

AN OPTIMALLY ADJUSTED TCUL CONTROLLER FOR THE
PROTECTION OF UNDERGROUND TRANSMISSION LINES

by

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DEDICATED

to Mother

Proverbs 22:6

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LIST OF SYMBOLS

<u>English Symbols:</u>	<u>Description</u>
A	a constant in the MVA equation
B	dimensionless matrix defined under (C.2)
C	connection matrix relating Z_p to Z_t
C_g	matrix relating V_g to V_G
$C_s = C_g$	
F	arbitrary function of time and a random variable in (D.1)
G_i	i-th entry of gradient vector
I_x	branch current (Appendix B); subscript denotes type of element R,L,C. etc.
I_{C1}, I_{C2}	vector of line charging capacitor currents
I_p, I_s	transformer primary and secondary current vectors
\tilde{I}	diagonal matrix of conjugate cable currents
ISA, ISO	secondary currents from adjoint and original network analyses
I	cable current vector, complex or magnitude
I_T	vector of magnitudes of secondary currents
I_{MAX}	vector of maximum permissible I_T
K	complex turns ratio matrix for current
MVA	megavolt - ampere
MVA_i	MVA of i-th load
N	complex turns ratio matrix for voltage; also arbitrary constant in Table 4.1-1

NS, NT	turns ratio S:P and T:S
P	designates primary winding
PFL_i	power factor of the i-th load
R	random variable from RANDU; resistance
Re	real operator
S	designates secondary winding; slack variable
T	designates tapped secondary winding; time in hours
T_{MAX}	maximum time in hours
U	identity matrix
$ V $	vector of load voltages (magnitudes)
V_x	branch voltage (Appendix B); subscript denotes type of element R,L,C,etc.
V_L	vector of complex load voltages
V_g	source voltage vector, 12X1
V_G	source voltage vector, 4X1
\hat{V}	vector of desired load voltage magnitudes
\tilde{V}	diagonal matrix of conjugate load voltages
V_p, V_s	transformer primary and secondary voltage vectors
VPA, VPO	primary voltages from adjoint and original network analyses
W_I	weighting matrix for currents in ϵ_T
W_M	weighting matrix for currents in ϵ
W_T	weighting matrix for currents I_T
W_V	weighting matrix for load voltages
W_{ij}	ij-th entry in W_I

X	reactance; arbitrary variable in Table 4.1-1; tap position in equation (4.3-1)
Y	arbitrary variable in Table 4.1-1
YC	diagonal matrix of line charging admittance
ZF	impedance of feeder cable, complex 12X12 diagonal matrix
ZL	load impedance, complex 4X4 diagonal matrix
ZS	transformer impedance, complex diagonal matrix
ZT	transmission cable impedance, complex 12X12 matrix
$Z_f = ZF$	
$Z_p = Z_1$	
$Z_t = ZT$	
Z_1	matrix of constant impedance, includes ZF and ZT
Z_2	matrix of impedance dependent on tap position and load, includes ZS and ZL

Greek Symbols:

ϵ	value of performance function
ϵ_I	part of ϵ related only to currents
ϵ_T	portion of ϵ used in iteration termination
ϵ_V	part of ϵ related only to voltages
ρ	any variable parameter
ϕ	branch current in the adjoint network
ϕ_I	current excitation vector for adjoint network
ϕ_i	phase angle (Appendix D)
ψ	branch voltages in the adjoint network
ψ_V	voltage excitation vector for adjoint network

I. INTRODUCTION

The problem considered here is the development of an automatic computer controlled system to provide a means for equalizing the currents in short parallel connected pipe-type underground transmission cables.

When several underground pipe-type cables supply a load center composed of several independent loads reliability may be increased by serving each independent load from several of the supply cables. When this is done the load on a particular cable is influenced by the several loads connected to it. As the individual loads fluctuate throughout the day the total load supplied by any given cable will also fluctuate. At times of peak load demand it is possible for one or more of the supply cables to become overloaded while others operate below maximum ratings. Therefore, to prevent costly cable damage and power outages, it is necessary to closely monitor the cable currents and to make appropriate adjustments to equalize the cable loading. The problem is further aggravated by the poor heat dissipation characteristics of underground pipe-type cables.

The problem considered here is based on a specific problem [1] from an electric utility which presently operates an extensive system of underground pipe-type cables. At present the balance of the cable currents is manually controlled and the operation is satisfactory except for the time required to implement the necessary adjustments.

Phase-angle-connected tap-changing-under-load (TCUL) transformers

are used to provide the means of equalizing the supply cable currents. The TCUL transformers were chosen because they also provide a means for regulating the load voltages.

As the loads increase it becomes necessary to more closely monitor the supply cable currents and to rapidly implement appropriate adjustments to maintain satisfactory operation. Therefore, automated control is required.

The phase-angle-connected transformers used here operate on the same well-known principles that govern the load distribution between two parallel connected alternators. The phase-angle connection provides an advance in phase angle together with an increase in the voltage magnitude. Therefore, a transformer can be made to assume more of both the real and reactive load power by increasing the tap position. Conversely, both real and reactive load can be shed by decreasing the tap position. By appropriate positioning of all the transformer taps it is possible to obtain approximate equalization of cable loading. By optimally selecting which of the transformers is to be adjusted, excessive tap changing and wear on the tap changer mechanism is avoided.

Optimization techniques have been applied to power systems mainly in the area of economic dispatch, optimal load flow, and control of generation. Sullivan and Elgerd [2] apply the Lagrange multiplier technique to a problem of minimally proportioning reactive generation in a two generator one transformer problem. The problem considered here appears to be unique in optimizing system operation by using phase-angle-connected transformers as the adjustable parameters.

II. APPROACHES TO A SOLUTION

In this chapter an optimal policy of control is defined and expressed in terms of a performance function. The effect of each term in the performance function is explained. The option of basing the adjustments on a computer model of the system or on measurements taken from the operating system is discussed.

2.1 An Optimal Policy of Control

In the previous work on the problem [3] an attempt was made to base the selection of an adjustment on the values of the controlled variables themselves. The adjustments generally turned out to be of a trial and error nature and the results, in general, were poor. This background work indicated that a means for selecting the "best" of the several possible adjustments was needed.

The system chosen for study contains twelve phase-angle-connected TCUL transformers. Therefore twelve adjustable parameters are available, i.e., the tap position of each transformer. Since each transformer is equipped with a tap changer having either 16 or 32 positions the large number of combinations of tap positions requires an orderly search for the combination which provides the best operating point for the system.

The best operating point for the system may be defined in a number of different ways depending on the objectives one has in mind. Here, the explicit objectives are protection from overload for system components and maintenance of nominal load voltages. Therefore, the operating point is defined in terms of cable and transformer currents and

load voltages.

The optimum operating point would be one where the cable currents are equal and at a minimum, the load voltages are within limits, and no transformer currents exceed the maximum specified values. However, due to the discrete nature of the transformer tap changer, it is not generally possible to reach the optimum operating point. The best operating point is, therefore, the one which most closely approaches the optimum.

A system performance function is defined in equation (2.2-1a). The value of the performance function, ϵ , indicates how close a particular operating point is to the optimum. The performance function is formulated so that ϵ is always positive. The smaller the value for ϵ , the more nearly optimum is the system operation. Since the value for ϵ depends on the magnitude of the cable currents (and the cable currents depend on the load conditions) ϵ may take on different values for equally optimal operating conditions. For this reason ϵ cannot be used for comparison or to determine when a specified degree of optimality has been attained. Therefore another performance function ϵ_T has been defined in equation (2.2-1b). ϵ_T is just ϵ with the cable current magnitude term removed. The value of ϵ_T is dependent on the degree of optimality and is equal to zero at the optimum operating point.

It is assumed that repeated adjustments are made until the best operating point is reached. In some applications it may be possible to define a "cut-off" value for ϵ_T . A cut-off value for ϵ_T would be chosen so as to insure that system operation was satisfactory even

though the best operating point had not been reached. Engineering judgment would be required in determining an appropriate cut-off value for ϵ_T for each particular application.

For some loading conditions it may not be possible to reduce ϵ_T to the cut-off value. In such a situation the adjustment procedure would be continued until all adjustable parameters had been tried. Those parameter values giving the best operating point would then be selected. Terminating the iterations this way prevents "hunting."

Now that a measure of system performance has been defined it is possible to consider a means for selecting the best of several possible adjustments. It seems reasonable that the best adjustment would be the one which would improve the system operation the most. Hence, the best adjustment would be the one which would reduce ϵ the most. One approach would be to try each of the 24 possible tap adjustments (12 up and 12 down) and to select the one which provides the smallest value of ϵ_T . If a large number of adjustments were required each time this method might be worth further consideration. However, it is assumed that the system would be sampled periodically and that the operating point would not be allowed to drift very far from the optimum. If a single adjustment could restore the system to the best operating point, it would be desirable to select that adjustment the first time.

Since the size of the parameter adjustment is fixed by the tap changer the rate of change of ϵ with respect to each parameter could be used as an indicator of the most promising adjustment. The parameter providing the greatest rate of change of ϵ is selected for adjustment. If the rate of change of ϵ is positive the parameter is adjusted in a negative

direction or if the rate of change is negative the parameter is adjusted in a positive direction. This procedure is just the negative gradient method of optimization applied to a single parameter. The vector consisting of the rate of change of ϵ with respect to each adjustable parameter is the gradient vector.

The method described above for selecting the parameter to be adjusted is the optimal policy of control used here. This method is optimal in the sense that it selects the best possible adjustment for improving the next operating point based on information obtained at the present operating point. The negative gradient method and calculation of the gradient vector is discussed further in Chapter IV.

2.2 The Performance Function

When minimized the performance function should provide the best possible operating conditions for the system. The requirements for this problem are 1) that the cable currents be approximately equal or more generally take on a specified ratio; 2) that the cable currents be minimized; 3) that the load voltages remain within specified upper and lower bounds and; 4) that transformer currents not exceed specified values. These objectives may be attained by proper choice of the weighting matrices in the performance functions ϵ and ϵ_T given below.

$$\epsilon = \frac{1}{2} \cdot |I|^T \cdot W_M \cdot |I| + \epsilon_T \quad (2.2-1a)$$

where

$$\begin{aligned} \epsilon_T = & \frac{1}{2} |I|^T \cdot W_I \cdot |I| + (|V| - \hat{V})^T \cdot W_V \cdot (|V| - \hat{V}) \\ & + (|I_T| - I_{\max})^T \cdot W_T \cdot (|I_T| - I_{\max}) \end{aligned} \quad (2.2-1b)$$

and I is the vector of controlled cable currents

W_M is a constant diagonal weighting matrix

ϵ_T is the performance function for termination

W_I is the matrix of weighting constants given in equation (2.2-4)

V is the vector of controlled load voltages

\hat{V} is the vector of desired load voltages

W_V is a constant diagonal weighting matrix

I_T is the vector of transformer secondary currents

I_{\max} is the vector of maximum transformer currents

W_T is a constant diagonal weighting matrix

If equality of the cable currents I_i is desired then equation (2.2-1b) would involve terms such as

$$[|I_i| - w_{ij}|I_j|]^2 = 0 \quad (2.2-2)$$

where w_{ij} is equal to unity. When the cable currents are equalized equation (2.2-2) is satisfied. In applications where the cables have different ratings it would be appropriate to operate the larger cables at higher currents. In order to satisfy equation (2.2-2) when $|I_i| \neq |I_j|$

then w_{ij} must take on the value

$$w_{ij} = |I_i| / |I_j| \quad (2.2-3)$$

If I_1 is taken as reference then W_I has the form

$$W_I = \begin{bmatrix} 3 & -w_{12} & -w_{13} & -w_{14} \\ -w_{12} & 2+w_{12}^2 & -w_{23} & -w_{24} \\ -w_{13} & -w_{23} & 1+w_{13}^2+w_{23}^2 & -w_{34} \\ -w_{14} & -w_{24} & -w_{34} & w_{14}^2+w_{24}^2+w_{34}^2 \end{bmatrix} \quad (2.2-4)$$

The entries in W_M in equation (2.2-1a) provide a means for emphasizing the minimization of a particular current or all currents in general. The entries will generally be equal unless non-unity ratios are specified for the cable currents. The entries in W_V are zero if the load voltage is satisfactory. When the j -th load voltage differs from the nominal 13800 volts by more than ± 400 volts, the j -th entry in W_V is set equal to an appropriate weighting factor λ_j . Here, λ_j would be chosen large to forcefully drive the load voltage back to an acceptable level. To prevent the load voltage from drifting towards the lower limit (under the influence of cable current minimization) the entries in W_V may be set equal to a smaller non-zero weighting factor at load voltages of, say, nominal ± 200 volts. The entries in W_T may be handled in a similar manner. For transformer currents below the specified maximum all entries in W_T are zero.

It should be noted that only the magnitudes of the controlled

variables are of concern in the performance function. The method used here would also be applicable to problems requiring the phase angle to be included in the performance function. The only difference would be a slight modification in the calculation of the gradient.

No constraint has been imposed on tap position in the performance function. Whenever a tap reaches an upper or lower limit any attempt to move the tap outside the permissible range is ignored and the next largest magnitude gradient entry is considered.

2.3 Digital Simulation Versus System Measurement

In this section two alternate means for implementing the controller on a physical system are discussed. Both methods require calculation of the gradient vector. They differ primarily in the methods used to perform the calculations. The first method would utilize a digital computer model of the system. The second method would make use of actual values of the controlled variables obtained from measurements on the physical system.

Digital Simulation

In this method the controlled variables are calculated from a model of the system. Measurements on the physical system would be used only to initiate control action and to provide load information for use in determining accurate load impedances for the model.

The gradient may be calculated by either of two methods: by manipulation of a matrix expression for the gradient and by the adjoint network method of Director and Rohrer [4]. The adjoint network method has

been used here since it appears to be easier to program and also provides other useful benefits at essentially no additional cost of complexity. It is shown later that a single analysis of the original network (inversion of the impedance matrix) plus a few matrix multiplications provides the following: 1) values of the controlled variables of the system from which ϵ_T can be found, and 2) a solution applicable to the adjoint network. The gradient can then be calculated from appropriate products of currents and voltages obtained from analyses of the original and adjoint networks.

Calculation of the gradient from the matrix expression requires considerable manipulation of matrices. Although this method has not been programmed it appears that it would require a somewhat larger program than the adjoint network method.

System Measurement

The system measurement approach would obtain the values of the controlled variables from measurements on the system. The gradient could be determined by either the matrix method or by the adjoint network method. Due to the complexity of the matrix expression for the gradient it would appear that the adjoint network method would require a smaller program. Also, measurement of the system variables would eliminate the need for an analysis of the original network in the adjoint network method.

Although this approach would use the actual values for the voltages and currents in gradient calculations the accuracy of the gradient would still be dependent on the stored impedance matrix.

This approach would also require more extensive monitoring and telemetry than the modeling approach. The magnitude and phase of all variables appearing in the performance function would be required. The increased telemetry requirements represent an expense over and above the cost of the computer facilities.

Factors Affecting a Choice

It appears that the digital simulation method has both operational features as well as financial factors in its favor. The major advantage of the modeling approach is the ability to verify the effect of a particular adjustment prior to making any adjustments on the physical system. This would be particularly important in those cases where the cut-off value of ϵ_T was not reached and all twelve transformers would be tried to determine the best adjustment. Since the computer requirements for the two approaches would be similar the added expense of telemetering the phase angle information appears to give the modeling approach a financial advantage also.

III. NETWORK MODEL

This chapter considers the network model, its characteristics and the assumptions that have been made in modeling the network. The single line equivalent for each network component is discussed briefly. Solution of the equations and the digital computer program are discussed.

3.1 Choice of the Network Configuration

The network configuration used in this study is shown in Fig. 3.1-1. This configuration provides a fairly complex and realistic system and was chosen to fit the general problem description provided. The network includes twelve TCUL transformers as control parameters and has eight controlled variables.

3.2 Characteristics of the Network

The network is assumed to operate as a balanced three-phase system in a steady-state condition. Transients resulting from the transformer tap adjustments are not considered. The transients appear to present no problems in the manually operated system and therefore should not be troublesome in an automatically controlled system. The only system parameters which vary significantly with time are the load impedances, and these vary slowly under normal operating conditions. The variation of the load impedance is quite slow as compared to the computation times of modern computers. Hence the steady-state assumption is valid.

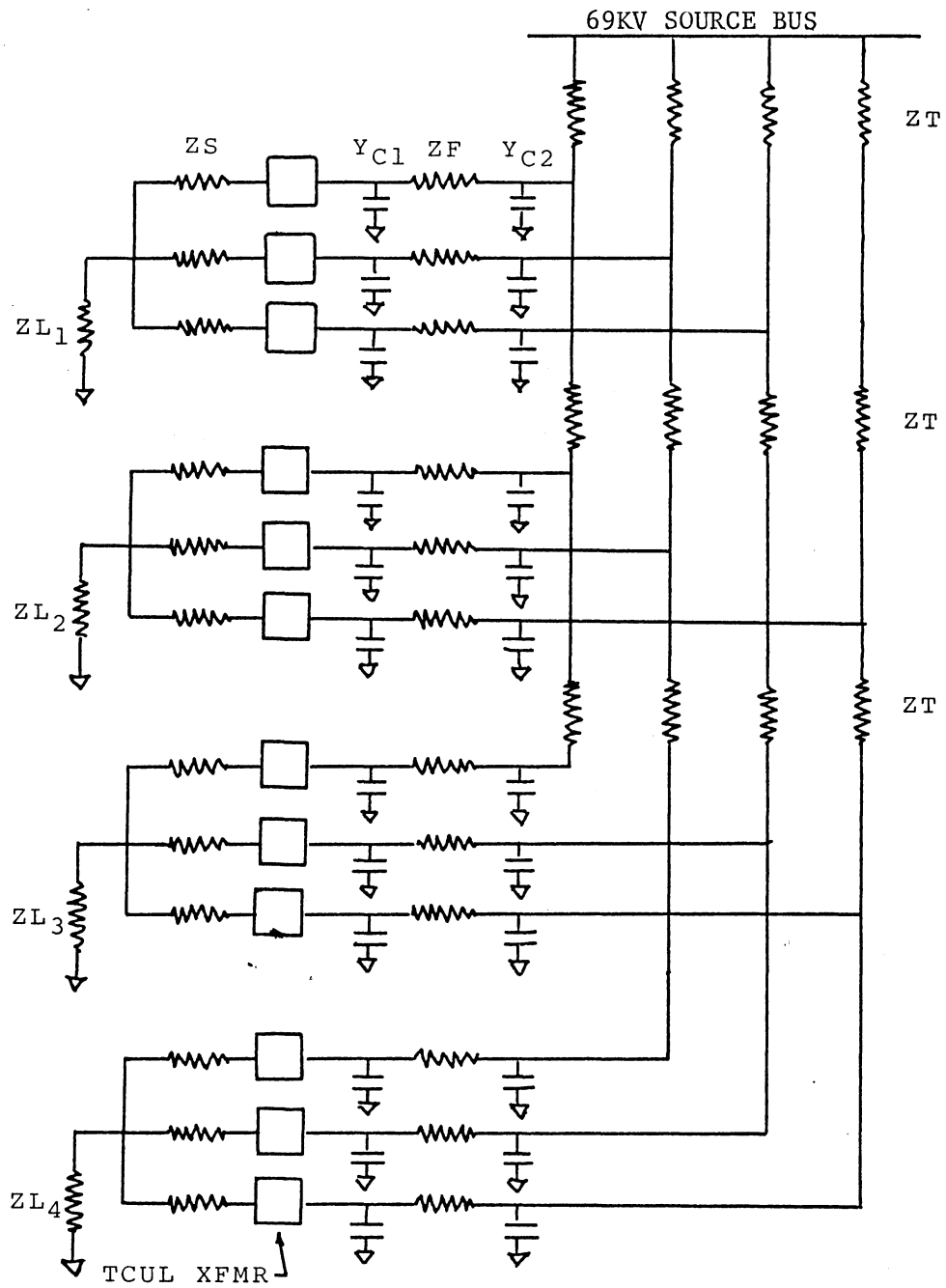


Fig. 3.1-1 Single Line Diagram

3.3 Single Line Network Representation

The single line equivalents used for each component are described in this section. Although per-unit representation is normally used in load flow studies, impedances expressed in ohms have been retained in this study.

Transmission Lines

Transmission lines have been represented by a pi-equivalent with one-half the total shunt cable capacitance at each end of the line. The per-mile line data is given in Table 3.3-1. Impedance values used in the network model were determined for the cable lengths shown in Fig. 3.1-1.

Transformers

The phase-angle-connected transformers used in the network have the phasor diagram and the connection diagram shown in Fig. 3.3-1. The primary, secondary, and tapped secondary windings are designated by P, S, and T respectively. The tapped winding has standards of 8 or 16 tap settings in both the phase lead (dashed) and phase lag (dotted) connections.

The transformer is represented by its series impedance in series with an ideal transformer having the complex voltage turns ratio N . The current turns ratio K is the complex conjugate of N .

Transformer impedance values were obtained from calculations based on data obtained from transformer data sheets provided by the manufacturer. Pertinent data is given in Table 3.3-2.

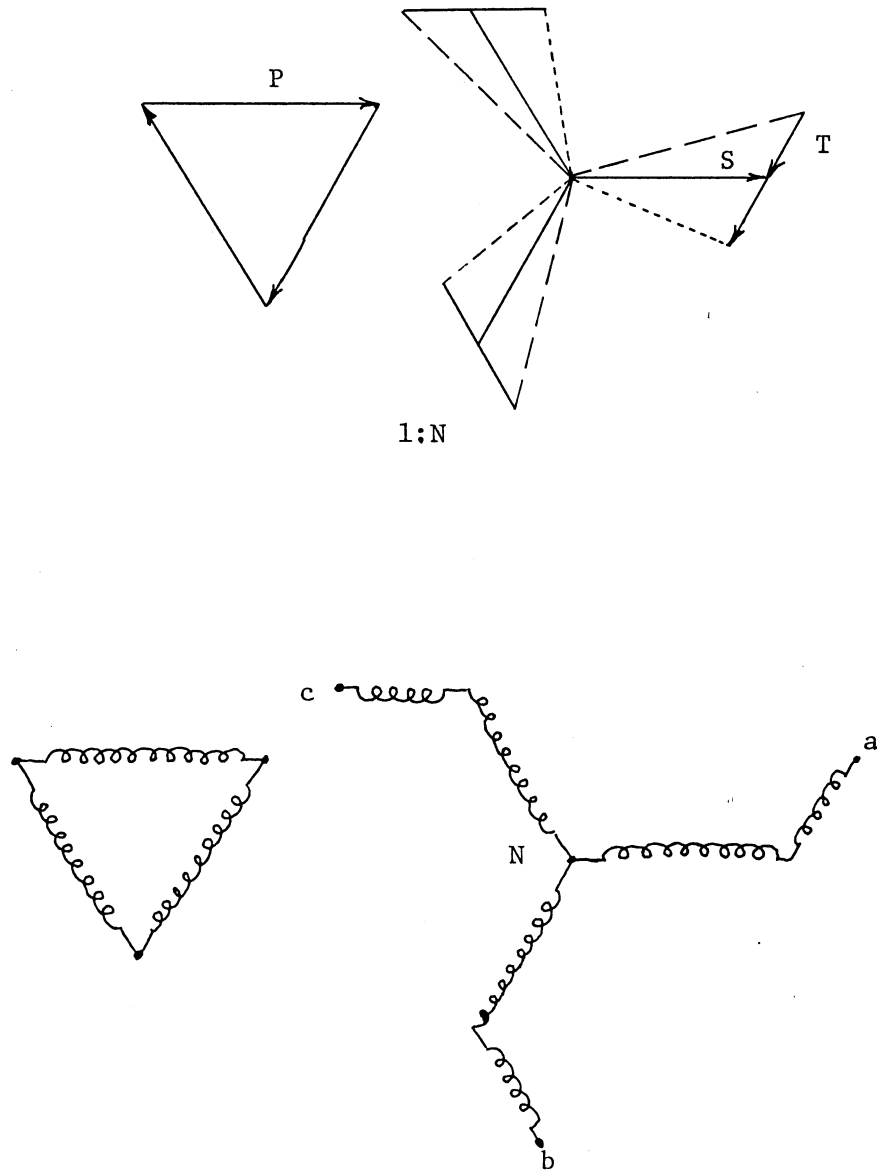
The transformer impedance is referred to the secondary winding. Since this impedance is a function of the tap position, the impedance

TABLE 3.3-1

Cable Impedance Data

	<u>R*</u>	<u>X*</u>
1500 MCM pipe type oil filled	0.128	0.502
500 MCM low pressure oil filled	0.262	0.447

* Percent impedance per mile on a 100 MVA base.



Parallel coils are on the same phase.

Fig. 3.3-1 Phasor and Connection Diagram
for a Phase-Angle-Connected Transformer

TABLE 3.3-2

Transformer Data

Unit #	KVA	Voltages	% Z	Load Loss Watts
1	20000	62725-13800	12.44	210580
1	20000	69000-13800	10.07	166970
1	20000	76665-13800	8.33	184220
2	20000	62725-13800	12.50	207692
2	20000	69000-13800	10.07	165140
2	20000	76665-13800	8.29	185747

values were calculated for the neutral and extreme leading and lagging tap positions. A second order curve was fitted to these three points and was used to determine impedance values for other tap positions. The variation of transformer impedance with tap position is shown in Fig. 3.3-2.

Variable Load Impedance

To properly represent the system it is necessary to simulate the variable power demands which cause variation of cable currents and load voltages. The Y equivalent load impedance is calculated from a load profile which is a function of time. The load power equation takes the form:

$$MVA_i(t) = A + F(R,t) \quad (3.3-1a)$$

$$PFL_i(t) = 0.75 + 0.25R \quad (3.3-1b)$$

where MVA_i is the megavolt-ampere function for the i -th load

A is a constant

F is any desired function of time and R

R is a uniformly distributed random number between 0 and 1

PFL_i is the power factor function for the i -th load

Source Representation

The four supply cables in Fig. 3.1-1 are assumed to be driven from an infinite bus. The Y equivalent source value is used and is held constant.

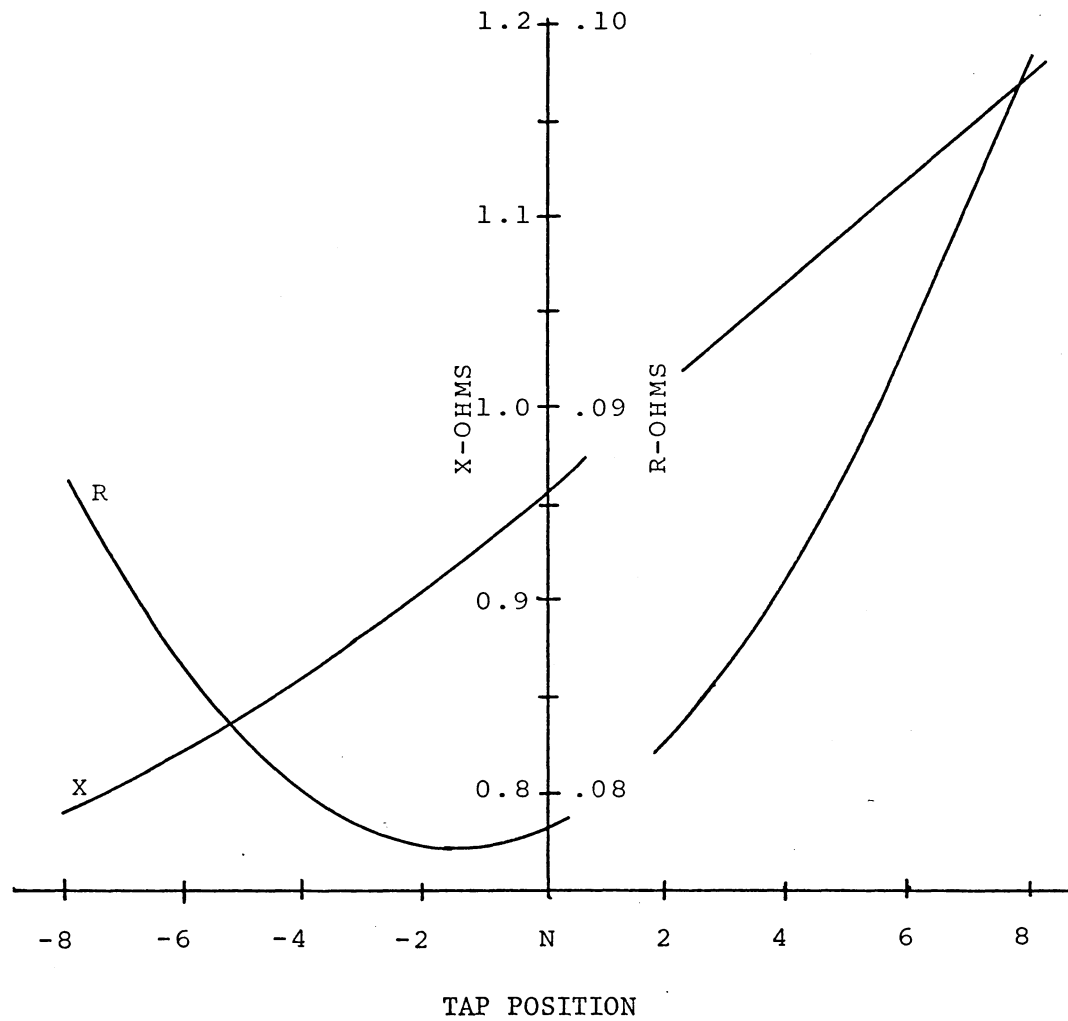


Fig. 3.3-2 Transformer Impedance versus Tap Position

3.4 Solution of the Network Equations

The number of equations required for a solution of the network can be obtained from an inspection of Fig. 3.1-1. If the ideal transformers and transformer impedances Z_S are replaced by equivalent pi networks the total element count for the network remains the same. Either 26 node equations or 34 loop equations are required for a complete solution. With reference to Fig. 3.1-1 it is obvious that each capacitor current may be expressed in terms of the corresponding transformer primary current. By doing this it is possible to reduce to 12 the number of loop equations which must be solved. Values for other unknowns may be readily obtained from the primary currents. This reduction in the size of the impedance matrix represents a significant savings in both computer time and core storage requirements. All the necessary equations have been formulated in Appendix A.

Since the adjoint network has the same topology as the original network, the same equations apply with only slight modifications. The equation for the primary current vector has the form

$$I_p = [Z_1 + A \cdot N^{-1} \cdot Z_2 \cdot K^{-1}]^{-1} V_g \quad (3.4-1)$$

where

- Z_1 is a constant symmetric impedance matrix
- A is discussed below
- N is the transformer voltage turns ratio
- Z_2 is a variable symmetric impedance matrix
- K is the complex conjugate of N
- V_g is the source vector

To make equation (3.4-1) apply to the adjoint network it is necessary only to interchange the N^{-1} and K^{-1} matrices and to apply the proper source vector. Interchanging the turns ratio matrices is equivalent to conjugating the turns ratio of the transformers in the adjoint network (see Appendix B).

Inspection of equation (3.4-1) shows that if the matrix A was an identity matrix then it would be necessary only to transpose the inverse impedance matrix in equation (3.4-1) to make it applicable to the adjoint network. The need for a second matrix inversion would be eliminated. It is shown in Appendix A that A is composed of an identity matrix U plus terms resulting from products with the line charging admittances YC. If the line charging admittances are neglected then A is just U and the transpose may be used directly without introducing an error.

In order to assess the magnitude of any error introduced two approximations were tested. First, YC was set equal to zero ($A = U$). The maximum error in I_p was approximately -1.0% in magnitude and 1.7 degrees in phase angle. In the second approximation YC was set equal to 0.0025 mho ($A \neq U$) and the inverse impedance matrix in equation (3.4-1) was simply transposed. The resulting error was approximately +0.5% in magnitude and 1.6 degrees in phase angle. When considering these approximations it should be recognized that a value of 0.0025 mho for YC is much larger than one would find in actual practice. Such a value corresponds to a line charging current of approximately 138 amperes per mile at 69 KV. More practical line charging currents would be of the

order of 20 amperes per mile or less. In view of these results it appears that quite satisfactory results would be obtained using either approximation.

3.5 Digital Computer Program

A computer program has been written which realizes the automatic controller being described. The essential features of the program are illustrated in the flow chart in Fig. 3.5-1.

The program was written in FORTRAN IV and requires approximately 70K bytes of core storage on an IBM 370 Model 155 computer. The analysis portion of the program handles the complex arithmetic in component form. Line charging capacitances have been ignored and the impedance matrix is inverted in both the original and adjoint network analyses. A large amount of printed output is generated making it easy to follow the control action.

An operational version of the program would require considerably less core storage. Core storage requirements can be reduced by eliminating most of the printed output and by storing the system impedance matrix in the complex mode. Overall computational efficiency of the program can be improved by using a more efficient means for solving the equations as discussed in Section 4.4.

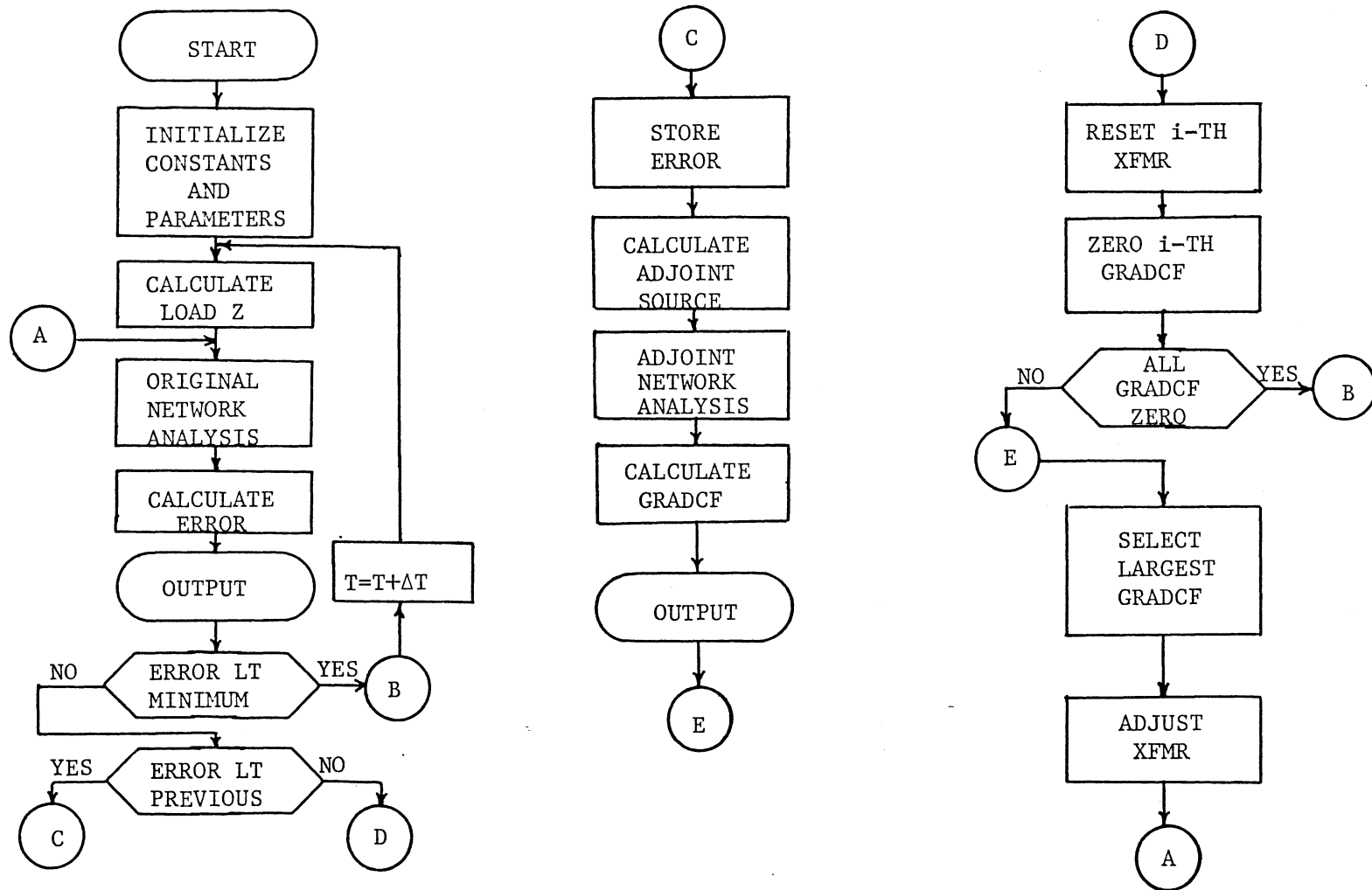


Fig. 3.5-1 Flow Chart for the Computer Program

IV. METHODS OF OPTIMIZATION

In this chapter two optimization methods are considered. Reasons for selecting the steepest descent gradient method are discussed. The adjoint network approach of Director and Rohrer [4] is extended to include phase-angle-connected transformers. Equations are developed for the analytic method of calculating the gradient. These two methods for calculating the gradient are compared and discussed.

4.1 Selection of an Optimization Method

It was mentioned in Chapter II that earlier work indicated the need for a method of selecting the best adjustment. Once this need was recognized the gradient and linear programming (LP) optimization techniques were considered.

Linear and Quadratic Programming

Formulation of an LP problem requires that the constraint equations and the objective function be linear. For this problem the currents are a nonlinear function of the transformer turns ratio and hence the LP method is not directly applicable. Quadratic programming is also not applicable because of the requirements of linear constraints.

The possibility of using linearized equations exists but the large number of constraint equations and slack variables increases the total number of equations which must be solved. Frazier [5] indicates that constraints of the form shown in Table 4.1-1 require the introduction of one, two, or three slack variables as shown. Equality of the four cable current magnitudes requires that the difference between each pair

of currents be driven to zero. Since all variables in LP problems must be positive the constraint equations would have the form

$$|I_i| - |I_j| + S_1 = 0 \tag{4.1-1}$$

$$|I_j| - |I_i| + S_2 = 0$$

The total number of equations required for this problem are shown in Table 4.1-2.

Since the LP method does not give a discrete result an additional procedure would still be required to obtain discrete tap settings. In view of these difficulties and the large number of equations required it was decided to pursue a gradient method.

The Steepest Descent Gradient Method

In the classical application of the steepest descent gradient method [6] the adjustable parameters are adjusted in the direction of the local negative gradient. The several variations of this method differ mainly in the way the parameter step size is selected. The steepest descent method is well known for its poor convergence characteristic near a minimum. This difficulty is caused by making the parameter step size proportional to the gradient. In this application the step size is fixed so that this type of convergence problem does not arise. The step size is pre-defined by the transformer tap-changer increment. Standard tap-changer increments are 5/4 and 5/8 percent change in turns ratio per step. A total change of ± 10 percent is standard. These step sizes are large compared to those normally used in classical applications. Experience with this problem has shown that an attempt to

TABLE 4.1-1

Slack Variable Requirements for
Equality-Inequality Constraints

<u>Type</u>	<u>Inequality</u>	<u>Equality</u>
I	$X + Y \leq N_1$	$S_1 + X + Y = N_1$
II	$X + Y = N_2$	$\bar{S}_1 + X + Y = N_2$
III	$X + Y \geq N_3$	$\bar{S}_1 - S_2 + X + Y = N_3$
IV	$N_4 \leq X + Y \leq N_5$	$\bar{S}_3 - S_4 + X + Y = N_4$ $S_5 + X + Y = N_5$

Note: The bar over a slack variable indicates that it is heavily weighted in the cost function and is excluded from the solution.

TABLE 4.1-2

Number of Equations Required for
an LP Formulation of this Program

<u>Variable</u>	<u>Type</u>	<u>No. Slack</u>	<u>No. Const. Eq.</u>	<u>No. Eq.</u>
Cable Current	Eq. (4.1-1)	2 ea.	2 PER PAIR	12
Load Voltage	IV	3 ea.	2 PER LOAD	8
XFMR Current	I	1 ea.	1 PER XFMR	12
Network Eq.	II	1 ea.	NONE	<u>24 (Real)</u>
TOTAL EQUATIONS				56

change several parameters at one time generally causes the iterative process to diverge. This appears to be caused by parameter changes moving the operating point through the minimum (for that parameter) and up the other side of the performance function surface. To insure convergence only one parameter is adjusted at a time. This parameter is selected according to the optimal policy discussed in Chapter II and is the parameter which corresponds to the gradient component having the greatest magnitude.

The steepest descent method is used in the optimization process because it is the simplest and most straightforward method and is easily programmed. Since the parameter step size is fixed in this application there are no convergence problems near the minimum. Also, the gradient may be readily calculated by the adjoint network method discussed in the next section.

4.2 Adjoint Network Method for Calculating the Gradient

The adjoint network method [4] for calculating the gradient of a performance function is best illustrated by means of Fig. 4.2-1. The derivation of an expression for the gradient of a phase-angle-connected transformer is given in Appendix B. The results of immediate interest are the equations expressing the components of the gradient in terms of the currents and voltages associated with the transformers, and the equations for the adjoint network excitation. The entry in the gradient vector for the i -th transformer is

$$G_i = -R_e (VPO \cdot ISA \cdot \underline{NS/60^\circ} + ISO \cdot VPA \cdot \underline{NS/-60^\circ}) \quad (4.2-1)$$

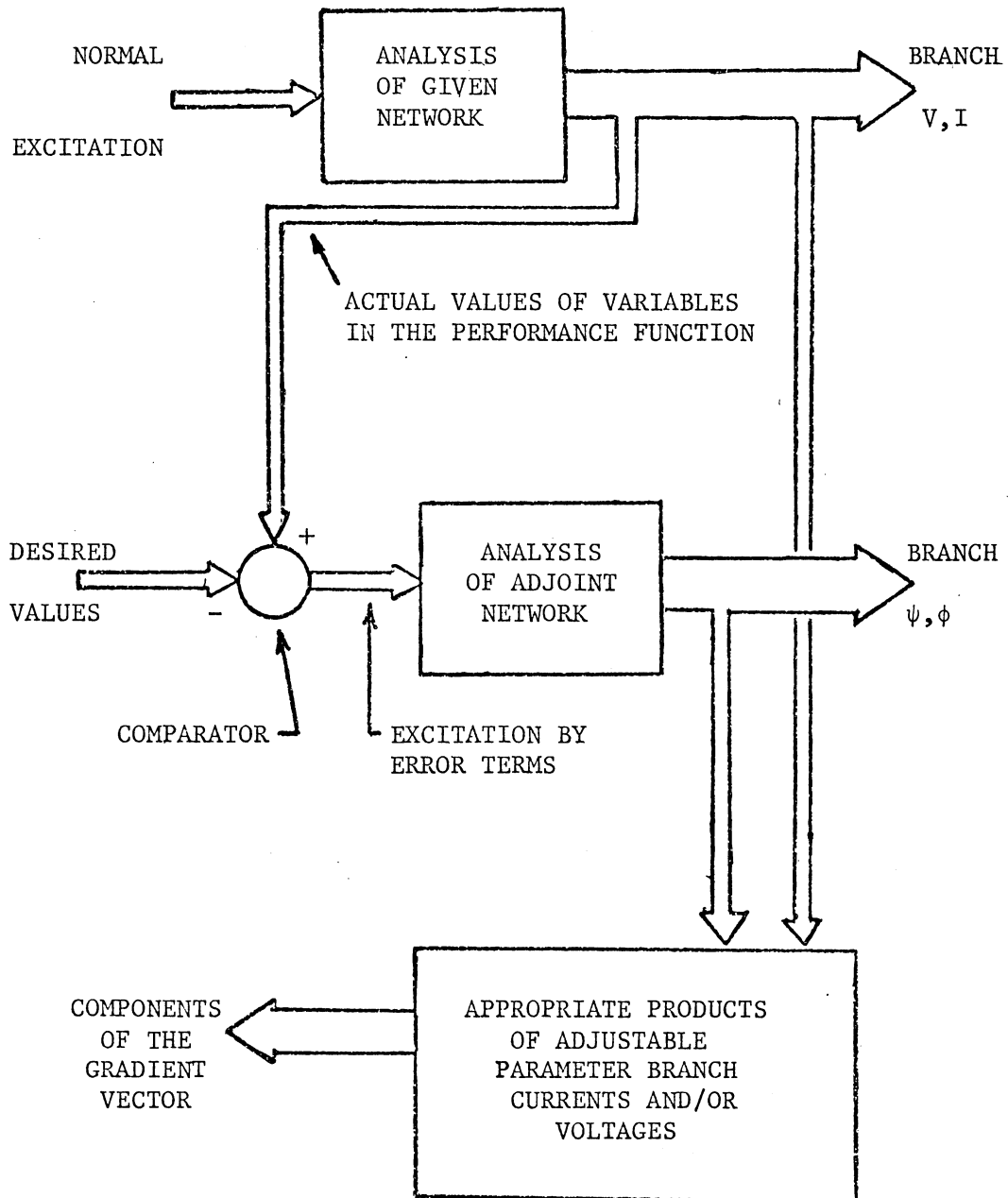


Fig. 4.2-1 Illustration of the Adjoint Network Method for Calculating the Gradient of a Performance Function

where R_e is the REAL operator
 VPO is the primary voltage in the original network
 VPA is the primary voltage in the adjoint network
 ISO is the secondary current in the original network
 ISA is the secondary current in the adjoint network
 NS is the secondary to primary turns ratio

The required adjoint network excitation vectors are

$$\phi_I = \tilde{V} \cdot W_V \cdot (|V| - \hat{V}) \quad (4.2-2)$$

$$\psi_V = -\tilde{I} \cdot W_I \cdot |I| \quad (4.2-3)$$

where ϕ_I is a vector of current sources used to monitor load voltages
 \tilde{V} is a diagonal matrix of conjugate voltages
 W_V is a diagonal constant weighting matrix
 V is the vector of controlled load voltages
 \hat{V} is a vector of desired load voltages
 ψ_V is a vector of voltage sources used to monitor cable currents
 \tilde{I} is the vector of controlled cable currents
 W_I is the sum of W_I and W_M in equation (2.2-1)
 I is the vector of controlled cable currents

In order to establish a common basis for comparison between the two methods for calculating the gradient, only those operations required

to calculate the gradient of ε_I (equation B.21) are counted. The number of matrix operations required for the adjoint network method are tabulated in Table 4.2-1.

4.3 The Matrix Method of Calculating the Gradient

The expression for the i -th entry in the gradient vector for the performance function ε_I given in equation (B.21) is given below. This equation is derived in Appendix C where the symbols are also defined.

$$G_i = R_e \{ V_g^T (Z_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot A^T)^{-1} K^{-1} \left[\frac{\partial K}{\partial X} \cdot K^{-1} \cdot Z_2 - \frac{\partial Z_2}{\partial X} + Z_2 \cdot N^{-1} \frac{\partial N}{\partial X} \right]$$

$$N^{-1} (A^T (Z_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot A^T)^{-1} \cdot (B_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot B_2) - B_2)$$

$$\cdot C_g \cdot \text{diag} \left[\frac{I_1^*}{|I_1|} \dots \frac{I_4^*}{|I_4|} \right] \cdot W_I \cdot |I| \} \quad (4.3-1)$$

Equation (4.3-1) must be evaluated for each of the twelve entries in the gradient vector. All terms in equation (4.3-1) external to the square brackets are constant for all twelve components of the gradient vector and need be calculated only once. The partial derivative terms inside the square brackets of equation (4.3-1) must be calculated for each component of the gradient and result in a unique matrix having non-zero values in only the i -th row and column.

The operations necessary to calculate the gradient vector are tabulated in Table 4.3-1. These figures do not include the operations required to form the impedance matrix or B_1 and B_2 . Formulation of the

Table 4.2-1

Matrix Operations Required to Calculate
the Gradient by the Adjoint Network Method

Matrix Inversion (Impedance Matrix)	(2)*	1
General Matrix Product		0
Matrix-Vector Product		4
Diag.-Diag. Product		7
Matrix Addition and Subtraction		3
Row/Column-Scaler Product		0

*Two inversions are required unless the approximations discussed earlier are used. Then only one inversion is required.

Table 4.3-1

Matrix Operations Required to Determine
the Gradient by the Matrix Method

Matrix Inversion (Impedance Matrix)	1
General Matrix Product	7
Matrix-Vector Product	9
Diag.-Diag. Product	1
Matrix Addition and Subtraction	4
Row/Column-Scalar Product	24

impedance matrix is required in both methods and B_1 and B_2 may be formed as a byproduct of the impedance matrix.

4.4 Comparison of the Two Methods for Calculating the Gradient

A comparison of Tables 4.2-1 and 4.3-1 shows that the use of the adjoint network method offers a substantial savings in numerical calculations. Since a general matrix product requires approximately N^3 scalar multiplications and additions the savings available is $7N^3$ complex operations. Considering that $N = 12$ in this problem and that a complex product requires four products rather than one the savings amount to approximately 48,000 multiplications and additions. The actual savings will be greater still since the load voltage and transformer current contribution to the gradient must also be considered.

Fig. 4.2-1 shows that the adjoint network source vectors are just the differences between the actual and desired values of the controlled variables. Therefore, the only additional terms required in the adjoint network method are the additional source terms. These would be similar to equations (4.2-2) and (4.2-3).

Adding terms to the matrix method to account for load voltage and transformer currents would require consideration of two equations similar to equation (B.30). The controlled variables in these equations would have to be expressed in terms of equation (C.1) by use of the relationships given in equations (A.7b) and (A.10). The end result would be a greater degree of added complexity for the matrix method as compared with the adjoint network method.

The computational efficiency of the adjoint network method may be further improved by using a method more efficient than matrix inversion for solving the network equations. Noble [7] demonstrates that approximately $N^3 + N^2$ operations are required to solve a set of N equations by matrix inversion but that triangular decomposition (LU factorization) requires approximately $1/3 N(N^2-1) + N^2$ operations. The LU method offers an additional savings on the order of $2/3 N^3$ operations. The LU method is discussed by Tinney and Walker [8] and Director [9] discusses its use in adjoint network calculations.

V. RESULTS AND CONCLUSIONS

This chapter gives the results of a computer run in which the model of the power system shown in Fig. 3.1-1 was operated both with and without the control scheme in operation. The load conditions for the test and the results of the test are presented graphically. A discussion of the results is followed by the conclusions.

5.1 Conditions for the Test

An attempt was made to provide a realistic test by providing a unique load for each of the four 13.8KV loads. The load MVA profiles for an arbitrary 24-hour period are shown in Fig. 5.1-1. Corresponding power factor profiles are shown in Fig. 5.1-2. The equations from which the MVA and power factor curves were obtained are given in Appendix D.

The constants in the load equations, (D.1 and D.2), were selected to provide a wide range of loading conditions. A worst case for cable overload occurs at approximately 8.30 and 14.00 hours when the three loads connected to a particular cable reach a peak simultaneously. Large variations in individual load level occurs at about 7.00 and 17.00 hours while the loads are rapidly changing at approximately 8.00 and 16.00 hours. In addition to the MVA variations the power factor of each load is varied randomly (between 0.75 and 1.0 lagging) on an hourly basis.

5.2 Results of the Test

In order to demonstrate the effectiveness of the control scheme, the system was operated both with and without the control applied. To

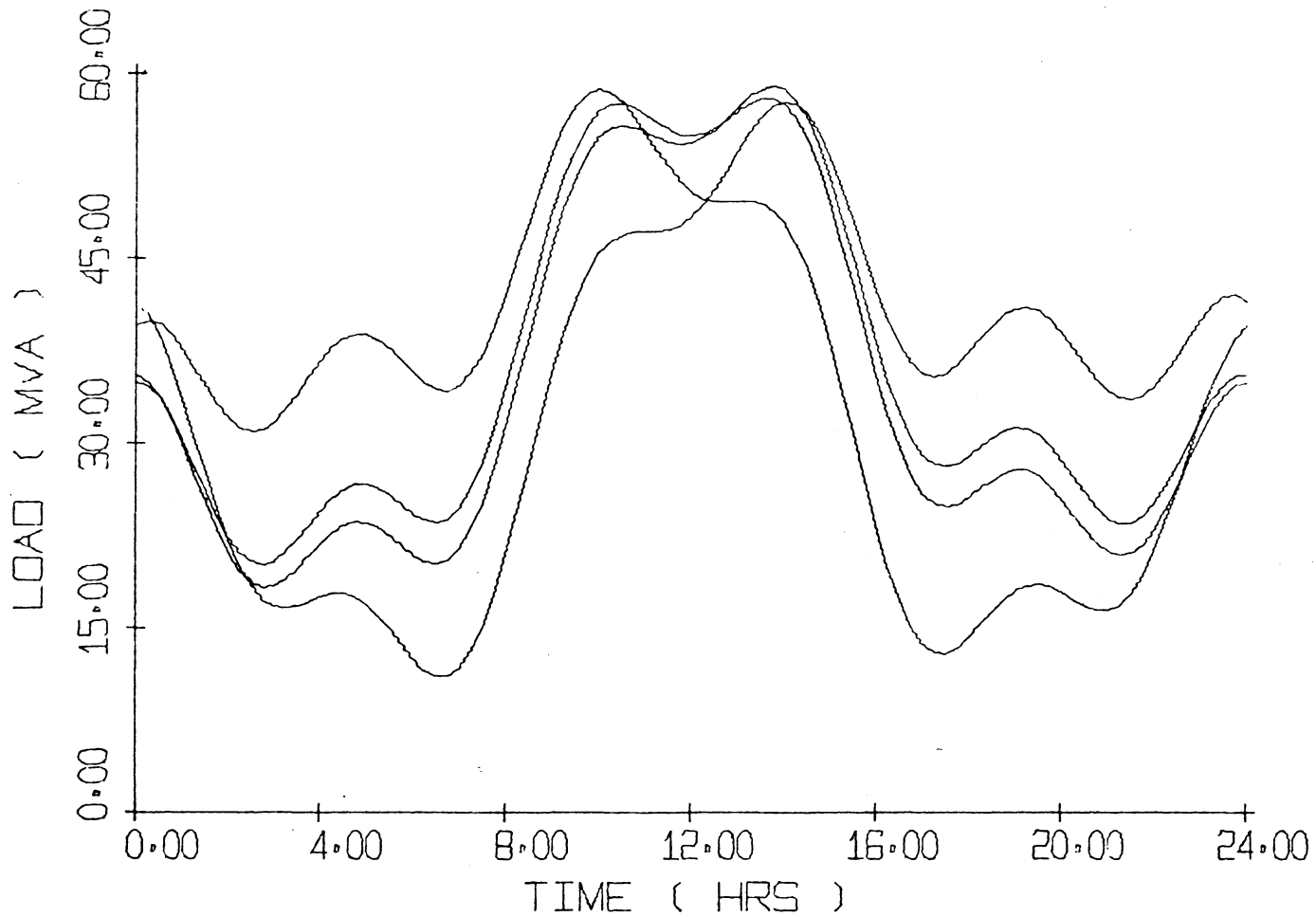


FIG 5.1-1 LOAD PROFILE VERSUS TIME

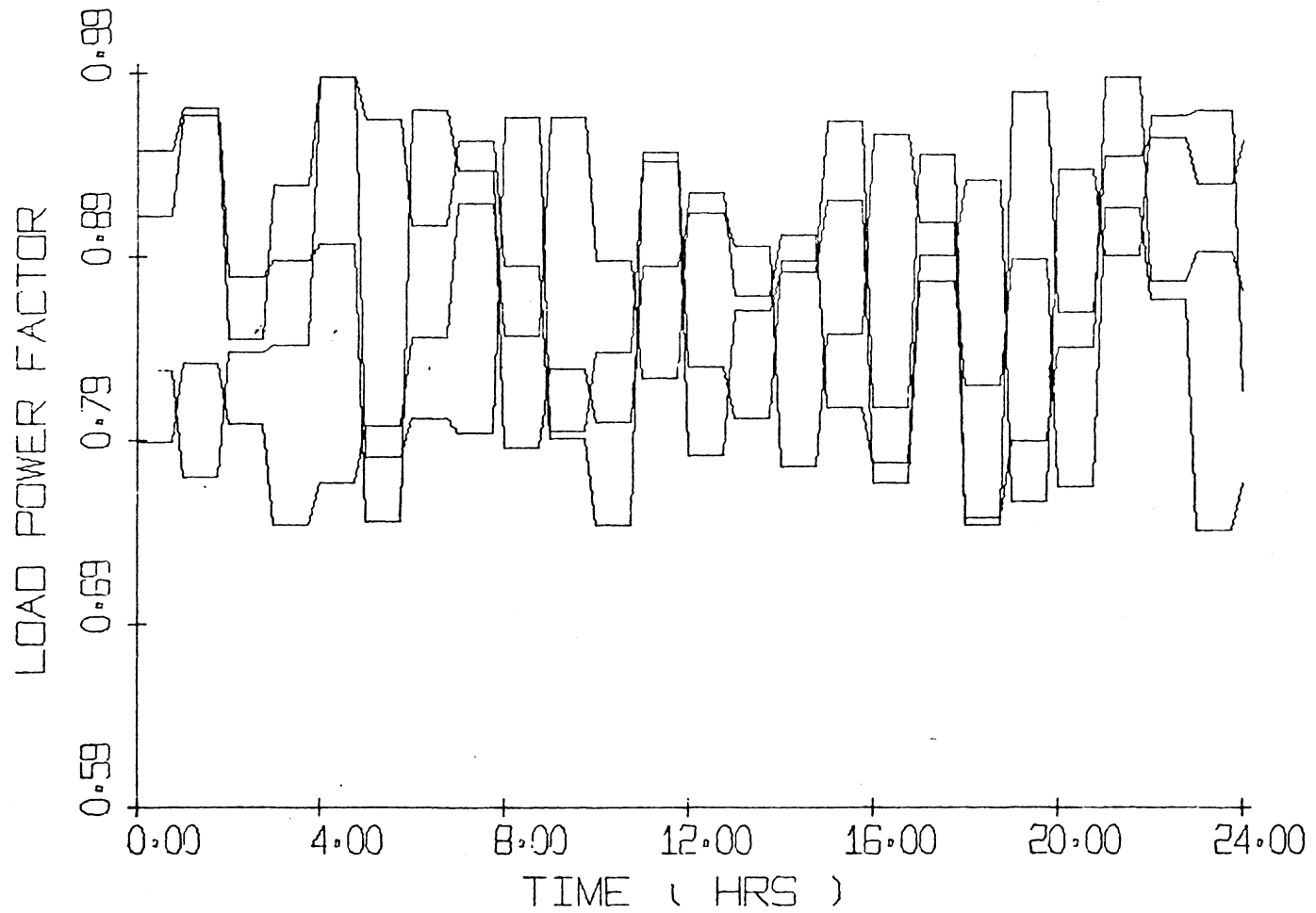


FIG 5.1-2 LOAD POWER FACTOR
VERSUS TIME

obtain a valid comparison, the system was adjusted one time before disabling the control to insure a common starting point for the two cases.

Cable Current Unbalance

The maximum unbalance in cable currents is shown in Fig. 5.2-1 for both cases. When the control was disabled, the current unbalance increased as the load changed and reached a maximum of approximately 65 amperes. With the control applied, the maximum current unbalance was approximately 7 amperes with an average unbalance of approximately 5 amperes. The control scheme significantly reduced the unbalance in the cable current.

Load Voltage Variation

Regulation of the load voltage without control was reasonably good over most of the 24-hour period. There were only four instances in which the nominal load voltage of 13.8 KV exceeded or fell below arbitrary limits of 14.2 and 13.4 KV (+2.9%). The extreme load voltages for the uncontrolled case are shown in Fig. 5.2-2.

With the control applied the extreme load voltages approached the upper and lower limits but never went outside the allowed range of voltages. Extreme load voltages for the controlled case are shown in Fig. 5.2-3.

Large Initial Unbalance

From Figs. 5.2-1 and 5.2-3 it is obvious that the control scheme was able to maintain very small current unbalance while simultaneously

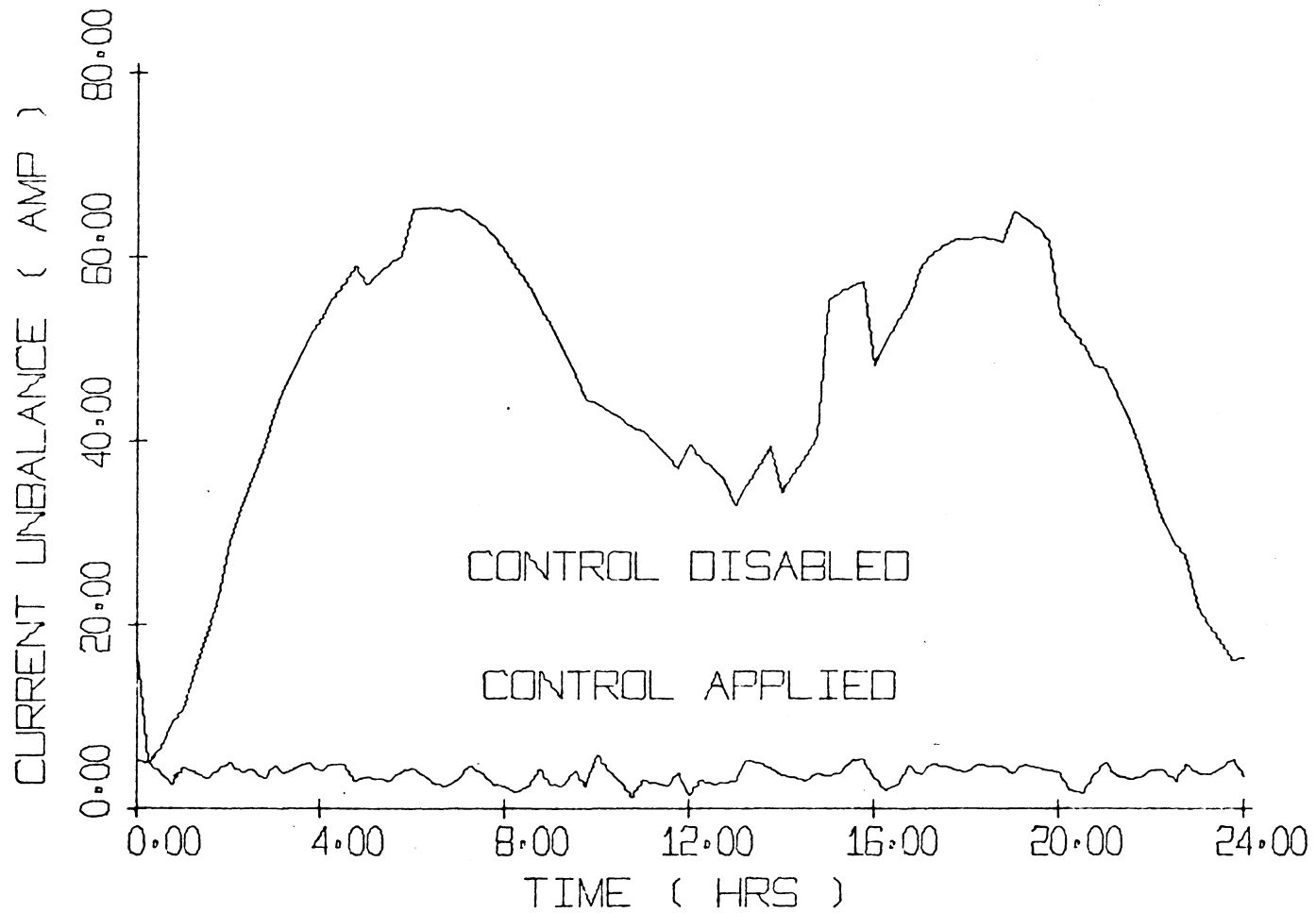


FIG 5.2-1 CABLE CURRENT VERSUS TIME

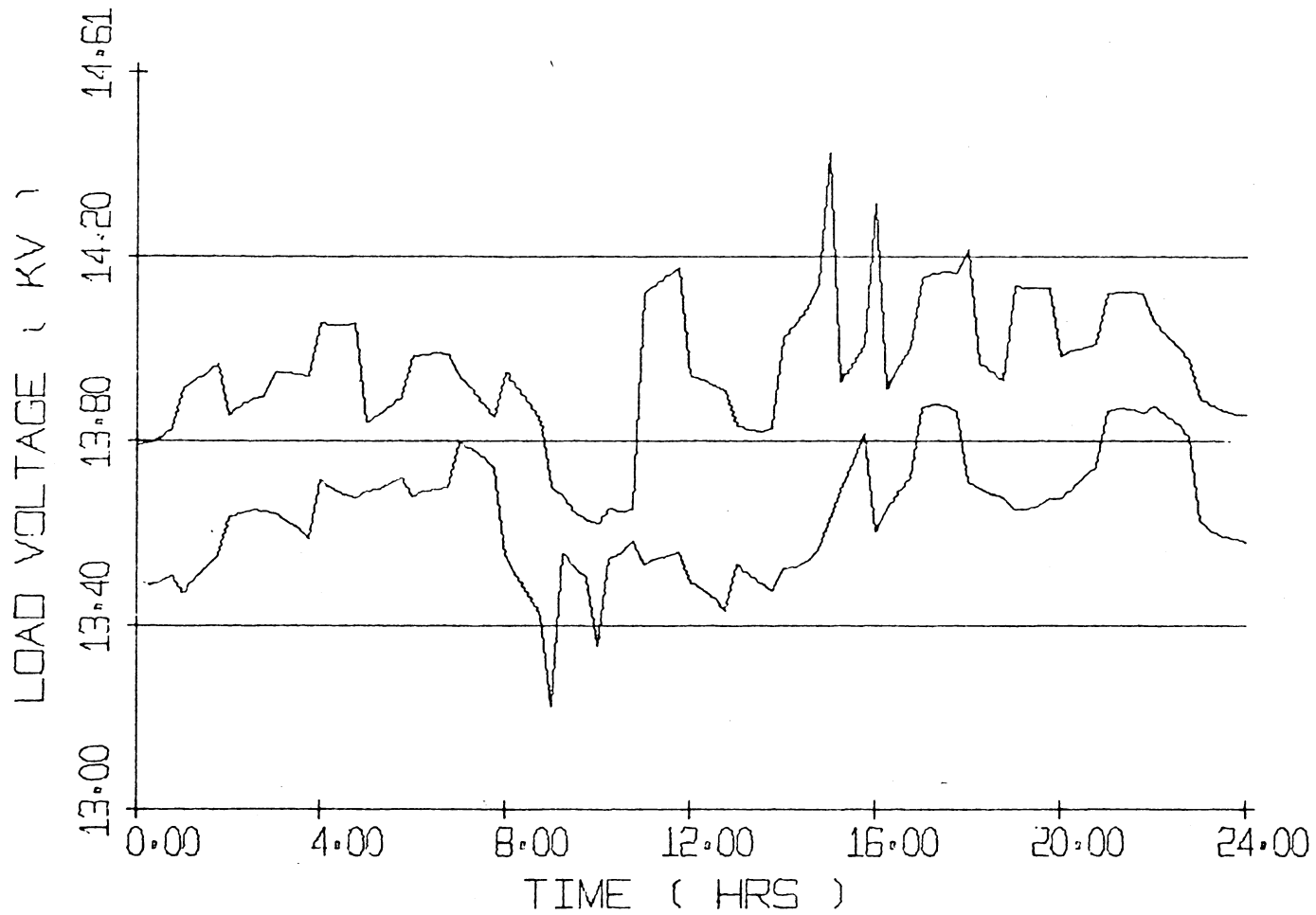


FIG 5.2-2 UNCORRECTED LOAD VOLTAGE
VERSUS TIME

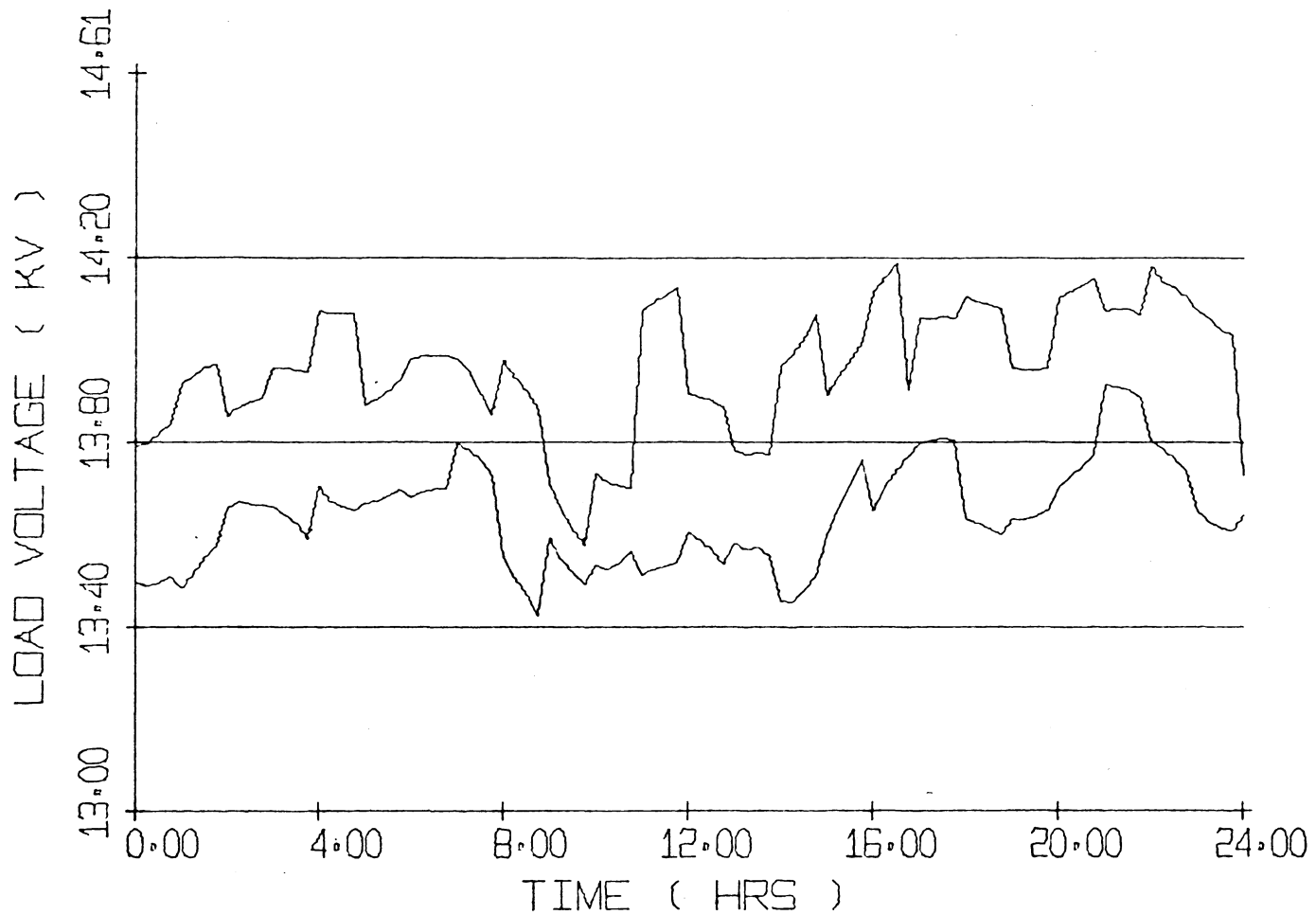


FIG 5.2-3 CORRECTED LOAD VOLTAGE
VERSUS TIME

limiting load voltage variations to ± 2.9 percent. In the above case the control was applied continuously from the beginning when the unbalance was not particularly severe. To demonstrate the capability of the control scheme to handle a large initial current unbalance, the application of control was delayed until the peak current unbalance occurred at 6.50 hours. Figs. 5.2-4 and 5.2-5 show the cable current unbalance and load voltage, respectively, when control is applied at 6.50 hours.

Comparison of Figs. 5.2-4 with 5.2-1 and Fig. 5.2-5 with 5.2-3 shows that the current unbalance and extreme load voltages are essentially identical over the interval when the control was applied. These results show that the control scheme is not adversely affected by large initial current unbalance.

5.3 Implementation

In Chapter II, two methods of implementing the control on a physical system were discussed briefly. Some of the factors which will influence the details of implementation are discussed here.

Sample Rate

The results presented above were based on a 15-minute sample time (the interval between adjustments). The system was also operated with a sample time of 30 minutes with essentially identical results. Therefore, the question of what constitutes an optimum sampling time arises. Ultimately, the sample time must be small enough so that the fastest changing parameter (load) cannot change greatly between samples. Determination of an optimum sampling time requires a familiarity with the

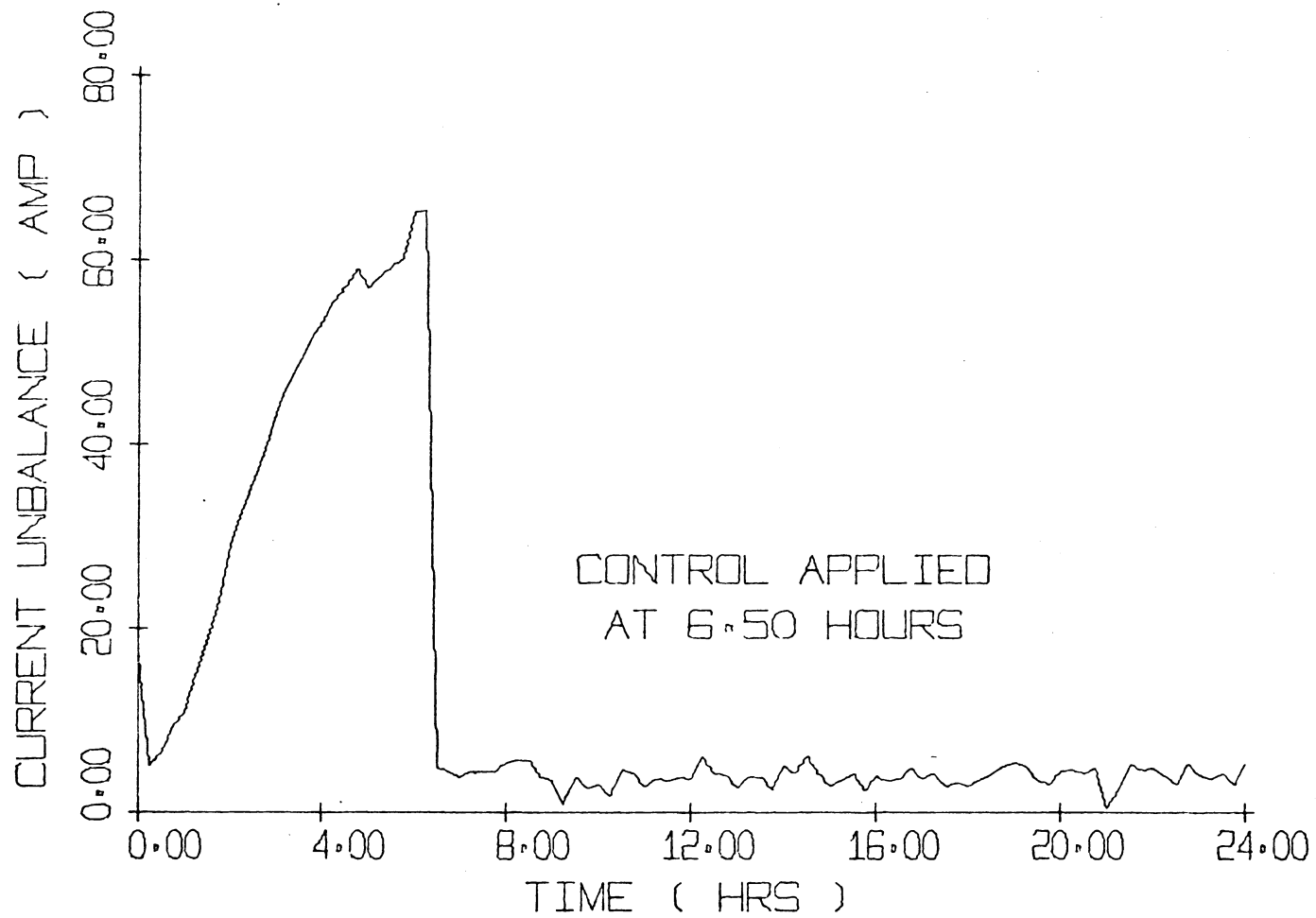


FIG 5.2-4 CABLE CURRENT VERSUS TIME

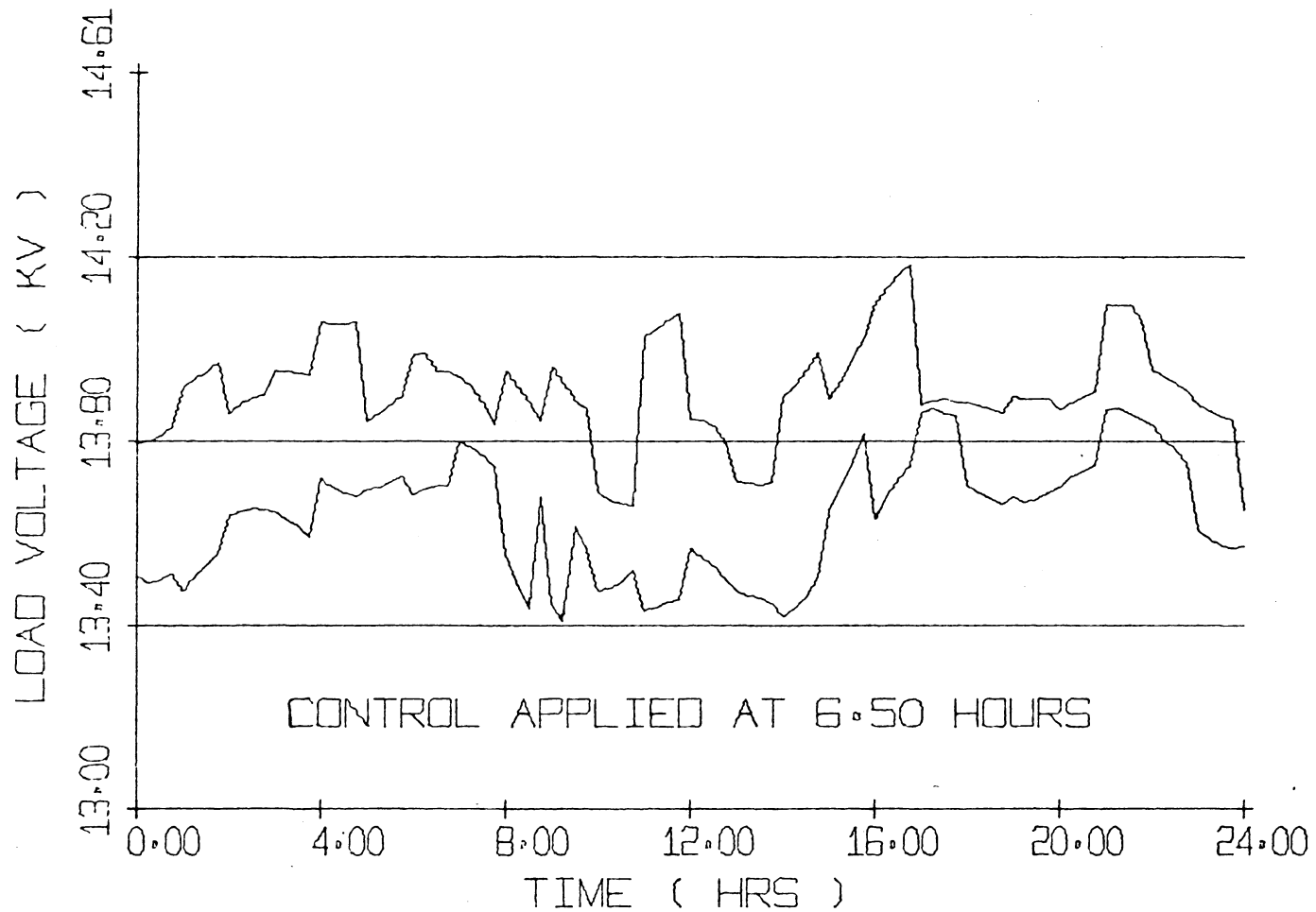


FIG 5.2-5 LOAD VOLTAGE VERSUS TIME

particular system to be controlled. A load demand history for such a system would be an aid in selecting a sampling interval.

An alternate approach could be based on a "performance threshold." This approach would require that the controlled variables be continuously monitored (with relatively simple logic circuits) so that whenever the performance drops below an acceptable level an "interrupt signal would be transmitted to the control computer indicating a need for adjustment. If this method were used alone, there would be no set sampling time. For greater reliability and fail-safe considerations, a combination of the two methods would be preferred.

Establishing the performance threshold is another problem requiring a knowledge of the particular system being controlled. Geographical, as well as performance considerations, would seem to dictate a need for individual threshold levels for current unbalance and the several load voltages.

Stability (Hunting)

In establishing a performance threshold one must reach a satisfactory compromise between performance and excessive control action. This is particularly true in selecting a value for ϵ_T , the iteration termination component of the performance function discussed in Chapter II. If ϵ_T is chosen too small, there will be load conditions for which the cut-off value of ϵ_T cannot be reached. Under these conditions, the optimization procedure would continue to try to reduce ϵ_T . A condition called "hunting," which is a form of instability, would exist. In the computer

program used here, the optimization procedure was allowed to try each adjustable parameter once. If a decrease in ϵ_T did not occur after all adjustments had been tried, the search was terminated because the system was at the best point attainable for the existing load conditions even though ϵ_T was not as small as desired.

A trade-off between the number of tap-changer increments and the degree of cable current balance must be made. A larger number of increments will provide a better current balance. In the example worked here a tap-changer increment of 5/8 percent provided an average unbalance of about 8 percent of the maximum uncontrolled unbalance.

Computer Selection

The computer requirements depend on the size of the system and the number of controlled variables. The storage requirements for the program used in this problem could be reduced considerably, perhaps to 40 or 50K, by eliminating the input/output simulated load features needed only for simulation purposes.

The selection of the computer best suited for a particular application will require a comparative study of the various minicomputer (or micro-computer) systems currently available. Many computers have full Fortran IV capability as well as other languages such as Basic and PL-1. In general, one must trade-off programming flexibility for computational efficiency. Other tasks the computer will be expected to handle will influence the final decision.

Fail-Safe Options

When the control of a large power system is automated, it is necessary to provide for failure of the control hardware. The controller described here considers only the primary problem of current balance and voltage regulation. An operational program would have to include various alarm functions as well as provisions for disabling the automatic control during fault or other abnormal conditions. A means for manually overriding the computer control should also be provided.

In modern applications, small computer controlled functions are operating as subsystems of larger computer-controlled networks. Proper coordination between the two (or more) computers should be provided to prevent inefficient operation.

5.4 Conclusions

A control scheme employing optimally adjusted TCUL phase-angle-connected transformers has been developed which will balance several cable currents and regulate several load voltages simultaneously. This scheme may be implemented using a digital computer and telemetry hardware to form a controller which will continuously and automatically maintain balanced currents in underground transmission lines. Such a controller would provide protection against cable overload due to normal load fluctuations.

The controller has been implemented on a digital computer model of the power system shown in Fig. 3.1-1. Simulated operation of this system over an arbitrary 24-hour period has demonstrated that the con-

troller is able to greatly reduce cable current unbalance while keeping load voltages within $\pm 2.9\%$ of the specified nominal load voltage for a wide range of load conditions.

A possible disadvantage of the method is that it requires an accurate model of the power system. A significant advantage of the method is that it is possible to verify the effect of a change in the position of any tap before a physical change in tap position is made. This makes it possible to predetermine an optimal sequence of adjustments and eliminates unnecessary tap changes. This is an important consideration from the maintenance point of view.

5.5 Further Study

If an automated adjustment procedure is to be used to maintain satisfactory system operation, information equivalent to that provided by the gradient should be available. In problems such as this one, the adjoint network is identical to the actual network except that phase-angle-connected transformers must have conjugate turns ratios. It would appear that there should be no reason to prevent construction of a hardware model of the adjoint. It would be more convenient, however, if adjoint information could be obtained from the actual system. It has been shown that certain approximations would allow a single network solution to apply to both the original and adjoint networks. Using superposition to obtain a solution for unit sources would allow one to reconstruct the actual and adjoint solutions from a properly weighted sum of the individual solutions. Relating such solutions to the physical net-

work may reveal simple relationships which would allow appropriate magnitude and phase factors to be applied to physical measurements on the actual system to obtain the adjoint solution and hence the gradient information.

Another interesting problem would be that of applying adaptive parameter identification techniques to this problem. This could be particularly useful in improving the computer model of the system by modifying the impedance matrix based on information obtained by comparing the computer solution with measurements made on the actual system. Successful application of this technique would enhance the gradient approach taken here.

In this study it was necessary to model the system on a digital computer and thereby avoid many of the problems involved in the real-time computer control of systems. It would seem desirable to combine the best features of the digital computer and the manually operated network analyzer. This could be accomplished by modifying a few of the basic components of the network analyzer in Patton Hall so that they could be controlled by the digital computer facilities in Whittemore Hall. The linking together of these two facilities would provide a powerful and flexible tool for the study of the real-time control of power systems.

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Appendix A

FORMULATION AND SOLUTION OF THE NETWORK EQUATIONS

The network equations are divided into two groups, the primary equations and secondary equations. The primary equations involve the elements between the transformer primary and the source. The secondary equations involve the transformer and load impedances. The transformer equations relate the voltages and currents in the primary and secondary equations. To extend the solution to the adjoint network, a vector current source I_{CS} is assumed to be connected in parallel with the load impedances to monitor the load voltages. I_{CS} is zero in the solution for the original network and corresponds to the adjoint source ϕ_I in the solution for the adjoint network. V_g takes on the values for ψ_V in the adjoint network solution.

Primary Equations

The four voltage sources, V_G , shown in Fig. 3.1-1 are replaced by a 12×1 source vector V_g for clarity in writing the equations.

$$V_g = C_g \cdot V_G \tag{A.1}$$

where

$$C_g^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{A.2}$$

where Y_{C1} is a diagonal admittance matrix for line charging
 capacitance C_1
 Y_{C2} is a diagonal admittance matrix for line charging
 capacitance C_2

Transformer Equations

The equations for an ideal phase-angle-connected transformer are expressed in inverted form.

$$V_P = N^{-1} \cdot V_S \quad (A.7a)$$

$$I_S = K^{-1} \cdot I_P \quad (A.7b)$$

where N is a diagonal complex turns ratio matrix
 K is the complex conjugate of N
 V_S is the transformer secondary voltage vector.
 I_S is the transformer secondary current vector.

A typical entry of matrix N has the form

$$N_i = NS + NS \cdot NT \cdot \underline{1/60^\circ} \quad (A.8)$$

where NS is the secondary to primary turns ratio
 NT is the tapped secondary to secondary turns ratio

Secondary Equations

The equations for the secondary are

$$V_S = Z_S \cdot I_S + C_S \cdot V_L \quad (A.9)$$

$$V_L = Z_L (C_S^T \cdot I_S - I_{CS}) \quad (\text{A.10})$$

where Z_S is a diagonal matrix of transformer impedances
 Z_L is a diagonal matrix of load impedances
 V_L is the vector of load voltages
 I_{CS} is the vector of adjoint network load voltage
 monitor sources (zero valued in the original network)
 C_S is a matrix relating secondary and load currents
 and is equal to C_g

Solution of the Equations

The equations may be solved for the primary current vector yielding

$$\begin{aligned} I_p = & [(Z_f + Z_p + Z_p \cdot Y_{C2} \cdot Z_f) + (U + Z_f \cdot Y_{C1} \\ & + Z_p (Y_{C1} + Y_{C2} + Y_{C2} \cdot Z_f \cdot Y_{C1})) \cdot (N^{-1} (Z_S + C_S \cdot Z_L \cdot C_S^T) K^{-1})]^{-1} \\ & [V_g + (U + Z_f \cdot Y_{C1} + Z_p (Y_{C1} + Y_{C2} + Y_{C2} \cdot Z_f \cdot Y_{C1})) (N^{-1} \cdot C_S \cdot Z_L \cdot I_{CS})] \end{aligned} \quad (\text{A.11})$$

where U is an identity matrix of appropriate size.

The controlled cable currents I_C are

$$I_C = C_g^T (I_p + I_{C1} + I_{C2}) \quad (\text{A.12})$$

Once I_p has been determined then the other currents and voltages may be calculated in the following order: I_S , V_L , V_S , V_p , I_{C1} , I_{C2} , and I_C .

Appendix B

DERIVATION OF THE ADJOINT NETWORK METHOD

In the following sections it is shown how Tellegen's Theorem may be used to relate the branch variables of an original and an adjoint network to the gradient of a performance function. This derivation follows that of Director and Rohrer [4] and extends their work by including the phase-angle-connected transformer which is a commonly used power system component.

Application of Tellegen's Theorem

The technique based on Tellegen's Theorem [10] requires only two network analyses (one for the original network and one for a so-called "adjoint network") to obtain the gradients of any network function with respect to all variable parameters.

Let V , I and ψ , ϕ be the branch voltages and branch currents for the original and adjoint networks respectively. For linear time-invariant networks Tellegen's Theorem states that

$$\sum_k (V_k \phi_k) = \sum_k (\psi_k I_k) = 0 \quad (\text{B.1})$$

where the subscript k implies summation over all source branches and all branches which contain variable parameters. It is assumed that the reference directions for corresponding elements in the original and adjoint networks are the same.

In the calculation of gradients we are interested in variations of network currents and voltages due only to parameter variations. Let

ΔV_k and ΔI_k represent variations of branch voltages and currents caused by a parameter variation of $\Delta \rho$ in the original network. The variation of a parameter value does not alter the topology of the network. Hence equation (B.1) still applies if V_k and I_k are replaced by $V_k + \Delta V_k$ and $I_k + \Delta I_k$ respectively. This substitution yields

$$\sum_k (V_k + \Delta V_k) \phi_k = \sum_k (I_k + \Delta I_k) \psi_k = 0 \quad (\text{B.2})$$

Subtracting equation (B.1) from equation (B.2) gives

$$\sum_k (\Delta V_k \phi_k) = 0 \quad (\text{B.3})$$

$$\sum_k (\Delta I_k \psi_k) = 0 \quad (\text{B.4})$$

Subtracting equation (B.4) from equation (B.3) yields

$$\sum_k (\Delta V_k \phi_k - \Delta I_k \psi_k) = 0 \quad (\text{B.5})$$

Equation (B.5) is subdivided into groups of terms according to the type of network elements (R, L, C, ..., T).

$$\begin{aligned} & \sum_V (\Delta V_V \phi_V - \Delta I_V \psi_V) + \sum_I (\Delta V_I \phi_I - \Delta I_I \psi_I) \\ & + \sum_R (\Delta V_R \phi_R - \Delta I_R \psi_R) + \dots + \sum_T (\Delta V_T \phi_T - \Delta I_T \psi_T) \\ & = 0 \end{aligned} \quad (\text{B.6})$$

The subscript denotes the type of network element. Ideal voltage and current sources are represented by subscripts V and I respectively.

By definition of an ideal source

$$\Delta V_V = \Delta I_I = 0 \quad (\text{B.7})$$

Placing only source terms on the lefthand side, equation (B.6) becomes

$$\begin{aligned} [\phi_I]^T [\Delta V_I] - [\psi_V]^T [\Delta I_V] = & - \sum_R (\Delta V_R \phi_R) \\ & + \sum_R (\Delta I_R \psi_R) + \dots - \sum_T (\Delta V_T \phi_T) + \sum_T (\Delta I_T \psi_T) \end{aligned} \quad (\text{B.8})$$

where matrix notation has been used on the lefthand side.

The defining equation for a resistive element is $V_R = R \cdot I_R$ from which

$$\Delta V_R = R \Delta I_R + \Delta R I_R \quad (\text{B.9})$$

A transformer having a complex turns ratio N has the defining equations

$$V_s = N \cdot V_p \quad (\text{B.10a})$$

$$I_p = - N^* \cdot I_s \quad (\text{B.10b})$$

where * denotes the complex conjugate

p,s denote primary and secondary respectively.

A perturbation ΔN in the turns ratio produces variations in currents and voltages. Neglecting second order terms gives

$$\Delta V_s = N \cdot \Delta V_p + \Delta N \cdot V_p \quad (\text{B.11a})$$

$$\Delta I_p = -N^* \cdot \Delta I_s - \Delta N^* \cdot I_s \quad (\text{B.11b})$$

Substituting equations (B.9) and (B.11) in equation (B.8) gives

$$\begin{aligned} [\phi_I]^T [\Delta V_I] - [\psi_V]^T [\Delta I_V] &= - \sum_R (R \cdot \Delta I_R + \Delta R \cdot I_R) \phi_R \\ &+ \sum_R (\Delta I_R \psi_R) + \dots - \sum_T (\Delta V_p \phi_p) + \sum_T (-N^* \cdot \Delta I_s - \Delta N^* \cdot I_s) \psi_p \\ &- \sum_T (N \cdot \Delta V_p + \Delta N \cdot V_p) \phi_s + \sum_T (\Delta I_s \psi_s) \end{aligned} \quad (\text{B.12})$$

where the transformer terms were included twice, once for the primary and again for the secondary.

Combining like terms and eliminating variations due to ΔI or ΔV terms provides a definition for the adjoint network parameters.

$$\begin{aligned} [\phi_I]^T [\Delta V_I] - [\psi_V]^T [\Delta I_V] &= + \sum_R (\psi_R - R \phi_R) \Delta I_R \\ &- \sum_R (I_R \phi_R) \Delta R + \dots - \sum_T (\phi_p + N \phi_s) \Delta V_p \\ &- \sum_T (V_p \phi_s) \Delta N + \sum_T (\psi_s - N^* \psi_p) \Delta I_s - \sum_T (I_s \psi_p) \Delta N^* \end{aligned} \quad (\text{B.13})$$

Terms involving ΔI_R , ΔV_p , and ΔI_s are eliminated by setting the coefficient terms equal to zero. Doing so provides a definition for the adjoint network elements.

$$\psi_R = R\phi_R \quad (\text{B.14})$$

$$\phi_P = -N\phi_S \quad (\text{B.15a})$$

$$\psi_S = N^*\psi_P \quad (\text{B.15b})$$

Equation (B.14) shows that the adjoint network element corresponding to a resistor R in the original network is an identical resistor. Comparison of equation (B.10) and (B.15) shows that the adjoint network element corresponding to a transformer in the original network is a transformer which has a conjugate turns ratio. In general, two terminal passive elements in the adjoint network are identical to those in the original network, however active elements are modified [4].

After the above terms are deleted, equation (B.13) takes the form

$$\begin{aligned} [\phi_I]^T [\Delta V_I] - [\psi_V]^T [\Delta I_V] = & - \sum_R (I_R \phi_R) \Delta R \\ & + \dots - \sum_T (V_P \phi_S) \Delta N - \sum_T (I_S \psi_P) \Delta N^* \end{aligned} \quad (\text{B.16})$$

In this particular application the turns ratio for transformers in the original network are of the form

$$N = NS + NS \cdot NT \cdot \underline{1/60^\circ} \quad (\text{B.17a})$$

$$N^* = NS + NS \cdot NT \cdot \underline{1/-60^\circ} \quad (\text{B.17b})$$

where NS is the turns ratio of secondary S to primary P

NT is the turns ratio of secondary T to secondary S

When NT is the variable parameter equation (B.16) may be written

$$\begin{aligned}
[\phi_I]^T [\Delta V_I] - [\psi_V]^T [\Delta I_V] &= - \sum_R (I_R \phi_R) \Delta R \\
+ \dots - \sum_T (V_P \cdot \phi_S \cdot \underline{NS/60^\circ} + I_S \cdot \psi_P \cdot \underline{NS/-60^\circ}) \Delta NT & \quad (B.18)
\end{aligned}$$

The vectors ΔV_I and ΔI_V may be expressed in the form

$$[\Delta V_I] = \left[\frac{\partial V_I^T}{\partial \rho} \right]^T [\Delta \rho] \quad (B.19a)$$

$$[\Delta I_V] = \left[\frac{\partial I_V^T}{\partial \rho} \right]^T [\Delta \rho] \quad (B.19b)$$

After making the above substitutions in equation (B.18) and operating on both sides of the equation with the real operator we obtain

$$\begin{aligned}
& \text{Re} \left\{ [\phi_I]^T \left[\frac{\partial V_I^T}{\partial \rho} \right]^T [\Delta \rho] - [\psi_V]^T \left[\frac{\partial I_V^T}{\partial \rho} \right]^T [\Delta \rho] \right\} \\
&= \text{Re} \left\{ - \sum_R (I_R \phi_R) \Delta R + \dots - \sum_T (V_P \phi_S \underline{NS/60^\circ} + I_S \psi_P \underline{NS/-60^\circ}) \Delta NT \right\} \quad (B.20)
\end{aligned}$$

The next section relates the gradient of the performance function to equation (B.20).

Gradient of the Performance Function

The performance function expressed in equation (2.2-1) is given below in a slightly modified form. Only two terms are included since the load voltage and transformer current terms are similar.

Let $\epsilon = \epsilon_I + \epsilon_V$ where ϵ_I is the performance function for current and ϵ_V is the performance function for voltage.

$$\epsilon_I = \frac{1}{2} \{ |I|^T \cdot W_I \cdot |I| \} \quad (\text{B.21})$$

$$\epsilon_V = \frac{1}{2} \{ [|V| - \hat{V}]^T \cdot W_V \cdot [|V| - \hat{V}] \} \quad (\text{B.22})$$

where I is an $m \times 1$ current vector, $m = 4$

V is a $q \times 1$ voltage vector, $q = 4$

\hat{V} is a $q \times 1$ vector of desired voltages

W_I is the sum of W_M and W_I in equation (2.2-1)

$$W_V = \text{diag} [l_{11} \quad l_{22} \quad l_{33} \quad l_{44}] \quad (\text{B.23})$$

and

l_{ij} is a constant weighting factor.

Consider $\frac{\partial \epsilon_I}{\partial \rho}$ and $\frac{\partial \epsilon_V}{\partial \rho}$ separately. Since W_I is constant and symmetric

$$\frac{\partial \epsilon_I}{\partial \rho} = \frac{\partial |I|^T}{\partial \rho} \cdot [W_I] \cdot [|I|] \quad (\text{B.24})$$

$\begin{matrix} nx1 & & nxm & & mxm & & mx1 \end{matrix}$

where ρ is an $n \times 1$ vector of adjustable parameters (tap position). Then

$$\frac{\partial \epsilon_I}{\partial \rho} = \begin{bmatrix} \frac{\partial |I_1|}{\partial \rho} \\ \vdots \\ \frac{\partial |I_m|}{\partial \rho} \end{bmatrix}^T \cdot [W_I] \cdot [|I|] \quad (\text{B.25})$$

$$\frac{\partial \epsilon_I}{\partial \rho} = \begin{bmatrix} \frac{\partial (I_1 I_1^*)^{1/2}}{\partial \rho} \\ \vdots \\ \frac{\partial (I_m I_m^*)^{1/2}}{\partial \rho} \end{bmatrix}^T \cdot [W_I] \cdot [|I|] \quad (\text{B.26})$$

$$\frac{\partial \epsilon_I}{\partial \rho} = \begin{bmatrix} \frac{1}{2} \left(\frac{\partial I_1}{\partial \rho} I_1^* + I_1 \frac{\partial I_1^*}{\partial \rho} \right) (I_1 I_1^*)^{-1/2} \\ \vdots \\ \frac{1}{2} \left(\frac{\partial I_m}{\partial \rho} I_m^* + I_m \frac{\partial I_m^*}{\partial \rho} \right) (I_m I_m^*)^{-1/2} \end{bmatrix}^T \cdot [W_I] \cdot [|I|] \quad (\text{B.27})$$

$$\frac{\partial \epsilon_I}{\partial \rho} = \text{Re} \begin{bmatrix} \frac{\partial I_1}{\partial \rho} \frac{I_1^*}{|I_1|} \\ \vdots \\ \frac{\partial I_m}{\partial \rho} \frac{I_m^*}{|I_m|} \end{bmatrix}^T \cdot [W_I] \cdot [|I|] \quad (\text{B.28})$$

Since both W_I and $|I|$ are real, they may be included inside the real

operator.

$$\frac{\partial \epsilon_I}{\partial \rho} = R_e \left\{ \frac{\partial I^T}{\partial \rho} \text{diag} \left[\frac{I_1^*}{|I_1|} \dots \frac{I_m^*}{|I_m|} \right] [W_I] [|I|] \right\} \quad (\text{B.29})$$

By a similar procedure obtain

$$\left[\frac{\partial \epsilon_V}{\partial \rho} \right] = R_e \left\{ \left[\frac{\partial V^T}{\partial \rho} \right] \text{diag} \left[\frac{V_1^*}{|V_1|} \dots \frac{V_q^*}{|V_q|} \right] [W_V] [|V| - \hat{V}] \right\} \quad (\text{B.30})$$

The gradient vector then becomes

$$\frac{\epsilon_V}{\partial \rho} = R_e \left\{ \left[\frac{\partial V^T}{\partial \rho} \right] [\tilde{V}] [W_V] [|V| - \hat{V}] + \left[\frac{\partial I^T}{\partial \rho} \right] [\tilde{I}] [W_I] [|I|] \right\} \quad (\text{B.31})$$

where

$$[\tilde{V}] = \text{diag} \left[\frac{V_1^*}{|V_1|} \dots \frac{V_q^*}{|V_q|} \right]$$

$$[\tilde{I}] = \text{diag} \left[\frac{I_1^*}{|I_1|} \dots \frac{I_m^*}{|I_m|} \right].$$

The variation of the performance function with respect to the network element perturbation vector $[\Delta \rho]$ and the gradient vector is given by

$$\Delta \epsilon = \left[\frac{\partial \epsilon}{\partial \rho} \right]_{1 \times n}^T [\Delta \rho]_{n \times 1} \quad (\text{B.32})$$

Substituting for the gradient vector yields the equation

$$\begin{aligned} \Delta \varepsilon = R_e \left\{ [|V| - \hat{V}]^T [W_V]^T [\tilde{V}]^T \left[\frac{\partial V^T}{\partial \rho} \right]^T \right. \\ \left. + [|I|]^T [W_I] [\tilde{I}]^T \left[\frac{\partial I^T}{\partial \rho} \right]^T \right\} [\Delta \rho] \end{aligned} \quad (\text{B.33})$$

A comparison of equations (B.32) and (B.33) defines the gradient vector in terms of the voltage and current vectors and weighting matrices in the performance function. Comparison of equations (B.20) and (B.33) shows that if the adjoint source vectors ϕ_I and ψ_V are defined as

$$\phi_I = \tilde{V} \cdot W_V \cdot (|V| - \hat{V}) \quad (\text{B.34})$$

$$\psi_V = -\tilde{I} \cdot W_I \cdot |I| \quad (\text{B.35})$$

then the gradient vector components may be calculated from the coefficients of the parameter variation ΔR , ..., ΔNT in equation (B.20).

For resistive elements the gradient vector components are obtained from

$$R_e (-I_R \phi_R) \quad (\text{B.36})$$

and for the phase-angle-connected transformer they are

$$-R_e (V_p \phi_s \underline{NS/60^\circ} + I_s \psi_p \underline{NS/-60^\circ}) \quad (\text{B.37})$$

Reference [4] tabulates the results for other active and passive network elements.

Appendix C

ANALYTIC DERIVATION OF THE GRADIENT OF THE PERFORMANCE FUNCTION

An expression is derived for the gradient of ϵ_I defined in equation (B.21). The purpose of the derivation is to provide the necessary expressions to permit a comparison of the analytic and adjoint network methods of computing the gradient vector.

Gradient of the Performance Function

The primary currents are expressed in a more compact form in equation (3.4-1) which is repeated below.

$$I_p = [Z_1 + A \cdot N^{-1} \cdot Z_2 \cdot K^{-1}]^{-1} V_g \quad (C.1)$$

In this equation only N , Z_2 , and K are functions of the tap position. Using the relationships derived in Appendix A an expression of the following form may be obtained for the cable currents.

$$I = C_g^T (B_1 + B_2 \cdot N^{-1} \cdot Z_2 \cdot K^{-1}) (Z_1 + A \cdot N^{-1} \cdot Z_2 \cdot K^{-1})^{-1} V_g \quad (C.2)$$

where I is a 4 x 1 vector of cable currents

C_g is defined in equation (A.2)

B_1 is $U + YC_2 \cdot Z_f$ (See Appendix A)

B_2 is $YC_1 + YC_2 + YC_2 \cdot Z_f \cdot YC_1$ (See Appendix A)

V_g is a 12 x 1 source vector

Equation (B.29) gives the gradient of ϵ_I . Let x represent ρ_i , the i -th adjustable parameter (tap position of the i -th transformer). Then the i -th entry in equation (B.29) may be written

$$\frac{\partial \epsilon_I}{\partial x} = R_e \left\{ \frac{\partial I^T}{\partial x} \cdot \text{diag} \left[\frac{I_1^*}{|I_1|}, \dots, \frac{I_4^*}{|I_4|} \right] \cdot W_I \cdot |I| \right\} \quad (C.3)$$

Matrices Z_1 , Z_2 , B_1 , B_2 , K^{-1} , and N^{-1} in equation (C.2) are symmetric. Therefore, the expression for the partial derivative in equation (C.3) becomes

$$\frac{\partial I^T}{\partial x} = V_g^T \frac{\partial [(Z_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot A^T)^{-1} (B_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot B_2)]}{\partial x} C_g \quad (C.4)$$

Let P denote the partial derivative in equation (C.4). Since Z_1 , A , B_1 , and B_2 are not functions of x equation (C.4) may be expanded using the relationship

$$\frac{\partial A^{-1}}{\partial x} = -A^{-1} \cdot \frac{\partial A}{\partial x} \cdot A^{-1} \quad (C.5)$$

The resulting expression for P is

$$P = (Z_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot A^T)^{-1} \cdot K^{-1} \cdot \left[\frac{\partial K}{\partial x} \cdot K^{-1} \cdot Z_2 - \frac{\partial Z_2}{\partial x} + Z_2 \cdot N^{-1} \frac{\partial N}{\partial x} \right] \cdot N^{-1} \cdot (A^T (Z_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot A^T)^{-1} (B_1 + K^{-1} \cdot Z_2 \cdot N^{-1} \cdot B_2) - B_2) \quad (C.6)$$

Since P is the i -th entry in a 12×1 vector, equation (C.6) must be evaluated twelve times to determine the complete gradient vector.

All terms in equation (C.6) except the partial derivatives are constant for a given calculation of the gradient. Since the entries in K^{-1} , Z_2 , and N^{-1} which are dependent on tap position lie along the diagonal, the derivative matrices are null except for the i -th diagonal element. The dependency of each of these matrices on the tap position is given below.

$$N = NS + NS \cdot x(.5 + j.866) \quad (C.7a)$$

$$K = NS + NS \cdot x(.5 - j.866) \quad (C.7b)$$

$$Z_2 = A_R x^2 + B_R x + C_R + j(A_X x^2 + B_X x + C_X)$$

where A_R , A_X , B_R , B_X , C_R , and C_X are the real coefficients in the second order approximation for the transformer impedance versus tap position relationship.

In equation (C.6) the first term in square brackets has non-zero entries only in the i -th row. The non-zero entry in the second term is the i -th diagonal element and the last term in the square brackets has non-zero entries in the i -th column.

Substituting these results into equation (C.3) yields the final expression for the i -th entry in the gradient vector.

$$\begin{aligned}
\frac{\partial \varepsilon_I}{\partial \mathbf{x}} &= \mathbf{R}_e \{ \mathbf{V}_g^T (\mathbf{Z}_1 + \mathbf{K}_1^{-1} \cdot \mathbf{Z}_2 \cdot \mathbf{N}^{-1} \cdot \mathbf{A}^T)^{-1} \cdot \mathbf{K}^{-1} \\
&\cdot \left[\frac{\partial \mathbf{K}}{\partial \mathbf{x}} \cdot \mathbf{K}^{-1} \cdot \mathbf{Z}_2 - \frac{\partial \mathbf{Z}_2}{\partial \mathbf{x}} + \mathbf{Z}_2 \cdot \mathbf{N}^{-1} \frac{\partial \mathbf{N}}{\partial \mathbf{x}} \right] \\
&\cdot \mathbf{N}^{-1} \cdot (\mathbf{A}^T (\mathbf{Z}_1 + \mathbf{K}_1^{-1} \cdot \mathbf{Z}_2 \cdot \mathbf{N}^{-1} \cdot \mathbf{A}^T)^{-1} (\mathbf{B}_1 + \mathbf{K}_1^{-1} \cdot \mathbf{Z}_2 \cdot \mathbf{N}^{-1} \cdot \mathbf{B}_2) - \mathbf{B}_2) \\
&\cdot \mathbf{C}_g \cdot \text{diag} \left(\frac{I_1^*}{|I_1|} \cdots \frac{I_4^*}{|I_4|} \right) \cdot \mathbf{W}_I \cdot |\mathbf{I}| \} \quad (\text{C.8})
\end{aligned}$$

Appendix D

LOAD MVA AND POWER FACTOR EQUATIONS

The equations from which the MVA and power factor versus time curves (Figs. 5.1-1 and 5.1-2) were obtained are given below.

$$\begin{aligned} \text{MVA}_i &= 35 - 15 \text{ SIN } (A+\phi_i) - 10 \text{ SIN } (2A+\pi/2) \\ &\quad + 5 \text{ SIN } (5A) \end{aligned} \quad (\text{D.1})$$

$$\text{PFL}_i = 0.75 + 0.25 R_i \quad (\text{D.2})$$

$$A = 2\pi (T+6)/\text{TMAX} \quad (\text{D.3})$$

$$\phi_i = (1.5+R_i)\pi \quad (\text{D.4})$$

where T is the time in hours

TMAX is the maximum time (24 hours)

R is a uniformly distributed random number obtained from RANDU with a starting value of 13579.

The first four random numbers are used in the calculation of the four ϕ_i . The ϕ_i remain constant for the entire 24-hour period. New random numbers are used in the calculation of the power factor at the beginning of each hour.

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AN OPTIMALLY ADJUSTED TCUL CONTROLLER FOR THE
PROTECTION OF UNDERGROUND TRANSMISSION LINES

by

Jesse Theron Hancock, Jr.

(ABSTRACT)

In areas supplied by underground transmission systems load division among several parallel connected cables may not be satisfactory. Phase-angle-connected TCUL transformers may be used to provide the phase shift necessary to equilibrate the cable loading and to provide voltage regulation at the load. A means for automating the TCUL adjustments to provide satisfactory current balance and voltage regulation was sought.

This problem was attacked by first expressing the system performance (the unbalance of cable currents and load voltage magnitude) in terms of a system performance function. An optimal policy of control is defined as the selection of the particular TCUL transformer which provides the greatest improvement in the system performance. A negative gradient optimization routine is used to determine the required adjustment.

A derivation of the adjoint network relationships for a phase-angle-connected transformer is given. These relationships provide a means for readily determining the gradient of the performance function. This approach is developed to provide a means for continuously monitoring and automatically controlling the system performance. A discussion of two ways in which this approach might be implemented physically is given.

The optimally adjusted TCUL controller was incorporated in a digital computer model of a small underground power system. Simulated operation of the system under widely varying load conditions demonstrated the ability of the controller to maintain very small cable current unbalance while simultaneously holding voltage variations to ± 2.9 percent of nominal.

A significant advantage of this approach is that the optimal sequence of adjustments required to restore the system to a satisfactory level of performance may be obtained and verified without making any actual adjustments to the TCUL transformers. This is important from a maintenance point of view. A possible disadvantage of the method is that an accurate model of the system must be available.