

RELIABILITY ANALYSIS OF SINGLE-HEADED ANCHOR BOLTS

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(ABSTRACT)

Several design equations for predicting the capacity of a single-headed anchor bolt embedded in plain concrete have been recommended in the United States. The capacities computed by these different recommendations, in some cases, differ significantly. The existing differences in current design criteria for anchor bolts subjected to tensile or shear loading is discussed with emphasis on the ACI, the PCI and the proposed Load and Resistance Factor Design (LRFD) equations. Available data from test results on the anchor bolts and welded studs were analyzed. Then, based on the analysis of these data and statistical information on basic design variables, a reliability analysis was performed. Using the advanced first-order second-moment reliability analysis method, risk levels implied in these design equations were computed for a dead and maximum live load combination.

It was found that there are inconsistencies in the levels of safety implied by both the ACI and the PCI design equations, and that the level of safety depends on the loading

and the failure mode under consideration. By comparing reliability indices for these design equations, it is thus possible to make an objective evaluation of current design criteria.

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Chapter I  
INTRODUCTION

1.1 BACKGROUND

In recent years, anchor bolts have been extensively used not only in ordinary types of structures, but also in more critical applications such as nuclear related structures. Several equations for the design of anchor bolts have been recommended in the United States. However, in some cases there is disagreement among these recommendations. This is due to the great variation in test results and uncertainties in the various parameters of these equations. Therefore, it is necessary to evaluate the widely used design equations by incorporating these uncertainties. One rational approach for dealing with these uncertainties is to perform a reliability analysis over a range of the mean values and the coefficient of variation of the design variables.

The use of semi probabilistic based design for headed anchor bolts has been proposed (40). However, this proposed design utilizes existing safety factors and load factors which do not consider the level of risk. Indeed, the consideration of risk is the main feature of the probability-based approach (3). The Load and Resistance Factor Design (LRFD) presented in Ref. [36] generalizes the use of the nominal

strength and the resistance factor of ordinary bolted connections for predicting the capacity of the anchor bolt governed by steel failure. This proposed design does not recommend any design equation and resistance factors for predicting the capacity of the anchor bolt governed by concrete failure.

It has been found that the reliability analysis can be utilized to evaluate load or safety factors used in the traditional design formats appropriately and consistently (3,11,15,20,37). The selection of load and resistance factors consists of choosing a target reliability from the observation of the level of safety implied in the existing design criteria. The evaluation of the implied level of safety in current design specifications for anchor bolt connection is thus an important first step in the development of probability-based design criteria.

## 1.2 OBJECTIVES AND SCOPE OF STUDY

Current equations for the design of anchor bolt are based on ultimate strength and working stress design which employ deterministic safety factors and load factors. These factors are determined subjectively on the basis of experience and judgement. Unfortunately, the results of tests on anchor bolts vary greatly resulting in different recommendations

for design equations and the corresponding strength reduction factors. It is the objective of this study to examine the level of safety implied in the existing design criteria for anchor bolts through reliability analysis. The results of this reliability analysis can be used later to determine the appropriate strength reduction factor and corresponding load factors.

For the purpose of this study, all available data from the previous test results on anchor bolts and welded studs are examined. This study deals with a single ASTM A307 headed anchor bolt embedded in plain normal weight concrete with ungrouted base subjected to monotonic shear or tensile loading. For shear loading, only the bolt which is sufficiently embedded will be considered, while for tensile loading, only the bolt with a sufficient edge distance to develop a full cone failure surface will be discussed. The load combination to be considered is dead and maximum live load, since in many cases this combination controls the structural design. Then, the level of safety implied in the design equations recommended by American Concrete Institute (ACI) in Ref. [10], Prestressed Concrete Institute (PCI) in Ref. [34] and the LRFD Committee in Ref. [36] are examined using the reliability analysis.

Since the available information is only sufficient to estimate the mean and variance of the design variables, the reliability analysis procedures considered in this study are the first-order second-moment method (FOSM). There are two different approaches in the FOSM which are known as the mean and the advanced FOSM. The mean FOSM has been proved to be lack of invariance, e.g. the reliability index obtained by this approach depends on a failure criterion used in the computation (22). In order to avoid the problem resulting from using a different formula for a failure criterion, the advanced FOSM (4,22,39) is employed in the analysis.

Chapter 2 presents the review of the previous studies on anchor bolts and their design recommendations, while a review of the reliability analysis techniques including the advanced FOSM procedures is presented in Chapter 3. The statistical information on basic design variables and the analysis of earlier test results are presented in Chapter 4. Using the available and derived statistical information, the reliability indices implied in current ACI, PCI and LRFED equations are examined, and the results are discussed in Chapter 5. Finally, Chapter 6 contains the summary and conclusions of the study.

## Chapter II

### REVIEW OF LITERATURE ON ANCHOR BOLTS

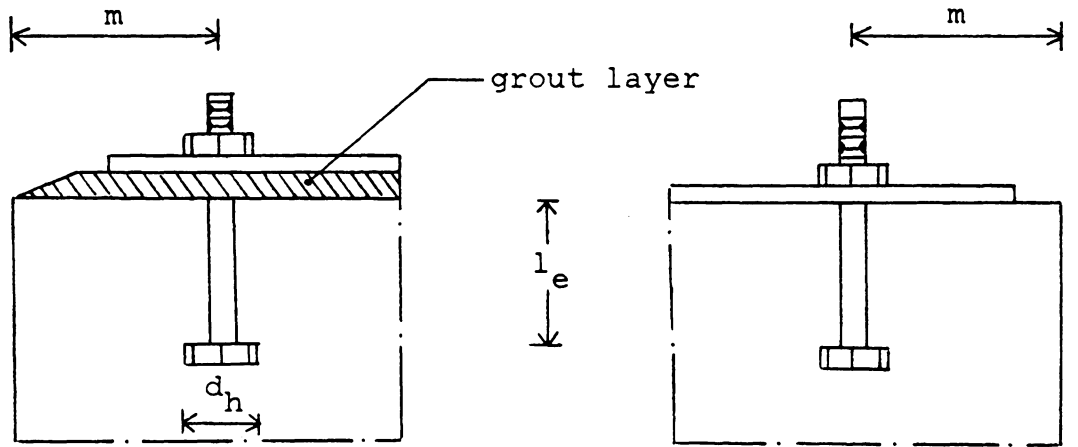
#### 2.1 GENERAL

One of fastening systems which is commonly used, when a connection between a steel or a precast concrete member and a concrete structure has to be made, is an anchor bolt connection. The connection is made by using a steel end plate welded to an attachment which is connected to the concrete structure with anchor bolts. The anchor bolt, which is installed prior to concrete placement, protrudes from the surface of the concrete, and the end plate can be placed directly on the surface of the concrete structure or on a one to two inch thick grout layer. These two types of base details are shown in Figure 2.1, in which  $l_e$ ,  $m$  and  $d_h$  denote the length of embedment, the edge distance and the bolt's head diameter, respectively.

Depending on the configuration, there are three types of anchor bolts which are frequently used (Figure 2.2):

- a. Headed anchor bolt
- b. J-shaped anchor bolt
- c. L-shaped anchor bolt

Studies on J-shaped bolts (6) and on L-shaped bolts (30,41) showed that they are likely to fail due to straightening of the hook rather than yielding of the bolt.



a. Grouted base detail

b. UngROUTED base detail

Figure 2.1 Typical Anchor Bolt Connection

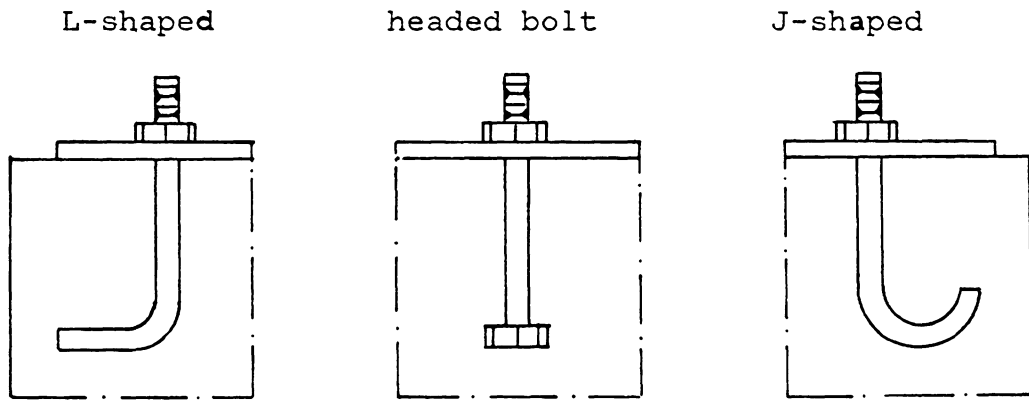


Figure 2.2 Three Types of Anchor Bolts

As a connector, an anchor bolt can be subjected to tension, shear and a combination of tension and shear loading. Then, the loads on the bolts are transferred to concrete by bonding, mechanical anchorage, and bearing. Depending on

the character of the loading, the materials used for the anchor bolts may vary from ASTM A307, A325 bolts to A36 steel bars.

In the following sections, tests on the headed anchor bolt and welded studs embedded in plain concrete are summarized. Also, recommended design equations on a single headed anchor bolt under shear or tensile loading are discussed with emphasis on the design equations used by ACI (10), PCI (34), and the LRFD (36). Finally, a sample calculation on the headed anchor bolt is presented to show differences in the results obtained using ACI, PCI and the LRFD equations.

## 2.2 TESTS ON A SINGLE-HEADED ANCHOR BOLT

Although the application of the anchor bolt may be different from that of a welded stud, the behavior of the anchor bolt with ungrouted base is similar to that of a stud with the contact plate mounted on the concrete surface. Several tests on headed anchor bolts and studs embedded in plain concrete have been conducted in the United States. These can be divided into two groups:

1. Tests on ASTM A307 headed anchor bolts with grouted base details. These studies concluded that the induced bending moment in grouted anchor bolts due to shear loading decreases the capacity of grouted an-

chor bolts compared to ungrouted base anchor bolts (1,2,42).

2. Tests on headed anchor bolts and welded studs with ungrouted base:

a) ASTM A307 headed anchor bolts under monotonic and cyclic shear loading. These tests investigated the effect of the edge distance on the shear capacity (2,5,26,42) and showed how hairpin reinforcement gives good ultimate load performance (5,26,42).

b) ASTM A307 headed anchor bolts under tensile and combined shear and tensile loading. This test investigated the effect of the edge distance and the length of embedment on the capacity of the anchor bolt (2).

c) Welded studs under shear (28,31,33,41), tensile (28,30,32,41) and combined shear and tensile loading (28,41). The objectives of these tests were to determine the optimum relationship for stud's dimension (32,41), to determine the strength of stud anchors embedded in lightweight concrete, to investigate the effect of the embedment's length and the effect of the edge distance on the capacity of the welded stud (28,33).

From these experiments, two basic failure modes were observed, which are: 1.) steel failure; and 2.) concrete failure. Steel failure depends on the cross sectional area ( $A_s$ ) and the ultimate tensile strength of the bolt ( $f_u$ ). On the other hand, the concrete failure depends on the length of embedment ( $l_e$ ), the edge distance ( $m$ ), the spacing of bolts, the diameter of anchor head ( $d_h$ ), and the compressive strength of concrete ( $f'_c$ ).

### 2.3 DESIGN EQUATIONS FOR A SINGLE-HEADED ANCHOR BOLT

As mentioned above, failure of the anchor bolt connection can be caused either by the failure of the bolt or failure of the concrete. Accordingly, the capacity of the connection is governed by two equations which predict the capacity of the concrete and the capacity of the bolt for each loading. Based on the experiments described above, several design equations have been recommended to predict the capacity of the anchor bolt connection. In this study, only equations for the prediction of the capacity of a single-headed anchor bolt under the shear or the tensile loading will be discussed.

### 2.3.1 Shear Capacity

In predicting the shear capacity governed by the steel failure, two totally different approaches can be found in design recommendations in the United States.

One of the approaches uses the shear-friction concept to compute the shear capacity of the bolt (2,10,36). This concept assumes that the tensile clamping force can be developed in the bolt under small shearing displacement. The ACI Committee 349 (10), which uses the shear-friction concept, recommends that the ultimate shear capacity of the bolt be computed as

$$V_s = \mu A_s f_y \quad (\text{Eq. 2.1})$$

in which  $A_s$  and  $f_y$  are the net tensile area and the minimum specified yield strength of the bolt, respectively, while  $\mu$  is the shear friction coefficient which is 0.7 for steel against concrete. ACI also recommends a strength reduction factor ( $\phi_s$ ) equal to 0.85 be used to compute the design shear capacity of the bolt. ACI (10), however, does not specify what value of  $f_y$  should be used for a bolt having no minimum specified yield strength. Ref. [9] recommends that if  $f_y$  is greater than 0.8 of the minimum specified tensile strength ( $f_u$ ), then  $0.8 f_u$  is used instead of  $f_y$  in Eq. 2.1. In this study, the limitation of  $f_y$  proposed in Ref. [9] is used along with the ACI equations. In fact, for ASTM A307

bolt, which has no definite tensile yield plateau, this recommendation makes the use of  $0.8 f_u$  unavoidable. Based on probabilistic analysis for ordinary bolted connection, Ref. [36] which proposes the LRFD uses the nominal strength equal to 45 ksi ( $0.75 f_u$ ) instead of  $f_y$  in Eq. 2.1, and  $\phi_c = 0.75$  is proposed to compute the design capacity. This proposed design also differs with the strength design in specifying load factors. Instead of 1.4 and 1.7, the LRFD uses 1.2 and 1.6 as dead and live load factors, respectively.

Another approach calculates the shear capacity of the bolt as the product of the cross sectional area of the bolt and the ultimate tensile (18,28,34) or shear strength (26) of the bolt. Using this approach, the PCI (34) design equation for predicting the ultimate shear capacity governed by steel failure is

$$V_s = A_s f_u \quad (\text{Eq. 2.2})$$

and  $\phi_s = 0.75$  is used to compute the design capacity.

The case of concrete failure is more complicated than the case of the steel failure. First, if the length of embedment is not sufficient to sustain the bolt tensile capacity, the connection will fail due to the bolt pulling out of the concrete (28,33), a process which is similar to the concrete failure mode for bolt in tension. To avoid this type of failure, the concrete pull out strength has to be greater than

the minimum specified tensile strength of the bolt (24). Using the ACI design equations (10), the minimum length of embedment to preclude this type of failure can be found by solving the equation

$$4\phi_c l_e (l_e + d_h) \sqrt{f'_c} = A_s f_u \quad (\text{Eq. 2.3})$$

in which  $d_h$  and  $f'_c$  is the diameter of the bolt's head and the concrete compressive strength, respectively. The second case of concrete failure is due to an insufficient edge distance. The critical edge distance for shear loading ( $m_s$ ) can be found by equating the shear capacity governed by concrete failure to the bolt's shear strength (10). Using the ACI design equations (10), the critical edge distance for shear loading is

$$m_s = d_b \sqrt{\frac{f_u}{11.43\phi_c \sqrt{f'_c}}} \quad (\text{Eq. 2.4})$$

in which  $d_b$  is the major thread diameter of the bolt.

For the bolt located beyond the critical edge distance, there are three different recommendations; 1.) concrete failure can be excluded (2,10,26); 2.) the concrete strength can be calculated by shear-friction concept (34); and 3.) the concrete strength is determined from empirical formula based on test results (18,28).

When the edge distance of the bolt is less than the critical edge distance, two different approaches have been re-

commended. One of the approaches (18,28,34) calculate the shear capacity of concrete using empirical formulas. PCI (34) which uses the empirical formula recommends that the ultimate shear capacity governed by concrete failure be computed as

$$V_c = 3250(m - 1)\sqrt{f'_c/5000} \quad (\text{Eq. 2.5})$$

and  $\phi_c = 0.85$  be used to compute the design shear capacity.

Another approach (2,10) assumes that at ultimate load the concrete failure surface can be idealized as a semiconical surface (Figure 2.3). The height of this semiconical surface is approximately equal to the edge distance. Also, the angle of failure surface ( $\alpha$ ) is assumed to radiate 45 degrees from the contact point of the bolt and the surface of the concrete in the direction of the applied force. In Ref. [42], it was observed that this angle is approximately 30 degrees for all cases examined. Actually, tensile stresses in the concrete failure surface vary from maximum ( $6$  or  $7\sqrt{f'_c}$ ) at the embedment to zero at the edge of the concrete (2,28). Conservatively, however, the average stress which equals to  $4\sqrt{f'_c}$  can be used instead of the varying stress (10). These stresses are assumed to act perpendicularly on the projection of the semiconical failure surface to the free edge. Based on these assumptions, ACI Committee 349 (10) recommends that the ultimate shear capacity of concrete be computed as

$$V_c = 2\pi m^2 \sqrt{f'_c} \quad (\text{Eq. 2.6})$$

and  $\phi_c = 0.85$  be used to compute the design shear capacity.

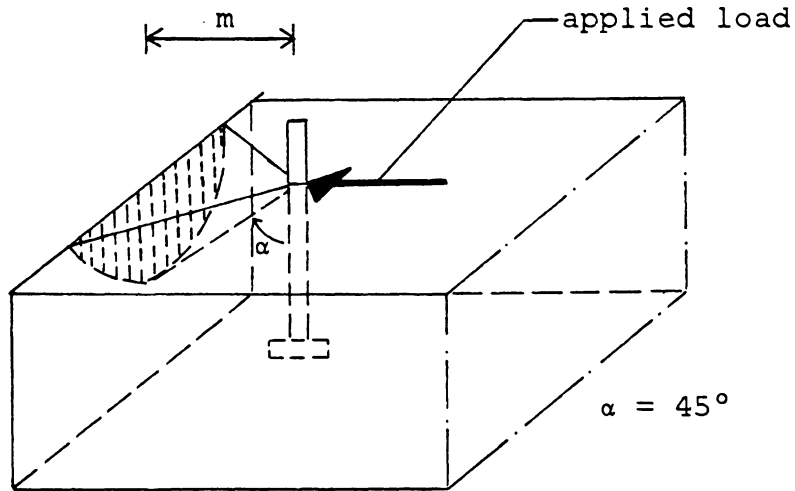


Figure 2.3 Semiconical Concrete Failure Surface

### 2.3.2 Tensile Capacity

Unlike the procedures for predicting the shear capacity, the procedures for predicting the tensile capacity of the anchor bolt only vary slightly.

Most procedures in predicting the tensile capacity governed by the steel failure (2,10,34,35) calculate the bolt tensile capacity as the product of the specified minimum yield strength and the cross sectional area of the bolt.

ACI (10) equation for predicting the bolt tensile resistance is

$$P_s = A_s f_y \quad (\text{Eq. 2.7})$$

and  $\phi_s = 0.90$  is used to compute the design tensile capacity. Furthermore, similar to the equation for predicting the shear capacity for steel failure,  $0.8 f_u$  is used instead of  $f_y$  for ASTM A307 bolt.

Although PCI (34) recommends the same equation as ACI does, there are differences in the values for  $f_y$  and  $\phi_s$ . PCI (34) recommends that  $f_y$  and  $\phi_s$  be taken as  $0.9 f_u$  and  $1.0$ , respectively. LFRD Committee (36), however, proposes that the nominal strength equivalent to  $0.75 f_u$  instead of  $f_y$  and  $\phi_s = 0.75$  be used to compute the design capacity using this equation.

In predicting the tensile capacity governed by concrete failure, all the procedures in the United States (2,10,18,34,35) assume that the failure surface can be idealized by a cone shaped surface. Depending on the distance of the bolt from a free edge, there are three types of concrete failure:

1. If the bolt is located too close to a free edge, a bursting failure of the concrete near the anchor head will occur (2,23).

The critical edge distance for tensile loading ( $m_t$ ) can be found by equating a lateral force, which conservatively equals  $0.25$  of the ultimate tensile

capacity of the bolt, to the concrete shear capacity (10). To prevent this type of failure, the minimum edge distance according to ACI (10) should be

$$m_t = d_b \sqrt{\frac{f_u}{85.33 \phi_c \sqrt{f'_c}}} \quad (\text{Eq. 2.8})$$

for a bolt having net tensile stress area equal to 0.75 of the gross area. However, ACI does not recommend any design equation. Furthermore, other available design recommendations (18,34,35) can not be applied for this type of failure as shown by Klinger and Mendonca (25).

2. If the bolt is located far from a free edge, then a full cone type of concrete failure surface will be developed (Figure 2.4a).

Similar to the assumption regarding the angle of failure surface in predicting the shear capacity for concrete failure, the angle of concrete failure surface can be assumed to radiate 45 degrees from the head of the bolt (10). Although all the procedures in computing the tensile capacity for concrete failure agree on the assumption of an idealized cone failure surface, some of them differ in the selection of the effective area of the cone.

Two procedures (2,10) use the projected surface area of the truncated cone with the average stress  $4\sqrt{f'_c}$  acting on it. ACI Committee 349 (10) recommends that the projected area of the anchor head be excluded in computing the ultimate tensile capacity of concrete resulting in the equation

$$P_c = 4\pi l_e (l_e + d_h) \sqrt{f'_c} \quad (\text{Eq. 2.9})$$

and  $\phi_c = 0.65$  be used to compute the design tensile capacity for the bolt embedded in plain concrete.

On the other hand, other procedures (18,34,35) calculate the tensile capacity as the product of tensile stresses acting parallel to the applied load and the surface area of the truncated cone. Using this assumption, PCI (34) calculates the ultimate tensile capacity of concrete by

$$P_c = 4\pi\sqrt{2} l_e (l_e + d_h) \sqrt{f'_c} \quad (\text{Eq. 2.10})$$

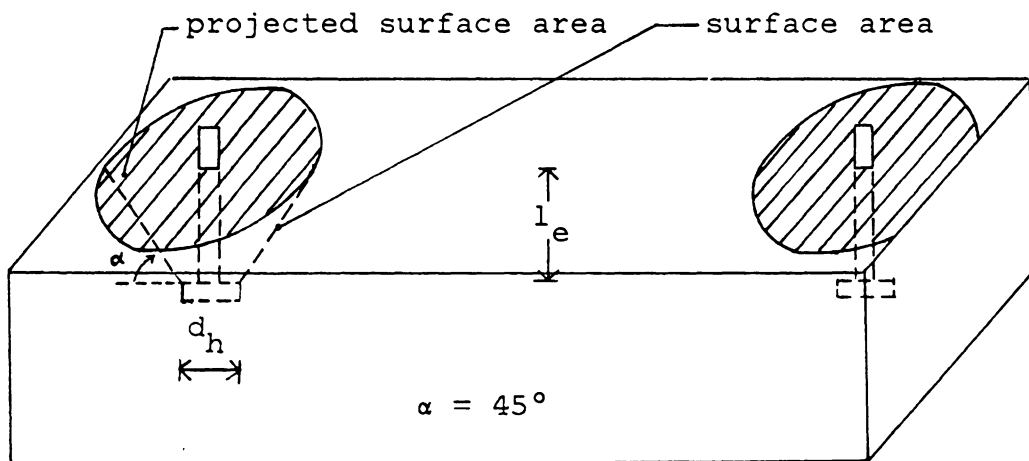
and  $\phi_c = 0.85$  is used to compute the design tensile capacity.

3. If the bolt is located at a point such that it can not develop the full cone failure surface, and is located beyond the critical edge distance, then a partial cone type failure surface will occur (Figure 2.4b). The tensile capacity of concrete for this case can be computed by reducing the full cone tensile capacity.

One procedure consists of reducing the full cone tensile capacity in proportion to the reduction of the surface area (35) or the projected surface area (2,10) of the cone. The reduced tensile capacity according to ACI Committee 349 (10) is

$$P'_c = P_c \left( \frac{\text{projected area of partial cone}}{\text{projected area of full cone}} \right) \quad (\text{Eq. 2.11})$$

Another procedure suggests that the reduction be computed in proportion to the ratio of the edge distance to the length of embedment (34). While still another procedure suggests that the reduction be made in proportion to the ratio of the surface area of a modified cone to the original cone (18).



a. Full cone

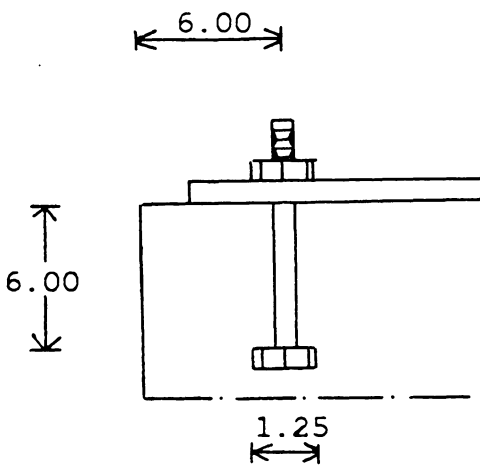
b. Partial cone

Figure 2.4 Cone Shaped Failure Surface

### 2.3.3 Sample Calculations

To clarify the existing differences among available design recommendations, the nominal and the design capacities of a single-headed anchor bolt embedded in plain concrete are calculated using the ACI (10), the PCI (34) and the LRFD (36) equations.

#### Design data



$$\begin{aligned}
 f'_c &= 3,000 \text{ psi} \\
 f_u &= 60,000 \text{ psi} \\
 f_y &= 54,000 \text{ psi} \\
 d_b &= 0.75 \text{ in} \\
 d_h &= 1.25 \text{ in} \\
 l_e &= 6.00 \text{ in} \\
 m &= 6.00 \text{ in} \\
 A_s &= 0.442 \text{ in}^2
 \end{aligned}$$

#### ACI equations

##### -Shear Capacity

Steel failure:  $0.8 f_u < f_y$  (9) and  $\mu = 0.7$

$$\begin{aligned}
 V_s &= \mu A_s f_y = 0.7 \times 0.442 (0.8 \times 60,000) \\
 &= 14,851 \text{ lbs}
 \end{aligned}$$

$$(V_s)_d = \phi_s V_s = 0.85 \times 14,851 = 12,624 \text{ lbs}$$

Concrete failure:

$$\begin{aligned}
 V_c &= 2\pi m^2 \sqrt{f'_c} = 2\pi \times 6^2 \times \sqrt{3,000} \\
 &= 12,389 \text{ lbs}
 \end{aligned}$$

$$(V_c)_d = \phi_c V_c = 0.85 \times 12,389 = 10,531 \text{ lbs}$$

**-Tensile Capacity**

Steel failure:

$$P_s = A_s f_y = 0.442(0.8 \times 60,000)$$

$$= 21,216 \text{ lbs}$$

$$(P_s)_d = \phi_s P_s = 0.9 \times 21,216 = 19,094 \text{ lbs}$$

Concrete failure:

$$P_c = \pi l_e (l_e + d_h) 4\sqrt{f'_c} = 6\pi(6 + 1.25) \times 4\sqrt{3,000}$$

$$= 29,941 \text{ lbs}$$

$$(P_c)_d = \phi_c P_c = 0.65 \times 29,940 = 19,461 \text{ lbs}$$

PCI equations**-Shear Capacity**

Steel failure:

$$V_s = A_s f_u = 0.442 \times 60,000$$

$$= 26,520 \text{ lbs}$$

$$(V_s)_d = 0.75 V_s = 19,890 \text{ lbs}$$

Concrete failure:

$$V_c = 3,250(m - 1)\sqrt{f'_c/5,000}$$

$$= 12,857 \text{ lbs}$$

$$(V_c)_d = 0.85 V_c = 10,928 \text{ lbs}$$

**-Tensile Capacity**

Steel failure:

$$P_s = A_s f_y = 23,868 \text{ lbs}$$

$$(P_s)_d = 1.0 P_s = 23,868 \text{ lbs}$$

Concrete failure:

$$P_c = 4\pi\sqrt{2} l_e (l_e + d_n) \sqrt{f'_c}$$

$$= 42,342 \text{ lbs}$$

$$(P_c)_d = 0.85 P_c = 35,990 \text{ lbs}$$

LRFD equations

-Shear Capacity

Steel failure:  $\mu = 0.7$

$$V_s = \mu A_s (0.75 f_u) = 0.7 \times 0.442 (0.75 \times 60,000)$$

$$= 13,917 \text{ lbs}$$

$$(V_s)_d = \phi_s V_s = 0.75 \times 13,917 = 10,437 \text{ lbs}$$

-Tensile Capacity

Steel failure:

$$P_s = A_s (0.75 f_u) = 0.442 (0.75 \times 60,000)$$

$$= 19,881 \text{ lbs}$$

$$(P_s)_d = \phi_s P_s = 0.75 \times 19,881 = 14,911 \text{ lbs}$$

The results show that PCI (34) gives higher nominal and design capacities for both shear and tensile resistances. There is a significant difference between the ACI and the PCI equations for the prediction the tensile capacity of concrete. While ACI and PCI specify  $\phi R_n \geq 1.4D_n + 1.7L_n$  as a design requirement, the LRFD specifies 1.2 and 1.6 for dead and live load factors, respectively.

Table 2.1 Capacities of the Anchor Bolt Computed Using  
ACI, PCI and LRFD Equations

Capacity	A C I		P C I		L R F D	
	Nominal (lbs)	Design (lbs)	Nominal (lbs)	Design (lbs)	Nominal (lbs)	Design (lbs)
<u>Shear</u>						
Steel	14,851	12,634	26,520	19,890	13,917	10,437
Concrete	12,389	10,531	12,857	10,928	---	---
<u>Tensile</u>						
Steel	21,216	19,094	23,868	23,868	19,881	14,911
Concrete	29,941	19,461	42,342	35,990	---	---

## Chapter III

### REVIEW OF LITERATURE ON RELIABILITY ANALYSIS

#### 3.1 GENERAL

In the last decade, there has been considerable interest in the development and application of reliability analysis, and numerous papers have been published on the subject. This is due to the fact that most problems in engineering design are non-deterministic and inevitably involve uncertainties brought about by the randomness of physical phenomena. This randomness arises from the inherent variability of design variables and the imperfect estimation of reality. To cope with these uncertainties, reliability analysis uses random variables instead of deterministic values.

Statistical information on the random variable is needed to describe its probability law. The required information can be obtained by gathering available data from test results. In practice, the data gathered may be insufficient, but this is not a reason to avoid the reliability analysis, since reliability analysis techniques also allow for the inclusion of objective information in the formulation (3).

Several proposals for reliability-based design have been made (16,36). In contrast to the traditional design method, in which safety is represented in terms of the safety factor

and the load factors, safety in the reliability-based design method is measured by the probability of failure. In practice, however, the probability of failure does not appear explicitly in the reliability-based design equations. Also, the form of the design equation is the same as the current ultimate strength design approach, which is

$$\phi R \geq \sum_{i=1}^n \gamma_i Q_i \quad (\text{Eq. 3.1})$$

in which  $R$  and  $Q_i$  are the resistance and load effects, while  $\phi$  and  $\gamma_i$  are the resistance and load factors, respectively. The distinctive feature of the probability-based approach is that the resistance and load factors, which are determined subjectively in traditional design approaches, are determined through reliability analysis.

Load and resistance factors based on the probabilistic approach have been determined for steel structures (37), concrete structures (12,13), and connectors (19). Also, studies covering all types of construction materials have been conducted (15,20). Instead of specifying a new level of reliability, current reliability analysis procedures estimate risk levels inherent in current design codes. This procedure is called code calibration (11,27). Then, after observing these levels of reliability in practical design situations, the target of reliability is chosen as a guide for the determination of probability-based load and resistance factors.

The advantages of the procedure just described are that the probability of failure need not to be specified, and the same level of safety is maintained as in the traditional approach. Thus, calibration is an important step in the development of probabilistic-based design procedures. It also gives an important overview of the level of reliability underlying current design practice.

For structural engineering problems, if a limit state can be modelled to be the function of resistance ( $R$ ) and load effects ( $Q_i$ ), then the reliability is defined as the probability that the load effects do not exceed the resistance of the structure. In other words, the reliability is the probability of non performing failure. Consequently, the probability of failure is the basic formulation in the reliability analysis. The term "failure" does not necessarily imply collapse; it should be regarded as a state which exceeds a predefined limit state.

In general, limit states can be grouped into two categories: 1.) ultimate limit states such as collapse; and 2.) serviceability limit states such as an excessive deflection. Mathematically, the limit state can be expressed as

$$f(R, Q_1, \dots, Q_n) = 0 \quad (\text{Eq. 3.2})$$

in which  $R$  and  $Q_i$  are resistance and load effect, respectively. Hence, associated probability of failure is

$$p_f = P(\text{failure}) = P(f \leq 0) \quad (\text{Eq. 3.3})$$

When the probability distributions of  $R$  and  $Q_i$  are known, the probability of failure is computed by

$$p_f = \int \dots \int f_x(R, Q_1, \dots, Q_n) \text{drdq}_1 \dots \text{dq}_n \quad (\text{Eq. 3.4})$$

where  $f_x$  is the joint probability distributions of  $R, Q_1, \dots, Q_n$ , and the integration is performed over the domain of  $f(R, Q_i) < 0$ .

In practice, however, the available information is frequently insufficient to obtain such probability distributions; it is just enough to estimate the means and the variances of  $R$  and  $Q_i$ . Furthermore, the integration may be very complicated when the function is non-linear. These difficulties have resulted in the development of an approximate method, which is the first-order second-moment method (FOSM) reliability analysis.

In the first application of the FOSM (3,11), the limit state equation was evaluated at the mean value of  $X_i$ . This procedure, which is called the mean value FOSM, results in a significant error when  $f(X)$  is non linear, and the computation of the reliability index depends on how the limit state is formulated (22). The problem posed by the mean FOSM has led to the development of the advanced FOSM (22,39), in which the limit state equation is evaluated at a point on the failure surface instead at the mean values of design variables.

### 3.2 ADVANCED FIRST-ORDER SECOND-MOMENT METHOD

The FOSM was developed mainly from the second-moment reliability or the safety index concept (3,11). Although this method which was developed by Cornell (11) is based on approximation, the reliability can be accurately assessed by the FOSM. As reflected in its name, the computation of the reliability in the FOSM involves only the means and variances of random variables, i.e. the first and the second moment, respectively. Therefore, the FOSM procedure does not require a knowledge of the distributions of design variables. Instead of measuring the reliability by the probability of failure, reliability in the FOSM is measured by a coefficient called the safety or reliability index ( $\beta$ ) which can be related to the probability of failure, if all probability distributions are known. This concept is described as follows.

Let  $f(R,Q) = R - Q$  be the performance function of a system with a probability density distribution  $f_{R-Q}$ . Let  $f(R,Q) < 0$  denote the failure of a system, whereas  $f(R,Q) > 0$  is the safe state of the system. Then,  $f(R,Q) = 0$  is known as the limit state equation for the system. If the probability density function is known, then the probability of failure of the system, which is the shaded area shown in Figure 3.1, can be computed. The probabilistic design approach requires

that the probability of failure be smaller than some acceptable values. In the second-moment concept, the requirement is substituted by a condition that the distance from  $\overline{R-Q}$  to the origin ( $R-Q = 0$ ) is greater than the product of a reliability index ( $\beta$ ) and the standard deviation of  $R-Q$  ( $\sigma_{R-Q}$ ). Mathematically, this can be expressed as

$$\overline{R-Q} \geq \beta \sigma_{R-Q} \quad (\text{Eq. 3.5})$$

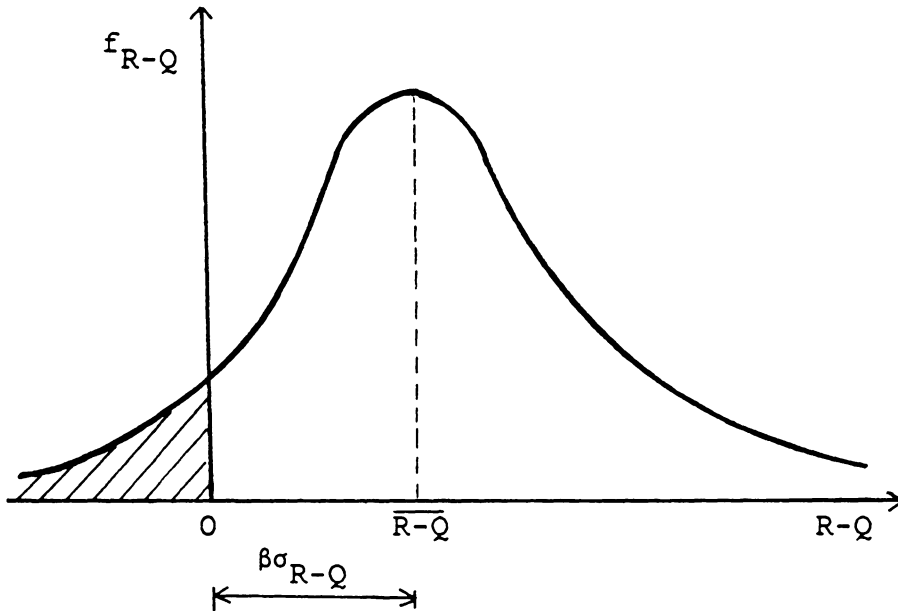


Figure 3.1 Geometric Interpretation of the Probability of Failure and the Reliability Index.

It can be seen from Fig. 3.1 that as the distance between  $\overline{R-Q}$  to the origin is increased, the shaded area will be decreased accordingly. As a matter of fact,  $\beta$  is a measure of the event of failure or  $f(R, Q) < 0$ . For example, as  $\beta$  gets larger, the probability of failure gets smaller. Thus, in-

stead of the probability of failure, the safety index is used to measure the reliability in the second-moment concept.

Introducing reduced variables with zero mean and unit variance

$$x_i = (X_i - \bar{X}_i) / \sigma_{X_i} \quad (\text{Eq. 3.6})$$

where  $X_i$  represents either  $R$  or  $Q$ , the second-moment concept measures the reliability by the distance from the failure surface to the origin of a reduced variables coordinate system (22). Figure 3.2 shows a limit state equation  $f(r,q) = 0$  in the coordinate system of the reduced variables  $r$  and  $q$ . Similar to the shift in the probability distribution function, the shift of limit state function from the origin changes the distance and the area of safe region  $f(r,q) > 0$  accordingly. Hence, the reliability of the system can be represented by the position of failure surface, which is defined as the minimum distance from the origin of the reduced variables coordinate system to the failure surface (4,39).

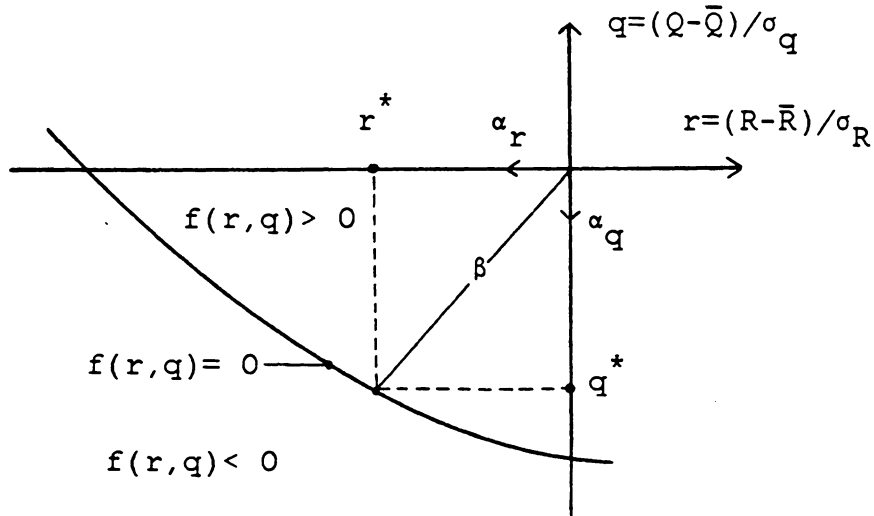


Figure 3.2 Limit State Function in a Two Reduced Variables Coordinate System.

Generally, the performance function of a system involves several basic variables such as

$$f(X) = f(X_1, \dots, X_n) \quad (\text{Eq. 3.7})$$

in which  $X_i$  represents either the resistance or the load variable in a structural engineering problem. These are usually assumed to be statistically independent. The limit state of the system is governed by  $f(X) = 0$  which is the failure surface of the system. For the performance function with  $n$  variables, the failure surface can be interpreted geometrically as an  $n$ -dimensional surface.

The first-order approximation is done by expanding  $f(X)$  in a Taylor series at a point  $X^*$  on the failure surface,

$$\begin{aligned}
f(X_1, \dots, X_n) &= f(X_1^*, \dots, X_n^*) + \sum_{i=1}^n (X_i - X_i^*) (\partial f / \partial X_i) \\
&+ \sum_{i=1}^n \sum_{j=1}^n (X_i - X_i^*) (X_j - X_j^*) / (\partial^2 f / \partial X_i \partial X_j) + \dots \quad (\text{Eq. 3.8})
\end{aligned}$$

and on the failure surface

$$f(X_1^*, \dots, X_n^*) = 0 \quad (\text{Eq. 3.9})$$

in which the derivatives are evaluated at point  $(X_1^*, \dots, X_n^*)$ . Considering Eq. 3.9, and then truncating the series at the first-order term, Eq. 3.8 becomes

$$f(X_1, \dots, X_n) = \sum_{i=1}^n (X_i - X_i^*) (\partial f / \partial X_i) \quad (\text{Eq. 3.10})$$

Furthermore, Eq. 3.6 gives

$$X_i - X_i^* = \sigma_{x_i} (x_i - x_i^*) \quad (\text{Eq. 3.11})$$

and

$$\partial f / \partial X_i = (\partial f / \partial x_i) (\partial x_i / \partial X_i) = (\partial f / \partial x_i) / \sigma_{x_i} \quad (\text{Eq. 3.12})$$

Hence,

$$f(X_1, \dots, X_n) = \sum_{i=1}^n (x_i - x_i^*) (\partial f / \partial x_i) \quad (\text{Eq. 3.13})$$

The geometric interpretation of this first-order approximation in a two reduced variables coordinate system is shown in Fig. 3.3. This approximation can be interpreted as linearizing the failure surface by a tangent plane to the failure surface at point  $x^*$  in the reduced variables coordinate system. By definition, the reliability index of the system is equivalent to the distance from the origin to the most probable failure point which is the point on the failure

surface with the minimum distance to the origin (4,39). Denoting the reliability index by  $\beta$ , then

$$x_i^* = -\alpha_i^* \beta \quad (\text{Eq. 3.14})$$

where  $\alpha_i^*$  is the direction cosines, which is derived by imposing the minimum distance and  $f(x) = 0$ . Using Lagrange multiplier, the direction cosines can be computed as (4,39)

$$\alpha_i^* = (\partial f / \partial x_i) / [\sum_{i=1}^n (\partial f / \partial x_i)^2]^{1/2} \quad (\text{Eq. 3.15})$$

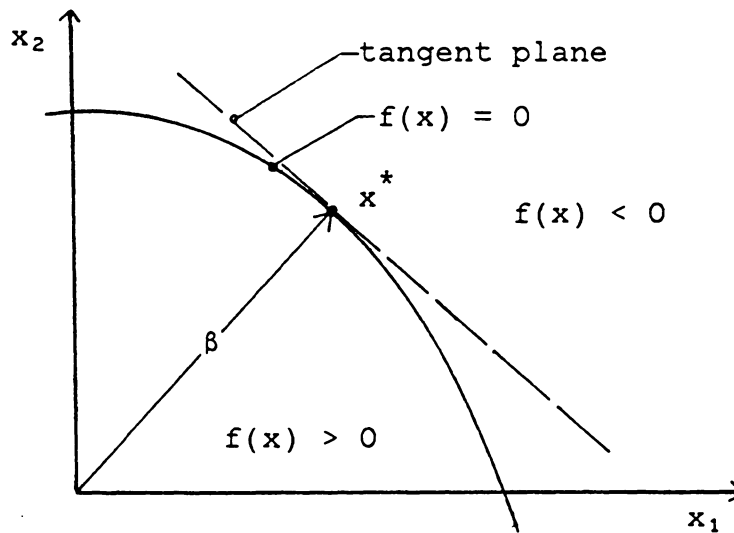


Figure 3.3 Linearization of the Failure Surface by a Tangent Plane at Point  $x^*$

Obviously, the computation of the reliability index requires that Eqs. 3.9, 3.14, and 3.15 be solved simultaneously, where  $\alpha_i^*$  is found by an iterative procedure to minimize  $\beta$ . Furthermore, the limit state function and its derivative are evaluated at point  $x^*$  on the failure surface.

Load and resistance factors can be obtained for a specified value of  $\beta$  by using a procedure similar to that described above. To get the equation for these factors, Eq. 3.6 is rewritten in terms of the reduced variables at point  $x^*$  on the failure surface resulting in

$$X_i = x_i^* \sigma_{x_i} + \bar{X}_i \quad (\text{Eq. 3.16})$$

where the value of  $x_i^*$  also has to satisfy Eq. 3.14, thus

$$X_i = \bar{X}_i - \alpha_i^* \beta \sigma_{x_i} \quad (\text{Eq. 3.17})$$

Since  $\sigma_{x_i}$  is equal to  $V_{x_i} \bar{X}_i$ , the following equation is obtained from Eq. 3.17

$$X_i = \bar{X}_i (1 - \alpha_i^* \beta V_{x_i}) \quad (\text{Eq. 3.18})$$

and denoting the coefficient in parentheses by  $\bar{\gamma}_i$ ,

$$\bar{\gamma}_i = 1 - \alpha_i^* \beta V_{x_i} \quad (\text{Eq. 3.19})$$

Thus,

$$X_i = \bar{\gamma}_i \bar{X}_i \quad (\text{Eq. 3.20})$$

The value of  $X_i$  has to satisfy the limit state equation  $f(X)$ , hence

$$f(\bar{\gamma}_1 \bar{X}_1, \dots, \bar{\gamma}_n \bar{X}_n) = 0 \quad (\text{Eq. 3.21})$$

Considering the last equation, it is clear that  $\bar{\gamma}_i$  is the partial safety factor on the mean value, which can be either a load or a resistance factor. To find the partial safety factor on the nominal value, the ratio of the nominal to the mean value is used,

$$\gamma_i = (\bar{X}_i / X_i) \bar{\gamma}_i \quad (\text{Eq. 3.22})$$

in which  $\gamma_i$  is the factor on the nominal value  $X_i$ . In short, these factors are determined from Eq. 3.19 after solving Eqs. 3.9, 3.14, 3.15 for a fixed value of  $\beta$ .

### 3.3 ANALYSIS OF UNCERTAINTY

#### 3.3.1 Estimation of Statistical Parameters

In reliability analysis, uncertainty must be prescribed by a quantitative measure. A convenient and a simple way to measure uncertainty is through the dispersion with respect to the mean value of a random variable (3,11). Therefore, statistical information on random variables is necessary to evaluate uncertainty.

A random variable is completely described, if its probability distribution and corresponding parameters are known. In the context of the FOSM, the random variable is described approximately by its mean and its coefficient of variation. In this sense, the mean constitutes the average value of the random variable, and the variance is a measure of dispersion with respect to the mean value.

In general, the resistance of a structural member is a function of several random variables such as the cross sectional area, the yield strength and so on. Mathematically, this can be written as

$$F = f(X_1, \dots, X_n) \quad (\text{Eq. 3.23})$$

and the mean of this function can be computed by (4)

$$\mu_f = E[F] = \int \dots \int f(x_1, \dots, x_n) f_x(x_1, \dots, x_n) dx_1 \dots dx_n \quad (\text{Eq. 3.24})$$

It is obvious that the computation of Eq. 3.24 is very complicated even though all probability distribution functions are known. Therefore, an approximate method called the first-order approximation is employed in finding the mean and the variance.

The approximation is done by expanding  $F$  in a Taylor series with respect to the mean values of variables,  $\mu_{x_1}, \dots, \mu_{x_n}$ , (4),

$$F = f(\mu_{x_1}, \dots, \mu_{x_n}) + \sum_{i=1}^n (X_i - \mu_{x_i}) (\partial f / \partial X_i) + 1/2 \sum_{i=1}^n \sum_{j=1}^n (X_i - \mu_{x_i}) (X_j - \mu_{x_j}) (\partial^2 f / \partial X_i \partial X_j) + \dots \quad (\text{Eq. 3.25})$$

in which the partial derivatives are evaluated at  $\mu_{x_i}$ . Then, neglecting the non linear terms from the series, the approximate mean value of  $F$  is

$$\mu_f = E[F] \approx f(\mu_{x_1}, \dots, \mu_{x_n}) \quad (\text{Eq. 3.26})$$

and the approximate variance of  $F$  is

$$\text{Var}(F) \approx \sum_{i=1}^n (\partial f / \partial X_i)^2 \text{Var}(X_i) + \sum_{i \neq j} (\partial f / \partial X_i) (\partial f / \partial X_j) \text{Cov}(X_i, X_j) \quad (\text{Eq. 3.27})$$

in which  $\text{Cov}(X_i, X_j)$  is the covariance of  $X_i$  and  $X_j$ . For statistically independent random variables, the  $\text{Cov}(X_i, X_j)$  is zero and the variance of  $F$  becomes

$$(\sigma_f)^2 = \text{Var}(F) \approx \sum_{i=1}^n (\partial f / \partial X_i)^2 \text{Var}(X_i) \quad (\text{Eq. 3.28})$$

On the basis of the variance, however, it is difficult to know whether the dispersion is large or small. In order to interpret the dispersion easily, the dispersion relative to the mean value called the coefficient of variation (cov) is usually used. The cov is a non dimensional coefficient, and can be expressed as

$$V_f = \sigma_f / \mu_f \quad (\text{Eq. 3.29})$$

### 3.3.2 Evaluation of Uncertainty

Uncertainties in engineering design can be grouped into two different types: 1.) those due to inherent variability; and 2.) those due to prediction errors. An example of uncertainty due to the inherent variability is the strength of the material. This type of uncertainty can not be reduced by obtaining additional data. On the other hand, prediction errors result from the imperfect prediction of the actual situation, such as an idealized mathematical equation in predicting structural capacity. This type of uncertainty can be reduced by refining the model and the prediction through

additional information gained by experience and testing (4,20).

Consider the case of a random variable. Mathematically, the variable  $X$  can be modelled by (3)

$$X = N \hat{X} \quad (\text{Eq. 3.30})$$

where  $\hat{X}$  is a random variable with mean  $\bar{X}$  and a cov  $V_{\hat{X}}$ .  $N$  is also a random variable representing errors in estimating  $\bar{X}$ , and has a mean equal to unity and a cov  $V_n$ . If  $\bar{X}$  is obtained from observational data of  $n$  sample values and only a sampling error is taken into account, then  $V_n$  represents uncertainty due to sampling, which is equal to (4,11)

$$V_n = V_{\hat{X}}/\sqrt{n} \quad (\text{Eq. 3.31})$$

Hence, the total uncertainty in  $X$  based on Eqs. 3.28 and 3.29 is

$$V_x = (V_n^2 + V_{\hat{X}}^2)^{1/2} \quad (\text{Eq. 3.32})$$

In addition to the sampling error, there are other prediction errors such as differences between field and laboratory test results. For example, the in-situ concrete compressive strength is not the same as the cylinder concrete compressive strength. This type of prediction error, if any, should be added to the total uncertainty in Eq. 3.32; it can be represented by a bias factor ( $v$ ). The mean value of this bias factor is obtained as the mean of the ratio of the test to the predicted strength, and its coefficient of variation is  $V_v$ .

Similarly, a function of several random variables can be modelled as

$$R = N_R f(X_1, \dots, X_n) \quad (\text{Eq. 3.33})$$

in which  $N_R$  is the ratio of the test result of  $R$  to the predicted value of  $R$ . In other words,  $N_R$  accounts for any errors in modelling  $R$  by an idealized mathematical equation,  $f(x)$ , which has a mean  $\bar{N}_R$  and a cov  $V_{N_R}$ . The function  $f(X)$  itself is a random variable with mean  $\bar{f}(X)$  and the cov  $V_f$ . Thus, the total uncertainty in  $R$  is

$$V_R = (V_f^2 + V_{N_R}^2)^{1/2} \quad (\text{Eq. 3.34})$$

$N_R$  is also a random variable. One source of uncertainty in  $N_R$  is due to variations in testing procedures such as inaccuracies in reading and gages. This uncertainty is represented by  $V_{\text{test}}$ . Another source of uncertainty is due to differences between the specified and the actual values used in the test such as between the specified strength and the actual strength of specimens. This uncertainty is represented by  $V_{\text{spec}}$ . As a matter of fact, the cov of the ratio of the measured to predicted  $R$  obtained directly from test results,  $V_{a/p}$ , includes all the above uncertainties (17,21). If  $V_{N_R}$  represents the true cov of  $N_R$ , then

$$V_{a/p}^2 = V_{N_R}^2 + V_{\text{test}}^2 + V_{\text{spec}}^2$$

Hence, the true cov of  $N_R$  can be computed by

$$V_{N_R} = (V_{a/p}^2 - V_{\text{test}}^2 - V_{\text{spec}}^2)^{1/2} \quad (\text{Eq. 3.35})$$

while the mean value of  $N_R$  is equal to the average value of the ratio of the measured R to the predicted R. Studies on reinforced concrete (21) indicate that  $V_{spec}$  can be approximately taken as 0.04, whereas  $V_{test}$  ranges from 0.02 to 0.04 (17). In this study, the value of  $V_{test}$  is assumed to be 0.04.

## Chapter IV

### STATISTICAL ANALYSIS OF DESIGN VARIABLES

#### 4.1 LOAD EFFECT VARIABLES

Considerable research on determining statistical information on loads and load effects for reliability analysis has been conducted. In this study, statistical information on load effect variables were obtained from Ref. [17]. The loads effects considered are the dead and lifetime maximum load effects. It is assumed that these load effects are mutually statistically independent.

Uncertainty in the load effect variables are due not only to inherent variability, but also due to prediction errors. For instance, the transformation of the load into the load effect is usually accomplished by idealizations in structural analysis process. Including these uncertainties, the statistics of load effects which are obtained from on-site surveys and measurements are as follows (17):

##### 1. Dead Load Effects

It was found that the ratio of the mean to the nominal dead load effects is 1.05, and the cov  $V_D$  is equal to 0.10.

##### 2. Lifetime Maximum Live Load Effects

The nominal live load according to ANSI Standard A58.1-1983 (7) is

$$L_n = L_o (0.25 + 15/\sqrt{A_I}) \quad (\text{Eq. 4.1})$$

in which  $L_o$  is a basic unreduced live load, and  $A_I$  is the influence area which is equal to two times the tributary area for beams and four times the tributary area for columns. Using this nominal live load and considering lifetime maximum live load for a 50 year period, the ratio of the mean to the nominal lifetime maximum live load is 1.00 and the cov  $V_L \approx 0.25$ .

#### 4.2 RESISTANCE VARIABLES

Resistance variables can be of two types. These are: 1.) basic resistance variables such as the tensile strength of the bolt; and 2.) resistance model variables such as the tensile capacity of the anchor bolt which are a function of the basic resistance variables. Some of the statistics of the basic resistance variables are obtained from previous studies, while the statistics for other basic resistance variables are obtained through a statistical analysis of available data combined with judgement. Statistics of the resistance model variables were derived from an analysis of actual test data. Test results were gathered from experiments on anchor bolts and welded studs conducted by several

investigators. Details of the tests are given in Refs. [2,5,26,28,30,31,32,42]. The predicted capacity was computed by using the observed values of the basic resistance variables. However, when these observed values are not specified in the test reports, the estimated mean values of the basic resistance variables were used in the computation of the predicted capacity.

The statistical analysis of design variables used in the prediction of the capacity of anchor bolts are given in the following sections. The statistical analysis was performed using the procedures described in Section 3.3. It is assumed that all resistance variables are mutually statistically independent. This assumption is based on the fact that the effect of correlation on the total cov of a resistance model variable consisting of several basic variables with small covs is negligible (37).

#### 4.2.1 Basic Resistance Variables

For steel failure, two basic resistance variables are the cross sectional area ( $A_s$ ) and the ultimate tensile strength of the bolt ( $f_u$ ). Statistical information on the tensile area and the tensile strength of ASTM A307 bolt is currently not available. Therefore, available information on other basic resistance variables similar to  $f_u$  and  $A_s$  is used to determine the statistics of ASTM A307 bolt.

A study on connectors (19) showed that when there is good control in the manufacturing of bolts, the cov of  $A_s$  ( $V_{as}$ ) can be taken as 0.05. In the same study, based on an analysis of test results, the ratio of the mean to the nominal tensile strength and the cov of the tensile strength of ASTM A325 bolts were found to be 1.20 and 0.07, while for ASTM A490 bolts these values were 1.07 and 0.02, respectively. Ellingwood (13) found that the total cov of the product of the steel area and the yield strength ( $A_s f_y$ ) was 0.11. In another study by Ellingwood (14), it was estimated that the cov of the yield strength of Grade 60 steel was 0.14 with the mean equal to 1.07 times its nominal value, and the cov of the reinforcing steel area was 0.036.

In this study, test results on the tensile capacity of welded studs and ASTM A307 bolt are analyzed and are summarized in Tables 4.1 and 4.2. The actual test capacity is denoted by  $P_{sa}$ , while the predicted capacity computed by the PCI design equation (Eq. 2.2) is denoted by  $P_s$ . The mean bias factor in the prediction equation ( $\bar{N}_{ps}$ ) is obtained by computing the mean of the ratio of  $P_{sa}$  to  $P_s$ . From Table 4.1, the mean bias factor for the tensile capacity of welded studs is 1.152, and the cov of this bias factor is 0.090. On the other hand, from Table 4.2, the mean of the ratio of the the actual test to the predicted tensile capacity of ASTM

A307 bolts is 1.4081 with the cov = 0.088, when the nominal tensile strength ( $f_u = 60$  ksi) is used to predict the tensile capacity. Based on this observation, the ratio of the mean to the nominal tensile strength of ASTM A307 bolt can be estimated by assuming that the mean bias factors in predicting the tensile capacity of A307 bolts and welded studs are the same, thus

$$1.152\bar{A}_s\bar{f}_u = 1.408A_s(0.9f_u)$$

where  $\bar{A}_s$  and  $\bar{f}_u$  are the mean values of  $A_s$  and  $f_u$ , respectively. Assuming that  $\bar{A}_s/A_s$  is equal to unity, the ratio of the mean to the nominal tensile strength is

$$\bar{f}_u/f_u = 1.100$$

On the basis of this information, the ratio of the mean to the nominal tensile strength for ASTM A307 bolt is assumed to be 1.10, and its cov is assumed to be 0.10. For the cross sectional area of the bolt, its cov is taken to be 0.04, and its mean is assumed to be equal to its nominal value.

Table 4.1 Analysis of Test Results on Tensile Capacity

## Welded Studs

No.	Ref.	Grade	$f_u$	$d$	$A_s$	$P_{sa}$	P C I	
			(ksi)	(in)	(in <sup>2</sup> )	(kips)	$P_s^*$	$P_{sa}/P_s$
1	28	Stud	70.9	0.750	0.4418	28.30	28.19	1.004
2	28	Stud	70.9	0.750	0.4418	28.50	28.19	1.011
3	28	Stud	70.9	0.750	0.4418	28.00	28.19	0.993
4	28	Stud	70.9	0.750	0.4418	31.50	28.19	1.117
5	28	Stud	70.9	0.750	0.4418	29.30	28.19	1.039
6	28	Stud	70.9	0.750	0.4418	29.30	28.19	1.039
7	28	Stud	70.9	0.750	0.4418	28.80	28.19	1.022
8	28	Stud	70.9	0.750	0.4418	31.50	28.19	1.177
9	28	Stud	70.9	0.750	0.4418	29.40	28.19	1.043
10	28	Stud	70.9	0.750	0.4418	27.30	28.19	0.968
11	28	Stud	70.9	0.750	0.4418	28.70	28.19	1.019
12	30	Stud	77.4	0.250	0.0491	3.70	3.42	1.082
13	30	Stud	79.7	0.375	0.1104	8.50	7.92	1.073
14	30	Stud	79.7	0.375	0.1104	8.70	7.92	1.098
15	30	Stud	76.0	0.500	0.1960	12.50	13.41	0.932
16	30	Stud	76.0	0.500	0.1960	15.00	13.41	1.118
17	30	Stud	73.0	0.625	0.3068	23.70	20.16	1.176
18	30	Stud	73.0	0.625	0.3068	25.00	20.16	1.240
19	30	Stud	73.0	0.625	0.3068	25.00	20.16	1.240
20	32	Stud	77.4	0.250	0.0491	3.38	3.42	0.988

No.	Refer.	Grade	$f_u$	$d$	$A_s$	$P_{sa}$	P C I	
			(ksi)	(in)	(in <sup>2</sup> )	(kips)	$P_s^*$	$P_{sa}/P_s$
21	32	Stud	77.4	0.250	0.0491	4.00	3.42	1.170
22	32	Stud	79.7	0.375	0.1104	10.60	7.92	1.338
23	32	Stud	79.7	0.375	0.1104	9.50	7.92	1.199
24	32	Stud	76.0	0.500	0.1960	16.30	13.41	1.216
25	32	Stud	76.0	0.500	0.1960	15.60	13.41	1.163
26	32	Stud	73.0	0.625	0.3068	23.80	20.16	1.180
27	32	Stud	73.0	0.625	0.3068	22.00	20.16	1.091
28	32	Stud	73.0	0.625	0.3068	24.40	20.16	1.210
29	32	Stud	75.2	0.750	0.4418	37.50	29.88	1.255
30	32	Stud	75.2	0.750	0.4418	38.10	29.88	1.275
31	32	Stud	76.8	0.875	0.6013	47.00	41.58	1.130
32	32	Stud	76.8	0.875	0.6013	45.00	41.58	1.082

$$\bar{N}_{ps} = (P_{sa}/P_s)_m = 1.152$$

$$(V_{a/p})_{ps} = 0.090$$

$$* P_s = A_s(0.9f_u)$$

Table 4.2 Analysis of Test Results on Tensile Capacity  
ASTM A307 Bolts

No.	Ref.	Grade	$f_u$	$d$	$A_s$	$P_{sa}$	P C I	
			(ksi)	(in)	(in <sup>2</sup> )	(kips)	$P_s^*$	$P_{sa}/P_s$
1	2"	A307	60.0	0.750	0.3345	26.10	18.06	1.445
2	2	A307	60.0	0.750	0.3345	26.20	18.06	1.451
3	2	A307	60.0	0.750	0.3345	25.40	18.06	1.406
4	2	A307	60.0	0.750	0.3345	26.30	18.06	1.456
5	2	A307	60.0	0.750	0.3345	29.60	18.06	1.639
6	2	A307	60.0	0.750	0.3345	23.20	18.06	1.285
7	2	A307	60.0	0.750	0.3345	24.40	18.06	1.351
8	2	A307	60.0	0.750	0.3345	21.00	18.06	1.163
9	2	A307	60.0	0.750	0.3345	26.00	18.06	1.439
10	2	A307	60.0	0.750	0.3345	26.10	18.06	1.445

$$(P_{sa}/P_s)_m = 1.152$$

$$(V_{a/p})_{ps} = 0.088$$

$$* P_s = A_s(0.9f_u)$$

" Threaded bolts

Four basic resistance variables are involved in predicting the capacity of the anchor bolt governed by concrete failure. These are: 1.) concrete compressive strength; 2.) the edge distance; 3.) the length of embedment; and 4.) the diameter of the bolt's head.

Due to the fact that the mean and the cov of concrete compressive strength may vary for different grades of concrete, this study is limited to the case of 4000 psi concrete ( $f'_c = 4000$  psi). Based on study on the strength of concrete (14,17,29), it was obtained that the mean concrete cylinder strength is 4700 psi for 4000 psi concrete (14), so that the ratio of the mean to the nominal concrete cylinder strength is

$$\bar{f}'_c / f'_c = 4700 / 4000 = 1.175$$

and the cov of in-situ concrete compressive strength is (29)

$$V_{fc} = (V_{ccyl}^2 + 0.0084)^{1/2} \quad (\text{Eq. 4.2})$$

in which  $V_{ccyl}$  is the cov of the concrete cylinder strength. It was found that  $V_{ccyl}$  ranges from 0.1 to 0.2 (29), and it can be taken as 0.15 for average control (17). Substitution of  $V_{ccyl}$  in Eq. 4.2 yields  $V_{fc} = 0.18$ .

The factors which contribute to the uncertainty in the edge distance are similar to those for bar placement. However, an anchor bolt can be installed more accurately in the field than a reinforcing bar. The cov of the edge distance

thus should be smaller than that of the effective depth of a reinforced concrete member. The cov of the effective depth is (13)

$$0.68/h_n$$

where  $h_n$  is the minimum member dimension in inches. It was shown in Ref. [13] that the cov of the dimensions of a concrete member is smaller than the cov of the effective depth. Considering this, the cov of the edge distance is assumed to be the same as the cov of a concrete member dimension, which is (13)

$$V_m = 0.4/h_n \quad (\text{Eq. 4.3})$$

and the mean value of  $m$  is assumed to be the same as its nominal value. In practice, the smallest dimension of a structural member is 12" so that  $V_m$  can be taken as

$$V_m = 0.4/12 = 0.03$$

Little or no data is available on the uncertainty in the embedment length ( $l_e$ ). In this study, the length of embedment is assumed conservatively to have a uniform probability distribution function with a deviation of -0.5" to + 0.5" from the specified embedded depth ( $l'_e$ ). This distribution is shown in Fig. 4.1, in which  $a$  and  $b$  denote ( $l'_e - 0.5$ ) and ( $l'_e + 0.5$ ), respectively. Using this distribution, the cov of  $l_e$  was found to be (4)

$$V_{l_e} = (b - a)/\sqrt{3}(a + b) \quad (\text{Eq. 4.4})$$

and the mean value of  $l_e$  is equal to its nominal value. For an embedment length ranging from 4" to 10", the value of  $V_{l_e}$  ranges from 0.058 to 0.029. Thus, conservatively  $V_{l_e}$  is assumed to be 0.05.

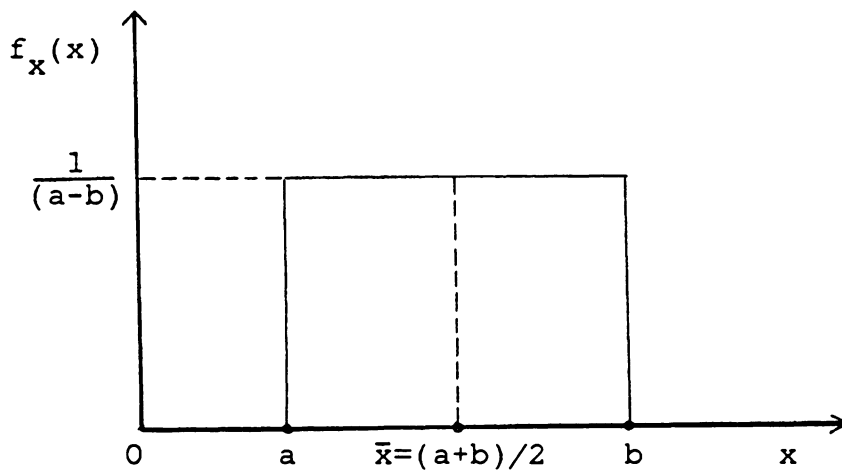


Figure 4.1 The Uniform Probability Distribution

Due to manufacturing process, there is some uncertainty in the diameter of a bolt's head ( $d_h$ ). However, there is very little variation in the head diameter due to good control in manufacturing process, and the contribution of this uncertainty is so small that it can be neglected as shown in the subsequent section.

#### 4.2.2 Shear Resistance

The results of 19 tests on anchor bolt governed by steel failure obtained from Refs. [5,26,42] are summarized in Table 4.3. Most tests were conducted on 3/4" diameter anchor bolts leading to the uneven representation of each diameter. However, it is shown in Ref. [33] that the shear capacity governed by steel failure is proportional to the cross sectional area of the bolt. Also, the uncertainty in the tensile strength of bolts with different diameters can be included in the cov of the tensile strength of the bolt.

For a bolt with the threaded part located in the shear failure plane, the net tensile area is used in computing the predicted capacity, this net tensile area is

$$A_s = 0.7854(d_b - 0.9743/n)^2 \quad (\text{Eq. 4.5})$$

in which  $d_b$  is the major thread diameter of the bolt and  $n$  is the number of threads per inch. Using Eqs. 2.1 and 2.2 to predict the resistance of the bolt ( $V_s$ ), the ratio of the actual test ( $V_{sa}$ ) to the predicted resistance were obtained for both the PCI and the ACI design equations, and this ratio is denoted by  $V_{sa}/V_s$  ( $N_{vs}$ ) in Table 4.3. The mean and the cov of these ratios, which are denoted by  $\bar{N}_{vs}$  and  $(V_{a/p})_{vs}$ , were computed; the results for the PCI equation are

$$\bar{N}_{vs} = 0.918, \quad (V_{a/p})_{vs} = 0.118$$

while the results for the ACI equation are

$$\bar{N}_{vs} = 1.638, (V_{a/p})_{vs} = 0.118$$

Due to the fact that  $(V_{a/p})_{vs}$  includes uncertainties in the testing procedures  $V_{test}$  and in values specified by test reports  $V_{spec}$ , which are assumed to be 0.04, the true uncertainty in  $N_{vs}$  is

$$V_{Nvs} = [(V_{a/p})_{vs}^2 - 0.04^2 - 0.04^2]^{1/2} \quad (\text{Eq. 4.6})$$

Also, there is uncertainty due to sampling which is a function of the sample size. This uncertainty can be represented by

$$V_n = V_{Nvs}/\sqrt{19}$$

Hence, the total uncertainty in  $N_{vs}$  is

$$(V_{Nvs})_t = V_{Nvs}(1 + 1/19)^{1/2} \quad (\text{Eq. 4.7})$$

Substituting the value of  $V_{Nvs}$  in Eq. 4.7 from Eq. 4.6, the coefficient of variation  $(V_{Nvs})_t$  for both the PCI and the ACI equations is 0.11.

Taking into account the bias in the design equation  $(N_{vs})$ , the shear resistance governed by steel failure can be modelled as

$$V_s = N_{vs} C A_s f_u \quad (\text{Eq. 4.8})$$

where C is a constant equal to 1.00 for the PCI equation and 0.56 for the ACI equation. Using the first-order approximation, the mean shear resistance governed by steel failure is

$$\bar{V}_s = \bar{N}_{vs} C \bar{A}_s \bar{f}_u \quad (\text{Eq. 4.9})$$

and the cov of  $V_s$  is

$$V_{vs} = [(V_{Nvs})_t^2 + V_{as}^2 + V_{fu}^2]^{1/2} \quad (\text{Eq. 4.10})$$

Substituting values for  $V_{as}$  and  $V_{fu}$  obtained in Section 4.2.1 in Eq. 4.10 results in

$$V_{vs} = [(V_{Nvs})_t^2 + 0.04^2 + 0.10^2]^{1/2} \quad (\text{Eq. 4.11})$$

The ratio of the mean to the nominal shear resistance for steel failure is

$$\bar{V}_s/V_s = \bar{N}_{vs} \bar{f}_u / f_u \quad (\text{Eq. 4.12})$$

Substitution of the values of  $\bar{N}_{vs}$ ,  $(V_{Nvs})_t$  and  $\bar{f}_u/f_u$  into Eqs. 4.11 and 4.12 gives

$$\bar{V}_s/V_s = 1.010, \quad V_{vs} = 0.152$$

for the PCI design equation, and

$$\bar{V}_s/V_s = 1.802, \quad V_{vs} = 0.152$$

for the ACI design equation.

To determine statistics of the shear resistance governed by concrete failure, the results of 25 tests on bolts with a sufficient length of embedment obtained from Refs. [5,26,42] were analyzed. The results are shown in Table 4.4. Test results on welded studs are excluded in this study, since all of them have insufficient length of embedment.

Equations 2.5 and 2.6 are used to predict the shear resistance governed by the concrete failure ( $V_c$ ), from which

the ratio of the actual test to the predicted capacity ( $N_{VC}$ ) is computed. The mean and the cov of  $N_{VC}$  for the PCI equation are

$$\bar{N}_{VC} = 1.396, (V_{a/p})_{VC} = 0.225$$

and for the ACI equation, these are

$$\bar{N}_{VC} = 1.573, (V_{a/p})_{VC} = 0.360$$

Since the specified concrete specimen strength may be different from the control cylinder strength,  $V_{spec}$  is taken as 0.04. Thus, with  $V_{test}$  taken as 0.04, the true cov of  $N_{VC}$  becomes

$$V_{Nvc} = [(V_{a/p})_{VC}^2 - 0.04^2 - 0.04^2]^{1/2} \quad (\text{Eq. 4.13})$$

and including uncertainty in sampling, the total uncertainty in  $N_{VC}$  can be written as

$$(V_{Nvc})_t = V_{Nvc}(1 + 1/25)^{1/2} \quad (\text{Eq. 4.14})$$

The shear resistance governed by the concrete failure for the PCI design equation can be written as

$$V_c = N_{VC} C(m - 1) \sqrt{f'_c} \quad (\text{Eq. 4.15})$$

in which  $C$  equals  $3250/\sqrt{5000}$ . Accordingly, the mean of  $V_c$  is

$$\bar{V}_c = \bar{N}_{VC} C(\bar{m} - 1) \sqrt{f'_c} \quad (\text{Eq. 4.16})$$

and the cov of  $V_c$  is

$$V_{vc} = \left[ \frac{\bar{m}^2}{(\bar{m} - 1)^2} V_m^2 + (V_{Nvc})_t^2 + 0.25 V_{fc}^2 \right]^{1/2} \quad (\text{Eq. 4.17})$$

Assuming that the smallest edge distance ( $m$ ) is 2", then

$$V_{VC} = [4V_m^2 + (V_{NVC})_t^2 + 0.25V_{fc}^2]^{1/2} \quad (\text{Eq. 4.18})$$

In a similar way, the mean shear resistance for concrete failure using the ACI equation is

$$\bar{V}_C = \bar{N}_{VC} C \bar{m}^2 \sqrt{\bar{f}'_C} \quad (\text{Eq. 4.19})$$

where  $C$  is a deterministic quantity equal to  $2\pi$ , and the cov of  $V_C$  is the same as given by Eq. 4.18.

Substituting values for  $V_m$  and  $V_{fc}$  in Eq. 4.18 results in

$$V_{VC} = [(V_{NVC})_t^2 + 0.0117]^{1/2} \quad (\text{Eq. 4.20})$$

whereas the ratio of the mean to the nominal shear resistance for concrete failure is

$$\bar{V}_C/V_C = \bar{N}_{VC} (\bar{f}'_C/f'_C)^{1/2} \quad (\text{Eq. 4.21})$$

The values of this ratio and its cov for the PCI design equation are

$$\bar{V}_C/V_C = 1.513, \quad V_{VC} = 0.247$$

and for the ACI design equation are

$$\bar{V}_C/V_C = 1.705, \quad V_{VC} = 0.378.$$

#### 4.2.3 Tensile Resistance

Forty two tests on the tensile capacity of anchor bolts and welded studs governed by the steel failure were obtained from Refs. [2,28,30,32]. These tests results are summarized in Table 4.5. Test results on the welded studs (28,30,32) are included as data for estimating the bias factor of the equation for predicting the tensile capacity.

The net tensile area for threaded bolts is computed from Eq. 4.5, and the predicted tensile capacity for the steel failure ( $P_s$ ) is computed using Eq. 2.7. The ratio of the test to the predicted strength ( $P_{sa}/P_s$ ) is denoted by  $N_{ps}$ . The mean value and the cov of these ratios for the PCI design equation are

$$\bar{N}_{ps} = 1.154, (V_{a/p})_{ps} = 0.108$$

and for the ACI design equation, these are

$$\bar{N}_{ps} = 1.299, (V_{a/p})_{ps} = 0.108$$

Assuming that both  $V_{spec}$  and  $V_{test}$  have the same value, 0.04, the true cov of  $N_{ps}$  can be computed by

$$V_{Nps} = [(V_{a/p})_{ps}^2 - 0.0032]^{1/2} \quad (\text{Eq. 4.22})$$

Taking into account a sampling error, the total cov of  $N_{ps}$  is

$$(V_{Nps})_t = V_{Nps} (1 + 1/42)^{1/2} \quad (\text{Eq. 4.23})$$

Using Eq. 2.7, the tensile capacity governed by the steel failure is modelled as

$$P_s = N_{ps} C A_s f_u \quad (\text{Eq. 4.24})$$

in which  $C$  is a constant equal to 0.9 for the PCI equation and 0.8 for the ACI equation. Then, by the first-order approximation, the mean of  $P_s$  can be expressed as

$$\bar{P}_s = \bar{N}_{ps} \bar{C} \bar{A}_s \bar{f}_u \quad (\text{Eq. 4.25})$$

and the cov of  $P_s$ , as

$$V_{ps} = [(V_{Nps})_t^2 + V_{as}^2 + V_{fu}^2]^{1/2} \quad (\text{Eq. 4.26})$$

Substituting values for  $V_{as}$  and  $V_{fu}$  in Eq. 4.26 gives

$$V_{ps} = [(V_{Nps})_t^2 + 0.0116]^{1/2} \quad (\text{Eq. 4.27})$$

while the ratio of the mean to the nominal tensile capacity for steel failure is

$$\bar{P}_s/P_s = \bar{N}_{ps}\bar{f}_u/f_u \quad (\text{Eq. 4.28})$$

The values of this ratio and its cov for the PCI design equation are

$$\bar{P}_s/P_s = 1.269, V_{ps} = 0.142$$

and for the ACI design equation, these are

$$\bar{P}_s/P_s = 1.429, V_{ps} = 0.142$$

Only 19 tests governed by concrete failure are available, since the test results on bolts and studs having insufficient edge distance were excluded from the analysis. These test results were obtained from Refs. [2,28,30] and are summarized in Table 4.6.

The predicted tensile capacity for concrete failure ( $P_c$ ) is computed by Eqs. 2.9 and 2.10, and the ratios of the test to the predicted strength ( $N_{pc}$ ) for the PCI and the ACI equation are obtained as shown in Table 4.6. Using these data, the mean and the cov of these ratios for the PCI equation are

$$\bar{N}_{pc} = 0.761, (V_{a/p})_{pc} = 0.232$$

and for the ACI equation, these are

$$\bar{N}_{pc} = 1.077, (V_{a/p})_{pc} = 0.232$$

Considering  $V_{\text{spec}}$  and  $V_{\text{test}}$ , which are taken to be 0.04, the true cov of  $N_{\text{pc}}$  is

$$V_{N_{\text{pc}}} = [(V_{a/p})_{\text{pc}}^2 - 0.0032]^{1/2} \quad (\text{Eq. 4.29})$$

Adding uncertainty due to sampling, the total uncertainty in  $N_{\text{pc}}$  can be expressed as

$$(V_{N_{\text{pc}}})_t = V_{N_{\text{pc}}}(1 + 1/19)^{1/2} \quad (\text{Eq. 4.30})$$

Taking into account the bias in the design equation, the ACI and the PCI design equations for predicting concrete tensile capacity can be written as

$$P_c = N_{\text{pc}} C l_e (l_e + d_h) \sqrt{f'_c} \quad (\text{Eq. 4.31})$$

in which  $C$  is a constant which equals to  $4\pi$  for the ACI equation and  $4\pi\sqrt{2}$  for the PCI equation. The corresponding mean of  $P_c$  is

$$\bar{P}_c = \bar{N}_{\text{pc}} C \bar{l}_e (\bar{l}_e + \bar{d}_h) \sqrt{f'_c} \quad (\text{Eq. 4.32})$$

and the cov of  $P_c$  is

$$V_{P_c} = \left[ (V_{N_{\text{pc}}})_t^2 + 0.25V_{f_c}^2 + \frac{(2\bar{l}_e + \bar{d}_h)^2}{(\bar{l}_e + \bar{d}_h)^2} V_{l_e}^2 + \frac{(\bar{d}_h)^2}{(\bar{l}_e + \bar{d}_h)^2} V_{d_h}^2 \right]^{1/2} \quad (\text{Eq. 4.33})$$

Assuming that  $\bar{d}_h$  equals 0.2  $\bar{l}_e$ , the cov of  $P_c$  is estimated to be

$$V_{P_c} = [(V_{N_{\text{pc}}})_t^2 + 0.25V_{f_c}^2 + 3.36V_{l_e}^2 + 0.0028V_{d_h}^2]^{1/2} \quad (\text{Eq. 4.34})$$

Neglecting the contribution of  $V_{d_h}$ , Eq. 4.34 becomes

$$V_{pc} = [(V_{Npc})_t^2 + 0.25V_{fc}^2 + 3.36V_{le}^2]^{1/2} \quad (\text{Eq. 4.35})$$

Finally, substituting values for  $V_{le}$  and  $V_{fc}$  in Eq. 4.35 yields

$$V_{pc} = [(V_{Npc})_t^2 + 0.0165]^{1/2} \quad (\text{Eq. 4.36})$$

whereas the ratio of the mean to the nominal tensile capacity for concrete failure is

$$\bar{P}_c/P_c = \bar{N}_{pc}(\bar{f}'_c/f'_c)^{1/2} \quad (\text{Eq. 4.37})$$

The values of this ratio and its cov for the PCI design equation are

$$\bar{P}_c/P_c = 0.825, \quad V_{pc} = 0.264$$

and for the ACI design equation, these are

$$\bar{P}_c/P_c = 1.167, \quad V_{pc} = 0.264$$

The results of the statistical analysis of the basic variables and the resistance of anchor bolts are summarized in Tables 4.7 and 4.8, respectively. Since the LRFD equations for predicting the capacity governed by steel failure are similar to the ACI design equations, the statistical analysis of the LRFD equations is not presented. The statistics for the LRFD equations can be obtained by multiplying the ratio of the mean to the nominal resistance for the ACI equation by (0.8/0.75) and using the same cov.

Table 4.3 Analysis of Test Results on Shear Capacity  
Steel Failure

No.	Ref.	Grade	$f_u$ (ksi)	d (in)	$A_s$ (in <sup>2</sup> )	$V_{sa}$ (kips)	P C I		A C I	
							$V_s$	$V_{sa}/V_s$	$V_s$	$V_{sa}/V_s$
1	5	A307	66.0	0.750	0.4418	28.16	29.16	0.966	16.33	1.724
2	5	A307	66.0	0.750	0.4418	30.80	29.16	1.056	16.33	1.886
3	5	A307	66.0	0.750	0.4418	30.36	29.16	1.041	16.33	1.859
4	5	A307	66.0	0.750	0.4418	30.80	29.16	1.056	16.33	1.886
5	5	A307	66.0	0.750	0.4418	27.72	29.16	0.951	16.33	1.697
6	5	A307	66.0	0.750	0.4418	29.26	29.16	1.003	16.33	1.792
7	5	A307	66.0	0.750	0.4418	29.92	29.16	1.026	16.33	1.832
8	5	A307	66.0	0.750	0.4418	27.06	29.16	0.928	16.33	1.657
9	5	A307	66.0	0.750	0.4418	27.06	29.16	0.928	16.33	1.657
10	26	A307	61.3	0.750	0.4418	23.80	27.08	0.879	15.17	1.569
11	26	A307	61.3	0.750	0.4418	24.50	27.08	0.905	15.17	1.615
12	26	A307	61.3	0.750	0.4418	22.80	27.08	0.842	15.17	1.503
13	26	A307	61.3	0.750	0.4418	25.50	27.08	0.942	15.17	1.681
14	26	A307	61.3	0.750	0.4418	25.00	27.08	0.923	15.17	1.648
15	26	A307	61.3	0.750	0.4418	25.50	27.08	0.942	15.17	1.681
16	26	A307	61.3	0.750	0.4418	23.00	27.08	0.849	15.17	1.516
17	26	A307	61.3	0.750	0.4418	23.00	27.08	0.849	15.17	1.516
18	42	A307	66.0	1.000	0.7854	34.60	51.84	0.667	29.03	1.192
19	42	A307	66.0	1.000	0.7854	35.40	51.84	0.683	29.03	1.219

$$\bar{N}_{vs} = (V_{sa}/V_s)_m = 0.918 \quad 1.638$$

$$(V_{a/p})_{vs} = 0.118 \quad 0.118$$

Table 4.4 Analysis of Test Results on Shear Capacity  
Concrete Failure

No.	Ref.	$f'_c$ (psi)	d (in)	m (in)	$l_e$ (in)	$V_{ca}$ (kips)	P C I		A C I	
							$V_c$	$V_{ca}/V_c$	$V_c$	$V_{ca}/V_c$
1	5	4815	0.75	4.00	6.0	19.12	9.57	1.998	6.98	2.739
2	5	4815	0.75	5.00	6.0	15.81	12.76	1.239	10.90	1.450
3	5	4850	0.75	6.00	6.0	22.62	16.00	1.414	15.75	1.436
4	26	4200	0.75	2.00	8.0	3.85	2.98	1.292	1.63	2.362
5	26	4200	0.75	2.00	8.0	4.00	2.98	1.342	1.63	2.454
6	26	4200	0.75	2.00	8.0	4.10	2.98	1.376	1.63	2.515
7	26	4200	0.75	4.00	8.0	6.75	8.94	0.755	6.52	1.035
8	26	4200	0.75	4.00	8.0	7.50	8.94	0.839	6.52	1.150
9	26	4200	0.75	6.00	8.0	14.50	14.89	0.974	14.66	0.989
10	26	4200	0.75	8.00	8.0	19.50	20.85	0.935	26.06	0.748
11	42	4050	1.00	4.00	9.1	16.50	8.78	1.880	6.40	2.578
12	42	4050	1.00	4.00	9.1	12.60	8.78	1.436	6.40	1.969
13	42	4390	1.00	4.00	9.1	11.70	9.14	1.281	6.66	1.757
14	42	4080	1.00	6.00	9.1	21.00	14.68	1.431	14.45	1.453
15	42	4080	1.00	6.00	9.1	21.00	14.68	1.431	14.45	1.453
16	42	4080	1.00	6.00	9.1	17.30	14.68	1.178	14.45	1.197
17	42	4080	1.00	6.00	9.1	19.90	14.68	1.356	14.45	1.377
18	42	4030	1.00	6.00	9.1	20.10	14.59	1.378	14.35	1.401
19	42	4080	1.00	6.00	9.1	18.60	14.68	1.267	14.45	1.287
20	42	4050	1.00	8.00	9.1	32.40	20.48	1.582	25.59	1.266
21	42	4050	1.00	8.00	9.1	32.40	20.48	1.582	25.59	1.266
22	42	4230	2.00	6.00	18.4	26.30	14.95	1.760	14.71	1.788
23	42	4230	2.00	6.00	18.4	25.40	14.95	1.699	14.71	1.727
24	42	4290	2.00	12.00	18.4	55.10	33.11	1.664	59.26	0.930
25	42	4290	2.00	12.00	18.4	60.00	33.11	1.812	59.26	1.012

$$\bar{N}_{vc} = (V_{ca}/V_c)_m = 1.396 \quad 1.573$$

$$(V_{a/p})_{vc} = 0.225 \quad 0.360$$

Table 4.5 Analysis of Test Results on Tensile Capacity  
Steel Failure

No.	Ref.	Grade	$f_u$ (ksi)	d (in)	$A_s$ (in <sup>2</sup> )	$P_{sa}$ (kips)	P C I		A C I	
							$P_s$	$P_{sa}/P_s$	$P_s$	$P_{sa}/P_s$
1	2*	A307	66.0	0.750	0.3345	26.10	19.87	1.314	17.66	1.478
2	2	A307	66.0	0.750	0.3345	26.20	19.87	1.319	17.66	1.484
3	2	A307	66.0	0.750	0.3345	25.40	19.87	1.278	17.66	1.438
4	2	A307	66.0	0.750	0.3345	26.30	19.87	1.324	17.66	1.489
5	2	A307	66.0	0.750	0.3345	29.60	19.87	1.490	17.66	1.676
6	2	A307	66.0	0.750	0.3345	23.20	19.87	1.168	17.66	1.314
7	2	A307	66.0	0.750	0.3345	24.40	19.87	1.228	17.66	1.382
8	2	A307	66.0	0.750	0.3345	21.00	19.87	1.057	17.66	1.189
9	2	A307	66.0	0.750	0.3345	26.00	19.87	1.308	17.66	1.472
10	2	A307	66.0	0.750	0.3345	26.10	19.87	1.314	17.66	1.478
11	28	Stud	70.9	0.750	0.4418	28.30	28.19	1.004	25.06	1.130
12	28	Stud	70.9	0.750	0.4418	28.50	28.19	1.011	25.06	1.137
13	28	Stud	70.9	0.750	0.4418	28.00	28.19	0.993	25.06	1.117
14	28	Stud	70.9	0.750	0.4418	31.50	28.19	1.117	25.06	1.257
15	28	Stud	70.9	0.750	0.4418	29.30	28.19	1.039	25.06	1.169
16	28	Stud	70.9	0.750	0.4418	29.30	28.19	1.039	25.06	1.169
17	28	Stud	70.9	0.750	0.4418	28.80	28.19	1.022	25.06	1.150
18	28	Stud	70.9	0.750	0.4418	31.50	28.19	1.177	25.06	1.324
19	28	Stud	70.9	0.750	0.4418	29.40	28.19	1.043	25.06	1.173
20	28	Stud	70.9	0.750	0.4418	27.30	28.19	0.968	25.06	1.089
21	28	Stud	70.9	0.750	0.4418	28.70	28.19	1.019	25.06	1.146
22	30	Stud	77.4	0.250	0.0491	3.70	3.42	1.082	3.04	1.217
23	30	Stud	79.7	0.375	0.1104	8.50	7.92	1.073	7.04	1.207
24	30	Stud	79.7	0.375	0.1104	8.70	7.92	1.098	7.04	1.235
25	30	Stud	76.0	0.500	0.1960	12.50	13.41	0.932	11.92	1.048
26	30	Stud	76.0	0.500	0.1960	15.00	13.41	1.118	11.92	1.258
27	30	Stud	73.0	0.625	0.3068	23.70	20.16	1.176	17.92	1.352

No.	Ref.	Grade	$f_u$ (ksi)	d (in)	$A_s$ (in <sup>2</sup> )	$P_{sa}$ (kips)	P C I		A C I	
							$P_s$	$P_{sa}/P_s$	$P_s$	$P_{sa}/P_s$
28	30	Stud	73.0	0.625	0.3068	25.00	20.16	1.240	17.92	1.395
29	30	Stud	73.0	0.625	0.3068	25.00	20.16	1.240	17.92	1.395
30	32	Stud	77.4	0.250	0.0491	3.38	3.42	0.988	3.04	1.111
31	32	Stud	77.4	0.250	0.0491	4.00	3.42	1.170	3.04	1.316
32	32	Stud	79.7	0.375	0.1104	10.60	7.92	1.338	7.04	1.505
33	32	Stud	79.7	0.375	0.1104	9.50	7.92	1.199	7.04	1.349
34	32	Stud	76.0	0.500	0.1960	16.30	13.41	1.216	11.92	1.368
35	32	Stud	76.0	0.500	0.1960	15.60	13.41	1.163	11.92	1.308
36	32	Stud	73.0	0.625	0.3068	23.80	20.16	1.180	17.92	1.328
37	32	Stud	73.0	0.625	0.3068	22.00	20.16	1.091	17.92	1.228
38	32	Stud	73.0	0.625	0.3068	24.40	20.16	1.210	17.92	1.362
39	32	Stud	75.2	0.750	0.4418	37.50	29.88	1.255	26.56	1.412
40	32	Stud	75.2	0.750	0.4418	38.10	29.88	1.275	26.56	1.434
41	32	Stud	76.8	0.875	0.6013	47.00	41.58	1.130	36.96	1.272
42	32	Stud	76.8	0.875	0.6013	45.00	41.58	1.082	36.96	1.217

$$\bar{N}_{ps} = (P_{sa}/P_s)_m = 1.154 \quad 1.299$$

$$(V_{a/p})_{ps} = 0.108 \quad 0.108$$

\* Threaded bolts

Table 4.6 Analysis of Test Results on Tensile Capacity  
Concrete Failure

No.	Ref.	$f'_c$ (psi)	d (in)	$d_h$ (in)	$l_e$ (in)	m (in)	$P_{ca}$ (kips)	P C I		A C I	
								$P_c$	$P_{ca}/P_c$	$P_c$	$P_{ca}/P_c$
1	2	3500	0.750	1.125	3.000	15.0	16.00	13.01	1.230	9.20	1.739
2	2	4315	0.750	1.125	4.000	15.0	25.40	23.93	1.061	16.92	1.501
3	2	4300	1.000	1.625	10.500	18.0	98.00	148.36	0.660	104.91	0.934
4	28	4900	0.750	1.250	7.625	12.0	43.00	84.19	0.511	59.53	0.722
5	28	5180	0.750	1.250	3.625	12.0	18.50	22.60	0.819	15.98	1.158
6	28	5180	0.750	1.250	3.625	12.0	18.50	22.60	0.819	15.98	1.158
7	28	5180	0.750	1.250	3.625	12.0	17.30	22.60	0.766	15.98	1.083
8	31	3000	0.500	1.000	3.688	12.0	7.80	16.83	0.463	11.90	0.655
9	31	3000	0.625	1.250	3.438	12.0	14.00	15.68	0.892	11.09	1.262
10	31	3000	0.625	1.250	3.438	12.0	12.40	15.68	0.790	11.09	1.118
11	31	3000	0.625	1.250	3.438	12.0	9.40	15.68	0.600	11.09	0.848
12	31	3000	0.750	1.500	3.438	12.0	14.00	16.52	0.847	11.68	1.199
13	31	3000	0.750	1.250	3.500	12.0	10.80	16.18	0.668	11.44	0.944
14	31	3000	0.750	1.250	3.500	12.0	10.80	16.18	0.668	11.44	0.944
15	31	3000	0.750	1.250	3.500	12.0	12.40	16.18	0.766	11.44	1.084
16	31	3000	0.750	1.250	3.500	12.0	11.70	16.18	0.723	11.44	1.023
17	31	3000	0.875	1.375	3.500	12.0	10.80	16.60	0.650	11.74	0.920
18	31	3000	0.875	1.375	3.500	12.0	12.40	16.60	0.747	11.74	1.056
19	31	3000	0.875	1.750	3.500	12.0	14.00	17.89	0.783	12.65	1.107

$$\bar{N}_{ps} = (P_{ca}/P_c)_m = 0.761 \quad 1.077$$

$$(V_{a/p})_{pc} = 0.232 \quad 0.232$$

Table 4.7 Summary of Statistics of Basic Variables

Design Variables	$\bar{X}_i/X_i$	cov
Dead load effect	1.050	0.100
Live load effect"	1.000	0.250
Cross-sectional area of bolt ( $A_s$ )	1.000	0.040
Tensile strength of bolt ( $f_u$ )	1.100	0.100
Concrete compressive strength ( $f'_c$ )*	1.175	0.180
Diameter of bolt's head ( $d_h$ )	---	---
Length of embedment ( $l_e$ )	1.000	0.050
Edge distance (m)	1.000	0.030

" Lifetime maximum live load

\* For 4000 psi concrete

Table 4.8 Summary of Statistics of Anchor Bolt Resistance

Design Variables	$\bar{X}_i/X_i$	cov
<u>PCI equations</u>		
Shear resistance:		
Steel failure ( $V_s$ )	1.010	0.152
Concrete failure ( $V_c$ )	1.513	0.247
Tensile resistance:		
Steel failure ( $P_s$ )	1.269	0.142
Concrete failure ( $P_c$ )	0.825	0.264
<u>ACI equations</u>		
Shear resistance:		
Steel failure ( $V_s$ )	1.802	0.152
Concrete failure ( $V_c$ )	1.705	0.378
Tensile resistance:		
Steel failure ( $P_s$ )	1.429	0.142
Concrete failure ( $P_c$ )	1.167	0.264
<u>LRFD equations</u>		
Shear resistance:		
Steel failure ( $V_s$ )	1.922	0.152
Tensile resistance:		
Steel failure ( $P_s$ )	1.524	0.142

## Chapter V

### CODE CALIBRATION

#### 5.1 GENERAL

The purpose of calibration is to determine the level of safety implied in the current design criteria on anchor bolts. Using the FOSM, this level of safety is measured by the reliability index ( $\beta$ ). In other words, the values of  $\beta$  corresponding to code specified loads and resistances are computed. The design criteria examined are the ACI (10), the PCI (34) and the LRFD (36) equations. The limit state considered is the ultimate limit state. In this study, only the combination of dead and live load is considered. The reliability index  $\beta$  is computed for different ratios of the unreduced live load to the nominal dead load ( $L_o/D_n$ ) and different influence areas ( $A_1$ ) in order to get an overview of the range of  $\beta$ .

The necessary equations and the algorithm to compute  $\beta$  for a specified limit state are presented in section 5.2. The results of the computation of reliability indices for the ACI, the PCI and the LRFD equations are presented in Section 5.3.

## 5.2 ALGORITHM FOR COMPUTATION OF RELIABILITY INDEX

Both ACI (8) and PCI (34) specify the same design requirement on the ultimate limit state criterion for a combination of dead and live load, which is

$$\phi R_n \geq 1.4D_n + 1.7L_n \quad (\text{Eq. 5.1})$$

where  $\phi$  depends on the type of loading (shear, tensile) and the type of failure (steel, concrete) under consideration as given in Section 2.3. For the LRFD equations, however, the load factors are 1.2 and 1.6 for dead and live load, respectively, and 1.4 for dead load acting alone.  $R_n$ ,  $D_n$ , and  $L_n$  are the nominal values of resistance, dead and live load, respectively. These values are equivalent to the code specified values. The nominal live load used in this study is that specified in ANSI A58.1-1983 for the lifetime maximum live load (Eq. 4.1). Using Eq. 5.1, the required nominal resistance  $R_n$  can be found for any value of  $L_n$  and  $D_n$ . Then, multiplying  $R_n$  by the appropriate ratio of the mean to the nominal resistance as given in Table 4.8, the required mean resistance  $\bar{R}$  is obtained.

Implicitly, the design requirement given in Eq. 5.1 constitutes a linear performance function; thus, the corresponding limit state equation can be written as

$$f(X) = R - Q = R - D - L \quad (\text{Eq. 5.2})$$

However, a negative value of  $\beta$  may be obtained when this li-

near performance function is used as a failure criterion, if the values of  $\beta\sigma_R$  and  $\beta\sigma_Q$  are larger than the means of  $R$  and  $Q$ , respectively (22). Since the value of  $\beta$  by the advanced FOSM does not depend on the failure criterion used in the computation and the problem of getting a negative  $\beta$  is avoided, a logarithmic performance function is used as the failure criterion. Thus, the limit state equation can be expressed as

$$f(X) = \ln R - \ln Q = \ln R - \ln(D + L) \quad (\text{Eq. 5.3})$$

Having determined this limit state equation, all variables are transformed into reduced variables by Eq. 3.6. For instance, the transformation of  $R$  into a reduced variable  $r$  is

$$R = \bar{R} + r\sigma_R \quad (\text{Eq. 5.4})$$

Thus, the limit state equation in terms of reduced variables is

$$f(x) = \ln(\bar{R} + r\sigma_R) - \ln(\bar{D} + d\sigma_D + \bar{L} + l\sigma_L) \quad (\text{Eq. 5.5})$$

Accordingly, the partial derivatives of  $f(x)$  with respect to the reduced variables are

$$\partial f / \partial r = \sigma_R / (\bar{R} + r\sigma_R) \quad (\text{Eq. 5.6})$$

$$\partial f / \partial d = -\sigma_D / (\bar{D} + d\sigma_D + \bar{L} + l\sigma_L) \quad (\text{Eq. 5.7})$$

$$\partial f / \partial l = -\sigma_L / (\bar{D} + d\sigma_D + \bar{L} + l\sigma_L) \quad (\text{Eq. 5.8})$$

The direction cosines of the reduced variables can be computed by using Eqs. 3.15, 5.6 to 5.8 as

$$\alpha_r^* = \sigma_R / Y(\bar{R} + r\sigma_R) \quad (\text{Eq. 5.9})$$

$$\alpha_d^* = -\sigma_D/Y(\bar{D} + d\sigma_D + \bar{L} + l\sigma_L) \quad (\text{Eq. 5.10})$$

$$\alpha_l^* = -\sigma_L/Y(\bar{D} + d\sigma_D + \bar{L} + l\sigma_L) \quad (\text{Eq. 5.11})$$

in which  $Y = [(\partial f/\partial r)^2 + (\partial f/\partial d)^2 + (\partial f/\partial l)^2]^{1/2}$ .

In addition,  $f(x)$  is zero for a point on the failure surface. The value of  $\beta$  satisfying these constraints can be obtained by an iterative procedure such as the Newton-Raphson method (38). By using this method,  $\beta$  is regarded as a root of  $f(x) = 0$ , and the iteration formula has the following form

$$\beta_{n+1} = \beta_n - f(x)/(\partial f/\partial \beta) \quad (\text{Eq. 5.12})$$

in which  $\beta_{n+1}$  and  $\beta_n$  are the  $(n+1)^{\text{th}}$  and the  $n^{\text{th}}$  value of  $\beta$ , respectively, while  $f(x)$  and the derivative of  $f(x)$  with respect to  $\beta$  ( $\partial f/\partial \beta$ ) are evaluated at  $\beta_n$ . The value of  $\partial f/\partial \beta$  can be computed by

$$\partial f/\partial \beta = \sum_{i=1}^n (\partial f/\partial x_i)(\partial x_i/\partial \beta) \quad (\text{Eq. 5.13})$$

in which  $x_i$  represents  $r$ ,  $d$ , and  $l$ , while  $\partial x_i/\partial \beta$  is obtained by differentiating Eq. 3.14,

$$\partial x_i/\partial \beta = -\alpha_i^* \quad (\text{Eq. 5.14})$$

Hence,

$$\partial f/\partial \beta = \sum_{i=1}^n -\alpha_i^* (\partial f/\partial x_i) \quad (\text{Eq. 5.15})$$

where  $\partial f/\partial x_i$  is the partial derivative as computed by Eq. 5.6 through 5.8.

With these equations, the reliability index  $\beta$  is computed using the following Rackwitz's algorithm (4,38):

1. Estimate a value of  $\beta$ .
2. Assume initial values of  $R$ ,  $D$ , and  $L$ . An estimate is obtained by taking the mean values as the initial values. Using these initial values, the values of  $r$ ,  $d$ , and  $l$  are computed by Eq. 3.6.
3. Evaluate the partial derivatives of  $f(x)$  and determine the direction cosines at point  $r$ ,  $d$ ,  $l$  using Eq. 5.6 to 5.11.
4. Find new values of  $r$ ,  $d$ ,  $l$  by Eq. 3.14 and compare them with their initial values.
5. Repeat steps 2 to 4 until the values of  $r$ ,  $d$ , and  $l$  converge.
6. Compute the derivative of  $f(x)$  with respect to  $\beta$  by Eq. 5.15 and then, compute a new  $\beta$  by Eq. 5.12.
7. Repeat steps 2 to 6 until the value of  $\beta$  converges.

### 5.3 RESULTS AND DISCUSSION

To get an overview of the risk levels implied by the PCI (34), the ACI (10) and the LRFD (36) equations, the computation of the reliability index is performed for  $L_o/D_n$  ranging from 0 to 2 and for the influence area ranging from 200 ft<sup>2</sup> to 2000 ft<sup>2</sup>. Reliability indices ( $\beta$ ) for the dead and live

load combination conforming to the ACI and the PCI design equations are shown in Tables 5.1 to 5.4. The values of  $\beta$  for shear capacity governed by steel failure and by concrete failure are shown in Table 5.1 and 5.2, respectively. For tensile capacity, these values are shown in Table 5.3 for steel failure and in Table 5.4 for concrete failure. For comparison, the reliability indices found in the LRFD equations for steel failure are presented in Table 5.5. Graphically, the values of  $\beta$  as a function of  $L_o/D_n$  are shown in Figs. 5.1, 5.2, and 5.5 for  $A_I = 1,600 \text{ ft}^2$ , while the values of  $\beta$  as a function of the influence area are shown in Figs. 5.3, 5.4 and 5.6 for  $L_o/D_n = 1.0$ .

It was found that the ranges of reliability indices for the ACI design equations are 4.1 to 4.4 for steel shear resistance, 1.6 to 1.8 for concrete shear resistance, 3.5 to 3.9 for steel tensile resistance, and 2.2 to 2.4 for concrete tensile resistance. For the PCI design equations, however, the ranges of reliability indices are 2.7 to 3.1 for steel shear resistance, 2.3 to 2.5 for concrete shear resistance, 2.7 to 3.0 for steel tensile resistance, and 0.8 to 1.2 for concrete tensile resistance. On the other hand, the ranges of reliability indices for the LRFD equations are 4.4 to 4.6 for steel shear resistance, and 4.1 to 4.3 for steel tensile resistance.

From Figs. 5.1 and 5.2, it can be observed that the values of  $\beta$  for the ACI and the PCI equations tend to increase as  $L_o/D_n$  increases. This increase is relatively large for  $L_o/D_n$  ranging from 0 to 0.5, and is nearly zero for  $L_o/D_n$  ranging from 0.5 to 2.0. The reason for this trend is that there is a limit on load reduction for large influence areas. In contrast to the increase of  $\beta$  for the ACI and the PCI equations,  $\beta$  for the LRFD equations decreases slightly for  $L_o/D_n$  ranging from 0 to 0.5 (Fig. 5.5). The reason is the use of different dead load factors for the case of dead load acting alone and for the combination of dead and live load, e.g. 1.4 and 1.2, respectively. On the other hand, Figs. 5.3, 5.4 and 5.6 show that the values of  $\beta$  are nearly constant over the range of the influence areas. This is due to the fact that the ANSI A58.1-83 nominal live load used in the computation of  $\beta$  is consistent with the actual reduction in the mean live load (12).

In all cases, except for the shear capacity governed by concrete failure, the reliability indices for the ACI design equations are higher than those for the PCI design equations. Compared to the reliability indices found in steel connections (19), however, the reliability indices of both the ACI and the PCI equations for ASTM A307 anchor bolt design are lower. For instance, the range of  $\beta$  for ASTM A325

bolt subjected to tensile loading is 4.7 to 6.8, while the reliability indices for the ACI equation for the tensile capacity of ASTM A307 anchor bolt ranges from 3.5 to 3.9. Since the target reliability ( $\beta_0$ ) used in the development of the LRFD is 4.5, it is not surprising that the values of  $\beta$  found in these design equations for steel resistances are close to 4.5. A slight deviation from the target reliability is due to the generalization of the LRFD equations for ordinary bolted connection to the anchor bolt, nevertheless, this deviation is still tolerable.

It was also found that the reliability indices for concrete failure are lower than those for steel failure. This is due to the fact that the covs in concrete resistances are higher than the covs in steel resistances. This indicates also that for a given load, failure in concrete is more likely than failure in steel.

In practice, a higher level of safety is commonly assigned to connectors. If concrete failure is regarded as the failure of a part of the connection, then the level of safety in concrete resistances should be the same as that for other steel connections. The computed reliability indices for the capacity of anchor bolt governed by concrete failure are however lower than the reliability indices observed in the design of reinforced concrete members for flexural and compression failure, which range from 2.6 to 3.5 (12,17).

Among the design equations for concrete failure, the PCI design equation for shear resistance and the ACI design equation for tensile resistance have reliability indices comparable to that of reinforced concrete members. The reliability indices for these equations are in the same range as those observed for the shear capacity of reinforced concrete, which ranges from 1.9 to 2.4 (17). In studies on the reliability of concrete members (12,20), it was concluded that current strength reduction factor for shear capacity is inadequate in comparison with strength reduction factor used for flexural capacity of reinforced concrete members.

Apparently, the low reliability indices obtained for the ACI equation in predicting the shear concrete capacity and for the PCI equation in predicting the tensile concrete capacity are due to prediction errors in modelling these capacities. The low reliability index in the ACI equation results from a significant difference in the ratio of the test to the predicted capacity for different values of edge distances ( $m$ ). For instance, while the ratios of the test to predicted capacity range from 2.3 to 2.5 for  $m = 2$ ", these ratios range from 1.2 to 1.8 for  $m = 6$ " (Table 4.4). For the PCI equation, on the other hand, the low reliability indices are due to the use of the cone failure surface in predicting the tensile concrete capacity, and the use of a

high value for the strength reduction factor ( $\phi_c = 0.85$ ) in computing the design tensile capacity. Eqs. 2.9 and 2.10 show that the tensile capacity computed by using the cone failure surface (Eq. 2.10) is  $\sqrt{2}$  higher than the result obtained by using the projected cone failure surface (Eq. 2.9).

Table 5.1 Reliability Indices for Shear Resistance  
Steel Failure

PCI (34): $V_s = A_s f_u$ with $\phi_s = 0.75$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	2.737	2.737	2.737	2.737	2.737	2.737
0.5	3.029	3.029	3.001	2.985	2.974	2.965
1.0	3.079	3.079	3.067	3.057	3.050	3.044
1.5	3.086	3.086	3.083	3.080	3.076	3.073
2.0	3.083	3.083	3.086	3.086	3.085	3.083

ACI (10): $V_s = 0.7A_s(0.8f_u)$ with $\phi_s = 0.85$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	4.141	4.141	4.141	4.141	4.141	4.141
0.5	4.333	4.333	4.314	4.303	4.296	4.290
1.0	4.367	4.369	4.359	4.352	4.347	4.343
1.5	4.375	4.375	4.372	4.369	4.367	4.364
2.0	4.375	4.375	4.376	4.375	4.374	4.372

Table 5.2 Reliability Indices for Shear Resistance  
Concrete Failure

PCI (34): $V_c = 3250(m-1)\sqrt{f'_c}/5000$ with $\phi_c = 0.85$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	2.309	2.309	2.309	2.309	2.309	2.309
0.5	2.449	2.449	2.431	2.422	2.415	2.411
1.0	2.496	2.496	2.480	2.471	2.464	2.460
1.5	2.518	2.518	2.505	2.497	2.492	2.487
2.0	2.530	2.530	2.520	2.513	2.508	2.505

ACI (10): $V_c = 2mm^2\sqrt{f'_c}$ with $\phi_c = 0.85$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	1.648	1.648	1.648	1.648	1.648	1.648
0.5	1.729	1.729	1.718	1.712	1.708	1.706
1.0	1.762	1.762	1.750	1.744	1.740	1.736
1.5	1.779	1.779	1.769	1.763	1.759	1.756
2.0	1.790	1.790	1.781	1.775	1.772	1.769

Table 5.3 Reliability Indices for Tensile Resistance  
Steel Failure

PCI (34): $P_s = A_s(0.9f_u)$ with $\phi_s = 1.00$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	2.659	2.659	2.659	2.659	2.659	2.659
0.5	2.986	2.986	2.957	2.939	2.927	2.918
1.0	3.034	3.034	3.024	3.015	3.008	3.002
1.5	2.035	3.035	3.036	3.034	3.032	3.030
2.0	3.027	3.027	3.034	3.036	3.037	3.036

ACI (10): $P_s = A_s(0.8f_u)$ with $\phi_s = 0.90$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	3.526	3.526	3.526	3.526	3.526	3.526
0.5	3.795	3.795	3.772	3.757	3.748	3.740
1.0	3.831	3.831	3.824	3.818	3.812	3.808
1.5	3.829	3.829	3.832	3.831	3.830	3.828
2.0	3.820	3.820	3.828	3.831	3.832	3.832

Table 5.4 Reliability Indices for Tensile Resistance  
Concrete Failure

PCI (34): $P_c = 4\pi\sqrt{2}l_e(l_e + d_h)\sqrt{f'_c}$ with $\phi_c = 0.85$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	0.826	0.826	0.826	0.826	0.826	0.826
0.5	1.058	1.058	1.027	1.011	1.000	0.993
1.0	1.142	1.142	1.113	1.097	1.086	1.078
1.5	1.184	1.184	1.159	1.145	1.135	1.127
2.0	1.208	1.208	1.187	1.175	1.166	1.159

ACI (10): $P_c = 4\pi l_e(l_e + d_h)\sqrt{f'_c}$ with $\phi_c = 0.65$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	2.178	2.178	2.178	2.178	2.178	2.178
0.5	2.308	2.308	2.291	2.282	2.276	2.272
1.0	2.353	2.353	2.337	2.329	2.323	2.318
1.5	2.375	2.375	2.362	2.354	2.349	2.345
2.0	2.388	2.388	2.377	2.370	2.366	2.362

Table 5.5 Reliability Indices for LRFD Equations  
Steel Failure\*

Shear resistance: $V_s = 0.7A_s(0.75f_u)$ with $\phi_s = 0.75$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	4.570	4.570	4.570	4.570	4.570	4.570
0.5	4.492	4.492	4.462	4.446	4.436	4.428
1.0	4.560	4.560	4.539	4.526	4.517	4.510
1.5	4.587	4.587	4.572	4.562	4.555	4.550
2.0	4.599	4.599	4.589	4.582	4.576	4.572

Tensile resistance: $P_s = A_s(0.75f_u)$ with $\phi_s = 0.75$						
Ratio	Influence Area (ft <sup>2</sup> )					
$L_o/D_n$	200	400	800	1200	1600	2000
0.0	4.300	4.300	4.300	4.300	4.300	4.300
0.5	4.192	4.192	4.155	4.134	4.120	4.110
1.0	4.271	4.271	4.248	4.233	4.222	4.213
1.5	4.295	4.295	4.282	4.273	4.265	4.259
2.0	4.304	4.304	4.297	4.291	4.286	4.282

\*  $\phi R_n \geq 1.4D_n$  or  $\phi R_n \geq 1.2D_n + 1.6L_n$

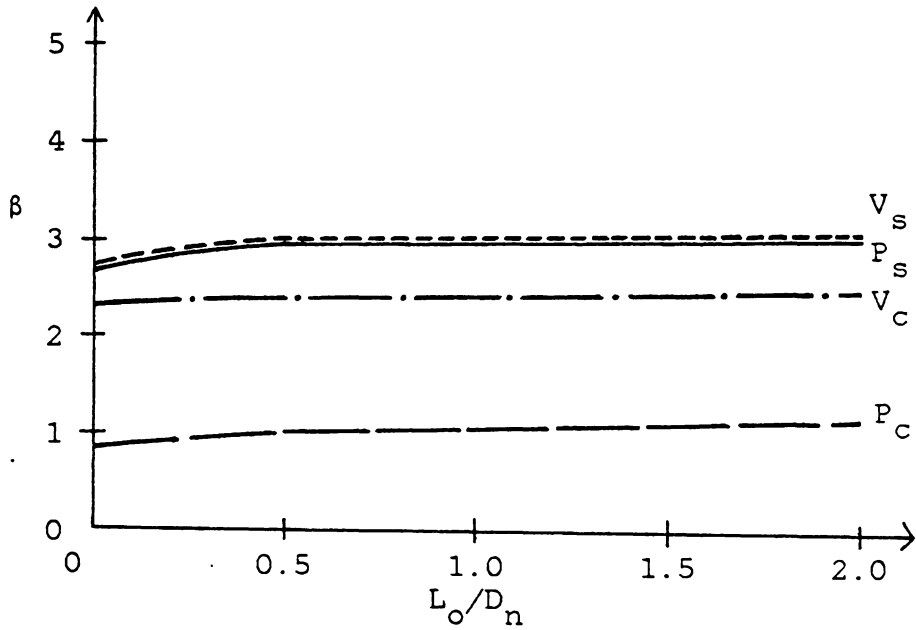


Figure 5.1 Reliability Indices for PCI Design Equations

$$A_I = 1,600 \text{ ft}^2$$

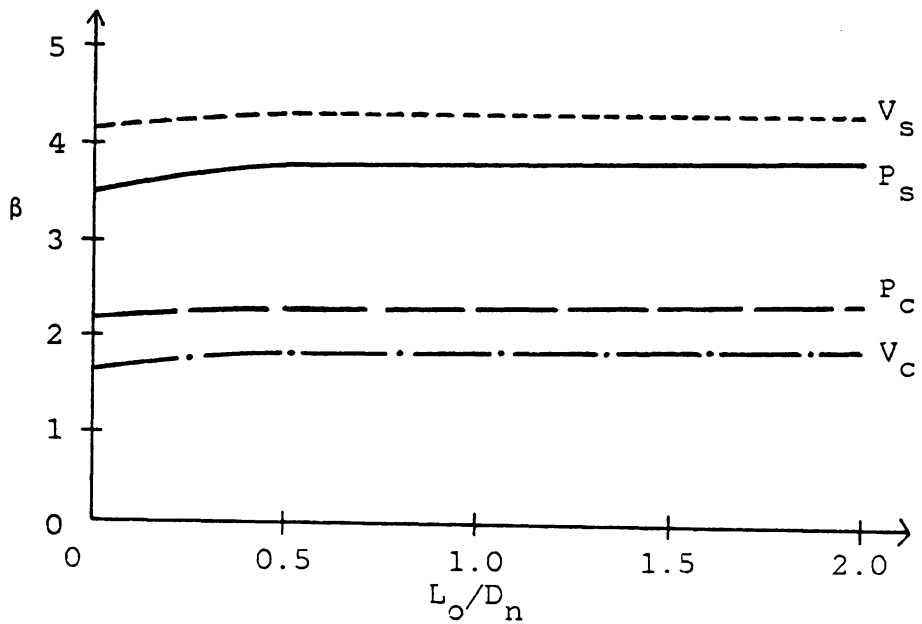


Figure 5.2 Reliability Indices for ACI Design Equations

$$A_I = 1,600 \text{ ft}^2$$

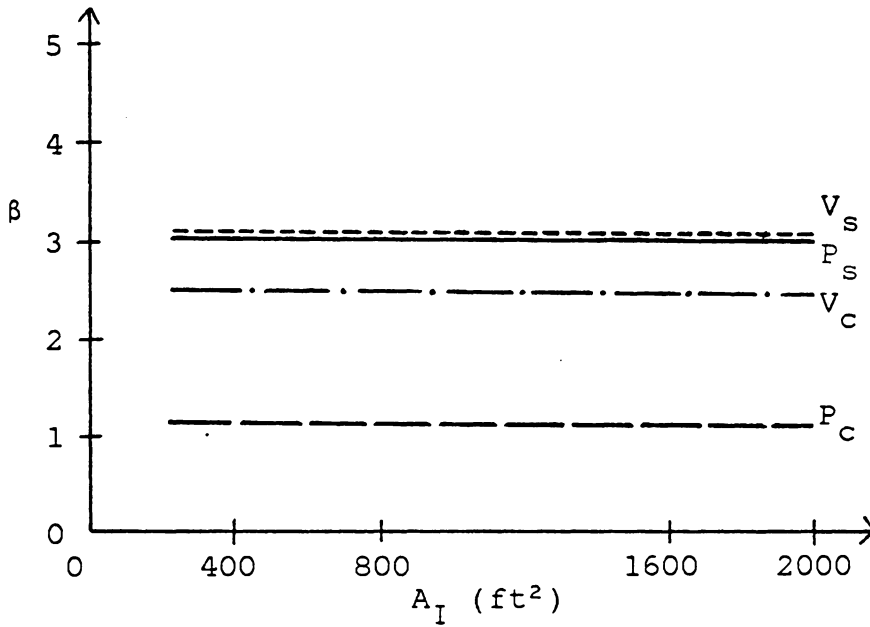


Figure 5.3 Reliability Indices for PCI Design Equations

$$L_o/D_n = 1.0$$

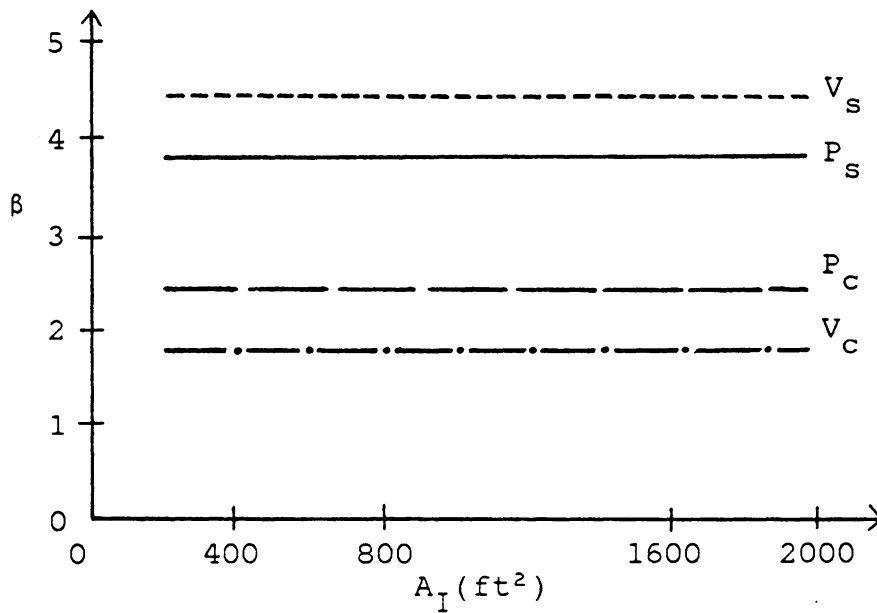


Figure 5.4 Reliability Indices for ACI Design Equations

$$L_o/D_n = 1.0$$

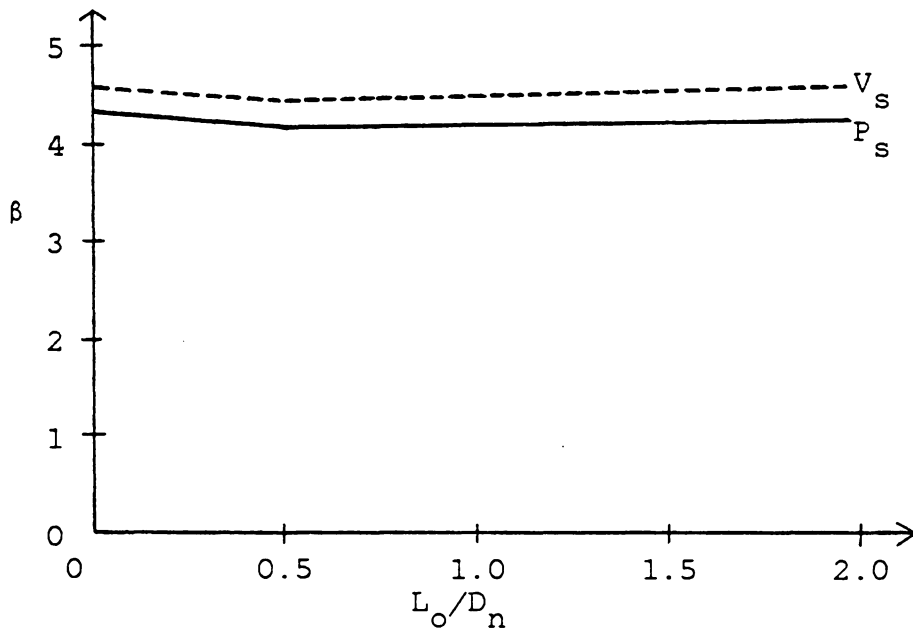


Figure 5.5 Reliability Indices for LRFD Equations

$$A_I = 1,600 \text{ ft}^2$$

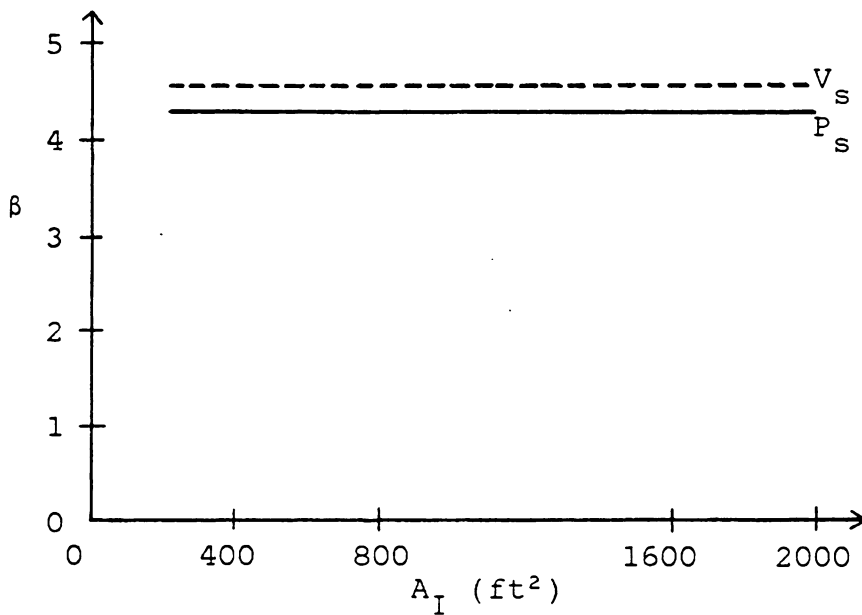


Figure 5.6 Reliability Indices for LRFD Equations

$$L_o/D_n = 1.0$$

## Chapter VI

### SUMMARY AND CONCLUSIONS

#### 6.1 SUMMARY

Previous studies related to the design of anchor bolts with emphasis on the ACI, the PCI, and the proposed LRFD methods are reviewed and are summarized to show the diversity in existing design criteria. Then, the risk levels implied in these widely used design equations for a single-headed anchor bolt embedded in plain concrete are evaluated using a reliability analysis. The loads considered are dead and live loads.

The reliability analysis method used in this study is the advanced first-order second-moment method. The procedure and the algorithm for the computation of the reliability index by this method are presented. The necessary statistical data are obtained from earlier studies on similar variables, and from the observation of test results on anchor bolt connections. Finally, a comparison of risk levels for the ultimate limit state implied in the ACI and the PCI design equations is presented. Reliability indices for the LRFD equations for steel resistance of the anchor bolt are also compared with those for ACI and PCI.

## 6.2 CONCLUSIONS

Based on the results of this study, the following conclusions can be made.

1. There is inconsistency in the levels of safety implied in both the ACI and the PCI design equations for a single headed anchor bolt. The level of safety depends on the failure mode and the loading under consideration.
2. Reliability analysis presented here can be used to evaluate different design equations in a consistent and a systematic way.
3. The generalization of the LRFD equation of ordinary bolted connections to the anchor bolt design is acceptable. For steel failure, it is suggested that the ACI and the proposed LRFD equations be used to predict shear capacity, and the proposed LRFD be used to predict tensile capacity in order to achieve the same reliability as for other steel connections.
4. It is considered good practice to design connections for a higher reliability than members in a structure. However, the risk levels computed for anchor bolts in this study were lower than those for other steel connections and reinforced concrete members.

5. Until further studies are conducted on anchor bolts, it is recommended that the capacity of a single-headed anchor bolt governed by concrete failure be computed using the ACI design equation for the tensile capacity and the PCI design equation for the shear capacity. The risk levels implied in these equations are comparable to current risk levels for flexural capacity of reinforced concrete members.
6. A correction factor should be applied to the ACI design equation for predicting the shear capacity governed by concrete failure in order to reduce the prediction error. One major source of error arises from the prediction of the angle of the failure surface, which is assumed to be constant (45 degrees).
7. The use of the PCI equation for predicting the tensile capacity of the anchor bolt should be avoided, unless past experience of the designer has proved the adequacy of this design equation.

Finally, to develop a probabilistic design methodology for anchor bolt, it is suggested that the following studies be made:

1. Further experiments on the strength of anchor bolt connections, especially for concrete failure modes and on the tensile strength of ASTM A307 bolt, are required.

2. The problem posed by the differences in the reliability indices for different design equations requires work by a standards committee to determine the appropriate design equations and the target reliability for the capacity of anchor bolt governed by concrete failure.

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