THE APPLICATION OF THYRISTORS TO INDUCTION MOTOR DRIVE SYSTEMS

by

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I INTRODUCTION

Historical Development

The basic industrial needs for rotating machinery fall into the two categories of constant speed and adjustable speed drives. Since the early electrical distribution systems were primarily direct-current systems, direct-current motors were applied almost universally in drive systems of each category. However, the shift of power distribution systems from direct to alternating current produced an industrial demand for simple, rugged drive systems which could operate directly from the available alternating current supply without power conversion and, as a result, the induction motor, patented by Nikola Tesla in 1886, eventually replaced the direct-current motor as the most universal constant speed drive.

Induction motors provide constant-speed industrial drives which are unsurpassed in simplicity, reliability and economy, but application of the induction motor to adjustable-speed systems has been a constant challenge to engineers. The intensive effort devoted to this problem during the past several decades has yielded a wide variety of schemes for altering the inherently constant-speed characteristics of induction motors which require either a means of rotor impedance switching or some form of incoming power conversion. Both functions can now be accomplished with completely static circuit elements, but the most economical scheme for limited duty cycle or limited low-speed torque drives has stemmed from adjustable-stator voltage control of induction motor torque. Few objections have been raised to the poor power factor and efficiency which results from subsynchronous operation since the lower initial cost and lower maintenance expense more than offset the cost of power loss for drives of less than 100 horsepower capacity.

The application of adjustable-voltage control of induction motors began to gain momentum in the early nineteen-fifties with thyratron-controlled excitation of saturable reactors placed in series with the stator. The relatively long time constants and low power gain of the reactors, coupled with the development of high current diodes, led to the substitution of magnetic amplifiers for saturable reactors. By 1963, the cost of thyristors (silicon controlled rectifiers) had dropped to a point where an economical drive could be marketed with silicon units as the primary control element. Adjustable-speed induction motor drives, utilizing thyristor control of stator voltage, are presently being employed to drive loads such as fans, pumps, traffic bridges, and overhead cranes. Thyristor-controlled hoist drives for overhead cranes up to 250 horsepower are now a standard product line. The prototype of this line was installed on a steel mill crane with a 100 horsepower hoist drive in November, 1964, and has proven to be relatively maintenance free in spite of a severe duty cycle.

General Problem Areas

Three different thyristor power circuits can be employed to accomplish the required functions in these applications, and each involves a different number of thyristors and a different set of output characteristics. The best choice of power circuit elements requires an understanding of the advantages and limitations posed by each scheme, and this is presently not available in published material. It is hoped that this thesis will fill some of this void by providing an investigation of the characteristics of each of the three potential control schemes and their application to adjustable-voltage control of variable-speed induction motor drives.

Consideration is first given to the principle of phase control using thyristors to control the power developed in single-phase loads with a discussion of the input power factor and output harmonic characteristics of this control. These characteristics are then extended to three-phase circuits where the output harmonic structure of each of the three power circuits is discussed. The torque-speed characteristics of an induction motor supplied from each of the three thyristor power circuits are presented along with comparative heating data which provide a basis for evaluating drive performance.

II REVIEW OF LITERATURE

The most familiar scheme for controlling the speed of an induction motor is that of switching the rotor resistance to enable the selection of a torque-speed characteristic which provides the desired speed for a given load torque and the steady-state motor current with this scheme seldom exceeds the rated value. The main objection to the use of this drive is customer acceptance of the complicated switching which compounds the maintenance expense. With adjustable-voltage control, the torque-per-ampere relationship for slips greater than 0.5 may be halved which results in excessive motor heating. Hence the operation of this drive at reduced speeds requires either a restriction of load torque with speed reduction such as a fan or pump load, or a limitation upon the length of time that the motor is run at slow speeds. Enthusiasm for this form of control was dampened by its limitations and application engineers were reluctant to suggest its usage. However, the simplicity, reliability and response of this drive utilizing thyristors has wide customer appeal and competition has increased in the past two years.

Hesitance to apply this drive for so many years has suppressed the number of articles written on the subject. Several excellent papers by Shepherd and Stanway[14,15,16] have appeared along with one by Heumann[7] since 1964. These papers describe many different power circuit arrangements and demonstrate the potential of this drive. Discussion of the effect which voltage harmonics have upon the motor performance is available in several ac machinery texts of which Langsdorf[11] provides the most thorough treatment. Most authors imply that superposition can be applied to the analysis of induction motor performance, but it has been found that an interaction between the motor and control occurs through some speed regions and the application of superposition in these regions does not give correct answers.

III THYRISTOR CONTROL OF SINGLE-PHASE POWER TO STATIC LOADS

Introduction

A basic knowledge of the operation and characteristics of the components required for phase control of power is necessary background for understanding the principle of phase-controlled regulator circuits. Only the peripheral concepts of thyristor operation are considered here since the principles of p-n-p-n junction devices are thoroughly treated in numerous texts. Thyristor control circuitry will be considered only as functional blocks with appropriate input-output notations since detailed circuitry necessitates a choice of one specific design approach.

The Power Diode

The first of the semiconductor devices to achieve the high voltage and current ratings necessary to promote their use in power circuitry was the simple p-n junction diode, and the sharp drop in the cost of this device which evolved as semiconductor manufacturers improved their processes for refining silicon encouraged the widespread use of silicon diodes in power applications. The diode is inherently a two-terminal unilateral device which exhibits a low impedance to current in the forward direction and a high impedance to current in the reverse direction. The standard diode symbol with the associated terminal

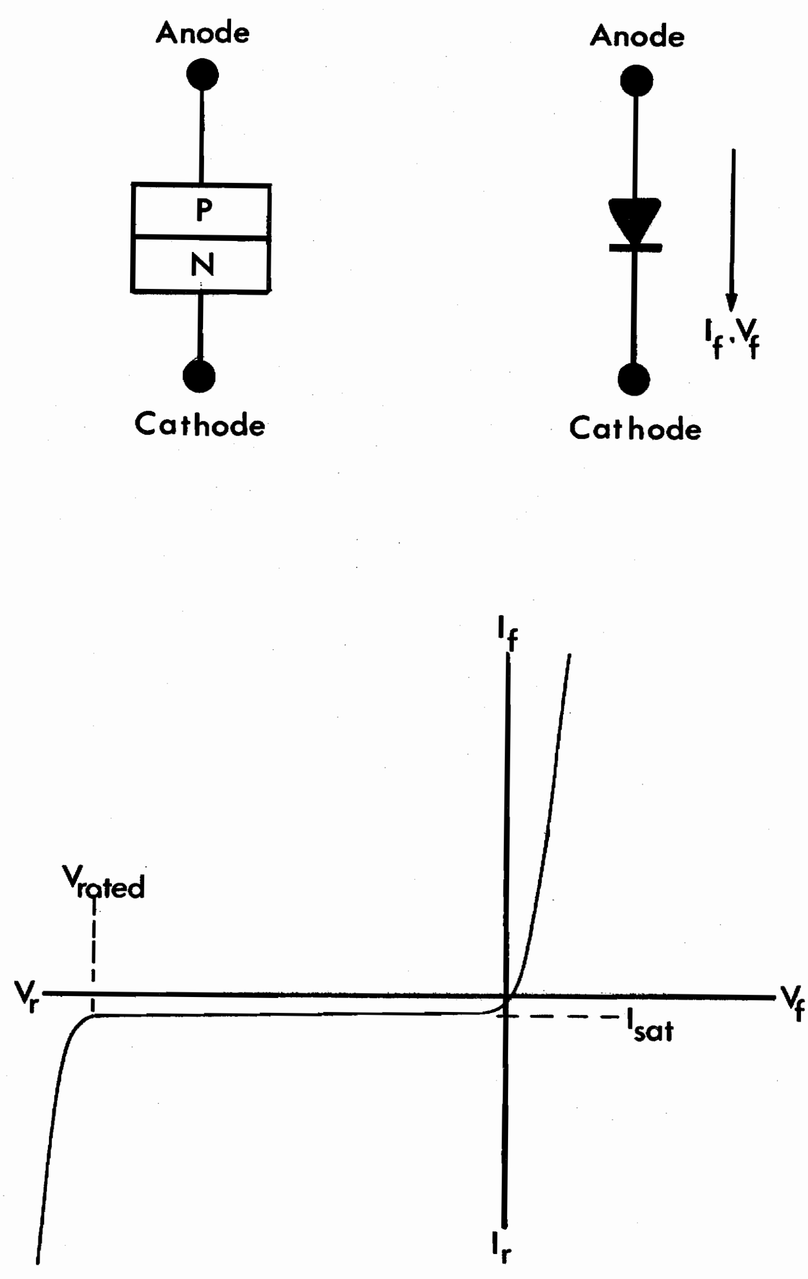


FIGURE 1. Standard symbols, terminal designation and volt-ampere characteristics of a diode.

designation and typical volt-ampere characteristics is shown in Figure 1. Recent developments in diode technology have produced a family of controlled-avalanche silicon devices which are virtually impossible to destroy with low energy reverse transients that could generate reverse voltages far in excess of the normal reverse voltage capability of the device. This provides a very reliable control element.

The Thyristor

The thyristor is a three-terminal unilateral device having a three-junction p-n-p-n structure. One of the two accepted symbols of this device resembles the standard diode symbol with one additional terminal which is designated as the gate (see Figure 2). One might correctly conclude, on the basis of this symbol, that a thyristor is a gate controllable rectifier; however, a more descriptive equivalent of the thyristor can be derived by slicing the structure diagonally across the two inner layers and connecting the two n-layers and p-layers with conductors as in Figure 3. The result is two complementary transistors arranged in a regenerative connection which demonstrates the latching nature of thyristors. Should a positive pulse be applied to the gate when the anode is positive with respect to the cathode, the collector current of the npn transistor will provide base drive to the pnp transistor and the resulting collector current of this transistor

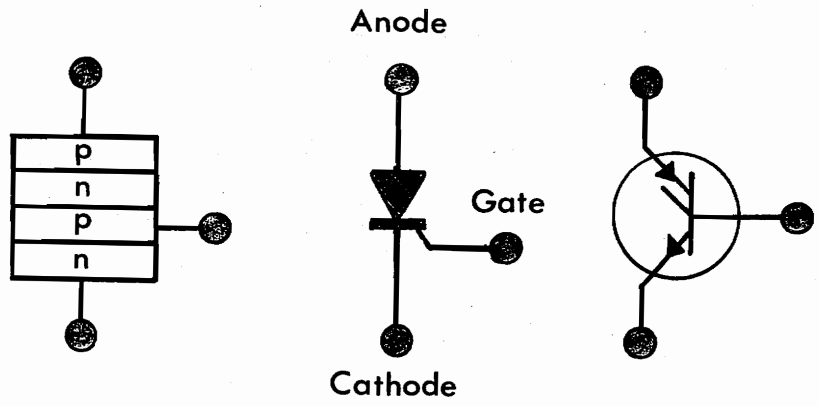


FIGURE 2. Layer structure of the thyristor and two accepted symbols.

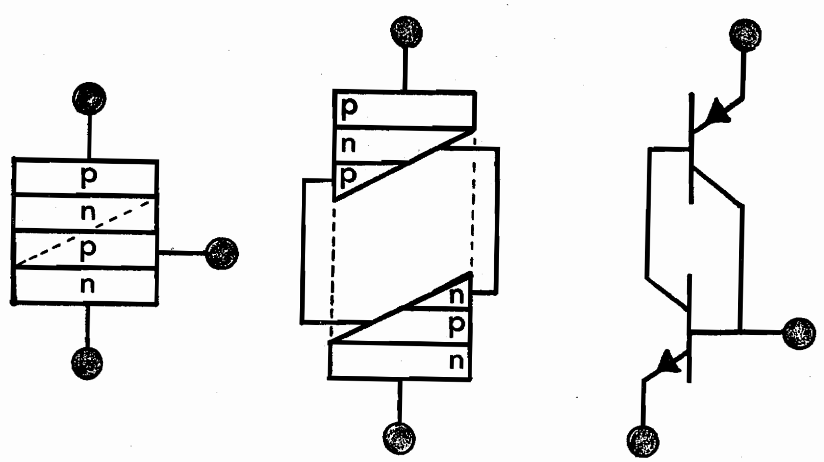


FIGURE 3. Development of the two transistor equivalent of the thyristor.

drives the base of the npn. Hence, the external gate drive can be removed without affecting the current of the device. It might be noted that the use of reverse bias on the emitter-base junction of the npn transistor to extinguish the current would require a reverse voltage in excess of the supply voltage which in most applications exceeds the reverse voltage capability of this junction and as a result, some other means must be found to commutate the current. In phase control applications where the collector supply voltage is alternately positive and negative, the circuit designer can rely upon this polarity reversal as a natural source of commutation. The pulse applied to the thyristor gate for triggering must be maintained long enough for the anode current to increase to a specified holding-current. This period is generally less than 30 microseconds when used with resistive loads. Phase control operation into inductive loads imposes additional restrictions upon the triggering pulse width which will be discussed later.

Single Phase Control

The circuit of Figure 4 represents a simple arrangement utilizing four diodes and one thyristor in a circuit which enables control of ac load power through control of the retard angle (θ_R). The diode bridge arrangement converts the thyristor to a pseudo-bilateral device which can be switched to the conductive state during both the positive

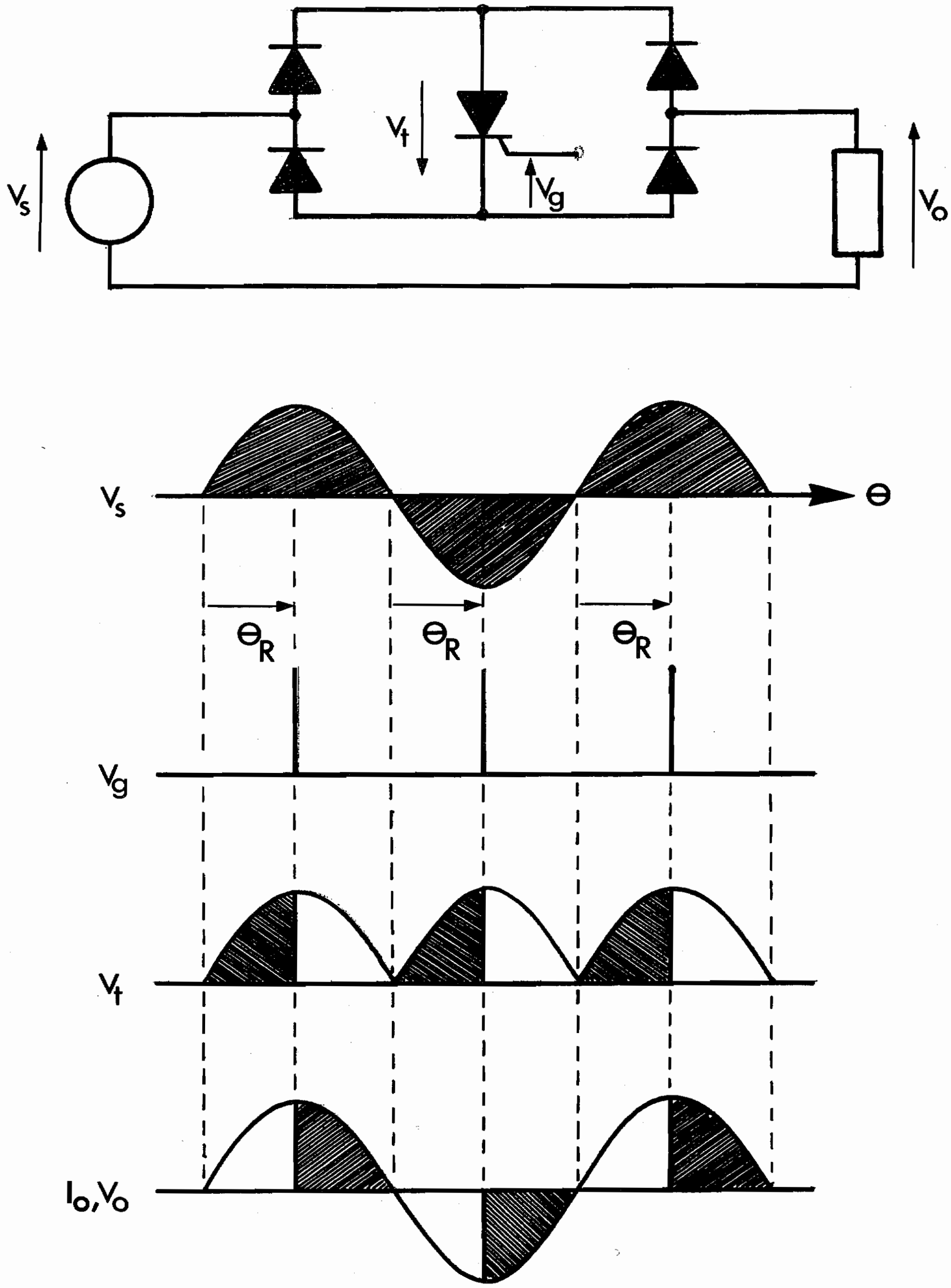


FIGURE 4. Thyristor control of load voltage using the principle of phase control.

and negative half cycles of supply voltage. The gate pulse (Vg) is supplied from a pulse generator with an adjustable phase delay (retard angle θ_R) synchronized to the zero crossover points of the supply voltage. From the wave forms of Figure 4 it can be seen that the instantaneous voltage during the interval $(0 < \theta < \theta_R)$ will appear as forward voltage across the thyristor while the instantaneous voltage during the interval $(\theta_R < \theta < \pi)$ will appear across the load.

The instantaneous load voltage will be:

$$V_o = 0 \qquad 0 \leq \theta < \theta_R$$

$$V_o = V_m \sin \theta \qquad \theta_R \leq \theta < \pi$$

And the effective voltage across the load would be:

$$V_o = \frac{V_m}{\sqrt{2}} \left[1 - \frac{\theta_R}{\pi} + \frac{1}{\pi} \sin \theta_R \cos \theta_R \right]^{\frac{1}{2}}$$

When an inductance is substituted for the load resistance of Figure 4, the necessary range of delay angle (θ_R) required to vary the load voltage from zero to the full supply value is reduced. This might best be understood by considering the required location of a narrow gate pulse to provide a sine wave of voltage across the load. Under these conditions, the current will lag the supply voltage by a lag angle (γ) of 90 degrees; hence, a minimum gate pulse retard angle (θ_R) of 90 degrees is required since the thyristor cannot switch until a source of anode current is available. As the retard angle is increased from 90 degrees to 180 degrees, the load voltage will be smoothly reduced to zero provided the pulse has a time-width adequate to enable the current to build up to the holding level. Should the retard angle be reduced to less than 90 degrees, half-wave direct current will appear in the load. If, for example, a retard angle of 10 degrees is provided, the thyristor will be triggered on the first half-cycle. The gate pulse during the next half cycle will appear while the thyristor is still conducting, but the current magnitude is diminishing. When the current reaches zero, about 70 degrees after the second gate pulse, the thyristor will switch to a blocking state until the next gate pulse appears. Hence, conduction will occur on alternate half-cycles resulting in unidirectional load current. In order to guard against this condition, a gate pulse width of 90 degrees could be used for retard angles less than 90 degrees, but a more practical solution is to employ a gate generator which will maintain a constant output voltage during the entire interval $(\theta_R \leqslant \theta < \pi)$. This form of gate drive, which is referred to as constant voltage, variable width, enables a smooth reversible transition of load voltage from zero to the maximum of the supply independent of the load resistance-reactance ratio. However, the gain of the controller will increase with increasing inductive lag angle (γ) since the range of gate signal phase delay required for full range control is reduced by the lag angle (γ).

Output Voltage Structure

One of the disadvantages to phase control is the non-sinusoidal nature of the load voltage waveforms giving rise to harmonics of the supply voltage. With the circuit of Figure 4 , the output voltage waveform exhibits half-wave symmetry with a time average of zero voltage, indicating that the Fourier series expression for the output will contain only the fundamental and its odd multiples. The relative magnitudes of these components vary with retard angle as shown in the curves of Figure 1A in the appendix. The harmonic structure of the output voltage becomes extremely important when the load is a three-phase induction motor, and a considerable portion of this discussion is devoted to the determination of the output voltage harmonics for various three-phase power circuit configurations and their effect upon motor performance.

Input Power Factor

Since the current waveforms of a thyristor controller are non-sinusoidal, any discussion of power factor requires an explicit definition. Most authors define power factor as either the cosine of the angle between voltage and current, or as the ratio of real power to apparent power. Both of these definitions give the same result when both the voltage and current are sinusoidal, but the results become somewhat ambiguous when the current and voltage waveshapes differ from one another. Before one chooses a definition, the need for the power factor should be considered. When a device is supplied through long lines, some means must be found for determining the line drop to insure that sufficient voltage is available at the load. The line drop is a function of the effective line current and the power factor enables one to determine the required line current if it relates power input to effective volt-amperes. Since line drop represents an energy loss, the power company bases its rate to industrial users upon the line power factor so the input power factor of devices consuming large amounts of power becomes an economic consideration to users. If the power company uses a moving-vane type of measuring instrument, the relatively low pass characteristics (approximately 100 Hz) renders them insensitive to any frequency higher than the 60 Hz component. The input power factor to a thyristor phase controller might then be considered from two viewpoints; first as the cosine of the angle between fundamental voltage and the fundamental component of line current, and secondly as the ratio of load power to effective volt-amperes. With either definition, the input power factor to a thyristor phase controller can be less than unity, even when the load is resistive (see Figure 5).

The average power of a waveform of period (2π) can be expressed as:

$$P = \frac{1}{2\pi} \int_{0}^{2\pi} e(\theta) i(\theta) d\theta$$

The non-sinusoidal current of Figure 4 can be expressed as the sum of the fundamental and its odd harmonics, as discussed previously, and when the reference is chosen at the line voltage zero, the input power would be:

$$P = \frac{1}{2\pi}\int_0^{2\pi} V_S(\theta) I_O(\theta) d(\theta)$$

$$P = \frac{1}{2\pi}\int_0^{2\pi} V_S \sin\theta \left[I_1 \sin(\theta + \gamma_1) + \sum_{n=2}^{\infty} I_k \sin(k\theta + \gamma_k) \right] d\theta$$

Where $k = (2n - 1)$

$n = 2, 3, 4, 5$

The fact that the integral over one cycle of the product of a fundamental frequency and a harmonic is zero eliminates all but the first term, and integration yields:

$$P = \frac{1}{2} V_S I_1 \cos \gamma_1$$

When the low pass instrumentation is used to measure power factor, the factor ($\cos \gamma_1$) will be the indicated power factor. Since the power input is a function of the fundamental components only, conventional wattmeters will indicate the total power. When the line losses must be determined, the effective current (I_{rms}) must be considered. Applying the second definition which considers power factor to be the ratio of input power to effective volt-amperes:

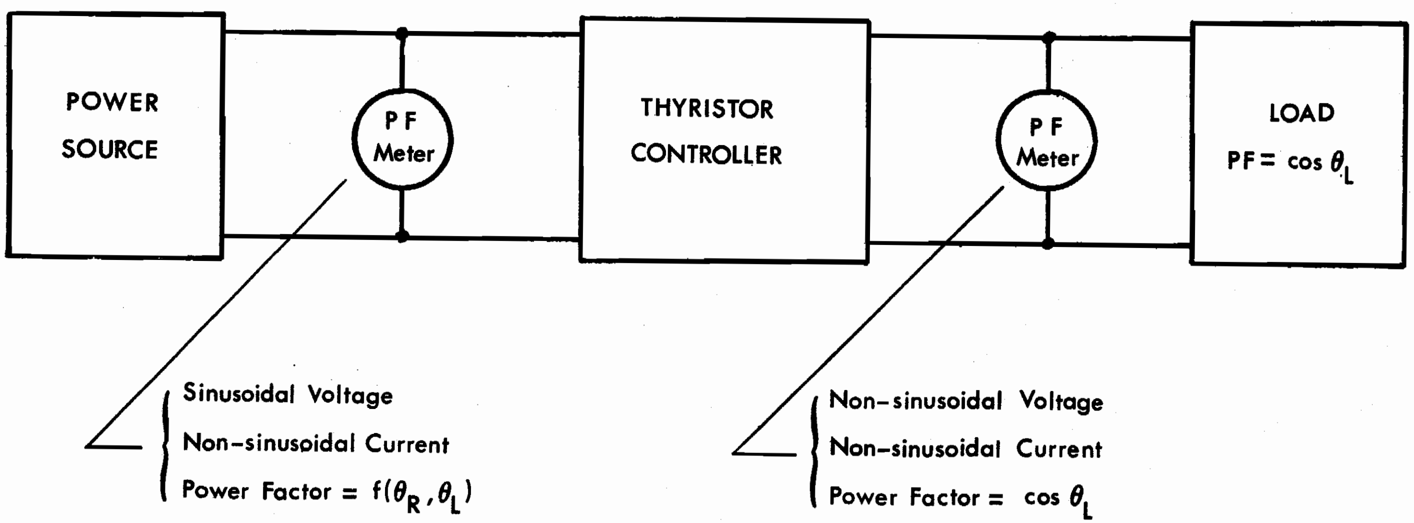


FIGURE 5. Diagrammatical summary of power factor considerations.

$$\text{Power Factor} = \frac{\frac{1}{2} V_S I_1 \cos \gamma_1}{\frac{V_S}{2}\left[I_1^2 + I_3^2 + I_5^2 + \ldots I_n^2\right]^{\frac{1}{2}}}$$

After cancelling terms, this reduces to:

$$\text{Power Factor} = \frac{I_1 \cos \gamma_1}{I_{rms}}$$

The per unit fundamental, third harmonic and fifth harmonic currents to a resistive load supplied by a phase controller are tabulated in Table I as a function of retard angle along with the term ($\cos \gamma_1$) and the power factor computed from the ratio of real power to effective volt-amperes. The input power factor, applying either of the two definitions, is reduced as retard angle increases.

The voltage developed across a resistor supplied from a thyristor phase-controller can be expressed by the series:

$$V_L(\theta) = \sum_{n=1}^{\infty} V_n \sin n\theta$$

TABLE I

θ_R (deg)	I_1 (PU)	I_3 (PU)	I_5 (PU)	$\cos \gamma_1$	Power Factor
0	1.000	0	0	1.000	1.000
20	0.9919	0.0372	0.0352	0.9993	0.9956
40	0.9437	0.1315	0.1046	0.9902	0.9667
60	0.8392	0.2387	0.1378	0.9587	0.8969
80	0.6837	0.3087	0.1146	0.8922	0.7810
90	0.5927	0.3183	0.1061	0.8436	0.7071
100	0.4974	0.3087	0.1146	0.7841	0.6245
120	0.3086	0.2387	0.1378	0.6336	0.4422
140	0.1469	0.1315	0.1046	0.4457	0.2559
160	0.0383	0.0372	0.0352	0.2303	0.0939
180	0	0	0	--	--

NOTE: The subscript refers to the harmonic order, and currents are expressed as the ratio of current magnitude to that of the fundamental with zero retard angle.

And since the load is resistive, the load current can be expressed by a similar series:

$$i_L(\theta) = \sum_{n=1}^{\infty} I_n \sin n\theta$$

The load power is then:

$$P = \frac{1}{2\pi} \int_0^{2\pi} \left(\sum_1^{\infty} V_n \sin n\theta \right) \left(\sum_1^{\infty} I_n \sin n\theta \right) d\theta$$

Since the integral of the cross products is zero, this reduces to:

$$P = \sum_{n=1}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} V_n I_n \sin^2 n\theta d\theta$$

$$P = \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n$$

The effective load voltage will be $(V_n/\sqrt{2})$. Since the load is linear, superposition can be applied to find the effective volt-amperes by summing the volt-amperes of each harmonic component individually:

$$VA = \sum_{n=1}^{\infty} \frac{V_n}{\sqrt{2}} \frac{I_n}{\sqrt{2}}$$

$$VA = \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n$$

The effective volt-ampere is equal to the load power which indicates that the power factor looking into a resistive load from a phase controller is unity.

When the load power factor is less than unity, the input power factor will be less than that indicated by Table I. The power factor measured between the controller and the load will be equal to the load power factor.[16]

The power factor measured at the input terminals to a phase controller will be equal to the load power factor when the retard angle is zero, and will decrease with increasing retard angle. In contrast to this variation, the power factor measured between a phase controller and the load will always be equal to the load power factor.

IV THYRISTOR CONTROL OF THREE-PHASE SYSTEMS

Harmonics In Three-Phase Systems

As a prelude to the transformation of the foregoing principles from single-phase to three-phase, a consideration of the additional information required to describe the harmonic voltages is important. Assuming a balanced three-phase system with a fundamental angular velocity (ω), the three line-to-neutral supply voltages can be defined as:

$$V_1 = V_s \sin \theta$$

$$V_2 = V_s \sin(\theta - \frac{2\pi}{3})$$

$$V_3 = V_s \sin(\theta - \frac{4\pi}{3})$$

Where: $\theta = \omega t$

which defines a 1-2-3 line voltage phase sequence. The nth harmonic of voltages would be:

$$V_{1,n} = V_s \sin n\theta$$

$$V_{2,n} = V_s \sin(n\theta - \frac{2\pi n}{3})$$

$$V_{3,n} = V_s \sin(n\theta - \frac{4\pi n}{3})$$

The most significant thing to note here is the consequence of substituting any multiple of three for the harmonic order (n). In this case the phase of all three voltages becomes identical leading one to conclude that any harmonic order which is divisible by three cannot exist as a line-to-line voltage in a balanced three-phase system and no current can result from these harmonics in the system if the load neutral is ungrounded. If the form of control does not exhibit the same effect upon each of the three phases, the harmonic voltages will not represent a balanced three-phase system and the harmonic orders divisible by three will circulate currents.

A second point of interest in these equations is the change of phase sequence which occurs with changing harmonic order. By allowing (n) to assume the value two, the expression for the second harmonic voltages is obtained.

$$V_{1,2} = V_s \sin 2\theta$$

$$V_{2,2} = V_s \sin(2\theta - \frac{4\pi}{3})$$

$$V_{3,2} = V_s \sin(2\theta - \frac{2\pi}{3})$$

Hence the second harmonic exhibits a phase sequence which is opposite to that of the fundamental. The phase sequence for the nth harmonic can be defined by the three expressions:

1-2-3 Sequence	$n = 1 + 3k$	$k = 0,1,2,3,$etc.
3-2-1 Sequence	$n = 2 + 3k$	$k = 0,1,2,3,$etc.
zero Sequence	$n = 3k$	$k = 1,2,3,$etc.

Three-Phase Circuit Requirements

In general, the principles involved in three-phase thyristor control are just extensions of those discussed in the preceding section for single phase circuits. When four wire systems are employed where the load is always connected in wye with a grounded neutral, each of the three phases can be treated on an individual basis and the analysis is no different from that of the single-phase circuit. However, four wire systems are becoming a rarity because little advantage is gained by the additional expense. Many power circuit arrangements are possible when the thyristors are located within the load legs, but these are not considered here since these arrangements invariably require additional wiring between the control panel and load. Therefore, the circuits considered will be restricted to those which insert some combination of semiconductor devices between the supply and load of any three-wire system operating into a balanced three-phase load with an ungrounded neutral. With the additional imposition that load voltage of each phase must be controllable between the limits of zero and the line voltage, only three different arrangements of the primary power elements are possible. The following three sections describe each of these circuits individually.

Symmetrical Anti-Parallel Thyristor Control

The power circuit configuration which produces the most symmetrical system requires the use of a control element, consisting of two thyristors connected in an antiparallel arrangement, in series with each of the three load phases. It might be remembered that the phase current of a balanced three-phase system supplying a balanced three-phase resistive load is in phase with the line-to-neutral voltage of the phase. Since the thyristor is fundamentally a current device, which in this circuit must be switching phase current, the proper synchronizing signals from which the retard angle (θ_R) should be referenced are the zero points of the line-to-neutral voltage of the phase in which the thyristors are located.

From the thyristor arrangement in Figure 6, it can be seen that no current can flow unless thyristors in two phases are conducting simultaneously. The three line-to-neutral voltages are shown in Figure 6 along with the gate signals for a retard angle of 150 degrees. For this retard angle, no current flows since no two thyristors are gated simultaneously. When the retard angle has been reduced to 120 degrees, an overlap of gate signals occurs, and each of the six thyristors will conduct during 60 degrees of the full 360 degree cycle. Hence, no value of load voltage between zero and that which corresponds to a retard angle of 120 degrees is obtainable.

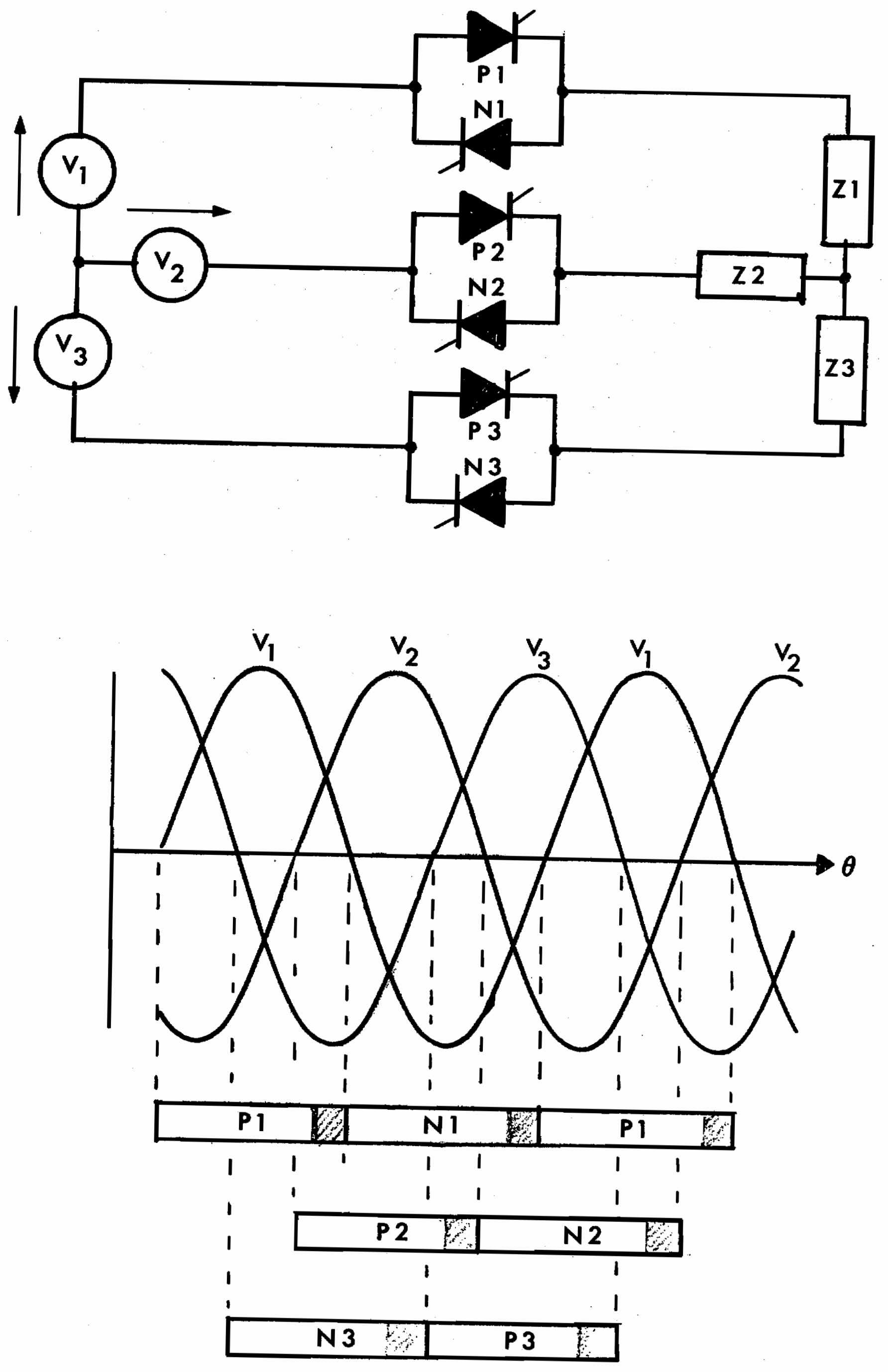


FIGURE 6. Power circuit arrangement for the symmetrical thyristor control circuit. The intervals during which a given thyristor can conduct are indicated with the shaded portion representing the interval during which gate voltage is applied to the thyristor for $\Theta_R = 150^o$.

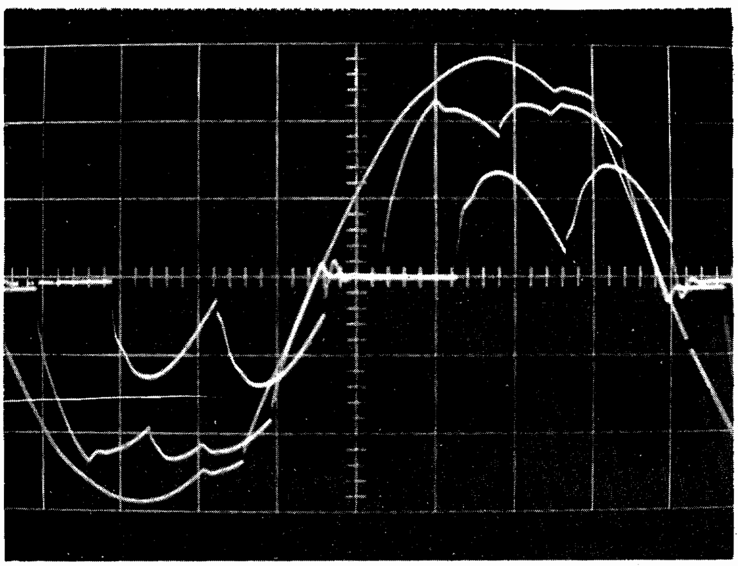


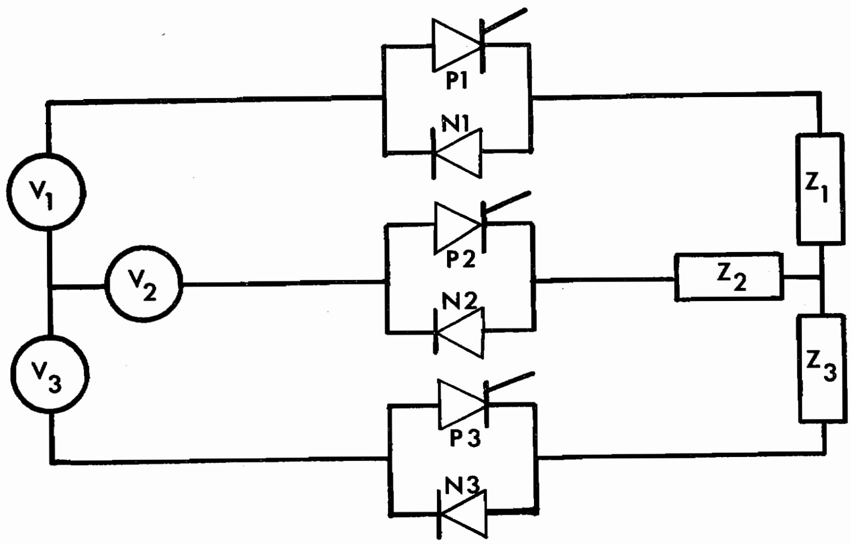
FIGURE 7.

Oscillogram of the phase current supplied to an RL load by a symmetrical thyristor circuit triggered at three different retard angles.

Figure 7 is an oscillogram of the phase current through a resistance-inductance load with three different retard angles. The half-wave symmetry of these currents coupled with the symmetrical gating of all three phases leads to a current waveform devoid of all harmonics lower than the fifth. The relative magnitudes of the fundamental, fifth and seventh harmonic voltages across one leg of a wye-connected resistance load are plotted in Figure 2A of the appendix with retard angle (θ_R) as the independent variable. Since the lowest harmonic contained in the output of this control is the fifth, the response of loads exhibiting power factors less than 0.9 (which provides 2:1 suppression of the fifth harmonic) to this control will be little different from that obtainable using a variable amplitude type of sine wave control.

Thyristor-Inverse Diode Control

The second arrangement of power elements to consider is the thyristor-inverse diode control which results from the substitution of a diode for one thyristor in each of the three control elements in the symmetrical anti-parallel thyristor control. The diodes must be arranged such that each is forward biased by the same polarity of the respective line-to-neutral voltage half-cycles. Since each thyristor-diode pair must conduct a phase current, trigger synchronization from the line-to-neutral voltages provides the necessary control span for



Note:. N1, N2, and N3 are diodes

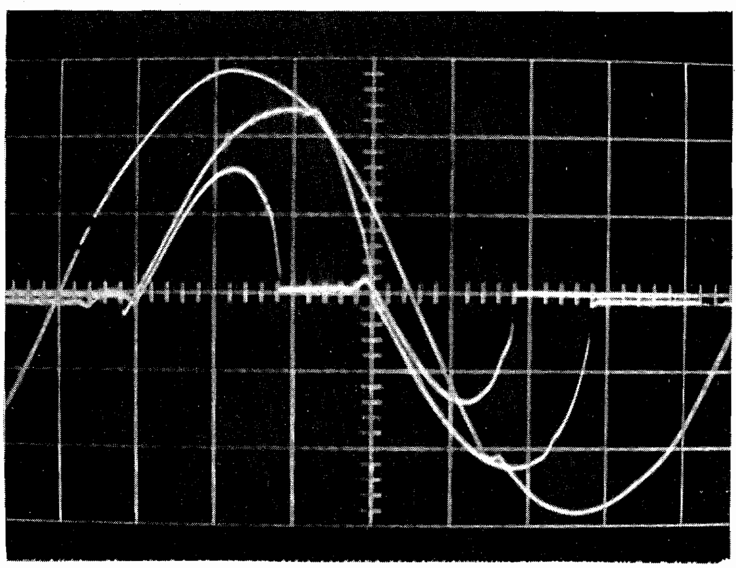


FIGURE 8. Power circuit arrangement of the thyristor-inverse diode controller with an oscillogram of the phase current to a lagging power factor load. Note the lack of half-wave symmetry: $f(\theta) \neq f(\theta + \pi)$.

referencing the retard angle. The power circuit diagram is included in Figure 8 along with the phase current waveforms for three values of retard angle supplied to a load with a power factor of 0.5.

A primary advantage of this control is the cost reduction available since the value of the diode is about one-fourth that of a thyristor with comparable current and voltage ratings. The three-phase currents are exactly alike with a 120 degree displacement maintaining a balanced system, but the phase currents do not exhibit half-wave symmetry indicating the presence of even as well as odd harmonics. Curves of the fundamental, second, and fourth harmonics as a function of retard angle are presented in Figures 5A and 6A of the appendix. This form of control does not exhibit the abrupt discontinuity of output voltage at a retard angle of 120 degrees characterized by the six-thyristor control, but rather, provides a smooth transition of voltage from full supply to zero.

Assymetrical Thyristor Control

The third power circuit arrangement to be considered is assymetrical thyristor control which is comprised of two antiparallel thyristor pairs connected in series with two of the load phases. The power circuit of this control, shown in Figure 9, is equivalent to placing a short circuit around one of the power control elements in the

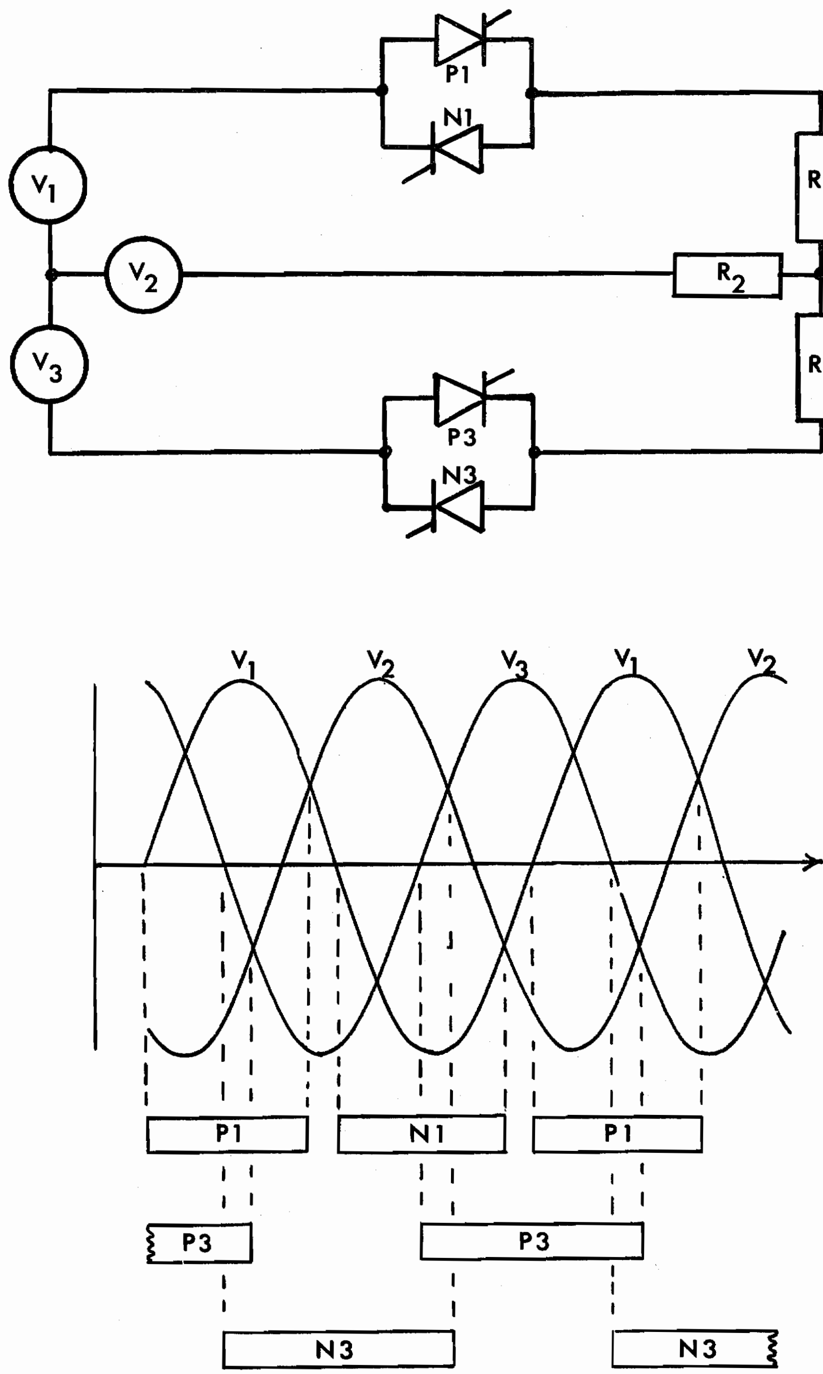


FIGURE 9. Power circuit arrangement of asymmetrical thyristor control with indicated intervals of potential conduction. The control range for N1-P1 must span 150° while that of P3-N3 spans 210°. No cell will actually conduct current for more than 180°.

symmetrical thyristor control; however, the necessary signal from which the retard angle is referenced requires some alteration if a circulating single-phase current is to be avoided with large retard angles. The intervals during which each of the four thyristors can conduct current is indicated in Figure 9 . If the gating signals are synchronized to the line-to-neutral voltages, considerable unbalance will result in the output voltage. With zero retard angle, full voltage will be available to the load. Increasing the retard angle will reduce the output of thyristors P1 and N1 at a greater rate than that of thyristors P3 and N3. When the retard angle reaches 150 degrees, thyristors P1 and N1 cease conduction while thyristors P3 and N3 are still conducting 60 degrees of line-to-line voltage (V_{32}), resulting in a single phase current. An additional increase of 30 degrees in the retard angle causes thyristors P3 and N3 to cease conduction when 30 degrees of this voltage is still available. Hence, a relatively large single phase current will circulate as the retard angle approaches 180 degrees.

In order to overcome this discontinuity and supply smooth voltage control to a load of unity power factor, a complicated scheme of retard angle control and synchronization is required. When the load power factor is lagging, reasonable balance can be achieved by phase shifting the synchronizing signal. In order for thyristors P3 and N3 to provide smooth control for large retard angles, the synchronizing signal requires a lag of 30 degrees, and with a load power factor angle of 30 degrees, full output will be obtained from this phase with a zero retard angle when synchronized to the lagging reference signal. Suppression of the relative magnitude of the single phase current requires that the synchronizing signal for thyristors P1 and N1 be advanced 30 degrees in phase. The maximum unbalance will now occur when the retard angles are small, or the output voltage large. Hence, the maximum percent unbalance is considerably reduced.

In addition to the direct effects which the unbalanced voltage characteristics of this type of control may have upon the load, the output-voltage harmonic structure is affected. Cancellation of the harmonic orders divisible by three was deduced with a balanced voltage assumption (see Page 27). Negation of this constraint results in the presence of all the odd harmonics, but the output voltage exhibits half-wave symmetry, and no even harmonics exist.

The relative magnitudes of the fundamental, third harmonics, and fifth harmonic currents in each of the three load phases supplied from an assymmetrical thyristor control are plotted in Figures 3A and 4A of the appendix. These values were measured with a commercial wave analyzer with the control synchronizing signals phase-shifted in the manner described. In addition to the unbalance in current magnitude indicated by these curves, the phase displacement of the three third harmonic voltages is not 120 degrees, but rather varies with retard angle. The method of symmetrical components applied on a point to point basis would reduce these to balanced systems with zero sequence components, but a more direct approach for induction motor loads is to measure the motor performance with varying retard angles.

V THYRISTOR CONTROL OF INDUCTION MOTOR TORQUE

The Induction Motor Principle

The essential feature which sets induction motors apart from other forms of electric motors is that the electrical energy of the rotor is provided solely by induction, as in a transformer, instead of being supplied by an external power source as in synchronous or dc machines.[2] The three primary or stator windings of a three-phase induction motor are wound to provide a nearly sinusoidal winding distribution with 120 electrical degrees of displacement between the magnetic axes of the coils. The excitation of the stator windings from a three-phase electrical source produces a revolving field of air gap flux rotating with a synchronous speed (N_S) of:

$$N_S = \frac{60f}{P} \quad \text{(rpm)}$$

where f = line frequency (Hz)

P = number of pole pairs

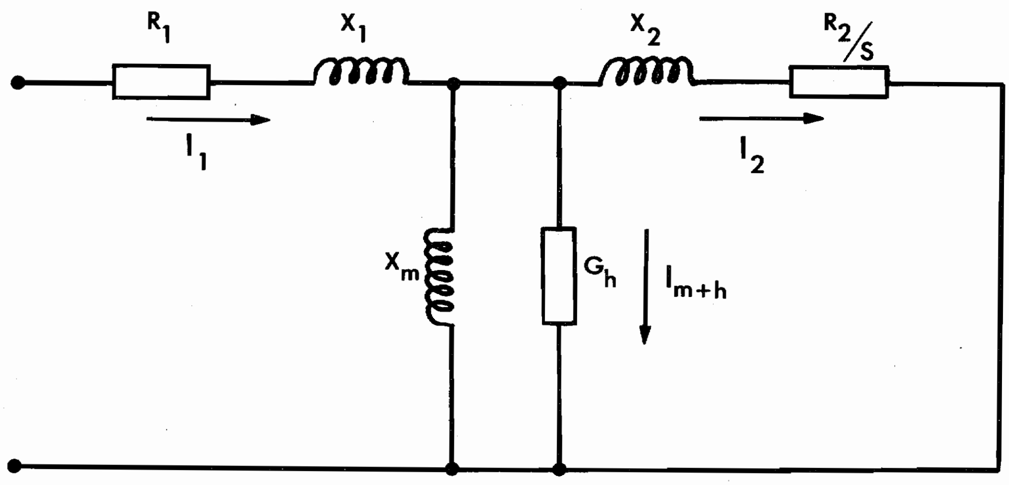
If the rotor speed (N_R) is different from the synchronous speed (N_S), a voltage will be induced into the rotor conductors circulating a current through the rotor. The interaction between rotor current and air-gap flux produces a tangential force developing torque. Since the rotor speed must be different from the rotating field speed to induce a rotor current, the induction motor can only develop torque when operating asynchronously. The difference in speed between the rotor and the stator field is usually expressed in terms of slip (S) which is defined by the relation:

$$S = \frac{N_S - N_R}{N_S} \quad \text{(per unit)}$$

Equivalent Circuit Constants

The induction motor equivalent circuit offers a convenient method for analyzing induction motor performance. The exact equivalent circuit, developed by Steinmetz, is shown in Figure 10 along with the conventional symbol definitions[2]. It is the same as a transformer equivalent circuit except for a speed variable load resistance (R_2/S). The equivalent circuit constants are usually based upon an assumed wye-connected stator which yields correct results when the line-to-neutral values of a balanced three-phase sine wave supply are used. As a result of the relatively large air-gap, the magnetizing inductance is smaller than that of a transformer resulting in a magnetizing component of stator current which is typically 40-50 percent of the rated current.

An approximate equivalent circuit which does not introduce large errors when the motor is operated at less than rated speed can be developed by moving the magnetizing inductance outside of the stator impedance and



R_1 = Stator Winding Resistance

X_1 = Stator Leakage Reactance

X_m = Magnetizing Reactance

G_h = Iron Loss Conductance

X_2 = Rotor Leakage Reactance*

R_2 = Rotor Resistance*

* Referred to stator

FIGURE 10. Definition of the induction motor equivalent circuit constants.

neglecting the iron loss. Under these conditions, the primary current (I_1) and rotor current (I_2) are equal, and the input current is the vector sum of the primary current (I_1) and magnetizing current (I_m). The primary current (I_1) of the approximate equivalent circuit becomes:

$$|I_1| = |I_2| = \frac{V}{\left[(R_1 + \frac{R_2}{S})^2 + (X_1 + X_2)^2\right]^{\frac{1}{2}}} \quad \text{(per unit current)}$$

The developed torque with all parameters expressed as per unit values can be expressed as:

$$T = \frac{I_1^2 R_2}{S} \quad \text{(per unit torque)}$$

Substituting for (I_1) in the torque equation gives:

$$T = \frac{V^2 \frac{R_2}{S}}{(R_1 + \frac{R_2}{S})^2 + (X_1 + X_2)^2} \quad \text{(per unit torque)}$$

It can be seen in the torque equation that the developed torque varies with the square of stator voltage. The power input to the rotor, which is approximately equal to the mechanical output power at rated speed when the windage and friction losses are neglected, would be:

$$P_r = \frac{I_2^2 R_2}{S} \quad \text{(per unit watts)}$$

The mechanical power output for a given torque varies linearly with speed:

$$P_o = \frac{I_2^{\,2} R_2}{S(1 - S_{rated})} (1 - S)$$

The mechanical output power is zero at stall (S = 1) and becomes negative when the motor is overhauled (S > 1). The difference between input power and output power must appear as heat in the rotor resistance, and in the case of a squirrel cage motor, this heat is dissipated within the motor frame. The problem is considerably alleviated in wound rotor motors by locating the resistance (R_2) outside the motor. This does not completely eliminate the heating problem. From the first torque equation (Page 43), it can be seen that the torque developed for a given (I_1) current magnitude decreases as slip increases. Operation at rated torque with speeds less than the rated speed requires more than one per unit stator and rotor current. This dissipates heat in both the stator and rotor winding resistances which may eventually result in an overheated motor unless some means is provided for removing this heat.

Restrictions Imposed By Motor Heating

The heat generated within the frame of a wound rotor motor varies directly with the square of stator current. For continuous operation of a wound-rotor motor at low speeds or in an overhauling mode without exceeding the rated rise of the machine, the stator current must not exceed one per unit. (In actual practice, the current would need to be kept somewhat below this magnitude since the motor dissipation factor decreases with speed.)

Solving the torque equation for current yields:

$$|I_1| = \left[\frac{ST}{R_2}\right]^{\frac{1}{2}} \qquad \text{(per unit amps)}$$

If the motor is to be operated at reduced speed without exceeding rated current, then the maximum torque for which the drive can be rated is:

$$T' = \frac{R_2}{S_{max}} \qquad \text{(per unit torque)}$$

where the prime indicates a rerated value. It can be seen that when the motor is rerated in this fashion for continuous operation in the region $(S \leqslant 1)$, the rerated torque where (S_{max}) is unity, will be equal to the per unit rotor resistance (R_2). The rerated speed (N'), which is the maximum speed obtainable at the new torque rating (T'), is plotted in Figure 11, along with the per unit power (P') obtainable from the motor as a function of torque rating (T') assuming a maximum slip of unity. This represents a considerable sacrifice in power rating since the maximum power rating must be less than 38 percent of basic motor rating. The power derating is generally reduced by one of three considerations:

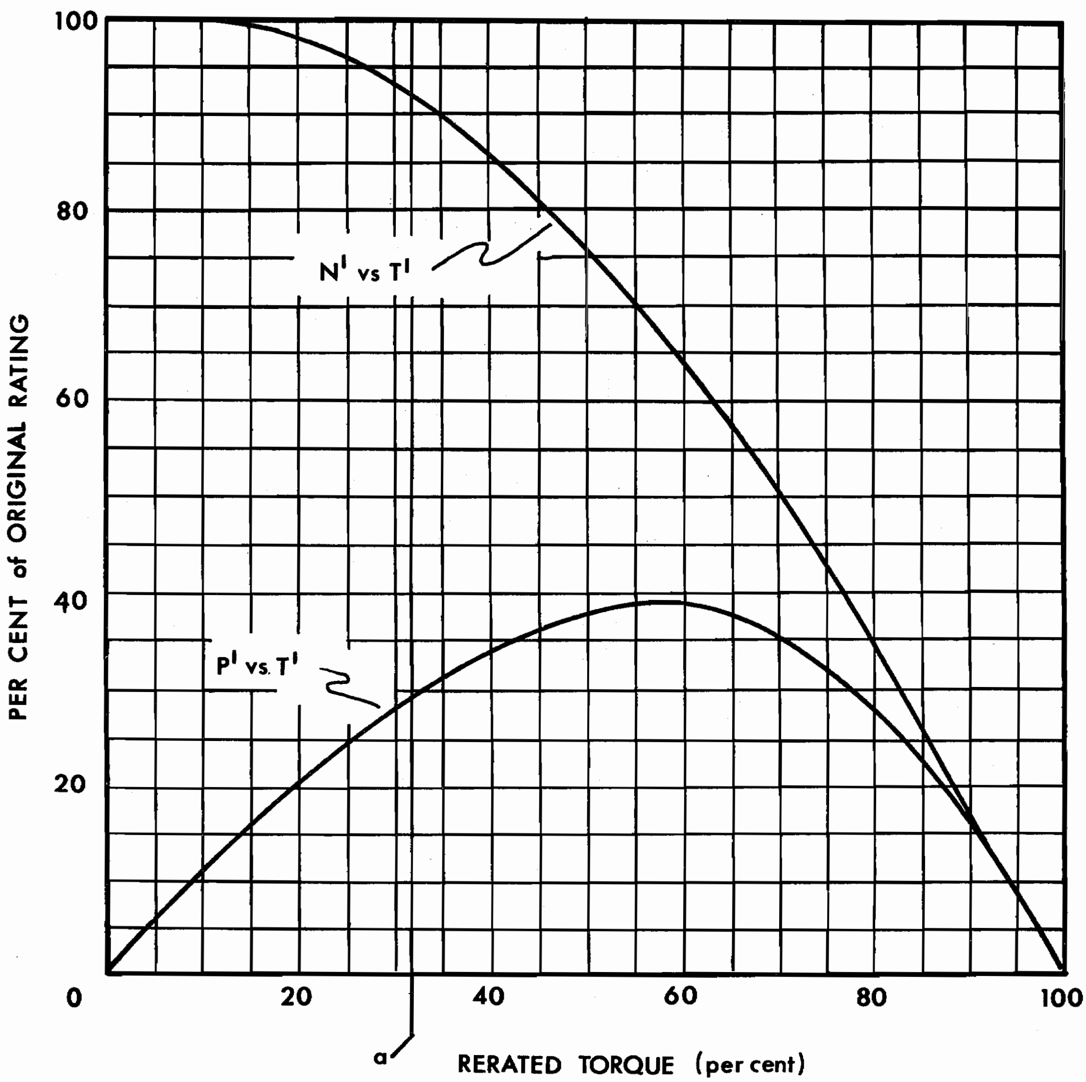


FIGURE 11. Rerating curves for continuous operation of an induction motor at a maximum slip of 1.0. If a rotor resistance of 0.32 PU is chosen, the drive would be continuously rated at 32 percent torque (a), 92 percent speed and 29 percent of the nameplate horsepower.

1. A large number of drives are supplied for fan and pump applications where the required motor torque varies with the square of speed. Very little rerating is required for these drives.

2. A second class of drives applied to cranes and traffic bridges only require reduced speed operation intermittently. The long thermal time constant of the motor provides a low pass characteristic which suppresses the rate of temperature rise and enables the designer to consider the average of dissipated power.

3. When sustained low speed operation at high torque levels is necessary, the motor can be supplied with a constant speed blower which considerably increases the dissipation factor. With this type of ventilation, rated torque can be developed at stall without overheating the motor.

Induction Motor Operation With Non-Sinusoidal Voltage

There are two sources of harmonics which appear in the operation of induction motors, (1) time harmonics which result from non-sinusoidal stator voltages and (2) space harmonics which result from both a non-sinusoidal winding distribution and the slotted magnetic construction. The control of space harmonics is important to motor design and are minimized by careful design. The time harmonics result from voltage harmonics applied to the stator. Standard motors are designed to perform within the nameplate rating when the power source exhibits a maximum harmonic distortion of 10 percent. Since both the magnitude of stator voltage distortion and motor speed variation are restricted in the majority of induction motor applications, the discussions of the effects of harmonics upon motor performance which appear in machine text books may not apply when the magnitudes of the stator voltage harmonics are large compared with the fundamental and the slip is varied over wide limits. The two most comprehensive treatments of this subject which could be found[4,11] concluded that superposition yields a valid approach to evaluating the effect of stator voltage harmonics.

When the impressed stator voltage (V_1) of an induction motor is non-sinusoidal, the voltage may be expressed as:

$$V_1 = V_{11} \sin(\theta + \gamma_1) + V_{12} \sin(2\theta + \gamma_2) + V_{13} \sin(3\theta + \gamma_3) + \ldots$$

With the assumed validity of superposition, a motor equivalent circuit for each component voltage can be established and the net torque found from the summation of the torques developed in each of the component equivalent circuits. The approximate equivalent circuit for the nth harmonic component is defined in Figure 12. The reactive components are equal to the product of the harmonic order (n) and the values determined at the fundamental frequency. The winding resistances are increased somewhat in the harmonic equivalent circuits as a result of skin effect. The two most significant changes in the equivalent circuit parameters are the magnitude of the slip (S_n) and the sign of the developed torque (T_n).

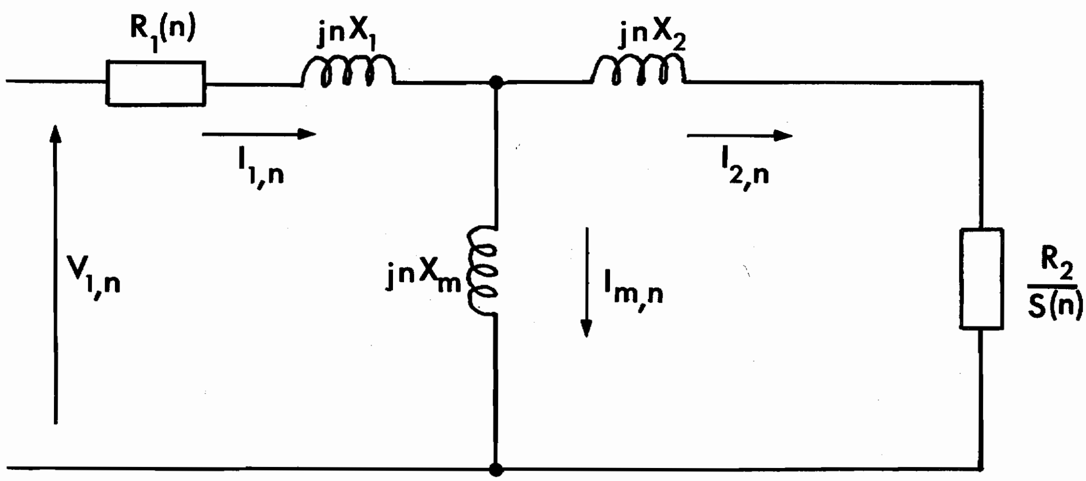
Synchronous speed for the harmonic motor would be:

$$N_S(n) = \frac{60nf}{P} \quad \text{(RPM)}$$

Where: f = fundamental frequency

P = number of pole pairs

The direction of the rotating magnetic field depends upon the phase sequence of the harmonic, and this sequence is uniquely defined by the harmonic order (n) as discussed previously (see page 28). In the case of the second harmonic, which exhibits a sequence opposite to that of the fundamental, the synchronous speed of the harmonic motor occurs when the



$$R_1(n) \approx R_1\left(1 + 0.18n\right)$$

$S(n) \longrightarrow$ defined in Table 2.

FIGURE 12. Equivalent circuit of Nth harmonic motor.

TABLE 2

n	S_n	Sequence	Torque Polarity
1	S	1-2-3	+
2	$\frac{1}{2}(3 - S)$	3-2-1	-
3	$\frac{1}{3}(S + 2) \quad S < 1$ $\frac{1}{3}(4 - S) \quad S > 1$	ZERO	+ -
4	$\frac{1}{4}(3 + S)$	1-2-3	+
5	$\frac{1}{5}(6 - S)$	3-2-1	-
6	$\frac{1}{6}(S + 5) \quad S < 1$ $\frac{1}{6}(7 - S) \quad S > 1$	ZERO	+ -
7	$\frac{1}{7}(6 + S)$	1-2-3	+
8	$\frac{1}{8}(9 - S)$	3-2-1	-

motor is running at twice rated speed in the reverse direction. The slip of the second harmonic motor (S_2) can be related to the fundamental slip (S) by the expression:

$$S_2 = \frac{1}{2}(3 - S)$$

Through the normal operating range ($0 < S < 2$), the slip of the second harmonic motor is restricted to the range ($0.5 < S_2 < 1.5$). Since synchronous speed for the higher harmonics is greater than that of the second, the minimum slip for these will be even less than that of the second harmonic. It can be seen then that the harmonic motor will always be operated at a relatively high slip. The slip (S_n) for each of the harmonic motors is expressed in terms of the fundamental slip (S) in Table 2. Special treatment is required for harmonics divisible by three since these harmonics exhibit a zero sequence. Like the single phase motor, the synchronous speed of these motors can occur for either direction of rotation.

The net torque developed by the motor is the sum of the component torques of all the harmonic motors. The torques of motors with the same sequence as the fundamental are positive while those with opposite sequences negative. The zero sequence motors must again be given special treatment since the torques of these motors will add to the fundamental torque through the slip range ($0 < S < 1$) and subtract from the fundamental torque through the range $(1 < S < 2)$. Since these torques result from zero sequence, no torque will be developed at zero speed. At the risk of confusion, it might be noted that the zero sequence torques can be braking torques for some values of secondary resistance which means that the torque polarities for the zero sequence harmonics will be opposite to those just discussed.

Based upon superposition, several conclusions can be drawn. Each of the harmonic components will develop a torque which may add or subtract from the fundamental torque. Since the negative sequence harmonics reduce the net torque, the magnitude of the fundamental voltage must be increased to produce a desired level of torque. The positive sequence harmonics develop a torque which add to the fundamental torque, but the relative slip for these components is large so that the torque per ampere relationship for these components is poor. When a distorted voltage is applied to the stator, the magnitude of effective current required to produce a given torque is greater than that which would be required from a sine wave voltage. Motor heating is proportional to the square of the effective current, therefore harmonics in the stator voltage increase the motor heating. Since the input impedance to an induction motor contains a reactive component, the effect of stator harmonics decrease with increasing harmonic order.

Motor Performance With Unbalanced Stator Voltages

The method of symmetrical components provides a scheme for reducing a system of three voltages of different magnitudes into a system of balanced voltages one of which has a positive sequence, one of a negative sequence and a third with a zero sequence. It has been shown that the zero-sequence term does not develop any torque in a motor with a floating neutral[2]. The net developed torque is then the sum of the torques developed by the positive and negative sequence terms. The net effect of unbalanced stator voltages is to require a larger current to develop a given torque with increased motor heating.

Characteristics Of The Test Motor

The motor used for all tests was a standard wound-rotor design with a wye-connected stator and a delta-connected rotor. The motor data provided by the manufacturer was:

HP - 3	FRAME - 254 U	No PHASES - 3
RPM - 1725	FREQUENCY - 60 Hz	VOLTAGE - 440/220

The equivalent circuit constants, also provided by the manufacturer, were:

$R_1 = 0.399$ ohms

$R_2 = 0.527$ ohms*

$X_1 = 0.65$ ohms

$X_2 = 0.65$ ohms*

$V_\phi = 127$ volts

$I_\phi = 9.0$ amperes

$X_m = 21.1$ ohms

*Referred to primary

These parameters can be converted to per unit values by the two expressions:

$$1 \text{ per unit ohms} = \frac{V_\phi}{I_\phi} = 14.1 \text{ ohms (primary side)}$$

$$1 \text{ per unit ohms} = \frac{249(HP)}{(I_2^{\,2})} = 9.22 \text{ ohms (secondary side)}$$

A wye-connected rotor resistance of 1.9 ohms/phase (0.206 PU) was used. The equivalent circuit constants expressed in per unit values then become:

$R_1 = 0.0283$ P.U.

$R_2 = 0.2464$ P.U.

$X_1 = 0.0464$ P.U.

$X_2 = 0.0464$ P.U.

$X_m = 1.497$ P.U.

One per unit torque is the torque required to develop rated horsepower at rated speed. One per unit torque is then:

$$1 \text{ P.U.} = \frac{\text{HP X 5250}}{\text{RATED SPEED}} = 9.13 \text{ lb-ft.}$$

A computer program (written in BASIC) which will provide the per unit torque-speed curves for any value of per unit stator voltage when the equivalent circuit constants are expressed in per unit values is included in the appendix (see page 92). This program will also provide the stator current and power factor as a function of slip. The torque-speed curves of the test motor with the specified rotor resistance are plotted in Figure 7A for the indicated values of stator voltage. These curves checked within ten percent the values measured on a dynamometer. The temperature variation of the equivalent circuit constants can result in this much variation. It can be seen in the curves of Figure 7A that the torque at a given speed varies with the square of stator voltage. If this drive were rated for continuous operation in the first or third motor quadrant ($S_{max} = 1$), the rerating curves of Figure 11 apply. The rerated torque is the ratio of per unit secondary resistance (0.246) to the maximum continuous slip (1.0). This drive would then be rated as 0.246 per unit torque, 0.96 per unit speed and 0.75 horsepower maximum for a constant torque load. From Figure 7A, it can be seen that the current required to develop this torque (0.25 P.U.) at stall (S = 1.0) is slightly greater than unity (1.05). This is the error introduced by neglecting the magnetizing current in developing the rerating curves of Figure 11.

Symmetrical Thyristor Motor Control

When the induction motor stator voltage is controlled through a thyristor phase control, the torque developed at a given speed is a function of the retard angle (θ_R). The adaptability of a given control scheme to a specific application depends to a great extent upon the shape of the torque-speed curves for fixed retard angles. The torque-speed curves of the test motor for fixed retard angles of the symmetrical thyristor control are plotted in Figure 8A. The shapes of these curves do not differ significantly from the sine wave voltage curves of Figure 7A. It might be concluded then that the dynamic performance of the motor obtainable with symmetrical thyristor control is equivalent to that obtainable with adjustable sine wave voltage control in the slip region ($0 < S < 2$).

Asymmetrical Thyristor Motor Control

The empirical torque-speed curves of the test motor obtained with an asymmetrical thyristor motor control are plotted in Figure 9A. The curves are continuous with no abrupt discontinuities, but when the retard angle is large ($\theta_R \approx 90^\circ$), a noticeable reduction in torque occurs as the slip is increased beyond unity. The torque then begins to increase once again when the slip exceeds 1.8. The maximum gain of the combination controller and motor ($\Delta T/\Delta\theta_R$) will occur at ($S = 1.8$)

with retard angle (θ_R) in the region of 90 degrees. This form of control can be used in speed-regulated drives in all four quadrants of the torque-speed plane.

The stator voltages which result from this form of control are unbalanced containing the harmonics divisible by three as well as the other odd harmonics, with the fundamental and third harmonic components dominating. The unbalanced fundamental components will produce the forward and backward field torque components shown qualitatively in Figure 13. The net fundamental torque which is the sum of these two torques shows a droop in torque in the region ($1 < S < 1.8$). The third harmonic field which is stationary in space will produce a positive torque for ($S < 1$) and a negative torque for ($S > 1$) as shown in Figure 14. When this component is added to the fundamental torque component, the first quadrant torque is increased and the torque droop in the fourth quadrant is compounded resulting in a torque-speed curve which has a characteristic shape similar to those obtained empirically.

Thyristor-Inverse Diode Motor Control

The most interesting of the speed-torque curves are those generated by the thyristor-inverse diode control and plotted in Figure 10A. The motor torque is smoothly controllable with retard angle throughout the

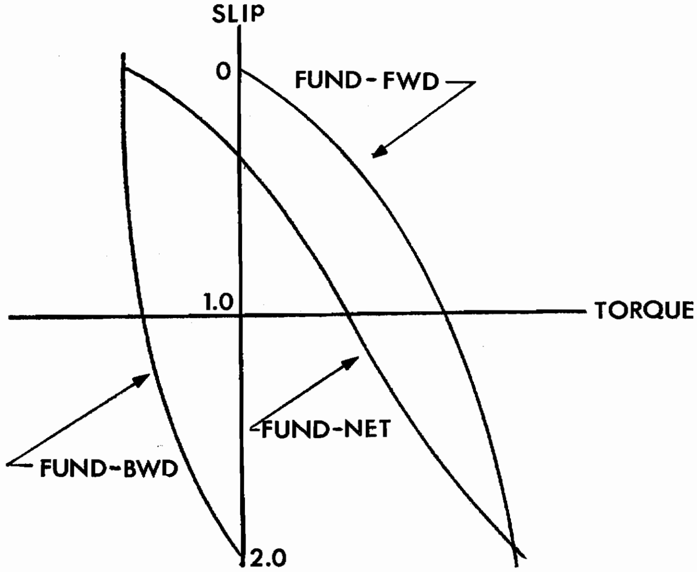


FIGURE 13. Addition of forward and backward fundamental field torques.

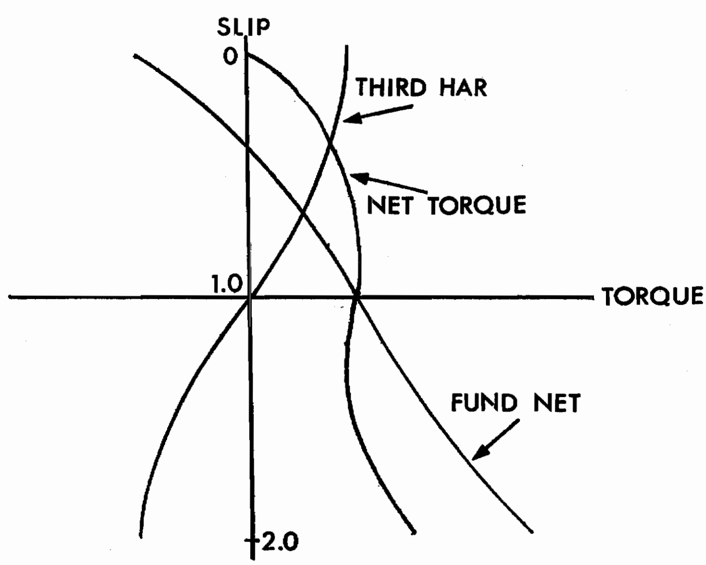


FIGURE 14. Torque resulting from the third harmonic is added to the net fundamental torque resulting in an ess-shaped characteristic.

speed range ($0 < S < 1.5$). When the retard angle approaches 100 degrees in the speed range ($1.5 < S < 2$) the torque begins to increase with increasing retard angle and has been found to exceed that obtainable with maximum voltage by several times. The torque maximum occurs with a retard angle of approximately 110 degrees and from Figure 6A, this is the retard angle which produces the maximum second harmonic voltage when the lag angle of the load is 40 degrees. The frequency of rotor current produced by the second harmonic is:

$$f_{R,2} = \frac{1}{2}(3 - S) \quad (120 \text{ Hz})$$

while that produced by the fundamental voltage is:

$$f_{R,1} = S \ (60 \text{ Hz})$$

At a slip of 1.5, the frequency of these two components are equal, and it was found that a retard angle which produced equal magnitudes of fundamental and second harmonic currents would result in no alternating rotor current when the slip was equal to 1.5. When the slip was increased beyond this value for the same retard angle, the torque began to increase rapidly, and a 30 Hz component appeared in the stator current. The 30 Hz current component can be seen from the symmetry which exists between the alternate cycles of phase currents and thyristor voltages in Figure 15.

A fundamental slip of 1.5 is synchronism for a 30 Hz field which has

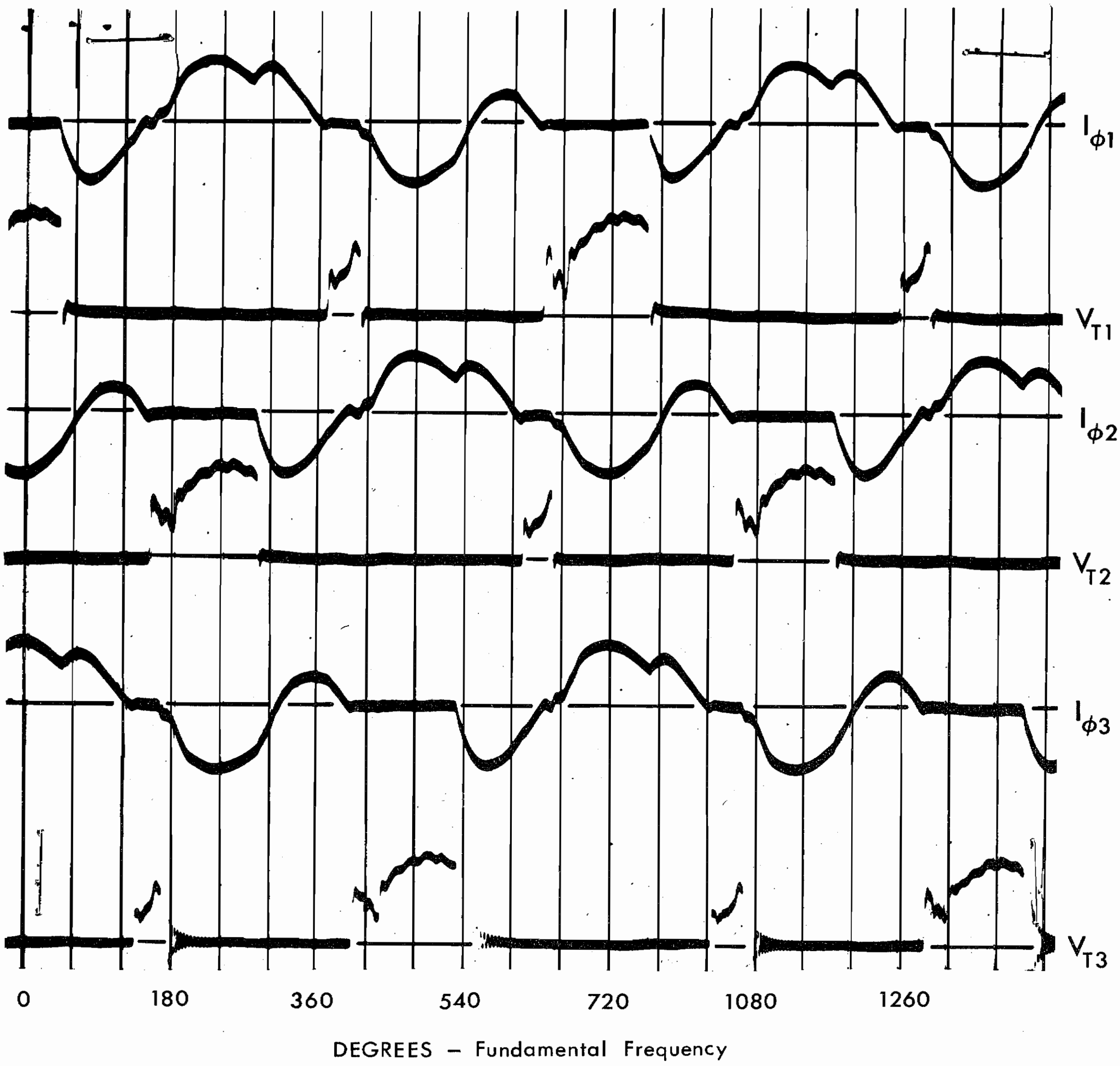


FIGURE 15. Phase currents and cell voltages of a thyristor-inverse diode thyristor controller supply an induction motor which is operating with a slip of 1.65. The symmetry which exists between alternate cycles of phase current indicate the presence of a 30 Hz component.

a phase rotation opposite to that of the fundamental. When the slip exceeds 1.5, this component produces a regenerative braking torque which would cause the net torque to increase sharply when the slip is increased beyond 1.5.

In the region where the 30 Hz component is large, the use of this circuit requires that a clamp be used to prevent the retard angle from exceeding 100 degrees when the motor slip is greater than 1.5. This circuit has been found to be acceptable for crane hoist controls where regulation of speed is only necessary in the region $(0.5 < S < 1.5)$. In applications where speed regulation over the entire torque-speed plane is essential, either the symmetrical or the asymmetrical thyristor control circuit is necessary.

Superposition And The Induction Motor

It was found that superposition yields acceptable qualitative results when the components of the stator currents are known, but it does not suggest the source of the subharmonic currents which appeared in the thyristor-inverse diode motor control. It was noted that these components began to appear with slips greater than that which produced an equivalence of fundamental and dominant harmonic slip frequencies. With the symmetrical thyristor control, the major harmonic is the fifth which produces a rotor frequency of:

$$f_{R,5} = \frac{1}{5}(6 - S) \quad (300\ \text{Hz})$$

This frequency is equal to the fundamental slip frequency (60 S) at a slip (S = 3). The drive was operated at a slip slightly greater than three with the symmetrical circuit and the presence of a 30 Hz component was noted with this circuit. This seems to suggest that two current components intermodulate which of course cannot occur in the linear network model required for superposition.

The principle of superposition was used to calculate the torque-speed curve of the thyristor-inverse diode control for a retard angle of 100 degrees. The harmonic component curves of Figures 5A and 6A were used in conjunction with the motor power factor curve of Figure 7A to obtain the magnitude of the stator voltage components as a function of slip. The motor equivalent circuit parameters were measured at 60 Hz, 120 Hz, and 240 Hz in order to develop an equivalent circuit for each of the three component motors. A computer was then used to calculate the net torque-speed curve, and the resultant curves were found to be in error by a factor of two.

Further investigation revealed that the measured harmonic components of the motor stator did not agree with those predicted by Figures 5A and 6A which were obtained using passive R-L loads. When the actual stator components were substituted, agreement within ten percent between calculated and measured torque-speed curves was obtained in the first quadrant.

The use of superposition to describe the performance of an induction motor operating with a non-sinusoidal waveform requires caution. If the performance is to be analytically evaluated, the model must be more descriptive than the conventional equivalent circuit.

Comparative Heating

Some consideration must be given to the additional motor heating which results from the non-sinusoidal voltage impressed upon the motor stator by the thyristor control. All of the motor field components other than the positive sequence fundamental field compound the heating problem, and an adequate design margin cannot be insured unless the heat generated by the other field components can be evaluated.

The heat developed within the motor frame is intimately related to the dissipated power which, on a per phase basis, is proportional to the square of effective phase current. The power dissipation with sine-wave control can be expressed as:

$$P_{ds} = 3K_h I_{sw}^2 \quad \text{(Watts)}$$

Where P_{ds} = power dissipation with sine wave voltages.

K_h = proportionality constant

I_{sw} = phase current with sine wave voltages.

The power dissipation with thyristor control of the stator voltage can be expressed as:

$$P_{dt} = K_h(I_{t1}^2 + I_{t2}^2 + I_{t3}^2) \quad \text{(Watts)}$$

Where P_{dt} = power dissipation with thyristor control

K_h = proportionality constant

I_{t1} = effective current of phase one, thyristor control.

A heating factor can now be defined by the relationship:

$$K_{hf} = \frac{P_{dt}}{P_{ds}}$$

The power dissipation which results from adjustable voltage control using sine-wave stator voltages can be calculated using the conventional equivalent circuit. However, the power dissipation with thyristor control (P_{dt}) is most readily obtainable from empirical data. The heating factor for each of the three control circuits

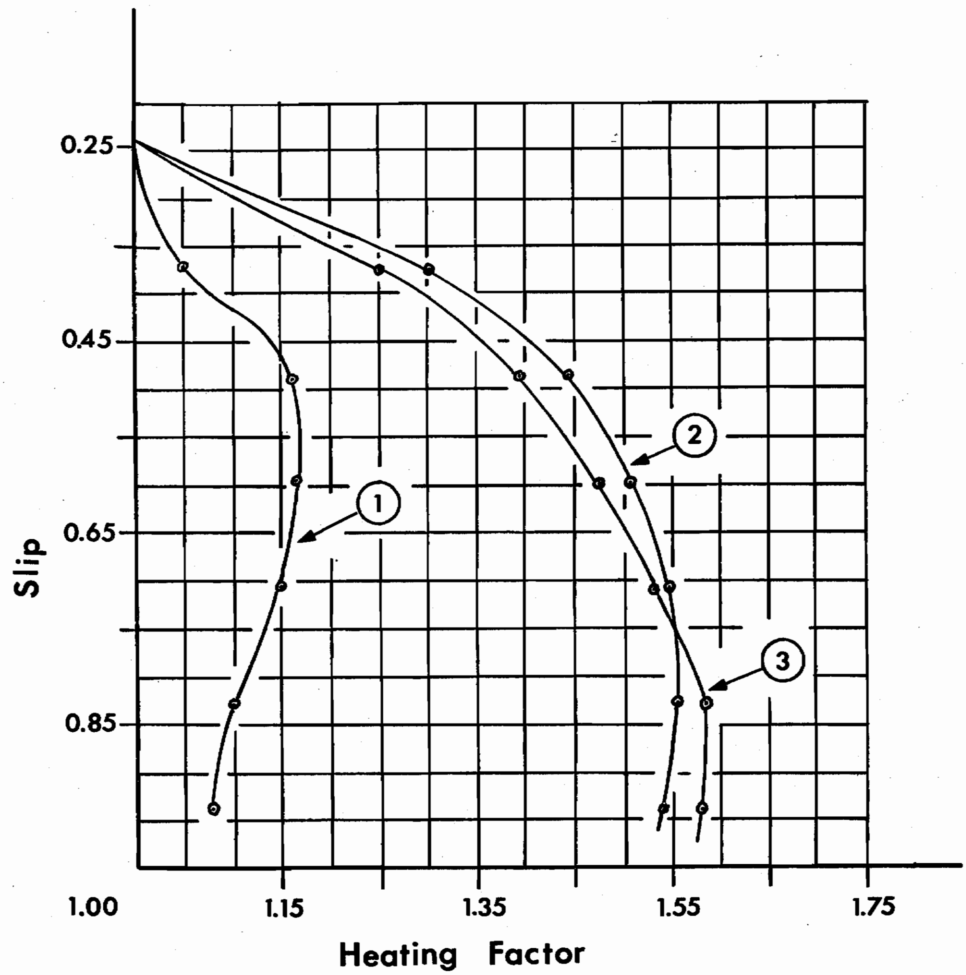


FIGURE 16. Heating factor versus slip for an induction motor (rotor resistance = 24%) at rated torque with (1) symmetrical thyristor control, (2) asymmetrical thyristor control, and (3) thyristor-inverse diode control.

is plotted as a function of slip for rated torque. From these curves, it can be seen that the additional power dissipation using symmetrical thyristor control in the region of rated torque is less than 18 percent while that of the other two circuits can exceed 50 percent.

The heating factor increases at lower torque levels since the relative magnitudes of the harmonics increase, but the sine-wave power dissipation (P_{ds}) decreases more rapidly such that (P_{dt}) diminishes with torque. The maximum values of the heating factor in the first quadrant at 50 percent torque were found to be:

K_{hf} (symmetrical thyristor) = 1.46

K_{hf} (asymmetrical thyristor) = 1.92

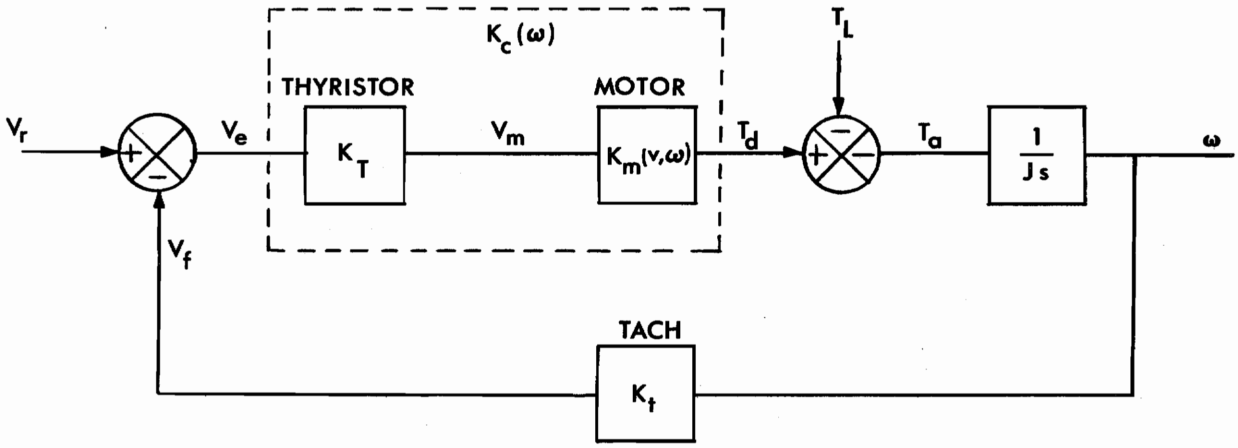
K_{hf} (thyristor-inverse diode) = 2.25

The most desirable arrangement of thyristors in the power circuit from the standpoint of motor heating is the symmetrical circuit.

VI THYRISTOR CONTROL OF MOTOR SPEED

Speed Regulated Induction Motor Drives

A speed-regulated drive system was built using the symmetrical thyristor power circuit in conjunction with tachometer feedback and the performance was reported in earlier work[9]. The block diagram of this drive (see Figure 17) contains two non-linear gain constants. The thyristor gain constant (K_T) relates the error voltage to motor stator voltage and cannot be expressed simply since the output harmonic structure as well as the voltage magnitude influence the motor characteristics. The second non-linearity is the relationship between motor stator voltage (V_m) and the developed torque (T_d). This constant varies with both stator voltage (V_m) and motor angular velocity (ω). It was found that the non-linearity of a diode volt-ampere characteristic could be incorporated into the trigger circuit to establish a relationship between retard angle (θ_R) and error voltage (V_e) such that the developed motor torque was nearly a linear function of error voltage for a fixed motor speed. This compensating non-linearity then enabled the replacement of the thyristor and motor blocks with a single block with a gain ($K_c(\omega)$). The remaining non-linearity ($K_c(\omega)$) can be eliminated at a given operating point (ω_o), and the theoretical frequency response (neglecting saturation of the thyristor



K_T = Gain constant thyristor control $\left[\frac{\Delta\theta_R}{\Delta V_e} \cdot \frac{\Delta V_m}{\Delta\theta_R}\right] \left(\frac{\text{Volts}}{\text{Volts}}\right)$

$K_m(v,\omega)$ = Gain constant of induction motor $\left[\frac{\Delta T_d}{\Delta V_m}\right]_{\substack{\omega = \omega_o \\ V_m = V_{mo}}} \left(\frac{\text{lb-ft}}{\text{Volt}}\right)$

$K_c(\omega)$ = Compensated gain of motor-thyristor combination $\left(\frac{\text{lb-ft}}{\text{Volt}}\right)$

T_d = Developed motor torque (lb-ft)

T_L = Load torque (lb-ft)

T_a = Accelerating torque (lb-ft)

K_t = Tachometer constant (volts/radian/second)

V_r = Reference signal (volts)

V_f = Feedback signal (volts)

V_e = Error signal (volts)

FIGURE 17. Block diagram constants of a thyristor-induction motor speed regulated drive.

controller) is plotted in Figure 18 at the operating point ($\omega_o = 0$). The indicated data points illustrate the good agreement between theoretical and empirical agreement which was obtained with this method.

The applications in which speed-regulated induction motor drives are used seldom require better than one percent speed regulation, and gear ratios are generally chosen such that the load inertia reflected to the motor shaft is equal to the motor inertia. Each of the three power circuits was used to build a drive consistent with these two constraints. The overshoot with varying magnitudes of input step functions was less than 25 percent with no additional compensation.

Open Loop Operation Of Thyristor Drives

The thyristor-induction motor combination may be operated as an open-loop speed-controlled drive, but the speed regulation may be poor for some types of loads. The first-quadrant torque envelope is shown in Figure 19 along with the torque-speed characteristic of the drive at some retard angle (θ_{R1}). This retard angle will develop the required load torque at some slip (S_{LC}). Should the load torque increase (ΔT_{LC}) to the higher level (T_{LC}'), the slip will be increased by (ΔS_{LC}) representing a large change in motor speed. The speed regulation with

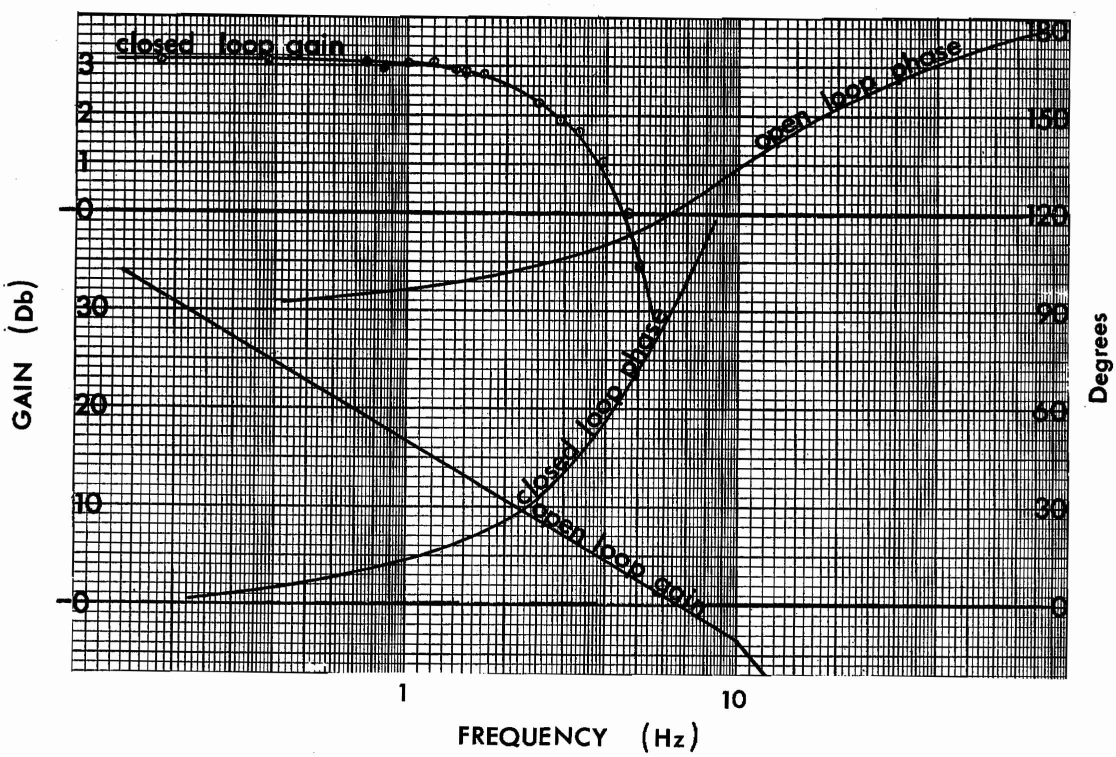


FIGURE 18. Theoretical frequency response of a speed-regulated induction motor drive using phase controlled thyristors as the power amplifier. The theoretical curves were developed using the compensating non-linearity described in the text and perturbation theory to derive a linear model about a speed operating point of zero. The empirical data points are shown on the closed-loop gain curve.

this type of load could be improved by increasing the rotor resistance forcing the breakdown torque point into the fourth quadrant. However, a larger motor frame is then required to develop the necessary horsepower and the added expense is greater than that required to close the speed loop with a tachometer. As a result, the closed-loop drive becomes the most economical for constant torque loads.

When the load torque varies with speed such as that of fans and pumps, the speed regulation obtainable with open-loop control is considerably better. The load-torque characteristic of a fan is plotted in Figure 19. When the load torque of the fan (T_{LF}) is increased by the same increment ($\Delta T_{LF} = \Delta T_{LL}$) at rated speed, the variation in motor slip (ΔS_{LF}) is considerably less than that of the constant torque load. Since this variation is usually within the requirements for fan and pump control, most drives for these types of loads are operated open- loop.

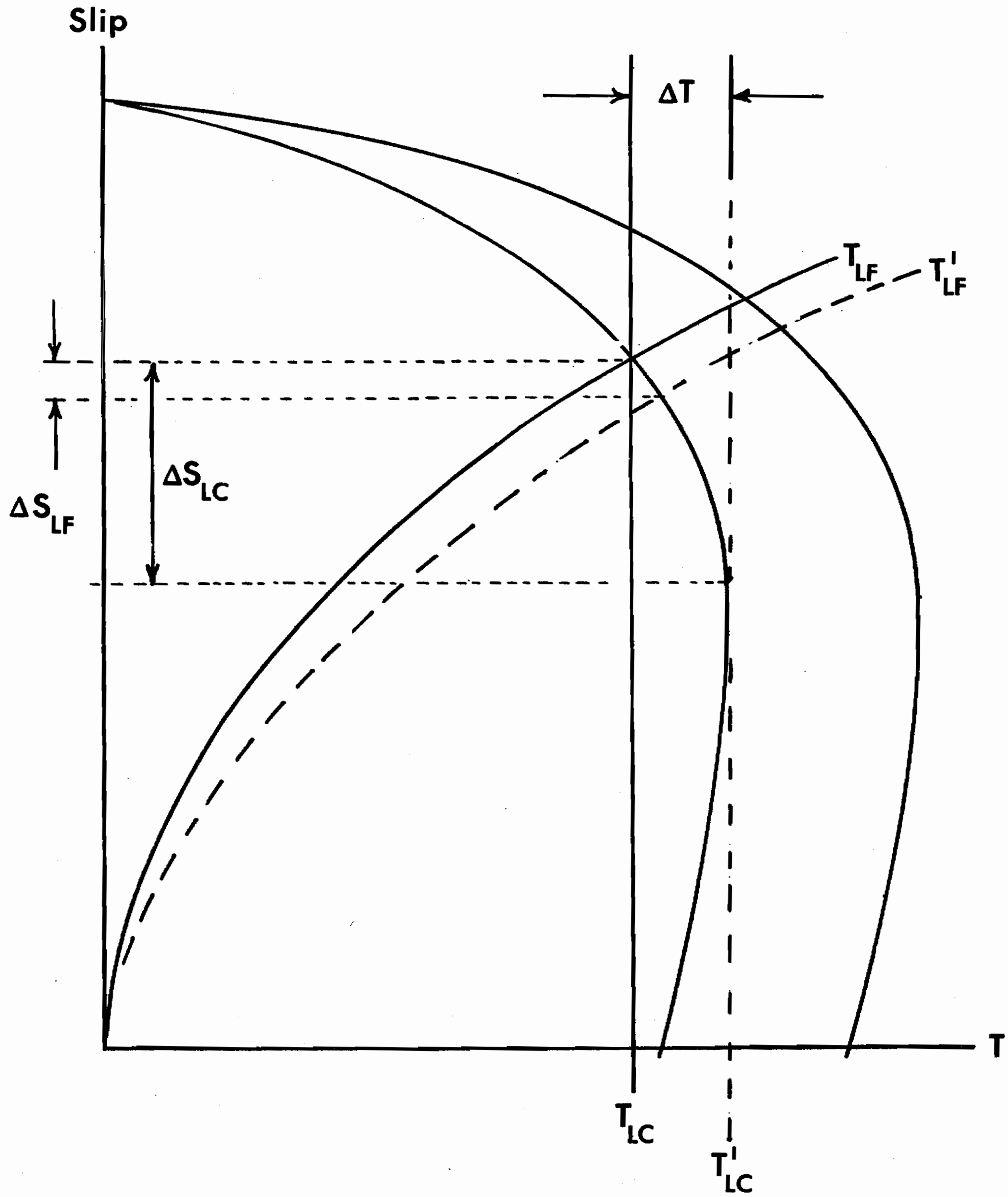


FIGURE 19. Speed regulation with a load torque which varies with the square of speed, and a load torque which is independent of speed.

SUMMARY AND CONCLUSIONS

This thesis provides a comparison of three different power circuit arrangements of thyristors for controlling the speed of an induction motor through control of the stator voltage. The output voltage harmonic structure is provided for each circuit along with a comparison of the motor torque-speed characteristics. Means for coping with the motor heating problem, which results from adjustable voltage control, are discussed and curves are provided to evaluate the additional heating which results from the non-sinusoidal stator voltage.

The principles of phase control are developed for single phase circuits and expressions are developed for calculating the input power factor which can be less than unity, even when the load is a resistance. These principles are then extended to three phase circuits where expressions for determining the phase sequence of the harmonics are developed. The applicability of superposition to the qualitative analysis of the effect of stator voltage harmonics upon motor performance is demonstrated, and the problems encountered in the quantitative application of superposition is discussed.

The symmetrical power circuit which consists of two thyristors arranged in antiparallel and connected in series with each of the motor phases was found to have the most desirable characteristics. The thyristor pair of one phase can be eliminated without sacrificing control range, but the unbalanced voltages which result increase the motor heating. A third control scheme can be developed by replacing one thyristor in each phase with a diode, but motor heating is increased and the shape of the torque-speed curves in the second and fourth quadrants limit the available speed range.

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VITA

Charles Edward Konrad was born on April 10, 1933, in Paden City, West Virginia. He attended Paden City Grade School and was graduated from Magnolia High School, New Martinsville, West Virginia, in May, 1951.

He worked for Fisher Body Corporation as a dispatcher before enlisting in the U. S. Army in 1953. He was graduated from the Armed Services Medical Equipment Maintenance Course in 1954 and worked on the installation and maintenance of X-ray, medical, and dental equipment throughout his military career. He was assigned for two years to the 2nd General Hospital in Landstuhl, Germany before being discharged in November, 1956. He then worked as a field serviceman for Picker X-ray Corporation in Cleveland, Ohio and enrolled in West Virginia University in September, 1957. He was awarded the Pennsylvania Electric Coil Scholarship and was elected to Tau Beta Pi, Eta Kappa Nu, and Helvetia. During summer vacations, he was employed as an engineering trainee by Ormet Corporation, Hannibal, Ohio. He completed the requirements for a Bachelor of Science in Electrical Engineering in May, 1961, and entered the Creative Engineering Program with the General Electric Company. In June, 1962, he assumed his present duties as development engineer for the Industry Control Department of General Electric Company.

He was married to Karen Lynne Batson in December, 1959, and has three wonderful children; Chip, Kurt, and Kristen.

Charles E. Konrad

APPENDIX

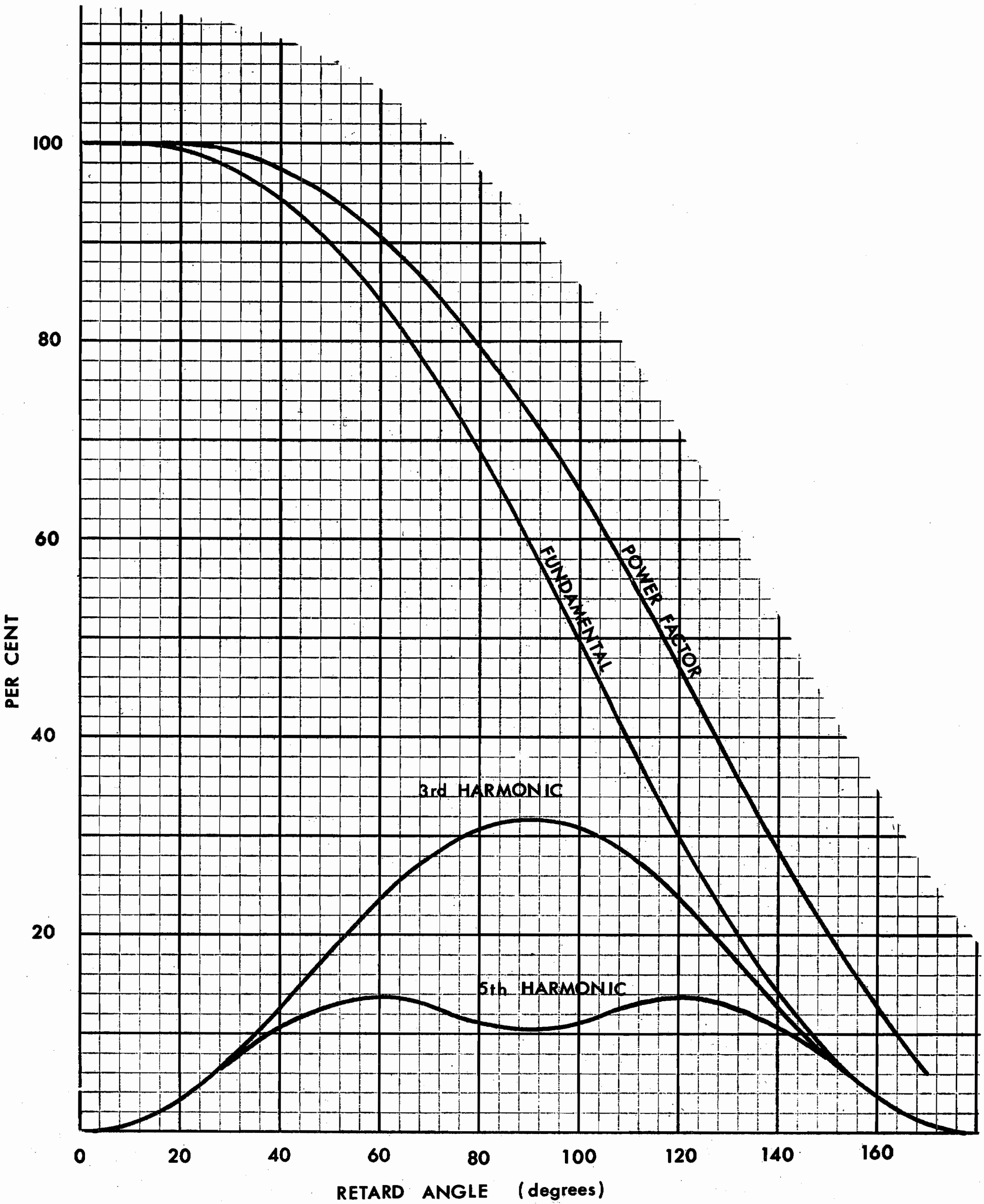


FIGURE 1A. Harmonic structure of load voltage developed across a resistor by a single-phase thyristor control. Magnitude is expressed in percent of the supply voltage.

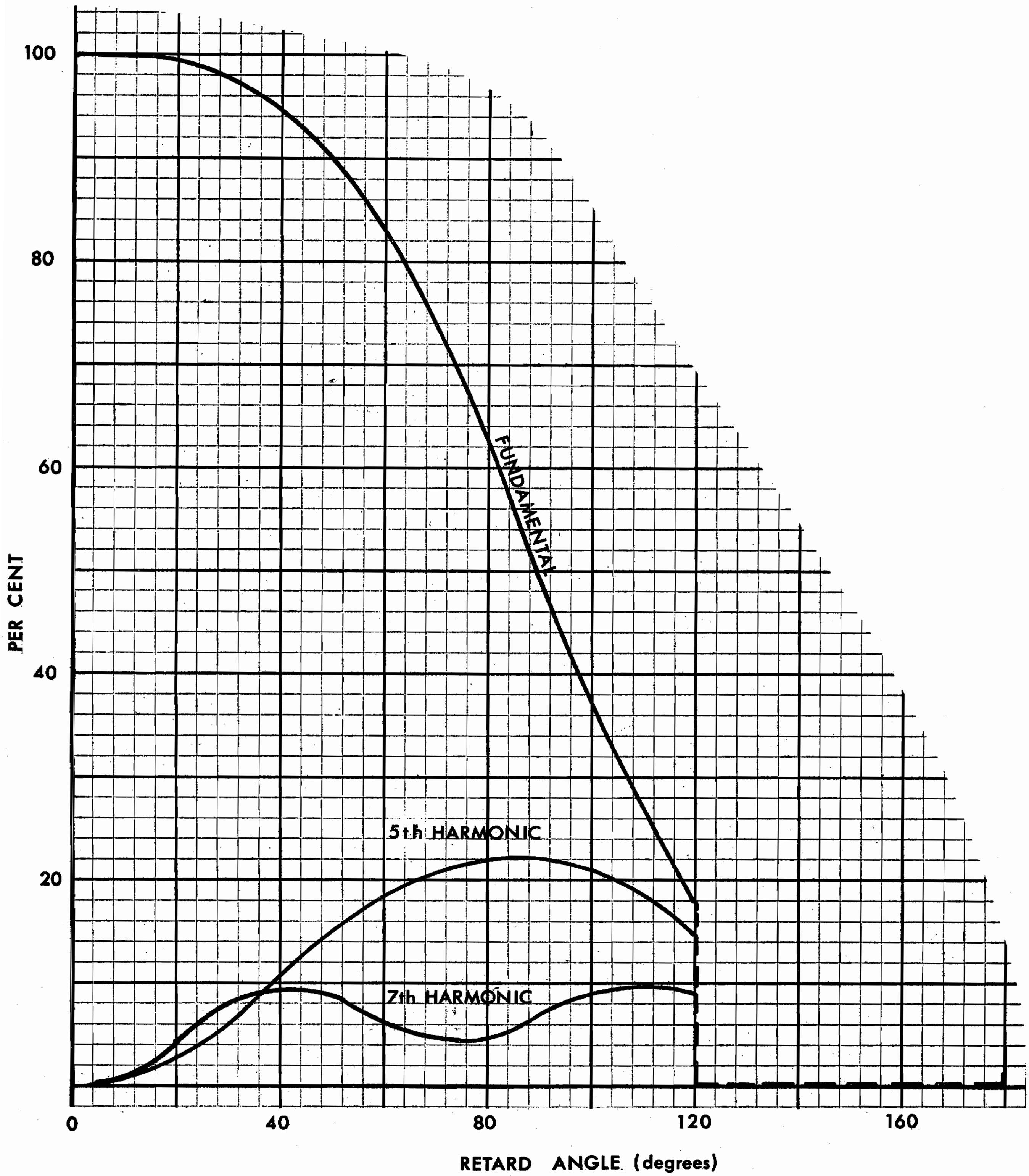


FIGURE 2A. Harmonic structure of line-to-neutral voltage developed across a resistor load with a symmetrical thyristor control. Magnitude is expressed as a percent of the supply line-to-neutral voltage.

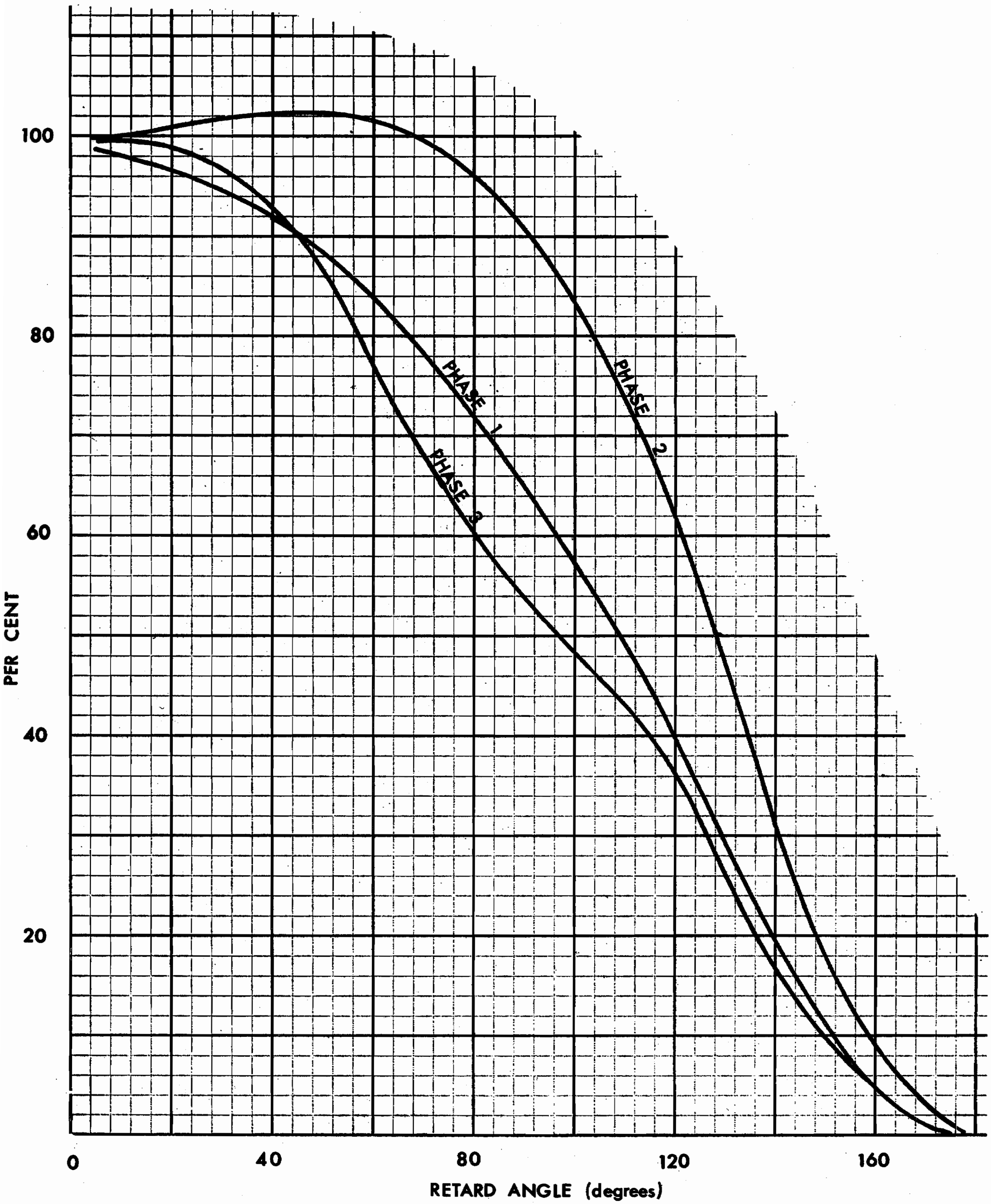
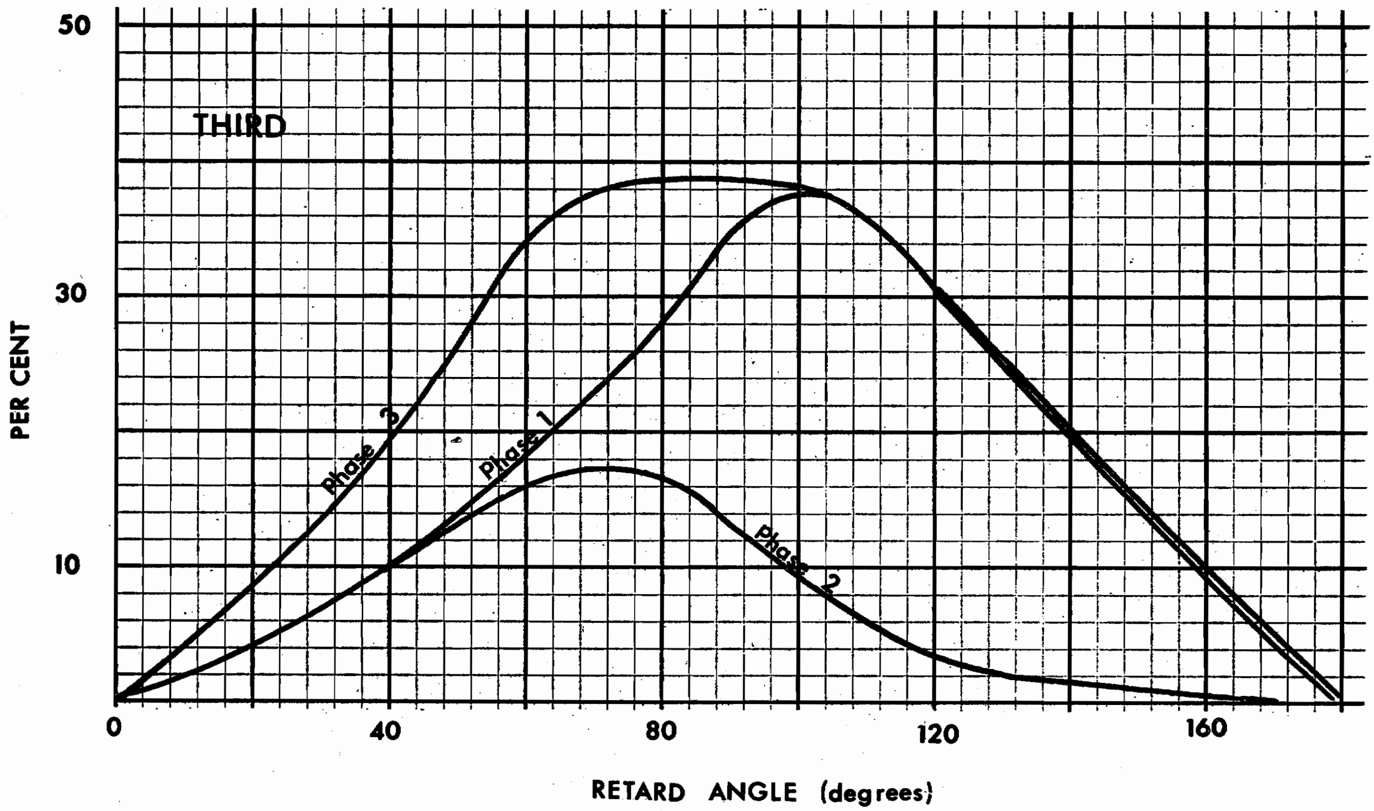


FIGURE 3A. Fundamental component of load voltage developed across each phase (line-to-neutral) of a balanced three phase load resistance by an asymmetrical control expressed in percent of the line-to-neutral supply voltage.



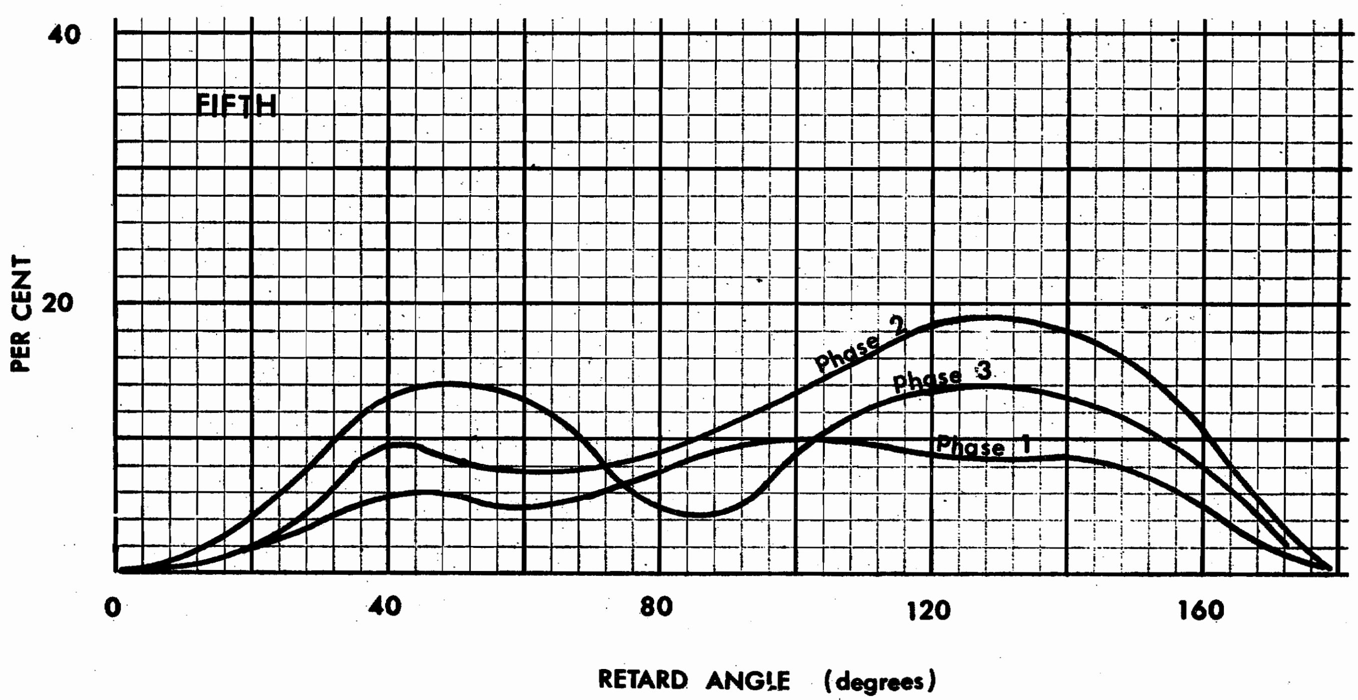


FIGURE 4A. Third and fifth harmonic components of line-to-neutral load voltage developed across each phase of a balanced load resistance by an asymmetrical control, expressed in percent of the line-to-neutral supply.

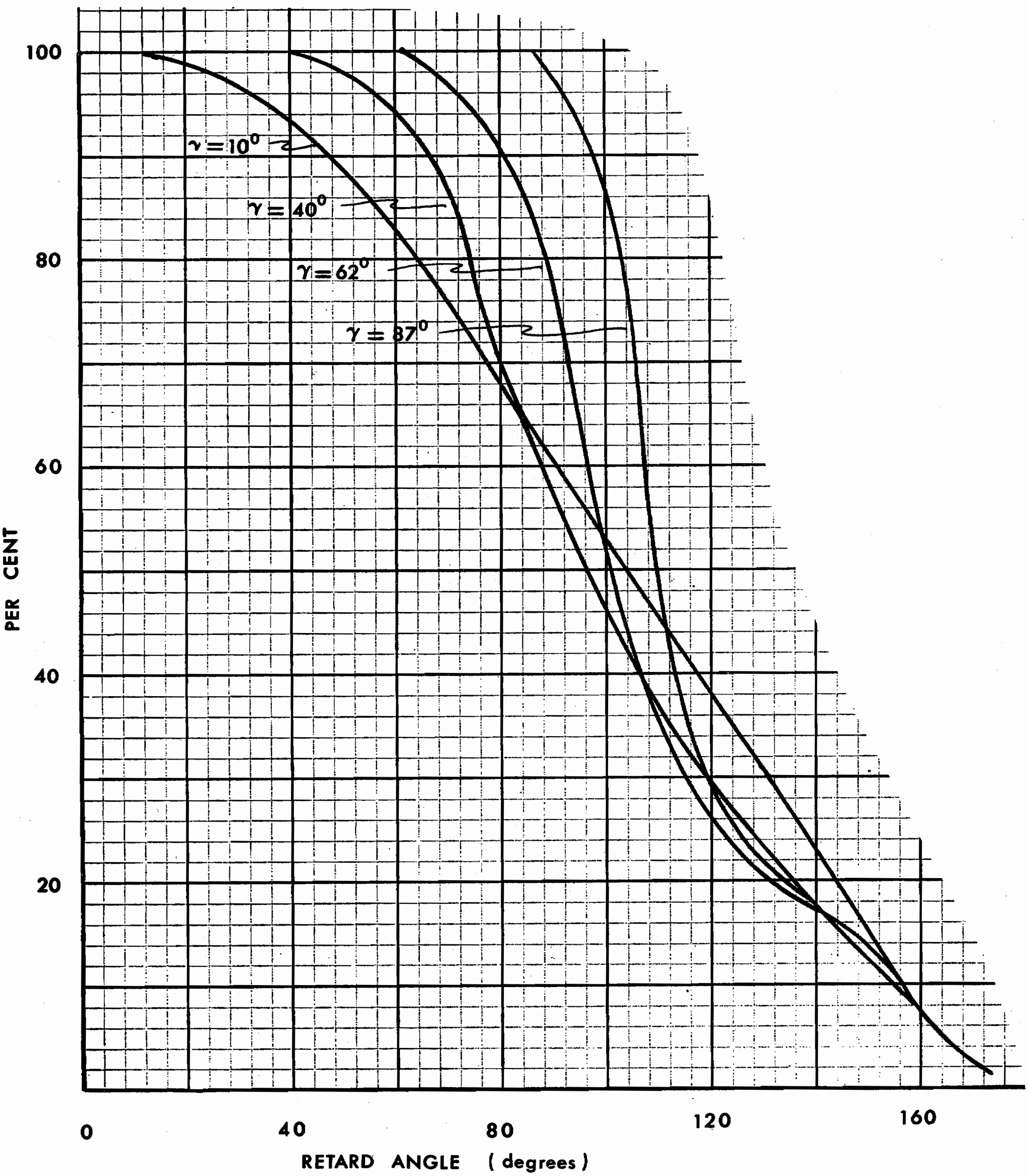
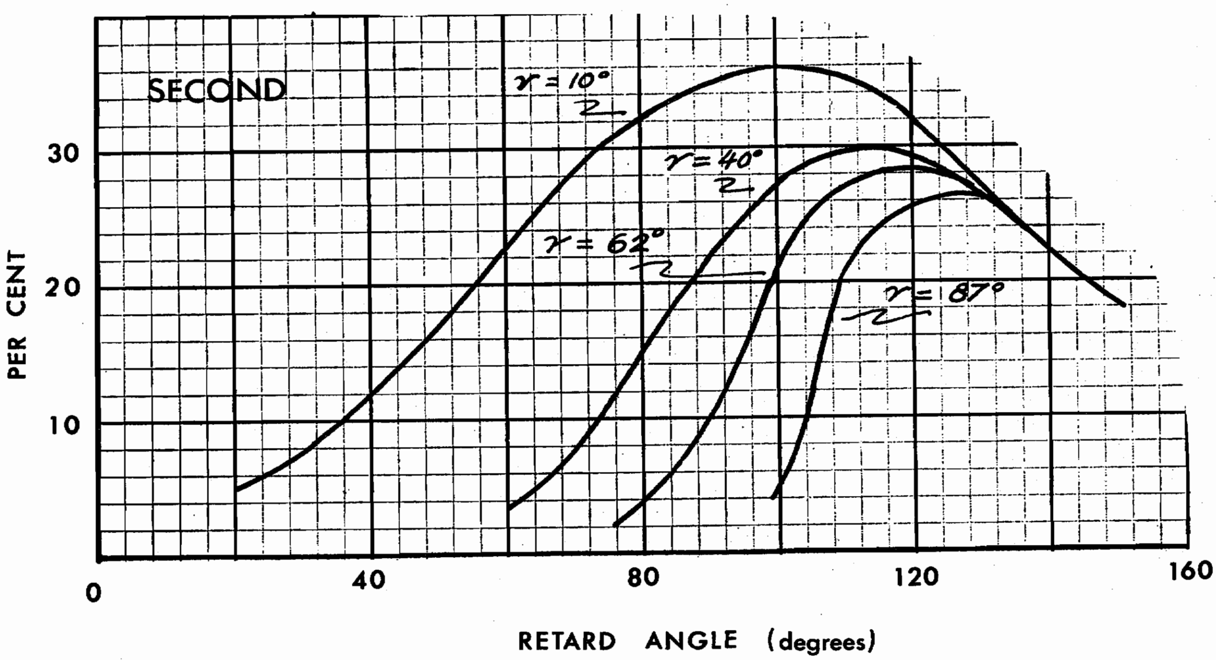


FIGURE 5A. Fundamental component of load voltage (line-to-neutral) developed by a thyristor-inverse diode control across a load of lagging power factor angle (γ), expressed in percent of the supply line-to-neutral voltage.



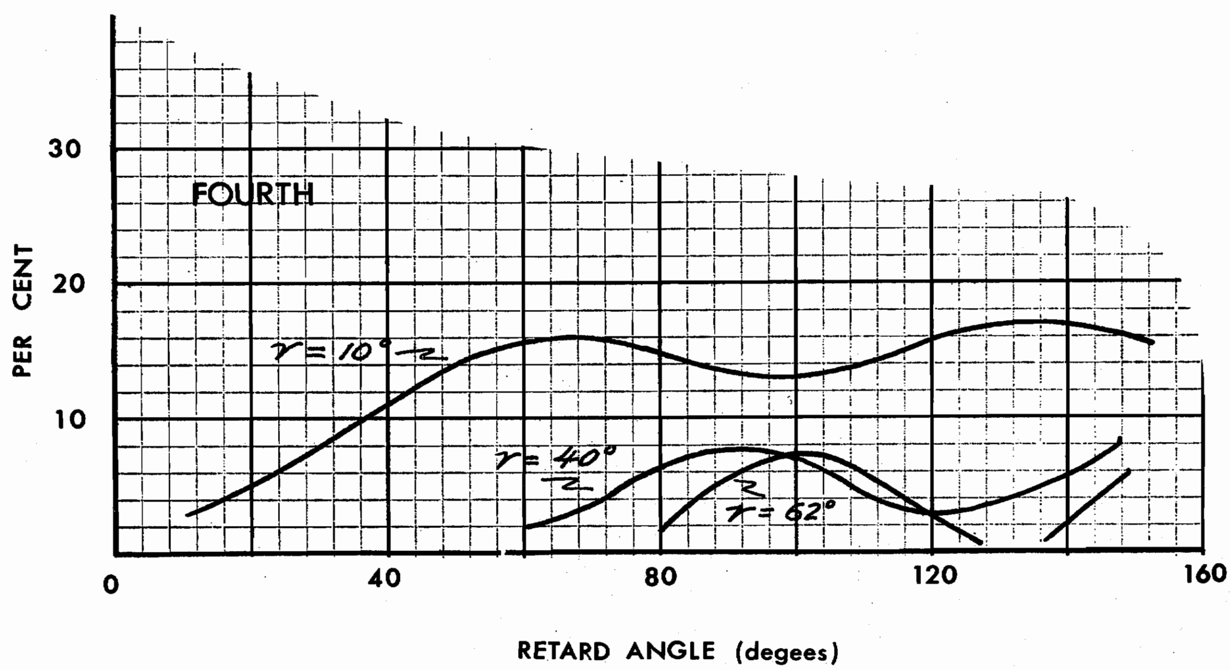


FIGURE 6A. Second and fourth harmonic components of load voltage (line-to-neutral) developed by a thyristor-inverse diode control across a load impedance of lagging power factor angle (γ).

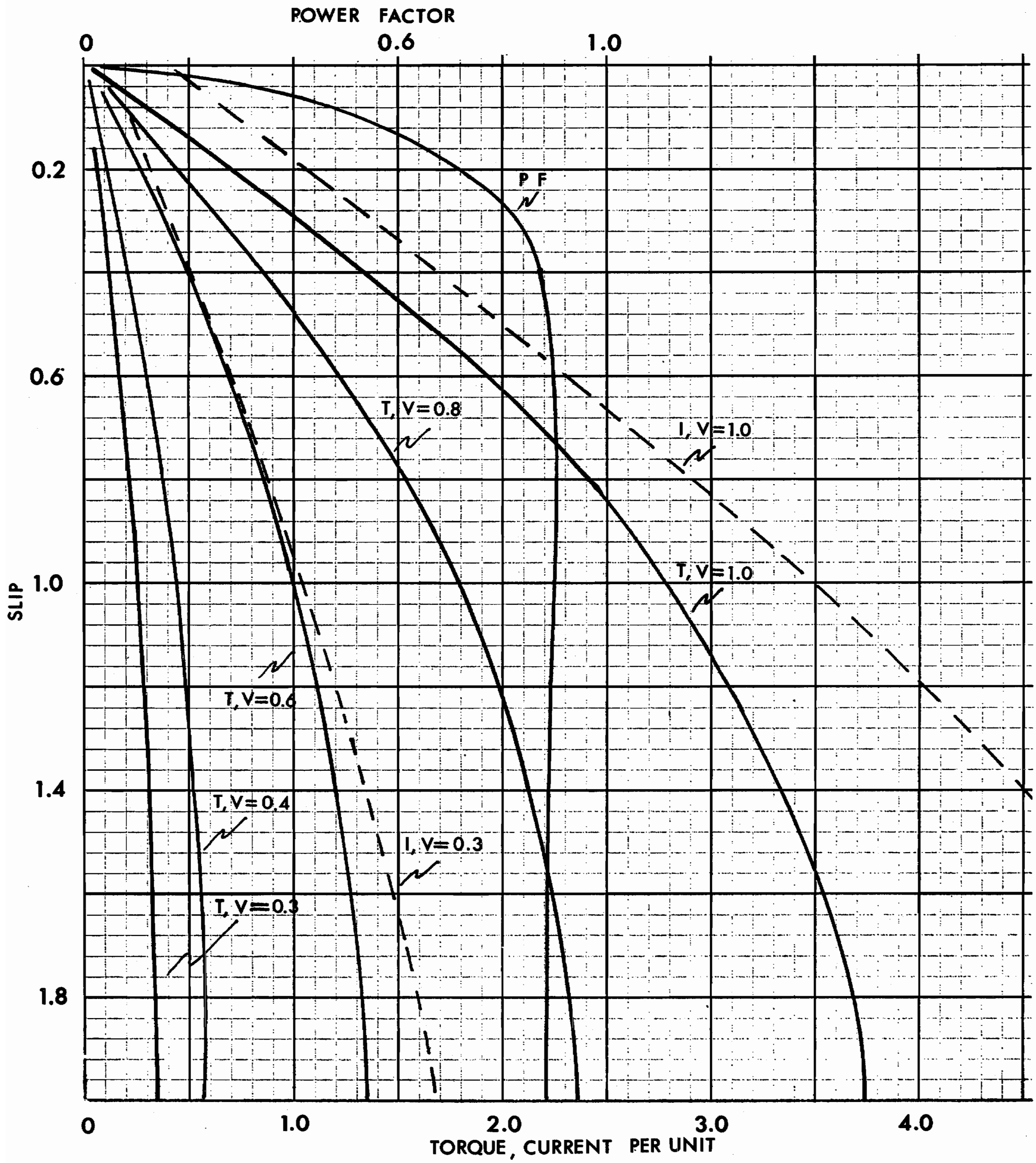


FIGURE 7A. Characteristics of the test motor calculated from the equivalent circuit constants.

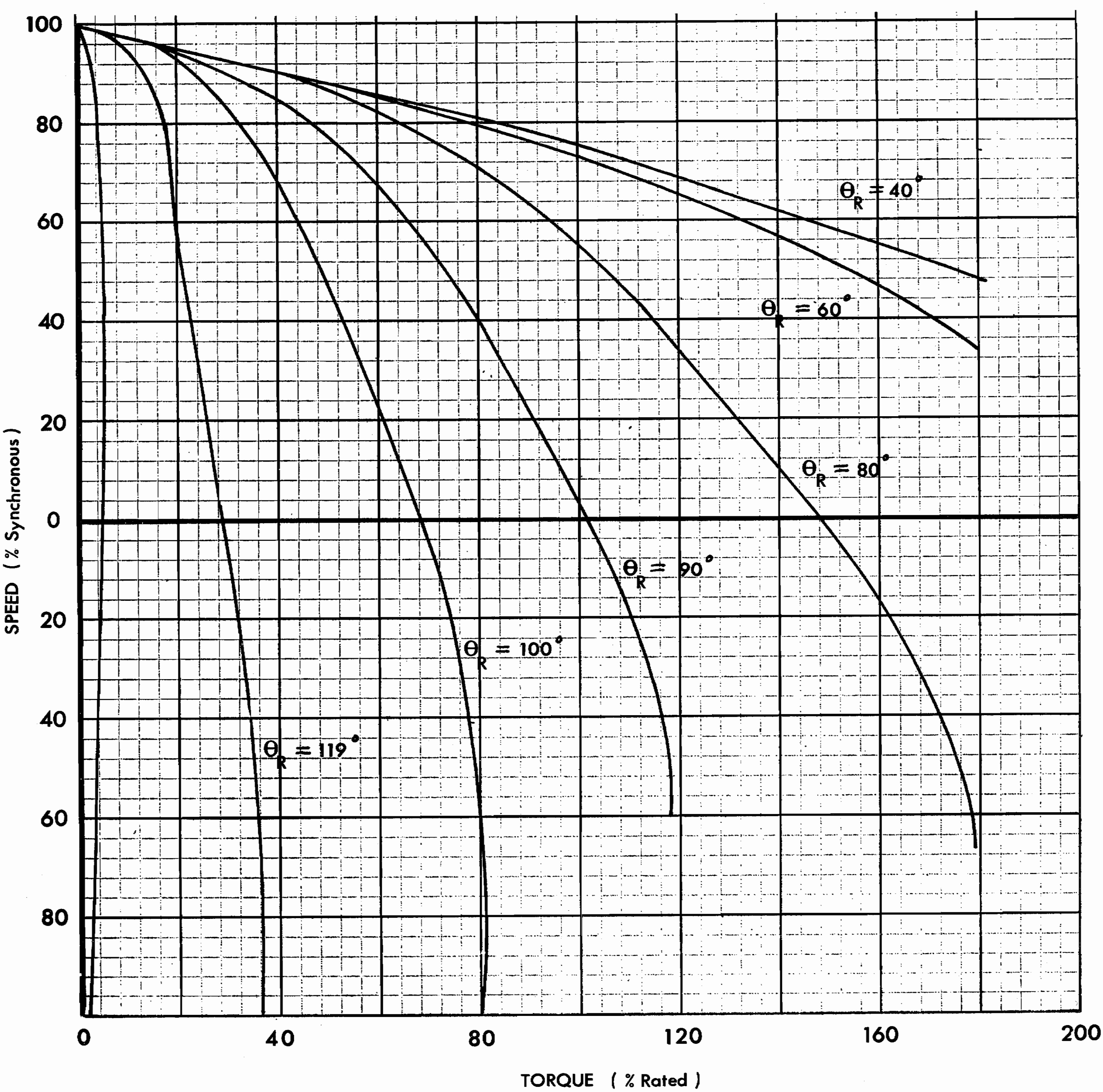


FIGURE 8A. Torque-speed curves of test motor supplied from symmetrical thyristor control at fixed retard angle (θ_R).

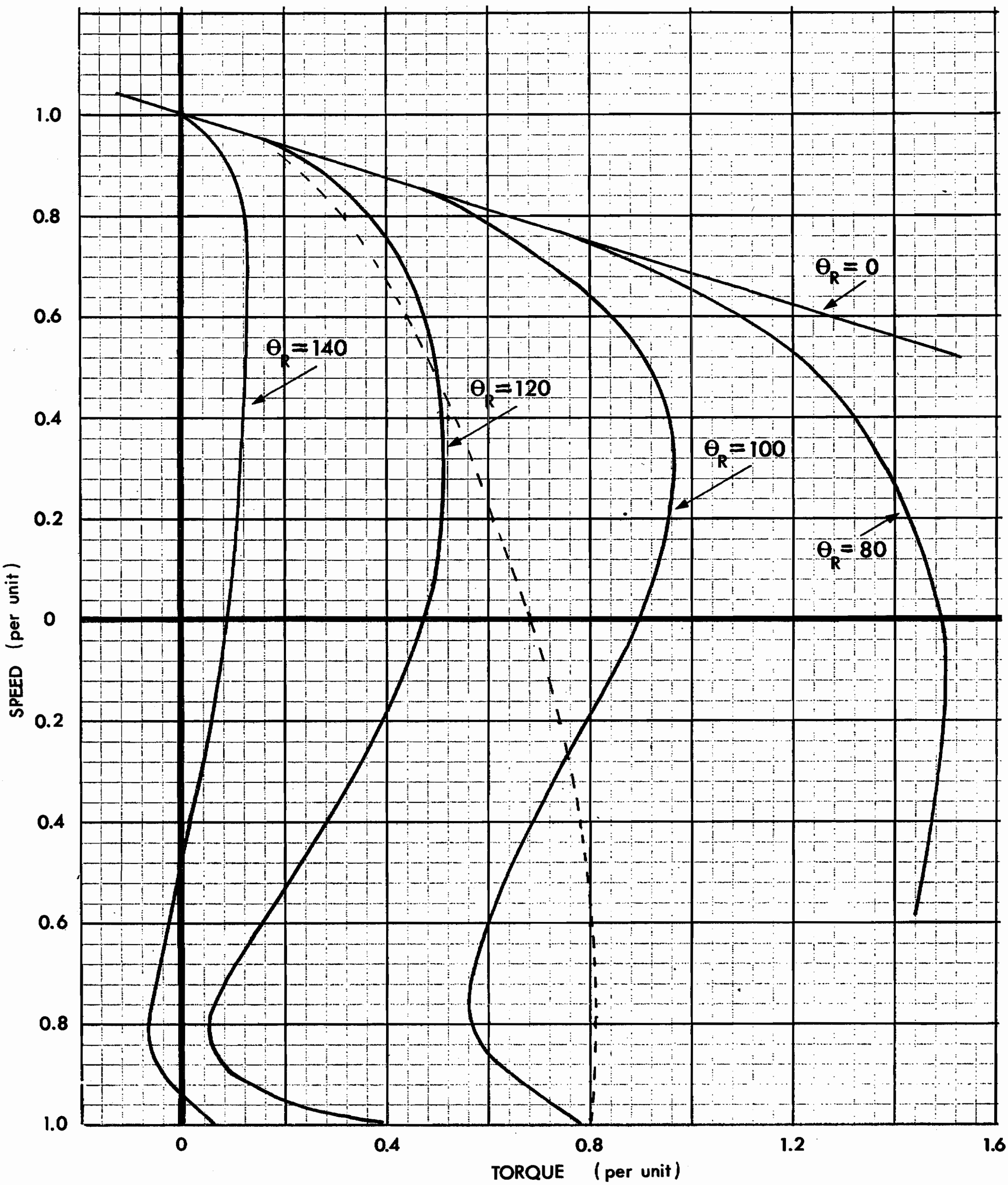


FIGURE 9A. Torque-speed curves of test motor supplied from asymmetrical thyristor control at fixed retard angle (θ_R).

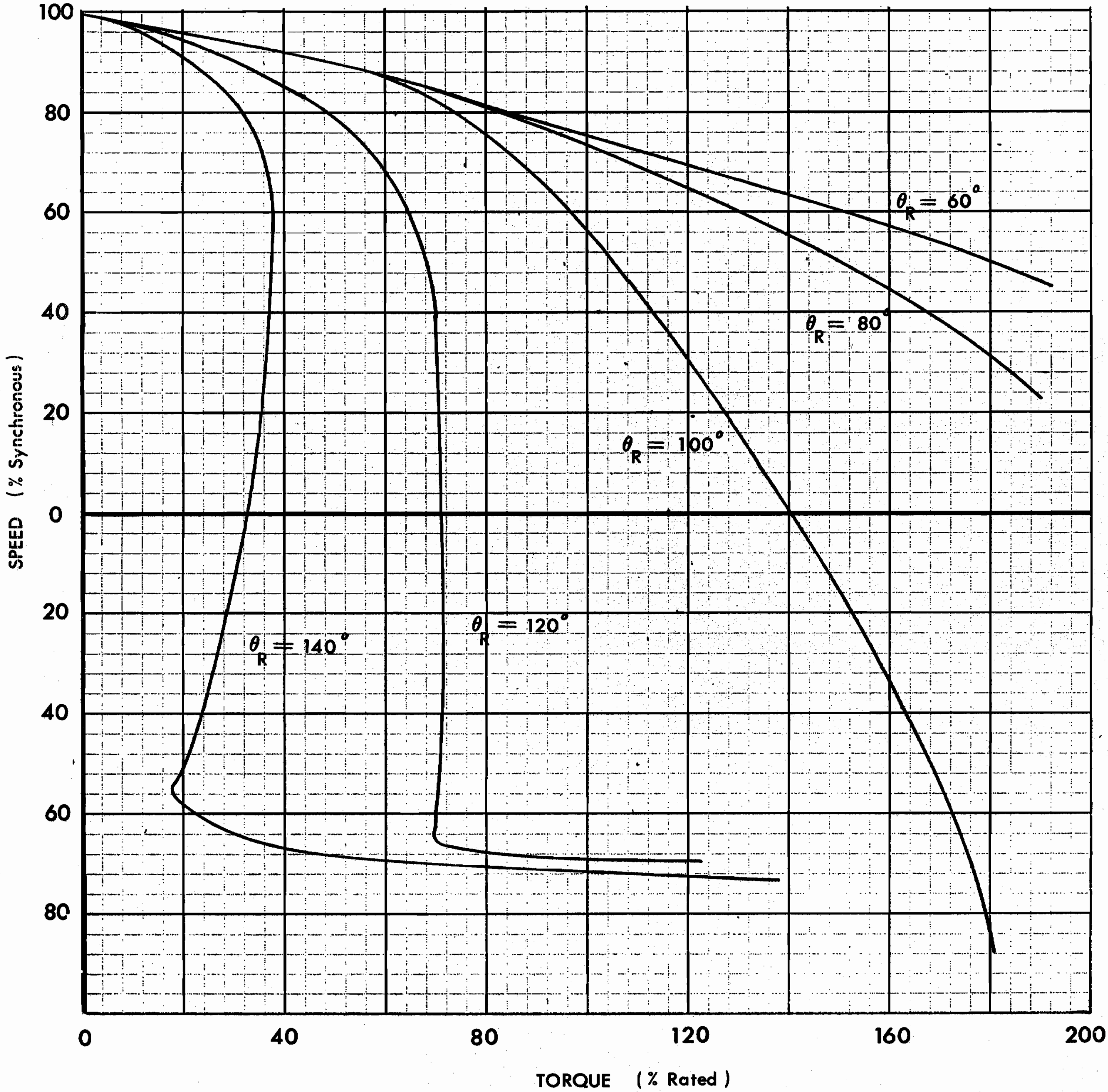


FIGURE 10A. Torque-speed curves of test motor supplied from thyristor-inverse diode control at fixed retard angle (θ_R).

```
10 REM  "THIS PROGRAM CALCULATES THE POWER FACTOR,"
11 REM  "STATOR CURRENT, AND TORQUE OF AN"
12 REM  "INDUCTION MOTOR IN THE FIRST AND FOURTH"
13 REM  "QUADRANTS.  ENTER DATA IN THE FORM:"
14 REM  "
15 REM  "196 DATA R1, R2, X1, XM, V1, V2, V3, ETC"
16 REM  "
17 REM  "WHERE R1, R2, X1, XM ARE PER UNIT, V(N) = PER UNIT"
18 REM  "STATOR VOLTAGES OF DESIRED CURVES"
20 READ R1, R2, X1, X3, V1
25 LET L = -X3*X1
110 PRINT
115 PRINT "STATOR VOLTS =" V1" PER UNIT"
116 PRINT
117 PRINT "SLIP", "PWR-FCT", "STA-CUR","TORQUE"
120 LET S = 0
125 FOR S=0.1 TO 2.1 STEP 0.2
130 LET B = X3*R2/S
135 LET C = R1*R2/S-2*X1*X3-X1↑2
140 LET D = X3*R1+X1*R1+X1*R2/S+X3*R2/S
145 LET A = ATN(B/L)-ATN(D/C)
150 LET H = SQR(L↑2+B↑2)/SQR(C↑2+D↑2)
155 LET I1= V1*SQR((1-H*COS(A))↑2+(H*SIN(A))↑2)/SQR(R1↑2+X1↑2)
160 LET P = COS(ATN((-H)*SIN(A)/(1-H*COS(A)))-ATN(X1/R1))
165 LET I2= V1*SQR((H*COS(A))↑2+(H*SIN(A))↑2)/SQR((R2/S)↑2+X1↑2)
170 LET T = (R2/S)*I2↑2
180 PRINT S,P,I1,T
185 NEXT S
190 READ V1
195 GO TO 110
196 DATA 0.0283,0.2464,0.0464,1.497,1.0,0.8,0.6,0.5
197 DATA 0.4, 0.3, 0.2
200 END
```

THE APPLICATION OF THYRISTORS TO INDUCTION MOTOR DRIVE SYSTEMS

by

Charles E. Konrad

ABSTRACT

This thesis provides a comparison of three different power circuit arrangements of thyristors for controlling the speed of an induction motor through control of the stator voltage. The output voltage harmonic structure is provided for each circuit along with a comparison of the motor torque-speed characteristics. Means for coping with the motor heating problem, which results from adjustable voltage control, are discussed and curves are provided to evaluate the additional heating which results from the non-sinusoidal stator voltage.

The principles of phase control are developed for single phase circuits and expressions are developed for calculating the input power factor, which can be less than unity, even when the load is a resistance. These principles are then extended to three phase circuits where expressions for determining the phase sequence of the harmonics are developed. The applicability of superposition to the qualitative analysis of the effect of stator voltage harmonics upon motor performance is demonstrated, and the problems encountered in the quantitative application of superposition are discussed.