5.6 Computational Experience

In this section, we present some computational experience on employing the algorithm of the foregoing section to solve problem EDLAP using twelve test problems. These test problems are randomly generated for this purpose using the problem generation scheme of Sherali and Tuncbilek (1990). A summary of the sizes of these test instances and their optimal/best found solutions is provided in Table 9.

Table 9. Test Problem Specification for Problem EDLAP.

TP	(n, m)	Known Optimum/ Best Value
1	(2, 2)	0.0
2	(2, 4)	22.959
3	(2, 4)	42.8814
4	(3, 5)	2.039
5	(3, 5)	7.8412
6	(3,9)	9.6773
7	(3,9)	39.6865
8	(4,8)	37.399
9	(5,15)	515.56
10	(5,20)	421.16491
11	(5,20)	49.637
12	(5,30)	819.48

We test two solution strategies, governed mainly by the lower bounding scheme. The first is the algorithm of Section 5.5 using the projected location-space lower bound ν_{LB1} defined in (5.48) of Proposition 1, and the partitioning Strategy #1 discussed in Section 5.5.3. Let us refer to this combination as *the Projected Location-space Bounding (PLSB)* approach. Table 10 provides computational results for this method using the test case of

Table 9. The second algorithmic strategy enhances this approach by employing the RLT based lower bound n_{LB2} as described in Section 5.5.3, and composing this with n_{LB1} to derive a lower bound LB = maximum{n_{LB1}, n_{LB2} } as recommended in Step 3 of the proposed branch-and-bound algorithm. As a partitioning strategy, after some preliminary investigation, we selected method #3 of Strategy #2 described in Section 5.5.3. Let us refer to this combination as the *RLT-bounding* (RLTB) approach. Table 11 presents computational results for this method using the test cases of Table 9. In these runs, we consider the following scaling of the test problems. Let

$x_i = sx$¢, $y_i = sy$¢ for some $s > 0$. Then, EDLAP becomes

$$\text{Minimize} = s\sum_i\sum_j c_{ij}\, w_{ij} \sqrt{(x_i' - \frac{a_j}{s})^2 + (y_i' - \frac{b_j}{s})^2}$$

which is an equivalent EDLAP having existing facilities at locations $(a_j / s\ ,\ b_j / s)\ \forall\ j$. If the original (a_j,b_j) coordinates lie in the box $0 \le x \le \bar{a}$, $0 \le y \le \bar{b}$, we select

$s = \sqrt{\bar{a}^2 + \bar{b}^2}$ and scale the (a_j, b_j) coordinates by replacing (a_j, b_j) by $(1/s)*(a_j,b_j)\ \forall\ j$.

After solving the scaled test problems using our proposed algorithm, we can obtain the optimal value of the objective function of the original EDLAP problem by multiplying the scale factor with the optimal value obtained for the objective function of the scaled problem. For the fathoming criterion, we set e′.=1 %. For the bounding step, we halt the execution of the algorithm prematurely if the cumulative execution time exceeded 600 cpu seconds, and for the cycle prevention method

Table 10. Computational Results for the Projected Location-Space Bounding Approach (PLSB).

TP	Initial Primal Incumbent	LB at Node 0	Best Solution Found	# of Nodes Enumerated	cpu secs
1	0	0	0	1	.14
2	22.959	10.829	22.959	5	.5
3	42.897	0	42.8814	11	1.01
4	2.039	0	2.039	73	5.07
5	48.6505	0	7.8412	142	10.73
6	18.02	0	9.6773	992	81.05
7	42.6169	0	39.6865	904	82.41
8	50.38	0	37.399	7660	194.74
9	515.560	0	515.560	1621	600 +
10	421.1649	0	421.1649	3877	600 +
11	49.637	0	49.637	13584	600+
12	819.48	0	819.48	9559	600 +

Table 11. Computational Results for the RLT Bounding Approach (RLTB).

TP	Initial Primal Incumbent	LB at Node 0	Best Solution Found	# of Nodes Enumerated	cpu sec.
1	0	0	0	1	.67
2	22.959	9.8	22.959	5	56.57
3	42.890	0	42.895	10	37.535
4	2.039	2.04	2.039	1	7.3
5	48.8739	0	7.8412	24	349
6	18.015	.995	18.015	20	600+

7	42.610	11.995	42.610	16	600+
8	50.38	15.97	50.380	27	600+
9	515.560	0	515.560	14	600+
10	421.164	0	421.164	20	600+
11	49.637	4.27	49.637	6	600+
12	819.48	85.76	819.48	4	600+

Examining the results in Tables 10 and 11, we see that for relatively small-moderate sized problems, the approach PLSB provides a competitive solution strategy. In particular, when testing *PLSB* on two test problems (of size $m = n = 5$) that are available in the literature (Selim, 1979), our algorithm has discovered significantly improved solution over the best solutions reported by Selim. These results are provided in Table 12.

Table 12. Computational Results for the Projected Location-Space Bounding Approach (PLSB) on the Test Problems from the Literature.

(n, m)	Initial Primal Incumbent	# of Nodes Enumerated	Best Solution Found	Known Optimal Solution	cpu sec.
(5, 5)	20.785	1839	2.0138	2.1	141.741
(5, 5)	19.138	2672	3.0644	7.8	242.546

We also notice from Tables 10 and 11 that as problem size increases, the performance of PLSB deteriorates, and the methodology clearly calls for stronger lower bounding algorithm strategy. For larger problem sizes, therefore, it becomes imperative to select RLTB over PLSB. However, the RLT-based lower bounding problem needs to be solved more efficiently (e.g. using a commercial package such as CPLEX on the linear variant of this problem) in order to make its use more competitive. Moreover, for larger sized problems than those experimented with here, for which Algorithm RLTB might not succeed in determining optimal solutions, we could use one of two strategies. First we could run RLTB with a generously high fathoming parameter in order to loosely explore the feasible region for good quality solutions that are guaranteed to lie within a tolerance of $\epsilon' . 100$ % of optimality. Second, we could exploit the fact that the RLT representation attempts to generate a tight convex outer approximation to the problem that drives an optimum to this relaxation toward the vicinity of a global optimum for EDLAP in order to generate good quality heuristic solutions based on either the initial relaxation itself, or based on a few enumerated nodes. Further computational testing and algorithmic refinements for deriving exact or heuristic solutions are relegated to future study.