

Modeling Transit Vehicle Travel Time Components for Use in Transit Applications

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ABSTRACT

Traffic congestion has continued to grow as a result of urbanization, which is associated with an increase in car ownership. As a way to improve the efficiency of the transportation system, emerging technologies including Connected Automated Vehicles (CAVs), loop detectors, Advanced Traveler Information Systems (ATISs), and Advanced Public Transportation Systems (APTSs) are being deployed. One of the successful techniques that has demonstrated benefits for system users, operators and agencies is Transit Signal Priority (TSP). TSP favors transit vehicles in the allocation of green times at traffic signals. A successful deployment of TSP depends on different factors including the prediction of various components of transit vehicle travel times to predict when a vehicle would arrive at a traffic signal. Current TSP state-of-the-art and state-of-practice disregards the impact of bus stops, transit vehicle characteristics, driver, and the prevailing traffic conditions on the predicted arrival time of transit vehicles at traffic signals. Considering these factors is important the success of TSP hinges on the ability to predict transit vehicle arrival times at traffic signals in order to provide these vehicles with priority service.

The main contribution of this research effort relates to the modeling of the various components of transit vehicle travel times. This model explicitly captures the impact of passengers, drivers and vehicle characteristics on transit vehicle travel times thus providing better models for use in various transit applications, including TSP. Furthermore, the thesis presents a comprehensive understanding of the determinants of each travel time component. In essence, the determinants of each component, the stochasticity in these determinants and the correlation between them are explicitly modeled and captured.

To achieve its contribution, the study starts by improving the current state-of-the-art and state-of-practice transit vehicle boarding/alighting (BA) models by explicitly accounting for the different factors that impact BA times while ensuring a relatively generalized formulation. Current formulations are specific for the localities and bus configurations that they were developed for. Alternatively, the proposed BA time model is independent of the transit vehicle capacity and transit vehicle configuration (except for the fact that it is only valid for two-door buses – a separate door for alighting and boarding the bus) and accounts for the

number of on-board passengers, boarding and alighting passengers. The model also captures the stochasticity and the correlation between the model coefficients with minimum computational requirements. Next the model was extended to capture the bus driver and vehicle impacts on the transit vehicle delay in the vicinity of bus stops, using a vehicle kinematics model with maximum speed and acceleration constraints to model the acceleration/deceleration delay. The validation of the model was done using field data that cover different driving conditions. Results of this work found that the proposed formulation successfully integrated the human and vehicle characteristics component in the model and that the new formulation improves the estimation of the total delay that transit vehicles experience near bus stops. Finally, the model was extended to estimate the time required to merge into the adjacent lane and the time required to traverse a queue upstream of a traffic signal. The final part of this study models the bus arrival time at traffic signal using shockwave and prediction model in a connected environment. This section aims to model the transit vehicle arrival time at traffic signal considering the impact of signal timing and the prevailing traffic conditions.

In summary, the proposed model overcomes the current state-of-the-art models in the following ways: 1) it accounts for the vehicle capacity and the number of on-board passengers on bus BA times, 2) it captures the stochasticity in the bus stop demand and the associated BA times, 3) it captures the impact of the traffic in modeling the delay at a bus stop , 4) it incorporates the driver and vehicle impact by modeling the acceleration and deceleration time, and 5) it uses shockwave analysis to estimate bus arrival times through the use of emerging technology data. Through statistical modeling and evaluation using field and simulated data, the model overcomes the current state-of practice and state-of art transit vehicle arrival time models.

Modelling Transit Vehicle Travel Time Components for Use in Transit Signal Applications

Taqwa Alhadidi

GENERAL AUDIENCE ABSTRACT

Traffic congestion grows rapidly causing increment in travel time, reducing travel time reliability, and reducing the number of public transportation riders. Using the Advanced Public Transportation Systems (APTS) technology with Advanced Traveler Information Systems (ATISs) helps in improving transportation network travel time by providing real-time travel information.

One of the successful techniques that has demonstrated benefits for system users, operators and agencies is Transit Signal Priority (TSP). A successful deployment of TSP depends on different factors including the prediction of various components of transit vehicle travel times to predict when a vehicle would arrive at a traffic signal. Current TSP state-of-the-art and state-of-practice disregards the impact of bus stops, transit vehicle characteristics, driver, and the prevailing traffic conditions on the predicted arrival time of transit vehicles at traffic signals.

The difficulty of modeling the various determinants of the transit vehicle travel time as explicit variables rather than include some of them are implicitly modeled due to two main reasons. First, there are various significant factors affecting estimating the transit vehicle arrival time including; the passenger demand at bus stop, driver characteristics, vehicle characteristics and the adjacent prevailing traffic conditions. Second, the stochasticity and the fluctuation nature of each variables as they differ spatiotemporally.

The research presented in this thesis provides a comprehensive investigation of the determinants of different transit vehicle travel time components of the transit vehicle arrival time at traffic signal leading to a better implementing of TSP. This study was initiated due to the noticeable overlooking of the different factors including human and vehicle behavior in the current state-of-practice and state-of-art which, as a result, fails to capture and incorporate the impact of these components on the implementing of TSP.

DEDICATION

“Incredible change happens in your life when you decide to take control of what you do have power over instead of craving control over what you don’t. For what it’s worth: it’s never too late or, in my case, too early to be whoever you want to be. There’s no time limit, stop whenever you want. You can change or stay the same, there are no rules to this thing. We can make the best or the worst of it. Change is good, remembering is wonderful and never forgetting where you came from is magnificent, but the most spectacular trait is to never stop moving forward or growing.”

TO MY BELOVED FATHER IBRAHIM, MY late MOTHER “HALA”, MY BELOVED SISTER FATIMA, AND MY SIBLINGS: YOU WILL ALWAYS HAVE A SPECIAL PLACE IN MY HEART AND BE MY BACKBONES.

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Chapter 1 **Introduction**

This chapter covers the background of the research topic and the research problem statement, followed by the research goal and objectives. Research contributions to the current state-of-art and practice are subsequently presented, and the chapter ends with a guide for readers to navigate the thesis.

1.1 INTRODUCTION

Growth in urbanization, increases in modernization and civilization, and changes to population density yield traffic congestion. Congestion causes increasing travel times and reduces both system infrastructure efficiency and its capability to meet growing demands. Several methods have been adopted to address urban traffic congestion, and results from the field show that the most promising option to handle this problem is deploying intelligent transportation system (ITS) applications. ITS incorporates any application that improves the efficient movement of individuals and goods by implementing different technologies.

As a result of their success, different ITS applications have been implemented globally. The most important of these are adaptive Traffic Signals (ATs) and improving the public transportation system via Advanced Public Transportation Systems (APTSs). ATs aims to improve intersection performance by minimizing delays at traffic signals, which is considered an important factor in network travel time. While APTS helps in attracting more riders to decrease the number of cars along the developed urban transportation network, this reduction helps in providing more accurate travel information for other modes of transportation. APTS applications show improvements in public transportation Level of Service (LOS) by providing real-time information (i.e. Bus arrival time, bus headway, automated vehicle location (AVL), Automated Passenger Counting (APC), bus travel time and trip travel time) for both passengers and agencies.

Travel time is a significant factor that can be used to evaluate the success of the implementation of any ITS application. Also, travel time is a Measure of Effectiveness (MOE), which is understandable for both the public and professionals. Improving the accuracy of travel time estimation helps transportation system users, professionals and operators. From the perspective of road users, travel time information helps in better scheduling their travel, planning trip modes and routes, and knowing their arrival times and any potential delays. From the professional's perspective, knowing the travel time helps in monitoring traffic congestion,

detecting incidents (i.e. lane closures, queuing spill back), monitoring congestion progression, updating route guidance and disseminating travelers' information. From the planner's viewpoint, travel time can be used to calibrate air quality emissions, fundamental diagrams, and travel demand forecasting models. System operators, on the other hand, use travel time information to control their fleet size and trip scheduling and manage their system operating cost.

1.2 Problem Statement

The accurate estimation of urban network travel times is a challenging task due to the complexity of the factors affecting urban network travelers. These factors include day-to-day and daily demand fluctuations and traffic demand compositions (including passenger cars, non-motorized passengers, and transit vehicles). Determining traffic demands is considered a crucial challenge because of the different conflicting factors, such as faster mobility and stopping times to serve passengers, which affects their arrival times at traffic signals. Using reliable travel times also allows users to estimate a more reliable arrival time at their destination.

Using reliable travel time data within Advanced Traveler Information Systems (ATISs) and Advanced Traffic Management Systems (ATMSs) helps transportation system users acquire better information to avoid traffic congestion. ATIS requires data collection from different sources including loop detectors, probe vehicles, connected vehicles or transit vehicles used as probe vehicles. On the one hand, the success of the implementation of any ATIS is measured by the transportation network data collection. On the other hand, the cost associated with the data collection is very high. Thus, the challenge becomes improving the operation of the current transportation system with minimum cost, as the budget is the constraint to prioritize the selection of any project. One of the key components to minimize the associated cost with the implementation of ATIS is implementing the APTS.

The APTS application helps in providing accurate travel time information for system users and operators. This requires an accurate prediction of transit vehicle travel times. The transit vehicle travel time on any link depends on different factors including the link's geometric features (e.g. dedicated transit lane versus non- dedicated transit lane), bus stop location (far-side, near-side, and midblock), bus stop configuration (number of loading areas), adjacent traffic conditions (volume and composition), and link control system (fixed traffic

signal, actuated traffic signal, or using TSP in an ATC system). Typically, bus travel time variability stems from the variations in passenger demands (day-to-day and bus-to-bus), traffic signal delays after stopping to serve passengers, wheelchair lift and bicycle rack usage, differences in bus operator experience, route length, and the number of stops and the location of these stops ([1], [2], [3]).

Transit vehicle travel time consists of three components: the transit vehicle travel time along a link, the elapsed time at a bus stop known as the dwell time (DT), and signalized intersection delay. However, the accurate estimation of the bus dwell time can be incorporated within the real-time ATCS to provide buses a higher priority at traffic signals. The key of implementing Transit Signal Priority (TSP) in the ATCS depends on the accurate prediction of bus arrival times at traffic signals. This requires knowing bus trajectories, traffic on the link and current traffic signal phase sequences. All of the current research works discuss the implementation of TSP within ATCS by considering the use of stop-to-stop transit travel time to estimate the transit arrival time at any intersection. However, this assumption is biased as it excludes many factors, including bus delays due to current queues at traffic signals, skipped stops or extra elapsed time at bus stops, and the acceleration and deceleration times of buses. All of these factors affect the bus travel speed between stops, which leads to a reduction in the bus travel speed and a waste in green time, which affects the total operation of the transportation system, especially in saturated and oversaturated networks ([4], [5], [6, 7]).

This research mainly fills the gap of the available literature in the implementation of TSP by focusing on improving the estimation of bus arrival times at traffic signals. This reduces the extra green time associated with the errors in estimating bus arrival times at traffic signals. The proposed approach attempts to compute the bus travel time components through the development of a valid framework that is applicable of functioning even within low system connectivity.

This research focuses on using different statistical and analytical techniques that improve the accurate estimation of the bus arrival times at traffic signals through using bus stops, transit vehicle information and prevailing traffic on the link and traffic signal data. These techniques help the developed methodology to calculate real-time bus locations, and arrival and departure times.

1.3 Research Contribution

Traffic congestion, with its associated low mobility and the increase in fuel consumption, is increasing in urban areas. In order to solve this problem, a real-time data collection and performance system is needed to improve the performance of the transportation system and maximize the efficiency of the optimization of the traffic signal control system.

The contributions made by this research entail improving transit system operations by enhancing real-time traffic information, including transit stop demands, and transit vehicle speeds and locations, in addition to the arrival and departure rates of vehicles at signalized intersections, as these can be used for delay and travel time estimation. Furthermore, the incorporation of this information in a TSP within ATCS helps to reduce the total system delay, decreasing transit vehicle travel time and subsequently improving schedule adherence. The anticipated benefits of this research work are not only in improving the estimation of transit vehicle travel times, improving the prediction of transit vehicle arrival times at bus stops and at traffic signals and predicting the elapsed time that transit vehicles spent at bus stops. But it also helps in efficiently operating the ATCS for all the agencies, even those who do not have a highly technologized communication system. The main contributions of this research can be categorized into two major parts: theoretical and practical contributions.

Theoretical Contributions

The theoretical contributions of this dissertation is the improvement of the models available to estimate components of transit vehicle travel times. The approach includes a combination of statistical and physical models to compute the various components of a bus travel time to better estimate and predict bus arrival times at traffic signals. The proposed approach improves the current state-of-the-art methods to estimate the various components of transit vehicle travel times. This is because, in part, this research considered different determinants of these transit travel time components that have been disregarded by previous methods. To this end, the results of this work address the shortcomings of previous models by accounting for the impact of transit vehicle ridership in predicting the boarding/alighting (BA) time, in addition to the model being formulated to be applicable to different bus capacities as long as a separate door for boarding and alighting. The BA model better fits empirical data. The second part of this work incorporates the impact of the vehicle driver and transit vehicle characteristics in modeling the time lost at a bus stop, which have also been disregarded by all

the available models of transit acceleration/deceleration time. The final contribution of this work is that it provides a method to compute the bus arrival time at a traffic signal using analytical and mathematical modeling techniques; these techniques were validated using simulated data generated by INTEGRATION.

Practical Contributions

The main practical contribution of this research work is to create a framework that collects surrounding traffic information in real-time to accurately estimate bus arrival times at bus stops and at traffic signals. The output of this work will help transit operators, traffic agencies, and transit riders. In regard to transit operators, BA modeling helps to update bus arrival and travel time. Using the updated travel time could help in reducing fleet size and operating costs and in improving transit service reliability. Traffic agencies can integrate the results of this work into their evaluation of transit services by comparing the change in demand with the estimated BA times and add services as needed. Moreover, the BA model helps in updating the transit vehicle arrival time not only at the next stop, but could also be used to update the transit vehicle arrival times at any bus stop by integrating historical data or the ATIS with the APTS information (in conjunction with the acceleration/deceleration model) to update the bus arrival time at bus stops. Transit riders can benefit from the results of this work by increasing their access to accurate information about their wait times at bus stops, travel times in transit, and updated arrival times.

1.4 Research Goal and Objectives

The primary goal of this research is to develop a general framework for the estimation of transit vehicle arrival times at traffic signals and bus stops using travel time data and traffic flow theory concepts. Specifically, the goal is to develop a framework to estimate transit vehicle arrival times at bus stops and traffic signals using APTS applications that can be extended to be used further in ATS. To achieve this primary research goal, the following secondary objectives need to be fulfilled:

- To validate the developed framework using different empirical data sets ,
- To investigate the use of various modelling techniques within the framework,
- To explicitly incorporate significant determinants of the various transit vehicle travel time components,

- To study the feasibility of using APTS data to estimate bus travel times and arrival times, and
- To investigate the impact of traffic signal timing and adjacent traffic on the bus arrival times at traffic signals and near-side bus stops.

This dissertation achieves its stated objectives and contributions through addressing each of the following points:

- Improve the current state-of-art and practice for estimating boarding/alighting times at bus stops by incorporating passenger demand variability, the transit bus ridership, and number of boarding and alighting passengers, and the transit vehicle capacity as explicit variables in the prediction model. The proposed model is a descent work as it outperforms the available models [8]. The novelty of this work comes from two sides. First, it includes more significant variables in the model. Second, to the extent possible the model is generalizable. Third, it captures the parameter stochasticity in a simple fashion. Fourth, we develop a Bayesian approach to apply the model to other localities that include different bus configurations.
- Explicitly model the effect of the adjacent traffic in estimating the bus dwell time at a bus stop. The effect was modeled considering the stochastic nature of the traffic density.
- Develop models for estimating acceleration and deceleration rates considering the driver and vehicle inputs/constraints. These models were used to analytically compute the acceleration and deceleration times and validate these times using empirical data.
- Provide a methodology to compute the bus arrival time at traffic signals using analytical and mathematical modeling techniques, and these techniques were validated using simulated data generated by INTEGRATION.

1.5 Thesis Structure

This dissertation constitutes 6 chapters. Chapter 1 provides an introduction that includes background, a statement of the problem, and research objectives along with the research contributions. That is followed, in Chapter two, by a review of the literature that is relevant to the topics covered in the dissertation. Indeed, the literature review covered the different components of the transit vehicle travel time: (1) overview of the transit travel time components

and their determinants and how each of these components is modeled, (2) limitations of the current state of art and practice in modeling each components of the travel time.

The following three chapters present the results of this work. In chapter 3, modeling the boarding alighting times is improved by explicitly considering several significant parameters that were ignored in the available state-of-the-art models while ensuring a relatively generalized formulation. Current formulations are specific for the localities and bus configurations that they were developed for. Alternatively, the proposed BA time model is independent of the transit vehicle capacity and transit vehicle configuration (except for the fact that it is only valid for two-door buses – a separate door for alighting and boarding the bus) and accounts for the number of on-board passengers, boarding and alighting passengers. The model also captures the stochasticity and the correlation between the model coefficients with minimum computational requirements.

In Chapter 4 models for the remaining delay components at a bus stop were developed. These three components include: merge delay, acceleration time and deceleration time is presented. The developed model is calibrated and validated using Blacksburg transit data. Model results showed that the proposed methodology is valid as it incorporates the driver and vehicle characteristics in modeling the acceleration and deceleration times. Modelling the acceleration/deceleration times was done to overcome the shortage in the using the kinematic shockwave to model the bus arrival time at traffic signal that is presented in the following chapter.

In Chapter 5, modeling the transit vehicle arrival time at traffic signals is presented using shockwave theory and prediction models. Although, using shockwave analysis to estimate the travel time is not novel, the model is unique in terms of its structure as it considers the bus speed, traffic signal information, and prevailing traffic information to estimate the bus arrival time at the traffic signal considering different traffic conditions. The adopted methodology was validated using generated data from INTEGRATION. The developed methodology results showed that the model is valid and outperforms current state-of-the-art models.

Finally, Chapter 6 presents the conclusions of this study and recommendations for future work.

In this chapter, the previous state of art of this research thesis is identified and presented. The literature review is organized in three sections that cover three independent research areas. The first section presents a comprehensive overview of transit vehicle travel times in which the most relevant models are described and presented. The second section relates to the transit data collection and the different transit vehicle travel time modeling techniques, which involved statistical and machine learning techniques. The last section covers the traffic signal controlling with emphasis on transit signal priority methods. Additionally, a special focus is dedicated to the shortage of the current models.

2.1 **Overview of Transit Vehicle Travel Times**

Modeling transit vehicle travel times was done along decades using different techniques and datasets. The transit vehicle travel time on any link depends on different factors including link geometric features (i.e. Dedicated transit lane versus non- dedicated transit lane), bus stop location (far-side, near-side, and midblock), bus stop configuration (number of loading areas), adjacent traffic conditions (volume and composition), and link control system (fixed traffic signal, ATC, or using TSP in an ATC system). Typically, bus travel time variability stems from the variations in passenger demands (day-to-day and bus-to-bus), traffic signal delays after stopping to serve passengers, wheelchair lift and bicycle rack usage, differences in bus operator experience, route length, and the number of stops and the location of these stops. The components of transit vehicle travel time are illustrated in Figure 1. In what follows, we present transit vehicle travel time components.

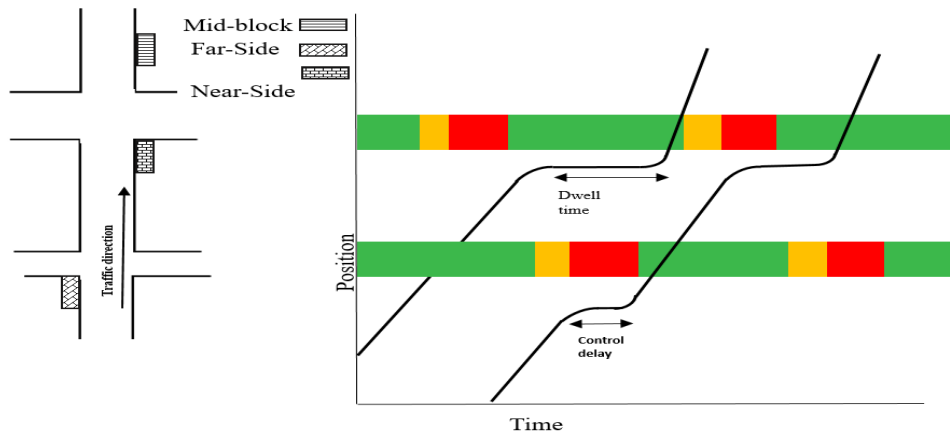


Figure 1: Transit Vehicle Travel Time

2.1.1 Dwell Time

Dwell Time (DT) is defined as the time when a transit vehicle stops at a station to serve passengers for boarding and/or alighting. This time is considered as the influencing factor of bus travel time estimation. This also infers that DT consists of the lost time when passengers board and alight, plus the time for opening-and-closing the door. Passenger boarding and alighting time depends on passenger demand and vehicle configuration (e.g., number of doors, floors and number of units/articulated versus single unit). DT can vary due to the variability in passenger demand between stops at different times of the day, vehicle load factor, passengers with mobility issues, and passenger with loads like bicycles, luggage and so forth [2].

The first attempt to predict dwell time was carried out by Levinson, who predicted bus dwell time based on an analysis of transit survey conducted in U.S. cities in 1980. He calculated DT as a linear relationship between passenger boarding and alighting time and door open and close time [1].

Guenther and Sinha used an elasticity concept to estimate the change in demand as a response to the change in service time. They used the natural logarithm regression to predict the dwell time for each passenger based on field data from the heaviest routes in Indiana [9].

Fricker computed dwell time for Purdue University's campus bus rapid transit (BRT) system using video records for two-door buses (front and side doors). BRT dwell time was computed by measuring five main factors: passenger leaving front door, passengers leaving side door, total passengers boarding, total passengers alighting, and number of standees. According to their observations, dwell time has a linear relationship with low and moderate values for passengers boarding and alighting, but as the number of passengers increases, this relationship increases rapidly in relation to the number of standee passengers [10].

Meng and Qu proposed a probabilistic approach to estimate dwell times of buses in a bus bay by incorporating the randomness caused by the interactions of bus bay layout, transit reactions on the bus stop, arriving passengers, and adjacent traffic [11].

Moreover, another study using smart cards in different bus configurations, including single decker, double decker and articulated buses, was conducted to model the impact of passengers' boarding/alighting activities on bus dwell time. The research results identified the specific effects of floor/entrance type, number of activities and occupancy on both boarding

and alighting dynamics. Also, results indicated that there is a linear relationship between the average boarding/ alighting times and the dwell time. Moreover, results indicated that the variability of boarding/alighting times decreases with the increase in the number of boarding/alighting passengers [12].

Another study was conducted to model bus dwell time using field data from four stops in Auckland, New Zealand. In this study, researchers conducted a comparison between Multiple linear regression (MLR) and decision trees (DTs), namely, classification and regression tree and chi-squared automatic interaction detector to model bus dwell time. The reason behind this study is that the assumptions of MLR cannot always be satisfied. Results of this study showed that the DT techniques eliminated the limitations of the MLR method and provided reliable and accurate estimations of bus dwell times [13] .

A study conducted in Auckland, New Zealand estimated DTs using manual field collected data from 22 bus stops. The study compared Gene Expression Programming (GEP) and Multiple Linear Regression (MLR) results to estimate and model DTs. The results indicated that the GEP model results were better than the MLR in modelling and accurately estimating DTs [14] . Another study was conducted to estimate DTs in traffic congestion using a total of 895 records. The collected data were analyzed using the Compound Poisson Service Time estimation Model (CPSTM), which takes into consideration the interaction between the arriving buses and number of boarding and/or alighting passengers. Different scenarios were used to estimate the total service time. The study results indicated that the accurate service time estimation does not require collecting real-time data [15].

Another study was conducted in China to estimate bus DTs and stop lost serving time; it expresses the dwell time as a function of dead time, acceleration and deceleration time, and Boarding/Alighting (B/A) times. The model was validated using data from 7 common stops. Results of model validation show that the model is able to describe more than 82% of the on-peak and off-peak stop times (dwell times and lost times), with a Mean Absolute Percentage Error (MAPE) of approximately 13% [16] . Xin and Chen used GPS data for downstream stations in China to predict bus transit dwell times using K-Near Neighborhood. They indicated that the bus dwell time at any stop depends on the bus dwell time at upstream stops [17].

Using different data sources to estimate DTs, a number of studies were recently conducted. Using video collected data, a model was developed for Bust Rapid Transit (BRT)

systems using a total of 877 observations from a full day service. These observations include the Bus Lost Time (BLT) as a component of the DTs. The study provides an accurate estimation of DT by incorporating the BLT into the DT estimation model. Using the developed model, it was possible to reduce the BRT capacity by 11% [18]. While a combination of stop-level and GPS data were used to prevent a bias estimation of DT, results of the study showed that the average time per passenger boarding or alighting decreases as the number of passengers boarding or alighting increases. Also, results indicated that DT for nearside or far-side stops is affected by the traffic queuing, or a signalized intersection delay; however, no evidence shows the impact of an intersection delay or the traffic queuing on the midblock stop DT [3]

2.1.2 Acceleration/Deceleration Times

Modelling the acceleration\deceleration was done over decades. Several works have been done for predicting maximum as well as typical acceleration levels, considering different vehicle types.

These models were developed based on out-of-date, limited data or using traditional measurement methods for providing an estimate of vehicle speeds ([19-25] [26], [27], [28], [29]). Yet, none of them focuses on transit vehicles based on field data due to the cost of collecting the required parameters from the field or some technical challenges, including data storage, data accuracy problems, or both of them.

2.1.3 Re-entry Delay

The Transit Capacity and Quality of Service Manual (TCQSM) defines re-entry delay as the time that the transit vehicle spent between the door closings and starting to leave the bus stop. This time mainly has two components, namely the startup time and the clearance time. The startup time is defined as the time needed to start up and travel on its path. It can also be referred to as the acceleration time. The clearance time includes the re-entry delay (merge-delay), which depends on the traffic volumes on the curb lane. Merge-delay time increases as traffic volume increases [2]. This time also depends on the queued traffic and the upstream traffic signal, which may result in long gaps in traffic that cause extra delays in this time [30]. Transit agencies don't recommend using the off-line bus stops to eliminate the merge-delay time; however, off-line stops reduce the near-end collision between the transit vehicle and the traffic in the adjacent lane. TCQSM suggests an average re-entry delay depending on the adjacent

lane volume based on 12 buses stopped at an un-signalized intersection where a critical gap of 7 seconds and random vehicle arrival times are assumed [2].

2.1.4 Transit vehicle arrival times at bus stops and at traffic signals

Modeling transit vehicle arrival times at bus stops and traffic signals has been accomplished using many methods. The importance of modeling this time is that it helps in improving the transit and enhancing the system efficiency. Different models have been deployed to predict and estimate the bus arrival times. In the literature, several methods were used to predict and estimate the arrival times, including historical average models, regression models, the Kalman filter model, and machine learning methods. Yet, none of them were considered as superior as these models assume that the traffic pattern stays stable over a long time period [4, 31, 32].

Lin and Zeng proposed a set of bus arrival time prediction algorithms for a transit traveler information system implemented in Blacksburg, Virginia. Four algorithms were introduced with different assumptions on input data and were shown to outperform several algorithms from the literature. Their algorithms, however, did not consider the effect of passengers at each bus stop, nor did they predict the bus arrival time at traffic signals [32]. Kidwell presented an algorithm for predicting bus arrival times based on real-time vehicle locations. The algorithm worked by dividing each route into zones and recording the time that each bus passed through each zone. Predictions were based on the most recent observation of a bus passing through each zone. This algorithm would not work in large cities where both travel time and dwell time could be subject to large variations[33].

2.2 Transit Data Collection and Modelling Techniques

In this subsection, an overview of the transit data type and collection methods are presented first. Afterwards, a brief discussion of the available transit travel time modelling techniques is presented.

2.2.1 Transit Data Collection

Transit data collection requires a large database, whose data includes several types, such as operational, spatial, and temporal data. Ultimately, the collected data are used in improving transit quality of service and evaluating transit agency operational performance. Thus, using

AVL and APC systems helps in collecting transit data, and enhancing scheduling, service planning, and quality monitoring [34].

In using GPS for transit data collection, the Hydraulics, Electrical and Mechanical (HEM) data corporation has recently provided a data logger that collects HEM vehicle records based on either a 2 Hz or 5 Hz frequency. HEM data provides several parameters of the vehicle characteristics and keeps these parameters to be used in modeling vehicle movement [35].

2.2.2 Modelling Techniques

Transit Capacity and Quality of Service Manual (TCQSM) suggests the three methods to estimate bus dwell time, including field measurements, default values by planners, and calculation models using field data. Field measurements can be conducted directly in the field or using archived automated vehicle location (AVL) and automated passenger counters (APC). In contrast with field measurements, default values are used when demand data is missing at any stop. Indeed, these default values can be used in cases of planning transit routes. These values depend on stop location (downtown, park-and-ride) and stop type (terminal station and transfer station). Finally, the calculation method is used to compute DT based on boarding and/or alighting, time for fare payment, passenger demand for each vehicle door and/or for each bus in cases where a transit stop has more than one loading area [2].

Traffic data is complex in nature, thus encouraging researchers to use different modeling techniques, including statistical and machine learning techniques. The results of different work indicated that there is no superior model to be used for modelling transit arrival times. The modeling technique depends on the data and the application of the modeling results. [4, 31-33]

2.3 Taxonomy of Traffic signal Controlling Systems

Different signal control types have been mentioned in various studies and implemented in field. These types are categorized based on traffic operation. With this in mind, traffic signals may be either pre-timed operation, semi-actuated controlled, fully-actuated control, or computer control signal. To further clarify the differences between these types, the main decisive factor between these types is the detected volume. For instance, pre-timed signals are those signals that have fixed cycle times, intervals, and phases. While semi-actuated intersections give the right of way (i.e., green time) to the main approach, but for minor approaches, detectors are

placed so that once volumes have been detected, a short green time is granted to the minor road. Not surprisingly, this method of controlling fails in operation performance for heavy traffic approaches. Rather, a method must be used to detect volume along all approaches to operate heavy volume intersections more efficiently. As this mentioned, fully-actuated intersections operate and manage the traffic by using detectors in each lane at every approach, which generate information for traffic volume along all approaches. As a result, green time is split based on the volume. A final method uses computer control signals, which are signals that have varied signal timing, phasing and splitting according to the detected volume [36].

2.3.1 Signal Cycle length

Computing signal cycle length is another factor that affects transportation system performance. This is due to the amount of served traffic at a specific intersection as well as for coordinated systems [37]. The estimation of cycle length is conducted using different methods, the most common of which are Webster (1951) and the Highway Capacity Manual (HCM)

The first attempt to estimate cycle length was done by Webster [38]. He developed a formula to compute the cycle length by differentiating the delay with respect to the cycle length. The optimum cycle length is associated with minimizing delay. The optimum cycle length can be computed based on the following formula:

$$c_o = \frac{1.5L + 5}{1 - Y} \quad (\text{II- 1})$$

Where:

C_o = is the optimal cycle length,

L =lost time and can be computed by $nL+R$,

n = number of phases,

L = lost time,

R = red time, and Y is the summation of the relation critical volumes and capacity.

However, most of practitioners prefer using the Highway Capacity Manual's (2010) procedure. Even still, this method does not take into account the most effective coordinated system contribution factors including downstream congestion level, high volume-to-capacity ration (v/c), and their impacts on the accommodated traffic. HCM suggests a set of formulas in

order to compute traffic signal settings using cycle length, effective green, and critical intersection volume-to-capacity ratio. These formulas are shown as follows:

$$X_c = \left(\frac{C}{C - L} \right) \sum_i y_{c,i} \quad (\text{II- 2})$$

$$C = \frac{L X_c}{X_c - \sum_i y_{c,i}}$$

Once the signal head turns green, vehicles start dissipating until the discharging rate reaches the maximum called “saturation flow.” The relationship between discharging rate with the cycle length is shown in Figure 2.

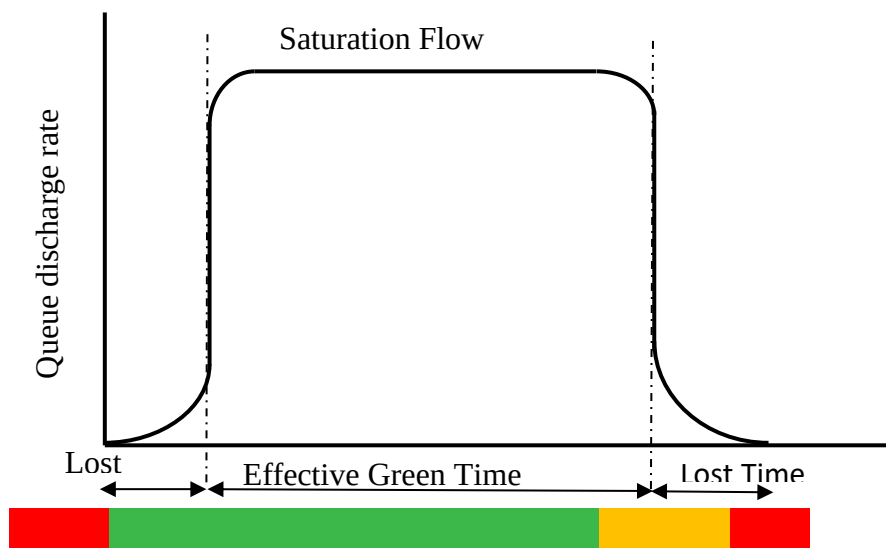


Figure 2: The Relationship between Rate of Discharge and Cycle Time

2.3.2 Optimization of traffic signal settings

Optimizing traffic settings may take decades to master. The optimization of traffic signal settings can be conducted either using mathematical approach or using simulation software. Yet, every work has its own uniqueness based on the traffic demand (oversaturated or under-saturated traffic conditions), network or subnetwork size (an isolated intersection, arterial corridor)—including networks with different scales (small, medium and large size), traffic demand composition, traffic control type (fixed, actuated, semi-actuated or adaptive system), and the selected objective function within its constraints. Methods of optimizing the isolated intersection were suggested by Webster and HCM to determining the cycle length and the effective green split based on minimizing delay. Optimization techniques for the isolated

intersection is not affected by using either the mathematical approach or the simulation approach.

2.3.3 Choosing the Objective Function

Selecting the objective function plays a major role in finding the optimum solution. Different measures of network performance are candidates to be selected either individually or using weighted sums of these measures. For instance, one may consider a delay, which presents extra elapsed time during the travel, or travel time, which expresses the time needed to move from origin to destination. Other factors may include but are not limited to: the queued vehicles at specific intersection, maximizing the completed trip, or the weighted number of completed trips [39].

2.3.4 Transit Signal Priority

Transit Signal Priority (TSP) is defined as the preferential operational treatment for transit vehicle movement along a network by providing extra time for passing through traffic-signal controlled intersections. Providing transit vehicles with TSP helps to improve transit schedule adherence and transit travel time efficiency—while minimizing impacts on normal traffic operations. This system requires establishing a set of components including: 1) a detection system that lets the TSP system know where the vehicle requesting signal priority is located. The detection system then communicates with a (2) priority request generator that alerts the traffic control system that the vehicle would like to receive priority. There is software that processes the request and decides whether and how to grant priority based on the programmed (3) priority control strategies. Finally, there is software that (4) manages the system, collects data, and generates reports [40].

2.3.5 Applying Optimized Traffic Signals for TSP treatment

Providing transit with special treatment helps in improving schedule adherence, increasing ridership, and decreasing delay for riders and network. As one of the treatments can be using dedicated bus lane or as known “dedicated lane”, it helps in providing transit vehicles with a separate physical infrastructure with eliminating the interaction with the other traffic composition.

Transit Signal Priority (TSP) system is used to facilitate the transit movements along intersections to get an efficient transit schedules [41]. TSP has been widely implemented in different areas by researchers and agencies around the world to evaluate the TSP system impacts on buses and general traffic as well. Most of the previous studies have shown significant improvements in terms of measures of effectiveness (MOES) using TSP such as transit travel time, delays, side-street queue length [42-44]. Some studies evaluated the TSP on different areas' types such as an isolated intersections and arterial areas. Also, some of the previous studies considered different signal control types such as fixed-time, actuated, and responsive or adaptive control [45].

Sam et al. [46] proposed a procedure for real-time signal control that considers transit interference and priority. The procedure generates a number of short-term alternative real-time phase sequences for various levels of transit priority. A traffic-responsive control was developed with transit priority by generating control strategies based on some priority lists [46]. The model tested in the afternoon peak hour for an intersection of Queen Street and Bathurst Streets. The results showed that there was a delay reduction compared with fixed-time operation.

A Stochastic mixed-integer nonlinear program (SMINP) model was proposed as the core component of a real-time Transit Signal Priority (TSP) control system [47]. The model used a novel approach to capture the impacts of the priority operation to other traffic using the deviations of the phase split times from the optimal background split times. The SMINP model was implemented in a VISSIM microscopic traffic model with an optimization solver. The aim of this study was to minimize the delay of bus system [47]. The results showed that there was a significant improvement with bus delay comparing with RCB- TSP in a single-bus case. Moreover, the bus priority using the proposed model was much more efficient under congested traffic conditions for a multiple-bus case.

Dion et al. Applied TSP along Columbia Pike arterial corridor, in Arlington, VA [42]. The study used the INTEGRATION microscopic traffic simulation model to evaluate the impact of different priority strategies on buses and general traffic during the morning peak and midday traffic periods. The transit priority strategies include regular and express buses. The priority logic that is considered in the study provides simple green extensions and green recalls within a fixed-time traffic signal control [42]. The simulation results showed that the buses provided with priority would benefit from TSP. The results also showed that when the less

demand is present, the overall negative impacts could be negligible due to the availability of spare capacity. The results also indicated that the buses experience reduced travel time, delay, stops, and fuel consumption. Another study applied TSP in Broad Street in Newark, New Jersey. This arterial is characterized by significant transit activity and heavy traffic volumes [44]. The study tested 12 different scenarios based on demand variations (a.m. Peak, p.m. Peak, and Future demand), control types, and the presence of transit priority operation. The study tested a traffic-adaptive TSP algorithm using WATSIM simulation model. The study goal was to quantify travel time impacts and transit operational benefits from using TSP [44]. The results showed that the travel time in both cross and major streets were reduced. Results also showed that the benefits of TSP can reduce the number of buses in the network. Another study evaluated the TSP impacts on the operations of a single signalized intersection within a coordinated arterial system [43]. The objective of this study is to isolate the impacts of various traffic, transit, and signal timing factors on the potential benefits of TSP. The results showed that TSP provides benefits to transit vehicles that receive priority [43]. The results also showed that the system-wide impact of TSP is dependent on the frequency of transit vehicles. As the transit vehicle frequency increases, larger system disbenefits are noticed. Another interesting finding is that the transit vehicle dwell times at nearside bus stops can have significant systemwide impacts on the benefits of TSP [43].

A green extension TSP was implemented to test the system-wide impacts on U.S. Route 1 in the Northern Virginia Area [48]. The corridor is controlled by a coordinated-actuated signal control with a cycle length of 180 s. The green time can extend 5 s if the system detects any transit vehicle that need more time than the normal green to pass the intersection. The extended green value is subtracted from the next phases in the same cycle and hence the cycle length will not be affected and it would be constant [48]. The results showed that the buses always benefited from the TSP but did not guarantee a system-wide benefit. The results also indicated that the use of green extension TSP does not improve the transit vehicle nor system-wide system. Another interesting finding in this study is that TSP operations at nearside bus stops specifically within the detection zone results in increasing the bus delays [48]. Kim et al. Studied the effect of nearside bus stops on the TSP system [49]. The study proposed a Weighted-Least-Squares (WLS) regression model to estimate the transit vehicle dwell time at the bus stops, and considered also the prediction interval. This study considered three different variables that could affect the dwell time such that schedule adherence, passenger loads, and bus headways. A t-test statistical analysis was conducted to test the statistical

significantly impact for each variable. The statistical analysis showed that the bus headways have a linear relationship with bus dwell time for all bus stops but not with schedule adherence and passenger loads [49]. The proposed model was tested on VISSIM model of an urban arterial section of Bellaire Boulevard in Houston, TX. The results showed that the dwell time for buses at nearside bus stops can be modeled using WLS regression. The delays for buses were improved using the proposed algorithm [49].

Another study addressed an appropriate synthesis for the TSP in small and medium-sized cities. The study tested and evaluated of alternatives TSP in Burlington, Vermont with a consideration of some impacts on bus travel time, delay, and side-street queue length [50]. The study used a 10 s green extension for the AM period in VISSIM traffic simulation model. The results indicated a significant delays for the buses in the opposite direction since they do not have the priority [50].

2.4 Limitation of the current state-of-art and Practice

This section summarizes the current state-of-art and practice of the conducted literature review. The limitation of the previous studies is categorized based on the covered studies of this research work.

2.4.1 Limitations of the previous models for estimating the dwell time

Revisiting the current state-of-art and state-of-practice to estimate the dwell time, all the current model expresses the dwell time as a function of the demand at the transit stop and disregards the impact of bus capacity in the estimating model building [1, 3, 9-11, 13, 14, 16, 17, 33, 51, 52]. Moreover, some of these models are not applicable in case there is only alighting passengers at the bus stop [1, 15].

The current state-of-practice reports the dwell time as the time difference between the bus departure time and the bus arrival time at the bus stop. In fact, this time has two components the merge-delay time and boarding alighting time. In essence, the estimation in the previous models is inaccurate especially in case of having off-curb transit stop. As such the dwell time at the off-curb transit stop becomes a function of stop demand the time as well that transit vehicle spends waiting to merge with the traffic in the adjacent lane.

2.4.2 Limitation of previous Studies for estimating the acceleration deceleration time

The previous modelling of acceleration/deceleration timing neglects the integrity of the impact of vehicle and driver in estimating the acceleration/deceleration time. Moreover, the data used in building these models are either out of date, or don't cover different driving conditions, or don't account the transit data in building these models. In fact, the struggles of covering these limitations comes from main two sources either data collection, or having a defined driving conditions which both reduces the complexity of considering the driver and the vehicle characteristics in modeling the acceleration/deceleration rate [19-22, 24, 25, 29, 53, 54].

2.4.3 Limitation of previous studies for estimating the bus arrival time at traffic signal

Most of the literature have used different statistical models including parametric and non-parametric models; all of these model used historical data to predict the transit vehicle arrival times at bus stops[4, 31-33, 47].

However, these models do not consider the influence of the traffic, nor the congestion level to predict the bus arrival time. The adjacent traffic plays a major role in predicting the transit vehicle arrival time at traffic signal. As the traffic state varies spatiotemporally, thus encourages the need to use a real-time dataset to predict the transit arrival time at traffic signal.

Chapter 3 **MODELING BUS PASSENGER BOARDING/ALIGHTING TIMES: A STOCHASTIC APPROACH**

This chapter is based on the following publication: Alhadidi T. and Rakha H.A. (2019),
“Modeling of Bus Passenger Boarding/Alighting Times: A Stochastic Approach,”
Transportation Research Interdisciplinary Perspectives.
<https://doi.org/10.1016/j.trip.2019.100027>.

3.1 ABSTRACT

This paper introduces two multilinear models that compute bus passenger boarding/alighting (BA) times at bus stops using empirical data (8,341 empirical observations) from King County Metro in Seattle, Washington. The first model is a classical frequentist model, while the second is a stochastic model developed using bootstrapping together with a Cholesky decomposition. Three variables are considered in the aforementioned models to estimate passenger BA times, namely; the number of alighting passengers, the number of boarding passengers, and the number of passengers on board the bus. These variables are normalized by the capacity of the bus to allow for use for different bus types assuming a separate door for boarding and alighting passengers. The models show that passenger BA times increase with an increase in all three variables (i.e. Positive model coefficients). The Cholesky decomposition technique was applied to bootstrapped data to capture the model coefficient correlations with the use of only nine parameters (coefficient means, variances, and correlations) while capturing the stochasticity observed in the empirical data.

Keywords: bus loading time, transit, bus stop, transit passengers, stochastic modeling, bootstrap, Cholesky decomposition.

3.2 INTRODUCTION

Intelligent Transportation System (ITS) applications attempt to enhance the efficiency of the transportation system through the use of emerging technology. One of many intelligent transportation system applications, advanced traffic management and information systems attempt to enhance the efficiency and resilience of the transportation system using real-time data collected from probe vehicles and stationary sensors, such as loop detectors, video cameras, or radar sensors[55].

However, predicting and estimating multimodal travel times is a challenging task due to the complexity of the factors affecting urban network travelers. These factors include day-to-day and within-day demand fluctuations and traffic demand compositions (including passenger cars, non-motorized passengers, and transit vehicles). Predicting transit vehicle travel time is necessary to allow travelers to avoid traffic congestion and travel more efficiently. Using reliable travel time data within Advanced Traveler Information Systems (ATISS) and Advanced Traffic Management Systems (ATMSS) helps transportation system users acquire better information to avoid traffic congestion. Using reliable travel times also allows users to estimate a more reliable arrival time at their destination.

Transit vehicle travel time on any link depends on different factors, including the link's geometric features (i.e., dedicated transit lane versus non-dedicated transit lane), bus stop locations (far-side, near-side, and midblock), bus stop configurations (number of loading areas), adjacent traffic conditions (volume and composition), and traffic control systems (fixed traffic signal, actuated traffic control, or transit signal priority in an actuated traffic control system). Transit vehicle travel times consist of three components: transit vehicle running times along roadways, dwell times (DTs) at bus stops, and signalized intersection delay.

Typically, bus travel time variability stems from the variations in passenger demands (day-to-day and bus-to-bus), traffic signal delays after stopping to serve passengers, wheelchair lift and bicycle rack usage, differences in bus operator experience, route length, and the number and location of stops [2]. The time elapsed when serving passenger demand at any station, known as the boarding/alighting (BA) time, constitutes the largest portion of DT.

Transit data collection requires a large database. Data types include operational, spatial, and temporal data. Ultimately, the collected data are used in improving transit quality of service, and in evaluating a transit agency's operational performance. Using Automatic Vehicle Location (AVL) and Automatic Passenger Count (APC) systems help in gathering transit data, enhance transit scheduling, and improve service planning and quality monitoring [34].

3.3 STUDY OBJECTIVES AND PAPER LAYOUT

The objective of this study is twofold. First, it attempts to identify critical variables and model bus passenger BA times using a deterministic frequentist regression modeling approach. Second, it develops a stochastic modeling approach by combining bootstrapping and a

Cholesky decomposition approach to capture empirically observed stochastic behavior with the use of only nine parameters.

The paper is laid out as follows. Initially, previous research findings are presented. Subsequently, the modeling approach is described followed by a description of the data gathering and validation procedures. Next a discussion of the frequentist regression statistical model that was used to model bus BA times is presented. Subsequently, the bootstrap approach to modeling the loading time is presented, followed by a comparison of the two approaches. The modeling of stochasticity in the model parameters using the Cholesky decomposition is presented next. Finally, the conclusions and recommendations of the paper are presented.

3.4 PREVIOUS FINDINGS

Dwell time is defined as the amount of time a transit vehicle remains stopped at a bus stop to serve passengers boarding and/or alighting. DT consists of the time it takes passengers to alight and board the bus (BA time), the time it takes to open and close the door(s), and the time needed to merge with traffic, as shown in Figure 3 and Figure 4 [2]. These combined times are critical stochastic factors that affect bus travel times. This variability arises from the fluctuation in the passenger demand between stops for different times of the day, different days of the week, vehicle load factors, passengers with mobility issues, and passengers loading bicycles, luggage, and so forth. Moreover, passenger BA times depend on passenger demand and the bus configuration (e.g., number of doors, floors, and number of units/articulated versus single unit). Consequently, the *Transit Capacity and Quality of Service Manual* (TCQSM) suggests three methods to estimate bus DTs, including field measurements using archived AVL and APC data, default values used by planners, and use of models based on field data considering BA counts, time for fare payment, passenger demand, vehicle configuration, and stop configuration [2]. Using archived AVL/APC data helps in designing the collection of real-time data, allowing more accurate computation of the time difference between the transit vehicle arrival time and the departure time at any bus stop [34].

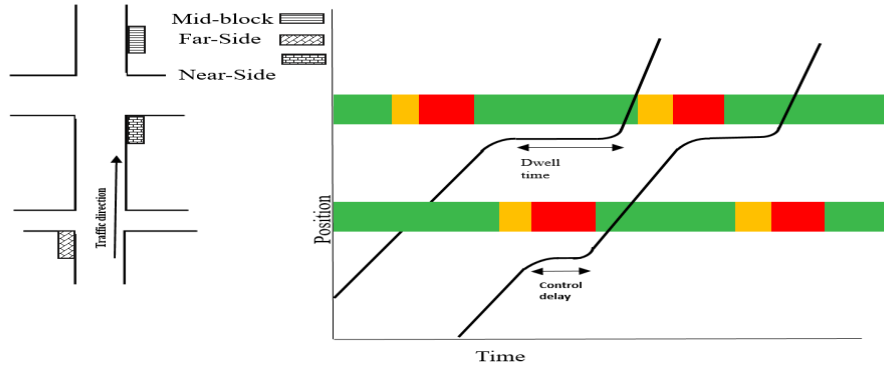


Figure 3: Transit travel time along link

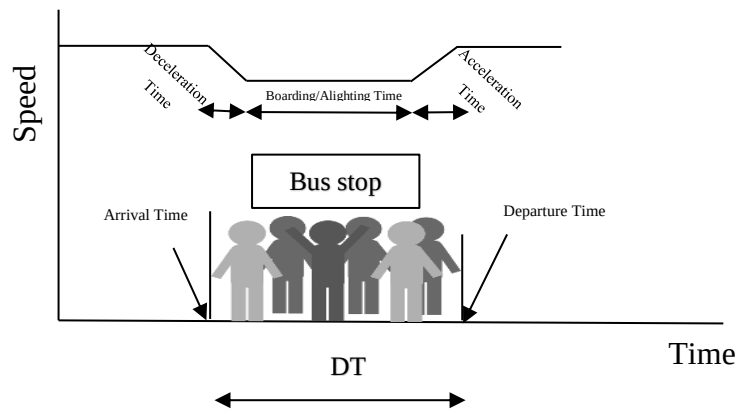


Figure 4: Transit travel time at bus stop

DT has been modeled for decades, and can be categorized into two main groups: ordinary-least squares regression models and probabilistic models. Several linear models have been used to estimate DTs, taking into consideration the number of boarding and alighting passengers ([9, 51, 56],[2, 10, 52]). The first attempt to predict DTs was made by Levinson in 1983 using transit survey data collected from U.S. cities in 1980. Mainly estimating DT as a function of the number of passengers boarding, Levinson found that that DT constitutes about 9–29% of transit vehicle travel time [56]. Considering other DT determinants like fare collection and bus configuration (i.e., number of doors and floors), other authors found that the fare method did not produce significant differences in DTs [51]. Lower bus floors was found to decrease DT by 13–15% [52]. Increasing the number of doors and faster fare collections was found to provide faster service, reducing DTs [1]. Ordinary least squares models for estimating DTs consider the number of served passengers as the most relevant factor [51, 52]. Another study showed that DT is affected by the fare method and the number

of doors, but not time-of-day [57]. A later study found that time of day and service type do not show a significant effect on bus dwell time estimation, and adding to the number of standing passengers results in a higher model coefficient of determination [31]. Subsequent studies developed several nonlinear and stochastic models as well as a non-stationary, high dimensional time series models to estimate bus travel times([31], [58], [59], [11])

A study conducted in Auckland, New Zealand estimated DTs using manual field collected data from 22 bus stops. The study compared Gene Expression Programming (GEP) and Multiple Linear Regression (MLR) results to estimate and model DTs. The results indicated that the GEP model results were better than the MLR in modelling and accurately estimating DTs [14] . Another study was conducted to estimate DTs in traffic congestion using a total of 895 records. The collected data were analyzed using the Compound Poisson Service Time estimation Model (CPSTM) that takes into consideration the interaction between the arriving buses and number of boarding and/or alighting passengers. Different scenarios were used to estimate the total service time. The study results indicated that the accurate service time estimation does not require collecting real-time data [15].

Another study was conducted in China to estimate bus DTs and stop lost serving time, it expresses the dwell time as a function of dead time, acceleration and deceleration time, and Boarding/Alighting (B/A) times. The model was validated using data from 7 common stops. Results of model validation show that the model is able to describe more than 82% of the on-peak and off-peak stop times (dwell times and lost times), with a Mean Absolute Percentage Error (MAPE) of approximately of 13% [16] .

Using different data sources to estimate DTs a number of studies were recently conducted. Using video collected data a model was developed for Bust Rapid Transit (BRT) systems using a total of 877 observations from a full day service. These observations include the Bus Lost Time (BLT) as a component of the DTs. The study provides an accurate estimation of DT by incorporating the BLT into DT estimation model. Using the developed model it was possible to reduce the BRT capacity by 11% [18]. While a combination of stop-level and GPS data were used to prevent a bias estimation of DT, results of the study showed that the average time per passenger boarding or alighting decreases as the number of passenger boarding or alighting increases. Also, results indicated that DT for nearside or far-side stops is affected by the traffic queuing, or signalized intersection delay; however, no

evidence shows the impact of intersection delay or the traffic queuing on the midblock stop DT [3].

3.5 METHODOLOGY

This section describes the two methodological approaches that were used to develop the proposed models, namely: the frequentist approach and the stochastic approach using the Cholesky decomposition.

3.5.1 Frequentist Modeling

As described in Wikipedia, *“In statistical modeling, regression analysis is a set of statistical processes for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables (or 'predictors'). More specifically, regression analysis helps one understand how the typical value of the dependent variable (or 'criterion variable') changes when any one of the independent variables is varied, while the other independent variables are held fixed.”* Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variables – that is, the average value of the dependent variable when the independent variables are fixed. Frequentist statistical models assume a constant variance of the error term. However, the error distribution depends on the distribution of the model variables and may not be necessarily normally distributed and have a constant variance. Consequently, model transformations may be needed in order to ensure that the error is both normally distributed and that the variance is constant. The tests on the residual error and the procedures used to transform the empirical data are described later in this paper.

3.5.2 Stochastic Modeling

The nature of explanatory variables in field measurements is stochastic because the explanatory variable in some studies may act like the response variable in another study. Several resampling methods have been discussed in the literature, including cross-validation, jackknifing, bootstrapping test, and randomization tests (which are widely used in different statistical inferences also it is known as permutation tests).

According to asymptotic theory (law of large numbers), as the sample size grows, the expected value of the variable coefficient converges to its mean because the variance converges to zero. Thus, a large sample of data is needed in order to estimate population distributions (i.e., mean, variance, and confidence intervals) accurately. For population distribution estimation, repeating large numbers of the sample size is an efficient way to express the population distribution ([60],[61]).

Nonparametric bootstrapping estimates the sampling distribution without the need to assume the population distribution. This assumption arises for the condition where a sample is drawn from the population with replacement $S_1^* = \{X_{11}^*, X_{12}^*, \dots, X_{1n}^*\}$; the reason for sampling with replacement is to produce a different sample of the original data set to generate a more accurate estimation of the model's statistics. Broadening the data through resampling it is possible to fit models and save the model coefficient values from each bootstrap sample; thus, not only can the coefficient distribution be generated, but the confidence intervals, covariance, quantiles, and error can be also be easily generated. Moreover, resampling allows for successful estimation of the model's uncertainty and the precision of the selected model ([60],[61]). Based on the normality assumption, the amount of bootstrapping resampling affects the model's confidence intervals; therefore, the minimum bootstrapping resampling is recommended to be 1,000 [61]. The direct use of bootstrapped data is data intensive because it requires the storage of large amounts of sample realizations. To overcome this drawback, the Cholesky decomposition can be used.

Cholesky decomposition is used widely as a quick mathematical numerical solution to a Monte Carlo simulation. The idea of using Cholesky is to solve the problem stochastically by using the law of large numbers, which uses the mean and the variance for the variable in order to draw a distribution function of the variable. In particular, Cholesky decomposition is used to decrease the effect of the multi-collinearity between variables. The reason for using the Cholesky decomposition is that this technique mainly depends on breaking a symmetric positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose. Moreover, using the positive-definite matrix yields exactly one solution for the matrix set.

Let $\mathbf{A}$ be a positive definite matrix. Then $\mathbf{A}$ can be decomposed in exactly one way into a product: $\mathbf{A} = \mathbf{R}^T \mathbf{R}$ such that $\mathbf{R}$ is an upper triangular matrix and has all main diagonal entries; this is called the Cholesky factor of $\mathbf{A}$.

Generally, Cholesky decomposition can be used to estimate β , which here is called the revised Cholesky least-squares estimator, and can be obtained using Equation III.1.

$$\beta(\mu) = S(\mu)^{-1}X^T \quad (\text{III.1})$$

To elaborate on this, suppose that $Z_0, \dots, Z_3$ are independent and identically distributed standard normal vectors (i.e., $Z_i \sim N(0,1)$), then the problem can be formulated using Equations III.2 and III.3.

$$\begin{aligned} \mu_{\beta_0} + c_{00}Z_0 &= \beta_0 \\ \mu_{\beta_1} + c_{10}Z_0 + c_{11}Z_1 &= \beta_1 \\ \vdots &\vdots \\ \mu_{\beta_4} + c_{40}Z_0 + c_{41}Z_1 + \dots + c_{44}Z_4 &= \beta_4 \end{aligned} \quad (\text{III.2})$$

$$\begin{pmatrix} \mu_{\beta_0} \\ \mu_{\beta_1} \\ \vdots \\ \mu_{\beta_4} \end{pmatrix} + \begin{pmatrix} c_{00} & 0 & 0 & 0 \\ c_{10} & c_{11} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ c_{40} & c_{41} & \dots & c_{44} \end{pmatrix} \begin{pmatrix} Z_0 \\ Z_1 \\ \vdots \\ Z_4 \end{pmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_4 \end{pmatrix} \Rightarrow \mu + CZ = \beta \quad (\text{III.3})$$

Here C is a lower triangular matrix that represents the coefficients that need to be determined. By taking the variance of both sides, and given that μ is constant ($\text{Var}(\mu) = 0$) and Z is standard normal ($\text{Var}(Z) = I$) one can compute the variance of the model coefficients. Equation III.4 shows the results of taking the variance of the left side of Equation III.3. In Equation III.5 the variance of the right-hand side of the equation is equated to that of Equation III.4 to solve for the variance matrix Σ . Matrix Σ can be calculated easily given the correlations between the parameters and their standard deviations.

$$\text{Var}(\mu + CZ) = CC^T \quad (\text{III.4})$$

$$\text{Var}(\beta) = \Sigma = \begin{pmatrix} \sigma_{\beta_0}^2 & \dots & \rho_{ij}\sqrt{\sigma_{\beta_i}^2\sigma_{\beta_j}^2} \\ \vdots & \ddots & \vdots \\ \rho_{ij}\sqrt{\sigma_{\beta_i}^2\sigma_{\beta_j}^2} & \dots & \sigma_{\beta_4}^2 \end{pmatrix} \Rightarrow \Sigma = CC^T \quad (\text{III.5})$$

3.6 DATA

The data used in the analysis were obtained from King County Metro, Seattle's main transit operator. King County Metro is the eighth-largest transit operator in the U.S., serving on average, more than 395,000 passengers daily from more than 200 routes. AVL and APC data were obtained from 163 buses out of a fleet of more than 1,540 buses. These buses are all single unit transit vehicles with two doors: one front door and one back door. The raw dataset contained more than 12,000 records. The data did not provide passenger characteristics (i.e., age, gender) nor the fare collection method; however, King County Metro uses mainly smart cards as a payment method. The operating transit vehicles analyzed in this research effort were one-floor vehicles with two doors: one front door for boarding passengers and one rear door for alighting passengers. The transit vehicles had a total capacity of 82 persons, with room for 39 seated passengers and 43 standees. The explanatory variables considered in the models are thus limited to the number of passengers boarding, passengers alighting, and passengers on board. In fact, all of the variables used in this model were normalized to bus capacity. In order to make the model more generalized regardless the bus capacity, the time-of-day factor and stop type factor (i.e., far-side, near-side, and midblock) were eliminated as explicit variables based on previous studies and because time and location do not matter. Indeed, these variables had a direct relationship with the number of boarding, alighting, and riding passengers [57].

3.6.1 Data Validation and Reduction

Data records were reduced to 8,341 data points, data were validated using a sequence of validation standards. Given that the data points come from the combination of AVL and APC data, the validation steps for APC data differs from the AVL data. The adopted steps for validating each data source are presented.

APC data used in this research recorded several parameters including the number of passengers boarding, alighting and passenger on board counts, stop name, stop ID, trip ID and route name. The validation of the APC data included the following sequence of steps:

- 1- Remove any demand response trip. This was done by checking the route ID, stop name and the stop ID to ensure that any stop belongs to the correct route. If this condition was not met, a trip was considered a demand responsive trip and removed from the analysis.

- 2- For each transit trip, the difference between the total number of the boarding passengers and the total number of the alighting passengers' must be less than 15% as recommended in the literature [62].
- 3- The sum of the boarding passengers and on board passengers at any stop could not be higher than the bus capacity.

AVL data included other parameters, some of which matched the APC data parameters, while others provided additional data including the actual bus arrival time, the scheduled bus arrival time, the bus departure time, the bus dwell time, and the bus loading time. The AVL data were only used within the vicinity of the bus stop.

Validation of the AVL data followed these procedures:

- 1- Check the reasonability of arrival and/or departure times (e.g., departure time record was later than the arrival time).
- 2- Remove records with missing APC data for that bus stop and stop ID.
- 3- Remove observations for which the distance between stops was inaccurate by more than 15%.
- 4- Remove trips for conditions in which the actual arrival time differed by more than 20 minutes from the scheduled arrival time.
- 5- Remove outliers where the DT was greater than 210 seconds, which is the recommended time for serving passengers with disabilities, or where DT was less than 4 seconds (i.e., less than the minimum recommended time for vehicles to open and close the door) [2].

3.7 MODEL DEVELOPMENT

This section describes the procedures used to develop the proposed models. In order to develop a generalized model that can be used for different bus capacities; the explanatory variables were normalized by dividing by the capacity of the bus. These normalized variables included the number of boarding passengers divided by the bus capacity, the number of alighting passengers divided by the bus capacity and the number of riders divided by the bus capacity. In this case all the explanatory variables are dimensionless and thus produce a more generalizable model.

3.7.1 Deterministic Model Development

This section presents the development of the deterministic bus BA time model using the King County Metro data. Data was split randomly into two sub-sets: 88-90% of the data points were used for model calibration and the remaining data set was used to validate the model. Model building has two conflicting objectives. The model has to be informative by including as many regressors as needed to predict the dependent variable, y ; however, the model has to decrease the variances in the measured predicted value $\hat{y}$ By involving the least number of regressors. Consequently, finding a compromise between these objectives results in the best regression model.

Choosing the most relevant candidate of regressors was the first step in the model development. Specifically, three variables for estimating the BA time were considered, namely: the number of boarding passengers, the number of passengers onboard the bus, and the number of alighting passengers. All three variables were normalized by dividing by the bus capacity to produce dimensionless explanatory variables (X_1 , X_2 , and X_3 , respectively).

Development of the BA time was done in three stages. First, data were used to fit some existing models. Then, the model was built based on an iterative transformation of the response and the regressors. Finally, the best model was evaluated based on model criteria selection and the minimum residual errors.

Over 100 models were considered and tested in the analysis, including existing and proposed models. These models included linear, polynomial, and exponential models. The consideration of variables was tested using both forward and backward stepwise regression. In the foreward regression the model starts with no coefficients and then adds explanatory variables as needed. In the backward regression approach the model starts with a full model and then removes variables as needed. Different variables and the various variable interactions was taken into consideration. Response and explanatory variables were transformed as needed to satisfy the normality error assumption. Considering all possible transformations, variables, and variable interactions resulted in the over 100 models. Failure of a large number of models is attributed to their inability to describe the variance in the data. A list of sample candidate models with their statistic metrics is presented in Table 1.

Table 1: Candidate Models with their Summary Statistics

ID	Model Variable	MS _{res}	PRESS	R ²	R ² _{adj}	C _p	AIC	BIC
1	Intercept	22.60	4251346	0.0000	0.0000	4242.7	75491.3	75505
2	Intercept, X ₁	20.43	3473767	0.1835	0.1834	3449.9	73807.1	73828
3	Intercept, X ₂	22.57	4239822	0.0029	0.0028	4208.7	75468.9	75490
4	Intercept, X ₃	22.17	4090529	0.0383	0.0381	4000.2	75168.5	75189
5	Intercept, X ₁ , X ₂	20.35	4239822	0.1898	0.1896	4208.7	73744.9	73773
6	Intercept, X ₁ , X ₃	20.33	3440498	0.1917	0.1915	3370.1	73724.8	73753
7	Intercept, X ₂ , X ₃	22.17	4090206	0.0386	0.0383	3900.5	75168.1	75196
8	Intercept, X ₁ , X ₂ , X ₃	20.17	3338031	0.2042	0.2040	3238.4	73596.8	73632
9	Intercept, X ₁ ²	20.43	3473767	0.1835	0.1834	3449.9	73807.1	73828
10	Intercept, ln(X ₁)	20.44	3476882	0.1827	0.1826	3450.3	73814.7	73836
11	Intercept, X ₁ ² , X ₂	20.35	3448031	0.1898	0.1896	33789.3	73744.9	73773
12	Proposed Model	0.85	5971	0.8895	0.8895	5.0	20851.3	20879

Several criteria were used for model selection. These included the sum of residuals, Mallows (C_p), the coefficient of determination (R^2), the adjusted coefficient of multiple determination (R^2_{Adj}), the residual mean square error (ms_{res}), the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the PRESS residuals.

Comparing the candidate models, the proposed model clearly outperforms the other models based on most of the selection criteria. The proposed model shows the lowest mean residual error, with the highest coefficient of multiple determination, which reflects the model's capability to model approximately 88.9% of the data compared with the other models. The model also has the best trade-off between the estimated parameters and the model goodness of fit using the BIC and AIC values. With this in mind, testing the model precision and avoiding model overfitting was done by comparing the Mallows's C_p Value for each model and choosing the lowest C_p Value.

Assuming that vehicle BA time is continuous and the demand data (number of passengers loading, alighting, and onboard the transit vehicle) are integer values, a general linear model was fit to the data considering a normal distribution of the form in Equation III.6.

$$\text{Ln}(T_{BA}) = \beta_1\sqrt{X_1} + \beta_2\sqrt{X_2} + \beta_3X_3 + \varepsilon \quad (\text{III.6})$$

Here T_{BA} is the duration of time the transit vehicle door is open (BA time), β_i 's are the model coefficients, X_1 Is the of the ratio of the number of boarding passengers to the bus

capacity, X_2 Is the ratio of the number of passengers on board the bus to the bus capacity, X_3 is the ratio of the alighting passengers to the bus capacity, and ε is the error term.

Transforming Equation III.6, the final equation for estimating the bus BA time is computed using Equation III.7. This equation puts a lower and upper bound on the estimated BA times to ensure that the values are realistic. Here min_{stop} and max_{stop} are the minimum and maximum stop durations, taken to be 0 and 210 secs in this study.

$$T_{BA} = \begin{cases} Min_{stop} & X_1 + X_3 = 0 \\ \text{Min} \left(e^{\beta_1 \sqrt{X_1} + \beta_2 \sqrt{X_2} + \beta_3 X_3}, Max_{stop} \right) & X_1 + X_3 > 0 \end{cases} \quad (III.7)$$

The model coefficient values along with their p values are summarized in Table 2. Furthermore, the calibrated model was validated using the validation data subset, with the model showing the ability to explain 88.4% of the total error of the randomly selected data. The table presents the dimensionless coefficients (normalized by the bus capacity of 82 passengers) and the non-dimensionless coefficients based on the actual number of boarding, on-board and alighting passengers.

Table 2: Statistical Model Calibration Results

Coefficients	Coefficient Values (Dimensionless)	Coefficient Values	Serror	P-value
B_1	8.57127	0.9465	0.00490	<0.0001
B_2	1.94351	0.2146	0.00650	<0.0001
B_3	3.84286	0.0469	0.00524	<0.0001

The model demonstrates that the expected BA time is set to min_{stop} (in our case zero) when there are no boarding and alighting passengers regardless of the number of passengers onboard the bus. Accordingly, the expected BA time increases in the range of 1.5 s to 3.7 s per unit increase in boarding passengers in the range between 1 and 10, which is similar to the recommended serving time for boarding passengers according to the TCQSM. Alternatively, the BA time only increases by 0.1 s for the addition of an alighting passenger in the range of 1 to 10 alighting passengers. Finally, the BA times increases by 0.2 s per unit increase in the number of passengers on-board the bus, without any change to the other variables included in Equation III.7. Figure 5 demonstrates that the BA times are more sensitive to the number of boarding passengers than to the number of alighting passengers.

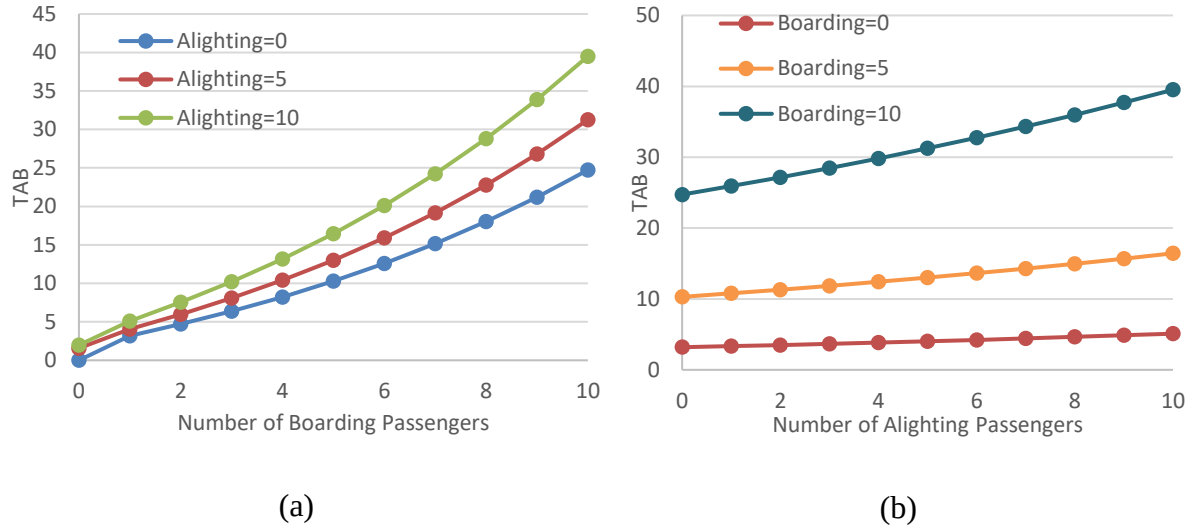


Figure 5: Variation in BA times as a function of the number of (a) boarding passengers and (b) the number of alighting passengers

The adequacy and validity of the models shown in Table 1 was done using the general linear model assumptions. Choosing the best model among a set of models means that the selected model should have a normal residual error, as shown in Figure 6

Figure 6 shows that the fitted values approximately follow the normal distribution, which is indicated by the solid blue line. The figure also shows that most of these values are bounded by the prediction interval, which is denoted by the dashed blue lines. However, it also shows that model results in large values at the tails of the distribution compared with standardized model assumption. This issue was resolved by setting a maximum and minimum stop duration, as demonstrated in Equation III.7. The maximum value is set at 210 s, which is the maximum recommended value by TCQSM. The minimum value is used when the sum of boarding and alighting passengers equals zero. In our case we assumed that the bus would not stop and thus set a value of zero.

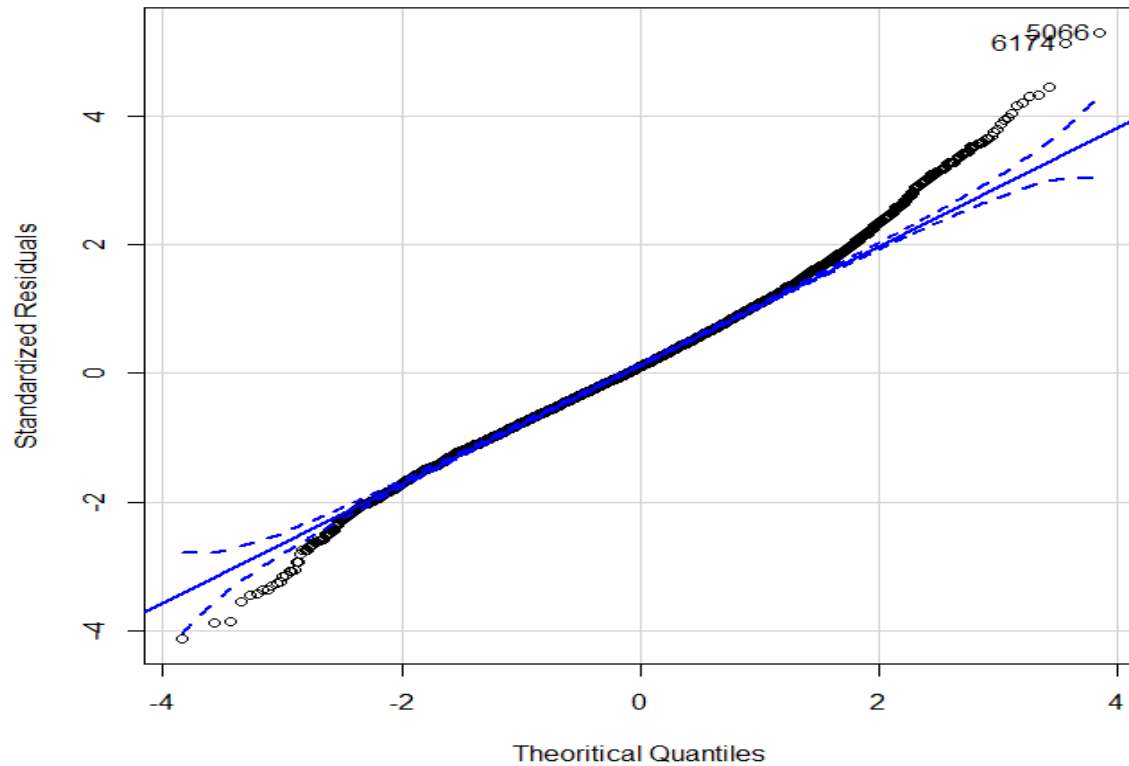


Figure 6: Normal Quantile plot of BA times

Another adequacy check of the model coefficient estimation was done to check the homoscedasticity between the residual error and the fitted error using a visual check and a statistical test. Figure 7 shows the visual homoscedasticity check. Based on the relationship between the residuals and the fitted errors value, it can be seen that the relationship between them is random and doesn't have a definite pattern. Also, the red line in the scatterplot is not curvy; instead it is almost straight and horizontal.

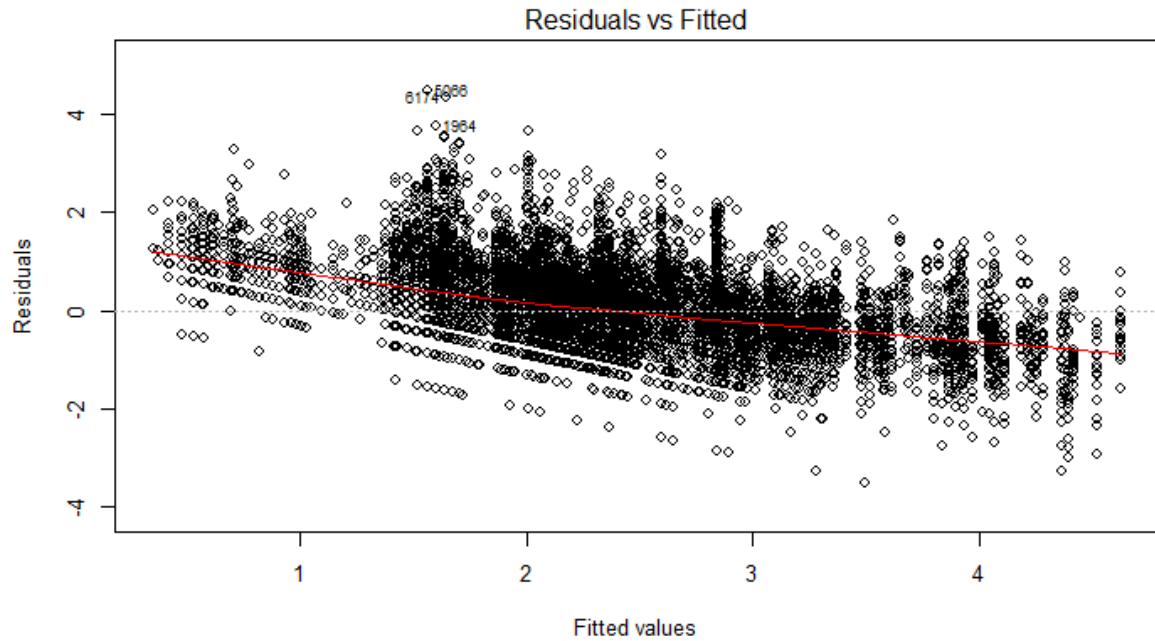


Figure 7: BA Model Homoscedasticity Check

Another check for the homoscedasticity was done using the Breusch–Pagan test to test the null hypothesis – that the variance is not constant in the residual means there is a heteroscedasticity in the residual errors. The results of Breusch–Pagan test is shown in Table 3.

Table 3: Breusch–Pagan Test Results

Statistic Parameter	Value
<i>DF</i>	1
<i>Chi2</i>	8.564
<i>Prob > Chi2</i>	0.00347

Table 3 shows the statistical summary of the Breusch–Pagan test. The p-value of the Breusch–Pagan test is less than 0.05, which indicated that we can accept the null hypothesis of the Breusch–Pagan test, which states that the variance is constant. As a result, this means that heteroscedasticity does not appear to exist.

3.7.2 Stochastic Model Development

This research deploys a nonparametric bootstrap technique to estimate the sampling distribution without the need to assume the population distribution. For this study, the case resampling technique was conducted using the model in Equation III.7 to capture the model coefficient distribution with the number of generated samples established as 100,000. The generated sample was checked using a subset of the large sample by a factor of 10, which results in sub-setting the sample by 10,000 observations after considering every 10th observation [60]. The bootstrapping approach that was implemented within R is an algorithm designed to sample from a distribution with an arbitrary density function, known only up to a constant of proportionality. This is needed for sampling from a complicated posterior distribution whose normalization constant is unknown.

A comparison between the original population parameters and the bootstrapped population parameters is presented in Table 4. This comparison was done to ensure that the resampling technique that was done was sufficient enough to replicate the original population.

Table 4: Population Parameter Summary Statistics

	Mean		Skewness	
	Original	Bootstrapped	Original	Bootstrapped
Boarding	0.0243	0.0288	1.8548	1.9096
Riders	0.2176	0.2176	3.4473	3.4639
Alighting	0.0230	0.0230	0.7987	0.8027
Loading Time	17.3409	17.2697	5.3786	5.4349

The results summarized in Table 4 demonstrate that there is no significant difference between the population parameters of the original dataset and the bootstrapped population.

The algorithm generates random samples and fits Equation III.7. Also, the algorithm is able to generate more parameters, including the mean, standard deviation, and median for

the model coefficients. The results for the generated bootstrapped samples are shown in Table 5.

Table 5: Bootstrap Coefficient Results

	R	Original	Bootbias	Bootse	Bootmed	Bootskew	Bootkurtosis
Boarding	10,000	8.5373	$-3.07e^{-4}$	0.0396	8.5374	-0.0285	-0.0092
Riders	10,000	1.9542	$-1.49e^{-4}$	0.0493	1.9544	0.0027	-0.0608
Alighting	10,000	3.7863	$1.65e^{-3}$	0.4124	3.7842	0.0201	-0.0316

Table 5 shows that bootstrapping gives the original coefficient values, while also computing the average statistics of the generated samples using the formula in Equation III.8.

$$\bar{T}^* = \hat{E}^*(T^*) = \sum_{b=1}^R \frac{T_b^*}{R} \quad (\text{III.8})$$

The bias of the bootstrapping was estimated by using $\hat{B}^* = \bar{T}^* - T$. The standard error was calculated for the bias using Equation III.9.

$$\widehat{SE}^*(T^*) = \sqrt{\frac{\sum_{b=1}^R (T_b^* - \bar{T}^*)^2}{R-1}} \quad (\text{III.9})$$

To compare the standard error values for the model coefficient using conventional modeling and the bootstrapping model, the values were used to construct the regression coefficient confidence interval based on the normality assumption as both of Bootstrap skewness and Kurtosis indicated that the distribution of these coefficients'' follow the normal distribution as these values tend to be close to zero.

A histogram of the model's coefficient distribution, kernel density, bootstrapping, and confidence intervals of the values was individually generated for each explanatory variable coefficient. Figure 8 shows the histograms for the bootstrapping case. The vertical dashed lines express the coefficient point-estimate in the conventional regression model (presented in Equation III. 7). Conversely, the horizontal lines express the bootstrap confidence interval for the coefficients, which is presented in. Interestingly, the figure shows that both of the

distribution functions follow a normal distribution and this distribution is very strong for all coefficients. z

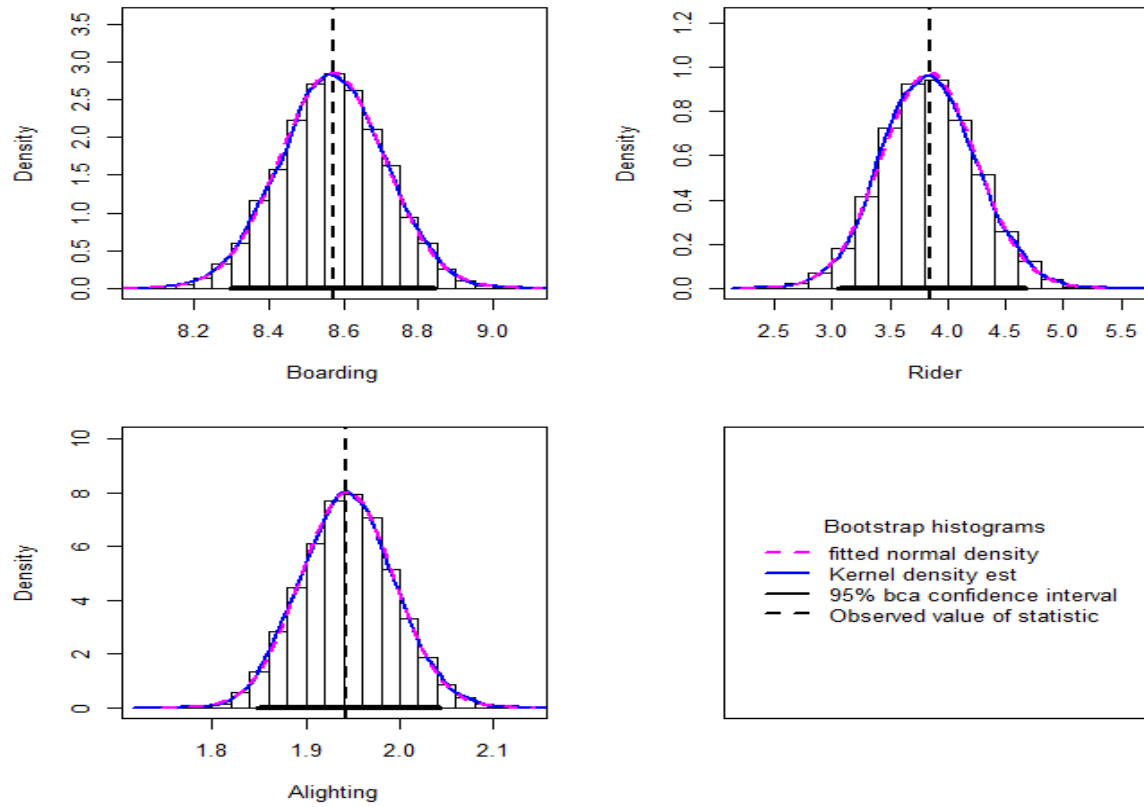


Figure 8: Bootstrap coefficients

Figure 8 subplots show the model parameter distribution using the bootstrap technique. The X-axis shows the parameter values in second/number of subplot name (i.e., boarding, Rider, Alighting) divided by the total capacity, while the y-axis shows the density for these values. The solid black line along the x-axis in each subplot indicates the confidence interval for these values. The solid black line and the dashed line values are not different from the values in Table 5 and Table 6. Checking the distribution of variables in the proposed model as stated in Equation III.7, the pink dashed line shows the fitted parameter density, while the blue dashed line shows the normal distribution for the parameters. In order to verify the independency in the resampling process, the coefficients' convergence was checked by testing whether the generated sample size was sufficient using the autocorrelation function (ACF), a tool for checking the randomness of the data points, considering the between-the-coefficients sample points, as shown in Figure 9.

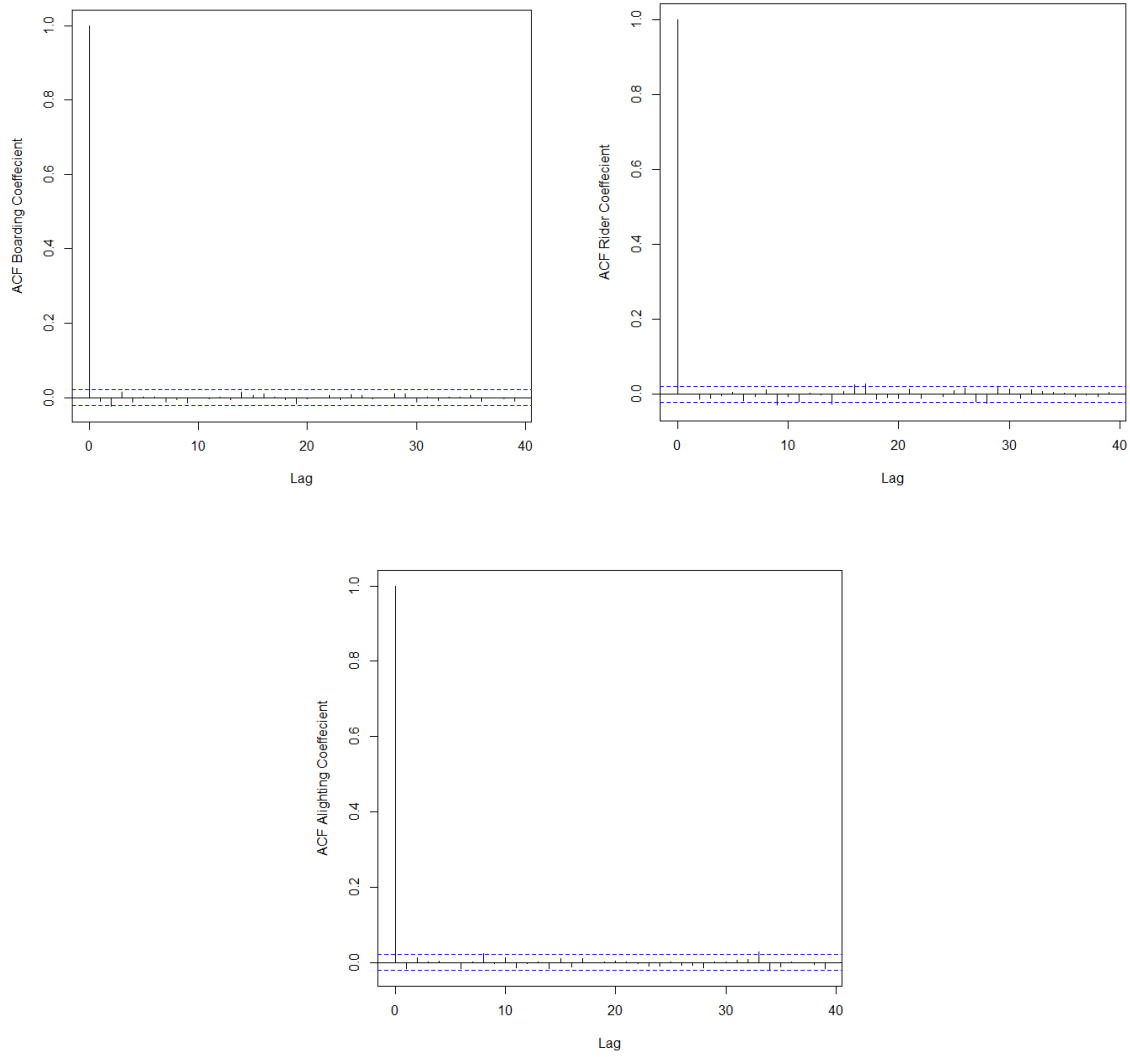


Figure 9: Variations in the autocorrelation function of the model parameters

Table 6: Confidence Intervals for the Coefficients

	2.5% Confidence Limit	97.5% Confidence Limit
Boarding	8.298	8.842
Riders	1.848	2.042
Alighting	3.060	4.662

If the data points are random, the ACF value should be near zero along all-time lag separations between them. The ACF predicts the value of a selected data point at time (t) given the prior value in time ($t - \Delta t$). Figure 9 also shows that the coefficients' values are

independent of each other. The dashed blue lines represent the 95% confidence limits, while the stem lines represent the ACF.

As expected based on bootstrapping a random sample point selection, an autocorrelation of 1.0 is observed for a lag of 0, which means that the probability of choosing the second sample would not be different from the previously selected sample. The values are within the confidence limits for larger lag times and close to zero. Figure 10 illustrates that there is interaction between the model parameters.

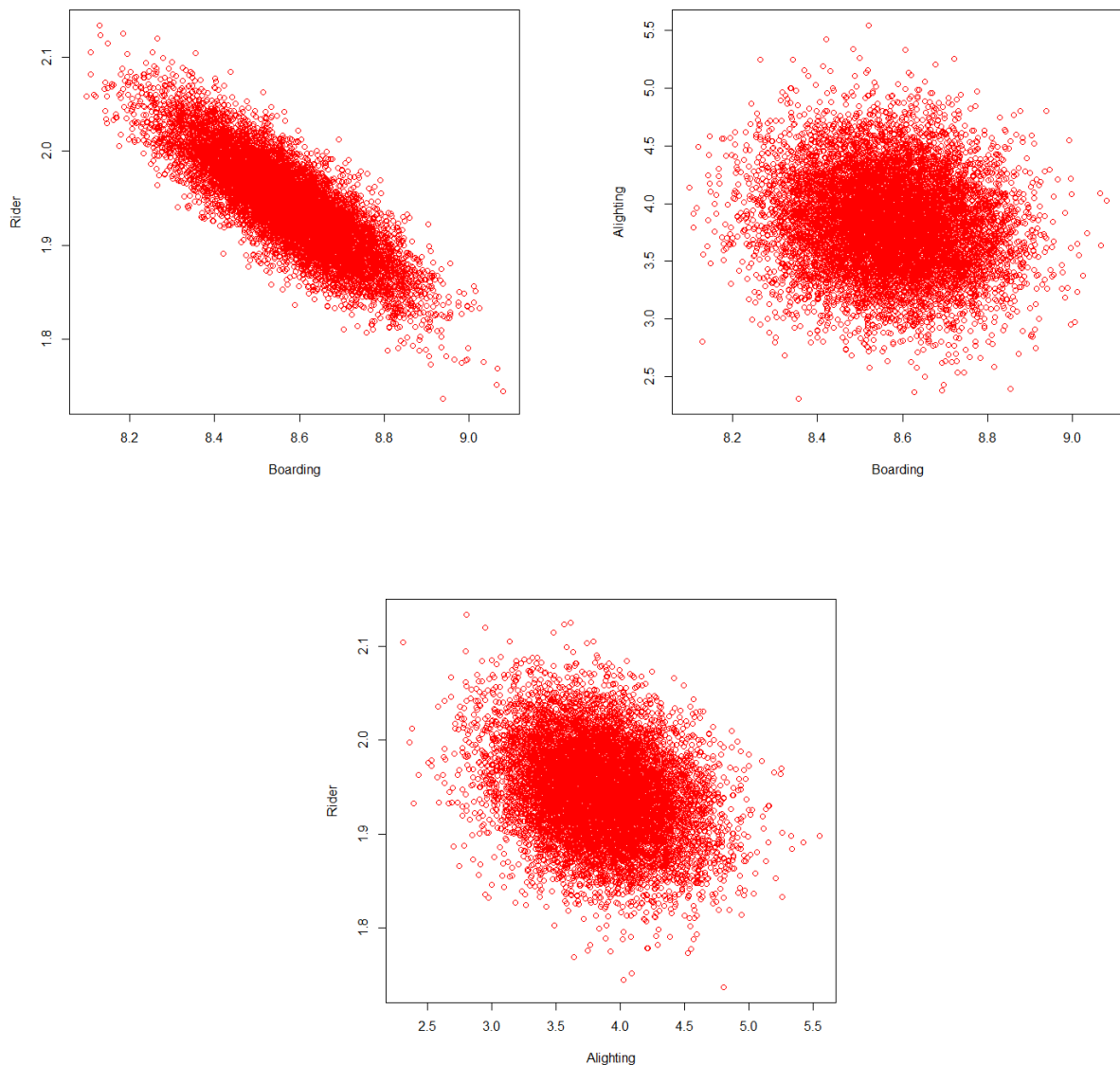


Figure 10: Scatterplots of model coefficient interdependencies

Figure 10 shows the interactions between the model coefficients (β_1 , β_2 , and β_3). Each subplot shows the relation between two coefficients, illustrating that the parameters are highly correlated.

The plots also show that the data points are intensively spread between the parameter confidence intervals for each subplot. Also, there is a clear linear relationship between β_1 versus β_2 , and β_2 versus β_3 . The relationship between β_1 versus β_3 can be modeled linearly, as the relationship between β_1 and β_2 is linear. The parameters' interaction models can be deduced and validated using another statistical technique (which is presented in the next section).

3.7.3 Model Application using Cholesky Decomposition

The Cholesky decomposition, described earlier, was used to capture the stochastic nature of the model coefficients while capturing the model coefficient correlations. The correlation coefficients were computed as $\rho_{12} = -0.79$; $\rho_{13} = -0.12$ and $\rho_{23} = -0.34$ with probabilities less than 0.001, demonstrating that all coefficients are correlated. Table 7 provides the standard deviation and various parameters for the model coefficients as derived from the Cholesky decomposition.

Comparing these statistics to those of the bootstrap results in Figure 10 reveals that the Cholesky approach is valid and can be used for BA data. Again, the inter-dependence of the generated 10,000 replications was plotted over those of the Cholesky model (shown in Figure 11), and both show similar results.

Table 7: Summary Statistics of the Cholesky Decomposition Model Parameters

Parameter	Mean (μ)	Quantiles		St. Dev. (σ)	Skewness (γ_3)	Kurtosis (γ_4)
		Q _{0.025}	Q _{0.975}			
β_1	8.570	8.072	8.665	0.141	-0.015	-0.079
β_2	1.947	0.392	2.213	0.041	-0.011	0.054
β_3	3.843	3.766	3.856	0.019	0.028	-0.015

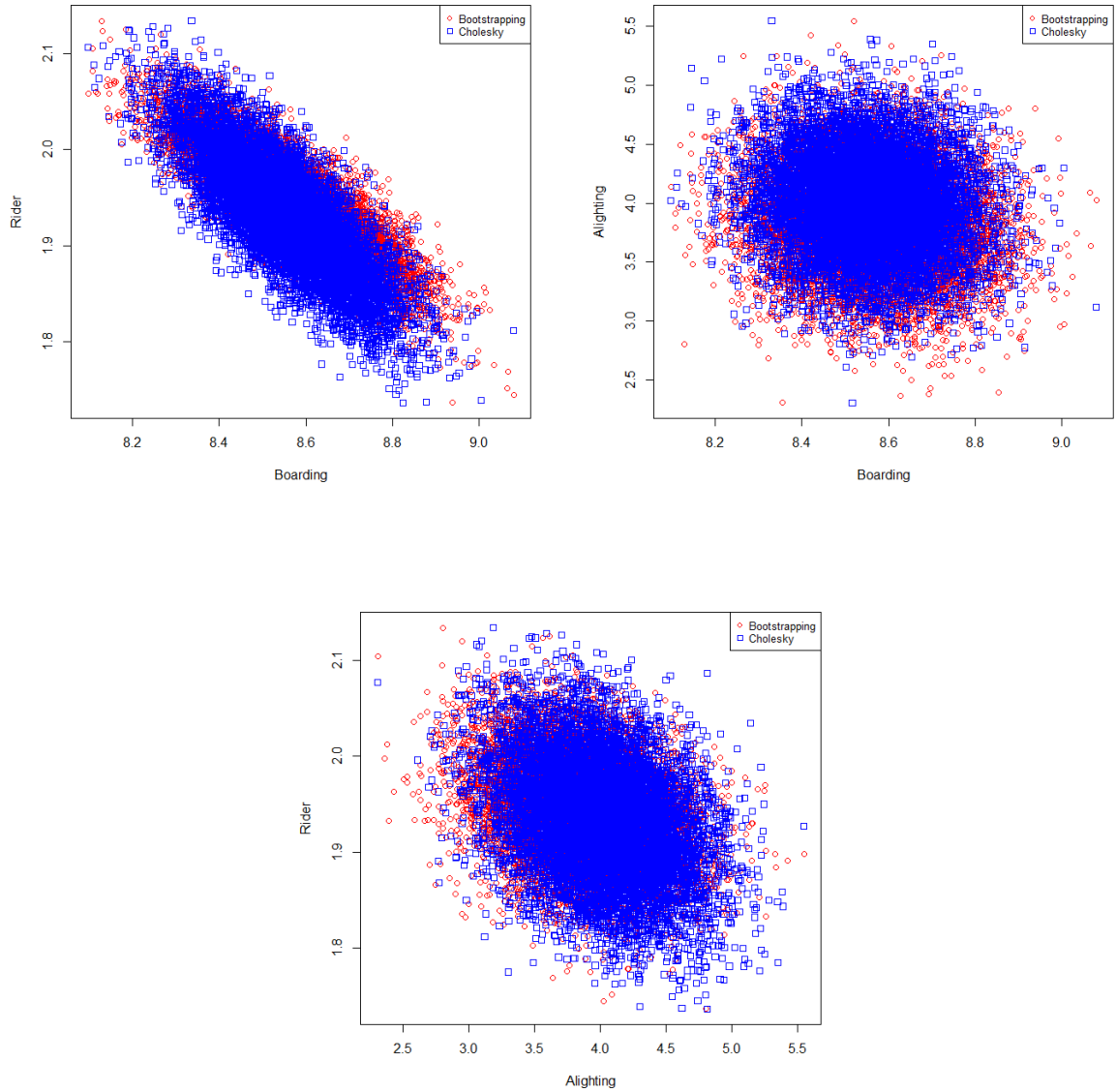


Figure 11: Interdependence of the Cholesky decomposition model parameters

3.8 Model Applicability to Different Local Conditions

This section discusses and presents an approach to apply the model to conditions not observed in the data used to develop the models (e.g. a bus that has a single door). Although the BA model was developed and validated using one data source (King County), the model was generalized by normalizing the model variables by the capacity of the bus. This allows the model to be used for any bus capacity as long as the bus has a similar configuration (i.e. has separate alighting and boarding doors) and that the method of payment is similar (85% of the

total passengers paid their fare using an automatic fare collection system). In the event that an agency wants to apply the model to conditions different from the data used to develop the models then a Bayesian approach could be used. No more details were available for passenger characteristics (i.e. cyclists with bicycles, elderly people, people with disabilities, etc.). Despite the fact that these limitations make the BA model conditionally applicable under certain conditions, the model is still able to be used in different conditions. The model's applicability is discussed below.

Transit agencies wanting to adapt the model may be categorized as one of two scenarios. First, the transit agency has the same conditions of King County. In this case, the developed model in Equation III.7 with its coefficients in Table 2 can be used directly. The second scenario is where the transit agencies have different local conditions (i.e. different number of bus doors) or different passenger characteristics (i.e. cyclists, strollers, elderly people, or passengers with disabilities), and different fare payment methods. If this is the case, the model coefficients estimated in Table 2 may no longer be accurate. However, the model will still be applicable if a Bayesian approach is implemented to update the model coefficients using Table 2 (the generalized one), as the prior. The agency would need to collect some limited local data, the model would then use the local data together with the prior coefficients to estimate the posterior of coefficients that are calibrated to the locality. In other words, the Bayesian approach can combine information from the existing model fitting results and locally collected data to generate posterior distribution of model parameters that reflect local conditions.

To be more specific, the posterior of the developed BA model can be used as a prior to develop locally specific model coefficients. This is based on the assumption that the multi-parameter posterior from previous studies can be accurately represented by independent marginal distributions of each parameter. Thus, a transit agency can use posterior of a previous study as the priors to reflect common characteristics of transit. When combined with the locally collected data, the output posterior reflects both common characteristics from existing studies and local specific information. .

Another reason to use the Bayesian approach for other localities is that the Bayesian method provides a more direct expression of uncertainty, including complete ignorance. In addition, the Bayesian approach is can leverage prior information to handle situations with smaller sample size. More importantly, the Bayesian method has the capability of handling

non-normal parameters, as Bayesian methods provide more accurate results because they can deal with asymmetric distributions.

Generally, the maximum likelihood estimation (MLE) is used in the classical frequentist inferences. MLE assumes model parameters that are fixed and known, and the bottom line of the selection of these parameters emerges from maximizing the likelihood function for the selected model. Notably, using informative knowledge about the process for both prior and posterior information about the process, using Bayes' theorem [63] in Equation (III.10). This equation can be expanded to achieve stochasticity among model parameters, which uses the conditional probability formula in Equation (III.11).

$$posterior \propto prior \times likelihood \quad (III.10)$$

$$P(\beta, \sigma | y, X, m) = \frac{P(\beta, \sigma | m) P(y | X, \beta, \sigma, m)}{p(y | m)} \quad (III.11)$$

Where:

$p(y | m)$ = the marginal distribution of y .

$P(\beta, \sigma | m)$ = the prior model distribution.

$P(y | X, \beta, \sigma, m)$ = the likelihood estimation.

$P(\beta, \sigma | y, X, m)$ = posterior distribution.

The selected model parameters can be calibrated using the Bayesian statistics method. The Markov Chain Monte Carlo (MCMC) approach can be used to generate the posterior distributions when these distributions cannot be generated analytically. The MCMC algorithm is designated for use in the distribution of an arbitrary density function to sample the data using a known constant of proportionality, which is needed to sample from a complicated posterior distribution whose normalization constant is unknown. The approach generates random samples from these distributions to estimate the posterior distribution or derived statistics (e.g., mean, median, and standard deviation).

The MCMC slice algorithm requires an initial solution for the model parameters, the prior distribution, and the residual variance. With these priors, the posterior distribution of model parameters conditionally depends on the response variable and the variance. Transit

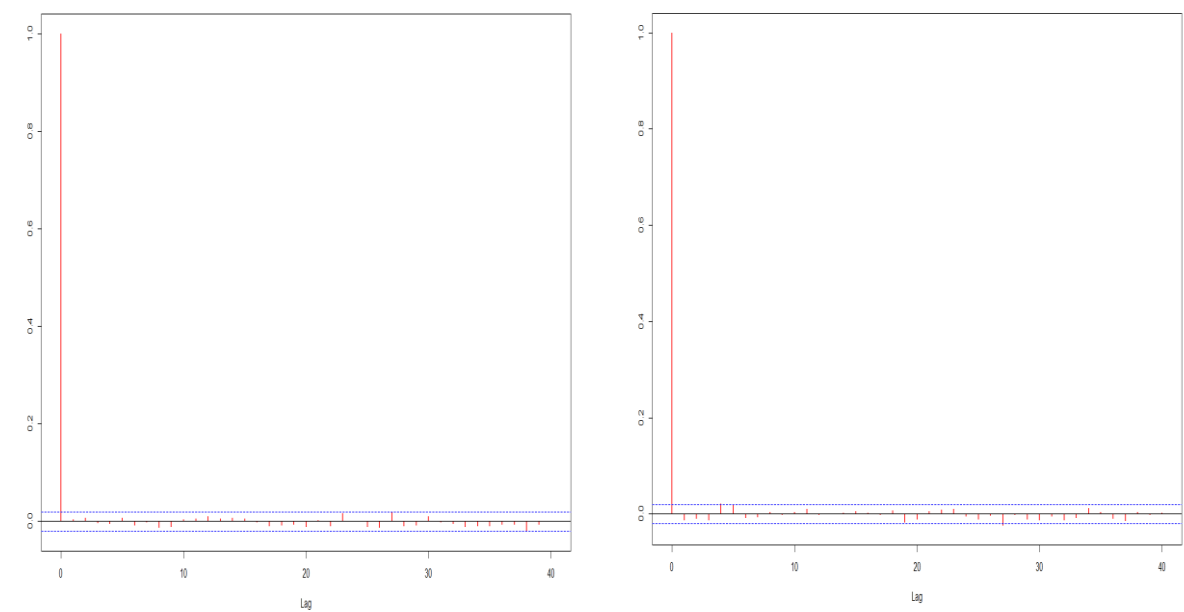
agencies can use this information to draw the marginal distribution using the Monte Carlo approach by first, drawing the variance from its marginal distribution.

Table 8 summarizes the different statistics for the 1,000 realizations for the model parameters. It can be seen that the mean value of all the model parameters are close to the parameter estimates from the classical frequentist general linear model (GLM).

Table 8: Summary Statistics of the Bayesian Model Parameters

Parameter	Mean (μ)	Quantiles		St. Dev. (σ)	Skewness (γ_3)	Kurtosis (γ_4)
		Q _{0.025}	Q _{0.975}			
B_1	8.573	8.489	8.658	0.126	0.027	2.988
β_2	1.943	1.909	1.975	0.407	-0.026	3.017
β_3	3.843	2.24	4.115	0.049	0.002	2.906
σ	0.847	0.812	0.851	0.006	-0.025	2.958

More than that, measuring the randomness between the generated samples as a function of time is done using the Autocorrelation Functions (ACF) for model the coefficients, which is presented in Figure 12. The solid horizontal lines represent the 95% confidence limits while the stem lines represent the autocorrelation function. As would be expected an autocorrelation of 1.0 is observed for a lag of 0 while the values are within the confidence limits for larger lag times.



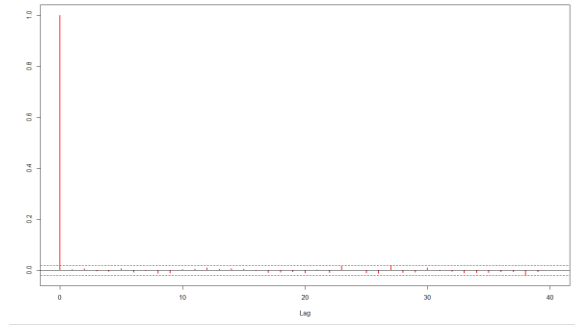


Figure 12: Variations in the Autocorrelation Function of the Model Parameters

In addition, a Kolmogorov-Smirnov goodness of fit test (K-S test) was run on the model parameter distributions and concluded that there was insufficient evidence to reject the null hypothesis that the model parameter distributions were different from the normal distribution at a level of significance of 0.05, as illustrated in Figure 13.

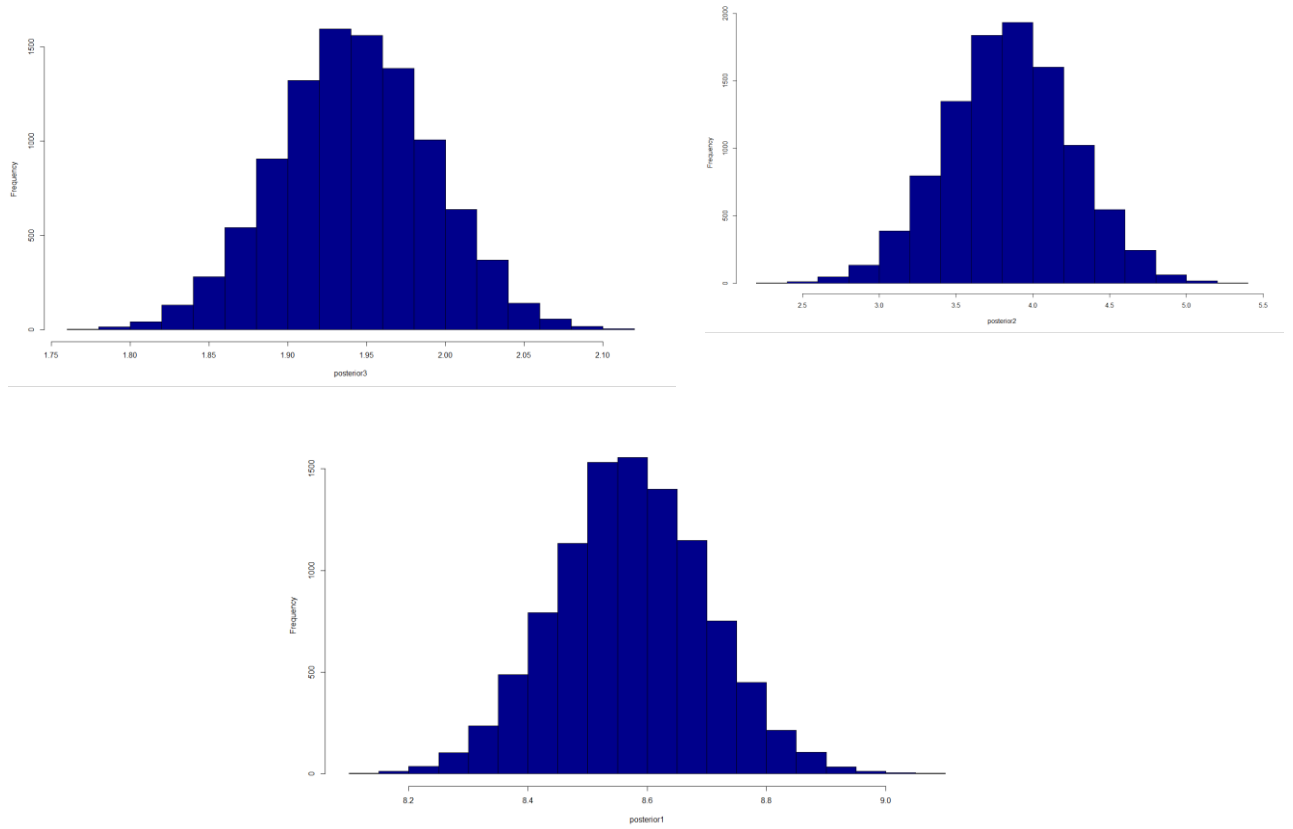


Figure 13: Histograms of the Model Parameter Distributions

3.9 CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

The contribution of this research is threefold. First, it proposes and provides standards to validate APC and AVL data. Second, it develops two statistical boarding/alighting time models

using a total of 8,341 AVL and APC empirical observations from King County Metro in Seattle, Washington. Third, it identifies the significant factors to estimate the boarding/alighting times and develops an approach to capture the stochasticity in the boarding/alighting times. The primary objective of this research was to develop two models that can predict boarding/alighting times as a function of boarding, on-board and alighting passengers.

The first model is a frequentist linear model and the second model is a stochastic model. Several models were tested and compared based on different statistical metrics including residual errors, goodness-of-fit, AIC and BIC. The best model was selected based on its ability to express the variation in the data with the lowest residual errors.

The stochastic bootstrapping technique showed consistency with the frequentist model with the addition of stochastic model coefficients, which can be utilized to conduct stochastic analyses. A key drawback of using the bootstrapping technique is that it requires large computational resources to store the thousands of model parameter realizations. As this study demonstrates, Cholesky decomposition is capable of capturing the desired stochasticity with the use of only $3 \times n$ parameters (mean, variance, and correlations), where n is the number of model coefficients. The Cholesky decomposition is demonstrated to yield results similar to that of the bootstrapping technique, with the use of minimum computational resources.

Inclusion of the boarding/alighting time models are important because they can be used to better estimate bus dwell times and bus travel times to enhance transit system operations. The stochastic boarding/alighting time can be used in further operational applications such as stochastic estimation of transit travel times and enhancing the estimation of bus arrival times at intersections for use in Transit Signal Priority (TSP) systems to better estimate green truncation and extension times. Finally, the results of this work can be useful for transit agencies to enhance bus operations and provide transit users with better estimates of bus arrival times.

Although the study achieved its objective of modeling bus passenger boarding/alighting times, further research is needed in the following two areas: (1) validating the model on other datasets, and (2) extending the model to different bus configurations. In our study the buses had a maximum capacity of 82 passengers (sitting and standing) and had two doors: a front door for boarding passengers and a rear door for alighting passengers. The model

can be strengthened if the passenger characteristics (i.e. Senior passengers and cyclists) are available and considered. A. Additional research is also needed to develop an evaluation tool for transit agency personnel that would allow for the evaluation BA and dwell times to assist in route restructuring and timing.

3.10 ACKNOWLEDGEMENTS

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3.11 AUTHOR CONTRIBUTION STATEMENT

The authors confirm contribution to the paper as follows: study conception and design: Alhadidi and Rakha; data reduction: Alhadidi; analysis and interpretation of results: Alhadidi and Rakha; draft manuscript preparation: Alhadidi and Rakha. All authors reviewed the results and approved the final version of the manuscript.

Chapter 4 **Estimating Transit Vehicle Travel Time Components in the Vicinity of Bus Stops**

This chapter is based on the following paper: Alhadidi T. and Rakha H.A., “Estimating Transit Vehicle Travel Time Components in the Vicinity of Bus Stops,” To be submitted.

4.1 Abstract

This work presents a quantitative approach to estimate the total time spent in the vicinity of a bus stop including the deceleration time, the acceleration time, and re-entry time (time required to merge onto the adjacent lane). Different statistical models were used to compute the deceleration, acceleration and merge time. Typical deceleration and acceleration levels were computed using kinematic equations. These rates were then used to compute both of deceleration and acceleration times. The adopted method to estimate both the deceleration time and the acceleration time was validated using transit data from Blacksburg Transit (BT) using the mean absolute percentage error (MAPE) and root mean square error (RMSE). The MAPE and RMSE values were calculated to be 0.3% and 13.3% for the deceleration time and 0.42% and 2.72% for the acceleration time, respectively. This indicates that the adopted method is valid and can be implemented to estimate transit vehicle deceleration/acceleration times.

The re-entry time was estimated to be a function of the adjacent roadway traffic density using both a multiple linear regression and Bayesian regression approach. Both methods showed consistency in estimating the merge-time model coefficients. The results of this study showed that the hypothesis of this study is valid and could be practically used at any stop location to update arrival times and departure times at bus stops. Further investigation can be done to quantify the impact of the pavement and weather condition on the results of this study.

Keywords: Bus dwell time, deceleration time, acceleration time, re-entry time, transit stopped delay, delay.

4.2 Introduction

Public transit buses, compared to private vehicles, provide an efficient transportation mode and help in reducing traffic congestion and energy/fuel consumption. However, the accurate estimation of travel time is still a challenging task due to different interacting factors that affect

it. These factors include the interaction with other transportation modes (i.e., cars, rails and bicycles), the variability in transit vehicle stop time at bus stops, the stochasticity of travel time between two stops, and the stochasticity in demand patterns.

The accurate estimation of bus travel times using emerging technology including automated vehicle location (AVL), detectors, and probe vehicles can assist in providing real-time information about bus arrival times for transit system users, agencies, and operators. This, in turn, would help in attracting more riders, improving users' satisfaction, and reducing travel time with its associated cost.

Estimation of the bus arrival time at any stop depends mainly on the time that a transit vehicle spends at a stop and the travel time between two stops. The travel time components between two stops have been quantified using data from different states in the 1980s, which are shown in Figure 14. The time at bus stops has been estimated using a conventional method (i.e., by considering only the boarding and alighting times and doors open-close time), which constitutes between 9-25% of the transit travel time [1]. However, a transit vehicle may spend longer than this anticipated time such that dwell time does not become only a function of boarding/alighting time, but also has other components as shown in Figure 15[2].

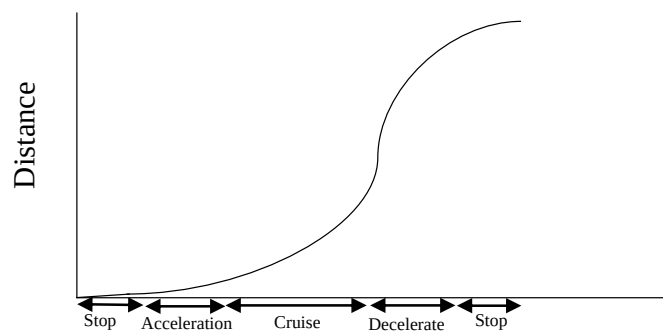


Figure 14: Transit travel time components between stops [1]

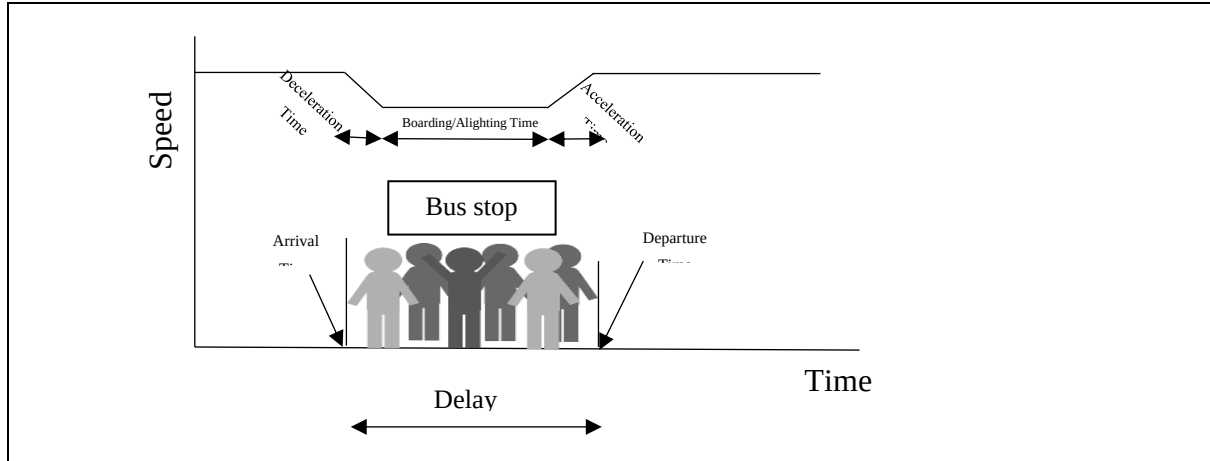


Figure 15: Transit travel time components at bus stop [2]

As can be seen in Figure 15, the delay at a bus stop components include the time it takes the transit vehicle to completely stop, the boarding/alighting time, the acceleration time, and the merge time. The final components are known as re-entry delay in the off-curb stops, according to the Transit Capacity and Quality of Service Manual (TCQSM) [2]. The re-entry delay expresses an imposed delay on the transit vehicle, which is caused by either the queued traffic in front of the transit vehicle or the traffic volume in the adjacent lane that prevents the transit vehicle from merging easily with traffic. Adding this time to the boarding/alighting time helps in accurate estimations of bus dwell times at bus stops.

Adding the transit vehicle deceleration/acceleration performance characteristics provides an in-depth understanding of vehicle deceleration/acceleration behavior that could be used to provide a more accurate estimation of buses' arrival times at bus stops and signal stops and update the current bus bay configuration requirements.

4.3 STUDY OBJECTIVES AND PAPER LAYOUT

The contribution of this paper to the literature is two-fold. First, it models transit vehicle deceleration/acceleration behavior considering a kinematic model based on field data by explicitly modeling the driver and the vehicle characteristics. Second, it estimates the transit vehicle re-entry time, which is a vital component of estimating the dwell time, especially at off-curb bus stops. Results of this work can be incorporated within micro-level simulations to estimate the total delay at bus stops and predict bus arrival and departure times at bus stops.

The results of this research can be extended to update bus bay design configurations, signal timings, bus schedules, and bus arrival times.

The paper is laid out as follows. Initially, previous research findings are presented. Subsequently, the description of the data collection and reduction is described followed by a fully detailed description of the adopted methodology. Next, a discussion of modeling the acceleration and deceleration later that was used later to analytically compute the acceleration and deceleration times is presented, followed by a discussion for modeling the merge duration using a classical frequentist statistical model and the stochasticity in the model parameters using the Bayesian regression. Subsequently, the results of modeling the different delay components at a bus stop are presented. A discussion of the study results is presented next. Finally, the conclusions of the paper are drawn and insights for future work are provided.

4.4 Previous Findings

The aim of this literature review is to provide a summary of current models that estimate the transit travel time components related to bus stops. The focus of this work is to estimate the lost time components at bus stops considering transit deceleration and acceleration times, in addition to re-entry delay. As such, the lost time between bus stopping time and the beginning of passenger boarding (or alighting) is beyond the scope of this work. Integrating the results of this work with boarding/alighting times will provide an accurate estimation of bus dwell times at bus stops.

4.4.1 Literature Review for Vehicle Acceleration/Deceleration Modeling

Modelling the acceleration/deceleration has been carried out for the past few decades. Several studies were conducted to predict maximum and typical acceleration levels, considering different vehicle types. These models are mainly categorized into two groups based on the driver's reaction towards the leading vehicle. These groups are: car-following models, where the driver's behavior towards the lead vehicle is considered (also known as kinematics models) and dynamics models, where the acceleration/deceleration behavior is not determined by considering the car-following behavior, but by considering the vehicle dynamics characteristics instead. However, both these models are computationally expensive due to

having to consider all the forces that act on vehicles, including tractive and resistive forces. These models are more complex in mathematical representation and development.

Several models have been developed to estimate deceleration/acceleration profiles for different vehicle types. These models were developed based on using outdated, limited data or using a traditional measurement method to provide an estimate of vehicle speed ([24], [22], [23], [20], [25], [21], [26], [27], [28], [29], [19]). Yet, none of them focus on transit vehicles based on field data due to the cost of collecting the required parameters from the field and some technical challenges, including data storage and/or data accuracy problems. Recently, the Hydraulics, Electrical and Mechanical (HEM) Data Corporation provided a data logger that collects HEM records based on either 2 Hz or 5 Hz frequency. This HEM data provides several parameters of vehicle characteristics and stores these parameters to be used in modeling vehicle movement.

4.4.1.1 Kinematic Models

As mentioned earlier, kinematic models focus on the relationship between the moving vehicle and the leader vehicle. In other words, these models consider the relationship between speed, traveled distance, and acceleration/deceleration rates without considering the forces that cause the motion. The simplest model uses a constant acceleration rate along the entire trip; however, this model is unable to express real vehicle movement because vehicle acceleration isn't constant, but it changes over trip time [27]. Other researchers argue that vehicles achieve a higher acceleration rate while they are traveling at low speed, compared with the associated acceleration rate at a higher speed. Considering this behavior, a dual-regime model that considers two acceleration rates was proposed; these rates are a constant, higher value at lower speeds and a lower value at higher speeds [20].

Another simple and commonly used mode is Drew's model, which is presented in Equation (IV.1). This model assumes that a vehicle starts at a speed equal to zero with the highest acceleration rate, and this rate decreases linearly as function of speed [27]. However, studying deceleration behavior of different vehicles using field data shows that the observed deceleration rate is lower than the suggested deceleration rate proposed by both the American Association of State Highway and Transportation Officials (AASHTO) and the Institution of Transportation Engineers (ITE). Rather, the observed rate increases as the initial speed increases ([27], [29]).

$$a(u) = \alpha - \beta (u) \quad (IV.1)$$

Where a = acceleration,
 U = speed,
 α = maximum acceleration rate,
 α / β = maximum attainable speed.

Describing the non-uniform behavior of deceleration rate, a polynomial relationship between acceleration and speed was proposed by Akçelik and Biggs ([19]). This model shows that a vehicle traveling at high speed covers a longer deceleration distance before reaching its target speed.

4.4.1.2 Dynamic Models

Searle's model was the first model that considered vehicle's engine power. However, it overlooks the impact of resistance forces [25]. Moreover, the model is valid for estimating the maximum vehicle acceleration rate based on using six vehicles' field data. This model is expressed by Equation (IV.2) and ((IV.3).

$$u^2(u) = u_0^2 + \frac{15.8 * \eta * Pmax}{M} t \quad (IV.2)$$

$$u^3(u) = u_0^3 + \frac{23.7 * \eta * Pmax}{M} x \quad (IV.3)$$

The results of Searle's model confirm that both the behavior and trend of the maximum acceleration is similar to the observed field data. However, Rakha and others ([24] ,[23]) developed a dynamic model to estimate the maximum vehicle acceleration assuming constant vehicle power; this model is presented in Equation (IV.4) through Equation (IV.8). The constant power model computes the maximum acceleration of the vehicle as the ratio of the resultant force to the vehicle mass, M (Equation(IV.4)). The resultant force is computed as the difference between the tractive force acting on the vehicle, F (Equation (IV.5)) and the sum of the resistive forces, namely, the aerodynamics resistance, R_a (Equation (IV.6)), the rolling resistance, R_r (Equation (IV.7)), and the grade resistance, R_g (Equation (IV.8)).

$$a = \frac{F - (R_a + R_r + R_g)}{M} \quad (IV.4)$$

With

$$F = \text{Min} \left(3,600 \eta \frac{P}{u}, 9.8066 \mu M_{ta} \right) \quad (\text{IV.5})$$

$$R_a = c_1 C_d C_h a u^3 \quad (\text{IV.6})$$

$$R_r = 9.8066 C_r (c_2 u + C_3) * \frac{M}{1000} \quad (\text{IV.7})$$

$$R_g = 9.8066 M G(t) \quad (\text{IV.8})$$

Where:

P = engine power,

μ = coefficient of friction between vehicle tires and pavement,

U = vehicle speed,

η = *power transmission efficiency*,

M_{ta} = *vehicle Mass on the tractive axle*,

$C_1 = 0.047285$,

C_d = *air drag coefficient*,

C_h = *correction factor for elevation*, $C_h = 1 - 0.000085 H$,

A = vehicle frontal area,

C_1, C_2 , and C_3 = rolling resistance coefficients, and

i = *pavement Grade*.

Another study conducted by other researchers used Global Positioning System (GPS) data from 100 passengers' vehicles to model their deceleration behavior at a controlled stop sign. However, results of this study do not include vehicle characteristics that affect deceleration values [29].

4.4.2 Literature Review for Re-Entry Delay

The Transit Capacity and Quality of Service Manual (TCQSM) defines re-entry delay as the time that the transit vehicle spends between door closing and the bus starting to leave the bus stop. This time mainly has two components, namely: the startup time and the clearance time. The startup time is defined as the time needed to begin moving and travel on its path, which can also be called acceleration time. Alternatively, the clearance time includes the re-entry delay (merge-delay), which depends on the traffic volumes in the curbside lane. Typically, merge-delay time increases as traffic volume increases [2]. This time depends also on the queued traffic and the upstream traffic signal, which may cause long gaps in traffic or extra delays [30]. Transit agencies do not recommend using off-lines bus stops to eliminate the

merge-delay time; however, off-line stops reduce rear-end collisions between transit vehicles and the traffic in the adjacent lane. The TCQSM suggests that an average re-entry delay depends on the adjacent lane volume based on 12 buses stopped at un-signalized intersection, where a critical gap of 7 seconds and random vehicle arrival times are assumed [2].

Due to the lack of the literature on this topic, the remainder of this subsection is built based on the gap acceptance model. Over the past few decades, gap acceptance modeling played a significant role in the literature due to its importance in modeling the micro-level movement of vehicles. These models can be categorized as either deterministic models or stochastic models. Most of the literature used deterministic models, where drivers are assumed to be consistent and homogenous [64]. Under this assumption a constant value for the critical gap is assumed for all drivers, which can be calibrated according to different conditions. However, the critical gap varies as a response to the change in vehicle speed, vehicle movement type, and type of vehicle. Different studies model this time as a function of capacity. Most of the studies define the merge-delay time as when the vehicle starts moving laterally from the current lane to reach the targeted lane ([65], [66]).

Several studies were conducted to estimate the value of the critical gap. Various studies tried to estimate the duration of the merge-delay time and reported that its ranges between 1 to 16.5 sec ([66], [65], [67]). Polus et al. analyzed gap acceptance at a roundabout. The results indicated that the waiting time is inversely proportioned with the critical gap and that this relationship follows an S curve ([2], [68], [69]).

Gap acceptance distribution provides details about the maneuver rather than having a unique value; however, it needs to consider different parameters and variables and it is computationally costly to determine. Reflecting the heterogeneity of the drivers' behavior and their impact on the intersection critical gap, Daganzo used the probit model to capture drivers' behavior. The study results indicated that this time differs not only between different drivers, but also that the same driver maneuvers differently in response to the same gap [70].

4.5 Methodology and Data Collection

The data used in this research included acceleration/deceleration times and re-entry delay time. This section handles each dataset separately, starting with a description of the collection, reduction, and processing, followed by the adopted methodology for each dataset.

4.5.1.1 Data Description

Here, the collection, reduction, and processing for each dataset is presented below.

4.5.1.1.1 *Deceleration and Acceleration Times*

The data for modeling acceleration/deceleration times were collected by Blacksburg Transit (BT) by driving buses around town of Blacksburg, VA. The data were collected using a HEM logger that records data with a frequency of 2 Hz. In total, about 47 parameters were collected, 6 of which were used in this study, namely: the time stamp, speed, ridership, and GPS location (i.e., longitude, latitude, and altitude). In order to cover a wide range of real-world driving conditions and to determine the uphill and downhill sections, a test was conducted on two road sections: US 460 business highway (with a speed limit of 65 mi/h [104 km/h]) and local streets (with the speed limit from 25 mi/h to 45 mph [40-72 km/h]). As described previously, the HEM logger records data with a frequency of 2 Hz. In order to generate a second-by-second record, the original data were combined by averaging the data points within each individual second. After generating second-by-second results, the null data where the longitude/latitude/altitude had a value of “0” were removed. Then, an approach was developed to extract the vehicle trajectories. The logic tracks second-by-second data and checks the trend in the recorder speed value. The speed trend changes were then split into either acceleration or deceleration, depending on the trend.

The extraction logic for managing the HEM data was built in-house. The input data was the HEM data with the second-by-second time stamp, GPS parameters, and bus speed. A program was used that checks the data compatibility with two-stage validation standards. These standards are either at the point level or at the entire trip level. Trip or point level data that failed to meet these standards were purged. At the point level, observations are deleted for the following reasons: (a) recorded “0” from the GPS data and/or (b) data points collected at garages or check points. After omitting the data points, the software split each group of points into either acceleration or deceleration events based on the speed trend. The validation steps at the trip level followed these steps: (a) split the trip into acceleration or deceleration based on the speed trend (i.e., if the speed trend is ascending that means the trip is acceleration trip and if the speed trend is descending that means the trip is deceleration trip) and (b) all the trips used in this work were either ascending from or descending to zero for either the acceleration or deceleration trips, respectively.

4.5.1.1.2 *Re-entry Delay time*

In order to estimate re-entry delay, different micro-level simulation scenarios were created using INTEGRATION. Different scenarios were run by varying the volume from 600 veh/hr up to 2,500 veh/hr at 50 veh/hr increments, speed variability changes from 0 to 0.015 at 0.025 increments, a fixed capacity equal to 2,500 veh/hr/ln, and the free-flow speed varying from 35 km/hr to 55 km/hr at 5 km/hr increments. A total of 1,365 simulation runs were performed in this study in order to capture different values of the re-entry delay time under different traffic conditions. Figure 16 shows the geometric configuration of the tested network. The network consists of three connected links, all of them with the same capacity and same traffic density at capacity. Link 2 has the bus stop and only allows the transit vehicle to use it. Thus, the bus is held for a pre-defined fixed stopping time to serve the demand at the bus stop, then the bus waits for a certain time to merge with the traffic in the adjacent lane to finish its trip.

The generated output file that tracks each vehicle traveling along each link second-by-second, checking for extra stopped time (i.e., the time that a transit vehicle spends beyond the pre-defined time) and the traffic passing the stopped bus until the end of link 2. In other words, the tracking was done based on the flow and the density in the bus stop's adjacent lane.

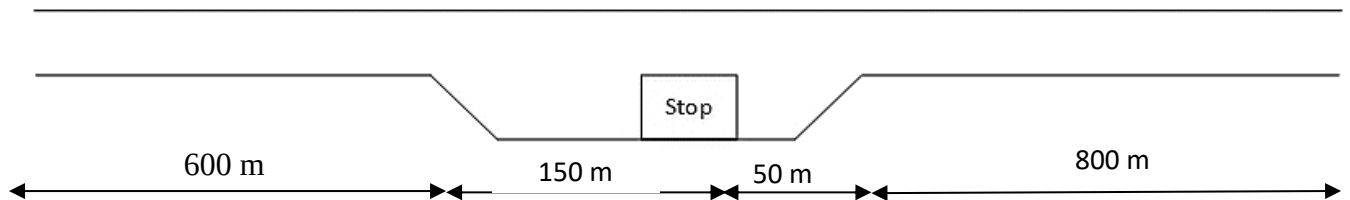


Figure 16: Network Configuration

4.5.1.2 *Methodology*

This section presents the research methodology. As was mentioned in the literature review, the time elapsed between the bus stopping and the beginning of passenger boarding, or alighting is beyond the scope of this work. Instead, the methodology described here handles the deceleration time, re-entry delay time, and the acceleration time.

4.5.1.2.1 Deceleration Time

Following the trend split, the deceleration trips were extracted. The extraction logic summarizes it via four parameters: trip time, deceleration rate, initial speed, and traveled distance.

Using vehicle kinematic equations, the deceleration rate was computed using Equation (IV.9). Then, the computed deceleration rate was used to estimate the deceleration rate as a function of the initial speed, as the final speed equals zero. Thereafter, to compute the deceleration time, the deceleration distance should be calculated first. The predicted deceleration rate, which is presented in Equation (IV.3), was applied to the well-known deceleration distance equation in order to compute the deceleration distance, which is presented in Equation (IV.4).

$$\text{Deceleration Rate} = \frac{V_0}{t} \quad (\text{IV.9})$$

$$a = \frac{B_1}{B_2 + B_3 V_0 + V_0^{-B_4}} \quad (\text{IV.3})$$

$$\text{Deceleration Distance} = \frac{V_0^2}{2 * g(a \mp G)} \quad (\text{IV.4})$$

Here, V_0 is the initial speed which is measured in (km/hr), t is the entire deceleration maneuver time, a is the deceleration level, g is the standard acceleration due to gravity (which equals to 9.81 m/sec²), and G is road grade, which is computed using Equation (IV.5).

$$G = \frac{\text{elevation}(t) - \text{elevation}(t - \Delta t)}{\sqrt{(D(t) - D(t - \Delta t))^2 - (\text{elevation}(t) - \text{elevation}(t - \Delta t))^2}} \quad (\text{IV.5})$$

Where $\text{elevation}(t)$ is the altitude (m) at time (t) and ($t - \Delta t$); and $D(t)$ is the distance (m) a bus traveled at the interval of Δt . The elevation was collected by HEM and the traveled distance was computed based on the collected speed data using Equation (IV.6).

$$\text{Distance} = V_0 * \Delta t \quad (\text{IV.6})$$

4.5.1.3 Re-entry Delay Modelling

In order to model the re-entry delay time, several scenarios were generated using INTEGRATION as a microscopic simulation software. INTEGRATION produces a second-

by-second probe vehicle output file, where each vehicle movement is tracked second by second on each link. At each generated data point, much information can be extracted from these data reports, such as the time stamp, vehicle location, speed, distance, delay, total delay, and vehicle emissions. In this study, the base scenario and the generated scenarios were run and the output data were extracted. In the base scenario, the network was run while it has only the transit vehicle. The generated scenarios were run separately according to the variations that were mentioned before. For each scenario, the transit travel time from leaving the bus stop until it reaches the end of the link in addition to the density on the adjacent lane were extracted. In this study, two statistical models were developed, one using classical frequentist modeling and another using stochastic modeling based on a Bayesian approach. In both of the two developed models, the re-entry delay was considered as the dependent variable since density is likely to influence the re-entry delay time.

4.5.1.3.1 *Re-Entry Delay Modelling Using Classic Frequentist Statistics.*

Modeling the re-entry delay using classic frequentist statistics (CFS) was done using the generated data from INTEGRATION using the generalized linear regression (GLR) for modeling the linear relationship between the dependent variable and independent variable(s). GLR assumes the linear relationship between model variables and the uses least squares error estimation (LSE) to estimate the model's coefficient. The general form of the GLR is presented in Equation (IV.7)

$$y = \beta'X + \epsilon \quad (\text{IV.7})$$

Where y is the dependent variable, X is a vector of explanatory variable(s), β is the vector of estimable parameters or coefficients, and ϵ is the random error term.

In LSE, the model parameter coefficients are estimated based on minimizing the sum of the squared difference between the observed value and the predicted value. Consequently, the GLR assumptions should be checked. Model assumptions can be checked using either graphical or appropriate diagnostic tools, including a homoscedasticity check and check for a normally distributed error term.

4.5.1.3.2 *Re-entry Delay Modelling Using Bayesian Approach.*

Using classical frequentist statistics (CFS) assumes that the model has a constant variance, such that any information about the model's variance is determined during model

building. Moreover, the variables in empirical studies, according to asymptotic theory, has a stochastic nature. This stochastic nature arises from the fact that the explanatory variable in some studies becomes a response variable in another study. Thus, CFS is unable to detect any change in the different statistical inferences.

Another key important fact to using Bayesian modeling for the re-entry delay parameters is that the Bayesian approach is able to use a prior knowledge for a certain case to be expanded to other different conditions including; different bus characteristics, different network configurations, and different prevailing traffic conditions. If this is the case, then the MLR model coefficients are no longer accurate. Instead, we can use the MLR model coefficients as the prior coefficient estimator. In this case, the agency needs to collect limited local data, the model would then use the local data together with the prior coefficients to estimate a posterior set of coefficients that are calibrated to the locality. In other words, the Bayesian approach can modify the model parameters to reflect the local conditions.

Another reason to use the Bayesian approach for other localities is that the Bayesian method provides a more direct expression of uncertainty, including complete ignorance. Also, in the Bayesian approach, there is no worry of using a smaller sample size, as it uses a smaller number of parameters. More importantly, the Bayesian method has the capability of handling non-normal parameters, as Bayesian methods provide more accurate results because they can deal with asymmetric distributions.

Generally, the maximum likelihood estimation (MLE) is used in the classical frequentist inferences. MLE assumes model parameters that are fixed and known, and the bottom line of the selection of these parameters emerges from maximizing the likelihood function for the selected model. Notably, using informative knowledge about the process for both prior and posterior information about the process, using Bayes' theorem [63] in Equation (IV.15). This equation can be expanded to achieve stochasticity among model parameters, which uses the conditional probability formula in Equation (IV.12).

$$\text{posterior} \propto \text{prior} \times \text{likelihood} \quad (\text{IV.15})$$

$$p(y | m) = \frac{P(\beta, \sigma | m) P(y|X, \beta, \sigma, m)}{P(\beta, \sigma | y, X, m)} \quad (\text{IV.12})$$

Where:

$p(y | m)$ = the marginal distribution of y .

$P(\beta, \sigma | m)$ = the prior model distribution.

$P(y|X, \beta, \sigma, m)$ = the likelihood estimation.

$P(\beta, \sigma | y, X, m)$ = posterior distribution.

The selected model parameters can be calibrated using the Bayesian statistics method. Associated with using the Bayesian method of data analysis is the Monte Carlo approach, which has been used to generate the posterior distributions when these distributions cannot be generated analytically. The Bayesian parameters can be modeled using the Markov Chain Monte Carlo (MCMC) slice algorithm implemented with the Bayesian algorithm in the R software. This algorithm is designated for use in the distribution of an arbitrary density function to sample the data using a known constant of proportionality, which is needed to sample from a complicated posterior distribution whose normalization constant is unknown. The approach generates random samples from these distributions to estimate the posterior distribution or derived statistics (e.g., mean, median, and standard deviation).

The MCMC slice algorithm requires an initial solution for the model parameters, the prior distribution, and the residual variance. With these priors, the posterior distribution of model parameters conditionally depends on the response variable and the variance. Transit agencies can use this information to draw the marginal distribution using the Monte Carlo approach by first, drawing the variance from its marginal distribution.

Moreover, Bayesian statistics are able to capture the uncertainty in selected parameters of the model and express them as random variables because of the maximum likelihood estimation (MLE) that is used in the classical frequentist inferences. MLE assumes model parameters are fixed and known and the bottom line of the selection of these parameters emerges from maximizing the likelihood function for the selected model.

4.6 Results

As this research aims to estimate the two main components of transit vehicle lost time at bus stops, the results are categorized into three main groups: deceleration profile, acceleration time, and clearance time.

4.6.1 Deceleration Profile

This section presents trip deceleration characteristics including deceleration rate, deceleration distance, and deceleration time.

4.6.1.1 Deceleration Rate

The idea of conducting this research arose from the fact that the power delivered by the engine during braking cannot be assumed to be the maximum power nor a constant percentage of the power along the deceleration trip. Moreover, a transit vehicle driver does not typically impose a high brake pedal input that fully stops the transit vehicle; rather, the driver must impose a gradual deceleration rate in order to minimize the vehicle jerk (rate of change of acceleration). Subsequently, this encouraged the research team to adopt vehicle kinematics equations, as they assume a constant acceleration/deceleration rate that estimate the deceleration rate as a function of the speed change over the entire trip; in other words, the research team agreed that the transit vehicle maintains a constant deceleration rate during stopping maneuvers so that passengers' stability is not affected by the vehicle. However, modeling the deceleration rate as a function of speed is a challenging task because, once the driver reaches their comfortable deceleration rate, the deceleration rate increases as the speed decreases to assure that the vehicle is completely stopped before passing the desired deceleration distance, and typically this rate is higher than the deceleration rate at a higher speed. In fact, the driver may start to decelerate from a certain speed by gradually pushing on the brake pedal until they reach a speed level at which they are assured the vehicle will stop; otherwise, they may only have to barely push on their brake pedal to make sure that the vehicle stops within the target distance.

The research team investigated several functions to model the deceleration rate as a function of the initial speed, since the quadratic model is unable to handle the model characteristics around the extreme points, and the linear regression model cannot produce a reasonable prediction (as the estimated value does not provide a good measure-of-fit). After a thorough investigation, the function in Equation (IV.17) was found to be the best fit to this research data. The proposed model shows its capability of handling the extreme points and has a degree of freedom around the extremes.

$$\text{Deceleration Rate} = \frac{2.7 * e^{-5}}{-2.6e^{-8} + 2.618e^{-6}V_0 + V_0^{-22.18}} \quad (\text{IV.17})$$

Model validity and adequacy were checked using different means, including graphical tools and appropriate diagnostic, as ways to check the selected model using the normal quantile plot and check the homoscedasticity. The chosen model has to show approximate residual errors in addition to having a random error pattern, as shown in Figure 17.

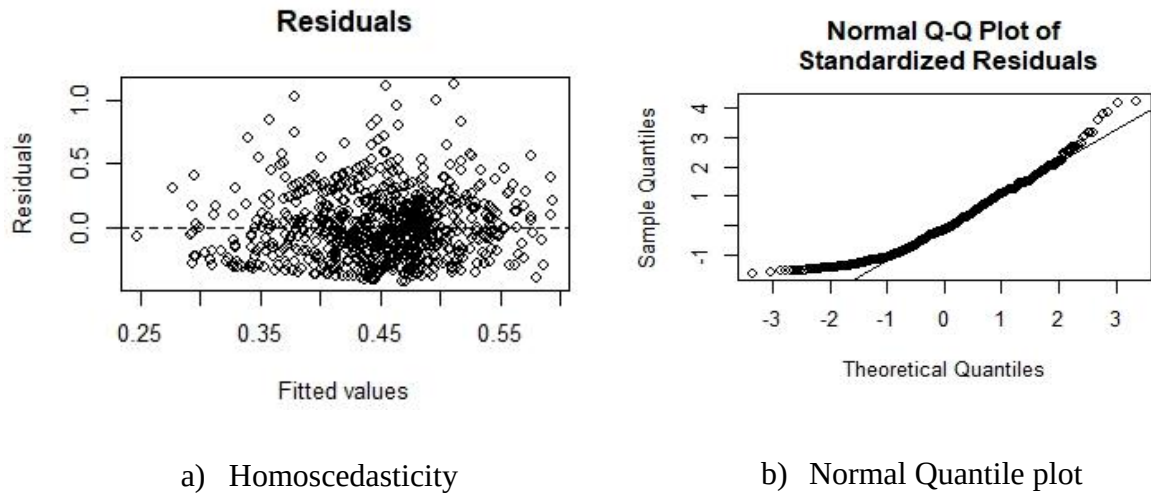


Figure 17 : Model Diagnostic

Figure 17.a shows the model's homoscedasticity check. It can be seen that the residual errors are independent from each other and are randomly distributed around 0. It also shows that these values do not exceed 1. Figure 17.b shows that the error term approximately follows normal distribution. However, it also shows that model results in large values at the tails of the distribution compared with standardized model assumptions. This issue was resolved by setting the minimum deceleration rate as 0, which corresponds with the value of 0 for speed change. In our case, while there was no change in speed, we assumed that the bus maintained a fixed acceleration rate and thus, set the deceleration value of zero.

4.6.1.2 Deceleration Distance

Deceleration distance was computed based on Equation (IV.18) using the estimated deceleration rate in Equation (IV.17). The computed distance was compared against the field distance, and both the mean absolute percentage error (MAPE) and root-mean-square-error (RMSE) were computed and found to be 2.2% and 28.36%, respectively.

4.6.1.3 Deceleration Time

The deceleration time was computed using the computed deceleration distance according to Equation (IV.6)(IV.4) and using the kinematic equation, which is shown in Equation (IV.18).

$$\text{Deceleration Time} = \frac{\text{Deceleration Distance}}{V_0} \quad (\text{IV.18})$$

Comparing the actual deceleration time with the estimated time was done using a graphical tool and then by computing MAPE and RMSE. The MAPE and RMSE were found to be 0.3% and 13.3%, respectively.

4.6.2 Clearance Time

This section presents the transit vehicle waiting time to merge with the traffic. As was mentioned before, this time has two components: the startup travel time and the re-entry delay time. This section discusses the re-entry delay, while the startup travel time was merged with the acceleration.

4.6.2.1 Re-Entry Delay

In this section, modeling the re-entry delay using classic frequentist statistics and the Bayesian approach are presented.

4.6.2.1.1 Modelling Re-entry Delay using Classic Frequentist Model

The re-entry delay time measures the extra time that a transit vehicle has to wait at the bus stop to merge with the traffic in the adjacent lane. This time is expressed as a function of the traffic density in the adjacent lane. Estimating the re-entry delay time as a function of density was done using generalized linear modeling (GLM). Assuming that the merge delay is continuous, and the traffic density is an integer variable, the general linear model was fit to the data, considering a normal distribution of the results from Equation (IV.19).

$$\text{Re - entry Delay} = B_0 + B_1 K \quad (\text{IV.19})$$

Where K is the density (Veh\km\ln); and B 's are the different model coefficients. Using LSE, the model coefficient values along with their p values are summarized in Table 9.

Table 9: Statistical Interference for Merge Delay Model

Parameter	Estimate	Standard Error	P-value
B_1	1.788	0.050	<0.001
B_2	0.318	0.004	<0.001

Table 9 shows the re-entry model parameters' statistical interferences. The transit vehicle has to wait at least 1.788 seconds to re-enter traffic. This time is close enough to the average perception time, which is 1.5 sec [71]. However, this time increases by 3.1 seconds per 10 vehicle/km/ln increase in density in the adjacent lane.

The model capability to explain the variance in the data was checked using goodness-of-fit was 92%. Moreover, the adequacy and validity of the model coefficient estimation was done using the general linear model assumptions. Choosing the best model among a set of models means that the selected model should have a normal residual error, as shown in Figure 18.

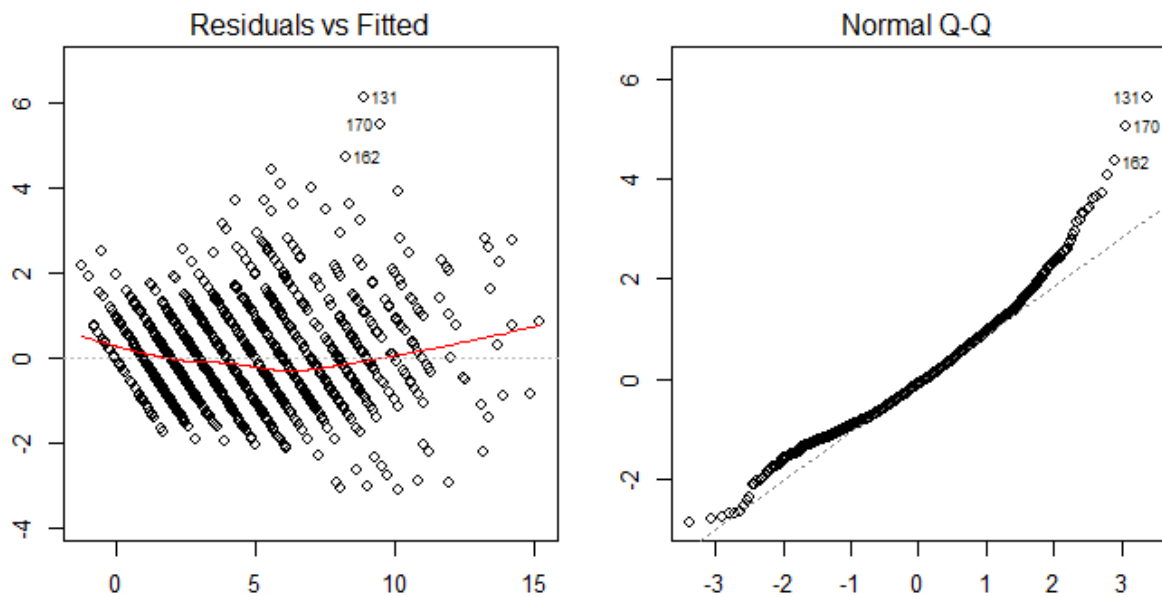
**Figure 18: Re-entry Delay Model Diagnostic**

Figure 18 shows the re-entry delay time model diagnostic for both the residual error and the normal quantile plot. Both the residual plot and normal quantile plot show that the residual errors are normally distributed around zero and follow normal distribution.

4.6.2.2 Acceleration Time

Estimation of the acceleration time was done using the predicted acceleration rate that is presented in Equation (IV.20). As a reminder, the acceleration rate was computed for the entire trip in addition to the second-by-second acceleration rate using the kinematic equation, which was presented in Equation (IV.9). Then, two acceleration rate values were extracted: the average acceleration rate and the maximum acceleration rate for the entire trip. The reason behind selecting these two values is that, during the entire trip, the acceleration rate does not stay constant, nor does the engine reach its maximum power. In fact, different engine powers with different driving conditions result in different maximum acceleration rates to reach the maximum desired speed. According to the dataset used, the maximum speed is 70.4 km/hr. In order to generalize the prediction model, the acceleration rate had to be transferrable to any transit vehicle and any driving condition. The acceleration rate prediction model is presented in Equation (IV.20).

$$\frac{a_{average}(U)}{a_{max}} = \left(1 - \left(\frac{x}{2.554}\right)^{2.519}\right) \quad (IV.20)$$

Where $a_{average}(U)$ is the average acceleration rate that can be obtained during the acceleration event; x is the ratio between the target speed and the maximum speed $= (V/70.4)$; and a_{max} is the maximum acceleration rate that can be obtained during the acceleration event.

The plot of the model of Equation (IV.20) is shown in Figure 19. The proposed model shows its capability to capture the variation in the data with a goodness-of-fit equal to 0.84.

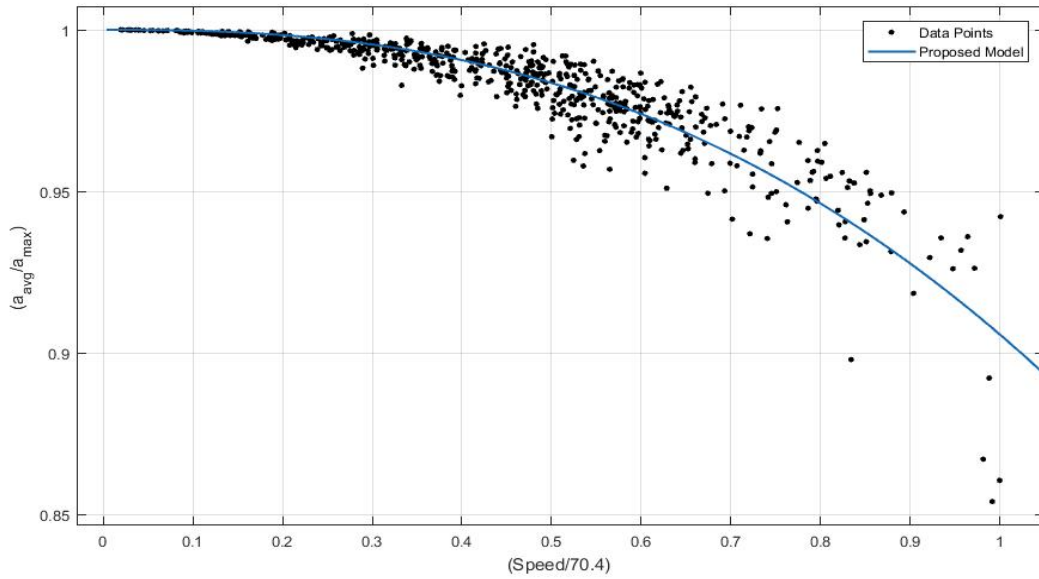


Figure 19: Acceleration rate prediction model.

In order to estimate the acceleration time, the predicted acceleration rate in Equation (IV.20) can be used in Equation (IV.21).

$$Acceleration\ Time = \frac{V_{target}}{a_{avg_predicted}} \quad (IV.21)$$

Where the V_{target} is the desired speed to which the transit vehicle wants to accelerate. Comparing the estimated acceleration time against the field value, both RMSE and MAPE were computed and found to be 2.72% and 0.42%, respectively.

4.6.3 Modelling Re-entry Delay Using Bayesian Approach

The selected model parameters can be calibrated using the Bayesian statistics method. Associated with using the Bayesian data analysis, the Monte Carlo approach has been used to generate the posterior distributions when these distributions cannot be generated analytically. More specifically, the Bayesian parameters can be modeled using the Markov Chain Monte Carlo (MCMC) slice algorithm implemented with the Bayesian algorithm in R software. This algorithm is designed to use a distribution of an arbitrary density function to sample the data using a known constant of proportionality, which is needed to sample from a complicated

posterior distribution whose normalization constant is unknown. The approach generates random samples from these distributions to estimate the posterior distribution or derived statistics (e.g., mean, median, and standard deviation).

The MCMC slice algorithm requires an initial solution for the model parameters. With these priors, the posterior distribution of the model parameters is conditionally dependent on the response variable and the variance. Then, the researchers used this information to draw the marginal distribution using the Monte Carlo approach by first drawing the variance from its marginal distribution. Notably, the posterior distribution of the variance distribution is scaled through an inverse Chi-square distribution with scaled deviation and $(n-k)$ degrees of freedom, where n is the number of observations and k is the number of model parameters. In this case, the number of observations is 1,365, and $(1,365-2=1,363)$. Table 10 summarizes the different statistics for the 1,000 realizations for the model parameters. It can be seen that the mean value of all the model parameters are close to the parameter estimates from the classical frequentist GLM model in Table 9.

Table 10 : Summary Statistics of the Different Model Parameters

Parameter	Mean (μ)	Quantiles		St. Dev. (σ)	Skewness (γ_3)	Kurtosis (γ_4)
		Q _{0.025}	Q _{0.975}			
B_0	1.789	1.604	1.823	0.050	-0.017	-0.030
B_1	0.3185	0.303	0.321	0.004	0.008	-0.016

Moreover, measuring the randomness between the generated samples as a function of time was done using autocorrelation functions (ACF) for modeling the coefficients, which is presented in Figure 20. As can be seen in the figure, the solid horizontal lines represent the 95% confidence limits, while the stem lines represent the autocorrelation function. As would be expected, an autocorrelation of 1.0 is observed for a lag of 0, while the values are within the confidence limits for larger lag times.

In addition, a Kolmogorov-Smirnov goodness of fit test (K-S test) was run on the model parameter distributions and concluded that there was insufficient evidence to reject the null hypothesis, as the model parameter distributions were different from the normal distribution at a level of significance of 0.05, as illustrated in Figure 21 .

Another important illustration of the model results is presented in Figure 22, which shows the interaction between the model parameters (β_0 , β_1). When comparing β_0 to β_1 , it is clear that a linear relationship between β_1 and β_0 can be deduced.

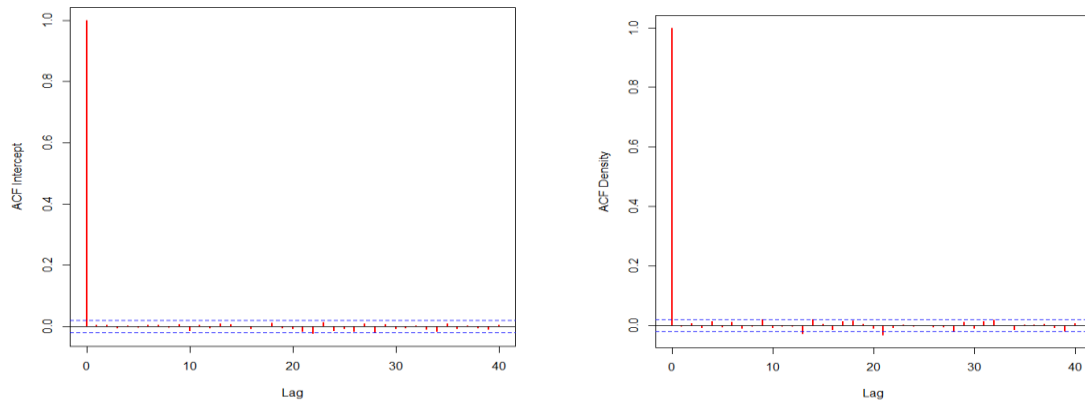


Figure 20 : Variations in the Autocorrelation Function of the Model Parameters

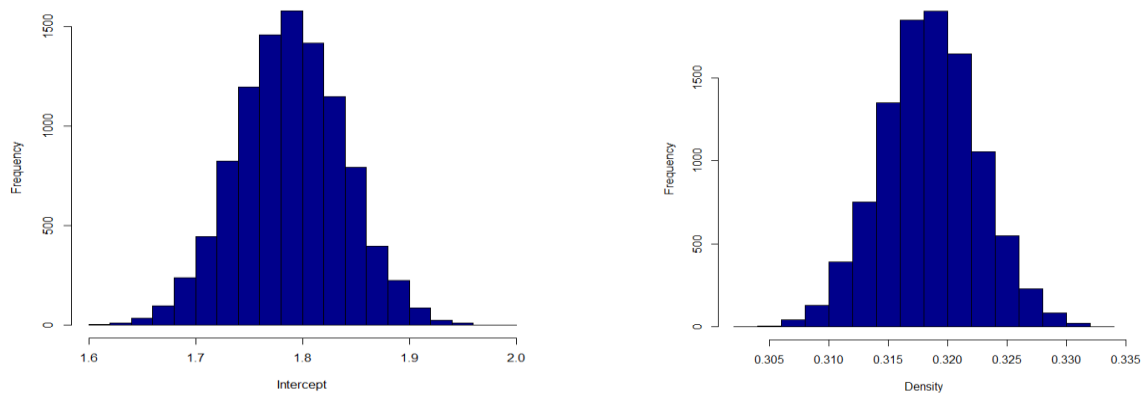


Figure 21: Histograms of the Model Parameter Distributions

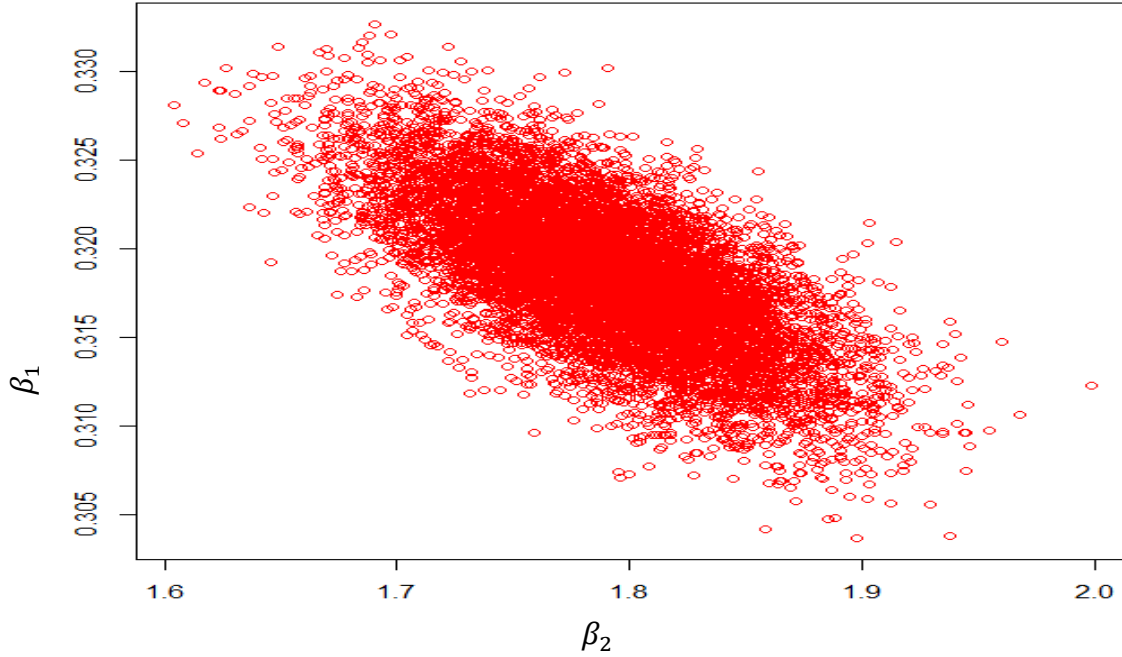


Figure 22: Inter-dependence of the Bayesian Model Parameters

4.7 Discussion

This work quantifies the total delay time components at bus stops. The total delay has three components: deceleration time, loading time, and the clearance time. This work primarily focuses on the deceleration time and the clearance time (as the loading time has been studied before by the researchers). The research team adopted vehicle kinematic equations to estimate the deceleration time. Also, the researchers computed the clearance time as a function of the link density and the targeted speed.

The results of this work show the validity of using the adopted methodology as it compared the estimated deceleration distance and time with the field values. Also, the results indicate that the re-entry delay variable confirms the hypothesis variable signs.

4.8 Conclusions and Recommendations

Modeling transit lost time has been done mainly considering only the dwell time; however, due to congestion, the dwell time does not reflect the full boarding/alighting passenger time. Instead, it has other components including the time required to completely stop the transit vehicle and the time required to merge with the traffic again.

To test the study hypothesis, the researchers started by adopting a kinematic equation to express the transit vehicle deceleration and acceleration characteristics due to the fact that the transit vehicle cannot maintain a constant acceleration/deceleration rate or a maximum acceleration/deceleration rate during for the entire maneuver. Hence, developing a mathematical model that is able to predict the acceleration/deceleration rate comes as the first priority. Consequently, the deceleration distance can be computed using the predicated deceleration rate, after which the deceleration time can be estimated more easily. That being said, the adopted methodology was compared against empirical data, and both the estimated deceleration distance and the deceleration time show their ability to replicate the field data with a reasonable error. The acceleration rate was computed as a function of vehicle and driver characteristics and, by revisiting the predicted acceleration model, the roadway characteristics are captured by the free-flow speed. Conversely, vehicle characteristics are measured by the maximum acceleration rate. Using the predicted acceleration rate and the change in speed, the acceleration time was computed using the kinematic equations.

Evaluating the adopted methodology for computing both the acceleration and deceleration time was done using the MAPE and RMSE. Overall, the study results show the validity of the adopted methodology to estimate the deceleration time; by comparing the estimated deceleration time with the field data, the MAPE and RMSE were computed to be 0.3% and 13.3%, respectively.

Also, the re-entry delay was estimated as a function of the traffic stream density in the vicinity of the bus stop and the change in transit vehicle speed, such that the total lost time at a bus stop includes the boarding/alighting time, in addition to the re-entry delay and the time required to decelerate and accelerate to its desired speed.

It is worth mentioning that the study results show that the vehicle kinematic equations help in estimating the deceleration time and acceleration time respectively, but also that the factors that affect the clearance time have a reasonable sign. The results of this study can be used to update the transit vehicle arrival and departure times at bus stops and provide a better estimation of bus arrival times at traffic signal. Also, it can be used to update traffic signal and transit stop configurations.

Although the study meets its objective, the study has some limitations. For example, the effect of pavement and weather conditions on the vehicle kinematics were not considered

in this study. This is because the main objective was to estimate the deceleration time alone. However, these factors may affect the deceleration rate.

4.9 ACKNOWLEDGEMENTS

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4.10 AUTHOR CONTRIBUTION STATEMENT

The authors confirm contribution to the paper as follows: study conception and design: Alhadidi, Rakha; data reduction: Alhadidi; analysis and interpretation of results: Alhadidi, Rakha; draft manuscript preparation: Alhadidi, Rakha. All authors reviewed the results and approved the final version of the manuscript.

Chapter 5 **Modelling Bus Arrival Time at Traffic Signals using Probe Vehicle Data: Shockwave Theory and Statistical Modeling**

5.1 Abstract

This chapter models the transit vehicle arrival time at traffic signals in a connected environment using modeling and data driven approaches. The analysis was done using different scenarios that were generated using the INTEGRATION software. The generated scenarios covered different driving conditions by varying the traffic volume and speed limit. The generated data includes vehicle speed, location, instantaneous travel time, traffic signal timing, traffic density, and traffic flow. In essence, the generated data are used to estimate bus arrival times at traffic signals using a shockwave analysis as a modeling approach and multiple linear regression (MLR) as a data driven approach to predict bus arrival times. Generally speaking, the shockwave analysis can be used to estimate bus arrival times at traffic signals by knowing the bus speed and the prevailing traffic conditions with a maximum difference that does not exceed 13.27%. This difference is mainly caused by the variation in headway between vehicles, vehicle speed, and vehicle kinematic characteristics. Finally, the statistical prediction model shows its capability to predict the transit vehicle arrival time as a function of the traffic flow and density with a coefficient of determination of 94%.

Keywords: arrival time, transit vehicle, delay, shockwave.

5.2 Introduction

Traffic congestion continues to be a major problem, especially within the non-homogeneous multimodal transportation system. Implementing advanced travelers' information systems (ATIs) helps in reducing traffic congestion by providing accurate travel information (i.e., travel times, arrival times, and selected mode to system users). In fact, it helps system users avoid traffic congestion by changing their departure time, mode, or route. Most importantly, ATIs help to improve public transportation reliability by not only in terms of schedule adherence, but also improving users' satisfaction, and reducing public transit travel time.

Recently, using different sources of information including probe vehicles, Advanced Public Transportation Systems (APTs) embedded tools (i.e., automated vehicle location (AVL) and

automated passenger counting (APC), automated fare collection (AFC)), and traffic signal timings has helped to manage the transportation system by providing real-time information for operators, agencies, and users. The integrity between all of this information helps to provide an effective real-time management of the multimodal transportation system and allows for the successful implementation of transit signal priority (TSP) as it depends on the accurate estimation of bus arrival time at traffic signals.

Bus arrival time at traffic signals depends on different factors, including the transportation network configuration, transit system layout, traffic management system, network heterogeneity, and the interaction between these factors. Several methods were used to estimate the bus arrival time at traffic signals using different data sources, including historical data, probe vehicles, and APTs. Many techniques have been used for estimating bus arrival time at traffic signals. These techniques include prediction techniques and analytical techniques.

Several studies had been conducted to provide arrival information based on a real-time bus information; however, none of these studies incorporate the impact of traffic signal delay in estimating the bus arrival time [31, 59, 72-75]. Rather, in these studies, delays were considered implicitly, using historical information of bus arrival times at a traffic signal and at the bus stop to model bus arrival times. Also, none of these studies mentioned the impact of queue and traffic signal timing on estimating bus arrival times. Another study used shockwave theory with the incorporation of different data sources, including traffic signal timing, estimated shockwave speed generated by the traffic signal, and the bus speed from GPS tracking. However, the methodology was not validated against any type of data [76]. Ironically, using the proper fundamental diagram (FD) affects the estimation of bus arrival time because it affects in estimating the traffic flow and traffic density (which directly affects the computation of the shockwave speed).

5.3 STUDY OBJECTIVES AND PAPER LAYOUT

The objective of this study is twofold. First, it provides a methodology for modeling the bus arrival time at traffic signals based on shockwave theory to estimate the transit vehicle arrival time using both traffic signal data and probe vehicle data. Second, it uses the probe vehicle data to build a statistical model that is able to predict the bus arrival time at traffic signals in case there is a malfunction in the connected environment.

The remainder of this chapter is organized as follows. The next section presents a literature review about the application of different techniques to model bus arrival time including prediction and estimation models. The following section describes both the adopted methodology and the datasets used to model bus arrival time. Then, the adopted methodology results are presented. After this, the adopted methodology results are discussed. The final section includes some concluding remarks and suggestions for future extensions.

5.4 Literature Review

Many studies have modeled bus arrival time at traffic signals using either historical data or real-time data. Different techniques were used to model bus arrival time and these techniques can be categorized into two main groups: (1) analytical and (2) prediction. In the analytical approach, traffic propagation, shockwaves, and traffic signals were used to estimate bus arrival time, while in the prediction approach, a statistical model is built using different statistical techniques. The main distinction between the analytical and predictive approaches is that the prediction method is data driven technique where the data is the main element to predict the bus arrival time. While the analytical technique uses a traffic signal, traffic data, and bus speed to compute the bus arrival time.

5.4.1 Modeling Transit Vehicle Arrival Time using an Analytical Approach

In the analytical approach, bus arrival time is modeled using shockwave theory. In order to use shockwave theory to compute the bus arrival time, these studies have used different information sources (i.e., traffic volume, speed, and traffic signal information). Modeling the bus arrival time at traffic signal using the aforementioned traffic data assumes that the traffic accelerates and decelerates suddenly. However, traffic conditions (including flow, density, and space mean speed) change suddenly.

A study was conducted to estimate the bus arrival time at traffic signals based on shockwave theory was done using the incorporation of dataset that included the pre-timed traffic signal information, shockwave speed, and bus speed [76]. Yet, the mentioned methodology was not validated against any type of data, nor was the evaluation of the method stated in the work.

5.4.2 Modeling Transit Vehicle Arrival Time Using Regression

Different techniques were used in modeling transit vehicle arrival times. These techniques can be categorized into different groups including: 1) historical data, 2) regression models, 3) the Kalman filtering model, 4) machine learning models, and 5) hybrid models.

1) Modeling Arrival Time Using the Historical Average Method

Modeling transit arrival using historical data is useful for predicting the dynamic real-time arrival time. Different studies were conducted to predict the transit vehicle arrival time using the historical data. In all of these studies, the traffic demand is assumed to be fixed and remains stationary. Moreover, the performance of these models are weak in fluctuating traffic patterns, especially for heterogeneous and congested areas [59, 73, 75].

2) Regression Models

In these studies, the arrival time is expressed as a function of a set of statistically significant independent variables. In contrast with historical average studies, regression models address the instability in traffic conditions [31, 74, 77]. A study was conducted to quantify the impact of various independent variables on arrival time. The selected independent variables included distance, number of stops, dwell times, boarding and alighting passengers, and weather descriptors while the dependent variable was the arrival time. The results of this study show that the weather is not statistically significant in the prediction of bus arrival time. Also, the study results showed that the developed models are able estimate the bus arrival time [74]. Other studies developed multiple linear models using different sets of variables, comparing the developed models with the new models; the results of these studies showed that the developed models outperformed other types of models [77, 78].

3) Kalman Filtering Model

Kalman filter (KF) models predict the future state of the dependent variable based on the current state of the dependent variable. KF uses a mathematical model that accommodates the fluctuation in the traffic state using the Kalman gain. The GPS data was collected from public transit buses on urban roadways in India. A prediction model using KF was done to predict the transit travel time under heterogeneous traffic conditions; the proposed model showed its capability to predict bus travel times and arrival times with an acceptable prediction error [75].

KF has been studied by different researchers [31, 72, 73, 75, 79]. Mainly, their KF models provide an accurate estimation of the arrival time based on the conditions of the previous time steps. The developed models are capable of improving the prediction accuracy because KF models are able to handle the noise.

4) Machine Learning Models

Using machine learning algorithms has become more popular nowadays. These models are able to deal with complex data and noise as they predict the nonlinear relationships between dependent and independent variables and are able to check the latent variable. However, dealing with these models is not easy, as sometimes they over fit the data. Different machine learning algorithms have been used to predict bus arrival times, the most popular and widely used algorithm is the artificial neural network (ANN). ANNs have shown a promising solution to predicting bus arrival time as they deal with complex and nonlinear data; however, the model is sensitive to location characteristics. This means that developed ANN models can't be transferred to another location because of location-based specifications (i.e., traffic, geometry, and traffic control). A study examined the impact of dwell time, schedule adherence, and the arrival time at the previous bus stop on the bus arrival time at a downstream station was done by using AVL data and the ANN [78].

Another study used the GPS data with road geometrical characteristics to develop an ANN model that is able to predict the bus arrival time. The results of this study showed the ability of the developed model to estimate bus speed and location with a high accuracy [80].

5.5 Methodology

This section presents the adopted methodology of this research work and presents the data collection and processing. As previously mentioned, the research effort has two components. First, it models bus arrival time using probe vehicle information using a statistical model. Second, it models the bus arrival time at the traffic signal using shockwave theory. This section starts with the description of dataset used in this work, refining and processing followed by the adopted methodology.

5.5.1 Data Set

The data used in this research effort was generated by INTEGRATION. Integration is a microscopic traffic assignment and a simulation software that was initially developed in the late 1980s [81, 82]. It tracks the movement of individual vehicles every one-tenth of a second. This allows for a detailed analysis of lane-changing movements and shockwave propagations and reports different measure of effectiveness. That being said, different simulation scenarios have been generated to accommodate the variation in different traffic conditions. The geometry of the tested scenarios is shown in Figure 23, while the variation in the simulated parameters are presented in Table 11. The intersection is a four legs intersection with equally length arms. The eastbound approach has a near-side bus stop that locates 300 m away from the traffic signal. The intersection is controlled with a two-phases fixed time traffic signal.

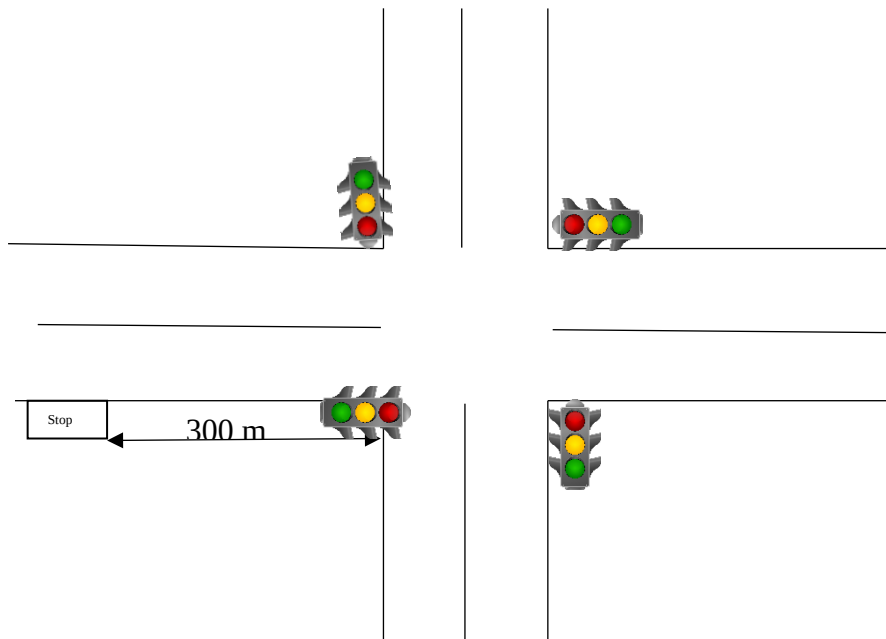


Figure 23: Testbed Configuration

Table 11: Scenario Characteristics

Parameter	Min	Max	Increment rate
Speed (km/hr)	35	55	5
Speed Variation	0	0.15	0.025
Flow (vph)	600	2000	100
Jam capacity	150 veh/km/lane		
Saturation Flow Rate	2,500 veh/hr/lane		
Traffic Signal	Total cycle length 60 seconds		
	50:50 phase split: 26 sec of effective green time, 34 sec of effective red time.		

5.5.2 Data Processing

INTEGRATION generates fully detailed vehicle characteristics every one-tenth of a second. The generated output file is a set of successive simulation time points (where transit vehicle location, speed, delay, stopped delay, and total delay every one-tenth second and other information are generated). A home-based logic was built to extract the reported parameters based on tracking the transit vehicle's actual departure time from the bus stop to reach the traffic signal. All the related parameters have been extracted from the actual departure time to the transit vehicle arrival time, and both flow and density were reported based on the detectors' output.

5.5.3 Model Development

The bus-arrival time prediction methodology consists of two approaches: a modeling approach and a statistical approach. The modeling approach was developed using shockwave and queuing theory. This model depends mainly on knowing the traffic pattern and traffic signal timing in addition to the bus location and speed. The modeling approach output is an equation to compute the bus arrival time based on shockwave and queuing theory and validated using the probe vehicle data. Conversely, the statistical modeling approach depends on data from the probe vehicles and the transit vehicle arrival time to build a statistical model that is

able to predict the relationship between the dependent variable (arrival time) and the independent variables (traffic stream flow and density in the vicinity of the traffic signal).

5.5.3.1 Modeling of Bus Arrival Time Using Shockwave Theory

A shockwave is where traffic status changes from state to another state on the fundamental diagram. The traffic condition is defined by its space, flow, density, and speed on the fundamental diagram.

Several possible bus trajectories might be obtained depending on the traffic condition and the traffic signal timing. The possible bus trajectories with their associated traffic signal timing are shown in Figure 24 and Figure 25 for both under-saturated and oversaturated traffic flow, respectively.

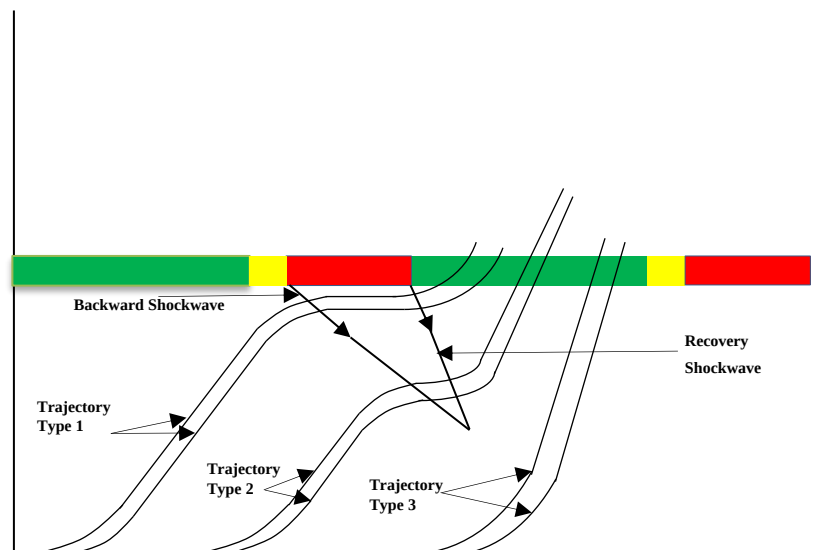


Figure 24: Time-space Diagram for Undersaturated Traffic Condition with Potential Bus Trajectories

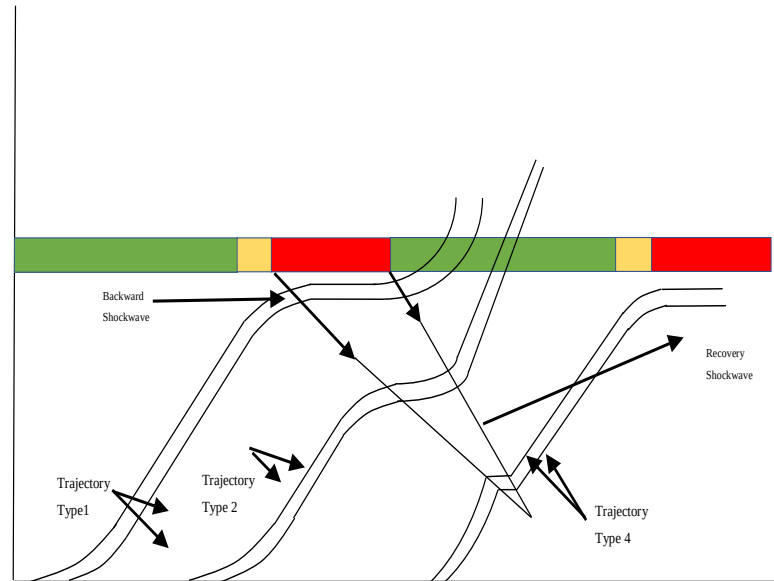


Figure 25: Time-space Diagram for Oversaturated Traffic Condition with Potential Bus Trajectories

Both Figure 24 and Figure 25 show the possible bus trajectories between the bus stop and the downstream traffic signal. The bus trajectory depends on the bus departure time, traffic signal timings, and the traffic in the adjacent lanes. For the under-saturated traffic condition (Figure 24), there are three possible trajectories depending on the traffic signal timings and the prevailing traffic conditions. Trajectory 1 presents the bus arrival time at traffic signal while it is on a red phase and the queue was discharged completely on the previous green phase. Sometimes, the available green time cannot discharge to total demand, and the bus may experience a delay due to the time that is needed to discharge the queue in front of the bus. The trajectory of this bus is presented in Trajectory 2. Trajectory 3 presents the bus that is passing the traffic signal without experiencing any delay. For the oversaturated conditions, another possible trajectory is presented in Figure 25, where the transit vehicle joins the intersection delay by the time it has backward propagating shockwave because it is unable to cross the intersection with the current green phase.

To compute the bus arrival time based on shockwave theory, different values should be estimated. These values are bus speed, shockwave speed, traffic signal timing, and estimating link travel time. Estimating bus speed is measured using real-time data from the GPS tracker.

While the traffic timing is obtained by either assuming a pre-timed or an adaptive traffic timing. Shockwave speed is determined using the probe vehicle data, which reports the flow, space mean speed, instantaneous speed, and the density.

5.5.4 Estimating Shockwave Speed

While the shockwave presents the boundary between two different traffic conditions, and using the conservation law at the shockwave, shockwave speed can be computed by using Equation (V.1).

$$U = \frac{\Delta q}{\Delta K} \quad (V.1)$$

Using the fundamental diagram that is suggested by Van Arde, which is shown in Figure 26, the two shockwaves that are associated with the traffic signal timing are presented as V_2 and V_3 , where V_2 is the backward forming shockwave due to the red phase and V_3 is the recovery shockwave that discharges the queue at the saturation flow rate of the traffic signal (another shockwave generates due to the generated queue during the green phase).

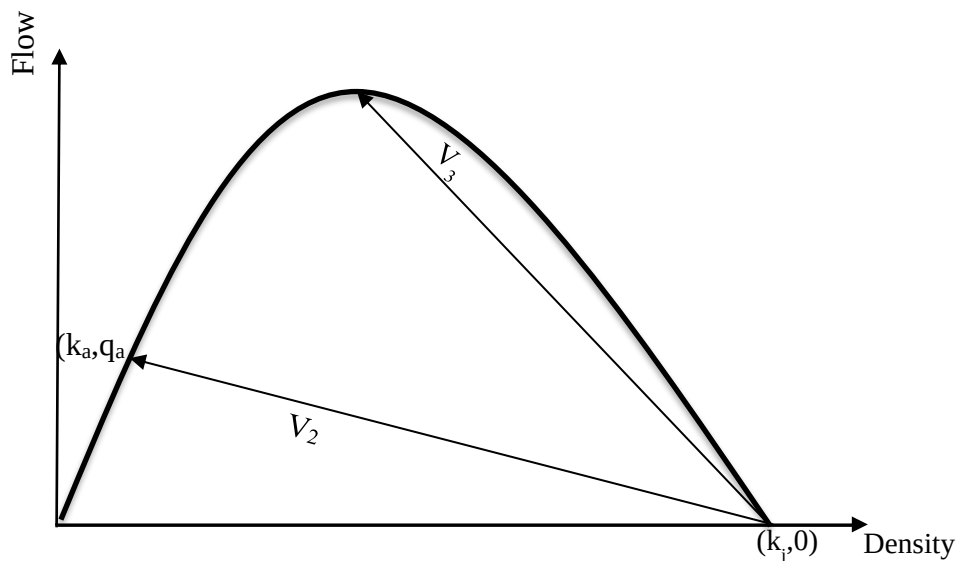


Figure 26: Fundamental Diagram with Shockwaves

Using the fundamental diagram that is suggested by Van Arde, which was shown in Figure 26. in associated with the the relationship between flow and density that has been proposed by . Lighthill and Whitham [83] and Richards [84] that explained the theory of one dimensional waves which becomes the way to analysis the traffic flow behavior. Their model is known as LWR, they used the conservation law to explain the fluid dynamics theory. Their model expressed the traffic behavior by explaining the relationship between flow, density, and speed as shown in Equation (V.2), the two shockwaves that are associated with the traffic signal timing are presented as V_2 and V_3 . Where V_2 is the backward shockwave and V_3 is the recovery shockwave.

$$Q = K.U \quad (V.2)$$

As was illustrated in both Figure 24 and Figure 25, the bus has four possible trajectories depending on the traffic signal timing, traffic condition, and bus speed.

For example, the bus leaves the near-side bus stop with a one of two possible traffic signal state:

Case 1: Traffic signal state is red when the bus departs the nearside bus stop.

Case 2: Traffic signal state is green when the bus leaves the nearside bus stop.

The formulation for each case is presented below, this formulation depends on bus speed, shockwaves propagation, traffic signal timing condition, and traffic volume.

5.5.4.1 Bus Departure Traffic Signal within Red time

This section presents the formulation of bus arrival time in case the bus leaves the nearside bus stop during a red phase time. Mainly, it tests weather the bus will arrive at the traffic signal within either the current red phase or next green time, or may need more than one cycle to cross the intersection. All of these scenarios depend on the queue that forms during the backward forming and the backward recovery shockwaves in addition to the saturation flow rate and the remaining queue from the previous green time. Speaking of traffic signal timing, it is assumed to contain only green and red timing, that the speed of the traffic and the shockwaves are homogeneous and constant, and that the bus accelerates and decelerates suddenly. This means this research considered the analysis traffic state that occupied the distance between the bus stop and the traffic signal by the time that the bus leaves the nearside

bus stop as it was shown in both Figure 24 and Figure 25 as the reference point for both of the figures is the bus stop.

Assuming the bus will join the intersection queue in the current phase, this happens if the distance traveled by the bus and the queue in front of it is greater than the distance between the bus stop and the traffic signal. The queue at the traffic signal has two sources, either the queue forms due to the backward forming shockwave that is caused by the red phase or the queue forms during the recovery wave that is caused by the green phase.

To be more specific, there are three different distances. The first distance is equal to the queue length during the backward forming shockwave, the length of this queue equals $V_{2,i} * E_i$. The second distance is equal to the queue forming when the signal turns green $V_{2,i} * (R_i - E_i)$. Finally, the third distance is the distance that is travelled by the bus during the time needed to join the queue, which is denoted by t_i . If the three distances are greater than or equal to the distance between the bus the traffic signal, then the bus will join the queue within this phase. This can be written as shown in Equation (V.3).

$$V_{2,i} * E_i + V_{2,i} * (R_i - E_i) + V_{1,i}(R_i - E_i) \geq L_k \quad (V.3)$$

Where $V_{2,i}$ is the backward forming shockwave in E_i , E_i is elapsed time (i.e., time from start of the red phase), R_i is the duration of the red phase, and L_k is the distance between the bus stop and the traffic signal.

In case the summation of the three distances was greater than 200 (the distance between the bus stop and the traffic signal), then the bus might cross the intersection in the next green phase, this case is illustrated later.

- A. Bus meets the backward shockwave propagation during the current red phase, which means the summation of the three distances equals at least the distance between the bus stop and the signalized intersection.

Such that we can compute the time that is elapsed until the bus joins the queue in front of it, Equation (V.3) can be rewritten as shown in Equation (V.4), where the time elapsed to join the back of the queue, which is t_i .

$$V_{2,i} * E_i + V_{2,i} * t_i + V_{1,i} * t_i = 200 \text{ m} \quad (V.4)$$

$$t_i = \frac{(200 - V_{2,i} * E_i)}{V_{2,i} + V_{1,i}}$$

Where V_{1i} is the bus speed when is leaving the downstream stop and t_i is the time to join the queue; the rest of the parameters were defined earlier.

The total arrival time is equal to the summation of the time to join the queue and the travel time in the queue. When the bus joins the queue in front of it, the bus stays stopped till the next green time. This time is expressed as $(R_i - (E_i + t_i))$. By the time traffic signal turns green, the queue starts to dissipate at the saturation flow rate. Bus travel time in the queue is expressed in Equation (V.5).

$$T_q = \text{bus waiting time to next green} + \text{queue dissipating time} \quad (\text{V.5})$$

$$T_q = [R_i - (E_i + t_i)] + \text{queue dissipating time}$$

Where T_q is the time that the transit vehicle spent in the queue; the rest of the parameters were defined earlier.

The time needed to dissipate the queue is computed as the ratio between the queue and the saturation flow rate $q_s(t)$, such that Equation (V.5)(V.5)(V.5)(V.5)(V.5) can be rewritten as presented in Equation (V.6). The number of vehicles in the queue is computed by Equation (V.7), assuming the average spacing between the vehicle in the queue is S_v .

$$T_q = \text{bus waiting time to next green} + \text{queue dissipating time} \quad (\text{V.6})$$

$$T_q = [R_i - (E_i + t_i)] + \frac{N}{q_s(t)}$$

$$N = \frac{V_{2i} * E_i + V_{2i} * t_i}{S_v} \quad (\text{V.7})$$

If Equation (V.3) is not met, that means case A is not satisfied. In this scenario, we need to check if case B is valid.

In case the summation of the three distances was greater than 200 (the distance between the bus stop and the traffic signal), then the bus might cross the intersection in the next green phase, this case is illustrated later.

- B. Bus does not meet the backward shockwave propagation during the current red phase, in other words, Equation (V.3) does not apply, which means the bus won't be able to join the queue and/or cross the intersection within the current red phase.

The time needed to dissipate the queue during the last red phase equals the time that is needed to dissipate the queue during the red phase in addition to the time needed to dissipate the queue that forms during the green phase. This time can be formulated as shown in Equation (V.8).

$$d_i = \frac{V_{2i} * R_i}{V_{3i} - V_{2i(G)}} \quad (V.8)$$

Where d_i is the time needed to dissipate the queue from the last red phase, $V_{2i(G)}$ is the backward forming shockwave due to the green phase, and V_{3i} is the recovery shockwave due to the green phase.

During the queue dissipating time, another queue will be forming due to the backward recovery shockwave until the recovery shockwave catches up to the previous backward forming shockwave due to the red phase. In this case, the bus might be able to catch the end of the queue during d_i . This might happen if the summation of the queue due to the backward forming shockwave, the recovery shockwave, the distance traveled by the bus during the time remaining for the signal to turn green, and the distance traveled by the bus during the green phase is greater than the distance between the nearside bus stop and the traffic signal. These distances are presented in Equation (V.9).

$$V_{1i} * (R_i - E_i) + V_{1i(G)}d_i + V_{2i} * R_i + V_{2i(G)} * d_i - V_{3i} * d_i \geq 200 \text{ m} \quad (V.9)$$

Where $V_{1i(G)}$ is the bus speed once the traffic signal turns green.

Once the traffic signal turns green, the queue starts dissipating during time d_i , but there is another queue forming due to the recovery shockwave; this queue stops forming by the time that the backward recovery shockwave meets the backward recovery shockwave. If the bus joins the back queue within this time, then the time that the bus needs to join the queue can be computed according to Equation (V.10).

$$V_{1i} * (R_i - E_i) + V_{1i(G)}t_i + V_{2i} * R_i + V_{2i(G)} * t_i - V_{3i} * t_i = 200 \text{ m} \quad (V.10)$$

$$t_i = \frac{(200 - V_{1i} * (R_i - E_i) - V_{2i} * R_i)}{V_{1i(G)} + V_{2i(G)} - V_{3i}}$$

The total arrival time is equal to the summation of the time to join the queue and the travel time in the queue. When the bus joins the queue in front of it, the bus stays stopped until the next green phase; this time is expressed as $(R_i - E_i)$. By the time traffic signal turns to green, the queue will start to dissipate at the saturation flow rate.

The time needed to dissipate the queue is computed as the ratio between the queued cars and the saturation flow rate $q_s(t)$. The number of vehicles in the queue is computed by Equation (V.11), assuming the average spacing between the vehicle in the queue is S_v . The time in queue can be computed using Equation (V.5).

$$N = \frac{V_{2i} * R_i + V_{2i(G)} * t_i - V_{3,i} * t_i}{S_v} \quad (V.11)$$

If Equation (V.10) is not met, that means case B is not satisfied. In this situation, case C might be applicable.

C. Bus does not meet the backward shockwave propagation during the current green phase; that means in this scenario $V_{1i} * (R_i - E_i) + V_{1i(G)} d_i + V_{2i} * R_i + V_{2i(G)} * d_i - V_{3i} * d_i < 200 \text{ m}$. Thus, bus maybe cross the intersection during the green phase but will not meet the queue during this green phase; but depending on the traffic condition and the signal timing bus might cross the intersection within the green phase, this happens only if Equation (V.12) is satisfied.

$$V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i \geq 200 \text{ m} \quad (V.12)$$

If Equation (V.12) was met, that means the bus will not face any queue in front of it such that the time to arrive the intersection can be estimated if the summation of the distances on the right-hand of Equation (V.12) equal 200m. In other words, Equation (V.12) can be rewritten as $V_{1i} * (R_i - E_i) + V_{1i(G)} * t_i = 200 \text{ m}$. So, the arrival time to can be computed using Equation (V.13), noting that there is no in-queue travel time.

$$t_i = \frac{200 - V_{1i} * (R_i - E_i)}{V_{1i(G)}} \quad (V.13)$$

In case Equation (V.12) is not met, then the traffic situation proceeds to scenario D.

D. The bus does not meet the backward shockwave propagation during the current red phase. Similar to scenario A, the bus will not cross the intersection within this green phase; however, there is a need to check the next red phase and the time that is needed to dissipate the queue that accumulates during the red phase.

In this scenario, $V_{1i} * (R_i - E_i) + V_{1i(G)}G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} \geq 200 m$, if yes, we follow the same technique in scenario A. If this is the case, then $V_{1i} * (R_i - E_i) + V_{1i(G)}G_i + V_{1,i+1} * t_{i+1} + V_{2,i+1} * t_{i+1} = 200 m$. The time that the bus needs to join the queue is computed by Equation (V.14).

$$t_{i+1} = \frac{200 - V_{1i} * (R_i - E_i) - V_{1i(G)}G_i}{V_{1,i+1} + V_{2,i+1}} \quad (V.14)$$

Where t_{i+1} is the time to arrive the queue during the next phase, $V_{1,i+1}$ is the bus speed when it leaves the bus stop during the next phase, $V_{2,i+1}$ is the V_{2i} is the backward forming shockwave in E_{i+1} , and E_{i+1} is elapsed time (i.e., the time from the start of red in the next phase).

Cruising travel time = $(R_i - E_i) + G_i + t_{i+1}$, while the travel time in queue equals to $T_{d,k} = [R_{i+1} - E_{i+1}] + \frac{N_{k,i+1}}{q_s(t)}$, and the number of cars in the queue is computed using $N_{k,i+1} = \frac{V_{2,i+1} * t_{i+1}}{S_v}$.

If the condition is not met, then scenario E becomes valid.

E. Similar to scenario B, the bus will not cross the intersection within this green phase, but it will face the queue within this green phase. This comes as $V_{1i} * (R_i - E_i) + V_{1i(G)}G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} < 200$. However, a need to check the next green phase and the time that is needed to dissipate the queue that accumulates during the last red phase can be computed using Equation (V.16).

$$d_{i+1} = \frac{V_{2,i+1} * R_{i+1}}{V_{3,i+1} - V_{2,i+1}(G)} \quad (V.15)$$

In this scenario, if $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{1,i+1(G)} * d_{i+1} + V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * d_{i+1} - V_{3,i+1} * d_{i+1} > 200 m$. Then, as scenario B, $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{1,i+1(G)} * t_{i+1} + V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * t_{i+1} - V_{3,i+1} * t_{i+1} = 200 m$. The time elapsed to join the queue is t_{i+1} which is computed by Equation (V.16).

$$t_{i+1} = \frac{200 - V_{1i} * (R_i - E_i) - V_{1i(G)} * G_i - V_{1,i+1} * R_{i+1} - V_{2,i+1} * R_{i+1}}{V_{1,i+1(G)} + V_{2,i+1(G)} - V_{3,i+1}} \quad (V.16)$$

Cruising travel time = $(R_i - E_i) + G_i + R_{i+1} + t_{i+1}$, while the travel time in queue equals to $T_{d,k} = \frac{N_{k,i+1}}{q_s(t)}$, where the number of cars in the queue is computed using $N_{k,i+1} = \frac{V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * t_{i+1} - V_{3,i+1} * t_{i+1}}{S_v}$.

F. The bus will not face the queue within this green phase. This happens when

$V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{1,i+1(G)} * d_{i+1} + V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * d_{i+1} - V_{3,i+1} * d_{i+1} < 200 m$. Then, we need to check the next green phase $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} + V_{1,i+1(G)} * G_{i+1} > 200 m$. If the later condition is satisfied, then to find the time needed to join the queue, $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} + V_{1,i+1(G)} * G_{i+1} = 200$.

$$t_{i+1} = \frac{200 - V_{1i} * (R_i - E_i) - V_{1i(G)} * G_i - V_{1,i+1} * R_{i+1} - V_{2,i+1} * R_{i+1}}{V_{1,i+1(G)}} \quad (V.17)$$

Cruising travel time = $(R_i - E_i) + G_i + R_{i+1} + t_{i+1}$, while the travel time in queue is equal to $T_{d,k} = 0$.

G. If $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * d_{i+1} + V_{2,i+1} * R_{i+1} + V_{2,i+1(G)} * d_{i+1} - V_{3,i+1} * d_{i+1} < 200 m$, the bus will not meet the queue in the current green phase.

Following scenario F, we need to check $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} + V_{1,i+1(G)} * G_{i+1} > 200 m$. If this condition is satisfied, then $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} + V_{1,i+1(G)} * t_{i+1} = 200 m$

$$t_{i+1} = \frac{200 - V_{1i} * (R_i - E_i) - V_{1i(G)} * G_i - V_{1,i+1} * R_{i+1} - V_{2,i+1} * R_{i+1}}{V_{1,i+1(G)}} \quad (V.18)$$

Cruising travel time = $(R_i - E_i) + G_i + t_{i+1}$, while the travel time in queue equals 0.

H. If $V_{1i} * (R_i - E_i) + V_{1i(G)} * G_i + V_{1,i+1} * R_{i+1} + V_{2,i+1} * R_{i+1} + V_{1,i+1(G)} * G_{i+1} < 200 m$, the bus has to wait more than one green phase to cross the intersection. This means repeating scenarios A through G until the bus is able to dissipate within a certain green phase.

5.5.4.2 Bus Departure Traffic Signal within Green Phase

This section presents the formulation of bus arrival time in the case that the bus leaves the nearside bus stop at a green phase. Mainly, it tests whether the distance traveled by bus during the remaining green time is longer than the distance between the nearside bus stop and the traffic signal, such that the bus will not encounter any queue in front of it. In other words, the bus will not be delayed due to the available queue and travel at a slower speed; rather, it will travel at its cruising speed. This case can be written mathematically by checking both conditions, $V_{1i} * (G_i - E_i) > 200$, and $E_i > d_i$. Thus, the arrival time can be computed as the travel time between the bus stop and the traffic signal $t_i = 200/V_{1i}$. However, sometimes a bus will join the back of a queue that might happen if $E_i < d_i$. If so, the bus will join the back of the queue if the following equation is satisfied: $V_{1i} * (d_i - E_i) + V_{2,i} * R_i + V_{2,i(G)} * E_i + V_{2,i(G)} * (d_i - E_i) - V_{3,i} * (d_i - E_i) > 200 \text{ m}$. Not surprisingly, the bus has two travel time components: cruising travel time and the travel time in queue.

The cruising travel time is estimated by $V_{1i} * t_i + V_{2,i} * R_i + V_{2,i(G)} * E_i + V_{2,i(G)} * t_i - V_{3,i} * t_i = 200 \text{ m}$. While the travel time in the queue is estimated by dividing the total number of cars in the queue by the saturation flow rate. Total number of cars in the queue is computed by $N = \frac{V_{2i} * R_i + V_{2i(G)} * t_i - V_{3,i} * t_i}{S_v}$.

In case that $V_{1i} * (d_i - E_i) + V_{2,i} * R_i + V_{2,i(G)} * E_i + V_{2,i(G)} * (d_i - E_i) - V_{3,i} * (d_i - E_i) < 200 \text{ m}$, the bus will not face the queue in this phase and the travel time is computed by the cruising travel time.

Note that for each green phase, we adjusted the distance to compute either t_i or t_{i+1} in the case of the presence of the residual queue, such as the 200 m becoming the 200 meters minus the residual queue.

Computing the saturation flow rate, shockwave speeds, and all the parameters in the equations for scenario 1 and 2 is done using detectors that report the flow and the detector occupancy every 3 seconds. The shockwave speeds were computed using the change in traffic state between the jam density and the corresponding traffic state according to Equation (V.19). The density of each state was computed using $k_i = \text{number of lanes} * k_{jam} * O_{i+1}$.

Where:

k_i : is the jam density at time step i.

O_{i+1} is the detector occupancy in the next time step.

$$v = \frac{\Delta q}{\Delta k} \quad (V.19)$$

5.5.5 Modeling Bus Arrival Time Using Statistical Approach

In this study, a multiple linear regression (MLR) was developed that considered the transit vehicle arrival time as the dependent variable. The candidate variables likely to influence the arrival time include the road network characteristics (e.g., number of lanes, lane widths, traffic signal timing) and traffic characteristics (like traffic density, free flow speed, and traffic mix). Within the study network, the number of lanes, lane widths, traffic signal timing, number of buses, and other geometric characteristics remain unchanged.

Developing a statistical model that is capable of predicting the transit vehicle arrival time and including the most relevant variables that affect the transit vehicle arrival time has to be done first. Model development starts with a stepwise regression where a model is built based on identifying the candidate explanatory variables. After choosing the candidate explanatory variables, choosing the best model depends on the model's ability to explain the variance in the data as well as meeting the model assumptions. The model is developed based on testing the relationship between the dependent variable (i.e., arrival time) and the independent variables (i.e., flow, density, space mean speed).

The general form of MLR is presented in Equation (V.20V.), the equation capturing the relationship between transit vehicle arrival time as the dependent variable and the candidate independent variables (including flow, density and space mean speed; namely X_1 , X_2 , and X_3 , respectively). It is given that there is a relationship between these parameters, according to the fundamental diagram; however, model evaluation follows the statistical performance index or visual checks.

$$y = \beta'X + \epsilon \quad (V.20)$$

Where y is the dependent variable, X is a vector of explanatory variable(s), β is the vector of estimable parameters or coefficients, and ϵ is the random error term.

All the candidate independent variables that the model has to include should be statistically significant at the 0.05 significance level. Moreover, the model has to meet the MLR assumption, including homoscedasticity and normally distributed error. More importantly, the independent variables should not have a multicollinearity between them.

5.6 Results

In this section the results of the bus arrival time according to the adopted methodology are presented. The results of this work are mainly divided into two parts: statistical and analytical. The analytical results cover the cases that were mentioned in the previous section, while the statistical results provide a prediction model for the bus arrival time based on probe vehicle data.

5.6.1 Statistical Modeling Results

As was mentioned earlier, INTEGRATION generates a second-by-second file that reports vehicle location, speed, delay, total delay, current link, and lane. Density, flow, and space mean speed using the traffic flow equation are presented below. The candidate models with their statistics summary are presented in Table 12.

Table 12: Candidate Model

Model	R^2	RMSE	AIC	BIC
X_1	0.64	11.44	2194.13	2205.077
X_2	0.44	14.16	2315.20	2326.15
X_3	0.01	18.91	2470.90	2481.9
X_1, X_2	0.90	6.00	1828.92	1843.519
X_1, X_3	0.64	11.42	2186.64	2201.24
X_2, X_3	0.45	14.04	2303.56	2318.15
X_1, X_2, X_3	0.90	5.89	1821.83	1840.05
Selected model	0.92	0.12	-375.46	-360.87

Comparing the candidate models, the proposed model clearly outperforms the other models based on most of the selection criteria. The proposed model shows the lowest mean residual error, with the highest coefficient of multiple determination, which reflects the model's

capability to model approximately 92% of the total error in the data compared with the other models. The model also has the best trade-off between the estimated parameters and the model goodness of fit using the BIC and AIC values. Assuming the bus arrival time is a constant and the candidate independent variables (density and flow) are also continuous values, a general linear model was developed to fit the data considering a normally distributed error of the form in Equation (V.21)

$$\sqrt{y} = B_0 + B_1 \ln(x_1) + B_2 \ln(x_2) + \varepsilon \quad (\text{V.21})$$

Here, y is the duration of time elapsed that the transit vehicle needs to arrive at the downstream intersection, B_i 's are the model coefficients, X_1 is the average traffic stream density (veh/hr/ln), X_2 is the traffic flow (veh/hr/lane), and ε is the error term. The model coefficient values along with their p values are summarized in Table 13.

Table 13: Statistical Model Calibration Results

Coefficients	Coefficient Values	SError	p -value
B_0	7.75105	0.15311	<0.0001
B_1	-1.57294	0.01376	<0.0001
B_2	1.63245	0.01878	<0.0001

According to their p -values, all the model parameters are significant. Moreover, the least expected arrival time is 49 seconds, while with the increase of the natural logarithmic of the traffic flow by 1 and without any change on the traffic density, the arrival time decreases by 2.5 seconds. Following the same concept, the arrival time is increased by 2.65 seconds with the increase of the natural logarithmic of the traffic density by 1 without any change to the traffic flow.

The model in Equation (V.21) was checked for MLR assumptions using a normal quantile plot. The corresponding normal quantile plot is shown in Figure 27. The blue dashed lines are the confidence intervals and the solid blue line is the normal distribution, while the points represent the error for each datapoint. It can be seen that the error follows the normal quantile distribution as most of the points fit on the solid blue line.

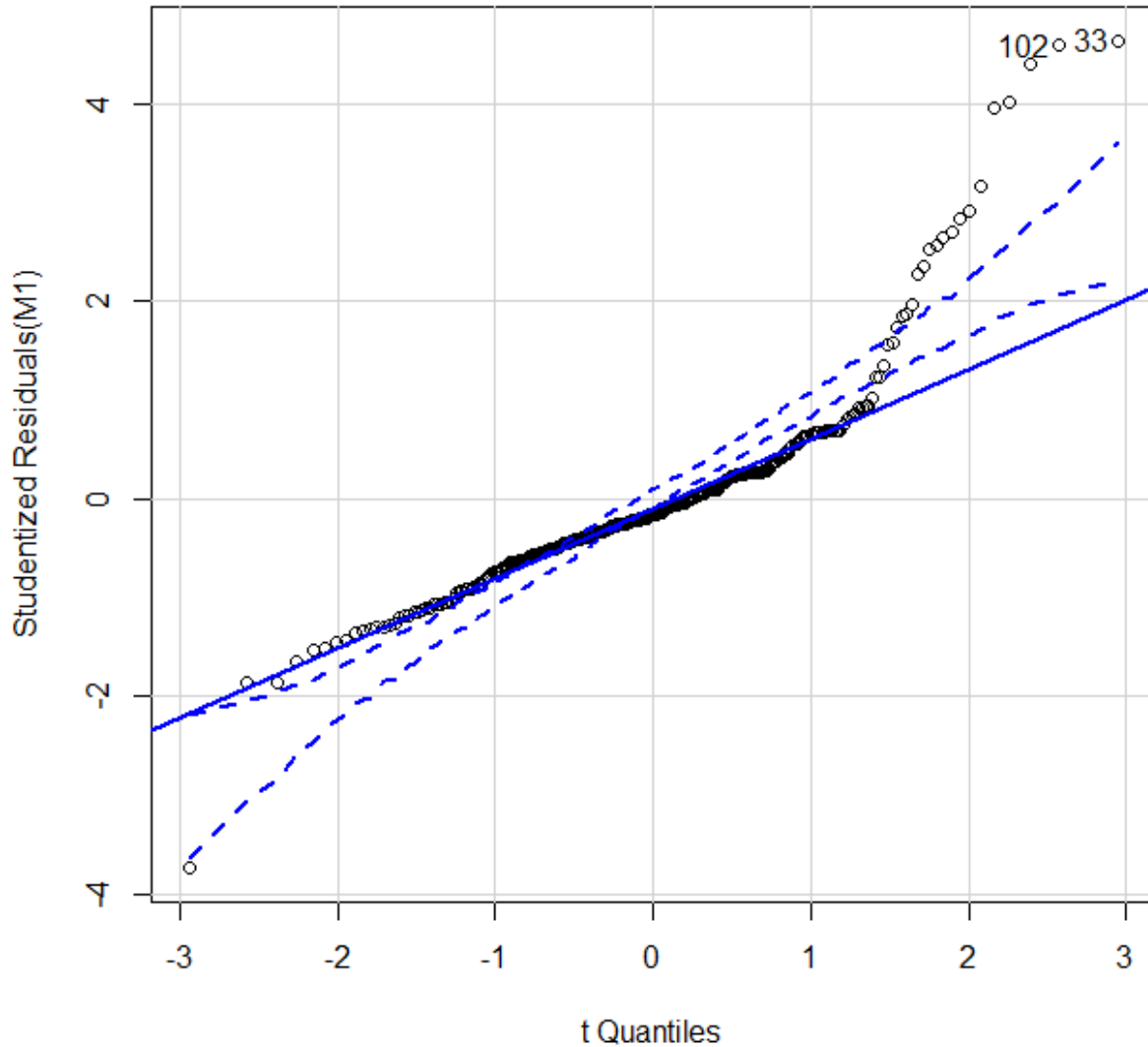


Figure 27: Normal Quantile Plot

5.6.2 Modeling Results

We assumed that there is a certain level of connectivity and thus, bus departure time, bus speed, and traffic signal can be obtained second-by-second. These data were used to validate the equations in this study.

As different scenarios were tested using different traffic volumes and free flow speed, different traffic conditions were obtained based on the prevailing traffic condition and traffic signal timings. Table 14 shows the different simulation scenario results where these parameters were reported by INTEGRATION. The computation and the validation of these results are described later on.

Table 14: Scenario 2 Simulation Results

ID	Di	V_{1i}	Q_{2i}	O_{2i}	$Q_{2i(G)}$	$O_{2i(G)}$
1-	9	31.7	720	52.6	720	52.9
2-	16	31.4	2160	169.5	360	52.2
3-	25	31	720	49.1	1440	101.7
4-	10	30.4	1440	88.1	2160	135.4
5-	30	29.3	720	68.3	720	43.9

Using the data in Table 15, density and shockwave speeds were computed using both Equation (V.22) and (V.23) and reported in Table 5-5.

$$K_{2i} = 2 * 150 * \frac{O_{2i}}{1000} \quad (V.22)$$

$$v_i = \frac{q_i - 0}{K_i - K_{jam}} \quad (V.23)$$

Table 15: Summary of Scenario 2 Parameter Calculation

ID	Density			Shockwave speeds		
	K_{2i}	$K_{2i(G)}$	K_{3i}	V_{2i}	$V_{2i(G)}$	V_{3i}
1-	15.78	15.87	75	5.36	5.36	35
2-	50.9	15.7	75	21.8	2.68	35
3-	14.7	30.5	75	5.32	12.1	35
4-	26.4	40.6	75	11.7	19.7	35
5-	20.5	13.2	75	5.56	5.26	35

Check scenario 1 when the bus departs at green time:

$d_i=9$ seconds, $E_i = 5$ seconds, $G_i = 26$ seconds, $R_i = 34$ seconds. The actual travel time is 20 seconds.

The following formula must be met to be satisfied:

$$V_{1i} * (G_i - E_i) > 200, \text{ and } E_i > d_i.$$

$$d_i = \frac{V_{2i} * R_i}{V_{3i} - V_{2i(G)}} = \frac{0.27 * 5.36 * 34}{0.27 * (35 - 5.36)} = 17.37 \text{ seconds.}$$

Thus, we need to check if $V_{1i} * (d_i - E_i) + V_{2,i} * R_i + V_{2,i(G)} * E_i + V_{2,i(G)} * (d_i - E_i) - V_{3,i} * (d_i - E_i) > 200 \text{ m}$.

$$0.27 * 31.7 * (17.36 - 5) + 0.27 * 5.36 * 34 + 0.27 * 5.36 * 5 + 0.27 * 5.36 * (17.36 - 5) - 0.27 * 35 * (17.36 - 5) = 63.35 \text{ m}$$

Since $63.35 \text{ m} < 200 \text{ m}$, the bus will not face the queue in this phase and the travel time will be computed by the cruising travel time using the following equation.

$$t_i = \frac{200}{0.27 * 31.7} = 23.36$$

The error for the arrival time is computed by using root mean squared error (RMSE).

$$RMSE = \frac{\text{Actual value} - \text{computed value}}{\text{actual value}} * 100\%$$

$$RMSE = \frac{|23 - 20|}{23} * 100\% = 12.83\%$$

Check scenario 3 when the bus departs at green time:

$d_i=16$ seconds, $E_i = 12$ seconds, $G_i = 26$ seconds, $R_i = 34$ seconds. The Actual travel time is 52 seconds.

The following formula must be met to be satisfied:

$$V_{1i} * (G_i - E_i) > 200, \text{ and } E_i > d_i.$$

$$d_i = \frac{V_{2i} * R_i}{V_{3i} - V_{2i(G)}} = \frac{0.27 * 21.8 * 34}{0.27 * (35 - 2.68)} = 22.9 \text{ seconds.}$$

Thus, we need to check if $V_{1i} * (d_i - E_i) + V_{2,i} * R_i + V_{2,i(G)} * E_i + V_{2,i(G)} * (d_i - E_i) - V_{3,i} * (d_i - E_i) > 200 \text{ m}$.

$$0.27 * 31.7 * (22.9 - 12) + 0.27 * 21.8 * 34 + 0.27 * 2.68 * 12 + 0.27 * 2.68 * (22.9 - 12) - 0.27 * 35 * (22.9 - 12) = 201.27 \text{ m}$$

$201.27 \text{ m} > 200 \text{ m}$,

As $E_i < d_i$, the cruising travel time is estimated by $t_i = \frac{(200 - V_{2i(G)} * E_i - V_{2i} * R_i)}{V_{1i(G)} + V_{2i(G)} - V_{3i}} =$
 $\frac{(200 - 0.27 * 2.68 * 12 - 0.27 * 21.8 * 34)}{31.3 + 2.68 - 35} = 33.27 \text{ seconds}$

$\text{total travel time} = \text{time to dissipate queue} + t_i = 33.27 + 22.9 = 56.18 \text{ seconds}$

The error for the arrival time is computed by using root mean squared error (RMSE).

$$RMSE = \frac{\text{Actual value} - \text{computed value}}{\text{actual value}} * 100\%$$

$$RMSE = \frac{|56.18 - 52|}{52} * 100\% = 8.04\%$$

Following the same concept for all the scenarios listed in Table 3 and 4, the RMSE was computed for all of them and was found to range between 2.6%-13.27%.

Check scenario 5 when the bus departs the bus stop at red timing:

$d_i = 30$ seconds, $E_i = 0$ seconds, $G_i = 26$ seconds, $R_i = 34$ seconds. The Actual travel time is 36 seconds.

The following formula must be met to be satisfied:

$$V_{2i} * E_i + V_{2i} * (R_i - E_i) + V_{1i}(R_i - E_i) \geq 200 \text{ m}$$

$$0.27 * 5.56 * 0 + 0.27 * 5.56 * 34 + 0.27 * 29.3 * 34 = 320 \text{ m}$$

320 m > 200 m; this is scenario A.

Computing the time to join the queue:

$$t_i = \frac{(200 - V_{2i} * E_i)}{V_{2i} + V_{1i}}$$

$$t_i = \frac{(200 - 0.27 * 5.56 * 0)}{0.27 * (5.56 + 29.3)} = 21.24 \text{ second}$$

$$\text{bus waiting time to next green} = [R_i - (E_i + t_i)] = 34 - 0 - 21.24 = 12.76 \text{ seconds}$$

$$\text{queue dissipating time} = \frac{N}{q_s(t)}$$

$$\text{queue dissipating time} = \frac{V_{2i} * E_i + V_{2i} * t_i}{S_v * q_s(t)}$$

$$\text{queue dissipating time} = \frac{0.27 * 5.56 * 0 + 0.27 * 5.56 * 21.24}{2.2 * 1440} = 0.01 \text{ sec}$$

Queue travel time = 12.76+0.01 = 12.77 seconds

Total travel time = 12.77+21.24=34.01 seconds.

$$RMSE = \frac{\text{Actual value} - \text{computed value}}{\text{actual value}} * 100\%$$

$$RMSE = \frac{|36 - 34.01|}{36} * 100\% = 5.56\%$$

5.7 Discussion

To improve the multimodal transportation system, improving the reliability of transit system travel time can be the most important factor. In fact, the accurate estimation of transit vehicle arrival time at traffic signals is the key point for improving the overall system travel time and reducing the total system delay. Two main variables affect the bus arrival time at a traffic signal in addition to its speed: the traffic flow in the adjacent lane and the traffic density. With this in mind, it is necessarily to quantify the impact of these variables on estimating the bus arrival time at traffic signals.

Subsequently, two methods were used to estimate the transit vehicle arrival time at traffic signals, namely analytical and prediction.

Two specific Measure of Effectiveness (MOEs) that are very sensitive to the accuracy of predicted accelerations, and quite important from an environmental perspective, are fuel consumption and emissions estimation. With this in mind, it seemed necessary to examine the behavior of the maximum acceleration distribution of the bulk dataset given its major impact on any fuel consumption or emissions calculation.

In other words, the results of this study are mainly divided into two parts, a formula that estimates bus arrival time and a prediction model that predicts the bus arrival time at the traffic signal. Overall, both models give significantly satisfactory results for predicting bus

arrival time at a traffic signal. However, the analytical model provides better insights for quantifying the impact of different significant factors on estimating bus arrival time at traffic signal. These insights, as well as the prediction model, can benefit the estimation of system travel time and delay.

5.8 Conclusions and Recommendations

This chapter presented a valid methodology for predicting transit vehicle arrival times at traffic signals using data from multiple sources including the traffic signal timings, transit vehicle data, and probe vehicle data. The findings of this study can be divided into two parts, as related to the modeling and statistical approaches. The findings provide valuable insights into the influences of different factors on bus arrival times. The main contributing factors such as traffic flow, traffic density, traffic signal timing, and bus speed were found to influence the speed of the transit vehicle's arrival time at traffic signals. In particular, these factors were used in modeling the transit vehicle arrival time based on a shockwave analysis. Speaking of the statistical model, as the flow increases, the expected arrival time increases; meanwhile, the negative impact of traffic density is explained by the relationship between the traffic flow and the density. These findings therefore have important implications for urban traffic operations and management.

Although the modeling methodology was developed to address the research question by using generated data, the model framework is general enough for application in other cases. The presented methodology is also useful for combining multiple sources of traffic data. Of course, the study had several limitations. For example, the influence of different buses was not captured in this study, and the queue jumper transit vehicle lane effect was not considered in this study.

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AUTHOR CONTRIBUTION STATEMENT

The authors confirm contribution to the paper as follows: study conception and design: Alhadidi, Rakha; data reduction: Alhadidi; analysis and interpretation of results: Alhadidi, Rakha; draft manuscript preparation: Alhadidi, Rakha. All authors reviewed the results and approved the final version of the manuscript.

The main contribution of this research effort relates to the modeling of the various components of transit vehicle travel times. This model explicitly captures the impact of passengers, drivers and vehicle characteristics on transit vehicle travel times thus providing better models for use in various transit applications, including TSP. To achieve its contribution, the study improved the current state-of-the-art and state-of-practice transit vehicle boarding/alighting (BA) models by explicitly accounting for the different factors that impact BA times while ensuring a relatively generalized formulation. Next the model was extended to capture the bus driver and vehicle impacts on the transit vehicle delay in the vicinity of bus stops, using a vehicle kinematics model with maximum speed and acceleration constraints to model the acceleration/deceleration delay. The validation of the model was done using field data that cover different driving conditions. Results of this work found that the proposed formulation successfully integrated the human and vehicle characteristics component in the model and that the new formulation improves the estimation of the total delay that transit vehicles experience near bus stops. Finally, the model was extended to estimate the time required to merge into the adjacent lane and the time required to traverse a queue upstream of a traffic signal. The final part of this study models the bus arrival time at traffic signal using shockwave and prediction model in a connected environment. In summary, the proposed model overcomes the current state-of-the-art models in the following ways: 1) it accounts for the vehicle capacity and the number of on-board passengers on bus BA times, 2) it captures the stochasticity in the bus stop demand and the associated BA times, 3) it captures the impact of the traffic in modeling the delay at a bus stop , 4) it incorporates the driver and vehicle impact by modeling the acceleration and deceleration time, and 5) it uses shockwave analysis to estimate bus arrival times through the use of emerging technology data. Through statistical modeling and evaluation using field and simulated data, the model overcomes the current state-of practice and state-of art transit vehicle arrival time models.

While the research presented in this dissertation is comprehensive, and met its objective, further research is needed in the following areas:

- In order to further generalize the BA time prediction model, further data from different agencies with different bus capacities and having different records including the impact of the cyclists, wheel-chaired passengers and kids on strollers would be needed. Furthermore, data from buses with different door

configurations would be ideal to expand the coverage of data. That would result in a more generalized model that accounts the impact of different boarding passenger types and vehicle capacities as determinants of estimating the BA times.

- This study disregards the impact of weather and pavement type on estimating the acceleration/deceleration time. A similar methodology could be adopted in order to investigate the impact of different pavement and weather conditions on transit vehicle acceleration/deceleration behavior. The outcome will be of significant importance for an accurate estimation of acceleration/deceleration times.
- This study disregards the impact of the variation in the traffic mix on transit vehicle arrival times. Further work is needed in this regard.

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