

THE TIME BEHAVIOR OF A SITE DIVERSITY SYSTEM

by

George Crosby Towner III

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APPROVED:

C. W. Bostian, Chairman

W. L. Stutzman

D. B. Hodge

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Chapter I
INTRODUCTION

1.1 HISTORICAL BACKGROUND

In the 1950's communications by radio had become widespread. This communication presented an abundance of interesting problems for radio engineers. One group of these problems was in the realm of radio wave propagation. It was found that radio waves could be channeled through layers in the atmosphere or "bounced" off of the ionosphere, a discovery which was used in communications over the horizon at frequencies not previously used for this type of communication. However, it was also found that these types of communications experienced variations in signal strength caused by changes in the transmission medium along with cancellations due to the reception of reflected copies of the same signal.

One method developed to combat this problem is called diversity. The idea behind diversity is to receive two, or more, uncorrelated copies of the same signal, and, because they are not correlated, the signals will not fade simultaneously.

There are many techniques that may be used to obtain the required copies of the signal. Space diversity is the most widely used diversity technique. It uses antennas separated spatially from each other to receive the signal. Frequency diversity entails receiving signals at two different frequencies with the same information on each. Polarization diversity uses this same principle with transmission being performed using two separate polarization states. One diversity technique is time diversity; this is useful for digital transmission. For each of these techniques the spacing between antennas, frequencies selected, the angular difference between polarization states, or the time between the signals themselves is determined by the degree of correlation allowable for the system to operate.

Once the copies of the signals are received there are many ways to process them. Perhaps the easiest of these is for a system to switch between the copies and use the first copy that is above a given threshold level for as long as it stays above the threshold; then the system continues checking the other copies [1]. Obviously this system has difficulty if all the copies are below the threshold, so a second switching method that uses the maximum of all of the copies is preferred. Combining all of the received signals in some prescribed fashion is another processing technique. This

method avoids wasting other useful signals [2]. Mack [3] states that the the output signal-to-noise ratios for the combined signals are better than that of the best single signal.

1.2 QUANTIFYING DIVERSITY BENEFITS

Because of the cost involved in the use of duplicate receiving equipment there is an obvious need to be able to evaluate the performance of a diversity system before actually installing it. This evaluation is performed by comparing a parameter common to both the single site and site diversity receiving systems.

In any communications system an important parameter is the received signal strength. Its importance naturally leads to a method for evaluating the effectiveness of a diversity system in terms of the received signal strengths of the diversity system and a competing single-site system with no diversity. This method uses the concept of diversity gain. Diversity gain is defined as the difference between the signal received when no diversity is used (single site signal, S_S) and the received diversity signal (derived using switching, combining or both) called the joint signal S_J . Usually the behavior of the joint signal and the single site signals are described by cumulative distributions which give

the percent of time that either of them are greater than a stated value. Examples of these distributions are shown in Figure 1-1. The diversity gain, G_{SD} , for these signal distributions is then the difference between the joint and single site signals at a given percentage of time P , described by

$$G_{SD} = S_J(P) - S_S(P) \quad (1)$$

This is shown graphically in Figure 1-2.

A second important factor to be considered when designing a communications system is the amount of time the received signal is above some given threshold value. A measure of this is the diversity improvement of the system. Diversity improvement is defined as the ratio of the percent of time that the joint signal exceeds a given threshold, and the percent of time that the single site signal exceeds the same threshold. Using the distributions of the signals, described earlier, the diversity improvement, I_D , may then be found using the ratio of the percent time the joint signal, P_J , exceeds a given signal level, S , and the percent time the single site signal, P_S , exceeds the same signal level S . This is described by

$$I_D = \frac{P_J(S)}{P_S(S)} \quad (2)$$

This is shown graphically in Figure 1-3.

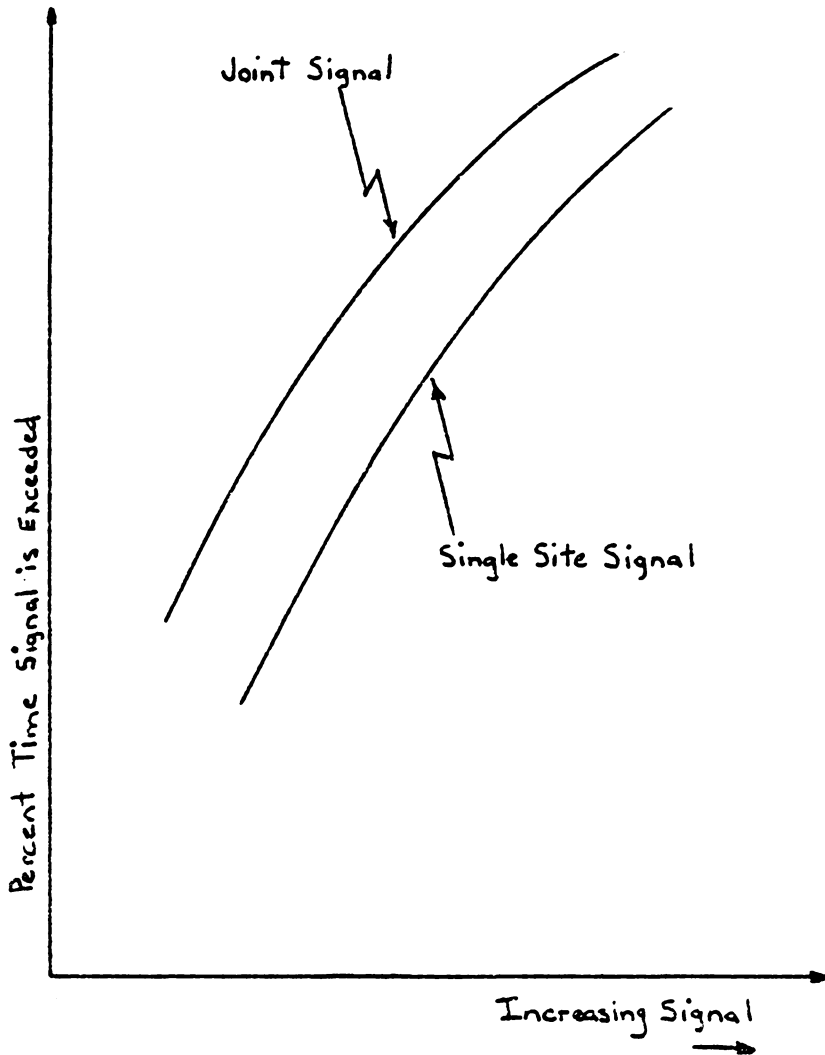


Figure 1-1. Hypothetical exceedance plot for single site and joint signal levels.

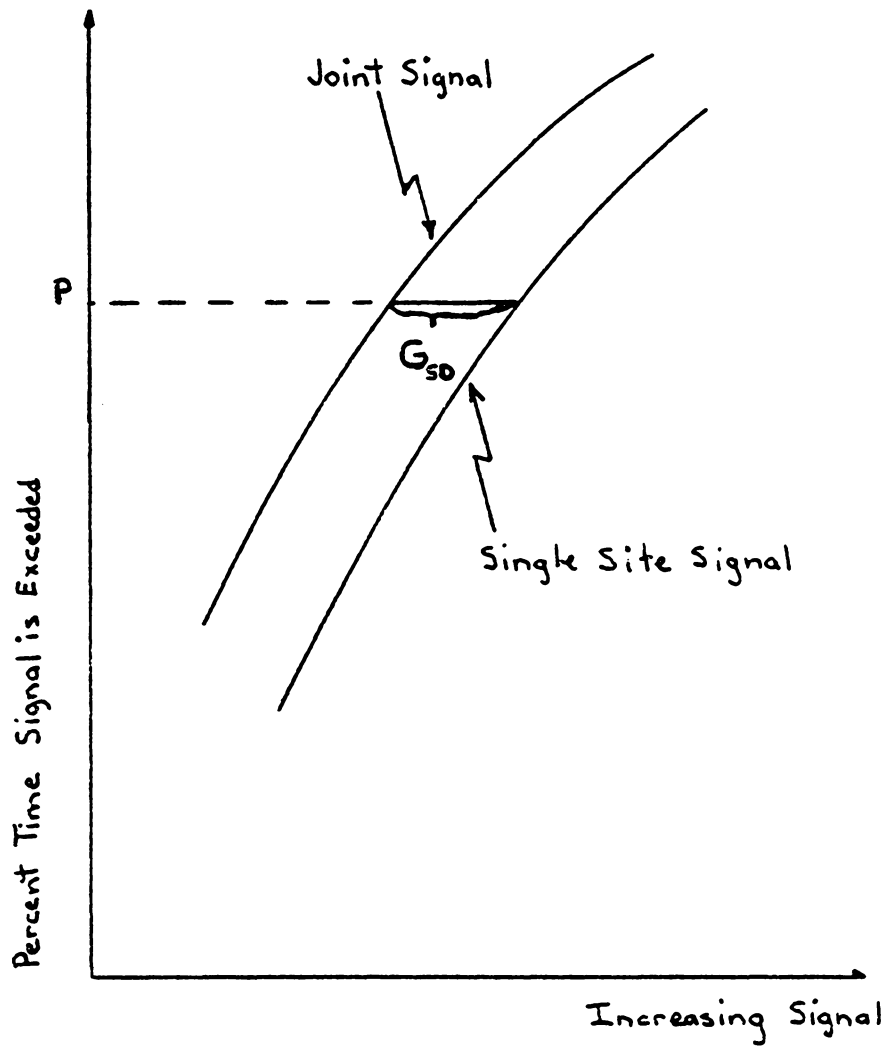


Figure 1-2. Graphical representation of signal diversity gain.

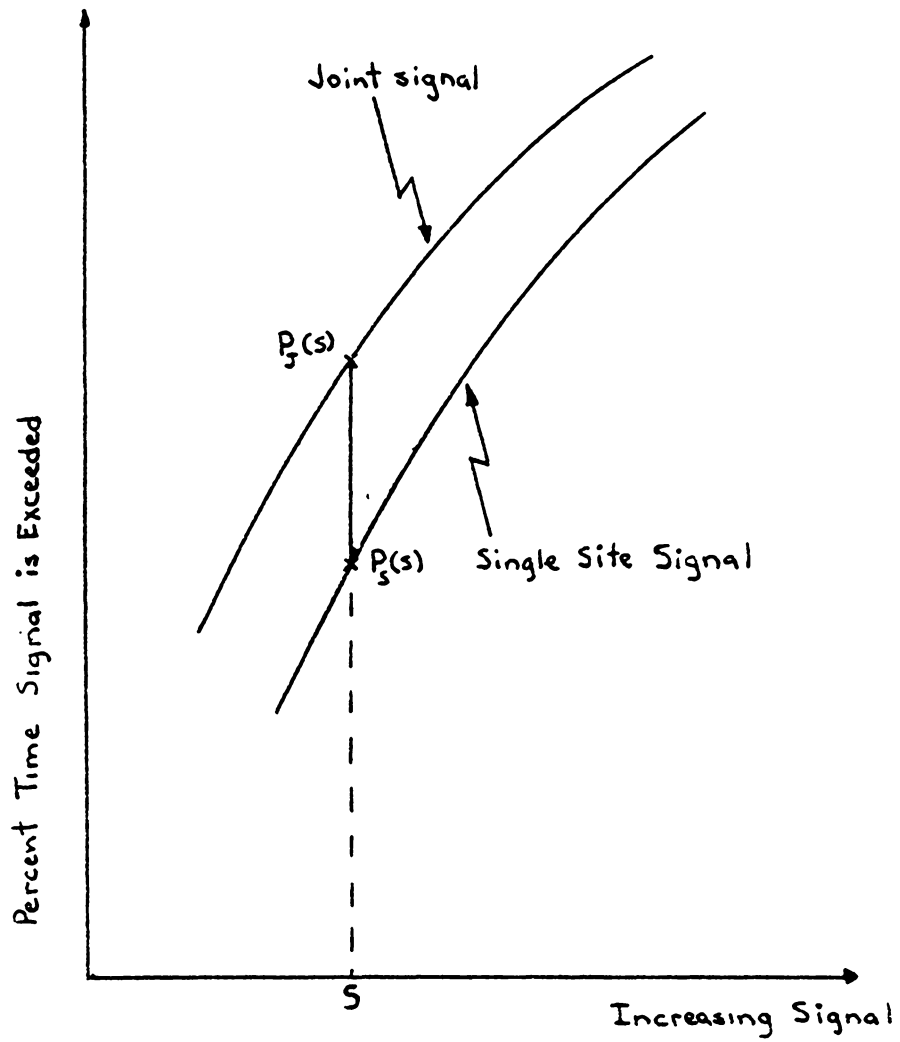


Figure 1-3. Graphical representation of diversity improvement.

A quantity that describes the benefit of a diversity system is the correlation between the received diversity signals. Diversity is successful when the received signals fade at different times. This is more likely to occur if the fading properties of the received signals are independent of each other. A measure of the independence between two variables is the correlation coefficient. It is defined in [4] as

$$r = \frac{E\{(x-m_x)(y-m_y)\}}{[E\{(x-m_x)^2\}E\{(y-m_y)^2\}]^{1/2}} \quad (3)$$

where m_x and m_y are the mean values of the signals x and y , and the expected value of a random variable z is defined in terms of the probability distribution of z , $f(z)$, by

$$E\{z\} = \int_{-\infty}^{\infty} zf(z)dz \quad (4)$$

A correlation coefficient of unity signifies that the signals are totally dependent (fades occur at the same time) and diversity would not be useful. A correlation coefficient of zero indicates that the signals are independent and diversity would be very helpful in this case. If a correlation coefficient of -1 was obtained then diversity would be extremely useful because in this case a fade of one signal corresponds to an increase in the other signal level. It was shown in [1] that if the magnitude of the correlation

coefficient $|r|$ is less than $1/e$ then the signals are essentially independent.

Chapter II

STATISTICAL ATTENUATION DIVERSITY GAIN

2.1 CONCEPTS AND DEFINITION

At the frequencies used for satellite communications one diversity technique has been studied extensively. This technique utilizes the fact that rain is spatially confined in both horizontal and vertical extent. Thus the rain forms cells whose size depends on the rain rate. This way by placing two (or more) separate earth stations large distances from each other (over one kilometer) the probability that there is an intense rain induced fade occurring simultaneously on both (or all) of the sites at the same time decreases from what it would be for a single site. This technique is known as site diversity. Any of the, previously mentioned, signal processing techniques may be used in conjunction with site diversity. To date, however, a large portion of the work on site diversity has been based on the assumption that the system switches to the maximum received signal. It is this assumption which is used throughout this thesis.

In the design of a satellite communications system many different parameters are used to analyze the overall system performance. This may be seen by looking at the transmission equation shown below.

$$(S/N)_{dB} = EIRP - (\text{Path Losses}) + G_r - 10 \log (kTB) \quad (5)$$

This equation is the sum of many terms. They are the signal to noise ratio, the transmit EIRP, the receiver antenna gain, the loss due to the system thermal noise and, the loss due to the transmission medium (path loss or attenuation). With satellite communications at frequencies above 10 GHz becoming a reality the path loss has recently been the focus of a great deal of study. This has yielded a large amount of information describing the attenuation (decrease in received signal level below the clear wather signal level) on a satellite communications link. One measure of a system's performance is the distribution of the attenuation over a given time period (percent time statistics), and it is by utilizing this parameter that the diversity gain of a space diversity system may be found.

Diversity gain found using signal levels in (1) may be extended to attenuation simply. The single site attenuation, A_S , is defined as the difference between the clear sky signal level, S_C , and the received single site signal level, S_S , and is given by

$$A_S = S_C - S_S \text{ (in dB)} \quad (6)$$

The joint attenuation, A_J , is defined as the minimum of all of the single site attenuations in a diversity system at each time t .

$$A_J = \text{Min} (A_1, A_2, \dots, A_N) \quad (7)$$

This leads to the definition of attenuation diversity gain for satellite communications systems. It is given in terms of the attenuation distributions for given percentages of time, P , by

$$G_{AD} = A_S(P) - A_J(P) \quad (8)$$

This is shown graphically in Figure 2-1. It must be noted however, that the single sites which compose the diversity system need not have the same clear sky signal levels. The use of attenuation normalizes the signals so that only signal changes are being analyzed and not relative differences in signal levels due to differences in receivers.

Using attenuation diversity gain a communications engineer may evaluate the performance of a diversity communications system as compared to a single site communications system in terms of a parameter which is meaningful to the design of the communications system. This is provided that attenuation data for a single site and diversity system are available for the region where the communications system is

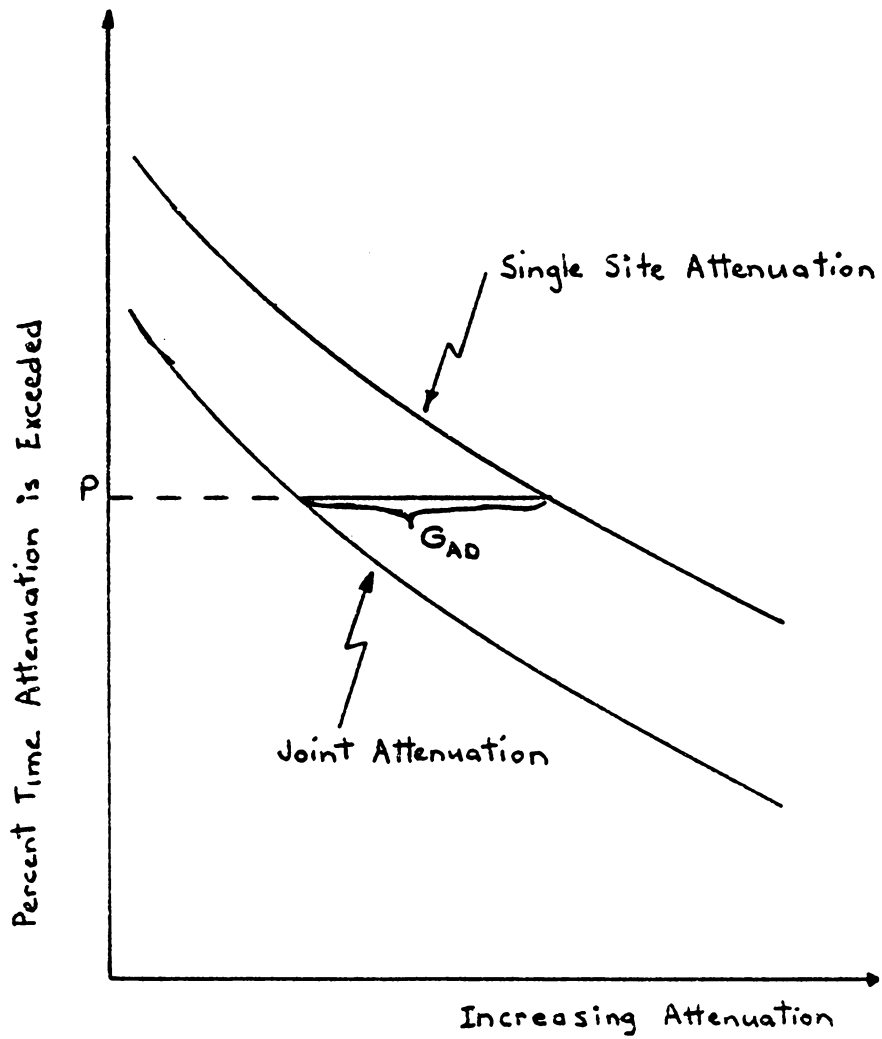


Figure 2-1. Graphical representation of attenuation diversity gain.

to be located. If these data are not available then the design engineer must look for other methods of evaluating diversity performance.

2.2 A MODEL

The design engineer must use measured data to evaluate the diversity gain of a site diversity system. This data measurement process is both expensive and time consuming, making the use of diversity gain unappealing to the design engineer. Because of this there has been a great deal of work done to model the behavior of satellite communications links in the presence of rain and ice. And in fact, there do exist several good models to predict attenuation at a single site [5],[6]. These models however are not useful for describing the joint attenuation found with a site diversity system. This, then, lead to the development, by Hodge, of an empirical model describing diversity gain [7].

The diversity data used to develop the Hodge model were taken with many different site separations. These data were then plotted as shown in Figure 2-2. It was observed that the behavior of these curves is given by the exponential below

$$G = a(1 - e^{-bd}) \quad (9)$$

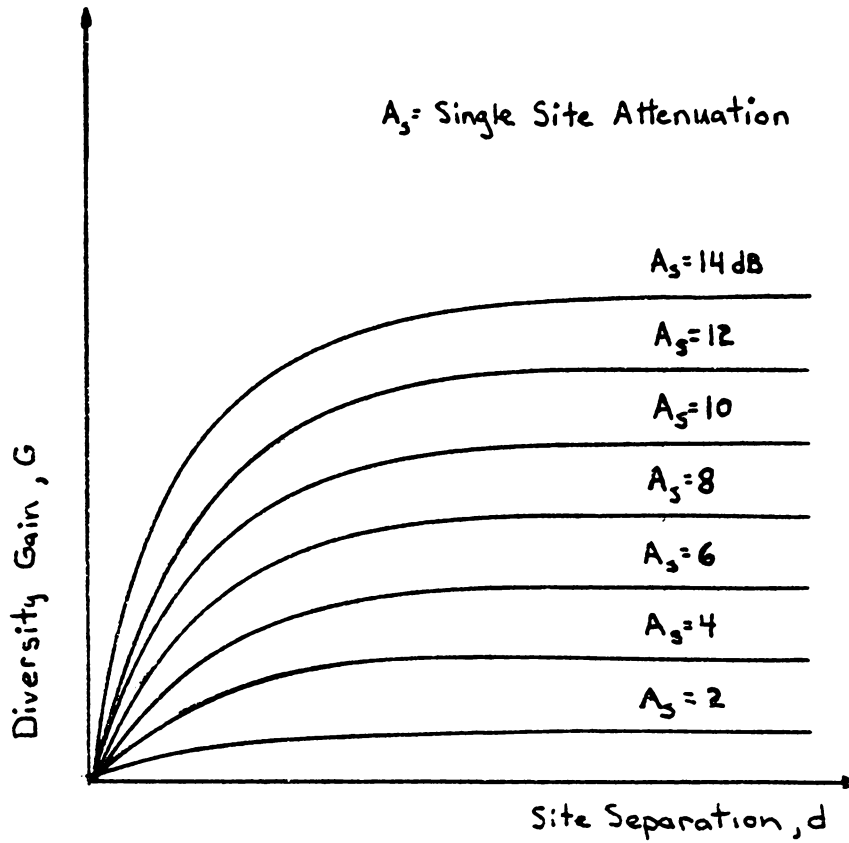


Figure 2-2. Hypothetical plot of diversity gain versus site separation with single-site attenuation as a parameter.

This lead to a least squares fit to the data resulting in equations for a and b given in terms of the single site attenuation A_S by

$$a = A_S - 3.6(1 - e^{-0.24A_S}) \quad (10)$$

and

$$b = 0.46(1 - e^{-0.26A_S}) \quad (11)$$

The gain G is expressed in dB as is the single site attenuation A_S . The site separation d is expressed in km.

The error of the Hodge model when compared to the original data was found to be at most 0.75 dB [7]. However as more site diversity experiments were performed the analyzed data showed that the model needed to be revised. The altered model includes the weaker dependences of diversity gain on frequency, baseline orientation angle, and elevation angle along with the strong dependence on site separation distance. The resulting model is shown below

$$G_H = G_d G_f G_\beta G_\Delta \quad (12)$$

with the gain due to orientation angle Δ (in degrees) given as

$$G_\Delta = 0.00177\Delta + 0.887; \quad 0 \leq \Delta \leq 90^\circ \quad (13)$$

and the gain due to the elevation angle β (in degrees) given as

$$G_{\beta} = 0.00492\beta + 0.834 \quad 0 \leq \beta \leq 90^{\circ} \quad (14)$$

and the gain due to frequency f (in GHz)

$$G_f = 1.64e^{-0.025f} \quad (15)$$

and the gain due to site separation d (in km) ,having the same form as (9) , given as

$$G_d = a(1-e^{-bd}) \quad (16)$$

with the coefficients a and b (re-evaluated to include the larger data set) given here in terms of single site attenuation, A .

$$a = 0.64A - 1.6(1 - e^{-0.11A}) \quad (17)$$

$$b = 0.585(1 - e^{-0.098A}) \quad (18)$$

This new model had an RMS error of 0.73 dB and was found to model the behavior of a site diversity system at large single site attenuations much better than the earlier version of the model [8].

The development of this model enables the design engineer to evaluate the behavior of a site diversity system in any

location without the need to collect attenuation data first. All the engineer need do is use an attenuation model, such as that of Stutzman and Dishman [5], to find the attenuation for a given percentage of time and then use that value of attenuation to calculate the diversity gain for the desired diversity system. The system parameters used in the model may even be adjusted to yield a maximum gain if that is desired.

2.3 TYPICAL BEHAVIOR OF A DIVERSITY SYSTEM

2.3.1 Experimental Data

The VPI&SU Satellite Communications Group has been operating two earth stations in a site diversity experiment since July 1, 1980. The received signal is at a frequency of 11.6 GHz and emanates from a beacon on the geosynchronous satellite SIRIO. The path to the satellite has an azimuth of 105.6° from north and is at an elevation angle of 10.7° . The site diversity system has a baseline of 7.3 km located at an angle of 54.4° south from the path of the satellite to the main site (site 1). The signals at each site are sampled by a PDP-11 computer once every second, and the signal levels are recorded if a change in signal amplitude of 0.7 dB has occurred since the last recorded sample. The data for each month are then reviewed and when it is determined

that a propagation event has occurred a 0 dB attenuation level for the event is estimated. The corresponding event attenuations are calculated for time intervals of 30 seconds and recorded on magnetic tape. All data not included on this event only attenuation tape are assumed to have a value of 0 dB attenuation (clear weather). This data base contains, at the present time, event data from first year of the experiment (July 1, 1980 - June 30, 1981). This represents a total of 90 events with attenuations ranging up to 27 dB. A plot of the attenuation distributions (including clear weather) for both site 1, and the remote site (site 2), for this period are shown in Figure 2-3.

The joint attenuation is also found when the data are being processed. It is defined to be the minimum of the two single site attenuations for each 30 second interval data point.

$$A_J(t) = \text{Min}[A_1(t), A_2(t)] \quad (19)$$

The distribution of the joint attenuation for the same data period as the single site attenuations above, is shown in Figure 2-4.

To find the attenuation diversity gain, G_{AD} , refer back to (8). The single site attenuation and joint attenuation distributions are all that are required. The two separate

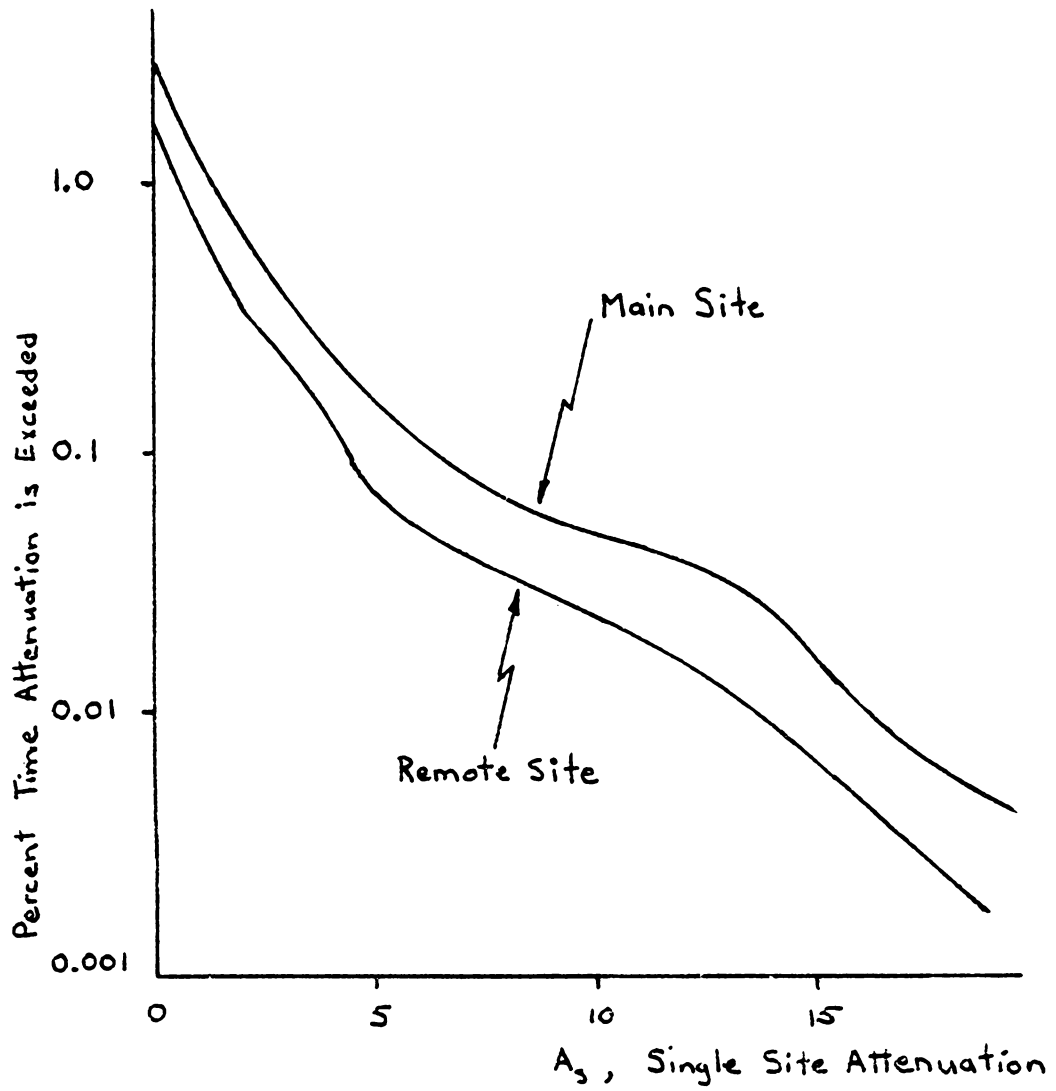


Figure 2-3. Single site attenuation exceedance plots of data measured using the VPI&SU site diversity experiment.

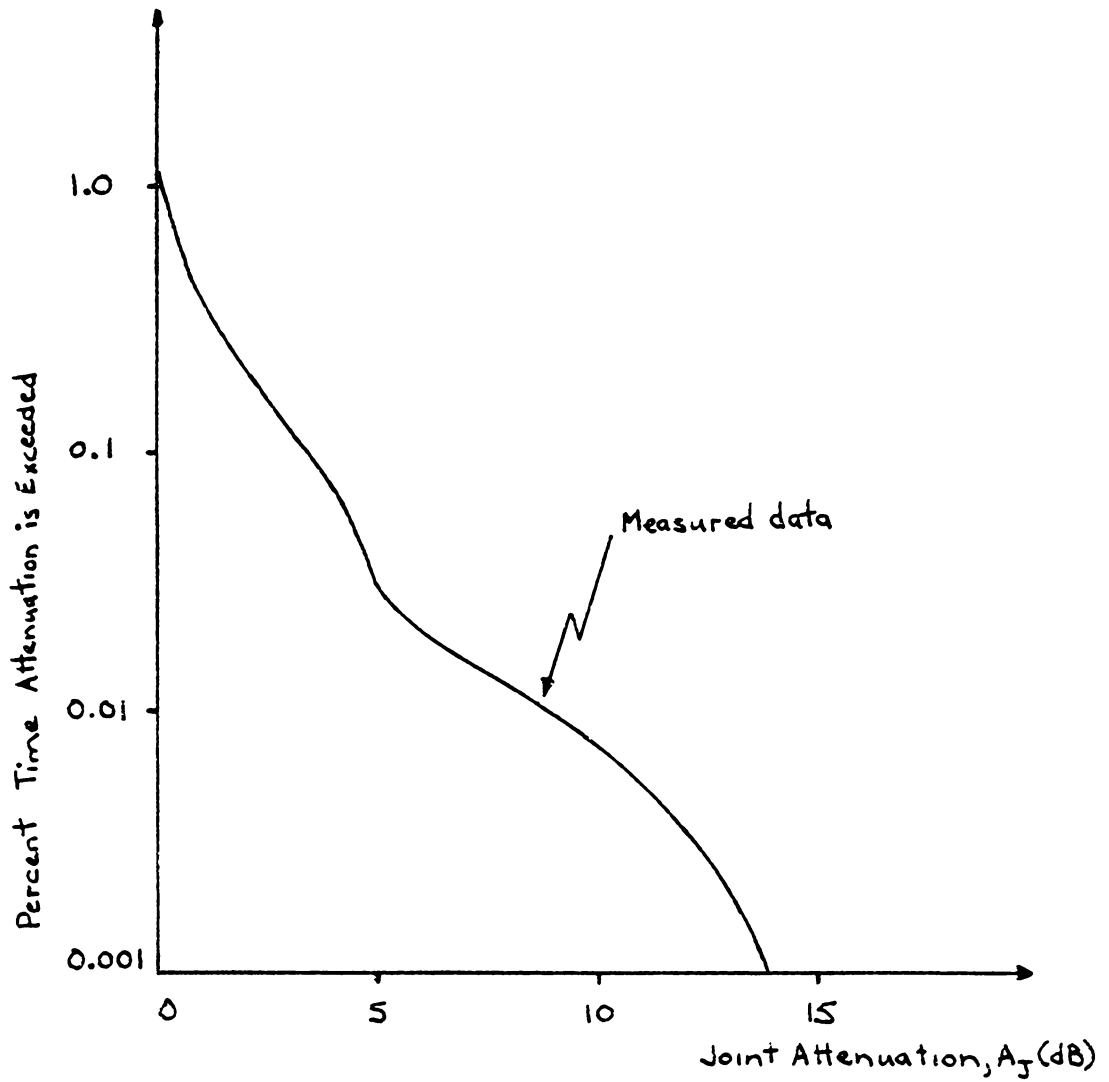


Figure 2-4. Attenuation exceedance plot of the joint attenuation measured using the VPI&SU site diversity experiment.

single site attenuation distributions must be averaged for given percentages of time, because there is no reason to use one distribution over the other, [9], along with the single site attenuation distributions from which it was derived. This is displayed along with the joint attenuation distribution in Figure 2-5 . The diversity gain for any given average single site attenuation is the difference between it and the joint attenuation occurring at the same percent of a time. A plot of measured diversity gain versus the average measured single site attenuation is shown in Figure 2-6. These results compare favorably with other collected data for values of average single site attenuation up to about 12 dB. At about 12 dB the diversity gain decreases slightly and then continues increasing. This phenomenon has not been observed in other experiments and the reasons for this are as yet unknown.

2.3.2 Modelled Predictions

The model of Hodge described by equations (12) thru (18) was derived by curve fitting to data from over 30 site diversity experiments, one of which was the VPI&SU site diversity experiment. To show the effectiveness of this model, the single site attenuation distribution of the VPI&SU experiment was used along with the parameters associated with

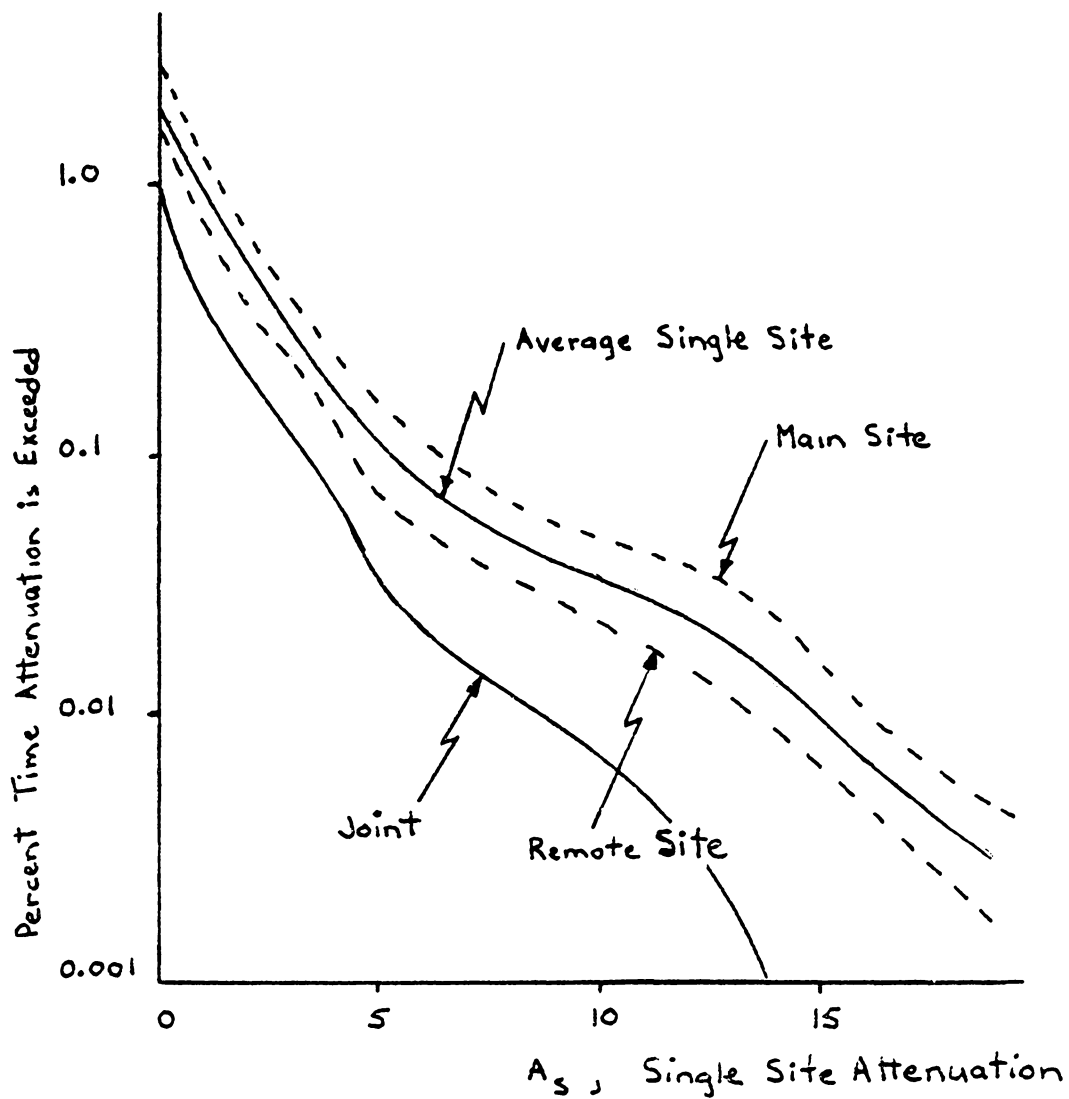


Figure 2-5. Attenuation exceedance plot of average single site attenuation and joint attenuation for data measured with VPI&SU site diversity experiment.

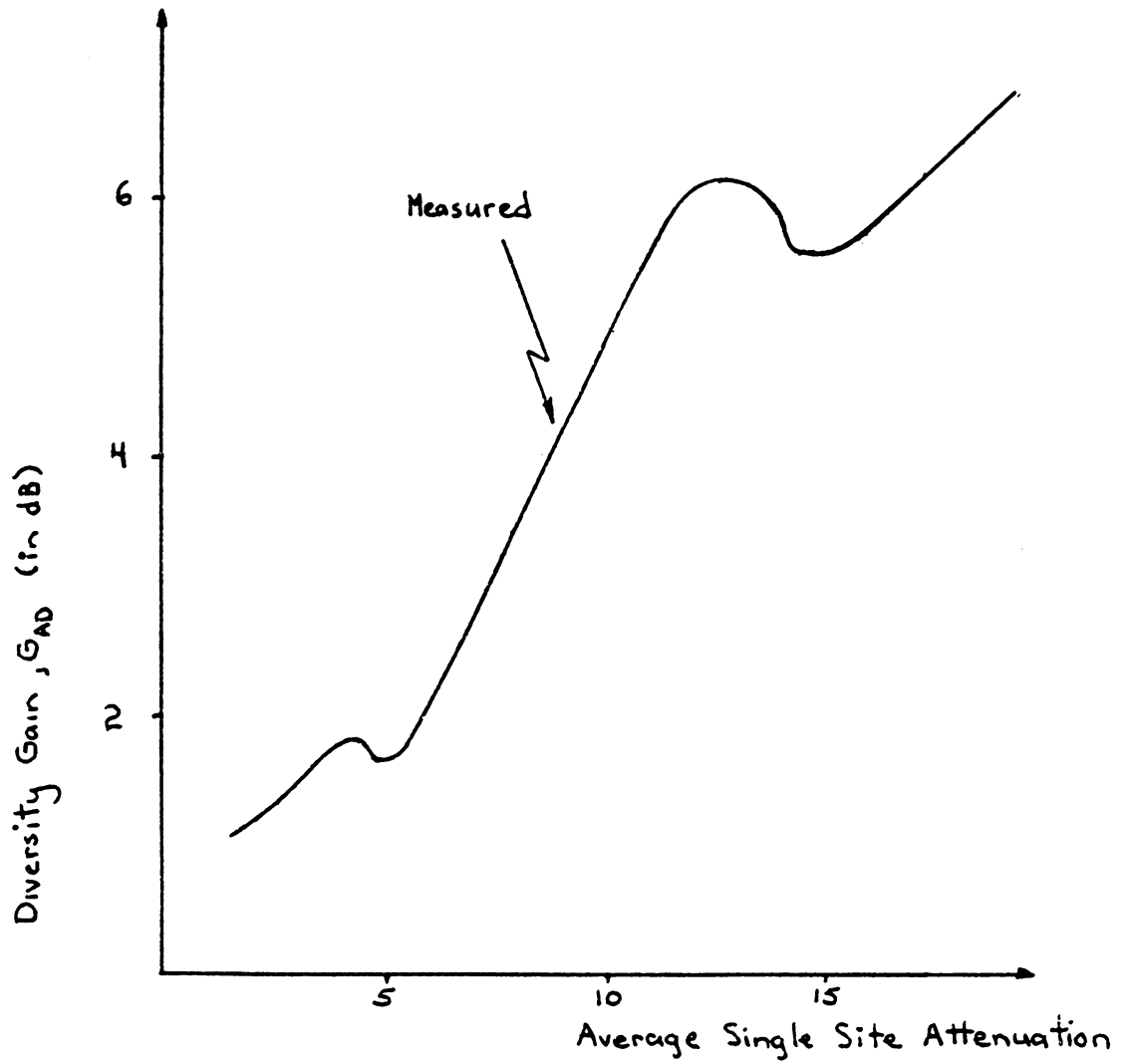


Figure 2-6. Plot of attenuation diversity gain versus average single site attenuation for data measured with the VPI&SU site diversity experiment.

FREQUENCY = 11.6 GHz

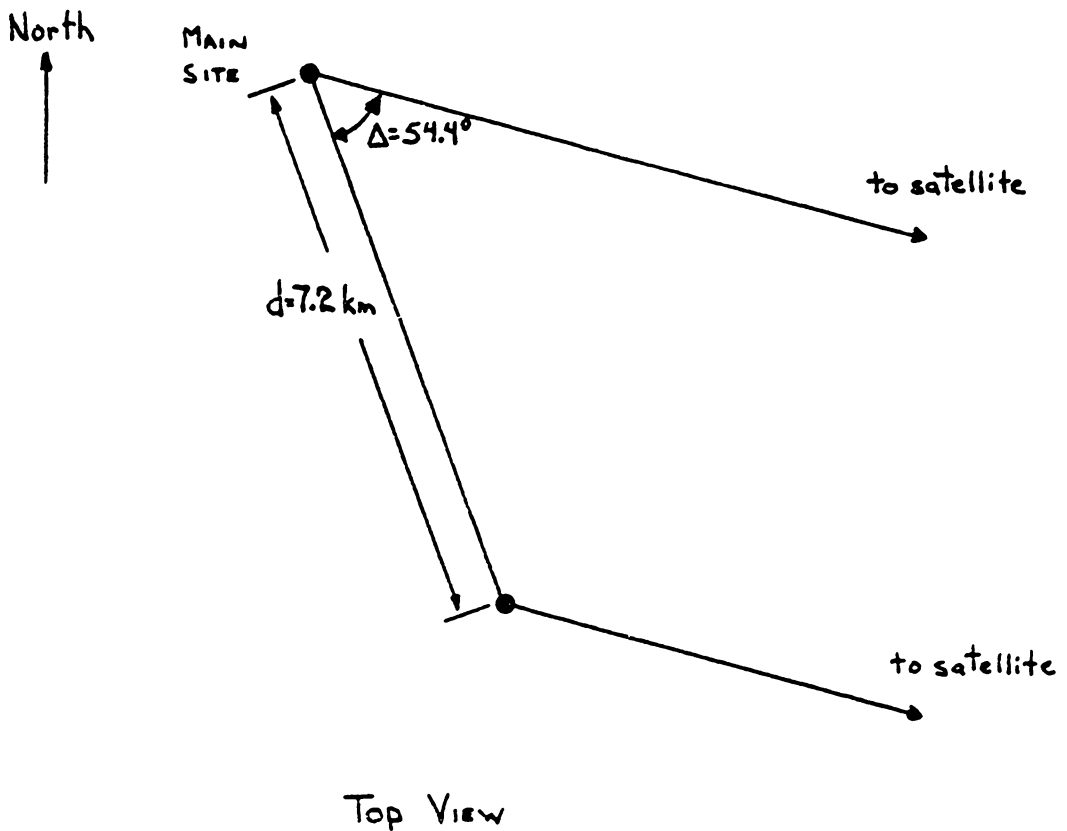
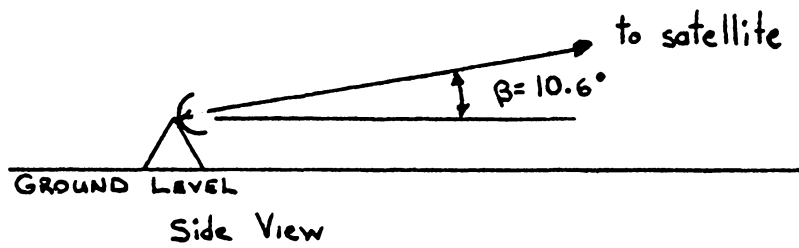


Figure 2-7. The VPI&SU site diversity experiment system parameters for use in the revised model of Hodge (8).

the VPI&SU site diversity system (shown in Figure 2-7) to find the modelled attenuation diversity gain, G_H . A plot of measured attenuation diversity gain and modelled attenuation diversity gain, versus single site attenuation, in Figure 2-8, show close agreement up to about 12 dB where the VPI&SU data diverges from the modelled data as it does when compared to other diversity data as described earlier. The results obtained using the model are good, with the errors occurring for attenuations that are exceeded a very small fraction of the year.

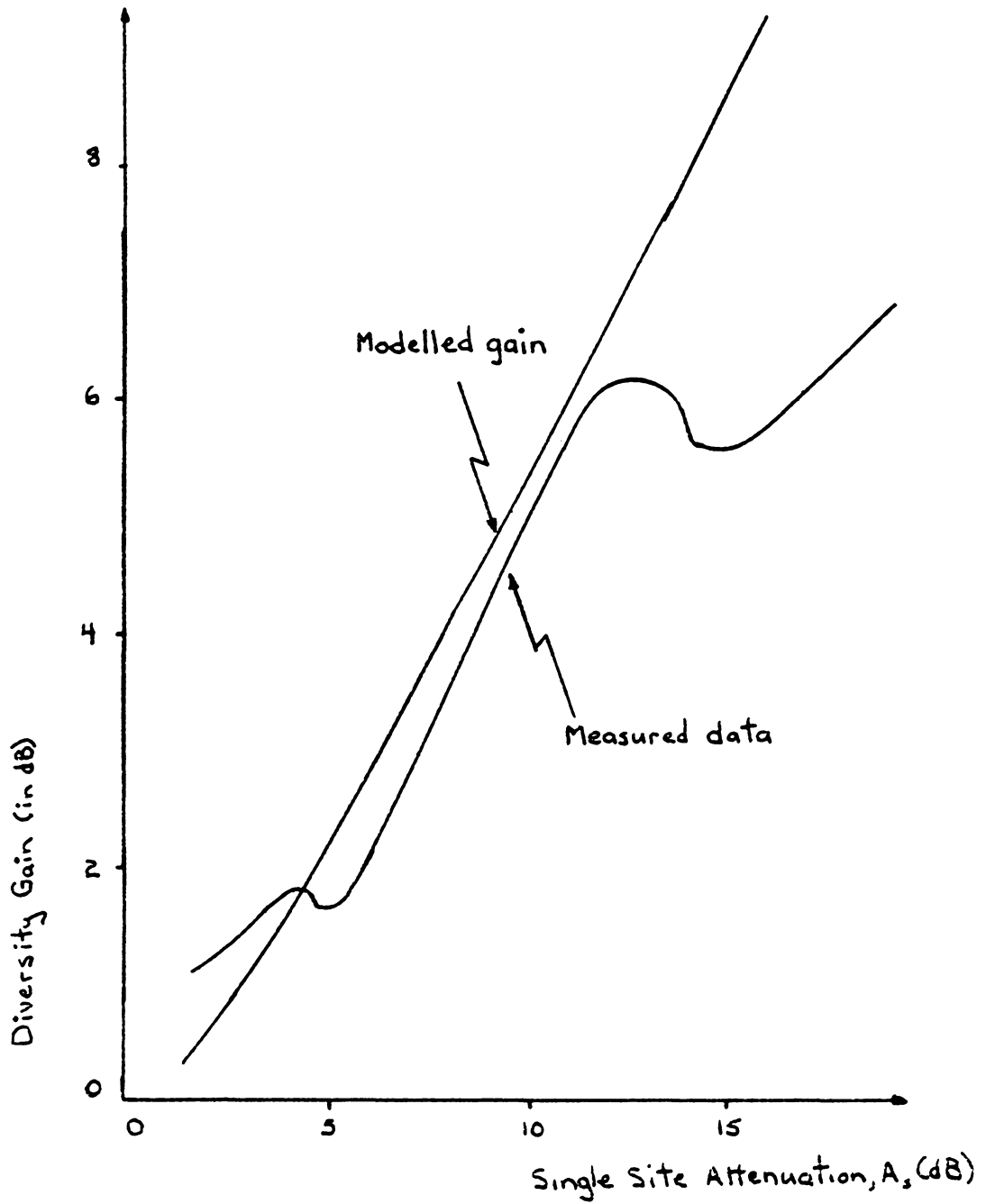


Figure 2-8. Plot of both modeled and measured attenuation diversity gain versus average single-site attenuation.

Chapter III

INSTANTANEOUS DIVERSITY GAIN

3.1 DEFINITION OF INSTANTANEOUS DIVERSITY GAIN.

One method for operating a site diversity system requires that each of the received signals be continuously sampled. The diversity system output signal, for each sample, is then simply the strongest signal available. The attenuation that it represents is called the joint attenuation. The instantaneous relationship between the fading at each of the various sites in the diversity system determines the instantaneous behavior of the joint attenuation. This means that the performance of a site diversity system may be described by analyzing the instantaneous fading at each of the single sites.

A logical choice for this description would be to extend statistical diversity gain to an instantaneous diversity gain. Instantaneous gain, G_S , is defined as the difference between simultaneous values of the attenuation found at a single site, $A_S(t)$, and the joint attenuation, $A_J(t)$.

$$G_S = A_S(t) - A_J(t) \quad (20)$$

The values of instantaneous gain derived from this equation describe the gain that may be achieved by using a site diversity system, with attenuation $A_J(t)$, instead of the single site system with attenuation described by $A_S(t)$. Thus the instantaneous gain describes the behavior of the site diversity system relative to the behavior of one element of the system.

But there is a flaw in this definition. The instantaneous diversity gain can be zero under some conditions when the sites of the diversity system are experiencing unequal fades. This is shown using, as an example, a two site diversity system.

The system has attenuations $A_1(t)$ and $A_2(t)$. The instantaneous gain, $G_1(t)$, is found using

$$G_1(t) = A_1(t) - A_J(t) \quad (21)$$

and the instantaneous gain, $G_2(t)$, is found from

$$G_2(t) = A_2(t) - A_J(t) \quad (22)$$

If for a time, t_1 , the attenuation $A_1(t_1)$ is less than $A_2(t_1)$ then the joint attenuation $A_J(t_1)$ will be equal to $A_1(t_1)$ and the gain $G_1(t_1)$ will be zero. The gain, $G_2(t_1)$, will however be nonzero and this value of the gain is obviously the gain available from the site diversity system at time t_1 .

This problem leads to the development of a better definition of instantaneous diversity gain, $G_I(t)$. This definition, like the previous one, is applicable to site diversity systems with 2 or more separate sites. It is described in terms of the largest fade experienced at an instant in time and the smallest fade at the same instant. This is shown by

$$A_I(t) = A_{MAX}(t) - A_{MIN}(t) \quad (23)$$

and will only be zero if all of the single site attenuations are the same. This equation is much more representative of the behavior of the diversity system.

The attenuations above, $A_{MAX}(t)$ and $A_{MIN}(t)$, are defined for an N-site diversity system as shown below.

$$A_{MAX}(t) = \text{MAX}\{A_1(t), A_2(t), \dots, A_N(t)\} \quad (24)$$

$$A_{MIN}(t) = \text{MIN}\{A_1(t), A_2(t), \dots, A_N(t)\} \quad (25)$$

3.2 BEHAVIOR OF INSTANTANEOUS DIVERSITY GAIN

For instantaneous diversity gain to be used in an efficient manner its behavior needs to be well understood. This requires the study of large amounts of data from many different site diversity experiments. At the present time we have available diversity data from one site diversity experiment which may be used to study the behavior of instantaneous diversity gain.

This is the same site diversity experiment that was analyzed previously for statistical diversity gain. The data used to study instantaneous diversity gain, however, includes only the periods during which propagation events occurred. This does not alter the significance of the results obtained; it just confines any comparisons with the results of statistical diversity gain to the same time period.

The analysis of the event-only site diversity data was performed in several steps using the computer programming language SAS (Statistical Analysis System). The first step was to calculate the value of the instantaneous diversity gain, using equation (23), for each 30 second sample. As expected for each value of attenuation A_{MAX} there exists a distribution of values of instantaneous diversity gain. The distributions of these instantaneous diversity gain values is then found for each value of A_{MAX} , as the next step.

These distributions yield very interesting results. Figure 3-1 is a plot of the distributions of instantaneous diversity gain versus A_{MAX} . This plot shows that for 10% of the time an event occurs all of the gain possible is achieved and also that for 10% of the time an event occurs little (1dB) or zero gain is achieved. These are unexpected results when considering the statistical diversity gain for the same event periods. Figure 3-2 shows the measured statistical diversity gain curve for the same period (events-only) superimposed on the distribution curve of instantaneous diversity gain. It is obvious that the statistical diversity gain curve underpredicts the mean and median values of instantaneous diversity gain. This means that an engineer using statistical diversity gain to set a system margin may require an excess amount of power to be transmitted for the system to be operational the desired amount. This can be costly.

3.3 BENEFITS OF INSTANTANEOUS DIVERSITY GAIN

The benefits of instantaneous diversity gain rest in its ability to describe the instantaneous behavior of the site diversity system with respect to the instantaneous behavior of a receiving system with only a single site. Statistical diversity gain is unable to do this because the distribu-

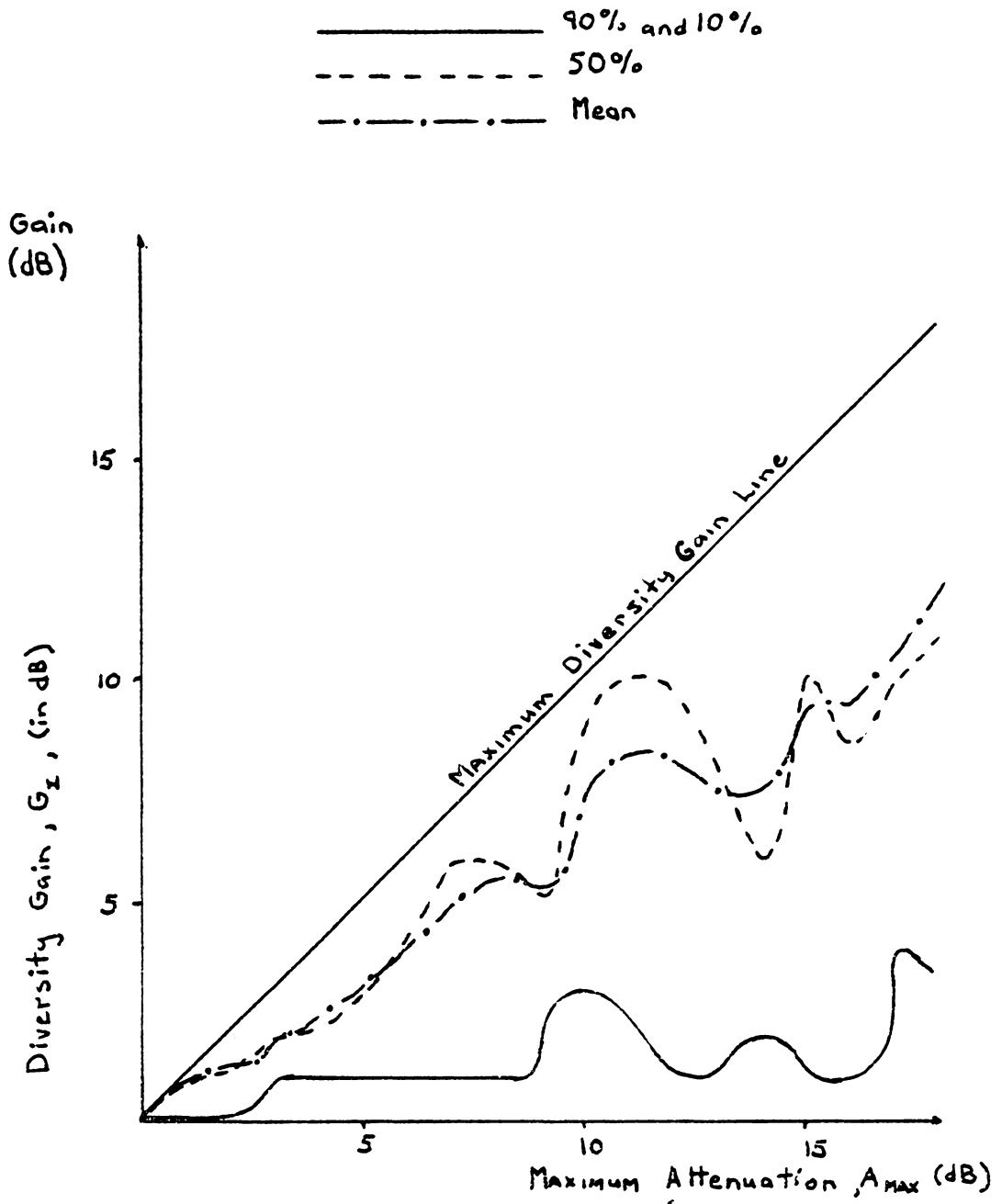


Figure 3-1. Plot showing the 90%, 50%, mean value, and 10% points of the distributions of instantaneous diversity gain versus maximum single site attenuation.

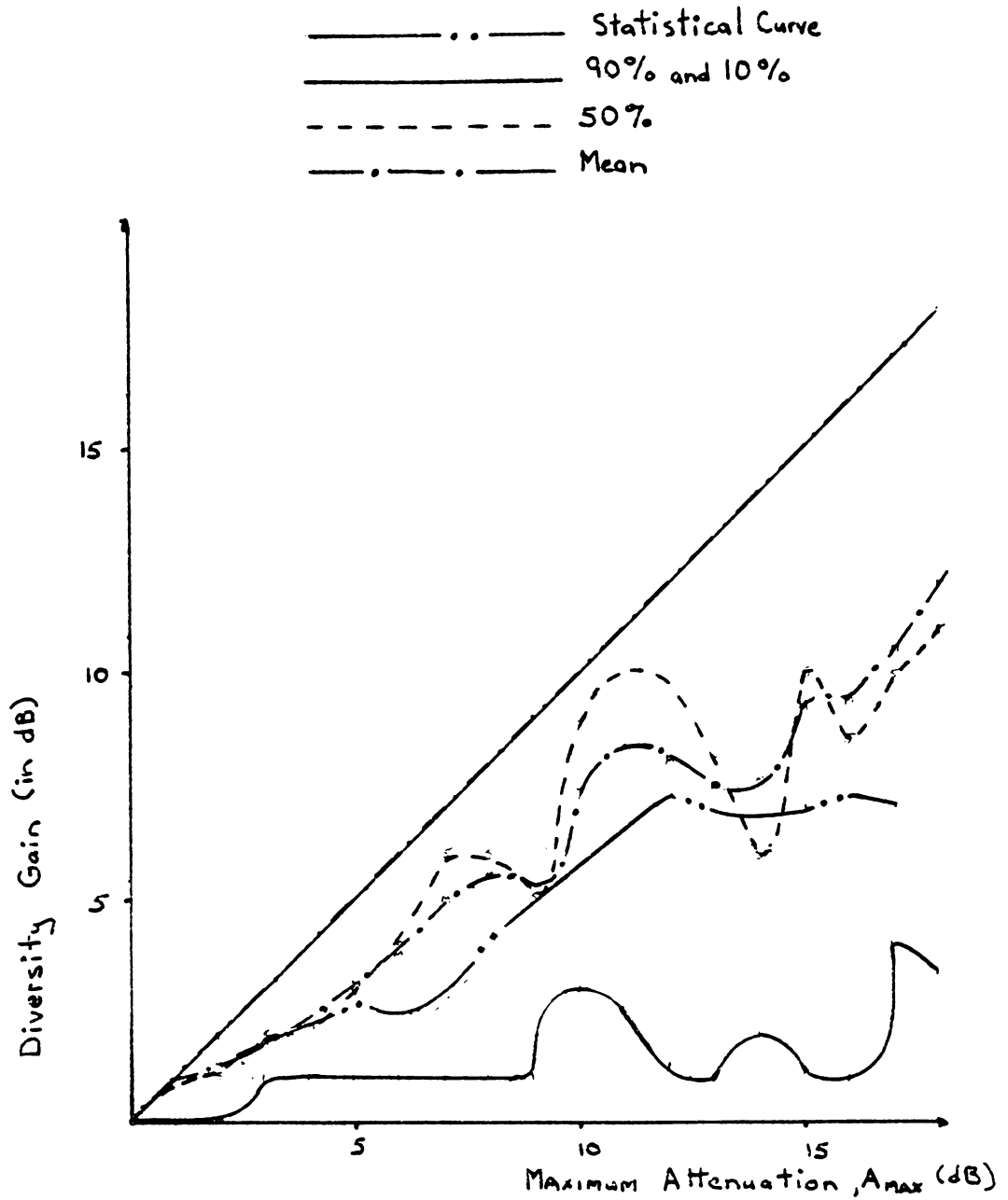


Figure 3-2. Same plot as Figure 3-1 with statistical diversity gain curve of Figure 2-6 superimposed on it for easy comparison.

tions used to evaluate it do not contain information on the instantaneous behavior of the system.

This can be shown by referring to Figure 3-2. The measured statistical diversity gain would seem to indicate that for a given single site attenuation only one value of diversity gain exists. The distributions of instantaneous diversity gain however show that this is not the case.

The lack of total characterization of a diversity system with statistical diversity gain may also be shown by the following examples. First, consider a two-site diversity system with instantaneous single site attenuations given by $A_1(t)$ and $A_2(t)$, and an instantaneous joint attenuation, $A_J(t)$, defined as the minimum of $A_1(t)$ and $A_2(t)$ for each time, t . This site diversity system can have the same statistical diversity gain for a given single site attenuation, A_1 , (and resulting probability P) and yet the instantaneous performance of the system depends on when the fading at one single site occurs with respects to the fading of the other single site.

Figure 3-3 shows hypothetical time histories for the single site and joint, and maximum attenuations of such a system. The statistical diversity gain is found to be

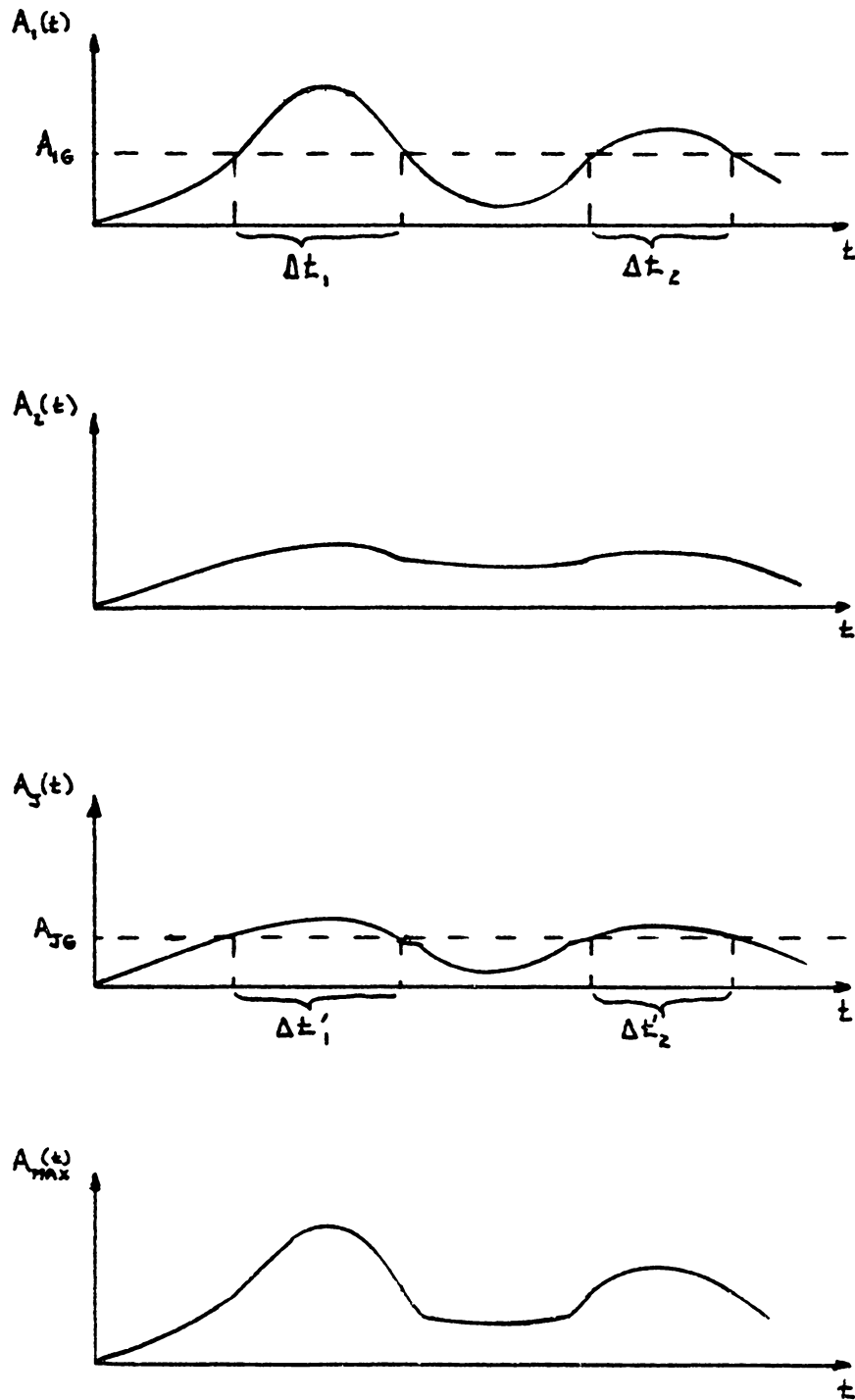


Figure 3-3. Hypothetical time histories of single sites, joint and, maximum attenuations for two site diversity system.

$$G_{AD} = A_{1G} - A_{JG} \quad (26)$$

where the probability that $A_1(t)$ exceeds A_{1G} , given by

$$\text{Prob}[A_{1G} < A_1(t)] = \sum_{i=1}^N (\Delta t_i) / T \quad (27)$$

equals the probability that $A_J(t)$ exceeds A_{JG} , given by

$$\text{Prob}[A_{JG} < A_J(t)] = \sum_{i=1}^N (\Delta t'_i) / T \quad (28)$$

Figure 3-4 shows a second example of the same type of time histories. The statistical diversity gain found here is identical to that of the previous example for the same probabilities. However, at each instant in time the gain achieved by the system of this example is much better. This is because the fading at each site occurs at different times, whereas in the first example the fades occurred virtually simultaneously.

These two examples, while they are hypothetical, describe conditions which might occur during single propagation events. Large experimental data sets contain a multitude of propagation events which are variations of these examples. This means that for any given event the statistical diversity gain may over or under predict the actual instantaneous behavior of the system.

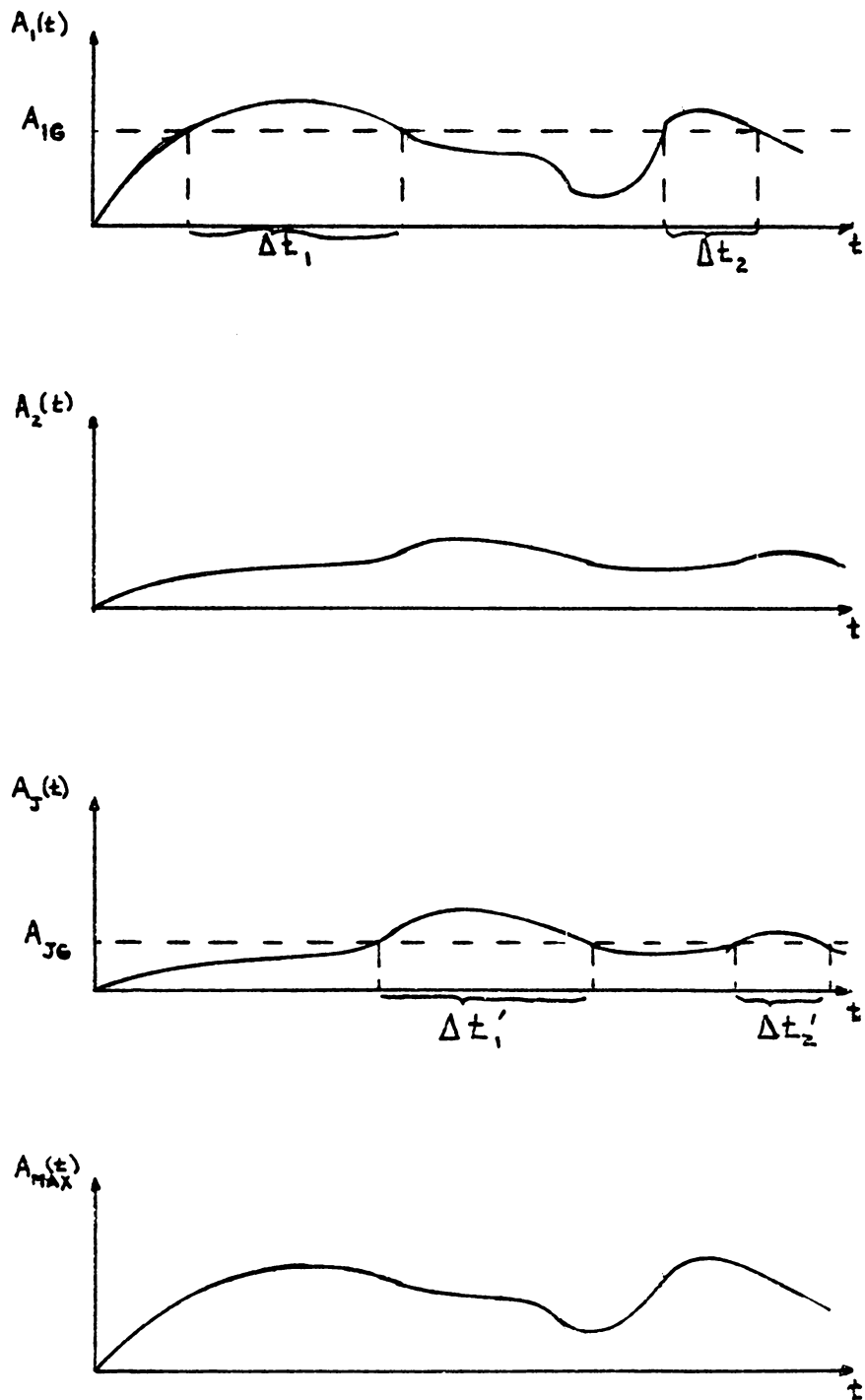


Figure 3-4. Second hypothetical set of the same time histories as in Figure 3-3.

A systems engineer must remember that when statistical diversity gain is used, about 50% of the time the system will not realize that gain. Using the distributions found for instantaneous diversity gain will give the engineer a much better idea of how the system will perform and allow the engineer to use a value of diversity gain in the margin calculations which will be achieved for a much larger period of time.

Chapter IV

USING THE CORRELATION COEFFICIENT TO STUDY DIVERSITY

4.1 INTRODUCTION

As mentioned earlier diversity systems of all types depend solely on how correlated the received signals are. Total correlation results in no diversity gain while non-correlation will result in maximum diversity gain. These two conditions are the extremes of diversity performance and it was shown earlier that the majority of the data lie in between these two extremes. Exactly where between the extremes depends on the diversity system and in particular on the degree of correlation between the single site signals over the long term operation of the system. A measure of this correlation is obviously the correlation coefficient.

The questions logically brought up now are: What relationship is there between diversity performance, specifically described by instantaneous diversity gain, and the correlation coefficient? And if there does exist such a relationship can it be found and expressed in a simple fashion?

4.2 EMPIRICAL STUDY

To compare the correlation coefficient with instantaneous diversity gain is not straightforward. This is because instantaneous diversity gain is represented by a set of curves displaying the distribution of the instantaneous diversity gain for many single site attenuations while the correlation coefficient is just a single number. However if each portion of the instantaneous diversity gain curve were represented by a straight line curve fit as shown in Figure 4-1, then the assumption that the correlation coefficient represents a measure of the slope of one of these curves would be a starting point.

This comparison is very easy to perform using the event-only data set analyzed in the study of instantaneous diversity gain. For this analysis, though, each event was analyzed separately. The slope of the mean values of the distributions were found by curve fitting to the instantaneous diversity gain data. The correlation coefficient between the two single site attenuations was found using a product-moment procedure. A plot of the slope, S , versus correlation coefficient, r , for each event is shown in Figure 4-2.

An initial, simple, guess of what the relationship between slope and correlation coefficient might be is that they

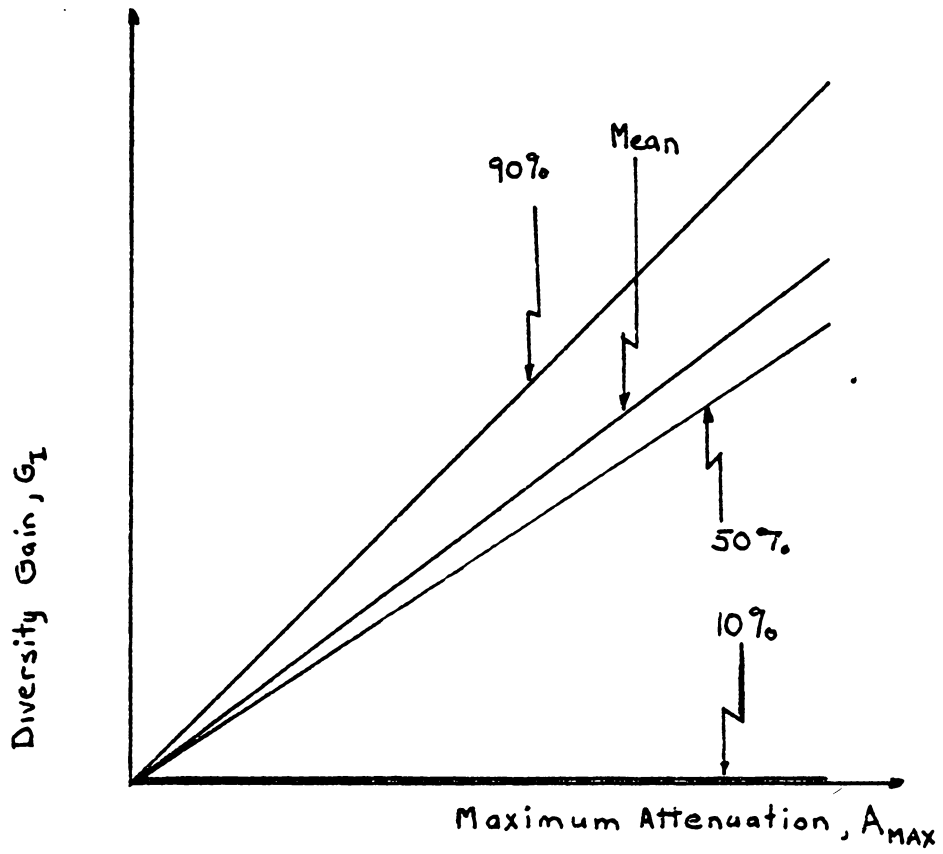


Figure 4-1. Example of linear interpretation of the distribution curves of Figure 3-1.

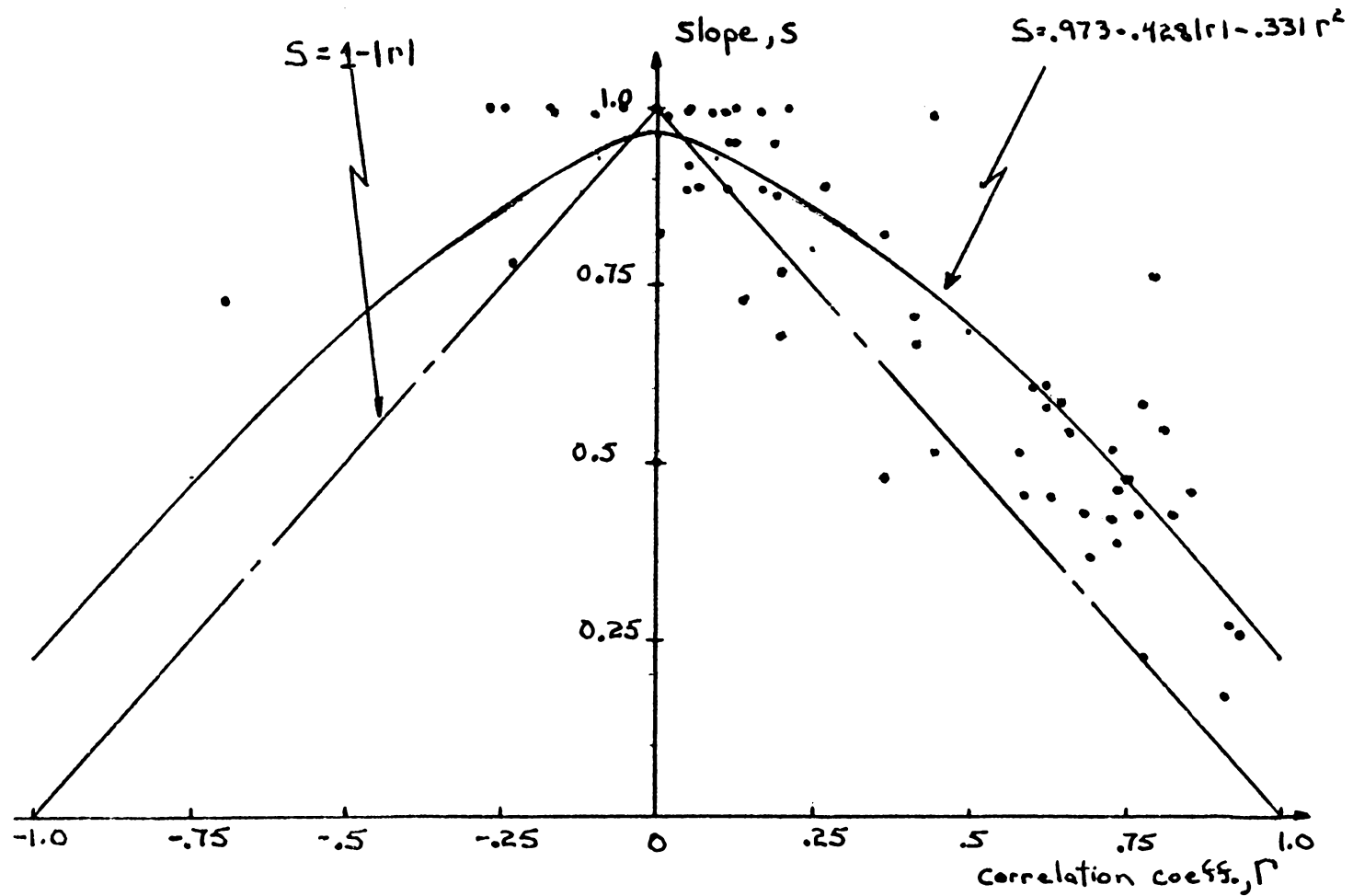


Figure 4-2. Scatter plot of the slope of linear curve fit to mean instantaneous diversity gain versus the correlation coefficient of the single site attenuations.

are linearly related. The straight lines shown in Figure 4-2 show the linear relationship

$$S = 1 - |r| \quad (29)$$

which at first seems reasonable. A curve fit to these points has shown, however, that a better relationship to be a quadratic of the form

$$S = .973 - .428|r| - .331|r|^2 \quad (30)$$

Thus for the one year of event only data from VPI&SU the relationship between the correlation coefficient and diversity gain appears quite simple. However, because this relationship was derived empirically from one data base its applicability to other data sets is obviously in question.

4.3 COMPARISON WITH OTHER WORK

Now that an empirical relationship between instantaneous diversity gain and the correlation coefficient has been found, the results need to be checked against the existing literature to see if the results are useful. Unfortunately this is not an easy task since only one paper seems to have been published in the site diversity field on the correlation between the received signals. This work is in reference [10] and is authored by Murita and Higuta.

Murita and Higuta assumed a log normal distribution of path averaged rain rate and found a relationship between the correlation coefficient of the path average rain rate at two sites of a site diversity system and site separation. They then plot this relationship with elevation angle as a parameter. They state that these correlation coefficients of path average rain rate are approximately equal to the correlation coefficient for the resulting attenuations. The correlation coefficient taken off of this curve was found to be approximately 0.72 for the VPI&SU site diversity experiment.

Because of the format of the VPI&SU diversity data base, calculation of a correlation coefficient to compare with that found from the work of Murita and Higuta would be very time consuming and difficult. This is because the data base contains events only, not clear weather. A correlation coefficient calculated for this set of data should be smaller in magnitude than 0.72 because the clear weather data not in the data set will obviously be highly correlated. The event-only, correlation coefficient calculated was 0.493 which supports this claim.

At this point we have derived an empirical relationship between the correlation coefficient of two fading signals and their corresponding instantaneous diversity gains. Then to show that this relationship may be assumed to be univer-

sal, we have used work by Murita and Higuta to find a system correlation coefficient (for long periods of time not just events) which is expectedly larger than the system correlation coefficient found from measured event only data. Now if we could show that the addition of the missing clear weather data to the VPI&SU data base would raise the system correlation coefficient to that found from the work of Murita and Higuta then the assumption that the correlation coefficient instantaneous diversity gain relationship may be universal is valid.

Murita and Higuta have plotted the percent time that the joint attenuation is exceeded versus correlation coefficient with the percent time that the single site attenuation is exceeded as a parameter. Using the VPI&SU single site attenuation exceedance curve as a base, the joint attenuation exceedance found from Figure 4-3 (taken from [10]) is plotted along with the measured joint attenuation in Figure 4-4. The agreement between the results is very good and, at least, that there is agreement between the data used to find (30) and the theory of Murita and Higuta. This allows us to make the assumption that the relationship between the correlation coefficient and instantaneous diversity gain is a "universal" relationship.

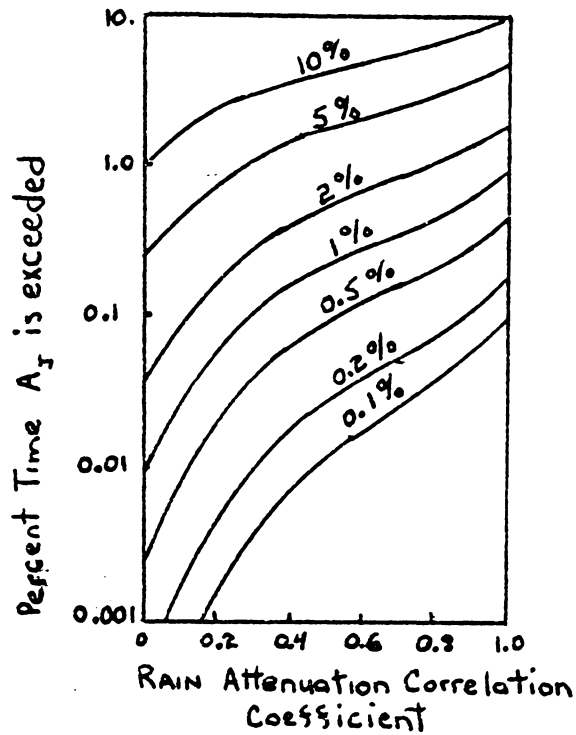


Figure 4-3. This is Figure 9 of Murita and Higuta [10] relating correlation coefficient and percent time the joint attenuation is exceeded with percent time the single site attenuation is exceeded as a parameter.

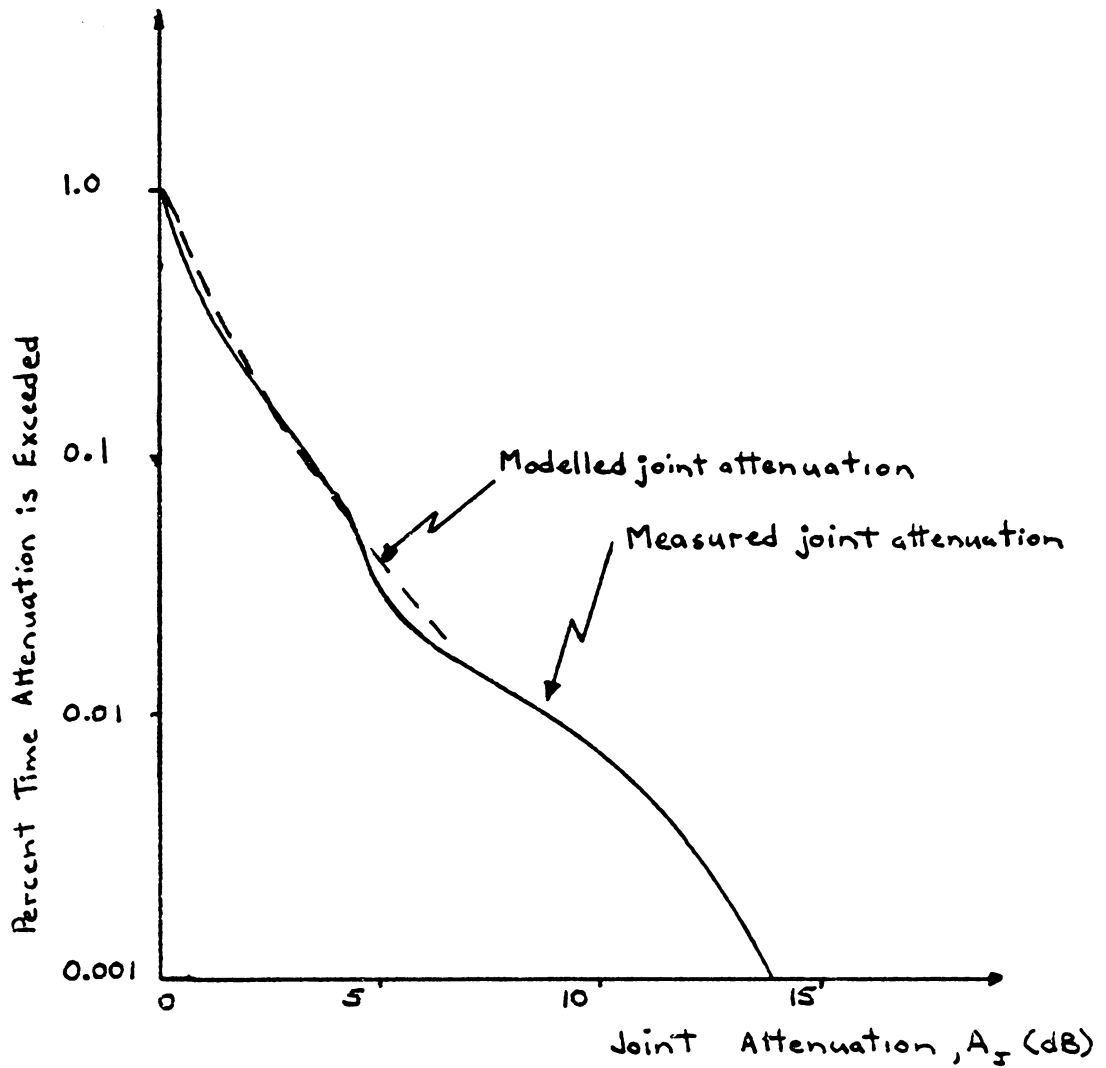


Figure 4-4. Exceedance plot of joint attenuation found using (10), (and a correlation coefficient of 0.72) and the actual measured joint attenuation.

4.4 A MODEL OF DIVERSITY GAIN

The assumption that the empirical relationship of equation (30) is universal suggests the development of a very simple model of diversity gain.

This model is a linear model with a slope dependent on the correlation coefficient according to the relationship of (30), with the correlation coefficient being dependent on site separation. The modeled gain, G_A , is shown below in terms of slope, s , and single site attenuation, A_S .

$$G_A = s(r)A_S \quad (31)$$

or using (30) for s ,

$$G_A = (.973 - .428|r| - .331|r|^2)A_S \quad (32)$$

This modeled gain, G_A , is shown plotted versus single site attenuation in Figure 4-5 for the VPI&SU site diversity experiment, using a values of $|r| = 0.72$ (from Murita and Higuta), and of $|r|=0.491$ (from the VPI&SU event-only data).

Also plotted is the Hodge model [8]. For attenuations up to 8dB the simple model, with $|r|=0.72$, agrees well with the Hodge model. The increasing error for attenuations above 8dB, is due to the fact that this correlation coefficient reflects the large amount of correlated clear weather during

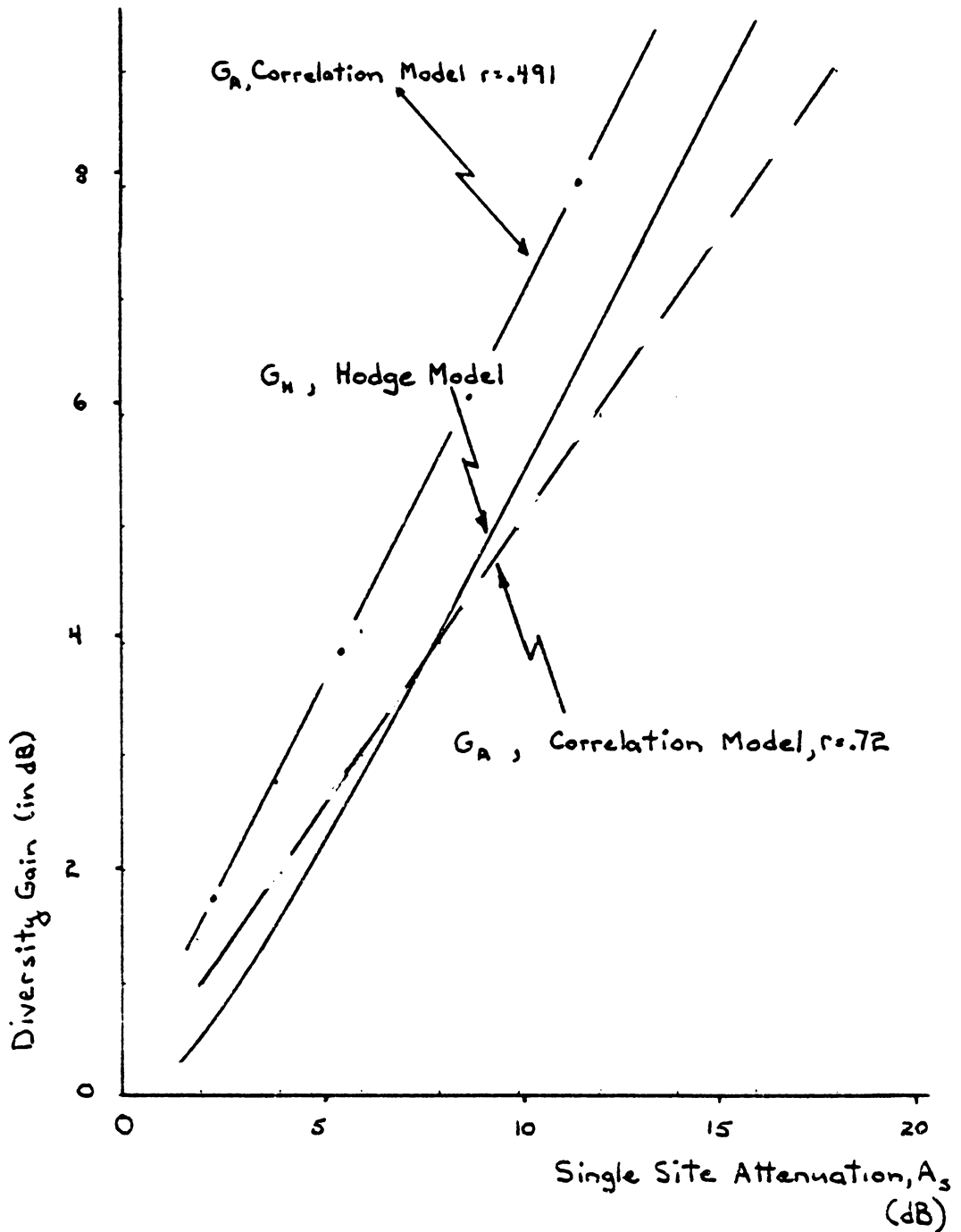


Figure 4-5. Plot of the diversity gains for the VPI&SU site diversity experiment found using the model of Hodge and the correlation coefficient model presented here.

a year. The simple model, with $|r|=0.491$, agrees well with the Hodge model for attenuations above 8dB. This is because it was found for events only and the large amount of clear weather was not included.

This is also reflected in Figure 4-6 where the gain is plotted versus site separation with single site attenuation as a parameter. For single site attenuations up to 8dB the two curves differ by 1dB at the most. Again the effect at higher attenuations is due to the biasing of the correlation coefficient by the large amounts of clear weather normally experienced.

While this kind of model is not suitable for use in designing systems with narrow margins it is not bad considering it was derived from a single data set of 1 years data. With an increase in the amount of data studied in this fashion this model could possibly be adjusted to yield suitable results. In addition this model has the added benefit that the correlation coefficient may be found from measurements of the path average rain rate so that there would be no need to actually measure an attenuation distribution.

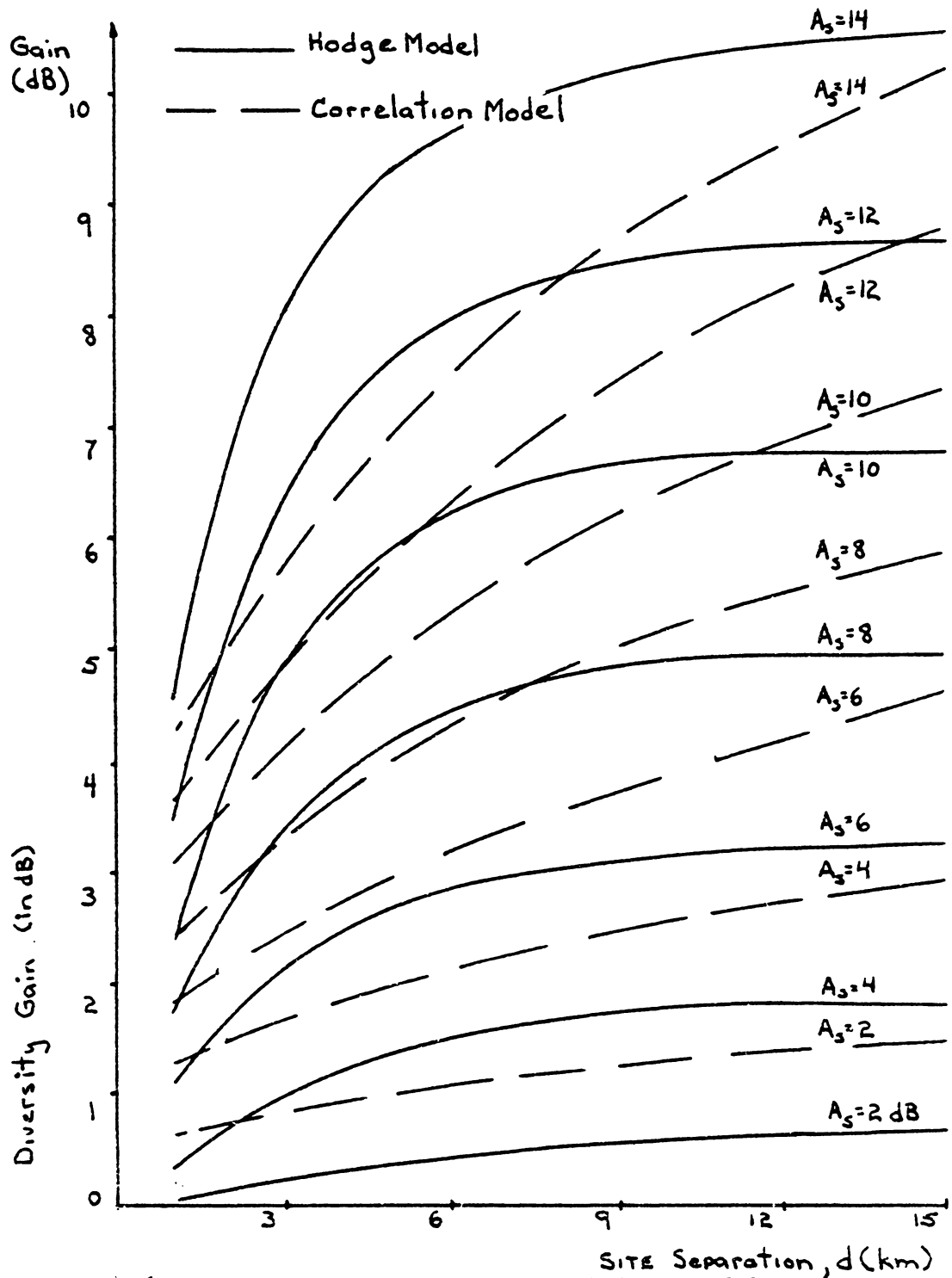


Figure 4-6. Plot displaying dependence of Hodge model, and correlation coefficient model, of diversity gain on site separation. Single site attenuation is also a parameter.

Chapter V
CONCLUSIONS

5.1 APPLICATIONS OF INSTANTANEOUS DIVERSITY GAIN

When satellite communications first started with 6/4 GHz transmit and receive frequencies the calculation of a system margin for propagation losses, other than free space, was fairly simple because there was little attenuation at those frequencies. However, with the current crowding in space it has been necessary to increase the transmit and receive frequencies for satellite communications. The new frequencies experience signal fading when there is rain in the path between the earth station and satellite. This fading can be quite severe.

One proposed solution to this problem is the use of site diversity. This entails using two or more separate earth stations to receive the same satellite transmission (or broadcast the same transmission to a satellite). The earth stations are physically remote from each other by distances in excess of 1 km. Because rain cells of large magnitude tend to be spatially small in size the separate earth stations will most probably not fade at the same time. The

overall site diversity system has a signal which is called the joint signal and is the strongest (smallest attenuation) signal at any time, t . The description of the behavior of the joint attenuation may also be done with an exceedance curve or by using the single site attenuation exceedance curve to find the attenuation expected, and a statistical attenuation diversity gain curve to determine the gain the system received by adding the other diversity sites. Either of these methods yield fairly good results, but they tend to be conservative and they do not fully describe what is actually happening as the diversity system operates. For each single site attenuation level experienced the site diversity system has a multitude of possible outputs each dependent on how the other sites in the system are behaving at that same time.

For this reason instantaneous diversity gain was defined. It evaluates the system performance at each instant in time and then displays this performance as distributions of the possible system gains for given single site attenuations. This enables an engineer to choose a less conservative value of diversity system gain for the margin calculations. This could save a great deal of money and effort when you consider the savings in the cost of a satellite when an extra 2dB of gain does not need to be added to a system.

Conversely, designing a system with a given margin and then having it not work at this margin for the required amount of time because the system does not achieve the designed gain often enough is costly too.

5.2 SAMPLE MARGIN CALCULATIONS

In an effort to show the usefulness of instantaneous diversity gain, several sample margin calculations have been performed, for both single site and site diversity receiving systems, using the VPI&SU site diversity experiment data, for six different system reliabilities. Tables I and II display the results of these calculations while Figures 2-5, 2-6, 2-8, 4-4, 3-1 show the curves of attenuation exceedance, statistical diversity gain, Hodge model diversity gain, correlation coefficient model gain ($|r|=0.72$) and instantaneous diversity gain, respectively, that were used to obtain these results.

The system margins were found based on the knowledge that the margin, for a given system reliability, is the difference between a clear weather signal-to-noise ratio and a minimum signal-to-noise ratio given by

Table I. Table of system margins calculated for single site and site diversity receiving systems for various reliabilities.

% TIME, P, A - IS EXCEEDED	SINGLE SITE MARGIN	DIVERSITY SYSTEM MARGINS		
		STATISTICAL DIV. GAIN	HODGE MODEL	CORRELATION MODEL
0.5	2.04	0.801	1.482	1.034
0.1	5.32	3.75	2.964	2.356
0.05	8.12	4.55	4.29	4.115
0.01	15.52	9.69	7.218	7.865
0.005	17.31	11.55	7.905	8.772
0.001	21.43	14.28	9.468	10.86

Table II. Table showing system margins calculated using instantaneous diversity gains occurring for 90%, 75%, 50% (and mean value) of the time that the single site attenuation exceeds P%.

% TIME, P, A _s IS EXCEEDED	SINGLE SITE MARGIN	INSTANTANEOUS DIVERSITY GAIN MARGINS			
		OCCURRING 90% OF P	OCCURRING 75% OF P	OCCURRING 50% OF P	MEAN VALUE
0.5	2.04	2.04	1.04	1.04	0.739
0.1	5.32	4.32	3.00	2.00	1.797
0.05	8.12	7.12	4.12	2.12	2.825
0.01	15.52	14.42	11.895	6.27	6.156
0.005	17.31	13.49	10.94	7.00	6.39
0.001	21.43	14.48	4.99	0.57	3.30

$$M(P) = (S/N)_{CW} - (S/N)_{REQ} \quad (33)$$

For a single site receiving system this is obviously the fade level which occurs at the desired reliability level.

$$M_S(P) = A_S(P) \quad (34)$$

The diversity system margin is then simply the difference between the single site margin and the diversity gain.

$$M_D(P) = A_S(P) - G_{DIV} \quad (35)$$

The diversity gain may be derived from the statistics, instantaneous data, or diversity gain models.

The system margins derived by these methods do not result, however, in the desired reliabilities. This fact becomes obvious after studying the distribution of instantaneous diversity gain for given single site attenuation levels. If the single site margin is found to be exceeded for 0.01 % of the time (as an example) then from the instantaneous diversity gain distributions there exist many values of diversity gain achieved by the system. These values occur for a certain percentage of the time that the single site attenuation occurs. Thus the reliability is reduced by the fraction of time that the designed for instantaneous diversity gain is not achieved. The actual reliability may be expressed by

$$R_{ACT} = R_S - (P_S)(P_I) \quad (36)$$

where R_{ACT} is the actual reliability, R_S is the single site reliability, P_S is the percent time the single site margin is exceeded and, P_I is the percent time that the instantaneous diversity gain is not achieved. Table III displays the actual reliabilities found for each of the given margins of Table I.

Clearly Tables II and III show that to achieve the desired reliability for a site diversity system, the margin must be larger than those found using statistics or models. If however, the reliability may be decreased by a small amount the results of Table II show that substantial decreases in the system margin will result.

Instantaneous diversity gain may be used to achieve the desired margin by designing the system for a reliability greater than is actually required. This way the actual reliability achieved by the diversity system, even though it will be less than it should be for the larger reliability, can be very close to the desired reliability. The 'closeness' to the desired reliability may be altered by the choice of instantaneous diversity gain. An example of this is shown in Figure 5-1.

Table III. Actual reliability of the margins in Table I.
 These reliabilities were found using the distributions
 of instantaneous diversity gain.

% TIME, P, A IS EXCEEDED	DESIRED RELIABILITY	ACTUAL RELIABILITY FOR GIVEN MARGINS			
		SINGLE SITE	STATISTICAL DIV. GAIN	HODGE MODEL	CORRELATION MODEL
0.5	99.5	99.5	99.22	99.461	99.45
0.1	99.9	99.9	99.88	99.875	99.867
0.05	99.95	99.95	99.9383	99.9378	99.937
0.01	99.99	99.99	99.986	99.985	99.9857
0.005	99.995	99.995	99.9939	99.9928	99.9931
0.001	99.999	99.999	99.99887	99.99885	99.99884

DESIRED RELIABILITY: 99.99%
SINGLE SITE MARGIN : 15.52 dB
DESIGN RELIABILITY : 99.995%
INSTANTANEOUS GAIN : 10.31 dB
(50%)
DIVERSITY MARGIN : 7 dB
ACTUAL RELIABILITY : 99.9925%

Figure 5-1. Sample margin calculation.

5.3 CONCLUDING REMARKS

In this thesis the instantaneous behavior of site diversity receiving systems has been described in two ways. First is instantaneous diversity gain. The calculation of this gain yields a distribution of gains achieved by the system for given single site attenuation. Second is the correlation coefficient. A relationship between the correlation coefficient and instantaneous diversity gain was established, for the VPI&SU data base, which lead to a simple model of diversity gain.

Instantaneous diversity gain yields a great deal more information about instantaneous system behavior than statistical diversity gain and is therefore much more beneficial in system design. The use of instantaneous diversity gain to analyze other existing diversity data should aid in the characterization of the distributions of gain for given single site attenuations. Once this is done, diversity systems may be designed that will achieve the desired reliabilities with minimum margin and cost.

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THE TIME BEHAVIOR OF A SITE DIVERSITY SYSTEM
by

George Crosby Towner III

(ABSTRACT)

The instantaneous performance of a site diversity system is analyzed. This analysis is performed using instantaneous diversity gain (a new parameter for describing diversity performance) and the correlation coefficient. Also a relationship between the correlation coefficient and instantaneous diversity gain was established.

In addition a review of statistical diversity gain and existing models of statistical diversity gain were also presented. Measured statistical diversity gain data from the VPI&SU site diversity experiment were also presented and compared with instantaneous diversity gain.

The relationship between the correlation coefficient and instantaneous diversity gain was used to present a crude model of diversity gain. This model was compared with the model of Hodge.

Margin calculations were performed to display the usefulness of instantaneous diversity gain. These were performed using the VPI&SU site diversity experiment data.