Use of the Karhunen-Loeve Expansion to Represent EDFA-ASE Noise

The primary source of additive noise in optically amplified systems is due to the amplified spontaneous emissions (ASE) produced by the optical amplifiers used as intermediate repeaters and as preamplifiers at the receiver end. The ASE noise mixes with the signal and produces beat noise components at the square-law receiver. The ASE noise is very broadband (~40 nm) and needs to be carefully analyzed to evaluate its degrading effect on system performance. In this appendix we discuss the use of the classical Karhunen Loeve expansion, as used in this dissertation, to represent ASE noise.

Most periodic waveforms can be expanded in terms of a Fourier series. However since the ASE noise is random and nonperiodic, it is not possible to directly use a conventional Fourier series representation. In this case, it becomes essential to find an orthonormal series where the coefficients are independent (uncorrelated), but identically distributed random variables. This form of a series, termed the Karhunen-Loeve expansion, is really an extension of the more general Fourier series, but with the basis functions defined so as to satisfy the uncorrelated coefficients condition. Hence the objective then becomes to find the orthonormal basis set $\phi_n(t)$.

The non-periodic ASE waveform $x(t)$ may be represented as

$$x(t) = \sum_{n=1} c_n \phi_n(t) \qquad\qquad a \le t \le b \qquad\qquad (A.1)$$

where the orthonormal set satisfies

$$\int_a^b \phi_n(t)\,\phi_m^*(t)\,\mathrm{d}t = \delta_{nm} \tag{A.2}$$

and the coefficients c_n are given by

$$c_n = \int_a^b x(t)\,\phi_n(t)\,\mathrm{d}t. \tag{A.3}$$

Here the coefficients are zero-mean Gaussian random variables, and $\delta_{nm} = 1$ is the Kronecker delta function. They are considered to be uncorrelated when

$$E\!\left[c_n c_m\right] = \lambda_n\,\delta_{nm} \tag{A.4}$$

and this happens when the set of orthonormal functions satisfies the condition

$$\lambda_n\,\delta_{nm} = \int_a^b R(t-t')\,\phi_n(t')\,\mathrm{d}t' \tag{A.5}$$

with $R(\ )$ being the autocorrelation function of $x(t)$. The constants λ_k and the functions $\phi_n(t)$ are the eigenvalues and eigenfunctions, respectively, for the integral equation in Eq. (A.5).

The technique described above is a fairly standard way of representing ASE noise that originates from optical amplifiers [71]. The use of the chi-square distribution is equivalent to assuming that there are m equal non-zero eigenvalues. For rectangular spectra, this has been shown to be a good approximation [60] for moderately large values of m.

A similar analysis, for deterministic signals was performed by Marcuse [44,45]. In his analysis he expands the ASE noise electric field $e(t)$ in a Fourier series representation as

$$e(t) = \sum_{\nu=0} c_\nu \exp\!\left(j\,\omega_\nu\,t\right) \tag{A.6}$$

where the frequency components

$$\omega_\nu = \frac{2\pi\nu}{T} \tag{A.7}$$

are periodic during the bit interval T. The expansion coefficients are of the noise field are then assumed to be zero-mean, independent Gaussian random variables.

Although the two expansions, described in Eqs. (A.5) and (A.6) are equivalent, since both are equivalent to the classical Karhunen-Loeve expansion, Marcuse has used them to analyze the case of a deterministic signal, whereas we have used the analysis to analyze the case of a spectrum-sliced system where the signal source is inherently noisy. Although the distributions are essentially similar, the results are very different. For example, for the deterministic systems treated by Marcuse, the receiver sensitivity calculated by the Gaussian approximation for even small values of $m = B_oT$ is fairly accurate. But for the noise-like case considered in this dissertation, the Gaussian approximation is extremely conservative for low values of m (See Section 3.3). Moreover, use of the exact analysis in the case of a spectrum-sliced system results in an optimum value of m, and we believe that this work is the first to report and analyze this optimum.