

MODELING GROWTH DYNAMICS OF JUVENILE
LOBLOLLY PINE PLANTATIONS

by

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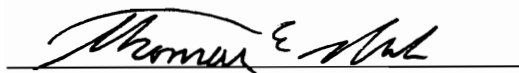
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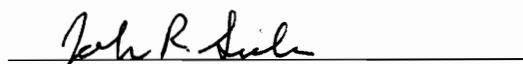
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Abstract

Modeling growth dynamics of juvenile loblolly pine plantations can help to address important management decisions that have to be made in young stands. The present study addressed diameter and height prediction as well as crown development analysis as functions of independent variables such as site index, relative spacing and age for trees younger than ten years old. It was found that height prediction for trees one and two years old was a function of the products of the variables site index and age (or square root of age) and relative spacing and age (or square root of age). For trees three years old and older these were also the independent variables used to explain height prediction but in this case age (or square root of age) was another significant independent variable. In regard to diameter prediction it was observed that groundline diameter was a function of age for trees one and two years old; while this variable was also a function of site index and relative spacing for trees three, four and five years old. For trees older than five years old diameter at breast height was a function of age, site index and relative spacing. The analysis of crown development showed that for trees five years old and older variables like diameter at breast height, total height, age, site index and relative spacing were significant when crown ratio was the dependent variable. For trees younger than five years old, groundline diameter and the other same independent variables (i.e. total height, age, site index and relative spacing) were again found significant when crown ratio was the dependent variable.

One of the most important factors of concern to foresters is the effect of density on growth. Spacing trials give valuable information on density effects on height and diameter growth. Data coming from trees planted at conventional spacings were used to make comparisons of the trends of characteristics such as diameter, height, groundline basal area and number of trees per area with published spacing studies of loblolly pine and other species. Similar patterns were observed when comparing trees planted at conventional spacings, seedlings planted at very close spacing, and published studies reporting on observations from spacing trials.

Although conventional spacing trials give valuable information about density effects on growth, there is a need to obtain this information in shorter periods of time. Seedlings grown at close spacings to accelerate the process of obtaining this information were used to scale seedlings stands to mature stands. Three methods of scaling were investigated. The most promising method involved using relative spacing (average distance between trees divided by average dominant tree height) as the x -axis.

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CHAPTER I Introduction

Background

Juvenile growth-density relationships are very helpful in understanding the later growth and development of trees. Important management decisions such as thinnings have to be made early in the development of stands (Maguire, 1982). This requires knowledge of characteristics of the stands early in their growth. Yet, little is known about stand dynamics in the first years (planting through 10 years old) of the stands. There is a need to model juvenile growth so stand characteristics at older ages can be predicted and the impact of early treatments on these stands characteristics can be understood. Density effects on characteristics such as diameter and height can be studied in young trees and the behavior of these young stands compared to older stands.

Although important management decisions must be made early in the development of the stands, most of the growth-yield models for southern pines have concentrated on trees older than 8-10 years old (Bacon, 1986; Farrar, 1989). To be most useful, growth models need to start at the time of regeneration. For planted stands, spacing trials are a good source of this type of information.

Conventional spacing trials provide important information regarding stand dynamics, but trees must be grown over long periods of time. Long-term trials may involve the increased risk of losing part or whole experiments due to catastrophic events. The possibility of using seedlings grown at close spacings should be considered as a

potential tool of getting information regarding stand dynamics characteristics in a shorter period of time.

An important consideration that has to be taken into account when using these seedlings is the scaling process from seedlings grown at close spacing to mature stands grown at conventional spacings.

Although scaling has been used extensively in physics and engineering sciences its use in biology has been limited. Part of the problem is that biological processes are relatively unpredictable so the definition of a scale model to characterize an original model can be challenging. A good understanding of the biological processes involved is very important.

There are, however, several examples of the use of scaling rules in biology. Some of these studies deal with animals (McMahon, 1973) or allometric relationships for animals (Hokkanen, 1986), while others are concerned with allometric measures of plant geometry (Weller, 1987 and Hutchings, 1983).

In forestry spacing trials, Smith and Hann (1984) studied the $-3/2$ rule of self-thinning both in seedlings and older trees. This important aspect of scaling should be further examined. Its use may mean savings in time and cost in forestry experimentation.

Objectives

The purpose of this study was to model the growth dynamics of juvenile loblolly pine plantations using data from conventional spacing trials. Using the same design but very close spacings, spacing trials were established for seedlings of loblolly pine. Comparisons of the trends between the two trials were made to examine whether the scaling of seedling stands to mature stands was possible. Thus the following objectives were pursued in this study:

1. To model juvenile growth of loblolly pine plantations.
2. To analyze growth relationships for seedlings planted at very close spacings.
3. To scale seedling stands to mature stands.

Data Description

Two experiments were used to accomplish the objectives of this study.

In the spring of 1983 four locations were selected in Virginia and North Carolina for the establishment of spacing trials. Two of these locations were situated in the piedmont region and the other two in the coastal plain region. All locations were cutover areas that received an operational chop and burn site preparation treatment prior to planting. (Amateis et al.,1988; Liu, 1992).

The planting stock used was genetically improved 1-0 loblolly pine seedlings. Only those healthy and vigorous seedlings lifted from the same nursery bed at the same

time were planted. Outplantings were established beside each block as replacements for first-year mortality. Three times during the first year each location was visited and mortality replaced with the outplantings. From establishment, both herbaceous and woody competing vegetation have been controlled using chemical herbicides. The herbicides have been applied annually as a directed spray using backpack sprayers. The chemical control helped achieve high initial survival of the pines and allowed the pines to gain control of the site quickly.

The experimental design chosen was a non-systematic design described by Lin and Morse (1975). Each measurement plot consists of 49 trees with guard trees bordering each plot. There are three replications in each of the four locations. Because of the poor survival in one of the locations (lower coastal plain) the three blocks of this location were reestablished one year after the initial planting at all locations.

For this study a spacing factor (F) of 4 feet (1.22 meters) was selected and four spacing levels of this factor (1F, 1.5F, 2F and 3F) were used in the design. Thus each block consists of sixteen plots arranged in a factorial way with the four spacing levels applied at random to rows and columns. The layout is shown in Figure 1.

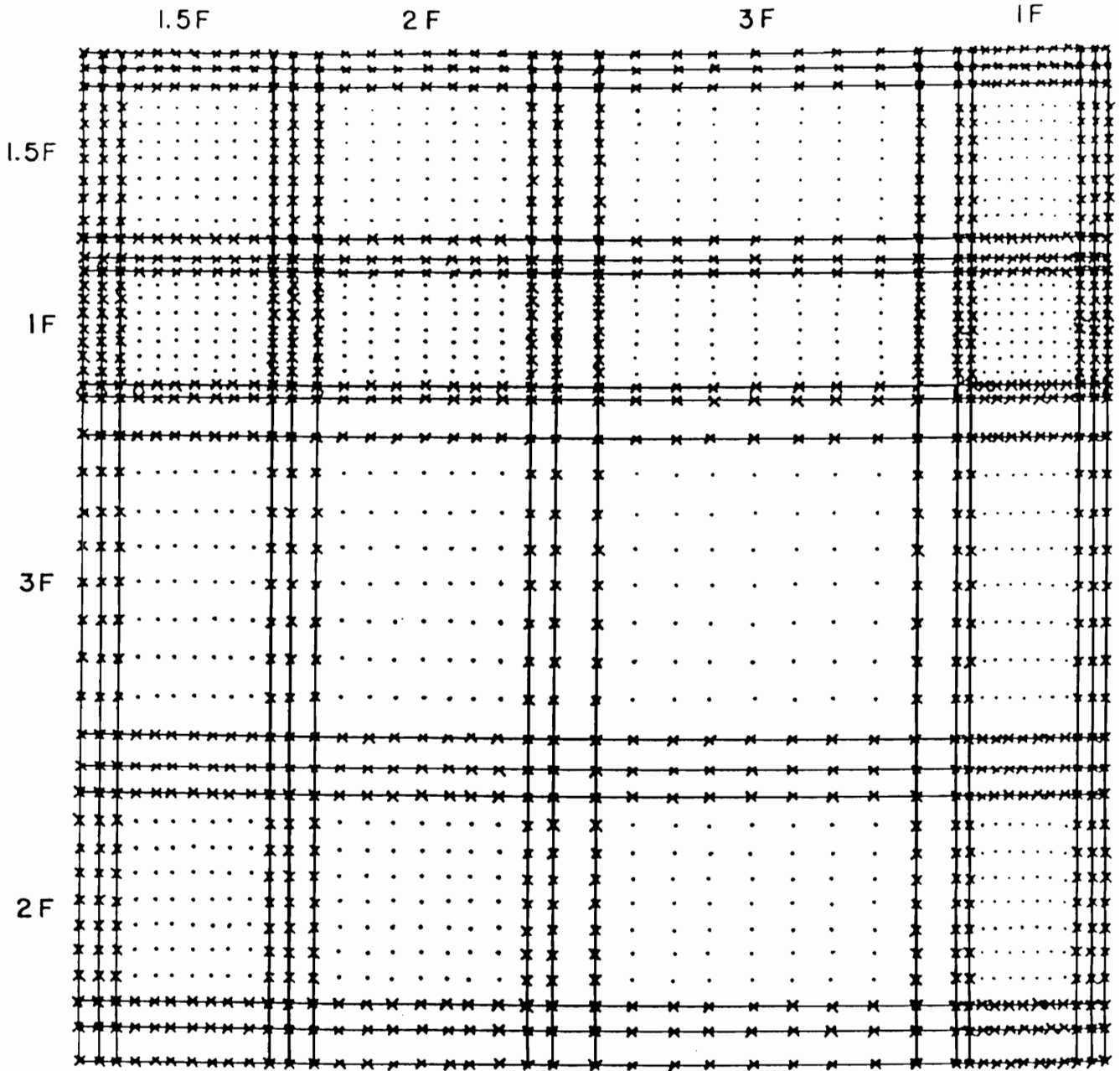


Figure 1. Example of a non-systematic block design. The spacing factor is F (Amateis et al., 1988).

The data collected, on an annual basis, included groundline diameter, total height, height to the base of the live crown (crown height), crown width within the rows, crown width between rows and maximum crown width. In 1988 diameter at breast height was measured in addition to groundline diameter; since 1988 diameter at breast height has been the only stem diameter measured. Also each tree was examined for signs of disease or damage and assigned an appropriate code. This dataset, referred to as conventional spacing study, was used to accomplish objective 1.

To accomplish objective 2, data coming from a seedling spacing study were used. In the spring of 1989 a miniature-scale loblolly pine spacing trial was established near the Ecosystem Simulation Laboratory on the VPI campus. The site was prepared to assure the environmental factors and site quality to be as uniform as possible. Measurements were taken at the beginning of each month from July to October of the first year (1989) and from April to October the next three years (1990-1992) on each measurement tree. Such measurements included groundline diameter, total height, height to the base of the live crown, crown width within the row and crown width between the rows. The same measurements annually collected from the spacing trial described to study objective 1 were taken in this miniature-scale study for four years. This commonality of measurements allowed comparisons between studies.

The same experimental design used for the spacing trial described in the data to be used to study objective 1 was used for the seedling study, but in this case the spacing factor (F) was 3 inches. There were also four levels of this factor: 1F, 1.5F, 2F and 3F. It is anticipated that the average height of the dominant trees in the miniature study will be approximately four feet after two growing seasons. In this way the ratio of the height to initial spacing in this study will be 16 (48 inches/3inches) for the closest

spacing. In the large-scale studying, assuming that the average height of dominant trees will be 60 feet by age 25, the height to initial spacing ratio would be 15 (60 feet/4 feet) for the closest spacing. Thus, the miniature-spacing study closely paralleled the spacing trials established at operational spacings (Amateis and Burkhart, 1987).

Both experiments, the conventional and the seedling spacing studies, were used to pursue objective 3 of this project.

CHAPTER II Spacing Studies and Juvenile Growth-Density Relationships

Introduction

The study of juvenile growth is of great importance in understanding the later development of the trees. Data from spacing trials are appropriate for analyzing one of the most important factors of concern to foresters, i.e. the effects of density and spacing on growth and development. Spacing trials give valuable information on density effects on height and diameter growth.

Most of the growth and yield models for southern pine have focused on stands greater than 8-10 years old (Bacon, 1986, Farrar, 1979). Important management decisions must be made for trees younger than 10 years old (Maguire, 1982). With models that start at the time of planting, it may be possible to estimate adequately the impact of silvicultural treatments early in the life of the stand on rotation-age yields.

Objectives

The purpose of this chapter is to present a model of the growth dynamics of juvenile loblolly pine plantations using data from conventional spacing trials established in the spring of 1983.

Literature Review

Numbers of trees planted per unit area greatly influences rates of growth and wood quality (Evert, 1971). Valuable data on juvenile growth and mortality relationships and density effects on diameter and height growth can be obtained from spacing trials measured at regular intervals after establishment (Burkhart, 1987).

Of all the decisions that foresters make at the time of tree planting, one of the most important is to determine the spacing at which trees will be placed (Balmer et al., 1975). The initial spacing influences not only the time when a thinning can be made, but also the size of the trees that will be removed as well as the stem quality and amount of wood available at that thinning (Campbell and Mann, 1974). To select the appropriate spacing, management objectives and biological relationships must be taken into account (Harms and Lloyd, 1981).

Many spacing trials in several forestry species have been established to study the effects of different planting densities on growth (Newnham, 1964). These spacing trials help in understanding the growth-density relationships in the early development of stands.

Spacing studies in Douglas-fir can be found in the literature. Some of these studies have been going on for over a period of 50 years (Eversole, 1955, Reukema, 1970, Reukema, 1979) which provide a valuable record of effects of initial spacing on stand development over a long period of time.

Spacing studies in lodgepole pine were reported by Johnstone and Pollack (1990)

and Yang (1991). The influence of regular spacing on growth of red pine was studied by Godman and Smith (1963) and by Stiel and Berry (1977). Studies in a 12-year-old ponderosa pine plantation at 5 different spacings were reported by Oliver (1979). He reported that spacing strongly influenced stem diameter at breast height, crown width, live crown ratio, and average branch diameter. The effects of spacing on the growth of slash pine were analyzed by Mann (1971), Bennett (1960), and Gilmore and Gregory (1974). Some of these results were from young plantations. Observations made sufficiently early in the stands help to define incipient density effects on crown and stem development.

The optimum spacing for loblolly pine has also been the subject of study (Ware and Stahelin, 1948, Campbell and Mann, 1974, Whitesell, 1974, Balmer et al., 1975, McClurkin, 1976). As Conrad et al. (1992) pointed out, planting density greatly influences the yield and value of a pine plantation. This is why a recommendation for optimum spacing is so important. Bailey (1986) reviewed the effects of stand density and age on average tree size and volume yield and studied their effects on relevant financial criteria such as internal rate of return and net present value. Studies by Little and Somes (1958) reported results 18 years after planting loblolly pine. Mann and Dell (1971) summarized the development and yield of loblolly pine 17 years after planting at five spacing over a broad spectrum of site classes.

A comparison between similar spacing studies was done by DeBell et al.(1989). They found that basal area and volume production in loblolly pine spacing trials in Hawaii were nearly double the average production in a similar trial in the piedmont of South Carolina (Balmer, 1975, Harms and Lloyd, 1981).

Kramer (1980) reported the influence of spacing in conifers in Germany. He emphasized that all aspects of the care of young plantations should be considered carefully.

Evert (1971) reviewed past spacing studies. An update of this review appeared in 1984 (Evert, 1984). Sjolte-Jorgensen (1967) also synthesized results from spacing studies. The influences of spacing on volume factors such as height, diameter, form factor were compared among several spacing studies. Also volume production, quality of wood and the economic importance of spacing were studied by these authors.

A large spacing study in Great Britain was reported by Hamilton and Christie (1975). One important aspect related to spacing trials is the selection of the experimental design suitable to apply in these experiments. This selection depends on the objectives of the study as pointed out by Mead (1979). Characteristics related to the statistical and practical advantages and disadvantages of randomized and systematic designs should be considered when selecting the design. The advantages of randomized block designs for spacing experiments are well known, but recently systematic designs have also been considered for these types of studies (Amateis et al., 1988). Systematic designs were first proposed by Nelder (1962) and then modified by Bleasdale (1967). Some characteristics of these designs are particularly advantageous to spacing studies in forestry. Evert (1971), in his review, showed some of these Nelder-type designs. A particular type of design that combines the characteristics of randomized and systematic design was proposed by Lin and Morse (1975). This design has been applied in spacing studies in loblolly pine (Amateis et al., 1988); some variants have also been applied to study density effects in poplar (Riitters et al., 1989).

Spacing studies, when measured at early ages, can help foresters to understand the growth and yield development of juvenile stands. Their study provides reliable information on juvenile growth (Burkhart, 1987).

Data Description

In the spring of 1983, four locations were selected in Virginia and North Carolina for the establishment of spacing trials. Two of these locations were situated in the piedmont region and the other two in the coastal plain region. All locations were cutover areas that received an operational chop and burn site preparation treatment prior to planting (Amateis et al., 1988, Liu, 1992).

The experimental design chosen was a non-systematic design described by Lin and Morse (1975). From the statistical perspective this arrangement provides unbiased estimates of the mean; analysis of variance techniques are possible as well as response surface and regression methods. From the practical point of view the unequal plot sizes with equal number of trees allows a compact block in the field. Compactness helps maintain uniformity of site conditions and reduces measurement costs associated with long term forestry spacing studies. In this particular design each measurement plot consists of 49 trees with three rows of guard trees bordering each plot. There are three replications in each of the four locations. Because of the poor survival in one of the locations (lower coastal plain) the three blocks of this location were reestablished one year after the initial planting at all locations.

For this study a spacing factor (F) of 4 feet (1.22 meters) was selected and four

spacing levels of this factor (1F, 1.5F, 2F and 3F) were used in the design. Thus each block consists of sixteen plots arranged in a factorial way with the four spacing levels applied at random to rows and columns.

The spacing levels chosen allow a broad range in initial densities from 303 to 2722 trees per acre. The data collected, on an annual basis, include groundline diameter, total height, height to the base of the live crown (crown height), crown width within the rows, crown width between rows, maximum crown width. In 1988 diameter at breast height was measured in addition to groundline diameter; since 1988, diameter at breast height has been the only stem diameter measured. Also each tree was examined for signs of disease or damage and assigned an appropriate code.

Analysis and Procedures

The relationship between height and some stand variables like age and relative spacing was examined. Using regression analysis height and diameter prediction were modeled separately as functions of variables such as age and relative spacing. Regression analyses of crown ratio on tree variables such as height and diameter and on stand variables such as age and number of trees were performed. The possibility of using the ratio of the length to the width of the crown as a dependent variable was also explored.

Thus, three different aspects were studied: height growth, diameter growth, and crown development.

Height prediction

The relationship between height and age was first studied. Two models were further developed, i.e.

$$H_t = H_0 + b_1 A^{b_2} \quad (2.1)$$

and

$$\ln H_t = b_1 \sqrt{A} \quad (2.2)$$

where:

H_t : total height

H_0 : initial height (in our dataset $H_0 = 1.0ft$).

$\ln H_t$: natural logarithm of total height.

A : years after planting.

Model 2.1 was proposed by Belli (1986). Model 2.2 was advanced as appropriate to stabilize the variance of height over age. Once these models were obtained, the next aspect studied was the influence of site index and competition on height growth. It was important to determine at what age these two variables, site index (S) and competition effect, measured by relative spacing (R_s), accounted for a significant portion of the variability in growth of height in these young trees. In order to obtain values of site index, only the latest measurements were used, i.e. measurements at age 10 at three locations and the measurements at age 9 of the other location were considered. An analysis was performed to establish if and when site index was independent of density, i.e. graphically it was established that for those plots with 48 sqft of more of growing space per tree site index was independent of density, thus

only these plots were considered in its evaluation. Each replication in each location was assigned a value of site index using the site index equation from Burkhardt et al. (1987).

To determine if the measure of site index accounted for a significant portion of the residual sum of squares in both models (2.1 and 2.2) the following regressions were considered:

$$H_t = H_0 + b_1 A^{b_2} + b_3 A^{b_2} S \quad (2.3)$$

and

$$\ln H_t = b_1 \sqrt{A} + b_2 \sqrt{A} S \quad (2.4)$$

where:

S : site index (ft at the age 25 years),

and the other variables are as defined before.

Models 2.3 and 2.4 were fitted using trees of age 1 through age 10. Also the same models were fitted using trees older than certain ages and the variable site index was tested to see if it was significant when only considering trees older than a certain age.

Finally the competition aspect was studied. The variable relative spacing (R_s) was added to Models (2.3) and (2.4) as follows:

$$H_t = H_0 + b_1 A^{b_2} + b_3 A^{b_2} S + b_4 A^{b_2} R_s \quad (2.5)$$

$$\ln H_t = b_1 \sqrt{A} + b_2 \sqrt{A} S + b_3 \sqrt{A} R_s \quad (2.6)$$

where:

R_s : relative spacing (average distance between trees divided by average height of the dominant trees),

and the other variables are as defined before.

The same methodology followed with the site index (S) variable was used with the relative spacing (R_s) variable. Models 2.5 and 2.6 were fitted first using all measurements, from years 1 to 10, and then the same models were developed using only trees older than a certain age. In this way the important aspects of site characteristics measured by site index and competition effect measured by relative spacing were studied in the juvenile development of the trees.

Diameter prediction

The relationship between groundline diameter (D_{gl}) for trees five years old and younger and age (A) was fitted:

$$D_{gl} = b_0 + b_1 A \quad (2.7)$$

where:

D_{gl} : groundline diameter.

A : years after planting.

An analysis to determine at what age site index (S) and density (measured by the variable relative spacing in this case (R_s)) became significant was performed by fitting the following equation:

$$D_{gl} = b_0 + b_1 A + b_2 S + b_3 R_s \quad (2.8)$$

where:

S : site index (ft at age 25 years).

R_S : relative spacing.

and the other variables are as defined before.

The relationship between diameter at breast height (D) for trees five years old and groundline diameter (D_{gl}) and height (H_t) of those trees, was also fitted:

$$D = b_0 + b_1 D_{gl} + b_2 H_t. \quad (2.9)$$

where all variables remain as defined before.

Equation 2.9 can be used to convert between the two diameters (groundline diameter and diameter at breast height).

The next important aspects studied were the influence of site (measured by the variable site index) and competition effect (measured by the variable relative spacing) on diameter at breast height (D), i.e. the following regression was fitted to trees five years old and older:

$$D = b_0 + b_1 A + b_2 S + b_3 R_s \quad (2.10)$$

where all the variables remain as defined before.

Crown development

As possible dependent variables that defined the shape of the trees the following ratios were computed: length of the live crown divided by the crown width between

rows (resultant variable: L_{wb}) and length of the live crown divided by the crown width within rows (L_{ww}). For trees five years old and younger the following regressions were performed:

$$L_{wb} = b_0 + b_1 D_{gl} + b_2 H_t \quad (2.11)$$

$$L_{wb} = b_0 + b_1 A + b_2 S + b_3 R_s \quad (2.12)$$

$$L_{wb} = b_0 + b_1 D_{gl} + b_2 H_t + b_3 A + b_4 S + b_5 R_s \quad (2.13)$$

where:

all the variables are as defined before.

Identical analyses were performed using crown ratio (C_r) as dependent variable.

The same analysis was performed for the years 5, 6, 7 and 8 using in this case D or diameter at breast height as the independent variable instead of diameter at groundline (D_{gl}).

Data Analysis

Height prediction

Using all the measurements from year 1 to year 10 the parameters estimates obtained for equation 2.1 were:

$$H_t = 1.0 + 1.198616 A^{1.401443} \quad (2.1a)$$

S_{yx} : 3.2012

The following was the result obtained when fitting equation 2.2 to the same dataset.

$$\ln H_t = 1.072868 \sqrt{A} \quad (2.2a)$$

$$S_{yx}: 0.4109$$

$$R^2: 0.9734 \text{ (computed using uncorrected sum of squares)}$$

The last equation as well as the next equations that do not have an intercept present R^2 values that can not be compared to the equations with an intercept.

The variable S was significant from the first year of development of these young trees. However when fitting equation (2.4) to trees one and two years old the sign of the coefficient of the variable age was negative and so the following equation was fitted to these trees aged one and two:

$$\ln H_t = b_2 \sqrt{A} S \quad (2.4.1)$$

and this was the result obtained:

$$\ln H_t = 0.009407 \sqrt{A} S \quad (2.4.1a)$$

$$S_{yx}: 0.48362$$

$$R^2: 0.7236 \text{ (computed using uncorrected sum of squares)}$$

The next step was to see if competition effect was important and to determine at what age this effect became significant. When introducing the variable R_s into the regression as a possible measure of competition, with all tree measurements from

year 1 to year 10 included, this variable was significant in the analysis. The following equation was obtained (using all the trees aged 1 through 10):

$$\ln H_t = 0.428975 \sqrt{A} + 0.009934 \sqrt{A} S - 0.023590 \sqrt{A} R_s \quad (2.6a)$$

S_{yx} : 0.39254

R^2 : 0.9737 (computed using uncorrected sum of squares)

When fitting the height-prediction equation for years one and two, the variable relative spacing was found to account for a significant reduction in the residual sum of squares. As it was done with the variable site index, a model that considered only the products of the variables site index and age, and relative spacing and age was fitted to trees one and two years old. Thus the final models for these young trees was the one that considered relative spacing, site index and age as independent variables but the form of the model was different for trees one and two years old, and for trees older than three years old. The equations obtained for these two cases were:

For trees one and two years old:

$$\ln H_t = 0.014271 \sqrt{A} S - 0.121217 \sqrt{A} R_s \quad (2.6.1a)$$

S_{yx} : 0.39592

R^2 : 0.8157 (computed using uncorrected sum of squares)

For trees three years old and older:

$$\ln H_t = 0.470462 \sqrt{A} + 0.009234 \sqrt{A} S + 0.003858 \sqrt{A} R_s \quad (2.6b)$$

$$S_{yx}: 0.22210$$

$$R^2: 0.9933 \text{ (computed using uncorrected sum of squares)}$$

The same methodology followed with equation (2.2) was applied to the study of the inclusion of site index and relative spacing to equation (2.1). Using all the trees aged one through ten and considering the inclusion of the variable site index resulted in:

$$H_t = 1.0 - 0.203488 A^{1.369854} + 0.021807 A^{1.369854} S \quad (2.3a)$$

$$S_{yx}: 2.5628$$

The variable site index was significant from the first year of development of the trees. When fitting equation (2.3) to trees one and two years old the sign of the variable age was negative so the same analysis followed for equation (2.4) was done in this case; i.e., the followed equation was fitted:

$$H_t = H_0 + b_1 A^{b_2} S \quad (2.3.1)$$

The result obtained for these young trees was:

$$H_t = 1.0 + 0.004461 A^{3.072288} S \quad (2.3.1a)$$

$$S_{yx}: 0.7742$$

The next step was to include the variable relative spacing, i.e. equation (2.5) was fitted to trees aged one through ten with the following result:

$$H_t = 1.0 - 0.461637 A^{1.389938} + 0.024574 A^{1.389938} S + 0.013094 A^{1.389938} R_s \quad (2.5a)$$

$$S_{yx}: 2.4416$$

The inclusion of relative spacing was significant when considering all the trees so equation (2.5) was fitted to trees aged one and two and this variable was found significant even for these young trees. When considering only the products of the variables site index and age and relative spacing and age (following the same methodology as in equation (2.6)) these were the results:

$$H_t = 1.0 + 0.005451 A^{2.889358} S - 0.014175 A^{2.889358} R_s \quad (2.5.1a)$$

$$S_{yx}: 0.7755$$

Thus for trees one and two years old models given by (2.5.1a) or (2.6.1a) may be used to obtain height prediction.

For trees three years old and older the following model together with the model given by (2.6b) are possible models to explain height prediction:

$$H_t = 1.0 - 0.513870 A^{1.340015} + 0.027161 A^{1.340015} S + 0.017809 A^{1.340015} R_s \quad (2.5b)$$

$$S_{yx}: 2.7002$$

Diameter prediction

In order to establish the relationship of groundline diameter (D_{gl}) and age (A), trees 5 years old and younger were used to fit the following equation:

$$D_{gl} = -0.451261 + 0.760349 A \quad (2.7a)$$

$$S_{yx}: 0.61095$$

$$R^2: 0.7564$$

Further analysis was done to establish at what age the variables site index and relative spacing became significant in equation (2.7a). When doing this it was found that the variables site index (S) and relative spacing (R_s) were significant in the analysis for trees aged three to five, giving the following result:

$$D_{gl} = -5.124574 + 0.859843 A + 0.056724 S + 0.613422 R_s \quad (2.8a)$$

$$S_{yx}: 0.64820 \quad R^2: 0.5551$$

For trees one and two years old groundline diameter prediction was a function of age; the fitted equation obtained follows:

$$D_{gl} = -.292349 + 0.637268 A \quad (2.7b)$$

$$S_{yx}: 0.29425$$

$$R^2: 0.5397$$

Equation 2.9 was fitted to trees five years old where both groundline diameter and diameter at breast height were measured. The result was as follows:

$$D = -0.873712 + 0.559251 D_{gl} + 0.104437 H_t \quad (2.9a)$$

$$S_{yx}: 0.22244$$

$$R^2 = 0.9181$$

The analysis with diameter at breast height for trees five years old and older was performed. Age, site index and relative spacing were used as independent variables to obtain the following:

$$D = -1.399554 + 0.332435 A - 0.014729 S + 0.092778 R_s \quad (2.10a)$$

$$S_{yx}: 0.43843$$

$$R^2 = 0.5431$$

Crown development

Equations 2.11, 2.12 and 2.13 were fitted using first the five first years of measurements. The same three equations were fitted using years 5, 6, 7 and 8 where diameter at breast height was available (for the first years groundline diameter was used as one of the independent variables). The same equations but with different dependent variables (C_r and L_{ww}) were fitted in the same way explained above. Following are the results obtained for all these regressions. Using groundline diameter as one of the independent variables:

$$L_{wb} = 0.417301 - 0.291922 D_{gl} + 0.068120 H_t \quad (2.11a)$$

where all the variables remain as defined before.

$$S_{yx}: 0.36680$$

$$R^2: 0.0905$$

All the variables were significant.

$$L_{wb} = 0.936901 - 0.012873 A - 0.007772 S - 0.050184 R_s \quad (2.12a)$$

where all the variables remain as defined before.

$$S_{yx}: 0.38039$$

$$R^2: 0.0220$$

All the variables were significant.

$$L_{wb} = 0.723690 - 0.307031 D_{gl} + 0.063434 H_t + 0.055085 A - 0.006367 S + 0.001713 R_s \quad (2.13a)$$

where all the variables remain as defined before.

$$S_{yx}: 0.36096$$

$$R^2: 0.1196$$

where all the variables except relative spacing were significant.

Using groundline diameter as one of the independent variables and L_{ww} as the dependent variable these were the results:

$$L_{ww} = 0.420123 - 0.299527 D_{gl} + 0.069532 H_t \quad (2.14)$$

where all the variables remain as defined before.

$$S_{yx}: 0.30341$$

$$R^2: 0.1325$$

All the variables were significant.

$$L_{ww} = 0.932515 - 0.013259 A - 0.007767 S - 0.047334 R_s \quad (2.15)$$

where all the variables remain as defined before.

$$S_{yx}:0.32107$$

$$R^2: 0.0299$$

All the variables were significant.

$$L_{ww} = 0.702712 - 0.315259 D_{gl} + 0.064519 H_t + 0.058381 A - 0.006189 S + 0.006034 R_s \quad (2.16)$$

where all the variables remain as defined before.

$$S_{yx}:0.29632$$

$$R^2: 0.1739$$

where all the variables except relative spacing were significant.

Using diameter at breast height as one of the independent variables and L_{wb} as the dependent variable these were the results:

$$L_{wb} = -1.762387 - 1.438132 D + 0.4338 H_t \quad (2.17)$$

where all the variables remain as defined before.

$$S_{yx}:1.67737$$

$$R^2: 0.4794$$

All the variables were significant.

$$L_{wb} = -8.466376 + 1.125284 A + 0.038506 S - 0.274932 R_s \quad (2.18)$$

where all the variables remain as defined before.

$$S_{yx}:1.74117$$

$$R^2: 0.4391$$

All the variables were significant.

$$L_{wb} = -4.883879 - 1.275371 D + 0.255339 H_t + 0.847264 A + 0.001577 S + 0.000131 R_s \quad (2.19)$$

where all the variables remain as defined before.

$$S_{yx}: 1.51454$$

$$R^2: 0.5757$$

where all the variables except site index and relative spacing were significant.

Using diameter at breast height as one of the independent variables and L_{ww} as the dependent variable these were the results:

$$L_{ww} = -1.746921 - 1.448221 D + 0.434484 H_t \quad (2.20)$$

where all the variables remain as defined before.

$$S_{yx}: 1.64271$$

$$R^2: 0.4841$$

All the variables were significant.

$$L_{ww} = -8.382264 + 1.120845 A + 0.037666 S - 0.277703 R_s \quad (2.21)$$

where all the variables remain as defined before.

$$S_{yx}: 1.73345$$

$$R^2: 0.4391$$

All the variables were significant.

$$L_{ww} = -4.700774 - 1.291089 D + 0.260298 H_t + 0.833272 A - 0.000399 S + 0.000288 R_s \quad (2.22)$$

where all the variables remain as defined before.

$$S_{yx}:1.50019$$

$$R^2: 0.5800$$

where all the variables except site index and relative spacing were significant.

Using groundline diameter as one of the independent variables and C_r as the dependent variable these were the results:

$$C_r = 0.822445 + 0.078980 D_{gl} - 0.017218 H_t \quad (2.23)$$

where all the variables remain as defined before.

$$S_{yx}:0.07642$$

$$R^2: 0.1562$$

All the variables were significant.

$$C_r = 0.629497 - 0.0000021 A + 0.003275 S + 0.008276 R_s \quad (2.24)$$

where all the variables remain as defined before.

$$S_{yx}:0.08009$$

$$R^2: 0.0732$$

All the variables except age were significant.

$$C_r = 0.765375 + 0.085296 D_{gl} - 0.013284 H_t - 0.031992 A + 0.001929 S - 0.006703 R_s \quad (2.25)$$

where all the variables remain as defined before.

$$S_{yx}:0.07136$$

$$R^2: 0.2641$$

where all the variables were significant.

Using diameter at breast height as one of the independent variables and C_r as the dependent variable these were the results:

$$C_r = 0.895953 + 0.109604 D - 0.029680 H_t \quad (2.26)$$

where all the variables remain as defined before.

$$S_{yx}: 0.10427$$

$$R^2: 0.5204$$

All the variables were significant.

$$C_r = 1.412096 - 0.075015 A - 0.003357 S + 0.031931 R_s \quad (2.27)$$

where all the variables remain as defined before.

$$S_{yx}: 0.10705$$

$$R^2: 0.4946$$

All the variables were significant.

$$C_r = 1.323049 + 0.080982 D - 0.011630 H_t - 0.073110 A - 0.002731 S + 0.013292 R_s \quad (2.28)$$

where all the variables remain as defined before.

$$S_{yx}: 0.08943$$

$$R^2: 0.6473$$

where all the variables were significant.

Conclusions

According to the analysis performed with these young trees, it can be seen that the variable site index is an important variable in the explanation of height growth from the first year after planting. Two different models were fitted and two forms of these two models were considered as possible equations to explain height prediction. For trees one and two years old the independent variables were the products of the variables site index and age (or square root of age) and relative spacing and age (or square root of age). For trees three years old and older these were also the independent variables used to explain height prediction but in this case age (or square root of age) was another significant independent variable. The fact that site index was significant even for trees as young as one year old has been reported for other species (Osborn, 1969).

In regard to diameter, for trees 5 years old and younger groundline diameter was analyzed and it was found that this variable was a function of age for trees one and two years old while the same variable was a function not only of age but also of site index and relative spacing for trees three, four and five years old. With respect to diameter at breast height this was a function of age, site index and relative spacing for trees five years old and older.

The analysis of crown development showed that for trees over four years old, variables such as diameter at breast height, total height, age, site index and relative spacing were significant when crown ratio was used as dependent variable. The regression analysis of the ratios of crown length to crown width between rows, and

crown length to crown width within rows used as dependent variables, particularly the last one showed low R^2 values. When the analysis was performed with trees five years old and younger, the regressions were better (higher R^2) when crown ratio was considered as the dependent variable and groundline diameter, total height, age, site index and relative spacing were used as independent variables. Thus, when considering possible models for crown development the use of crown ratio as a dependent variable seems more reasonable for these young trees.

CHAPTER III Use of seedlings to study growth relationships

Introduction

The study of juvenile growth-density relationships is of great importance in understanding these relationships later in the development of the trees. Data from spacing trials are appropriate for analyzing one of the most important factors of concern to foresters, i.e. the effects of density and spacing on growth and development. Spacing trials give valuable information on density effects on height and diameter growth.

While trees growing at conventional spacings provide essential information, the trees must be grown over relatively long time periods. The use of seedlings grown at close spacings to accelerate the process of obtaining stand-dynamics information should be considered as a possible tool. Seedlings planted at very close spacing may offer insight into stand development that would require a much longer time to acquire using conventional methods.

Literature Review

The use of seedlings to observe the way growth and yield relationships can be affected by density and spacing may help foresters understand the effects of these relationships in mature stands planted at operational spacings.

The savings in time and cost are among the advantages of using seedlings in closely

planted spacing studies. Spacing trials using seedling would be continued over short periods of time as compared to those consisting of conventional spacings, thus diminishing the risk of losing plots because of catastrophic events such as fire, insect outbreaks, ice or snow storms. Furthermore closely spaced seedlings would require less field area which means less environmental variability and greater possibilities for the researcher to have as much local control as possible.

The patterns observed in height, diameter and crown development in seedling spacing trials can be compared to those observed in mature stands to see how similar these trends are.

Smith and Hann (1984) studied relationships in seedlings and compared the results to older trees. These authors developed a non-linear model based on the $-3/2$ power rule of self-thinning for seedlings of red alder growing at three initial densities and on two soil types. This same relationship was modelled for 20-50 year old unthinned red pine growing at six initial densities and site index 27 m at 50 years. The authors showed that the size-density model described may be valid over a wide range of initial densities and ages.

Although the possibility of using seedling stands to obtain data in a short time to help in understanding growth and development in older stands is attractive, caution must be exercised. Any inferences necessitate an understanding of the physiological and environmental transformations going on while the trees are growing.

Data Description

To accomplish objective 2, data from a seedling spacing study were used. In the spring of 1989 a miniature-scale loblolly pine spacing trial was established near the Ecosystem Simulation Laboratory on the VPI campus. The site was prepared to assure the environmental factors and site quality were as uniform as possible. Measurements were taken at 28-day intervals throughout the growing seasons on each measurement tree. The measurements included groundline diameter, total height, height to the base of the live crown, crown width within the row and crown width between the rows. The measurements taken in the miniature-scale study over four growing seasons are the same as those collected annually from the trials planted at conventional spacings. This allowed comparisons to be made between the two studies.

The experimental design chosen was a non-systematic design described by Lin and Morse (1975). From the statistical perspective this arrangement provides unbiased estimates of the mean; analysis of variance techniques are possible as well as response surface and regression methods. From the practical point of view the unequal plot sizes with equal number of trees allows a compact block in the field. Compactness helps maintain uniformity of site conditions and reduces measurement costs associated with long term forestry spacing studies. In this particular design each measurement plot consists of 49 trees with three rows of guard trees bordering each plot.

For this study a spacing factor (F) of 3 inches was selected and four spacing levels of this factor ($1F$, $1.5F$, $2F$ and $3F$) were used in the design. Thus each block consists of sixteen plots arranged in a factorial way with the four spacing levels applied at random to rows and columns. In the conventionally-spaced trials, the spacing factor F

is 4 feet. Thus the miniature-scale trial spacing factors are 1/16 those of the full-scale trial. The layout is shown in Figure 1 of chapter I.

Analysis and Procedures

Height, diameter, crown ratio and relative spacing trends of the seedling growth were compared to the patterns observed in the older trees planted at typical tree densities. Also, cross sectional area at groundline per unit area was analyzed. Mortality trends were also studied.

Data Analysis

The following variables were studied: arithmetic mean groundline diameter, quadratic mean groundline diameter, height, height to the live crown, crown length, crown ratio, relative spacing, cross sectional area at groundline, number of trees per unit area and mortality.

The patterns observed in the miniature-scale spacing study were compared to those in the conventional spacing study. The age of the Virginia Tech conventional spacing trials is limited, and trends in the miniature scale study were also compared to patterns observed in several published spacing studies. Table 1 shows a list of the studies used for making comparisons.

Table 1: Published spacing studies used for comparisons.

Tree species	Author	Age (years)
Loblolly pine	Little and Somes (1958)	18
Loblolly pine	Hansbrough et.al. (1964)	12
Loblolly pine	Hansbrough (1968)	18
Loblolly pine	Mann and Dell (1971)	17
Loblolly pine	Campbell and Mann (1974)	17
Loblolly pine	Gilmore and Gregory (1974)	20
Loblolly pine	Shepard (1974)	25
Loblolly pine	Whitesell (1974)	11
Loblolly pine	Balmer et.al. (1975)	15
Loblolly pine	McClurkin (1976)	20
Loblolly pine	Harms and Lloyd (1981)	20
Loblolly pine	De Bell et.al. (1989)	
Douglas-fir	Munger (1946)	26
Jack pine	Ralston (1953)	25
Jack pine	Guilkey and Westing (1956)	15
Douglas-fir	Reukema (1959)	32
Slash pine	Bennett (1960)	7
Red pine	Godman and Smith (1963)	51
Douglas-fir	Curtis and Reukema (1970)	43
Douglas-fir	Reukema (1970)	40
Red pine	Stiell and Berry (1977)	20
Ponderosa pine	Oliver (1979)	12
Douglas-fir	Reukema (1979)	50
Virginia pine	Russell (1979)	15
Douglas-fir, Western hemlock and Western redcedar	Reukema and Smith (1987)	25
Lodgepole pine	Johnstone and Pollack (1990)	20
Lodgepole pine	Yang (1991)	7

Diameter

Figure 2 shows arithmetic mean groundline diameter versus age for each of the initial spacings in the conventional spacing study. Only the square plots were considered in the following analysis, i.e.:

Plot 1	4×4 feet
Plot 6	6.5×6.5 feet
Plot 11	8×8 feet
Plot 16	12×12 feet

Figure 3 shows arithmetic mean groundline diameter versus measurement by spacing for the miniature-scale study. Only the square plots were considered in the following analysis, i.e.:

Plot 1	3×3 inches
Plot 6	4.5×4.5 inches
Plot 11	6×6 inches
Plot 16	9×9 inches

Plot 11 in the seedling spacing study had a drainage problem and several trees were replanted after a few months, thus this plot was dropped from further analysis. The next set of figures (Figures 4, 5, 6 and 7) show arithmetic mean groundline diameter

versus Julian Date by spacing for each year of the measurements of the miniature spacing study (1989, 1990, 1991 and 1992). In all the years arithmetic mean groundline diameter increased as the growing space increased. In general the pattern was the same as the one observed in Figure 2 corresponding to the spacing trials at typical tree density. The latter figure showed the average diameter over several plots, thus trends are “smoother” than the graphs for the miniature-scale study which are based on only one plot each. When looking at other studies where average diameter at breast height was reported, the patterns were the same as the ones observed for mean groundline diameter in the conventional and miniature spacing trials. Hansbrough (1964) pointed out that the increase in diameter as spacing increased was much evident since age 5. The same patterns were reported by Little and Sommes (1958), Hansbrough (1968), Shepard (1974), Whitesell (1974), Balmer et al. (1975) and Harms and Lloyd (1981). All these studies were done in loblolly pine. Comparisons were also made with spacing trials in other species. Spacing studies of as early as 1946 were observed. The same trend in diameter was reported by Munger (1946) in Douglas-fir, Guilkey and Westing (1956) in jack pine, Reukema (1979) in Douglas-fir plantations established in 1925 at the Wind River Experimental Forest near Carson, Washington. Same results were reported by Bennett (1960) in slash pine in a 7 year spacing study, Godman and Smith (1963) and Stiell and Berry (1977) in red pine plantations, Oliver (1979) in ponderosa pine, Russell (1979) in Virginia pine, Reukema and Smith (1987) in Douglas-fir, western hemlock and western redcedar, Johnstone and Pollack (1990) and Yang (1991) in lodgepole pine.

Quadratic mean diameter

In their comparison between spacing studies in Hawaii and South Carolina, De Bell et al. (1989) showed that quadratic mean diameter increased as spacing increased. Johnstone and Pollack's (1990) study of lodgepole pine showed the same pattern. These trends were observed for the conventional spacing study for quadratic mean groundline diameter (Figure 8). For the miniature-scale spacing study the same trend was observed (Figures 9, 10, 11, 12 and 13). Regression analysis of quadratic mean groundline diameter on age was performed in both studies; the results were similar to those reported by Johnstone and Pollack (1990) for quadratic mean diameter in lodgepole pine, i.e. the regression in each spacing accounted for over 97 % of the variation in mean diameter. Results are shown in Tables 2 and 3 in the Appendix.

Height

The average height increased as spacing increased in the conventional spacing study (Figure 14) except for the first four years where height in all the spacings seems to be the same. For the last four years this is also the pattern for plots 11 and 16. Figures 15 through 19 show the trends for the miniature spacing study. For this study the same pattern observed in the conventional spacing study was observed in the last three measurements (Figures 17, 18 and 19). In the first year (Figures 16) this trend was different, i.e. the tallest trees were not the ones in the widest spacing. Little and Sommes (1958) reported that average height increased only slightly as spacing increased. Hansbrough et al. (1964) reported an increase in height as distance between trees increased. The same pattern was observed by Gilmore and Gregory (1974). These authors reported that the effect of stand density on height growth

became more pronounced as trees got older. Similar patterns about height were also observed by Shepard (1974), Balmer et al. (1975), Harms and Lloyd (1981). Comparisons with the trends in height observed in spacing trials in other species were made. Increase in height as spacing increased were reported in Douglas-fir by Munger (1946), and in lodgepole pine by Johnstone and Pollack (1990). No effect of spacing on height was observed in jack pine by Ralston (1953) and in slash pine by Bennett (1960). As performed by Johnstone and Pollack (1990) linear regression analysis of mean stand height on years after planting were done in both the conventional and the miniature-scale studies. For each spacing simple linear equations accounted for over 95 % of the variation in mean height (Tables 4 and 5). These results are similar to the ones reported by Johnstone and Pollack (1990).

Crown development

Height to the live crown

Harms and Lloyd (1981) reported that the height to the live crown averaged about the same regardless of the spacing considered. In our case there was a decrease of height to the live crown for the conventional spacing study (Figure 20) as spacing increased. This trend was observed in the first year of measurements of the miniature-scale spacing study (Figures 21, 22). For the next two years of this study (Figures 23 and 24) height to the live crown did not follow a definite trend and for the last year (1992) the average height to the live crown was the same regardless of the spacing.

Crown length

Harms and Lloyd (1981) reported that crown length increased as spacing increased. This same pattern was observed for the conventional spacing study (Figure 25) after age 3. In the miniature-scale spacing study this trend was observed for the last three years of measurements (Figures 28, 29 and 30). The first year of this study did not show this trend (Figures 26, 27).

Crown ratio

The trend of increased crown ratio with increased spacing was reported by Harms and Lloyd (1981) and Bennett (1960). This same trend was observed in the conventional spacing study (Figure 31) and the miniature-scale study (Figures 32 through 35). Looking at the plots for each year and spacing it can be observed that for 1989 there was an increase in crown ratio within a measurement year except for the closest spacing where there was first an increase, then a decrease and finally a small increase. For 1990 it can be observed that for all the spacings there was first a decrease, then an increase and then an apparent leveling off at the end of the year. For 1991 there was an increase and then a decrease during the year. These patterns were probably due to flushing during the growing seasons.

Relative spacing

Relative spacing versus age by spacing is shown in Figures 36 through 41. Figure 36 shows the trend in the conventional spacing study while the last five figures correspond to the miniature spacing study. In the conventional spacing study as well as in the first two years of the miniature spacing study (Figures 38 and 39) relative spacing

increased as spacing increased. In these three figures it can be observed that relative spacing decreased with time for all the spacings. The last two years in the miniature-scale study (Figures 40 and 41) did not show a definite trend for relative spacing due to high rates of mortality.

Cross sectional area at groundline

Various authors have reported increased basal area per unit area with decreased spacing (Hansbrough et al. 1964, Hansbrough 1968, Gilmore and Gregory 1974, Balmer et al. 1975, Harms and Lloyd 1981 all for loblolly pine, Reukema 1959 in Douglas-fir, Stiell and Berry 1977 in red pine, Russell 1979 in Virginia pine and Johnstone and Pollack 1990 in lodgepole pine). The same trend was observed for cross sectional area at groundline in the conventional spacing study (Figure 42). For the miniature-scale spacing study a similar trend was observed for the first two years (Figures 43, 44 and 45). The last two years showed that the closest spacing had the lowest cross sectional area at groundline (Figures 46 and 47). Hansbrough et al. (1964) reported that for the last year of their study (12-year study) basal area in the widest spacing was greater than basal area at some of the closest spacings.

Number of trees per unit area

In the conventional spacing study there was no evidence of mortality as of age 8 (Figure 48). In the miniature-scale study the first year showed this trend (Figures 49, 50); the second year showed mortality at the closest spacing (Figure 51). The third and fourth year showed mortality in all except the widest spacing (Figures 52 and 53).

Conclusions

In general the patterns observed for the trees in the miniature spacing trial were similar to the ones in the conventional spacing study. These patterns were also similar to the trends observed in other spacing studies in the literature. Table 1 showed some of the published spacing studies in different species to which the patterns observed in the miniature-scale spacing study and the conventional spacing study were compared. Reviews of spacing studies and the trends observed are presented by Sjolte-Jorgensen (1967), Hamilton and Christie (1974) and Evert (1971, 1984). Sjolte-Jorgensen reported results in diameter, height and basal area comparable in general to the trends observed in this report, i.e. diameter and height increased as spacing increased while basal area decreased as spacing increased. The same general conclusion with respect to diameter and basal area was reported by Evert (1984). With respect to height the latter author pointed out that some of the experiments reported increase of height as spacing increased but there were some studies that reached the opposite conclusion.

The next important aspect to be considered is to what extent the trends observed in the miniature-scale study can be scaled to older trees; this will be the subject of the next chapter in this volume.

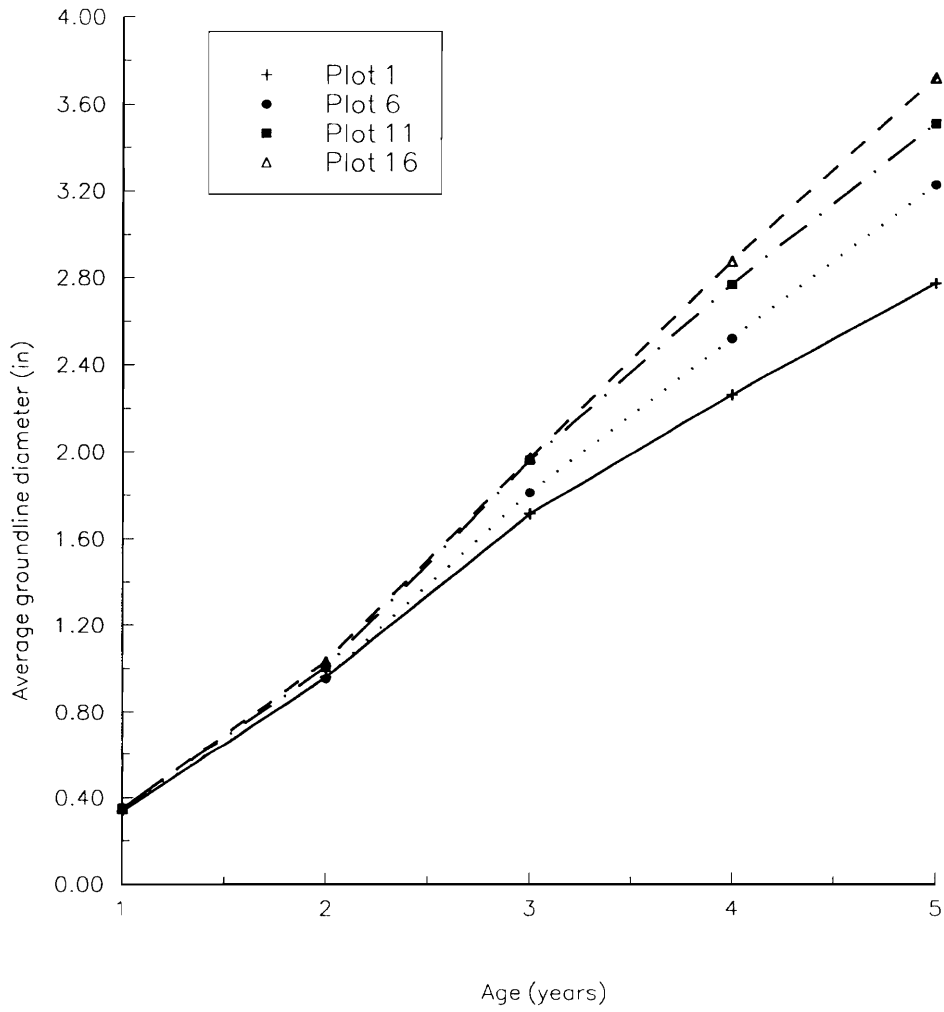


Figure 2: Average groundline diameter vs. Age for the conventional spacing study planted with loblolly pine.

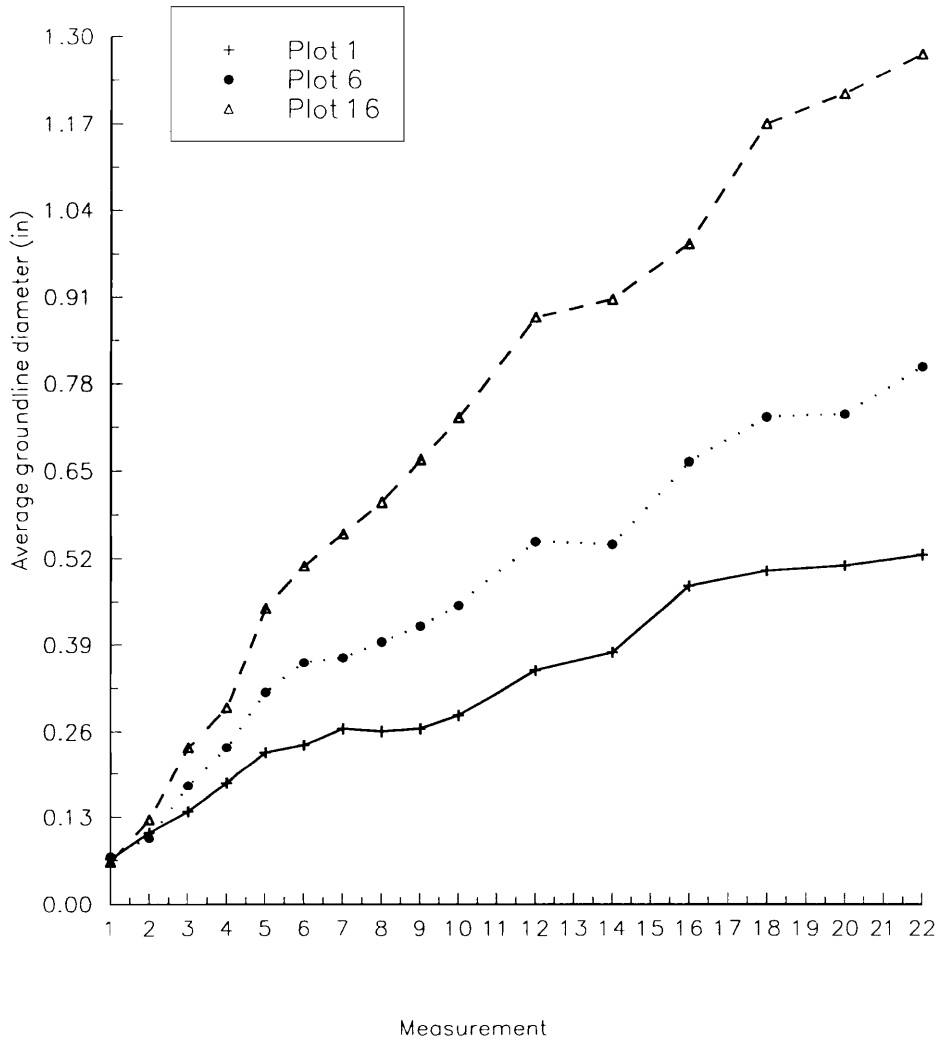


Figure 3: Average groundline diameter vs. Measurement for the miniature spacing study planted with loblolly pine.

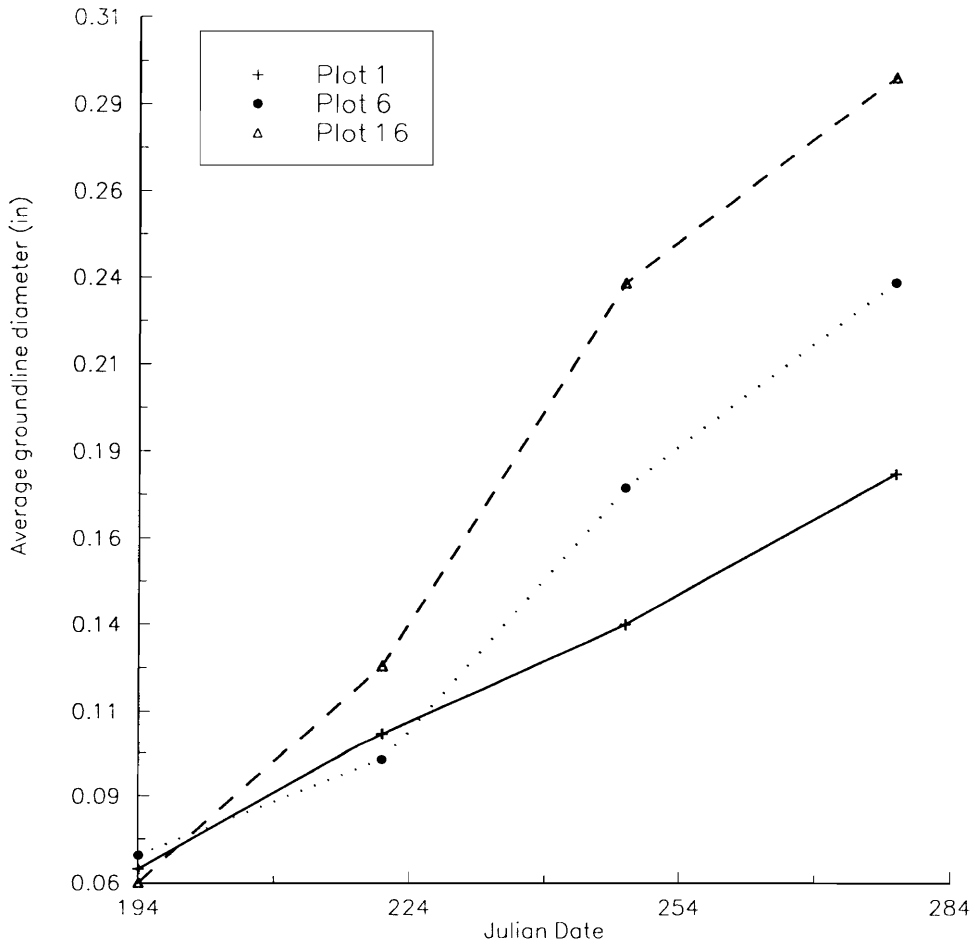


Figure 4: Average groundline diameter vs. Time (1989) for the miniature spacing study planted with loblolly pine.

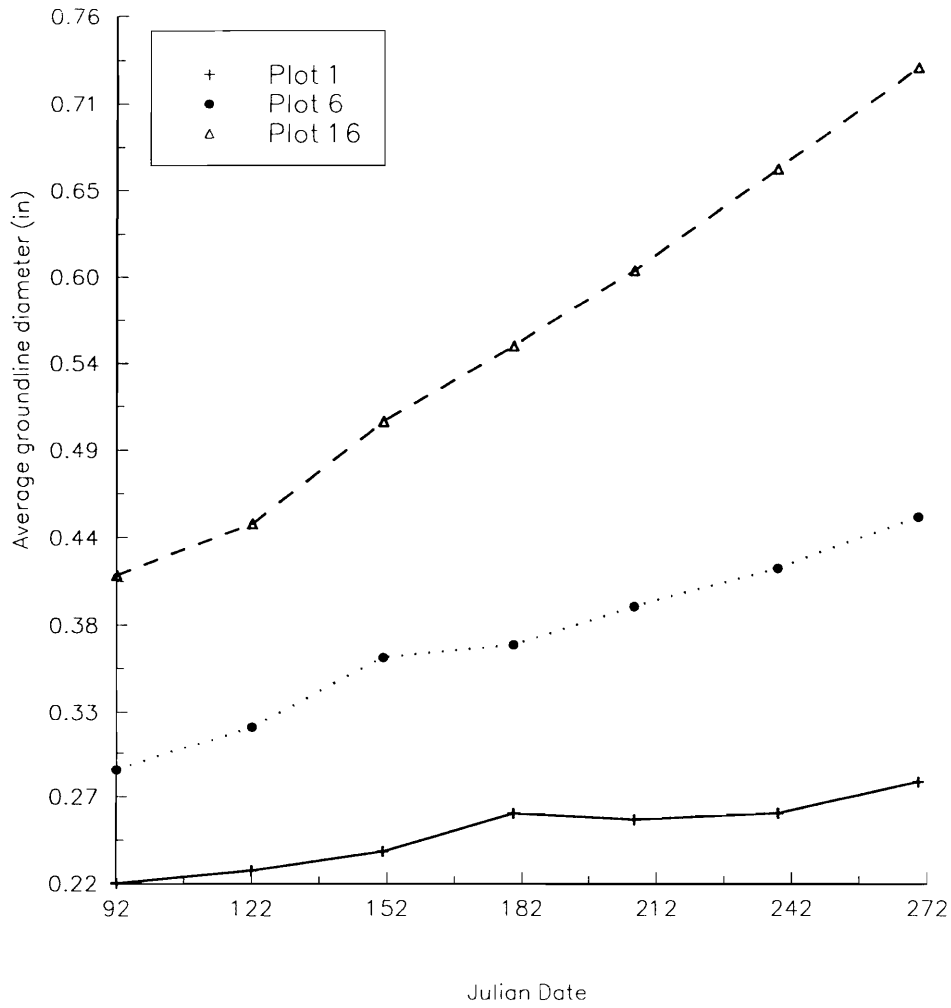


Figure 5: Average groundline diameter vs. Time (1990) for the miniature spacing study planted with loblolly pine.

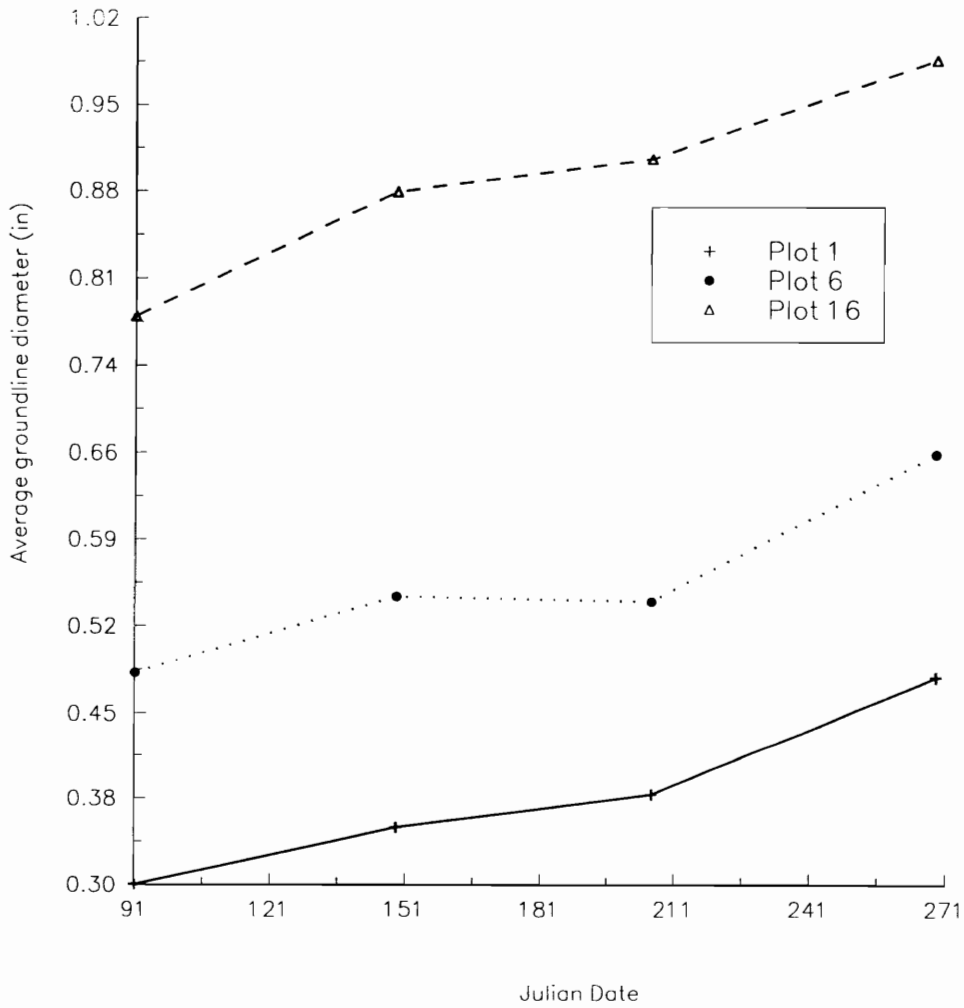


Figure 6: Average groundline diameter vs. Time (1991) for the miniature spacing study planted with loblolly pine.

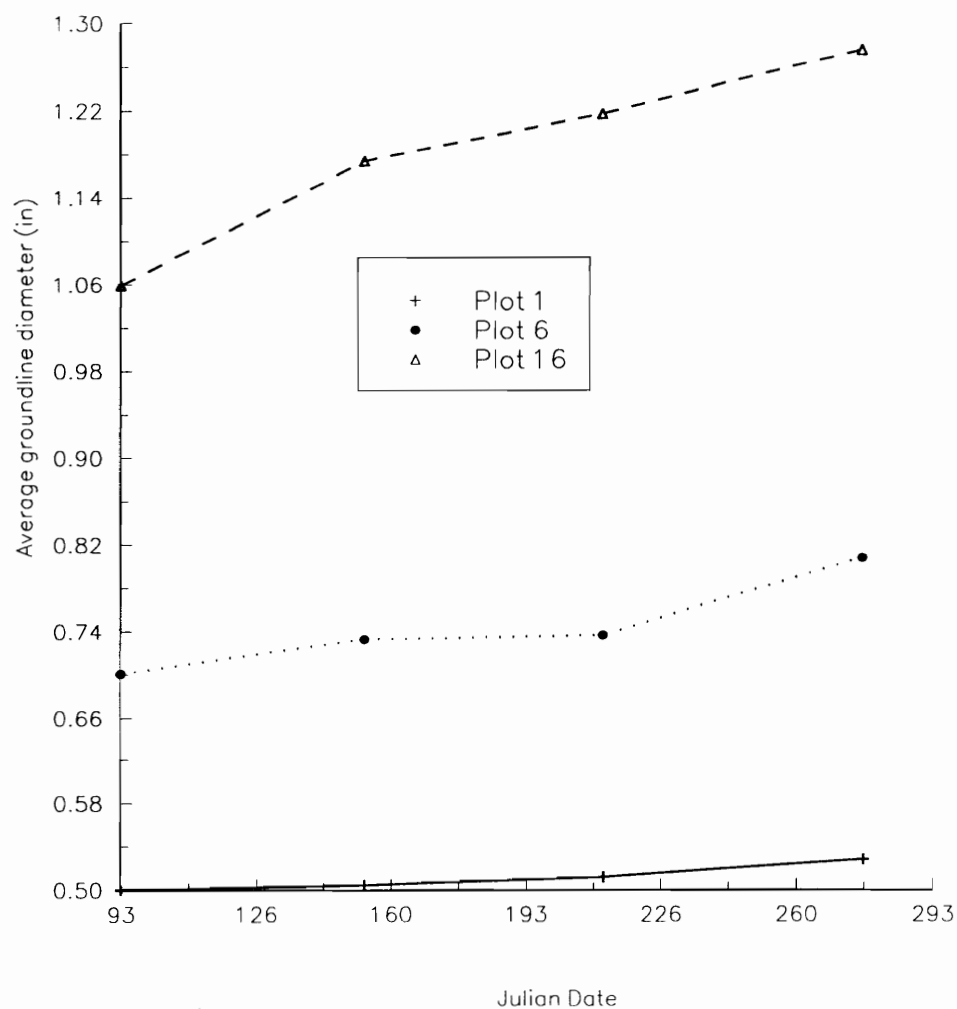


Figure 7: Average groundline diameter vs. Time (1992) for the miniature spacing study planted with loblolly pine.

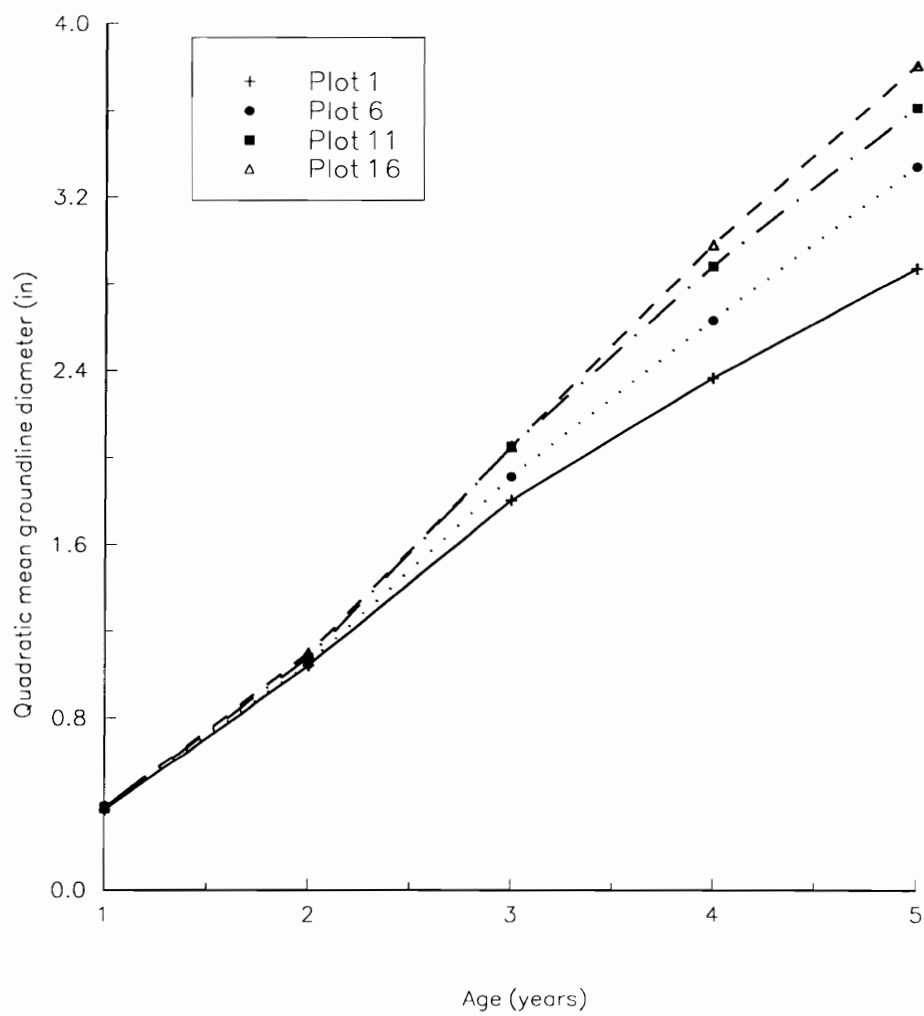


Figure 8: Quadratic mean groundline diameter vs. Age for the conventional spacing study planted with loblolly pine.

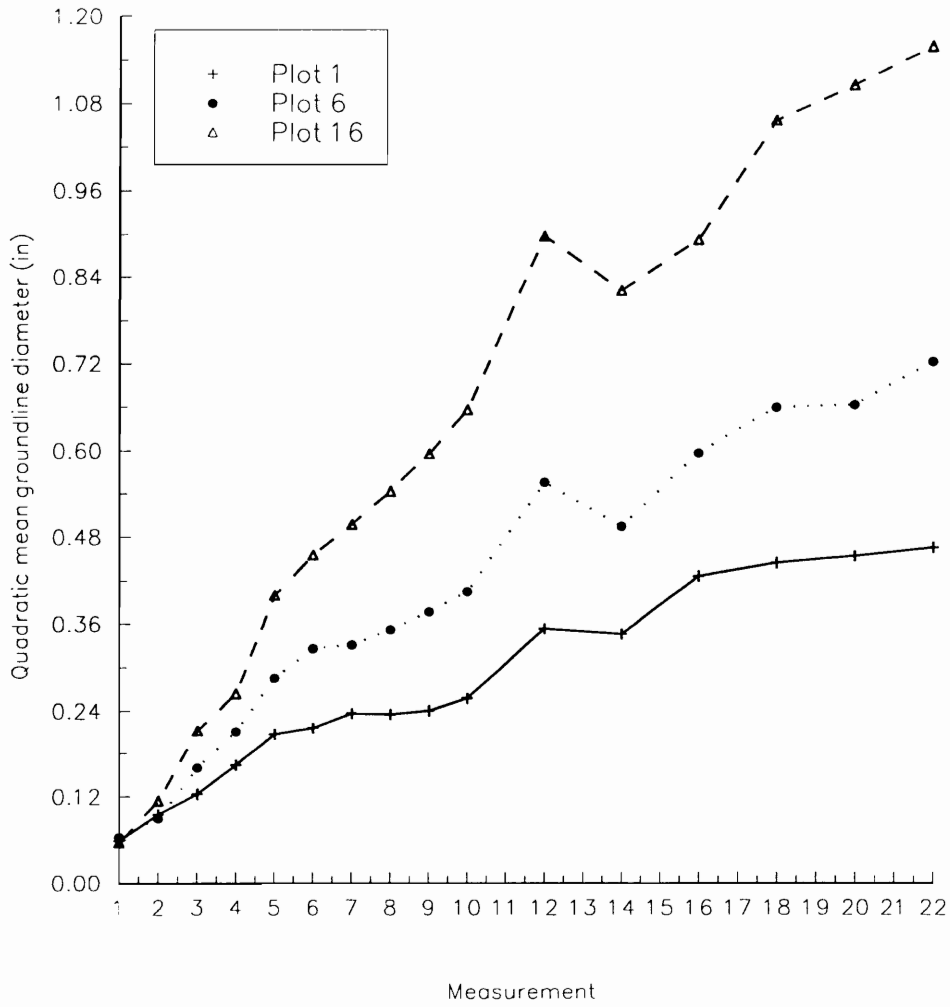


Figure 9: Quadratic mean groundline diameter vs. Measurement for the miniature spacing study planted with loblolly pine.

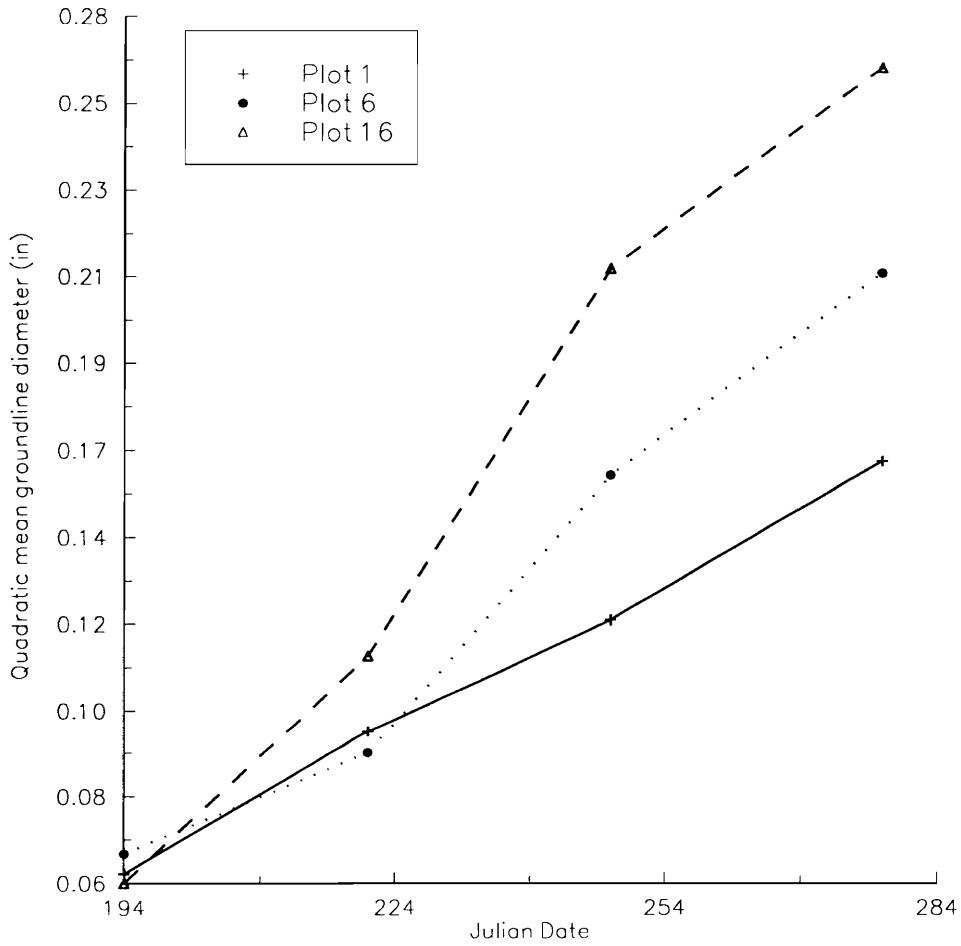


Figure 10: Quadratic mean groundline diameter vs. Time (1989) for the miniature spacing study planted with loblolly pine.

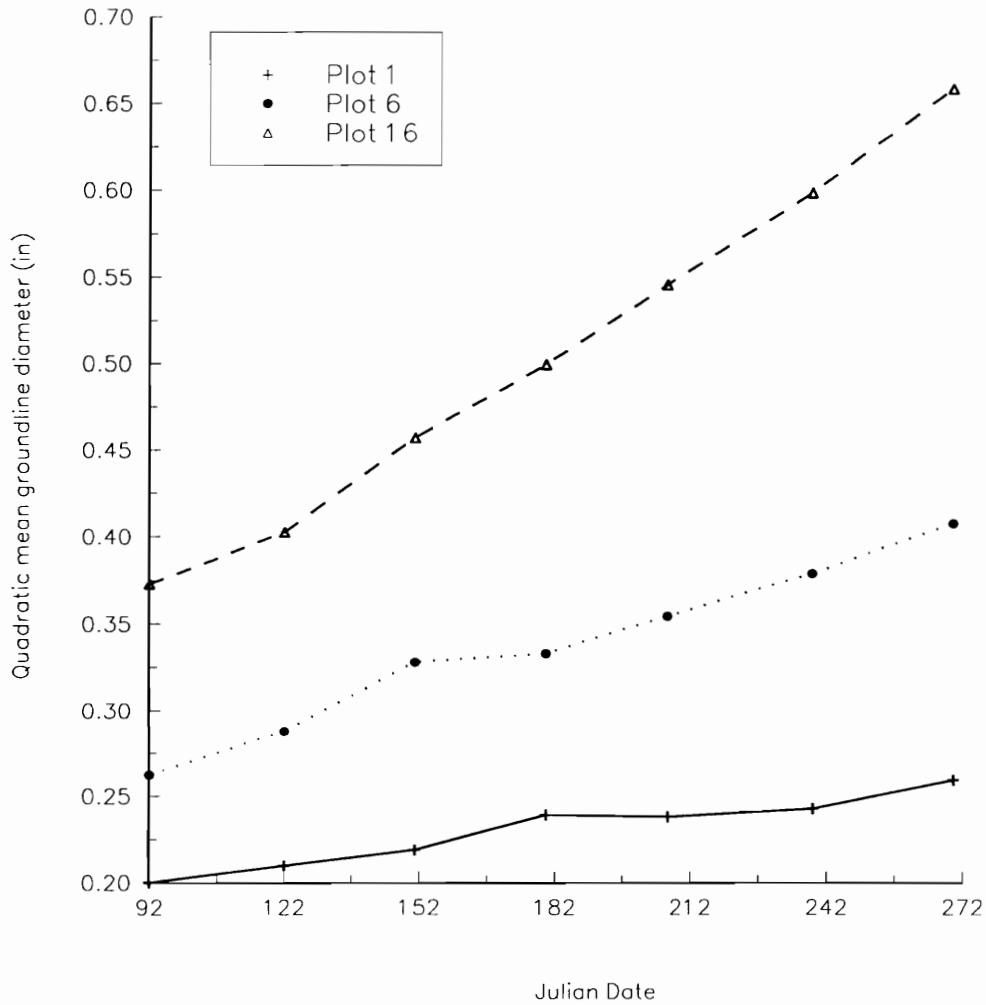


Figure 11: Quadratic mean groundline diameter vs. Time (1990) for the miniature spacing study planted with loblolly pine.

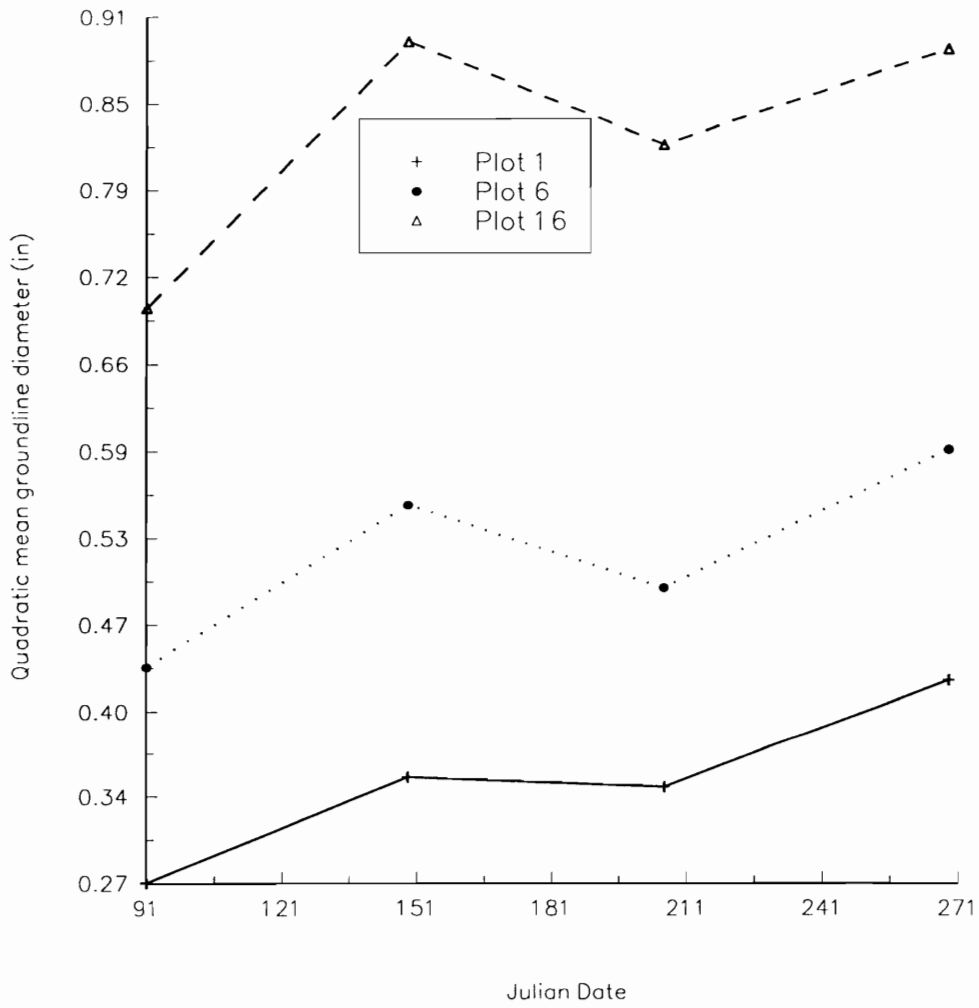


Figure 12: Quadratic mean groundline diameter vs. Time (1991) for the miniature spacing study planted with loblolly pine.

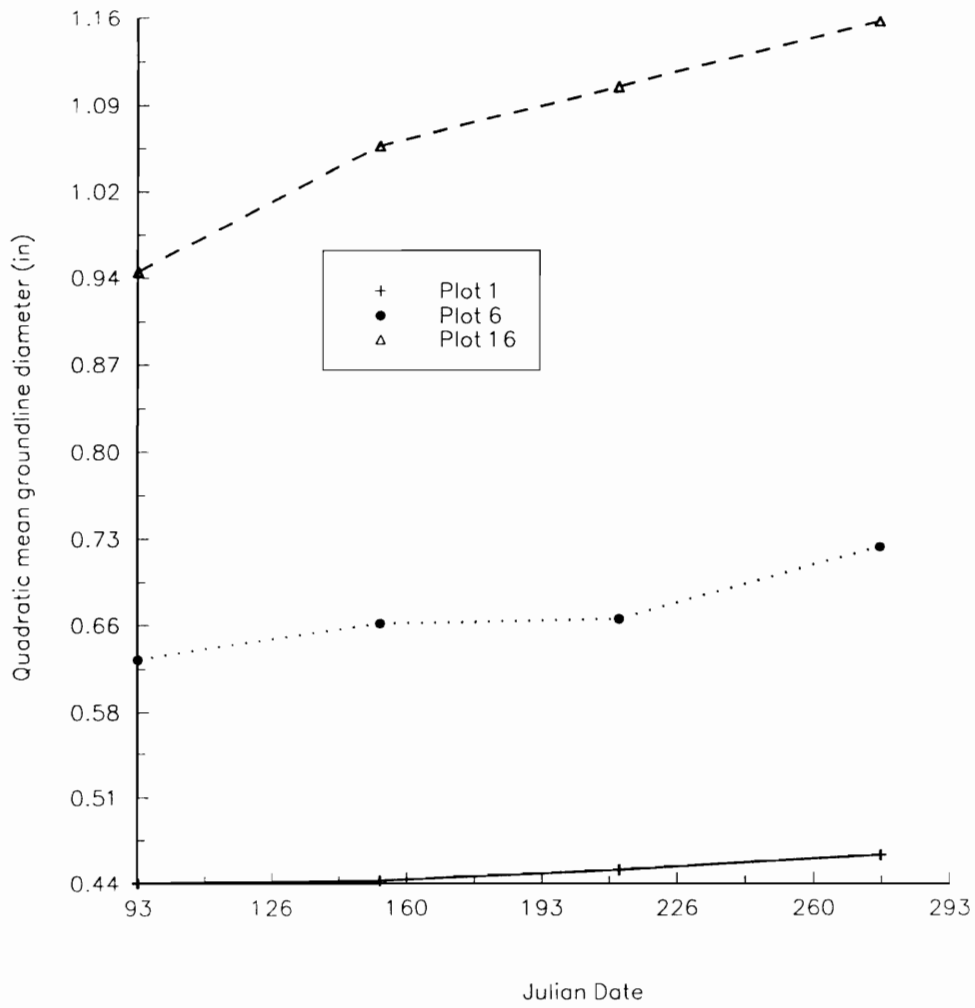


Figure 13: Quadratic mean groundline diameter vs. Time (1992) for the miniature spacing study planted with loblolly pine.

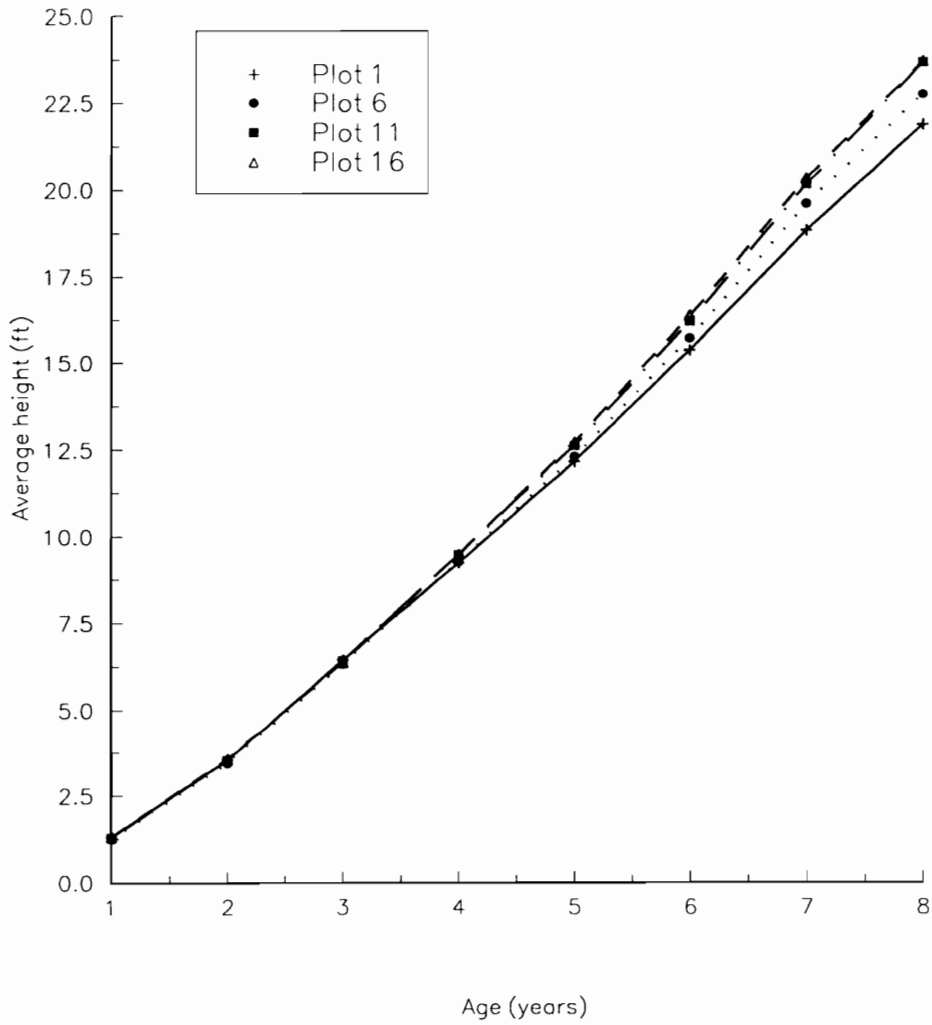


Figure 14: Average height vs. Age for the conventional spacing study planted with loblolly pine.

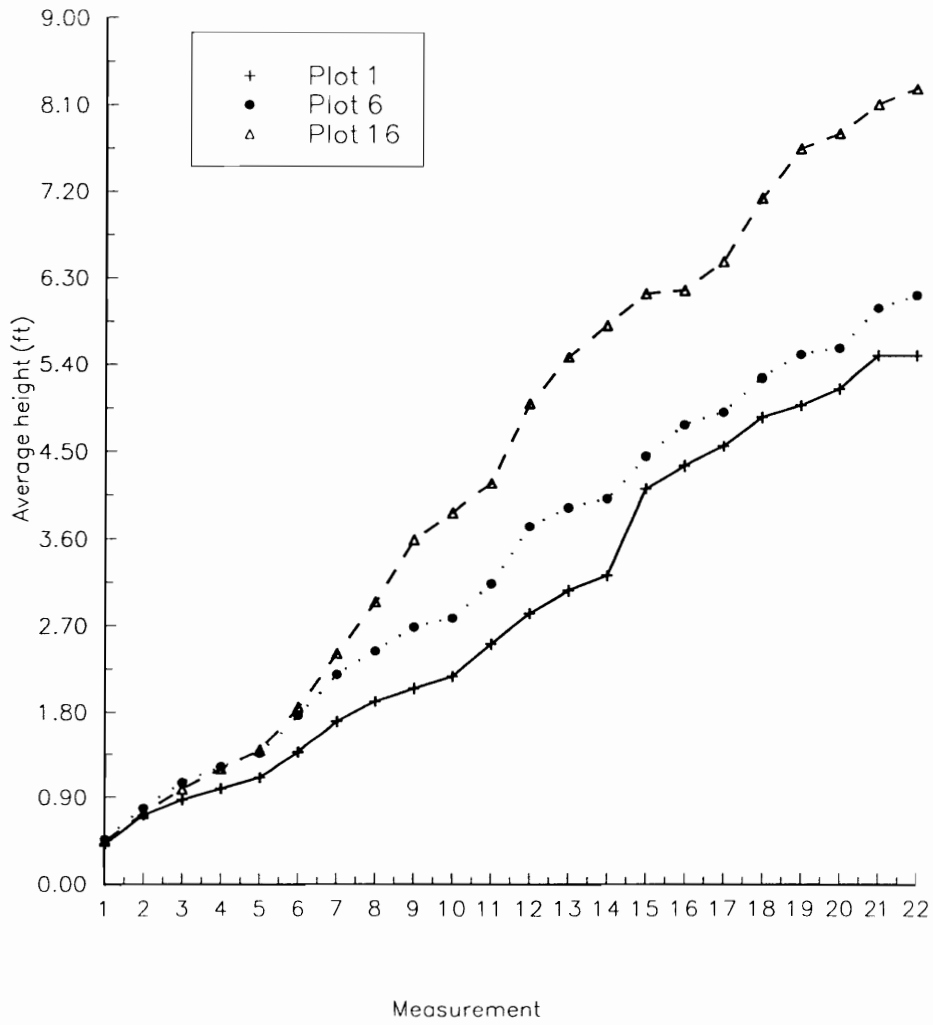


Figure 15: Average height vs. Measurement for the miniature spacing study planted with loblolly pine.

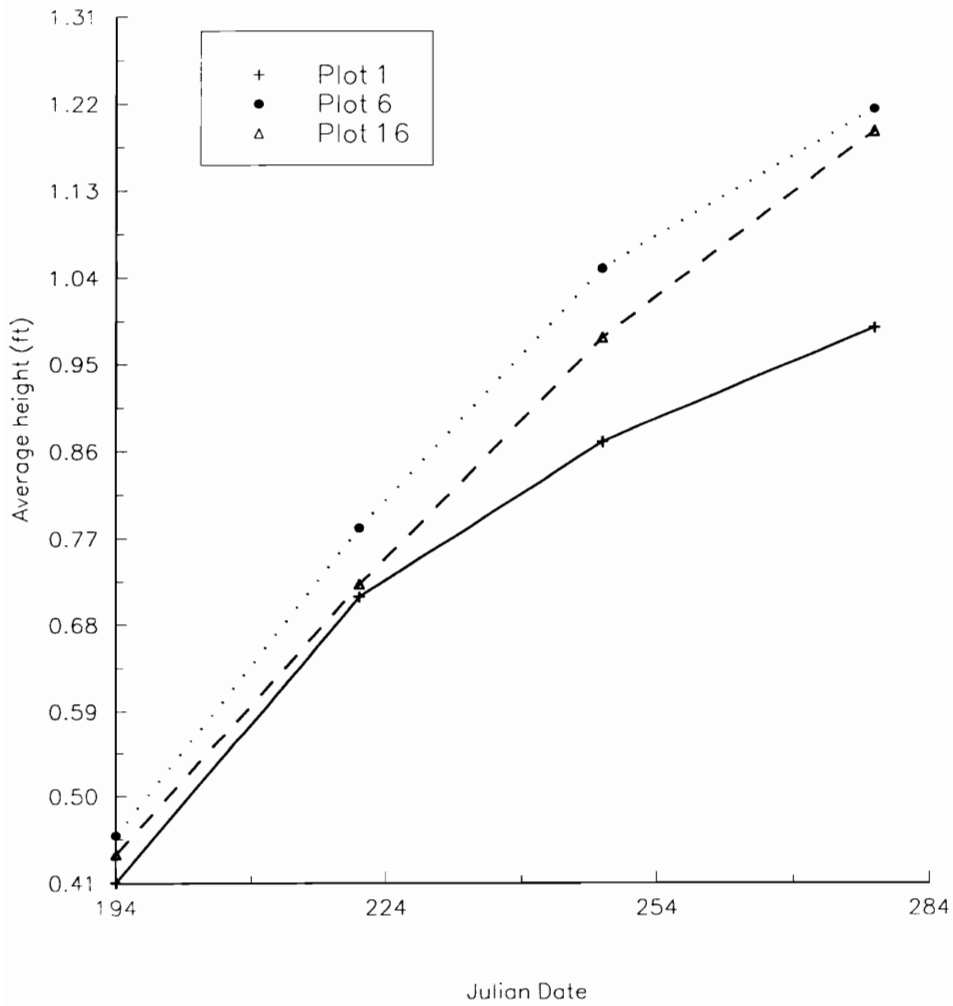


Figure 16: Average height vs. Time (1989) for the miniature spacing study planted with loblolly pine.

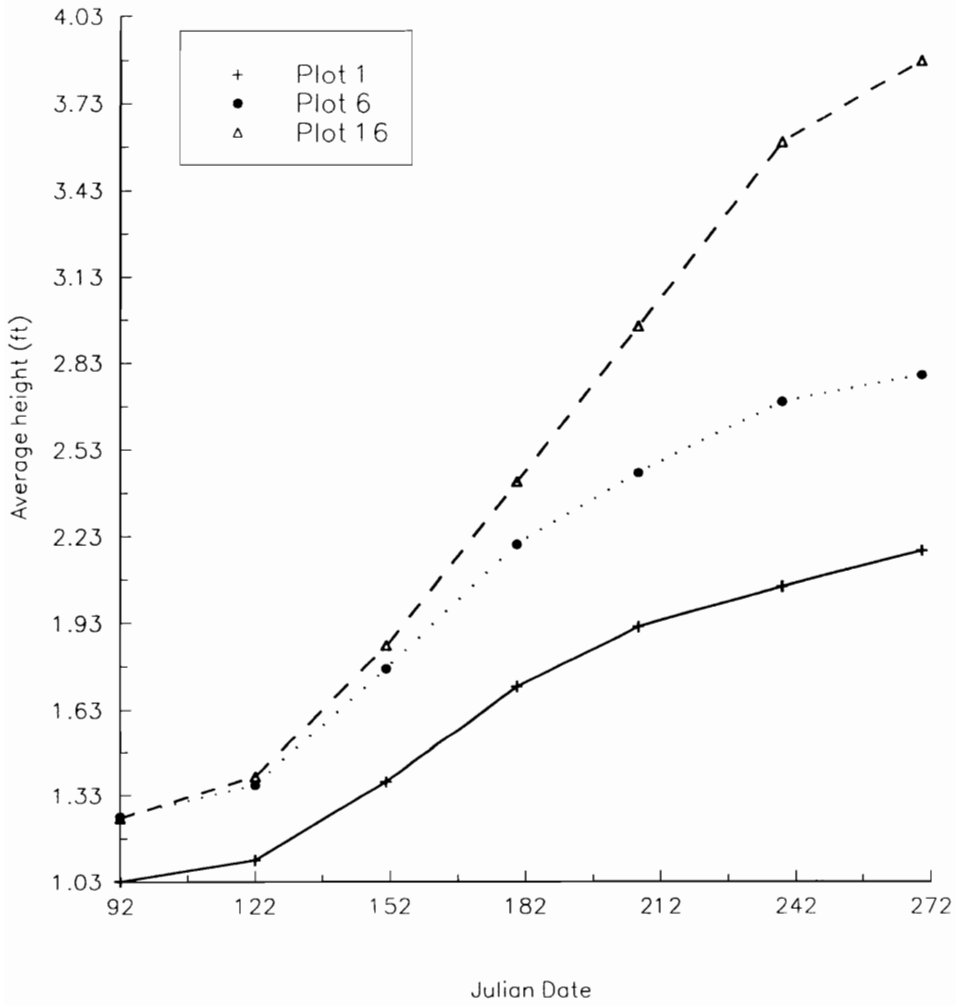


Figure 17: Average height vs. Time (1990) for the miniature spacing study planted with loblolly pine.

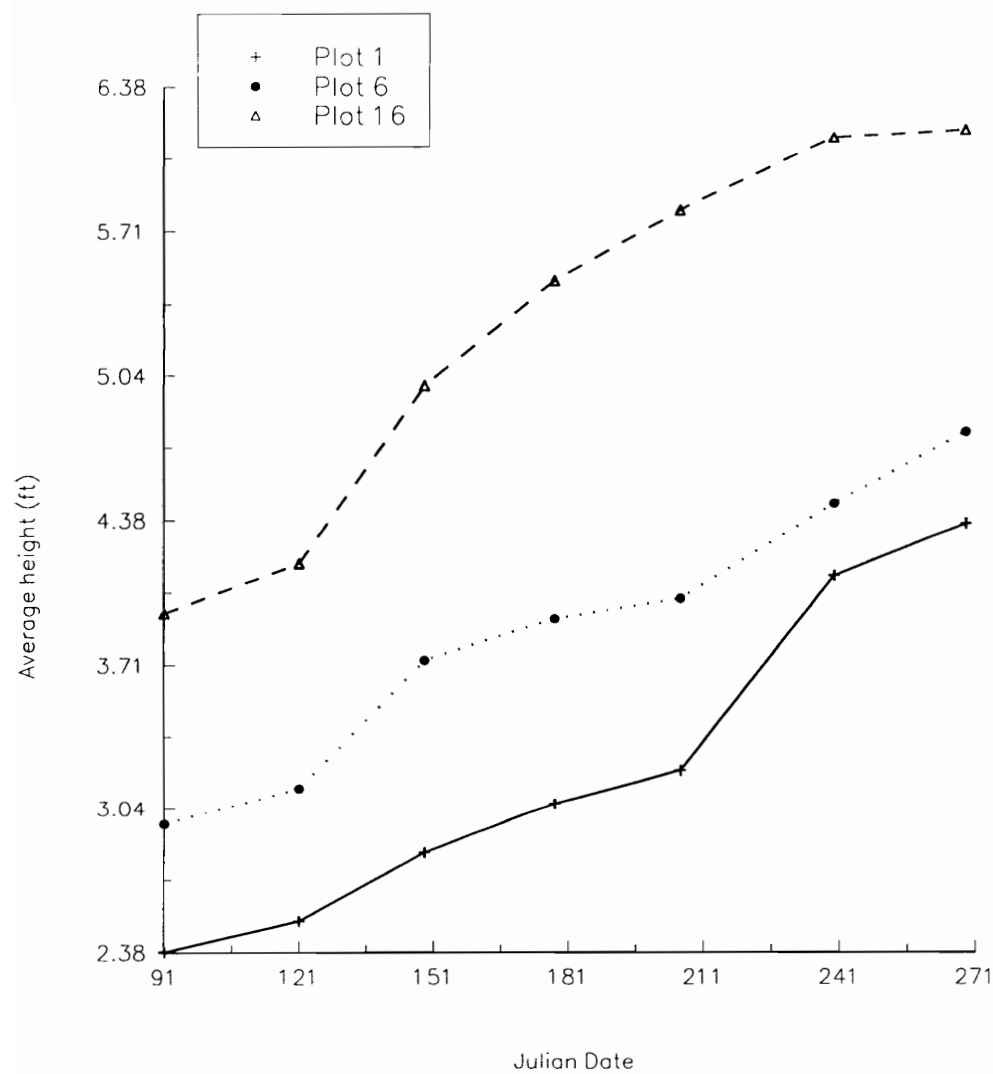


Figure 18: Average height vs. Time (1991) for the miniature spacing study planted with loblolly pine.

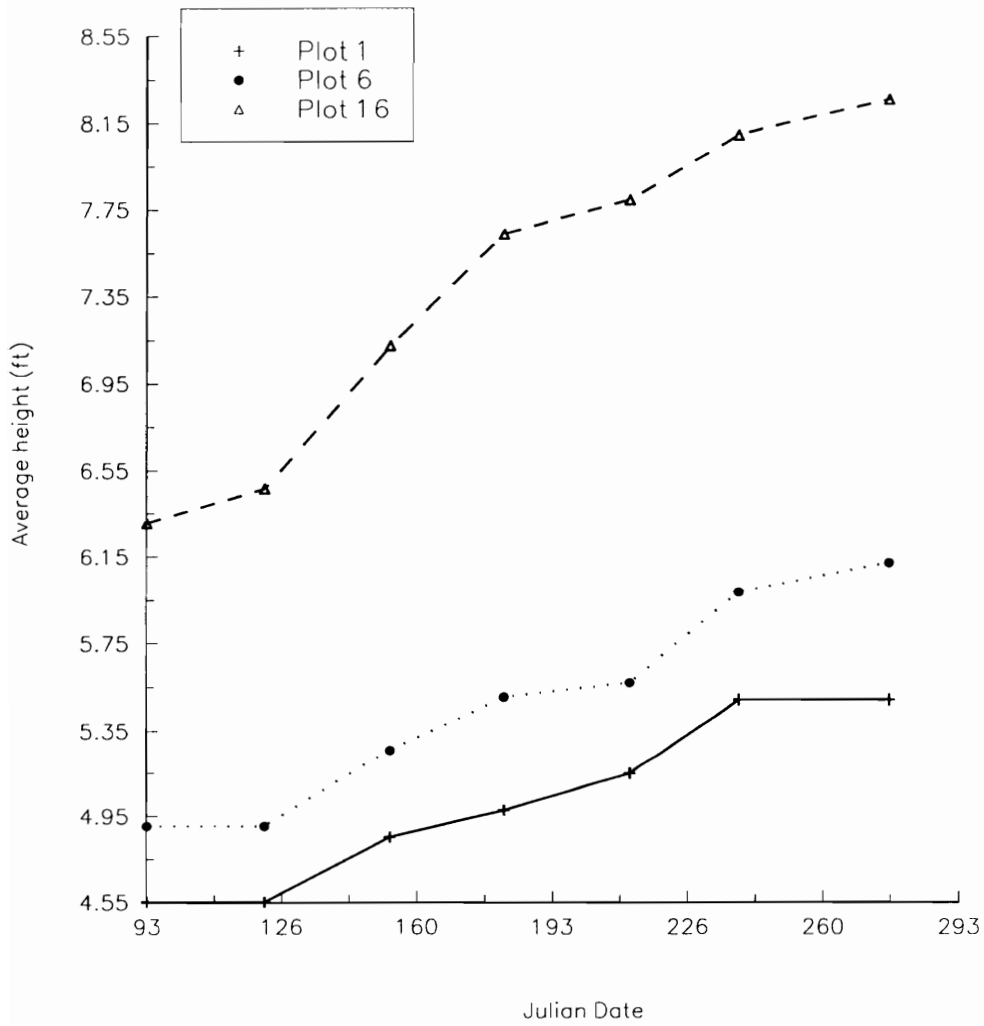


Figure 19: Average height vs. Time (1992) for the miniature spacing study planted with loblolly pine.

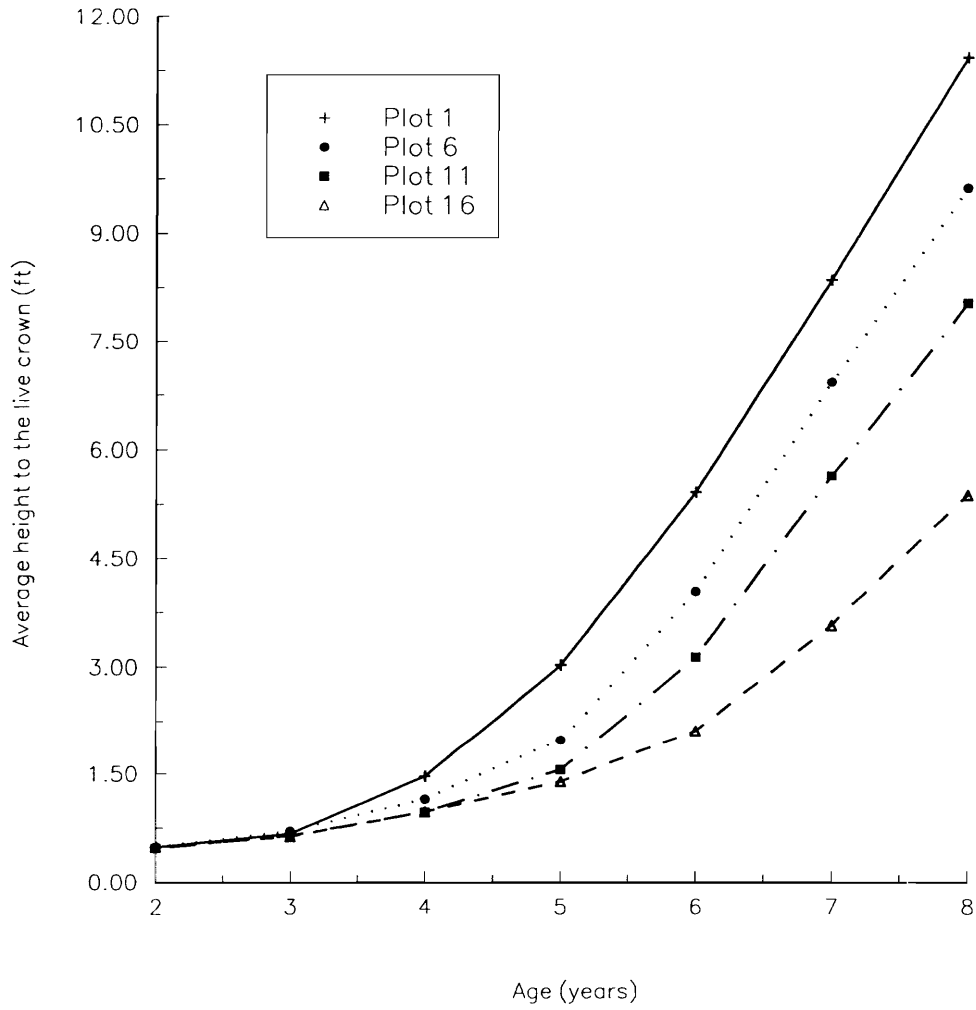


Figure 20: Average height to the live crown vs. Age for the conventional spacing study planted with loblolly pine.

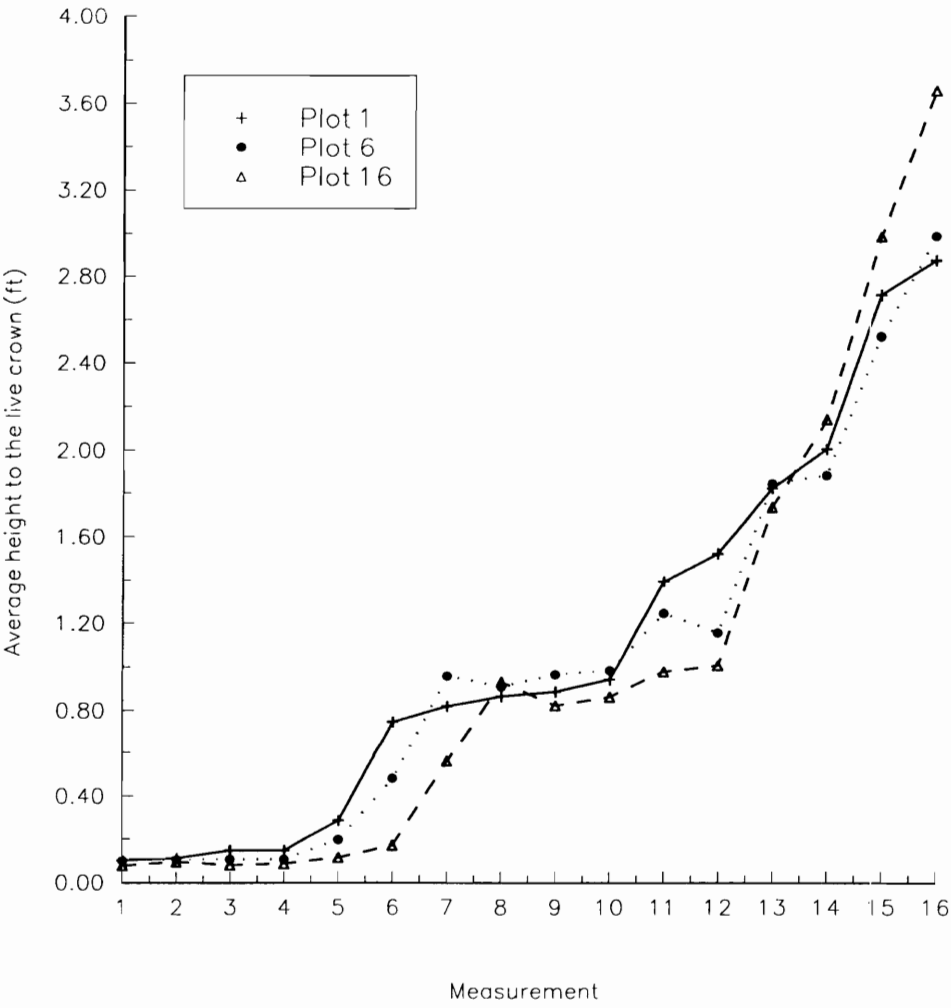


Figure 21: Average height to the live crown vs. Measurement for the miniature spacing study planted with loblolly pine.

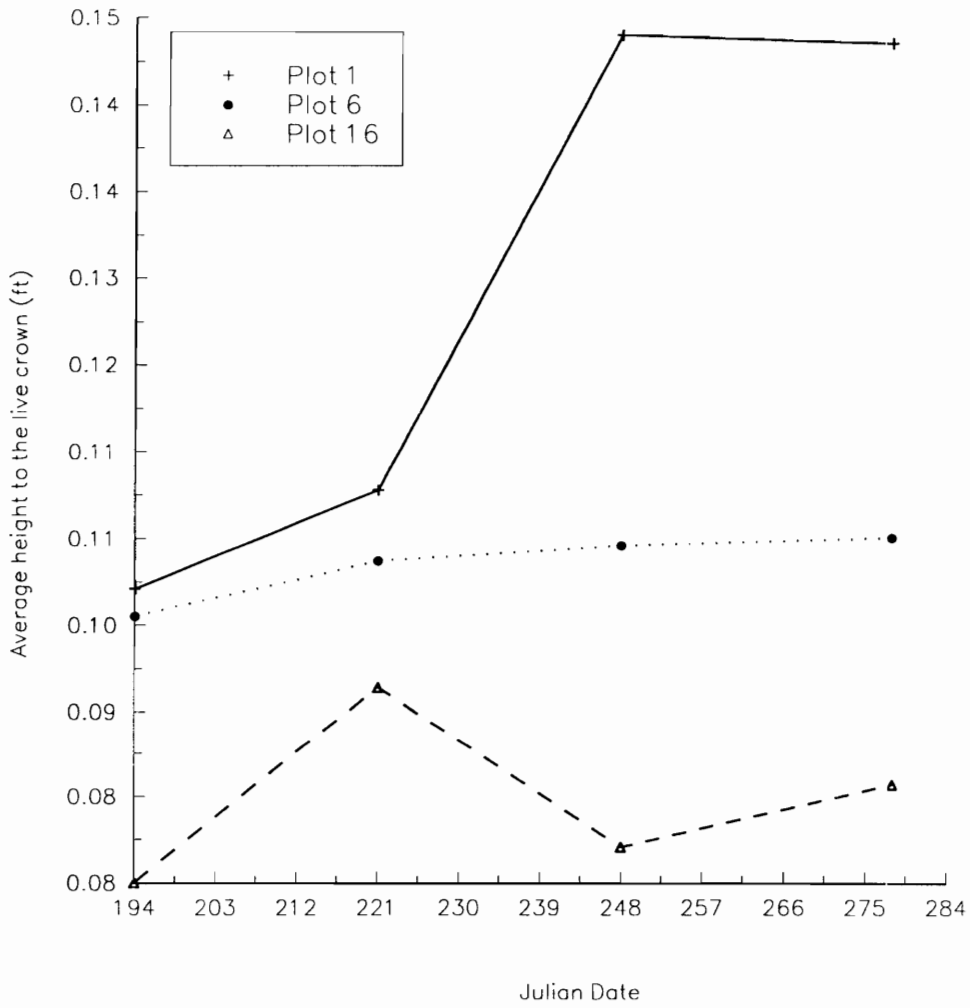


Figure 22: Average height to the live crown vs. Time (1989) for the miniature spacing study planted with loblolly pine.

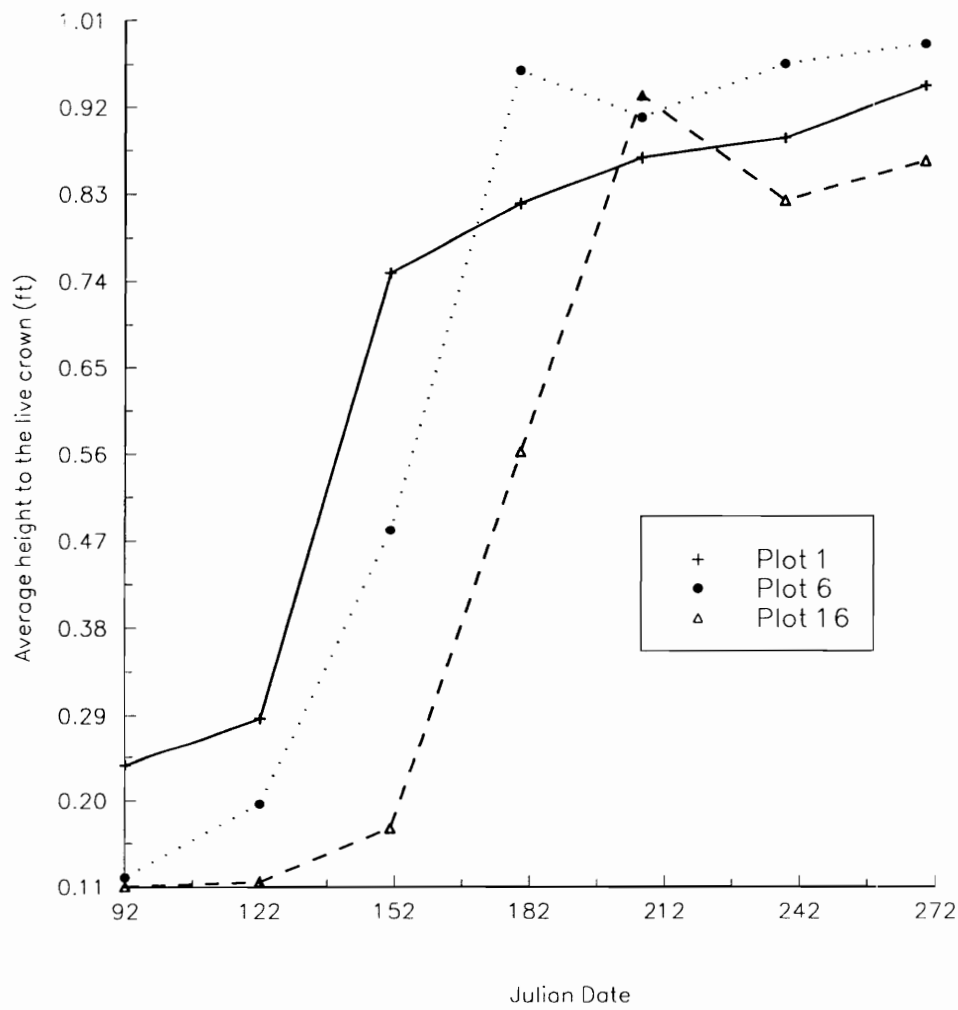


Figure 23: Average height to the live crown vs. Time (1990) for the miniature spacing study planted with loblolly pine.

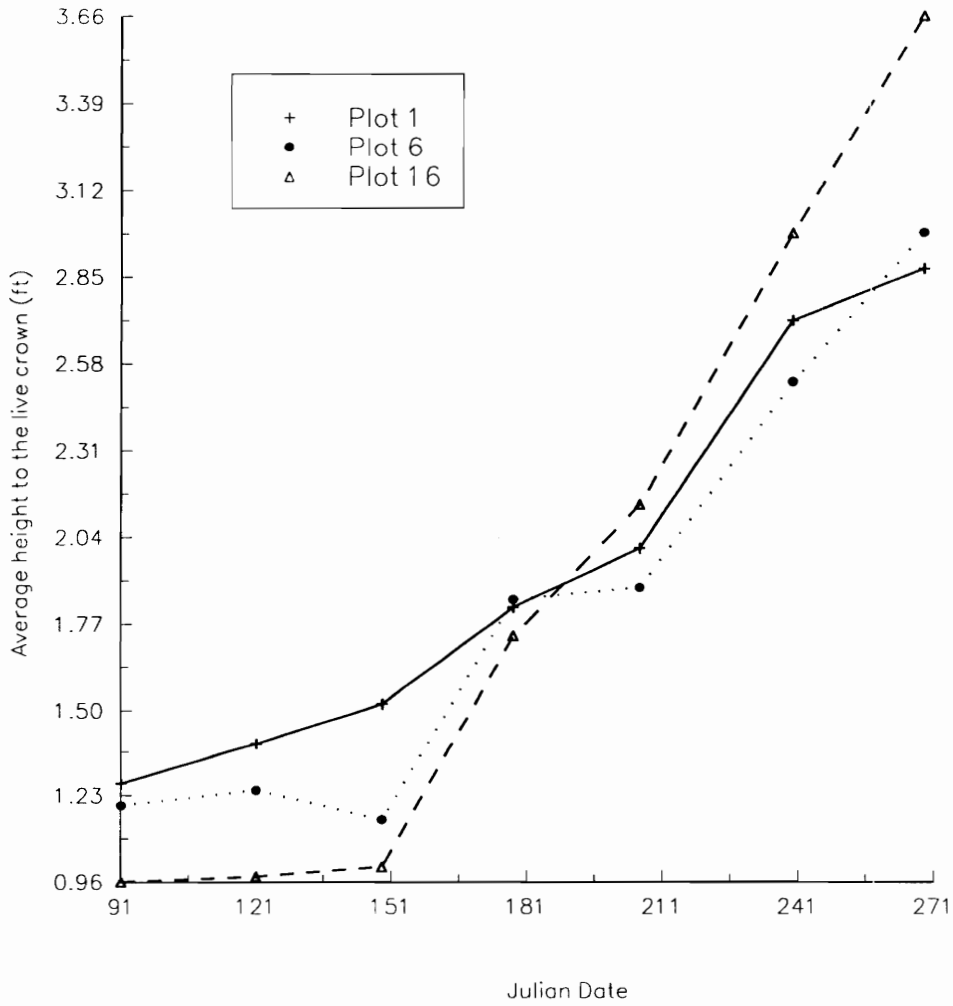


Figure 24: Average height to the live crown vs. Time (1991) for the miniature spacing study planted with loblolly pine.

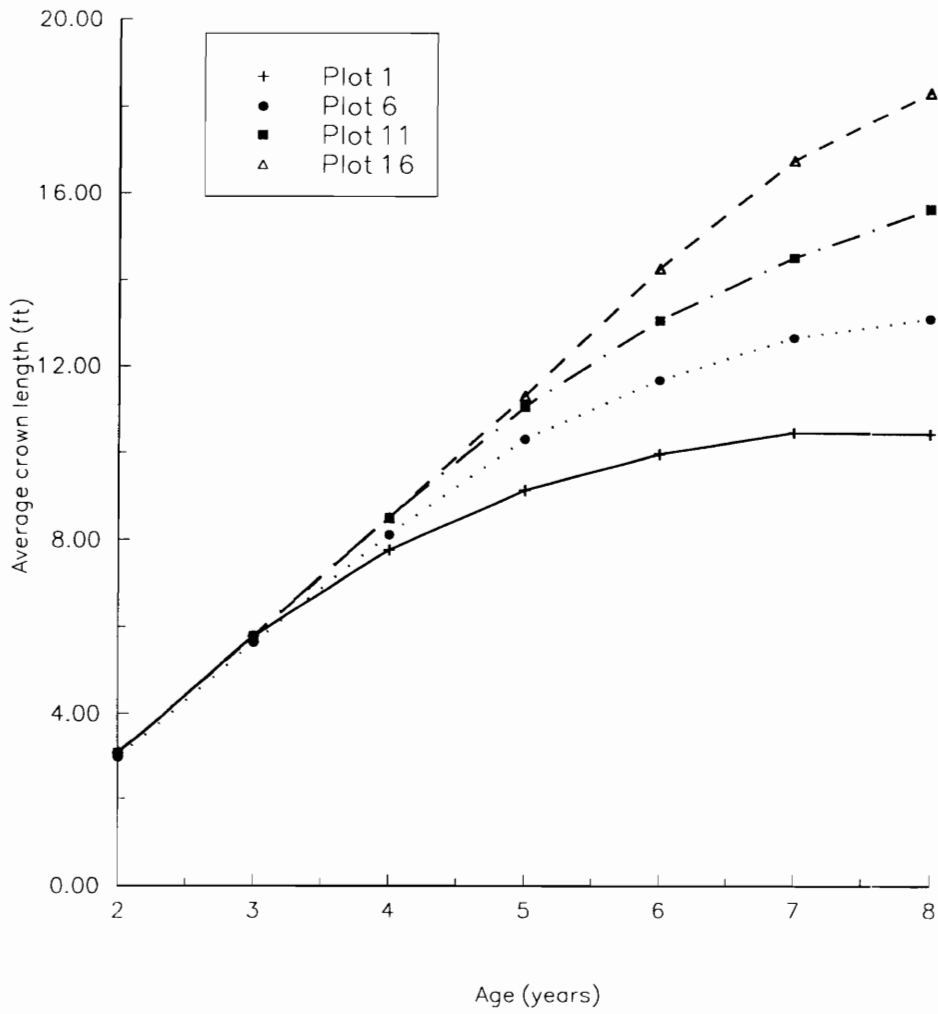


Figure 25: Average crown length vs. Age for the conventional spacing study planted with loblolly pine.

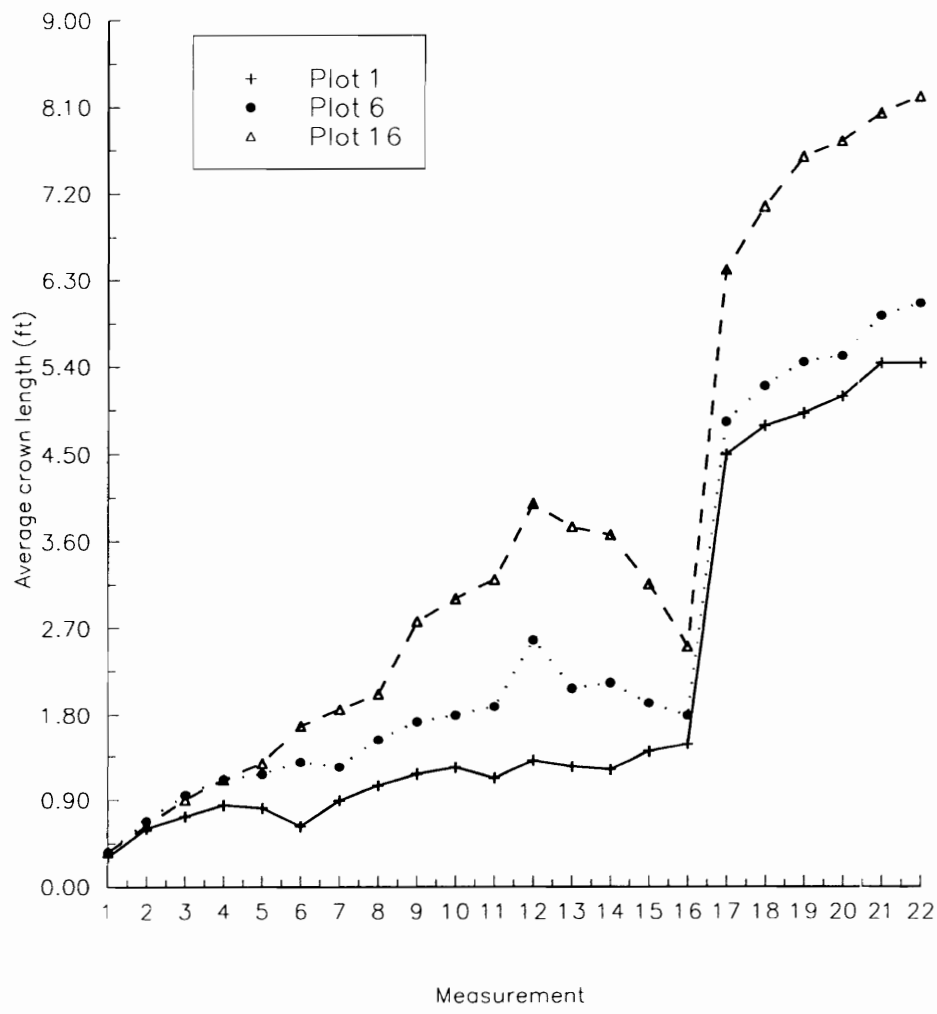


Figure 26: Average crown length vs. Measurement for the miniature spacing study planted with loblolly pine.

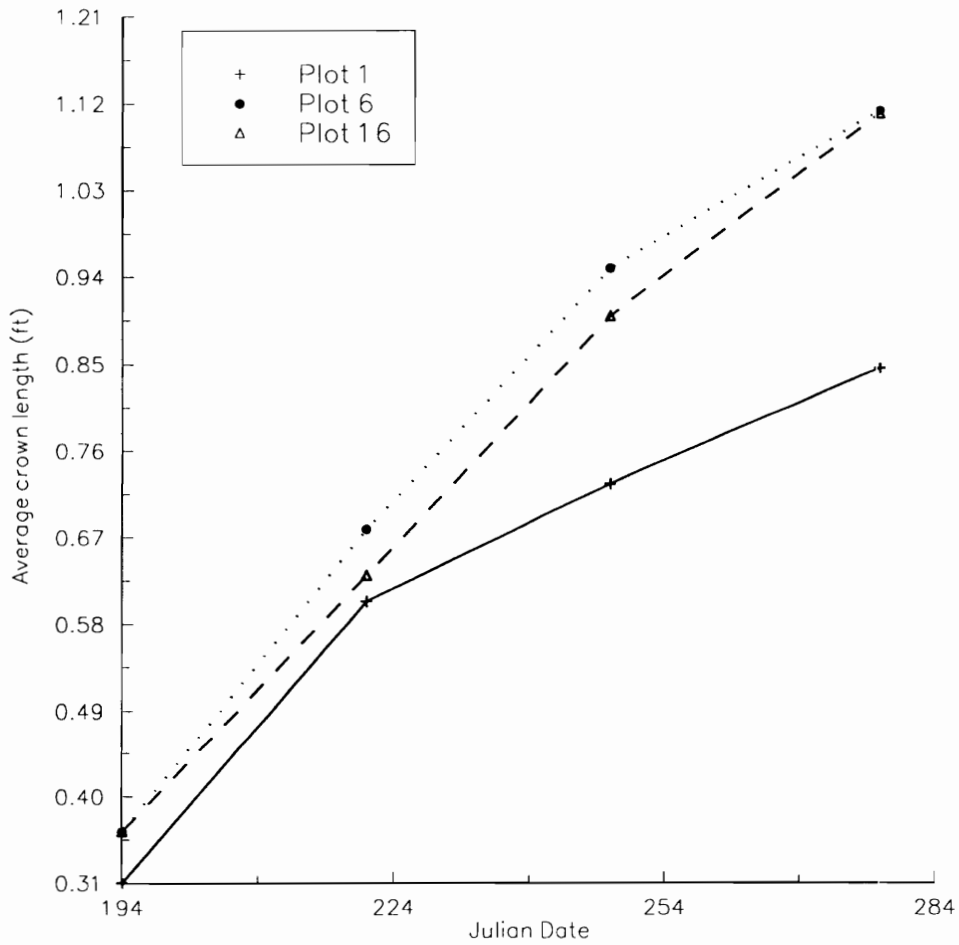


Figure 27: Average crown length vs. Time (1989) for the miniature spacing study planted with loblolly pine.

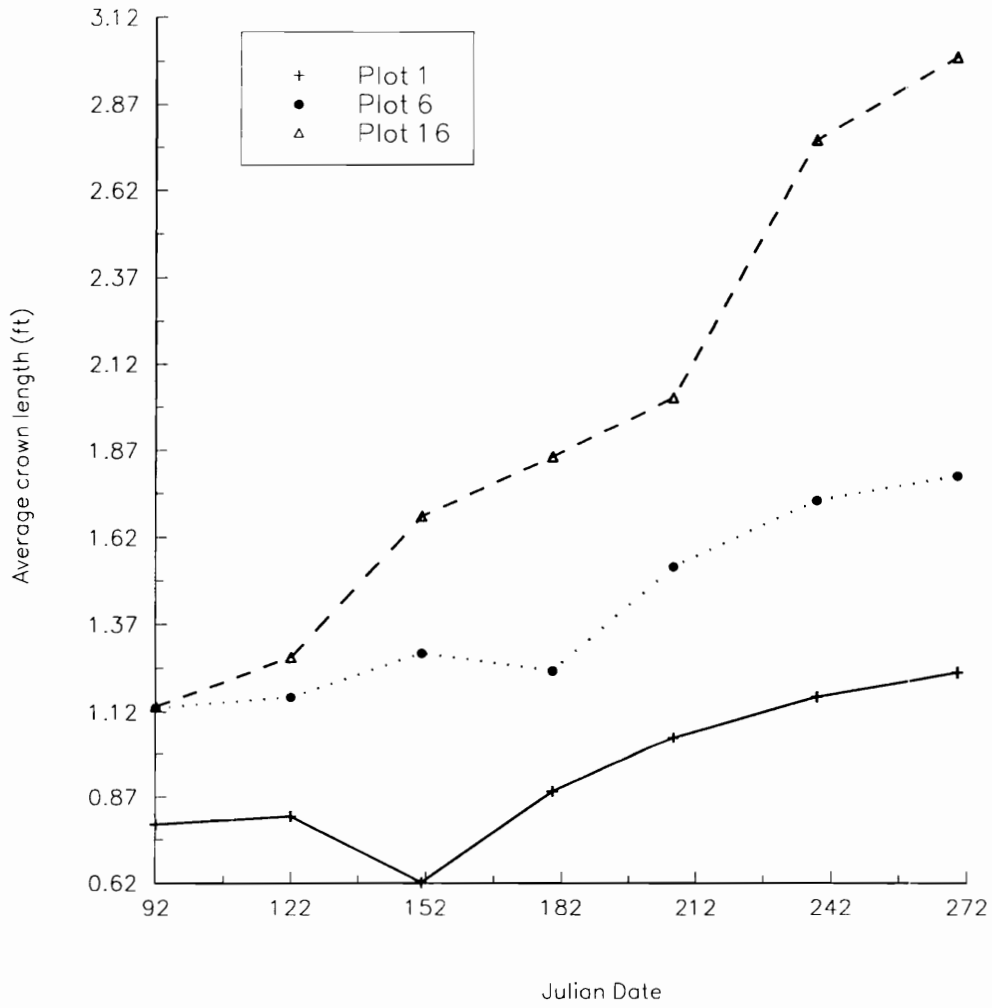


Figure 28: Average crown length vs. Time (1990) for the miniature spacing study planted with loblolly pine.

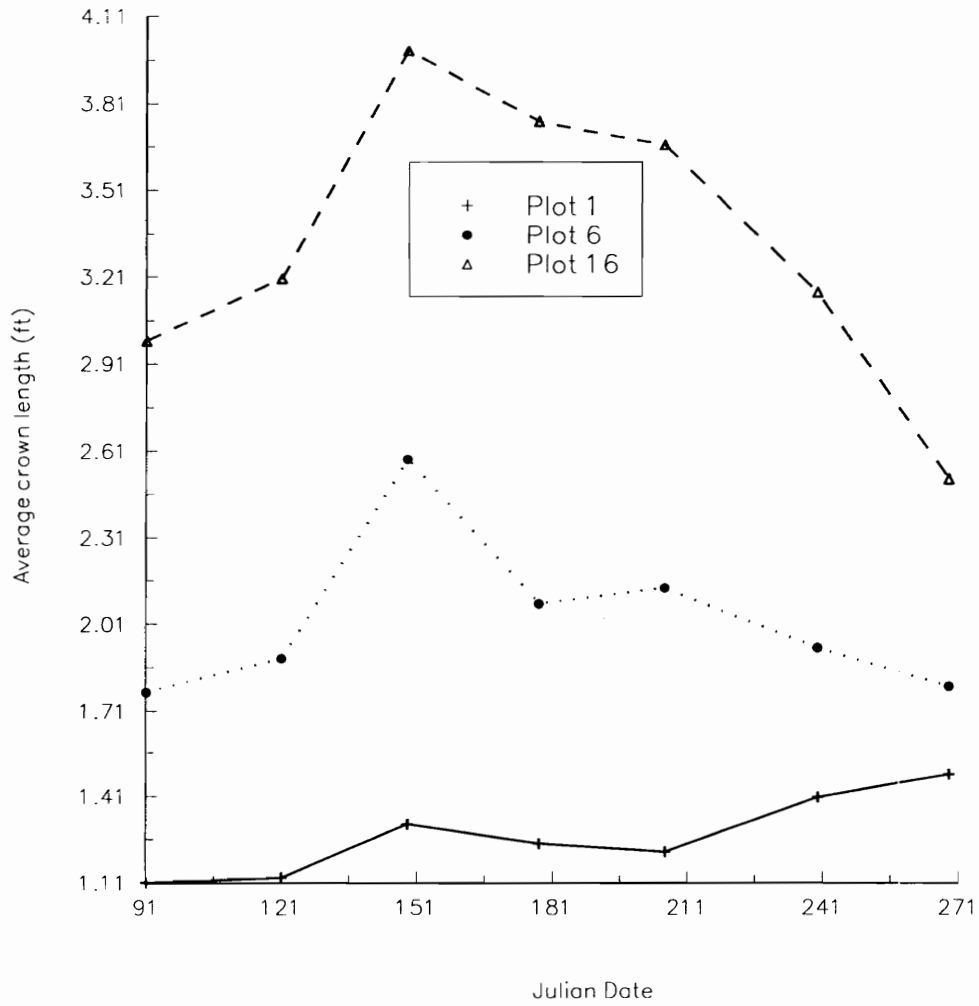


Figure 29: Average crown length vs. Time (1991) for the miniature spacing study planted with loblolly pine.

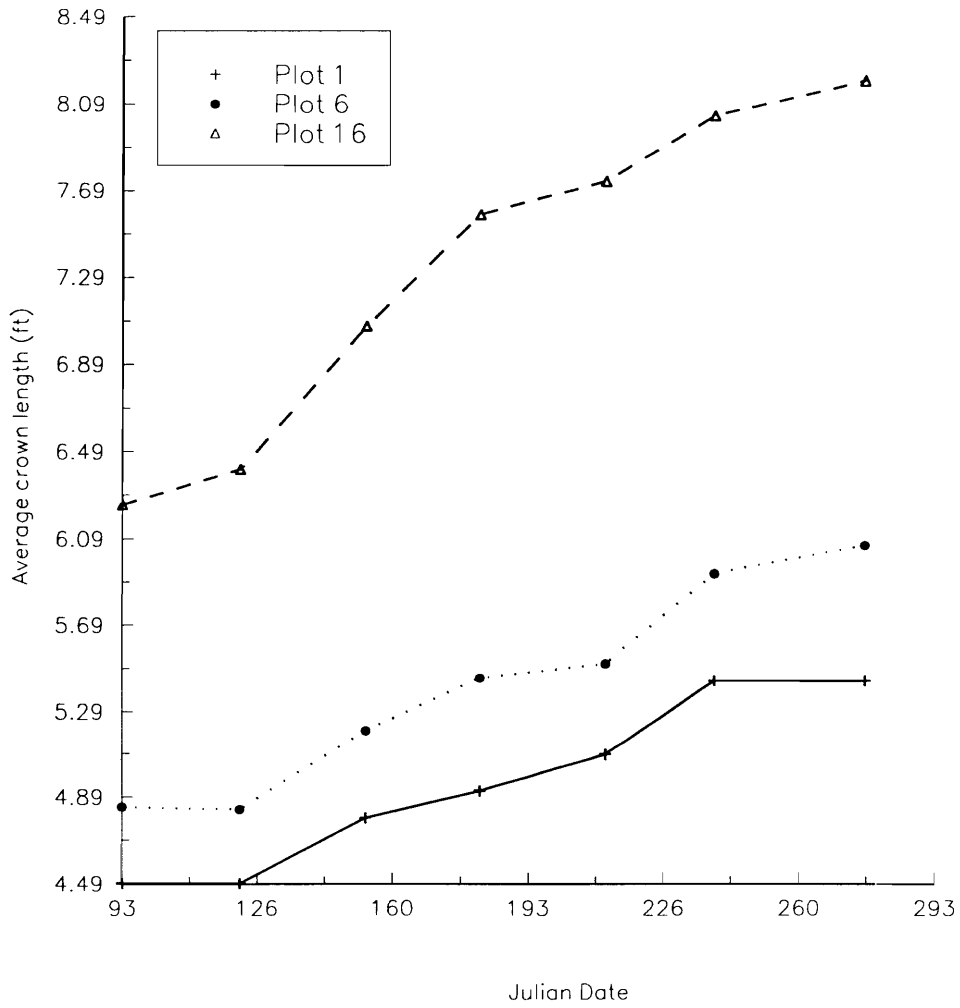


Figure 30: Average crown length vs. Time (1992) for the miniature spacing study planted with loblolly pine.

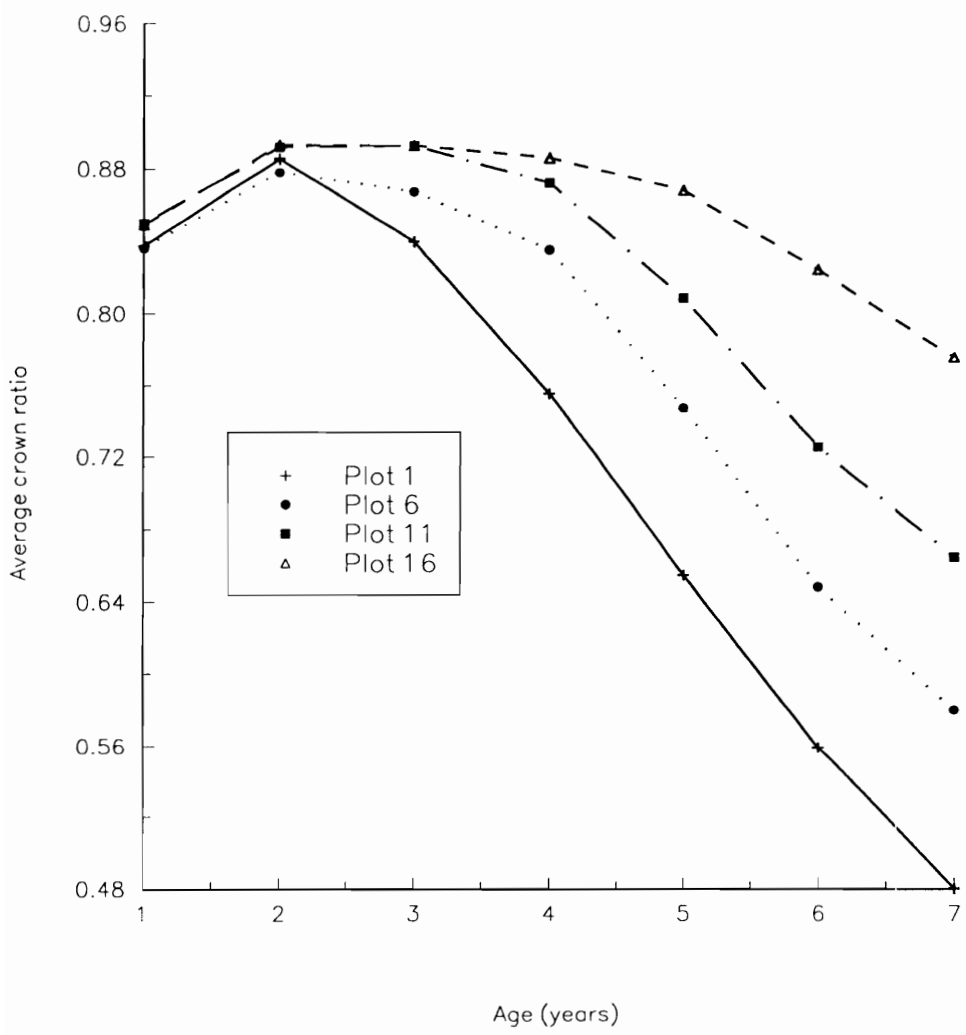


Figure 31: Average crown ratio vs. Age for the conventional spacing study planted with loblolly pine.

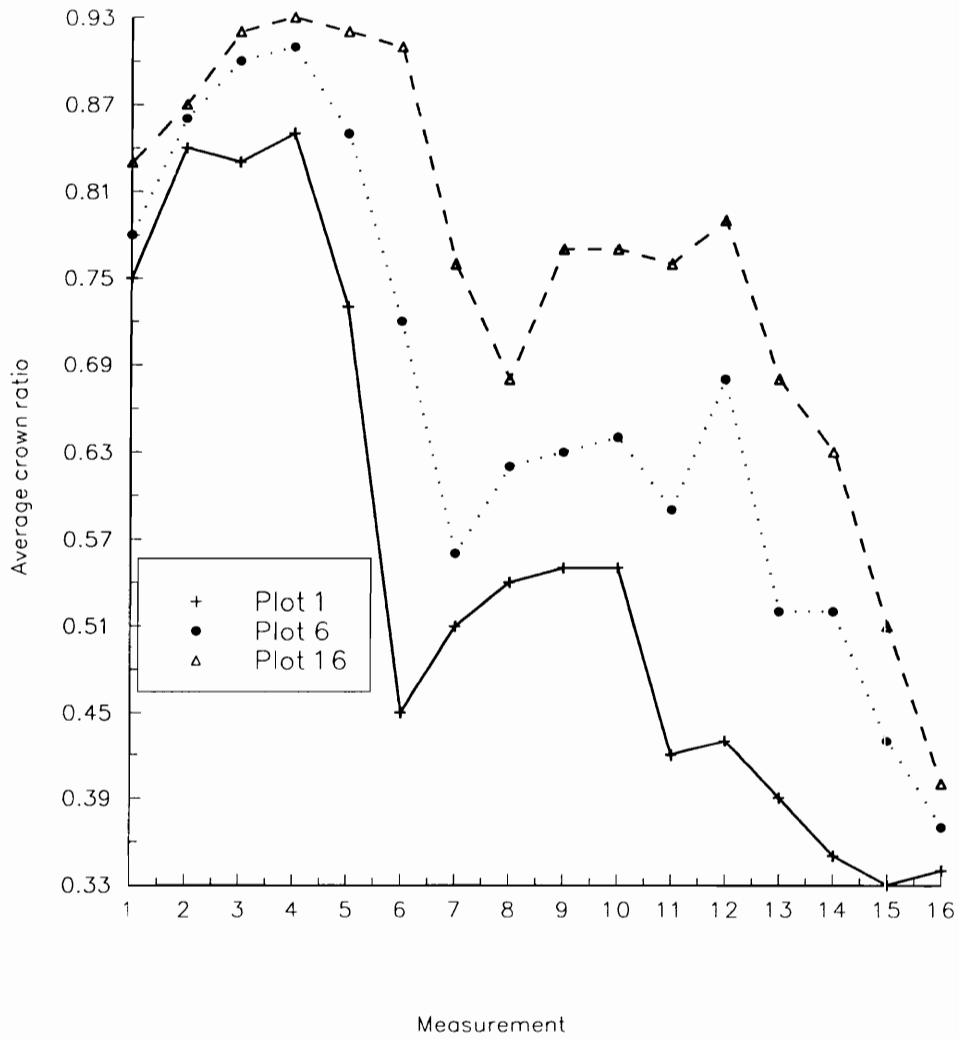


Figure 32: Average crown ratio vs. Measurement for the miniature spacing study planted with loblolly pine.

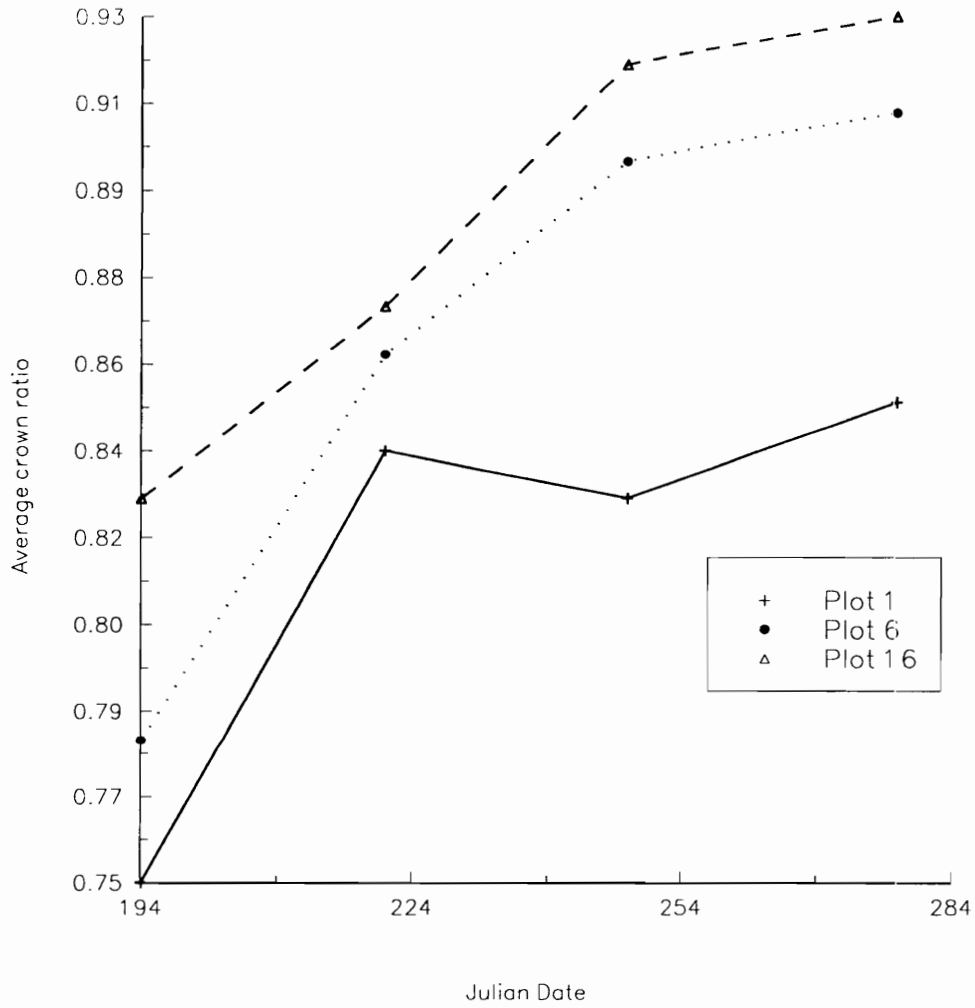


Figure 33: Average crown ratio vs. Time (1989) for the miniature spacing study planted with loblolly pine.

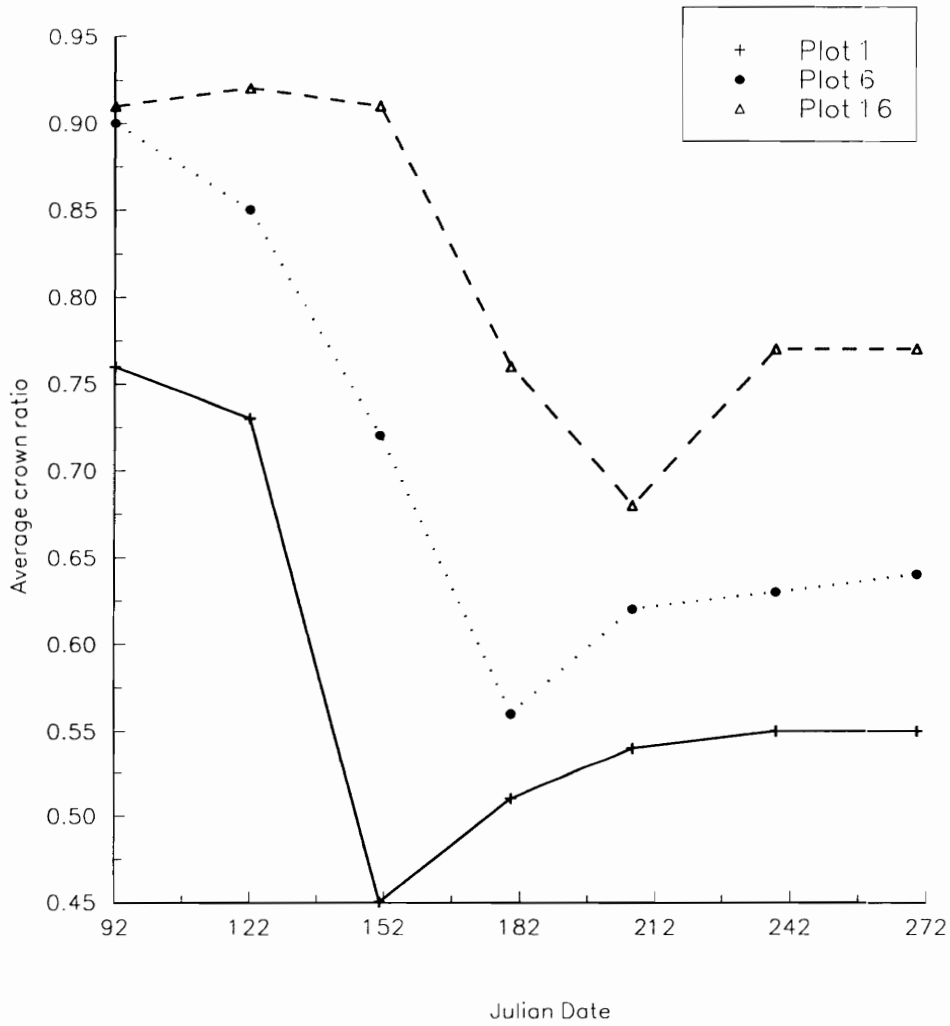


Figure 34: Average crown ratio vs. Time (1990) for the miniature spacing study planted with loblolly pine.

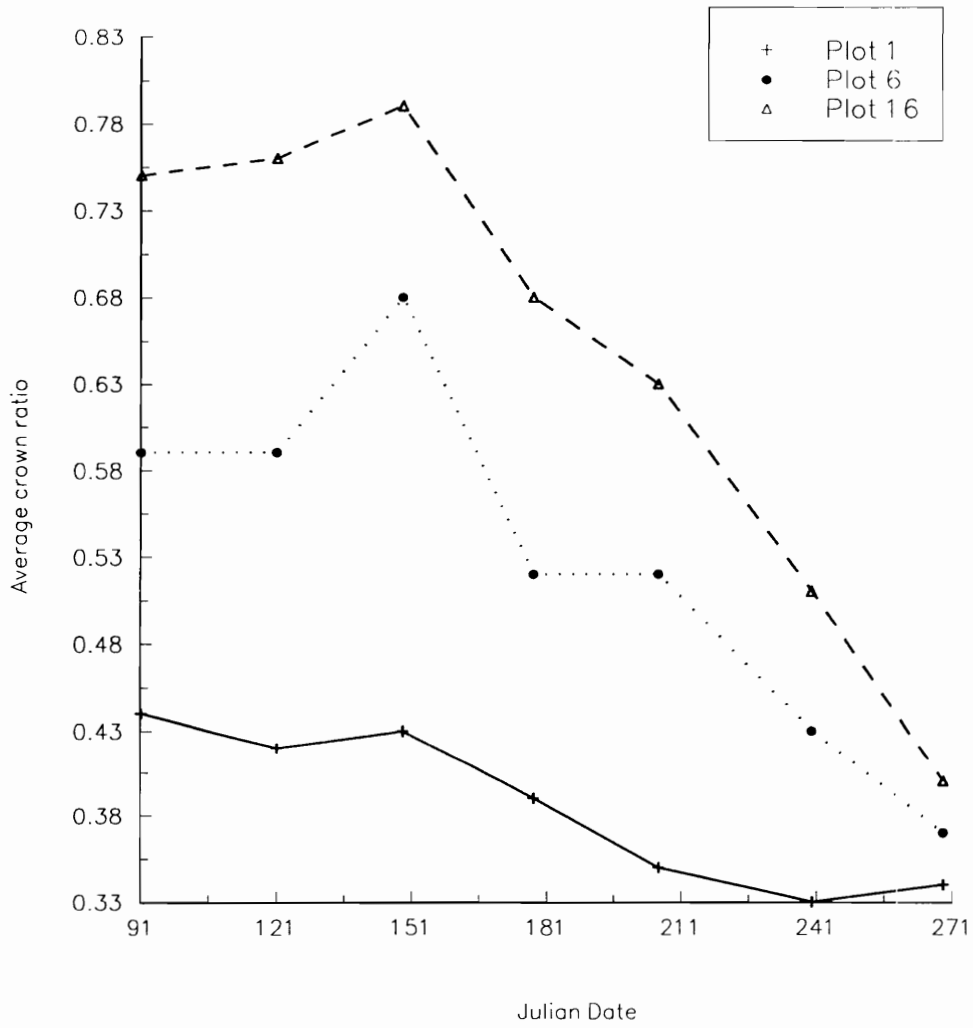


Figure 35: Average crown ratio vs. Time (1991) for the miniature spacing study planted with loblolly pine.

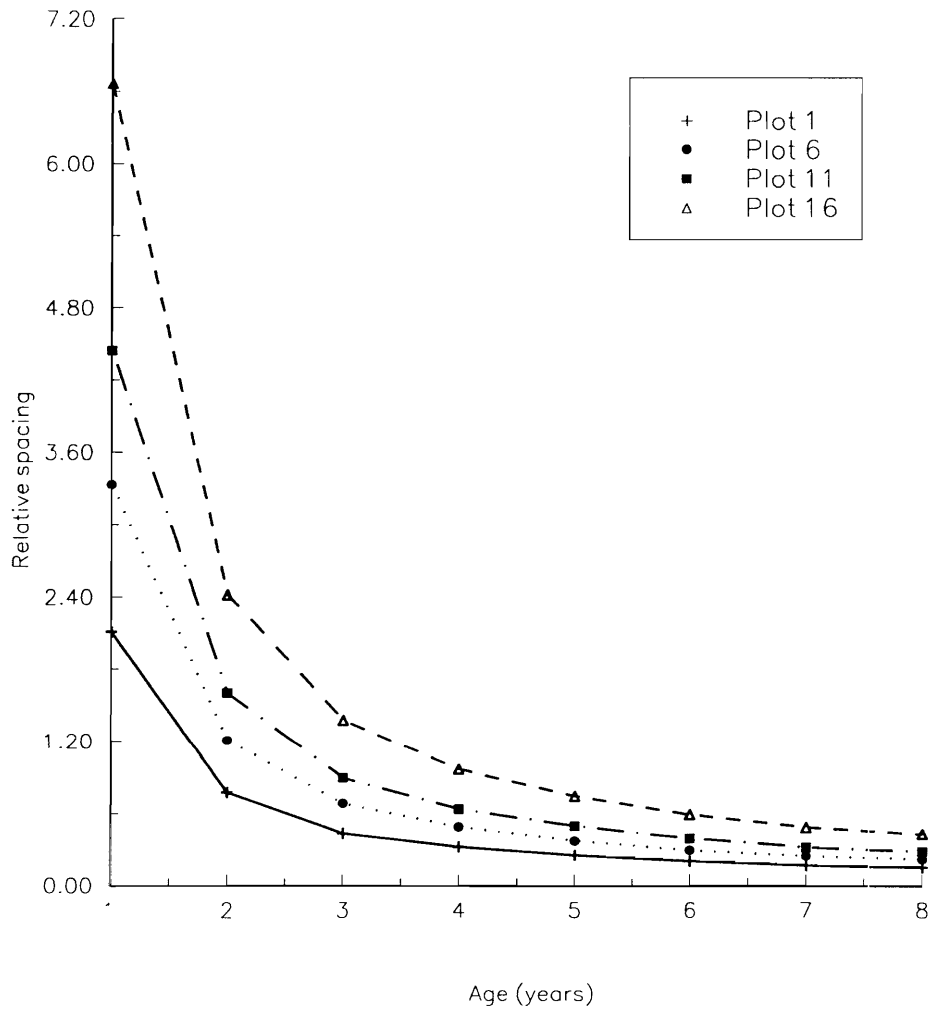


Figure 36: Relative spacing vs. Age for the conventional spacing study planted with loblolly pine.

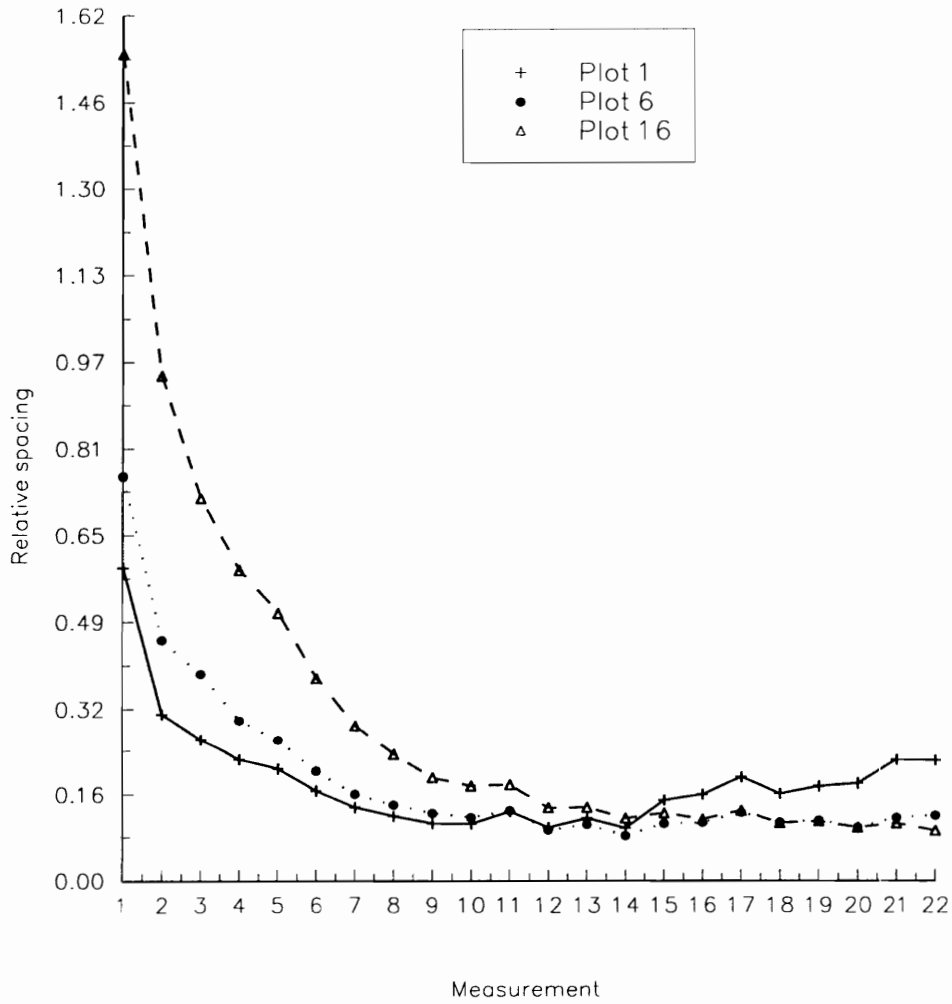


Figure 37: Relative spacing vs. Measurement for the miniature spacing study planted with loblolly pine.

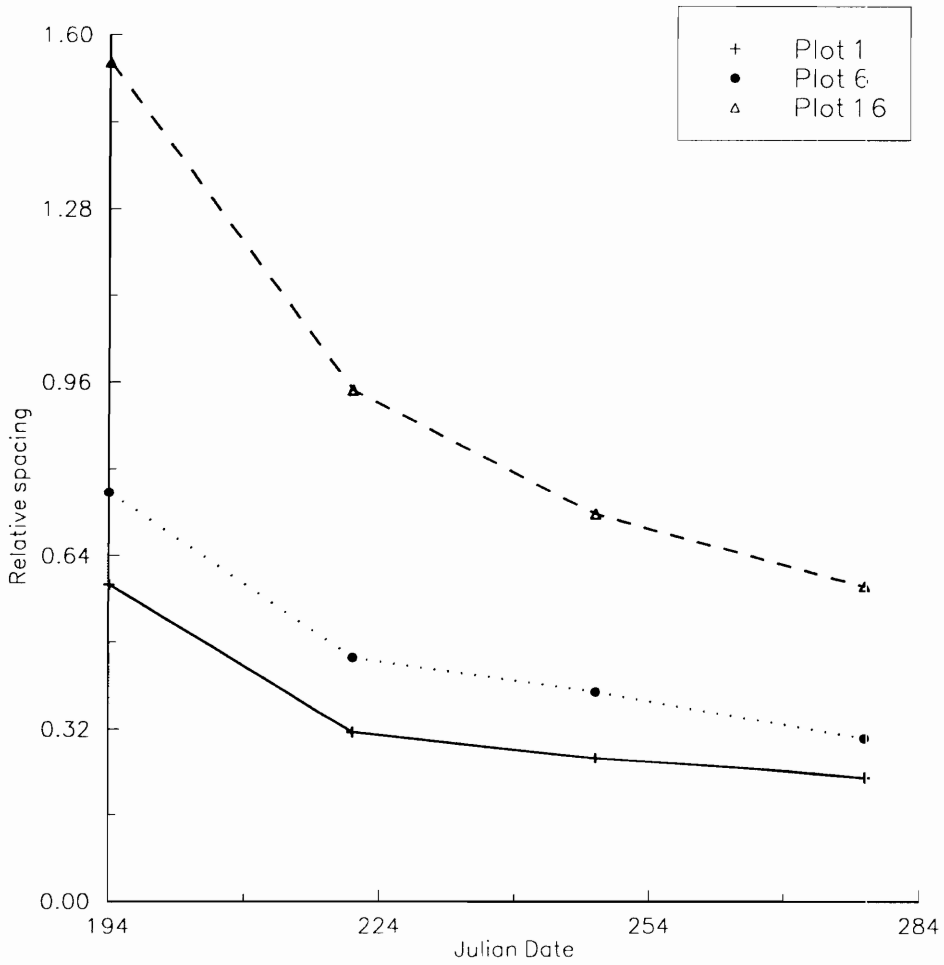


Figure 38: Relative spacing vs. Time (1989) for the miniature spacing study planted with loblolly pine.

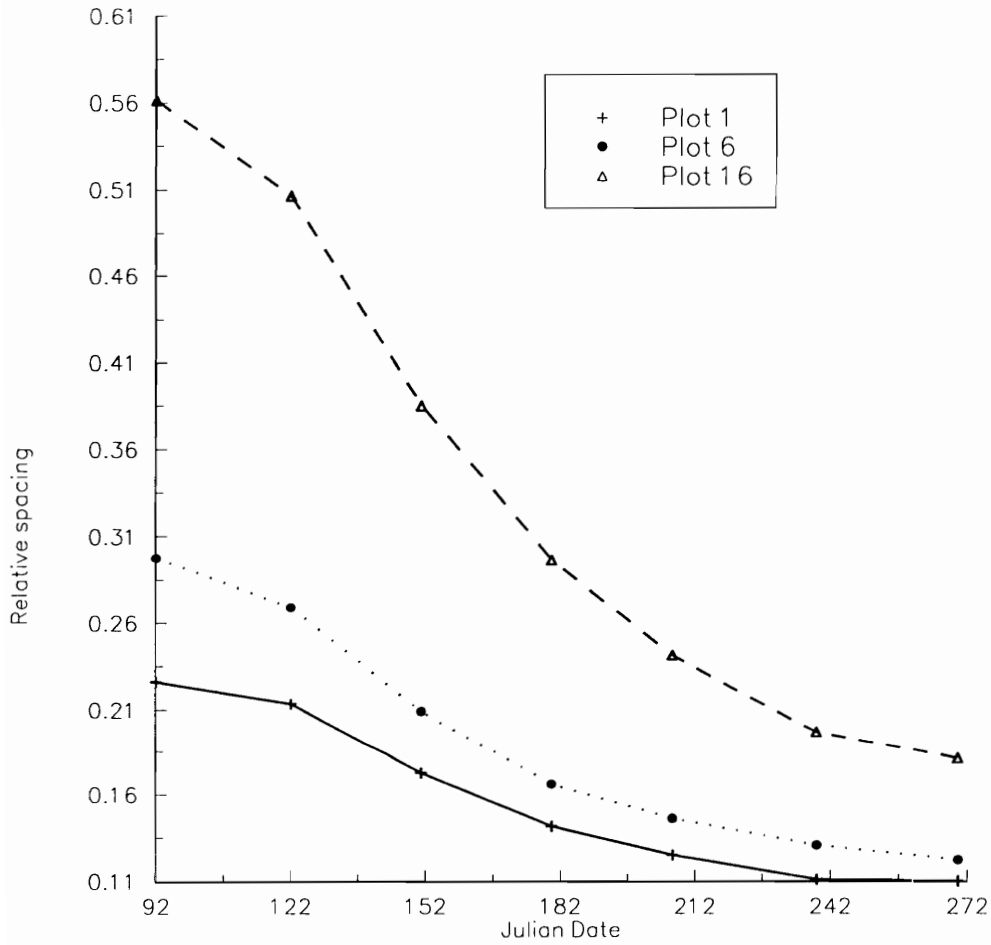


Figure 39: Relative spacing vs. Time (1990) for the miniature spacing study planted with loblolly pine.

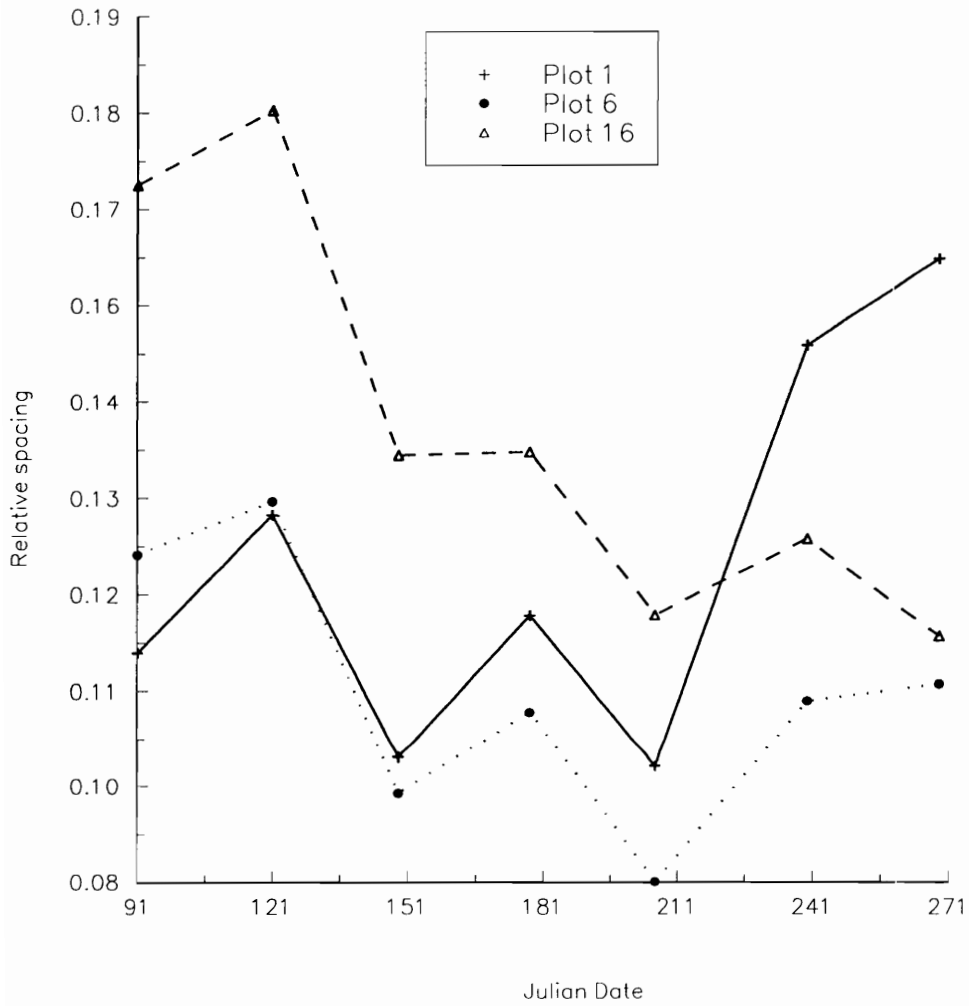


Figure 40: Relative spacing vs. Time (1991) for the miniature spacing study planted with loblolly pine.

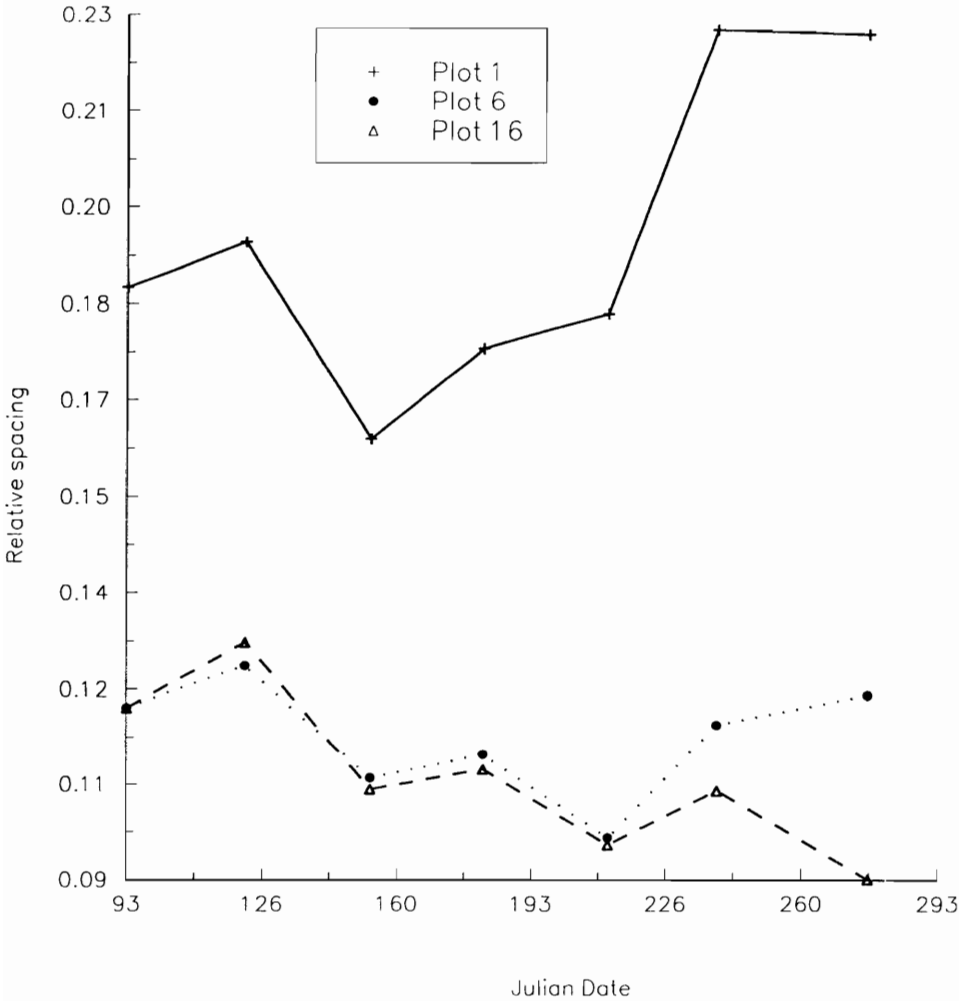


Figure 41: Relative spacing vs. Time (1992) for the miniature spacing study planted with loblolly pine.

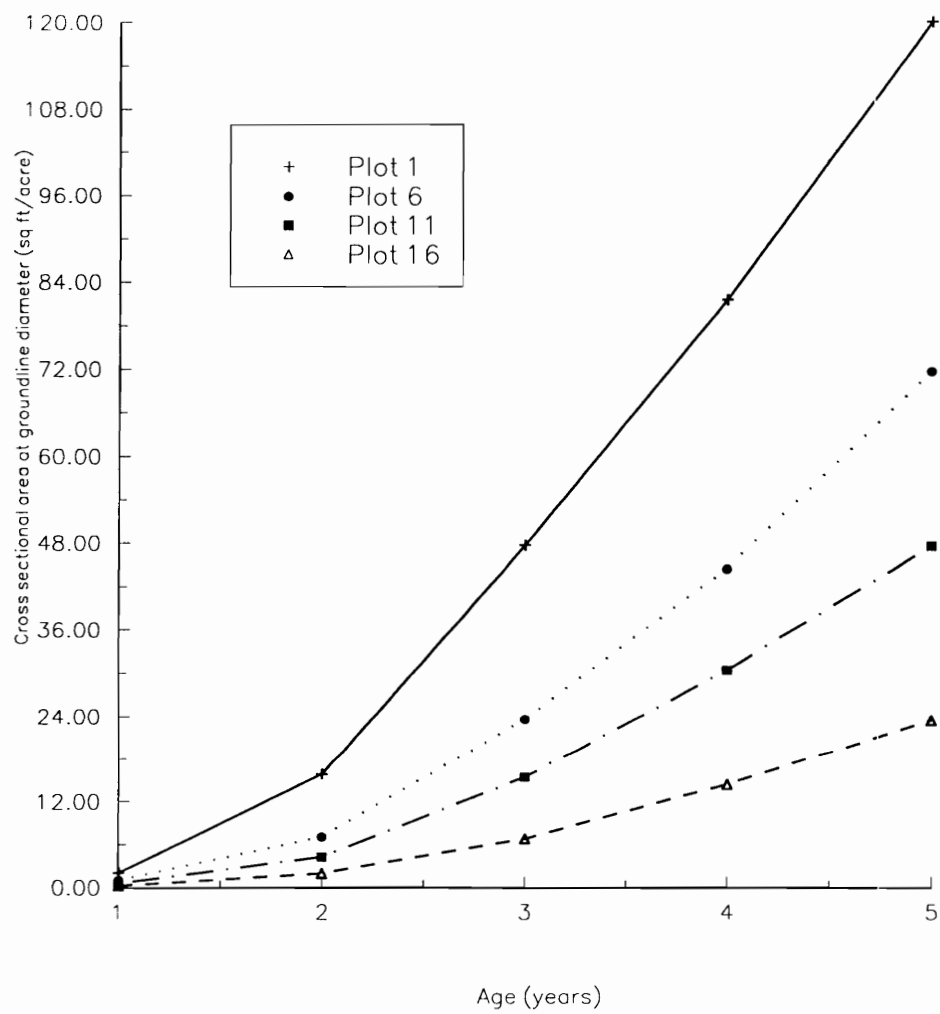


Figure 42: Cross sectional area at groundline vs. Age for the conventional spacing study planted with loblolly pine.

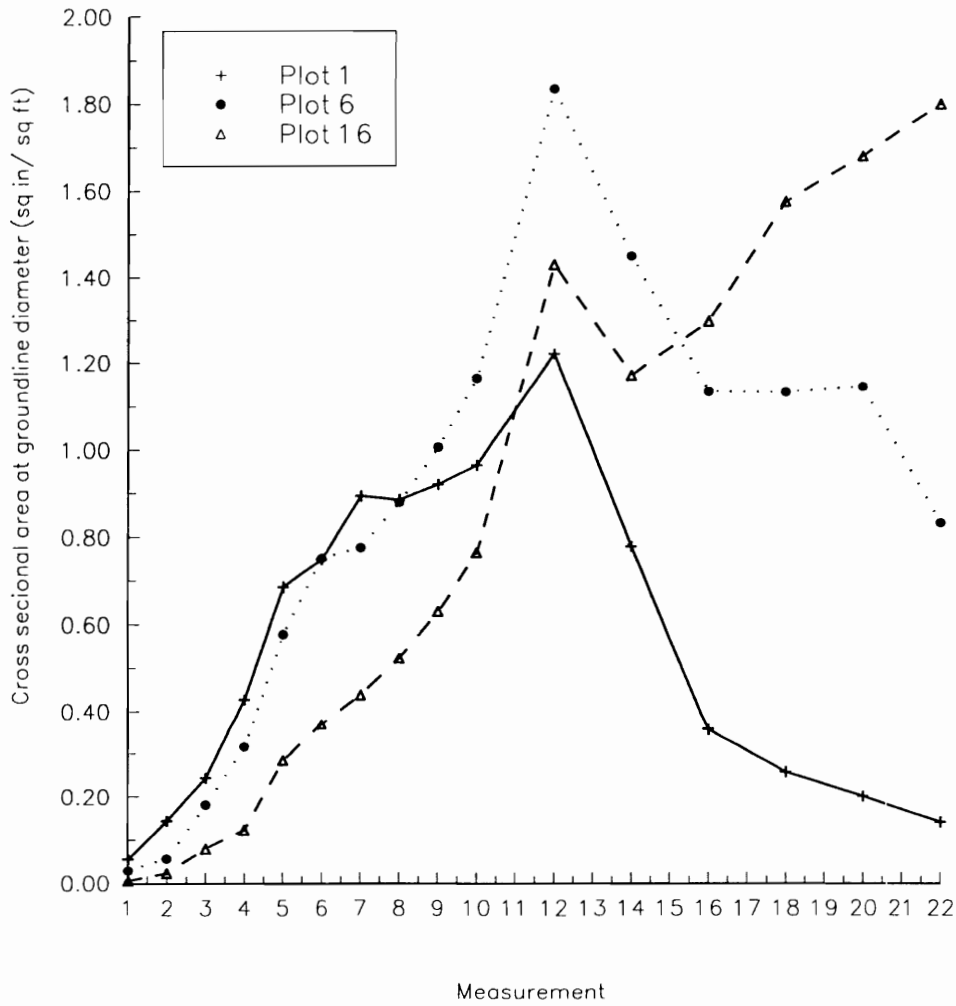


Figure 43: Cross sectional area at groundline vs. Measurement for the miniature spacing study planted with loblolly pine.

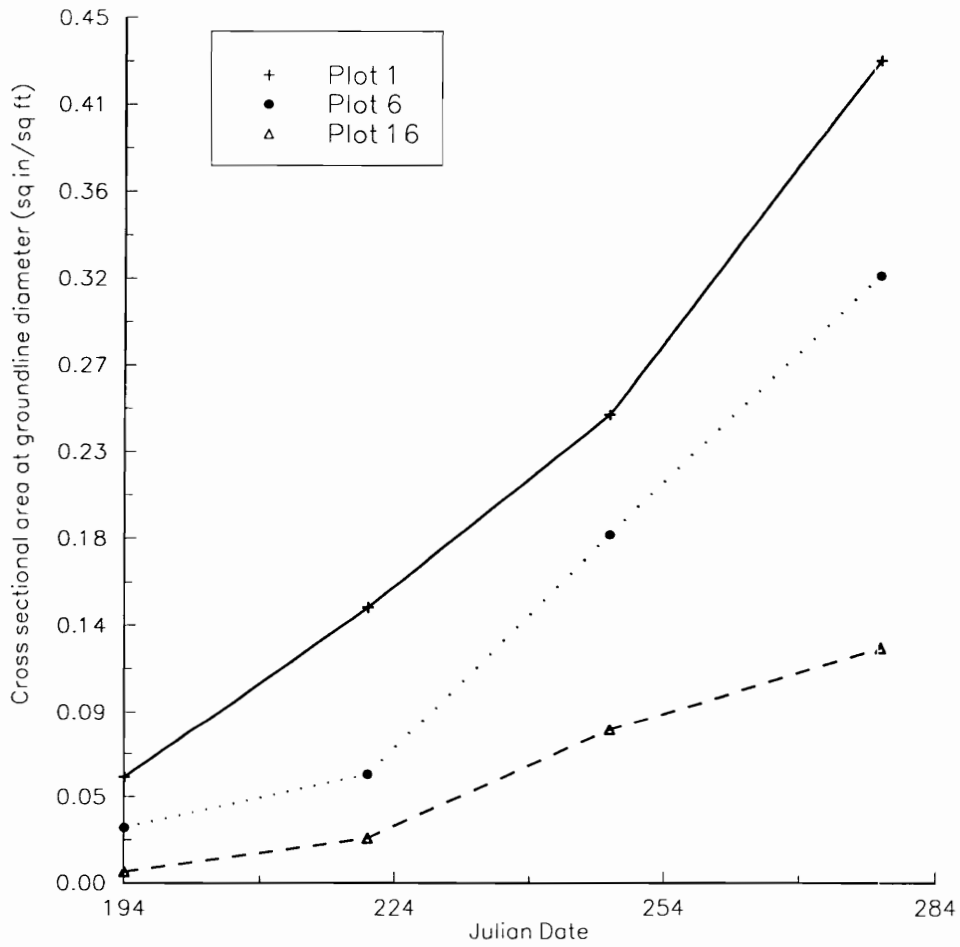


Figure 44: Cross sectional area at groundline vs. Time (1989) for the miniature spacing study planted with loblolly pine.

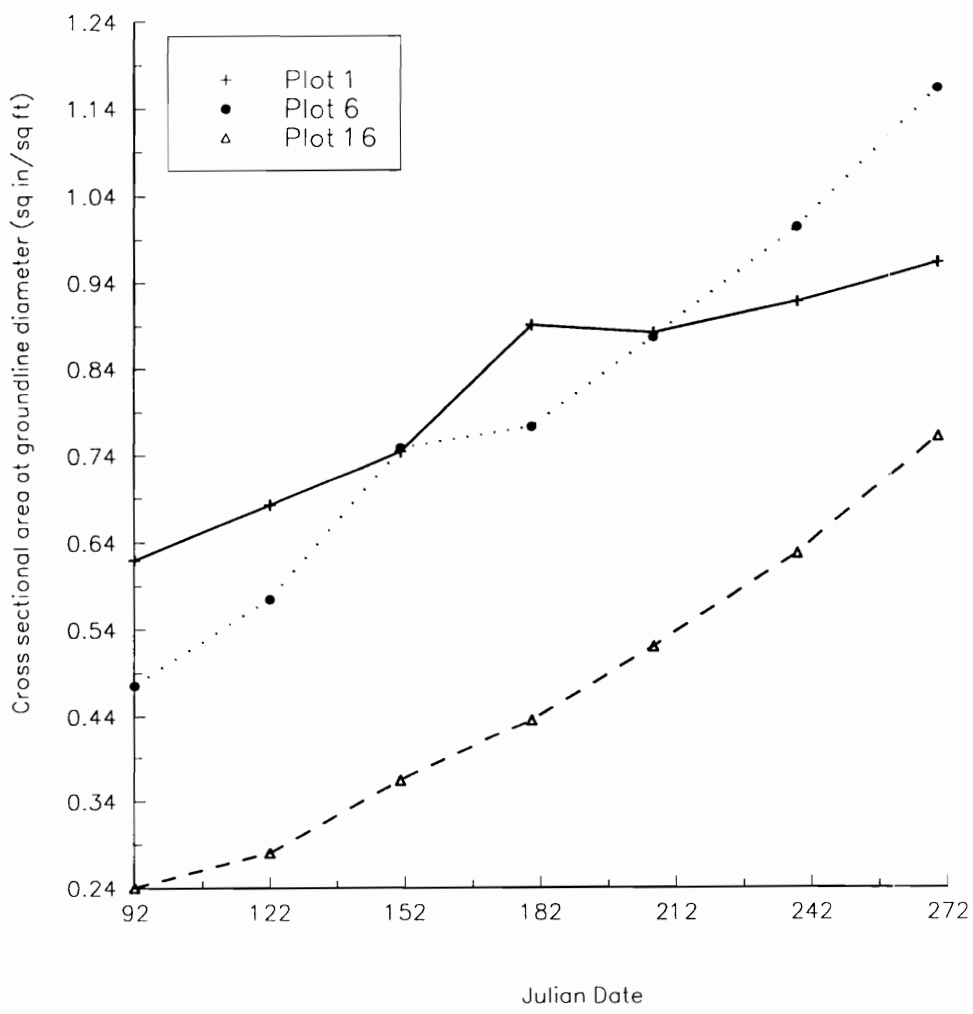


Figure 45: Cross sectional area at groundline vs. Time (1990) for the miniature spacing study planted with loblolly pine.

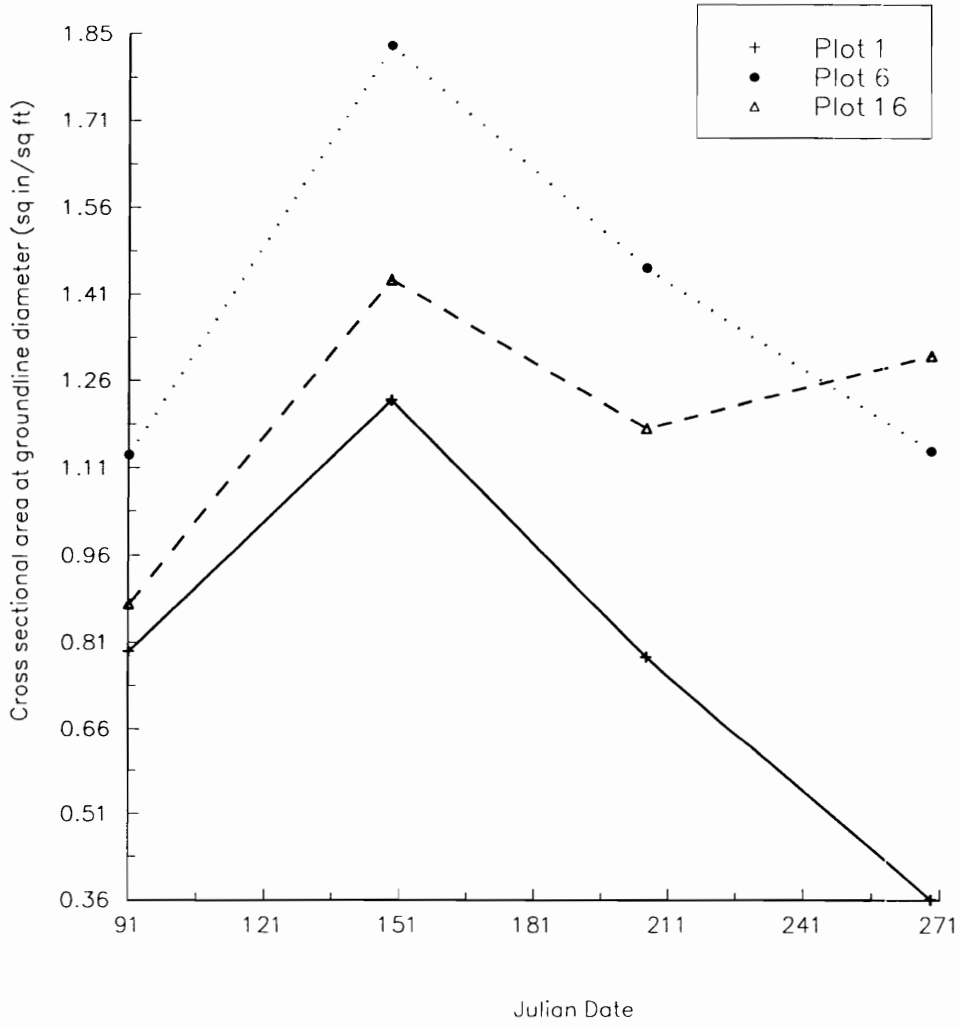


Figure 46: Cross sectional area at groundline vs. Time (1991) for the miniature spacing study planted with loblolly pine.

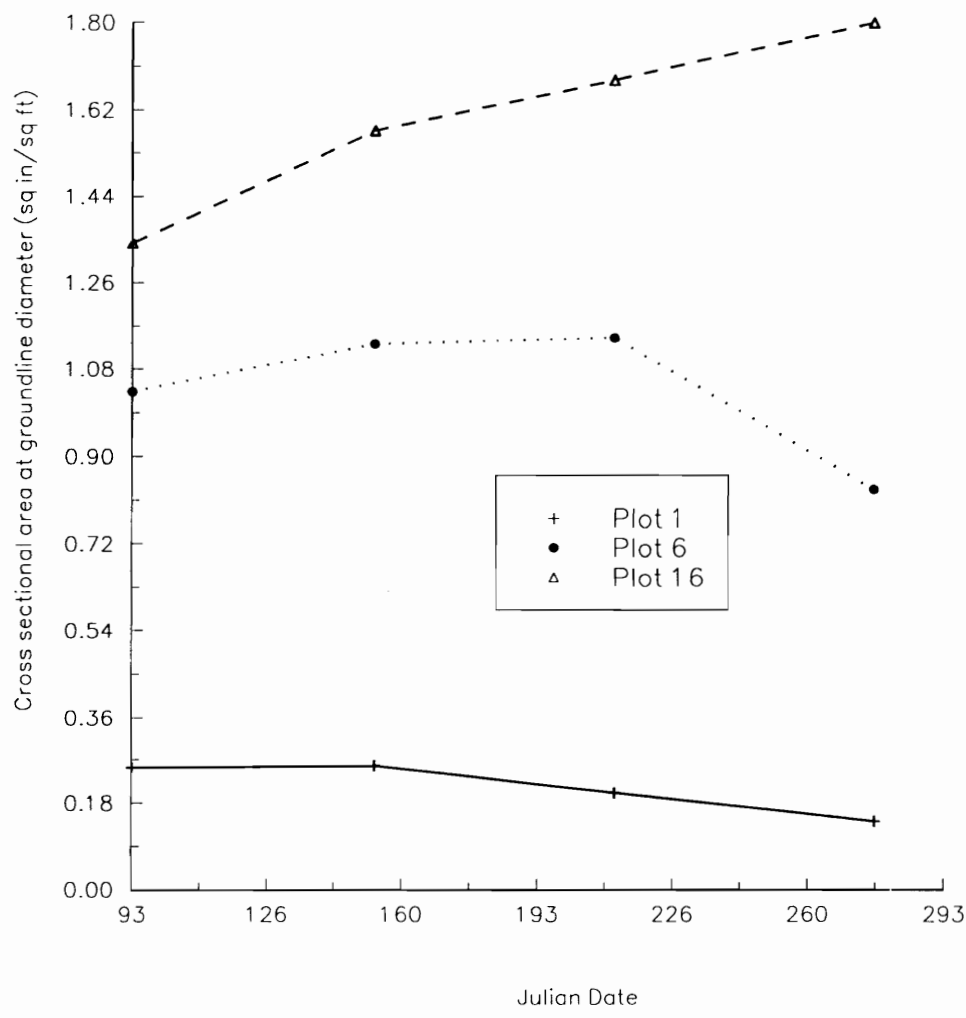


Figure 47: Cross sectional area at groundline vs. Time (1992)for the miniature spacing study planted with loblolly pine.

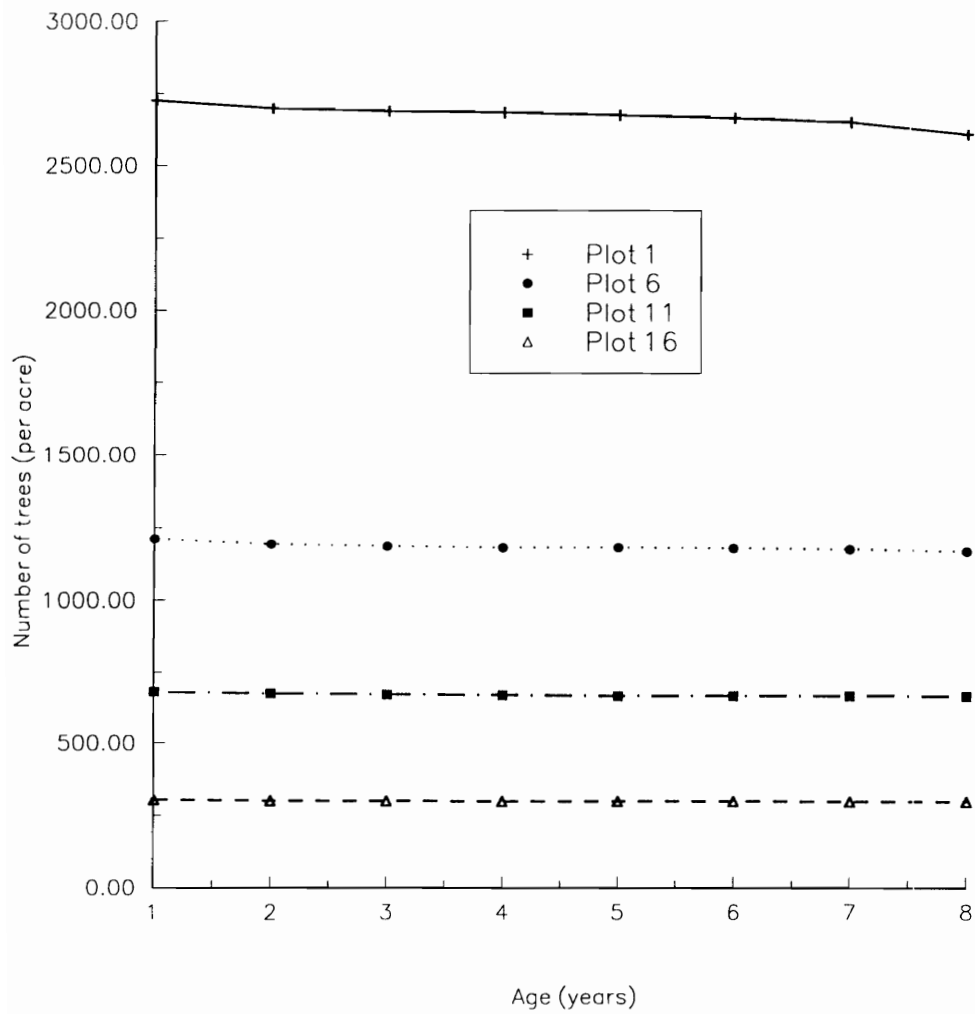


Figure 48: Number of trees vs. Age for the conventional spacing study planted with loblolly pine.

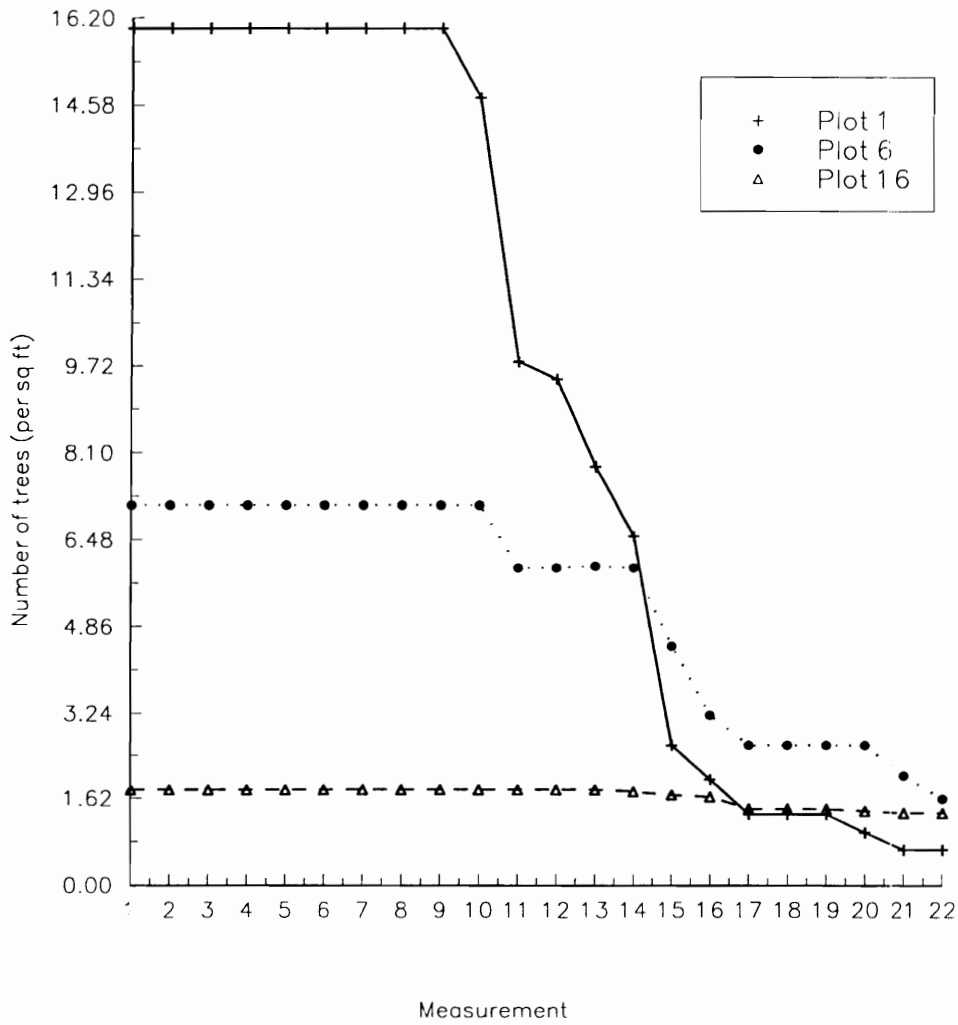


Figure 49: Number of trees vs. Measurement for the miniature spacing study planted with loblolly pine.

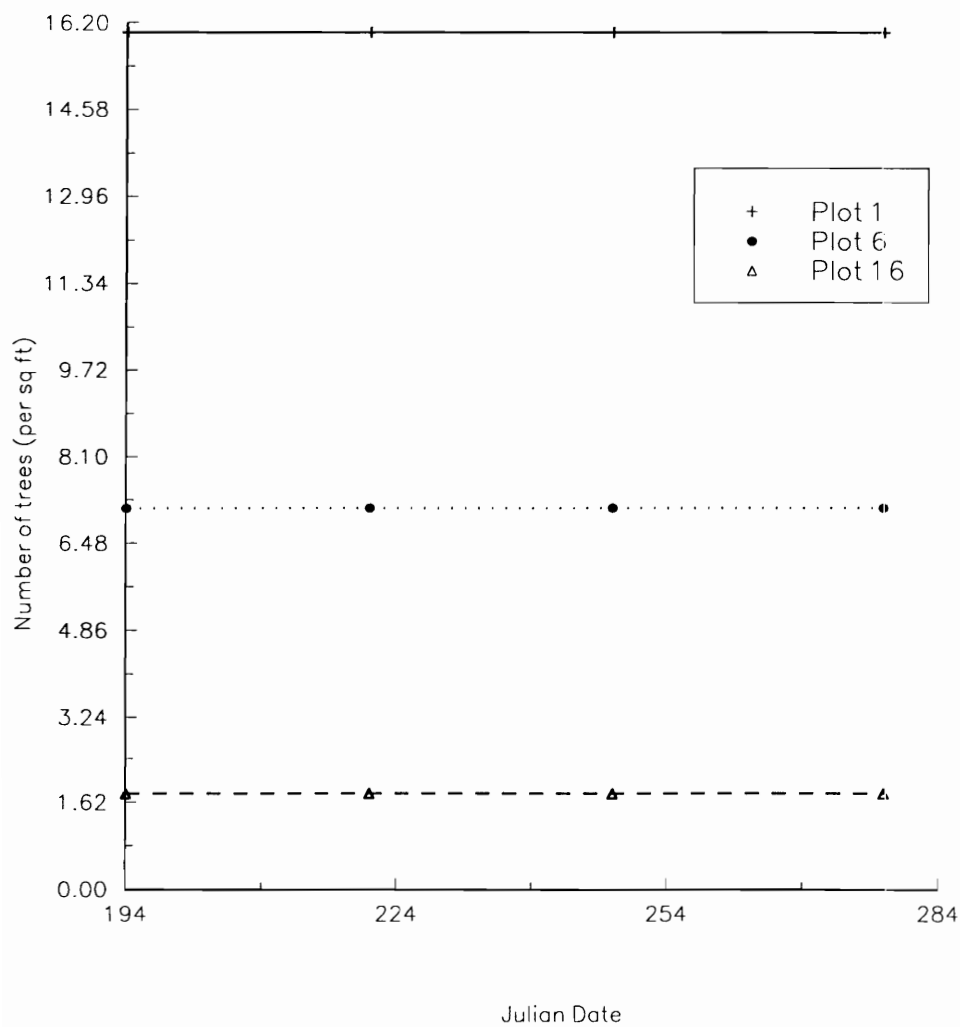


Figure 50: Number of trees vs. Time (1989) for the miniature spacing study planted with loblolly pine.

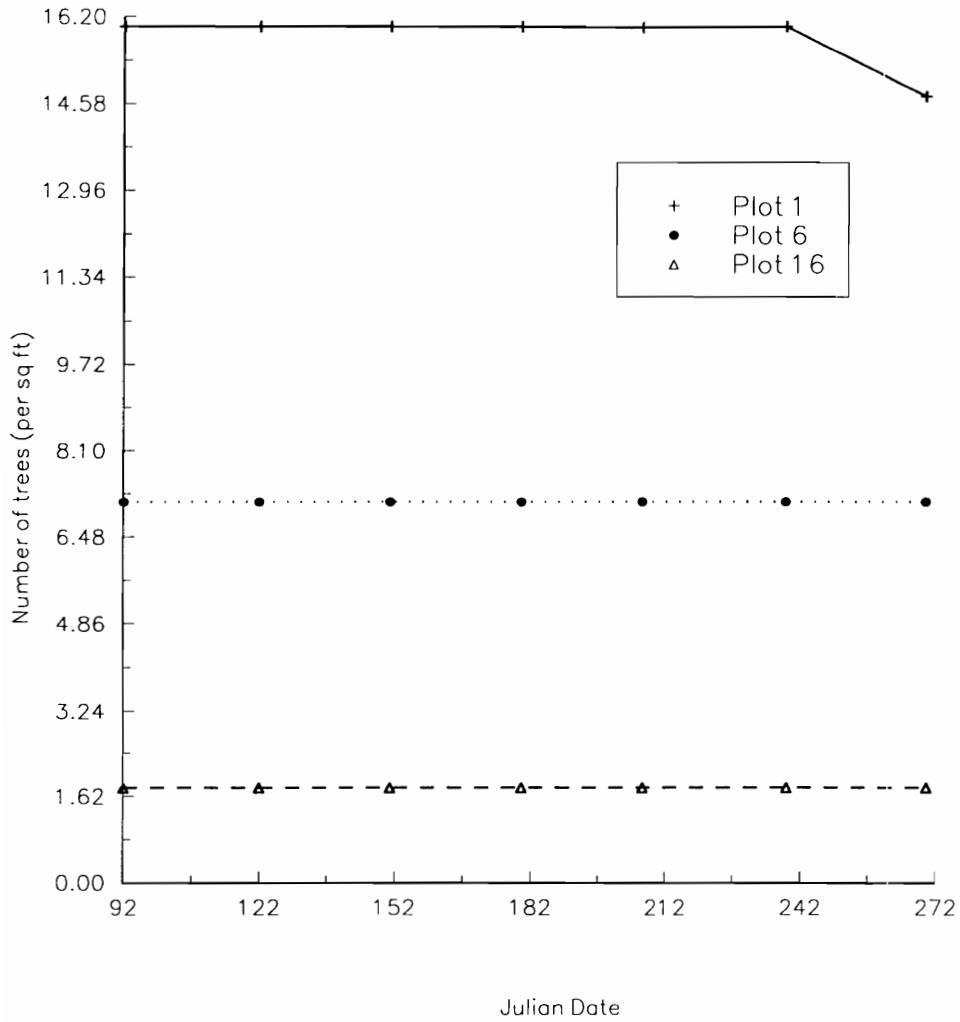


Figure 51: Number of trees vs. Time (1990) for the miniature spacing study planted with loblolly pine.

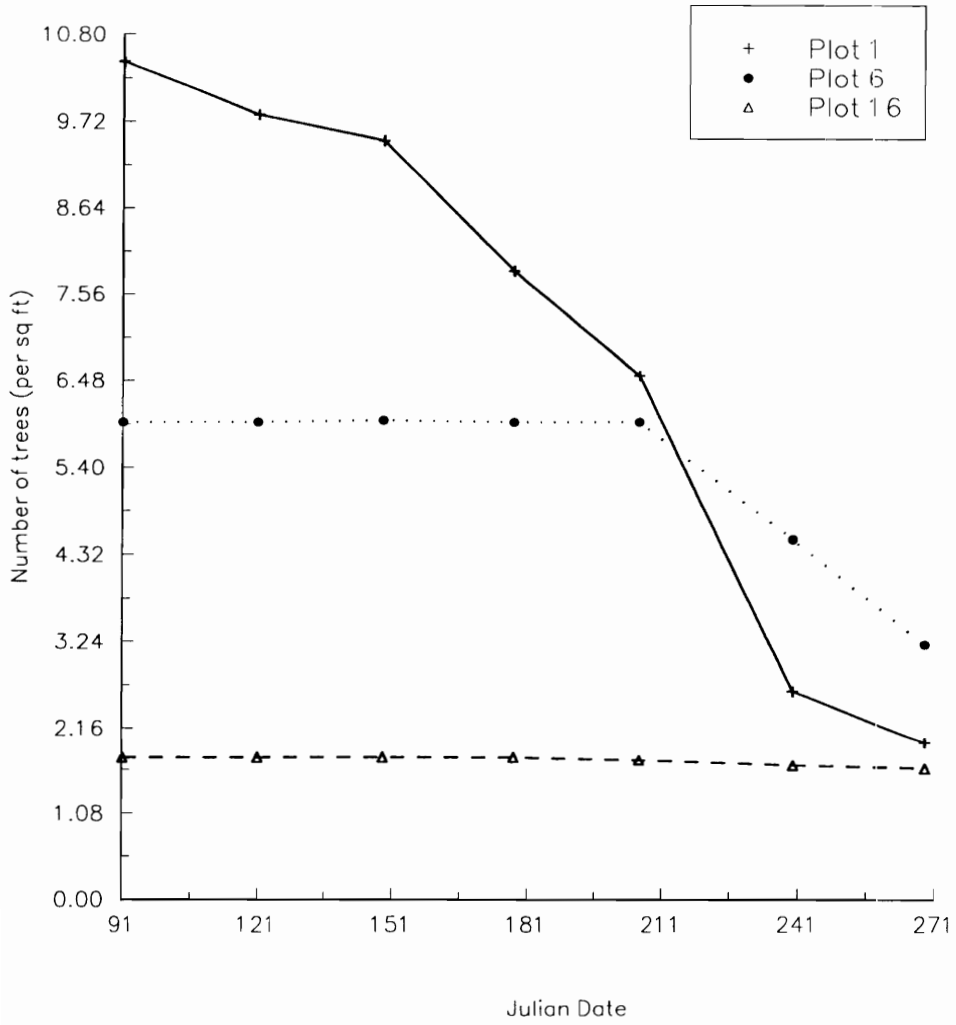


Figure 52: Number of trees vs. Time (1991) for the miniature spacing study planted with loblolly pine.

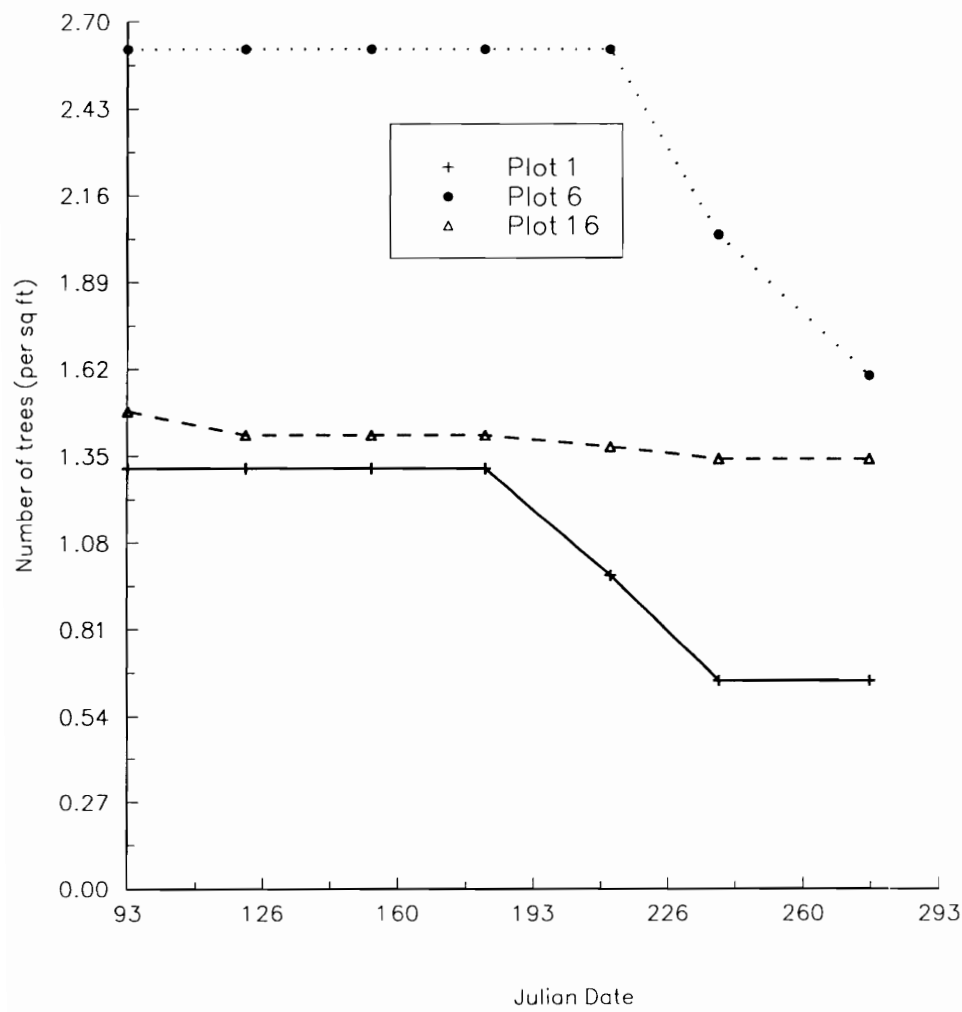


Figure 53: Number of trees vs. Time (1992) for the miniature spacing study planted with loblolly pine.

CHAPTER IV Scaling studies

Introduction

Trees growing at conventional spacings provide essential information to understand the influence of density on growth dynamics, but these trees must be grown over relatively long time periods. The use of seedlings grown at close spacings to accelerate the process of obtaining stand-dynamics information should be considered. Seedlings planted at very close spacing may offer insight into stand development that would require a much longer time to acquire using conventional methods.

The opportunity of using seedling stands brings up the question to what extent can we scale seedling stands to mature stands. Scaling has been used extensively in physics and engineering sciences. Its use in biology has been limited. Biological systems are very complex and unpredictable, making it extremely difficult to define a scale model similar to the original or prototype system. Potential uses of scale stands should be considered when studying pollution or other treatment effects on the growth of the trees.

McMahon (1973) tried to obtain general rules for changing proportions of idealized trees as a function of scale and later applied these rules to animals. Allometric relationships for animals were developed by Hokkanen (1986). For even-aged plant populations the relationship of the $-3/2$ power rule of self-thinning and allometric measures of plant geometry were studied by Weller (1987) and by Hutchings (1983).

In forestry spacing trials, Smith and Hann (1984) studied the $-3/2$ rule of self-thinning both in seedlings and older trees. This important aspect of scaling should be further examined. Its use may mean savings in time and cost in forestry experimentation; a good understanding of the biological process nonetheless is essential.

Objectives

The objective of this study was to develop methods to scale mensurational relationships from very closely spaced seedlings to more widely spaced trees.

Literature Review

Spacing studies at operational spacings are costly and results are not acquired quickly. Several years pass before intraspecific competition begins; this period may involve risk of losing whole spacing trials or part of them because of catastrophic events. Furthermore, the cost involved in the amount of land and management of spacing trials is significant. Large field plots have environmental variability which sometimes is hard to quantify by the researcher and isolate from the spacing effects. One possibility to diminish these effects is the establishment of small-scale spacing trials planted at very close spacings. Analysis of data from these closely-spaced seedlings should provide information regarding mature trees planted at operational densities.

For many years small-scale studies have been applied in engineering sciences and physics. Through scaling of all influential quantities, the original phenomenon or prototype is transformed into a scale model or similar system which preserves the relative values and proportions of the prototype even when it may require less space or time to develop (Schuring, 1977).

The principles of similarity have been around for long time. In 1878 Fourier analyzed the dimensions of the units that enter into the analytical theory of heat. In his work in order to measure the quantities that enter in the laws of movement of heat in solids he analyzed and compared five different kinds of units: the units of length, time, temperature, weight and the unit that measure quantities of heat. He applied the dimensional analysis to several equations he presented in this work. This dimensional analysis was then studied by Buckingham (1914) and contributed to the development of the theory of similarity.

Skoglund (1967) defined two systems to be similar if their corresponding variables are proportional at corresponding locations and times.

Schuring (1977) considered four important reasons for a scaled reproduction of a physical phenomenon or an engineering system. The first reason was related to the experimental work involved and the other three reasons were related to scale modelling of manageable proportions of a real system that allowed the investigator to work easily (for example a bridge may be too large for direct experimentation); less time involved to obtain results was a third reason mentioned by this author and lastly scaling required a good understanding of the phenomenon under investigation.

Although many examples of using small scale studies can be found in sciences like geophysics, engineering, aerospace, etc, the use of this type of study in biology is rarely found. Part of the reason may be due to the complexity and stochastic nature of biological systems. Studies in the pharmacological sciences involve the use of animal species to test some drugs that later will be used in humans; this represents the introduction of the difficulty of working with different species in the original and similar models and thus principles of data extrapolation from one system to another are needed (Yates and Kugler, 1986). This same problem of the role of the similarity principles in data extrapolation was analyzed by Rosen (1983). He pointed out the important role that mathematical models have in developing the concepts of similarity between two systems which he called the model and prototype systems. He further discussed dimensional analysis as a possible way to work out "similarities" between two systems. The important aspect of extrapolating data from animal models to human models as it happened so often is also addressed by this author.

In the particular case that we are concerned with, forestry spacing trials, the important mensurational variables such as height, diameter and number of stems per unit area of trees or groups of trees will be considered. The influence of spacing and density on tree and stand characteristics will be measured through these variables. Spacing trials of seedlings planted at very close spacing are looked at as potentially similar to stands of older trees. In this way, small scale studies in forestry may give insight into the evaluation of the effects of density and spacing on mature trees. The growth and yield relationships observed in mature trees planted at operational densities can be compared with these relationships examined in miniature scale spacing studies. Smith and Hann (1984) studied relationships in seedlings and compared the

results to older trees. These authors developed a non-linear model based on the $-3/2$ power rule of self-thinning for seedlings of red alder grown at three initial densities (8 x 8, 4 x 4 and 2 x 2 cm) and on two soil types (a forest soil collected from an alder site and a Willamette Valley river loam). There were seven destructive harvest dates over 525 growing days. This same relationship was modelled for 20-50 year old unthinned red pine growing at six initial densities (1.2 x 1.2, 1.4 x 1.4, 1.8 x 1.8, 2.1 x 2.1, 2.4 x 2.4 and 3.0 x 3.0 m) and site index 27 m at 50 years. The authors showed that the size-density model described may be valid over a wide range of initial densities and age.

Although the possibility of using seedling stands to obtain data in a short time to help in understanding growth and development in older stands is attractive, caution must be exercised. Any inferences necessitate an understanding of the physiological and environmental transformations going on while the trees are growing.

Data Description

Two ongoing experiments will be used to accomplish the objective of this study.

In the preceding chapters these two experiments were explained in detail. To accomplish objective 3, both spacing trials already described to analyze objectives 1 and 2 will be used.

The experimental design used for both spacing trials was a non-systematic design described by Lin and Morse (1975). The spacing factor (F) of 4 feet was selected for

the conventional spacing study. Four levels of this factor (1F, 1.5F, 2F and 3F) were used in the design. For the seedling spacing study the spacing factor (F) was 3 inches. There were also four levels of this factor: 1F, 1.5F, 2F and 3F. It is anticipated that the average height of the dominant trees in the miniature study will be approximately four feet after two growing seasons. In this way the ratio of the height to initial spacing in this study will be 16 (48 inches/3 inches) for the closest spacing. In the large-scale study, assuming that the average height of dominant trees will be 60 feet by age 25, the height to initial spacing ratio would be 15 (60 feet/4 feet) for the closest spacing. Thus, the miniature-spacing study will closely parallel the spacing trials established at operational spacings (Amateis and Burkhart, 1987)

In the seedling study measurements were taken at the beginning of each month from July until October for the first year (1989) and at the beginning of each month from April to October (7 measurements) for the subsequent three years (1990-1992). In this way the measurements were taken throughout the growing seasons on each measurement tree. The measurements included groundline diameter, total height, height to the base of the live crown, crown width within the row, crown width between rows and maximum crown width. The measurements taken in the miniature-scale study over four growing seasons are the same as those collected annually from the trials planted at conventional spacings. In 1988 diameter at breast height was measured in addition to groundline diameter in the conventionally-spaced experiment; since 1988, diameter at breast height has been the only stem diameter measured.

Analysis and Procedures

The use of scaling in biology is not as common as its use in the physical and engineering areas. The possibility of scaling seedling stands planted at close spacing to mature stands planted at conventional spacing gives the opportunity of gaining information about growth development in a relatively short period of time. Three different approaches were taken:

Time as x -axis

The possibility of using time as x -axis and scaling seedling stands to mature stands was explored. The values of the ratio

$$R_y(t) \doteq \frac{\frac{dy(t)}{dt}}{y(t)} \quad (4.1)$$

at times $t = t_1, \dots, t_{n-1}$ were approximated by

$$\tilde{R}_y(t_j) \doteq \frac{y(t_{j+1}) - y(t_j)}{y(t_j)(t_{j+1} - t_j)} \quad (4.2)$$

where $y(t_{j+1})$ and $y(t_j)$ are the values of a given characteristic -such as diameter, height or volume- at times t_{j+1} and t_j , respectively. In the sequel, we shall call this procedure Scheme I. The quotient defined by (4.1) can be considered as relative diameter, height or volume growth. Volume for the trees in the conventional spacing study was computed after converting diameter at breast height to groundline diameter and using the formula of a cone. For the conventional spacing study the measurements were taken annually so the difference $t_{j+1} - t_j$ is one year while for the seedling spacing study the measurements were taken monthly for four years, so this difference is one

month. The main reason for considering the ratio (4.1) is that it offers a simple way to extract the information from the data which is independent of any possible linear scaling factor. In fact, note that if $y_1(t), y_2(t)$ are two differentiable functions and $y_1(t) = cy_2(t)$ for a certain constant $c \neq 0$ then $R_{y_1}(t) = R_{cy_2}(t) = \frac{\frac{d}{dt}cy_2(t)}{cy_2(t)} = \frac{\frac{d}{dt}y_2(t)}{y_2(t)} = R_{y_2}(t)$

Plots of the quotient $\tilde{R}_y(t_j)$ defined by (4.2) were obtained for both studies.

By noting that $R_y(t) = \frac{\frac{d}{dt}y(t)}{y(t)} = \frac{d}{dt} \ln y(t)$ we see that another scheme results from directly approximating the time derivative $\frac{d}{dt} \ln y(t)$ by the corresponding incremental ratio. In doing so we obtain

$$R_y(t_j) \approx \hat{R}_y(t_j) \doteq \frac{\ln y(t_{j+1}) - \ln y(t_j)}{t_{j+1} - t_j} = \frac{\ln \left(\frac{y(t_{j+1})}{y(t_j)} \right)}{t_{j+1} - t_j}$$

or simply

$$\hat{R}_{y(t_j)} \doteq \ln \left(\frac{y(t_{j+1})}{y(t_j)} \right) \quad (4.3)$$

since $t_{j+1} - t_j = 1$ for all j 's. We call this approximating procedure Scheme II. Trends for both studies were also obtained using this scheme. Results from Scheme I and Scheme II were compared.

Relative spacing as x -axis

Instead of using time as x -axis other relative quantities that can be used were explored. One possibility that may be particularly useful is the use as x -axis relative spacing, defined as the average distance between trees divided by the average height of

the dominant trees. Trends of height, diameter growth and groundline basal area with respect to relative spacing were observed and compared in both studies (conventional and seedling spacing study).

Use of ratios

Another possible approach to avoid the simple x -axis scaling procedures was the use of dimensional analysis (Stansfield, 1976, Szucs, 1980) among different variables in the two systems, seedlings and older trees. This important tool has been widely used, particularly in the field of physics. There are, however, some examples in forestry such as the development of growth and yield models using dimensional analysis (Amateis and McDill, 1989). In the Amateis and McDill paper, the authors show that dimensional analysis can be used to develop dimensionally compatible models from a particular set of variables. They also discussed the possibility of using this tool to advantage when dealing with complex modelling problems that involve many independent variables.

In this study, the use of dimensionless ratios between variables was explored. Ratios such as height to spacing, crown dimensions, etc, were evaluated. These ratios were then plotted over time. Trends were compared in both experiments. The following ratios were computed:

- Crown ratio (C_r) defined as length of live crown divided by total height, i.e.:

$$C_r = \frac{H_t - H_c}{H_t} \quad (4.4)$$

where:

C_r : crown ratio

H_t : total height

H_c : height to the live crown

- Crown length divided by the crown width between rows (C_{lb}), i.e.:

$$C_{lb} = \frac{C_l}{C_{wb}} = \frac{H_t - H_c}{C_{wb}} \quad (4.5)$$

where:

C_l : crown length

C_{wb} : crown width between rows

H_t : total height

H_c : height to the live crown

- Crown length divided by crown width within rows (C_{lw}), i.e.:

$$C_{lw} = \frac{C_l}{C_{ww}} = \frac{H_t - H_c}{C_{ww}} \quad (4.6)$$

where:

C_{ww} : crown width within rows

and the other variables are as defined before.

- Height to spacing ratio, i.e.:

$$H_s = \frac{H_t}{S_p} \quad (4.7)$$

where:

H_t : total height

S_p : initial spacing (4, 6 and 12 ft for the conventional study and 3, 4.5 and 9 inches for the seedling study).

Data Analysis

Time as x -axis

In this section we present results obtained by using Schemes I and II as previously described.

Figures 54 and 55 show the graph of relative groundline diameter growth over time for the conventional spacing study over 4 years. Figure 56 corresponds to the miniature study over a period of 2 years (11 measurements). The square plots 1, 6 and 16 were considered in the analysis. The square plot 11 in the miniature study had a drainage problem and several trees were replanted after a few months. For this reason, this plot was not considered for further analysis. Two rectangular plots (plots 4 and 7) were also included to see if the rectangularity effect was the same in both studies (Figures 55 and 57). A similar pattern in relative diameter growth can be observed in both studies although looking at the y -axis the conventional spacing study seemed to show relative growth two times that of the miniature study at the initial time of measurement. Figures 58 through 61 show the graphs of relative height growth over time for both studies (8 years in the conventional spacing study and 2 years in the miniature spacing study). Here again, similar patterns for the analyzed square and rectangular plots were observed. The relative height growth at the initial measurement for the conventional spacing study was a little more than twice as much

as the initial measurement for the miniature study. Figures 62 through 65 show the graphs of relative volume growth over time. These volumes were computed as volume of a cone to be consistent in the conventional and miniature spacing studies. In the conventional spacing study the cone formula was applied after diameter at breast height was converted to groundline diameter. Looking at these graphs not only the patterns were similar in both cases but also the values of relative volume growth were quite correspondent to each other in the square and rectangular plots analyzed.

Figures 66 through 77 show the plots obtained when Scheme II was used. The results observed were very similar to the ones observed when Scheme I was used. The similarities of the results obtained with Scheme I and Scheme II are not surprising since both were derived by approximating time derivatives, in one case of $y(t)$ and in the other case of $\ln y(t)$. This can also be seen in the resulting approximation schemes. In fact, letting $\xi_j = \frac{y(t_{j+1})}{y(t_j)}$ we see that in Scheme I we have $\tilde{R}_y(t_j) = \xi_j - 1$ while for Scheme II $\hat{R}_y(t_j) = \ln(\xi_j)$. Finally, note that $\xi - 1$ corresponds to the first two terms in a Taylor series expansion of $\ln \xi$ around $\xi = 1$

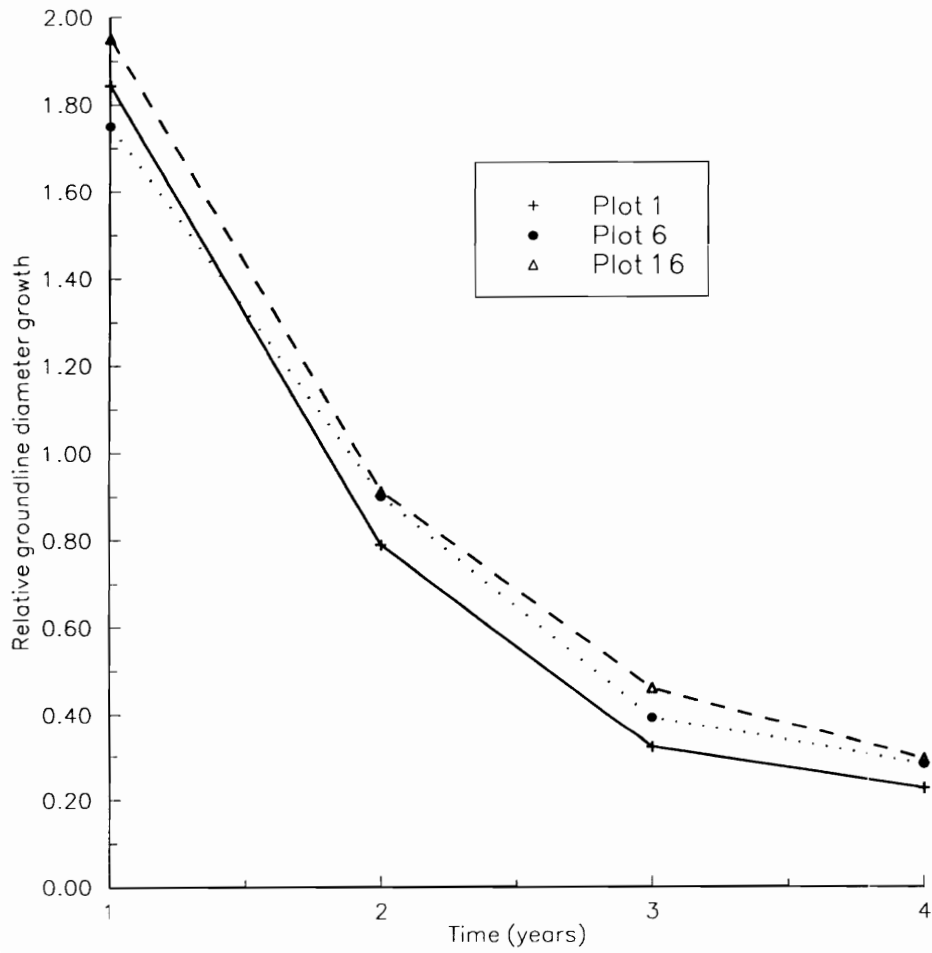


Figure 54: Relative groundline diameter growth for square spacings in the conventional spacing study (Scheme I).

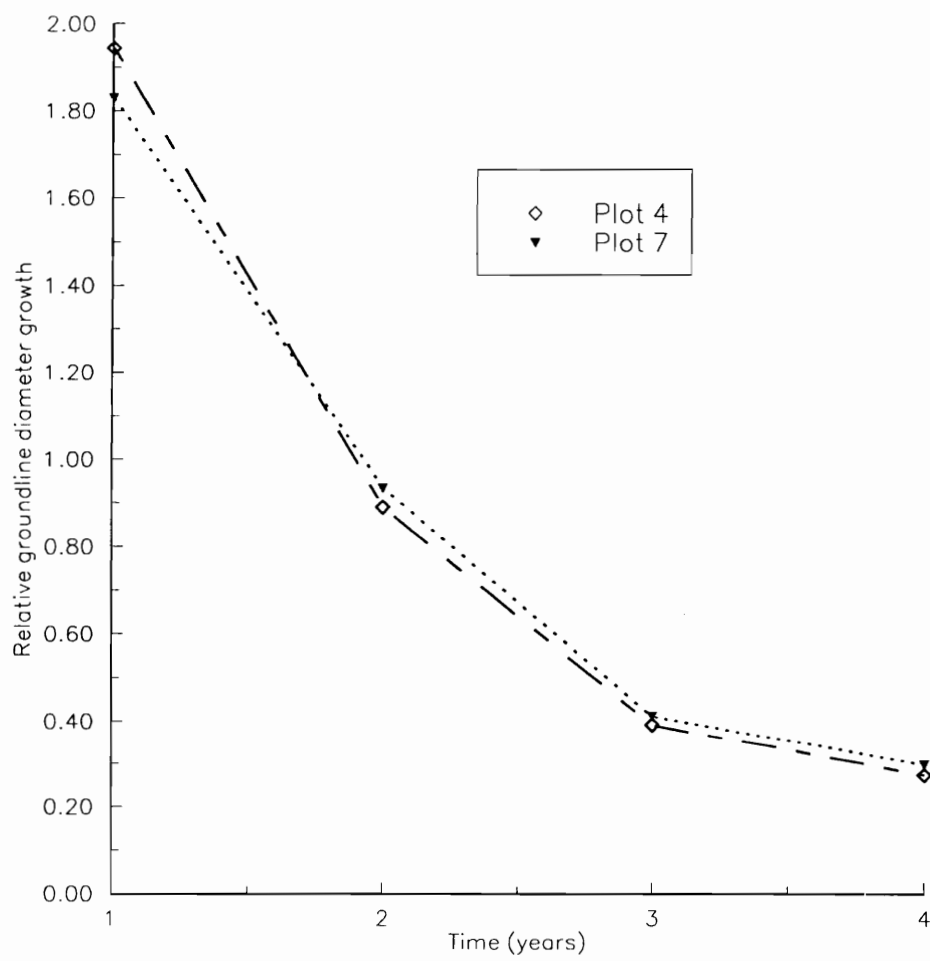


Figure 55: Relative groundline diameter growth for rectangular spacings in the conventional spacing study (Scheme I).

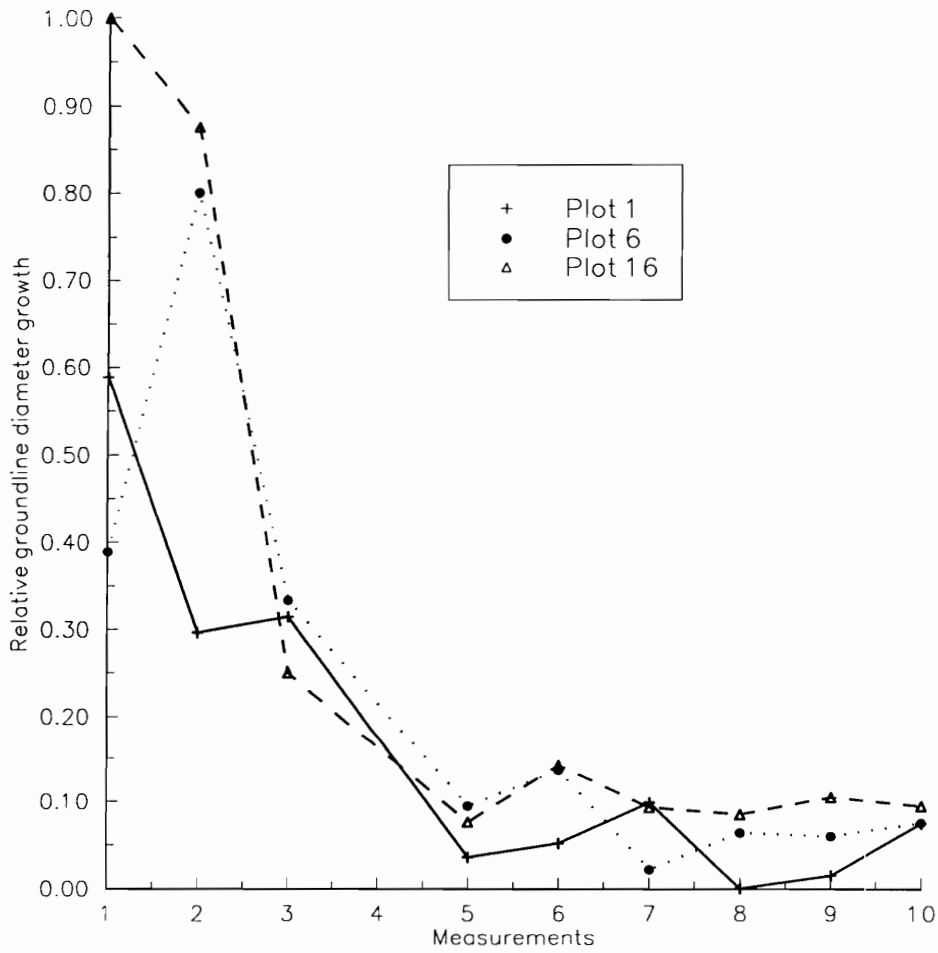


Figure 56: Relative groundline diameter growth for square spacings in the miniature spacing study (Scheme I).

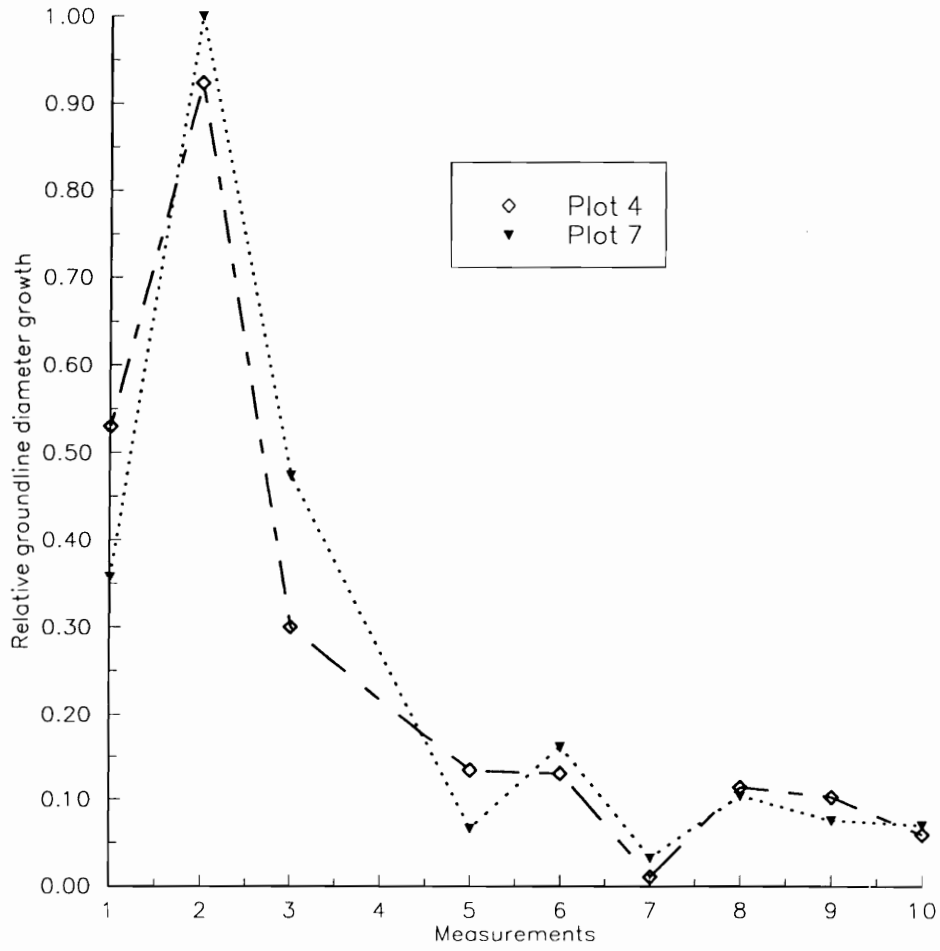


Figure 57: Relative groundline diameter growth for rectangular spacings in the miniature spacing study (Scheme I).

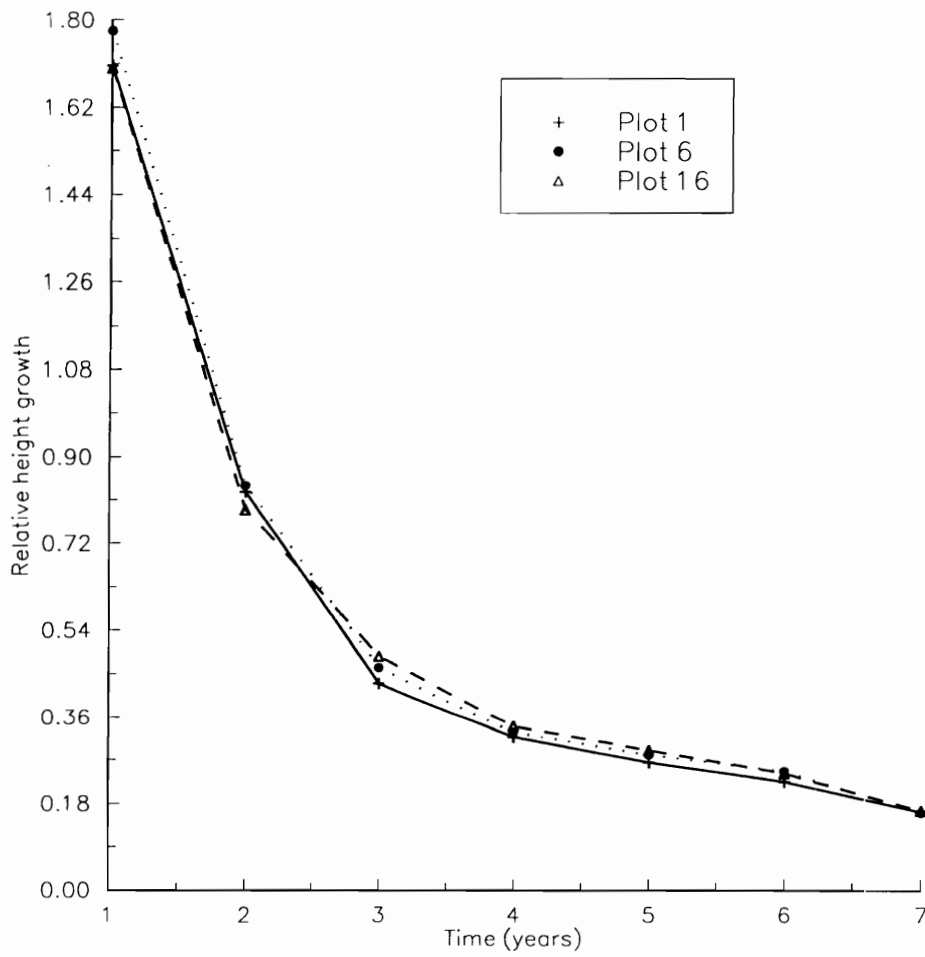


Figure 58: Relative height growth for square spacings in the conventional spacing study (Scheme I).

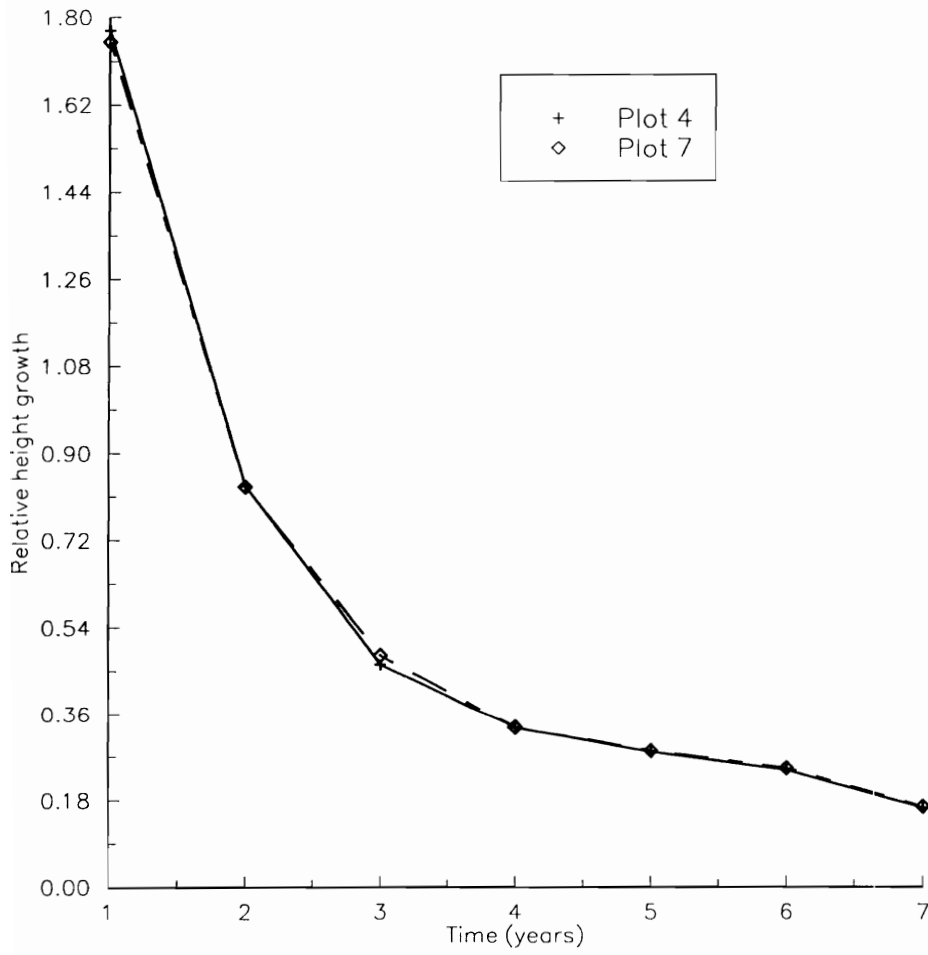


Figure 59: Relative height growth for rectangular spacings in the conventional spacing study (Scheme I).

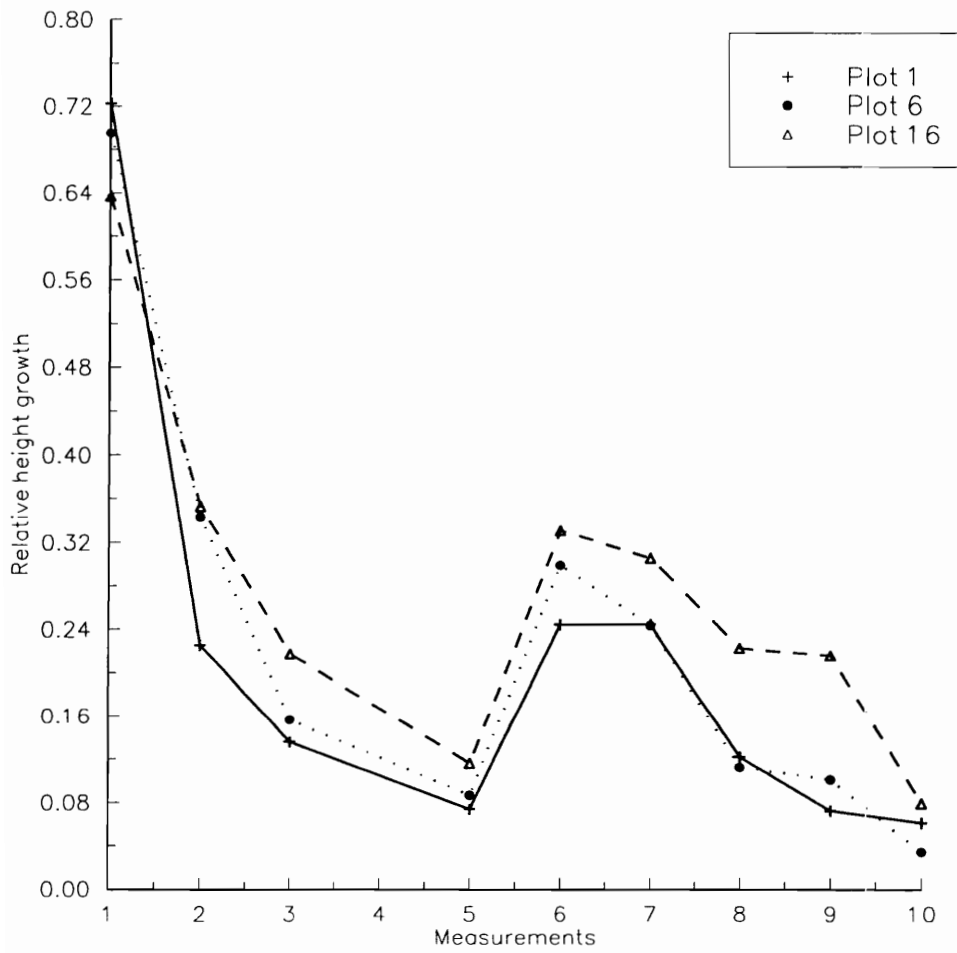


Figure 60: Relative height growth for square spacings in the miniature spacing study (Scheme I).

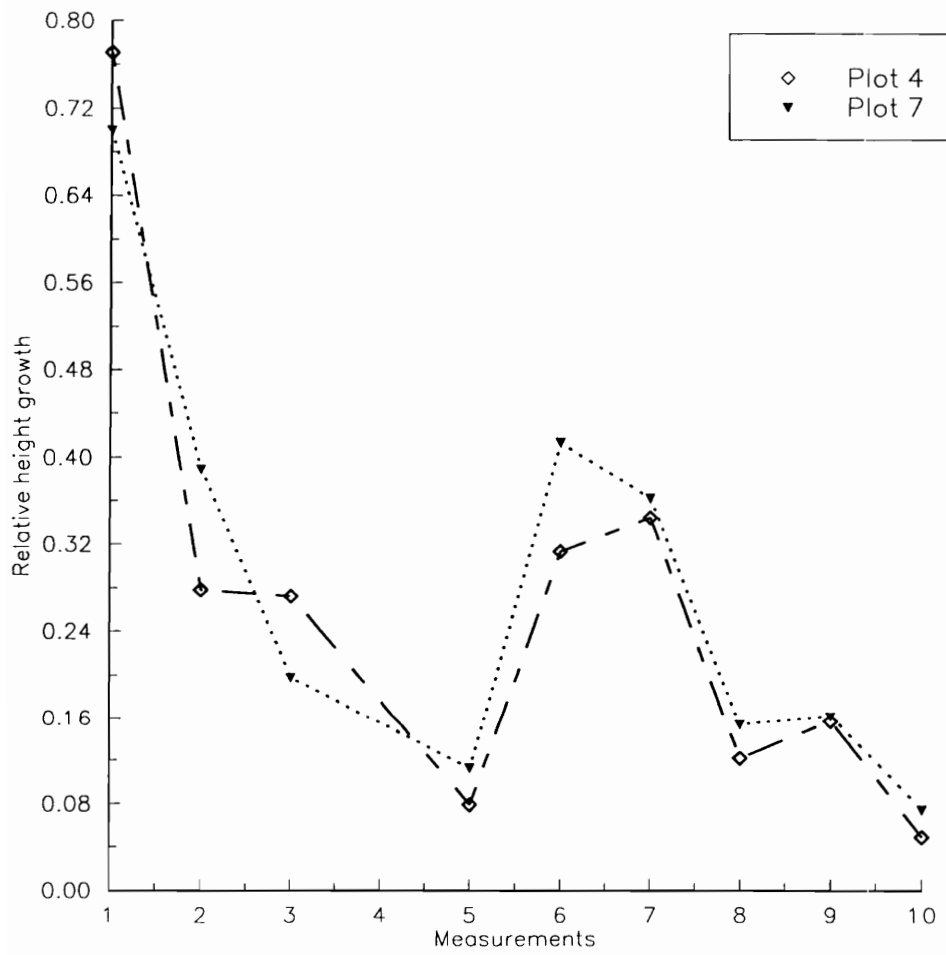


Figure 61: Relative height growth for rectangular spacings in the miniature spacing study (Scheme I).

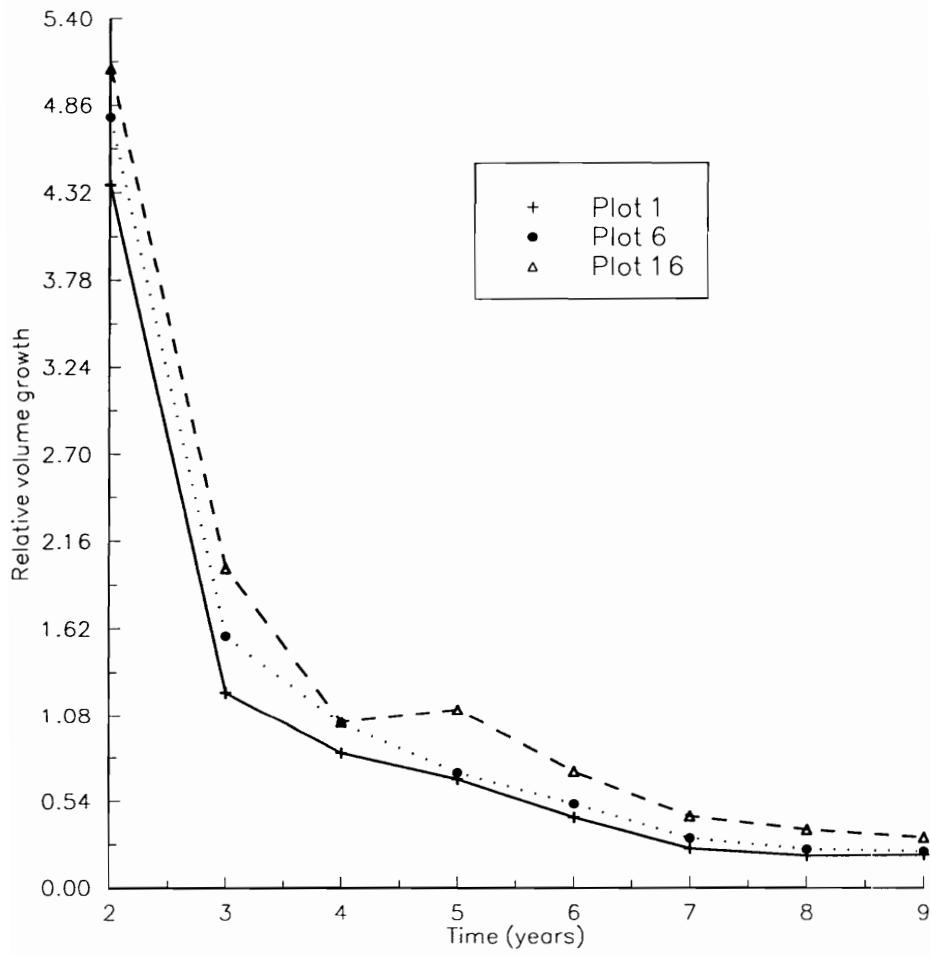


Figure 62: Relative volume growth for square spacings in the conventional spacing study (Scheme I).

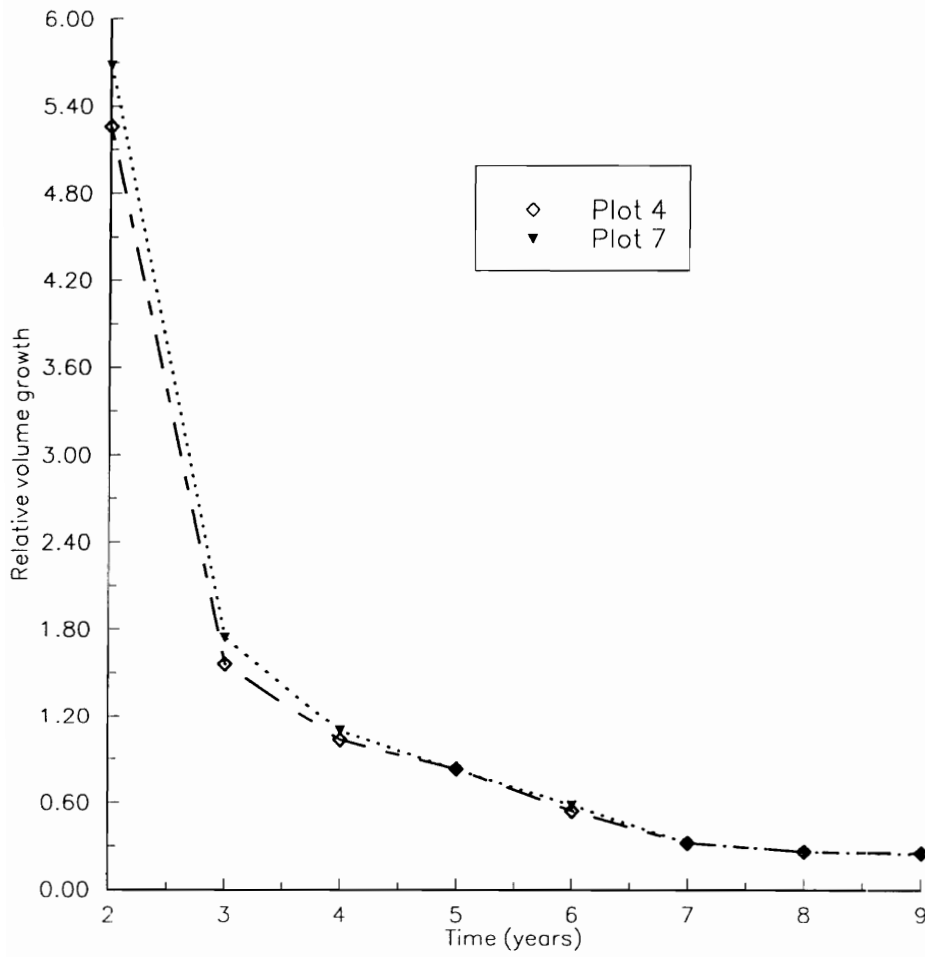


Figure 63: Relative volume growth for rectangular spacings in the conventional spacing study (Scheme I).

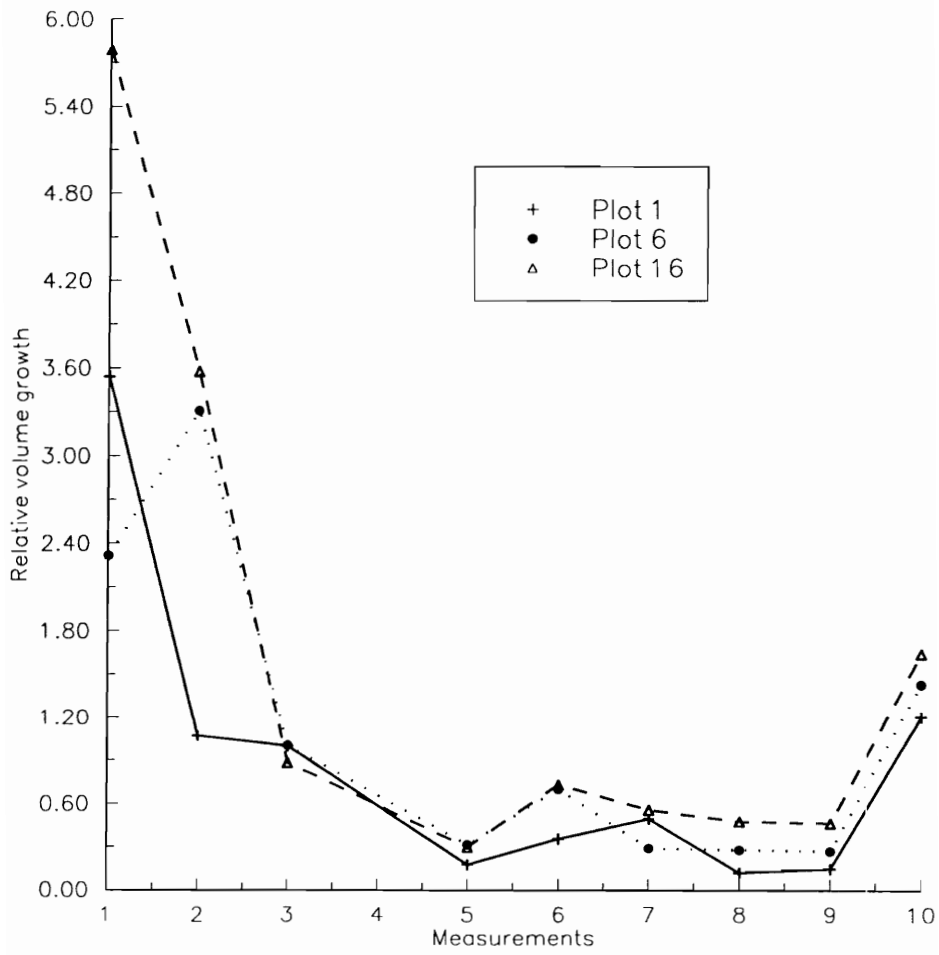


Figure 64: Relative volume growth for square spacings in the miniature spacing study (Scheme I).

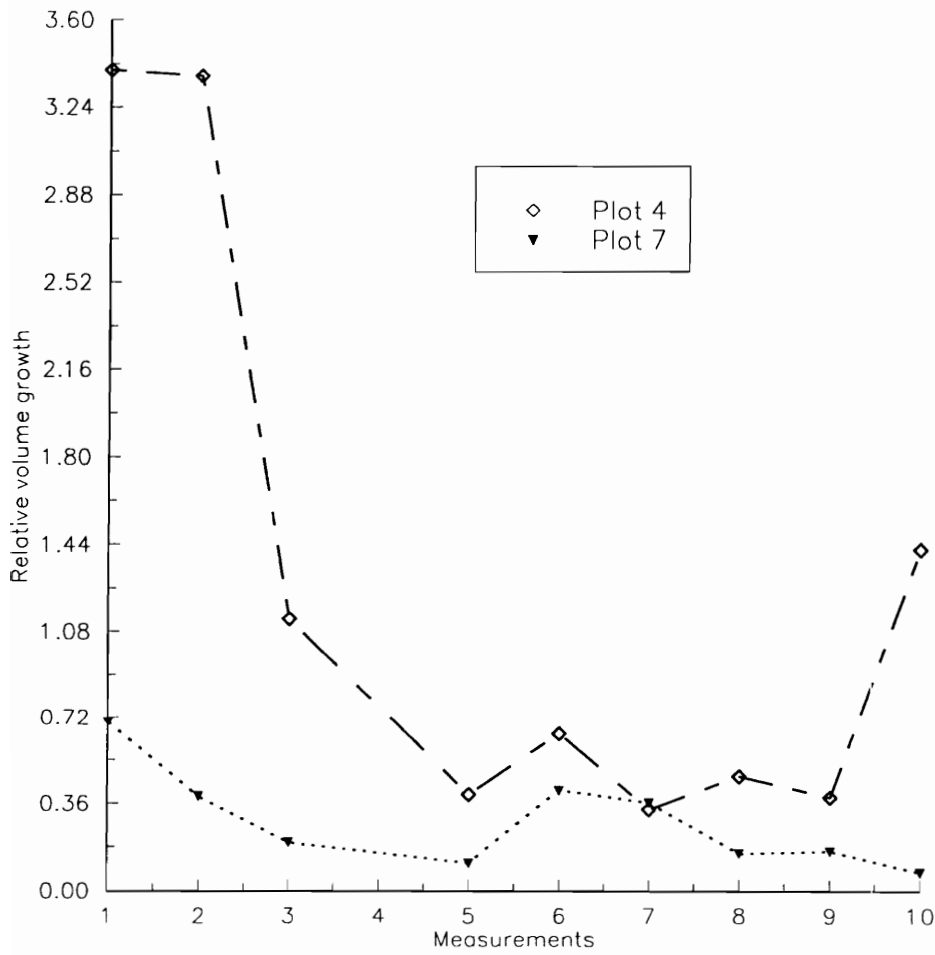


Figure 65: Relative volume growth for rectangular spacings in the miniature spacing study (Scheme I).

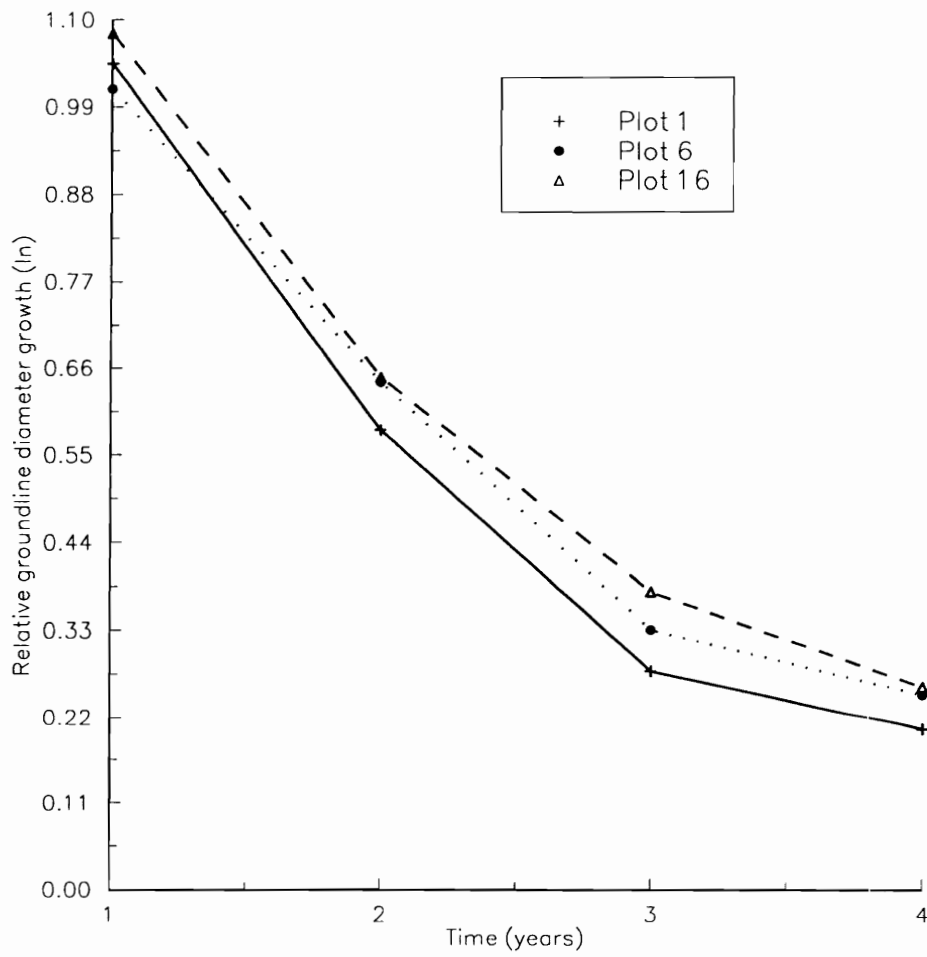


Figure 66: Relative groundline diameter growth (ln) for square spacings in the conventional spacing study (Scheme II).

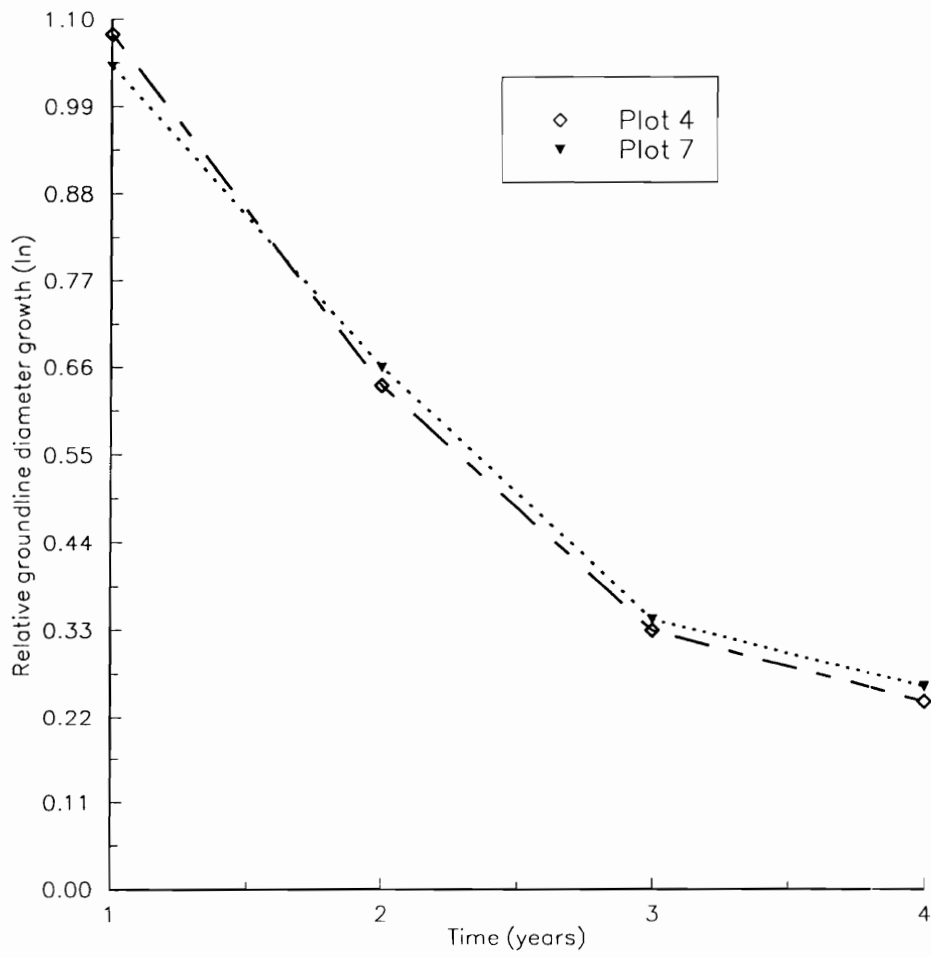


Figure 67: Relative groundline diameter growth (ln) for rectangular spacings in the conventional spacing study (Scheme II).

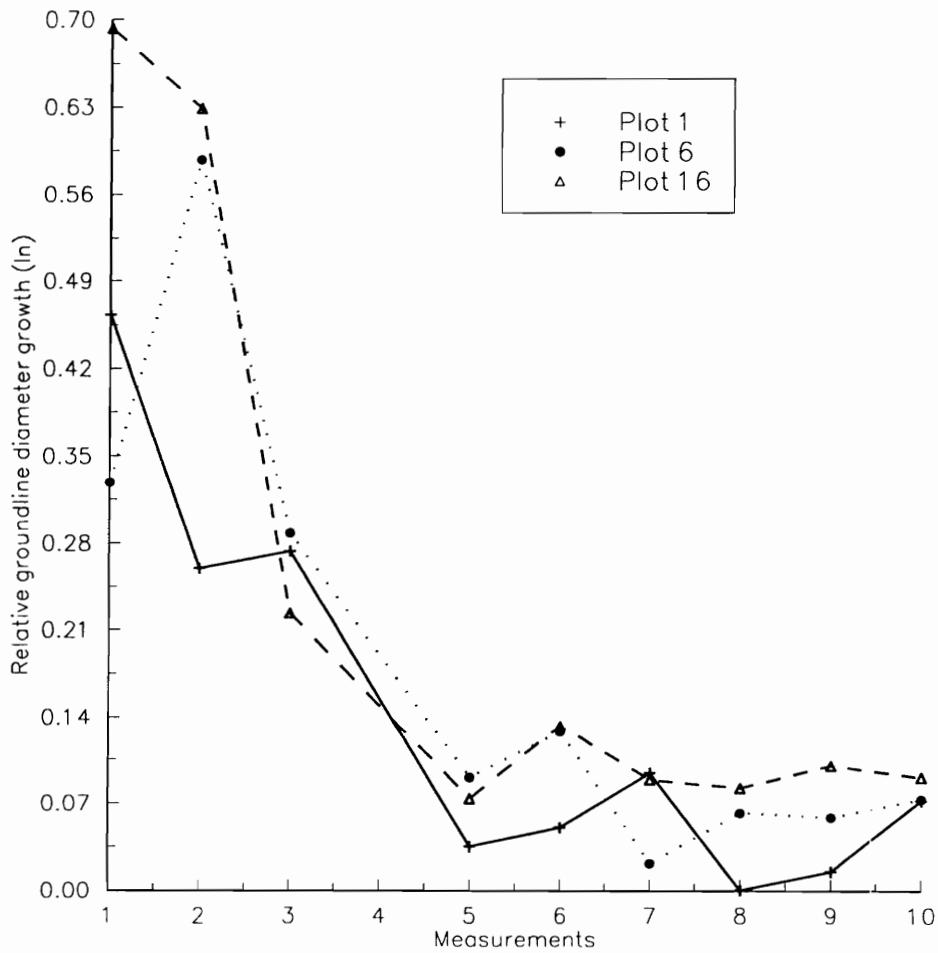


Figure 68: Relative groundline diameter growth (ln) for square spacings in the miniature spacing study (Scheme II).

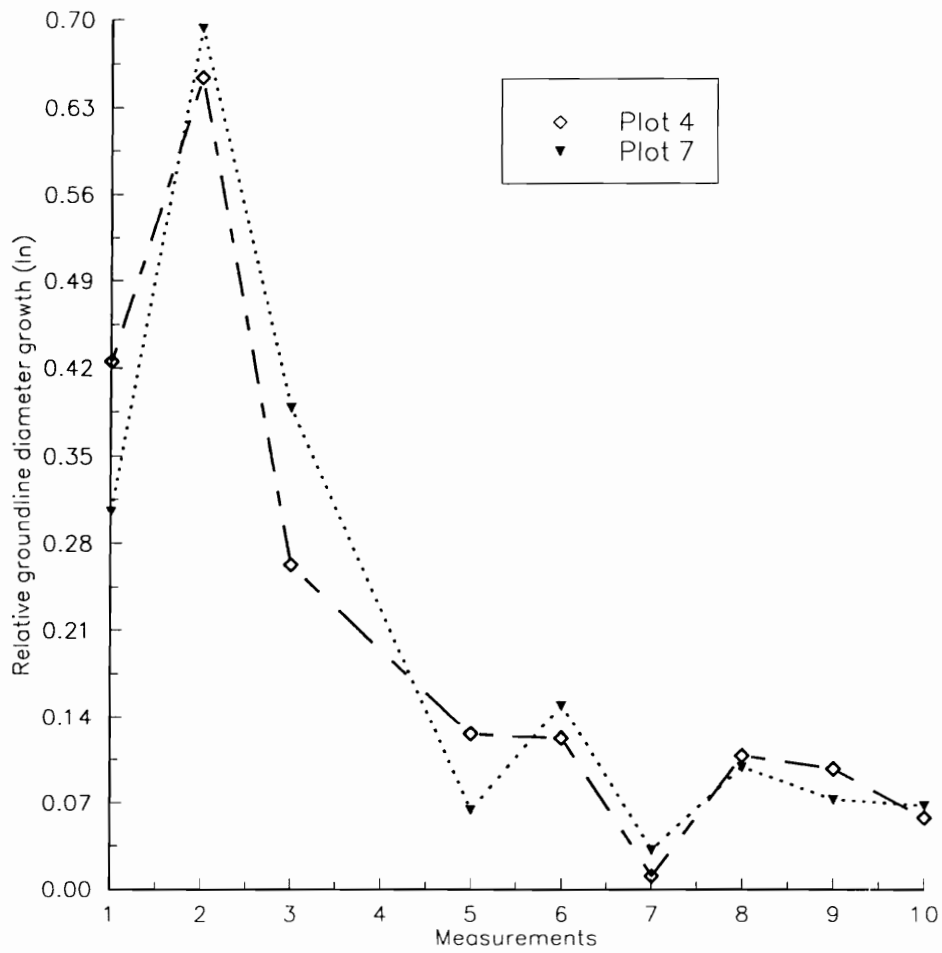


Figure 69: Relative groundline diameter growth (ln) for rectangular spacings in the miniature spacing study (Scheme II).

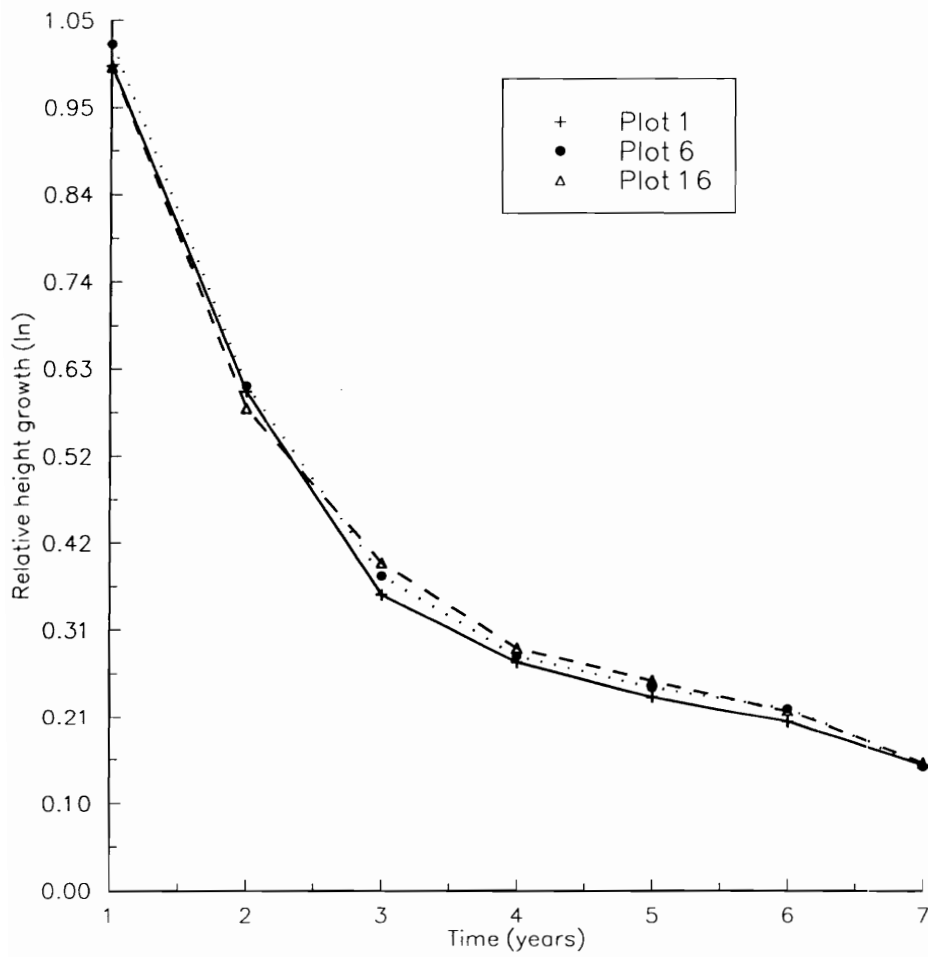


Figure 70: Relative height growth (ln) for square spacings in the conventional spacing study (Scheme II).

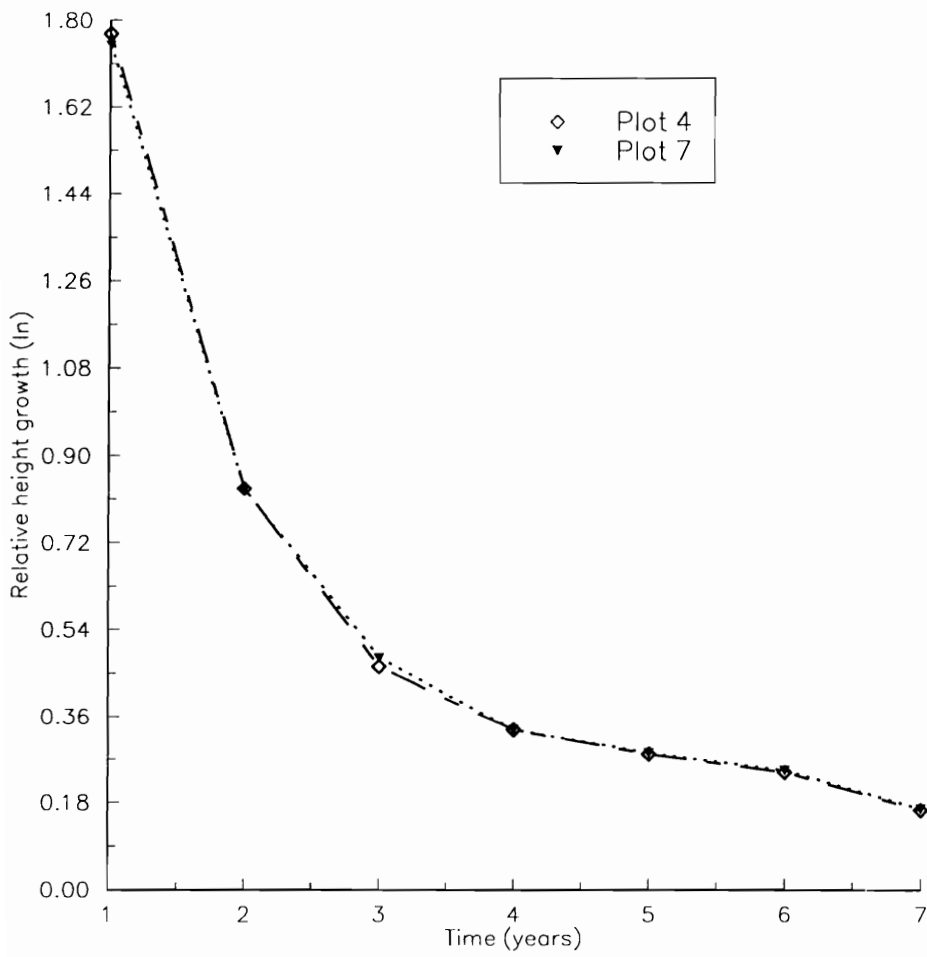


Figure 71: Relative height growth (ln) for rectangular spacings in the conventional spacing study (Scheme II).

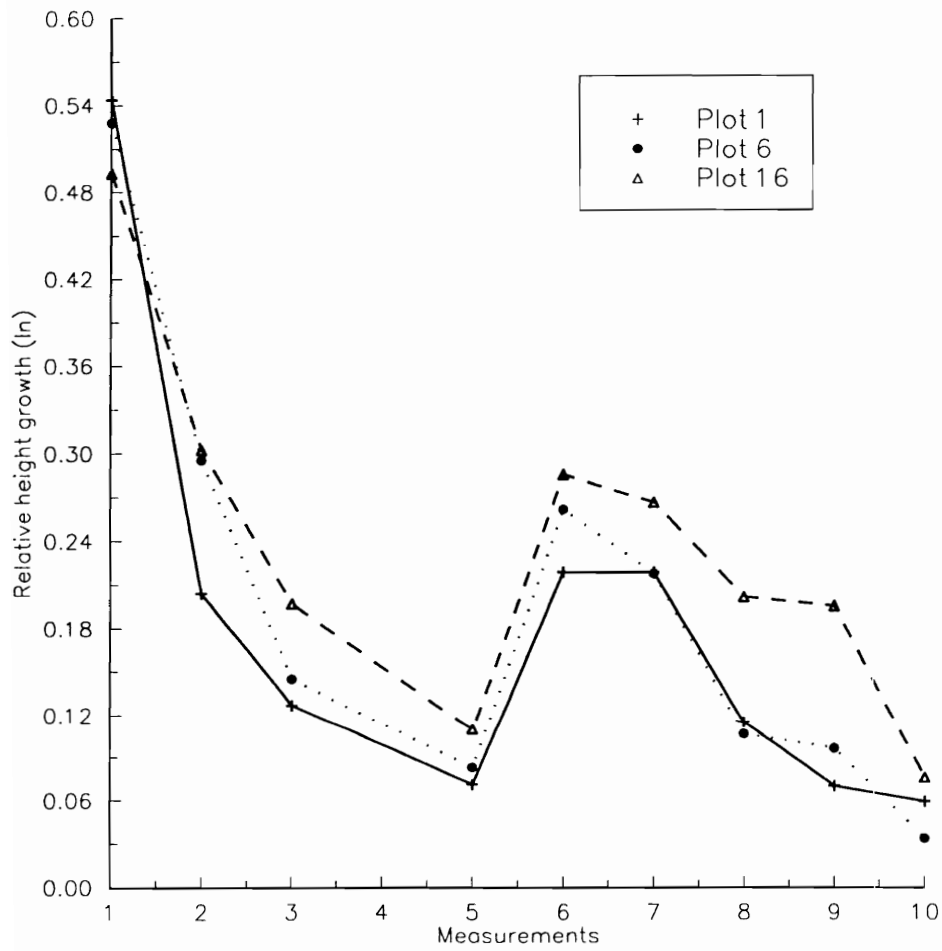


Figure 72: Relative height growth (ln) for square spacings in the miniature spacing study (Scheme II).

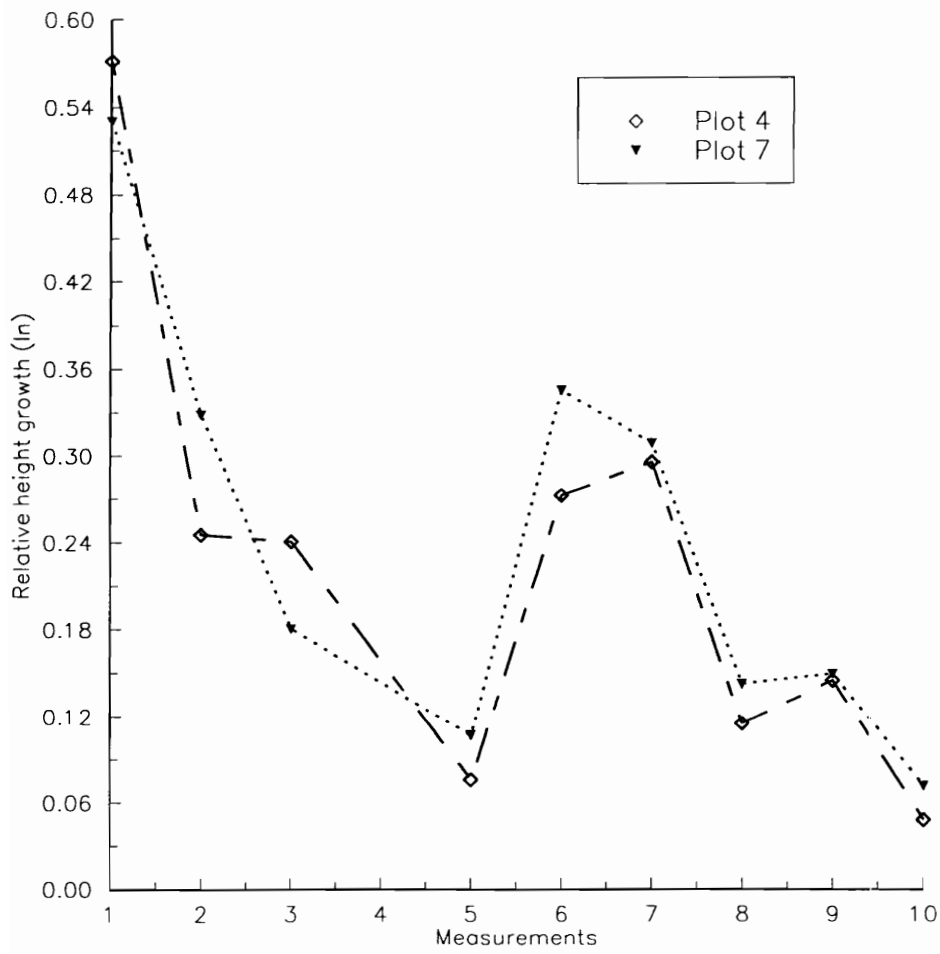


Figure 73: Relative height growth (ln) for rectangular spacings in the miniature spacing study (Scheme II).

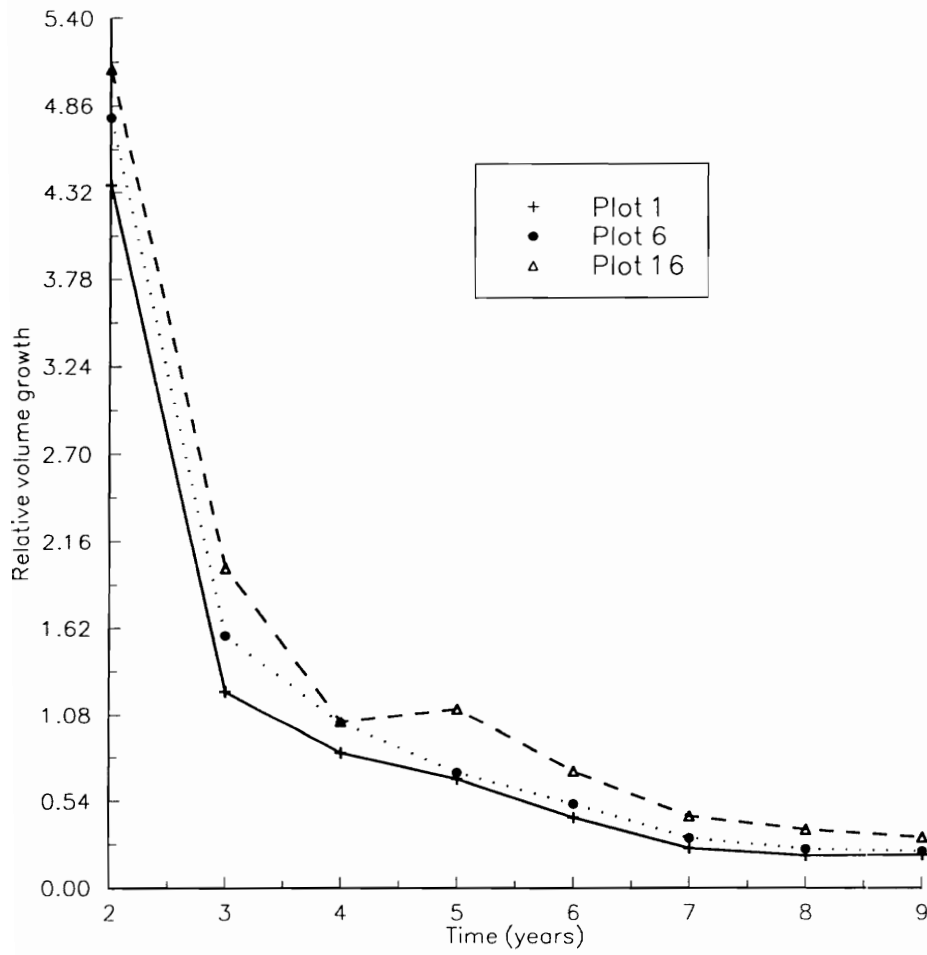


Figure 74: Relative volume growth (ln) for square spacings in the conventional spacing study (Scheme II).

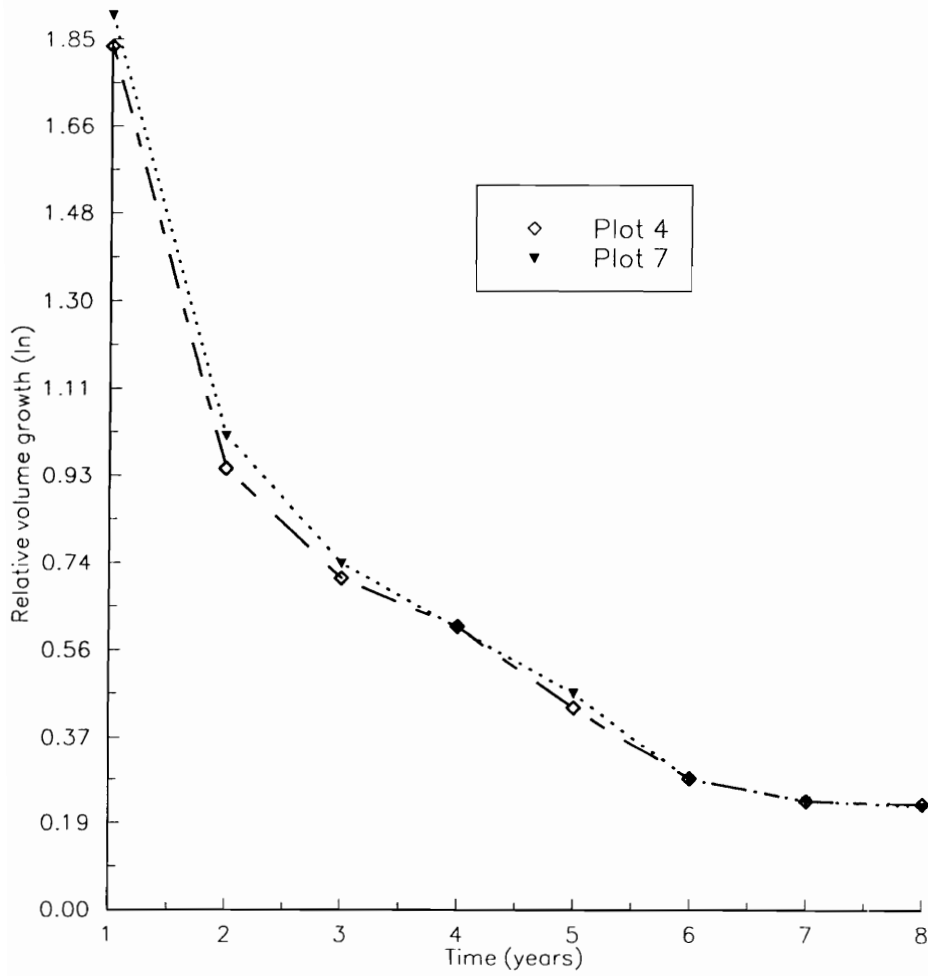


Figure 75: Relative volume growth (ln) for rectangular spacings in the conventional spacing study (Scheme II).

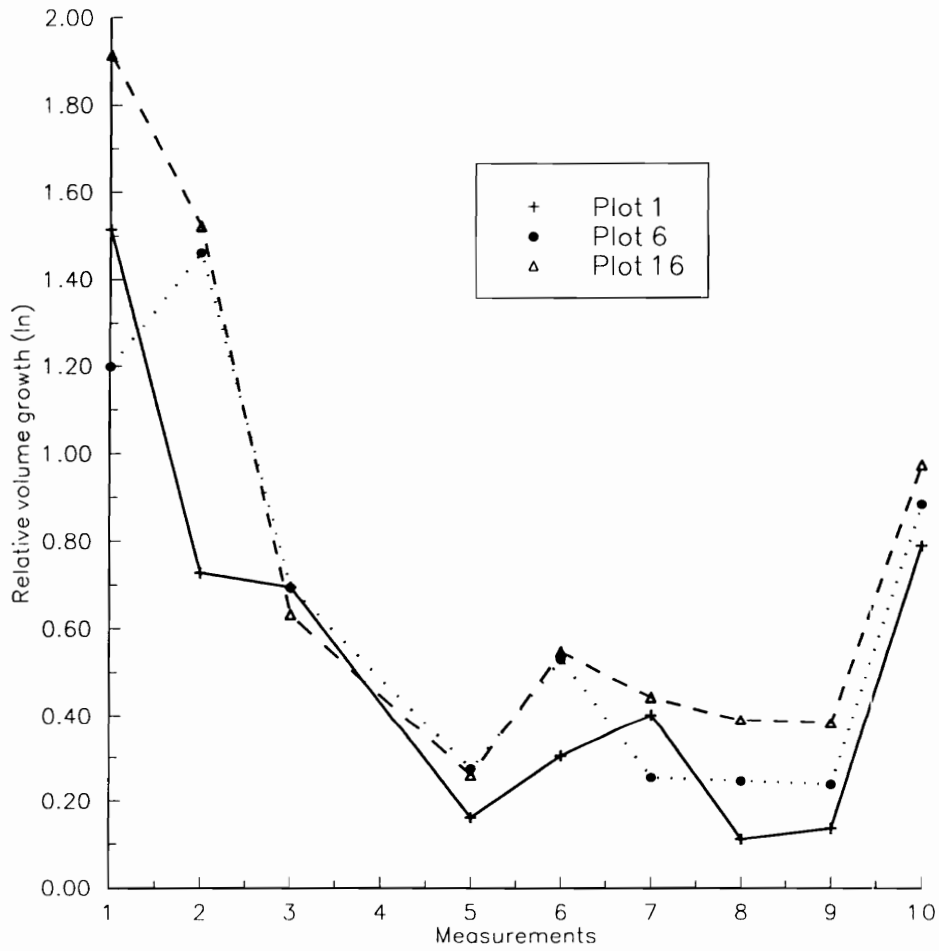


Figure 76: Relative volume growth (ln) for square spacings in the miniature spacing study (Scheme II).

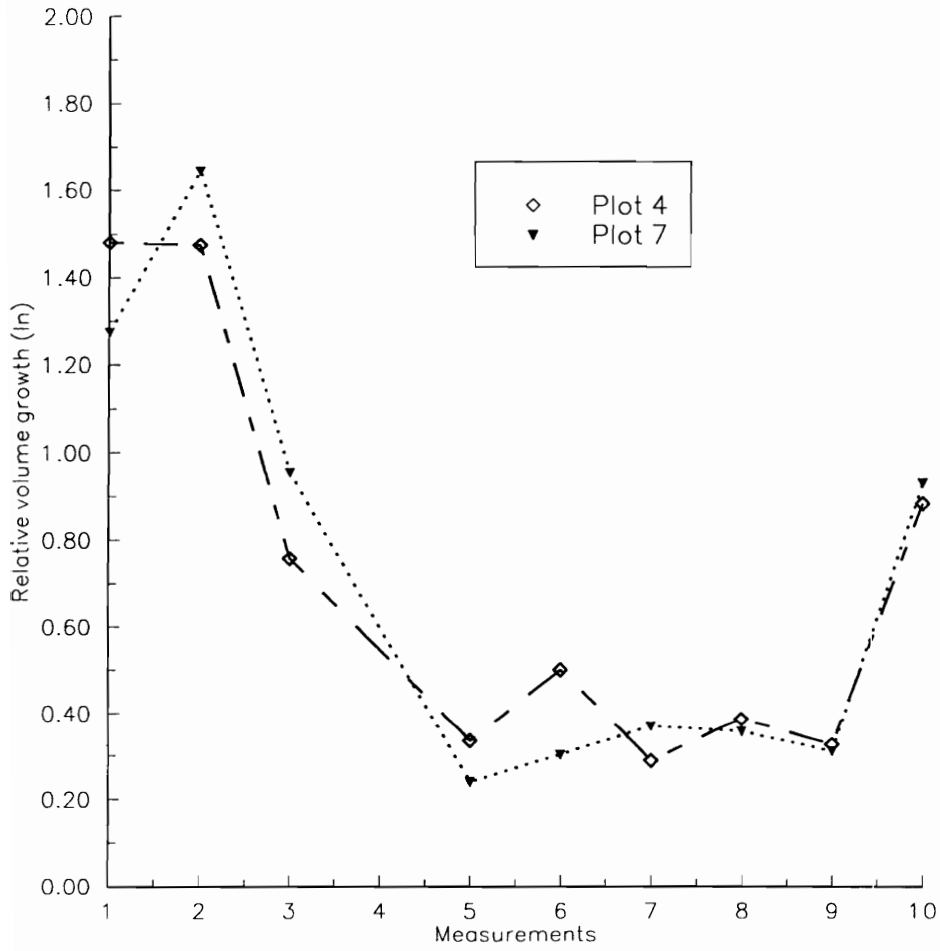


Figure 77: Relative volume growth (ln) for rectangular spacings in the miniature spacing study (Scheme II).

Relative spacing as x -axis

Figures 78 through 83 show the trends of diameter, height and groundline basal area when they are plotted against relative spacing as x -axis. These trends were compared in both studies, and only the square plots 1, 6 and 16 were included. Looking at the patterns in both studies it can be seen that they were quite similar. Observing the graphs of diameter and height, plot 1 of the miniature study did show a similar pattern to plot 1 in the conventional study except for the last five measurements of the fourth growing season. This was probably due to heavy mortality occurring between two measurement times in the same growing season. In both studies the diameter in each of the plots tended to approach an asymptote which in the conventional spacing study was 0.4in while in the miniature study was 0.1in. Total height also showed similar behavior as the one pointed out for diameter. With respect to groundline basal area, plot 1 of the miniature study did not show a similar pattern to plot 1 in the conventional study. The other two plots compared, plots 6 and plot 16, showed similar patterns in both studies.

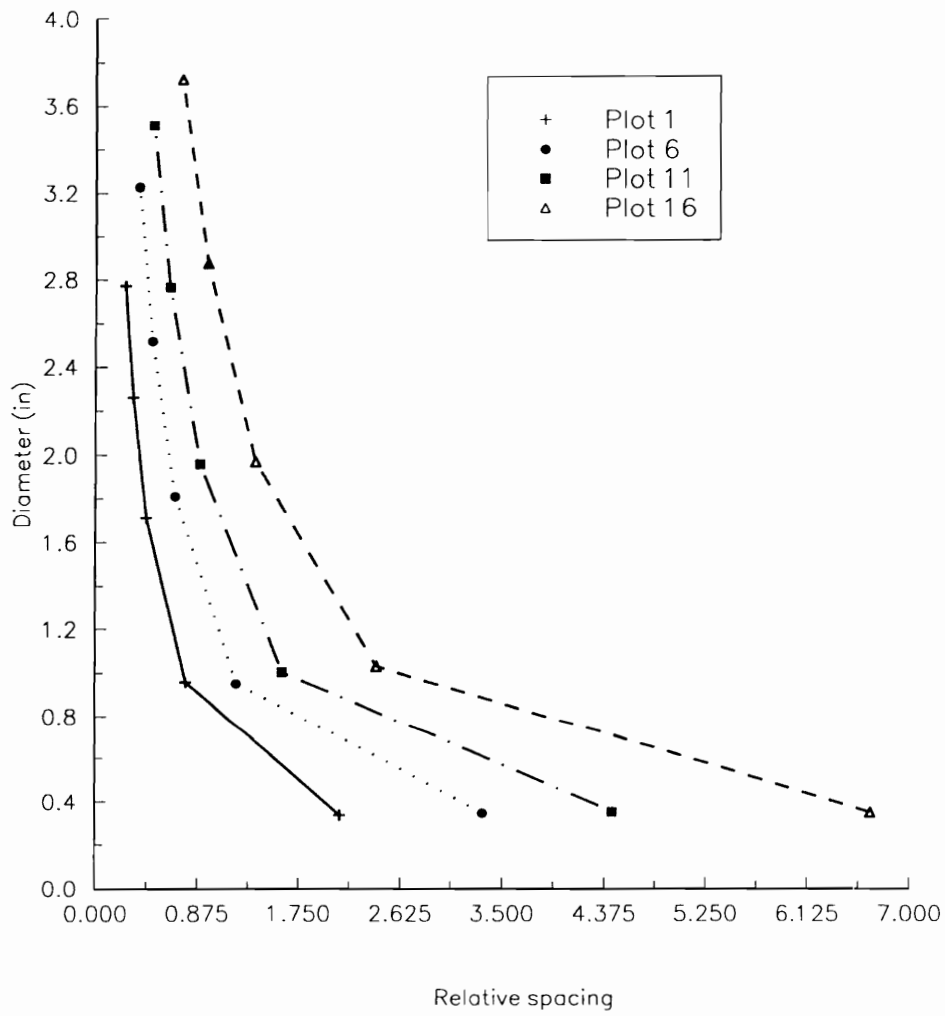


Figure 78: Groundline diameter vs. relative spacing for square spacings in the conventional spacing study.

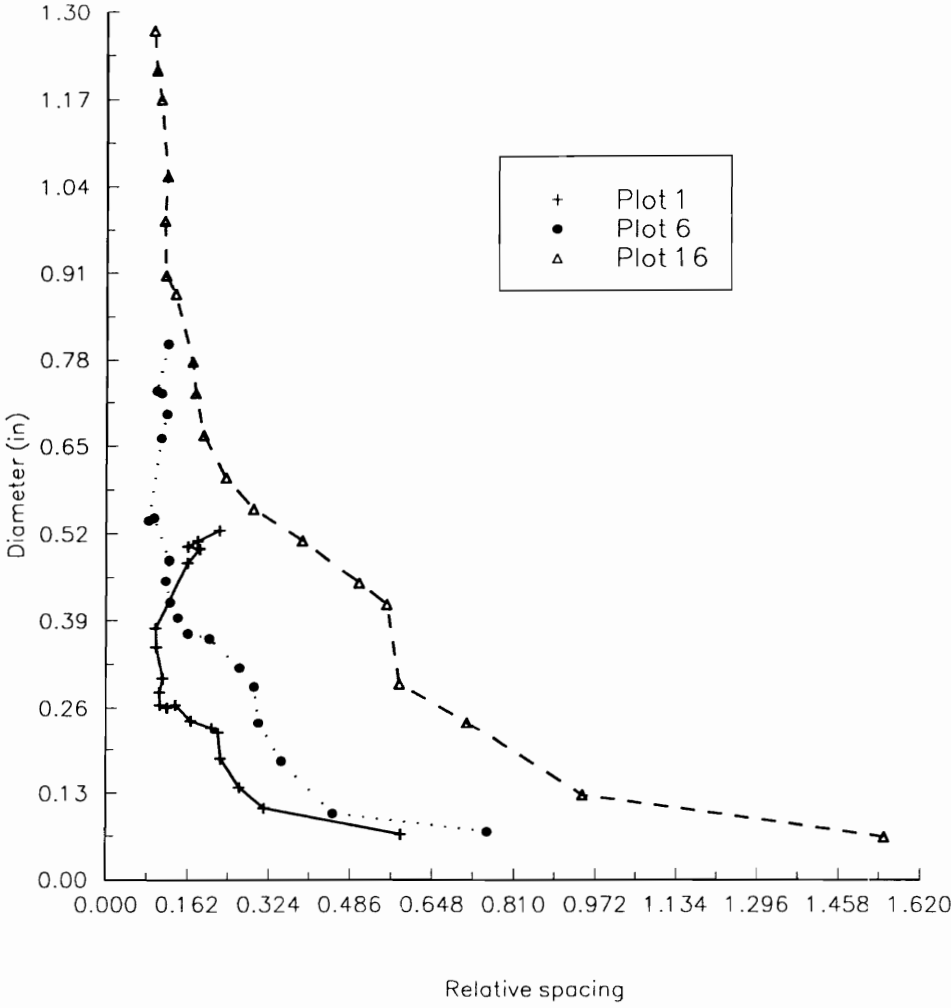


Figure 79: Groundline diameter vs. relative spacing for square spacings in the miniature spacing study.

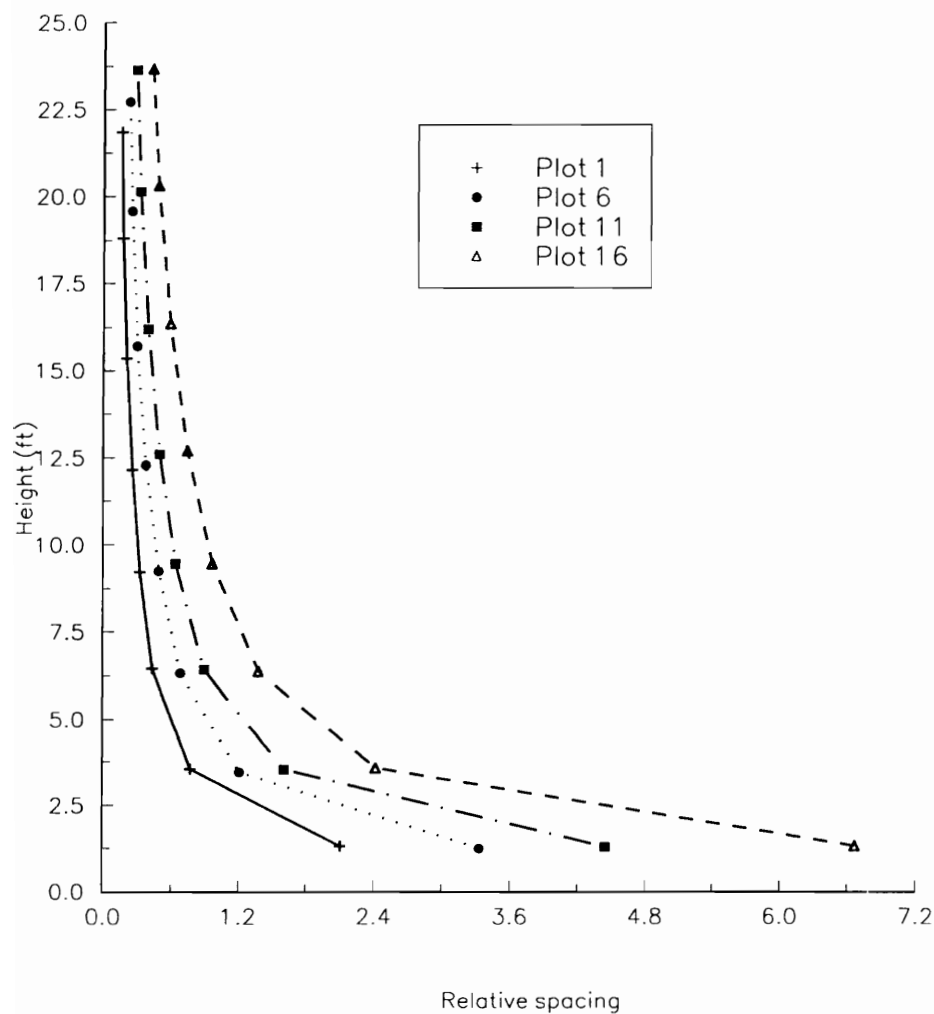


Figure 80: Height vs. relative spacing for square spacings in the conventional spacing study.

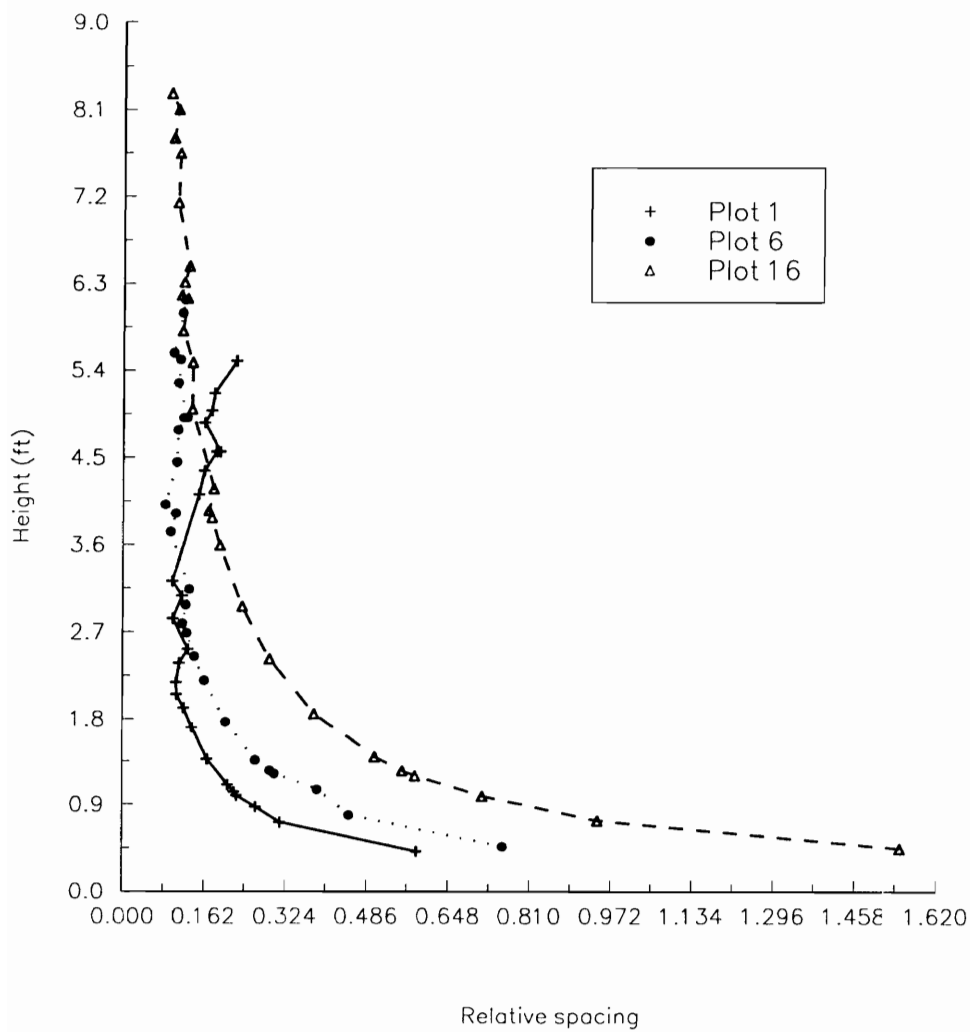


Figure 81: Height vs. relative spacing for square spacings in the miniature spacing study.

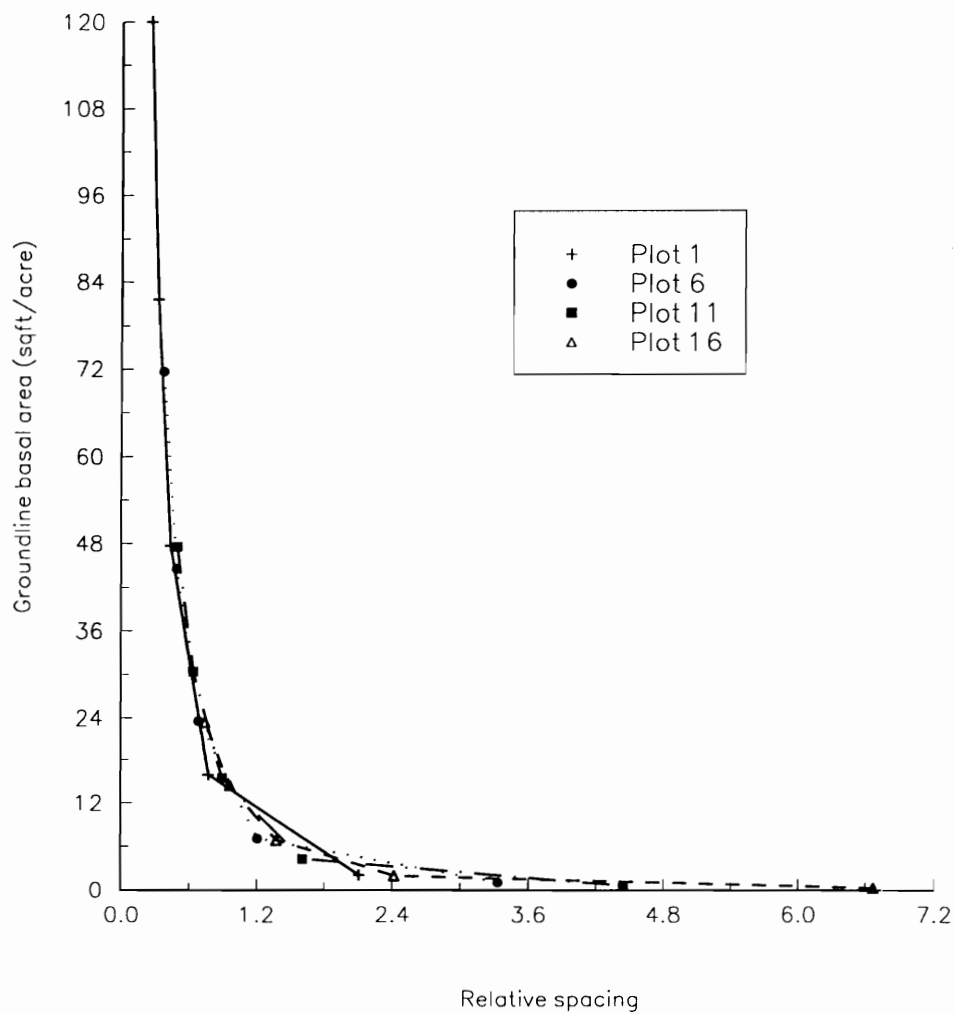


Figure 82: Groundline basal area vs. relative spacing for square spacings in the onventional spacing study.

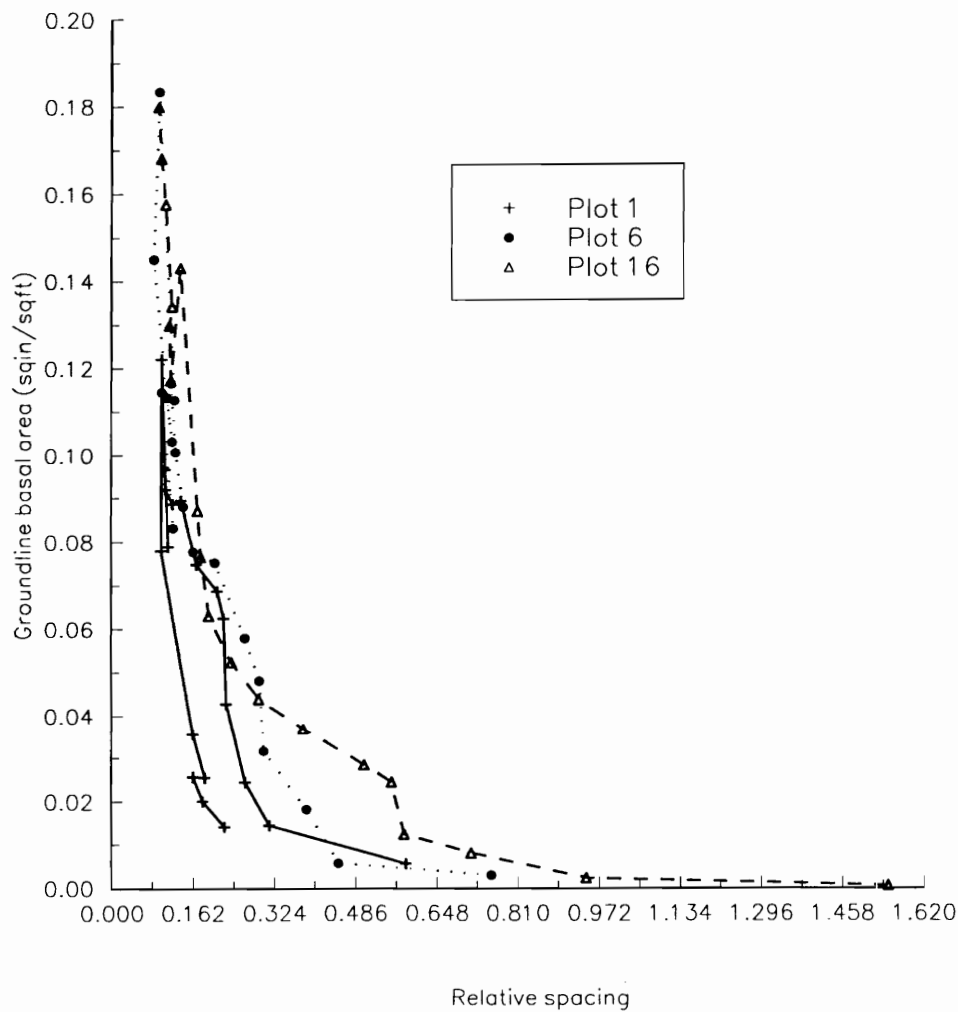


Figure 83: Groundline basal area vs. relative spacing for square spacings in the miniature spacing study.

Use of ratios in both studies

The ratios described in the section Analysis and Procedures were computed. Figures 84 through 93 show the graphs of these ratios versus time for both studies. In these figures the x -axis time in the miniature spacing study showed the first four measurements of the first year (1989) and the last 6 measurements of the second year (1990). The first ratio computed height to spacing (H_s), for the three square plots (plots 1, 6 and 16) showed a similar pattern in both, the conventional and the miniature study. More years of measurements in the conventional study would probably show a better relationship; these experiments were designed so that these ratios would be approximately the same by the time the conventional study is 25 years old.

The other ratio computed was C_r (crown ratio). In this case also the rectangular plots 4 and 7 were included in the analysis. Also in this case it can be observed that similar trends were followed by both studies, particularly before one year and a half of the miniature study. By the last half of the second year in the seedling study, an increment in C_r of the trees can be observed. With respect to the other two ratios computed, i.e. C_l/C_{ww} and C_l/C_{wb} (already explained), there were different responses in the trends, with the closest spacing switching with the widest spacing in terms of patterns comparing both studies. Again with these two ratios we observed the same phenomena as the one observed for the last half of the second year with C_r , i.e. an increase of the ratios of the trees.

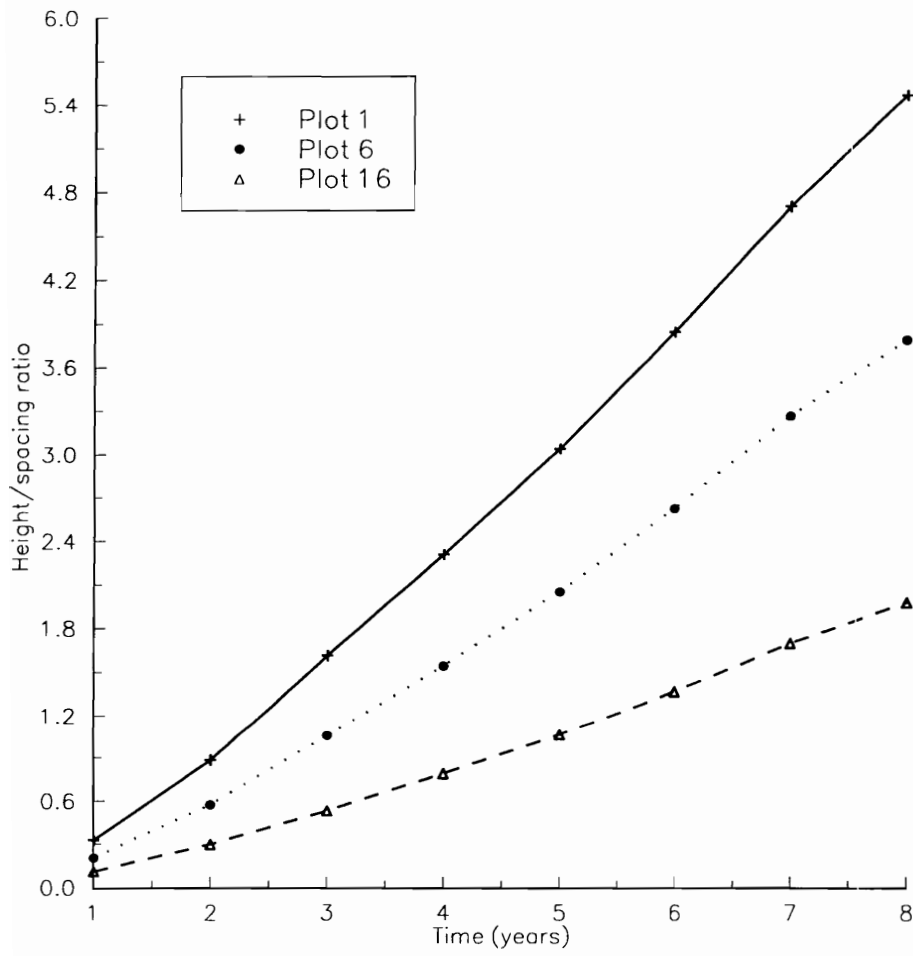


Figure 84: Height/spacing ratio for the conventional spacing study.

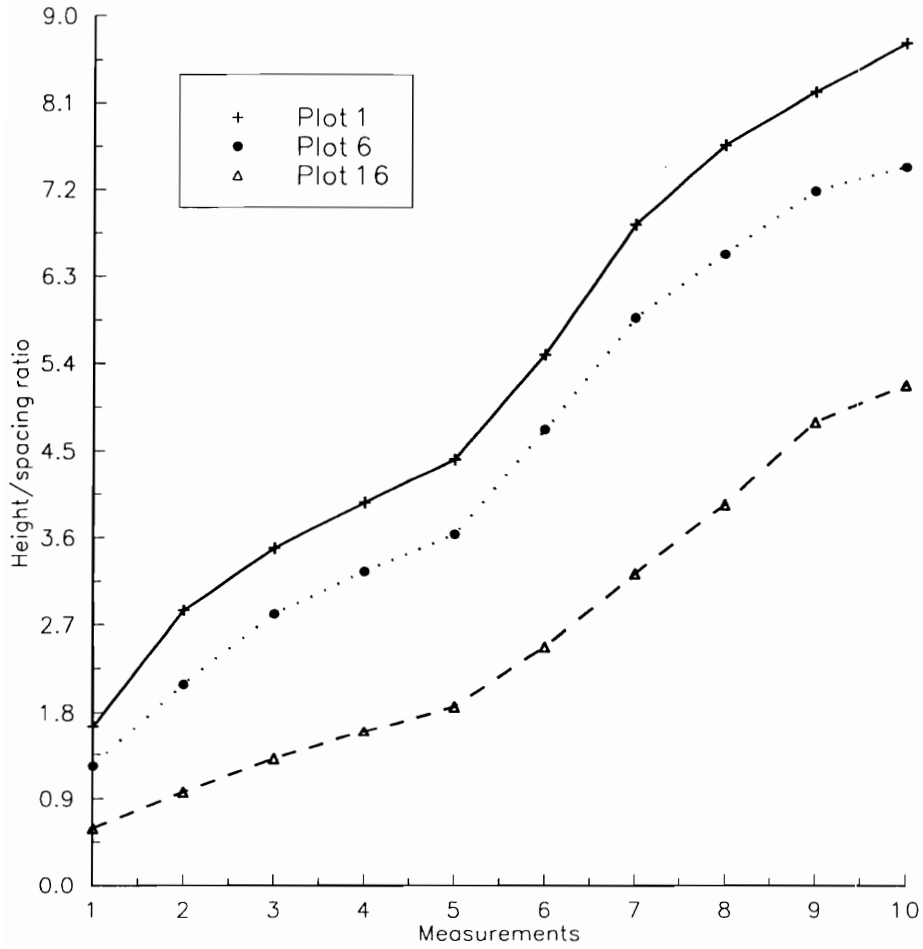


Figure 85: Height/spacing ratio for the miniature spacing study.

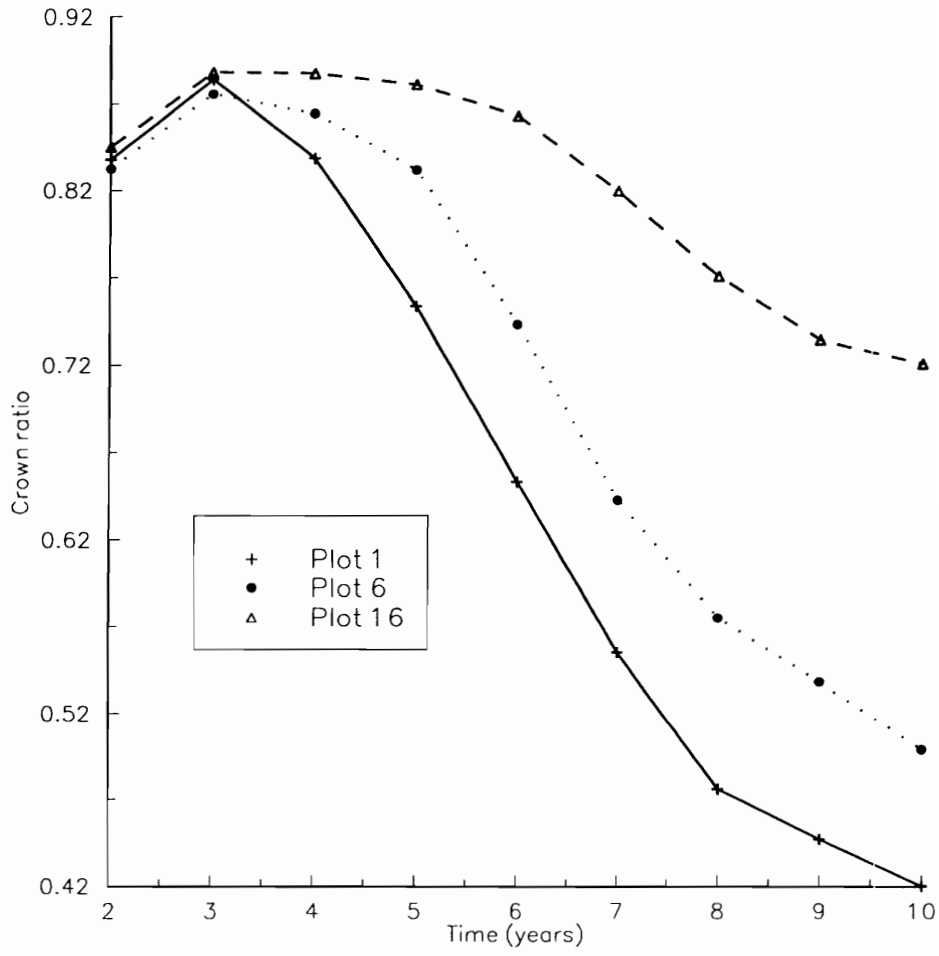


Figure 86: Crown ratio for square spacings in the conventional spacing study.

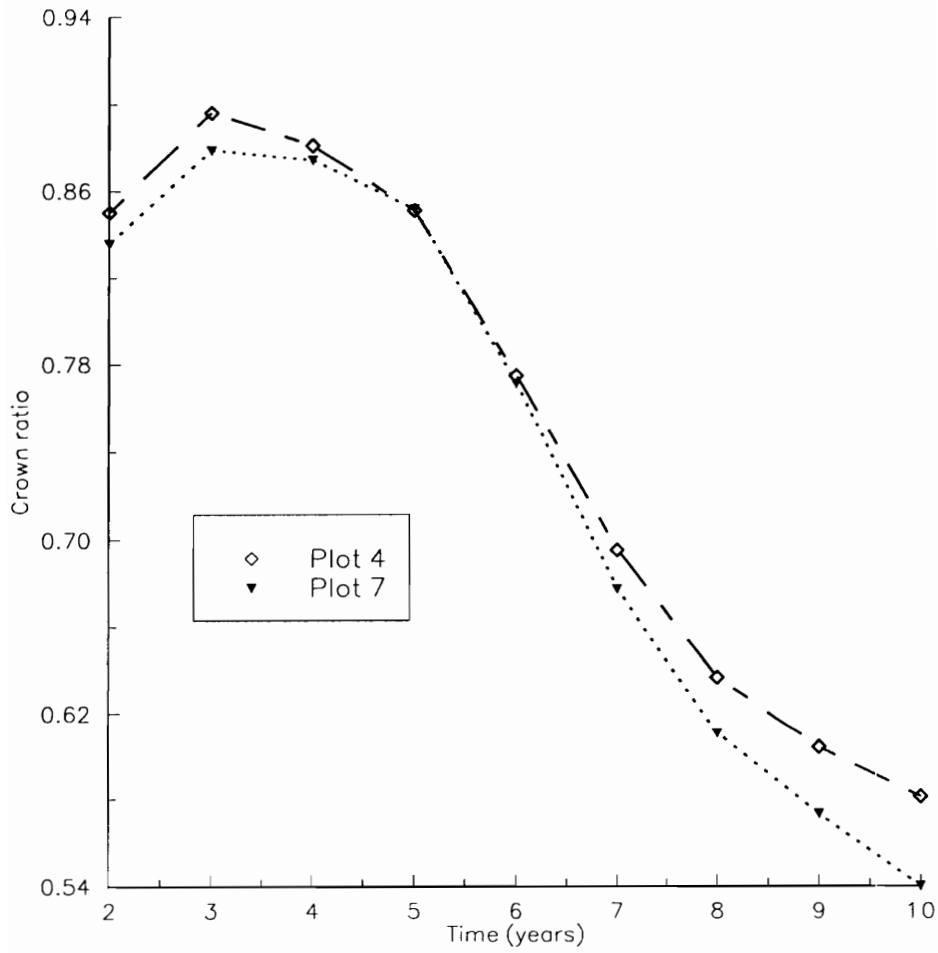


Figure 87: Crown ratio for rectangular spacings in the conventional spacing study.

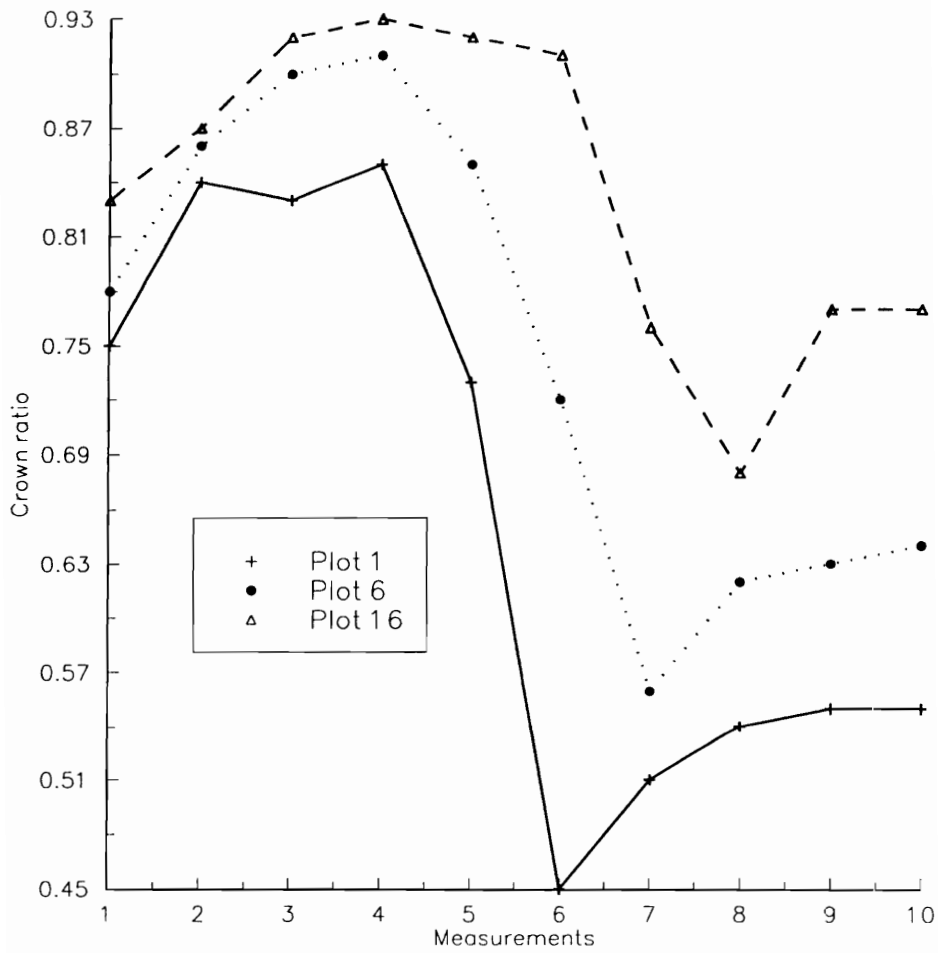


Figure 88: Crown ratio for square spacings in the miniature spacing study.

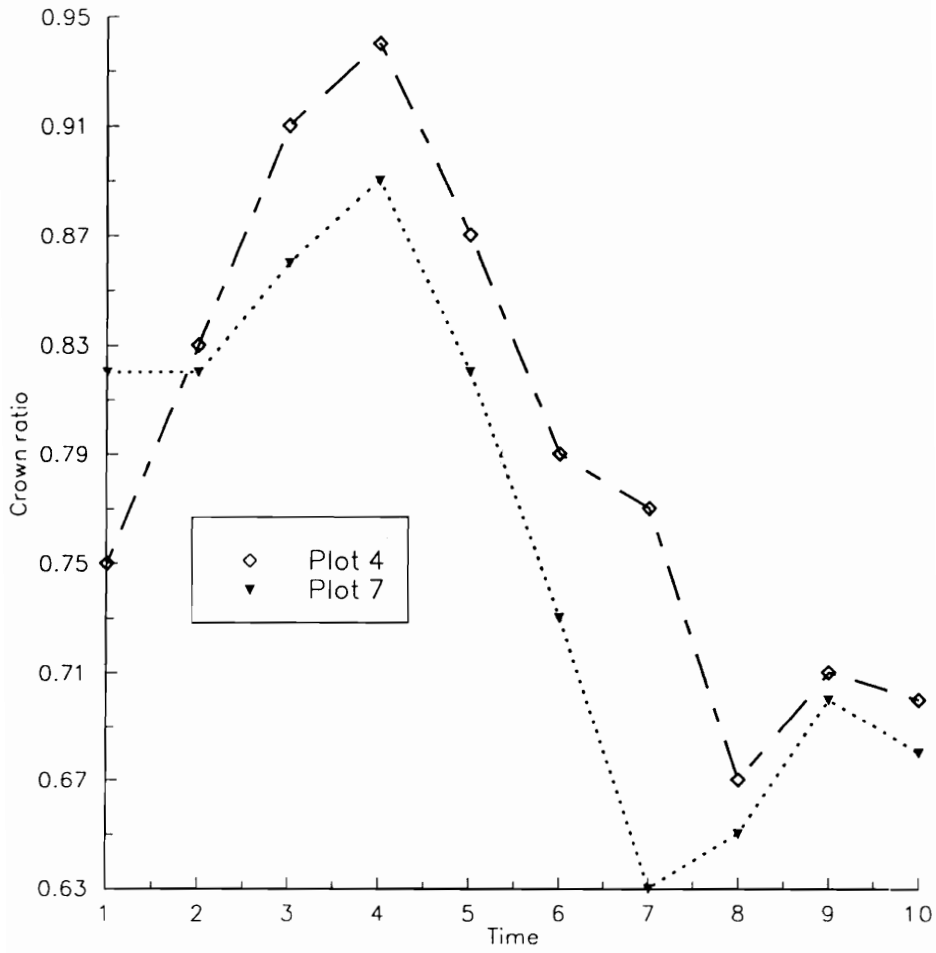


Figure 89: Crown ratio for rectangular spacings in the miniature spacing study.

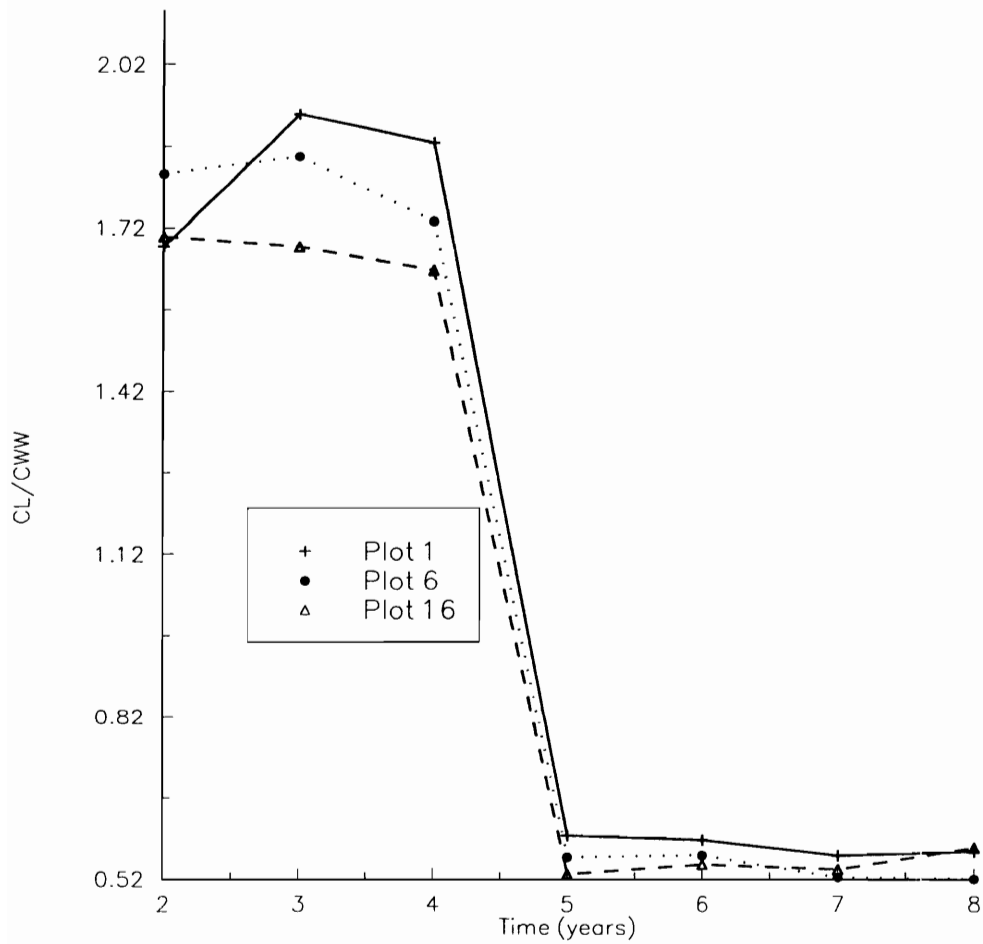


Figure 90: Crown length/Crown width within rows for square spacings in the conventional spacing study.

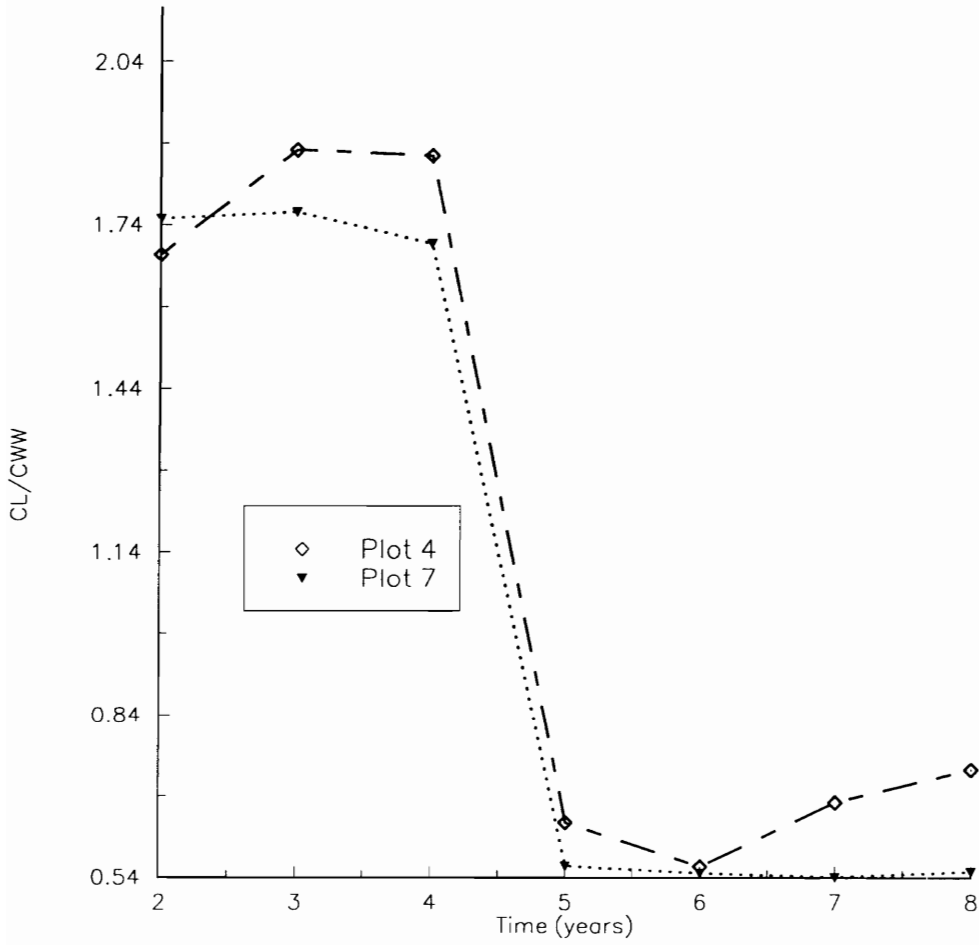


Figure 91: Crown length/Crown width within rows for rectangular spacings in the conventional spacing study.

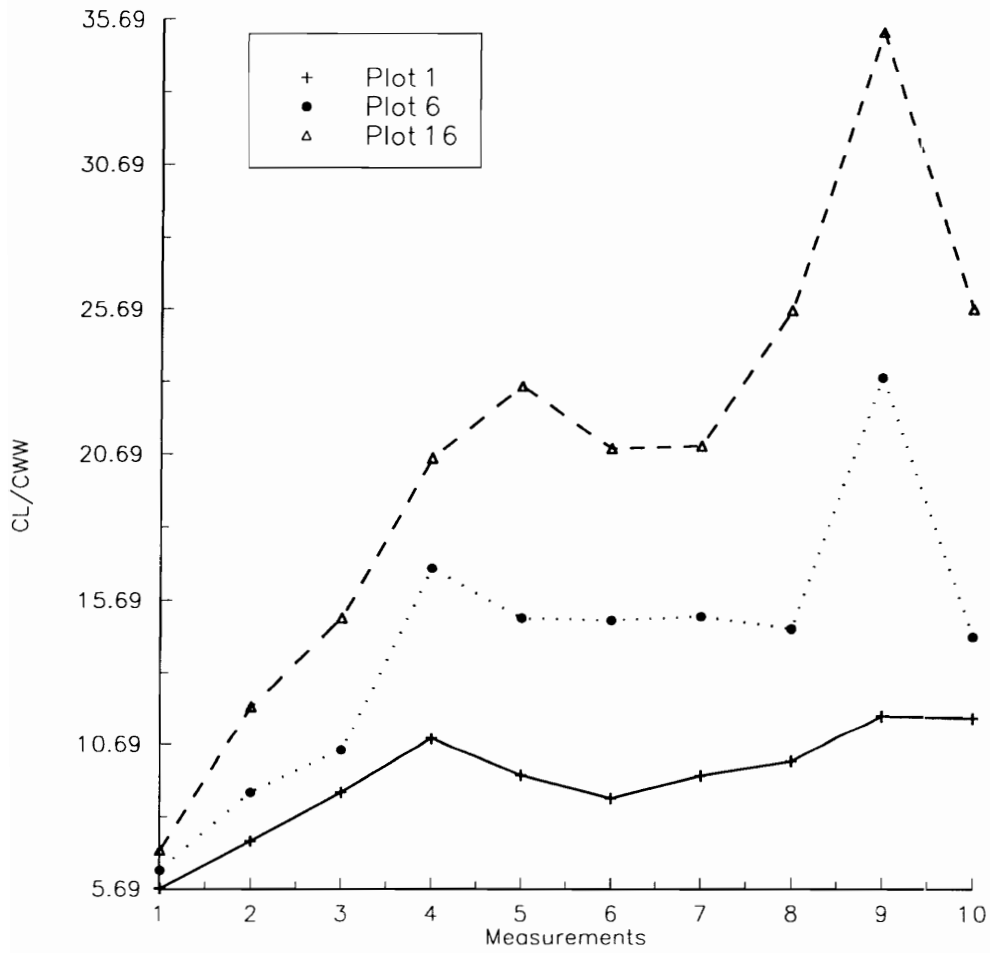


Figure 92: Crown length/Crown width within rows for square spacings in the miniature spacing study.

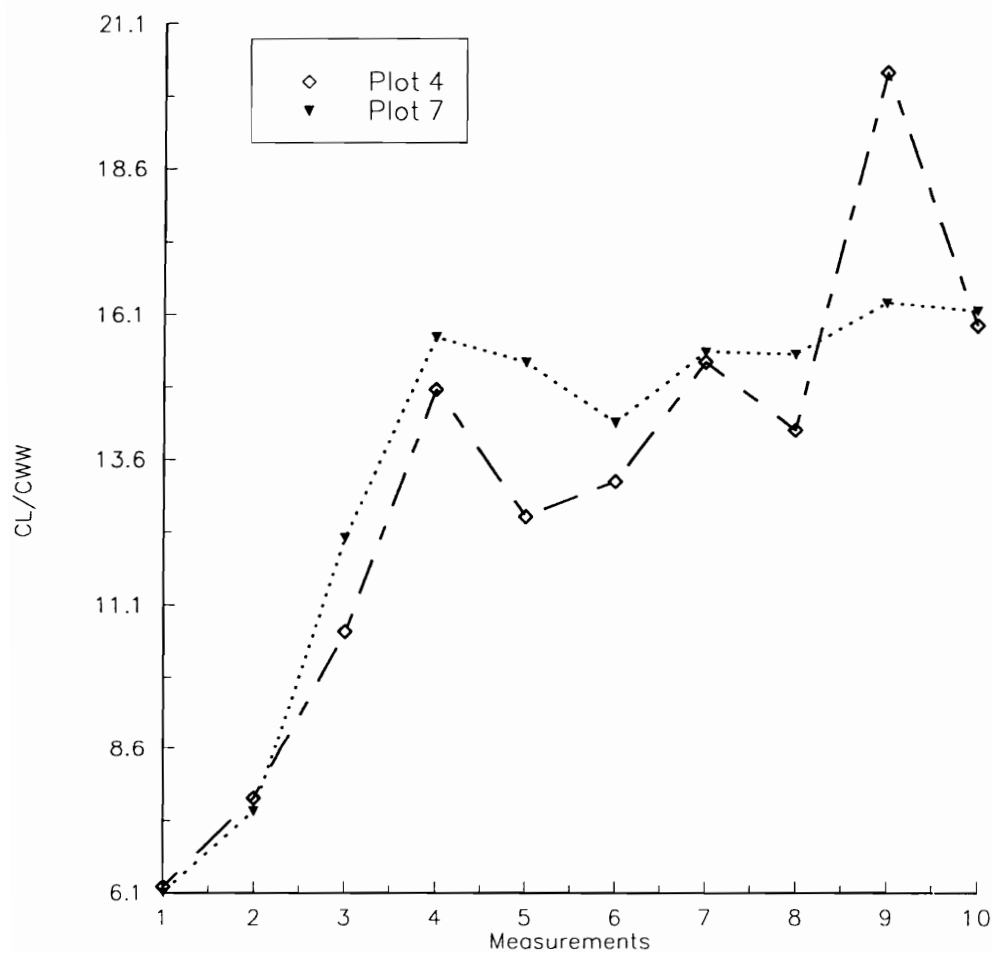


Figure 93: Crown length/Crown width within rows for rectangular spacings in the miniature spacing study.

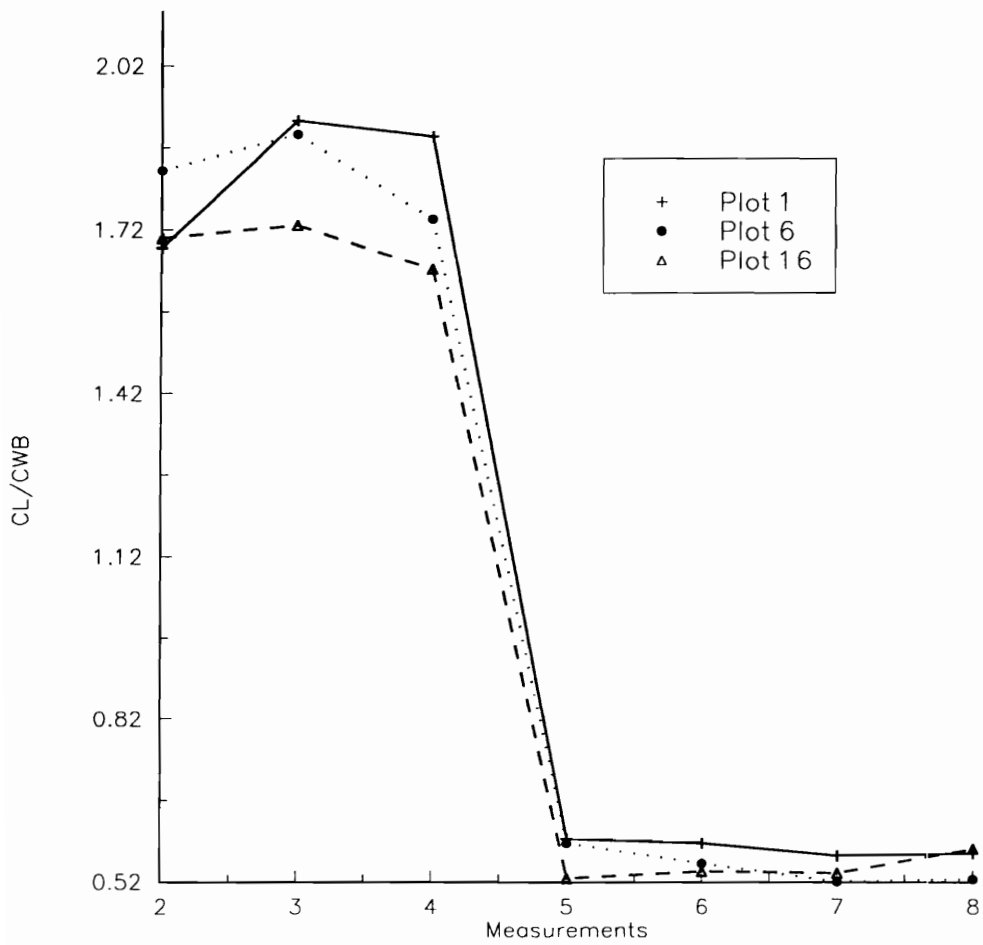


Figure 94: Crown length/Crown width between rows for square spacings in the conventional spacing study.

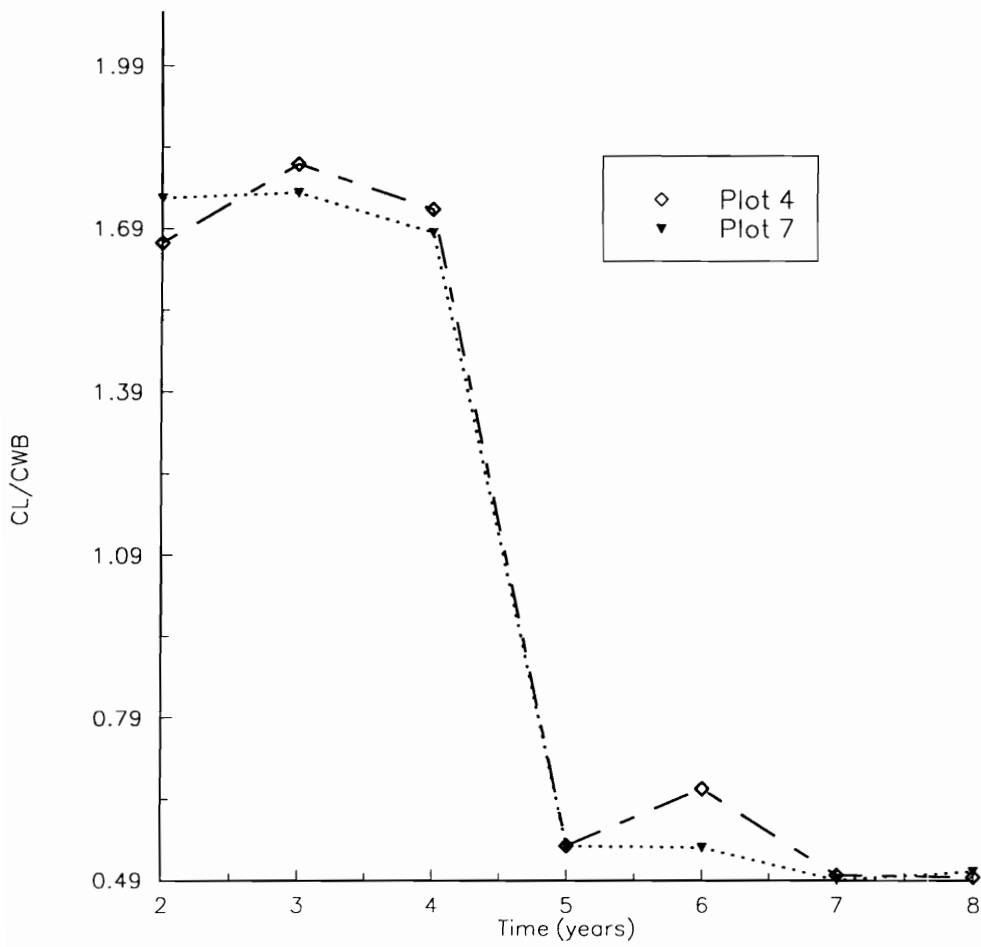


Figure 95: Crown length/Crown width between rows for rectangular spacings in the conventional spacing study.

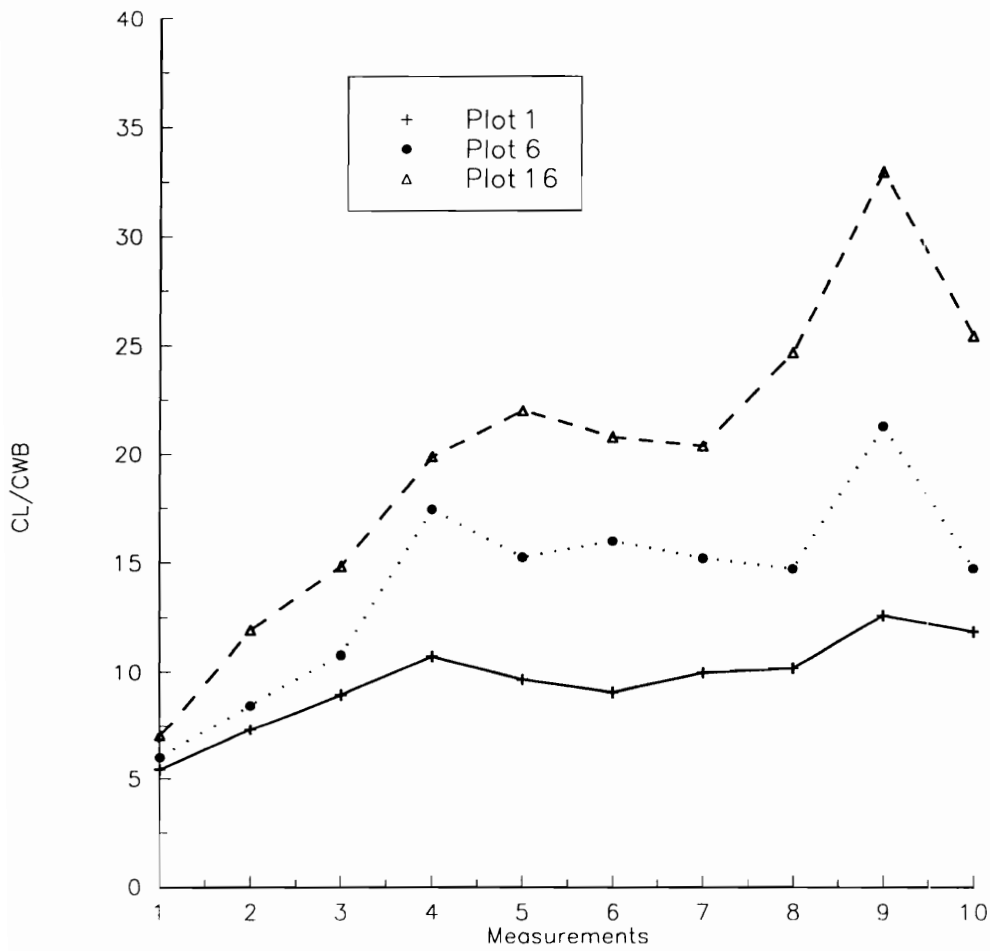


Figure 96: Crown length/Crown width between rows for square spacings in the miniature spacing study.

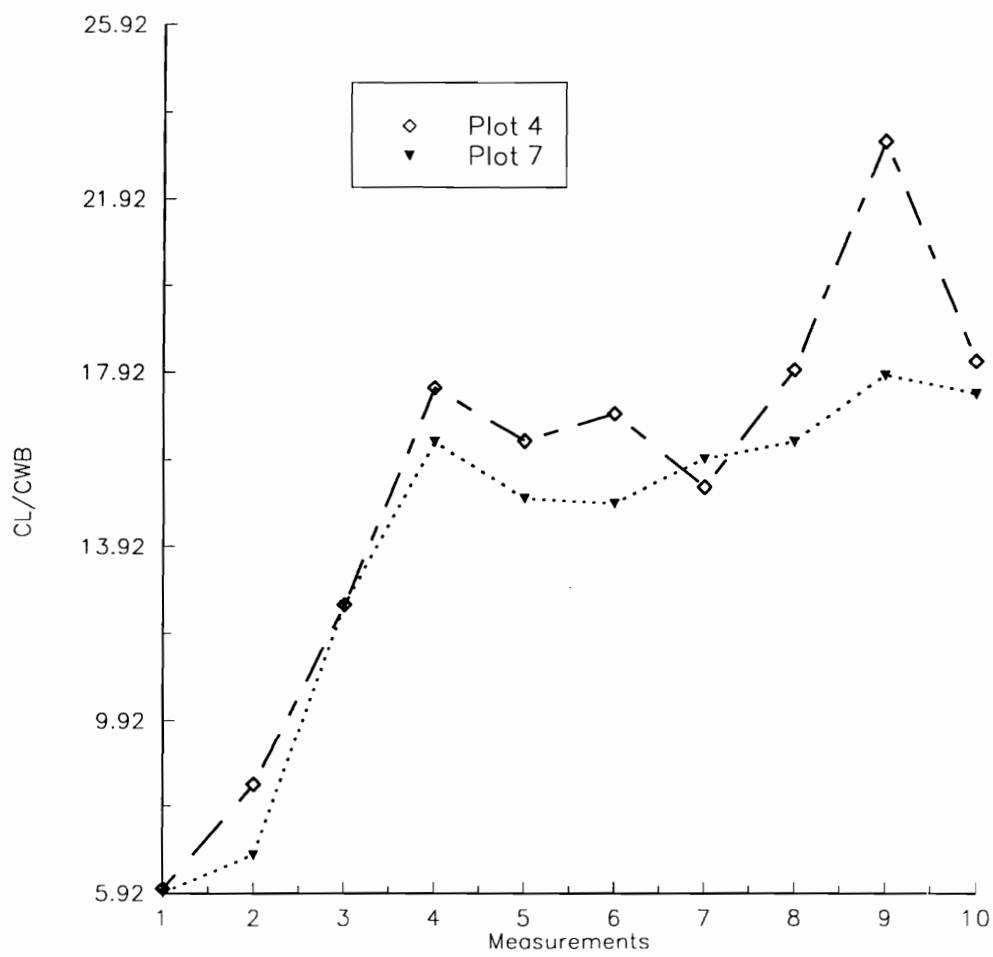


Figure 97: Crown length/Crown width between rows for rectangular spacings in the miniature spacing study.

Conclusions

It seems reasonable to conclude at this point that more years of measurements for the conventional spacing study are necessary for a more in depth analysis of the scaling problem. Also, more frequent measurements would certainly provide more accurate information for estimating the relative growth of characteristics such as diameter, height or volume and the corresponding growth rates (time derivatives) from which a reasonable scaling procedure could be derived.

Observing Figures 54 through 77 we could conjecture the existence of a scaling factor $k > 0$ such that

$$R_{y_c}(t) = kR_{y_m}(t) \quad (4.8)$$

instead of the equality $R_{y_c}(t) = R_{y_m}(t)$ initially sought. In (4.8) y_c and y_m are characteristics such as diameter, height or volume in the conventional and miniature spacing studies respectively. Although further research in this area is needed to decide the extent to which equation (4.8) is accurate for a given characteristic, this study provides an interesting starting point in this direction. For instance, assuming that (4.8) is true for a certain $k > 0$ it follows that

$$\frac{d}{dt} \ln y_c(t) = k \frac{d}{dt} \ln y_m(t).$$

Integrating this expression from t_0 to t , where t_0 is any given initial time, we obtain

$$\ln y_c(t) = k \ln y_m(t) + a$$

for a certain constant a , or equivalently,

$$y_c(t) = by_m^k(t). \quad (4.9)$$

where b is another positive constant.

An interesting problem for further research would be to estimate the values of b and k for a given characteristic that best fits the experimental data. From Figures 54-77, there is no reason to believe that $k = 1$. Moreover, for the case of the height a value of $k > 2$ is to be expected with the data collected until now (see Figures 58, 59, 60 and 61).

The scaling process from the data of a miniature stand to the corresponding “estimated scaled mature stand” for a certain given characteristic y could not only be done from the use of equation (4.9) once b and k have been estimated, but also from the experimental data, as follows.

$$k\hat{R}_{ym}(t) \approx \hat{R}_{yc}(t) \approx R_{yc}(t) = \frac{d}{dt} \ln y_c(t).$$

Integrating the above expression from t_0 to t , where t_0 is a given initial time, we get that

$$\ln y_c(t) - \ln y_c(t_0) \approx \int_{t_0}^t k\hat{R}_{ym}(s) ds$$

and therefore

$$y_c(t) \approx y_c(t_0)e^{k\left(\int_{t_0}^t \hat{R}_{ym}(s) ds\right)}$$

The right hand side on the above expression depends only on the initial value of y_c at time $t = t_0$ and on the data of the seedling stand for the same characteristic. The integral $\int_{t_0}^t \hat{R}_{ym}(s) ds$ will have to be approximated since we only have pointwise values for $\hat{R}_{ym}(s)$.

Finally, we want to emphasize that although the study of this scaling problem seems to be very complex, many other things remain to be done in order to further explore

its feasibility. Other types of differential equations could provide an important tool to extract different nonlinear dependence relations from the data.

The similarities observed in the patterns of groundline diameter and height growth in the figures when relative spacing was used as x -axis indicates that the possibility of using this type of scale instead of time is promising. Here again as more measurements are made in the conventional spacing study, especially when getting towards the rotation age, these patterns may be much more helpful in trying to scale seedling stands to mature stands.

CHAPTER V Summary, Conclusions and Future plans

In this study we have established functions to predict diameter, height and crown development for loblolly pine trees younger than ten years old. According to the analysis performed the variable site index was an important variable in the explanation of height prediction from the first year of planting. Two different models were fitted. In both models the variables site index and relative spacing were significant independent variables to explain height prediction. However, a different form of these two models was used for trees one and two years old and for trees three years old and older. In the first case the independent variables were the product of the variables site index and age and the product of relative spacing and age. For trees three years old and older these products were also significant independent variables but in this case the variable age or square root of age was used as another independent variable.

With respect to diameter, two aspects were considered: groundline diameter for trees younger than five years old and diameter at breast height for trees older than five years old. For trees three to five years old groundline diameter was found to be a function of site index and relative spacing. For trees one and two years old it was a function of age. For trees five years old and older diameter at breast height was a function of age, site index and relative spacing.

Crown development analysis showed that possible independent variables were diameter, either groundline diameter (trees five years old and younger) or diameter at breast height (of trees older than five years old). Other independent variables considered in the analysis were total height, site index, age and relative spacing.

Using data coming from trees planted at conventional spacings, comparisons of trends of characteristics such as diameter, height, crown development, groundline basal area, relative spacing and number of trees per unit area were made with seedling and other published spacing studies. In general, the patterns observed for the trees planted at conventional spacings were similar to the seedlings grown at very close spacing. These trends were also similar to the observed trends in published spacing studies.

The next important aspect considered was the use of seedlings grown at close spacings to scale seedling to mature stands. Although the scaling problem seems to be more complex than initially thought, the preliminary results obtained in this study provide a solid basis for further developments and studies. At this point, it seems reasonable to suggest that replications of the seedling trial as well as more measurements in the conventional spacing trial are needed for a more in-depth study. This information would not only result in better estimates of the scaling factors for different characteristics but it would also enhance our knowledge of the scope of the models by providing new data that could be used to validate results of this study.

There is much room for further study. Some areas for continued research are:

- 1) Use of data coming from other spacing studies that have been grown over longer periods of time for improving and validating estimates of the scaling factors.
- 2) Replication of seedlings spacing trials for a more accurate estimation of the scaling factors " k ".
- 3) Use of other differential equations to capture nonlinear and higher order scaling factors. An adequate use of these powerful mathematical tools together with our

previous knowledge on the subject could not only result in more accurate models but it could also provide us with very helpful information about how growth in seedlings might be projected to mature trees.

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APPENDIX

Table 2: Linear regressions of quadratic mean groundline diameter on age for the square spacings in the conventional spacing study.

----- PLOT=1 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	25.94354	12.97177	767.898	0.0013
Error	2	0.03379	0.01689		
C Total	4	25.97732			
Root MSE	0.12997	R-square	0.9987		
Dep Mean	4.28897	Adj R-sq	0.9974		
C.V.	3.03036				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-1.202258	0.27875764	-4.313	0.0498
AGE	1	2.181955	0.21243186	10.271	0.0093
AGE2	1	-0.095876	0.03473632	-2.760	0.1100
----- PLOT=6 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	36.15718	18.07859	660.502	0.0015
Error	2	0.05474	0.02737		
C Total	4	36.21192			
Root MSE	0.16544	R-square	0.9985		
Dep Mean	4.72980	Adj R-sq	0.9970		
C.V.	3.49786				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-1.028736	0.35483322	-2.899	0.1012
AGE	1	1.947844	0.27040651	7.203	0.0187
AGE2	1	-0.007727	0.04421619	-0.175	0.8774

Table 2: continued.

----- PLOT=11 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	44.08488	22.04244	379.910	0.0026
Error	2	0.11604	0.05802		
C Total	4	44.20092			
Root MSE	0.24087	R-square	0.9974		
Dep Mean	5.07829	Adj R-sq	0.9947		
C.V.	4.74321				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-1.302283	0.51661638	-2.521	0.1279
AGE	1	2.169698	0.39369603	5.511	0.0314
AGE2	1	-0.011684	0.06437619	-0.181	0.8727
----- PLOT=16 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	49.30857	24.65429	628.751	0.0016
Error	2	0.07842	0.03921		
C Total	4	49.38700			
Root MSE	0.19802	R-square	0.9984		
Dep Mean	5.23600	Adj R-sq	0.9968		
C.V.	3.78188				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-1.174227	0.42470351	-2.765	0.1097
AGE	1	2.005669	0.32365232	6.197	0.0251
AGE2	1	0.035747	0.05292281	0.675	0.5690

Table 3: Linear regressions of quadratic mean groundline diameter on age for the square spacings in the miniature spacing study.

----- PLOT=1 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	181.91236	90.95618	278.949	0.0001
Error	16	5.21708	0.32607		
C Total	18	187.12944			
Root MSE	0.57102	R-square	0.9721		
Dep Mean	6.98632	Adj R-sq	0.9686		
C.V.	8.17345				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	0.077314	0.56634906	0.137	0.8931
AGE	1	0.011684	0.00169727	6.884	0.0001
AGE2	1	-0.000002173	0.00000107	-2.024	0.0599
----- PLOT=6 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	450.23517	225.11758	386.840	0.0001
Error	16	9.31104	0.58194		
C Total	18	459.54621			
Root MSE	0.76285	R-square	0.9797		
Dep Mean	10.16684	Adj R-sq	0.9772		
C.V.	7.50331				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-1.320950	0.75660562	-1.746	0.1000
AGE	1	0.020544	0.00226745	9.060	0.0001
AGE2	1	-0.000004850	0.00000143	-3.382	0.0038

Table 3: continued.

----- PLOT=11 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	985.12456	492.56228	436.777	0.0001
Error	16	18.04354	1.12772		
C Total	18	1003.16809			
Root MSE		1.06194	R-square	0.9820	
Dep Mean		11.02053	Adj R-sq	0.9798	
C.V.		9.63604			
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-0.576095	1.05324958	-0.547	0.5919
AGE	1	0.012005	0.00315645	3.803	0.0016
AGE2	1	0.000004767	0.00000200	2.388	0.0296
----- PLOT=16 -----					
Model: MODEL1					
Dependent Variable: QM					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	2	1295.87562	647.93781	441.883	0.0001
Error	16	23.46099	1.46631		
C Total	18	1319.33661			
Root MSE		1.21091	R-square	0.9822	
Dep Mean		15.68316	Adj R-sq	0.9800	
C.V.		7.72111			
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-3.864726	1.20100171	-3.218	0.0054
AGE	1	0.035060	0.00359925	9.741	0.0001
AGE2	1	-0.000008364	0.00000228	-3.674	0.0021

Table 4: Linear regressions of mean stand height on age for the square spacings in the conventional spacing study.

----- PLOT=1 -----					
Model: MODEL1					
Dependent Variable: HEIGHT					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	34.49417	34.49417	2246.695	0.0001
Error	6	0.09212	0.01535		
C Total	7	34.58629			
Root MSE		0.12391	R-square	0.9973	
Dep Mean		3.37854	Adj R-sq	0.9969	
C.V.		3.66752			
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-0.699589	0.09654865	-7.246	0.0004
AGE	1	0.906250	0.01911948	47.399	0.0001
----- PLOT=6 -----					
Model: MODEL1					
Dependent Variable: HEIGHT					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	38.02191	38.02191	1271.806	0.0001
Error	6	0.17938	0.02990		
C Total	7	38.20129			
Root MSE		0.17290	R-square	0.9953	
Dep Mean		3.45023	Adj R-sq	0.9945	
C.V.		5.01140			
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-0.831360	0.13472618	-6.171	0.0008
AGE	1	0.951464	0.02667976	35.662	0.0001

Table 4: continued.

----- PLOT=11 -----					
Model: MODEL1					
Dependent Variable: HEIGHT					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	40.91080	40.91080	1046.924	0.0001
Error	6	0.23446	0.03908		
C Total	7	41.14526			
Root MSE	0.19768	R-square	0.9943		
Dep Mean	3.55400	Adj R-sq	0.9934		
C.V.	5.56217				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-0.887270	0.15403053	-5.760	0.0012
AGE	1	0.986948	0.03050259	32.356	0.0001
----- PLOT=16 -----					
Model: MODEL1					
Dependent Variable: HEIGHT					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	41.33114	41.33114	1031.230	0.0001
Error	6	0.24048	0.04008		
C Total	7	41.57162			
Root MSE	0.20020	R-square	0.9942		
Dep Mean	3.57197	Adj R-sq	0.9933		
C.V.	5.60471				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-0.892058	0.15599343	-5.719	0.0012
AGE	1	0.992005	0.03089130	32.113	0.0001

Table 5: Linear regressions of mean stand height on age for the square spacings in the miniature spacing study.

----- PLOT=1 -----					
Model: MODEL1					
Dependent Variable: HEIGHT					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	60555.29667	60555.29667	582.018	0.0001
Error	23	2393.00333	104.04362		
C Total	24	62948.30000			
Root MSE	10.20018	R-square	0.9620		
Dep Mean	87.66000	Adj R-sq	0.9603		
C.V.	11.63607				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-18.100994	4.83528893	-3.744	0.0011
AGE	1	0.131793	0.00546290	24.125	0.0001
----- PLOT=6 -----					
Model: MODEL1					
Dependent Variable: HEIGHT					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	68387.03940	68387.03940	859.129	0.0001
Error	23	1830.81100	79.60048		
C Total	24	70217.85040			
Root MSE	8.92191	R-square	0.9739		
Dep Mean	101.52800	Adj R-sq	0.9728		
C.V.	8.78763				
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-10.864248	4.22933936	-2.569	0.0172
AGE	1	0.140056	0.00477830	29.311	0.0001

Table 5: continued.

----- PLOT=11 -----

Model: MODEL1
Dependent Variable: HEIGHT

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	114751.91861	114751.91861	476.799	0.0001
Error	23	5535.44299	240.67143		
C Total	24	120287.36160			

Root MSE

Dep Mean

C.V.

15.51359

102.05600

15.20106

R-square

Adj R-sq

0.9540

0.9520

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-43.533292	7.35405662	-5.920	0.0001
AGE	1	0.181424	0.00830859	21.836	0.0001

----- PLOT=16 -----

Model: MODEL1
Dependent Variable: HEIGHT

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	143086.14201	143086.14201	646.780	0.0001
Error	23	5088.25639	221.22854		
C Total	24	148174.39840			

Root MSE

Dep Mean

C.V.

14.87375

132.99200

11.18395

R-square

Adj R-sq

0.9657

0.9642

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	1	-29.580934	7.05074927	-4.195	0.0003
AGE	1	0.202588	0.00796592	25.432	0.0001

VITA

Olga Beatriz Avila was born on February 25, 1960 in Buenos Aires, Argentina. She attended the National University of Santiago del Estero, Argentina where she received a Bachelor of Science degree in Agronomical Engineering in December 1983. Upon graduation she was hired by the National University of Santiago del Estero where she worked for three years as an instructor and research engineer in statistics applied to biology. During 1985 the National Institute of Statistical Research of Tucumán, Argentina granted her a fellowship to take graduate courses in applied statistics. In June 1988 she entered Virginia Polytechnic Institute and State University where she earned the M.S. degree in Forest Biometrics in August 1990 and the M.S. degree in Statistics in April 1992. She received the Ph.D degree in Forest Biometrics in September 1993.

Olga Anita de Spies