The second objective is to study the effect of traffic flow variations on path flows (Origin-Destination [O-D] demands) as a function of time-of-day (TOD) and day-of-the-week (DOW).

In terms of the paper layout, initially the characteristics of the study network and the data collection timeframe are presented, followed by an overview of the study procedure, with a complete analysis of traffic flow and demand variations. The details of the Analysis of Variance (ANOVA)(Crow, Davis et al. 1960) tests are then described, followed by the study conclusions.

### 4.2 STUDY DESCRIPTION

### 4.2.1 Network Configuration

A 16-mile portion of the eastbound I-66 freeway in the state of Virginia was considered in this study as shown in Figure 4.1. This section, which travels from Manassas west to Vienna east, is equipped with loop detectors (LDs) that are typically spaced every 0.5 miles. The study area begins at Exit 46 and ends at Exit 62 and the present study focuses on the eastbound lanes heading toward Washington, DC. This section is composed of four lanes, including a left HOV-2 lane on weekdays from 5:30 a.m. to 9:30 a.m. There are seven on-ramps (one approaches from the north and the other six approach from the south) and seven off-ramps (all to the south), each having one or two lanes. In order to manage heavy traffic or special conditions, the right shoulder lane in some portions of the study area is open to traffic periodically, but there are no detectors covering this "shoulder lane." I-66 is a major freeway that serves the Northern Virginia Area.

A total of 31 LD stations were located along the study area (Figure 4.2) when the analysis was conducted; each station covered all lanes in both directions. In addition, there are several LDs located on on- and off-ramps. Of the 31 LDs eastbound, 20 are located in mainstream locations where there are no ramp effects (basic freeway sections). The remainder LDs covers both mainstream and on/off-ramps. The spacing between LD stations is variable, varying between 0.5 to 1 miles in length.

### 4.2.2 Data Coverage

VDOT collects data 24 hours a day, 7 days a week for all detectors. However, for several reasons common to this type of equipment, the data obtained does not cover all the days of the week, hours of the day, and locations in the study period. For this specific study, the analysis period covered a 3-month duration, from May 1, 2002 to July 31, 2002. The data included 23 days in May, 9 days in June, and 12 days in July. This dataset amounted to a total of 44 days of 1-minute aggregated data. The detailed data coverage information can be found in Table 4.1.

The analysis period included four Mondays, nine Tuesdays, eight Wednesdays, seven Thursdays, six Fridays, five Saturdays, and five Sundays. While data were available for some holidays (e.g. May 27, Memorial Day), these data were excluded from the dataset because they are not representative of typical traffic patterns.

The LDs measured and logged traffic stream time-mean speed, volume, and occupancy for each lane. The field data included the station number, LD number, speed, volume, occupancy, location, and several other elements in a standard text file. The raw field data were compiled in minute averages, and were aggregated into 15 -minute averages for purposes of this analysis. This step allowed for the creation of a manageable dataset while still capturing the temporal variation in traffic. For the purpose of this paper, traffic flow data were defined as the sum of the traffic flows across all lanes at the station under consideration. It should also be noted that in addition to the field data analysis, the network was simulated using the INTEGRATION software(Assoc. 2002) .

### 4.3 DATA ANALYSIS

An analysis of the traffic data is presented in order to assess the spatiotemporal variation in traffic conditions during typical weekdays. The analysis in this paper defines Monday through Friday as weekdays and Saturdays and Sundays as weekend days. Additionally, path flows (O-D tables) are generated using the QueensOD software (Assoc. 2002). An
analysis of the O-D data is presented in the paper in order to assess the variation in traffic demand by DOW and TOD. The analysis that is presented in this paper extends on a previous publication by Rakha and Van Aerde (Rakha and Van Aerde 1995) by introducing path flow and total demand variations to the analysis.
Since traffic variation is expected to depend more on the DOW than the day of the month, the analysis focuses on two comparisons. The first comparison assesses the variation of link and path flows between weekdays and weekend days, and the second comparison assesses the variation within weekdays.

The mean link flow for a specific DOW $k$ is computed as

$$
\begin{equation*}
\bar{q}_{i j k}=\frac{\sum_{d=1}^{N_{k}} q_{i j k d}}{N_{k}} \quad \forall i, j, k, \tag{1}
\end{equation*}
$$

Where: $i$ is the location index (ranges from 1 to 31 ), $j$ is the time index (ranges from 1 to 96 ), and $d$ is the day index number for a specific DOW (e.g. ranges from 1 to 4 for Mondays and 1 to 9 for Tuesdays).
Similarly, the average path flow (O-D demand) between zones $l$ and $m$ for time interval $i$, and day-of-the week $k$ can be computed as

$$
\begin{equation*}
\bar{T}_{i l m k}=\frac{\sum_{d=1}^{N_{k}} T_{i l m k d}}{N_{k}} \quad \forall i, l, m, k \tag{2}
\end{equation*}
$$

The link flow standard deviation can computed as

$$
\begin{equation*}
\sigma_{i j k}^{q}=\sqrt{\frac{\sum_{d=1}^{N_{k}}\left(q_{i j k d}-\bar{q}_{i j k}\right)^{2}}{N_{k}-1}} \quad \forall i, j, k \tag{3}
\end{equation*}
$$

Similarly, the path flow standard deviation can computed as

$$
\begin{equation*}
\sigma_{i j k}^{T}=\sqrt{\frac{\sum_{d=1}^{N_{k}}\left(T_{i l m k d}-\bar{T}_{i l m k}\right)^{2}}{N_{k}-1}} \quad \forall i, l, m, k \tag{4}
\end{equation*}
$$

Finally, the coefficient of variation (CV) can be computed as the standard deviation divided by the mean ( $C V^{q}{ }_{i j k}$ and $C V^{T}{ }_{i l m k}$ ). Consequently, the CV provides a normalized measure of data dispersion and thus can be used for comparison across different roadway segments. As part of this research effort, the link and path flow mean and CV are computed for each 15-min interval within a day, location (in the case of link flows) or O-D pair (in the case of path flows), and DOW.

### 4.3.1 Traffic Flow Variation

To study the flow variation, six representative station detectors were selected from the 20 mainstream stations. These six stations included locations on a basic freeway section, within the influence area of an on-ramp, and within a weaving section, as shown in Figure 4.2. Consequently, these stations provide good insight into the behavior of the study section.
Figure 4.3 illustrates the temporal variation in link flows at each of the six detectors for weekdays and weekends. In the case of weekdays (Figure 4.3a), the figure illustrates a noticeable peak during the morning peak period (4:30 to 10:00 a.m.), with a maximum flow of $8,000 \mathrm{veh} / \mathrm{h}$. Traffic volumes decay after the morning peak and remain fairly constant before increasing slightly for the afternoon peak ( $4: 00$ to $6: 00 \mathrm{p} . \mathrm{m}$.). Subsequently, the volumes decrease sharply after 6:00 p.m. Alternatively, the weekend flows (Figure 4.3b) illustrates a steady increase in flows between 4:30 a.m. and 12:00 p.m., with a maximum flow of $6,000 \mathrm{veh} / \mathrm{h}$ at noon that remains fairly constant from noon to $6: 00 \mathrm{p} . \mathrm{m}$. and decreases afterwards. In both cases, the maximum flow is observed at location 5, which is located at the west end of the corridor, followed by location 3. In general lower flows are observed at the east end of the corridor at location 0 (less than the flows at location 5 by approximately $4000 \mathrm{veh} / \mathrm{h}$ ).
When the temporal variation of flow within a day is plotted for each weekday, similar trends are observed (Figure 4.4), with Mondays showing the lowest morning peak flow among the various weekdays. In general, variations between the six station locations is less noticeable during the peak morning hours. When the data are analyzed for a specific DOW
(seven Tuesdays) for each detector, the flows follow the same trend (Figure 4.5). Stations 0 and 1 show a more pronounced morning peak with lower flows from 10:00 a.m. to 6:00 p.m. when compared to other locations. The flow variations among different Tuesdays for a specific location is almost non existent for these two locations and very small for other locations.

In order to study the variability in traffic flow a combined CV for all 6 detector locations was computed as

$$
\begin{equation*}
\overline{C V}_{i k}^{q}=\frac{\sum_{j=1}^{n} C V_{i j k}^{q}}{n} \tag{5}
\end{equation*}
$$

where $n$ is the number of locations.
The temporal variation in the average CV demonstrates a peaking from 3:00 a.m. to 5:00 a.m. (40\%) on weekdays and remains relatively low otherwise (less than $20 \%$ ), as illustrated in Figure 4.6. The peak experience at midnight can be explained by the lower traffic volumes (lower denominator) during this portion of the day. For the common driver it means that the historical data is less reliable during the morning peak than for the rest of the day. In the case of weekends, the CV experiences some peaks during midnight and from $5 \mathrm{a} . \mathrm{m}$. to $8 \mathrm{a} . \mathrm{m}$. Otherwise the CV oscillates between $10 \%$ and $20 \%$.

When the data are analyzed for each of the weekdays, Tuesdays and Wednesdays experienced the most variability with values up to $80 \%$ around 4:00 a.m. However, if not for some localized peaks the flow variability is less than $40 \%$ for most of the day. Alternatively, analyzing Saturdays and Sundays separately, it is clear that the majority of CV variations occur on Saturdays, however, the CV values remain under $20 \%$, and are very stable from 10:00 a.m. to 6:00 p.m. (less than $10 \%$ ).

### 4.3.2 Lane Usage

In analyzing the traffic flow variation across lanes, a specific day (Tuesday, May 7, 2002) was analyzed in more detail. Detectors 2, 5, 9, 15, and 19 were selected for the analysis (as shown on Figure 4.2). The first three locations were selected to study lane variations on a basic freeway section; the fourth location was selected because of its proximity to an entrance and exit ramp ( 700 m distance) mimicking a weaving section; and detector 19 was selected because of its proximity to an on-ramp ( 250 m downstream of the on-ramp). All locations are four-lane sections; Lane 1 is the inner left lane (median), and Lane 4 is the outer right lane. It should be noted that from 5:30 a.m. to 9:30 a.m. Monday through Friday, Lane 1 is utilized as an HOV-2 lane.
As shown in Figure 4.7, the lane volume distribution for LDs 2 , 5 , and 9 are very similar. Specifically, Lane 1 carries the lowest traffic volume during the entire day, when operating as an HOV-2 lane and normal lane. Lanes 2 and 3 are highly utilized at all times, especially during the morning peak hours. Lanes 2,3 , and 4 are equally utilized during the evening hours when traffic is low. For LD 15, Lane 4 carries the majority of traffic. This can be explained by the fact that the location of LD 15 is between Exits 53 and 54 and is behaving as a weaving section. The four-lane volume variations for LD 19 are very similar to LD 2, with Lane 2 and 3 carrying the higher traffic volumes. The heavy congestion with extremely low average speed, might explain the significant flow drop for all lanes around 8:00 a.m.

### 4.3.3 Path Flow Variation

The path flows or O-D demands were estimated using the QueensOD software (Assoc. 2002) by numerically solving(Aerde, Rakha et al. 2003)

$$
\begin{equation*}
\text { Max. } \quad T_{i k} \ln \left(\frac{T_{i k}}{t_{i k}}\right)-\sum_{l m} T_{i l m k} \ln \left(\frac{T_{i l m k}}{t_{i l m k}}\right)-\sum_{l m}\left(\lambda_{i l m k} \cdot 2\left(\sum_{a}\left(V_{a i k} \cdot p_{i l m k}^{a}\right)-\left(\sum_{a} p_{i l m k}^{a}\left(\sum_{x y} T_{i x y k} p_{i x y k}^{a}\right)\right)\right)\right) \quad \forall i, k, \tag{6}
\end{equation*}
$$

where $T_{i l m k}$ is the estimated path flow between zones $l$ and $m$ for time interval $i$ and DOW $k, t_{i l m k}$ is the seed path flow between zones $l$ and $m, T_{i k}$ is the summation of all path flows $\left(T_{i l m k}\right)$ across all O-D combinations (summation of $(l \times m-l)$ path flows), $t_{i k}$ is the summation of all path flows $t_{i l m k}$ across all O-D combinations, $p^{a}{ }_{i l m k}$ is the probability of path flow $T_{i l m k}$ utilizing link $a$, and $V_{\text {aik }}$ is the actual observed link volume on link $a$ during time interval $i$ for DOW $k$.

The numerical solution begins by building a minimum path tree and performing an all-or-nothing traffic assignment of the seed matrix. A relative or absolute link-flow error is computed depending on user input. Using the link-flow errors,

O-D adjustment factors are computed and utilized to modify the seed O-D matrix. The adjustment of the O-D matrix continues until one of two criteria is met: the change in O-D error reaches a user-specified minimum, or the number of iterations criterion is met. If additional trees are to be considered, the model builds a new set of minimum-path trees and shifts traffic gradually to the second minimum-path tree. The minimum objective function for two trees is computed in a similar fashion, as described for the single tree scenario. The process of building trees and finding the optimum solution continues until all possible trees have been explored (Assoc. 2002). In the case of the I-66 study section given that each O-D pair had a single path, a total of 44 path flows were generated for each interval/DOW combination using the QueensOD software.

### 4.3.3.1 Total Demand

The total demand is defined as the arithmetical summation of all 44 path flows generated by the QueensOD software. The total demand mean and CV were compared for each 15-minute interval and DOW combination. The results demonstrate that during the morning peak period (from 5:00 a.m. to 10:00 a.m.) a total demand of approximately $15,000 \mathrm{veh} / \mathrm{h}$ is observed for a typical weekday, as illustrated in Figure 4.8. Alternatively, the weekend demand increases slowly at 7:00 a.m., reaching its peak around 11:00 a.m. and maintaining this peak demand for approximately 6 hrs, reaching values up to $11,000 \mathrm{veh} / \mathrm{h}$.

When each day is analyzed in more detail (Figure 4.8b), similar trends are observed with almost identical values for each weekday. A higher variability in traffic demand is observed on Tuesdays, while Saturdays show a higher demand when compared to Sundays for some portion of the day (Figure 4.8c). When the entire data is plotted by DOW, Figure 4.9 is obtained, showing that the variability among the same DOW is almost non existent. The results demonstrate that one specific day-of-the-week, for example, Wednesday is a very good representation of what can be expected on a different Wednesday. For weekdays, the CV remains relatively constant between 5:00 a.m. and 11:00 p.m. (10-20\%), reaching higher values during the early morning hours (from 3:00 a.m. to 5:00 a.m., 60\%), as illustrated in Figure 4.10. In the case of weekends, a peak is experienced during 5:00 a.m. to 10:00 a.m. and remains relatively constant for the remainder of the day.

When each day is analyzed in detail (Figure 4.10b), it is observed that the demand CV is very similar to the flow CV. Tuesday experiences more variability during the daylight hours, but higher CV values (approximately $80 \%$ ) are observed on Mondays and Fridays during the early morning hours. In the case of the weekends, the demand CV is very similar to the link flow CV maintaining constant values around $10 \%$ for most of the day and experiencing some localized peaks from midnight to 6: $00 \mathrm{a} . \mathrm{m}$.

### 4.3.3.2 Path Flows

Each of the 44 path flows was analyzed to assess the variation within the different DOWs. Analysis of the data revealed that flow paths $1,2,25,35$, and 40 were significantly higher than the other path flows and represent approximately $75 \%$ of the total demand. These path flows (O-D flows) represent O-D \#1: west origin to east end; O-D \#2: west origin to exit 52 Lee Hwy; O-D \#25: Sully Rd on-ramp to Exit 55 Fairfax County Pkwy; O-D \#35: Fairfax County Pkwy on-ramp to the east end; and O-D \#40: Lee Jackson Memorial Hwy to the east end. Consequently, these demands are analyzed in further detail.

By comparing weekday and weekend path flows, the morning peak for the weekends starts later than weekdays (9:30 a.m.), afterwards the pattern is similar. When each day is analyzed, Monday experiences the lowest peak demand 1,400 $\mathrm{veh} / \mathrm{h}$, but higher variability in the different path flows in comparison to the other weekdays. Tuesdays experience a peak of up to $2,000 \mathrm{veh} / \mathrm{h}$ with a low variability for each path flow. For the remainder of the weekdays' path flow values of up to $2,500 \mathrm{veh} / \mathrm{h}$ are reached.

When the CV for the mean weekday and weekend is analyzed, a significant difference between weekdays and weekends is observed. Specifically, there is more variability in the CV during weekend days, however higher values of the CV (less reliability) are typical of weekdays (up to $300 \%$ ). When each day is analyzed separately the results demonstrate significant differences among the different days of the week.

### 4.3.4 Statistical Analysis

### 4.3.4.1 ANOVA Test

Crow et al. (Crow E.L., Davis F.A. et al. 1960) state that "the data obtained from an experiment involving several levels
of one or more factors are analyzed by the technique of analysis of variance. This technique enables us to break down the variance of the measured variable into the portions caused by the several factors, varied singly or in combination, and a portion caused by experimental error. More precisely, analysis of variance consists of (1) partitioning of the total sum of squares of deviations from the mean into two or more component sums of squares, each of which is associated with a particular factor or with experimental error, and (2) a parallel partitioning of the total number of degrees of freedom." The assumptions of the ANOVA technique are that the dependent variable populations have the same variance and are normally distributed (LIttell, Freund et al.). The distinction between normality and non-normality of data and what should be done in the case non-normality exists are two controversial issues. Some researchers, on the one hand, believe that the ANOVA technique is robust and can be used with data that do not conform to normality. On the other hand, other researchers believe that non-parametric techniques should be used whenever there is a question of normality. Research has shown that data that do not conform to normality due to skewness and/or outliers can cause an ANOVA to report more type 1 and type 2 errors (Ott L. 1988). Conover (Conover W. J. 1980) recommends use of ANOVA on raw data and ranked data in experimental designs where no non-parametric test exists. The results from the two analyses can then be compared. If the results are nearly identical then the parametric test is valid. If the rank transformed analysis indicates substantially different results than the parametric test, then the ranked data analysis should be used.

In an attempt to characterize the observed variations in link flow, path flow and total demand and to develop dependency relationships several ANOVA tests were conducted on the data. To complete this analysis, three datasets were used, including the following: 1) a dataset of link flows as a function of the DOW, TOD, and location variables; 2) a dataset of demand as a function of DOW and TOD factors; 3) and a dataset of path flows as a function of the DOW, TOD, and path-flow number variables. Due to space limitations only the first analysis is presented in detail here, however, all results are summarized in Table 4.2.

The data were firstly checked for any abnormal traffic conditions, such as vehicle detector failures or missing data. The selection process resulted in 34 weekdays and 10 weekend days being considered. For each effective day, three matrices of demand, flow, and path flows were generated, the matrices are $i$ rows (number of time interval in the day) and $j$ columns (number of flow locations or number of path flows).
To investigate whether the variability of traffic conditions between weekdays (Monday to Sunday) is statistically significant, a single-factor ANOVA was conducted. Core weekdays were considered to be Tuesday through Thursday, as it was initially not evident if Mondays or Fridays could be considered as core weekdays. The ANOVA tested if the root mean square error (RMSE) between different day spatiotemporal surface contours were different from core weekday surfaces when compared to within day differences as

$$
\begin{equation*}
R M S E=\sqrt{\frac{\sum_{i=1}^{I} \sum_{j=1}^{J}\left(\bar{q}_{i j k}-\bar{q}_{i j, k=2 \sim 4}\right)^{2}}{I \times J}} \quad \forall k, \tag{7}
\end{equation*}
$$

where $i$ is the number of 15 -minute periods in the day and $j$ is the number of locations (in the case of link flows) or the number of path flows (in the case of path flows). It should be noted that all comparisons are made to the core weekdays ( $k=2,3$, and 4).

These results demonstrate that there is no statistical difference between the observations for Tuesdays, Wednesdays and Thursday at the $95 \%$ confidence level, as summarized in Table 4.2. Consequently, the data for these days are grouped together as core weekdays. When the link flows are analyzed, Mondays and Fridays are also not statistically different from the core weekdays. Similarly, the path flow and total demand analysis demonstrate that there is no statistical difference between the observations for the core weekdays at the $95 \%$ confidence level. However, the Monday total demand is found to be statistically different from the core weekdays at a level of significance of $95 \%$, while Fridays are found to be similar to core weekdays. When the path flows are considered the results indicate that Mondays are not statistically different from core days at a level of significance of $95 \%$, considering demand is a summation of all O-D pairs, this fact shows that the distribution pattern of path flows is similar; however, the total demand is statistically different. The results demonstrate that the Friday path flows are statistically different from the core weekday path flows. Consequently, in this case the distribution of path flows is different while the total demand is similar to core weekdays. The results for Saturdays and Sundays are consistent across all three factors: link flows, path flows, and total demand.
In order to examine homogeneity of the variance assumption within the ANOVA procedures, the variation in residuals as
a function of the estimated values is plotted in Figure 4.13 (Stewart J. A., Rakha H. et al. 1995). The Studentized residuals are used because it is convenient to refer them against a $t$ contribution. The residual errors for the total demand, link flows, and path flows were all within two standard deviations as illustrated in Figure 4.13.

### 4.3.4.2 Sum of Squared Error Measure

In comparing traffic conditions across different days, a regression measure similar to $R^{2}$ that was proposed by Rakha and Van Aerde (Rakha and Van Aerde 1995) is utilized. For each of the three variables (link flows, path flows, and total demand) the squared error about the average core weekday surface is estimated as the difference for each cell (combination of time interval and location) from the average core weekday surface. For example, in the case of the link flow variable the error for day $k$ is computed as

$$
\begin{equation*}
S_{1 k}=\sum_{i=1}^{I} \sum_{j=1}^{J}\left(\bar{q}_{i j k}-\bar{q}_{i j, k=2 \sim 4}\right)^{2} \quad \forall k . \tag{8}
\end{equation*}
$$

The sum of the link flow squared error for each day about its respective overall mean is then estimated as

$$
\begin{equation*}
S_{t k}=\sum_{i=1}^{I} \sum_{j=1}^{J}\left(\bar{q}_{i j k}-\bar{q}_{k}\right)^{2} \quad \forall k \quad \text { where } \bar{q}_{k}=\frac{\sum_{j=1}^{J} \sum_{i=1}^{I} \bar{q}_{i j k}}{I \times J} . \tag{9}
\end{equation*}
$$

The sum-of-squared error explained by the average link flow spatiotemporal surface for day $k\left(S_{2 k}\right)$ can be estimated as the difference between $S_{t k}$ and $S_{l k}$. Thus, an analogous $R^{2}$ measure can be estimated as the amount of error captured by the average weekday surface. An $R^{2}$ of 1 means that the average surface explains $100 \%$ of the squared error and a value of 0 means the average surface does not explain any of the error. Similar measures were computed for the path flow and the total demand variables.
The variation of the analogous $R^{2}$ parameter over the 44 days ( 34 workdays and 10 weekend days) is presented in Figure 4.14. It appears that the $R^{2}$ for the total demand for all weekdays is considerably high (between 80 and $100 \%$ ), but the fact that Mondays have slightly lower values than core weekdays is consistent with the ANOVA results. The Saturday and Sunday demand surface $R^{2}$ differs considerably from weekdays as would be expected. The variation of $R^{2}$ over the 44 days for the link flow variable ranges between 60 and $80 \%$ without any notable differences among the weekdays. As would be expected, Saturday and Sunday link flow measures are very low (ranging from 0 to $40 \%$ ). When the path flows are considered the $R^{2}$ appears to be more scattered with values ranging between 40 to $80 \%$ for weekdays and 10 to $60 \%$ for weekend days.

### 4.3.4.3 Success Measure

Another measure proposed by Rakha and Van Aerde (Rakha and Van Aerde 1995) is termed the success measure. This measure counts the number of spatial/temporal observations that fall inside the typical day confidence limits to provide a measure of success. The estimation of the typical day confidence limits requires defining the variable distribution. Given that the original loop detector measurements were made at 1 -minute intervals and aggregated into 15 -minute observations. Using the central limit theorem, it can be assumed that each of these 15 -minute observations may become normally distributed because the 15 -minute observation on one day should not be correlated with the same observation to another day. To verify this assumption, a 15 -minute estimate of link flows for a day was stratified into bins. The observed probabilities were then tested using a chi-square goodness-of-fit test in order to establish whether the normal distribution was valid as shown on Figure 4.15 . The analysis showed that the observed 15 -minute flows were not statistically different from the expected outcome of a normal distribution.

An average proportion of cells within the core weekday confidence limits can be computed by estimating the upper and lower bounds assuming a normal distribution as

$$
\begin{equation*}
q_{i j}^{u}=\bar{q}_{i j, k=2 \sim 4}+z_{\alpha / 2} \times \sigma_{i j, k=2 \sim 4}^{q} \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
q_{i j}^{l}=\bar{q}_{i j, k=2 \sim 4}-z_{\alpha / 2} \times \sigma_{i j, k=2 \sim 4}^{q} \tag{11}
\end{equation*}
$$

for the link flow variable, where the standard deviation is computed using Equation [3]. The proportion of successful variables is then estimated as

$$
p_{k d}=\frac{n_{k d}^{s}}{n_{k d}}
$$

where $n_{k d}^{s}$ is the number of variables within the confidence limits for day $d$ of DOW $k, n_{k d}$ is the number of spatiotemporal observations for day $d$ of DOW $k$ (typically $I \times J$ ) if all cells have data, and $p_{k d}$ is the probability of success.

In computing the lower bound success rate, the average success rate for the core weekdays (Tuesdays, Wednesdays, and Thursdays) is computed as

$$
\begin{equation*}
\bar{p}=\frac{\sum_{k=2}^{4} \sum_{d=1}^{N_{k}} p_{k d}}{\sum_{k=2}^{4} N_{k}} \tag{12}
\end{equation*}
$$

The lower bound success rate for day $d$ of DOW $k$ is computed as

$$
\begin{equation*}
p_{k d}^{l}=\bar{p}-3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n_{k d}}} \quad \forall k, d \tag{13}
\end{equation*}
$$

Figure 4.16 shows how flow $p_{k d}^{l}$ varies for the different days that were analyzed. The weekday link flow success rate fell within the confidence limits for all observations except for a couple of observations on Friday, an observation on Tuesday, and one on Thursday. For Saturdays and Sundays, none of the success rates fell within the confidence range. The results for the path flows provide a better success rate for weekdays, however, differences between weekend and weekdays is noticeable.

### 4.4 Study Conclusions

Many ITS technologies attempt to explore the fact that traffic conditions on one day may be quite different from a similar previous day. This paper attempts to quantify these similarities and differences by considering link flows, path flows, and total demand. Several conclusions are drawn from this study that can be generalized given that I-66 highway appears to be reflective of typical traffic conditions (recurring congestion) on North American urban freeways.

The spatiotemporal variation of link flows within the different days of the week (Monday through Friday) appears to be highly similar and consistent. As would be expected, the analysis demonstrates that weekend spatiotemporal behavior is different from weekday behavior. For both weekdays and weekends, the flow coefficient of variation within a day of the week is very low for most days, with higher link flow coefficients of variation during extremely low flows (early morning hours).

Regarding the lane distribution on four-lane freeways along basic freeway sections that are not in the influence area of merges, diverges, or weaving sections, the middle lanes; Lane 2 and Lane 3 are highly utilized by vehicles. Lane 4 (shoulder lane) occupation is similar to that of Lanes 2 and 3 only during light flow conditions. Lane 1 (median lane) is always under utilized. However, it should be noted that the median lane acted as an HOV-2 lane during the a.m. peak period, which could affect the results for the congested periods.
The mean demand for weekdays and weekends has a very similar trend after 10:00 a.m. The CV value of demand is very low for both weekdays and weekends showing that the total demand does not overtly change from day to day. Demand conditions within core weekdays appear to be highly similar and consistent. The study demonstrates that in terms of path flows Fridays appear to be different from core weekdays. As was the case with link flows Saturdays and Sundays are different from core weekdays.
The residual errors for demand, link flow, and path flow are all within two standard deviations supporting the homogeneity of variance assumption for ANOVA. The ANOVA results demonstrate that Monday traffic demand conditions differ from core weekdays. In addition, Friday path flows are different from core weekdays. Consequently, the analysis concludes that only Tuesdays, Wednesdays, and Thursdays should be considered as core weekdays.
The success measure of traffic condition parameters can be used to distinguish statistically between significant and insignificant variations from typical traffic conditions. The method could be developed further to operate in real-time as part of an online travel time estimation system, while values inside the confidence limits indicate that historical data can
be used, a p-value outside the confidence range would indicate suspicious observations, further failures can then trigger the need to abandon the use of historical data for travel-time prediction.

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Table 4.1: Data Availability on I-66

| Week | Monday <br> Date |  | Tuesday <br> Date |  | Wednesday Date |  | Thursday Date |  | Friday Date |  | Saturday <br> Date |  | Sunday <br> Date |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  | ) | 0\% | ) | 90\% |  | 100\% | 4-May | 100\% | , | 00\% |
| 2 | 6- | 90\% | 7-Ma | 100\% | 8-May | 80\% | 9-May | 0\% | 10-May | 0\% | 11-May | 0\% | 12 | 0\% |
| 3 | 13-May | 0\% | 14-May | 0\% | 15-May | 90\% | 16-May | 85\% | 17- | 75\% | 18-May | 100\% |  | 60\% |
| 4 | 20 | 90\% | 21-M | 55\% | 22 | 85\% | 23-M | 95\% | 2 | 0\% | 25-May | 99\% | 26-May | 40\% |
| 5 | 27-May | 0\% | 28-May | 50\% | 29-May | 90\% | 30-May | 85\% | 31-May | 40\% | 1-Jun | 95\% | -Jun | 65\% |
| 6 | 3-Jun | 95\% | 4-Jun | 35\% | -Jun | 0\% | 6-Jun | 100\% | 7-Jun | 75\% | 8-Jun | 0\% | 9-Jun | \% |
| 7 | 10-Jun | 0\% | 11-Jun | 0\% | 12-Jun | 0\% | 13-Jun | 0\% | 14-Jun | 0\% | 15-Jun | 0\% | 16-Jun | 0\% |
| 8 | 17-Jun | 0\% | 18-Ju | 70\% | 19-Jun | 95\% | 20-Jun | 0\% | 21-Jun | 0\% | 22-Jun | 0\% | 23-Jun | 0\% |
| 9 | 24-Jun | 0\% | 25-Jun | 0\% | 26-Jun | 0\% | 27-Jun | 0\% | 28-Jun | 0\% | 29-Jun | 0\% | 30-Jun | 0\% |
| 10 | 1-Jul | 0\% | -Jul | 0\% | -Jul | 0\% | -Jul | 0\% | 5-Jul | 0\% | 6-Jul | 0\% | 7-Jul | 0\% |
| 11 | 8-Jul | 0\% | 9-Jul | 98\% | 10-Jul | 100\% | 11-Jul | 100\% | 12-Jul | 100\% | 13-Jul | 0\% | 14-Jul | 0\% |
| 12 | 15-Jul | 0\% | 16-Jul | 0\% | 17-Jul | 0\% | 18-Jul | 75\% | 19-Jul | 100\% | 20-Jul | 98\% | 21-Jul | 100\% |
| 13 | 22-Jul | 95\% | 23-Jul | 60\% | 24-Jul | 0\% | 25-Jul | 0\% | 26-Jul | 0\% | 27-Jul | 0\% | 28-Jul | 0\% |
| 14 | 29-Jul | 0\% | 30-Jul | 95\% | 31-Jul | 95\% |  |  |  |  |  |  |  |  |

Note: $\quad \%$ : Percentage of data coverage

Table 4.2: Single-factor ANOVA Results

| Description | ANOVA groups | DF | F | F-critical | Sig(95\%) |
| :---: | :--- | :---: | :---: | :---: | :---: |
| Link Flow | Tue vs. Wed vs. Thur | 21 | 3.06 | 3.47 | No |
|  | Core-weekday vs. Mon. | 26 | 0.19 | 4.23 | No |
|  | Core-weekday vs. Fri. | 28 | 4.09 | 4.20 | No |
|  | Core-weekday vs. Sat. | 27 | 93.5 | 4.21 | Yes |
|  | Core-weekday vs. Sun. | 27 | 193.4 | 4.21 | Yes |
| Total Demand | Tue vs. Wed vs. Thur | 21 | 2.51 | 3.47 | No |
|  | Core-weekday vs. Mon. | 26 | 5.54 | 4.23 | Yes |
|  | Core-weekday vs. Fri. | 28 | 0.01 | 4.20 | No |
|  | Core-weekday vs. Sat. | 27 | 221.8 | 4.21 | Yes |
|  | Core-weekday vs. Sun. | 27 | 391.1 | 4.21 | Yes |
| Path Flow | Tue vs. Wed vs. Thur | 21 | 0.13 | 3.47 | No |
|  | Core-weekday vs. Mon. | 26 | 0.12 | 4.23 | No |
|  | Core-weekday vs. Fri. | 28 | 5.68 | 4.20 | Yes |
|  | Core-weekday vs. Sat. | 27 | 15.4 | 4.21 | Yes |
|  | Core-weekday vs. Sun. | 27 | 30.7 | 4.21 | Yes |



Figure 4.1: Map of I-66 test bed section


Figure 4.2: I-66 East Bound Network Description
a. Mean flows for weekdays

b. Mean flows for weekend days

The mean for weekend


Figure 4.3: Mean of 6 location flows for weekdays and weekend days






Figure 4.4: Mean of 6 location flows for each Day-of-the-Week


Figure 4.5: Average flow over 7 Tuesdays for Flow \#0, \#1, \#2, \#3, \#4, and \#5, respectively
a: Average CV flow for weekdays and weekends

b: CV Flows for each day of weekday

b: CV Flows for weekends


Figure 4.6: Flow CVs for each Day-of-the-Week


Figure 4.7: Volume Distribution for different lanes
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b: Demand for Monday, Tuesday, Wednesday, Thursday, and Friday

c: Demand for Saturday and Sunday


Figure 4.8: Demands for each Day-of-the-Week


Figure 4.9: Total Demand for each Day-of-the-Week, all samples. Monday through Sunday
a: Demand CV for Weekday and Weekend

b: Demand CV for Monday, Tuesday, Wednesday, Thursday, and Friday

c: Demand CV for Saturday and Sunday


Figure 4.10: Demand CVs for each Day-of-the-Week
a: Variation in residual errors as a function of RMSE estimate for Demands

b: Variation in residual errors as a function of RMSE estimate for Flows

c: Variation in residual errors as a function of RMSE estimate for ODs


Figure 4.11: Variation in residual errors as a function of RMSE estimate for demands, link flows, and path flows
a: $R^{2}$ Variation for Demands

b: $\mathrm{R}^{2}$ Variation for Flows

c: $R^{2}$ Variation for ODs


Figure 4.12: R2 Variations for Demands, Flows and ODs


Figure 4.13: Observed and normal distribution in 15 min flow estimates for $\mathbf{2 4}$ core weekdays
a: Variation of $p$ for Demands

b: Variation of p for Flows

c: Variation of $p$ for ODs


Figure 4.14: Variations of $p$ for Demands, Flows, and ODs

CHAPTER 5. CONSISTENCY OF SHOCKWAVE AND QUEUING THEORY IN DELAY ESTIMATION

# Consistency of Shock-wave and Queuing Theory Procedures for Analysis of Roadway Bottlenecks 

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#### Abstract

A number of literatures have described queuing and shock-wave analyses as separate tools for solving bottleneck problems. Although some interrelationships between the two methods have been described in the literature, a number of these literatures have claimed that deterministic queuing theory and shock-wave analysis are fundamentally different producing different delay estimates (McShane and Roess (1990)(McShane and Roess 1990); Nam and Drew (1998)(Nam D.H. 1998)). For example, Nam and Drew (1998)(Nam D.H. 1999) mention that "deterministic queuing analysis always underestimates the overall magnitude of delays compared to shock-wave analysis." Furthermore, Nam and Drew claim that "the area between the demand and capacity curves in a queuing diagram is analytically equivalent to the total vehicle-hours of travel in congestion as opposed to the widely accepted total vehicle-hours of delay." Alternatively, this paper demonstrates the consistency in delay estimates that are derived from queuing theory and shock-wave analysis and highlights the common errors that are made in the literature with regards to shock-wave analysis delay estimation. Furthermore, the paper demonstrates that the area between the demand and capacity curves can represent the total delay or the total vehicle-hours of travel if the two curves are spatially offset (i.e. the count locations are at different spatial locations on the highway).


## Key words:

## Shockwave, Queuing theory, Delay

### 5.1 INTRODUCTION

A cumulative plot is the graph of a function $N(t)$ that gives the cumulative number of vehicles (or other moving objects) that pass an observer at time $t$ starting from an arbitrary initial count (Daganzo (2000)(Daganzo 2000)) and are used to analyze the flow of items past a number of bottlenecks. Their usefulness in hydrologic synthesis was recognized for over a century and form the basis for a technique known as 'mass curve analysis' for determining capacity of reservoirs. Cumulative plots were introduced to the transportation arena by Moskowitz (1963)(Moskowitz and Newman 1963) and then again by Gazis and Potts (1965)(Gazis and Potts 1965), but Newell demonstrated its full potential as an analysis tool (Daganzo (2000)(Daganzo 2000)). These cumulative plots are used to describe how items compete for service time through a node.

Traffic flow can be characterized using flow, density and speed through an analogy with fluid dynamics. Lighthill and Whitham (1955)(LightHill and Whitham 1955), as well as Richards (1956)(Richards 1956), made the first successful attempts at such a description. They both demonstrated the existence of traffic shock-waves and proposed a first theory of one-dimensional waves that could be applied to the prediction of highway traffic flow behavior. Equations 1 and 2 represent their model. The first equation defines the relation between flow, density, and speed that has been developed from the application of fluid dynamics theory. Using Equation 1, Equation 2 was then developed to describe the speed at which a change in traffic characteristics, or a shock-wave, propagates along a roadway considering the conservation of flow at the shock-wave.

$$
\begin{equation*}
q_{i}=k_{i} \cdot u_{i} \tag{1}
\end{equation*}
$$

[^1]\[

$$
\begin{equation*}
w_{i j}=\frac{q_{j}-q_{i}}{k_{j}-k_{i}} \tag{2}
\end{equation*}
$$

\]

The main difference between shock-wave and deterministic queuing models is in the way vehicles are assumed to queue upstream of a bottleneck. While queuing analysis assumes vertical queuing, shock-wave analysis considers the spatial dimension of queues. The consideration of the horizontal extent of a queue enables the capturing of more realistic queuing behavior and the determination of the maximum queue reach, which is not possible with deterministic queuing models, as these models only track the number of queued vehicles, not their spatial location.

Delay (Chin 1996) is a measure of the additional travel time associated with traversing a specific distance relative to some base case travel time. Several literatures have claimed that queuing theory and shock-wave analysis yield inconsistent delay estimates. For example, McShane and Roess (1990) (McShane and Roess 1990) demonstrated that shock-wave analysis over-estimates average vehicle delays by as much as 60 percent in comparison with queuing theory; Chin (1996) (Chin 1996) estimated differences in delay estimates in the range of 5 percent but concluded that because the differences were not too large that it could be argued that they were consistent. Finally, Nam and Drew (1998) (Nam D.H. 1999) mention that "deterministic queuing analysis always underestimates the overall magnitude of delays compared to shock-wave analysis." Furthermore, Nam and Drew claim that "the area between the demand and capacity curves in a queuing diagram is analytically equivalent to the total vehicle-hours of travel in congestion as opposed to the widely accepted total vehicle-hours of delay." Unfortunately, these claimed differences result from inconsistencies in computing the base case travel time, which is used to compute the delay.

The main objective of this paper is to demonstrate, contrary to what is suggested in the some literature, the consistency in delay estimates between queuing theory and shock-wave analysis. Furthermore, the paper demonstrates that the claim by Nam and Drew that the area between the demand and capacity curves in a queuing diagram is equivalent to the total travel time as opposed to total delay is partially correct. The paper demonstrates when this claim is valid and when it is not valid. Finally, the paper corrects the equations that were derived in Nam and Drew's publication and demonstrates the consistency of the two formulations considering a fixed-capacity time-varying arrival rate bottleneck and a variable-capacity bottleneck.

In terms of the paper layout, the first section describes the traffic dynamics approach that was proposed by Nam and Drew (1998(Nam D.H. 1998)) together with an example application of the model. Subsequently, corrections are made to the Nam and Drew proposed vehicle dynamics approach. The models are applied to two bottleneck problems, one for a constant capacity bottleneck with time-varying arrivals and another for a temporally varying capacity bottleneck. Finally, the conclusions of the paper are presented.

### 5.2 NAM AND Drew Traffic Dynamics Approach

### 5.2.1 Traffic Dynamics Analysis

Nam and Drew (1998)(Nam D.H. 1998) presented an example in which traffic flow on a freeway exceeds the capacity of a bottleneck. In this case an observer will observe two flows; the arrival rate and the queuing discharge rate. The flow and density of the approaching traffic stream are denoted as $q_{n 1}$ and $k_{n 1}$, respectively, and those of the queuing flow are denoted as $q_{q}$ and $k_{q}$, respectively, as illustrated in Figure 5.1. For a roadway segment between $x_{1}$ and $x_{2}$, the length of the queue discharge flow regime and arrival flow regime at time $t$ are $l_{1}$ and $l_{2}$, respectively. After some time $\Delta t$, the domains of the traffic regimes change to $l_{3}$ and $l_{4}$, respectively. Consequently, over the time interval $\Delta t$ the congestion domain grows backward by $l_{3}-l_{l}$.

Using the principle of conservation of vehicles the queuing rate can be computed as
$\left(q_{n 1}-q_{q}\right) \Delta t$
The change in the number of vehicles traveling on the segment is given as

$$
\begin{equation*}
\left(k_{q} l_{3}+k_{n 1} l_{4}\right)-\left(k_{q} l_{1}+k_{n 1} l_{2}\right) \tag{4}
\end{equation*}
$$

Recognizing that the total length is constant (i.e. $l_{1}+l_{2}=l_{3}+l_{4}$ ), [4] can be written as

$$
\begin{equation*}
\left(k_{q}-k_{n 1}\right)\left(l_{3}-l_{1}\right) \tag{5}
\end{equation*}
$$

Utilizing the principle of conservation of vehicles, [3] and [5] are equal

$$
\begin{equation*}
\left(q_{n 1}-q_{q}\right) \Delta t=\left(k_{q}-k_{n 1}\right)\left(l_{3}-l_{1}\right) \tag{6}
\end{equation*}
$$

Re-arranging the terms of [6] the following relationship can be obtained:

$$
\begin{equation*}
\frac{l_{1}-l_{3}}{\Delta t}=\frac{q_{q}-q_{n 1}}{k_{q}-k_{n 1}} \tag{7}
\end{equation*}
$$

Here $\frac{l_{1}-l_{3}}{\Delta t}$ is the speed of frontal boundary between the two flow regimes (i.e. it is the speed of the shock-wave between the arrival and queue departure regimes). In other words the speed of the shock-wave is equivalent to the difference in flow divided by the difference in density between the two flow regimes, as follows:

$$
\begin{equation*}
w_{u}=\frac{q_{q}-q_{n 1}}{k_{q}-k_{n 1}} \tag{8}
\end{equation*}
$$

which is consistent with Equation 2 that was presented earlier.
If $n$ is the number of vehicles in queue, the queuing rate in $\Delta t$ is

$$
\begin{equation*}
\frac{\Delta n}{\Delta t}=\frac{k_{q}\left(l_{3}-l_{1}\right)}{\Delta t} \tag{9}
\end{equation*}
$$

Using [7] and [8], Nam and Drew manipulated [9] to derive

$$
\begin{equation*}
\frac{\Delta n}{\Delta t}=\left(q_{n 1}-q_{q}\right)+\frac{l_{3}-l_{1}}{\Delta t} k_{n 1}=\left(q_{n 1}-q_{q}\right)-w_{u} k_{n 1} \tag{10}
\end{equation*}
$$

Nam and Drew demonstrated that [10] is identical to the queuing rate obtained from shock-wave analysis. Using [8] they demonstrated that [10] is equivalent to

$$
\begin{equation*}
\frac{\Delta n}{\Delta t}=\left(q_{n 1}-q_{q}\right)\left(1+\frac{k_{n 1}}{k_{q}-k_{n 1}}\right) \tag{11}
\end{equation*}
$$

which implies that deterministic queuing analysis always underestimates the queue length by a factor of $\frac{k_{n 1}}{k_{q}-k_{n 1}}$.
After the conclusion of the peak demand, the traffic demand decreases below the capacity of the bottleneck and the queue length starts to decrease in size. Considering $q_{n 2}$ and $k_{n 2}$ to represent the flow and density of the reduced approaching flow, respectively, the discharging rate is opposite to the queuing rate and can be estimated as

$$
\begin{equation*}
-\frac{\Delta n}{\Delta t}=\left(q_{q}-q_{n 2}\right)+w_{d} k_{n 2}=\left(q_{q}-q_{n 2}\right)\left(1+\frac{k_{n 2}}{k_{q}-k_{n 2}}\right) \tag{12}
\end{equation*}
$$

where $w_{d}=\frac{q_{n 2}-q_{q}}{k_{n 2}-k_{q}}$ is the speed of the forward recovery shock-wave. Equation 12 demonstrates that deterministic queuing theory underestimates the queue recovery rate by a factor of $\frac{k_{n 2}}{k_{q}-k_{n 2}}$.

### 5.5.2 Comparative Analysis

Nam and Drew then compared queuing theory and shock-wave analysis procedures using a constant capacity bottleneck example illustration, as will be described in this section.

In a queuing diagram the queuing rate is equal to $q_{n 1}-q_{q}$ during the peak period, and the discharge rate is equal to
$q_{q}-q_{n 2}$ during the discharge period, as illustrated in Figure 5.2. In the case of queuing theory, the maximum queue occurs when the traffic demand drops from $q_{n 1}$ to $q_{n 2}$ at time $t_{1}$ given that queuing theory considers that vehicles queue in a fictitious vertical queue. Congestion disappears at time $t_{2}$ when the queue is served. The spatial extent of the queue can be estimated by scaling the queuing diagram and dividing by the density of traffic in the queue $k_{q}$.

Considering a time-space diagram Nam and Drew constructed the spatial formation of queues considering that the queues build up at a rate of $\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right) / k_{q}$ and that the queue dissipates at a rate of $\left(q_{q}-q_{n 2}+w_{d} k_{n 2}\right) / k_{q}$, as demonstrated in Figure 5.2. Nam and Drew demonstrated that the queue reaches its maximum rate at time $t_{l}-t_{0}$, where $t_{0}$ is the travel time in the absence of congestion from the tail of the physical queue upstream of the bottleneck to the bottleneck. Similarly, the equivalent queuing diagram was constructed by multiplying by the traffic density within the congested regime $k_{q}$.

Using the geometry of Figure 5.2, the maximum queue length by shock-wave analysis is computed as

$$
\begin{equation*}
l_{\max }=-w_{u}\left(t_{1}-t_{0}\right) \tag{13}
\end{equation*}
$$

where

$$
\begin{align*}
& t_{0}=t_{1}-\left[\frac{w_{d}}{w_{d}-w_{u}}\right] t_{2} \text { and }  \tag{14}\\
& t_{2}=\left[1+\frac{q_{n 1}-q_{q}}{q_{q}-q_{n 2}}\right] t_{1} \tag{15}
\end{align*}
$$

The total travel time ( $T T T$ ) is computed as the area of the time-space domain of congestion multiplied by the density of traffic under congestion as

$$
\begin{equation*}
T T T=\frac{l_{\max } t_{2}}{2} k_{q} \tag{16}
\end{equation*}
$$

Substituting [13] in [16] Nam and Drew derived

$$
\begin{equation*}
T T T=-\frac{w_{u}}{2} k_{q}\left(t_{1}-t_{0}\right) t_{2} \tag{17}
\end{equation*}
$$

Using the modified arrival and departure curves in Figure 5.3, Nam and Drew computed the area as

$$
\begin{align*}
& \frac{1}{2}\left[Q_{1} t_{2}-Q_{2}\left(t_{1}-t_{0}\right)\right]=\frac{1}{2}\left[\left(q_{n 1}-w_{u} k_{n 1}\right)\left(t_{1}-t_{0}\right) t_{2}-q_{q} t_{2}\left(t_{1}-t_{0}\right)\right] \\
& =\frac{1}{2}\left(q_{n 1}-w_{u} k_{n 1}-q_{q}\right)\left(t_{1}-t_{0}\right) t_{2} \tag{18}
\end{align*}
$$

Subtracting [18] from [17] Nam and Drew demonstrated that the difference was zero as follows:

$$
\begin{align*}
& {\left[-\frac{w_{u}}{2} k_{q}\left(t_{1}-t_{0} t_{2}\right)\right]-\left[\frac{1}{2}\left(q_{n 1}-w_{u} k_{n 1}-q_{q}\right)\left(t_{1}-t_{0}\right) t_{2}\right]} \\
& =-\frac{1}{2}\left(w_{u} k_{q}+q_{n 1}-w_{u} k_{n 1}-q_{q}\right)\left(t_{1}-t_{0}\right) t_{2}  \tag{19}\\
& =-\frac{1}{2}\left[w_{u}\left(k_{q}-k_{n 1}\right)-\left(q_{q}-q_{n 1}\right)\right]\left(t_{1}-t_{0}\right) t_{2}=0
\end{align*}
$$

Consequently, Nam and Drew concluded that the area between the arrival and departure curves (or demand and capacity curves) in a deterministic queuing diagram is equivalent to the total vehicle-hours of travel in congestion as opposed to the widely accepted total vehicle-hours of delay.

Nam and Drew then derived formulations for the various descriptive variables based on queuing theory and shock-wave analysis, as summarized in

Table 5.1and Table 5.2, respectively.

### 5.3 Traffic Dynamics

### 5.3.1 Overview of Traffic Dynamics

Daganzo (2000)(Daganzo 2000) mentions that "since horizontal separations in the ( $t, N$ ) diagram represent time and vertical separations represent accumulation, it should not come as a surprise that the area of the region enclosed by $A(t), D(t)$ and any two vertical lines, $t=t_{0}$ and $t=t_{1}\left(t_{0}<t_{1}\right)$, should be the total wait time done in the system in the interval $\left(t_{0}, t_{l}\right)$." In this analysis $A(t)$ represents the cumulative arrivals at any instant $t$ at some count station while $D(t)$ represents the cumulative departures at any instant $t$ at another downstream count station. Similarly, Daganzo demonstrates that the wait done by items $N_{0}+1$ through $N_{l}$, $\left(N_{0}<N_{l}\right)$ in a First-In-First-Out (FIFO) system is given by the area of the region enclosed by curves $A(t)$ and $D(t)$ and by the horizontal lines, $N=$ $N_{0}$ and $N=N_{l}$. The second deduction does not hold for systems with passing except if the arrival and departure curves touch one another when $N=N_{0}$ and $N=N_{1}$, as is the case in Figure 5.3 because all items that entered must have departed within the identified time interval $\left(t_{0}, t_{l}\right)$. Daganzo also mentions that the result is approximately true if the combined wait of items that only arrived or only departed in the observation period is a small fraction of the total wait.

It is important to note that delay is not necessarily equal to the time spent between the two count stations (time spent within the system). Specifically, the time spent within the system includes the time required to travel between the upstream and downstream count stations in the absence of congestion plus any additional time spent in the system as a result of congestion (known as delay). Here a virtual arrival curve can be introduced by shifting the arrival curve to the right by the free-flow travel time (distance $d_{q} / v_{f}$ as shown in Figure 5.4). Therefore, for $N_{0}, t_{0}$ the distance between arrival curve $A(t)$ and departure curve $D(t)$ represents the total travel time within the system, while the distance between the virtual arrival curve $V(t)$ and the departure curve $D(t)$ represents the total delay within the system. It is important to note that if the arrival and departure curves touch one another at any instant then the area between the two curves represents the total delay and not total travel time given that there is no temporal offset between the two curves. Alternatively, if the two curves are temporally offset (e.g. located at different locations along a highway) the area between the two curves represents the total travel time.

The travel time between the arrival and departure count stations during the onset of congestion can be computed as

$$
\begin{equation*}
t_{q}=\frac{d_{q}}{v_{q}} . \tag{20}
\end{equation*}
$$

Here $d_{q}$ is defined as the distance traveled in queue and $v_{q}$ is the travel speed in queue. As discussed before the delay can be computed as

$$
\begin{equation*}
w=t_{q}-\frac{d_{q}}{v_{f}}, \tag{21}
\end{equation*}
$$

where $v_{f}$ is the free-flow speed. Here the assumption is that the base travel is at free-speed. Consequently, the distance $\&$ time in queue is estimated as $d_{q}=\frac{w}{\frac{1}{v_{q}}-\frac{1}{v_{f}}}$ and $t_{q}=\frac{w}{1-\frac{v_{q}}{v_{f}}}$, respectively.
The above computations assume that the base-case (no congestion) scenario involves travel at free-flow speed, which may not necessarily be true. Consequently, as part of this research effort we generalize the formulation to consider multiple traffic regimes for the base case, as described in the next section.

### 5.3.2 Proposed Modifications to the Nam and Drew Formulation

As was demonstrated in the previous section, the area between the arrival and departure curves can either represent the total travel time or the total delay depending on whether there is a spatial separation between the arrival and departure measurements. Consequently, Nam and Drew's conclusion that the area between the demand and the capacity curves in a queuing diagram is analytically equivalent to the total hours of travel in congestion as opposed
to the widely accepted total vehicle-hours of delay is partially correct and only holds if the arrival and capacity curves are offset spatially, as was demonstrated by Daganzo (2000)(Daganzo 2000). Alternatively, if the arrival and departure curves are not spatially offset, the area between these two curves represents the total delay. Daganzo demonstrated that the delay can be computed by offsetting the arrival curve a distance equal to the free-flow travel time to the right to create a virtual arrival curve. The area between the virtual arrival curve and the departure curve then represents the total delay. However, Daganzo's approach is valid only if the base case for which we are computing delay relative to involves travel at the free-speed.

In the case of the Nam and Drew derivation, because the demand and capacity curves (also known as arrival and departure curves) of Figure 5.3 coincided with each other at both ends of the figure, the area between these curves represents the total delay and not the total travel time as was suggested by Nam and Drew.

Furthermore, Nam and Drew claimed that the queue extent derived from queuing theory and shock-wave analysis in Figure 5.2 are different and thus they concluded that the delay estimates from both models would differ. However, it is important to note at this point that the queue extent that is derived from the queuing model represents the number of vehicles that would have been seen directly upstream of the restriction by time $t$ if the physical extent of the queue had been eliminated and thus should not be compared with the shock-wave queue extent estimates. In other words the queuing theory queue extent is a fictitious queue extent. In order to compare the queue extents for the identical conditions, the queuing theory queue extent should be scaled by the ratio of the total travel time (TTT) to the total delay $(T D)$. In doing so the extent of the queues are identical and thus the delays computed are also identical, as will be demonstrated in the following section.

Nam and Drew also made a third error in their analysis while computing the total delay using shock-wave analysis procedures. Specifically, they derived the total delay as

$$
\begin{equation*}
\frac{-w_{u} t_{2}}{2}\left(t_{1}-t_{0}\right)\left(k_{q}-k_{n 2}\right) \tag{22}
\end{equation*}
$$

Implicit in Equation 22 is the assumption that the base case traffic stream density in the absence of congestion is $k_{n 2}$, as demonstrated in Figure 5.5. However, given that the initial arrival rate travels at a density of $k_{n 1}$ for the duration of $t_{1}$ and subsequently changes to $k_{n 2}$, the base case condition involves two regimes with densities $k_{n 1}$ and $k_{n 2}$, as illustrated in Figure 5.5. Consequently, with this in mind the total delay should be computed as

$$
\begin{equation*}
\frac{-w_{u}\left(t_{1}-t_{0}\right)}{2}\left[t_{2}\left(k_{q}-k_{n 2}\right)-t_{1}\left(k_{n 1}-k_{n 2}\right)\right] . \tag{23}
\end{equation*}
$$

Equation 23 expands on Daganzo's procedures by considering a base case composed of two traffic regimes instead of the single regime that involves travel at free-flow speed.

In addressing the errors of the Nam and Drew (1998)(Coifman 1998) formulations, we have developed new formulations for estimating delay using shockwave analysis and queuing theory, as summarized in

Table 5.1 and Table 5.2. In the following sections, we shall demonstrate that the delay estimates from the shock-wave analysis and queuing theory are identical using the proposed formulations.

### 5.4 Example Applications

In this section we compare shock-wave and queuing theory delay estimates for two scenarios. The first scenario involves a time-varying arrival rate at a constant capacity bottleneck. This example is identical to the example presented by Nam and Drew. In this example we demonstrate the error in the Nam and Drew formulation and further demonstrate the consistency between shock-wave and queuing theory delay estimates. The second scenario compares shock-wave and queuing theory delay estimates for a constant arrival rate at a time-varying capacity
bottleneck.

### 5.4.1 Variable Arrival Rate at a Constant Capacity Bottleneck

We use the example used by Nam and Drew in their paper. Specifically, consider a bottleneck on a six-lane (three in each direction) urban freeway. The capacity of the bottleneck is $1800 \mathrm{veh} / \mathrm{h} /$ lane and density of the congestion domain is $120 \mathrm{veh} / \mathrm{mile} / \mathrm{lane}$. The approaching flow is $6000 \mathrm{veh} / \mathrm{h}$ and its density is $120 \mathrm{veh} / \mathrm{mile}$ for the first 2 hours, and then the approaching flow changes to $4500 \mathrm{veh} / \mathrm{h}$ with a density of $75 \mathrm{veh} / \mathrm{mile}$.

### 5.4.1.1 Nam and Drew Solution:

First, we present the solution that Nam and Drew presented in an earlier publication (Nam and Drew (1998)). The solutions are based on the formulations that were presented in
Table 5.1 and Table 5.2.
$w_{u}=\frac{q_{n 1}-q_{q}}{k_{n 1}-k_{q}}=\frac{\left(\frac{6000}{3}-1800\right)}{40-120}=-2.5 \mathrm{mph}$
$w_{d}=\frac{q_{q}-q_{n 2}}{k_{q}-k_{n 2}}=\frac{\left(1800-\frac{4500}{3}\right)}{120-25}=3.158 \mathrm{mph}$
The time variables are computed as
$t_{1}=2.0 \mathrm{~h}$ (Given),
$t_{2}=\left[1+\frac{q_{n 1}-q_{q}}{q_{q}-q_{n 2}}\right] t_{1}=\left[1+\frac{\frac{6000}{3}-1800}{1800-\frac{4500}{3}}\right] \times 2.0=3.333 \mathrm{~h}$, and
$t_{0}=t_{1}-\left[\frac{w_{d}}{w_{d}-w_{u}}\right] t_{2}=2.0-\frac{3.158}{3.158-(-2.500)} \times 3.333=0.140 \mathrm{~h}$.
The maximum queue length based on shock-wave analysis is computed as
$l_{\max }=\left[\frac{q_{n 1}-q_{q}-w_{u} k_{n 1}}{k_{q}}\right]\left(t_{1}-t_{0}\right)=\left(\frac{6000-5400-(-2.5) \times 120}{360}\right) \times(2-0.14)=4.65 \mathrm{mi}$.
Alternatively, the maximum queue length based on queuing theory is computed as
$l_{\max }^{\prime}=\left[\frac{q_{n 1}-q_{q}}{k_{q}}\right] t_{1}=\left[\frac{6000-5400}{360}\right] \times 2.0=3.333 \mathrm{mi}$.
The total travel time (TTT) and total delay (TD) that are computed using shock-wave analysis are
$T T T_{s}=\frac{\left(t_{1}-t_{0}\right)^{2}}{2}\left(1-\frac{w_{u}}{w_{d}}\right)\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right)=\frac{(2-0.14)^{2}}{2}\left(1-\frac{-2.5}{3.158}\right)(6000-5400+2.5 \times 120)=2789$ veh.h

$$
T D_{s}=\frac{-w_{u} t_{2}}{2}\left(t_{1}-t_{0}\right)\left(k_{q}-k_{n 2}\right)=\frac{2.5 \cdot 3.333}{2}(2-0.14) \cdot(360-75)=2208.53 \text { veh.h }
$$

Alternatively, the total travel time (TTT) and total delay (TD) based on queuing theory are computed as
$T T T_{q}=\frac{t_{1}^{2}}{2}\left(q_{n 1}-q_{q}\right)\left(1+\frac{q_{n 1}-q_{q}}{q_{q}-q_{n 2}}\right)=\frac{2^{2}}{2}(6000-5400)\left(1+\frac{6000-5400}{5400-4500}\right)=2000$ veh.h, and
$T D_{q}=\frac{t_{1} t_{2}}{2 k_{q}}\left(q_{n 1}-q_{q}\right)\left(k_{q}-k_{n 2}\right)=\frac{2 \times 3.333}{2 \times 360}(6000-5400)(360-75)=1583.2$ veh.h.

### 5.4.1.2 Proposed Model Solution:

Nam and Drew's formulation assumed the density of the base case to be constant and equal to the density of the second arrival rate, however, the base case density involves two density regimes for each of the arrival rates. Consequently, the proposed solution that is developed as part of this research effort and presented in

Table 5.1 and Table 5.2 is summarized as follows.
The total travel time (TTT) and total delay (TD) by shock-wave analysis is computed as

$$
\begin{aligned}
& T T T_{s}^{\prime}=\frac{\left(t_{1}-t_{0}\right)^{2}}{2}\left(1-\frac{w_{u}}{w_{d}}\right)\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right)=\frac{(2-0.14)^{2}}{2}\left(1-\frac{-2.5}{3.158}\right)(6000-5400-(-2.5) \cdot 120) \\
& =2789 \text { veh.h } \\
& T D_{s}^{\prime}=\frac{-w_{u}\left(t_{1}-t_{0}\right)}{2}\left[t_{2}\left(k_{q}-k_{n 2}\right)-t_{1}\left(k_{n 1}-k_{n 2}\right)\right]=\frac{2.5 \cdot(2-0.14)}{2}(3.333 \cdot(360-75)-2 \cdot(120-75)) \\
& =1999.3 \cong 2000 \text { veh.h }
\end{aligned}
$$

Alternatively, the total travel time (TTT) and total delay (TD) estimated by queuing theory is as follows:

$$
\begin{aligned}
& T T T_{q}^{\prime}=\frac{t_{1}^{2}}{2}\left(1+\frac{q_{n 1}-q_{q}}{q_{q}-q_{n 2}}\right)\left(q_{n 1}-q_{q}\right)=\frac{2^{2}}{2}\left(1+\frac{6000-5400}{5400-4500}\right)(6000-5400)=2000 \text { veh.h and } \\
& T D_{q}^{\prime}=\frac{t_{1}^{2}}{2}\left(q_{n 1}-q_{q}\right)\left(1+\frac{q_{n 1}-q_{q}}{q_{q}-q_{n 2}}\right)=\frac{2^{2}}{2}(6000-5400)\left(1+\frac{6000-5400}{5400-4500}\right)=2000 \text { veh.h }
\end{aligned}
$$

Note that the total travel time and total delay estimates are identical for the queuing theory formulation because the arrival and departure curves are not spatially offset. The example demonstrates that both approaches produce identical delay estimates and that the differences in delay estimates that were reported by Nam and Drew are attributed to the fact that their computation failed to recognize the fact that the base case scenario includes two density regimes. A summary of the various computations are provided in Table 5.3.

### 5.4.2 Constant Arrival and Time Varying Capacity Bottleneck

In addition to analyzing a constant capacity bottleneck we include an additional example which includes a time-varying capacity bottleneck. In this example we consider a signalized intersection with single lane approaches. The traffic signal timings include a cycle length of 60 seconds and an effective green time of 40 seconds, which corresponds to an effective red time of 20 seconds. The approach flow is $900 \mathrm{veh} / \mathrm{h}$ with a density of $15 \mathrm{veh} / \mathrm{km}$. The saturation flow rate is $1800 \mathrm{veh} / \mathrm{h}$ with a density of $50 \mathrm{veh} / \mathrm{km}$. The jam density is assumed to be $100 \mathrm{veh} / \mathrm{km}$.

### 5.4.2.1 Shock-wave Analysis:

Based on Figure 5.6 the speeds of the various shock-waves can be computed as

$$
\begin{aligned}
& w_{C B}=\frac{q_{C}-q_{B}}{k_{C}-k_{B}}=\frac{900-0}{15-100}=-10.6 \mathrm{~km} / \mathrm{h} \\
& w_{B A}=\frac{q_{B}-q_{A}}{k_{B}-k_{A}}=\frac{0-1800}{100-50}=-36 \mathrm{~km} / \mathrm{h}, \text { and } \\
& w_{A C}=\frac{q_{A}-q_{C}}{k_{A}-k_{C}}=\frac{1800-900}{50-15}=25.7 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

The time for the backward forming and backward recovery shock-waves to meet is computed as

$$
w_{C B}\left(r+t_{m}\right)=w_{B A} t_{m}=d
$$

$$
t_{m}=8.35 \mathrm{~s} ; d=-\frac{w_{B A} \times t_{m}}{3.6}=83.5 \mathrm{~m} ; \quad t_{c}=\frac{d}{w_{A C}}=11.7 \mathrm{~s}
$$

Finally, the total delay is computed as the area of the congested triangle multiplied by the difference between the
congested and base densities, as

$$
T D_{s}=\frac{d \times r}{2} * k_{B}+\frac{d \times\left(t_{m}+t_{c}\right)}{2} \times k_{A}-\frac{d \times\left(r+t_{m}+t_{c}\right) \times k_{C}}{2}=100.2 \mathrm{veh} \cdot \mathrm{~s} .
$$

### 5.4.2.2 Queuing Analysis:

Using Figure 5.7 the arrival rate is computed as

$$
q=\frac{q_{C}}{3600}=\frac{900}{3600}=0.25 \mathrm{veh} / \mathrm{s}
$$

while the saturation rate is computed as

$$
s=\frac{q_{A}}{3600}=\frac{1800}{3600}=0.5 \mathrm{veh} / \mathrm{s} .
$$

The maximum number of vehicles in queue is computed as

$$
Q_{m}=q r=0.25 \times 20=5 \mathrm{veh}
$$

and the time to clear the queue is computed as

$$
T=\frac{Q_{m}}{s-q}=\frac{5}{0.50-0.25}=20 \mathrm{~s}
$$

The total delay is computed as

$$
T D_{q}=\frac{1}{2}(r+T) Q_{m}=\frac{(20+20) \times 5}{2}=100 \mathrm{veh} \cdot \mathrm{~s} .
$$

Again, the delay estimates as was the case with the variable arrival/constant capacity scenario, estimated by the queuing theory and shock-wave analyses are practically identical. Consequently, we demonstrate the consistency of both formulations in estimating delay at bottlenecks.

### 5.5 CONCLUSIONS

The delay computations using shock-wave analysis and queuing theory were compared for two example applications, namely (a) time varying arrival rate at a constant capacity bottleneck and (b) a constant arrival rate at a time varying capacity bottleneck. The results demonstrate the consistency between shock-wave analysis and queuing theory. Furthermore, the paper highlights the error in the Nam and Drew (1998)(Coifman 1998; Coifman and Ergueta 2003) computation and corrects the equations that were derived by Nam and Drew. In summary, the paper demonstrates that queuing theory provides a simple and accurate technique for estimating delay at highway bottlenecks.

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Table 5.1: Queuing Theory Formulations

| Variables | Nam and Drew Formulation | Proposed Formulation |
| :--- | :--- | :--- |
| Maximum number of vehicles in <br> queue | $\left(q_{n 1}-q_{q}\right) t_{1}$ | $\left(q_{n 1}-q_{q}\right) t_{1}$ |
| Maximum queue length | $\left[\frac{q_{n 1}-q_{q}}{k_{q}}\right] t_{1}$ | $\left[\frac{q_{n 1}-q_{q}}{k_{q}}\right] t_{1}$ |
| Total delay | $\frac{t_{1} t_{2}}{2 k_{q}}\left(q_{n 1}-q_{q}\right)\left(k_{q}-k_{n 2}\right)$ | $\frac{t_{1}^{2}}{2}\left(q_{n 1}-q_{q}\right)\left(1+\frac{q_{n 1}-q_{q}}{q_{q}-q_{n 2}}\right)$ |
|  |  | When arrival and departure counts are <br> at different locations. |
| Total travel time (in congestion) | $\frac{t_{1}^{2}}{2}\left(1+\frac{q_{n 1}^{2}-q_{q}}{q_{q}-q_{n 2}}\left(q_{n 1}-q_{q}\right)\left(1+\frac{q_{n 1}-q_{q}}{q_{q}-q_{n 2}}\right)\right.$ |  |
|  |  | When arrival and departure counts are <br> at different locations. |
| Average individual delay | $\frac{t_{1}}{2 q_{q}}\left(q_{n 1}-q_{q}\right)$ |  |

Table 5.2: Shock-wave Analysis Formulations

| Variables | Nam and Drew Formulation | Proposed Formulation |
| :--- | :--- | :--- |
| Maximum number of <br> vehicles in queue | $\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right)\left(t_{1}-t_{0}\right)$ | $\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right)\left(t_{1}-t_{0}\right)$ |

Maximum queue length $\quad\left[\frac{q_{n 1}-q_{q}-w_{u} k_{n 1}}{k_{q}}\right]\left(t_{1}-t_{0}\right) \quad\left[\frac{q_{n 1}-q_{q}-w_{u} k_{n 1}}{k_{q}}\right]\left(t_{1}-t_{0}\right)$

Total delay

$$
\frac{-w_{u} t_{2}}{2}\left(t_{1}-t_{0}\right)\left(k_{q}-k_{n 2}\right) \quad \frac{-w_{u}\left(t_{1}-t_{0}\right)}{2}\left[t_{2}\left(k_{q}-k_{n 2}\right)-t_{1}\left(k_{n 1}-k_{n 2}\right.\right.
$$

Total travel time (in congestion)

$$
\frac{\left(t_{1}-t_{0}\right)^{2}}{2}\left(1-\frac{w_{u}}{w_{d}}\right)\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right) \quad \frac{\left(t_{1}-t_{0}\right)^{2}}{2}\left(1-\frac{w_{u}}{w_{d}}\right)\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right)
$$

Average individual delay $\quad-\frac{w_{u}}{2 q_{q}}\left(t_{1}-t_{0}\right)\left(k_{q}-k_{n 2}\right) \quad \frac{-w_{u}\left(t_{1}-t_{0}\right)}{2 q_{q} t_{2}}\left[t_{2}\left(k_{q}-k_{n 2}\right)-t_{1}\left(k_{n 1}-k_{n 2}\right.\right.$
$\begin{array}{lc}\begin{array}{l}\text { Average individual travel } \\ \text { time (in congestion) }\end{array} & \frac{1}{2 q_{q}}\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right)\left(t_{1}-t_{0}\right)\end{array} \quad \frac{1}{2 q_{q}}\left(q_{n 1}-q_{q}-w_{u} k_{n 1}\right)\left(t_{1}-t_{0}\right)$

Table 5.3: Comparison of Queuing and Shock-wave Analysis for Example 4.1

| Descriptive Items | Queuing <br> Theory | Shock-wave <br> Analysis | Difference | Units |
| :--- | :--- | :--- | :--- | :--- |
| Queuing Rate | 600 | 900 | 300 | $\mathrm{veh} / \mathrm{h}$ |
| Discharging rate | 900 | 1136.85 | 236.85 | $\mathrm{veh} / \mathrm{h}$ |
| Physical queuing rate | 1.67 | 2.33 | 0.66 | mph |
| Physical discharging rate | 2.5 | 3.5 | 1 | mph |
| Maximum number of vehicles in queue | 1200 | 1674 | 474 | veh |
| Maximum physical queue length | 3.33 | 4.65 | 1.32 | mile |
| Total veh-h of delay | 2000 | 2000 | 0 | veh-h |
| Total veh-h of travel in congestion | 2000 | 2790 | 790 | veh-h |
| Average individual delay | 6.67 | 6.67 | 0 | min <br> Average individual travel time |
|  | 7.43 | 9.3 | 1.87 |  |



Figure 5.1: Schematic Representation of Congestion Buildup over $\Delta t$ (Source: Nam and Drew, 1998)


Figure 5.2: Superimposition of Time-Space Diagrams from Queuing and Shock-wave Analysis


Figure 5.3: Superimposition of Queuing Theory and Shock-wave Analysis Diagrams


Figure 5.4: Arrival, Departure, and Virtual Arrival Curves for a Single Bottleneck with Variable Arrivals


Figure 5.5: Traffic Flow Regimes with and without a Bottleneck


Figure 5.6: Time-Space Diagrams from Shock-wave Analysis (Constant Arrival Rate with Variable Capacity Bottleneck)


Figure 5.7: Queuing Diagram (Constant Arrival Rate with Variable Capacity Bottleneck)

## Chapter 6. Estimation of Space-Mean Speed

# Estimating Traffic Stream Space-Mean Speed and Reliability From Dual and Single Loop Detectors 

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#### Abstract

The relationship between time-mean and space-mean speed that was derived by Wardrop (1)(Wardrop 1952) and presented in several textbooks (e.g. Hobbs and Richardson (2)(Hobbs and Richardson 1967); May (3)(May 1990); Garber and Hoel (4)(Garber and Hoel 2002)) is suitable for estimating time-mean speeds from space-mean speeds. However, in most cases it is desired to estimate the space-mean speed from time-mean speed measurements. Consequently, the paper develops a new formulation, which utilizes the variance about the time-mean speed as opposed to the variance about the space-mean speed, for the estimation of space-mean speeds. The paper demonstrates that the space-mean speeds are estimated within a margin of error from 0 to 1 percent. Furthermore, the paper develops a relationship between the space and time-mean speed variances and between the space-mean speed and the spatial travel time variance.

In addition, the paper demonstrates that both the Hall and Persaud (5)(Hall and Persaud 1988) and the Dailey (6)(Dailey 1999) formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation computes the space-mean speed (harmonic mean) while the Dailey formulation computes the time-mean speed (arithmetic mean).


### 6.1 INTRODUCTION

### 6.1.1 Background

Lighthill and Witham (7)(LightHill and Whitham 1955) derived the classical steady-state traffic flow relationship between the traffic stream flow rate $(q)$, the traffic stream density $(k)$, and the traffic stream space-mean-speed $\left(\bar{u}_{s}\right)$ as

$$
\begin{equation*}
q=k \cdot \bar{u}_{s} . \tag{1}
\end{equation*}
$$

Traffic stream speeds are typically measured in the field using a variety of spot speed measurement technologies. The most common of these spot speed measurement technologies is a presence-type loop detector, which identifies the presence and passage of vehicles over a short segment of roadway (typically 5 to 20 meters long). When a vehicle enters the detection zone, the sensor is activated and remains activated until the vehicle leaves the detection zone. These surveillance detectors measure the traffic stream flow rate (number of actuations per unit time), traffic stream speed (in the case of dual loop detectors), and percentage of time that the detector is occupied (detector occupancy). The traditional practice for estimating speeds from single loop detectors is based on the assumption of a constant average effective vehicle length. Studies, however, have shown that this assumption provides speed estimates that are sufficiently inaccurate as to severely limit the usefulness of these speed estimates for real-time traffic management and traveler information systems (Hellinga (8)(Hellinga 2002)). In addressing these issues researchers have investigated the use of filtering techniques. For example, Dailey (6)(Dailey 1999) developed a Kalman filter on vehicle length estimates while Hellinga (8)(Hellinga 2002) used exponentially smoothed adjacent dual loop detector vehicle length measurements to

[^2]enhance the speed estimates of single loop detectors. Hellinga demonstrated that the exponential smoothing of 20-s average vehicle length measurements from adjacent dual loop detectors enhanced the accuracy of the speed estimates by approximately 20 percent. Wang and Nihan (9)(Wang and Nihan 2000) using screening procedures to remove intervals with long vehicles and space-mean speed estimates were derived from the intervals with passenger cars only. Alternatively, researchers have investigated the use of median as opposed to mean statistics in order to enhance the robustness of the statistics by ensuring that the measures are not influenced by outlier observations. For example, Lin et al. (10)(Lin, Dahlgren et al. 2004) used the median vehicle passage time as opposed to the mean passage time to estimate speeds from single loop detectors. Similarly, Coifman et al. (11)(Coifman, Dhoorjaty et al. 2003) computed the median speed from the median occupancy in order to reduce speed estimate errors when a wide range of vehicle lengths are present in the traffic stream.

Dailey (6)(Dailey 1999) and Wang and Nihan (12)(Wang and Nihan 2003) claimed that the traditional speed estimation method proposed by Hall and Persaud (5)(Hall and Persaud 1988) is biased. However, Coifman (13)(Coifman, Dhoorjaty et al. 2003) refuted this claim and demonstrated that the speed estimates are not biased. This paper demonstrates that the conclusions of Dailey ( 6 (Dailey 1999) and Hall and Persaud (5)(Hall and Persaud 1988) are both valid and that the differences in the conclusions result from the use of time-mean versus space-mean speeds, as will be discussed in detail in the paper.

The average traffic stream speed can be computed in two different ways: a time-mean speed and a space-mean speed. The difference in speed computations is attributed to the fact that the space-mean speed reflects the average speed over a spatial section of roadway, while the time-mean speed reflects the average speed of the traffic stream passing a specific stationary point. Specifically, Daganzo (14)(Daganzo 2000) demonstrates that the space-mean speed is a density weighted average speed, while the time-mean speed is a flow weighted average speed. Given that a stationary observer will observe faster vehicles more often than slower vehicles while an aerial photograph would show more slow moving vehicles than faster vehicles over a fixed roadway length, it should come as no surprise that the time-mean speed is greater than or equal to the space-mean speed.

### 6.1.2 Paper Objectives and Layout

The objectives of this paper are two-fold. First, the paper modifies the Wardrop (1)(Wardrop 1952) formulation to estimate the space-mean speed as a function of the time-mean speed. Second, the paper derives the relationship between the time-speed and space-speed variances, as well as the relationship between space-speed and travel time variance. Consequently, the paper provides a means for estimating the reliability of travel times for use within the context of traveler information systems. Subsequently, the paper presents two formulations that are documented in the literature for estimating traffic stream speed from single loop detectors that may appear to be inconsistent at first glance. The paper demonstrates that both formulations are correct and that differences in the formulations arise from the fact that the Hall and Persaud (5)(Hall and Persaud 1988) formulation estimates the traffic space-mean speed while the Dailey (6)(Dailey 1999) formulation estimates the time-mean speed.

The significance of this research effort is three-fold. First, because the reality is that Traffic Management Center (TMC) controllers are designed to estimate time-mean speeds the proposed formulation provides an efficient approach for estimating space-mean speeds. Space-mean speed, as opposed to time-mean speed, is used within state-of-the-practice traffic stream models and thus is critical to the accurate modeling of traffic stream behavior. Second, the paper provides a means for quantifying the reliability of space-mean speed and travel time estimates. Third, the paper demonstrates the consistency and differences between the Hall and Persaud (5)(Hall and Gunter 1986) and Dailey (0)(Dailey 1999) formulations for estimating traffic stream speed from single loop detectors.

Initially, the problems with the Wardrop formulation are discussed and the relationship between space-mean speed and time-mean speed is derived using the statistics of the estimates. Subsequently, two formulations for estimating traffic stream speed from single loop detector volume to occupancy measurements are presented. Subsequently, we demonstrate that both formulations are consistent and that differences arise because of differences in estimating time versus space-mean speeds. Finally the conclusions of the paper are presented.

### 6.2 Relationship between Time-MEan And Space-Mean Speeds

As was mentioned earlier, time-mean speed is the arithmetic mean of the speeds of vehicles passing a point on a highway
during an interval of time. Alternatively, the space-mean speed is the harmonic mean of the speeds of vehicles passing a point on a highway during an interval of time. The space-mean speed is a traffic density speed estimate and reflects the spatial dimension of speed and thus is utilized in the standard speed-flow-density relationships.

### 6.2.1 State-of-Practice Relationships

Wardrop (1)(Wardrop 1952) derived the relationship between the time-mean speed $\left(\bar{u}_{T}\right)$ and the space-mean speed $\left(\bar{u}_{S}\right)$ as

$$
\begin{equation*}
\bar{u}_{T}=\bar{u}_{s}+\frac{\sigma_{s}^{2}}{\bar{u}_{s}} \tag{2}
\end{equation*}
$$

where $\sigma_{S}^{2}$ is the variance in vehicle speeds about the space-mean speed. Consequently, Equation 2 is applied to estimate the time-mean speed from the space-mean speed. However, in most cases the time-mean speed, as opposed to the space-mean speed, is available and it is desirable to estimate the space-mean speed from the time-mean speed. Because TMCs do not measure/estimate space-mean speeds (harmonic mean), but instead measure/estimate time-mean speeds, there is a need to compute space-mean speeds from time-mean speeds. The importance of space-mean speed lies in the fact that Equation 1 requires the use of this variable as opposed to time-mean speed. Consequently, a new formulation is required to address this need.
Studies have shown that the difference between time-mean speed and space-mean speed estimates are on the order of 1 to 5 percent with greater differences occurring when the coefficient of variation (CV) is large and the mean speed is small (May (3)(May 1990)). Figure 6.11 and Figure 62 illustrate the difference between time-mean speed and space-mean speed measurements for data gathered from a dual loop detector located on the I-880 freeway in Los Angeles (Coifman et al., (15)(Coifman, Lyddy et al. 2000)). The data are unique in that they include individual detector activations at a resolution of $1 / 60^{\text {th }}$ of a second as opposed to aggregated 20 - or $30-\mathrm{s}$ estimates. The data were gathered over an entire day on the median lane (lane 1) and the lane adjacent to the shoulder lane (lane 4). The data demonstrates that while the differences between space- and time-mean speeds are typically in the range of 1 to 5 percent, larger differences can be observed when the traffic stream speed is lower (during congestion). Specifically, differences in the range of 10 to 30 percent are not uncommon.
Garber and Hoel (4)(Garber and Hoel 2002) describe a more direct relationship between time-mean and space-mean speed as

$$
\begin{equation*}
\bar{u}_{T}=0.966 \bar{u}_{S}+3.541 \tag{3}
\end{equation*}
$$

However, the model parameters are specific to the local roadway and traffic stream characteristics. For example, data from the I- 880 freeway resulted in different model parameters when a regression line was fit to the data. Specifically, the optimum model constant was 2.389 , as opposed to 3.541 , and the model slope was 0.986 , as opposed to 0.966 . Consequently, the model proposed by Garber and Hoel would require calibration to local roadway and traffic conditions and could not be generalized.
In this paper, we shall demonstrate that the relationship between time-mean and space-mean speed that was derived by Wardrop (1)(Wardrop 1952) and presented in several textbooks (e.g. Hobbs and Richardson (2)(Hobbs and Richardson 1967); May (3)(May 1990); Garber and Hoel (4)(Garber and Hoel 2002)) produces an error in the range of 0 to 1 percent in time-mean speed estimates. We also propose a new formulation for estimating space-mean speeds from time-mean speeds with a similar margin of error (within 0 to 1 percent). Specifically, we use the statistics of the estimates to derive

$$
\begin{equation*}
\bar{u}_{S} \approx \bar{u}_{T}-\frac{\sigma_{T}^{2}}{\bar{u}_{T}} \tag{4}
\end{equation*}
$$

It should be noted that after developing this relationship it was recognized that Khisty and Lall (16)(Khisty and Lahh 2003) presented a similar relationship and demonstrated it was valid using 3 observations; however, Khisty and Lall provided no description of how the relationship was derived analytically. Consequently, the paper expands the state of knowledge by deriving this relationship from the statistics of measurements.
To test the Wardrop and proposed formulation, the aforementioned I- 880 data samples are utilized. The I- 880 section that is analyzed is a five-lane section with lanes numbered in ascending order from median to shoulder lane. Using these data, the speed and length of each individual vehicle was computed, as illustrated in Figure 3 and Figure 4. The solid black line represents a moving average of 50 observations while the light grey lines represent the actual field measurements.

The median lane data clearly demonstrates a high degree of variability in speed and vehicle length measurements; however the mean vehicle length appears to remain fairly constant throughout the entire day. The median lane only had 1.19 percent of the vehicles with lengths that exceeded 8 meters. Considering a threshold of 8 meters for the classification of trucks, the median lane (lane 1) only had a 1 percent truck volume. Alternatively, second rightmost lane (lane 4) was composed of approximately a 12 percent truck volume considering a vehicle length threshold of 8 meters. The data of the median lane and lane 4 also demonstrates the onset of congestion during the PM peak period with a significant decrease in vehicle speeds. Using these raw field data, 5 -minute time-mean (arithmetic mean) and space-mean (harmonic mean) speeds were computed. In addition, the speed variance about the time-mean and space-mean speeds was computed. Furthermore, using these aggregated data, an estimate of the time-mean and space-mean speeds was made using Equations 2 and 4, respectively. The results of Figure 5 and Figure 6 demonstrate a high degree of correlation between the measured and computed time-mean speeds ( $\mathrm{R}^{2}$ in excess of $99 \%$ ) on the median lane and lane 4, respectively. A further analysis of the estimate errors revealed that the error increased as a function of the speed coefficient of variation, as illustrated in Figure 7. However, the speed estimate errors did not exceed $4 \mathrm{~km} / \mathrm{h}$ for the entire range of speed coefficients of variation. These CVs ranged from 0 to 50 percent as illustrated in Figure 7 with a higher speed CV in the median lane compared to the inner lanes (lane 4).
The results that were presented demonstrate that the proposed formulation maintains the accuracy of parameter estimates while estimating the space-mean speed from the time-mean speed. Consequently, if current loop detector technologies were to store not only the mean speed within a polling interval but also the speed variance, it would be possible to estimate the space-mean speed from the time-mean speed to a high degree of accuracy, as demonstrated in Equation 4.

### 6.2.2 Proposed Model for Estimating Space-Mean Speed

In deriving the proposed relationship between the time-mean speed and the space-mean speed, we will consider the statistics of estimates similar to an earlier publication by Dailey (6)(Dailey 1999) in which he attempted to estimate the traffic stream speed from single loop detectors as is presented later in this paper.
The speed of the $j^{\text {th }}$ vehicle in a polling interval can be computed as

$$
\begin{equation*}
u_{j}=\frac{D}{t_{j}} \tag{5}
\end{equation*}
$$

Equation 5 assumes that the distance of travel between the two reference points $(D)$ is sufficiently long enough that differences in vehicle lengths can be ignored in computing the vehicle speed ( $u_{j}$ ). Specifically, the speed of vehicle $j$ within a polling interval is computed as the travel distance ( D ) divided by the time it takes the vehicle to travel between the two reference points $\left(t_{j}\right)$.
The time-mean speed is computed as the expected speed over all observations within the polling interval as

$$
\begin{equation*}
\bar{u}_{T}=E\left\{u_{j}\right\}=3.6 \cdot E\left\{\frac{D}{t_{j}}\right\}=[3.6 \cdot D] \cdot E\left\{\frac{1}{t_{j}}\right\}=d \cdot E\left\{\frac{1}{t_{j}}\right\}, \tag{6}
\end{equation*}
$$

where $\mathrm{E}\{*\}$ is the expectation operator. The operator $E$ is the expectation over all realizations within the polling interval. It should be noted that the constant 3.6 is used to convert the speed from units of $\mathrm{m} / \mathrm{s}$ to $\mathrm{km} / \mathrm{h}$. In summary, Equation 6 demonstrates that the time-mean speed is equal to the product of the distance between the two observation points and the geometric mean of the travel times between these two reference points within the polling interval $i$.
We can express the travel time measurements as the expected value (mean) and some deviation $\left(\Delta t_{j}\right)$ that occurs for this observation $j$,

$$
\begin{equation*}
t_{j}=\bar{t}+\Delta t_{j} \tag{7}
\end{equation*}
$$

where the statistics of the deviation term are selected such that the $\mathrm{E}=0$.
Substituting Equation 7 in Equation 6, we get

$$
\begin{equation*}
\bar{u}_{T}=d \cdot E\left\{\frac{1}{\bar{t}_{i}+\Delta t_{j}}\right\}=\frac{d}{\bar{t}} \cdot E\left\{\frac{1}{1+\frac{\Delta t_{j}}{\bar{t}}}\right\} \tag{8}
\end{equation*}
$$

Expanding the right-hand-side (RHS) using the power series we get

$$
\begin{equation*}
\bar{u}_{T}=\frac{d}{\bar{t}} \cdot E\left\{1-\frac{\Delta t_{j}}{\bar{t}}+\frac{\Delta t_{j}^{2}}{\bar{t}^{2}}-\frac{\Delta t_{j}^{3}}{\bar{t}^{3}}+\cdots\right\} \tag{9}
\end{equation*}
$$

Alternatively, the space-mean speed is computed as the distance of travel divided by the expected travel time, as follows:

$$
\begin{equation*}
\bar{u}_{s}=\frac{d}{E\left\{t_{j}\right\}}=\frac{d}{E\left\{\bar{t}+\Delta t_{j}\right\}}=\frac{d}{\bar{t}} \tag{10}
\end{equation*}
$$

Inserting Equation 10 in Equation 9 and approximating the series for the first three terms, we get

$$
\begin{equation*}
\bar{u}_{T}=\bar{u}_{s} \cdot E\left\{1-\frac{\Delta t_{j}}{\bar{t}}+\frac{\Delta t_{j}^{2}}{\bar{t}^{2}}-\frac{\Delta t_{j}^{3}}{\bar{t}^{3}}+\cdots\right\} \approx \bar{u}_{s} \cdot E\left\{1+\frac{\Delta t_{j}^{2}}{\bar{t}^{2}}\right\} . \tag{11}
\end{equation*}
$$

Dailey (6)(Dailey 1999) demonstrated that the vehicle length and speed observations could be considered independent (coefficient of correlation of 0.018 ) using sample field data. If we consider no differences in vehicle lengths (i.e. all vehicles are of equal lengths), the travel time and speed measurements are highly negatively correlated (Pearson product moment correlation coefficient of -0.975 in the case of the sample data that were described earlier). However, if we consider potential differences in vehicle lengths that are independent of the variability in vehicle speeds, the travel time and speed measurements while continuing to be negatively correlated are less correlated. For example, considering a detection length of 5 meters, an average vehicle length of 8 meters, and a vehicle length coefficient of variation of 0.25 , results are a Pearson product moment correlation coefficient of -0.70 . Consequently, we may assume that the speed and travel time observations are negatively but not highly correlated and thus relate the travel times to the time-mean speed as

$$
\begin{equation*}
t_{j}=\frac{d}{u_{j}} \Rightarrow \bar{t}+\Delta t_{j}=\frac{d}{\bar{u}_{T}-\Delta u_{j}} \tag{12}
\end{equation*}
$$

Equation 12 considers $\Delta u_{j}$ as the deviation is vehicle specific speeds about the time-mean speed and is selected such that $\mathrm{E}\left\{\Delta u_{j}\right\}=0$. It should be noted that in the case that the deviations are zero $\left(\Delta u_{j}=0\right)$, Equation 12 reverts to Equation 10 given that the time-mean and space-mean speeds would be equal in magnitude. In addition, if we assume vehicle travel times and speeds to be highly negatively correlated, we may use the space-mean speed instead of the time-mean speed and thus derive the Wardrop formulation that was presented in Equation 2.
Re-arranging Equation 10, the mean travel time for polling interval $i$ can be computed as

$$
\begin{equation*}
\bar{t}=\frac{d}{\bar{u}_{s}} . \tag{13}
\end{equation*}
$$

Inserting Equation 13 in Equation 12 the deviation in vehicle speed can be approximated for

$$
\begin{equation*}
\Delta t_{j}=\frac{d}{\bar{u}_{T}-\Delta u_{j}}-\bar{t}=\frac{d}{\bar{u}_{T}-\Delta u_{j}}-\frac{d}{\bar{u}_{S}}=d \cdot\left[\frac{\left(\bar{u}_{S}-\bar{u}_{T}\right)}{\left(\bar{u}_{T}-\Delta u_{j}\right)}+\frac{\Delta u_{j}}{\bar{u}_{S}\left(\bar{u}_{T}-\Delta u_{j}\right)}\right] . \tag{14}
\end{equation*}
$$

Recognizing that the difference between the space-mean and time-mean speeds is minor ( 1 to $5 \%$ ) and that the deviation in speed is typically small relative to the mean speed, we can approximate Equation 14 for

$$
\begin{equation*}
\Delta t_{j} \approx d \cdot\left[\frac{\Delta u_{j}}{\bar{u}_{s} \cdot \bar{u}_{T}}\right]=\frac{d}{\bar{u}_{s}} \cdot \frac{\Delta u_{j}}{\bar{u}_{T}} . \tag{15}
\end{equation*}
$$

Incorporating Equation 13 in Equation 15, we get

$$
\begin{equation*}
\frac{\Delta t_{j}}{\bar{t}} \approx \frac{\Delta u_{j}}{\bar{u}_{T}} \tag{16}
\end{equation*}
$$

Inserting Equation 16 in Equation 11 and solving for the expectation, we get

$$
\begin{equation*}
\bar{u}_{T} \approx \bar{u}_{S} \cdot E\left\{1+\frac{\Delta u_{j}^{2}}{\bar{u}_{T} \cdot \bar{u}_{T}}\right\}=\bar{u}_{S}+\frac{\bar{u}_{S}}{\bar{u}_{T}} \cdot \frac{E\left\{\Delta u_{j}^{2}\right\}}{\bar{u}_{T}} \approx \bar{u}_{S}+\frac{\sigma_{T}^{2}}{\bar{u}_{T}} \tag{17}
\end{equation*}
$$

where $\frac{\bar{u}_{S}}{\bar{u}_{T}} \approx 1.0$.
It should be noted that the variance $\left(\sigma_{T}{ }^{2}\right)$ in Equation 17 is the variance with respect to the time-mean speed for all realizations within the polling interval. Alternatively, Equation 17 can be written as Equation 4 by substituting the space-mean speed in the denominator for the time-mean speed and solving for the space-mean speed. The advantage of Equation 4 is that all terms on the RHS are computed using the time-mean speed for the computation of the space-mean speed.

### 6.2.3 Relationship between Time-Mean and Space-Mean Speed Variance

In this paper, we also attempt to derive the relationship between the time-mean speed and space-mean speed variance. Specifically, considering the formulation of the space-mean speed variance as

$$
\begin{equation*}
\sigma_{s}^{2}=E\left(u_{j}-\bar{u}_{s}\right)^{2}, \tag{18}
\end{equation*}
$$

the space-mean speed variance can be formulated as

$$
\begin{equation*}
\sigma_{s}^{2} \approx E\left(u_{j}-\bar{u}_{T}+\frac{\sigma_{T}^{2}}{\bar{u}_{T}}\right)^{2}=E\left(u_{j}-\bar{u}_{T}\right)^{2}+E\left(\frac{\sigma_{T}^{2}}{\bar{u}_{T}}\right)^{2}+2 E\left[\left(u_{j}-\bar{u}_{T}\right) \frac{\sigma_{T}^{2}}{\bar{u}_{T}}\right] . \tag{19}
\end{equation*}
$$

Equation 19 substitutes the space-mean speed for the relationship that was proposed earlier in Equation 4. Recognizing that $E\left(u_{j}-\bar{u}_{T}\right)=0$, Equation 19 can be reduced to

$$
\begin{equation*}
\sigma_{S}^{2}=\sigma_{T}^{2}+\left(\frac{\sigma_{T}^{2}}{\bar{u}_{T}}\right)^{2} \tag{20}
\end{equation*}
$$

Using the I-880 field data space-speed, variances were computed and compared against the estimates of Equation 20, as illustrated in Figure 8. The figure clearly demonstrates a high degree of correlation between the field-computed and estimated variances (slope of 0.99 and $\mathrm{R}^{2}$ in excess of 99 percent). Equation 20 demonstrates that the space-speed variance is typically greater than the time-speed variance.

### 6.3 Estimating Traffic Stream Speed from Single Loop Detectors

Unfortunately, a significant number of loop detectors in Freeway Traffic Management Systems (FTMSs) are single loop detectors. Consequently, these detectors do not measure vehicle speeds; instead, the detectors measure the traffic volume that passes the detection station and the occupancy (percentage of time the detector is occupied) of detectors. This section describes a number of procedures, documented in the literature, for estimating traffic stream speed from single loop detector volume and occupancy measurements.

### 6.3.1 Hall and Persaud Procedures

Hall and Persaud (5)(Hall and Persaud 1988) developed a procedure for estimating traffic stream speed from single loop detector volume and occupancy measurements. These procedures are discussed in detail in this section.
Specifically, Hall and Persaud computed the speed of the $j^{\text {th }}$ vehicle within polling interval as

$$
\begin{equation*}
u_{j}=3.6\left(\frac{I_{j}+I_{D}}{t_{j}}\right) \tag{21}
\end{equation*}
$$

Equation 21 assumes that the vehicle length $\left(l_{j}\right)$ and detection zone length $\left(l_{D}\right)$ are in meters while the time that the loop detector is activated $\left(t_{j}\right)$ is in units of seconds. The speed of the vehicle $\left(u_{j}\right)$ is then computed in $\mathrm{km} / \mathrm{h}$ by multiplying by the constant 3.6.
Summing up the time that loop detector is occupied within the polling interval

$$
\begin{equation*}
\sum_{j=1}^{N_{i}} t_{j}=3.6\left(\bar{l}+I_{D}\right) \sum_{j=1}^{N} \frac{1}{u_{j}} \tag{22}
\end{equation*}
$$

the occupancy for a polling interval can be computed as

$$
\begin{equation*}
O=\frac{\sum_{j=1}^{N} t_{j}}{T}=3.6\left(\frac{\bar{l}+I_{D}}{T}\right) \sum_{j=1}^{N} \frac{1}{u_{j}} . \tag{23}
\end{equation*}
$$

Substituting the vehicle speeds for the distance divided by the travel time and assuming the vehicle lengths are constant, we get

$$
\begin{equation*}
O=3.6\left(\frac{\bar{l}+I_{D}}{T}\right) \sum_{j=1}^{N} \frac{1}{u_{j}}=3.6\left(\frac{\bar{l}+I_{D}}{T}\right) \frac{\sum_{j=1}^{N} t_{j}}{d} . \tag{24}
\end{equation*}
$$

Multiplying and dividing Equation 24 by the number of observations within the polling interval $i$, we get

$$
\begin{equation*}
O=3.6\left(\frac{\bar{l}+I_{D}}{T}\right) \frac{\sum_{j=1}^{N_{i}} t_{j}}{d} \cdot \frac{N}{N}=3.6\left(\frac{\bar{l}+I_{D}}{T}\right) \frac{\sum_{j=1}^{N} t_{j}}{d N} \cdot N \tag{25}
\end{equation*}
$$

where: $E\left\{t_{j}\right\}=\bar{t}=\frac{\sum_{j=1}^{N} t_{j}}{N}$.
Replacing the distance divided by the mean travel time for the space-mean speed, we get

$$
\begin{equation*}
N_{i}=3.6\left(\frac{\bar{I}_{i}+I_{D}}{T}\right) \cdot \frac{\bar{t}}{d} N=3.6\left(\frac{\bar{l}+I_{D}}{T}\right) \cdot \frac{N}{\bar{u}_{S}} . \tag{26}
\end{equation*}
$$

Solving for the space-mean speed, we get

$$
\begin{equation*}
\bar{u}_{s}=3.6\left(\frac{\bar{l}+I_{D}}{T}\right) \cdot \frac{N}{O} \tag{27}
\end{equation*}
$$

It should be noted that Equation 27 estimates the space-mean speed from a single loop detector as a constant multiplied by the volume to occupancy ratio within a given polling interval. This constant may vary depending on the average vehicle length within a polling interval.

### 6.3.2 Dailey Procedures

Dailey (6)(Dailey 1999) explicitly considered the statistics of estimates from loop detector measurements, including volume $(N)$ and occupancy $(O)$. Specifically, using the relationship between occupancy and the $j^{\text {th }}$ vehicle speed $\left(u_{j}\right)$ and length $\left(l_{j}\right)$ the occupancy for a polling interval can be computed as follows, as was described earlier:

$$
\begin{equation*}
O=\frac{3.6}{T} \cdot \sum_{j=1}^{N} \frac{I_{j}+I_{D}}{u_{j}} \tag{28}
\end{equation*}
$$

Dailey expressed the speed and length observations as the expected value (mean) for the polling interval plus some deviation $\left(\Delta l_{j}, \Delta u_{j}\right)$ around that mean. By substituting the sum of the vehicle length and the constant detection length $\left(l_{j}+\right.$ $l_{D}$ ) for $L_{j}$, the following can be derived:

$$
\begin{align*}
& L_{j}=I_{j}+I_{D}=\left[\bar{I}+I_{D}\right]+\Delta l_{j}=\bar{L}+\Delta l_{j}  \tag{29}\\
& u_{j}=\bar{u}+\Delta u_{j} \tag{30}
\end{align*}
$$

It should be noted that in Equation 30, the mean speed is the time-mean speed for a polling interval given that it is the expectation of the speed over all realizations within the polling interval.
The expected occupancy within a polling interval can be computed using the expected value operator $(E\{*\})$, as follows:

$$
\begin{equation*}
E\{O\}=E\left\{\frac{3.6}{T} \cdot \sum_{j=1}^{N_{i}} \frac{I_{j}+I_{D}}{u_{j}}\right\}=E_{i}\left\{\frac{3.6}{T} \cdot \sum_{j=1}^{N_{i}} \frac{L_{j}}{u_{j}}\right\}=3.6 \cdot \frac{N}{T} \cdot E\left\{\frac{L_{j}}{u_{j}}\right\} \tag{31}
\end{equation*}
$$

It should be noted that each measurement produces a pair of volume $(N)$ and occupancy values $(O)$. Dailey denoted $E$ as the expectation over all realizations that have the volume $N$ to compute the expected occupancy for a polling interval, as follows:

$$
\begin{equation*}
E\{O\}=3.6 \frac{N}{T} \cdot E\left\{\frac{L_{j}}{u_{j}}\right\} \tag{32}
\end{equation*}
$$

Inserting Equations 29 and 30 into Equation 32, we get

$$
\begin{equation*}
E\{O\}=3.6 \frac{N}{T} \cdot E_{i}\left\{\frac{\bar{L}+\Delta l_{j}}{\bar{u}+\Delta u_{j}}\right\}=3.6 \frac{N}{T} \cdot E_{i}\left\{\frac{\bar{L}}{\bar{u}+\Delta u_{j}}+\frac{\Delta l_{j}}{\bar{u}+\Delta u_{j}}\right\} \tag{33}
\end{equation*}
$$

where the statistics of the deviation terms are selected such that $E\left\{\Delta l_{j}\right\}=E\left\{\Delta u_{j}\right\}=0$. Consequently, Equation 33 can be simplified assuming that $\Delta l_{j}$ and $\Delta u_{j}$ are independent

$$
\begin{equation*}
E\{O\}=3.6 \frac{N}{T} \cdot E\left\{\frac{\bar{L}}{\bar{u}+\Delta u_{j}}\right\}=3.6 \frac{N}{T} \cdot \frac{\bar{L}}{\bar{u}} E\left\{\frac{1}{1+\frac{\Delta u_{j}}{\bar{u}}}\right\} \tag{34}
\end{equation*}
$$

Dailey demonstrated that the correlation coefficient between $\Delta l_{j}$ and $\Delta u_{j}$ was very low using some sample field data ( $\mathrm{r}=$ 0.018 ) and thus, concluded that such an assumption of independency was reasonable. Dailey then expanded the RHS of Equation 34 using a power series to derive

$$
\begin{equation*}
E\{O\}=3.6 \frac{N}{T} \cdot \frac{\bar{L}}{\bar{u}} \cdot E\left\{1-\frac{\Delta u_{j}}{\bar{u}}+\frac{\Delta u_{j}^{2}}{\bar{u}^{2}}-\frac{\Delta u_{j}^{3}}{\bar{u}^{2}}+\ldots\right\} \tag{35}
\end{equation*}
$$

Noting that $\mathrm{E}\left\{\Delta u_{j}\right\}=0$ and approximating the series for three terms, Dailey derived the following:

$$
\begin{equation*}
E\{O\}=3.6 \frac{N}{T} \cdot \frac{\bar{L}}{\bar{u}} \cdot\left[1+\frac{E\left\{\Delta u_{j}^{2}\right\}}{\bar{u}^{2}}\right] \tag{36}
\end{equation*}
$$

Substituting $E\left\{\Delta u_{j}^{2}\right\}$ for the speed variance within the polling interval $i\left(\sigma_{i}^{2}\right)$, Dailey derived

$$
\begin{equation*}
E\{O\}=3.6 \frac{N}{T} \cdot \frac{\bar{L}}{\bar{u}} \cdot\left[1+\frac{\sigma^{2}}{\bar{u}^{2}}\right] \tag{37}
\end{equation*}
$$

Rearranging the terms and solving for $N$ Dailey derived

$$
\begin{equation*}
N=\frac{\bar{u} T}{3.6 \bar{L}} \cdot E\{O\} \cdot\left[\frac{\bar{u}^{2}}{\sigma^{2}+\bar{u}^{2}}\right] \tag{38}
\end{equation*}
$$

If the measurements over the different polling intervals are considered to have a constant mean, then the expected occupancy for a polling interval can be expressed as

$$
\begin{equation*}
E\{O\}=E\{\bar{O}+\Delta O\}=\bar{O} \tag{39}
\end{equation*}
$$

Considering the polling interval occupancy to be an estimate of the expected polling interval occupancy, the traffic stream speed over the polling interval can be computed as

$$
\begin{equation*}
\bar{u}=3.6\left(\frac{\bar{L}}{T}\right) \cdot \frac{N_{i}}{O_{i}} \cdot\left[\frac{\sigma^{2}+\bar{u}^{2}}{\bar{u}^{2}}\right]=3.6\left(\frac{\bar{I}+I_{D}}{T}\right) \cdot \frac{N}{O} \cdot\left[1+\frac{\sigma^{2}}{\bar{u}^{2}}\right] . \tag{40}
\end{equation*}
$$

Dailey mentioned that "previous authors have asserted that a ratio of measured volumes and occupancies converted to density by a constant can be used to estimate speed" (Hall and Persaud (5)(Hall and Persaud 1988); Persaud and Hurdle (17)(Persaud and Hurdle 1988); Hall and Gunter (18)(Hall and Persaud 1988); Ross (19)(Ross 1988)). Dailey then concluded that "an estimate which does not consider the variability of the speed contained in the variance ( $\sigma^{2}$ ) has a bias." While this conclusion is correct, the speed that is estimated in this formulation is the time-mean speed and not the space-mean speed, as is typically utilized in traffic stream analysis.

### 6.3.3 Comparison of Procedures

As was mentioned earlier, the speed that is computed in Equation 40 is the time-mean speed and not the space-mean speed. Consequently inserting Equation 27 (Hall and Persaud's derivation of space-mean speed) in Equation 40, we obtain

$$
\begin{equation*}
\bar{u}_{T}=\bar{u}_{S} \cdot\left[1+\frac{\sigma_{T}^{2}}{\bar{u}_{T} \cdot \bar{u}_{T}}\right] \approx \bar{u}_{S}+\frac{\sigma_{T}^{2}}{\bar{u}_{T}} \tag{41}
\end{equation*}
$$

In conclusion, we demonstrate that both the Hall and Persaud (5)(Hall and Persaud 1988) and the Dailey (6)(Dailey 1999) formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation compute the space-mean speed while the Dailey formulation computes the time-mean speed.

### 6.4 Relationship Between Space-Mean Speed and Travel Time Reliability

Typically, space-mean speed is measured at specific locations along a highway in order to estimate roadway travel times. Alternatively, travel times can be measured using license plate recognition cameras or Automatic Vehicle Identification (AVI) technologies and the desire is to not only estimate average travel speeds but also the reliability of these speeds. Consequently, this section attempts to relate space-mean speed variability to travel time variability in order to estimate either travel time or travel speed confidence limits.

The variance of travel times for all observations $j$ within a polling interval can be computed as

$$
\begin{equation*}
\sigma_{t}^{2}=\sum_{j} \frac{\left(t_{j}-\bar{t}\right)^{2}}{n} \tag{42}
\end{equation*}
$$

where $t_{j}=\frac{D}{u_{j}}$ and $\bar{t}=\frac{D}{\bar{u}}$. It should be noted that the mean speed $(\bar{u})$ is the space-mean speed. Expanding Equation 42 we derive

$$
\begin{equation*}
\sigma_{t}^{2}=\sum_{j} \frac{\left(t_{j}-\bar{t}\right)^{2}}{n}=\sum_{j} D^{2} \frac{\left(\frac{1}{u_{j}}-\frac{1}{\bar{u}}\right)^{2}}{n}=\frac{D^{2}}{n} \sum_{j} \frac{\left(\bar{u}-u_{j}\right)^{2}}{\bar{u}^{2} u_{j}^{2}} \tag{43}
\end{equation*}
$$

Recognizing that the speed coefficient of variation (standard deviation divided by mean) is typically small (less than 10 percent), we can ignore differences in speeds with minimum effect on the formulation to derive

$$
\begin{equation*}
\sigma_{t}^{2} \approx \frac{D^{2}}{\bar{u}^{4}} \sum_{j} \frac{\left(u_{j}-\bar{u}\right)^{2}}{n} \tag{44}
\end{equation*}
$$

Re-arranging the terms of Equation 44, the travel time variance can be related to the variance in speeds about the
space-mean speed as

$$
\begin{equation*}
\sigma_{t}^{2} \approx \frac{\bar{t}^{2}}{\bar{u}^{2}} \sum_{j} \frac{\left(u_{j}-\bar{u}\right)^{2}}{n}=\frac{\bar{t}^{2}}{\bar{u}^{2}} \sigma_{s}^{2} \tag{45}
\end{equation*}
$$

Solving Equation 45 we derive

$$
\begin{equation*}
\sigma_{t} \approx \frac{\bar{t}}{\bar{u}} \sigma_{s} \tag{46}
\end{equation*}
$$

Consequently, the final relationship relates the travel time and space-mean speed coefficients of variations as $C V_{t} \approx C V_{s}$.
Equation 47 demonstrates that the coefficient of variation of space-mean speeds is approximately equal to the coefficient of variation of travel times. In other words, a standard deviation in vehicle speeds of 10 percent the space-mean speed results in a standard deviation of roadway travel times that is 10 percent the mean travel time.

### 6.5 CONCLUSIONS

The paper demonstrates that the relationship between time-mean and space-mean speed that was derived by Wardrop (1)(Wardrop 1952) and presented in several textbooks (e.g. May (3)(May 1990)) produces an error in the range of 1 percent in time-mean speed estimates. However, the formulation estimates the time-mean speed from the space-mean speed, which is typically the reverse of what is required. Specifically, the objective is to estimate the space-mean speed from the time-mean speed. Consequently, using the statistics of the estimates, the paper derives a modified relationship between space-mean speed and time-mean speed which computes space-mean speed as a function of time-mean speed. The paper demonstrates that the proposed formulation, which utilizes the variance about the time-mean speed as opposed to the variance about the space-mean speed, produces an estimate error to within 0 to 1 percent, as is the case for the Wardrop formulation.
In addition, the paper demonstrates that both the Hall and Persaud (5)(Hall and Persaud 1988) and the Dailey (6)(Dailey 1999) formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation computes the space-mean speed (harmonic mean) while the Dailey formulation computes the time-mean speed (arithmetic mean).
Finally, the paper demonstrates that the space-mean speed coefficient of variation (standard deviation divided by mean) is approximately equal to the coefficient of variation of roadway travel times. Using this relationship it would be possible to estimate travel speed confidence limits based on field measurements of travel times.

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Figure 6.1 Comparison between Time-Mean and Space-Mean Speed Estimates (Median Lane).


Figure 6.2 Comparison between Time-Mean and Space-Mean Speed Estimates (Lane 4).


Figure 6.3 Temporal Variation in Vehicle Speed and Length Measurements on the Median Lane.


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Figure 6.8 Correlation between Estimated and Measured Space Speed Variance (Median Lane (left) and Lane 4 (right)).

Chapter 7. Travel Time Estimation based on Loop Detectors

# Evaluation of Freeway Travel Time Estimation Approaches based on Spot Speed Measurements 

Wang Zhang ${ }^{7}$ and Hesham Rakha ${ }^{8}$<br>Word count: $\quad 6,547$<br>Tables and figures: $\quad 3,000$<br>Total words: $\quad 9,547$


#### Abstract

The paper focuses on freeway travel time estimation (TTE) algorithms that are based on spot speed measurements. Several TTE approaches are introduced including a traffic dynamics TTE algorithm that is documented in the literature. This traffic dynamics algorithm is analyzed highlighting some of its drawbacks, followed by some proposed corrections to the traffic dynamics formulation. The proposed approach estimates traffic stream density from occupancy measurements as opposed to flow measurements at the onset of congestion. Next, the paper validates the proposed model using field data from I-880 and simulated data. Comparison of five different TTE algorithms is conducted. The comparison demonstrates that the proposed approach is superior to the TTE traffic dynamics approach. Particularly, a multi-link simulation network is built to test spot-speed-measurement TTE performance on multi links, as well as data smoothing technique's effect on TTE accuracy. In summary, the paper has found a feasible TTE procedure that is adaptive to various traffic conditions.


### 7.1 INTRODUCTION

Accurately estimating travel time on urban freeways with a certain device or method, or Travel Time Estimation (TTE), is a critical component of Advanced Traveler Information Systems (ATIS) and Advanced Traffic Management Systems (ATMS). For instance, correct instantaneous TTE outcomes can be used by Dynamic Traffic Assignment (DTA), by knowing when and where commuters are on the road network. It enables traffic management centers not only to react to existing conditions, but also to anticipate problems before they occur. Also, instantaneous travel time information can be transmitted to on-road and off-road commuters that would help reduce existing congestion and optimize the entire road system's mobility.

This paper discusses the different TTEs based on spot measurements such as loop detectors that are broadly installed on freeways. By using I-880 field data and micro-simulation data, several TTE approaches are first introduced and analyzed, corrections are made, and then a proposed approach is presented with both field and simulation validations. Lastly, conclusions are addressed based on comparisons of five different approaches.

### 7.2 BACKGROUND

### 7.2.1 Vehicle Re-Identification Approach

Vehicle Re-Identification (VRI, Coifman (Coifman 2003)) tries to obtain travel time by matching the same vehicle based on its length at both upstream and downstream detector stations. Dual-loop detector are required by the VRI approach because it requires vehicle length measurements, and VRI can only do vehicle matching for one specific lane. The approach requires a matching matrix that contains vehicle length sequences at both upstream and downstream locations; a matching algorithm is applied to find a matched sequence of vehicles; and finally the TTE is successfully found by

[^3]studying matched vehicle's passing times at upstream and downstream locations.
One advantage of VRI is that it does not need spot-measured speeds as a necessary input of TTE, as it is well known that a loop detector cannot provide accurate speed measurements under congested conditions, and this fact would lead any TTE approach based on spot speed measurements to erroneous travel time estimates. The disadvantage of VRI is that it is lane specific. This creates problems in estimating traffic flow with frequent lane-changing maneuvers. Moreover, due to the detector's resolution of only $1 / 60$ s, vehicle-length estimation accuracy decreases while vehicle speed increases. These two factors increase estimate errors for free-flow condition cases where vehicles have longer spacing headway, more lane changing, and higher noise on vehicle-length measurements. Therefore, for the free-flow condition, the VRI is more likely to depend on long vehicle-length matching, such as trucks. On the other hand, it is VRI's advantage to estimate travel time under congested conditions in which more precise vehicle-length measurements are obtained and vehicles seldom make lane changes, so long matched vehicle sequences are easily found and TTEs are obtained with high confidence.

The travel time profiles from the matched vehicles are regarded as a solid resource of ground truth travel time, in the following discussion relating to the I-880 field case, VRI outcomes are utilized as the ground truth for comparing other approaches.

### 7.2.2 Dynamics Approach

A traffic dynamics TTE approach proposed in the literature(Nam D.H. 1998) is a method evolved from traffic dynamics to evaluate link travel time based on loop-detector data. The logic behind this approach is to estimate traffic density as the difference between the number of vehicles entering the link and leaving it during the same time interval, which corresponds to the change in the number of vehicles traveling on the link. Using this approach it is feasible to obtain total travel time during that interval and then further to estimate the average travel time of all vehicles within a polling interval.


Figure 7.1 Traffic Dynamics, Link TTE from Cumulative Flow
For example, two loop detectors at upstream and downstream placements can represent ends of a link, and the link travel time could be obtained by calculating cumulative flows and interval traffic volumes of both detectors along with link length. The procedure to evaluate travel time is shown below:

$$
\begin{align*}
& k\left(t_{n}\right)=\frac{Q\left(x_{1}, t_{n}\right)-Q\left(x_{2}, t_{n}\right)}{\Delta x}  \tag{1}\\
& m=Q\left(x_{2}, t_{n}\right)-Q\left(x_{1}, t_{n-1}\right) \tag{2}
\end{align*}
$$

$$
\begin{equation*}
t_{j}=\frac{\Delta x}{2} \frac{\left[q\left(x_{1}, t_{n}\right) k\left(t_{n-1}\right)+q\left(x_{2}, t_{n}\right) k\left(t_{n}\right)\right]}{q\left(x_{1}, t_{n}\right) q\left(x_{2}, t_{n}\right)}, \quad \text { when } \mathrm{m}>=0 \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
t=\frac{\Delta x}{2} \frac{k\left(t_{n-1}\right)+k\left(t_{n}\right)}{q\left(x_{2}, t_{n}\right)}, \quad \text { when } \mathrm{m}<0 \tag{4}
\end{equation*}
$$

Where:
1: upstream index
2: downstream index
$\mathrm{t}_{\mathrm{n}}$ : time interval n
Q: cumulative flow counts (veh)
q : traffic volume per interval (veh)
$\Delta \mathrm{X}$ : link length ( m )
k: density ( $\mathrm{veh} / \mathrm{m}$ )
t : link travel time
m : domain factor, when $\mathrm{m} \geq 0$ it represents normal flow condition (vehicles can enter and exit the link during the same interval), when $\mathrm{m}<0$ it represents congested flow condition (vehicles can enter but cannot exit the link during the same interval), according to the $m$ value, different travel time calculations apply.
As shown above, the main procedures of the Dynamics TTE are to calculate densities from cumulative flows and then calculate travel time from the densities and interval volumes. Dynamics is another approach to estimate travel time without spot speed measurements and for the roadway section. However, this approach heavily relies on cumulative flow so any detector malfunction or error results in serious estimate errors. Therefore, the detector data cleaning and adjustment is very critical to the Dynamics TTE approach.

### 7.2.3 Enhancements of Dynamics Approach

In order to obtain better performance of the Dynamics TTE approach, Vanajakshi and Rilett (Vanajakshi L. 2004) developed an enhanced Dynamics approach; the main differences from original Dynamics approach are:

1) Apply a set of data-cleaning procedures (Vanajakshi L. 2004), because any malfunction on a detector, spatially or temporally, could cause error in cumulative flow counts and others.

- Maximum speed as $150 \mathrm{~km} / \mathrm{h}$.
- Maximum density as $125 * 5=625 \mathrm{veh} / \mathrm{km}$ where 5 is the number of lanes.
- Maximum vehicle length as 30 m .
- Upstream cumulative flow volume is always larger than or equal to downstream cumulative flow volume in any interval. (Note: all data were cleaned for the model testing).

2) Use occupancy instead of cumulative flow to estimate density.
$k=52.8 \times \frac{O}{\left(\overline{L_{v}}+L_{d}\right)}$
k density (veh/mi)
$\overline{L_{v}}$ : average vehicle length (ft)
$L_{d}$ : detection zone length ( ft )
$O$ : percentage of occupancy
3) Different travel time calculation for normal flow condition.

$$
\begin{equation*}
t_{j}=m_{p} \frac{\Delta x}{2} \frac{\left[q\left(x_{1}, t_{n}\right) k\left(t_{n-1}\right)+q\left(x_{2}, t_{n}\right) k\left(t_{n}\right)\right]}{q\left(x_{1}, t_{n}\right) q\left(x_{2}, t_{n}\right)}+\left(1-m_{p}\right) \frac{\Delta x}{2}\left[\frac{k\left(t_{n-1}\right)+k\left(t_{n}\right)}{q\left(x_{2}, t_{n}\right)}\right] \tag{6}
\end{equation*}
$$

where $m_{p}$ is the ratio of $m$ to the inflow value.
It is claimed that the original approach only considers vehicles which enter and leave the link in the same interval, but
this modification applies the normal flow model to a portion of such vehicles and applies congested flow model to those vehicles that are not able to exit in the same interval.
4) Considering the speed on a freeway is insensitive to flow in the low to moderate range, the enhanced approach does not use Dynamics TTE, but uses spot speed measurements to estimate travel time. The threshold is set to 40 vehicles per 2 min , and the Dynamics TTE approach is only applied when flow condition is higher than this threshold.

### 7.3 Corrections and Problems with the Dynamics Approach

### 7.3.1 Correction to Dynamics TTE for congested conditions

In both the original and enhanced Dynamics approaches, the travel time equation under congested condition is as shown in equation [4], the deduction of this equation is as the following (Nam D.H. 1999):


Figure 7.2 TTE from Cumulative Flow under Congestion, Nam and Drew (Nam D.H. 1998)
In Figure 7.2, the X -axis is time and the Y -axis is cumulative flow, and the two trajectories represent the cumulative/time relationships at upstream $\left(Q\left(x_{1}\right)\right)$ and downstream $\left(Q\left(X_{2}\right)\right)$ locations. Time $t_{n-1}$ and $t_{n}$ are interval start and end times, $t^{\prime}$ is the expected time of departure from the link of the last vehicle that enters the link during the interval, and t " is the expected time of departure from the link the first vehicle enters the link during the interval, therefore:

$$
\begin{align*}
t^{\prime} & =t_{n-1}+\frac{k\left(t_{n-1}\right) \Delta x}{q\left(x_{2}, t_{n}\right)}  \tag{7}\\
t^{\prime \prime} & =t_{n-1}+\frac{k\left(t_{n}\right) \Delta x}{q\left(x_{2}, t_{n}\right)} \tag{8}
\end{align*}
$$

Knowing that total travel time is:

$$
\begin{equation*}
\frac{1}{2}\left[\left(t^{\prime \prime}-t_{n-1}\right)+\left(t^{\prime}-t_{n}\right)\right]\left[Q\left(x_{1}, t_{n}\right)-Q\left(x_{1}, t_{n-1}\right)\right] \tag{9}
\end{equation*}
$$

Vehicle travel time under congested condition is:

$$
\begin{equation*}
t t\left(t_{n}\right)=\frac{\Delta x}{2} \frac{k\left(t_{n-1}\right)+k\left(t_{n}\right)}{q\left(x_{2}, t_{n}\right)} \tag{10}
\end{equation*}
$$

According to conservation of vehicles:

$$
\begin{equation*}
\frac{q\left(x_{1}, t_{n}\right)-q\left(x_{2}, t_{n}\right)}{\Delta x}=\frac{k\left(t_{n}\right)-k\left(t_{n-1}\right)}{\Delta t} \tag{11}
\end{equation*}
$$

The Dynamics further claims that the relationships between travel time and the two flow rates are:

$$
\begin{gather*}
\frac{\partial t t}{\partial q_{1}}=\frac{\Delta t}{2 q_{2}}  \tag{12}\\
\frac{\partial t t}{\partial q_{2}}=-\frac{k\left(t_{n-1}\right) \Delta x+q_{1} \Delta t}{2 q_{2}} \tag{13}
\end{gather*}
$$

However, this deduction is not correct from the very beginning. The right time/cumulative flow relationship is as the following (Figure 7.3):


Figure 7.3 TTE from Cumulative Flow under Congestion, Proposed
Figure 7.3 demonstrates that in congested condition, vehicles spend travel time longer than an interval length to traverse the link, which means that a vehicle entering a link cannot exit the link in the same interval.

$$
\begin{align*}
& t^{\prime \prime}=t_{n-1}-\frac{k\left(t_{n-1}\right) \Delta x}{q\left(x_{2}, t_{n-1}\right)}  \tag{14}\\
& t^{\prime}=t_{n}-\frac{k\left(t_{n}\right) \Delta x}{q\left(x_{2}, t_{n}\right)} \tag{15}
\end{align*}
$$

Consequently, the correct equation to evaluate travel time in the congested condition is:

$$
\begin{equation*}
t t\left(t_{n}\right)=\frac{\Delta x}{2}\left[\frac{k\left(t_{n-1}\right)}{q\left(x_{2}, t_{n-1}\right)}+\frac{k\left(t_{n}\right)}{q\left(x_{2}, t_{n}\right)}\right] \tag{16}
\end{equation*}
$$

Applying equation [11], it transforms to:

$$
\begin{align*}
& t t\left(t_{n}\right)=\frac{\Delta x}{2}\left[\frac{k\left(t_{n-1}\right)}{q\left(x_{2}, t_{n-1}\right)}+\frac{q\left(x_{1}, t_{n}\right)-q\left(x_{2}, t_{n}\right)}{q\left(x_{2}, t_{n}\right)} \frac{\Delta t}{\Delta x}+\frac{k\left(t_{n-1}\right)}{q\left(x_{2}, t_{n}\right)}\right] \\
& =\frac{k\left(t_{n-1}\right) \Delta x}{2 q\left(x_{2}, t_{n-1}\right)}+\frac{k\left(t_{n-1}\right) \Delta x}{2 q\left(x_{2}, t_{n}\right)}+\frac{q\left(x_{1}, t_{n}\right)-q\left(x_{2}, t_{n}\right)}{2 q\left(x_{2}, t_{n}\right)} \Delta t  \tag{17}\\
& \approx \frac{2 k\left(t_{n-1}\right) \Delta x+\left[q\left(x_{1}, t_{n}\right)-q\left(x_{2}, t_{n}\right)\right] \Delta t}{2 q\left(x_{2}, t_{n}\right)}
\end{align*}
$$

Therefore, the correct relationships between travel time and the two flow rates are:

$$
\begin{equation*}
\frac{\partial t t}{\partial q_{1}}=\frac{\Delta t}{2 q_{2}} \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial t t}{\partial q_{2}}=-\frac{2 k\left(t_{n-1}\right) \Delta x+q_{1} \Delta t}{2 q_{2}} \tag{19}
\end{equation*}
$$

### 7.3.2 Interval Volume Impact on Dynamics Approach

The primary problem of the Nam and Drew traffic Dynamics TTE approach occurs under low-flow conditions. Typically during uncongested conditions the average travel time should be fairly constant; however, the travel time estimates based on I-880 field data show an underlying oscillating travel time function in the middle of the night, as illustrated in Figure 7.4. For this $548-\mathrm{m}$ link the free-flow speed travel time should be approximately 20 s , however, as is evident from the figure the underlying travel time ranges from 0 to 50 s , with errors in the range of $50 \%$. Given that the input to the travel time estimation algorithm include the traffic stream density in the previous polling interval, the temporal variation in density estimates were plotted, as illustrated in Figure 7.5.


Figure 7.4 Mid-night (00:00-04:00) TTE results of I-880, Dynamics method


Figure 7.5 00:00-04:00 density profiles of I-880, Dynamics method
This figure also shows that the traffic stream density estimates oscillate significantly as was the case with the travel time estimates. The correlation factor between the traffic stream density and the travel time is 0.708 , which indicates a strong positive correlation between these two variables. In an attempt to investigate if density is the primary cause for these travel time errors, a simple sensitivity analysis was run by varying the density in range of 0 to $350 \mathrm{veh} / \mathrm{km}$ ( 70 veh $/ \mathrm{km} / \mathrm{lane}$ ) considering two flow levels of 900 and $1,200 \mathrm{veh} / \mathrm{h}$ ( 180 to $240 \mathrm{veh} / \mathrm{h} / \mathrm{lane}$ ) at the upstream and downstream locations, respectively. In addition the analysis is repeated for a 10-fold higher flow rate (1800 to 2400 veh/h/lane), as illustrated in Figure 7.6. Figure 7.6 clearly demonstrates that the travel time is highly dependent on the traffic stream density level at the low flows and less sensitive at the high flow rates. Specifically, and 5 -vehicle differential in density ( $1 \mathrm{veh} / \mathrm{km} / \mathrm{lane}$ ) results in an approximate 10 -second travel time differential for the 180 and 240 $\mathrm{veh} / \mathrm{h} /$ lane flow rates, and only a 1 -second differential for the 1800 and $2400 \mathrm{veh} / \mathrm{h} /$ lane flow rates.


Figure 7.6 Density/Travel Time profiles
Given that the Dynamics TTE model computes density from the cumulative flow difference between upstream and downstream detectors. Under free-flow traffic conditions for this 548 -meter long test bed, a $5 \mathrm{veh} / \mathrm{km}$ density differential represents just a 3 -vehicle differential per interval ( 2 min ). In other words, if the cumulative flow difference between the upstream and downstream detector stations changes by more than three vehicles from one interval to another, TTE for these two intervals could result in a 10 -second difference representing a $50 \%$ variation from normal conditions. In the case of the I-880 test bed from 00:00 to 04:00 with 2 min as an interval, $57 \%$ of the interval's cumulative flow differences between upstream and downstream are found to be larger than 3 vehicles, ranging from 3 to 14 . Under free-flow conditions, a small change in cumulative flow would have observable effects on density, which then impacts TTE considerably. This explains the high variability in of travel times shown in Figure 7.4. Undoubtedly, using cumulative flow in a Dynamics TTE model shows obvious estimate errors under low-flow conditions.

### 7.3.3 Traffic Simulation Dataset Validation

The percentage change in traffic volume from one interval to the next interval is illustrated graphically in Figure 7.7 while Figure 7.8 illustrates the change in travel time from one time interval to the next.


Figure 7.7 Detector's traffic-volume variance from one interval to the previous one, I-880


Figure 7.8 TTE variance from one interval to the previous one, I-880
Qualitatively, both figures are analogous. Further proof of similarity is found by conducting an auto-correlation analysis of flows and travel times. The auto-correlation factor is -0.651 and -0.771 for flow and travel time, respectively. The negative correlation indicates that a reduction in a variable in one time interval is followed by an increase in the next. During the period from 00:00 to 04:00 a.m., 50 and $45 \%$ of the 2 -minute intervals have a volume variation greater than $20 \%$ from one interval to the next at the upstream station and downstream stations, respectively.
A similar network was constructed in the INTEGRATION (Assoc. 2002) software in which probe vehicles provided true travel time measurements. The TTE travel time estimates were compared to the actual travel times. For these "perfect data," true travel time was obtained from the probe vehicles (red line) and two detectors were setup in the simulation network. Therefore, the data collected by the simulated detectors was used to estimate travel time (blue line, Dynamics), which is shown in Figure 7.9. Similar to the I-880 field data, the TTEs experience an oscillating trend during the midnight period.


Figure 7.9 True travel time vs. estimated travel time, 00:00-01:00, simulation test bed
The variability in detector measurements from one time interval to another is similar to the field data that were presented earlier, as illustrated in Figure 7.10 and 7.11.


Figure 7.10 Detector's traffic volume variance from an interval to the previous one, simulation


Figure 7.11 TTE variance from an interval to the previous one, simulation
The auto-correlation for volume and travel-time were similar to the I-880 field data. Specifically, the auto-correlation factor for the upstream detector was -0.701 and -0.765 for the downstream detector. These correlation factors are consistent with the I-880 field data.

### 7.3.4 Impacts from " $m$ "

As discussed above, Nam and Drew's original Dynamics TTE model and Vanajakshi and Rilett's enhanced Dynamics TTE model both use " $m$ " $\left(m=Q\left(x_{2}, t_{n}\right)-Q\left(x_{1}, t_{n-1}\right)\right)$ to define two different traffic flow conditions. If $m$ is larger than or equal to 0 the traffic condition is deemed as uncongested, in which the vehicle is able to enter and leave the link within a single time interval. If $m$ is less than 0 the traffic condition is deemed as congested, in which a vehicle entering a link is not able to exit within the same time interval. For these two traffic conditions the Dynamics TTE model chooses different formulations to estimate travel time.
The I-880 and simulation data temporal variation in $m$ relationships are presented in Figure 7.12 and Figure 7.13:


Figure 7.12 Profile of " $m$ " in 24 hours, I-880 case


Figure 7.13 Profile of " $\mathbf{m}$ " in $\mathbf{2 4}$ hours, simulation case
According to the I-880 speed profile, a heavy congestion is found from 16:00-19:00, (the lower bound of speed profile is $20-30 \mathrm{~km} / \mathrm{h}$ ). However, in the I-880 field data m -factor remains positive during the p.m. peak period, which means in true congested conditions, the Dynamics approach still uses the normal condition formulation to estimate travel times. This is because that m's value is also determined by the length of the link and the length of the polling interval. For example, first assume that a vehicle is traveling at a constant speed of $u$, for a link with length of 1 and a polling interval of duration $t$, the condition that it can enter and exit the link during the same interval is:

$$
\begin{equation*}
u \geq \frac{l}{t} \tag{20}
\end{equation*}
$$

For the $\mathrm{I}-880$ case study, $\mathrm{l}=548 \mathrm{~m}, \mathrm{t}=120 \mathrm{~s}$, consequently the minimum speed to meet this requirement is $\mathrm{u}=16 \mathrm{~km} / \mathrm{h}$. Basically, for an interval, if the traffic stream average traveling speed is greater than or equal to $16 \mathrm{~km} / \mathrm{h}$, m is larger than or equal to 0 , and vice versa. However, this threshold depends on the link length and the duration of the polling interval. If a time interval is reset to 1 min , then the threshold speed changes to $32 \mathrm{~km} / \mathrm{h}$. If the time interval is reset to 4 min , the threshold speed changes to $8 \mathrm{~km} / \mathrm{h}$. Thus, it is demonstrated that a change in the link length and/or the duration of the polling interval can change the congestion/non-congestion regime definition. Another parameter impacted by the polling interval duration is " $m$," as described above the " $m$ " is used to determine regimes that apply different equations in estimating travel time. The m/time profiles (simulation data) with different interval sizes ( $30 \mathrm{~s}, 60 \mathrm{~s}$, and 122 s ) are shown in Figure 7.14 below.


Figure 7.14 Profile of $\mathbf{m}$ on different interval sizes
The figure demonstrates the inconsistency in TTE between different interval sizes for $m$ value. For the same traffic condition, m's value (negative or positive) is subject to the interval's duration (06:00 to 10:00), but not the true traffic flow conditions.

The problem for the simulation's case is that m is negative at midnight according to detector measurements, but the traffic condition at midnight is always under free-flow condition so there is no way for m to appear as a negative value. The reason is that m's value is heavily affected by cumulative flow, and in low-flow condition it is possible to create a negative $m$ value as long as the vehicle counts have a sudden change from one interval to the next.
This issue brings out a question about how to determine normal condition and congested condition. In Dynamics TTE model, vehicle speeds actually determine it ( $16 \mathrm{~km} / \mathrm{hr}$ ), but speeds are further subject to lengths of interval and link, therefore using " $m$ " as the boundary of normal and congested conditions is questionable, because once " $m$ " does not represent the correct condition, wrong formulations would be applied.

### 7.3.5 Summary

The original Dynamics approach travel-time estimation under congested conditions is incorrect, as addressed above. Moreover, under the free-flow condition, the Dynamics TTE approach cannot provide accurate estimates primarily because of the impact from cumulative flow errors that are widely found on detectors. Though the cumulative problem can be reduced by data cleaning, the interval volume is another factor where very small variations between intervals would cause considerable errors in the TTE. The last factor, " m " is subject to change by different interval sizes and link lengths; this theoretically leads to incorrect formulation selection in TTE.

### 7.4 Proposed Model

The above discussion has indicated that using " $m$ " as a regime divider is inefficient since it cannot always identify the true flow condition and is affected by the polling interval size. To find a reliable way to identify the correct flow condition is significant, it is proposed to use a density threshold as the boundary between normal and congested conditions. The speed/flow and speed/density relationships of the link are primarily calibrated from upstream and downstream detector data, and then the density at maximum flow is regarded as the density at capacity whose value is utilized as the threshold ( $25 \mathrm{veh} / \mathrm{km} /$ lane for I-880 field data).


Figure 7.15 Proposed density profile on different interval sizes
Figure 7.15 demonstrates the density/time profiles with different interval sizes. It is shown that they are almost identical. With the black dashed line as the density threshold, if the density is lower than the line then the normal condition is applied, and vice versa. There is no indication that the interval size would have an impact on the traffic condition choice; the formulation for selecting between normal and congested flow conditions is consistent regardless of the interval size. Such advantage effectively avoids impacts brought on by interval size and link length.

Moreover, considering the cumulative method's density estimating drawbacks, the proposed approach utilizes occupancy to estimate density, and the average vehicle length is updated every interval. In summary, the proposed approach is presented as follows:

$$
\begin{array}{ll}
t_{j}=\frac{\Delta x}{2} \frac{\left[q\left(x_{1}, t_{n}\right) k\left(t_{n-1}\right)+q\left(x_{2}, t_{n}\right) k\left(t_{n}\right)\right]}{q\left(x_{1}, t_{n}\right) q\left(x_{2}, t_{n}\right)}, \text { when } \mathrm{k}<125 \mathrm{veh} / \mathrm{km}(25 \mathrm{veh} / \mathrm{km} / \mathrm{lane}) \\
t=\frac{\Delta x}{2}\left[\frac{k\left(t_{n-1}\right)}{q\left(x_{2}, t_{n-1}\right)}+\frac{k\left(t_{n}\right)}{q\left(x_{2}, t_{n}\right)}\right], & \text { when } \mathrm{k}>125 \mathrm{veh} / \mathrm{km}  \tag{22}\\
k=52.8 \times \frac{O}{\left(\overline{L_{v}}+L_{d}\right)} &
\end{array}
$$

repeat: [5]

Note: the value of density threshold ( $125 \mathrm{veh} / \mathrm{km}$ ) requires calibration for different test beds.

### 7.5 Field Validation

### 7.5.1 Different Travel Time Algorithms

The I-880 test bed contains field data of two loop station (dual loop detector) measurements including individual vehicle length, individual vehicle speed, flow volume, and occupancy.

- Link length: 548.64 m
- Time interval: 2 min
- Total time span: 24 hrs
- True travel time: obtained by VRI (Vehicle Re-Identification ${ }^{[6]}$ ).
- Note: All results presented have been smoothed.
- It is necessary to run a data cleaning before conducting the Dynamics TTE method. The cleaning target is set to meet some thresholds ${ }^{[3]}$ :

Five approaches are compared here on the same test bed:
Approach 1: Nam and Drew's Approach, uses cumulative flow to estimate density and two regimes determined by "m", equation [3] and [4].
Approach 2: Nam and Drew's Approach, uses occupancy to estimate density (equation [5]) and two regimes determined
by " $m$ ", equation [3] and [4].
Approach 3: Vanajakshi and Rilett's Approach ${ }^{[5]}$, uses occupancy to estimate density (equation [5]) and two regimes determined by " $m$ " (different formula in estimating travel time under normal condition, equation [6] for normal condition and [4] for congested condition.
Approach 4: Proposed approach uses occupancy to estimate density (equation [5]) and two regimes determined by a density threshold developed using the flow/speed relationship, equation [3] and [16].
Approach 5: Spot speed measurement approach uses the speed measured by a loop detector to assume spot speeds as representative of the link speed and then calculates the link travel time based on the spot speed measurement, equation [22] and [23].

### 7.5.2 I-880 Test Bed Results Analysis

Figure 7.16 illustrates the travel time profiles of the five algorithms in comparison to the actual travel time estimates based on the VRI technique.


Figure 7.16 Five TTE outcomes comparison, I-880
It is observed that Approach 1 has the lowest estimation accuracy among the five. While the other four approaches display very similar estimates to each other, only Approach 5 shows some variation from the other three during the midnight period. Noted that the true travel time value is higher than approaches 2, 3, 4, and 5 from 00:00 to 14:00; one possible reason is that the VRI algorithm estimates travel time mainly depending on long-length vehicle measurements when traffic flow is in free-flow condition. In this way the true travel time profile primarily represents long-length vehicle (truck) travel times, but not the entire traffic flow travel time. In congested conditions (14:30-18:30), Approaches $2,3,4$, and 5 present trustable travel time estimates. In order to see how well each approach matches the true travel time, Figure 7.17 is presented:


Figure 7.17 Five TTE outcomes comparison II, I-880
If true travel time (VRI) is plotted on the X -axis and five approach travel times are plotted on the Y-axis, the figure above is obtained. On this figure, a good estimate approach would show point positions closer to the diagonal line; if the point is above the diagonal line it represents over-estimation, and vice versa. Obviously, Approach 1 does not estimate travel time well when the true travel time is less than 50 s (normal traffic conditions). Under the same condition, the other four approach results are very close and show the same trend: they under-estimate the travel time when the VRI travel time is 25 to 50 s (partly due to the long-length vehicle's slower traveling speed). Contrarily, when true travel time is higher than 50 s (in congestion condition), Approach 1 has better estimation accuracy with a bit of over-estimation. The other four approaches show ideal estimation accuracy in that their positions are very near the diagonal line.


Figure 7.18 Five TTE outcomes comparison under PM peak hours, I-880
Figure 7.18 displays only travel time outcomes for the p.m. peak period (14:30-18:30). When true travel time is less than 50 s , Approaches 2, 3, 4, and 5 underestimate the travel time. When the true travel time is higher than 50 s (in heavy congestion), these four approaches display better estimation. As discussed before, Approach 2 and 3 use " m " to determine two regimes and then utilize different equations to calculate travel time. In this period, $m$ values are all greater than 0 in each interval, which means that the normal condition travel time equation is used under the congestion condition. Since Approaches 2 and 4 use different regime dividers but the same equations, it is possible to find in one interval different traffic conditions were applied ( $45 \%$ of intervals are found to use different regimes during this period for Approaches 2 and 4). However, the results show no obvious difference in TTE between Approach 2 (using normal condition equation) and Approach 4 (using the congestion condition equation). This reveals that different equations of normal and congested condition do not bring out distinctly different values in TTE. Additionally, Approaches 2 and 3 use the same regime divider but different travel time equations under the normal condition; unable to observe differences
between the outcomes of each indicates that different travel-time calculations for normal conditions have no obvious impact on the travel time estimates.

As stated before, the density estimation is very critical for the accurate estimation of travel times. The main difference between Approach 1 and the others is the density estimation procedure. Above results not only indicate that the cumulative-flow estimation error has great impacts on the travel time accuracy, but also show that estimating density from occupancy is a more feasible procedure than cumulative flow.

### 7.6 Model Testing using Simulated Data

### 7.6.1 Comparison of Approaches

In order to avoid data failure and detector malfunction impacts, particularly on Approach 1, some simulation scenarios were run to test the algorithms on error free data. The INTEGRATION was used for the study using the same O-D demand as the field data. Two loop detectors were placed on a link of 548.64 m length, in order to create a congestion period as to mimic I-880's AM and PM peak hours, a two-lane on-ramp was introduced downstream of the link with periodic volume loading, creating a bottleneck. Therefore, this simulation test bed contains the complete periods of free-flow condition, pre-peak condition with traffic increasing, peak-hour condition with maximum traffic, and post-peak condition with traffic decreasing. True travel time is obtained by the simulation probe vehicles and all results presented below are after data-smoothing processing ${ }^{[7]}$.


Figure 7.19 Five TTE outcomes comparison I, simulation
Consistency is observed in the figure above that Approach 1 shows oscillating estimates during the free-flow condition, and as explained above it is because of the flow-count variance from one interval to the next.


Figure 7.20 Five TTE outcomes comparison II, simulation
Figure 7.20 illustrates the relation between the true travel times and estimated travel times from 0 to 100 s . Beside Approach 1's oscillating pattern under the free-flow condition shown, it over-estimates travel time slightly (consistent to the I-880 case). Approaches 2, 3, and 4 have similar performance with a slight under-estimation of travel time in range of 40 to 100 s . Approach 5 has the similar tendency with more underestimates compared to 2 , 3 , and 4 . Approach 1 performs much better with the simulated data than with the I-880 field data because the simulation case has truly accurate cumulative flow counts that guarantee Approach 1's effectiveness. Also, under free-flow conditions (20- to 40-second travel time), Approaches 2, 3, 4, and 5 can estimate travel time very precisely.

If only the AM peak results are considered, the results look similar, as illustrated in Figure 7.21. It is observed that the travel time in the AM peak hours varies from 20 to 70 s . Approach 1 over-estimates travel times during the entire peak period. Approaches 2, 3, 4, and 5 estimate travel times correctly if the travel time value is less than 40 s , with increasing of the true travel time, these four methods show an increasing trend of under-estimate travel times.


Figure 7.21 Five TTE outcomes comparison under AM peak hours, simulation
Results for the heavy-congestion condition are displayed in Figure 7.22. It is observed that Approach 1 underestimates and overestimates travel times and Approach 2, 3, 4, and 5 over-estimate travel times. This is the opposite of what Figure 7.21 shows. Moreover, no difference is found between Approaches 2 and 4 although they apply different estimate regimes.


Figure 7.22 Five TTE outcomes comparison under heavy congestion, simulation
Table 7.1 displays statistically TTE performance of each approach under the same scenario of simulation test-bed, results of RMSE between estimated travel time and true travel time of five approaches show the consistency between graphic results and numerical results. Propose model (approach 4) has the best estimation accuracy for the whole simulation period and congested period, oppositely, Dynamics (approach 1) provides the highest RMSE value as the lowest accuracy. It is also observed that spot speed TTE (approach 5) can provide a fairly good TTE under the free-flow condition.
Table 7.1. RMSE of each approach on simulation test-bed comparing to probe travel time

| TTE RMSE | Approach 1 | Approach 2 | Approach 3 | Approach 4 | Approach 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 24 hours data | 173200 | 123400 | 126200 | 98266 | 160300 |
| Free-flow condition | 7625 | 595 | 606 | 586 | 403 |
| Congested condition | 165600 | 122800 | 125600 | 97600 | 159900 |

### 7.6.2 Interval Length Sensitivity Analysis

Interval length can be set from link free-flow travel time to 5 min according to Nam \& Drew's Dynamics TTE method. As discussed before, the interval size would impact the " $m$ " value. In order to see how interval length affects the travel time estimates, Approach 1 is tested with simulation data, and the interval changes from 30 to 60 to 120 s.


Figure 7.23 Interval size impacts on Dynamics method
For the constant traffic condition (constant travel time), the results show oscillating TTEs, for shorter time intervals. Moreover, with interval sizes shortening, Approach 1 more likely underestimates travel time, and such character is found very obvious on the 30 s figure. Such results prove that the Dynamics TTE approach is sensitive to interval sizes.

### 7.6.3 Multi-Link TTE based on Spot Speed Measurements

A TTE for a section containing multiple links is more practical than just for single link, in all five spot-measurement approaches discussed, Approach 5 is particularly simple theoretically and the outcomes turn out to be acceptable when comparing to other options. Therefore, a multi-link INTEGRATION simulation network is developed to test the approach. This is an 8.5 -kilometer long section with composed of 5 lanes and link \#3 and \#9 are set as bottlenecks with only 4 lanes.


Figure 7.24 Multi-link test bed of INTEGRATION simulation
Spot speed is measured at both ends of each link. In addition, time stamp of each vehicle passage is recorded to provide the actual travel times experienced by a vehicle. The time-mean speed (TMS) and space-mean speed (SMS) are computed. These speeds are used as the spot speed measurements to evaluate link travel time. Two methods to calculate link travel time are used.

$$
\begin{equation*}
t=\frac{1}{2}\left(\frac{\Delta x}{u_{1}}+\frac{\Delta x}{u_{2}}\right) \quad(\text { Method } 1) \tag{23}
\end{equation*}
$$

$$
\begin{equation*}
t=\frac{\Delta x}{\left(u_{1}+u_{2}\right) / 2} \quad(\text { Method } 2) \tag{24}
\end{equation*}
$$

Where:

$$
\Delta \mathrm{x}: \text { length of link, (m) }
$$

$u_{i}$ : spot speed measurement at position $i,(m / s)$
t : travel time of line i, (s)
Total travel time is calculated by offsetting the travel times at downstream links temporally, which represents the time lag when calculating link travel time for downstream links. This process considers the instantaneous travel time variation that makes long-section TTEs more reliable and reasonable.
The entire section's total travel time calculation that introduces errors:

$$
\begin{equation*}
T T T_{j}=\sum_{n=1}^{N} t_{i, j} . \tag{25}
\end{equation*}
$$

The correct total travel time calculation that is applied in the study:

$$
C T T_{1, j}=t_{1, j}
$$

$$
\begin{align*}
& C T T_{i, j}=\left\{\begin{array}{c} 
\\
\left.C T T_{i-1, j}+t_{i, j} \ldots \ldots . \text { if } \quad C T T_{i-1, j} \leq S\right) \\
C T T_{i-1, j}+t_{i, j+1} \ldots \ldots . \\
\text { if } \left.\quad S \leq C T T_{i-1, j} \leq 2 \cdot S\right) \\
C T T_{i-1, j}+t_{i, j+N} \cdots \ldots . . \\
\text { if } \left.\quad N \cdot S \leq C T T_{i-1, j} \leq(N+1) \cdot S\right)
\end{array} \quad i \geq 2\right.  \tag{26}\\
& T T T_{j}=C T T_{i, j}, \quad i=N \tag{27}
\end{align*}
$$

Where:
$t_{i, j}$ : travel time of link i at interval $\mathrm{j},(\mathrm{s}) ;$
$C T T_{i, j}$ : cumulative travel time from link 1 to i at interval $\mathrm{j},(\mathrm{s}) ;$
n : number of links of the section;
N : total number of links;
S: interval size;
$T T T_{j}$ : total travel time of the section at interval $\mathrm{j},(\mathrm{s})$

Also, TMS and SMS are utilized to calculate travel time separately. True travel time is obtained from the probe vehicles within the simulation (sample size of $100 \%$ ), only data within the $5^{\text {th }}$ and $95^{\text {th }}$ percentile are considered in computing the average travel time in an attempt to screen outlier data. The traffic volumes are loaded, with demands exceeding the capacity of the bottlenecks periodically, in order to create congestion build-up, congestion, and congestion decay. Another point to address is that, during simulation, $5 \%$ variation factor applies to Origin-Destination values, in order to mimic traffic patterns realistically.


Figure 7.25 Travel time of methods based on TMS and TMS, Multi-link test bed
The thick yellow line above represents the true travel time of the entire 9 -link section based on probe vehicle data. Dashed and solid lines represent the TTE ( 2 methods) based on spot measured SMSs and TMSs, respectively. As shown in Figure 7.25, under free-flow conditions, TTE based on TMS travel time estimates are more accurate, though both the TMS and SMS methods tend to over-estimate the travel time. Under congested conditions, methods based on TMS tend to under-estimate travel time, while methods based on SMS are typically consistent with actual travel times. As for SMS methods, method \#2 has better accuracy than \#1 for both free-flow and congested conditions.


Figure 7.26 Travel time of methods based on TMS and TMS, Multi-link test bed, Long section
For the same test bed, if the link length is doubled, TTE outcomes are shown in Figure 7.26. In this trail the spot speeds are only recorded at five locations with an average spacing of 2 km . It is observed that TTE outcomes based on SMS and TMS are unable to capture the temporal variation in travel times given the sparse recording locations. This finding demonstrates that the detector spacing can have a significant impact on the travel time estimates.
Data smoothing techniques (Hardle 1991) are utilized here to improve estimation accuracy. Kernel smoothers provide mathematically tractable algorithms for smoothing data. Two smoothing weights are utilized on this test bed.
Epanechnikov kernel $\quad \frac{3}{4}\left(1-\mu^{2}\right),(|\mu| \leq 1)$
Triweight kernel

$$
\begin{equation*}
\frac{35}{32}\left(1-\mu^{2}\right)^{3},(|\mu| \leq 1) \tag{28}
\end{equation*}
$$



Figure 7.27 TTE of methods based on SMS using data smoothing
SMS measurements at each spot have been smoothed using an Epanechnikov and Triweight kernel and illustrated in Figure 27. Moreover, smoothed speeds are utilized in two TTE methods as described above. Therefore Figure 27 illustrates six TTE outcomes including two methods based on SMS, two methods based on Epanechnikov smoothing, and two methods based on Triweight smoothing. As shown, under free-flow conditions, all estimations are observed to be over-estimated, while all four smoothed methods illustrate better estimation under congested conditions. The Root Mean Squared Error (RMSE) is calculated between each method and the true travel time, and the lowest value represents the best estimation of travel time; results are summarized in Table 7.2.
Table 7.2: RMSE of six TTEs on Multi-Link Test Bed

|  | All based on space-mean speed measured on spots |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Method <br> $\mathbf{1}$ | Method <br> $\mathbf{2}$ | Method 1 (Smoothing) |  | Method 2 (Smoothing) |  |
|  | 97.6 | 98.6 | Epanechnikov | Triweight | Epanechnikov | Triweight |

From Table 7.2, it is demonstrated that data smoothing techniques improve the estimation quality by decreasing the RMSE, especially for congested conditions. Among the two methods smoothed Method 2 with Epanechnikov kernel smoothing offers the best travel time estimate.

### 7.7 CONCLUSIONS

After reviewing several TTE approaches, inaccuracies in the Dynamics approach and drawbacks are first addressed using field and simulation data. Subsequently, the paper presents the proposed approach including correction of the TTE equation for congested conditions and proposes a new procedure to distinguish between congested and uncongested conditions. Overall, the Dynamics TTE model cannot provide accurate TTE under free-flow conditions because the traffic stream density estimates during uncongestion are prone to errors. The paper demonstrates that a minor change in the flow rate within an interval greatly affects the density estimates and thus impacts TTEs further. These problems are observed for both the field and simulation data. The research also demonstrates that the " m " parameter is impacted by the polling interval size. It is observed that utilizing " m " to define regimes is prone to error. On the other hand, the proposed regime threshold is more practical for identifying the onset of congestion. The polling interval size is demonstrated to
have significant impacts on the travel time estimates.
Second, five TTE approaches are compared using field and simulation data. Approach 1 over-estimates travel time for free-flow conditions. It is noted that any approach applying occupancy to estimate density brings out ideal TTE outcomes, with ideal estimations under free-flow and congested conditions and a slight under-estimation during the transition of traffic condition from free-flow to congested conditions. Moreover, since Approaches 2, 3, and 4 apply different estimation equations and different regime dividers, their estimate outcomes are so close that it is demonstrated that the travel time equations for normal condition and congested conditions are similar.
Lastly, Approach 5 is tested on a multi-link simulation network to test how the spot speed measurement approach works. The results demonstrate that approaches based on spot SMS provide very good matches to true travel times, while the methods based on spot TMS under-estimate travel time in congested conditions. In addition, data smoothing techniques can definitely improve estimation accuracy that helps to reduce estimate errors under congested conditions.
To provide traffic management center guidance on choosing an appropriate approach under different traffic conditions, spot measured SMS approach is proved to be a suitable TTE under free-flow condition, while in congested condition the proposed approach is suggested given its best performance among others. As traffic condition has to be identified before applying the proper TTE approach, average flow speed can be used as reference. Because of the simplicity of average flow speed estimation at traffic management center as well as speed measurement's acceptable accuracy, it is fast and precise to identify traffic conditions via speed. Therefore, if average flow speed is higher than 50 mph , spot measured SMS approach would be adopted as it is deemed under free-flow condition; otherwise the proposed approach is selected.

In summary, the Dynamics TTE approach is a feasible approach to estimate link travel time, provided using occupancy to estimate density. The best performance of the Dynamics TTE approach is obtained when applying the proposed $\mathrm{q} / \mathrm{k}$ regime divider to define traffic conditions and the proposed equation for estimating congested conditions. Spot measured SMS is proven to yield very satisfying TTE outcomes for a link. By utilizing data smoothing techniques the multi-link, TTE accuracy is enhanced.

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## CHAPTER 8. CONCLUSIONS AND FUTURE RESEARCH

### 8.1 CONCLUSIONS

Given that the dissertation is divided into four major parts the conclusions of each section is presented separately.

### 8.1.1 Traffic Variability

The dissertation analyzed 3 months of regular weekday loop traffic data from I-66 in Northern Virginia, attempting to quantify the similarities and differences between days of the week by considering link flows, path flows, and total demand. Several conclusions are drawn from this study that can be generalized given that the I-66 highway appears to be reflective of typical traffic conditions (recurring congestion) on North American urban freeways.

The spatiotemporal variation of link flows within the different days of the week (Monday through Friday) appears to be highly similar and consistent. As would be expected, the analysis demonstrates that weekend spatiotemporal behavior is different from weekday behavior. For both weekdays and weekends, the flow coefficient of variation within a day of the week is very low for most days, with higher link-flow coefficients of variation during extremely low flows (e.g., early morning hours).

Regarding the lane distribution on four-lane freeways along basic freeway sections that are not in the influence area of merges, diverges, or weaving sections, the middle lanes: Lane 2 and Lane 3, are highly utilized by vehicles. Lane 4 (shoulder lane) occupation is similar to that of Lanes 2 and 3 only during light flow conditions. Lane 1 (median lane) is always under utilized. However, it should be noted that the median lane acted as an High Occupancy Vehicle 2 (HOV-2) lane during the morning peak period, which could affect the results for the congested periods.

The mean demand for weekdays and weekends has a very similar trend after 10:00 a.m. The Coefficient of Variance value of demand is very low for both weekdays and weekends showing that the total demand does not overtly change from day to day. Demand conditions within core weekdays appear to be highly similar and consistent. The study demonstrates that in terms of path flows, Fridays appear to be different from core weekdays. As was the case with link flows, Saturdays and Sundays are different from core weekdays.

The residual errors for demand, link flow, and path flow are all within two standard deviations supporting the homogeneity of variance assumption for ANOVA. The ANOVA results demonstrate that Monday traffic demand conditions differ from core weekdays. In addition, Friday path flows are different from core weekdays. Consequently, the analysis concludes that only Tuesdays, Wednesdays, and Thursdays should be considered core weekdays.
The success measure of traffic condition parameters can be used to distinguish statistically between significant and insignificant variations from typical traffic conditions. The method could be developed further to operate in real-time as part of an online TTE system, while values inside the confidence limits indicate that historical data can be used: a p-value outside the confidence range would indicate suspicious observations, further failures can then trigger the need to abandon the use of historical data for travel-time prediction. Also for serving as TTE's reference, it should be noted that three separate databases should be constructed. Data for Tuesday, Wednesday, and Thursday can be combined together as a regular weekday, Monday and Friday should have their own databases.

### 8.1.2 Consistency between Shockwave and Queuing Theory in Delay Estimation

The delay computations using shock-wave analysis and queuing theory were compared for two example applications, namely (a) time varying arrival rate at a constant-capacity bottleneck and (b) a constant arrival rate at a time-varying capacity bottleneck. The results demonstrate the consistency between shock-wave analysis and queuing theory. Furthermore, the paper highlights the error in the Nam and Drew (Nam D.H. 1998) computation and corrects the equations that were derived by Nam and Drew. In summary, the paper demonstrates that queuing theory provides a simple and accurate technique for estimating delay at highway bottlenecks. The discussion of this chapter is inspired by Nam and Drew's paper on traffic Dynamics internal algorithm, therefore, the findings will
serve as a foundation for estimating travel time under congested conditions in the following proposed TTE approach discussed in Chapter 6.

### 8.1.3 Estimating Space-Mean Speed from Spot Speed Measurements

This dissertation demonstrates that the relationship between time-mean and space-mean speed that was derived by Wardrop (Wardrop 1952) and presented in several textbooks (e.g. May (May 1990)) produces an error in the range of 1 percent in time-mean speed estimates. However, the formulation estimates the time-mean speed from the space-mean speed, which is typically the reverse of what is required. Specifically, with the higher interest from application point of view, the objective is to estimate the space-mean speed from the time-mean speed. Consequently, using the statistics from the estimates, Chapter 5 derives a modified relationship between space-mean speed and time-mean speed that computes space-mean speed as a function of time-mean speed. It demonstrates that the proposed formulation, which utilizes the variance about the time-mean speed as opposed to the variance about the space-mean speed, produces an estimate error to within 0 to 1 percent, as is the case for the Wardrop formulation. This discovered relationship has a very positive and beneficial application to traffic engineers and ITS analysts who want to get space-mean speed data from surveillance systems that can only provide time-mean speed estimates.

In addition, this dissertation demonstrates that both the Hall and Persaud (Persaud and Hurdle 1988) and the Dailey (Dailey 1999) formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation computes the space-mean speed (harmonic mean) while the Dailey formulation computes the time-mean speed (arithmetic mean).
Finally, this dissertation demonstrates that the space-mean speed coefficient of variation (standard deviation divided by mean) is approximately equal to the coefficient of variation of roadway travel times. Using this relationship it would be possible to estimate travel speed confidence limits based on field measurements of travel times.

### 8.1.4 Travel Time Estimation from Spot Speed Measurements

This portion of the research effort first addresses the inaccuracies in the Dynamics approach and its drawbacks using field and simulated data. Subsequently, the dissertation presents the proposed approach including correction of the TTE equation for congested conditions and proposes a new procedure to distinguish between congested and uncongested conditions. Overall, the Dynamics TTE model cannot provide accurate TTE under free-flow conditions because the traffic stream density estimates during uncongestion are prone to errors. The dissertation demonstrates that a minor change in the flow rate within an interval greatly affects the density estimates and thus impacts TTEs further. These problems are observed with both the field and simulation data. The research also demonstrates that the " $m$ " parameter is impacted by the polling interval size. It is observed that utilizing " $m$ " to define regimes is prone to error. On the other hand, the proposed regime threshold is more practical for identifying the onset of congestion. The polling interval size is demonstrated to have significant impacts on TTE.
The comparison of five TTE approaches on field and simulation data indicates that Approach 1 over-estimates travel time in free-flow conditions. It is noted that any approach applying occupancy to estimate density brings out acceptable TTE outcomes, with ideal estimations under free-flow and congested conditions and a slight under-estimation during the transition of traffic condition from free-flow to congested conditions. Moreover, since Approaches 2, 3, and 4 apply different estimation equations and different regime dividers, their estimate outcomes are so close that it is demonstrated that the travel time equations for normal and congested conditions are similar, quantitatively.
The spot speed approach is tested on a multi-link simulation network, and the results demonstrate that approaches based on spot SMS provide very good matches to true travel times, while methods based on spot TMS under-estimate travel time in congested conditions. In addition, data smoothing techniques can definitely improve estimation accuracy that helps to reduce estimate errors under congested conditions.

In summary, the Dynamics TTE approach is a feasible approach to estimate link travel time, provided it uses occupancy to estimate density. The best performance of the Dynamics TTE approach is obtained when applying the
proposed $\mathrm{q} / \mathrm{k}$ regime divider to define traffic conditions and the proposed equation for estimating congested conditions. Spot-measured SMS is proven to yield very satisfying TTE outcomes for a link. By utilizing data smoothing techniques for the multi-link, TTE accuracy is enhanced.

### 8.2 FUTURE RESEARCH

The research has identified a number of challenges that need to be addressed that can be broadly categorized into data, modeling, and prediction issues. The data challenges include issues in data cleaning and fusion. The modeling and prediction challenges include the prediction of traffic states in the near future based on historical and real-time data, the estimation of travel time reliability measures, the integration of multi-type data sources for the provision of multi-modal traffic information, the dissemination of traffic information to the traveling public, and modeling the response of the traveling public to the disseminated information. The ability to predict "proactive" as opposed to "reactive" travel times poses a significant challenge especially if one considers non-recurring congestion events, where historical information may not be available. Another major challenge is the "on-line" processing challenge, which requires that decisions be made quickly (within a couple of minutes) and thus less accurate but computationally more efficient, adaptive, self-correcting algorithms may be considered. Consequently, it is recommended that research be conducted to address a number of research issues that include (a) investigating and comparing alternative macroscopic, mesoscopic, and microscopic traffic stream modeling schemes (continuous, discrete event simulation, and cellular automata), (b) testing alternative parallel processing schemes in an attempt to enhance the computational efficiency of traffic modeling approaches, and (c) investigating various model scaling approaches, similar to the scaling of physical models.

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