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ARTICLE

Incorporating historical weather information in crop insurance rating

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Abstract

Crop insurance programs rely on conditional predictive distributions of loss random variables (e.g., yield, revenue, loss costs, etc.) to determine probabilities and magnitudes of loss. The loss variables may be related to stochastic variables that are not known at the time the policy is priced. Such is the case for weather; weather is stochastic, realizations are not known when the crop insurance policy is sold, and there is often additional historical information on weather relative to the loss variable itself. We provide a Bayesian methodology for incorporating historical weather information in crop insurance rating. We apply the method in empirical applications to county-level U.S. corn yields and loss cost ratios in the Midwest. The historical weather-conditioned distributions differ from those based on shorter samples. In the yield distribution setting, additional temporal weather information leads to economic gains relative to other rating approaches; the magnitude of these gains increases with the amount of historical weather information included in the analysis.

KEYWORDS
Bayesian statistics, crop insurance, historical information, loss cost ratio, premium rates, weather

JEL CLASSIFICATION
Q18, Q14, C11

1 | INTRODUCTION

Crop insurance is the most expensive agricultural policy in the United States with over $110 billion in liability in 2020. Subsidized crop insurance programs are becoming more popular worldwide in both developed and developing countries (Barnett 2014; Mahul & Stutley 2010). In the U.S. federal crop insurance program, a key principle in the design of crop insurance policies is that they should be actuarially fair: The expected indemnity under the policy should be equal to the premium. Achieving this objective requires accurate pricing of policies; accurate pricing is dependent on accurate modeling of the stochastic variables causing losses.

Traditionally, historical yield data or historical loss cost data have been used to estimate a conditional predictive distribution of yields or loss costs from which expected losses and probabilities of loss can be obtained (Coble et al. 2010).[i] Loss probabilities and expected losses are then used to calculate premiums. In the case of yields, the conditional predictive distribution of yields can be written as $p(Y^*|Z)$, where Y^* is the predicted yield and Z is a vector of fixed variables known at the time the policy is sold. For instance, if one is interested in predicting the yield in time period $T+1$, then Z might simply consist of t (i.e., the predictive distribution of yields is conditioned on time). The conditional predictive distribution of loss costs can be modeled in a similar way. Many rating procedures exclusively use deterministic variables when constructing the conditional predictive distribution of the loss variable.

However, it is widely recognized that a large part of the observed variation in yields and loss costs is due to changes in weather and other stochastic variables (Belasco et al. 2020). The conditional predictive distribution of the loss variables can be amended to incorporate these additional stochastic variables, which—in contrast to the fixed variables—are not known at the time the policy is sold. In the federal crop insurance program, historical weather information is incorporated through post hoc rate adjustments (Rejesus et al. 2015). County base rates are contingent on expected loss costs, which in turn are based on a probability density function of loss costs. Similar to the case of the conditional yield distribution, the probability density of loss costs can be expressed as a function of deterministic and stochastic variables. Weather information is also frequently used by reinsurers when evaluating crop insurance portfolios and portfolio risk (Zhu et al. 2019). Weather-conditioned distributions have been shown to roughly approximate yield distributions based on observed yields and several studies have discussed the potential benefits of using weather or climate information in crop insurance rating (Goodwin 2008; Nadolnyak et al. 2008; Tack & Ubilava 2015; Tack et al. 2018; Yi et al. 2020).

In addition to potential gains from conditioning loss distributions on weather, weather data are often available over a greater period of time compared to yield data or loss cost data. This is especially the case at the farm level where yield records are notoriously short, in counties where production is sporadic, or for crops with limited historical production. Although all historical yield data are used in area policies in the federal crop insurance program, new shallow loss programs only use data from 1991 onward. Similarly, the federal crop insurance program experienced rapid growth in the 1980s; representative loss cost data are only available for the past 30 years.

If weather data are informative for the conditional predictive distributions of yields and loss costs, then incorporating historical weather information may provide additional rating accuracy. This idea is analogous to the argument that rating accuracy can be improved by bringing information across time or space to bear (Ker et al. 2016; Park et al. 2019). Studies of spatial or temporal pooling usually use data with observations on the loss variables in all periods. In contrast, our approach uses observations where the loss variables are missing. The inclusion of historical weather data necessarily involves observations with missing dependent variables; this setting can be viewed as a missing data problem.

We implement a Bayesian approach for incorporating historical weather information in crop insurance rating. We treat the cases of weather information as a stochastic predictor of both crop yields and loss cost ratios. In the case of yields, we use county-level corn yields from seven states in the Midwest. For loss cost ratios, we use county-level corn and soybean loss cost ratios in Illinois and Iowa for the federal crop insurance program. In both cases, the response variable is a function of temperature and precipitation over the growing season. Because the loss cost ratios are bounded between zero and one, we utilize a Bayesian approach to beta regression. In both cases, the models are embedded in a Bayesian algorithm that uses historical weather information in estimating the required conditional distributions. One advantage of the Bayesian approach is that missing data problems are easily tackled; even if the conditional yield or loss cost distributions cannot be derived analytically, they can be obtained through Markov chain Monte Carlo techniques.

In the case of yields, we show that (1) a private insurer incorporating weather information can advantageously select against the government, (2) this advantage is strengthened when additional historical weather information is accounted for, and (3) this advantage is slightly stronger at lower coverage levels. In the case of loss costs, we demonstrate that (1) historical weather-conditioned loss cost distributions differ modestly from those without historical information and (2) weather weighting can be done through a single procedure using distributional regression. The approach is widely applicable and can serve as a basis for incorporating historical weather information in a variety of crop insurance settings.

2 | LITERATURE ON CROP INSURANCE RATING

There is an extensive literature on estimation of yield distributions for crop insurance rating. These studies have varied from consideration of different parametric and nonparametric distributions of yields (Goodwin & Ker 1998; Ker & Coble 2003; Ker & Goodwin 2000; Ozaki et al. 2008; Racine & Ker 2006; Sherrick et al. 2004; Wu & Zhang 2020; Zhang 2017), to methods of detrending (Finger 2010, 2013; Zhu et al. 2011) and heteroskedasticity adjustments (Harri et al. 2011; Ker & Tolhurst 2019). Another line of literature has examined smoothing of crop yield densities across time or space with the idea that the incorporation of more information in the derivation of premiums for any single unit improves insurance rates. The spatial setting includes Park et al. (2019) and Ker et al. (2016), whereas Liu and Ker (2021) also smooth in the time dimension.

Tsiboe and Tack (2021) incorporate soil and topographic information in crop insurance rating. They find that such information improves rating accuracy primarily for farms with limited yield information. But for farms with full yield information, improvements are more modest. Soil information is a deterministic variable known at the time the policy is sold and incorporating this information is conceptually similar to dealing with time trends and other fixed determinants of yields or loss costs. Tsiboe and Tack (2021) also explain the connection between farm-level rates and county rates in the federal crop insurance program. Because farm-level rates are based on county-level base rates, improvements in county-level rates are relevant for farm-level insurance. Farm-level insurance is the most popular type of insurance sold through the federal crop insurance program.

In contrast to the literature previously discussed, which largely estimated yield distributions using only historical data on yields, some studies have investigated linkages between crop insurance rating and weather phenomena. Discussion of issues arising from climate variability can be found in Goodwin (2008) and Odening and Shen (2014). Nadolnyak et al. (2008) studied the value of including information on the El Niño Southern Oscillation (ENSO) in constructing climate-based yield forecasts. The effects of ENSO were further investigated by Tack and Ubilava (2015) and Yi et al. (2020). Of these, only Yi et al. (2020) incorporates the uncertain nature of ENSO forecasts; this is done in a quasi-Bayesian framework. Outside of multiple-peril crop insurance, weather and other risk management instruments have been discussed by Turvey (2001), Xu et al. (2010), and Dalhaus et al. (2020). These studies are mostly in the context of weather derivatives or weather index insurance where weather is the loss variable.

A small number of articles have considered weather information in the U.S. federal crop insurance program and in the modeling of loss cost ratios. Rejesus et al. (2015) discussed incorporation of historical weather information as implemented in the program. They modeled loss cost ratios as a function of climate variables and adjusted premiums based on binning the weather outcomes. Borman et al. (2013) used likelihood weighting to adjust for both historical weather and acreage. Their application was again to loss costs, and they also suggested weighting of priors as a second

Bayesian approach for using historical weather data. Tack et al. (2018) estimated the impact of warming on crop insurance rates by modeling yields as a function of weather. Weather variables were then integrated out to obtain the requisite conditional yield distribution. However, Tack et al. (2018) did not compare the economic gains that could accrue from using weather conditioned distributions, nor did they make use of historical weather data.

A recent paper by Belasco et al. (2020) discusses a weather-based disaster program as an alternative to the federal crop insurance program. They find that such a program would result in significant savings to the federal government but would introduce basis risk for participants if sold at the county level. There are some similarities between their approach and our model, but they do not incorporate historical weather information. Nor do they make a comparison to yield distributions currently estimated in the federal crop insurance program. Nonetheless, they show that there are benefits to using now-abundant weather data in crop insurance or agricultural disaster programs.

The Bayesian missing data approach for conditional density forecasting is set out in a—perhaps under-recognized—study by Griffiths et al. (2010). The Bayesian paradigm has the advantage of reflecting uncertainty from all unknowns, whereas conditioning only on information that is known. Bayesian density forecasts are broadly consistent in that a single probability model underlies any quantity of interest such as predicted values, stochastic regressors, fixed regressors, and more. As well, they may also incorporate information in the form of priors if desirable. Shen et al. (2016) show that incorporating expert knowledge can lead to more accurate crop insurance rates, particularly in cases where available data are limited. This can be viewed as an advantage for most crop insurance settings where data on yields or loss costs are constrained. A more extensive review of Bayesian hierarchical concepts and spatial data analysis is available in Banerjee et al. (2003).

We make two primary contributions to the crop insurance literature. Our first contribution is to implement a theoretically consistent Bayesian approach for incorporating historical weather data into the estimation of conditional predictive yield distributions. We show that incorporating historical weather information results in economic gains for private insurers. That is, we demonstrate the efficacy of the proposed approach. Our second contribution is to implement the same approach for loss cost distributions. This involves a single algorithm for bounded loss costs, and we find that the historical weather-conditioned distributions differ modestly from empirical distributions based on observed loss costs.

The results have implications for the design of crop insurance programs both in the United States and worldwide. This study suggests that increasingly large, and often disparate, data sets can be combined and used to improve agricultural policy (Coble et al. 2018). As measurement and modeling of weather and crop production continue to evolve, so will crop insurance products and actuarial methodologies. By developing rates that reflect heterogeneous risk exposure across locations, the methods developed here may encourage increased program participation and minimize adverse selection.

3 | INFORMATION FROM INCOMPLETE OBSERVATIONS

The problem of incorporating historical weather information is a missing data problem because some observations are incomplete. Although we have weather data, the dependent variable (yield or loss cost) may not be available. This differs from the typical missing data problem where one or more independent variables are missing. In any event, these incomplete observations can still be of use in estimating the conditional predictive distribution of the loss variable. Suppose we have the following setup where a linear process is assumed to generate the sample observations according to

$$Y = Z\beta + \epsilon, \tag{1}$$

where Y is an $(N \times 1)$ vector of observations on the dependent variable, Z is an $(N \times K)$ matrix of observations on K regressors, β is a vector of unknown coefficients, and $\epsilon \sim N(0, \sigma^2)$ is a random error vector. Furthermore, let θ be the parameter vector that collects β and σ^2 or all parameters in the model. Equation 1 is the classical linear model with no missing data and without stochastic regressors; it defines a distribution for Y.

If Y is the crop yield, then pricing a yield insurance policy requires predicting Y^*, which is the value of Y in period $N+1$. Rating involves prediction of not only Y^*, but the entire conditional distribution $p(Y^*|Z)$. For yield insurance policies in the federal crop insurance program, the expected loss under a policy is given by

$$E[\text{Loss}] = \int_0^{\lambda\overline{Y^*}} \frac{\left(\lambda\overline{Y^*} - Y^*\right)}{\lambda\overline{Y^*}} p(Y^*|Z)dY^*, \tag{2}$$

where λ is a coverage level between 0 and 1, and $\overline{Y^*}$ is the mean of $p(Y^*|Z)$. Pricing in the federal crop insurance program is intended to be actuarially fair: the premium on a policy is equal to the expected loss. The premium is then given by the ratio of the expected loss to total liability where total liability is $\lambda\overline{Y^*}$.

We can also consider the case where Y is a loss cost ratio. County base rates in the federal crop insurance program are calculated according to

$$E[\text{Loss Cost Ratio}] = \int_0^1 Y^* p(Y^*|Z)dY^*, \tag{3}$$

which shows that the county base rates are determined by the expected loss cost given by the mean of the conditional predictive distribution of loss costs. Rejesus et al. (2015) arrives at the expected loss cost ratio using a two-step approach. Expected loss cost ratios from fractional regression feed into a binning procedure (variable width histogram) that is used to derive weather weights. Other adjustments, such as catastrophic loading are used in the federal crop insurance program, but we abstract from those procedures here. The key point is that we ultimately require the conditional distribution $p(Y^*|Z)$ regardless of whether we intend to construct rates in a yield or loss cost setting.

The matrix Z usually includes fixed regressors such as time trends, but can also include stochastic regressors. Suppose that Z can be partitioned into two components (Z_D, Z_S) where Z_D is an $(N \times K_D)$ matrix of fixed or deterministic regressors and Z_S is an $(N \times K_S)$ matrix of stochastic regressors. Clearly, $K_D + K_S = K$. Letting Z^* denote the vector of regressors at time $N+1$, Z_D^* is known at the time that the prediction for Y^* is made, whereas Z_S^* is not. Z_S could include other stochastic factors impacting crop yields or loss cost ratios such as weather, pest infestation, or disease.

Including historical weather data further partitions Y and Z. In particular, we have a subset of data where Y is unknown but Z is known (this is in contrast to the prediction for Y^* where both Y^* and Z_S^* are unknown). We denote the known and unknown Y as Y_K and Y_U respectively, and the corresponding matrices of regressors as $Z_{D,K}$, $Z_{D,U}$, $Z_{S,K}$, and $Z_{S,U}$. Along these lines, let Z_K denote the $(N_K \times K)$ matrix of regressors corresponding to observations with known Y and Z_U the $(N_U \times K)$ matrix of regressors corresponding to the cases with unknown Y. By definition $N_K + N_U = N$.

The question is whether including $Z_{S,U}$ in the analysis in any way improves prediction of Y^* and $p(Y^*|Z)$. As noted in Knight et al. (1998), if the regressors are all fixed, then there is no gain from including the observations with missing responses. However, if the regressors are stochastic, the general rule is that these observations should not be deleted from the analysis. This can be seen in the likelihood, which is given by

$$L(\theta \mid Y, Z) = \prod_{i=1}^{N} f(Y_i, Z_i|\theta) = \prod_{i=1}^{N} f(Y_i|Z_i, \theta) f(Z_i|\theta)$$

and if Y_i is not observed, $f(Z_i|\theta)$ remains to contribute to the likelihood. Although the observations in Z_U cannot provide any information about the relationship between Z and Y, they do tell us something about the distribution of Z_S.[ii]

The intuition is that more observations of Z reduce our uncertainty about the distribution of Z. A simple example is illustrative. Suppose that the single stochastic variable is a drought indicator and for a 10-year period, no drought is observed. Droughts are associated with low yields with probability one, whereas normal periods of precipitation are associated with high yields with probability one. Then the empirical distribution of drought is a point mass with probability one of no drought. Suppose that we observe a drought in the eleventh year. The empirical distribution of drought is then Bernoulli with probability 1/11. Because droughts are associated with lower yields, newfound recognition of the probability of drought should factor into yield predictions for the twelfth year. If the relationship between drought and yield is not stochastic (as in the above example), then a myopic analyst might assume that yields are always good in the first scenario.

4 | ECONOMETRIC APPROACH

Our econometric approach is based on the preceding formulation of the problem as one of missing data. Bayesian statistics provide a straightforward paradigm for incorporating observations with either missing responses or incomplete explanatory variables. The general idea is to augment the parameter vector θ with the missing data. For the sake of clarity, we first proceed with a simple example in which Z contains a single deterministic regressor and a single stochastic regressor, which are related to Y as in Equation 1. The stochastic regressor is assumed to be distributed according to $p(Z_S|\rho)$ where ρ is a vector of parameters. Remember that, from an insurance rating perspective, our objective is to arrive at

$$\begin{aligned} p\left(Y^*|Y_K,Z,Z_D^*\right) = \iiiint & p(Y^* \mid Y_K,Z,Z^*)p\left(Z_S^*|\rho\right)p(\rho|Z_S) \\ \times p\left(\sigma^2|Y_K,Z_K,\beta\right)p(\beta|Y_K,Z_K)\,d\beta\, d\sigma^2\, d\rho\, dZ_S^*, \end{aligned} \tag{4}$$

which is the predictive density at time $N+1$ conditioned on all of the information that is available at time N. That is to say, we desire the predictive density of Y^* (the yield or loss cost ratio at time $N+1$) based on Y_K (historical yield or loss cost ratio observations), Z (all historical deterministic and stochastic terms affecting yield), and Z_D^* (the deterministic terms in time period $N+1$). Note that although there may be some situations where the density in Equation 4 is of a known form, it is usually unknown and cannot be derived analytically. The density can be formed by integrating over the parameters as well as the stochastic variables at time $N+1$ (Z_S^*). Of the parameters, ρ is the vector of parameters for the distribution of the stochastic weather variables (Z_S) whereas β and σ^2 are parameters of the model relating the deterministic and stochastic terms (Z) to the yield or loss cost ratio (Y).

The conditional distribution of Equation 4 is not restricted to taking a certain form. It can accommodate non-normal features as suggested by Harri et al. (2009) and Atwood et al. (2002). Moreover, non-normal behavior can be driven by the relationship between weather and yields as suggested in Tolhurst and Ker (2015). Multimodal and skewed distributions, which have been cited as evidence of mixtures of underlying distributions of extreme and normal weather, are consistent with this approach. Although there are decisions to be made about how to appropriately model the elements of the conditional predictive distribution, the approach permits a degree of flexibility. As shown below, the individual elements of the model can be motivated by existing literature.

As the integrals associated with the predictive distribution of Equation 4 are usually intractable, Bayesian sampling methods can be used to draw from the distribution in the following manner:

1. Draw β from $p(\beta|Y_K,Z_K,\sigma^2)$
2. Draw σ^2 from $p(\sigma^2|Y_K,Z_K,\beta)$
3. Draw ρ from $p(\rho|Z_S)$
4. Draw Z_S^* from $p(Z_S^*|\rho)$
5. Draw Y^* from $p(Y^* \mid Y_K,Z,Z^*)$

Depending on the assumptions that are made on the distribution of the regressors and the prior distributions for the regression parameters, the five steps for drawing from the density given by Equation 4 can be condensed. There are essentially three elements to the resulting algorithm. First, the regression parameters must be drawn; second, the missing stochastic regressors are drawn; last, the predicted value of the response is drawn. The complexity of the first step depends on the likelihood function relating the weather variables to yields or loss cost ratios and associated priors. The second step depends on the distribution of the weather variables and associated priors. The last step again depends on the model relating the stochastic variables to yields or loss cost ratios.

We stress again that the predictive distribution of interest in Equation 4 is generally of an unknown form. Several other points are worth noting. First, Steps 1 and 2 only use observations with observed dependent variables. Observations with weather data, but without observed yields or loss cost ratios, are not informative for the parameters of the model relating weather and the dependent variable. Second, historical weather data is informative for the distributional model for weather. Therefore, it is also informative for draws of Y^* through Steps 3, 4, and 5. In this way, historical weather information is directly incorporated into the estimation of the conditional predictive distribution of yields or loss cost ratios.

Having provided a general approach, we now define two specific econometric models based on commonly available weather variables and specifications that have appeared in the literature. We use these models in the simulation and empirical analyses that follow. The first model is for a conditional predictive distribution of crop yields. Suppose now that the stochastic regressors are distributed as $Z_S \sim MVN(\chi,\Sigma)$ with $\chi \sim N(\mu,\Omega)$ and $\Sigma \sim IW(a,b)$ where $IW(\cdot)$ is the inverse Wishart distribution. The inverse Wishart distribution is a standard prior for a variance–covariance matrix (Reich & Ghosh 2019). The stochastic regressors follow a multivariate normal distribution with unknown mean vector and variance–covariance matrix. Crop yields are related to the deterministic and stochastic regressors by a linear model with

$$log(Y_t) = \beta_0 + \beta_1 t + \beta_2 low_t + \beta_3 med_t + \beta_4 high_t + \beta_5 prec_t + \beta_6 prec_t^2 + \epsilon_t, \tag{5}$$

where *low*, *med*, and *high* are degree day bins measured over the growing season and precipitation is total precipitation as in Tack et al. (2012). Equation 5 can be amended to include quadratic or other nonlinear deterministic and stochastic regressors as needed. However, this basic specification has been shown to accommodate a variety of nonlinear behaviors in the weather/yield relationship (Ramsey et al. 2021; Schlenker & Roberts 2009; Tack et al. 2017; Tack et al. 2012). Other weather specifications along the lines of those in Rejesus et al. (2015) and Belasco et al. (2020) could also be used. The model is estimated using only data from the county or geographical unit in question, and we do not pursue potential gains for pooling information across insurance units, although such gains have been demonstrated in a host of other applications (Cho & Brorsen, 2021; Liu & Ker 2021; Park et al. 2019; Wu et al. 2021).

The rating algorithm proceeds as follows with a detailed description provided in the Appendix S1. We first draw from $p(Z_S^*|\rho)$. This is accomplished in two steps by first drawing from the full conditional distributions for χ and Σ, which are

$$\chi \mid \Sigma, Z_S \sim MVN(\chi_n, \Sigma_n), \tag{6}$$

$$\Sigma \mid \chi, Z_S \sim IW(a_n, b_n). \tag{7}$$

The parameters χ_n, Σ_n, a_n, and b_n, denoted with a n subscript, parameterize the conditional distributions of χ and Σ, and their formulas are given in detail in the Appendix S1. The missing regressors are then drawn from the distribution $p\left(Z_S^* | \chi, \Sigma\right)$, which is the multivariate normal distribution given above. These steps follow the usual Gibbs sampling scheme for a multivariate normal distribution with unknown mean and variance–covariance matrix that is described in more detail in Hoff (2009).

Having obtained draws of the missing stochastic regressors, draws of Y^* can be obtained from the posterior predictive distribution. We assume constant priors on $\theta = (\beta, \sigma^2)$ (i.e. completely uninformative priors on the regression coefficients and error variance), so that the resulting draws are from

$$p(Y^* \mid Y, Z, Z^*) \propto t\left(Z^* b, \left(\left(\nu s^2/(\nu-2)\right)\left(1 + Z^*(Z\prime Z)^{-1} Z^{*\prime}\right), \nu\right)\right), \tag{8}$$

which is a t distribution defined in terms of mean, variance, and degrees of freedom, where

$$\begin{aligned} \nu &= N_K - K, \\ b &= (Z_K \prime Z_K)^{-1} Z_K{}' Y_K, \\ s^2 &= (Y_K - Z_K b)'(Y_K - Z_K b)/\nu. \end{aligned} \tag{9}$$

The derivations to arrive at Equation 8 are simply those for a linear model where the priors are constants following the usual treatment available in Judge et al. (1988). The full algorithm is available in detail in the Appendix S1 but does not involve anything more than estimating and drawing from multivariate normal and linear models in succession.

The second model is for the conditional predictive distribution of loss cost ratios. The loss cost ratio is bounded between zero and one. The previous model is clearly inappropriate because the conditional predictive distribution of loss cost ratios would not have a bounded support. There are two popular approaches for modeling dependent variables with bounded support: fractional regression and distributional regression. Fractional regression uses a logistic transformation to model the mean of the dependent variable (Papke & Wooldridge 1996). Beta regression is commonly used to model responses on the open interval (0,1) (Cepeda-Cuervo 2015). In fact, time-varying beta distributions (though not restricted to the unit interval) have been used to model yields by Zhu et al. (2011).

We implement beta regression as the model relating the deterministic and stochastic variables to loss cost ratios. We maintain the same distributions and priors for the stochastic regressors such that $Z_S \sim MVN(\chi, \Sigma)$ with $\chi \sim N(\mu, \Omega)$ and $\Sigma \sim IW(a, b)$. Loss cost ratios are assumed to be distributed $Y \sim Beta(\alpha_1, \alpha_2)$ where $\mu_b = \alpha_1/(\alpha_1 + \alpha_2)$ and $\nu_b = \alpha_1 + \alpha_2$. In this case, α_1 and α_2 are shape parameters of the beta distribution, whereas μ_b is the mean of the beta distribution and ν_b is the variance. In terms of beta regression, it is often easier to parameterize the mean and variance as opposed to the shape parameters. We parameterize the mean and variance as

$$\begin{aligned} logit(\mu_b) &= \beta_{\mu 0} + \beta_{\mu 1} low_t + \beta_{\mu 2} med_t + \beta_{\mu 3} high_t + \beta_{\mu 4} prec_t + \beta_{\mu 5} prec_t^2, \\ log(\nu_b) &= \beta_{\nu 0} + \beta_{\nu 1} low_t + \beta \nu 2 med_t + \beta_{\nu 3} high_t + \beta_{\nu 4} prec_t + \beta_{\nu 5} prec_t^2. \end{aligned} \tag{10}$$

The major change from the yield model is that the loss cost ratio is assumed to follow a beta distribution, and the mean and precision of the beta distribution are functions of weather variables. As long as we can sample the parameters of the beta distribution, it is then possible to recover the conditional predictive distribution of loss cost ratios. In principle, the explanatory variables for the two parameters given above could vary. However, past work has usually taken the same explanatory variables for each component of the model whether it be applied to loss cost ratios or yields (Rejesus et al. 2015; Tack et al. 2012).

The likelihood function is nonstandard and is given in Liu and Kong (2015) and the vector $\theta = \beta$. In keeping with the uninformative priors in the model for yields, we assume that all the parameters are independent. The βs are all assumed to have $N(0,0.001)$ priors where 0.001 is the precision. In other words, the coefficients have very diffuse normal priors. This Bayesian approach to beta regression is discussed in more detail in Cepeda-Cuervo (2015).

Drawing from the second model proceeds much in the same way as the first. The stochastic regressors are first drawn from their distribution following Equations 6 and 7. The beta regression parameters are then estimated via slice sampling. As discussed by Griffiths et al. (2010), a variety of nonlinear models for the relationship between weather and the dependent variable can be accommodated in this framework. It is not necessary to have a model that is amenable to Gibbs sampling. The draws of the stochastic weather variables are then used to make draws from the conditional predictive distribution of loss cost ratios. In this way, the required distribution of loss cost ratios is recovered and can be used for rating.

5 | APPLICATION TO CROP YIELDS

We first demonstrate the efficacy of this approach in the case of crop yields. Whether in the U.S. or elsewhere, yields are the fundamental loss variable underpinning many crop insurance programs. An approach for incorporating historical weather information in the construction of yield distributions is useful in many settings. To establish the basic properties of this approach, we first conduct a simulation exercise under varying levels of error variance and increasing amounts of historical weather information.

5.1 | Simulation study

We have in mind data on crop yields and weather from a single county over a period of 75 years. The setup is similar to Equation 5 except that level yields are modeled instead of logarithmic yields. Crop yield is assumed to be a linear function of four stochastic variables (weather variables) and two deterministic variables (intercept and year trend). Furthermore, $[low, med, high, prec] = Z_S \sim MVN(\chi, \Sigma)$ and $\epsilon \sim N(0, \sigma^2)$. The true parameters are as follows

$$\chi = \begin{bmatrix} 1543 \\ 1595 \\ 18 \\ 58 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 3451 & 15467 & 608 & 232 \\ 15467 & 86823 & 4153 & 900 \\ 608 & 4153 & 371 & -20 \\ 232 & 900 & -20 & 185 \end{bmatrix}, \quad \beta = \begin{bmatrix} -3.420963e+03 \\ 1.679168e+00 \\ 1.077635e-01 \\ 2.828317e-02 \\ -9.477178e-01 \\ -7.359430e-02 \end{bmatrix}, \tag{11}$$

where we vary the error variance between 5, 15, and 30. These values are intended to approximate the estimated weather distribution and coefficients for the full sample in the following empirical application. These hypothetical distributions are also similar to results for nonirrigated corn in Tack et al. (2018).

The true predictive yield distribution is then recovered by drawing 10,000 times from the data generating process with the time trend fixed at 2018. We arbitrarily choose 2018 as the year to forecast yields. The true distribution is then compared with three alternative predictive distributions. The first is based on the weather-conditioned model detailed above but uses no historical weather information. The second is the same model but with historical weather information. Last, we compare with an approach based only on the use of observed yields.

The true predictive distribution is shown in Figure A1 of the Appendix S1 by fitting a kernel density estimate to the draws from the data generating process. The distribution has mean of 158 bushels/acre. The shape of this distribution is notable in that, as mentioned in the preceding sections, it does not follow a specific parametric form. The distribution has a small degree of negative skew with skewness coefficient of −0.09. This small amount of skewness is driven by the weather distribution as well as the nonlinear relationship between temperature and yields implied by Equation 5.

We then conduct a rating simulation with varying sample sizes and error variances. For each error variance, we draw weather and yield histories of 15, 30, and 50 years. We assume that 75 total years of weather data are available in all cases. Three estimates of the conditional yield distribution are then obtained: the weather-conditioned model without historical weather information, the model using all historical information, and a simple empirical estimate. This procedure is repeated 1000 times for each sample size/error variance combination. At each iteration, we use the estimated distributions to compute rates at three coverage levels and compare this to the rates from the true distribution. The rates from the yield distributions are calculated according to

$$\text{Premium Rate} = \frac{p\left(Y^* < \lambda \overline{Y}^*\right)\left(\lambda \overline{Y}^* - E\left(Y^* | Y^* < \lambda \overline{Y}^*\right)\right)}{\lambda \overline{Y}^*}, \tag{12}$$

where $0 < \lambda < 1$ is the coverage level and $\overline{Y}^*$ is the mean of the conditional predictive yield distribution, that is, the expected yield. All probabilities and expectations are taken from the conditional predictive distribution.

The results of the simulation are shown in Table 1. In general, the weather-conditioned approaches perform better when the error standard deviation is smaller. This makes some sense as larger standard deviations imply that more of the variation in yields is unaccounted for by the regression model. The models that account for historical weather data also perform better than those without at lower error variances. As expected, all models perform better at larger sample sizes. Results are supportive of the weather-based approach, with the weather conditioned models outperforming the empirical approach except in the case of very large error standard deviations.

The simulation results provide a slightly idealized view of the performance of these models as they are based on a known distribution and an underlying data-generating process that follows exactly the weather conditioned model. Whether the weather conditioned approach is empirically more efficient compared to a yield-only approach is the subject of the next section. Nonetheless, the results of the simulation emphasize the importance of a model that is capable of accurately relating weather to yields. A natural question is whether a weather-conditioned model is likely to provide any improvement in modeling if the weather variables are not informative for yields or loss costs. Indeed, if the distribution of weather or the model relating weather to yields is grossly uninformative, then the weather-conditioned model is likely to underperform a purely empirical approach (Table 1).

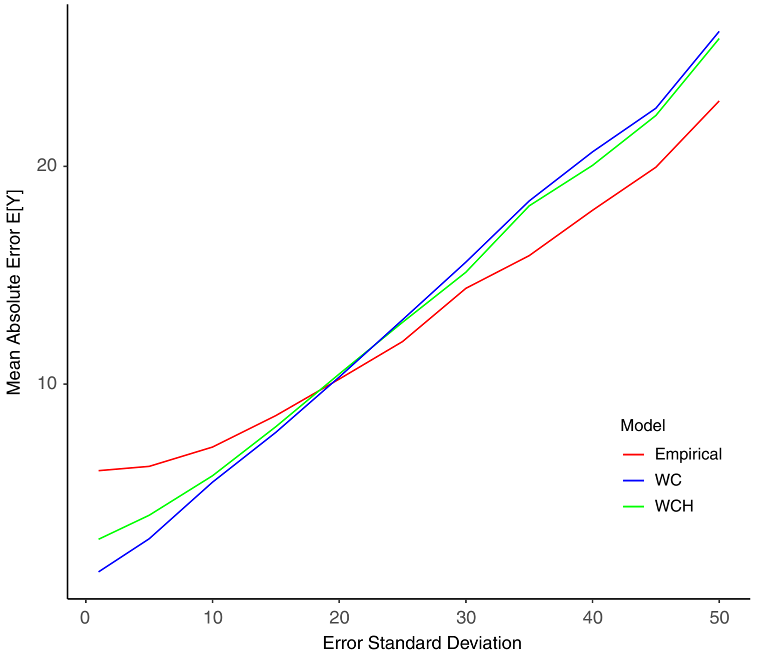
TABLE 1 Simulation results

Model	Sample size	Error SD	Mean absolute error from true distribution			
			E[Y]	Rate 75	Rate 85	Rate 90
Emp	15	5	6.36	0.29	0.53	1.45
Emp	15	15	8.78	0.59	1.31	2.56
Emp	15	30	14.39	2.75	4.00	5.40
WC	15	5	3.67	0.04	0.19	0.62
WC	15	15	8.25	0.89	1.81	2.96
WC	15	30	15.79	4.99	6.53	7.67
WCH	15	5	2.84	0.03	0.17	0.59
WCH	15	15	7.97	1.06	2.11	3.39
WCH	15	30	16.07	5.73	7.41	8.67
			Mean absolute error from true distribution			
Model	Sample size	Error SD	E[Y]	Rate 70	Rate 80	Rate 90
Emp	50	5	3.21	0.04	0.19	0.68
Emp	50	15	4.38	0.24	0.07	0.79
Emp	50	30	7.65	1.07	2.85	4.67
WC	50	5	2.00	0.01	0.05	0.27
WC	50	15	3.84	0.24	0.04	0.98
WC	50	30	7.23	1.59	2.19	6.12
WCH	50	5	1.87	0.00	0.05	0.23
WCH	50	15	3.73	0.00	0.07	0.77
WCH	50	30	7.24	3.93	0.63	5.79

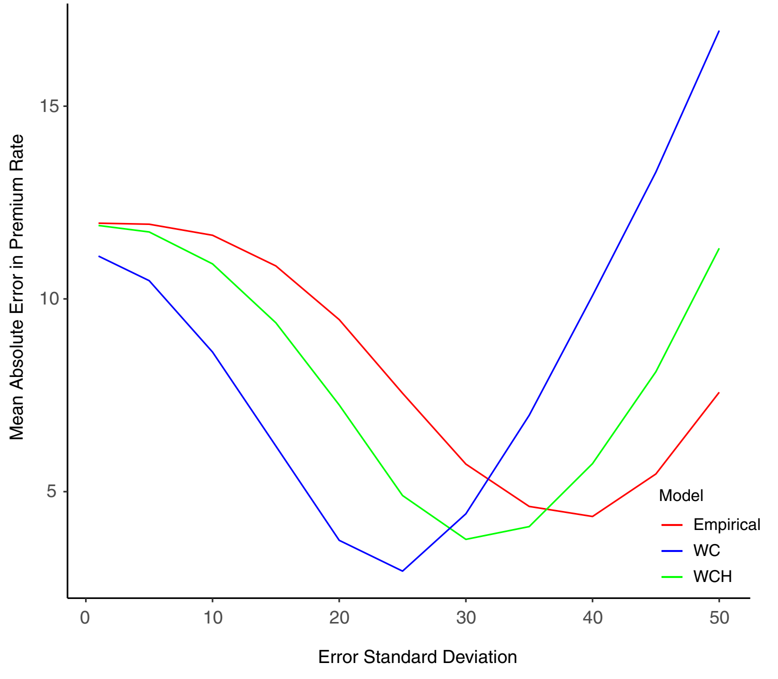
Note: True mean is approximately 158 bushels/acre. True rates vary by error variance. EMP denotes an RMA-like model using the empirical distribution of detrended and heteroskedasticity adjusted yields. WC is the weather conditioned Bayesian model. WCH is the weather conditioned Bayesian model using all available historical weather data.

To investigate this issue, we run the same simulation with an observed sample of 15 years of yields and vary the error standard deviation from 1 to 50 in increments of 5. Again, we draw 10,000 times from the data generating process and compute the mean absolute error for each approach across the 10,000 draws. The results are shown in Figure 1 with the top plot showing the mean absolute error for the expected yield and the bottom plot showing the mean absolute error for the rate at the 90% coverage level. We find that, consistent with our expectations, when there is a large degree of uncertainty in the relationship between weather and yields (given by the error variance), the performance of the weather-conditioned models degrades. For the expected yield, the error standard deviation at which the empirical and weather-conditioned models cross is 20, whereas it is 30 to 35 for the rates at the 90% coverage level.

It should be emphasized that the preceding simulation is dependent on the assumed true model. However, it does raise the question of the impact of a model for weather and the dependent variable that is not informative. In those cases, we would not suggest weather conditioning, in line with Rejesus et al. (2015). We also argue that there has been substantial work in these sorts of production models such that the data and empirical techniques exist to accurately predict yields or loss cost ratios. A common critique of such models is that they may not adequately capture changes in the relationship between weather and yields arising from technological change or change in management practices. The Bayesian approach is somewhat insulated from this critique because only recent observations (those with yield and weather data) are used to estimate this production relationship.



(a)



(b)

FIGURE 1 Mean absolute errors from simulated data (a) Mean absolute error for E[Y]. (b) Mean absolute error for premium rate at 90% coverage. Empirical denotes an RMA-like model using the empirical distribution of detrended and heteroskedasticity adjusted yields. WC is the weather conditioned Bayesian model. WCH is the weather conditioned Bayesian model using all available historical weather data.

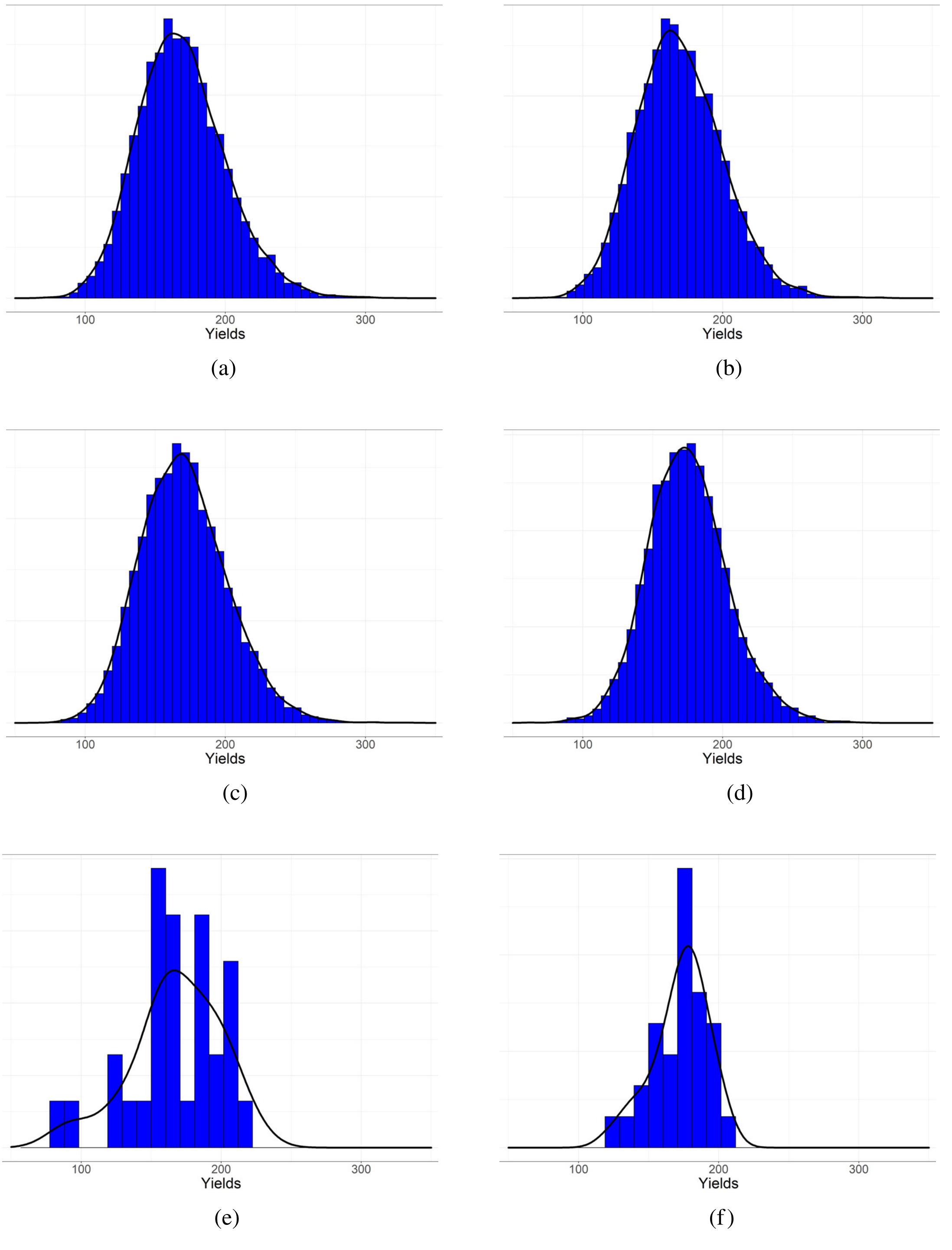


FIGURE 2 Estimated 2018 conditional yield densities with 30-year cutoff (a) Bayesian unrestricted, Illinois, Brown. (b) Bayesian unrestricted, Iowa, Adams. (c) Bayesian restricted, Illinois, Brown. (d) Bayesian restricted, Iowa, Adams. (e) RMA empirical, Illinois, Brown. (f) RMA empirical, Iowa, Adams.

Historical weather data are primarily used to update estimates of the distribution of weather. These historical observations are then essentially now casted. Ultimately, the suitability of the methodological approaches is an empirical question, which we examine shortly.

5.2 | Empirical application

We assess the economic consequences of the weather-conditioned rating approach through games and resampling tests. Versions of these tests have been used in the crop insurance literature (Liu & Ker 2021; Park et al. 2019; Shen et al. 2016). The data are county-level corn yields from Illinois, Iowa, Michigan, Minnesota, Missouri, Ohio, and Wisconsin from 1955 to 2017. We delete any county where over 10% of harvested acreage is irrigated. Weather data are the same as those of Schlenker and Roberts (2009), and we assume that the growing season starts April 1 and concludes on September 30.

We use the same weather variable construction and regression specification as in Tack et al. (2018), although we vary the specification in succeeding robustness checks. Our approach deviates

TABLE 2 Bayesian unrestricted vs RMA empirical, 90% coverage

State	N of ctys	Retained by pvt(%)	Loss ratio Govt	Loss ratio Pvt	Loss ratio Difference	Game 1 *p*-value	Game 2 *p*-value
30-year cutoff							
Illinois	73	23.6	1.522	0.359	1.163	0.0059	0.0577
Iowa	91	14.3	1.142	0.243	0.899	0.0002	0.0207
Michigan	30	11.8	0.915	0.205	0.710	0.0000	0.0013
Minnesota	57	4.8	0.597	0.101	0.496	0.0000	0.0577
Missouri	25	19.6	1.621	0.445	1.176	0.0002	0.0207
Ohio	58	30.7	1.241	0.651	0.563	0.0002	0.0002
Wisconsin	48	13.6	1.005	0.353	0.652	0.0002	0.0059
25-year cutoff							
Illinois	73	18.6	1.552	0.460	1.092	0.0000	0.0207
Iowa	91	20.1	1.131	0.386	0.745	0.0002	0.1316
Michigan	30	9.3	0.933	0.404	0.529	0.0000	0.0207
Minnesota	57	7.7	0.675	0.041	0.634	0.0000	0.0013
Missouri	25	22.6	1.617	0.618	0.999	0.0013	0.0207
Ohio	58	16.2	1.322	0.430	0.892	0.0002	0.0013
Wisconsin	48	9.5	1.113	0.191	0.922	0.0000	0.0207
20-year cutoff							
Illinois	73	17.5	1.823	0.219	1.604	0.0000	0.0013
Iowa	91	14.0	1.084	0.624	0.460	0.0000	0.1316
Michigan	30	2.5	1.100	1.752	−0.652	0.0002	0.0059
Minnesota	57	3.7	0.753	0.030	0.723	0.0000	0.2517
Missouri	25	16.0	1.674	0.717	0.957	0.0002	0.1316
Ohio	58	6.9	1.406	0.130	1.276	0.0000	0.0207
Wisconsin	48	5.0	1.383	0.196	1.187	0.0000	0.0059

Note: Games conducted over 20 years (1998–2017). Game 1 is the adverse selection game of Ker and McGowan (2000), and Game 2 is the efficiency game of Ker et al. (2016). Reported *p*-values for both are binomial and based on annual Bernoulli trial with number of trials equal to 20 and binomial parameter equal to 0.5 (under the null).

from Tack et al. (2018) in that we use a model at the county level as in Equation 5. This implicitly allows the weather coefficients to vary by county. Our weather model is in line with the way that rates are constructed in the federal crop insurance program in that only data from the county in question is utilized. However, an interesting extension might be to consider borrowing information across counties in the estimation of the model relating weather to yields. This could provide additional strength to the weather conditioned approach in cases where the yield history is short with the qualification that such a model would not account for heterogeneity in weather effects.

The first game follows Ker and McGowan (2000), and is intended to mimic the situation faced by private insurers deciding to cede or retain policies sold in the federal crop insurance program. Private insurers do not determine rates for federal crop insurance policies and are mandated to sell all policies provided by the Federal Crop Insurance Corporation (FCIC) in locations in which they operate. The USDA-FCIC Standard Reinsurance Agreement provides a means for private insurers to cede certain policies and achieve asymmetric underwriting gains. Basically, private insurers can develop their own rating procedures and use such procedures to adversely select against the FCIC. The game thus provides a means of economically evaluating the benefits of one actuarial approach

TABLE 3 Bayesian restricted versus RMA empirical, 90% coverage

State	N of ctys	Retained by pvt(%)	Loss ratio Govt	Loss ratio Pvt	Loss ratio Difference	Game 1 p-value	Game 2 p-value
30-year cutoff							
Illinois	73	19.3	1.467	0.293	1.174	0.0013	0.0207
Iowa	91	13.0	1.155	0.121	1.034	0.0000	0.0577
Michigan	30	15.3	0.960	0.192	0.768	0.0000	0.0013
Minnesota	57	5.3	0.603	0.091	0.512	0.0000	0.0577
Missouri	25	18.6	1.596	0.521	1.075	0.0000	0.0577
Ohio	58	28.9	1.211	0.654	0.557	0.0000	0.0013
Wisconsin	48	15.0	0.997	0.435	0.562	0.0013	0.0059
25-year cutoff							
Illinois	73	18.8	1.541	0.476	1.065	0.0000	0.0577
Iowa	91	23.0	1.177	0.332	0.845	0.0000	0.0207
Michigan	30	10.8	0.927	0.521	0.406	0.0000	0.0059
Minnesota	57	8.2	0.674	0.052	0.622	0.0000	0.0059
Missouri	25	23.4	1.657	0.619	1.038	0.0000	0.0207
Ohio	58	14.1	1.331	0.219	1.112	0.0002	0.0059
Wisconsin	48	13.1	1.138	0.299	0.839	0.0000	0.0059
20-year cutoff							
Illinois	73	20.3	1.702	0.800	0.902	0.0000	0.0059
Iowa	91	21.8	1.113	0.633	0.480	0.0000	0.1316
Michigan	30	2.5	1.129	0.850	0.279	0.0000	0.1316
Minnesota	57	7.6	0.742	0.474	0.268	0.0000	0.4119
Missouri	25	23.2	1.680	1.070	0.610	0.0000	0.1316
Ohio	58	5.2	1.387	0.090	1.297	0.0000	0.0013
Wisconsin	48	7.3	1.415	0.195	1.220	0.0000	0.0013

Note: Games conducted over 20 years (1998–2017). Game 1 is the adverse selection game of Ker and McGowan (2000), and Game 2 is the efficiency game of Ker et al. (2016). Reported p-values for both are binomial and based on annual Bernoulli trial with number of trials equal to 20 and binomial parameter equal to 0.5 (under the null).

TABLE 4 Bayesian unrestricted versus RMA empirical, 80% coverage

State	N of ctys	Retained by pvt(%)	Loss ratio Govt	Loss ratio Pvt	Loss ratio Difference	Game 1 *p*-value	Game 2 *p*-value
30-year cutoff							
Illinois	73	32.2	2.203	0.271	1.932	0.0000	0.0059
Iowa	91	21.2	1.058	0.212	0.846	0.0000	0.0013
Michigan	30	22.3	1.046	0.175	0.871	0.0000	0.0000
Minnesota	57	10.6	0.202	0.043	0.159	0.0000	0.0002
Missouri	25	24.0	1.992	0.542	1.450	0.0000	0.0059
Ohio	58	40.0	1.317	0.670	0.647	0.0000	0.0013
Wisconsin	48	29.9	1.215	0.227	0.988	0.0000	0.0002
25-year cutoff							
Illinois	73	22.7	2.025	0.374	1.651	0.0000	0.0207
Iowa	91	23.0	1.013	0.284	0.729	0.0000	0.0013
Michigan	30	13.7	0.730	0.691	0.039	0.0000	0.0013
Minnesota	57	12.0	0.225	0.045	0.180	0.0002	0.0000
Missouri	25	22.8	1.884	0.609	1.275	0.0002	0.0013
Ohio	58	21.3	1.567	0.344	1.223	0.0000	0.0002
Wisconsin	48	16.8	1.056	0.244	0.812	0.0002	0.0207
20-year cutoff							
Illinois	73	18.8	2.462	0.024	2.438	0.0000	0.0059
Iowa	91	12.3	0.989	0.398	0.591	0.0000	0.0059
Michigan	30	5.7	1.062	0.951	0.111	0.0000	0.0059
Minnesota	57	6.1	0.255	0.024	0.231	0.0000	0.0013
Missouri	25	15.4	2.071	0.470	1.601	0.0002	0.0577
Ohio	58	9.6	1.629	0.190	1.439	0.0000	0.0013
Wisconsin	48	6.7	1.393	0.072	1.321	0.0000	0.0059

Note: Games conducted over 20 years (1998–2017). Game 1 is the adverse selection game of Ker and McGowan (2000), and Game 2 is the efficiency game of Ker et al. (2016). Reported *p*-values for both are binomial and based on annual Bernoulli trial with number of trials equal to 20 and binomial parameter equal to 0.5 (under the null).

or rating scheme compared to another. The null hypothesis in Game 1 is that the choices of private insurers are no different from randomization.

The second game is similar to the first but recognizes that private insurers have an inherent advantage regardless of the rating approach. Private insurers react to the rates set by the FCIC. Ker et al. (2016) developed an alternative test that is similar in spirit to the first game of Ker and McGowan (2000). However, the second mover advantage enjoyed by the private insurers is nullified in their alternative test. The null hypothesis tested in Game 2 is that the proposed method used by the private insurers and that used by the FCIC are equally efficient. Details of this approach are available in Ker et al. (2016). Both games are essentially resampling type tests.

We compare three alternative rating systems. The first is an approximation of the rating approach currently employed by the FCIC. We assume that the FCIC first estimates a linear trend for each county using available yield data and then adjusts for heteroskedasticity following Harri et al. (2011). Adjusted yields are then used to generate the empirical premium rate for the next period following the approach detailed in Liu and Ker (2021). Although the FCIC uses two-step ahead forecasts in practice, without loss of generality, we consider one-step ahead forecasts in the empirical analysis. The second and third rating systems are versions of the Bayesian weather conditioned model: one that uses the same amount of restricted data as the FCIC model and one that uses all available historical weather data.

We consider three data cutoffs: 20, 25, and 30 years.[iii] The FCIC rating methodology only uses available yield data (i.e. up to the cutoff) to estimate the conditional distribution of yields and

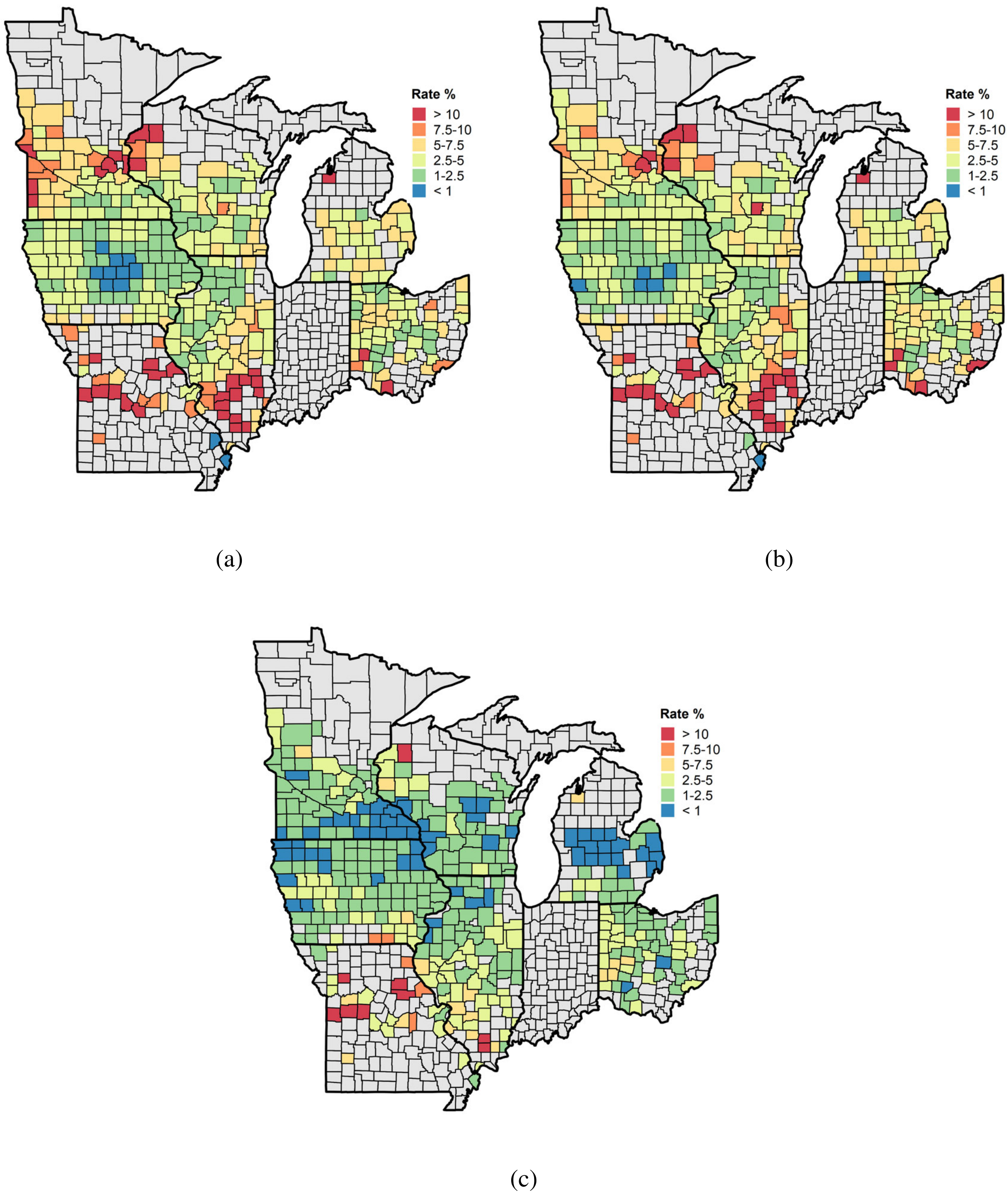


FIGURE 3 Corn premium rates (2018): 90% coverage level (a) Bayesian unrestricted. (b) Bayesian restricted. (c) RMA empirical. All three types of rates are one-year ahead forecasts (2018) based on most recent 30-year of available yields (1988–2017). Bayesian unrestricted uses all available historical weather information; Bayesian restricted uses the same 30-year cutoff on weather information; and RMA empirical uses no weather information.

determine premium rates. The FCIC methodology is compared to our Bayesian approach, and thus we assume that the private insurers in the games use the Bayesian weather conditioned rating schemes. Our approach differs in the incorporation and use of historical weather information. Although we only use yield data up to the cutoff, we incorporate all available weather data. For instance, at the 30 year cutoff, both methods use yield data from the previous 30 years. The Bayesian method also uses any additional weather data beyond the 30 year period.

The games are based on periods of 20 years. As the game advances, additional data are available in the form of historical weather information. Based on the rates for FCIC and the private insurer,

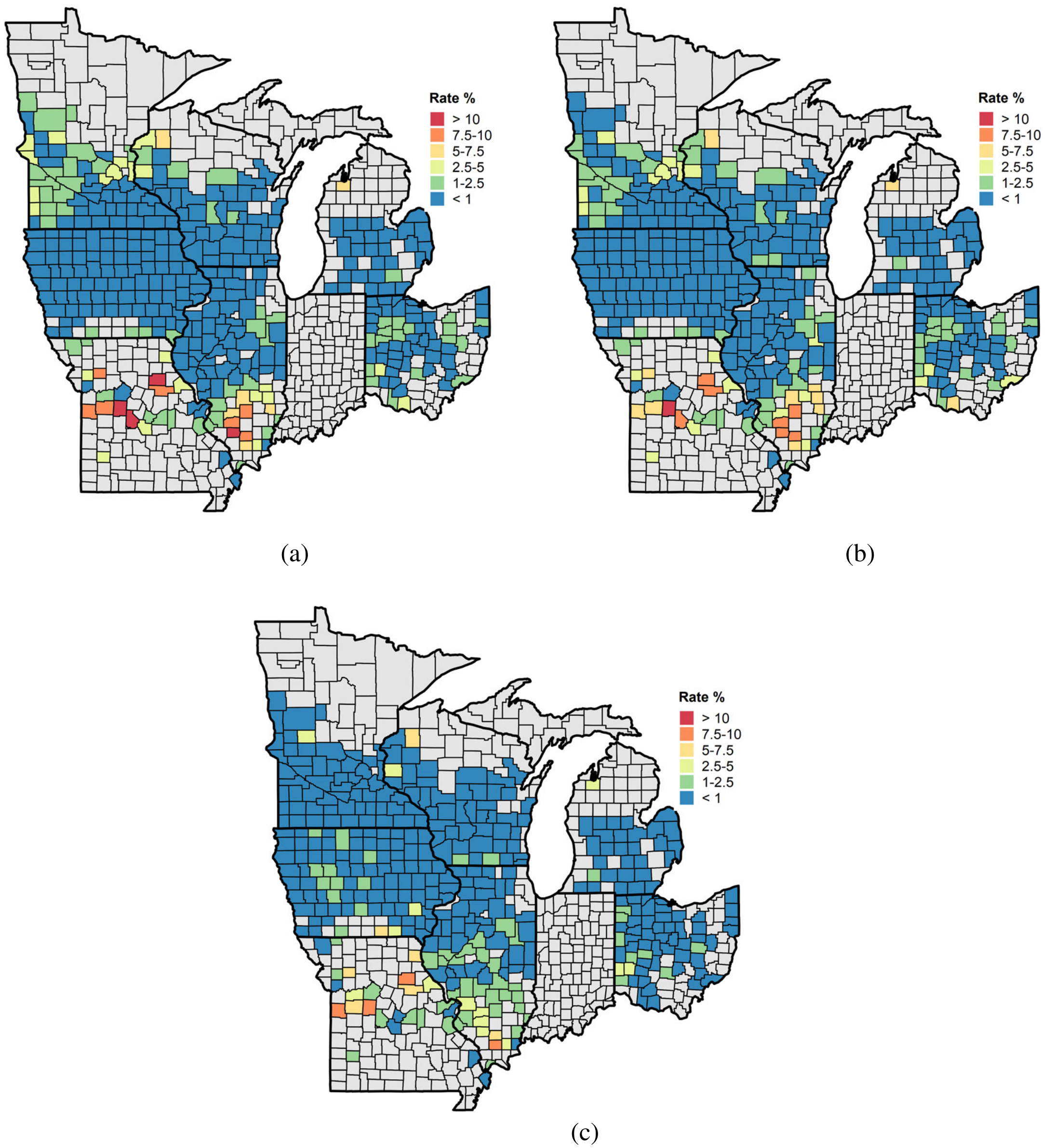


FIGURE 4 Corn premium rates (2018): 70% coverage level (a) Bayesian unrestricted. (b) Bayesian restricted. (c) RMA empirical. All three types of rates are 1-year ahead forecasts (2018) based on most recent 30-year of available yields (1988–2017). Bayesian unrestricted uses all available historical weather information; Bayesian restricted uses the same 30-year cutoff on weather information; and RMA empirical uses no weather information.

the private insurer decides which contracts to retain. Underwriting gains and losses for the various pools of retained or ceded contracts are then calculated based on the actual yield from the projected year. The process is repeated each year between 1998 and 2017, and loss ratios for the retained and ceded contracts are obtained. We then examine whether the loss ratio in a year from the retained contracts is less than the loss ratio from a private insurer that randomly retains contracts. If the private insurer randomly retains contracts, then they are indifferent between the systems used to generate rates.

In all cases, we consider 7640 contracts (382 counties across 20 years) for county-level yield insurance. Example yield distributions are shown in Figure 2 (more examples are shown in Figures A2 and A3 in the Appendix S1). The Bayesian unrestricted model uses all available weather information, whereas the Bayesian restricted model uses the restricted data set (only observations where yields are available). Figure 2 contains histograms and kernel densities of the samples from the Bayesian predictive distributions and the predictive distribution based on empirical adjusted yields. As evidenced in the plots, the empirical distributions are multimodal, and single observations can be very important in determining the shape of the yield distribution. In contrast, the weather conditioned distributions are generally smoother.

TABLE 5 Bayesian restricted versus RMA empirical, 80% coverage

State	N of ctys	Retained by pvt(%)	Loss ratio Govt	Loss ratio Pvt	Loss ratio Difference	Game 1 p-value	Game 2 p-value
30-year cutoff							
Illinois	73	27.6	1.987	0.311	1.676	0.0000	0.0013
Iowa	91	18.1	1.074	0.112	0.962	0.0000	0.0013
Michigan	30	24.7	1.131	0.168	0.963	0.0000	0.0002
Minnesota	57	13.0	0.190	0.112	0.078	0.0000	0.0013
Missouri	25	21.8	1.955	0.548	1.407	0.0000	0.0013
Ohio	58	36.9	1.234	0.679	0.555	0.0000	0.0013
Wisconsin	48	31.0	1.243	0.271	0.972	0.0002	0.0059
25-year cutoff							
Illinois	73	22.2	2.006	0.403	1.603	0.0000	0.0207
Iowa	91	25.1	0.984	0.384	0.600	0.0000	0.0013
Michigan	30	15.0	0.813	0.478	0.335	0.0000	0.0059
Minnesota	57	13.8	0.229	0.057	0.172	0.0002	0.0013
Missouri	25	26.0	1.891	0.925	0.966	0.0002	0.0013
Ohio	58	18.1	1.542	0.091	1.451	0.0000	0.0002
Wisconsin	48	17.6	1.061	0.264	0.797	0.0013	0.0013
20-year cutoff							
Illinois	73	19.2	2.271	0.503	1.768	0.0000	0.0059
Iowa	91	15.7	1.013	0.430	0.583	0.0000	0.0207
Michigan	30	4.3	1.100	0.489	0.611	0.0000	0.0207
Minnesota	57	9.6	0.264	0.000	0.264	0.0000	0.0002
Missouri	25	22.8	1.964	1.350	0.614	0.0000	0.0577
Ohio	58	7.6	1.589	0.166	1.423	0.0002	0.0000
Wisconsin	48	7.6	1.424	0.009	1.415	0.0000	0.0059

Note: Games conducted over 20 years (1998–2017). Game 1 is the adverse selection game of Ker and McGowan (2000), and Game 2 is the efficiency game of Ker et al. (2016). Reported p-values for both are binomial and based on annual Bernoulli trial with number of trials equal to 20 and binomial parameter equal to 0.5 (under the null).

Results from both games with a linear time trend and an unrestricted Bayesian model at the 90%, 80%, and 70% coverage levels are reported in Tables 2, 4, and 6. The same results versus the Bayesian restricted model are available in Tables 3, 5, and 7. The results are strongly supportive of the Bayesian models that incorporate weather information. In Tables A1, A2, and A3 of the Appendix S1, we also test the unrestricted model against the restricted Bayesian model. The performance of the model with historical weather information is again superior, although the difference is not as significant as in the comparisons to the empirical approach. The Game 1 p-values are all statistically significant at conventional levels of significance, and the p-values for Game 2 are largely consistent with this result.

Comparison across Tables 2, 4, and 6 shows increasing advantage for the Bayesian approach at lower coverage levels. The inclusion of historical negative weather events may also place more mass in the lower tail of the distribution and better reflect the true probability of low yield realizations. Figures 3 and 4 show maps of premium rates for all three models, in 2018, and at the 90% and 70% coverage levels respectively. The biggest difference in rates is observed at the 90% coverage level, as expected, and there are several patterns in the maps.[iv] The premium rates obtained from the models using historical weather information are generally larger than those obtained from the empirical

TABLE 6 Bayesian unrestricted versus RMA empirical, 70% coverage

State	N of ctys	Retained by pvt(%)	Loss ratio Govt	Loss ratio Pvt	Loss ratio Difference	Game 1 p-value	Game 2 p-value
30-year cutoff							
Illinois	73	39.1	4.342	0.423	3.919	0.0000	0.0000
Iowa	91	22.7	1.270	0.069	1.201	0.0000	0.0000
Michigan	30	30.3	1.979	0.241	1.738	0.0000	0.0000
Minnesota	57	21.1	0.030	0.031	- 0.001	0.0000	0.0000
Missouri	25	30.2	2.668	0.651	2.017	0.0000	0.0000
Ohio	58	40.4	2.061	0.536	1.525	0.0002	0.0013
Wisconsin	48	42.1	2.747	0.167	2.580	0.0000	0.0002
25-year cutoff							
Illinois	73	27.0	3.788	0.396	3.392	0.0000	0.0000
Iowa	91	20.7	1.071	0.230	0.841	0.0000	0.0000
Michigan	30	24.8	1.127	0.278	0.849	0.0000	0.0002
Minnesota	57	18.6	0.048	0.000	0.048	0.0000	0.0000
Missouri	25	27.2	2.415	0.563	1.852	0.0000	0.0000
Ohio	58	25.8	2.040	0.597	1.443	0.0000	0.0002
Wisconsin	48	30.2	1.517	0.146	1.371	0.0000	0.0002
20-year cutoff							
Illinois	73	19.8	4.225	0.185	4.040	0.0000	0.0002
Iowa	91	12.3	1.026	0.278	0.748	0.0000	0.0002
Michigan	30	12.7	1.025	0.550	0.475	0.0000	0.0013
Minnesota	57	16.3	0.013	0.000	0.013	0.0000	0.0002
Missouri	25	18.0	2.779	0.166	2.613	0.0000	0.0207
Ohio	58	12.5	2.011	0.257	1.754	0.0000	0.0000
Wisconsin	48	15.7	1.714	0.021	1.693	0.0000	0.0002

Note: Games conducted over 20 years (1998–2017). Game 1 is the adverse selection game of Ker and McGowan (2000), and Game 2 is the efficiency game of Ker et al. (2016). Reported p-values for both are binomial and based on annual Bernoulli trial with number of trials equal to 20 and binomial parameter equal to 0.5 (under the null).

TABLE 7 Bayesian restricted versus RMA empirical, 70% coverage

State	N of ctys	Retained by pvt(%)	Loss ratio Govt	Loss ratio Pvt	Loss ratio Difference	Game 1 *p*-value	Game 2 *p*-value
30-year cutoff							
Illinois	73	33.7	3.797	0.412	3.385	0.0000	0.0000
Iowa	91	21.2	1.248	0.041	1.207	0.0000	0.0000
Michigan	30	32.8	2.461	0.224	2.237	0.0000	0.0000
Minnesota	57	21.3	0.033	0.029	0.004	0.0000	0.0000
Missouri	25	30.2	2.740	0.722	2.018	0.0000	0.0000
Ohio	58	39.1	1.928	0.567	1.361	0.0000	0.0013
Wisconsin	48	41.2	2.704	0.168	2.536	0.0000	0.0002
25-year cutoff							
Illinois	73	25.9	3.637	0.547	3.090	0.0000	0.0000
Iowa	91	20.5	0.963	0.339	0.624	0.0000	0.0000
Michigan	30	29.0	1.399	0.203	1.196	0.0000	0.0000
Minnesota	57	19.6	0.051	0.000	0.051	0.0000	0.0000
Missouri	25	33.4	2.600	0.965	1.635	0.0000	0.0000
Ohio	58	24.4	2.052	0.485	1.567	0.0000	0.0002
Wisconsin	48	33.1	1.737	0.134	1.603	0.0000	0.0059
20-year cutoff							
Illinois	73	20.1	3.764	0.608	3.156	0.0000	0.0000
Iowa	91	14.1	1.104	0.245	0.859	0.0000	0.0000
Michigan	30	14.8	1.134	0.396	0.738	0.0000	0.0002
Minnesota	57	21.2	0.014	0.000	0.014	0.0000	0.0000
Missouri	25	26.8	2.738	1.205	1.533	0.0000	0.0013
Ohio	58	12.1	2.039	0.195	1.844	0.0000	0.0000
Wisconsin	48	17.4	1.679	0.065	1.614	0.0000	0.0013

Note: Games conducted over 20 years (1998–2017). Game 1 is the adverse selection game of Ker and McGowan (2000), and Game 2 is the efficiency game of Ker et al. (2016). Reported *p*-values for both are binomial and based on annual Bernoulli trial with number of trials equal to 20 and binomial parameter equal to 0.5 (under the null).

distributions or the Bayesian unrestricted model. This accords with our earlier observation that the inclusion of historical weather information may bring more extreme negative weather events into the data. Higher rates generally align with counties on the edge of the production area. Such counties may be subject to more adverse weather events being, in general, less suitable for dry land corn production.

We also considered robustness to differences in forecast timing and in the time trend. The results in Table A4 of the Appendix S1 use a squared time trend in all specifications at the 90% coverage level. We find somewhat different results in the sense that the empirical distributions outperform the Bayesian approach in both Ohio and Wisconsin at the 20-year cutoff. Results for Game 2 in several other states are also not significant at conventional levels. But as the results are similar for most other states, we conclude that the method is relatively robust to changes in the time trend. As with any approach, the choice of functional form can be determined by out-of-sample comparisons and predictive accuracy. In terms of choosing time trends for underlying models, specifications with and without different trend components can be compared against one another. In terms of forecast timing (predicting 2 years out instead of 1), we essentially find no differences from previous results.

TABLE 8 Mean loss costs and probable maximum loss

County	Crop	Empirical	Bayesian restricted	Bayesian unrestricted
Mean loss cost ratio				
Adams, IL	Corn	0.070	0.079	0.086
Peoria, IL	Corn	0.035	0.035	0.032
Adair, IA	Corn	0.048	0.054	0.058
Montgomery, IA	Corn	0.038	0.042	0.051
Adams, IL	Soybeans	0.040	0.043	0.043
Peoria, IL	Soybeans	0.023	0.024	0.022
Adair, IA	Soybeans	0.031	0.034	0.038
Montgomery, IA	Soybeans	0.032	0.036	0.044
1 in 5 PML				
Adams, IL	Corn	0.128	0.113	0.123
Peoria, IL	Corn	0.030	0.048	0.044
Adair, IA	Corn	0.064	0.075	0.083
Montgomery, IA	Corn	0.071	0.058	0.074
Adams, IL	Soybeans	0.056	0.055	0.055
Peoria, IL	Soybeans	0.030	0.032	0.028
Adair, IA	Soybeans	0.053	0.046	0.051
Montgomery, IA	Soybeans	0.051	0.048	0.061
1 in 10 PML				
Adams, IL	Corn	0.140	0.155	0.171
Peoria, IL	Corn	0.072	0.068	0.059
Adair, IA	Corn	0.165	0.104	0.114
Montgomery, IA	Corn	0.097	0.083	0.105
Adams, IL	Soybeans	0.078	0.065	0.066
Peoria, IL	Soybeans	0.049	0.039	0.034
Adair, IA	Soybeans	0.107	0.059	0.067
Montgomery, IA	Soybeans	0.086	0.060	0.081

This may not be surprising as it is not clear that one model would perform better than another at extreme coverage levels.

In addition to the empirical approach, we also conducted the game against specifications where the government is assumed to use a beta or nonparametric distribution to construct rates. These results are shown in the Appendix S1. Also in the Appendix S1, we considered specifications where weather variables were in logarithms, and there was no squared term in precipitation. In all of these situations, the weather-conditioned model (we only consider the model with historical weather information in these checks) performs well against the other candidate models. There are a few states where the p-values for Game 2 are not significant at conventional levels, but overall there is no systematic evidence against the performance of the Bayesian approaches.

In summary, a private insurer using the Bayesian method that incorporates historical weather data is able to extract significant rents. This conclusion holds across almost all specifications and cut-offs considered. Incorporating historical weather information has the potential to result in more accurate premiums across counties (as counties vary in their weather distributions). Moreover, the inclusion of additional data allows the extreme weather events to be incorporated in the model even if yield data are not available. The result is a rating methodology that allows a private insurer to adversely select against the government in a rating game. We must emphasize that in employing the rating game, we are assuming a particular criterion for the performance of the models.

6 | APPLICATION TO LOSS COST RATIOS

To demonstrate the applicability of the approach, we apply the method to the modeling of loss cost ratios. This is conceptually similar to the work of Rejesus et al. (2015) and Borman et al. (2013) in the sense that the loss cost ratios are related to historical weather information. We focus on loss costs for corn and soybean policies in Illinois and Iowa for the federal crop insurance program. We choose these states and crops because most liability is in yield or revenue policies where loss costs are likely to be determined by adverse weather events. Loss cost data were obtained from the Risk Management Agency's Summary of Business and are available from 1988 to 2017.

Unlike previous work, which has used weather data at the climate division level, we continue to use the county-level regressors from Schlenker and Roberts (2009), previously discussed. The weather data are—as before—available from 1955 onward. As these weather variables have been shown to be informative for yields, they should also be informative for loss costs of insurance policies based on yield shortfalls. We take the loss cost ratio in a county as our dependent variable and assume that the loss cost ratios follow a beta distribution. The mean and precision of the beta distribution are linear functions of the stochastic weather variables as in Equation 10. No time trend is included in the model for the loss cost ratios, which is consistent with Rejesus et al. (2015). A model for the loss cost ratio is estimated for each county; we do not consider spatial pooling with respect to the weather distributions or the model relating weather to loss cost ratios. The application is similar to our yield application except the dependent variable follows a beta distribution.

We compare three methods for obtaining conditional predictive distributions of loss cost ratios. The mean of this distribution is the relevant statistic for rating in the federal crop insurance program as shown in Equation 3. Additional discussion of the exact rating procedures is available in Tsiboe and Tack (2021) and Coble et al. (2010). It suffices that the final premiums are functions of the distributions of loss costs and crop yields. We compare the empirical distribution with the mean from the Bayesian approach where—as in the application to yields—we either restrict the amount of weather information or allow the use of the full historical weather data. We also consider quantiles of the loss cost ratio distributions, which give probable maximum loss. The probable maximum loss (PML) is the loss cost ratio that is expected to be exceeded over a certain number of years. The PML is equivalent to value at risk and is a measure of the probability of extreme loss costs.

We first consider two counties in detail to give a sense of the suitability of the beta regressions with the given weather specifications. Figure 5 shows the actual loss cost ratios against the predicted loss cost ratios from the beta regression. The beta regression is informative for predicting loss cost ratios for both counties considered. For Carroll, IL corn, the model adequately predicts the maximum observed loss cost ratio. We can also see predicted loss cost ratios for prior years where historical weather information is available. Some of the predicted historical loss costs differ in distribution from observed loss costs. The weather variables are reasonable predictors of the loss cost ratios in most counties.

Figure 6 shows the loss cost densities for six counties for the unrestricted and restricted Bayesian models. These are predictive distributions from which the mean loss cost ratios can be obtained. In four cases, the unrestricted (historical weather-conditioned) Bayesian model has more density in the tail, whereas the opposite is observed in one county. Little difference from historical weather information is apparent for Perry, IL soybeans. We expect to find that there may be some systematic differences from the inclusion of historical weather data, but the systematic nature of any differences is likely driven by the fact that we choose counties from states near one another and subject to similar weather conditions. In this sense, the weather series would be similar suggesting some commonalities in the impacts of weather across counties.

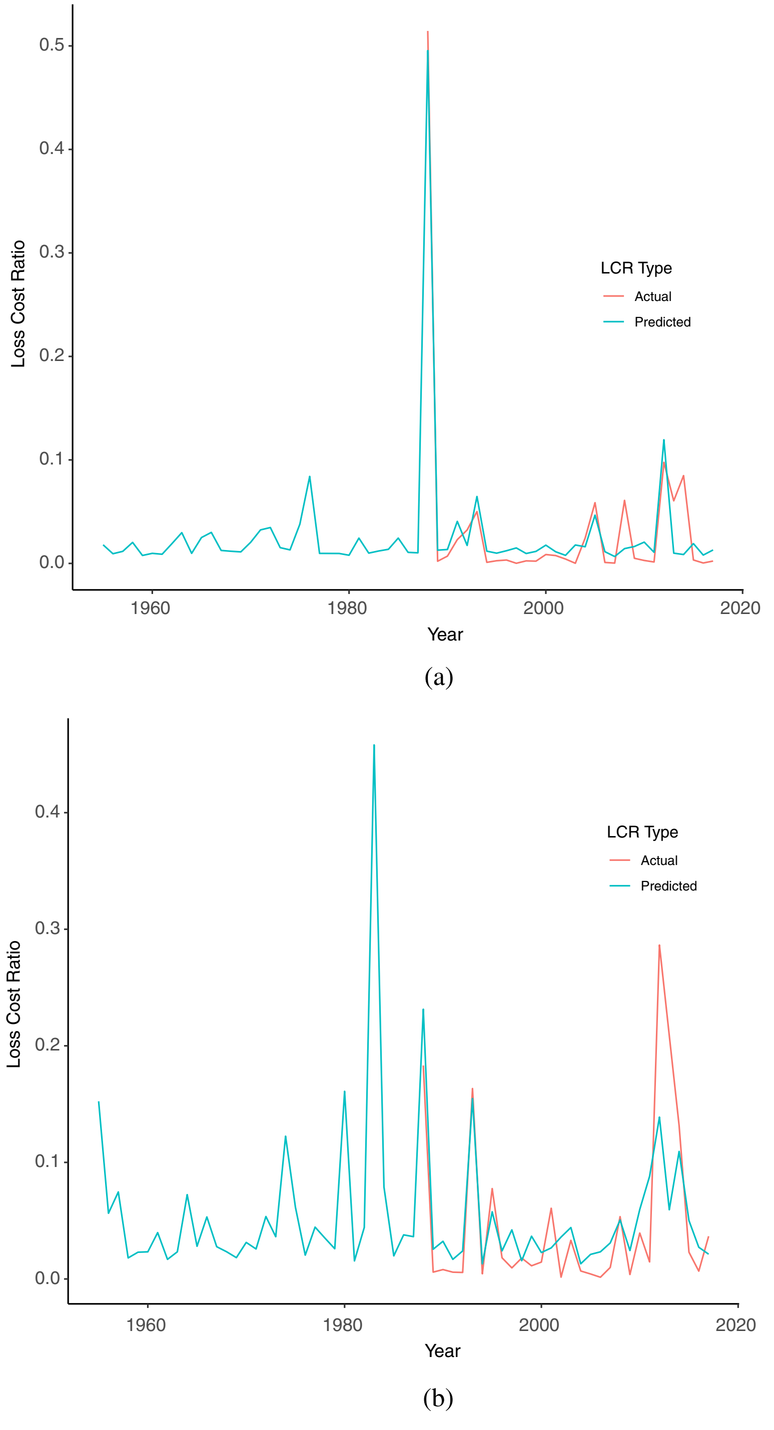


FIGURE 5 Actual and predicted loss cost ratios (a) Carroll, IL Corn. (b) Adair, IA Corn.

Table 8 shows average loss costs with and without historical weather as well as the probable maximum loss (i.e., the loss cost ratios at various quantiles of the predictive loss cost ratio distribution). The differences in mean loss costs across models are only moderate, and this is consistent with

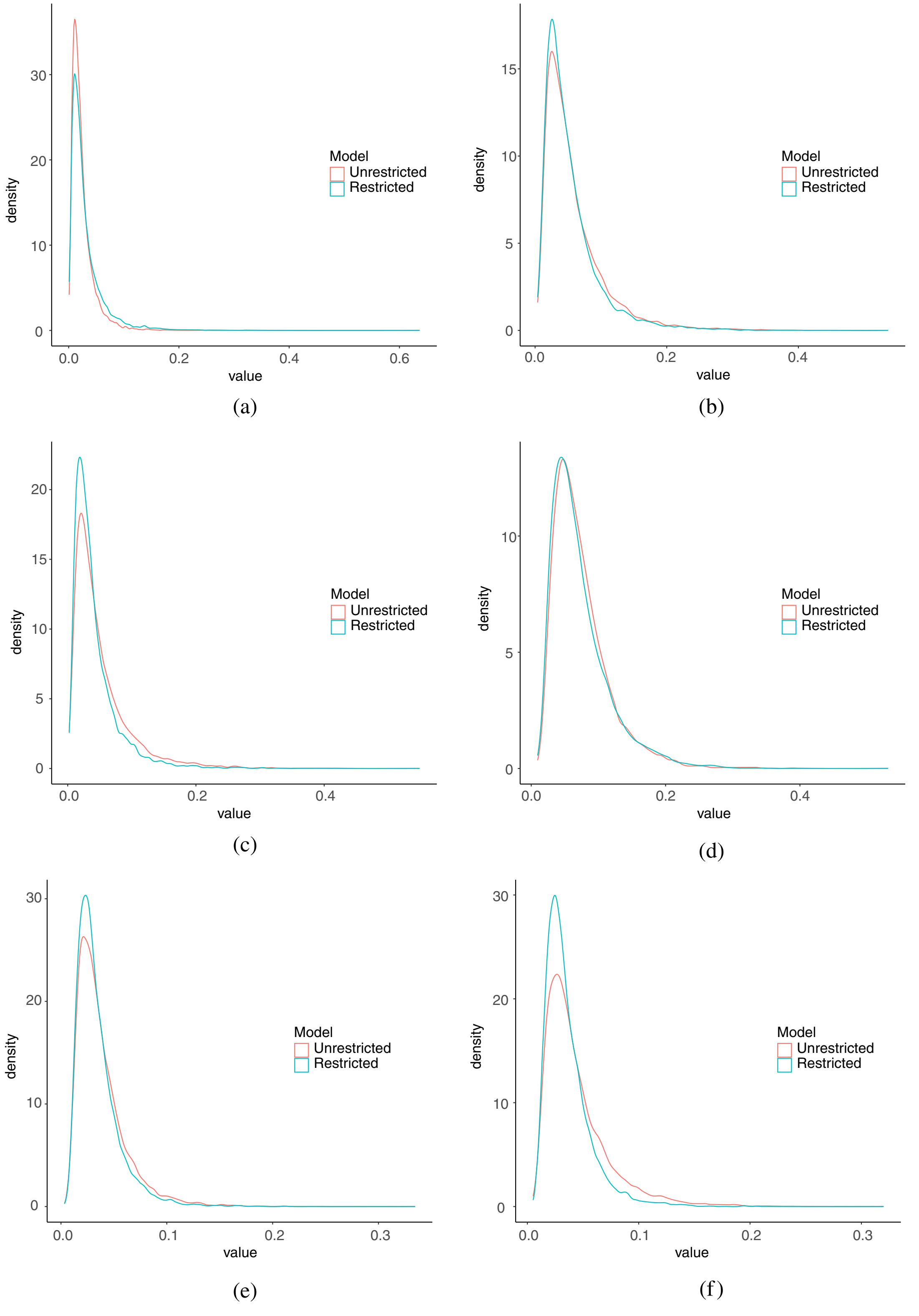


FIGURE 6 Estimated loss cost ratio densities (a) Carroll, IL Corn. (b) Adair, IA Corn. (c) Montgomery, IA Corn. (d) Perry, IL Soybeans. (e) Adair, IA Soybeans. (f) Montgomery, IA Soybeans. Restricted is the weather conditioned Bayesian model. Unrestricted is the weather conditioned Bayesian model using all available historical weather data.

findings in Borman et al. (2013). There are also modest differences in the 1 in 5 and 1 in 10 probable maximum loss.

The mean loss cost ratios for all counties in the sample are plotted in Figure 7. The mean ratio is generally higher for the weather conditioned estimates (restricted weather in green and unrestricted shown in blue). The differences are most notable for counties with high loss cost ratios. However, we cannot suggest that this observation for Illinois and Iowa corn and soybeans be extrapolated beyond

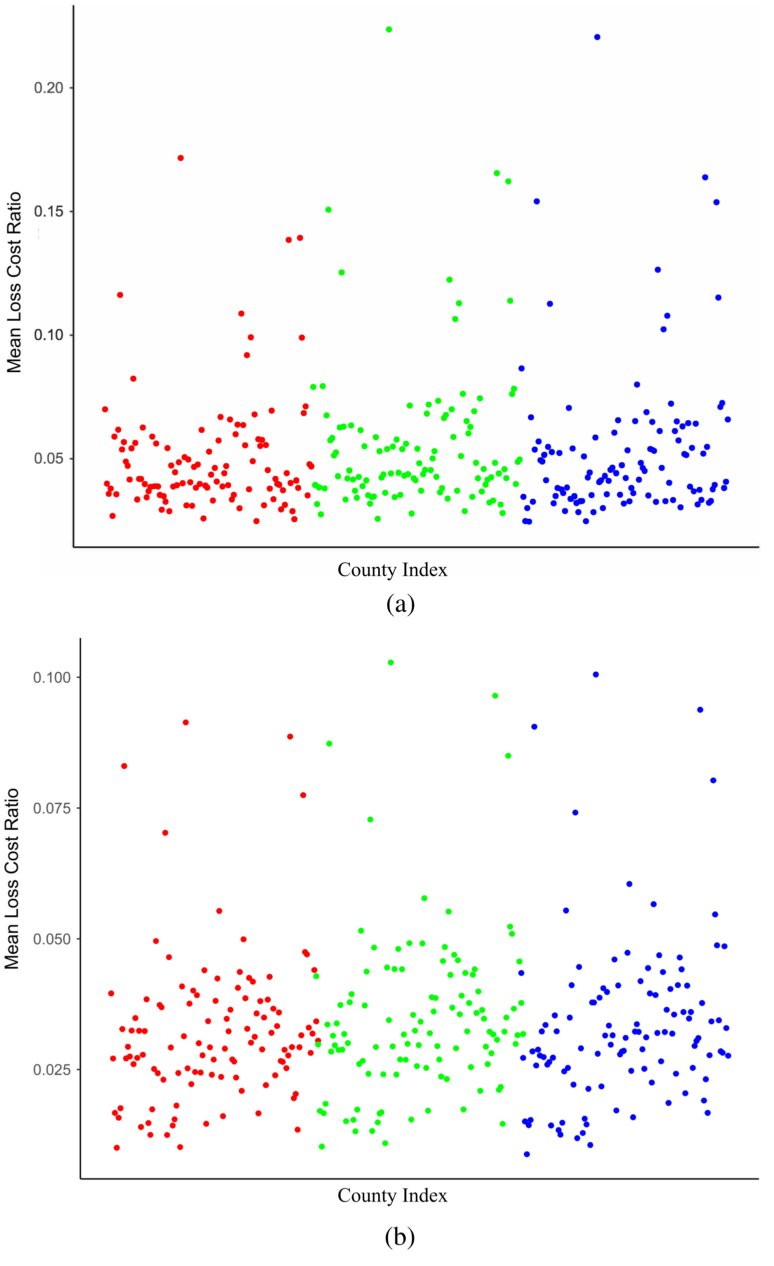


FIGURE 7 Predicted mean loss costs (a) Corn. (b) Soybeans. Red points are predicted mean loss costs for all counties from an empirical distribution. Green points and blue points are from Bayesian restricted and Bayesian unrestricted models, respectively. X-axis arranges counties by county identification number.

this example. The effect of including historical weather information is an empirical question. We could just as easily have a situation where historical weather information lowers the mean of the loss cost ratio (i.e. extreme observations are more extreme in retrospect), and this effect is shown for several counties in Table 8. The loss cost distribution for any county/crop combination is ultimately determined by differences in the contemporaneous and historical weather distributions for the Bayesian models.

This application presents a Bayesian approach for incorporating historical weather information into the modeling of loss cost ratios. As noted in Rejesus et al. (2015), modeling of loss costs is inherently problematic because characteristic data for the federal crop insurance program only go back to the late 1980s. Extreme events, such as a 100-year loss, are unlikely to be observed in the available loss cost data. Or we observe extreme events, such as the 2012 drought that occurred in the Midwest, but cannot assign an accurate probability to the occurrence of this event. The method proposed here incorporates additional weather data to improve estimates of weather probabilities and the associated loss costs. This is done through a single procedure that makes use of distributional regression. Most (though not all) differences in mean loss cost ratios are modest in this application.

7 | CONCLUSION

Most crop insurance actuarial methods—aside from those that are explicitly intended for weather index insurance—eschew additional weather information. Although the federal crop insurance program accounts for historical weather data, it does so at the level of loss costs by making ex-post rate adjustments. Recent advances in measurement of weather variables and yield modeling now make it possible to explicitly incorporate historical weather information in the estimation of conditional yield distributions. Historical weather data can also be used in modeling of loss cost ratios at increasingly fine levels of aggregation. Including additional weather information has the potential to lead to more accurate rates, especially in situations where underlying yield data or loss cost data are scarce.

We implement a Bayesian approach to estimate conditional distributions of crop yields and loss cost ratios while accounting for historical weather data. The approach is applied in a missing data setting (i.e., where the dependent variable is not observed for some observations) and produces insurance rates that allow private insurers to extract rents in a rating game. The extent to which the private insurers are able to do so increases at lower coverage levels and with additional weather information. The efficacy of the suggested approach shows that historical weather information can lead to more accurate rates.

Possible disadvantages of our proposed method are that it requires detailed weather information, and the model relating weather to yields or loss costs should be accurate. There is an inherent tradeoff between making use of observed yields—at least somewhat directly—and using them primarily in the weather model. Adding too distant weather and yield information introduces error if the model is not capable of accounting for any changes in the production environment. The error is likely to grow in the amount of additional historical information included in the model. If the model is capable of capturing these changes, then the analyst could add as much weather and yield data as necessary. Certainly, this implies the need for yield (or loss cost) models that are capable of capturing changes in the relationship between weather and yield (or loss costs). We note that this is a slightly different issue compared to adding historical weather information alone. If the model of the production technology is accurate, then adding additional historical weather information will generally not be a problem.

Our empirical results suggest that in the context of the federal crop insurance program, the use of historical weather information for yield distributions ultimately results in improved rating. With respect to the disadvantages mentioned above, we argue that advances in weather modeling and production economics make these less of a concern. Detailed weather information for the entire continental United States is available through a variety of projects. As weather data have been made more accessible, a voluminous literature has emerged on the statistical modeling of crop yields.

There are several additional areas of research that could build on this study. Additional improvements in yield modeling or in weather data would improve the accuracy of the model. Extensions could be made to nonlinear yield functions or more complicated stochastic representations of weather data. It might also prove useful to find ways to smooth the distributions across space or by similarity. We assume that the yield model coefficients differ by county, but it may be possible to use a larger subset of the data to estimate these coefficients. We have also assumed a stationary distribution of weather; our argument is that the rate of change of the distribution is not likely to be large enough to impact estimates at the time scales of interest in this study. However, dynamic models for weather could be incorporated into this model quite readily, provided that the distribution can be estimated in a Bayesian framework.

Incorporation of historical weather information in crop insurance rating has the potential to improve rates and also partially negate problems of information asymmetry. Several studies have noted problems of adverse selection in crop insurance markets (Goodwin & Smith 1995; He et al. 2019; Just et al. 1999; Ker & McGowan 2000; Richards & Mischen 1998). By augmenting short yield histories with additional weather information, the Bayesian approach may partially remedy the problem of asymmetric information.

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ENDNOTES

[i] The loss cost ratio is the ratio of indemnities to liability. The mean of the distribution of the loss cost ratios can be considered an estimate of the actuarially fair premium rate.

[ii] The preceding discussion assumes that the distribution of weather ($f(Z_i|\theta)$) is stable and that the production technology ($f(Y_i|Z_i,\theta)$) adequately captures the relationship between yield and weather. There may be some justification for using trimmed samples in estimating the production technology when yield distributions are subject to major changes over time. This concept is discussed in depth by Liu and Ker (2020).

[iii] We choose these cutoff points to roughly mimic yield cutoffs for shallow loss programs in the federal crop insurance program. Area programs use all available yield information; county-level yield series are often longer than 30 years. However, shallow loss policies only use yields from 1991 onward. Our approach remains applicable for area policies in the sense that weather data are available for a longer amount of time compared to yield data depending on the county and crop.

[iv] Differences in rates are largest at 90% coverage partly because rates at higher coverage levels are larger in absolute terms. However, this does not imply that the largest advantage in the rating game will accrue at 90% coverage. The results of the game are based on a binary decision rule that does not take into account the magnitude of rate differences.

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