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Appendix A: Sample FORTRAN Programs

A.1. Regular Springs

```
C          *****ASSUMING MASSLESS MOORING LINES*****
      INTEGER MXPARAM,N
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (MXPARAM=50,N=12)
C          *****SPECS FOR PARAMETERS*****
      INTEGER MSTEP, MSOLVE
      PARAMETER (MSTEP=10000000,MSOLVE=2)
C          *****SPECS FOR LOCAL VARIABLES*****
      INTEGER IDO, NOUT
      DIMENSION Y(12),PARAM(50),A(1,1)
C          *****SPECS FOR SUBROUTINES*****
      EXTERNAL DIVPAG,SSET
C          *****SPECS FOR FUNCTIONS*****
      EXTERNAL FCN, FCNJ
C
      CALL UMACH (2, NOUT)

      open(unit=1,file='pd50.dat',status='unknown')

C          *****SET INITIAL CONDITIONS*****
      T=0.D00
      Y(1)=0.D00
      Y(2)=0.D00
      Y(3)=2.83840079416846045D00
      Y(4)=0.D00
      Y(5)=0.D00
      Y(6)=0.D00
      Y(7)=0.D00
      Y(8)=0.D00
      Y(9)=0.D00
      Y(10)=0.D00
      Y(11)=0.D00
      Y(12)=0.D00
      F1=0.D00
      F2=0.D00
      F3=0.D00
      FX=0.D00

C          *****SET MAX TIME*****
      TMAX=300.D00
C          *****SET ERROR TOLERANCE*****
      TOL=.0000000001D00
C          *****SET PARAM TO DEFAULTS***
      CALL SSET (MXPARAM, 0.0, PARAM, 1)
C          *****SET MAXIMUM # OF STEPS ALLOWED*****
      PARAM(4)=MSTEP
C          *****SELECT CHORD METHOD AND A DIVIDED DIFF JACOBIAN***
      PARAM(13)=MSOLVE
C          ***PRINT HEADER***
      WRITE (NOUT, 99996)
      WRITE (1,99997)
```

```

C      ***PRINT INITIAL CONDITIONS*****
      WRITE (NOUT, '(7D10.3)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
      WRITE (1, '(7E14.5)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
C      ***BEGIN DIVPAG LOOP***
      IDO =1
      STEP=0.D00
10     CONTINUE
      STEP = STEP+0.03D00
      TEND = STEP
C      *****A(1,1) IS NOT USED*****
      CALL DIVPAG (IDO, N, FCN, FCNJ, A, T, TEND, TOL, PARAM, Y)
      IF (STEP.LE.TMAX) THEN
          WRITE (NOUT, '(7D10.3)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
          WRITE (1, '(7E14.5)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
C      *****FINAL CALL TO RELEASE WORKSPACE*****
          IF (STEP.EQ.TMAX) IDO=3
          GOTO 10
      END IF
      WRITE (NOUT, 99998) PARAM(35)
C      *****HEADER*****
99996 FORMAT (7X, 'TIME', 11X, 'Y1', 12X, 'Y3', 12X, 'Y5', 12X, 'Y7', 12X, 'Y9', 12X
           $, 'Y11')
99997 FORMAT (7X, 'TIME', 11X, 'Y1', 12X, 'Y3', 12X, 'Y5', 12X, 'Y7', 12X, 'Y9', 12X
           $, 'Y11')
C      *****SHOW # OF FUNTION CALLS***
99998 FORMAT (4X, 'NUMBER OF FCN CALLS WITH DIVPAG =', F6.0)
      END
      SUBROUTINE FCN (N,T,Y,YPRIME)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DIMENSION YPRIME(12),Y(12)
C      *****SPECS FOR ARGUMENTS*****
C      *****THE FOLLOWING ARE NONDIMENSIONAL*****
      R=1.D00
      ZMC=1.D00
      ZWC=1.D00
      ZL=6.D00
      ZIXX=.5D00*ZMC*R**2.D00
      ZIYY=.25D00*ZMC*R**2.D00+(1.D00/12.D00)*ZMC*ZL**2.D00
      ZIZZ=ZIYY
      ZK1=49.9999999999992457967645D00
      ZK2=ZK1
      ZK3=ZK1
      ZK4=ZK1
      C1=0.1D00
      C2=C1
      C3=C1
      C4=C1
      C5=C1
      C6=C1
      ZL1=4.D00
      ZL2=ZL1
      ZL3=ZL1
      ZL4=ZL1
      ZA=2.D00
      ZB=2.D00
      A11=ZL/2.D00
      A12=0.D00

```

```

A13=R
A21=A11
A22=A12
A23=-A13
A31=-A11
A32=A12
A33=A13
A41=-A11
A42=A12
A43=-A13

C *****STOP WHEN Y(3) NEG*****

IF (Y(3).LT.0.D00) THEN
  STOP 00000
END IF

C *****FORCES*****
PI=3.14159265358979323846264338327950288419717D00
EPSILON=30.D00*PI/180.D00
OMEGA=.82D00
TF=2.D00*PI/OMEGA
TX=5.D00
TY=TX+0.25D00*TF
FO=0.35D00
V=0.5D00

IF (T.LE.TX) THEN
  FX=(3.D00*FO/(TX**2.D00))*T**2.D00-
$(2.D00/(TX**3.D00))*FO*T**3.D00
ELSE
  FX=FO*DCOS(OMEGA*(T-TX))
END IF

IF (T.LE.TY) THEN
  F2=(3.D00*V*FO/(TY**2.D00))*T**2.D00
$(2.D00*V*FO/(TY**3.D00))*T**3.D00
ELSE
  F2=V*FO*DCOS(OMEGA*(T-TY))
END IF

F1=FX*DSIN(EPSILON)

F3=FX*DCOS(EPSILON)

C *****CONNECTION TO GROUND*****
X1=ZL/2.D00+ZA
X2=ZL/2.D00+ZA
X3=-ZL/2.D00-ZA
X4=-ZL/2.D00-ZA
Y1=0.D00
Y2=0.D00
Y3=0.D00
Y4=0.D00
Z1=ZB+R
Z2=-ZB-R
Z3=ZB+R

```

Z4=-ZB-R

C *****SIMPLIFICATIONS*****

C7=DCOS(Y(7))
C9=DCOS(Y(9))
C11=DCOS(Y(11))
S7=DSIN(Y(7))
S9=DSIN(Y(9))
S11=DSIN(Y(11))
C92=C9**2.D00
S92=S9**2.D00
A=ZIXX*S92+ZIYY*C92
B11 = 1.D00/(ZIYY*C92)
B12 = S9*B11
B22 = A*B11/ZIXX

C *****EQUATIONS OF MOTION*****

YPRIME(1)=Y(2)

YPRIME(2)=-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*
\$(S9*C11*C7+S11*S7)-X1)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11*S9*
\$C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7*C9+
\$A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+
\$(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)*
\$(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)
\$-X1)-ZK2*(Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*
\$C7+S11*S7)-X2)+ZK2*ZL2*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*
\$C11)+A23*(S9*C11*C7+S11*S7)-X2)**2.D00+(Y(3)+A21*S7*C9+A22*
\$(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)**2.D00+(Y(5)-
\$A21*S9+A22*S11*C9+A23*C9*C11-Z2)**2.D00)**(-.5D00)*(Y(1)+A21*
\$C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)-ZK3*
\$(Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)
\$-X3)+ZK3*ZL3*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*
\$C11*C7+S11*S7)-X3)**2.D00+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*
\$C7)+A33*(S9*C11*S7-S11*C7)-Y3)**2.D00+(Y(5)-A31*S9+A32*S11*C9
\$+A33*C9*C11-Z3)**2.D00)**(-.5D00)*(Y(1)+A31*C9*C7+A32*(S11*S9
\$*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)-ZK4*(Y(1)+A41*C9*C7+
\$A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)+ZK4*ZL4*
\$((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*
\$S7)-X4)**2.D00+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9
\$*C11*S7-S11*C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11
\$-Z4)**2.D00)**(-.5D00)*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)
\$+A43*(S9*C11*C7+S11*S7)-X4)+F1-C1*Y(2)

YPRIME(3)=Y(4)

YPRIME(4)=-ZK1*(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9
\$*C11*S7-S11*C7)-Y1)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11*S9*C7-
\$S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7*C9+
\$A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+
\$(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)*
\$(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)
\$-Y1)-ZK2*(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*
\$S7-S11*C7)-Y2)+ZK2*ZL2*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*
\$C11)+A23*(S9*C11*C7+S11*S7)-X2)**2.D00+(Y(3)+A21*S7*C9+A22*
\$(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)**2.D00+(Y(5)-
\$A21*S9+A22*S11*C9+A23*C9*C11-Z2)**2.D00)**(-.5D00)*(Y(3)+A21*

$$\begin{aligned}
& \$S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)-ZK3* \\
& \$ (Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7) \\
& \$-Y3)+ZK3*ZL3*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9* \\
& \$C11*C7+S11*S7)-X3)**2.D00+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11* \\
& \$C7)+A33*(S9*C11*S7-S11*C7)-Y3)**2.D00+(Y(5)-A31*S9+A32*S11*C9 \\
& \$+A33*C9*C11-Z3)**2.D00)**(-.5D00)*(Y(3)+A31*S7*C9+A32*(S7*S11 \\
& \$*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)-ZK4*(Y(3)+A41*S7*C9+ \\
& \$A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)+ZK4*ZL4* \\
& \$((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11* \\
& \$S7)-X4)**2.D00+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43* \\
& \$ (S9*C11*S7-S11*C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+A43* \\
& \$C9*C11-Z4)**2.D00)**(-.5D00)*(Y(3)+A41*S7*C9+A42*(S7*S11*S9+ \\
& \$C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)+F2-C2*Y(4)+ZWC
\end{aligned}$$

YPRIME(5)=Y(6)

$$\begin{aligned}
& YPRIME(6)=-ZK1*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)+ZK1*ZL1 \\
& \$*((Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11* \\
& \$S7)-X1)**2.D00+(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13* \\
& \$ (S9*C11*S7-S11*C7)-Y1)**2.D00+(Y(5)-A11*S9+A12*S11*C9+A13*C9* \\
& \$C11-Z1)**2.D00)**(-.5D00)*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11- \\
& \$Z1)-ZK2*(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)+ZK2*ZL2*((Y(1) \\
& \$+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2) \\
& \$**2.D00+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7 \\
& \$-S11*C7)-Y2)**2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)** \\
& \$2.D00)**(-.5D00)*(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)-ZK3* \\
& \$ (Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)+ZK3*ZL3*((Y(1)+A31*C9 \\
& \$*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**2.D00+ \\
& \$ (Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7) \\
& \$-Y3)**2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**2.D00)** \\
& \$(-.5D00)*(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)-ZK4*(Y(5)- \\
& \$A41*S9+A42*S11*C9+A43*C9*C11-Z4)+ZK4*ZL4*((Y(1)+A41*C9*C7+ \\
& \$A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)**2.D00+ \\
& \$ (Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11* \\
& \$C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)**2.D00) \\
& \$**(-.5D00)*(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)+F3-C3*Y(6)
\end{aligned}$$

$$\begin{aligned}
& EQ4=ZIXX*(Y(12)*Y(10)*C9-2.D00*Y(8)*Y(10)*S9*C9)+ZIYY*2.D00* \\
& \$Y(10)*Y(8)*C9*S9-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+ \\
& \$A13*(S9*C11*C7+S11*S7)-X1)*(-A11*C9*S7-A12*(S11*S9*S7+C7*C11) \\
& \$-A13*(S9*C11*S7-S11*C7))-ZK1*(Y(3)+A11*(S7*C9)+A12*(S7*S11*S9 \\
& \$+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)*(A11*C9*C7+A12*(C7*S11*S9 \\
& \$-C11*S7)+A13*(S9*C11*C7+S11*S7))+ZK1*ZL1*((Y(1)+A11*C9*C7+A12 \\
& \$*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+ \\
& \$A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)** \\
& \$2.D00+(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)** \\
& \$(-.5D00)*((Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11* \\
& \$C7+S11*S7)-X1)*(-A11*C9*S7-A12*(S11*S9*S7+C7*C11)-A13*(S9*C11 \\
& \$*S7-S11*C7))+(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9* \\
& \$C11*S7-S11*C7)-Y1)*(A11*C9*C7+A12*(C7*S11*S9-C11*S7)+A13*(S9* \\
& \$C11*C7+S11*S7)))-ZK2*(Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+ \\
& \$A23*(S9*C11*C7+S11*S7)-X2)*(-A21*C9*S7-A22*(S11*S9*S7+C7*C11) \\
& \$-A23*(S9*C11*S7-S11*C7))-ZK2*(Y(3)+A21*(S7*C9)+A22*(S7*S11*S9 \\
& \$+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)*(A21*C9*C7+A22*(C7*S11*S9 \\
& \$-C11*S7)+A23*(S9*C11*C7+S11*S7))+ZK2*ZL2*((Y(1)+A21*C9*C7+A22 \\
& \$*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)**2.D00+(Y(3)+ \\
& \$A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)**
\end{aligned}$$

$\$2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)**2.D00)**$
 $\$(-.5D00)*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*$
 $\$C7+S11*S7)-X2)*(-A21*C9*S7-A22*(S11*S9*S7+C7*C11)-A23*(S9*C11*$
 $\$*S7-S11*C7))+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*$
 $\$C11*S7-S11*C7)-Y2)*(A21*C9*C7+A22*(C7*S11*S9-C11*S7)+A23*(S9*$
 $\$C11*C7+S11*S7)))-ZK3*(Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+$
 $\$A33*(S9*C11*C7+S11*S7)-X3)*(-A31*C9*S7-A32*(S11*S9*S7+C7*C11)$
 $\$-A33*(S9*C11*S7-S11*C7)))-ZK3*(Y(3)+A31*(S7*C9)+A32*(S7*S11*S9$
 $\$+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)*(A31*C9*C7+A32*(C7*S11*S9$
 $\$-C11*S7)+A33*(S9*C11*C7+S11*S7))+ZK3*ZL3*((Y(1)+A31*C9*C7+A32$
 $\$*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**2.D00+(Y(3)+$
 $\$A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)**$
 $\$2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**2.D00)**$
 $\$(-.5D00)*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*$
 $\$C7+S11*S7)-X3)*(-A31*C9*S7-A32*(S11*S9*S7+C7*C11)-A33*(S9*C11*$
 $\$*S7-S11*C7))+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*$
 $\$C11*S7-S11*C7)-Y3)*(A31*C9*C7+A32*(C7*S11*S9-C11*S7)+A33*(S9*$
 $\$C11*C7+S11*S7)))-ZK4*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+$
 $\$A43*(S9*C11*C7+S11*S7)-X4)*(-A41*C9*S7-A42*(S11*S9*S7+C7*C11)$
 $\$-A43*(S9*C11*S7-S11*C7)))-ZK4*(Y(3)+A41*(S7*C9)+A42*(S7*S11*S9$
 $\$+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)*(A41*C9*C7+A42*(C7*S11*S9$
 $\$-C11*S7)+A43*(S9*C11*C7+S11*S7))+ZK4*ZL4*((Y(1)+A41*C9*C7+A42$
 $\$*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)**2.D00+(Y(3)+$
 $\$A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)**$
 $\$2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)**2.D00)**$
 $\$(-.5D00)*((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*$
 $\$C7+S11*S7)-X4)*(-A41*C9*S7-A42*(S11*S9*S7+C7*C11)-A43*(S9*C11*$
 $\$*S7-S11*C7))+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*$
 $\$C11*S7-S11*C7)-Y4)*(A41*C9*C7+A42*(C7*S11*S9-C11*S7)+A43*(S9*$
 $\$C11*C7+S11*S7))$

$EQ6=ZIXX*Y(8)*Y(10)*C9-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*$
 $\$C11)+A13*(S9*C11*C7+S11*S7)-X1)*(A12*(C11*S9*C7+S7*S11)-A13*$
 $\$(S9*S11*C7-S7*C11))-ZK1*(Y(3)+A11*(S7*C9)+A12*(S7*S11*S9+C11*$
 $\$C7)+A13*(S9*C11*S7-S11*C7)-Y1)*(A12*(S7*C11*S9-S11*C7)-A13*$
 $\$(S9*S11*S7+C11*C7))-ZK1*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-$
 $\$Z1)*(A12*C11*C9-A13*C9*S11)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11$
 $\$*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7$
 $\$*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+$
 $\$(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)*(($
 $\$(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)$
 $\$-X1)*(A12*(C11*S9*C7+S7*S11)-A13*(S9*S11*C7-S7*C11))+(Y(3)+$
 $\$A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)*$
 $\$(A12*(S7*C11*S9-S11*C7)-A13*(S9*S11*S7+C11*C7))+(Y(5)-A11*S9+$
 $\$A12*S11*C9+A13*C9*C11-Z1)*(A12*C11*C9-A13*C9*S11))-ZK2*(Y(1)+$
 $\$A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)*$
 $\$(A22*(C11*S9*C7+S7*S11)-A23*(S9*S11*C7-S7*C11))-ZK2*(Y(3)+A21$
 $\$*(S7*C9)+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)*$
 $\$(A22*(S7*C11*S9-S11*C7)-A23*(S9*S11*S7+C11*C7))-ZK2*(Y(5)-A21$
 $\$*S9+A22*S11*C9+A23*C9*C11-Z2)*(A22*C11*C9-A23*C9*S11)+ZK2*ZL2$
 $\$*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*$
 $\$S7)-X2)**2.D00+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9$
 $\$*C11*S7-S11*C7)-Y2)**2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11$
 $\$-Z2)**2.D00)**(-.5D00)*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*$
 $\$C11)+A23*(S9*C11*C7+S11*S7)-X2)*(A22*(C11*S9*C7+S7*S11)-A23*$
 $\$(S9*S11*C7-S7*C11))+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+$
 $\$A23*(S9*C11*S7-S11*C7)-Y2)*(A22*(S7*C11*S9-S11*C7)-A23*(S9*$

$$\begin{aligned}
& \$(S11*S7+C11*C7))+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)*(A22* \\
& \$(C11*C9-A23*C9*S11))-ZK3*(Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7* \\
& \$(C11)+A33*(S9*C11*C7+S11*S7)-X3)*(A32*(C11*S9*C7+S7*S11)-A33* \\
& \$(S9*S11*C7-S7*C11))-ZK3*(Y(3)+A31*(S7*C9)+A32*(S7*S11*S9+C11 \\
& \$(C7)+A33*(S9*C11*S7-S11*C7)-Y3)*(A32*(S7*C11*S9-S11*C7)-A33* \\
& \$(S9*S11*S7+C11*C7))-ZK3*(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11- \\
& \$(Z3)*(A32*C11*C9-A33*C9*S11)+ZK3*ZL3*((Y(1)+A31*C9*C7+A32* \\
& \$(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**2.D00+(Y(3)+ \\
& \$(A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3) \\
& \$(**2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**2.D00)** \\
& \$(-.5D00)*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9* \\
& \$(C11*C7+S11*S7)-X3)*(A32*(C11*S9*C7+S7*S11)-A33*(S9*S11*C7- \\
& \$(S7*C11)))+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11 \\
& \$(S7-S11*C7)-Y3)*(A32*(S7*C11*S9-S11*C7)-A33*(S9*S11*S7+C11* \\
& \$(C7)))+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)*(A32*C11*C9-A33 \\
& \$(C9*S11))-ZK4*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43* \\
& \$(S9*C11*C7+S11*S7)-X4)*(A42*(C11*S9*C7+S7*S11)-A43*(S9*S11* \\
& \$(C7-S7*C11))-ZK4*(Y(3)+A41*(S7*C9)+A42*(S7*S11*S9+C11*C7)+ \\
& \$(A43*(S9*C11*S7-S11*C7)-Y4)*(A42*(S7*C11*S9-S11*C7)-A43*(S9* \\
& \$(S11*S7+C11*C7))-ZK4*(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)* \\
& \$(A42*C11*C9-A43*C9*S11)+ZK4*ZL4*((Y(1)+A41*C9*C7+A42*(S11* \\
& \$(S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)**2.D00+(Y(3)+A41* \\
& \$(S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)** \\
& \$(2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)**2.D00)** \\
& \$(-.5D00)*((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9* \\
& \$(C11*C7+S11*S7)-X4)*(A42*(C11*S9*C7+S7*S11)-A43*(S9*S11*C7- \\
& \$(S7*C11)))+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11 \\
& \$(S7-S11*C7)-Y4)*(A42*(S7*C11*S9-S11*C7)-A43*(S9*S11*S7+C11* \\
& \$(C7)))+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)*(A42*C11*C9-A43 \\
& \$(C9*S11))
\end{aligned}$$

YPRIME(7)=Y(8)

YPRIME(8)=B11*EQ4+B12*EQ6-C4*Y(8)

YPRIME(9)=Y(10)

$$\begin{aligned}
& \$(YPRIME(10)=(-ZIXX*Y(12)*Y(8)*C9+ZIXX*(Y(8))**2.D00*C9*S9- \\
& \$(ZIIYY*(Y(8))**2.D00*C9*S9-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7- \\
& \$(S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)*(-A11*C7*S9+A12*S11*C9*C7+ \\
& \$(A13*C9*C11*C7)-ZK1*(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13 \\
& \$(S9*C11*S7-S11*C7)-Y1)*(-A11*S7*S9+A12*S7*S11*C9+A13*C9*C11* \\
& \$(S7)-ZK1*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)*(-A11*C9-A12* \\
& \$(S11*S9-A13*S9*C11)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7 \\
& \$(C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7*C9+A12* \\
& \$(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+(Y(5)- \\
& \$(A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)*((Y(1)+A11 \\
& \$(C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)* \\
& \$(-A11*C7*S9+A12*S11*C9*C7+A13*C9*C11*C7)+(Y(3)+A11*S7*C9+A12* \\
& \$(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)*(-A11*S7*S9+A12 \\
& \$(S7*S11*C9+A13*C9*C11*S7)+(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11- \\
& \$(Z1)*(-A11*C9-A12*S11*S9-A13*S9*C11))-ZK2*(Y(1)+A21*C9*C7+A22* \\
& \$(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)*(-A21*C7*S9+A22 \\
& \$(S11*C9*C7+A23*C9*C11*C7)-ZK2*(Y(3)+A21*S7*C9+A22*(S7*S11*S9+ \\
& \$(C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)*(-A21*S7*S9+A22*S7*S11*C9+ \\
& \$(A23*C9*C11*S7)-ZK2*(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)* \\
& \$(-A21*C9-A22*S11*S9-A23*S9*C11)+ZK2*ZL2*((Y(1)+A21*C9*C7+A22*
\end{aligned}$$

```

$(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)**2.D00+(Y(3)+
$A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)**
$2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)**2.D00)**
$(-.5D00)*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*
$C7+S11*S7)-X2)*(-A21*C7*S9+A22*S11*C9*C7+A23*C9*C11*C7)+(Y(3)
$+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)*
$(-A21*S7*S9+A22*S7*S11*C9+A23*C9*C11*S7)+(Y(5)-A21*S9+A22*S11
$*C9+A23*C9*C11-Z2)*(-A21*C9-A22*S11*S9-A23*S9*C11))-ZK3*(Y(1)
$+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)*
$(-A31*C7*S9+A32*S11*C9*C7+A33*C9*C11*C7)-ZK3*(Y(3)+A31*S7*C9+
$A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)*(-A31*S7*S9
$+A32*S7*S11*C9+A33*C9*C11*S7)-ZK3*(Y(5)-A31*S9+A32*S11*C9+A33
$*C9*C11-Z3)*(-A31*C9-A32*S11*S9-A33*S9*C11)+ZK3*ZL3*((Y(1)+
$A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**
$2.D00+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-
$S11*C7)-Y3)**2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**
$2.D00)**(-.5D00)*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*
$(S9*C11*C7+S11*S7)-X3)*(-A31*C7*S9+A32*S11*C9*C7+A33*C9*C11*
$C7)+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11
$*C7)-Y3)*(-A31*S7*S9+A32*S7*S11*C9+A33*C9*C11*S7)+(Y(5)-A31*
$$S9+A32*S11*C9+A33*C9*C11-Z3)*(-A31*C9-A32*S11*S9-A33*S9*C11))
$-ZK4*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+
$S11*S7)-X4)*(-A41*C7*S9+A42*S11*C9*C7+A43*C9*C11*C7)-ZK4*
$(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)
$-Y4)*(-A41*S7*S9+A42*S7*S11*C9+A43*C9*C11*S7)-ZK4*(Y(5)-A41*
$$S9+A42*S11*C9+A43*C9*C11-Z4)*(-A41*C9-A42*S11*S9-A43*S9*C11)+
$ZK4*ZL4*((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*
$C7+S11*S7)-X4)**2.D00+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+
$A43*(S9*C11*S7-S11*C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+
$A43*C9*C11-Z4)**2.D00)**(-.5D00)*((Y(1)+A41*C9*C7+A42*(S11*S9
$*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)*(-A41*C7*S9+A42*S11*C9
$*C7+A43*C9*C11*C7)+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43
$*(S9*C11*S7-S11*C7)-Y4)*(-A41*S7*S9+A42*S7*S11*C9+A43*C9*C11*
$$S7)+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)*(-A41*C9-A42*S11*
$$S9-A43*S9*C11)))/ZIYY-C5*Y(10)

```

YPRIME(11)=Y(12)

YPRIME(12)=B12*EQ4+B22*EQ6-C6*Y(12)

RETURN

END

SUBROUTINE FCNJ(N,T,Y,DYPDY)

DOUBLE PRECISION DYPDY(50,50),Y(12)

INTEGER N

RETURN

END

A.2 Compressionless Springs

```
C          *****ASSUMING MASSLESS MOORING LINES*****
INTEGER MXPARM,N
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
PARAMETER (MXPARM=50,N=12)
C          *****SPECS FOR PARAMETERS*****
INTEGER MSTEP, MSOLVE
PARAMETER (MSTEP=10000000,MSOLVE=2)
C          *****SPECS FOR LOCAL VARIABLES*****
INTEGER IDO, NOUT
DIMENSION Y(12),PARAM(50),A(1,1)
C          *****SPECS FOR SUBROUTINES*****
EXTERNAL DIVPAG,SSET
C          *****SPECS FOR FUNCTIONS*****
EXTERNAL FCN, FCNJ
C
CALL UMACH (2, NOUT)

open(unit=1,file='pd50.dat',status='unknown')

C          *****SET INITIAL CONDITIONS*****
T=0.D00
Y(1)=0.D00
Y(2)=0.D00
Y(3)=2.83840079416846045D00
Y(4)=0.D00
Y(5)=0.D00
Y(6)=0.D00
Y(7)=0.D00
Y(8)=0.D00
Y(9)=0.D00
Y(10)=0.D00
Y(11)=0.D00
Y(12)=0.D00
F1=0.D00
F2=0.D00
F3=0.D00
FX=0.D00

C          *****SET MAX TIME*****
TMAX=300.D00
C          *****SET ERROR TOLERANCE*****
TOL=.0000000001D00
C          *****SET PARAM TO DEFAULTS***
CALL SSET (MXPARM, 0.0, PARAM, 1)
C          *****SET MAXIMUM # OF STEPS ALLOWED*****
PARAM(4)=MSTEP
C          *****SELECT CHORD METHOD AND A DIVIDED DIFF JACOBIAN***
PARAM(13)=MSOLVE
C          ***PRINT HEADER***
WRITE (NOUT, 99996)
WRITE (1,99997)
C          ***PRINT INITIAL CONDITIONS***
WRITE (NOUT, '(7D10.3)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
WRITE (1, '(7E14.5)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
```

```

C      ***BEGIN DIVPAG LOOP***
      IDO =1
      STEP=0.D00
10  CONTINUE
      STEP = STEP+0.03D00
      TEND = STEP
C      *****A(1,1) IS NOT USED*****
      CALL DIVPAG (IDO, N, FCN, FCNJ, A, T, TEND, TOL, PARAM, Y)
      IF (STEP.LE.TMAX) THEN
          WRITE (NOUT, '(7D10.3)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
          WRITE (1, '(7E14.5)')T,Y(1),Y(3),Y(5),Y(7),Y(9),Y(11)
C      *****FINAL CALL TO RELEASE WORKSPACE*****
          IF (STEP.EQ.TMAX) IDO=3
          GOTO 10
      END IF
      WRITE (NOUT, 99998) PARAM(35)
C      *****HEADER*****
99996 FORMAT (7X, 'TIME', 11X, 'Y1', 12X, 'Y3', 12X, 'Y5', 12X, 'Y7', 12X, 'Y9', 12X
$, 'Y11')
99997 FORMAT (7X, 'TIME', 11X, 'Y1', 12X, 'Y3', 12X, 'Y5', 12X, 'Y7', 12X, 'Y9', 12X
$, 'Y11')
C      *****SHOW # OF FUNTION CALLS***
99998 FORMAT (4X, 'NUMBER OF FCN CALLS WITH DIVPAG =', F6.0)
      END
      SUBROUTINE FCN (N,T,Y,YPRIME)
          IMPLICIT DOUBLE PRECISION (A-H, O-Z)
          DIMENSION YPRIME(12),Y(12)
C      *****SPECS FOR ARGUMENTS*****
C      *****THE FOLLOWING ARE NONDIMENSIONAL*****
      R=1.D00
      ZMC=1.D00
      ZWC=1.D00
      ZL=6.D00
      ZIXX=.5D00*ZMC*R**2.D00
      ZIYY=.25D00*ZMC*R**2.D00+(1.D00/12.D00)*ZMC*ZL**2.D00
      ZIZZ=ZIYY
      C1=0.1D00
      C2=C1
      C3=C1
      C4=C1
      C5=C1
      C6=C1
      ZL1=4.D00
      ZL2=ZL1
      ZL3=ZL1
      ZL4=ZL1
      ZA=2.D00
      ZB=2.D00
      A11=ZL/2.D00
      A12=0.D00
      A13=R
      A21=A11
      A22=A12
      A23=-A13
      A31=-A11
      A32=A12
      A33=A13

```

```

A41=-A11
A42=A12
A43=-A13

C      *****STOP WHEN Y(3) NEG*****

IF (Y(3).LT.0.D00) THEN
  STOP 00000
END IF

C      *****FORCES*****
PI=3.14159265358979323846264338327950288419717D00
EPSILON=60.D00*PI/180.D00
OMEGA=.75D00
TF=2.D00*PI/OMEGA
TX=5.D00
TY=TX+0.25D00*TF
FO=0.7D00
V=0.5D00

IF (T.LE.TX) THEN
  FX=(3.D00*FO/(TX**2.D00))*T**2.D00-
$(2.D00/(TX**3.D00))*FO*T**3.D00
ELSE
  FX=FO*DCOS(OMEGA*(T-TX))
END IF

IF (T.LE.TY) THEN
  F2=(3.D00*V*FO/(TY**2.D00))*T**2.D00
$(2.D00*V*FO/(TY**3.D00))*T**3.D00
ELSE
  F2=V*FO*DCOS(OMEGA*(T-TY))
END IF

F1=FX*DSIN(EPSILON)

F3=FX*DCOS(EPSILON)

C      *****CONNECTION TO GROUND*****
X1=ZL/2.D00+ZA
X2=ZL/2.D00+ZA
X3=-ZL/2.D00-ZA
X4=-ZL/2.D00-ZA
Y1=0.D00
Y2=0.D00
Y3=0.D00
Y4=0.D00
Z1=ZB+R
Z2=-ZB-R
Z3=ZB+R
Z4=-ZB-R

C      *****SIMPLIFICATIONS*****
C7=DCOS(Y(7))
C9=DCOS(Y(9))
C11=DCOS(Y(11))
S7=DSIN(Y(7))

```

```

S9=DSIN(Y(9))
S11=DSIN(Y(11))
C92=C9**2.D00
S92=S9**2.D00
A=ZIXX*S92+ZIYY*C92
B11 = 1.D00/(ZIYY*C92)
B12 = S9*B11
B22 = A*B11/ZIXX

C *****COMPRESSIONLESS SPRINGS*****
ALPHA1=Y(1)+A11*C9*C7+A13*(S9*C11*C7+S11*S7)-X1
ALPHA2=Y(1)+A21*C9*C7+A23*(S9*C11*C7+S11*S7)-X2
ALPHA3=Y(1)+A31*C9*C7+A33*(S9*C11*C7+S11*S7)-X3
ALPHA4=Y(1)+A41*C9*C7+A43*(S9*C11*C7+S11*S7)-X4

BETA1=Y(3)+A11*S7*C9+A13*(S9*C11*S7-S11*C7)
BETA2=Y(3)+A21*S7*C9+A23*(S9*C11*S7-S11*C7)
BETA3=Y(3)+A31*S7*C9+A33*(S9*C11*S7-S11*C7)
BETA4=Y(3)+A41*S7*C9+A43*(S9*C11*S7-S11*C7)

GAMMA1=Y(5)-A11*S9+A13*C9*C11-Z1
GAMMA2=Y(5)-A21*S9+A23*C9*C11-Z2
GAMMA3=Y(5)-A31*S9+A33*C9*C11-Z3
GAMMA4=Y(5)-A41*S9+A43*C9*C11-Z4

Q1=(ALPHA1**2.D00+BETA1**2.D00+GAMMA1**2.D00)**0.5D00
Q2=(ALPHA2**2.D00+BETA2**2.D00+GAMMA2**2.D00)**0.5D00
Q3=(ALPHA3**2.D00+BETA3**2.D00+GAMMA3**2.D00)**0.5D00
Q4=(ALPHA4**2.D00+BETA4**2.D00+GAMMA4**2.D00)**0.5D00

IF (Q1.LE.ZL1) THEN
  ZK1=0.D00
ELSE
  ZK1=49.9999999999992457967645D00
END IF

IF (Q2.LE.ZL2) THEN
  ZK2=0.D00
ELSE
  ZK2=49.9999999999992457967645D00
END IF

IF (Q3.LE.ZL3) THEN
  ZK3=0.D00
ELSE
  ZK3=49.9999999999992457967645D00
END IF

IF (Q4.LE.ZL4) THEN
  ZK4=0.D00
ELSE
  ZK4=49.9999999999992457967645D00
END IF

C *****EQUATIONS OF MOTION*****
YPRIME(1)=Y(2)

```


$$\begin{aligned}
& \$(S9*C11*S7-S11*C7)-Y1)**2.D00+(Y(5)-A11*S9+A12*S11*C9+A13*C9* \\
& \$(C11-Z1)**2.D00)**(-.5D00)*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11- \\
& \$(Z1)-ZK2*(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)+ZK2*ZL2*((Y(1) \\
& \$(+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2) \\
& \$(**2.D00+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7 \\
& \$(S11*C7)-Y2)**2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)** \\
& \$(2.D00)**(-.5D00)*(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)-ZK3* \\
& \$(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)+ZK3*ZL3*((Y(1)+A31*C9 \\
& \$(S7*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**2.D00+ \\
& \$(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7) \\
& \$(S7-C7)-Y3)**2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**2.D00)** \\
& \$((-.5D00)*(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)-ZK4*(Y(5)- \\
& \$(A41*S9+A42*S11*C9+A43*C9*C11-Z4)+ZK4*ZL4*((Y(1)+A41*C9*C7+ \\
& \$(A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)**2.D00+ \\
& \$(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11* \\
& \$(S7-C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)**2.D00) \\
& \$(**(-.5D00)*(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)+F3-C3*Y(6)
\end{aligned}$$

$$\begin{aligned}
& EQ4=ZIXX*(Y(12)*Y(10)*C9-2.D00*Y(8)*Y(10)*S9*C9)+ZIYY*2.D00* \\
& \$(Y(10)*Y(8)*C9*S9-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+ \\
& \$(A13*(S9*C11*C7+S11*S7)-X1)*(-A11*C9*S7-A12*(S11*S9*S7+C7*C11) \\
& \$(S7-C11)*C7)+A13*(S9*C11*S7-S11*C7))-ZK1*(Y(3)+A11*(S7*C9)+A12*(S7*S11*S9 \\
& \$(S7+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)*(A11*C9*C7+A12*(C7*S11*S9 \\
& \$(S7-C11*S7)+A13*(S9*C11*C7+S11*S7))+ZK1*ZL1*((Y(1)+A11*C9*C7+A12 \\
& \$(S7*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+ \\
& \$(A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)** \\
& \$(2.D00+(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)** \\
& \$((-.5D00)*((Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11* \\
& \$(S7+S11*S7)-X1)*(-A11*C9*S7-A12*(S11*S9*S7+C7*C11)-A13*(S9*C11 \\
& \$(S7-S11*C7)))+(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9* \\
& \$(S9*C11*S7-S11*C7)-Y1)*(A11*C9*C7+A12*(C7*S11*S9-C11*S7)+A13*(S9* \\
& \$(S9*C11*C7+S11*S7))-ZK2*(Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+ \\
& \$(A23*(S9*C11*C7+S11*S7)-X2)*(-A21*C9*S7-A22*(S11*S9*S7+C7*C11) \\
& \$(S7-C11)*C7)+A23*(S9*C11*S7-S11*C7))-ZK2*(Y(3)+A21*(S7*C9)+A22*(S7*S11*S9 \\
& \$(S7+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)*(A21*C9*C7+A22*(C7*S11*S9 \\
& \$(S7-C11*S7)+A23*(S9*C11*C7+S11*S7))+ZK2*ZL2*((Y(1)+A21*C9*C7+A22 \\
& \$(S7*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)**2.D00+(Y(3)+ \\
& \$(A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)** \\
& \$(2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)**2.D00)** \\
& \$((-.5D00)*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11* \\
& \$(S7+S11*S7)-X2)*(-A21*C9*S7-A22*(S11*S9*S7+C7*C11)-A23*(S9*C11 \\
& \$(S7-S11*C7)))+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9* \\
& \$(S9*C11*S7-S11*C7)-Y2)*(A21*C9*C7+A22*(C7*S11*S9-C11*S7)+A23*(S9* \\
& \$(S9*C11*C7+S11*S7))-ZK3*(Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+ \\
& \$(A33*(S9*C11*C7+S11*S7)-X3)*(-A31*C9*S7-A32*(S11*S9*S7+C7*C11) \\
& \$(S7-C11)*C7)+A33*(S9*C11*S7-S11*C7))-ZK3*(Y(3)+A31*(S7*C9)+A32*(S7*S11*S9 \\
& \$(S7+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)*(A31*C9*C7+A32*(C7*S11*S9 \\
& \$(S7-C11*S7)+A33*(S9*C11*C7+S11*S7))+ZK3*ZL3*((Y(1)+A31*C9*C7+A32 \\
& \$(S7*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**2.D00+(Y(3)+ \\
& \$(A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)** \\
& \$(2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**2.D00)** \\
& \$((-.5D00)*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11* \\
& \$(S7+S11*S7)-X3)*(-A31*C9*S7-A32*(S11*S9*S7+C7*C11)-A33*(S9*C11 \\
& \$(S7-S11*C7)))+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9* \\
& \$(S9*C11*S7-S11*C7)-Y3)*(A31*C9*C7+A32*(C7*S11*S9-C11*S7)+A33*(S9* \\
& \$(S9*C11*C7+S11*S7))-ZK4*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+ \\
& \$(A43*(S9*C11*C7+S11*S7)-X4)*(-A41*C9*S7-A42*(S11*S9*S7+C7*C11)
\end{aligned}$$

$$\begin{aligned}
& \$-A43*(S9*C11*S7-S11*C7))-ZK4*(Y(3)+A41*(S7*C9)+A42*(S7*S11*S9 \\
& \$+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)*(A41*C9*C7+A42*(C7*S11*S9 \\
& \$-C11*S7)+A43*(S9*C11*C7+S11*S7))+ZK4*ZL4*((Y(1)+A41*C9*C7+A42 \\
& \$*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)**2.D00+(Y(3)+ \\
& \$A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)** \\
& \$2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)**2.D00)** \\
& \$(-.5D00)*((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11* \\
& \$C7+S11*S7)-X4)*(-A41*C9*S7-A42*(S11*S9*S7+C7*C11)-A43*(S9*C11 \\
& \$*S7-S11*C7))+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9* \\
& \$C11*S7-S11*C7)-Y4)*(A41*C9*C7+A42*(C7*S11*S9-C11*S7)+A43*(S9* \\
& \$C11*C7+S11*S7)))
\end{aligned}$$

$$\begin{aligned}
& EQ6=ZIXX*Y(8)*Y(10)*C9-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7* \\
& \$C11)+A13*(S9*C11*C7+S11*S7)-X1)*(A12*(C11*S9*C7+S7*S11)-A13* \\
& \$*(S9*S11*C7-S7*C11))-ZK1*(Y(3)+A11*(S7*C9)+A12*(S7*S11*S9+C11* \\
& \$C7)+A13*(S9*C11*S7-S11*C7)-Y1)*(A12*(S7*C11*S9-S11*C7)-A13* \\
& \$*(S9*S11*S7+C11*C7))-ZK1*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11- \\
& \$Z1)*(A12*C11*C9-A13*C9*S11)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11 \\
& \$*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7 \\
& \$*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+ \\
& \$*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)* \\
& \$*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7) \\
& \$-X1)*(A12*(C11*S9*C7+S7*S11)-A13*(S9*S11*C7-S7*C11))+(Y(3)+ \\
& \$A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)* \\
& \$*(A12*(S7*C11*S9-S11*C7)-A13*(S9*S11*S7+C11*C7))+(Y(5)-A11*S9+ \\
& \$A12*S11*C9+A13*C9*C11-Z1)*(A12*C11*C9-A13*C9*S11))-ZK2*(Y(1)+ \\
& \$A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)* \\
& \$*(A22*(C11*S9*C7+S7*S11)-A23*(S9*S11*C7-S7*C11))-ZK2*(Y(3)+A21 \\
& \$*(S7*C9)+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)* \\
& \$*(A22*(S7*C11*S9-S11*C7)-A23*(S9*S11*S7+C11*C7))-ZK2*(Y(5)-A21 \\
& \$*S9+A22*S11*C9+A23*C9*C11-Z2)*(A22*C11*C9-A23*C9*S11)+ZK2*ZL2 \\
& \$*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11* \\
& \$S7)-X2)**2.D00+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9 \\
& \$*C11*S7-S11*C7)-Y2)**2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11 \\
& \$-Z2)**2.D00)**(-.5D00)*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7* \\
& \$C11)+A23*(S9*C11*C7+S11*S7)-X2)*(A22*(C11*S9*C7+S7*S11)-A23* \\
& \$*(S9*S11*C7-S7*C11))+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+ \\
& \$A23*(S9*C11*S7-S11*C7)-Y2)*(A22*(S7*C11*S9-S11*C7)-A23*(S9* \\
& \$S11*S7+C11*C7))+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)*(A22* \\
& \$C11*C9-A23*C9*S11))-ZK3*(Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7* \\
& \$C11)+A33*(S9*C11*C7+S11*S7)-X3)*(A32*(C11*S9*C7+S7*S11)-A33* \\
& \$*(S9*S11*C7-S7*C11))-ZK3*(Y(3)+A31*(S7*C9)+A32*(S7*S11*S9+C11 \\
& \$*C7)+A33*(S9*C11*S7-S11*C7)-Y3)*(A32*(S7*C11*S9-S11*C7)-A33* \\
& \$*(S9*S11*S7+C11*C7))-ZK3*(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11- \\
& \$Z3)*(A32*C11*C9-A33*C9*S11)+ZK3*ZL3*((Y(1)+A31*C9*C7+A32* \\
& \$*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**2.D00+(Y(3)+ \\
& \$A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3) \\
& \$**2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**2.D00)** \\
& \$(-.5D00)*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9* \\
& \$C11*C7+S11*S7)-X3)*(A32*(C11*S9*C7+S7*S11)-A33*(S9*S11*C7- \\
& \$S7*C11))+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11 \\
& \$*S7-S11*C7)-Y3)*(A32*(S7*C11*S9-S11*C7)-A33*(S9*S11*S7+C11* \\
& \$C7))+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)*(A32*C11*C9-A33 \\
& \$*C9*S11))-ZK4*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43* \\
& \$*(S9*C11*C7+S11*S7)-X4)*(A42*(C11*S9*C7+S7*S11)-A43*(S9*S11* \\
& \$C7-S7*C11))-ZK4*(Y(3)+A41*(S7*C9)+A42*(S7*S11*S9+C11*C7)+ \\
& \$A43*(S9*C11*S7-S11*C7)-Y4)*(A42*(S7*C11*S9-S11*C7)-A43*(S9*
\end{aligned}$$

$$\begin{aligned} & \$S11*S7+C11*C7))-ZK4*(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4) * \\ & \$ (A42*C11*C9-A43*C9*S11)+ZK4*ZL4*((Y(1)+A41*C9*C7+A42*(S11* \\ & \$S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4) **2.D00+(Y(3)+A41* \\ & \$S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4) ** \\ & \$2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4) **2.D00) ** \\ & \$(-.5D00)*((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9* \\ & \$C11*C7+S11*S7)-X4)*(A42*(C11*S9*C7+S7*S11)-A43*(S9*S11*C7- \\ & \$S7*C11))+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11 \\ & \$*S7-S11*C7)-Y4)*(A42*(S7*C11*S9-S11*C7)-A43*(S9*S11*S7+C11* \\ & \$C7))+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)*(A42*C11*C9-A43 \\ & \$*C9*S11)) \end{aligned}$$

YPRIME(7)=Y(8)

YPRIME(8)=B11*EQ4+B12*EQ6-C4*Y(8)

YPRIME(9)=Y(10)

$$\begin{aligned} & YPRIME(10)=(-ZIXX*Y(12)*Y(8)*C9+ZIXX*(Y(8)) **2.D00*C9*S9- \\ & \$ZIIYY*(Y(8)) **2.D00*C9*S9-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7- \\ & \$S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)*(-A11*C7*S9+A12*S11*C9*C7+ \\ & \$A13*C9*C11*C7)-ZK1*(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13 \\ & \$*(S9*C11*S7-S11*C7)-Y1)*(-A11*S7*S9+A12*S7*S11*C9+A13*C9*C11* \\ & \$S7)-ZK1*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)*(-A11*C9-A12* \\ & \$S11*S9-A13*S9*C11)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7 \\ & \$*C11)+A13*(S9*C11*C7+S11*S7)-X1) **2.D00+(Y(3)+A11*S7*C9+A12* \\ & \$ (S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1) **2.D00+(Y(5)- \\ & \$A11*S9+A12*S11*C9+A13*C9*C11-Z1) **2.D00) **(-.5D00)*((Y(1)+A11 \\ & \$*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1) * \\ & \$(-A11*C7*S9+A12*S11*C9*C7+A13*C9*C11*C7)+(Y(3)+A11*S7*C9+A12* \\ & \$ (S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)*(-A11*S7*S9+A12 \\ & \$*S7*S11*C9+A13*C9*C11*S7)+(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11- \\ & \$Z1)*(-A11*C9-A12*S11*S9-A13*S9*C11))-ZK2*(Y(1)+A21*C9*C7+A22* \\ & \$ (S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)*(-A21*C7*S9+A22 \\ & \$*S11*C9*C7+A23*C9*C11*C7)-ZK2*(Y(3)+A21*S7*C9+A22*(S7*S11*S9+ \\ & \$C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)*(-A21*S7*S9+A22*S7*S11*C9+ \\ & \$A23*C9*C11*S7)-ZK2*(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2) * \\ & \$(-A21*C9-A22*S11*S9-A23*S9*C11)+ZK2*ZL2*((Y(1)+A21*C9*C7+A22* \\ & \$ (S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2) **2.D00+(Y(3)+ \\ & \$A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2) ** \\ & \$2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2) **2.D00) ** \\ & \$(-.5D00)*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11* \\ & \$C7+S11*S7)-X2)*(-A21*C7*S9+A22*S11*C9*C7+A23*C9*C11*C7)+(Y(3) \\ & \$+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2) * \\ & \$(-A21*S7*S9+A22*S7*S11*C9+A23*C9*C11*S7)+(Y(5)-A21*S9+A22*S11 \\ & \$*C9+A23*C9*C11-Z2)*(-A21*C9-A22*S11*S9-A23*S9*C11))-ZK3*(Y(1) \\ & \$+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3) * \\ & \$(-A31*C7*S9+A32*S11*C9*C7+A33*C9*C11*C7)-ZK3*(Y(3)+A31*S7*C9+ \\ & \$A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)*(-A31*S7*S9 \\ & \$+A32*S7*S11*C9+A33*C9*C11*S7)-ZK3*(Y(5)-A31*S9+A32*S11*C9+A33 \\ & \$*C9*C11-Z3)*(-A31*C9-A32*S11*S9-A33*S9*C11)+ZK3*ZL3*((Y(1)+ \\ & \$A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3) ** \\ & \$2.D00+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7- \\ & \$S11*C7)-Y3) **2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3) ** \\ & \$2.D00) **(-.5D00)*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33* \\ & \$ (S9*C11*C7+S11*S7)-X3)*(-A31*C7*S9+A32*S11*C9*C7+A33*C9*C11* \\ & \$C7)+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11 \end{aligned}$$

```

$*C7)-Y3)*(-A31*S7*S9+A32*S7*S11*C9+A33*C9*C11*S7)+(Y(5)-A31*
$S9+A32*S11*C9+A33*C9*C11-Z3)*(-A31*C9-A32*S11*S9-A33*S9*C11)
$-ZK4*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+
$S11*S7)-X4)*(-A41*C7*S9+A42*S11*C9*C7+A43*C9*C11*C7)-ZK4*
$(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)
$-Y4)*(-A41*S7*S9+A42*S7*S11*C9+A43*C9*C11*S7)-ZK4*(Y(5)-A41*
$S9+A42*S11*C9+A43*C9*C11-Z4)*(-A41*C9-A42*S11*S9-A43*S9*C11)+
$ZK4*ZL4*((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*
$C7+S11*S7)-X4)**2.D00+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+
$A43*(S9*C11*S7-S11*C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+
$A43*C9*C11-Z4)**2.D00)**(-.5D00)*((Y(1)+A41*C9*C7+A42*(S11*S9
$*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)*(-A41*C7*S9+A42*S11*C9
$*C7+A43*C9*C11*C7)+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43
$*(S9*C11*S7-S11*C7)-Y4)*(-A41*S7*S9+A42*S7*S11*C9+A43*C9*C11*
$S7)+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)*(-A41*C9-A42*S11*
$S9-A43*S9*C11)))/ZIYY-C5*Y(10)

```

```
YPRIME(11)=Y(12)
```

```
YPRIME(12)=B12*EQ4+B22*EQ6-C6*Y(12)
```

```
RETURN
```

```
END
```

```
SUBROUTINE FCNJ(N,T,Y,DYPDY)
```

```
DOUBLE PRECISION DYPDY(50,50),Y(12)
```

```
INTEGER N
```

```
RETURN
```

```
END
```

A.3. Poincaré - Compressionless Springs

```
C          *****ASSUMING MASSLESS MOORING LINES*****
INTEGER MXPARM,N
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
PARAMETER (MXPARM=50,N=12)
C          *****SPECS FOR PARAMETERS*****
INTEGER MSTEP, MSOLVE
PARAMETER (MSTEP=10000000,MSOLVE=2)
C          *****SPECS FOR LOCAL VARIABLES*****
INTEGER IDO, NOUT
DIMENSION Y(12),PARAM(50),A(1,1)
C          *****SPECS FOR SUBROUTINES*****
EXTERNAL DIVPAG,SSET
C          *****SPECS FOR FUNCTIONS*****
EXTERNAL FCN, FCNJ
C
CALL UMACH (2, NOUT)

open(unit=1,file='pd50.dat',status='unknown')

C          *****SET INITIAL CONDITIONS*****
T=0.D00
Y(1)=0.D00
Y(2)=0.D00
Y(3)=2.83840079416846045D00
Y(4)=0.D00
Y(5)=0.D00
Y(6)=0.D00
Y(7)=0.D00
Y(8)=0.D00
Y(9)=0.D00
Y(10)=0.D00
Y(11)=0.D00
Y(12)=0.D00
F1=0.D00
F2=0.D00
F3=0.D00
FX=0.D00

C          *****SET MAX TIME*****
OMEGA2=.75D00
PI2=3.14159265358979323846264338327950288419717D00
TF2=2.D00*PI2/OMEGA2
TMAX=1001.D00*TF2
C          *****SET ERROR TOLERANCE*****
TOL=.0000000001D00
C          *****SET PARAM TO DEFAULTS***
CALL SSET (MXPARM, 0.0, PARAM, 1)
C          *****SET MAXIMUM # OF STEPS ALLOWED*****
PARAM(4)=MSTEP
C          *****SELECT CHORD METHOD AND A DIVIDED DIFF JACOBIAN***
PARAM(13)=MSOLVE
C          ***PRINT HEADER***
WRITE (NOUT, 99996)
WRITE (1,99997)
```

```

C      ***BEGIN DIVPAG LOOP****
      IDO =1
      STEP=99.D00*TF2
10  CONTINUE
      STEP = STEP+TF2
      TEND = STEP
C      *****A(1,1) IS NOT USED****
      CALL DIVPAG (IDO, N, FCN, FCNJ, A, T, TEND, TOL, PARAM, Y)
      IF (STEP.LE.TMAX) THEN
          WRITE (NOUT, '(7D10.3)')T,Y(1),Y(2),Y(3),Y(4),Y(7),Y(8)
          WRITE (1, '(7E14.5)')T,Y(1),Y(2),Y(3),Y(4),Y(7),Y(8)
C      *****FINAL CALL TO RELEASE WORKSPACE*****
          IF (STEP.EQ.TMAX) IDO=3
          GOTO 10
      END IF
      WRITE (NOUT, 99998) PARAM(35)
C      *****HEADER*****
99996 FORMAT (7X, 'TIME', 11X, 'Y1', 12X, 'Y2', 12X, 'Y3', 12X, 'Y4', 12X, 'Y7', 12X
$, 'Y8')
99997 FORMAT (7X, 'TIME', 11X, 'Y1', 12X, 'Y2', 12X, 'Y3', 12X, 'Y4', 12X, 'Y7', 12X
$, 'Y8')
C      *****SHOW # OF FUNTION CALLS***
99998 FORMAT (4X, 'NUMBER OF FCN CALLS WITH DIVPAG =', F6.0)
      END
      SUBROUTINE FCN (N,T,Y,YPRIME)
          IMPLICIT DOUBLE PRECISION (A-H, O-Z)
          DIMENSION YPRIME(12),Y(12)
C      *****SPECS FOR ARGUMENTS*****
C      *****THE FOLLOWING ARE NONDIMENSIONAL*****
      R=1.D00
      ZMC=1.D00
      ZWC=1.D00
      ZL=6.D00
      ZIXX=.5D00*ZMC*R**2.D00
      ZIYY=.25D00*ZMC*R**2.D00+(1.D00/12.D00)*ZMC*ZL**2.D00
      ZIZZ=ZIYY
      C1=0.1D00
      C2=C1
      C3=C1
      C4=C1
      C5=C1
      C6=C1
      ZL1=4.D00
      ZL2=ZL1
      ZL3=ZL1
      ZL4=ZL1
      ZA=2.D00
      ZB=2.D00
      A11=ZL/2.D00
      A12=0.D00
      A13=R
      A21=A11
      A22=A12
      A23=-A13
      A31=-A11
      A32=A12
      A33=A13

```

```

A41=-A11
A42=A12
A43=-A13

C *****STOP WHEN Y(3) NEG*****

IF (Y(3).LT.0.D00) THEN
  STOP 00000
END IF

C *****FORCES*****
PI=3.14159265358979323846264338327950288419717D00
EPSILON=90.D00*PI/180.D00
OMEGA=.75D00
TF=2.D00*PI/OMEGA
TX=5.D00
TY=TX+0.25D00*TF
FO=0.35D00
V=0.5D00

IF (T.LE.TX) THEN
  FX=(3.D00*FO/(TX**2.D00))*T**2.D00-
$(2.D00/(TX**3.D00))*FO*T**3.D00
ELSE
  FX=FO*DCOS(OMEGA*(T-TX))
END IF

IF (T.LE.TY) THEN
  F2=(3.D00*V*FO/(TY**2.D00))*T**2.D00
$(2.D00*V*FO/(TY**3.D00))*T**3.D00
ELSE
  F2=V*FO*DCOS(OMEGA*(T-TY))
END IF

F1=FX

F3=0.D00

C *****CONNECTION TO GROUND*****
X1=ZL/2.D00+ZA
X2=ZL/2.D00+ZA
X3=-ZL/2.D00-ZA
X4=-ZL/2.D00-ZA
Y1=0.D00
Y2=0.D00
Y3=0.D00
Y4=0.D00
Z1=ZB+R
Z2=-ZB-R
Z3=ZB+R
Z4=-ZB-R

C *****SIMPLIFICATIONS*****
C7=DCOS(Y(7))
C9=DCOS(Y(9))
C11=DCOS(Y(11))
S7=DSIN(Y(7))

```

```

S9=DSIN(Y(9))
S11=DSIN(Y(11))
C92=C9**2.D00
S92=S9**2.D00
A=ZIXX*S92+ZIYY*C92
B11 = 1.D00/(ZIYY*C92)
B12 = S9*B11
B22 = A*B11/ZIXX

C *****COMPRESSIONLESS SPRINGS*****
ALPHA1=Y(1)+A11*C9*C7+A13*(S9*C11*C7+S11*S7)-X1
ALPHA2=Y(1)+A21*C9*C7+A23*(S9*C11*C7+S11*S7)-X2
ALPHA3=Y(1)+A31*C9*C7+A33*(S9*C11*C7+S11*S7)-X3
ALPHA4=Y(1)+A41*C9*C7+A43*(S9*C11*C7+S11*S7)-X4

BETA1=Y(3)+A11*S7*C9+A13*(S9*C11*S7-S11*C7)
BETA2=Y(3)+A21*S7*C9+A23*(S9*C11*S7-S11*C7)
BETA3=Y(3)+A31*S7*C9+A33*(S9*C11*S7-S11*C7)
BETA4=Y(3)+A41*S7*C9+A43*(S9*C11*S7-S11*C7)

GAMMA1=Y(5)-A11*S9+A13*C9*C11-Z1
GAMMA2=Y(5)-A21*S9+A23*C9*C11-Z2
GAMMA3=Y(5)-A31*S9+A33*C9*C11-Z3
GAMMA4=Y(5)-A41*S9+A43*C9*C11-Z4

Q1=(ALPHA1**2.D00+BETA1**2.D00+GAMMA1**2.D00)**0.5D00
Q2=(ALPHA2**2.D00+BETA2**2.D00+GAMMA2**2.D00)**0.5D00
Q3=(ALPHA3**2.D00+BETA3**2.D00+GAMMA3**2.D00)**0.5D00
Q4=(ALPHA4**2.D00+BETA4**2.D00+GAMMA4**2.D00)**0.5D00

IF (Q1.LE.ZL1) THEN
  ZK1=0.D00
ELSE
  ZK1=49.9999999999992457967645D00
END IF

IF (Q2.LE.ZL2) THEN
  ZK2=0.D00
ELSE
  ZK2=49.9999999999992457967645D00
END IF

IF (Q3.LE.ZL3) THEN
  ZK3=0.D00
ELSE
  ZK3=49.9999999999992457967645D00
END IF

IF (Q4.LE.ZL4) THEN
  ZK4=0.D00
ELSE
  ZK4=49.9999999999992457967645D00
END IF

C *****EQUATIONS OF MOTION*****
Y(5)=0.D00
Y(6)=0.D00

```

$Y(9)=0.D00$
 $Y(10)=0.D00$
 $Y(11)=0.D00$
 $Y(12)=0.D00$

$YPRIME(1)=Y(2)$

$YPRIME(2)=-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*$
 $\$(S9*C11*C7+S11*S7)-X1)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11*S9*$
 $\$C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7*C9+$
 $\$A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+$
 $\$(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)*$
 $\$(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)$
 $\$-X1)-ZK2*(Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*$
 $\$C7+S11*S7)-X2)+ZK2*ZL2*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*$
 $\$C11)+A23*(S9*C11*C7+S11*S7)-X2)**2.D00+(Y(3)+A21*S7*C9+A22*$
 $\$(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)**2.D00+(Y(5)-$
 $\$A21*S9+A22*S11*C9+A23*C9*C11-Z2)**2.D00)**(-.5D00)*(Y(1)+A21*$
 $\$C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)-ZK3*$
 $\$(Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)$
 $\$-X3)+ZK3*ZL3*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*$
 $\$C11*C7+S11*S7)-X3)**2.D00+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*$
 $\$C7)+A33*(S9*C11*S7-S11*C7)-Y3)**2.D00+(Y(5)-A31*S9+A32*S11*C9$
 $\$+A33*C9*C11-Z3)**2.D00)**(-.5D00)*(Y(1)+A31*C9*C7+A32*(S11*S9$
 $\$*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)-ZK4*(Y(1)+A41*C9*C7+$
 $\$A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)+ZK4*ZL4*$
 $\$((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*$
 $\$S7)-X4)**2.D00+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9$
 $\$*C11*S7-S11*C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11$
 $\$\$-Z4)**2.D00)**(-.5D00)*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)$
 $\$+A43*(S9*C11*C7+S11*S7)-X4)+F1-C1*Y(2)$

$YPRIME(3)=Y(4)$

$YPRIME(4)=-ZK1*(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9$
 $\$*C11*S7-S11*C7)-Y1)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11*S9*C7-$
 $\$S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7*C9+$
 $\$A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+$
 $\$(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)*$
 $\$(Y(3)+A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)$
 $\$-Y1)-ZK2*(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*$
 $\$S7-S11*C7)-Y2)+ZK2*ZL2*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*$
 $\$C11)+A23*(S9*C11*C7+S11*S7)-X2)**2.D00+(Y(3)+A21*S7*C9+A22*$
 $\$(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)**2.D00+(Y(5)-$
 $\$A21*S9+A22*S11*C9+A23*C9*C11-Z2)**2.D00)**(-.5D00)*(Y(3)+A21*$
 $\$S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)-ZK3*$
 $\$(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)$
 $\$-Y3)+ZK3*ZL3*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9*$
 $\$C11*C7+S11*S7)-X3)**2.D00+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*$
 $\$C7)+A33*(S9*C11*S7-S11*C7)-Y3)**2.D00+(Y(5)-A31*S9+A32*S11*C9$
 $\$+A33*C9*C11-Z3)**2.D00)**(-.5D00)*(Y(3)+A31*S7*C9+A32*(S7*S11$
 $\$*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3)-ZK4*(Y(3)+A41*S7*C9+$
 $\$A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)+ZK4*ZL4*$
 $\$((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9*C11*C7+S11*$
 $\$S7)-X4)**2.D00+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*$
 $\$(S9*C11*S7-S11*C7)-Y4)**2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*$
 $\$\$C9*C11-Z4)**2.D00)**(-.5D00)*(Y(3)+A41*S7*C9+A42*(S7*S11*S9+$

$$\$C11 * C7) + A43 * (S9 * C11 * S7 - S11 * C7) - Y4) + F2 - C2 * Y(4) + ZWC$$

$$YPRIME(5) = Y(6)$$

$$YPRIME(6) = 0.D00$$

$$\begin{aligned} EQ4 = & ZIXX * (Y(12) * Y(10) * C9 - 2.D00 * Y(8) * Y(10) * S9 * C9) + ZIYY * 2.D00 * \\ & \$Y(10) * Y(8) * C9 * S9 - ZK1 * (Y(1) + A11 * C9 * C7 + A12 * (S11 * S9 * C7 - S7 * C11) + \\ & \$A13 * (S9 * C11 * C7 + S11 * S7) - X1) * (-A11 * C9 * S7 - A12 * (S11 * S9 * S7 + C7 * C11) \\ & \$ - A13 * (S9 * C11 * S7 - S11 * C7)) - ZK1 * (Y(3) + A11 * (S7 * C9) + A12 * (S7 * S11 * S9 \\ & \$ + C11 * C7) + A13 * (S9 * C11 * S7 - S11 * C7) - Y1) * (A11 * C9 * C7 + A12 * (C7 * S11 * S9 \\ & \$ - C11 * S7) + A13 * (S9 * C11 * C7 + S11 * S7)) + ZK1 * ZL1 * ((Y(1) + A11 * C9 * C7 + A12 \\ & \$ * (S11 * S9 * C7 - S7 * C11) + A13 * (S9 * C11 * C7 + S11 * S7) - X1) ** 2.D00 + (Y(3) + \\ & \$A11 * S7 * C9 + A12 * (S7 * S11 * S9 + C11 * C7) + A13 * (S9 * C11 * S7 - S11 * C7) - Y1) ** \\ & \$2.D00 + (Y(5) - A11 * S9 + A12 * S11 * C9 + A13 * C9 * C11 - Z1) ** 2.D00) ** \\ & \$(-.5D00) * ((Y(1) + A11 * C9 * C7 + A12 * (S11 * S9 * C7 - S7 * C11) + A13 * (S9 * C11 * \\ & \$C7 + S11 * S7) - X1) * (-A11 * C9 * S7 - A12 * (S11 * S9 * S7 + C7 * C11) - A13 * (S9 * C11 \\ & \$ * S7 - S11 * C7)) + (Y(3) + A11 * S7 * C9 + A12 * (S7 * S11 * S9 + C11 * C7) + A13 * (S9 * \\ & \$C11 * S7 - S11 * C7) - Y1) * (A11 * C9 * C7 + A12 * (C7 * S11 * S9 - C11 * S7) + A13 * (S9 * \\ & \$C11 * C7 + S11 * S7)) - ZK2 * (Y(1) + A21 * C9 * C7 + A22 * (S11 * S9 * C7 - S7 * C11) + \\ & \$A23 * (S9 * C11 * C7 + S11 * S7) - X2) * (-A21 * C9 * S7 - A22 * (S11 * S9 * S7 + C7 * C11) \\ & \$ - A23 * (S9 * C11 * S7 - S11 * C7)) - ZK2 * (Y(3) + A21 * (S7 * C9) + A22 * (S7 * S11 * S9 \\ & \$ + C11 * C7) + A23 * (S9 * C11 * S7 - S11 * C7) - Y2) * (A21 * C9 * C7 + A22 * (C7 * S11 * S9 \\ & \$ - C11 * S7) + A23 * (S9 * C11 * C7 + S11 * S7)) + ZK2 * ZL2 * ((Y(1) + A21 * C9 * C7 + A22 \\ & \$ * (S11 * S9 * C7 - S7 * C11) + A23 * (S9 * C11 * C7 + S11 * S7) - X2) ** 2.D00 + (Y(3) + \\ & \$A21 * S7 * C9 + A22 * (S7 * S11 * S9 + C11 * C7) + A23 * (S9 * C11 * S7 - S11 * C7) - Y2) ** \\ & \$2.D00 + (Y(5) - A21 * S9 + A22 * S11 * C9 + A23 * C9 * C11 - Z2) ** 2.D00) ** \\ & \$(-.5D00) * ((Y(1) + A21 * C9 * C7 + A22 * (S11 * S9 * C7 - S7 * C11) + A23 * (S9 * C11 * \\ & \$C7 + S11 * S7) - X2) * (-A21 * C9 * S7 - A22 * (S11 * S9 * S7 + C7 * C11) - A23 * (S9 * C11 \\ & \$ * S7 - S11 * C7)) + (Y(3) + A21 * S7 * C9 + A22 * (S7 * S11 * S9 + C11 * C7) + A23 * (S9 * \\ & \$C11 * S7 - S11 * C7) - Y2) * (A21 * C9 * C7 + A22 * (C7 * S11 * S9 - C11 * S7) + A23 * (S9 * \\ & \$C11 * C7 + S11 * S7)) - ZK3 * (Y(1) + A31 * C9 * C7 + A32 * (S11 * S9 * C7 - S7 * C11) + \\ & \$A33 * (S9 * C11 * C7 + S11 * S7) - X3) * (-A31 * C9 * S7 - A32 * (S11 * S9 * S7 + C7 * C11) \\ & \$ - A33 * (S9 * C11 * S7 - S11 * C7)) - ZK3 * (Y(3) + A31 * (S7 * C9) + A32 * (S7 * S11 * S9 \\ & \$ + C11 * C7) + A33 * (S9 * C11 * S7 - S11 * C7) - Y3) * (A31 * C9 * C7 + A32 * (C7 * S11 * S9 \\ & \$ - C11 * S7) + A33 * (S9 * C11 * C7 + S11 * S7)) + ZK3 * ZL3 * ((Y(1) + A31 * C9 * C7 + A32 \\ & \$ * (S11 * S9 * C7 - S7 * C11) + A33 * (S9 * C11 * C7 + S11 * S7) - X3) ** 2.D00 + (Y(3) + \\ & \$A31 * S7 * C9 + A32 * (S7 * S11 * S9 + C11 * C7) + A33 * (S9 * C11 * S7 - S11 * C7) - Y3) ** \\ & \$2.D00 + (Y(5) - A31 * S9 + A32 * S11 * C9 + A33 * C9 * C11 - Z3) ** 2.D00) ** \\ & \$(-.5D00) * ((Y(1) + A31 * C9 * C7 + A32 * (S11 * S9 * C7 - S7 * C11) + A33 * (S9 * C11 * \\ & \$C7 + S11 * S7) - X3) * (-A31 * C9 * S7 - A32 * (S11 * S9 * S7 + C7 * C11) - A33 * (S9 * C11 \\ & \$ * S7 - S11 * C7)) + (Y(3) + A31 * S7 * C9 + A32 * (S7 * S11 * S9 + C11 * C7) + A33 * (S9 * \\ & \$C11 * S7 - S11 * C7) - Y3) * (A31 * C9 * C7 + A32 * (C7 * S11 * S9 - C11 * S7) + A33 * (S9 * \\ & \$C11 * C7 + S11 * S7)) - ZK4 * (Y(1) + A41 * C9 * C7 + A42 * (S11 * S9 * C7 - S7 * C11) + \\ & \$A43 * (S9 * C11 * C7 + S11 * S7) - X4) * (-A41 * C9 * S7 - A42 * (S11 * S9 * S7 + C7 * C11) \\ & \$ - A43 * (S9 * C11 * S7 - S11 * C7)) - ZK4 * (Y(3) + A41 * (S7 * C9) + A42 * (S7 * S11 * S9 \\ & \$ + C11 * C7) + A43 * (S9 * C11 * S7 - S11 * C7) - Y4) * (A41 * C9 * C7 + A42 * (C7 * S11 * S9 \\ & \$ - C11 * S7) + A43 * (S9 * C11 * C7 + S11 * S7)) + ZK4 * ZL4 * ((Y(1) + A41 * C9 * C7 + A42 \\ & \$ * (S11 * S9 * C7 - S7 * C11) + A43 * (S9 * C11 * C7 + S11 * S7) - X4) ** 2.D00 + (Y(3) + \\ & \$A41 * S7 * C9 + A42 * (S7 * S11 * S9 + C11 * C7) + A43 * (S9 * C11 * S7 - S11 * C7) - Y4) ** \\ & \$2.D00 + (Y(5) - A41 * S9 + A42 * S11 * C9 + A43 * C9 * C11 - Z4) ** 2.D00) ** \\ & \$(-.5D00) * ((Y(1) + A41 * C9 * C7 + A42 * (S11 * S9 * C7 - S7 * C11) + A43 * (S9 * C11 * \\ & \$C7 + S11 * S7) - X4) * (-A41 * C9 * S7 - A42 * (S11 * S9 * S7 + C7 * C11) - A43 * (S9 * C11 \\ & \$ * S7 - S11 * C7)) + (Y(3) + A41 * S7 * C9 + A42 * (S7 * S11 * S9 + C11 * C7) + A43 * (S9 * \\ & \$C11 * S7 - S11 * C7) - Y4) * (A41 * C9 * C7 + A42 * (C7 * S11 * S9 - C11 * S7) + A43 * (S9 * \\ & \$C11 * C7 + S11 * S7)) \end{aligned}$$

$$\begin{aligned}
& EQ6=ZIXX*Y(8)*Y(10)*C9-ZK1*(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7* \\
& \$C11)+A13*(S9*C11*C7+S11*S7)-X1)*(A12*(C11*S9*C7+S7*S11)-A13* \\
& \$(S9*S11*C7-S7*C11))-ZK1*(Y(3)+A11*(S7*C9)+A12*(S7*S11*S9+C11* \\
& \$C7)+A13*(S9*C11*S7-S11*C7)-Y1)*(A12*(S7*C11*S9-S11*C7)-A13* \\
& \$(S9*S11*S7+C11*C7))-ZK1*(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11- \\
& \$Z1)*(A12*C11*C9-A13*C9*S11)+ZK1*ZL1*((Y(1)+A11*C9*C7+A12*(S11 \\
& \$*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7)-X1)**2.D00+(Y(3)+A11*S7 \\
& \$*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)**2.D00+ \\
& \$(Y(5)-A11*S9+A12*S11*C9+A13*C9*C11-Z1)**2.D00)**(-.5D00)*(\\
& \$(Y(1)+A11*C9*C7+A12*(S11*S9*C7-S7*C11)+A13*(S9*C11*C7+S11*S7) \\
& \$-X1)*(A12*(C11*S9*C7+S7*S11)-A13*(S9*S11*C7-S7*C11))+(Y(3)+ \\
& \$A11*S7*C9+A12*(S7*S11*S9+C11*C7)+A13*(S9*C11*S7-S11*C7)-Y1)* \\
& \$(A12*(S7*C11*S9-S11*C7)-A13*(S9*S11*S7+C11*C7))+(Y(5)-A11*S9+ \\
& \$A12*S11*C9+A13*C9*C11-Z1)*(A12*C11*C9-A13*C9*S11))-ZK2*(Y(1)+ \\
& \$A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11*S7)-X2)* \\
& \$(A22*(C11*S9*C7+S7*S11)-A23*(S9*S11*C7-S7*C11))-ZK2*(Y(3)+A21 \\
& \$*(S7*C9)+A22*(S7*S11*S9+C11*C7)+A23*(S9*C11*S7-S11*C7)-Y2)* \\
& \$(A22*(S7*C11*S9-S11*C7)-A23*(S9*S11*S7+C11*C7))-ZK2*(Y(5)-A21 \\
& \$*S9+A22*S11*C9+A23*C9*C11-Z2)*(A22*C11*C9-A23*C9*S11)+ZK2*ZL2 \\
& \$*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7*C11)+A23*(S9*C11*C7+S11* \\
& \$S7)-X2)**2.D00+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+A23*(S9 \\
& \$*C11*S7-S11*C7)-Y2)**2.D00+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11 \\
& \$-Z2)**2.D00)**(-.5D00)*((Y(1)+A21*C9*C7+A22*(S11*S9*C7-S7* \\
& \$C11)+A23*(S9*C11*C7+S11*S7)-X2)*(A22*(C11*S9*C7+S7*S11)-A23* \\
& \$(S9*S11*C7-S7*C11))+(Y(3)+A21*S7*C9+A22*(S7*S11*S9+C11*C7)+ \\
& \$A23*(S9*C11*S7-S11*C7)-Y2)*(A22*(S7*C11*S9-S11*C7)-A23*(S9* \\
& \$S11*S7+C11*C7))+(Y(5)-A21*S9+A22*S11*C9+A23*C9*C11-Z2)*(A22* \\
& \$C11*C9-A23*C9*S11))-ZK3*(Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7* \\
& \$C11)+A33*(S9*C11*C7+S11*S7)-X3)*(A32*(C11*S9*C7+S7*S11)-A33* \\
& \$(S9*S11*C7-S7*C11))-ZK3*(Y(3)+A31*(S7*C9)+A32*(S7*S11*S9+C11 \\
& \$*C7)+A33*(S9*C11*S7-S11*C7)-Y3)*(A32*(S7*C11*S9-S11*C7)-A33* \\
& \$(S9*S11*S7+C11*C7))-ZK3*(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11- \\
& \$Z3)*(A32*C11*C9-A33*C9*S11)+ZK3*ZL3*((Y(1)+A31*C9*C7+A32* \\
& \$(S11*S9*C7-S7*C11)+A33*(S9*C11*C7+S11*S7)-X3)**2.D00+(Y(3)+ \\
& \$A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11*S7-S11*C7)-Y3) \\
& \$**2.D00+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)**2.D00)** \\
& \$(-.5D00)*((Y(1)+A31*C9*C7+A32*(S11*S9*C7-S7*C11)+A33*(S9* \\
& \$C11*C7+S11*S7)-X3)*(A32*(C11*S9*C7+S7*S11)-A33*(S9*S11*C7- \\
& \$S7*C11))+(Y(3)+A31*S7*C9+A32*(S7*S11*S9+C11*C7)+A33*(S9*C11 \\
& \$*S7-S11*C7)-Y3)*(A32*(S7*C11*S9-S11*C7)-A33*(S9*S11*S7+C11* \\
& \$C7))+(Y(5)-A31*S9+A32*S11*C9+A33*C9*C11-Z3)*(A32*C11*C9-A33 \\
& \$*C9*S11))-ZK4*(Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43* \\
& \$(S9*C11*C7+S11*S7)-X4)*(A42*(C11*S9*C7+S7*S11)-A43*(S9*S11* \\
& \$C7-S7*C11))-ZK4*(Y(3)+A41*(S7*C9)+A42*(S7*S11*S9+C11*C7)+ \\
& \$A43*(S9*C11*S7-S11*C7)-Y4)*(A42*(S7*C11*S9-S11*C7)-A43*(S9* \\
& \$S11*S7+C11*C7))-ZK4*(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)* \\
& \$(A42*C11*C9-A43*C9*S11)+ZK4*ZL4*((Y(1)+A41*C9*C7+A42*(S11* \\
& \$S9*C7-S7*C11)+A43*(S9*C11*C7+S11*S7)-X4)**2.D00+(Y(3)+A41* \\
& \$S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11*S7-S11*C7)-Y4)** \\
& \$2.D00+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)**2.D00)** \\
& \$(-.5D00)*((Y(1)+A41*C9*C7+A42*(S11*S9*C7-S7*C11)+A43*(S9* \\
& \$C11*C7+S11*S7)-X4)*(A42*(C11*S9*C7+S7*S11)-A43*(S9*S11*C7- \\
& \$S7*C11))+(Y(3)+A41*S7*C9+A42*(S7*S11*S9+C11*C7)+A43*(S9*C11 \\
& \$*S7-S11*C7)-Y4)*(A42*(S7*C11*S9-S11*C7)-A43*(S9*S11*S7+C11* \\
& \$C7))+(Y(5)-A41*S9+A42*S11*C9+A43*C9*C11-Z4)*(A42*C11*C9-A43 \\
& \$*C9*S11))
\end{aligned}$$

```
YPRIME(7)=Y(8)

YPRIME(8)=B11*EQ4+B12*EQ6-C4*Y(8)

YPRIME(9)=Y(10)

YPRIME(10)=0.D00

YPRIME(11)=Y(12)

YPRIME(12)=0.D00

RETURN
END

SUBROUTINE FCNJ(N,T,Y,DYPDY)
DOUBLE PRECISION DYPDY(50,50),Y(12)
INTEGER N
RETURN
END
```

Vita

Juan Carlos Archilla was born on June 26, 1975 in Hato Rey, Puerto Rico. In May 1993 he graduated from South Lakes High School in Reston, Virginia. He later completed his Bachelor of Science degree in Civil Engineering at Virginia Polytechnic Institute and State University in May 1997. After being awarded a GEM M.S. Engineering Fellowship by the National Consortium for Graduate Degrees for Minorities in Engineering and Science, Inc., he decided to attend graduate school at Virginia Tech where he completed his Master of Science degree in Civil Engineering in April 1999. Shortly after finishing his graduate studies, Juan will begin working for Bechtel Power Corporation in Frederick, Maryland.