YIELD-LINE ANALYSIS AND EXPERIMENTAL STUDY OF REINFORCED CONCRETE SLABS CONTAINING OPENINGS

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Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Civil Engineering

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December, 1986

Blacksburg, Virginia

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(ABSTRACT)

Four rectangular, isotropically reinforced concrete slabs were constructed and loaded until collapse. All slabs were fixed on three edges with the fourth edge free. Three slabs contained openings at various locations while the fourth remained solid. The magnitudes of deflections were measured during loading and the final yield pattern and ultimate load were compared to those predicted by simple and advanced yield-line theory.

An analytical computer program was developed and is presented for quick evaluation of the ultimate load and collapse mode of many types of uniformly loaded slabs by simple yield-line theory. Short specialized programs were also formulated to analyze the experimental slabs, considering the presence of simple corner levers and edge loads around the openings. This resulted in more accurate theoretical predictions and produced estimates of the percent difference between simple and advanced theory predictions. Analysis of the results showed excellent agreement between the advanced theory predictions and the experimental results.

Acknowledgements

The author would like to give his utmost thanks to the chairman of his advisory committee Dr. Richard M. Barker for his guidance, assistance, and suggestions which made this work possible.

Special thanks are extended towards Dr. Richard E. Weyers for providing space and equipment for the experimental portion as well as valuable advice; Glen Thomas and his staff for their technical support; Brian Roberts, Jim Leeuwrik, and Bernard Deneke for their help during the pouring and finishing of the slabs; and Al Sehn for his time and help with the data aquisition system.

The funding for the experimental portion of this thesis was provided by the Department of Civil Engineering for which the author extends his appreciation.

Sincere gratitude is especially expressed to the author's parents, Rita and Gregory, for their love and support.

Table of Contents

List of Illustrations

List of Tables

Chapter I

INTRODUCTION

Reinforced concrete slabs are one of the worlds' most widely used structural systems. In the past, elastic methods have generally been used for their design. These methods, though giving safe designs, have their drawbacks. It is of growing concern in today's balance of strength and economy that elastic methods cannot give an accurate estimate of the factor of safety of a particular structure. Limitations on when elastic methods may be used can also become troublesome. It is because of concerns like these that yield-line theory is gaining respect from designers as an alternative method.

Yield-line theory is a helpful tool for analyzing the ultimate strength of existing slabs, and may be used for design as well. It provides a reasonable estimate of the ultimate load so one can determine a slab's factor of safety against collapse. This is opposed to designing by methods which provide a safe design, yet no indication of how safe. Among the great advantages of the yield-line theory is the wide variety of problems to which the theory may be applied. Elastic methods are not only complicated and often require the use of a computer, but are also restricted in the type of slabs for which they can be used. The yield-line method has the versatility to analyze slabs of any shape, regardless of edge and loading conditions, usually with little difficulty. Moreover, it is accepted that the theory can be applied just as easily to slabs which contain openings.

Denmark and Sweden are among the leading countries in this field and have allowed design by the yield-line method for many years. British codes later adopted it in 1957 with restrictions. It is required that the support to span moment ratio be similiar to that needed by elastic distribution, suggesting that a value between 1.0 and 1.5 is used. (Park & Gamble 1980) Presently, the ACI code states that " A slab system may be designed by any procedure satisfying conditions of equilibrium and geometric compatibility if shown that the design strength at every section is at least equal to the required strength considering Sections 9.2 and 9.3, and that all serviceability conditions, including specified limits on deflections, are met." (ACI 1983) The only statement mentioning the redistribution of moments allows an increase of a maximum of ten percent as per Section 13.6.7 . It will be shown that the yield-line theory does provide adequate strength although the crack widths and deflections must be checked.

In this work, the history and basis of yield-line theory will be presented with special attention directed to the implications of openings within the slab. An algorithm is developed to evaluate ultimate loads and failure modes of rectangular, orthotropically reinforced concrete slabs containing openings. To test the theory, four slabs were constructed and tested to failure. The results of these tests were analyzed and compared to predictions by simple and advanced yield-line theory.

Chapter II

YIELD-LINE THEORY

2.1 Basic Theory

Yield-line theory originated in the 1920's by Ingerslev in his works on limit analysis of slabs. This theory was significantly advanced by Knud W. Johansen in his thesis which was presented in Danish in 1943. Johansen, who subsquently became recognized as the pioneer of yield-line theory, pointed out errors in the original theory and sought to correct them by the invention of nodal forces, which will be mentioned later. He not only significantly advanced the theory and layed out its rules but also helped it gain acceptance by working and publishing numerous examples with governing equations. He also presented in-depth analysis of earlier extensive tests. (Jones & Wood 1967) His theory was first presented in English in 1953 in a summary by Hognestad. Since then it been followed and advanced upon by Jones, Wood, Morely and others. Though many points of confusion have been cleared up and many aspects advanced, the major concepts have stayed intact. (Hughes 1980)

The premise of the theory is that failure of a slab occurs when a mechanism consisting of a series of idealized yield-lines is reached. Due to the great redundancy of a slab, moments can be redistributed after first yielding, allowing yield-lines to grow. At each point along the yield-line, the reinforcement is assumed to be fully yielded and a discontinuity in rotation developed. Collapse occurs once the yield-lines form a valid mechanism and instablity occurs. The slab is assumed to undergo rigid-plastic behavior where the elastic deformations are much less than the plastic deformations and are therefore ignored. An idealized moment-curvature relation is assumed as shown in Figure 1. (Jones & Wood 1967) The rotations are assumed to be concentrated along the yield-lines, and regions bounded by these axes of rotation are considered to remain planar. To better visualize this, it can be seen that the analysis of a one-way slab by yield-line theory is much like the plastic analysis of a beam.

Yield-line theory provides an upper-bound solution; that is, the solution will be either correct or unsafe. The critical solution will be one which maximizes the ratio of the ultimate moment capacity to ultimate load. For use in analysis of existing slabs, the ultimate moment is assumed to be known, leaving the ultimate load to be determined. The critical pattern will now be the one for which the load capacity is minimized. The correctness of the computed ultimate load will depend greatly upon the yield pattern chosen. An illogical choice will result in a much higher predicted load capacity, and consequently less safe, than that of an educated choice. It will be shown later that regardless of the pattern chosen, the exact solution can rarely be predicted.

The most fundamental assumption of yield-line theory is that the strength of the slab is determined strictly by its flexural strength. This is an important assumption as it would be impossible to analyze critical combinations of bending, shear, torsion, and axial load together at each point on the slab. The combinations of these effects have been more readily observed during the testing of beams. Interaction diagrams have been plotted showing the effect of ultimate flexural resistance when the beams are also subjected to various magnitudes of

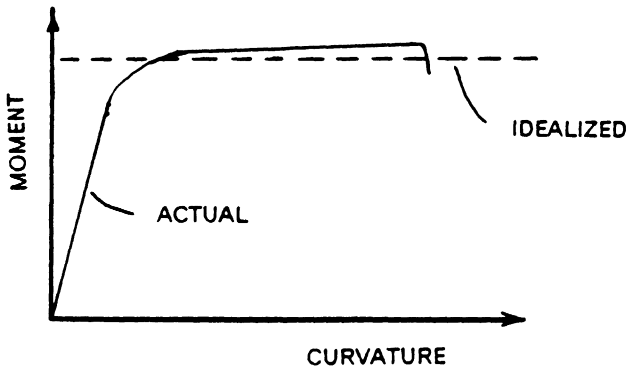


Figure 1. Idealized Moment-Curvature Relation

shear, torsion, etc. (Jones & Wood 1967). From these tests and diagrams, the assumption has generally been considered conservative, and other factors can usually be neglected.

The yield condition for which to compute the normal moments is usually considered to be as presented in Johansen's "stepped yield criterion" (Jones & Wood 1967). This criterion is repeated below and is aided by Figure 2.

1. The normal and twisting moments along a yield-line can be found by summing the effects of each individual band of reinforcement.

2. Each band of reinforcement at a yield-line may be considered to be broken into small steps, parallel and perpendicular to the reinforcement.

3. All reinforcement crossing a yield-line is assumed to yield.

4. All reinforcement stays straight as it yields, no "kinking" or change in horizontal direction occurs at the yield-line.

5. For each band of reinforcement, on steps perpendicular to the reinforcement there is only a normal moment per unit length, and on steps parallel to the reinforcement there is neither normal nor twisting moment.

6. The values of the normal and twisting moments on the yield-line are such that they are equal to that of the components of the normal moment along the steps.

To assess the value of this normal moment, the idea of a moment key line is introduced. A moment key line for a set of reinforcement is a line normal to the actual reinforcing bars. The ultimate bending strength about sections parallel to this line is taken as "m"; that is, the normal moment if the yield-line had been perpendicular to the reinforcement. When the

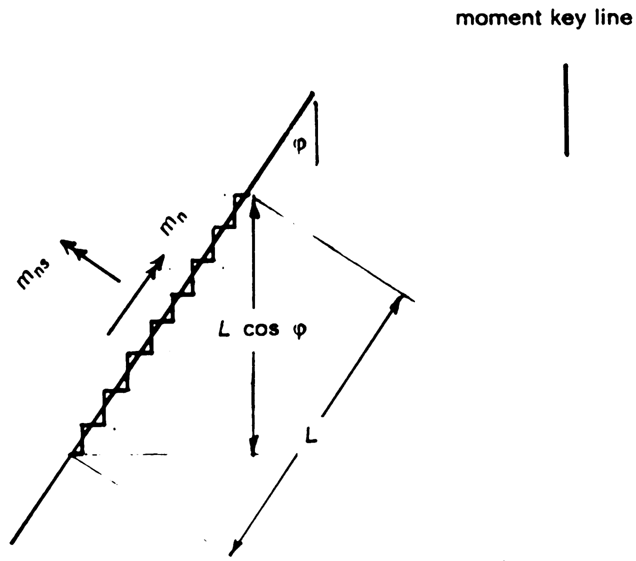


Figure 2. Stepped yield criterion

yield-line occurs at some angle, phi, measured clockwise from the moment key line, the normal moment can be written as :

$$m_n = m \cos^2\varphi$$

and the twisting moment as :

$$m_{ns} = m \sin\varphi \cos\varphi$$

Likewise, if there is reinforcement in two perpendicular directions with strengths m_1 *and* m_2, the following would result,

$$m_n = m_1 \cos^2\varphi + m_2 \cos^2\{\frac{\pi}{2} + \varphi\}$$

$$= m_1 \cos^2\varphi + m_2 \sin^2\varphi$$

$$m_{ns} = m_1 \sin\varphi \cos\varphi + m_2\{\frac{\pi}{2} + \varphi\} \cos\{\frac{\pi}{2} + \varphi\}$$

$$= (m_1 - m_2) \sin\varphi \cos\varphi$$

It follows that for a slab which is isotropically reinforced, the moment resistances, $m_1 = m_2 = m_n$ *and* $m_{ns} = 0$ regardless of the orientation of the yield-line.

Obviously, the determination of these moments is more difficult if the slab is orthotropically reinforced. To simplify these cases, Johansen developed affinity theorems which allowed these orthotropic slabs to be treated with greater ease. These theorems allowed orthotropically reinforced slabs to be analyzed as isotropic by transforming the shape and loading of the original slab by special rules. The original theorems were simplified as well as expanded upon by Jones and Wood (1967) to be applicable to more types of slabs. The rules and limitations will not be discussed here, for further reference see Jones and Wood (1967).

Once the tools for calculating the moment resistances are understood, the yield-line method may be applied. In any approach, a yield pattern must first be postulated for which certain simplified rules must be followed. (Jones & Wood 1967)

1. Yield-lines are generally straight and end at the slab boundaries, though they may be curved for concentrated loads.

2. Yield-lines between two rigid regions must pass through the intersection of their axes of rotation, assuring the compatibility of deformations.

3. The axes of rotation of the rigid regions usually lie along the lines of support and pass over columns.

From these simplified rules, it is fairly simple to postulate a reasonable failure pattern for a given slab and so the collapse load for that pattern can be calculated. Methods of calculating the collapse load and adjusting its pattern to its critical position will be presented shortly. Examples of valid mechanisms along with slab notation are shown in Figure 3.

It must be pointed out that this is only simple theory. After the critical pattern is found according to these rules, a more complex pattern will still exist, yielding a smaller value for the ultimate load. This complex pattern occurs with the phenomenon of corner levers. As the slab is loaded, the edges in the corners will have the tendency to lift up off of the support. Strong torsional moments occur in the corners and the yield-line will tend to fork away from the corner in either direction instead of extending into the corner. If these edges are held down, A negative yield-line will form at a diagonal to the two edges between the ends of the two positive yield-lines, leaving the region in the corner undeformed. This results in a pattern similiar to that shown in Figure 4. If the negative yield-lines at the corner levers are assumed to take on a circular shape, the pattern becomes more critical; and when the negative yield-lines form in a parabolic or elliptical shape the ultimate load may become lower yet (Jones

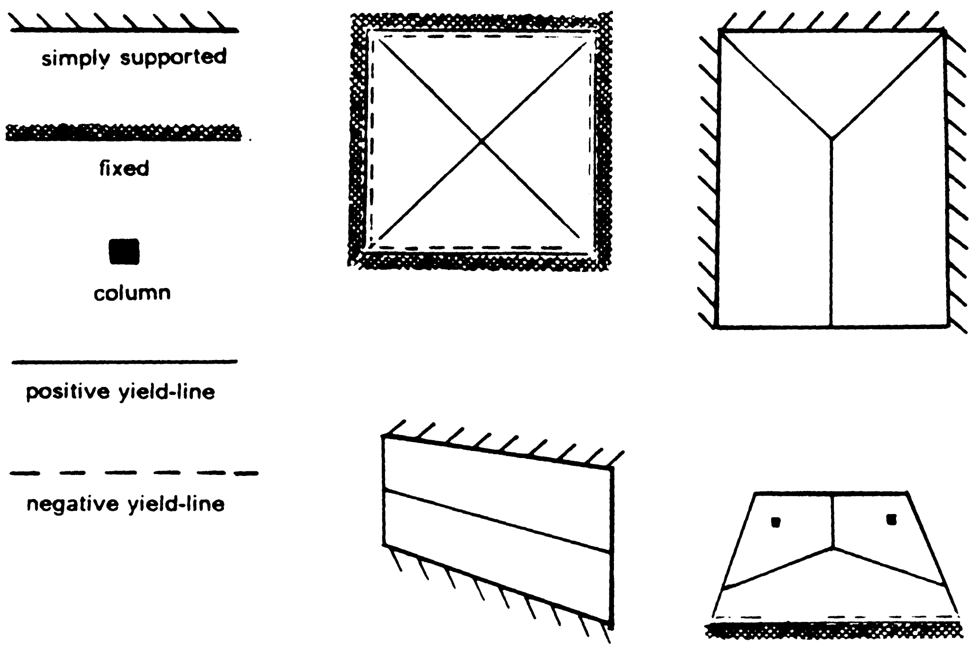


Figure 3. Slab notation and Example mechanisms

& Wood 1967). The worst actual layout is virtually impossible to find, and the analysis of any shape other than a simple corner lever can be extremely difficult. When considering the assumptions made and the small possibility of determining the critical pattern, it is shown that the exact solution may only be obtained for a few cases such as one-way slabs.

Fortunately, it is not necessary to rigorously calculate the worst pattern, as the use of simple corner levers generally predicts loads only up to two percent higher than that of the critical case. Simple yield-line theory generally predicts up to ten percent higher for rectangular slabs. These percentages may vary greatly depending on the shape and loading of the slab (Demsky & Hatcher 1969). Mills (1970) has found that for shapes such as triangular slabs, simple theory can predict up to 20 percent higher than that of the complex fan shapes of the critical pattern. These studies will be discussed later. It is suggested that for most cases it is most practical to use either simple theory or include simple corner levers and make approximate adjustments.

To avoid violating fundamental assumptions, several points must be considered during analysis and design. In order to achieve the large rotations necessary to arrive at the collapse mechanism, the slab must have adequate ductility. This is not usually a problem because slabs inherently have a generous thickness to limit deflections. It is considered that a steel percentage of one percent or lower must be maintained for concrete slabs. In addition, because the slab is assumed to fail by flexure, shear must be carefully checked when large concentrated loads are present. Small areas which are heavily loaded may fail by punching shear. If this is not considered, the theory's prediction may be naturally incorrect. Studies on slabs carrying large concentrated load have been conducted by Nylander. (Jones & Wood 1967)

There are a few points that may have an effect on strength which the yield-line theory ignores. The assumed yield criterion states that kinking of the reinforcement steel does not occur. However, when a yield-line forms at an angle to the direction of the reinforcement, the

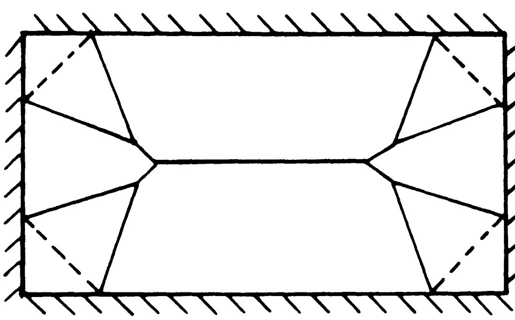


Figure 4. Corner lever pattern

reinforcement will have the tendency to kink towards a position normal to the yield-line giving the section additional moment resistance. Theoretically up to 40% increases may occur, however the bearing of the concrete will not allow this. A study by Kwieincki (1965) has shown that increases up to 16 % may occur, yet analysis of more in-depth testing by others have come to conclude that the effect is insignificant. Mills (1975) later reviewed these studies and determined that the differences may have been caused by an uncertainity of the number of bars crossing the yield-line. He directed special tests in which there would be no ambiguity. His results concluded that in the worst instance, a maximum increase in strength of nine percent may occur due to kinking. In most instances, however, the effect was considered small enough to be neglected.

Yield-line theory also neglects the additional strength which may be developed by membrane action as the deflections grow large. This is one area which tends to make the theory swing from unsafe to conservative. Researchers have been trying to incorporate this effect into yield-line theory, with work being done by Wood, Jones, Morely, Park & Gamble, Eyre & Kemp, and Breastrup. Unfortunately, this work is dependent on knowing the degree of horizontal restraint present, making it difficult to apply to actual structures. The extent of the possible effects of membrane action may be best demonstrated in tests made by Ockleston in 1955 where the collapse load double of that predicted was attributed to membrane action (Cope & Clark 1984). Park and Gamble (1980) cite numerous studies of slabs with stiff horizontal restraints showing that significant increases of strength may occur due to membrane action. Although the magnitude of the effects of membrane action is not presently easily estimated, the additional conservatism it lends has helped build confidence in the theory.

Among other alternative theories being used, the Hillerborg strip method of design is quite popular. This is a lower-bound theory which can also be applied to slabs of strange shapes or slabs containing openings. In this method, the load is separated into portions which is to be carried by each direction of reinforcement. Each load is evaluated as being carried by a series of strips for which the amount of reinforcement is chosen. This procedure has been used to attempt to determine the most economical reinforcement pattern. Although the method is useful for design, it cannot be used to analyze existing slabs. In comparison, the yield-line theory does not determine how to best arrange the reinforcement; however, it can be used for both design and analysis. The yield-line theory treats the reinforcement pattern as of little importance, as the moments are assumed to be redistributed until the slab has failed. Muspratt (1970) has suggested that even when using Monte Carlo techniques of generating the distribution of a given quantity of reinforcement, there is little change of the ultimate strength of the slab. It should be repeated that great ductility would be needed in order to fully redistribute moments in a slab with a radical reinforcement distribution.

2.2 Methods

Johansen presented two methods of evaluating slabs, the equilibrium method and the work method. The equilibrium method, which is only mentioned here briefly, makes use of general equilibrium equations with the addition of his so-called nodal force theory. In this theory, unknown forces along the yield-lines can be represented as forces at points on each panel where the yield lines meet. These forces are usually zero and general equilibrium methods can be used. However, there are situations where these forces are not zero and complicated rules must be followed to determine the magnitude of these. This had been a source of confusion for a long while and was not worked out until Jones, Wood and others helped clarify it. For additional information refer to Jones & Wood (1967). The work method uses the principle of virtual work, and excepting cases which involve a great deal of complicated algebra, is usually much simpler to apply. The work method is discussed below and is used hereafter.

In the work method, a failure mechanism is first postulated. By introducing a virtual displacement to the system, the ultimate load can be solved by equating the internal and external work. Since the regions are considered planar, the assumption of the magnitude of deflection at one point will enable one to easily find the relative deflection at any point on the slab. The work done by the external forces acting on the slab due to the virtual displacement can be shown as :

$$W_e = \iint w\delta \, dxdy$$

$$\text{where} \quad \begin{aligned} w &= \text{external loads} \\ \delta &= \text{virtual displacement} \end{aligned}$$

The internal work results from the normal moments along each of the yield-lines. This can be shown as :

$$W_i = \sum m_n l \theta$$

$$\text{where} \quad \begin{aligned} m_n &= \text{normal moment capacity of the yield} - \text{line per unit length} \\ l &= \text{length of yield} - \text{line} \\ \theta &= \text{rotation of yield} - \text{line} \end{aligned}$$

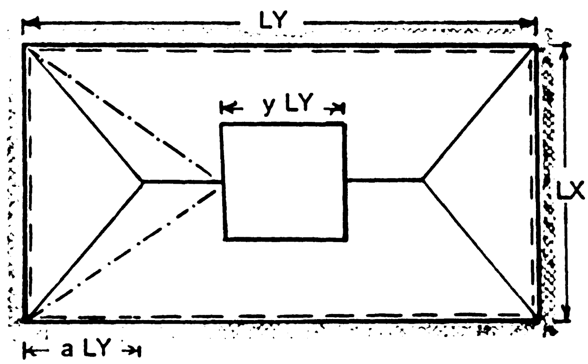
It is now necessary to find the pattern which minimizes the collapse load. This can be done in several ways. One may use the arithmetic method, solving several postulated patterns chosen by trial and error and picking the critical one. An algebraic method may also be used by writing the work equation in terms of one variable defining the position of the pattern. For this case, plugging in several values for the variable can result in a plot of ultimate load versus the variable value. The minimum ultimate load can be easily picked off the graph with little error. The third method, which is not always valid, is the algebraic method used with the addition of differential calculus. When the work equation is in terms of the load and one variable, the ratio of moment capacity to load (M/W), can be differentiated with respect to this variable. When this is equal to zero, the stationary maximum solution is obtained and the pattern is critical.

It is important to note that this method is not always valid and should be used with caution. If there is a discontinuity in the slab or in the loading, the stationary maximum may not be within the valid limits for which the work equation was set up. For example, in Figure 5, the pattern for a slab with a central opening is defined by the value of "a". The minimum ultimate load is to be found when "a" is between 0 *and* $\frac{1-y}{2}$. A stationary maximum solution of "a" $> \frac{1-y}{2}$ would be incorrect because it assumes a pattern which is not consistent with the pattern defined in the work equation.

Eyre and Kemp (1983) found mathematical solutions by differentiation without ignoring invalid patterns. Though their solution may often be correct, they were forced to state that when the resulting pattern was found to be invalid, the problem must be resolved by another method. This limited the usefulness of their method.

If there are more than one variable defining the pattern, the algebraic method becomes more difficult. A family of curves must be plotted to find the critical pattern. When using the third method, the equation must be differentiated with respect to each of the variables. This will result in enough simultaneous equations to solve.

There are advantages to each method and each is more effectively used in different situations. The choice will depend on how often this type of problem arises and on how difficult the algebra is. Generally, if the algebra is fairly easy or if it is a problem which arises often, it is worth going through the algebraic technique. However, algebraic solutions are already well known for many of the common cases, leaving only those for which the shape or loading are unique. These problems are unlikely to reappear, and along with those containing complex algebra, may be solved easiest with arithmetic techniques. Although the arithmetic method may seem weak in comparision to algebraic with calculus, it should be noted for its attributes. Instead of returning one value as the solution, its process shows how sensitive the solution is to variation of the pattern, giving the designer a better understanding of the solution. (Jones & Wood 1967)



$$0 \leq a \leq \frac{1-y}{2}$$

Figure 5. Valid range of variables

When dealing with slabs containing openings, not only is there a problem with the solution falling within valid limits after differentiation, the differentiation may become extremely complex. Use of the algebraic method aided by a microcomputer to compute systematic trial and error computations may be the simplest method to solve any problem. Time must be taken to write the work equation regardless of the method being used. Creating a code which solves the general work equation for all valid patterns may often be less work than differentiating or trial and error plotting. Special steps must always be taken to ensure that all modes are accounted for and that invalid patterns are ignored. This can be accomplished by only solving the algebraic equation for values of the variable within its valid range. This method is greatly utilized in the program presented later, letting the computer do the work and avoiding difficult differentiation.

2.3 Previous Studies

Many experimental and theoretical studies have been performed to find the results of simple theory, the effect of corner levers, and the effects of openings. In the theoretical studies on the effects of corner levers mentioned earlier, Mills (1970) used an iterative computer analysis, varying the size and orientation of the corner levers of triangular slabs to show that its effect can be much greater than for that of rectangular slabs. Demsky and Hatcher's (1969) study which was limited to slabs with three sides fixed, also varied the size and orientation of the simple corner levers.

Experimental studies of solid slabs have shown that actual collapse loads are consistently higher than those predicted by yield-line theory. Tests on an actual structure by Ockleston (1956), showed theoretical predictions underestimating the collapse load by an average of 20% for one-way slabs and by much greater for two-way slabs. Tests by Soare and Petcu (1968) found yield-line theory predictions 5.4 to 21.5% less than experimental. Metz's (1965) testing of 16 slabs, resulted in collapse loads from 5 to 40% higher than predicted. Gupta and Kanungoe (1976) concluded that yield-line predictions may be from 2 to 4 times lower than actual collapse loads. In this study, they also added predictions of membrane enhancement by Ockleston and Christensen to arrive at predictions within 10 percent of the actual collapse load.

In most studies concerned with slabs containing openings, corner lever effects are not included. Islam & Park (1971) presented a theoretical study disregarding corner levers in which they sought to develop design charts. They set up numerical equations for slabs with four edges supported under a uniform load and with many locations for openings. By limiting the configuration of the opening and the ratio of the reinforcement, they were able to gather their results to create design charts. Families of curves were plotted showing how the ultimate load varied with respect to one variable as another variable was set to different values. For a particular slab, the appropriate curve could be selected and the minimum load easily picked off. From these charts, it was shown that the openings had more effect on the strength of simply supported slabs than of fixed slabs, as was expected. His results showed that up to 22% decrease of strength may occur when openings are at a corner or along the long side. Only up to 11 and 13% decrease of strength may occur for slabs with openings along the short side and central openings respectively. Zavlasky, (1967) in a limited theoretical study of slabs with rectangular central openings, confirmed that up to 12% reduction occurs.

A few experimental studies on slabs containing openings in India have been published. Moondra and Sharma (1980) conducted testing on seven circular slabs with varying shapes and sizes of openings. The tests showed that the slab could carry 37 to 70 percent higher loads than predicted by theory, and that there was a reduction of strength with increased opening size. Narasimhan and Verreyya (1978) tested square slabs with rectangular openings. All slabs collapsed at loads greater than predicted. They suggest that a "moment multipling factor" (MMF) could be used to aid analysis of this type. This factor could be applied to the theoretical ultimate strength of a solid slab to predict the increase or decrease occuring when the slab has an opening. By presenting graphs showing the MMF with varying ratios of opening to slab dimensions, the MMF can be picked off of the graph and applied to the ultimate strength of a solid slab.

Through extensive studies and experimentation, yield-line theory has shown itself to be a reasonable alternative method for the analysis of reinforced concrete slabs. Ability to perform analysis by quick hand calculations as well as having the versitility of analyzing unique slabs are among yield-line theory's many appealing features. The effects of membrane action have been shown to make the theory's predictions conservative. Careful application of this theory, including the maintainance of adequate ductility, checking servicability, and the awareness of elastic moment distribution, will result in good designs and a better understanding of the behavior of slabs.

Chapter III

COMPUTER PROGRAM

3.1 Purpose

A computer program has been written to analyze uniformly loaded, orthotropically reinforced rectangular slabs with rectangular openings. This was included to ensure the ability to obtain quick, easy, and accurate analysis of the ultimate strength of many common slabs. The program makes use of simple yield-line theory, neglecting the effects of corner levers and membrane action.

The program, "SLAB.BAS", was written in BASIC for use on a personal computer so that it could be interactive, alerting the user to errors made during the input process. A program could be written to analyze any particular shaped slab; however, to include several odd shapes with their possible edge conditions would be impractical unless those particular shapes were dealt with often. Therefore, this program is limited to the most commonly occuring slab, the rectangular slab. The three sets of edge conditions which are most likely to occur with a rectangular slab were chosen to be developed. These were four sides fixed, three sides fixed with the remaining side free, and two opposing sides fixed with the other sides free. It will be shown later that the program will also be able to analyze simply supported slabs and some slabs which have a mixture of both simply supported and fixed edges.

It is recognized that openings in a slab may be necessary for the passage of ducts and pipes. For this reason, several rectangular opening locations were considered for each set of edge conditions. Central openings, openings at the center of each fixed side, and openings at the center of the free side were included. To limit the scope of the program, the type of loading and type of reinforcement had to be restricted. Since yield-line theory assumes that the slab fails in bending, point loadings were not considered because they tend to induce failure by punching shear. Since most slabs are designed for a specific uniform load, only uniformly loaded slabs are considered. Although slabs with varied reinforcement could have possibly been included, it was beyond the scope of this paper; this program is only capable of analyzing slabs which are orthotropically reinforced.

3.2 Input

The input is short and simple. It is also aided by a sketch of the desired slab if a color/graphics monitor is available. This sketch shows the slab with dimension lines and variable names for the dimensions which are to be input. If a color/graphics monitor is not available, the user must refer to Figure 6 for aid in inputting data. The user must input the type of monitor, the edge conditions of the slab, the slab dimensions, the opening location and dimensions, and the positive and negative moment capacities in each direction. The designation for the slab dimensions are X (vertical) and Y (horizontal) and must be entered in feet. The opening dimensions are A and B and are in feet also. The moment capacities are designated by MXT, MXB, MYT, and MYB, and are entered in kip-feet per unit foot. The M means moment capacity, the second letter describes the direction in which the reinforcement runs, and the last letter designates whether the reinforcement is on the top or bottom. For example, the negative moment capacity along a line parallel to the x-axis would be determined by the top reinforcement running along the y-direction and would be input as MYT.

To avoid confusion and to clarify further points, it is necessary to inform the user that some of the variables used during input have a different meaning within the program. For example, the dimensions X and Y are only used as slab dimensions during the input of data; however, within the program, the dimensions become LX and LY. A and B are internally transformed into a ratio of the opening dimensions to their corresponding sides' lengths. For example, the vertical opening dimension input as B will now be equal to the product of X and LX. The variables X and Y will now have values between 0 and 1 which are multiplied with either LX or LY. The same applies to A and B, they were input as opening dimensions but within the program they will designate ratios. This should be carefully noted as several items further along will be in reference to the length of their corresponding sides as well. Although the program will show all variables in capitals, the governing equations at the end of the chapter will label all transformed ratios (a, b, x, & y) in lowercase.

To avoid incorrect output due to physically impossible input, the input is checked before the analysis is run. All dimensions must be positive, the opening dimensions must be smaller than their corresponding slab dimensions, and the moment capacities must be non-negative. The user is alerted to any errors, as well as given the opportunity to check input values for other errors. The user can then change the input if necessary. It is noted that the value of zero for an opening dimension will not be accepted; however, if the analysis of a slab with no openings is desired, very small numbers may be entered with the same result. Once the input is entered, the slab is analyzed. The output returned to the user will consist of an echo of the input information as well as the ultimate uniform load and failure pattern number. The configuration of each collapse pattern is shown at the end of the chapter with their corresponding mode numbers.

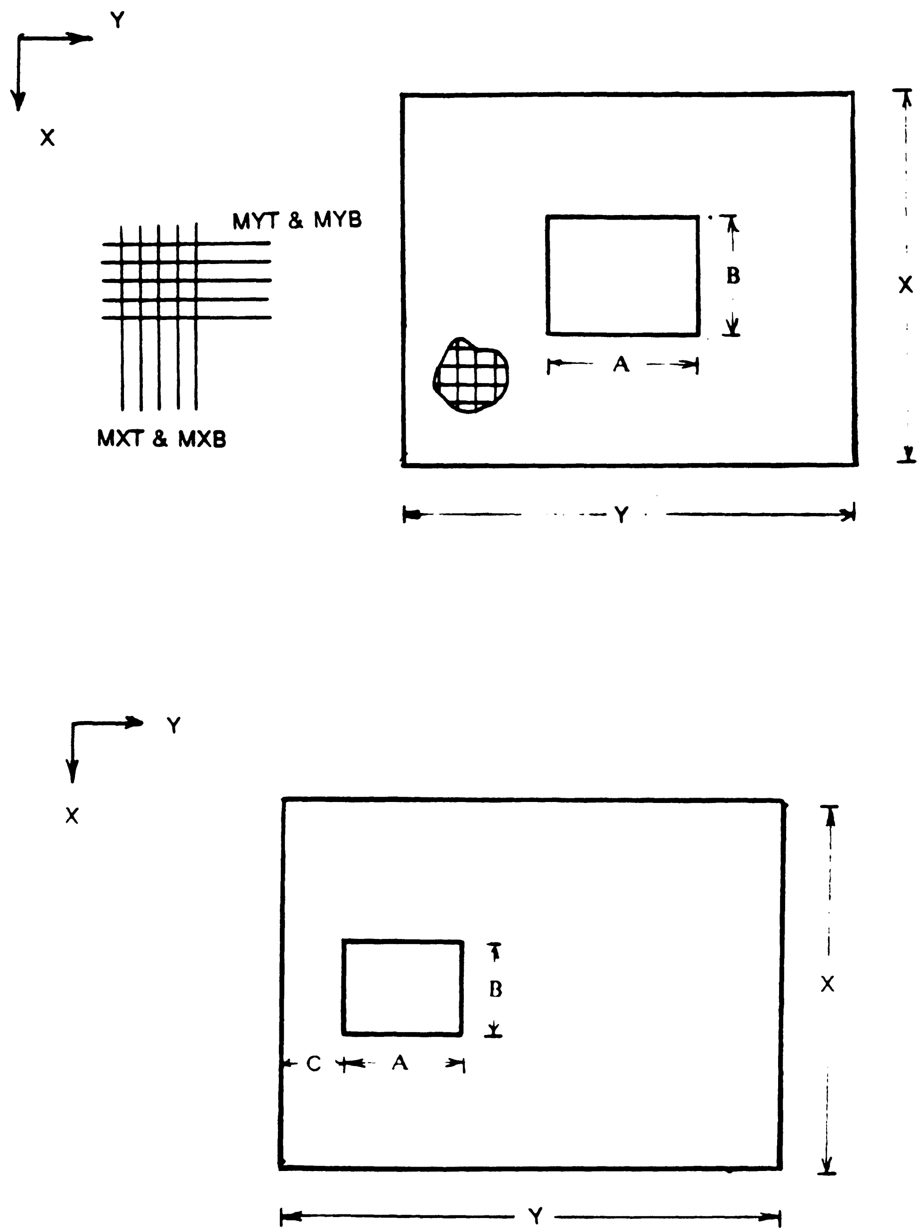


Figure 6. Aid for inputting - A) Three and Four sides fixed, B) Two sides fixed

3.3 Method

The analysis portion of this program uses the principle of virtual work to solve for the ultimate load. For each set of end conditions and opening locations, a set of all possible general failure patterns or modes was postulated. Figure 7 shows the set of failure modes for a slab with three sides fixed and an opening at the center of the free edge. These general patterns are defined by the configuration of the yield-lines and pattern dimension variables. The pattern dimension shows the position of key points on the yield pattern such as where two or more yield-lines meet or where a yield-line meets the edge of the slab. These dimensions are variable and have a specific range for each mode of failure. The correct position within this range will be the one for which the ultimate load is minimized. One will notice that each mode represents a range of yield patterns for which the pattern variable can vary while the shape of the pattern remains similiar. The set of all modes make up all of the possible failure patterns. Only rarely will two modes represent similar failure patterns. As shown in Figure 7, if "a" is equal to "x" in mode 2 and "a" is equal to "y" in mode 4, the patterns are generally the same. All sets of failure patterns follow simple yield-line theory as described earlier and do not take into account the presence of corner levers and more complex fan shapes. In practice, openings are usually small relative to total slab area. Therefore, the patterns were also based on the assumption that the entire slab would collapse. Caution must be taken if the openings are extremely large, as local failure may occur and the critical mode will not be analyzed.

The ultimate load can be determined when the external work done by the loading moving through an arbitrary displacement is equated with the work dissipated by the slab as it fails by that particular pattern. Being an upper bound solution, the critical pattern will be the one that fails under the smallest load. This work equation is originally set up in its most general terms. The ultimate uniform load (W) is isolated and equated with a formula that is in terms

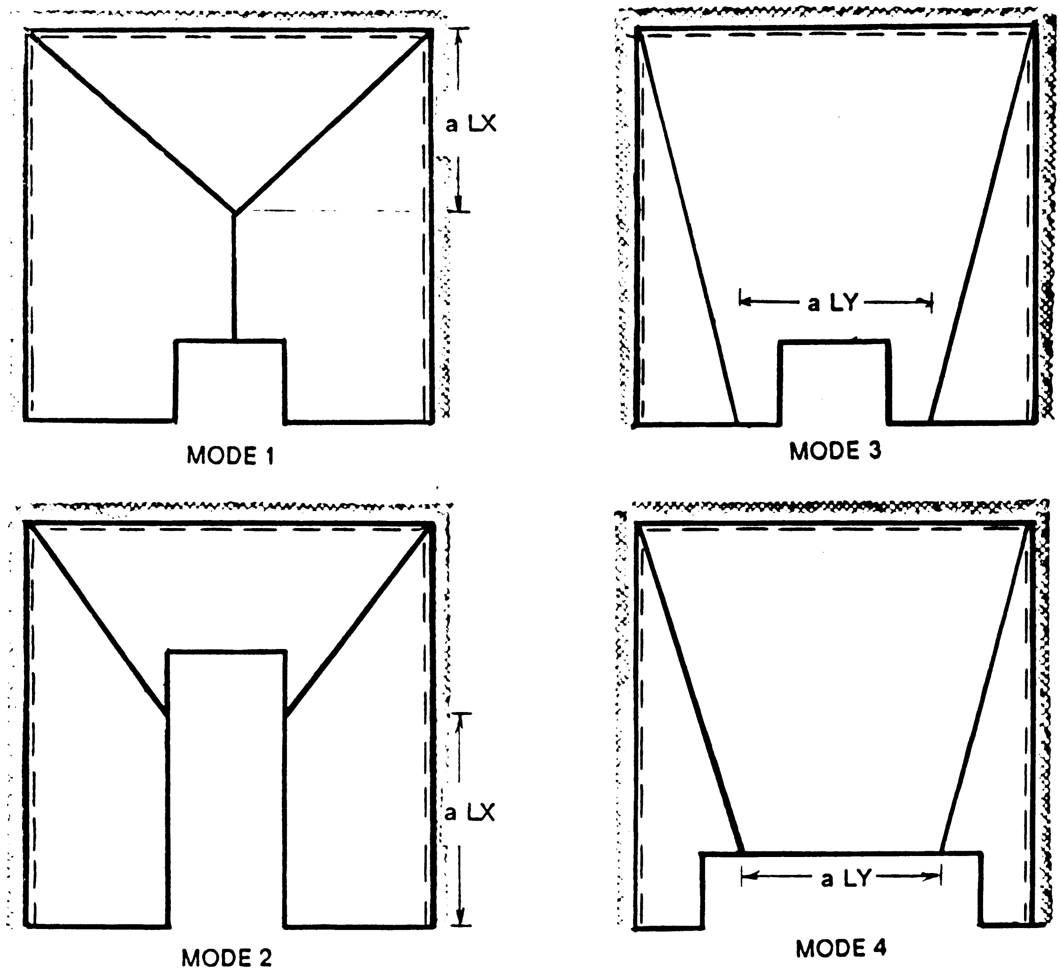


Figure 7. Set of postulated yield patterns

of the slab and opening dimensions (LX, LY, xLX, & yLY), the moment capacities (MXT, MXB, MYT, & MYB), and the pattern dimension variables (a, b, & d) as shown in Figure 8.

After the input process, the slab and opening dimensions as well as the moment capacities are known, leaving the ultimate load and the pattern dimension variables to be determined. For each mode, the values of the pattern dimension variables have certain constraints. The range of possible pattern dimensions will be dictated by the yield pattern and size of the opening. In Figure 9, the pattern dimension ratio "a" defines the perpendicular distance between the fixed edge and the intersection of yield-lines. It can be shown that for a central hole "a" must be between $\frac{1 + x}{2(1 - y)}$ and 1. In some cases a particular pattern may not be possible. For example, if a slab has three sides fixed and a large central opening as in the bottom of Figure 9, this mode cannot exist. If it is found that due to large opening dimensions that the lower bound of "a" is greater than the length of its corresponding side as in Figure 9, the pattern is geometrically impossible; the arbitary value of 1,000,000 k-ft/ft is given for the failure load of that mode. This value will take on no meaning, as another mode which is possible will yield a lower value.

An iterative process was developed for finding the values of the pattern dimension variables for which the ultimate load is minimized. When possible values for "a" exist, the value of "W" is calculated for each value of "a" as it is varied from its lower to the upper limits. It is varied in increments of one one-hundredth of the length of that corresponding side. As "a" is varied through its range, the smallest value of the load is saved as the critical value for that mode. Each mode is analyzed in the same manner. After each possible mode has been analyzed, the critical values for each mode are compared. The smallest value of the load and its corresponding mode number are given in the output. The choice of increment step was found to be satisfactory. As the increment of the interval is decreased by the factor of ten, the computing time is increase by a factor of ten while the increase of accuracy is insignificant. Increasing the interval by a factor of ten, cuts the computing time by 90% while losing only 0.4% in accuracy.

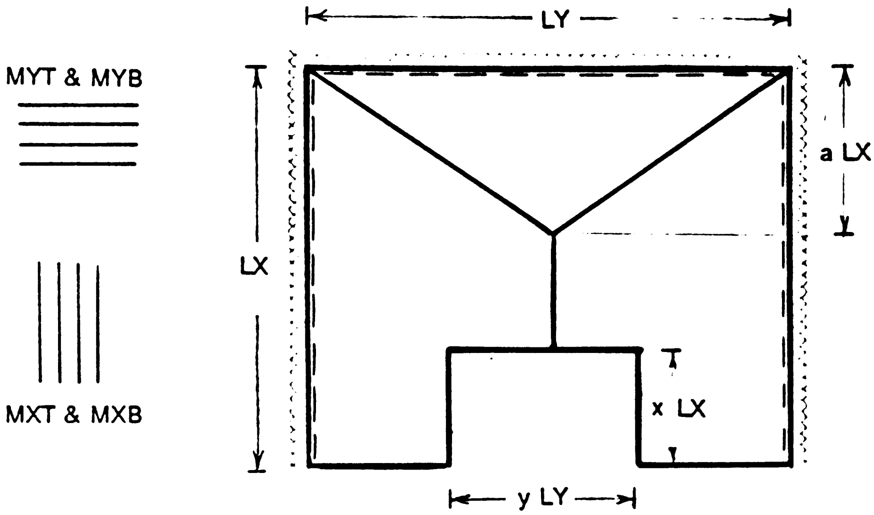


Figure 8. Example of terms

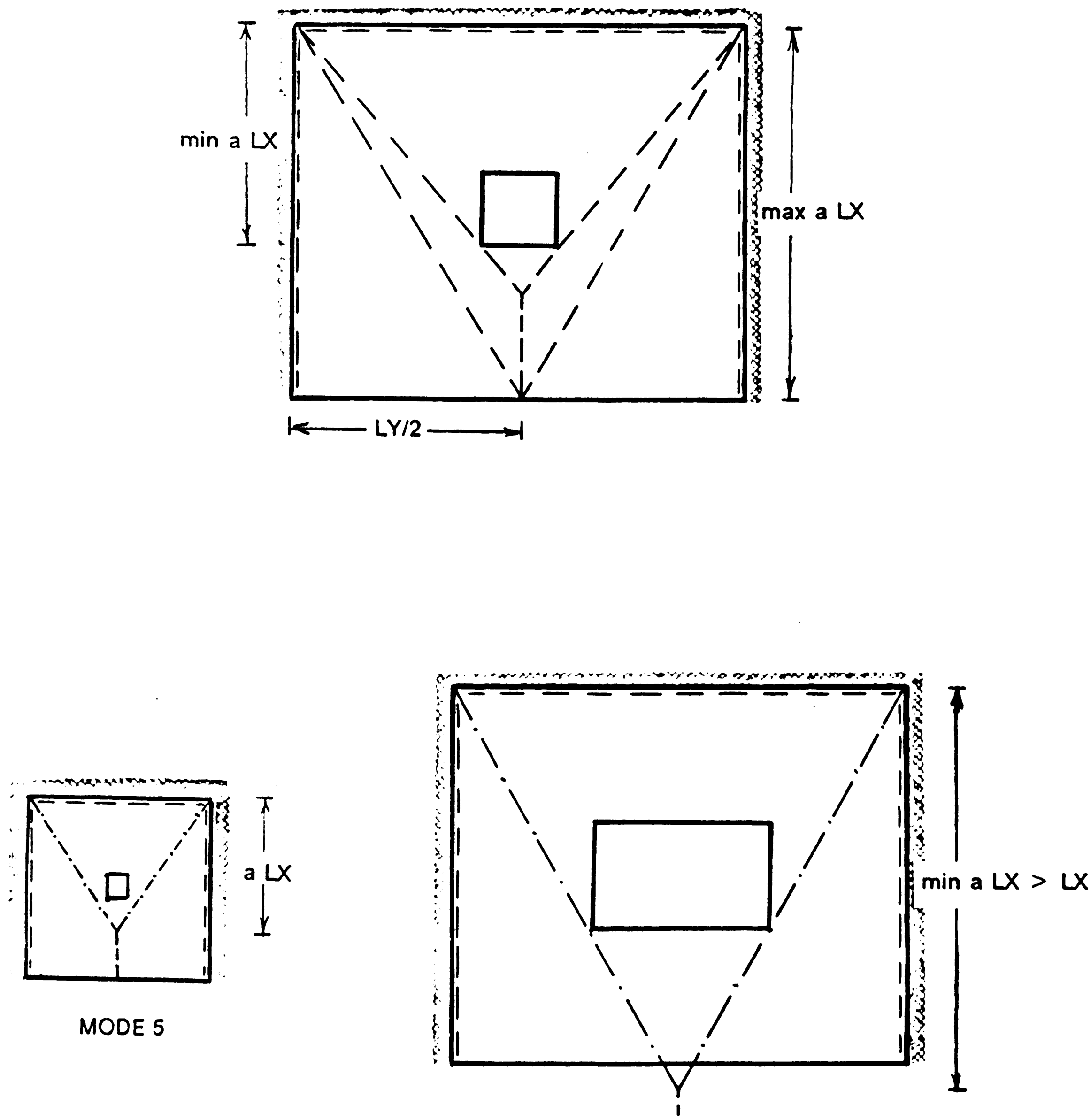


Figure 9. Possible patterns and impossible pattern

In some cases, there may be two or three pattern dimensions, each with their own constraints. Moreover, the constraints will often be dependent on values of other pattern dimensions. For example in the mode described in Figure 10, the pattern dimensions are "a" and "b" and their ranges are $0 < a < (1 - b)$ *and* $(1 - x) < b < (1 - a)$. Although the value of "x" will be known from the input, the upper limit of "a" depends on the value of "b" and the upper value of "b" is dependent on the value of "a". In these cases, the value of one of the variables ("b" in this case) is automatically selected to be a reasonable value within its valid limits. The limits of "a" are now defined. Keeping "b" as a constant, "a" is varied within its limits and its value at which the load is minimized is saved. For this new value of "a", the new constraints for "b" are established and "b" is varied in the same manner. This process is repeated once again, beginning with the last value of "b" set as a constant. After this set of trials, the new values for "a" and "b" are compared to their previous values from the first trial. The solution is considered correct when the change of each variable is less than three percent of the length of their respective sides. If the pattern dimensions change less than the three percent, the load for that mode is saved and the next mode is analyzed. If the values for both "a" and "b" have not yet stabilized, this process is repeated until it does. Varying the value for the convergence test may or may not affect the computing time depending on the problem and the magnitude of the change. Decreasing the value for convergence leads to a negligible increase in accuracy. A sample of input and output is given in Figure 11.

3.4 Limitations

As mentioned before, the slab must be uniformly loaded and orthotropically reinforced. Openings located along an edge are considered centered with respect to the two adjacent sides. The program is limited to the following edge condition and opening location combinations. Four sides fixed : openings are central, and center of a fixed side. Three sides fixed :

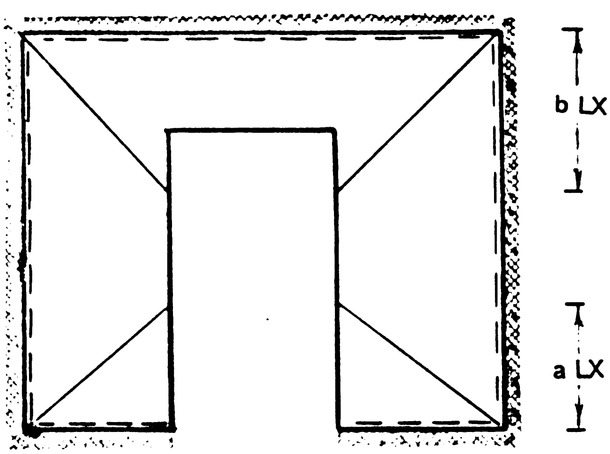


Figure 10. Example of two pattern variables

```
Please refer to thesis for location of dimensions on figures.

Please enter value for horizontal side dimension, Y (ft) ? 25
Please enter value for vertical side dimension, X (ft) ? 20
Please enter value for opening dimension, A (ft) ? 0.00001
Please enter value for opening dimension, B (ft)?  0.00001
Enter negative moment capacity for bending about the x-axis MYT (k-ft/ft)? 0
Enter positive moment capacity for bending about the x-axis MYB (k-ft/ft)? 8
Enter negative moment capacity for bending about the y-axis MXT (k-ft/ft)? 0
Enter positive moment capacity for bending about the y-axis MXB (k-ft/ft)? 16


Your values are now the following:

A = .00001 ft.   B = .00001 ft.    Y = 25 ft.   X = 20 ft.  C = 0
ft. (IF APPLICABLE)
MYT = 0 k-ft/ft    MYB = 8 k-ft/ft    MXT = 0 k-ft/ft    MXB = 16 k-ft/ft


Would you like to change any of these values? (Y/N)?




   ***   RECTANGULAR SLAB -- THREE SIDES FIXED -- CENTRAL OPENING   ***
                  with the following dimensions :

A = .00001 ft.   B = .00001 ft.    Y = 25 ft.   X = 20 ft.
C = 0 ft. (IF APPLICABLE)
MYT = 0 k-ft/ft    MYB = 8 k-ft/ft    MXT = 0 k-ft/ft    MXB = 16 k-ft/ft



                 The ultimate load is  .2617343  KIPS/SQ.FT

                       The failure is by mode # 1



Would you like to analyze another slab (y/n)??
```

Figure 11. Example input and output

openings are central, center of the horizontal fixed side, and center of the horizontal free side. Two sides fixed : openings are interior, fixed edge, symmetrical interior, and symmetrical fixed edge openings. Although the last two conditions may seem odd, their analysis was originally included due to the possibility of testing these situations. When three sides fixed are desired, the free edge is considered to be horizontal. When two sides fixed are desired, the vertical edges are considered fixed. The condition of three sides fixed with an opening at the center of the vertical side was left out due to the large number of modes and large number of pattern variables per mode. Its analysis was extremely lengthy and due to space was excluded.

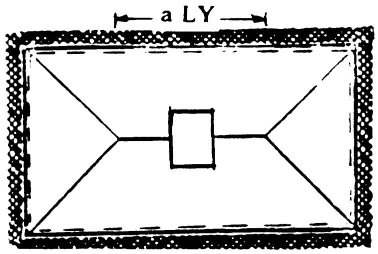
As briefly mentioned earlier, simply supported slabs may also be analyzed but with some limitations. Simply supported edges may be analyzed similarly to fixed edge because deflections are not allowed along the edge in either case. Therefore, the failure patterns and their governing equations will be similar. Only the amount of energy dissipated will change. If there is any negative reinforcement, the side is considered fixed and energy is dissipated. When negative reinforcement does not exist, deflection will be zero but rotation will not be restrained and the energy dissipated due to that side will be zero. Entering the negative moment capacity in a certain direction as zero will result in those two opposite sides being considered simply supported. Similarly a slab with four sides simply supported can be modeled by entering all negative moment capacities as equal to zero. It can be easily seen that in this program it is not possible to model the condition where one side is fixed while its opposing side is simply supported. For example when MXT is entered, the reinforcement spans the entire length giving each opposing side a value of either zero or some positive value for its moment capacity. For similar reasons, additional free edges cannot be modelled by this program. The restraint against deflection would be violated and a different set of yield patterns would have to be considered.

It should be stressed that these patterns will not be critical, in that corner levers were not considered. In comparing a slab with consideration of corner levers to the program's simple theory predictions, it will be found that the presence of corner levers will reduce the strength predicted. This does not necessarily mean that these values are non-conservative. It should be remembered that membrane action is also excluded and the prediction could possibly be conservative.

Similiar programs may be written to facilitate design. The work equations can be written in terms of the major reinforcement and desired ratios of other reinforcement entered by the designer along with the desired design load and slab dimensions. Iteration routines can be written to maximize the ultimate moment capacity, yielding the minimum reinforcement to be used.

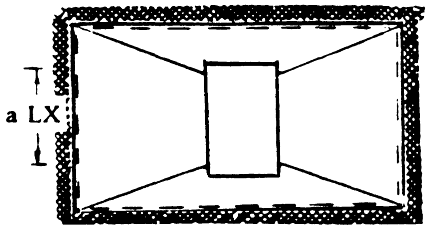
3.5 Work Equations

For each pair of end conditions and opening locations, each mode shape and its respective governing equation are shown in Figures 12 through 23. The negative yield-lines shown near the edges are assumed to occur at along the edges. Symmetry in yield pattern occurs in most situations. The program listing is presented in its entirety in Appendix A.



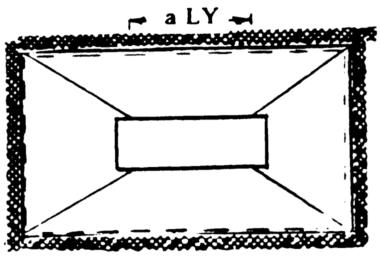
MODE 1

$$W = \left[\frac{24}{LX^2LY^2(1-a)}\right]\left[\frac{LX^2(MYT + MYB) + (1-a)LY^2(MXT + (1-y)MXB)}{2 + a - 6xy + 3x^2y}\right]$$



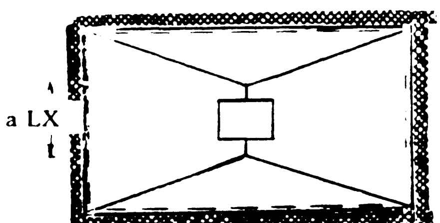
MODE 2

$$W = 24\left[\frac{LX^2(1-a)(MYT + (1-a)MYB) + LY^2(1-y)(MXT + (1-y)MXB)}{\{(1-y)LX^2LY^2\}\ \{2(1-a)^2(1-y) + 3a(1-a)(1-y) + 3y(1-x)^2\}}\right]$$



MODE 3

$$W = 24\left[\frac{LX^2(1-x)(MYT + (1-x)MYB) + LY^2(1-a)(MXT + (1-a)MXB)}{\{LX^2LY^2(1-x)\}\ \{2(1-a)^2(1-x) + 3a(1-a)(1-x) + 3x(1-y)^2\}}\right]$$

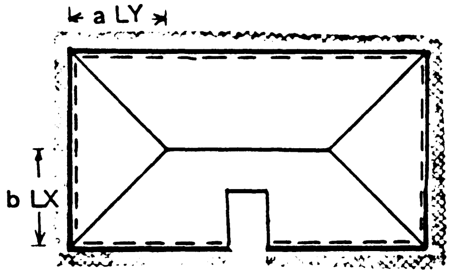


MODE 4

$$W = \left[\frac{24}{(1-a)LX^2LY^2}\right]\left[\frac{LY^2(MXT + MXB) + (1-a)LX^2(MYT + (1-x)MYB)}{2 + a - 6xy + 3xy^2}\right]$$

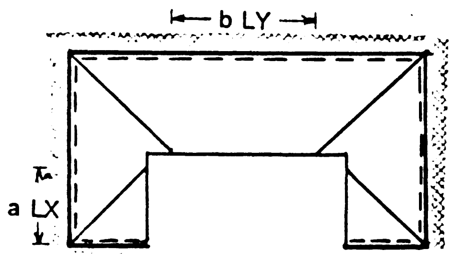
Figure 12. Four sides fixed - Central opening

MODE 1



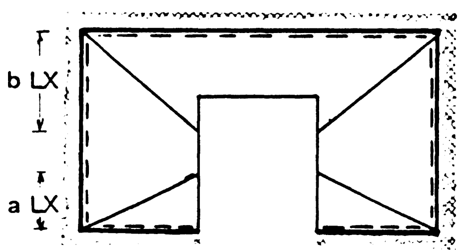
$$W = 6\left[\frac{2b(1-b)LX^2(MYT + MYB) + abLY^2(MXT + MXB) + a(1-b)LY^2(MXB + (1-y)MXT)}{\{a(1-b)LX^2LY^2\}\ \{3b - 2ab - 3x^2y\}}\right]$$

MODE 2



$$W = 6\left[\frac{LY^2(1-x)(1-y)^2(MXT + MXB) + LY^2a(1-b)(MXT + (1-b)MXB) + 4a(1-x)LX^2(MYT + (1-x+a)MYB)}{\{a(1-x)LX^2LY^2\}\ \{2a(1-y)^2 + 3(1-y)^2(x-a) + 2(1-x)(1-b)^2 + 3b(1-b)(1-x)\}}\right]$$

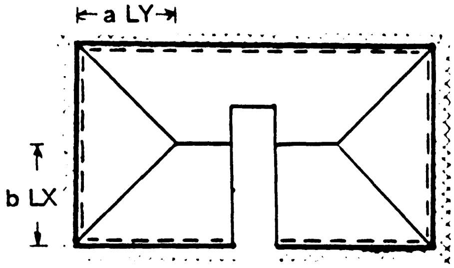
MODE 3



$$W = 6\left[\frac{4abLX^2(MYT + (a+b)MYB) + a(1-y)LY^2(MXT + (1-y)MXB) + b(1-y)^2LY^2(MXT + MXB)}{\{a(1-y)LX^2LY^2\}\ \{2b(a+b)(1-y) + 3b(1-a-b)(1-y) + 3y(1-x)^2\}}\right]$$

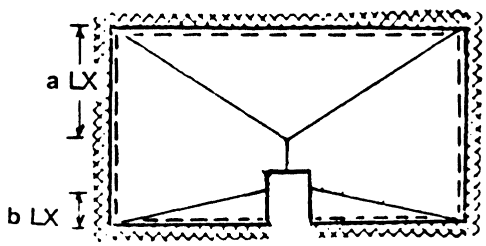
Figure 13. Four sides fixed - Opening on fixed edge

MODE 4



$$W = 6\left[\frac{2b(1-b)LX^2(MYT + MYB) + a(1-b)(1-y)LY^2(MXT + MXB) + abLY^2(MXT + (1-y)MXB)}{\{abLX^2LY^2\}\ \{4a(1-b) + 3(1-b)(1-2a) - 3by(1-b) - 3y(x-b)(2-x-b)\}}\right]$$

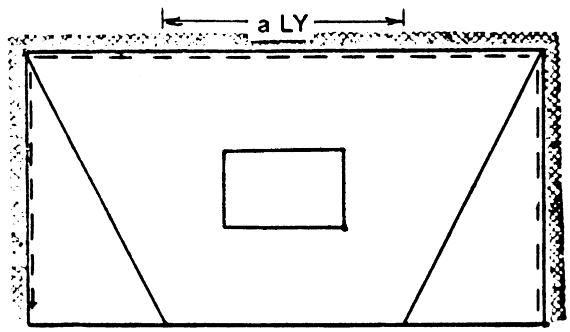
MODE 5



$$W = \left[\frac{6}{abLX^2LY^2}\right]\left[\frac{4abLX^2(MYT + (1-x-b)MYB) + a(1-y)^2LY^2(MXT + MXB) + bLY^2(MXT + MXB)}{3 - a - 3x + 3(1-y)^2(x-b) + 2b(1-y)^2}\right]$$

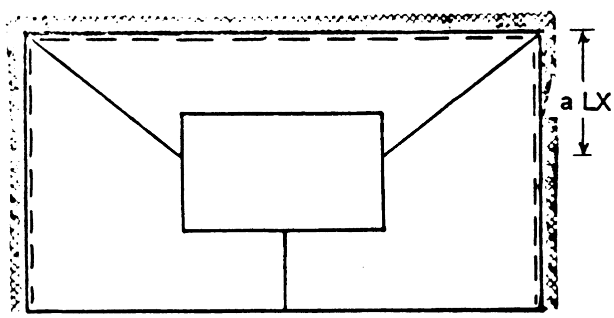
Figure 14. Four sides fixed - Opening on fixed edge

MODE 1



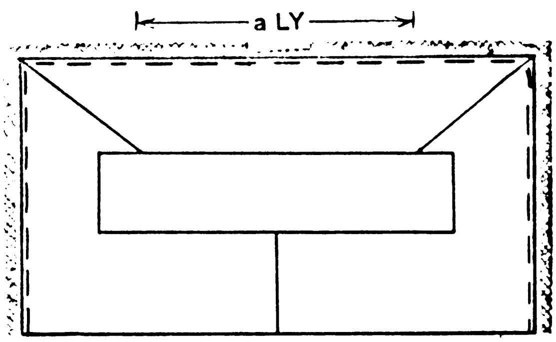
$$W = \left[\frac{6}{(1-a)LX^2LY^2}\right]\left[\frac{4LX^2(MYT + MYB) + (1-a)LY^2(MXT + (1-a)MXB)}{2 + a - 3xy}\right]$$

MODE 2



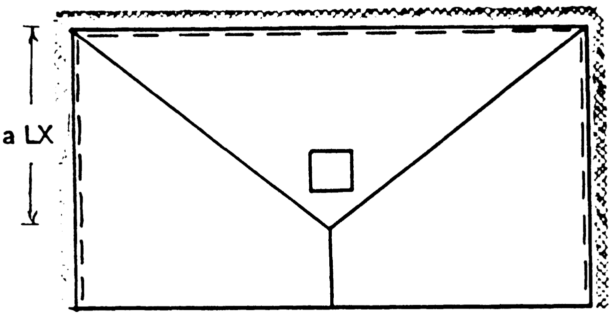
$$W = \left[\frac{24}{LX^2LY^2}\right]\left[\frac{2aLX^2(2MYT + (1 - x + 2a)MYB) + LY^2(1-y)(MXT + (1-y)MXB)}{8a^2(1-y)^2 + 3y(1-y)(1-x)^2 + 6a(1-x) + 6a(1-y)^2(1 + x - 2a)}\right]$$

MODE 3



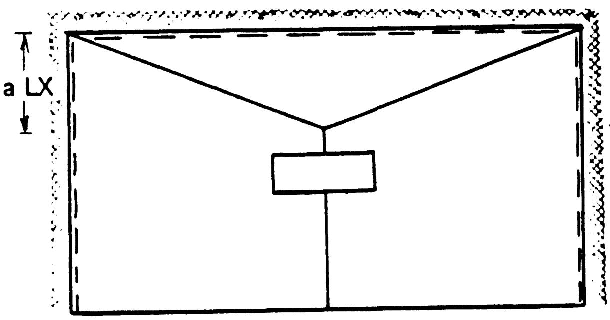
$$W = 24\left[\frac{4(1-x)LX^2(MYT + (1-x)MYB) + 2LY^2(1-a)(MXT + (1-a)MXB)}{\{(1-x)LX^2LY^2\}\ \{4(1-x)(1-a)^2 + 6a(1-x)(1-y) + 12x(1-y)^2 + 6(1-x)\}}\right]$$

Figure 15. Three sides fixed • Central opening



MODE 4

$$W = \left[\frac{6}{LX^2 LY^2}\right] \left[\frac{4aLX^2(MYT + MYB) + LY^2(MXT + MXB)}{3a - a^2 - 3xy}\right]$$

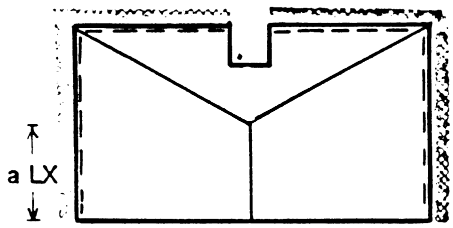


MODE 5

$$W = \left[\frac{6}{aLX^2 LY^2}\right] \left[\frac{4aLX^2(MYT + (1 - x)MYB) + LY^2(MXT + MXB)}{3 - a - 3xy(2 - y)}\right]$$

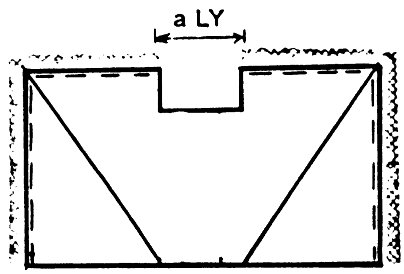
Figure 16. Three sides fixed - Central opening

MODE 1



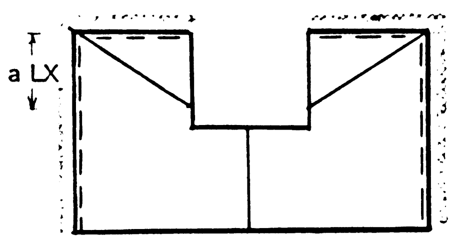
$$W = \left[\frac{6}{LX^2 LY^2}\right]\left[\frac{4(1-a)LX^2(MYT + MYB) + LY^2(MXB + (1-y)MXT)}{2(1-a)^2 + 3a(1-a) - 3x^2 y}\right]$$

MODE 2



$$W = \left[\frac{6}{(1-a)LX^2 LY^2}\right]\left[\frac{4LX^2(MYT + MYB) + (1-a)LY^2((1-a)MXB + (1-y)MXT)}{2(1-a) + 3a - 3x^2 y}\right]$$

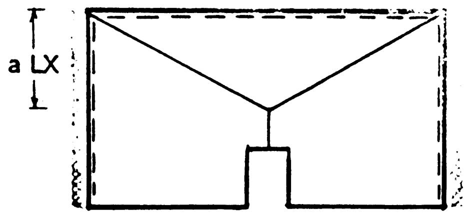
MODE 3



$$W = \left[\frac{6}{aLX^2 LY^2}\right]\left[\frac{4aLX^2(MYT + (1-x+a)MYB) + (1-y)^2 LY^2(MXT + MXB)}{3(1-x) + 2a(1-y)^2 + 3(x-a)(1-y)^2}\right]$$

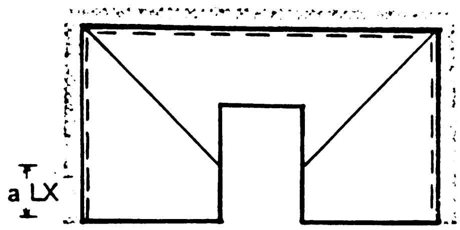
Figure 17. Three sides fixed - Opening on fixed side

MODE 1



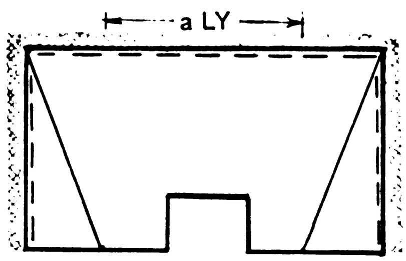
$$W = \left[\frac{6}{aLX^2LY^2}\right]\left[\frac{LY^2(MXT + MXB) + 4aLX^2(MYT + (1 - x)MYB)}{3 - a - 3xy(2 - y)}\right]$$

MODE 2



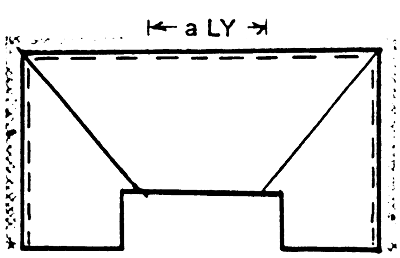
$$W = 6\left[\frac{4(1 - a)LX^2(MYT + (1 - a)MYB) + (1 - y)LY^2(MXT + (1 - y)MXB)}{\{(1 - y)LX^2LY^2\}\ \{3a(1 - a)(1 - y) + 2(1 - a)^2(1 - y) + 3y(1 - x)^2\}}\right]$$

MODE 3



$$W = \left[\frac{6}{(1 - a)LX^2LY^2}\right]\left[\frac{4LX^2(MYB + MYT) + (1 - a)LY^2(MXT + (1 - a)MXB)}{2 + a - 3xy(2 - x)}\right]$$

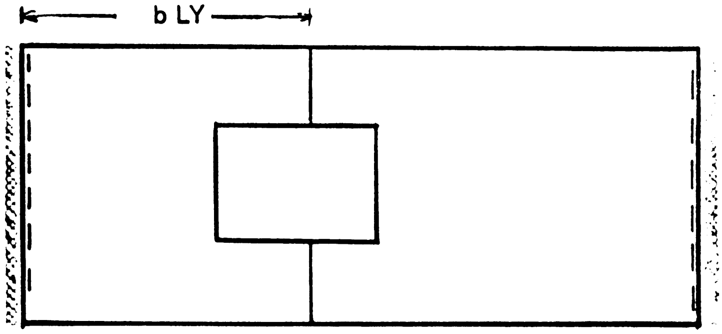
MODE 4



$$W = 6\left[\frac{4LX^2(1 - x)(MYT + (1 - x)MYB) + (1 - a)LY^2(MXT + (1 - a)MXB)}{\{(1 - x)LX^2LY^2\}\ \{2(1 - x)(1 - a)^2 + 3a(1 - a)(1 - x) + 3x(1 - y)^2\}}\right]$$

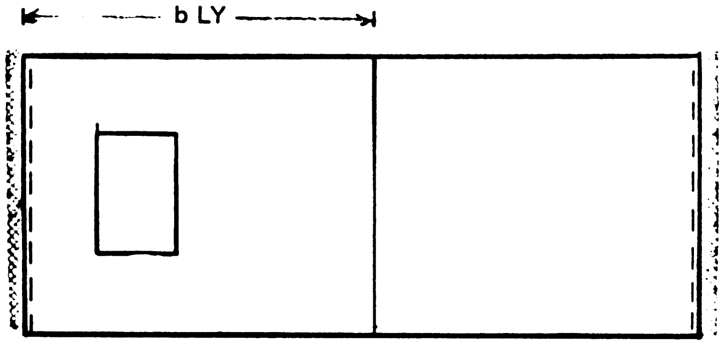
Figure 18. Three sides fixed - Opening on free side

MODE 1



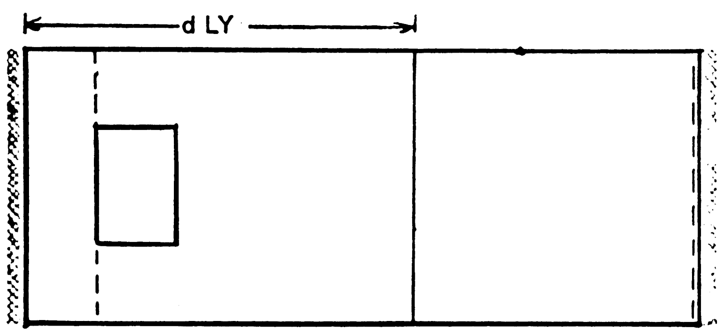
$$W = \left[\frac{2}{LY^2}\right]\left[\frac{(MYT + (1 - x)MYB)}{b(1 - b)(1 - x) + (1 - b)xc^2 + xb(1 - c - y)^2}\right]$$

MODE 2



$$W = \left[\frac{2}{(1 - b)LY^2}\right]\left[\frac{(MYT + MYB)}{b - xy(2c + y)}\right]$$

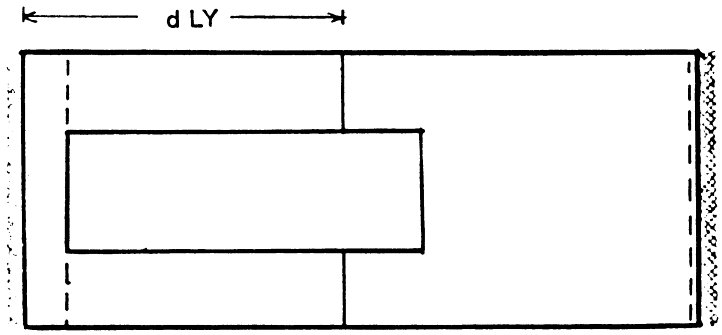
MODE 3



$$W = \left[\frac{2}{(1 - d)LY^2}\right]\left[\frac{(1 - d)(MYB + (1 - x)MYT) + (d - b)(MYT + MYB)}{(d - b)(1 - b) - x(c + y - b)^2}\right]$$

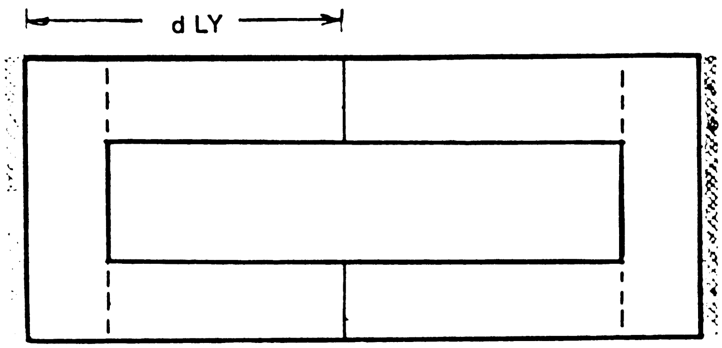
Figure 19. Two sides fixed - Interior opening

MODE 4



$$W = \left[\frac{2}{(d-c)LY^2}\right]\left[\frac{(1-x)(1-d)(MYT+MYB)+(d-c)(MYT+(1-x)MYB)}{(1-c)(1-d)-x(1-d)(d-c)-x(c+y-d)(2-d-c-y)}\right]$$

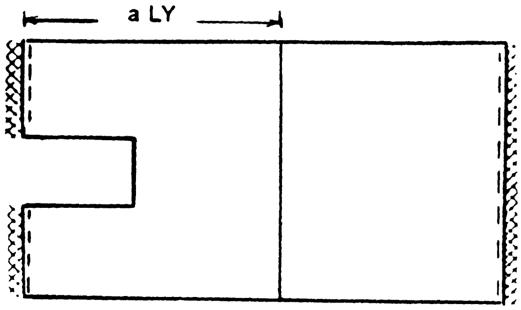
MODE 5



$$W = \left[\frac{2}{LY^2}\right]\left[\frac{MYT+MYB}{(d-c)(c+y-d)}\right]$$

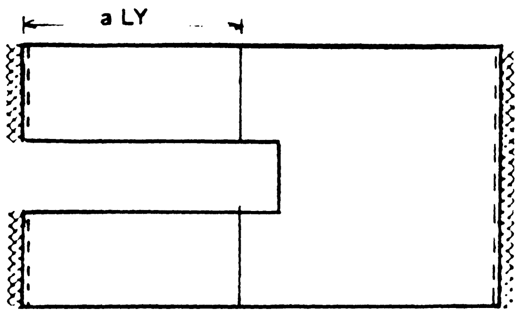
Figure 20. Two sides fixed - Interior opening

MODE 1



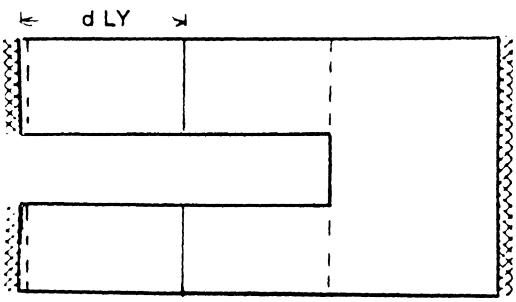
$$W = \left[\frac{2}{(1-a)LY^2}\right]\left[\frac{(1-a)(MYB + (1-x)MYT) + a(MYB + MYT)}{a + xy^2}\right]$$

MODE 2



$$W = \left[\frac{2}{aLY^2}\right]\left[\frac{(1-a)(1-x)(MYT + MYB) + a(MYT + (1-x)MYB)}{(1-a) - ax(1-a) - x(y-a)(2-a-y)}\right]$$

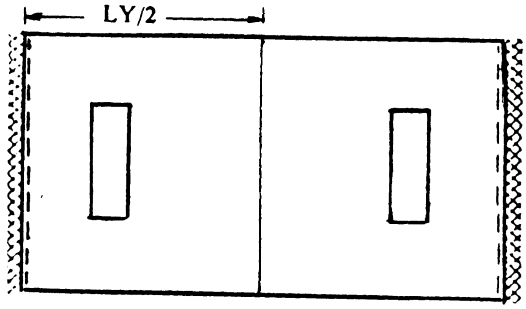
MODE 3



$$W = \left[\frac{2(MYT + MYB)}{d(y-d)LY^2}\right]$$

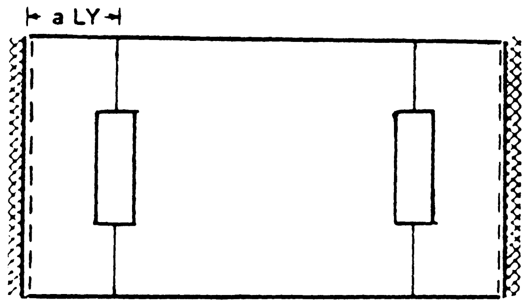
Figure 21. Two sides fixed - End opening

MODE 1



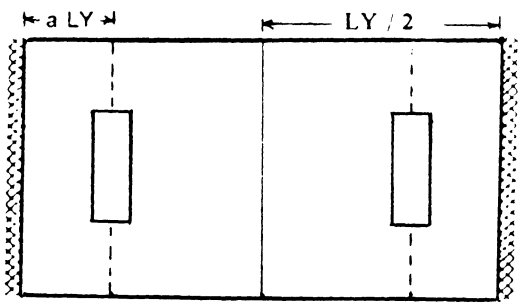
$$W = \left[\frac{8}{LY^2}\right] \left[\frac{(MYT + MYB)}{1 - 4xy(2c + y)}\right]$$

MODE 2



$$W = \left[\frac{2}{LY^2}\right] \left[\frac{(MYT + (1 - x)MYB)}{a - a^2 - 2ax(c + y - a) - x(a - c)(c + a)}\right]$$

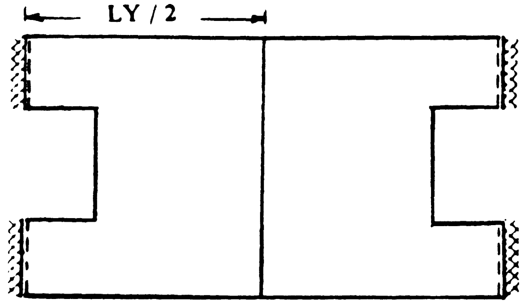
MODE 3



$$W = \left[\frac{8}{LY^2}\right] \left[\frac{(MYB + (1 - x)MYT)}{(1 - 2c)^2 - 4xy^2}\right]$$

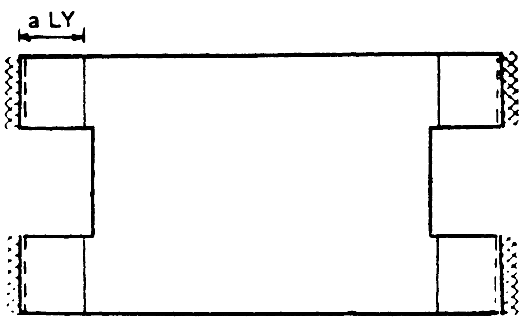
Figure 22. Two sides fixed - Symmetrical interior openings

MODE 1



$$W = \left[\frac{8}{LY^2}\right]\left[\frac{(MYB + (1 - x)MYT)}{1 - 4xy^2}\right]$$

MODE 2



$$W = \left[\frac{2(1 - x)}{aLY^2}\right]\left[\frac{(MYT + MYB)}{a(1 - x) + (1 - 2a) - 2x(y - a)}\right]$$

Figure 23. Two sides fixed - Symmetrical end openings

Chapter IV

EXPERIMENTAL STUDY

4.1 Introduction

Four slabs were constructed and tested to compare the ultimate loads and yield patterns to those predicted by simple yield-line theory. The first contained no opening and was used as a control slab while the other three contained openings in various locations. A testing apparatus was built to achieve fixity for three edges of the slab. An airbag was placed under the slab and inflated to apply a uniform lateral load on the slab. Deflections, ultimate loads, and yield patterns were observed and analyzed. In order to reduce costs, available materials from other experimental programs were used whenever possible.

4.2 Testing Apparatus

To effectively fix the edges of the slab, a sandwich of wide flange steel beams was used. Using six available W 10 X 12 beams, it was possible to fix three sides of the slab. Three beams were positioned as three sides of a rectangle and anchored onto a concrete floor. Twenty-two 1/2 inch diameter "Thunderstud" anchor bolts were used to prevent the beams from lifting from the floor due to the upward and torsional load applied by the airbag acting on slab. These three beams were tightly fitted with 10 inch pieces of 2 by 4's between the flanges acting as stiffeners to prevent rotation of the flanges. The slab was placed on top of these beams and the remaining beams were placed upon the slab, positioned directly over the anchored beams. Three pairs of 1/2 inch diameter Grade 5 bolts for each fixed side were placed in pre-drilled holes through the slab and adjoining flanges. The bolts were tightened, effectively fixing the slab between the flanges of the steel beams. An airbag available from another experimental program was used to apply lateral pressure to the slab. The 30 psi capacity of the airbag was well above the estimated five to seven psi load needed to fracture the slab. The dimensions of the slab as well as the location of the beams were dictated by the size of the airbag. Since the plan dimensions of the airbag were 64 in. by 50 in., the loaded portion of the slab was chosen to be five feet by four feet. The airbag was placed below the slab and elevated by plywood over a 2 X 4 grillage to rest closer to the bottom surface of the slab. This was necessary because the slab rested ten inches above the floor while the airbag was thought to have the capability of expanding a maximum of six inches. The photo in Figure 24 shows the testing apparatus.

Figure 24. Testing apparatus

4.3 Slab Materials and Design

The slabs were constructed of reinforced concrete. Each slab was approximately 6′-0″ by 4′-6″ with a loaded surface of 5′-0″ by 4′-0″. The additional length allowed for the connection of the slab to the testing apparatus as well as allowing for sufficient development lengths of the reinforcement. It was necessary to minimize the slab thickness in order to achieve low weight, low cost, and easier handling; therefore, the range of one to two inches was considered. The thickness of one and one-half inches when reinforced with #2 rebar was found to be most desirable with respect to several considerations including ductility, clear cover, and ultimate load. The two central layers of #2 bars having diameters of 1/4 inch, left a clear cover of 1/2 inch. This clear cover dictated that 3/8 inch maximum aggregate be used. A mesh of smooth straight #2 bars spaced at four inches was used as reinforcement and was tied together with soft iron wire. Deformed #3 bars would have been difficult to use due to ductility considerations. Also, the weight involved in giving the slab an adequate thickness was too great for easy handling; and #3 bars could not be readily bent by hand. Although smooth bars are not generally recommended for ultimate load testing, several steps were taken to improve their performance. To improve bonding characteristics, all bars were washed to remove grease, they were then wetted and left to rust for three days. In addition to providing adequate development lengths, all bars were given a three to six inch 90 degree bend at the fixed sides. The four inch spacing for the mesh provided adequate ductility as well as an acceptable ultimate strength. Figure 25 shows the typical reinforcement pattern. Smaller spacings would have made the ductility inadequate, would have been much more work to construct, and would have been more difficult to pour concrete through. A larger spacing on the other hand would give an unacceptably small ultimate strength and would provide too few bars for an experiment of this scale. A ultimate load capacity of the slab of approximately five psi was considered desirable. This reduced the possibility of problems with the airbag and required fewer

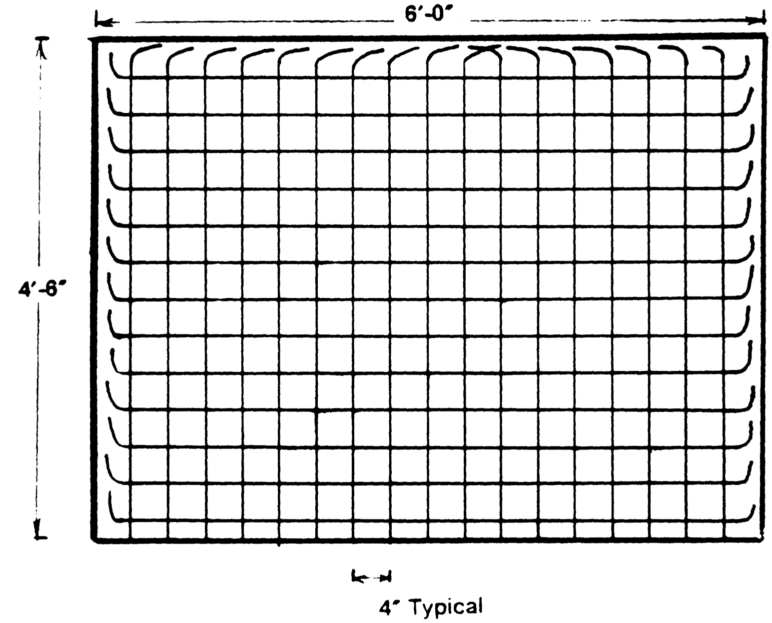


Figure 25. Typical reinforcement pattern

holes to be drilled through the concrete floor for anchorage. It was also large enough to reduce the ratio of percent increment of load to ultimate load to a desirable value.

The three locations of the openings were chosen so that they could be evaluated by the program. A central opening, an opening in the center of the free side, and an opening in the center of the opposing fixed side were used. The sizes of the openings were to be kept relatively small due to the possible effects of the airbag bubbling through the holes. It was originally assumed that with small holes, the openings and their edges would recieve no loading. The exact dimensions of the openings were restricted due to considering the accuracy of the moment capacities. Yield-line analysis considers moment capacities as moment per unit length, while an actual slab has steel spaced at a certain distance. Therefore, opening sizes should be selected so that they would take away a representative length of concrete with respect to the steel within. For example, since the bars are spaced at four inches, an opening should extend to two inches past the last rebar cut, so that one bar would be cut with every four inches of concrete. One can see that this will limit the sizes of the openings to multiples of four inches. However, due to the symmetry constraints in the program which centers the opening, the opening dimension in the y-direction of each had to be either 4″, 12″, 20″, etc. (Refer to Figure 26). Likewise the length in the x-direction of the central opening had to be 8″, 16″, etc., while the length in the x-direction of the remaining two slabs could be any multiple of 4″. Possible opening sizes were run through the program and those with the most distinctive yield patterns were chosen. The following sizes were selected. The opening on the long fixed edge was 1′-0″ by 1′-0″, the opening on the long free edge was 1′-0″ by 1′-0″, and the central opening was 1′-0″ by 1′-4″. From this point forward the slabs will be designated by the following numbers.

- Slab #1 - Solid slab

- Slab #2 - Slab with opening on the fixed edge

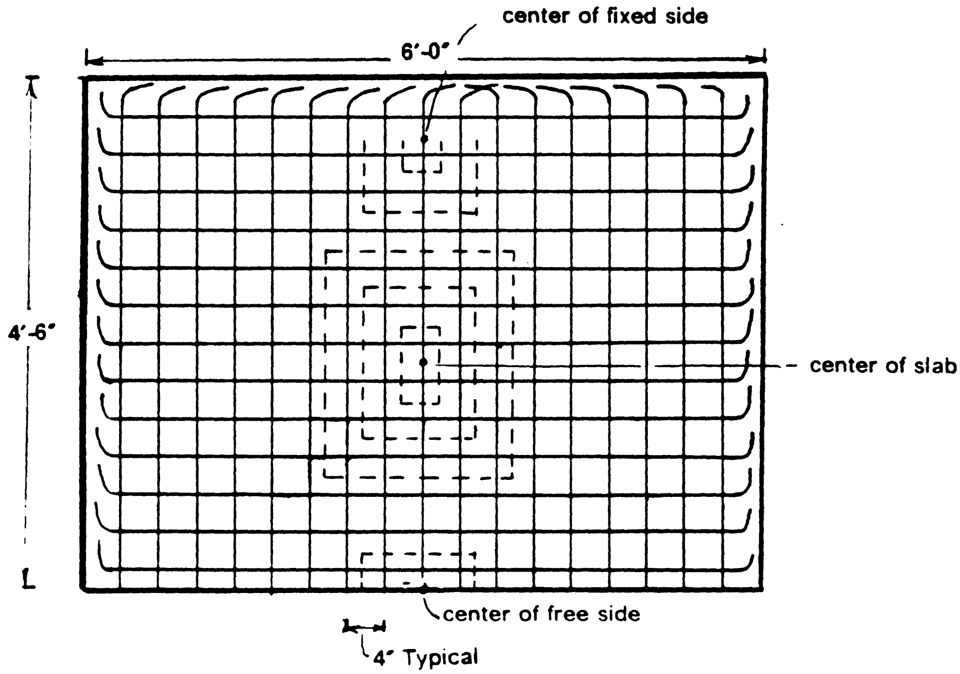


Figure 26. Restriction of opening sizes

- Slab #3 - Slab with opening on the free edge

- Slab #4 - Slab with central opening

Although it is done in practice, no extra reinforcement was added around the openings in order to avoid additional strength.

4.4 Formwork

The forms for the four slabs were built adjacent to each other to save on materials. It consisted first of platform supports built of 2 by 4's which could be jacked up to level the slab on the uneven floor. It was nailed together to be 12'-0" by 9'-3" with supports along lines where the plywood would meet. Refer to Figure 27 for a sketch of the formwork. 2 by 4's were also placed across the third points of each sheet of plywood to prevent sagging under the concrete weight. 3/4 inch tounge and groove plywood was nailed on to the supports to become the bottom of the form. Finally, 1 by 2's were nailed on top of the plywood to form the sides of the slabs. These were placed along the edges and across the the middle separating the area into four portions. This not only defined the shape of the slab but also gave it its proper one and one-half inch height. Scrap 2 by 4's were nailed around the outside, connecting the 1 by 2's and the platform. This strengthened 1 by 2's and prevented their falling off or bowing under the lateral concrete loads.

4.5 Construction

Sheets of thin plastic were used to cover the bottom and sides of the forms to prevent them from bonding to the concrete. The reinforcement was bent, tied into the mesh, and placed into

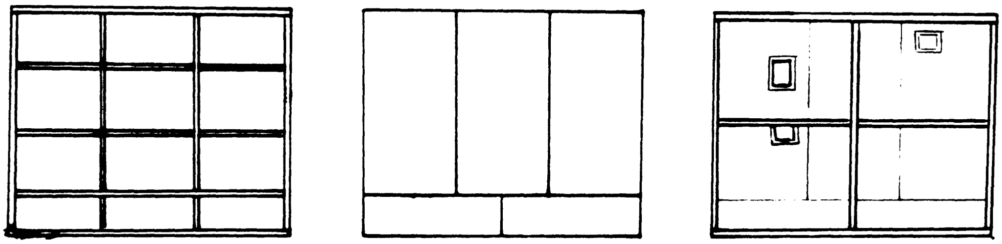


Figure 27. Formwork

the forms. Small pieces of one-half inch diameter dowel were placed under the bottom layer of reinforcement, lifting it off of the forms and assuring the proper clear distance. The dowels were located under every bar along edges which were to be fixed since this would be outside of the loaded area and would not affect the strength of the slab. A few extra pieces were necessary within the slab to minimize the sagging of the steel under its own weight and the weight of the concrete during pouring. The reinforcement at the proper opening locations were cut with bolt cutters and 1 by 2's, which had been soaked in oil to prevent bonding, were built to the size of the openings and nailed down.

The concrete was to be mixed in the laboratory; however, due to the relatively small mixer the pouring would be long and difficult and would probably result in a non-uniform slab. Ready-mixed concrete was ordered to insure a uniform mix as well as a shorter pouring period. A compressive strength of 4000 psi was desired along with a slump of four inches for adequate workability. The concrete was poured, vibrated, and finished in approximately two hours. Special care had to be taken so that it was vibrated enough to get a nice uniform surface yet not too much so that separation occurred. After the slab had sufficently hardened, it was covered with wet burlap sacks and plastic sheeting over top. The slab was kept moist for 28 days to attain full strength. Refer to Figure 28 for a photo of the cured slabs on their forms.

4.6 Instrumentation

The slabs were instrumented with sixteen linear potentiometer deflection gages. These were mounted along the free edge and the center line of the slab as shown by the photo in Figure 29. The numbering scheme for the gages is shown in Figure 30. Due to the nature of the loading and edge conditions, all deflections should be symmetrical about the center line, and the maximum deflection should in most cases occur along this line. The deflection may

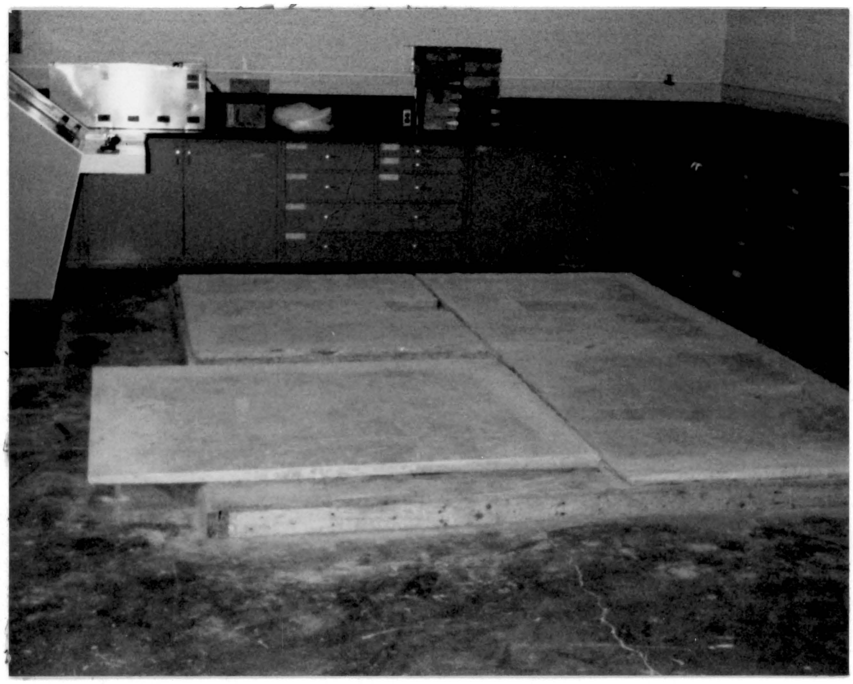


Figure 28. Cured slabs

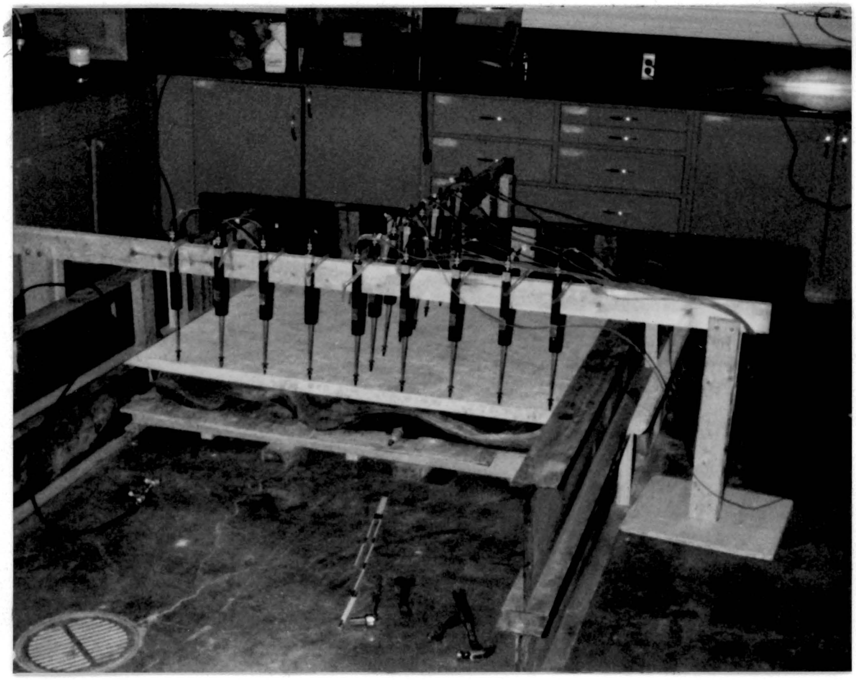


Figure 29. Instrumentation set-up

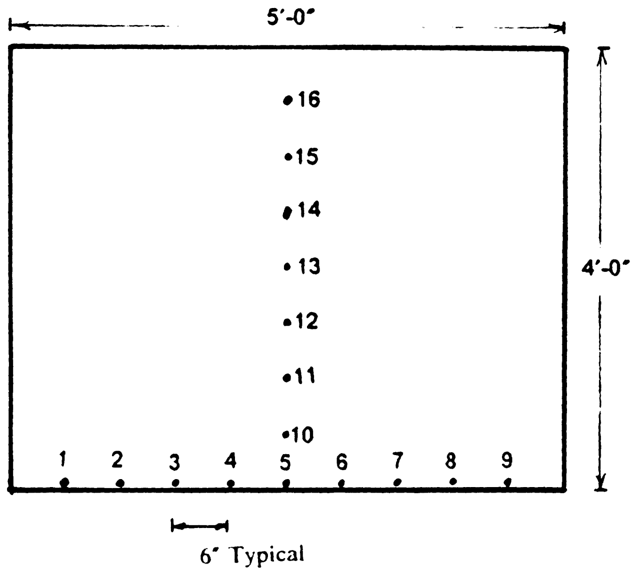


Figure 30. Numbering scheme for deflection gages

not be along this line if there is an opening located at the center of the free edge. The airbag was instrumented with a double-U shaped manometer which is calibrated to read to the nearest hundredth of a psi. The manometer stood approximately nine feet tall and contained water in the two chambers, having the capacity of measuring pressures above seven psi.

4.7 Concrete and Steel Test Results

Five steel reinforcing bars were tested on a Tinius Olsen machine. Several diameter measurements were taken with a micrometer and it was determined that the bars had an average area of 0.0503 square inches. It was found that the average yield stress was 89.94 ksi and the average stress at failure was 92.29 ksi.

Two concrete cylinders were cast in 6" by 12" cylinders. In order to estimate the slab dead load, the cylinders were weighed and were found to average 149 pcf. After 39 days they were tested, fracturing at 123.5 k and 129.5 k. The average compressive stress was 4475 psi. Tables 1 and 2 summarize the material test results.

4.8 Testing Procedure

Once the slab was in place on the testing apparatus and was instrumented, inflation could begin. Compressed air was provided in the test room and was supplied through a pressure line with a throttling knob to control flow. The outflow from the throttling device was connected to a "T" fixture, directing the flow to both the manometer and the airbag. The airbag was first inflated to 0.13 psi to counteract the dead weight of the slab and initial deflection readings were taken. The airbag was thereafter inflated in increments of approximately one fifth of a psi. At each increment, the manometer was read and recorded, and the gages were read by

Table 1. Steel Tensile Test Results

Specimen	Ultimate Load (lb)	Yield Load (lb)	Yield Stress (ksi)
1	4580	4370	86.9
2	4630	4460	88.7
3	4720	4630	92.1
4	4630	4570	90.9
5	4650	4580	91.1

Table 2. Concrete Compression Test Results

Specimen	Compressive Load (kips)	Compressive Stress (ksi)
1	123.5	4.37
2	129.5	4.58

the data aquisition system every 30 seconds and stored on disk. This continued past first yielding and to failure. Crack patterns during loading were observed and recorded by photographs at every one psi interval. A photo of a slab nearing failure is shown in Figure 31. Loading was stopped when the slab showed no sign of increased strength over a few loadings. Though the slabs could have been loaded further to make the results more defined, the tests were stopped to preserve the testing apparatus.

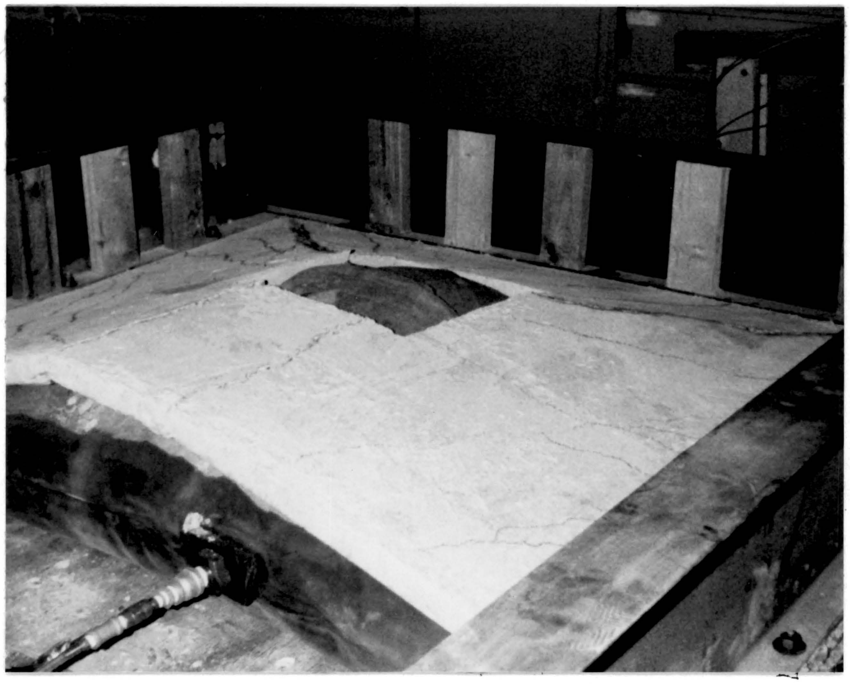


Figure 31. Slab nearing collapse

Chapter V

DISCUSSION OF RESULTS

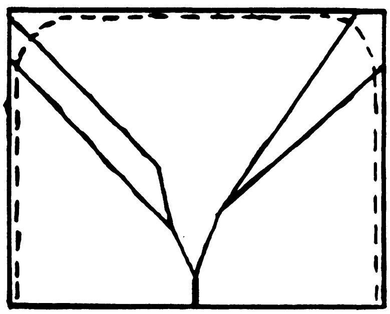
In this section, a discussion of the test results is presented. Comments are made on the nature and accuracy of the observations. A comparison between predicted and experimental load capacity is given.

The raw data collected was adjusted to take into account the angle of the manometer and the subtraction of the dead load. The revised pressure readings were then matched with the deflection readings, tabulated and plotted. In reviewing the deflection data, it was discovered that one of the disks was bad and recorded only one-half of the deflection readings for slab #2. The ultimate load and yield pattern however were obtained.

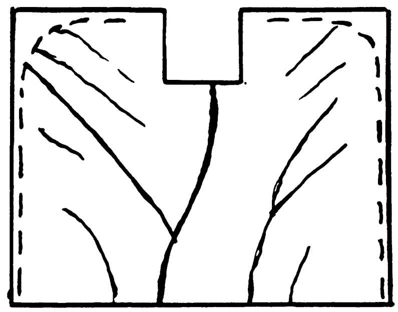
During the testing, It was observed that the inflated airbag did not quite reach the free edge of the slab, and therefore a fully uniform load was not achieved. This effect was considered small and was neglected. First cracking typically occured between 1.5 and 2.0 psi. Major crack widths at the end of testing were generally between 1/8 inch and 1/4 inch with the largest being 1/2 inch. Because of the small crack widths, it was not apparent whether kinking of the reinforcement had occured. At higher loadings and greater deflections, the concrete on the outside of the beams cracked and popped off in chunks exposing some reinforcement. It was observed that instead of the top flanges of the anchored beams being twisted outwards from moment at the support, they were pulled inward from in-plane forces. At the greatest loading, the length of the long side decreased 1" from 4'-11" to 4'-10". At no time was there a noticable rotation along the supports.

There was some difficulty reading the manometer to accurately correspond to the deflection gages. Each had great precision, the manometer to the hundredth of a psi and the gages to the thousandth of an inch. However, there may be some question as to how to match the two. Once the throttle was shut off, the concrete would continue to deflect for a short period of time, giving the airbag more room to expand and subsequently achieve a drop in pressure. These post-loading changes were usually quite small so the readings shortly after the shutting of the valve produced good results. The deflection versus loading plots show that these readings were quite reasonable. Figure 32, shows the failure patterns and ultimate loads for the slabs. The following Tables and Figures will be presented at the end of the chapter: Tables 5 through 12 tabulate the deflections versus loading, plots of these are shown in Figures 33 through 40 (comparing gage readings from opposite sides of the centerline show that symmetry in deflection is fairly well maintained), profiles of deflection along the instrumented lines are shown in Figures 41 through 44, and a typical rotation versus load plot is shown in Figure 45.

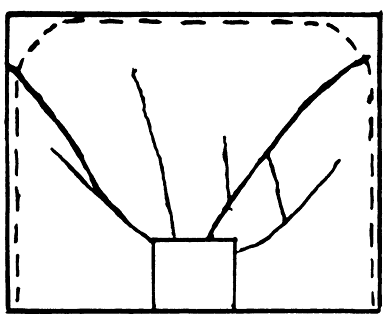
The yield-line patterns were fairly consisent with those predicted by yield-line theory. Slabs 1, 3, and 4 showed distinct failure lines with near-rigid regions contained within. Slab 2, however, did not develop the distinct pattern expected. Because of an early formation of a full center crack line, the slab tended to rotate as two regions about the two opposing fixed edges. As the boundary conditions resisted this type of rotation, the areas on the fixed edge near the opening underwent serious spalling. A series of smaller regions were formed on either side of the center yield-line and no other prominent cracks appeared.



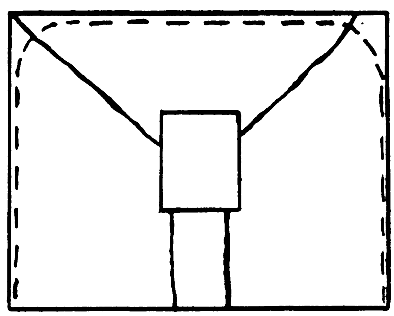
Slab #1 6.00 psi



Slab #2 6.00 psi



Slab #3 5.65 psi



Slab #4 5.53 psi

Figure 32. Failure patterns and Ultimate Loads

It was expected that Slab 4 would contain a single yield-line from the opening to the free edge. A pair of yield-lines formed instead, each developing along a line of reinforcement. Analysis showed that the theoretical difference between these patterns is negligible.

The magnitude of deflections was much larger than expected with a maximum of 8-1/2 inches. This may at first give the impression that yield-line theory is of little practical use. However, it can be seen that the test slabs had a fairly large span to depth ratio, practical designs with thicker slabs and longer spans will not have as serious of a deflection problem. While the moment is a function of the length squared, the slab's moment of inertia is a function of the depth of the section cubed; therefore, the ratio of deflection to span length should be much less. These tests do show, however, that the deflection may present some problem and should be checked when using yield-line theory for design.

When comparing the actual collapse load to theoretical predictions, it should be remembered that the ultimate strength of a slab is dependent on the properties and dimensions of the slab and its materials. Since these properties and dimensions vary, a precise value for the ultimate strength is not possible, only an approximate value based on averages can be given. Values for the ultimate moment capacity of the slab was calculated by: $M = A_s f_y \left[d - \frac{a}{2} \right]$. Several thickness measurements were taken from the slab and the reinforcement and were averaged as were the concrete compressive strengths and the steel tensile strengths. These averages were used to determine the following moment capacities in the various directions : MYT = 0.664 k ft/ft, MYB = 0.821 k ft/ft, MXT = 0.946 k ft/ft, and MXB = 0.539 k ft/ft.

The computer program was used to analyze the ultimate strength and the yield pattern of each slab by simple theory. Supplemental programs were written for the test cases to include the effects of corner levers. They were run for each slab to compare the percent difference in capacity. This showed that the presence of corner levers reduced the ultimate strength by over seven percent. It is noted that the supplemental program did not attempt to analyze all possible cases. Only simple corner levers were considered and an additional reduction of up to two percent may exist. Table 3 tabulates the simple yield-line theory predictions, the complex yield-line predictions including corner levers, and the experimental results of the slabs. The values in parentheses denote the percent difference between each of the theoretical predictions and the experimental results.

The results show that the experimental load was from slightly to fairly lower than those predicted by yield-line theory with simple corner levers. Predictions were from 1.2 to 16.0 percent high. Although these predictions will be slightly lower due to patterns with complex fan shapes, this does not seem to account for the differences that are apparent.

It is interesting to notice that the solid slab and the slab with the opening on the fixed edge give agreeable results between the theoretical and the experimental, both being within four percent, while the two remaining slabs had failed at a load much lower than predicted by yield-line theory. The presence of membrane action or kinking would have only raised the theoretical predictions, making them less believable.

It becomes apparent that one of the original assumptions was incorrect. It was originally assumed that the slab was being uniformly loaded on all portions except for over the openings. If the forces from the exposed area of the airbag were being distributed to the edges of the opening instead of to the in-plane forces in the airbag, this could explain the low loads for slabs 3 & 4 where the openings were located where the deflection was large. The solid slab would obviously not be affected, and the slab with the opening on the fixed edge would be only slightly affected as its opening was located at a point of low deflection.

With this in mind, the set of specialized programs were altered to include an edge load around the opening from a uniform load acting on an area the size of the opening. The work done by this concentrated edge load acting through an approximate deflection was added to the energy expression. It was estimated that the average deflections were 17, 90, and 80

Table 3. Predictions and Results

	Ultimate Load (psi)		
Slab	**Simple**	**Complex**	**Experimental**
#1 Solid	6.69 (+10.3)	6.25 (+ 4.0)	6.00
#2 Fixed	6.55 (+ 8.4)	6.07 (+ 1.2)	6.00
#3 Free	6.85 (+17.5)	6.32 (+10.6)	5.65
#4 Central	7.13 (+22.4)	6.58 (+16.0)	5.53

percent of the maximum deflection of slabs 2, 3, and 4 respectively. The results of this analysis are presented in Table 4. The results are now excellent, all falling within four percent. The theoretical values were scattered between 1.4 percent low to 4.0 percent high.

Because membrane action increases load carrying capacity, it is wondered what part membrane action had to play. Membrane action was apparent by the shortening of the long edge. However, the membrane action could not take full effect due to the obvious lack of stiff horizontal restraint. The popping off of the concrete reduced the amount of bond and may have allowed some slippage. Also, as shown by Narsimhan and Verreyya (1978), because the openings cut up to 33 percent of the reinforcement, a good portion of membrane action may have been prevented. These are several reasons why membrane action did not take full effect. It appears that only a small amount of membrane action took place, leaving the final results intact.

Table 4. Revised Predicitons Considering Edge Loads

	Ultimate Load (psi)		
Slab	Old Complex	Complex w/ edge loads	Experimental
#1 Solid	6.25 (+ 4.0)	6.25 (+4.0)	6.00
#2 Fixed	6.07 (+ 1.2)	5.93 (-1.2)	6.00
#3 Free	6.32 (+10.6)	5.57 (-1.4)	5.65
#4 Central	6.58 (+16.0)	5.69 (+2.8)	5.53

Table 5. Table of Deflections - Slab #1 - (Gages 1-8)

Load	Deflection (in)							
(psi)	1	2	3	4	5	6	7	8
0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.16	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
0.76	0.03	0.05	0.07	0.08	0.08	0.08	0.07	0.02
1.09	0.05	0.08	0.10	0.12	0.12	0.12	0.11	0.05
1.49	0.07	0.12	0.16	0.18	0.19	0.18	0.16	0.09
1.81	0.11	0.18	0.23	0.28	0.29	0.27	0.24	0.14
1.98	0.13	0.23	0.29	0.31	0.34	0.30	0.28	0.16
2.18	0.16	0.25	0.34	0.42	0.46	0.42	0.37	0.23
2.39	0.18	0.32	0.43	0.52	0.55	0.51	0.44	0.28
2.48	0.21	0.36	0.50	0.61	0.65	0.62	0.52	0.34
2.79	0.26	0.44	0.59	0.76	0.81	0.77	0.64	0.42
2.94	0.28	0.48	0.67	0.82	0.89	0.83	0.69	0.45
3.07	0.32	0.56	0.78	0.95	1.02	0.96	0.78	0.52
3.27	0.37	0.64	0.90	1.09	1.17	1.10	0.89	0.59
3.47	0.40	0.71	1.00	1.21	1.30	1.21	0.98	0.65
3.71	0.45	0.79	1.12	1.34	1.44	1.34	1.08	0.72
3.94	0.50	0.89	1.25	1.52	1.61	1.50	1.21	0.81
4.15	0.55	0.99	1.40	1.68	1.80	1.67	1.34	0.91
4.35	0.60	1.09	1.54	1.85	1.98	1.83	1.48	0.99
4.60	0.68	1.23	1.72	2.08	2.22	2.04	1.63	1.10
4.74	0.71	1.30	1.83	2.21	2.37	2.16	1.73	1.17
4.87	0.76	1.39	1.96	2.37	2.54	2.32	1.85	1.25
5.06	0.81	1.50	2.12	2.56	2.76	2.50	1.99	1.35
5.24	0.86	1.59	2.25	2.73	2.96	2.67	2.12	1.43
5.43	0.92	1.70	2.41	2.94	3.19	2.87	2.27	1.53
5.58	0.97	1.81	2.57	3.13	3.42	3.04	2.41	1.63
5.69	1.03	1.93	2.75	3.37	3.71	3.27	2.58	1.74
5.82	1.11	2.08	2.97	3.67	4.04	3.55	2.77	1.87
5.98	1.18	2.22	3.18	3.95	4.41	3.84	2.99	2.01
6.00	1.25	2.38	3.43	4.28	4.83	4.18	3.24	2.18

Table 6. Table of Deflections - Slab #1 - (Gages 9-16)

Load	Deflection (in)							
(psi)	9	10	11	12	13	14	15	16
0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.16	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
0.76	0.03	0.07	0.06	0.05	0.05	0.04	0.03	0.02
1.09	0.05	0.11	0.10	0.08	0.08	0.06	0.05	0.02
1.49	0.06	0.17	0.16	0.13	0.12	0.10	0.07	0.04
1.81	0.10	0.26	0.24	0.21	0.18	0.15	0.11	0.07
1.98	0.12	0.31	0.26	0.24	0.22	0.18	0.13	0.08
2.18	0.15	0.41	0.38	0.32	0.28	0.23	0.17	0.10
2.39	0.18	0.51	0.45	0.39	0.33	0.27	0.20	0.11
2.48	0.21	0.60	0.55	0.48	0.42	0.34	0.25	0.14
2.79	0.25	0.76	0.69	0.61	0.52	0.42	0.30	0.17
2.94	0.26	0.83	0.76	0.67	0.57	0.45	0.33	0.19
3.07	0.30	0.95	0.88	0.77	0.66	0.52	0.38	0.21
3.27	0.33	1.10	1.02	0.91	0.78	0.61	0.45	0.25
3.47	0.36	1.22	1.14	1.01	0.87	0.68	0.50	0.27
3.71	0.40	1.36	1.27	1.13	0.98	0.76	0.54	0.29
3.94	0.45	1.54	1.43	1.28	1.10	0.86	0.61	0.33
4.15	0.50	1.71	1.60	1.42	1.23	0.96	0.69	0.36
4.35	0.54	1.88	1.75	1.57	1.35	1.05	0.76	0.40
4.60	0.61	2.11	1.96	1.75	1.51	1.18	0.84	0.45
4.74	0.64	2.24	2.09	1.87	1.62	1.26	0.89	0.47
4.87	0.68	2.42	2.25	2.00	1.74	1.35	0.96	0.51
5.06	0.73	2.62	2.43	2.17	1.88	1.46	1.04	0.55
5.24	0.77	2.81	2.60	2.33	2.01	1.56	1.11	0.59
5.43	0.82	3.03	2.80	2.50	2.16	1.68	1.19	0.63
5.58	0.87	3.24	2.99	2.67	2.30	1.79	1.27	0.67
5.69	0.92	3.50	3.23	2.87	2.48	1.92	1.36	0.71
5.82	1.00	3.83	3.52	3.12	2.70	2.09	1.48	0.77
5.98	1.06	4.18	3.85	3.39	2.92	2.26	1.59	0.83
6.00	1.13	4.59	4.22	3.71	3.19	2.46	1.73	0.90

Table 7. Table of Deflections - Slab #2 - (Gages 1-8)

Load (psi)	Deflection (in) 1	2	3	4	5	6	7	8
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
0.31	0.01	0.01	0.02	0.02	0.00	0.00	0.02	0.01
0.66	0.02	0.04	0.05	0.06	0.00	0.00	0.06	0.04
0.93	0.04	0.06	0.08	0.09	0.03	0.01	0.09	0.07
1.15	0.07	0.11	0.15	0.18	0.11	0.07	0.15	0.11
1.35	0.09	0.14	0.19	0.24	0.17	0.12	0.19	0.15
1.37	0.09	0.15	0.20	0.25	0.18	0.13	0.20	0.15
1.58	0.12	0.19	0.26	0.32	0.27	0.22	0.27	0.20
1.71	0.13	0.22	0.29	0.36	0.31	0.27	0.32	0.23
1.82	0.14	0.24	0.31	0.40	0.35	0.30	0.35	0.26
1.88	0.16	0.28	0.37	0.47	0.42	0.37	0.41	0.30
1.98	0.18	0.30	0.42	0.54	0.49	0.44	0.47	0.34
2.22	0.21	0.36	0.50	0.63	0.59	0.54	0.56	0.41
2.36	0.23	0.39	0.53	0.67	0.64	0.59	0.60	0.43
2.55	0.26	0.47	0.66	0.83	0.79	0.72	0.71	0.52
2.73	0.31	0.54	0.76	0.96	0.94	0.86	0.84	0.60
2.84	0.33	0.59	0.83	1.05	1.04	0.96	0.93	0.66
3.03	0.37	0.67	0.95	1.20	1.20	1.11	1.06	0.76
3.27	0.42	0.76	1.08	1.37	1.38	1.29	1.21	0.86
3.43	0.47	0.86	1.22	1.56	1.60	1.50	1.38	0.98
3.59	0.51	0.93	1.32	1.68	1.73	1.62	1.48	1.05
3.79	0.58	1.08	1.54	1.96	2.02	1.88	1.68	1.19
3.89	0.62	1.13	1.63	2.07	2.14	1.99	1.77	1.25
3.98	0.68	1.26	1.82	2.34	2.43	2.24	1.97	1.40
4.16	0.75	1.38	1.99	2.56	2.67	2.44	2.14	1.51
4.28	0.79	1.48	2.12	2.75	2.86	2.60	2.26	1.61
4.40	0.84	1.57	2.26	2.93	3.05	2.77	2.39	1.69
4.53	0.92	1.72	2.48	3.22	3.35	3.01	2.58	1.83
4.71	0.99	1.88	2.71	3.52	3.66	3.25	2.77	1.97
4.88	1.07	2.02	2.92	3.79	3.94	3.49	2.95	2.09
4.93	1.14	2.16	3.13	4.07	4.22	3.71	3.12	2.20
5.10	1.24	2.35	3.41	4.44	4.61	4.02	3.38	2.37
5.19	1.32	2.51	3.66	4.77	4.95	4.32	3.62	2.53
5.45	1.43	2.76	4.05	5.27	5.48	4.76	3.97	2.76
5.59	1.50	2.92	4.27	5.55	5.79	5.04	4.19	2.91
5.66	1.56	3.07	4.51	5.87	6.14	5.33	4.43	3.05
5.80	1.63	3.23	4.77	6.20	6.52	5.64	4.69	3.22
5.92	1.66	3.39	5.01	6.45	6.88	5.95	4.93	3.38
5.93	1.71	3.59	5.34	6.84	7.15	6.33	5.27	3.62
5.97	1.79	3.78	5.63	7.35	7.69	6.64	5.53	3.81
6.00	1.86	3.93	5.86	7.73	8.06	6.94	5.78	3.99

Table 8. Table of Deflections - Slab #2 - (Gages 9-16)

Load (psi)	Deflection (in) 9	10	11	12	13	14	15
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.07	0.00	0.00	0.00	0.00	0.01	0.01	0.00
0.31	0.01	0.02	0.00	0.02	0.02	0.01	0.02
0.66	0.02	0.64	0.02	0.04	0.05	0.04	0.04
0.93	0.04	0.09	0.05	0.08	0.08	0.08	0.07
1.15	0.07	0.18	0.14	0.16	0.16	0.13	0.12
1.35	0.08	0.23	0.19	0.21	0.20	0.17	0.15
1.37	0.08	0.24	0.20	0.21	0.20	0.18	0.15
1.58	0.12	0.33	0.28	0.30	0.27	0.24	0.20
1.71	0.14	0.37	0.32	0.33	0.30	0.26	0.22
1.82	0.16	0.40	0.36	0.37	0.33	0.28	0.24
1.88	0.18	0.48	0.44	0.45	0.41	0.34	0.29
1.98	0.21	0.55	0.51	0.52	0.47	0.40	0.32
2.22	0.24	0.65	0.61	0.62	0.56	0.47	0.39
2.36	0.26	0.71	0.66	0.67	0.61	0.51	0.42
2.55	0.30	0.85	0.79	0.80	0.72	0.59	0.48
2.73	0.35	0.98	0.93	0.93	0.82	0.68	0.55
2.84	0.39	1.08	1.03	1.02	0.90	0.75	0.60
3.03	0.45	1.25	1.18	1.18	1.04	0.87	0.71
3.27	0.50	1.42	1.36	1.34	1.19	0.99	0.82
3.43	0.56	1.63	1.55	1.53	1.35	1.13	0.94
3.59	0.59	1.76	1.68	1.65	1.46	1.23	1.01
3.79	0.67	2.03	1.95	1.91	1.68	1.42	1.17
3.89	0.70	2.15	2.07	2.02	1.78	1.50	1.24
3.98	0.79	2.44	2.33	2.26	1.97	1.66	1.37
4.16	0.85	2.67	2.56	2.47	2.17	1.82	1.52
4.28	0.89	2.86	2.74	2.64	2.31	1.94	1.63
4.40	0.95	3.06	2.93	2.82	2.48	2.07	1.74
4.53	1.02	3.35	3.21	3.10	2.72	2.28	1.90
4.71	1.09	3.64	3.50	3.37	2.96	2.48	2.07
4.88	1.15	3.92	3.77	3.62	3.18	2.67	2.24
4.93	1.20	4.20	4.03	3.87	3.40	2.87	2.41
5.10	1.29	4.58	4.39	4.21	3.69	3.13	2.63
5.19	1.37	4.91	4.71	4.50	3.96	3.36	2.85
5.45	1.47	5.41	5.19	4.94	4.37	3.69	3.16
5.59	1.54	5.73	5.46	5.23	4.62	3.91	3.36
5.66	1.59	6.04	5.76	5.50	4.87	4.12	3.57
5.80	1.67	6.40	6.09	5.81	5.15	4.37	3.81
5.92	1.75	6.75	6.40	6.11	5.41	4.61	4.00
5.93	1.85	7.16	6.80	6.46	5.73	4.88	4.22
5.97	1.94	7.49	8.22	6.74	5.96	5.10	4.38
6.00	2.03	7.81	8.49	7.00	6.21	5.29	4.55

Table 9. Table of Deflections - Slab #3 - (Gages 1-8)

Load	Deflection (in)						
(psi)	1	2	3	4	6	7	8
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.07	0.00	0.00	0.01	0.01	0.00	0.00	0.00
0.60	0.02	0.04	0.06	0.07	0.06	0.06	0.03
1.08	0.05	0.08	0.11	0.13	0.13	0.11	0.07
1.31	0.06	0.10	0.13	0.17	0.17	0.13	0.09
1.49	0.08	0.13	0.18	0.23	0.21	0.17	0.11
1.68	0.11	0.20	0.28	0.34	0.31	0.25	0.16
1.88	0.15	0.27	0.38	0.46	0.42	0.34	0.22
2.06	0.17	0.31	0.45	0.56	0.51	0.41	0.26
2.19	0.21	0.38	0.55	0.69	0.50	0.33	0.18
2.42	0.24	0.45	0.64	0.80	0.73	0.59	0.38
2.58	0.28	0.52	0.75	0.93	0.84	0.67	0.45
2.71	0.35	0.64	0.92	1.15	1.03	0.83	0.55
2.91	0.42	0.76	1.10	1.39	1.24	1.00	0.66
3.03	0.49	0.89	1.30	1.65	1.49	1.20	0.81
3.22	0.53	0.97	1.42	1.80	1.65	1.32	0.88
3.32	0.57	1.07	1.55	1.97	1.85	1.47	0.99
3.60	0.66	1.23	1.79	2.28	2.19	1.75	1.17
3.73	0.70	1.31	1.92	2.43	2.38	1.90	1.27
3.95	0.76	1.42	2.07	2.63	2.61	2.07	1.39
4.15	0.82	1.55	2.26	2.87	2.87	2.28	1.52
4.31	0.87	1.65	2.40	3.05	3.07	2.43	1.63
4.48	0.94	1.78	2.59	3.29	3.33	2.64	1.77

Table 10. Table of Deflections - Slab #3 - (Gages 9-16)

Load	Deflection (in)						
(psi)	9	11	12	13	14	15	16
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.60	0.03	0.00	0.05	0.04	0.03	0.03	0.00
1.08	0.05	0.06	0.10	0.09	0.07	0.04	0.01
1.31	0.06	0.09	0.13	0.11	0.09	0.06	0.01
1.49	0.08	0.15	0.18	0.15	0.12	0.08	0.03
1.68	0.10	0.27	0.28	0.23	0.18	0.13	0.05
1.88	0.13	0.39	0.39	0.32	0.25	0.17	0.08
2.06	0.15	0.50	0.47	0.39	0.30	0.20	0.09
2.19	0.18	0.64	0.59	0.49	0.38	0.26	0.13
2.42	0.21	0.76	0.70	0.57	0.44	0.30	0.15
2.58	0.24	0.89	0.81	0.66	0.51	0.35	0.18
2.71	0.29	1.10	1.00	0.82	0.63	0.43	0.22
2.91	0.36	1.34	1.25	1.00	0.76	0.52	0.27
3.03	0.44	1.57	1.42	1.16	0.89	0.62	0.31
3.22	0.48	1.72	1.55	1.26	0.96	0.67	0.35
3.32	0.53	1.90	1.71	1.41	1.08	0.74	0.39
3.60	0.63	2.24	2.01	1.66	1.26	0.87	0.45
3.73	0.68	2.41	2.16	1.78	1.35	0.93	0.48
3.95	0.74	2.62	2.35	1.94	1.47	1.02	0.52
4.15	0.81	2.87	2.57	2.12	1.62	1.11	0.57
4.31	0.86	3.07	2.88	2.26	1.73	1.19	0.62
4.48	0.93	3.33	2.96	2.45	1.88	1.29	0.67
4.57	0.98	3.52	3.12	2.60	1.98	1.37	0.70
4.78	1.07	3.82	3.38	2.80	2.14	1.48	0.76
4.95	1.15	4.12	3.64	3.02	2.32	1.59	0.82
5.10	1.23	4.45	3.93	3.25	2.50	1.72	0.88
5.32	1.35	4.84	4.27	3.54	2.73	1.87	0.96
5.47	1.45	5.21	4.64	3.79	2 94	2.00	1.03
5.52	1.50	5.41	4.82	4.01	3.06	2.12	1.09

Table 11. Table of Deflections - Slab #4 - (Gages 1-8)

Load	Deflection (in)							
(psi)	1	2	3	4	5	6	7	8
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.37	0.02	0.03	0.04	0.04	0.04	0.00	0.00	0.03
0.55	0.03	0.04	0.05	0.07	0.07	0.00	0.01	0.04
0.82	0.05	0.07	0.09	0.11	0.11	0.00	0.04	0.07
1.02	0.06	0.09	0.11	0.13	0.14	0.00	0.05	0.08
1.21	0.07	0.11	0.14	0.16	0.17	0.03	0.08	0.10
1.37	0.10	0.15	0.18	0.22	0.23	0.09	0.13	0.13
1.55	0.13	0.20	0.25	0.30	0.32	0.17	0.19	0.18
1.67	0.14	0.21	0.28	0.33	0.36	0.21	0.22	0.20
1.81	0.16	0.25	0.32	0.38	0.41	0.25	0.25	0.23
1.92	0.17	0.27	0.35	0.43	0.46	0.31	0.30	0.26
2.05	0.21	0.34	0.44	0.54	0.58	0.42	0.38	0.32
2.21	0.25	0.40	0.53	0.66	0.70	0.53	0.47	0.38
2.40	0.27	0.45	0.61	0.76	0.81	0.62	0.54	0.43
2.55	0.30	0.49	0.66	0.83	0.89	0.70	0.60	0.47
2.68	0.36	0.61	0.82	1.03	1.10	0.90	0.75	0.57
2.80	0.39	0.65	0.89	1.17	1.19	0.97	0.81	0.62
2.94	0.42	0.70	0.96	1.21	1.29	1.07	0.89	0.68
3.09	0.45	0.75	1.02	1.29	1.38	1.16	0.96	0.72
3.24	0.48	0.80	1.10	1.40	1.50	1.27	1.05	0.78
3.40	0.51	0.87	1.20	1.51	1.63	1.39	1.14	0.84
3.58	0.56	0.95	1.30	1.66	1.78	1.53	1.25	0.92
3.75	0.60	1.02	1.42	1.81	1.95	1.69	1.37	1.00
3.87	0.65	1.12	1.56	1.97	2.10	1.83	1.48	1.07
4.04	0.71	1.25	1.74	2.21	2.35	2.05	1.64	1.19
4.24	0.78	1.37	1.94	2.45	2.60	2.27	1.81	1.30
4.36	0.85	1.51	2.14	2.71	2.85	2.50	1.98	1.42
4.47	0.91	1.61	2.29	2.91	3.08	2.71	2.14	1.53
4.62	0.97	1.75	2.50	3.18	3.37	2.98	2.34	1.66
4.70	1.05	1.92	2.74	3.48	3.74	3.35	2.64	1.88
4.83	1.15	2.10	3.01	3.83	4.17	3.78	2.96	2.10
4.98	1.27	2.35	3.39	4.32	4.70	4.27	3.33	2.35
5.01	1.36	2.55	3.70	4.72	5.10	4.60	3.57	2.51
5.06	1.46	2.77	4.05	5.17	5.55	4.99	3.85	2.69
5.10	1.61	3.10	4.57	5.83	6.16	5.50	4.30	2.96
5.37	1.71	3.36	5.00	6.39	6.69	5.93	4.61	3.17
5.53	1.78	3.56	5.32	6.82	7.11	6.24	4.86	3.33
5.45	1.86	3.84	5.78	7.44	7.67	6.65	5.11	3.53

Table 12. Table of Deflections - Slab #4 - (Gages 9-16)

Load	Deflection (in)				
(psi)	9	10	11	15	16
0.00	0.00	0.00	0.00	0.00	0.00
0.06	0.00	0.00	0.00	0.00	0.00
0.37	0.01	0.02	0.00	0.01	0.01
0.55	0.02	0.04	0.00	0.03	0.01
0.82	0.04	0.08	0.03	0.04	0.02
1.02	0.05	0.11	0.05	0.05	0.03
1.21	0.06	0.14	0.08	0.07	0.04
1.37	0.07	0.20	0.14	0.09	0.05
1.55	0.11	0.29	0.22	0.13	0.08
1.67	0.12	0.32	0.25	0.15	0.08
1.81	0.13	0.37	0.30	0.16	0.10
1.92	0.15	0.42	0.35	0.20	0.11
2.05	0.18	0.54	0.47	0.25	0.13
2.21	0.21	0.66	0.59	0.30	0.16
2.40	0.25	0.76	0.69	0.35	0.19
2.55	0.26	0.84	0.77	0.38	0.21
2.68	0.32	1.06	0.99	0.49	0.27
2.80	0.34	1.14	1.07	0.52	0.28
2.94	0.38	1.24	1.16	0.56	0.31
3.09	0.40	1.32	1.25	0.60	0.33
3.24	0.43	1.44	1.36	0.66	0.36
3.40	0.46	1.56	1.48	0.71	0.39
3.58	0.50	1.72	1.63	0.78	0.42
3.75	0.54	1.87	1.78	0.85	0.46
3.87	0.59	2.03	1.93	0.91	0.50
4.04	0.64	2.26	2.17	1.02	0.55
4.24	0.70	2.52	2.40	1.13	0.61
4.36	0.76	2.77	2.65	1.24	0.67
4.47	0.81	2.99	2.87	1.34	0.72
4.62	0.88	3.28	3.14	1.47	0.80
4.70	0.98	3.63	3.48	1.62	0.87
4.83	1.08	4.03	3.87	1.82	0.97
4.98	1.19	4.53	4.37	2.05	1.10
5.01	1.26	4.92	4.75	2.23	1.19
5.06	1.34	5.35	5.17	2.43	1.29
5.10	1.45	6.07	5.81	2.79	1.48
5.37	1.52	6.56	6.30	2.99	1.59
5.53	1.61	6.97	6.69	3.16	1.67
5.45	1.67	7.51	7.25	3.43	1.82

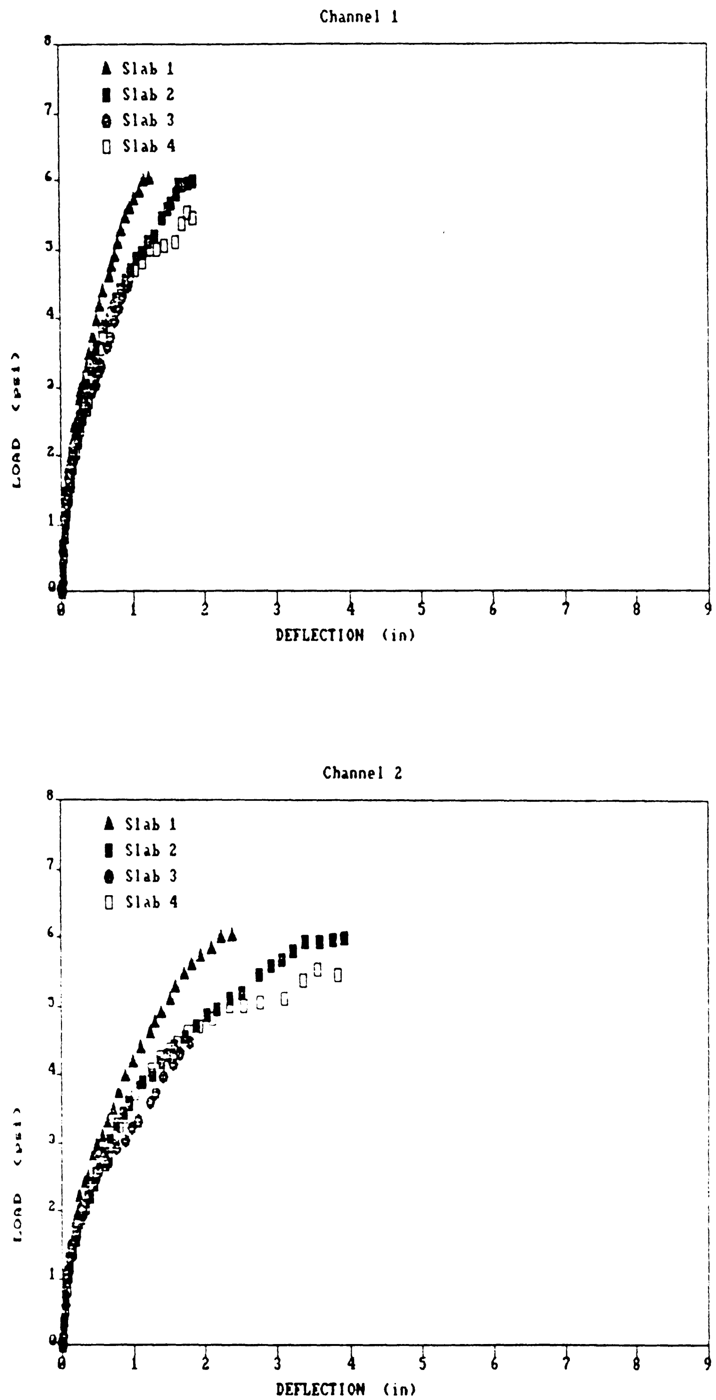
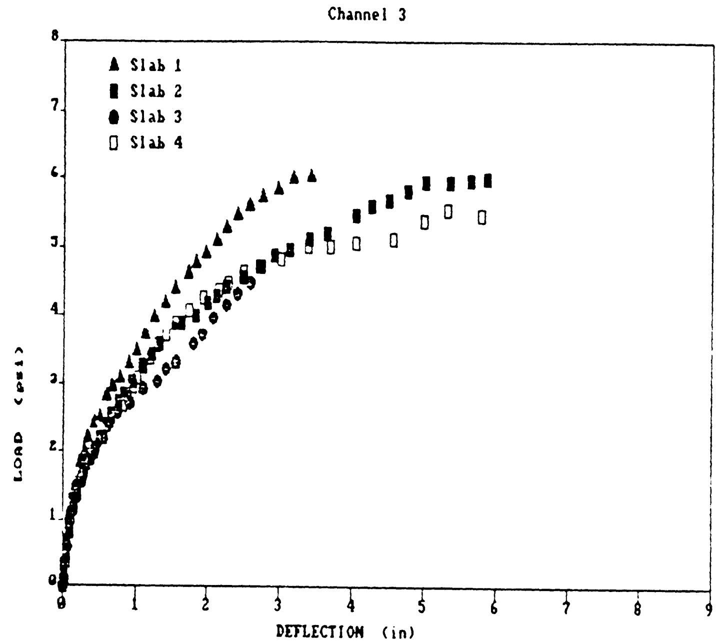


Figure 33. Deflection versus Load - Gages 1 & 2



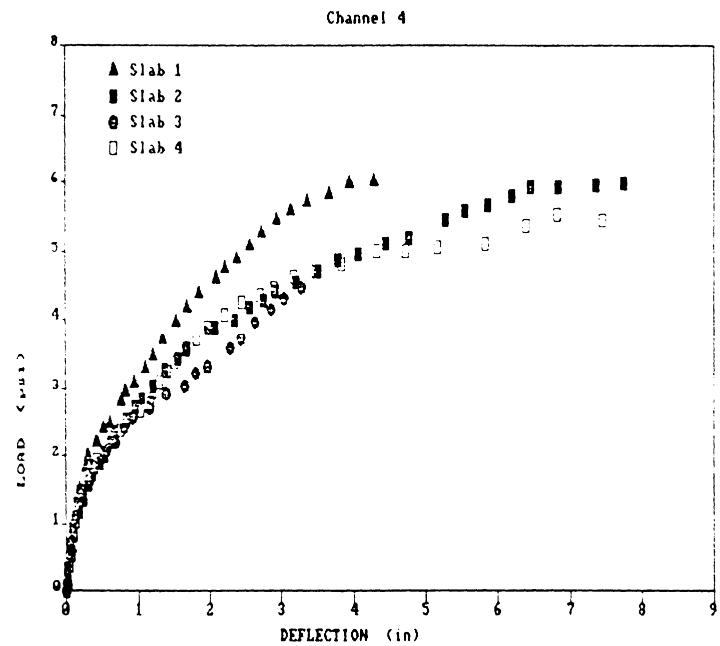
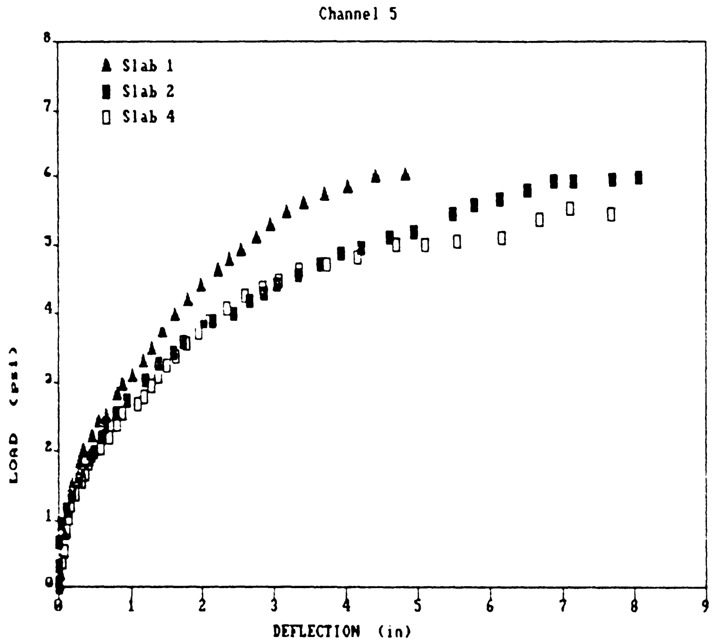


Figure 34. Deflection versus Load - Gages 3 & 4



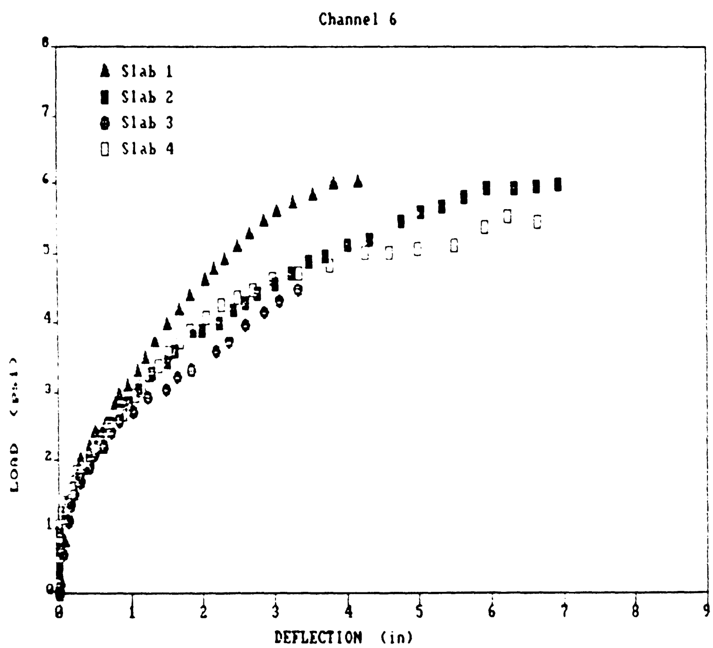
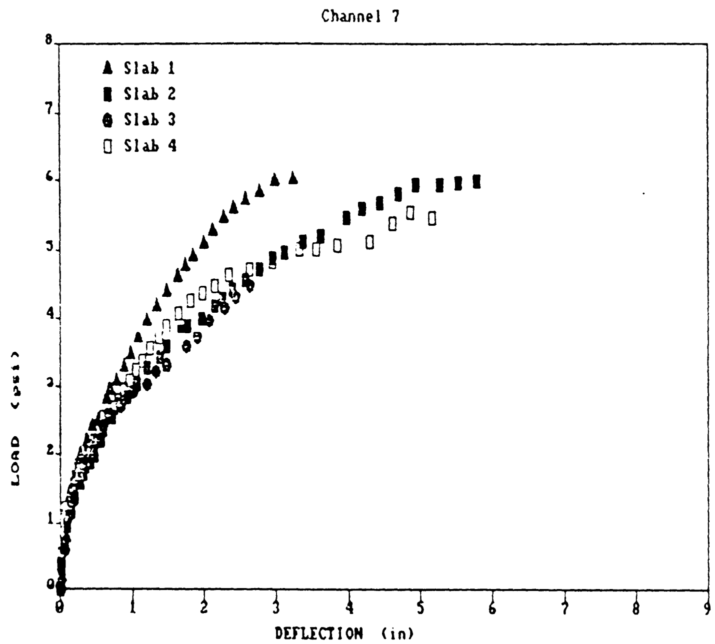


Figure 35. Deflection versus load - Gages 5 & 6



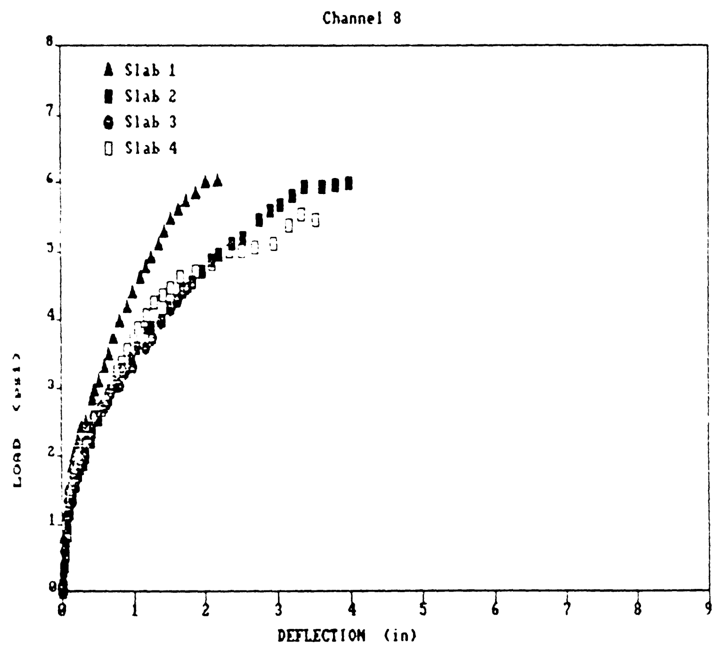
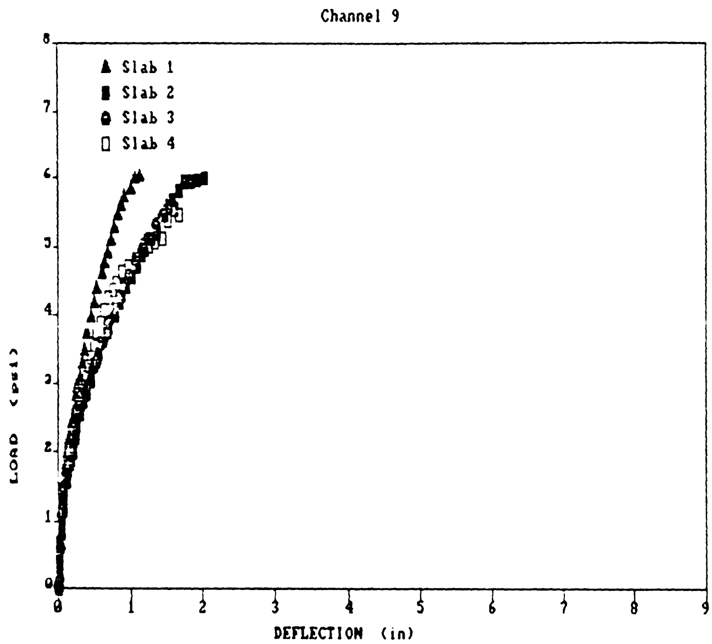


Figure 36. Deflection versus load - Gages 7 & 8



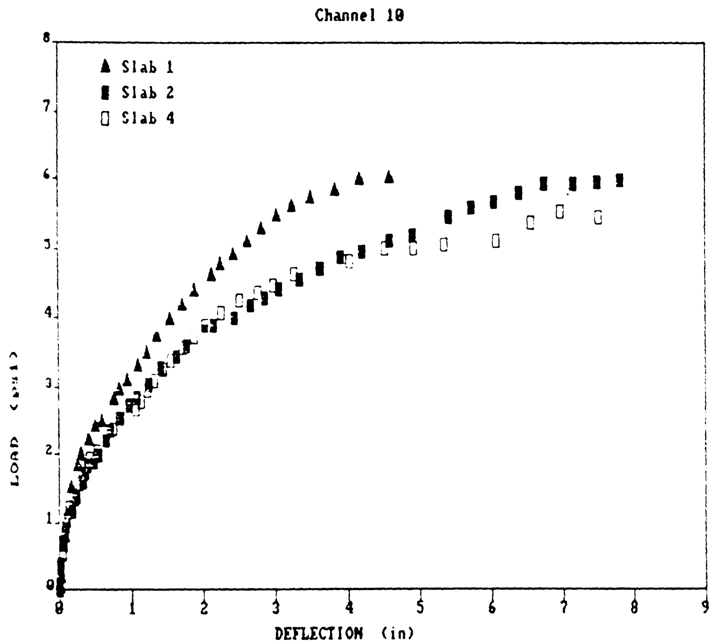
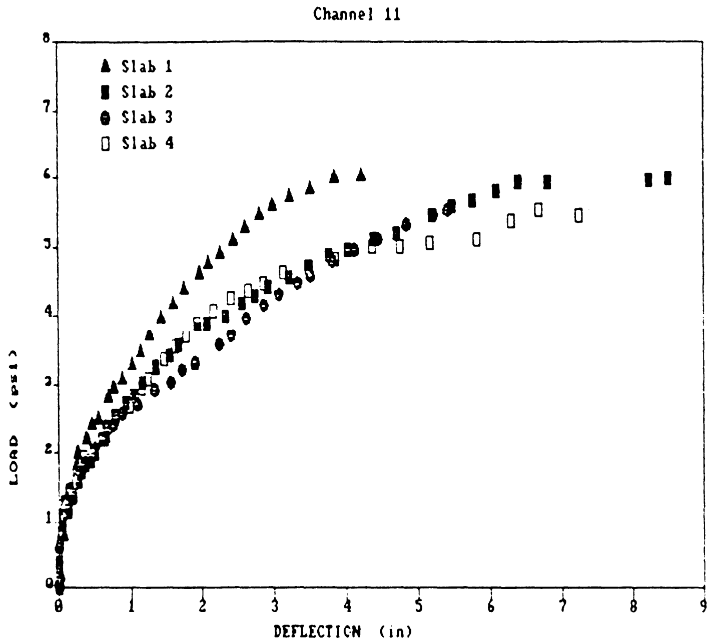


Figure 37. Deflection versus load - Gages 9 & 10



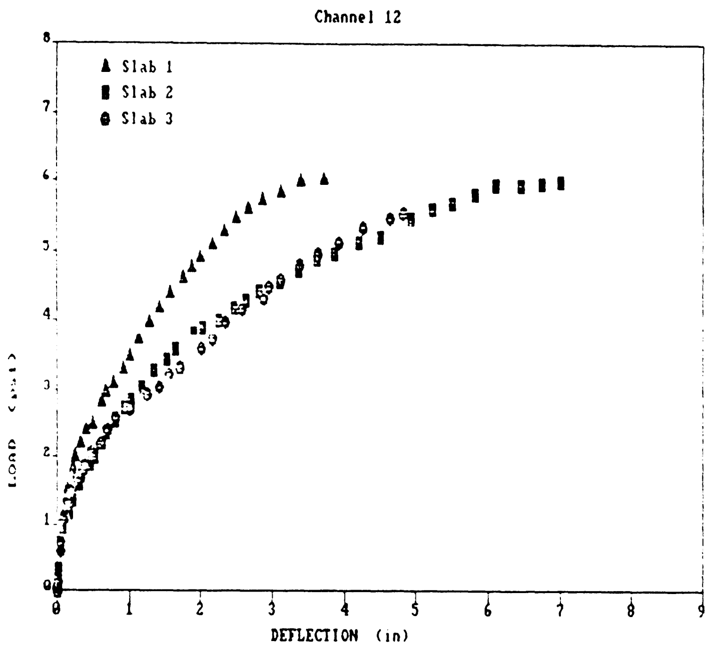
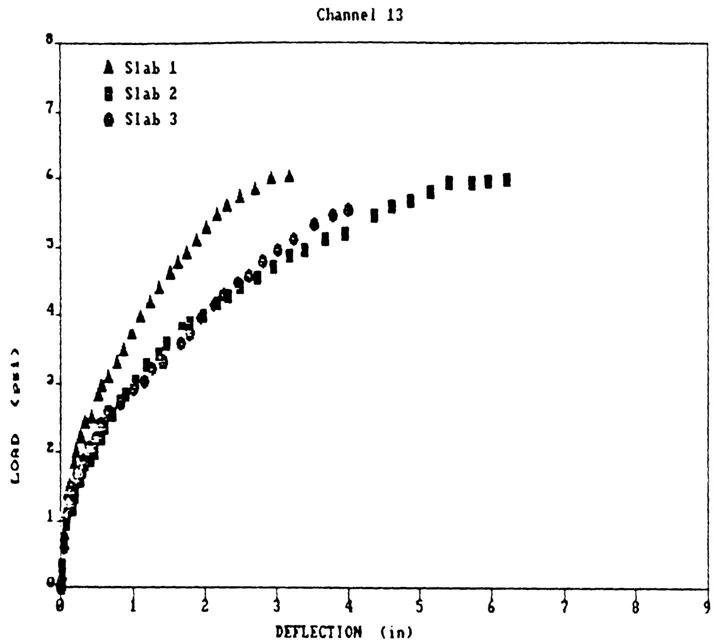


Figure 38. Deflection versus load - Gages 11 & 12



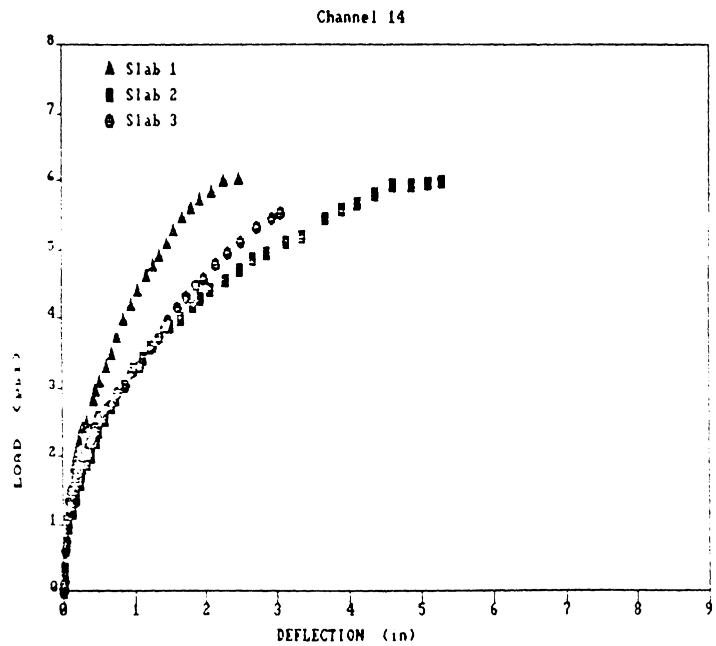
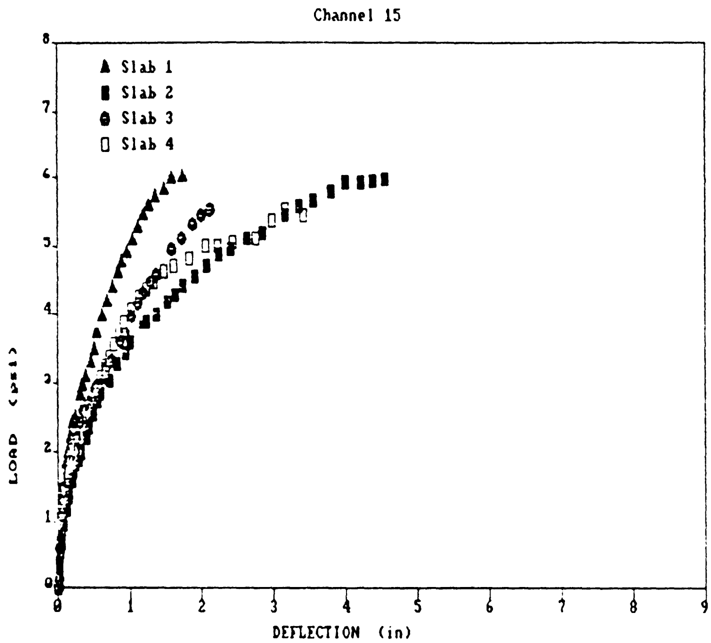


Figure 39. Deflection versus load - Gages 13 & 14



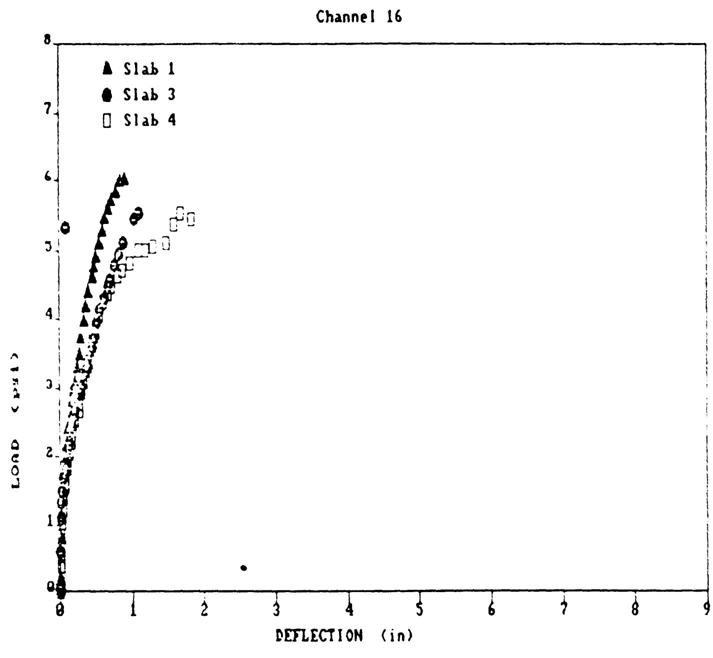
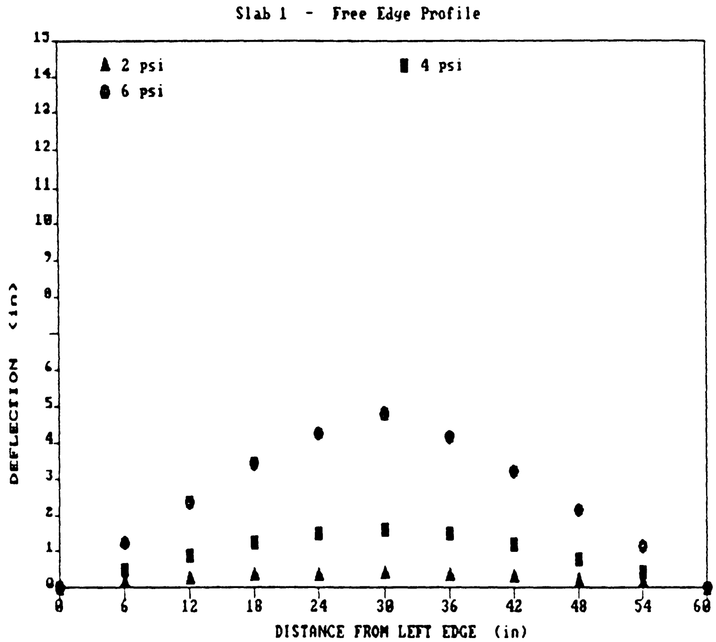


Figure 40. Deflection versus load - Gages 15 & 16



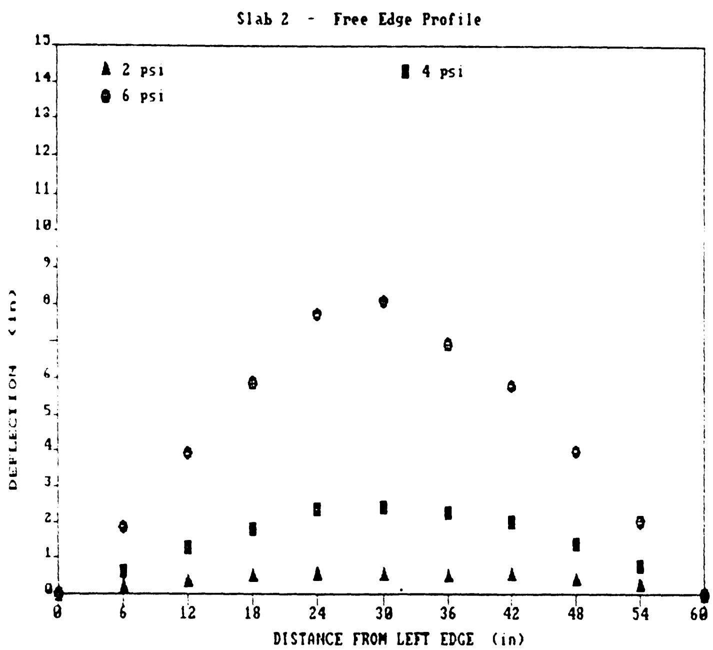


Figure 41. Free edge profiles - Slabs 1 & 2

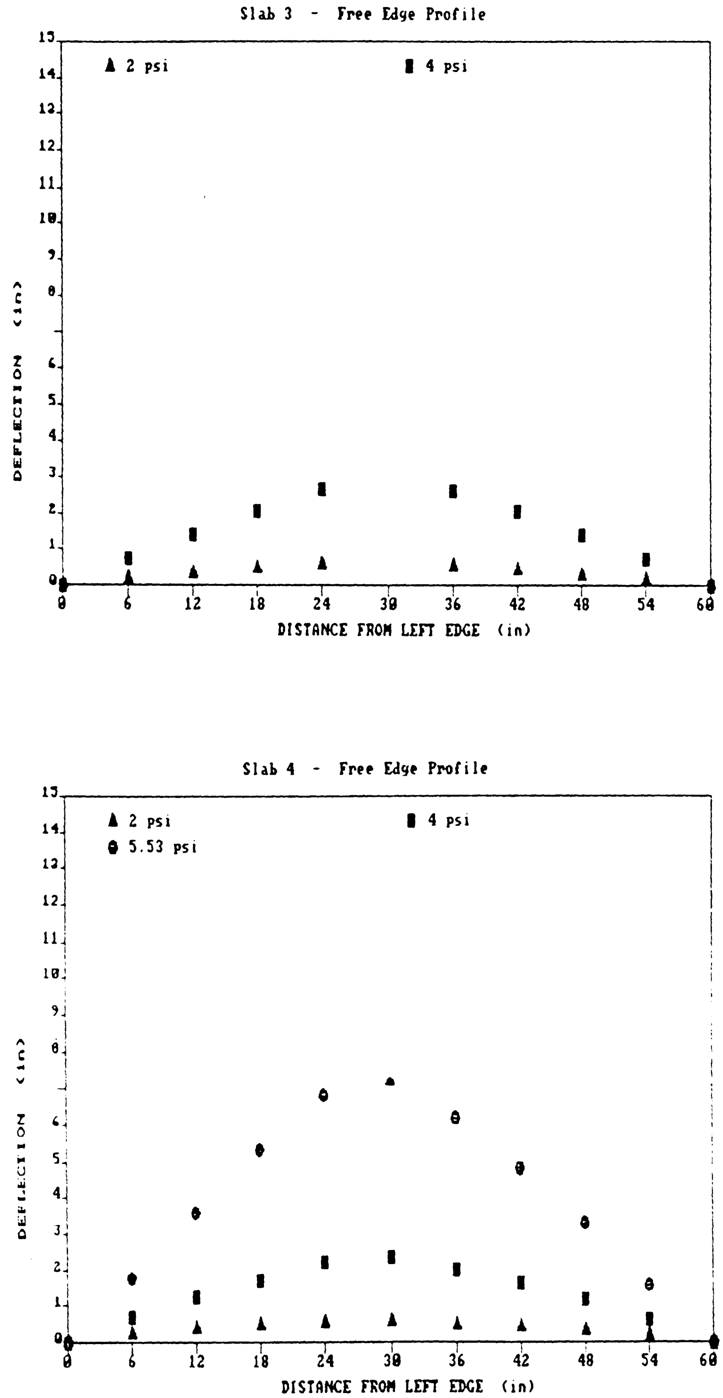
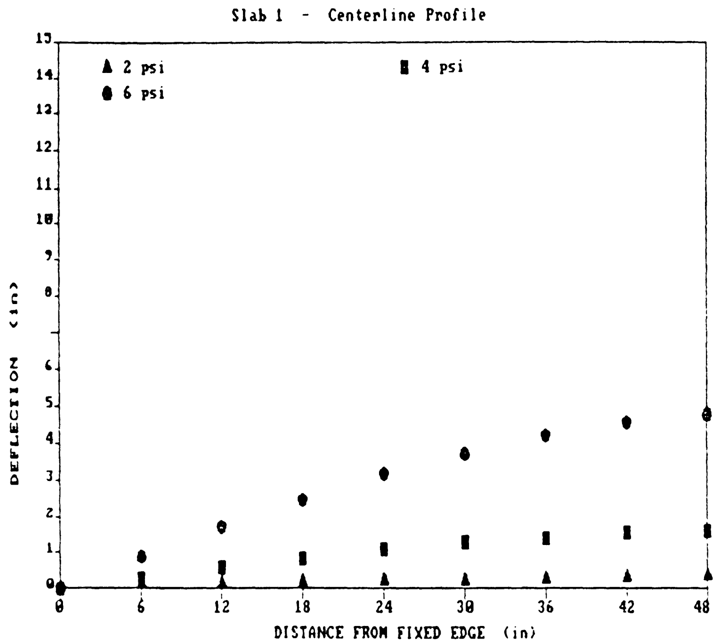


Figure 42. Free edge profiles - Slabs 3 & 4



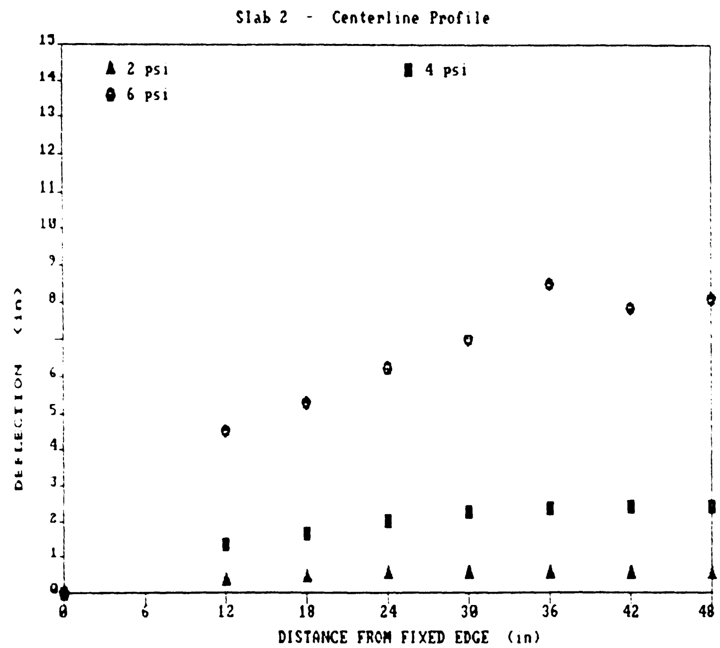
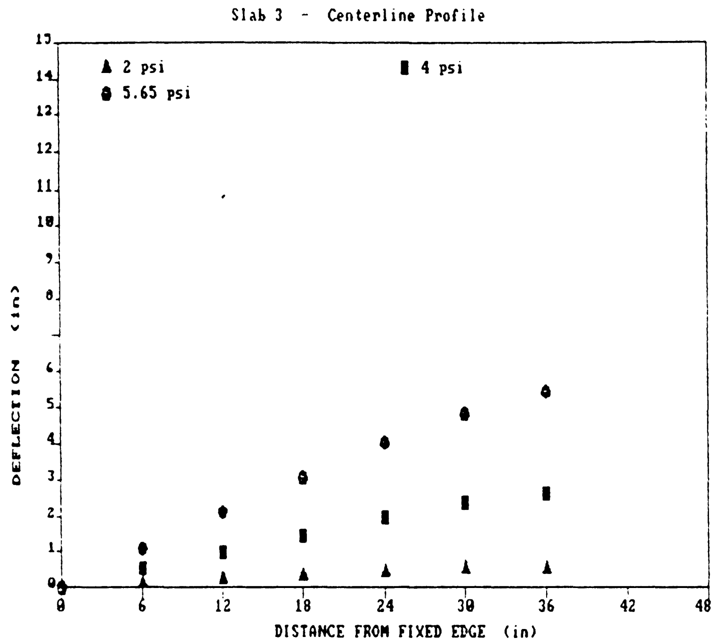


Figure 43. Centerline profiles - Slabs 1 & 2



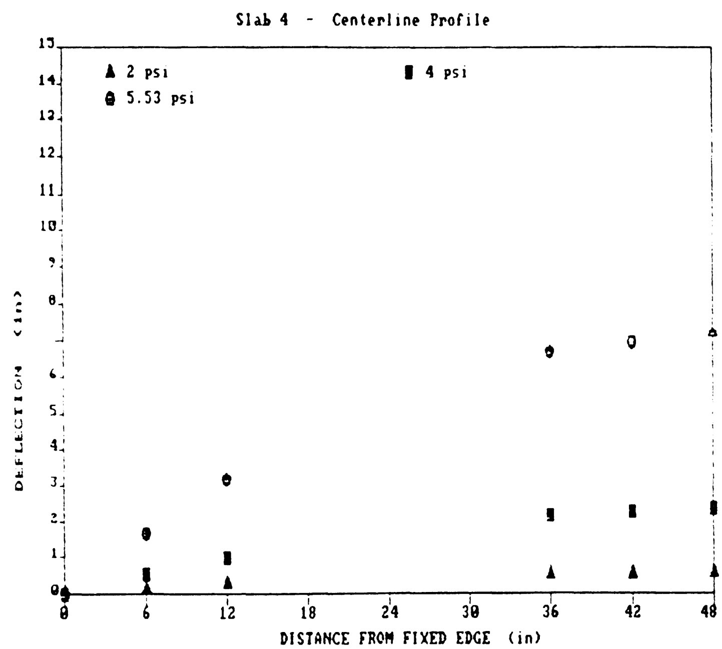


Figure 44. Centerline profiles - Slabs 3 & 4

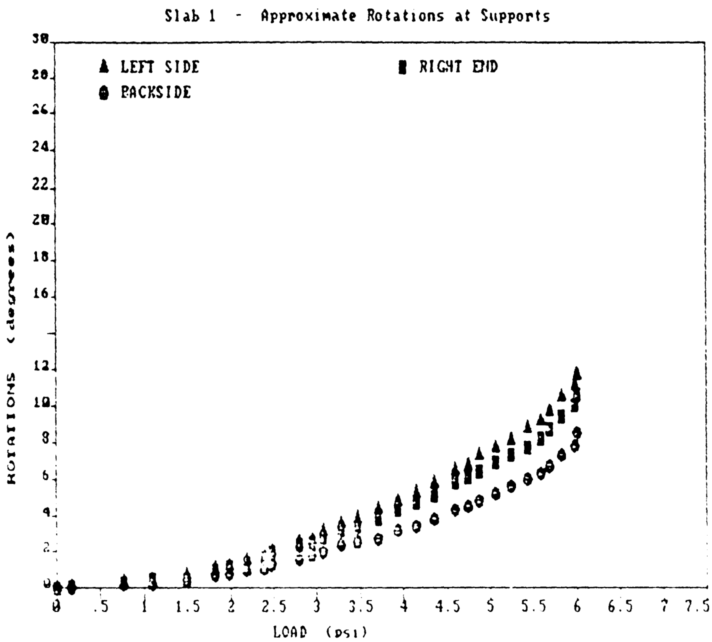


Figure 45. Typical rotation versus load

Chapter VI

CONCLUSIONS

From the results of these experiments, it is concluded that the ultimate strength of reinforced concrete slabs with and without openings can be reasonably predicted by yield-line theory. Although a test program with a limited number of slabs cannot be entirely conclusive, the additional evidence from past tests confirm that safe predictions are obtained. Although it has been shown that yield-line theory predicts the ultimate strength and failure mode, its use for design must also consider servicability requirements.

More research will be required before the theory will be recommended by the ACI Code. Ongoing research on membrane action is needed to better understand its effect and to provide accurate means of adjusting yield-line theory's predictions. This should be especially true when applied to slabs containing openings, as they may have much less membrane effects, thus minimizing the theory's conservatism.

Overall, the yield-line theory is an excellent tool for understanding the behavior of slabs. It can be applied to most slabs requiring no more than simple hand calculations. Its method can be used for slabs of unusual shapes where common methods may not be applied.

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APPENDIX I - PROGRAM LISTING

The computer program analyzing the ultimate strength of orthotropically reinforced concrete slabs is listed in its entirety. This interactive program written in BASIC is titled "SLAB.BAS".

```
10 '
20 '
30 '
40 '    ***********************************************************************
50 '    ********************   PROGRAM  STATEMENT   ***********************
60 '    ***********************************************************************
70 '
80 '
90 '
100     CLS
110     KEY OFF  : PRINT :PRINT :PRINT :PRINT
120     PRINT "     The following is a program in BASIC developed by Stephen"
130     PRINT "G. Ahart as a portion of his thesis.  The program is developed"
140     PRINT "to determine the collapse load of various orthotropically"
150     PRINT "reinforced, uniformly loaded rectangular concrete slabs"
160     PRINT "with or without rectangular openings."
170     PRINT : PRINT "The slab may be :"
180     PRINT
190     PRINT "               1. fixed on four sides
200     PRINT "               2. fixed on three sides, one side free
210     PRINT "               3. fixed on two sides, two opposite sides free
220     PRINT
230     PRINT "The openings can be:
240     PRINT
250     PRINT "               1. at the center of the slab
260     PRINT "               2. at the middle of a fixed edge
280     PRINT "               3. at the middle of the free edge (where applicable)
290     PRINT :PRINT :PRINT
300     INPUT "Do you have a color/graphics monitor";ANS$
310         IF ANS$="Y" OR ANS$="y" THEN GR=1
320         IF ANS$="N" OR ANS$="n" THEN GR=2
330         IF GR<>0 THEN 580
340     PRINT "ERROR -- ENTER Y OR N"
350     PRINT : PRINT : GOTO 300
360 '
370 '
380 '
390 '
400 '    ***********************************************************************
401 '    ********************   VARIABLE DEFINITIONS   *********************
402 '    ***********************************************************************
403 '
404 '
405 '
406 '   LX    -  Dimension of the vertical side
407 '   LY    -  Dimension of the horizontal side
408 '   X     -  Opening dimension (in input) - Ratio of opening/side length
409 '   Y     -  Opening dimension (in input) - Ratio of opening/side length
410 '   C     -  Opening offset
411 '   MXT   -  Negative ultimate moment resistance of section normal to x-axis
```

```
412 '   MXB   - Positive ultimate moment resistance of section normal to x-axis

413 '   MYT   - Negative ultimate moment resistance of section normal to y-axis

414 '   MYB   - Positive ultimate moment resistance of section normal to y-axis

415 '   A     - Pattern dimension variable
416 '   B     - Pattern dimension variable
417 '   D     - Pattern dimension variable
418 '   GR    - Signals type of monitor
419 '   TYPE  - Denotes edge conditions
420 '   OPEN  - Denotes opening location
421 '   ATOP  - Maximum permissible value for pattern dimension variable
422 '   BTOP  -    "        "        "    "    "       "        "
423 '   DTOP  -    "        "        "    "    "       "        "
424 '   ABOT  - Minimum permissible value for pattern dimension variable
425 '   BBOT  -    "        "        "    "    "       "        "
426 '   DBOT  -    "        "        "    "    "       "        "
427 '   W     - Ultimate load for pattern being examined
428 '   WU    - Current critical value of ultimate load for mode examined
429 '   WSAVE - Current critical value of ultimate load of all modes examined
430 '   ALAST - Pattern memory for convergence test
431 '   BLAST -    "       "     "     "        "
432 '   DLAST -    "       "     "     "        "
433 '   MODE  - Designation for mode being examined
434 '   FLAG  - Signal to direct proper input instructions
435 '   CHANGE - Signal to repeat input process
440 '
450 '
460 '
470 '
500 '
510 '   *********************************************************************
520 '   **************************   MAIN MENUS   *************************
530 '   *********************************************************************
540 '
550 '
560 '
570 '
580     CLS : DEFSNG B : DEFSNG A : C=0
590     PRINT :PRINT :PRINT :PRINT
600     PRINT "*************************************************************"
610     PRINT :PRINT :PRINT :PRINT :PRINT
620     PRINT "Would you like to analyze a rectangular slab which is :
630     PRINT :PRINT
640     PRINT "              1. fixed on four sides
650     PRINT "              2. fixed on three sides, one side free
660     PRINT "              3. fixed on two sides, two opposite sides free
670     PRINT : PRINT : PRINT : PRINT :PRINT :
680     PRINT "*************************************************************"
690     INPUT "Please enter selection (i.e. 1,2, or 3) "; TYPE
```

```
700       IF TYPE = 1 THEN GOSUB 1000
710       IF TYPE = 2 THEN GOSUB 1300
720       IF TYPE = 3 THEN GOSUB 1600
730       IF TYPE<>1 AND TYPE<>2 AND TYPE<>3 THEN GOTO 810
740    INPUT "Would you like to analyze another slab (y/n)?";ANS$
750       IF ANS$ = "y"  OR ANS$ = "Y" THEN 500
760       IF ANS$ = "n" OR ANS$ = "N" THEN 790
770    PRINT " ERROR  --  PLEASE ENTER Y OR N !"
780    GOTO 740
790    CLS
800    END
810    CLS : PRINT : PRINT : PRINT
820    PRINT "Please enter the number 1,2,or 3"
830    GOTO 610
840 '
850 '
1000 '
1010 '
1020 '  ********************************************************************
1030 '  *************   FOUR SIDES FIXED - CHOOSE OPENING   ****************
1040 '  ********************************************************************
1050 '
1060 '
1070 '
1080   CLS : PRINT  : PRINT  : PRINT  : PRINT
1090   PRINT "Would you like this this slab analyzed with :"
1100   PRINT : PRINT
1110   PRINT "           1.  A central opening
1120   PRINT "           2.  An opening on the center of an edge
1140   PRINT :PRINT :PRINT
1150   INPUT "Please enter selection (1,or 2)"; OPEN1
1160      IF OPEN1 = 1 THEN GOSUB 3000
1180      IF OPEN1 = 2 THEN GOSUB 6000
1190      IF OPEN1<>1 AND OPEN1<>2  THEN GOTO 1230
1200   RETURN
1210 '
1220 '
1230   CLS : PRINT  :PRINT  : PRINT
1240   PRINT " ERROR  :  Please enter the number 1,or 2"
1250   PRINT : PRINT : PRINT
1260   GOTO 1090
1270 '
1280 '
1290 '
1300 '
1310 '
1320 '
1330 '  ********************************************************************
1340 '  *************   THREE SIDES FIXED  -  CHOOSE OPENING   *************
1350 '  ********************************************************************
1360 '
```

```
1370 '
1380 '
1390 '
1400   CLS : PRINT : PRINT :PRINT
1410   PRINT "Would you like this slab analyzed with :"
1420   PRINT : PRINT
1430   PRINT "             1.  A central opening
1440   PRINT "             2.  An opening on the fixed horizontal side
1450   PRINT "             3.  An opening on the free horizontal side
1460   PRINT : PRINT : PRINT
1470   INPUT "Please enter selection (1,2,or 3)"; OPEN2
1480      IF OPEN2 = 1 THEN GOSUB 8000
1490      IF OPEN2 = 2 THEN GOSUB 10000
1500      IF OPEN2 = 3 THEN GOSUB 12000
1510      IF OPEN2<>1 AND OPEN2<>2 AND OPEN2<>3 THEN GOTO 1540
1520   RETURN
1530 '
1540   CLS : PRINT : PRINT : PRINT
1550   PRINT "  ERROR  -  Please enter number 1,2,or 3 "
1560   PRINT : PRINT : PRINT
1570   GOTO 1410
1580 '
1590 '
1600 '
1610 '
1620 '
1630 ' ****************************************************************
1640 ' **************   TWO FIXED SIDES  -  CHOOSE OPENING   **************
1650 ' ****************************************************************
1660 '
1670 '
1680 '
1690 '
1700   CLS : PRINT  : PRINT  : PRINT
1710   PRINT "Would you like this slab analyzed with:
1720   PRINT : PRINT
1730   PRINT "              1.  An interior opening
1740   PRINT "              2.  An opening on its end
1750   PRINT "              3.  Symmetrical interior openings
1760   PRINT "              4.  Symmetrical end openings
1770   PRINT : PRINT : PRINT : PRINT
1780   INPUT "Please enter selection (1,2,3,or 4) "; OPEN3
1790      IF OPEN3 = 1 THEN GOSUB 14000
1800      IF OPEN3 = 2 THEN GOSUB 16000
1810      IF OPEN3 = 3 THEN GOSUB 18000
1820      IF OPEN3 = 4 THEN GOSUB 20000
1830      IF OPEN3<>1 AND OPEN3<>2  AND OPEN3<>3 AND OPEN3<>4 THEN 1860
1840   RETURN
1850 '
1860   CLS : PRINT  : PRINT : PRINT
1870   PRINT " ERROR  -  Please enter number 1 or 2 "
```

```
1880    PRINT : PRINT : PRINT
1890    GOTO 1710
1900 '
3000 '
3010 '
3020 '
3030 '
3040 '  ********************************************************************
3050 '  **************   FOUR SIDES FIXED  -  CENTRAL OPENING   **************
3060 '  ********************************************************************
3070 '
3075 '
3080 '
3085 '
3090      IF GR=1 THEN 3130
3100    GOSUB 35000
3110    GOTO 3270
3120 '
3130    GOSUB 50000
3140    GOSUB 58300
3150    GOSUB 50400
3160    GOSUB 30000
3170    CLS
3180    GOSUB 50000
3190    GOSUB 58300
3200    GOSUB 50400
3210    GOSUB 30690
3220    GOSUB 30820
3230 '
3250 '
3260 '
3270 '  *** MODE 1 ***
3280 '
3290 '
3300    WU = 1000000!
3310    ATOP =.9990001
3320    ABOT = Y
3330       FOR A=ABOT TO ATOP STEP .01
3340            W=24*(LX*LX*(MYT+MYB)+LY*LY*(1-A)*(MXT+(1-Y)*MXB))/(LX*LX*LY*LY*(
             1-A)*(2+A-6*X*Y+3*X*X*Y))
3350               IF W<WU THEN WU = W
3360       NEXT
3370    WU1 =WU
3380    WSAVE = WU1 : MODE = 1
3390 '
3400 '
3410 '  *** MODE 2 ***
3420 '
3430 '
3440    WU=1000000!
3450    ATOP=X
```

```
3460    ABOT=.001
3470       FOR A=ABOT TO ATOP STEP .01
3480          W=24*(LX*LX*(1-A)*(MYT+(1-A)*MYB)+LY*LY*(1-Y)*(MXT+(1-Y)*MXB))/(LX
            *LX*LY*LY*(1-Y)*(2*(1-A)^2*(1-Y)+3*A*(1-A)*(1-Y)+3*Y*(1-X)^2))
3490             IF W<WU THEN WU=W
3500       NEXT
3510    WU2=WU
3520       IF WU2<WSAVE THEN WSAVE= WU2
3530       IF WSAVE= WU2 THEN MODE = 2
3540 '
3550 '
3560 '  *** MODE 3 ***
3570 '
3580 '
3590    WU=1000000!
3600    ATOP=Y
3610    ABOT=.001
3620       FOR A=ABOT TO ATOP STEP .01
3630          W=24*(LX*LX*(1-X)*(MYT+(1-X)*MYB)+LY*LY*(1-A)*(MXT+(1-A)*MXB))/(LY
            *LY*LX*LX*(1-X)*(2*(1-X)*(1-A)^2+3*X*(1-Y)^2+3*A*(1-X)*(1-A)))
3640             IF W<WU THEN WU=W
3650       NEXT
3660    WU3=WU
3670       IF WU3<WSAVE THEN WSAVE=WU3
3680       IF WSAVE=WU3 THEN MODE = 3
3690 '
3700    WU = 1000000!
3710    ATOP =.9990001
3720    ABOT = X
3730       FOR A=ABOT TO ATOP STEP .01
3740           W=24*(LY*LY*(MXT+MXB)+LX*LX*(1-A)*(MYT+(1-X)*MYB))/(LX*LX*LY*LY*(
            1-A)*(2+A-6*X*Y+3*X*Y*Y))
3750              IF W<WU THEN WU = W
3760       NEXT
3770    WU4 =WU
3780    IF WSAVE>WU4 THEN WSAVE=WU4
3785    IF WU4=WSAVE THEN MODE=4
3790 '
3800 '
3810 '
3900 '
3910 '  *** OUTPUT ***
3920 '
3930    CLS
3960    PRINT :PRINT :PRINT
3970    PRINT "   ***   RECTANGULAR SLAB  --  FOUR SIDES FIXED  --  CENTRAL OPEN
ING  ***"
3980    GOSUB 40000
3990    RETURN
4000 '
6000 '
```

```
6010 '
6020 '
6030 '  ******************************************************************
6040 '  ******   FOUR FIXED SIDES  ---  OPENING ON FIXED EDGE  ******
6050 '  ******************************************************************
6055 '
6060 '
6065 '
6070 '
6080      IF GR=1 THEN 6130
6100    GOSUB 35000
6110    GOTO 6250
6120 '
6130    GOSUB 50000
6140    GOSUB 58300
6150    GOSUB 52000
6160    GOSUB 30000
6170    CLS
6180    GOSUB 50000
6190    GOSUB 58300
6200    GOSUB 52000
6210    GOSUB 30690
6220    GOSUB 30820
6230 '
6235 '
6240 '
6250 '  *** MODE 1 ***
6260 '
6270 '
6280    B = (1+X)/2
6290    BLAST=3 : ALAST=3
6300    WU = 1000000!
6310    J=B*(1-Y)/(2*X)
6320      IF J<.5 THEN ATOP=J ELSE ATOP=.5
6330    ABOT=.001
6340      FOR A=ABOT TO ATOP STEP .01
6350        W=6*(2*B*(1-B)*LX*LX*(MYT+MYB)+A*B*LY*LY*(MXT+MXB)+A*(1-B)*LY*LY*
             (MXB+(1-Y)*MXT))/(A*(1-B)*LX*LX*LY*LY*(3*B-2*A*B-3*Y*X*X))
6360            IF WU>W THEN WU=W
6370            IF WU=W THEN AKEEP=A
6380      NEXT
6390    A=AKEEP
6400    BTOP=.999
6410    BBOT=(2*X*A)/(1-Y)
6420    WU=1000000!
6430      FOR B=BBOT TO BTOP STEP .01
6440        W=6*(2*B*(1-B)*LX*LX*(MYT+MYB)+A*B*LY*LY*(MXT+MXB)+A*(1-B)*LY*LY*
             (MXB+(1-Y)*MXT))/(A*(1-B)*LX*LX*LY*LY*(3*B-2*A*B-3*Y*X*X))
6450            IF WU>W THEN WU=W
6460            IF WU=W THEN BKEEP=B
6470      NEXT
```

```
6480    B=BKEEP
6490       IF ABS(B-BLAST)>.03 THEN 6520
6500       IF ABS(A-ALAST)>.03 THEN 6520
6510    GOTO 6540
6520    BLAST=B : ALAST=A
6530    GOTO 6300
6540    WU1=WU
6550    WSAVE=WU1 : MODE = 1
6560 '
6570 '
6580 '  *** MODE 2 ***
6590 '
6600 '
6610    B=Y/2
6620    BLAST=3 : ALAST=3
6630    WU = 1000000!
6640    ATOP = X
6650    ABOT=.001
6660       FOR A=ABOT TO ATOP STEP .01
6670          W=6*(LY*LY*(1-X)*(1-Y)^2*(MXT+MXB)+4*A*(1-X)*LX*LX*(MYT+(1-X+A)*
           MYB)+A*LY*LY*(1-B)*(MXT+(1-B)*MXB))/(A*(1-X)*LX*LX*LY*LY*(2*A*(1-
           Y)^2+3*(X-A)*(1-Y)^2+2*(1-X)*(1-B)^2+3*B*(1-X)*(1-B)))
6680             IF WU>W THEN WU=W
6690             IF WU=W THEN AKEEP=A
6700       NEXT
6710    A=AKEEP
6720    BTOP=Y
6730    BBOT=.001
6740    WU=1000000#
6750       FOR B=BBOT TO BTOP STEP .01
6760          W=6*(LY*LY*(1-X)*(1-Y)^2*(MXT+MXB)+4*A*(1-X)*LX*LX*(MYT+(1-X+A)*
           MYB)+A*LY*LY*(1-B)*(MXT+(1-B)*MXB))/(A*(1-X)*LX*LX*LY*LY*(2*A*(1-
           Y)^2+3*(X-A)*(1-Y)^2+2*(1-X)*(1-B)^2+3*B*(1-X)*(1-B)))
6770             IF WU>W THEN WU=W
6780             IF WU=W THEN BKEEP=B
6790       NEXT
6800    B=BKEEP
6810       IF ABS(B-BLAST)>.03 THEN 6840
6820       IF ABS(A-ALAST)>.03 THEN 6840
6830    GOTO 6860
6840    BLAST=B : ALAST=A
6850    GOTO 6630
6860    WU2=WU
6870       IF WU2<WSAVE THEN WSAVE=WU2
6880       IF WSAVE=WU2 THEN MODE = 2
6890 '
6900 '
6910 '  *** MODE 3 ***
6920 '
6930 '
6940    B=1-X
```

```
6950  BLAST=3 : ALAST=3
6960  WU=1000000!
6970  ATOP=1-B
6980  ABOT=.001
6990     FOR A=ABOT TO ATOP STEP .01
7000        W=6*(B*LY*LY*(1-Y)^2*(MXT+MXB)+4*A*B*LX*LX*(MYT+(A+B)*MYB)+A*LY*LY
             *(1-Y)*(MXT+(1-Y)*MXB))/(A*(1-Y)*LY*LY*LX*LX*(2*B*(A+B)*(1-Y)+3*B
             *(1-Y)*(1-A-B)+3*(1-X)^2*Y))
7010           IF WU>W THEN WU=W
7020           IF WU=W THEN AKEEP=A
7030     NEXT
7040  A=AKEEP
7050  BTOP=1-A
7060  BBOT=1-X
7070  WU=1000000!
7080     FOR B=BBOT TO BTOP STEP .01
7090        W=6*(B*LY*LY*(1-Y)^2*(MXT+MXB)+4*A*B*LX*LX*(MYT+(A+B)*MYB)+A*LY*LY
             *(1-Y)*(MXT+(1-Y)*MXB))/(A*(1-Y)*LY*LY*LX*LX*(2*B*(A+B)*(1-Y)+3*B
             *(1-Y)*(1-A-B)+3*(1-X)^2*Y))
7100           IF WU>W THEN WU=W
7110           IF WU=W THEN BKEEP=B
7120     NEXT
7130  B=BKEEP
7140     IF ABS(B-BLAST)>.03 THEN 7170
7150     IF ABS(A-ALAST)>.03 THEN 7170
7160  GOTO 7190
7170  BLAST=B : ALAST=A
7180  GOTO 6960
7190  WU3=WU
7200     IF WU3<WSAVE THEN WSAVE=WU3
7210     IF WSAVE=WU3 THEN MODE = 3
7220 '
7230 '
7240 '  *** MODE 4 ***
7250 '
7260 '
7270  B=X/2
7280  BLAST=3 : ALAST=3
7290  WU = 1000000!
7300  ATOP = (1-Y)/2
7310  ABOT=.001
7320     FOR A=ABOT TO ATOP STEP .01
7330        W=6*(A*(1-B)*LY*LY*(1-Y)*(MXT+MXB)+A*B*LY*LY*(MXT+(1-Y)*MXB)+2*B*(
             1-B)*LX*LX*(MYT+MYB))/(A*B*LY*LY*LX*LX*(4*A*(1-B)+3*(1-B)*(1-2*A-
             Y)+3*Y*(1-X)^2))
7340           IF WU>W THEN WU=W
7350           IF WU=W THEN AKEEP=A
7360     NEXT
7370  A=AKEEP
7380  BTOP=X
7390  BBOT=.001
```

```
7400    WU=1000000!
7410       FOR B=BBOT TO BTOP STEP .01
7420          W=6*(A*(1-B)*LY*LY*(1-Y)*(MXT+MXB)+A*B*LY*LY*(MXT+(1-Y)*MXB)+2*B*(
            1-B)*LX*LX*(MYT+MYB))/(A*B*LY*LY*LX*LX*(4*A*(1-B)+3*(1-B)*(1-2*A-
           Y)+3*Y*(1-X)^2))
7430             IF WU>W THEN WU=W
7440             IF WU=W THEN BKEEP=B
7450       NEXT
7460    B=BKEEP
7470       IF ABS(B-BLAST)>.03 THEN 7500
7480       IF ABS(A-ALAST)>.03 THEN 7500
7490    GOTO 7520
7500    BLAST=B : ALAST=A
7510    GOTO 7290
7520    WU4=WU
7530       IF WU4<WSAVE THEN WSAVE=WU4
7540       IF WSAVE=WU4 THEN MODE = 4
7550 '
7560 '
7600 '  *** MODE 5 ***
7610 '
7620 '
7630    B=X/2
7640    BLAST=3 : ALAST=3
7650    WU=1000000!
7660    ATOP=1-X
7670    ABOT=.01
7680       FOR A = ABOT TO ATOP STEP .01
7690          W=6*(4*A*B*LX^2*(MYT+(1+B-X)*MYB)+A*(1-Y)^2*LY^2*(MXT+MXB)+B*LY^2*
               (MXT+MXB))/(A*B*LX^2*LY^2*(3-A-3*X+3*(1-Y)^2*(X-B)+2*B*
               (1-Y)^2))
7700             IF WU>W THEN WU=W
7705             IF WU=W THEN AKEEP=A
7710       NEXT
7715    A=AKEEP
7720    BTOP=X
7725    BBOT=.01
7730    WU=1000000!
7735       FOR  B = BBOT TO BTOP STEP .01
7740          W=6*(4*A*B*LX^2*(MYT+(1+B-X)*MYB)+A*(1-Y)^2*LY^2*(MXT+MXB)+B*LY^2*
               (MXT+MXB))/(A*B*LX^2*LY^2*(3-A-3*X+3*(1-Y)^2*(X-B)+2*B*
               (1-Y)^2))
7745             IF WU>W THEN WU=W
7750             IF WU=W THEN BKEEP=B
7755       NEXT
7760    B=BKEEP
7765    IF ABS(B-BLAST)>.03 THEN 7780
7770    IF ABS(A-ALAST)>.03 THEN 7780
7775    GOTO 7790
7780    BLAST=B : ALAST=A
7785    GOTO 7650
```

```
7790    WU5=WU
7795       IF WU5<WSAVE THEN WSAVE=WU5
7800       IF WSAVE=WU5 THEN MODE=5
7805 '
7810 '
7815 '
7870 '  *** OUTPUT ***
7880 '
7890    CLS
7930 '
7940    PRINT :PRINT :PRINT
7950    PRINT "   ***   RECTANGULAR SLAB  --  FOUR SIDES FIXED  --  OPENING ON L
ONG SIDE   ***"
7960    GOSUB 40000
7970    RETURN
7980 '
7990 '
8000 '
8010 '
8020 '  ***********************************************************************
8030 '  *************    THREE SIDES FIXED  --  CENTRAL OPENING   ************
8040 '  ***********************************************************************
8050 '
8060 '
8070 '
8080 '
8120       IF GR=1 THEN 8160
8130    GOSUB 35000
8140    GOTO 8300
8150 '
8160    GOSUB 50000
8170    GOSUB 58240
8180    GOSUB 50400
8190    GOSUB 30000
8200    CLS
8210    GOSUB 50000
8220    GOSUB 58240
8230    GOSUB 50400
8240    GOSUB 30690
8250    GOSUB 30820
8260 '
8265 '
8270 '
8280 '  *** MODE 1 ***
8290 '
8300 '
8310    WU = 1000000!
8320    ATOP = .999
8330    J=1-(2*(1-Y)/(X+1))
8340       IF J>0 THEN ABOT=J ELSE ABOT=.001
8350       FOR A=ABOT TO ATOP STEP .01
```

```
8360         W=6*(4*LX*LX*(MYT+MYB)+(1-A)*LY*LY*(MXT+(1-A)*MXB))/((1-A)*LY*LY*
              LX*LX*(2+A-3*X*Y))
8370            IF W<WU THEN WU = W
8380       NEXT
8390    WU1 =WU
8400    WSAVE = WU1 : MODE = 1
8410 '
8420 '
8430 '  *** MODE 2 ***
8440 '
8450 '
8460    WU=1000000!
8470    ATOP=(1+X)/2
8480    ABOT=(1-X)/2
8490       FOR A=ABOT TO ATOP STEP .01
8500         W=24*(2*A*LX*LX*(2*MYT+(2*A+1-X)*MYB)+LY*LY*(1-Y)*(MXT+(1-Y)*MXB))
              /(LX*LX*LY*LY*(8*A*A*(1-Y)^2+6*A*(1-2*A+X)*(1-Y)^2+6*A*(1-X)+3*Y*
              (1-X)^2*(1-Y)))
8510            IF W<WU THEN WU=W
8520       NEXT
8530    WU2=WU
8540       IF WU2<WSAVE THEN WSAVE= WU2
8550       IF WSAVE= WU2 THEN MODE = 2
8560 '
8570 '
8580 '  *** MODE 3 ***
8590 '
8600 '
8610    WU=1000000!
8620    ATOP=Y
8630    ABOT=.001
8640       FOR A=ABOT TO ATOP STEP .01
8650         W=24*(4*(1-X)*LX*LX*(MYT+(1-X)*MYB)+2*LY*LY*(1-A)*(MXT+(1-A)*MXB))
              /((1-X)*LY*LY*LX*LX*(4*(1-X)*(1-A)^2+6*(1-X)+12*X*(1-Y)^2+6*A*(1-X
              )*(1-Y)))
8660            IF W<WU THEN WU=W
8670       NEXT
8680    WU3=WU
8690       IF WU3<WSAVE THEN WSAVE=WU3
8700       IF WSAVE=WU3 THEN MODE = 3
8710 '
8720 '
9050 '  *** MODE 4 ***
9060 '
9070 '
9080    WU=1000000!
9090    ATOP=.9990001
9100    J=(X+1)/(2*(1-Y))
9110       IF J>1 THEN 9160 ELSE ABOT=J
9120       FOR A=ABOT TO ATOP STEP .01
```

```
9130         W=6*(4*A*LX*LX*(MYT+MYB)+LY*LY*(MXT+MXB))/(LY*LY*LX*LX*(3*A-A*A-3*
              X*Y))
9140              IF W<WU THEN WU=W
9150        NEXT
9160     WU4=WU
9170        IF WU4<WSAVE THEN WSAVE=WU4
9180        IF WSAVE=WU4 THEN MODE = 4
9190 '
9200 '
9210 '   *** MODE 5 ***
9220 '
9230 '
9240     WU=1000000!
9250     ATOP=(1-X)/2
9260     ABOT=.001
9270        FOR A=ABOT TO ATOP STEP .01
9280          W=6*(4*A*LX*LX*(MYT+(1-X)*MYB)+LY*LY*(MXT+MXB))/(A*LY*LY*LX*LX*(3-
              A-3*X*Y*(2-Y)))
9290              IF W<WU THEN WU=W
9300        NEXT
9310     WU5=WU
9320       IF WU5<WSAVE THEN WSAVE=WU5
9330       IF WSAVE=WU5 THEN MODE = 5
9340 '
9350 '
9360 '   *** OUTPUT ***
9370 '
9380 '
9390     CLS
9430 '
9440     PRINT :PRINT
9450     PRINT "   ***   RECTANGULAR SLAB  --  THREE SIDES FIXED  --  CENTRAL OPE
NING   ***"
9460     GOSUB 40000
9470     RETURN
9480 '
10000 '
10010 '
10020 '
10030 '
10040 '  **********************************************************************
10050 '  ********   THREE SIDES FIXED -- FIXED HORIZONTAL OPENING   ***********
10060 '  **********************************************************************
10070 '
10080 '
10085 '
10090 '
10100       IF GR=1 THEN 10150
10110    GOSUB 35000
10120    GOTO 10270
10130 '
```

```
10150   GOSUB 50000
10160   GOSUB 58240
10170   GOSUB 52000
10180   GOSUB 30000
10190   CLS
10200   GOSUB 50000
10210   GOSUB 58240
10220   GOSUB 52000
10230   GOSUB 30690
10240   GOSUB 30820
10250 '
10255 '
10260 '
10270 ' *** MODE 1 ***
10280 '
10290 '
10300   WU = 1000000!
10310   J=1-(X/(1-Y))
10320      IF J<0 THEN 10390
10330   ATOP=J
10340   ABOT=.001
10350      FOR A=ABOT TO ATOP STEP .01
10360         W=6*(4*(1-A)*LX*LX*(MYT+MYB)+LY*LY*(MXB+(1-Y)*MXT))/(LY*LY*LX*LX*
           ((2+A)*(1-A)-3*X*X*Y))
10370            IF W<WU THEN WU = W
10380      NEXT
10390   WU1 =WU
10400   WSAVE = WU1 : MODE = 1
10410 '
10420 '
10430 ' *** MODE 2 ***
10440 '
10450 '
10460   WU=1000000!
10470   J=1-((1-Y)/X)
10480      IF J>0 THEN ABOT=J ELSE ABOT=.001
10490   ATOP=.999
10510      FOR A=ABOT TO ATOP STEP .01
10520         W=6*(4*LX*LX*(MYT+MYB)+LY*LY*(1-A)^2*MXB+LY*LY*(1-A)*(1-Y)*MXT)/
           (LY*LY*LX*LX*(1-A)*(2+A-3*X*X*Y))
10530            IF W<WU THEN WU=W
10540      NEXT
10550   WU2=WU
10560      IF WU2<WSAVE THEN WSAVE= WU2
10570      IF WSAVE= WU2 THEN MODE = 2
10580 '
10590 '
10600 ' *** MODE 3 ***
10610 '
10620 '
10630   WU=1000000!
```

```
10640  ATOP=X
10650  ABOT=.001
10660    FOR A=ABOT TO ATOP STEP .01
10670       W=6*(4*A*LX*LX*(MYT+(1-X+A)*MYB)+LY*LY*(1-Y)^2*(MXT+MXB))/(A*LY*LY
         *LX*LX*(3-3*X+2*A*(1-Y)^2+3*(X-A)*(1-Y)^2))
10680         IF W<WU THEN WU=W
10690    NEXT
10700  WU3=WU
10710    IF WU3<WSAVE THEN WSAVE=WU3
10720    IF WSAVE=WU3 THEN MODE = 3
10730 '
10740 '
11060 '
11070 '
11080 ' *** OUTPUT ***
11090 '
11100 '
11110  CLS
11140 '
11150  PRINT :PRINT :PRINT
11160  PRINT "   ***   RECTANGULAR SLAB  --  THREE SIDES FIXED  --  OPENING ON
FIXED LONG SIDE  ***"
11170  GOSUB 40000
11180  RETURN
11190 '
11200 '
12000 '
12010 '
12020 '
12030 '  ************************************************************************
12040 '  ********   THREE SIDES FIXED -- FREE HORIZONTAL OPENING   ************
12050 '  ************************************************************************
12060 '
12070 '
12080 '
12090 '
12100    IF GR=1 THEN 12150
12110  GOSUB 35000
12120  GOTO 12270
12130 '
12150  GOSUB 50000
12160  GOSUB 58240
12170  GOSUB 54000
12180  GOSUB 30000
12190  CLS
12200  GOSUB 50000
12210  GOSUB 58240
12220  GOSUB 54000
12230  GOSUB 30690
12240  GOSUB 30820
12250 '
```

```
12255 '
12260 '
12270 ' *** MODE 1 ***
12280 '
12290 '
12300   WU = 1000000!
12310   ATOP=(1-X)
12320   ABOT=.001
12330      FOR A=ABOT TO ATOP STEP .01
12340          W=6*(4*A*LX*LX*(MYT+(1-X)*MYB)+LY*LY*(MXT+MXB))/(A*LY*LY*LX*LX*(3-
            A-3*X*Y*(2-Y)))
12350            IF W<WU THEN WU = W
12360      NEXT
12370   WU1 =WU
12380   WSAVE = WU1 : MODE = 1
12390 '
12400 '
12410 ' *** MODE 2 ***
12420 '
12430 '
12440   WU=1000000!
12450   ATOP=X
12460   ABOT=.001
12470      FOR A=ABOT TO ATOP STEP .01
12480         W=6*(4*(1-A)*LX*LX*(MYT+(1-A)*MYB)+(1-Y)*LY*LY*(MXT+(1-Y)*MXB))/((
            1-Y)*LY*LY*LX*LX*(3*A*(1-A)*(1-Y)+2*(1-Y)*(1-A)^2+3*Y*(1-X)^2))
12490           IF W<WU THEN WU=W
12500      NEXT
12510   WU2=WU
12520      IF WU2<WSAVE THEN WSAVE= WU2
12530      IF WSAVE= WU2 THEN MODE = 2
12540 '
12550 '
12560 ' *** MODE 3 ***
12570 '
12580 '
12590   WU=1000000!
12600   ATOP=.999
12610   ABOT=Y
12620      FOR A=ABOT TO ATOP STEP .01
12630         W=6*(4*LX*LX*(MYT+MYB)+(1-A)*LY*LY*(MXT+(1-A)*MXB))/((1-A)*LY*LY*
            LX*LX*(2+A-3*X*Y*(2-X)))
12640           IF W<WU THEN WU=W
12650      NEXT
12660   WU3=WU
12670      IF WU3<WSAVE THEN WSAVE=WU3
12680      IF WSAVE=WU3 THEN MODE = 3
12690 '
12700 '
12710 ' *** MODE 4 ***
12720 '
```

```
12730 '
12740  WU=1000000!
12750  ATOP=Y
12760  ABOT=.001
12770    FOR A=ABOT TO ATOP STEP .01
12780       W=6*(4*LX*LX*(1-X)*(MYT+(1-X)*MYB)+LY*LY*(1-A)*(MXT+(1-A)*MXB))/((
        1-X)*LY*LY*LX*LX*(3*X*(1-Y)^2+2*(1-A)^2*(1-X)+3*A*(1-A)*(1-X)))
12790         IF W<WU THEN WU=W
12800    NEXT
12810  WU4=WU
12820    IF WU4<WSAVE THEN WSAVE=WU4
12830    IF WSAVE=WU4 THEN MODE = 4
12840 '
12850 '
12860 ' *** OUTPUT ***
12870 '
12880 '
12890  CLS
12920 '
12930  PRINT :PRINT :PRINT
12940  PRINT "   ***   RECTANGULAR SLAB  --  THREE SIDES FIXED  --  OPENING ON
FREE LONG SIDE  ***"
12950  GOSUB 40000
12960  RETURN
12970 '
12980 '
12990 '
13000 '
14000 '
14010 '  ********************************************************************
14020 '  *************   TWO SIDES FIXED  --  INTERIOR OPENING   *************
14030 '  ********************************************************************
14032 '
14034 '
14036 '
14040 '
14050  FLAG = 1
14060 '
14070    IF GR=1 THEN 14120
14080  GOSUB 35000
14090  GOTO 14250
14100 '
14120  GOSUB 50000
14130  GOSUB 55000
14140  GOSUB 30000
14150  CLS
14160  GOSUB 50000
14170  GOSUB 55000
14180  GOSUB 30690
14190  GOSUB 30850
14220 '
```

```
14230 '
14240 '
14250 ' *** MODE 1 ***
14260 '
14270 '
14280  WU=1000000!
14290  BTOP=C+Y
14300  BBOT=C
14310     FOR B=BBOT TO BTOP STEP .01
14320        W=2*(MYT+(1-X)*MYB)/(LY*LY*(B*(1-B)-X*(B-C)*(1-B)*(C+B)-X*B*(Y-B+C
          )*(2-B-C-Y)))
14330           IF W<WU THEN WU=W
14340     NEXT
14350  WU1=WU
14360  WSAVE=WU1 : MODE = 1
14370 '
14380 '
14390 ' *** MODE 2 ***
14400 '
14410 '
14420  WU=1000000!
14430  BTOP=.999
14440  BBOT=C+Y
14450     FOR B=BBOT TO BTOP STEP .01
14460        W=2*(MYT+MYB)/(LY*LY*(1-B)*(B-X*Y*(2*C+Y)))
14470           IF W<WU THEN WU=W
14480     NEXT
14490  WU2=WU
14500     IF WU2<WSAVE THEN WSAVE=WU2
14510     IF WSAVE=WU2 THEN MODE = 2
14520 '
14530 '
14540 ' *** MODE 3 ***
14550 '
14560 '
14680  DTOP=.9990001
14690  DBOT=C+Y
14700  WU=1000000!
14710     FOR D=DBOT TO DTOP STEP .01
14720        W=2*((D-C)*(MYT+MYB)+(1-D)*(MYB+(1-X)*MYT))/((1-D)*LY*LY*((1-C)*(D
          -C)-X*(Y)^2))
14730           IF W<WU THEN WU=W
14750     NEXT
14820  WU3=WU
14830     IF WU3<WSAVE THEN WSAVE=WU3
14840     IF WSAVE=WU3 THEN MODE = 3
14850 '
14860 '
14870 ' *** MODE 4 ***
14880 '
14890 '
```

```
14900  WU=1000000!
14910  DTOP=C+Y
14920  DBOT=C
14930    FOR D=DBOT TO DTOP STEP .01
14940      W=2*((1-X)*(1-D)*(MYT+MYB)+(D-C)*(MYT+(1-X)*MYB))/(LY*LY*(D-C)*((1
       -C)*(1-D)-X*(D-C)*(1-D)-X*(C+Y-D)*(2-D-C-Y)))
14950        IF W<WU THEN WU=W
14960    NEXT
14970  WU4=WU
14980    IF WU4<WSAVE THEN WSAVE=WU4
14990    IF WSAVE=WU4 THEN MODE = 4
15000 '
15010 '
15020 ' *** MODE 5 ***
15030 '
15040 '
15050  WU=1000000!
15060  DTOP=C+Y
15070  DBOT=C
15080    FOR D=DBOT TO DTOP STEP .01
15090      W=2*(MYT+MYB)/(LY*LY*(D-C)*(C+Y-D))
15100        IF W<WU THEN WU=W
15110    NEXT
15120  WU5=WU
15130    IF WU5<WSAVE THEN WSAVE=WU5
15140    IF WSAVE=WU5 THEN MODE = 5
15150 '
15160 '
15170 ' *** OUTPUT ***
15180 '
15190  CLS
15220 '
15230  PRINT :PRINT :PRINT
15240  PRINT "   ***   RECTANGULAR SLAB  --  TWO SIDES FIXED  --  INTERIOR OPEN
ING   ***"
15250  GOSUB 40000
15260  RETURN
15270 '
15280 '
15290 '
16000 '
16010 '
16020 ' ***********************************************************************
16030 ' *************   TWO SIDES FIXED  --  END OPENING   *******************
16040 ' ***********************************************************************
16042 '
16044 '
16046 '
16048 '
16050    IF GR=1 THEN 16100
16060  GOSUB 35000
```

```
16070  GOTO 16230
16080 '
16100  GOSUB 50000
16110  GOSUB 56000
16120  GOSUB 30000
16130  CLS
16140  GOSUB 50000
16150  GOSUB 56000
16160  GOSUB 30690
16170  GOSUB 30850
16200 '
16210 '
16220 '
16230 ' *** MODE 1 ***
16240 '
16250 '
16260  WU=1000000!
16270  ATOP=.9990001
16280  ABOT=Y
16290     FOR A=ABOT TO ATOP STEP .01
16300        W=2*(A*(MYT+MYB)+(1-A)*(MYB+(1-X)*MYT))/((1-A)*LY*LY*(A+X*Y*Y))
16310           IF W<WU THEN WU=W
16320     NEXT
16330  WU1=WU
16340  WSAVE=WU1 : MODE = 1
16350 '
16360 '
16370 ' *** MODE 2 ***
16380 '
16390 '
16400  WU=1000000!
16410  ATOP=Y
16420  ABOT=.001
16430     FOR A=ABOT TO ATOP STEP .01
16440        W=2*((1-X)*(1-A)*(MYT+MYB)+A*(MYT+(1-X)*MYB))/(A*LY*LY*(A*(1-X)*(1
          -A)+(1-A)^2-(Y-A)*X*(2-A-Y)))
16450           IF W<WU THEN WU=W
16460     NEXT
16470  WU2=WU
16480     IF WU2<WSAVE THEN WSAVE=WU2
16490     IF WSAVE=WU2 THEN MODE = 2
16500 '
16510 '
16520 ' *** MODE 3 ***
16530 '
16540 '
16550  WU=1000000!
16560  DTOP=Y
16570  DBOT=.001
16580     FOR D=DBOT TO DTOP STEP .01
16590        W=2*(MYT+MYB)/(LY*LY*D*(Y-D))
```

```
16600           IF W<WU THEN WU=W
16610      NEXT
16620   WU3=WU
16630      IF WU3<WSAVE THEN WSAVE=WU3
16640      IF WSAVE=WU3 THEN MODE = 3
16650 '
16660 '
16670 '
16680 ' *** OUTPUT ***
16690 '
16700 '
16710   CLS
16740 '
16750   PRINT :PRINT :PRINT
16760   PRINT "   ***   RECTANGULAR SLAB  --  TWO SIDES FIXED  --  END OPENING
 ***"
16770   GOSUB 40000
16780   RETURN
16790 '
16800 '
16810 '
18000 '
18010 '
18020 ' **********************************************************************
18030 '  *******   TWO SIDES FIXED  --  SYMMETRICAL INTERIOR OPENINGS   *******
18040 ' **********************************************************************
18042 '
18044 '
18046 '
18048 '
18050   FLAG=2
18060      IF GR=1 THEN 18130
18080   GOSUB 35000
18100   GOTO 18300
18110
18130   GOSUB 50000
18140   GOSUB 57000
18150   GOSUB 30000
18170   CLS
18180   GOSUB 50000
18190   GOSUB 57000
18200   GOSUB 30690
18210   GOSUB 30850
18220 '
18230 '
18240 '
18270 ' *** MODE 1 ***
18280 '
18290 '
18300   WU1=8*(MYT+MYB)/(LY*LY*(1-4*(*Y*(2*C+Y)))
18310   WSAVE=WU1 : MODE = 1
```

```
18320 '
18330 '
18340 ' *** MODE 2 ***
18350 '
18360 '
18370   WU=1000000!
18380   ATOP=C+Y
18390   ABOT=C
18400      FOR A=ABOT TO ATOP STEP .01
18410         W=2*(MYT+(1-X)*MYB)/(LY*LY*(A-A*A-2*A*X*(C+Y-A)-X*(A-C)*(A+C)))
18420            IF W<WU THEN WU=W
18430      NEXT
18440   WU2=WU
18450      IF WU2<WSAVE THEN WSAVE=WU2
18460      IF WSAVE=WU2 THEN MODE = 2
18470 '
18480 '
18490 ' *** MODE3 ***
18500 '
18510 '
18520 '
18530 '
18540 '
18550 '
18560   WU3=8*(MYB+(1-X)*MYT)/(LY*LY*((1-2*C)^2-4*X*(Y)^2))
18570 '
18580 '
18590 '
18600      IF WU3<WSAVE THEN WSAVE=WU3
18610      IF WSAVE=WU3 THEN MODE = 3
18620 '
18630 '
18640 ' *** OUTPUT ***
18650 '
18660 '
18670   CLS
18700 '
18710   PRINT :PRINT :PRINT
18720   PRINT "***  RECTANGULAR SLAB  -  TWO SIDES FIXED  -  SYMMETRICAL INTERI
OR OPENINGS   ***"
18730   GOSUB 40000
18740   RETURN
18750 '
18760 '
18770 '
18780 '
20000 '
20010 '  *******************************************************************
20020 '  **********   TWO FIXED SIDES  --  SYMMETRICAL END OPENINGS   *********
20030 '  *******************************************************************
20035 '
```

```
20040 '
20045 '
20046 '
20050   FLAG=3
20060      IF GR=1 THEN 20110
20070   GOSUB 35000
20080   GOTO 20240
20090 '
20110   GOSUB 50000
20120   GOSUB 58000
20130   GOSUB 30000
20140   CLS
20150   GOSUB 50000
20160   GOSUB 58000
20170   GOSUB 30690
20180   GOSUB 30850
20240 '
20250 '
20260 '
20270 ' *** MODE 1 ***
20280 '
20290 '
20300   WU1=8*(MYB+(1-X)*MYT)/(LY*LY*(1-4*X*Y*Y))
20310   WSAVE=WU1 : MODE = 1
20320 '
20330 '
20340 ' *** MODE 2 ***
20350 '
20360 '
20370   WU=1000000!
20380   ATOP=Y
20390   ABOT=.001
20400      FOR A=ABOT TO ATOP STEP .01
20410         W=2*(1-X)*(MYT+MYB)/(A*LY*LY*(1+A*X-A-2*X*Y))
20420            IF W<WU THEN WU=W
20430      NEXT
20440   WU2=WU
20450      IF WU2<WSAVE THEN WSAVE=WU2
20460     IF WSAVE=WU2  THEN MODE = 2
20465 '
20470 '
20480 '
20490 ' *** OUTPUT ***
20500 '
20510 '
20520   CLS
20550 '
20560 '
20570   PRINT :PRINT :PRINT
20580   PRINT "  ***   RECTANGULAR SLAB  --  TWO SIDES FIXED  --  SYMMETRICAL E
ND OPENINGS   ***"
```

```
20590   GOSUB 40000
20600   RETURN
20610 '
20620 '
20630 '
20640 '
30000 '
30010 '
30020 '
30030 '
30040 '
30050 '  ***********************************************************************
30060 '  **************   INPUT SUBROUTINE  -- OPENING DIMENSIONS   ***********
30070 '  ***********************************************************************
30080 '                  *********   WITH GRAPHICS   ***********
30090 '
30100 '
30110 '
30120 '
30130 '
30140   LOCATE 14,2 : INPUT "Please enter value for horizontal side dimension, Y
 (ft) " ;LY
30150   LOCATE 15,2 : INPUT "Please enter value for vertical side dimension, X (
ft) ";LX
30160      IF LY<=0 OR LX<=0 THEN LOCATE 17,2 ELSE 30260
30170   PRINT "ERROR -- PLEASE ENTER POSITIVE VALUES."
30180   LOCATE 19,2 : INPUT "PRESS ENTER TO RE-ENTER VALUES.";ANS
30190   GOSUB 60040
30200   GOTO 30140
30260   LOCATE 16,2 : INPUT "Please enter value for opening dimension, A (ft) ";
 A
30270      IF A>0 THEN 30320
30280   LOCATE 18,2 : PRINT "A MUST BE GREATER THAN 0 !"
30290   LOCATE 20,2 : INPUT "PRESS ENTER TO RE-ENTER VALUES.";ANS
30300   GOSUB 60070
30310   GOTO 30260
30320      IF A<LY THEN 30370
30330   LOCATE 18,2 : PRINT "ERROR -- VLAUE FOR A MUST BE SMALLER THAN Y !"
30340   LOCATE 20,2 : INPUT "PRESS ENTER TO RE-ENTER VALUES.";ANS
30350   GOSUB 60070
30360   GOTO 30260
30370      IF FLAG=2 AND A>=(LY/2) OR FLAG=3 AND A>=(LY/2) THEN LOCATE 18,2 ELSE
        30420
30380   PRINT "ERROR -- VALUE FOR A MAY NOT EXCEED Y/2 !"
30390   LOCATE 20,2 : INPUT "PRESS ENTER TO RE-ENTER VALUES.";ANS
30400   GOSUB 60070
30410   GOTO 30260
30420   LOCATE 17,2 : INPUT "Please enter value for opening dimension, B (ft)";
        B
30430      IF B<=0 THEN LOCATE 19,2  ELSE 30480
30440   PRINT "ERROR -- VALUE FOR B MUST BE GREATER THAN 0 !"
```

```
30450   LOCATE 21,2 : INPUT "PRESS ENTER TO RE-ENTER VALUES.";ANS
30460   GOSUB 60080
30470   GOTO 30420
30480      IF B>=LX THEN LOCATE 19,2  ELSE 30530
30490   PRINT "ERROR -- VALUE FOR B MUST BE SMALLER THAN VALUE FOR X !"
30500   LOCATE 21,2 : INPUT "PRESS ENTER TO RE-ENTER VALUES.";ANS
30510   GOSUB 60080
30520   GOTO 30420
30530      IF FLAG=0 OR FLAG=3 THEN 30640
30540   GOSUB 58420
30545   INPUT "Please enter value for C (ft) ";C
30546   C1=C
30550      IF C>LY-A THEN 30590
30560      IF C>LY-A-C THEN C=LY-A-C
30570      IF FLAG=1 AND C>=0 AND C<(LY-C-Y) THEN 30630
30580      IF FLAG=2 AND C>=0 AND (C/LY)<(.5-(A/LY)) THEN 30630
30590   LOCATE 20,2 : PRINT "ERROR  --  INVALID VALUE FOR C"
30600   LOCATE 22,2 : PRINT "PRESS ENTER TO RE-ENTER VALUE.";ANS
30610   GOSUB 60090
30620   GOTO 30560
30630   C=C/LY
30640   IF CHANGE<>1 THEN 30680
30650   CLS
30660   GOSUB 30690
30670   GOSUB 30820
30680   RETURN
30690   LOCATE 14,2 : INPUT "Enter negative moment capacity for bending about th
e x-axis MYT (k-ft/ft)"; MYT
30700   LOCATE 15,2 : INPUT "Enter positive moment capacity for bending about th
e x-axis MYB (k-ft/ft)"; MYB
30710   LOCATE 16,2 : INPUT "Enter negative moment capacity for bending ab
out the y-axis MXT (k-ft/ft)"; MXT
30720   LOCATE 17,2 : INPUT "Enter positive moment capacity for bending about th
e y-axis MXB (k-ft/ft)"; MXB
30730      IF MYT>=0 AND MYB>=0 AND MXT>=0 AND MXB>=0 THEN 30780
30740   LOCATE 19,2 : PRINT "MOMENT CAPACITIES MAY NOT BE NEGATIVE !"
30750   LOCATE 21,2 : INPUT "PRESS ENTER TO RE-ENTER VALUES.";ANS
30760   GOSUB 58420
30770   GOTO 30690
30780   X=B/LX
30790   Y=A/LY
30800   RETURN
30810   PRINT : PRINT
30820   CHANGE = 0
30830   PRINT
30840   PRINT "Your values are now the following:
30850   PRINT
30860   PRINT "A ="A"ft.   B ="B"ft.    Y ="LY"ft.   X ="LX"ft.  C ="C1*LY"ft. (
IF APPLICABLE)
30870   PRINT "MYT ="MYT"k-ft/ft    MYB ="MYB"k-ft/ft    MXT ="MXT"k-ft/ft    MX
B ="MXB"k-ft/ft
```

```
30880   PRINT : PRINT
30890   INPUT "Would you like to change any of these values? (Y/N)"; ANS$
30900      IF ANS$="y" OR ANS$="Y" THEN 30910 ELSE 30930
30910   CLS
30920   CHANGE =1 : GOTO 30130
30930      IF ANS$="n" OR ANS$="N" THEN 30960
30940   PRINT "ERROR -- Please answer Y or N !"
30950   PRINT : GOTO 30890
30960   FLAG=0 : CHANGE = 0
30970   CLS :PRINT  : PRINT "WORKING"
30980   RETURN
30990 '
31000 '
31010 '
35000 '
35010 '
35020 '
35030 '  *******************************************************************
35040 '  **************   INPUT SUBROUTINE  -- OPENING DIMENSIONS   ***********
35050 '  *******************************************************************
35060 '                    *******   NO GRAPHICS   ********
35070 '
35080 '
35090 '
35100 '
35110 '
35120   CLS : PRINT : PRINT
35130   PRINT "Please refer to thesis for location of dimensions on figures."
35140   PRINT
35150   INPUT "Please enter value for horizontal side dimension, Y (ft) " ;LY
35160   INPUT "Please enter value for vertical side dimension, X (ft) ";LX
35170      IF LY<=0 OR LX<=0 THEN PRINT "ERROR -- PLEASE USE POSITIVE VALUES."
           ELSE 35210
35180   GOTO 35150
35210   INPUT "Please enter value for opening dimension, A (ft) "; A
35220      IF A<=0 THEN PRINT "INVALID VALUE FOR A!" ELSE 35250
35230   INPUT "Enter new value for A "; A
35240   GOTO 35220
35250      IF A>=LY THEN PRINT "ERROR -- Value for A must be smaller than the va
lue for Y! " ELSE 35280
35260   INPUT "Enter new value for A "; A
35270   GOTO 35220
35280      IF FLAG=2 AND A>=(LY/2) OR FLAG=3 AND A>=(LY/2) THEN PRINT "A MAY NOT
 EXCEED Y/2" ELSE 35300
35290   GOTO 35210
35300   INPUT "Please enter value for opening dimension, B (ft)"; B
35310      IF B<=0 THEN PRINT "INVALID VALUE FOR B!" ELSE 35340
35320   INPUT "Enter new value for B "; B
35330   GOTO 35310
35340      IF B>=LX THEN PRINT "ERROR -- Value for B must be smaller than the va
lue for X! " ELSE 35370
```

```
35350   INPUT "Enter new value for B "; B
35360   GOTO 35310
35370      IF FLAG=0 OR FLAG=3 THEN 35460
35380   INPUT "Please enter value for offset of opening, C (ft)";C
35385   C1=C
35390      IF C>LY-A THEN 35430
35400      IF C>LY-A-C THEN C=LY-A-C
35410      IF FLAG=1 AND C>=0 AND C<(LY-C-Y) THEN 35450
35420      IF FLAG=2 AND C>=0 AND (C/LY)<(.5-(A/LY)) THEN 35450
35430   PRINT : PRINT "ERROR  --  INVALID VALUE FOR C"
35440   PRINT : GOTO 35380
35450   C=C/LY
35460 '
35470   INPUT "Enter negative moment capacity for bending about the x-axis MYT (
k-ft/ft)"; MYT
35480   INPUT "Enter positive moment capacity for bending about the x-axis MYB (
k-ft/ft)"; MYB
35490   INPUT "Enter negative moment capacity for bending about the y-axis MXT (
k-ft/ft)"; MXT
35500   INPUT "Enter positive moment capacity for bending about the y-axis MXB (
k-ft/ft)"; MXB
35510      IF MYT>=0 AND MYB>=0 AND MXT>=0 AND MXB>=0 THEN 35550
35520   PRINT "ERROR --  MOMENT CAPACITIES MAY NOT BE NEGATIVE !"
35530   PRINT
35540   GOTO 35470
35550   X=B/LX
35560   Y=A/LY
35570   PRINT : PRINT
35580   PRINT "Your values are now the following:
35590   PRINT
35600   PRINT "A ="A"ft.   B ="B"ft.    Y ="LY"ft.   X ="LX"ft.  C ="C1*LY"ft. (
IF APPLICABLE)
35610   PRINT "MYT ="MYT"k-ft/ft    MYB ="MYB"k-ft/ft    MXT ="MXT"k-ft/ft    MX
B ="MXB"k-ft/ft
35620   PRINT : PRINT
35630   INPUT "Would you like to change any of these values? (Y/N)"; ANS$
35640      IF ANS$="y" OR ANS$="Y" THEN 35110
35650      IF ANS$="n" OR ANS$="N" THEN 35680
35660   PRINT "ERROR -- Please answer Y or N !"
35670   PRINT : GOTO 35630
35680   FLAG=0
35690   CLS :PRINT  : PRINT "WORKING"
35700   RETURN
35710 '
35720 '
40000 '
40005 '
40006 '
40010 '
40015 ' ***********************************************************************"
40020 ' *********************   OUTPUT SUBROUTINE   ***********************
```

```
40025 ' *******************************************************************"
40030 '
40035 '
40040 '
40045 '
40046  PRINT "                    with the following dimensions :"
40047  PRINT
40050  PRINT "A ="Y*LY"ft.   B ="X*LX"ft.    Y ="LY"ft.   X ="LX"ft.
40055  PRINT "C ="C*LY"ft. (IF APPLICABLE)
40060  PRINT "MYT ="MYT"k-ft/ft    MYB ="MYB"k-ft/ft    MXT ="MXT"k-ft/ft    MX
B ="MXB"k-ft/ft
40080  PRINT
40090  PRINT : PRINT : PRINT : PRINT
40100  PRINT "                 The ultimate load is "WSAVE" KIPS/SQ.FT
40110  PRINT : PRINT "                      The failure is by mode #"MODE
40120  PRINT  : PRINT : PRINT
40130  RETURN
40140 '
40150 '
50000 '
50005 '
50010 '
50015 '
50020 '
50025 ' *******************************************************************"
50030 ' *********************    COMMON GRAPHICS    ***********************
50035 ' *******************************************************************"
50040 '
50045 '
50050 '
50055 '
50060 '
50070  SCREEN 2
50080  CLS
50090  LINE (42,17)-(267,86),,B
50100  LINE (42,4)-(266,4)
50110  LINE (287,18)-(287,85)
50120  LINE (42,3)-(42,5)
50130  LINE (267,3)-(267,5)
50140  LINE (285,17)-(289,17)
50150  LINE (285,86)-(289,86)
50160  LOCATE 1,19 : PRINT "Y"
50170  LOCATE 7,38 : PRINT "X"
50180 '
50190  LOCATE 4,45 :PRINT "Y-DIR REINFORCEMENT
50200  LINE (520,22)-(590,22)
50210  LINE (520,25)-(590,25)
50220  LINE (520,28)-(590,28)
50230  LINE (520,31)-(590,31)
50240  LOCATE 8,45 : PRINT "X-DIR REINFORCEMENT"
50250  LINE (530,40)-(530,80)
```

```
50260  LINE (537,40)-(537,80)
50270  LINE (544,40)-(544,80)
50280  LINE (551,40)-(551,80)
50290  LINE (558,40)-(558,80)
50300     FOR P=3 TO 11
50310        LOCATE P,5
50320        PRINT CHR$(219);
50330     NEXT
50340     FOR P=3 TO 11
50350        LOCATE P,35
50360        PRINT CHR$(219);
50370     NEXT
50380     RETURN
50390 '
50400 '
50410 '
50420 '
51000 '
51010 '
51020 ' *******   CENTER OPENING GRAPHICS   *********
51030 '
51040 '
51050  LINE (126,42)-(177,62),1,BF
51060  LINE (127,36)-(176,36)
51070  LINE (192,43)-(192,61)
51080  LINE (126,35)-(126,37)
51090  LINE (177,35)-(177,37)
51100  LINE (190,42)-(194,42)
51110  LINE (190,62)-(194,62)
51120  LOCATE 4,19 : PRINT "A"
51130  LOCATE 7,26 : PRINT "B"
51140  RETURN
51150 '
51160 '
51170 '
52000 '
52010 '
52020 '
52030 ' ********   OPENING ON FIXED EDGE GRAPHICS   ********
52040 '
52050 '
52060  LINE (126,17)-(177,37),1,BF
52070  LINE (127,43)-(176,43)
52080  LINE (192,18)-(192,36)
52090  LINE (126,42)-(126,44)
52100  LINE (177,42)-(177,44)
52110  LINE (190,37)-(194,37)
52120  LOCATE 7,19 : PRINT "A"
52130  LOCATE 4,26 : PRINT "B"
52140  RETURN
52150 '
```

```
52160 '
54000 '
54010 '
54020 '
54030 ' ******   OPENING ON FREE EDGE GRAPHICS   **************
54040 '
54050 '
54060  LINE (126,66)-(177,86),1,BF
54070  LINE (127,60)-(176,60)
54080  LINE (192,67)-(192,85)
54090  LINE (126,59)-(126,61)
54100  LINE (177,59)-(177,61)
54110  LINE (190,67)-(194,67)
54120  LINE (190,85)-(194,85)
54130  LOCATE 7,19 : PRINT "A"
54140  LOCATE 10,26 : PRINT "B"
54150  RETURN
54160 '
54170 '
54180 '
55000 '
55010 '
55020 '
55030 ' ******** TWO SIDES FIXED - INTERIOR OPENING GRAPHICS   *************
55040 '
55050 '
55060  LINE (96,42)-(177,62),1,BF
55070  LINE (97,36)-(176,36)
55080  LINE (96,35)-(96,37)
55090  LINE (177,35)-(177,37)
55100  LOCATE 4,18 : PRINT "A"
55110  LINE (200,43)-(200,61)
55120  LINE (198,42)-(202,42)
55130  LINE (198,62)-(202,62)
55140  LOCATE 7,24 : PRINT "B"
55150  LINE (42,36)-(96,36)
55160  LOCATE 4,9 : PRINT "C"
55170  RETURN
55180 '
55190 '
56000 '
56010 '
56020 '
56030 ' ********   TWO SIDES FIXED - END OPENING GRAPHICS   *************
56040 '
56050 '
56060 '
56070 '
56080 '
56090 '
56100  LINE (42,42)-(92,62),1,BF
```

```
56110  LINE (43,36)-(91,36)
56120  LINE (92,35)-(92,37)
56130  LOCATE 4,9 : PRINT "A"
56140  LINE (117,43)-(117,61)
56150  LINE (115,42)-(119,42)
56160  LINE (115,62)-(119,62)
56170  LOCATE 7,14 : PRINT "B"
56180  RETURN
56190 '
56200 '
56210 '
57000 '
57010 '
57020 ' *****   TWO SIDES FIXED - SYM. INTERIOR OPENINGS GRAPHICS   *****
57030 '
57040 '
57050 '
57060 '
57070 '
57080 '
57090  LINE (82,42)-(122,62),1,BF
57100  LINE (187,42)-(227,62),1,BF
57110  LINE (150,43)-(150,61)
57120  LINE (148,42)-(152,42)
57130  LINE (148,62)-(152,62)
57140  LOCATE 7,18 : PRINT "B"
57150  LINE (83,36)-(121,36)
57160  LINE (82,35)-(82,37)
57170  LINE (122,35)-(122,37)
57180  LOCATE 4,14 : PRINT "A"
57190  LINE (42,36)-(82,36)
57200  LOCATE 4,8 : PRINT "C"
57210  RETURN
57220 '
57230 '
58000 '
58010 '
58020 '
58030 ' ******   TWO SIDES FIXED - SYM. END OPENINGS GRAPHICS   ******
58040 '
58050 '
58060 '
58070 '
58080 '
58090 '
58100  LINE (42,42)-(82,62),1,BF
58110  LINE (227,42)-(267,62),1,BF
58120  LINE (150,43)-(150,61)
58130  LINE (148,42)-(152,42)
58140  LINE (148,62)-(152,62)
58150  LOCATE 7,18 : PRINT "B"
```

```
58160   LINE (43,36)-(81,36)
58170   LINE (82,35)-(82,37)
58180   LINE (42,35)-(42,37)
58190   LOCATE 4,8 : PRINT "A"
58200   RETURN
58210 '
58215 '
58220 '
58225 ' ************   THREE SIDES FIXED  --  GRAPHICS   *************
58230 '
58235 '
58240 '
58250   LOCATE 2,5
58260      FOR P=1 TO 31
58270         PRINT CHR$(220);
58280      NEXT
58290      RETURN
58300 '
58305 '
58310 '
58315 ' ************   FOUR SIDES FIXED  --  GRAPHICS   **************
58320 '
58325 '
58330   LOCATE 12,5
58340      FOR P=1 TO 31
58350         PRINT CHR$(223);
58360      NEXT
58370   LOCATE 2,5 :
58360      FOR P=1 TO 31
58390         PRINT CHR$(220);
58400      NEXT
58410      RETURN
58420 '
58430 '
58440 '
58450 '
60000 '
60010 ' **************** CLEARING SUBROUTINE ************************
60020 '
60025 '
60030 '
60035 '
60040    LOCATE 13,1 : PRINT "
                               "
60050    LOCATE 14,1 : PRINT "
                               "
60060    LOCATE 15,1 : PRINT "
                               "
60070    LOCATE 16,1 : PRINT "
                               "
```

```
60080    LOCATE 17,1 : PRINT "
                                        "
60090    LOCATE 18,1 : PRINT "
                                        "
60100    LOCATE 19,1 : PRINT "
                                        "
60110    LOCATE 20,1 : PRINT "
                                        "
60120    LOCATE 21,1 : PRINT "
                                        "
60130    LOCATE 22,1 : PRINT "
                                        "
60140    LOCATE 23,1 : PRINT "
                                        "
60150    LOCATE 24,1 : PRINT "
                                        "
60160    RETURN
```

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