A SIMPLIFIED METHOD FOR DETERMINING HOB OFFSET VALUES IN THE DESIGN OF NONSTANDARD SPUR GEARS

by

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LIST OF SYMBOLS

Symbol	Description
a	Horizontal length from apex to load; addendum
b	Tooth thickness; Y intercept; dedendum
c	Distance from neutral axis; clearance
C	Standard center distance; cord length; constants
C'	Nonstandard center distance
ΔC	Change in center distance
d	Beam length
e	Hob offset
e_T	Total hob offset
E	Modulus of elasticity
F_n	Normal force
F_t	Tangential force
h	Beam height; distance measured from tooth tip
h_t	Depth of cut
I	Moment of inertia
k	Constant (full-depth system $k = 1.00$)
L	Taper beam length
m	Slope
m_p	Contact ratio
M	Bending moment
N	Number of teeth
p	Circular pitch
P	Diametral pitch

LIST OF SYMBOLS (Continued)

Symbol	Description
R	Cutting pitch radius
R'	Operating pitch radius
R_b	Base radius
R_o	Outside radius
R'_o	Nonstandard outside radius
R_d	Dedendum radius
R'_d	Nonstandard dedendum radius
s	Bending stress
S	Arc length
t	Standard tooth thickness
t'	Thickness on operating pitch radius
X	Horizontal coordinate
y	Lewis factor
Y	Vertical coordinate
Z'	Nonstandard length of action
ϕ	Cutting pressure angle
ϕ'	Operating pressure angle
δ	Deflection due to bending and shear
α	Arc angle
τ	Taper beam tooth angle
β	Angle between the normal and tangential load
μ	Poisson's ratio
θ	Tooth angle
ω	Angular velocity

I. INTRODUCTION

One problem in using the involute system of gearing is the possibility of interference between the tip of the gear tooth and the flank of the pinion tooth when the number of teeth in the pinion is reduced below the minimum allowable for that system of gearing, (e.g. 12 teeth for 25 degree full depth teeth, 14 teeth for 20 degree stub teeth, 18 teeth for 20 degree full depth teeth, and 32 teeth for 14½ degree full depth teeth). Other problems which can occur when using standard involute spur gears include poor contact ratio (average number of teeth in contact) and inadequate strength of the pinion teeth. There also exists the problem of using standard cutters to cut gears for application to nonstandard center distances.

In the case of interference between the tip of the gear tooth and the flank of the pinion tooth there exists a well established relationship to give the mathematically correct value that the cutter should be withdrawn (or in some cases moved inward) so that the addendum line of the hob just passes through the interference point of the pinion (the point at which the line of action is tangent to the base circle of the pinion). This equation will be given in a later section along with the rules for its application to problems involving interference between the tip of the gear tooth and the flank of the pinion tooth.

A typical problem which a mechanical designer might encounter concerns the design of a set of spur gears for a predetermined center distance which is not one of the values obtained by dividing the sum of the number of pinion and gear teeth by two times one of the standard diametral pitches, (e.g. 12, 14, 16, 18, 20, etc.). The design of these gears can be accomplished by determining the value that a standard cutter should be moved in or withdrawn from its normal cutting position such that when the pinion and gear are cut, they will operate at the required center distance when meshed together. The design problem, however, is trying to establish what values of hob (cutter) offset should be used to cut the pinion and the gear so that the standard cutter may be used.

Several authors have shown the derivation for a mathematically correct relationship which will give the <u>sum</u> of the hob offsets for the pinion and the gear. The major problem is that there is no second mathematical expression whereby it is possible to solve explicitly for either the hob offset of the pinion or the hob offset of the gear.

Accordingly, the object of this investigation was to develop a method for determining the hob offset of the pinion and of the gear so that the gears would operate properly at the required predetermined center distance. A secondary object of this investigation was to develop a series of design charts for determining hob offset values that could be used with relative ease and would cover a wide range of problems. An attempt also was made at developing a series of equations so that it would be possible to solve explicitly for either the hob offset of the pinion or the hob offset of the gear.

II. REVIEW OF THE LITERATURE

Several authors have discussed the generation of non-standard gears with standard hobs. Spotts' (1) and Hirschhorn's (5) texts contain brief discussions of nonstandard gears. Steeds (2) and Mabie and Ocvirk (4) each devoted a full chapter to the design of gears for operation at non-standard center distances in their texts. Kinsman's (3) entire doctoral thesis is concerned with the design of nonstandard gears.

Each of the above authors shows the derivation for obtaining the sum ($e_1 + e_2$) of the hob offsets required for cutting a set of gears to operate at a nonstandard center distance.

Many of these authors have given guidelines about a second equation relating e_1 and e_2 so that it would be possible to solve explicitly for the hob offset of the pinion or the hob offset of the gear. Steeds (2) states that an arbitrary relationship between the quantities e_1 and e_2 is to let $e_1 = ke_2$ where k is an arbitrary constant. The value of k used should be chosen so as to reduce the undercutting of the teeth of the pinion to a minimum when the number of teeth in the pinion is small compared to that of the gear.

Kinematically, Kinsman (3) states, it makes no difference how the value of total hob offset $e_1 + e_2$ is divided into e_1 (hob offset of pinion) and e_2 (hob offset of the gear), as long as the two values are selected such that undercutting (removal of interfering metal from the flank of the pinion tooth by the cutter) and pointing (withdrawing the hob a distance such that the tooth becomes pointed) is avoided, if possible. From a strength standpoint, Kinsman states that the following should be considered: a.) select values of e_1 and e_2 so that neither the pinion teeth nor the gear teeth are pointed or undercut. b.) The corrected Lewis form factors (a purely geometrical property of the size and shape of the tooth) for the pinion and the gear must be adequate. c.) If a and b are unsatisfactory, choose e_1 and e_2 so that the pitch point bisects the path of contact.

Mabie and Ocvirk (4) indicate that often e_1 and e_2 are assumed to vary inversely with the number of teeth in order to insure a strong pinion. If e_1 and e_2 are negative, e_1 and e_2 should vary directly with the number of teeth. This is the usual practice followed in the United States.

Hirschhorn (5) states that, in Britain, the following cutter-setting corrections have been adopted by the majority of gear manufacturers as standard practice. If $N_1 + N_2 \geq 60$ teeth then

$$e_1 = \frac{0.4}{P}\left(1 - \frac{N_1}{N_2}\right) \text{ or } e_1 = \frac{0.02}{P}(30 - N_1)$$

whichever is the greater and $e_2 = e_1$ if $N_1 = N_2$. If $N_1 + N_2 < 60$ then

$$e_1 = \frac{0.02}{P}(30 - N_1)$$

and
$$e_2 = \frac{0.02}{P}(30 - N_2)$$

choosing the one that favors the pinion the most.

Some literature that will be pertinent to the results of this investigation, but not directly related to nonstandard gears, was also reviewed. This literature deals primarily with the static deflections in spur gear teeth.

An experimental investigation was performed by Furrow (8) on the static deflection of standard spur gear teeth loaded at any point along the line of action. In his work, Furrow proposed a specific manner in which to apply the equations of Timoshenko and Baud (6), whose approach was based on replacing the gear tooth with a tapered cantilever beam. A paper by Furrow and Mabie (9) gives a review of the results of Furrow's experimental work along with information concerning the computer program used to apply Timoshenko and Baud's equation for the deflection of gear teeth. Mention must also be made of Caldwell (7), who derived the necessary equations for determining the value of the angle of application of the load at any point along the line of action.

III. INVOLUTE SPUR GEAR THEORY

This section will review some important definitions and some of the basic kinematic equations which describe the action of a spur gear. The reader is probably well informed on the basic concepts of gear design and gear nomenclature. However, some of the basic concepts will be reviewed briefly, primarily to acquaint the reader with the nomenclature used here.

A <u>spur gear</u> is one having teeth parallel to the axis of the gear. The nomenclature is shown in Figure 1.

The diametral pitch P is defined as the ratio of the number of teeth to the number of inches of pitch diameter. The diametral pitch equals the number of gear teeth to each inch of pitch diameter.

Referring to Figures 1 and 2, the base pitch, which remains the same for both a standard and a nonstandard gear is given by

$$p_b = \frac{2\pi R_b}{N} = \frac{\pi \cos \phi}{P} \qquad (1)$$

The diametral pitch of a standard gear is given by

$$P = \frac{N}{2R} \qquad (2)$$

The standard circular pitch is given by

$$p = \frac{2\pi R}{N} = \frac{\pi}{P} \qquad (3)$$

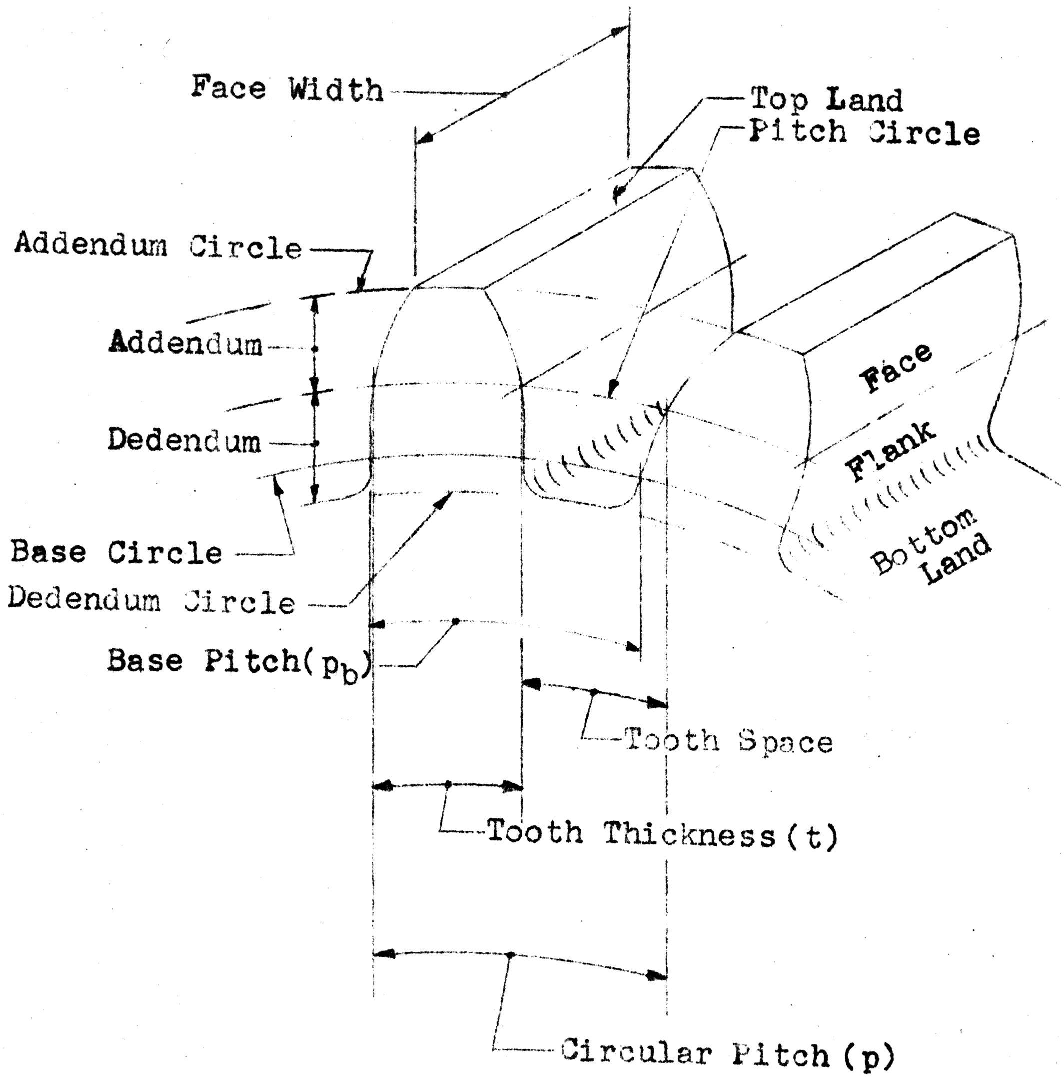

FIGURE 1. NOMENCLATURE OF GEAR TEETH

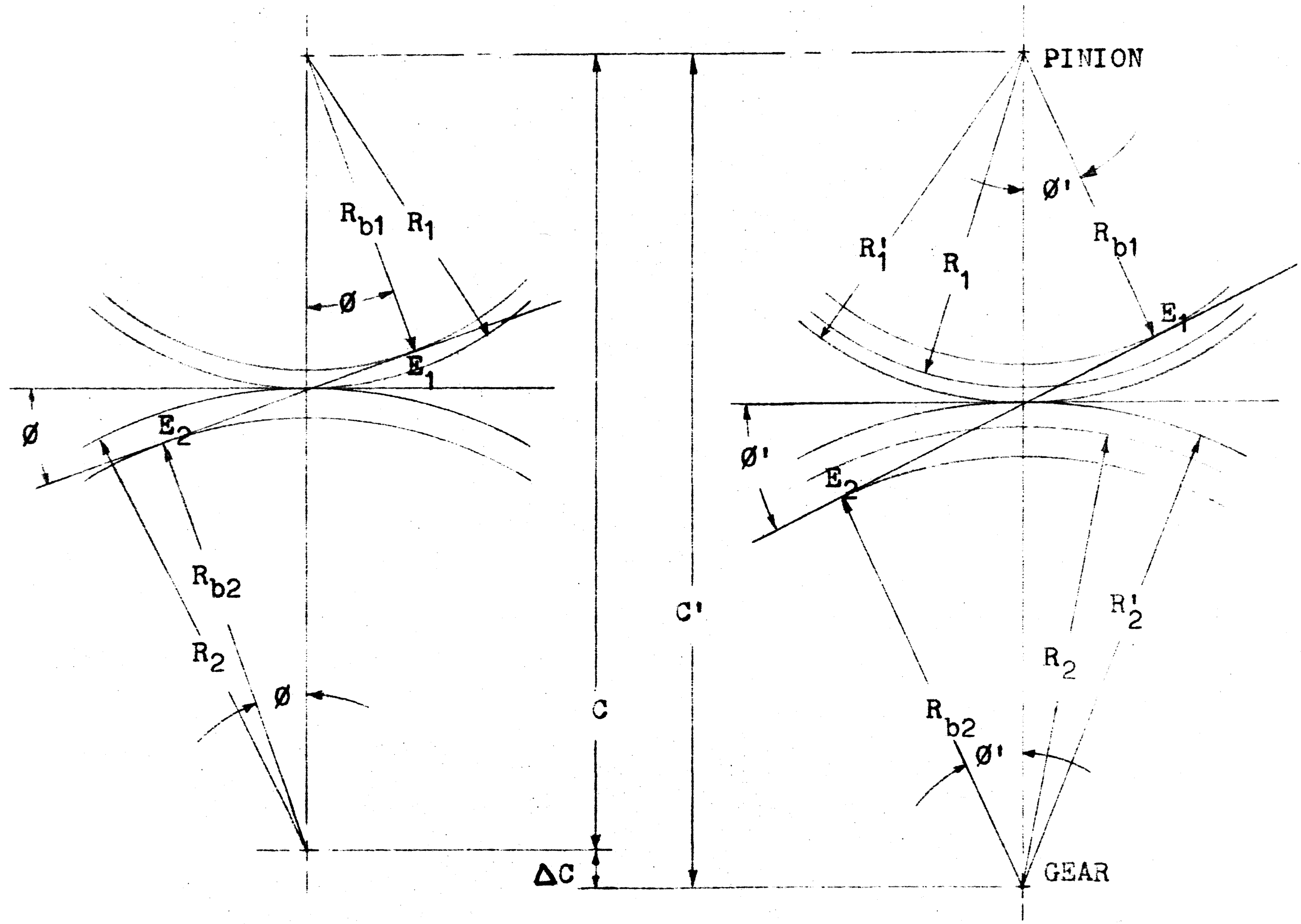

FIGURE 2. CENTER DISTANCE CHANGE

The thickness of the standard gear tooth is equal to one-half of the standard circular pitch.

$$t = \frac{\pi R}{N} = \frac{\pi}{2P} = \frac{p}{2} \tag{4}$$

A <u>standard gear</u> is one where the ratio of the number of teeth to pitch diameter is one of the standard values of diametral pitch. The term standard gear also means the tooth thickness must be equal to the tooth space, which equals one-half of the circular pitch.

For the purpose of this thesis, a <u>nonstandard gear</u> is defined as an involute gear whose teeth are not of standard thickness. That is, on the standard pitch circle of the gear, defined by the relationship that its radius is equal to the number of teeth in the gear divided by twice the diametral pitch, the arc tooth thickness is <u>not</u> equal to one-half of the circular pitch.

With reference to Figure 2, the following quantities can be defined:

Standard center distance with zero backlash

$$C = R_1 + R_2 = \frac{N_1 + N_2}{2P} \tag{5}$$

Nonstandard center distance

$$C' = R'_1 + R'_2 = C\,\frac{\cos \phi}{\cos \phi'} \tag{6}$$

where the operating pressure angle ϕ' is established only after the pinion and the gear have been meshed together.

Base circle radii

$$R_{b1} = R_1 \cos \phi = R_1' \cos \phi' \quad (7)$$

$$R_{b2} = R_2 \cos \phi = R_2' \cos \phi' \quad (8)$$

Standard cutting pitch radii

$$R_1 = \frac{N_1}{2P} \quad (9)$$

$$R_2 = \frac{N_2}{2P} \quad (10)$$

Operating pitch circle radii

$$R_1' = \left(\frac{N_1}{N_1 + N_2}\right) C' \quad (11)$$

$$R_2' = \left(\frac{N_2}{N_1 + N_2}\right) C' \quad (12)$$

Standard outside radii

$$R_{o1} = R_1 + a \quad (13)$$

$$R_{o2} = R_2 + a \quad (14)$$

Standard dedendum radii

$$R_{d1} = R_1 - b \quad (15)$$

$$R_{d2} = R_2 - b \quad (16)$$

The thickness at any point on the involute surface of the tooth can be found if the thickness at some other point is known by the following equation:

$$t_B = 2R_B \left(\frac{t_A}{2R_A} + \text{inv } \phi_A - \text{inv } \phi_B\right) \quad (17)$$

where the involute of an angle is defined as

$$\text{inv } \phi_A = \tan \phi_A - \phi_A \tag{18}$$

Figure 3 shows a portion of a rack that could be used in the cutting of both standard and nonstandard gears. When cutting a standard gear the hob offset e is zero, and, therefore, the cutting pitch line and standard pitch line are identical. The thickness of the gear tooth on the standard pitch line is given by Equation (4). If it is desired to cut a nonstandard gear, a standard rack can be used to cut the gear by withdrawing or retracting the rack a distance e. From Figure 3 it is noted that the standard pitch line and the cutting pitch line are then no longer the same. The reason for this is that the standard pitch line of the rack is no longer tangent to the cutting pitch circle of the gear; therefore, the standard pitch line cannot serve as the cutting pitch line. A new line on the rack must be established which is tangent to the cutting pitch circle of the gear as is required for generating the gear. It should also be noted that the cutting pressure angle ϕ of the rack is constant and does not change. It is only after two gears have been meshed together that the operating pressure angle ϕ' is established. The thickness on the cutting pitch circle can now be seen to be given by:

$$t = 2e \tan \phi + p/2 \tag{19}$$

where e is positive if the rack is withdrawn and negative if the rack is pushed into the gear blank.

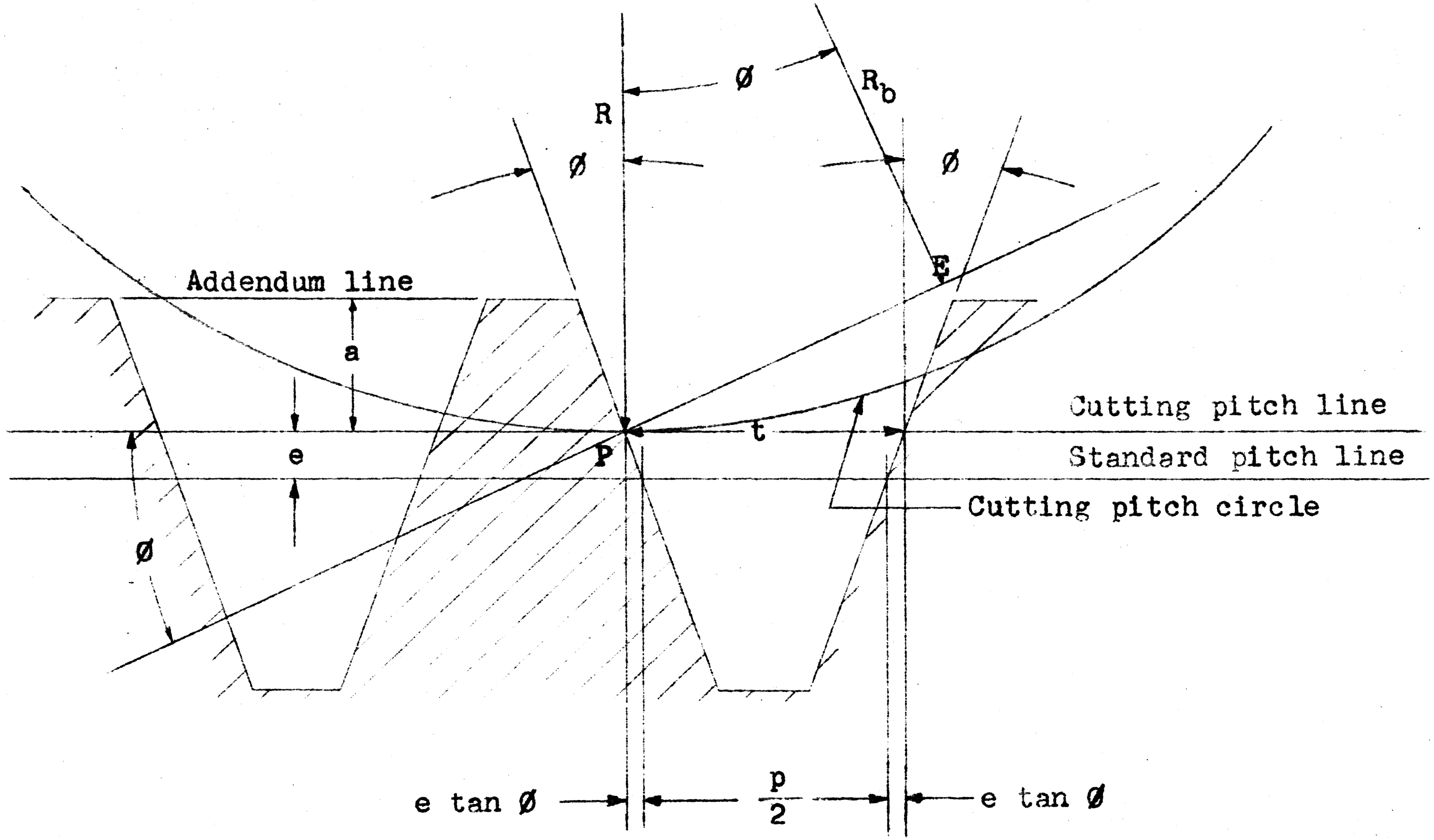

FIGURE 3. GEAR CUTTING RACK

If the rack is withdrawn just enough so that the addendum line passes through the interference point (point E_1 on Figure 2) of the pinion, it is possible to develop an equation so that the hob offset e can be determined so that the addendum line passes through the interference point E_1 (See Mabie and Ocvirk (4) p. 128 for complete derivation). This equation is found to be:

$$e = \frac{1}{P}\left(k - \frac{N}{2}\sin^2 \phi\right) \qquad (20)$$

It must be emphasized that Equation (20) is good only if the rack addendum passes through the interference point E_1 of the pinion.

Figure 4 shows two gears in mesh which have been cut with hob offsets e_1 and e_2 respectively. An equation will now be derived from Mabie and Ocvirk (4) which will give the sum, $e_1 + e_2$, of the hob offset. The fundamental law of gearing states:

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1}$$

If Equation (12) is divided by Equation (11):

$$\frac{N_2}{N_1} = \frac{R'_2}{R'_1}$$

Therefore

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{R'_2}{R'_1} \qquad (21)$$

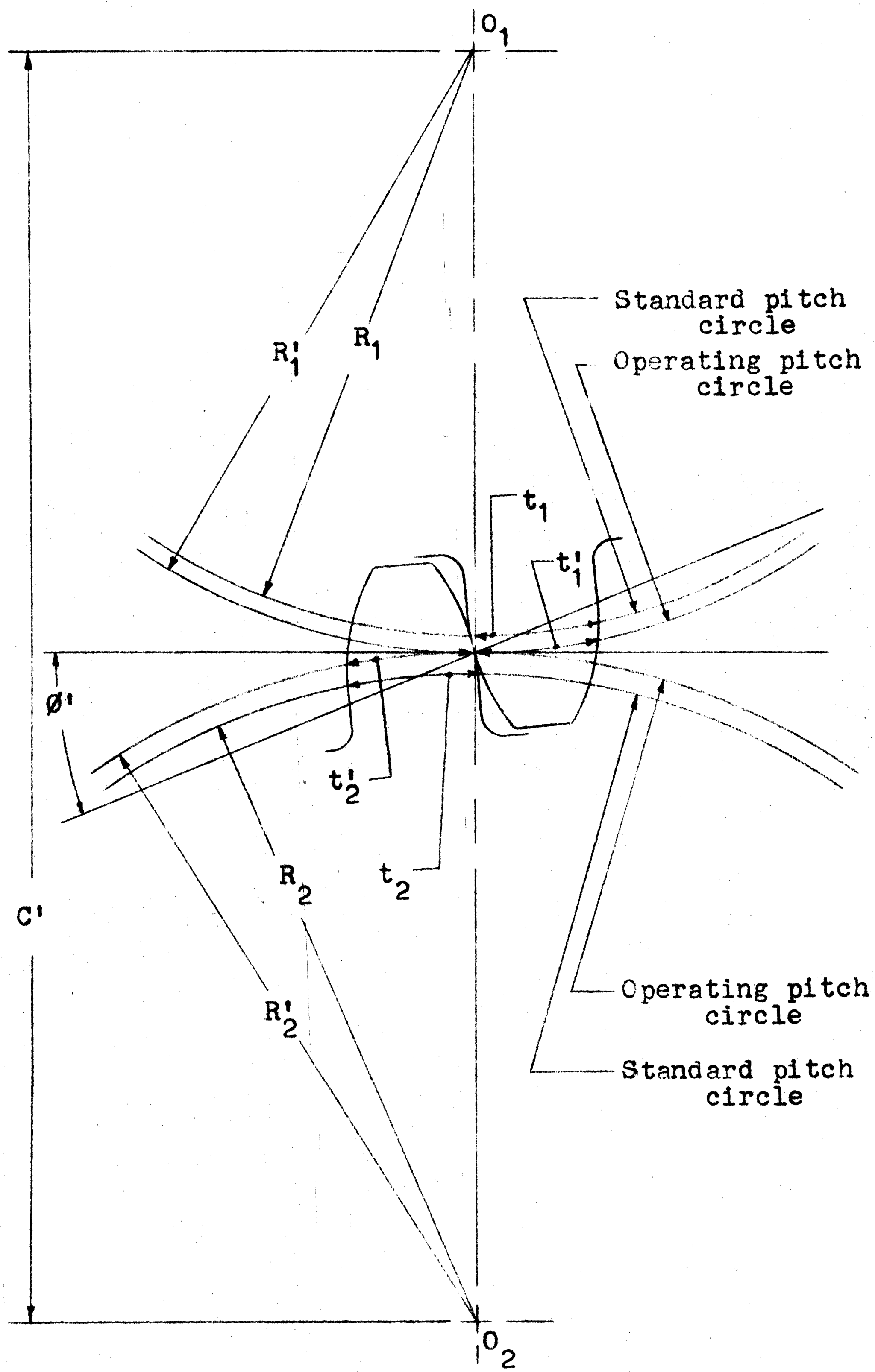

FIGURE 4

NONSTANDARD GEAR MESH

From Figure 4, assuming no backlash

$$t_1' + t_2' = \frac{2\pi R_1'}{N_1} = \frac{2\pi R_2'}{N_2} \tag{22}$$

From Equation (17):

$$t_1' = 2R_1' \left(\frac{t_1}{2R_1} + \text{inv } \emptyset - \text{inv } \emptyset'\right)$$

$$t_2' = 2R_2' \left(\frac{t_2}{2R_2} + \text{inv } \emptyset - \text{inv } \emptyset'\right)$$

Substituting these into Equation (22):

$$2R_1' \left(\frac{t_1}{2R_1} + \text{inv } \emptyset - \text{inv } \emptyset'\right) + 2R_2' \left(\frac{t_2}{2R_2} + \text{inv } \emptyset - \text{inv } \emptyset'\right) = 2\frac{\pi R_1'}{N_1}$$

and dividing by $2R_1'$:

$$\left(\frac{t_1}{2R_1} + \text{inv } \emptyset - \text{inv } \emptyset'\right) + \frac{R_2'}{R_1'} \left(\frac{t_2}{2R_2} + \text{inv } \emptyset - \text{inv } \emptyset'\right) = \frac{\pi}{N_1}$$

or $$\frac{t_1}{2R_1} + \frac{R_2'}{R_1'}\frac{t_2}{2R_2} = \frac{\pi}{N_1} + \left(\frac{R_2'}{R_1'} + 1\right) (\text{inv } \emptyset' - \text{inv } \emptyset)$$

Substituting Equation (21) into this yields

$$\frac{t_1}{2R_1} + \frac{N_2}{N_1}\frac{t_2}{2R_2} = \frac{\pi}{N_1} + \left(\frac{N_2}{N_1} + 1\right) (\text{inv } \emptyset' - \text{inv } \emptyset)$$

From Equations (9 and 10):

$$2R_1 = \frac{N_1}{P} \text{ and } 2R_2 = \frac{N_2}{P}$$

Therefore,

$$\frac{t_1 P}{N_1} + \frac{t_2 P}{N_1} = \frac{\pi}{N_1} + \frac{N_2 + N_1}{N_1} (\text{inv } \emptyset' - \text{inv } \emptyset)$$

Multiplying by N_1/P

$$t_1 + t_2 = \frac{\pi}{P} + \frac{N_1 + N_2}{P} (\text{inv } \phi' - \text{inv } \phi) \qquad (23)$$

From Equation (19)

$$t_1 = 2e_1 \tan \phi + p/2$$

$$t_2 = 2e_2 \tan \phi + p/2$$

Substituting these into Equation (23):

$$2(e_1 + e_2) \tan \phi + p = \frac{\pi}{P} + \frac{N_1 + N_2}{P} (\text{inv } \phi' - \text{inv } \phi)$$

Substituting Equation (3) into the last result and simplifying

$$e_1 + e_2 = \frac{(N_1 + N_2)(\text{inv } \phi' - \text{inv } \phi)}{2P \tan \phi} \qquad (24)$$

which is the desired result.

In Chapter II, the following guideline was suggested for use with Equation (24): If $e_1 + e_2$ is positive, let e_1 and e_2 vary inversely with the number of teeth; if negative, let e_1 and e_2 vary directly with the number of teeth. In equation form:

$$\frac{e_1}{e_2} = \frac{N_2}{N_1} \qquad \text{for positive } \Delta C$$

Letting $e_T = e_1 + e_2$ (25)

$$e_1 = (e_T - e_1) \frac{N_2}{N_1}$$

$$e_1 (1 + \frac{N_2}{N_1}) = e_T \frac{N_2}{N_1}$$

$$e_1(N_1 + N_2) = e_T N_2$$

or
$$\frac{e_1}{e_1 + e_2} = \frac{N_2}{N_1 + N_2} \qquad (26)$$

and similarly,

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} \qquad \text{for negative } \Delta C$$

$$e_1 = (e_T - e_1)\frac{N_1}{N_2}$$

$$e_1\left(1 + \frac{N_1}{N_2}\right) = e_T \frac{N_1}{N_2}$$

$$e_1(N_1 + N_2) = e_T N_1$$

or
$$\frac{e_1}{e_1 + e_2} = \frac{N_1}{N_1 + N_2} \qquad (27)$$

Equations (26) and (27) will be useful for comparing the results of this investigation with the suggested guideline to obtain e_1 or e_2 from Equation (24).

Three other equations that will be needed in the derivations to follow are the equations for the outside radii, dedendum circle radii, and the equation for the depth of cut. (The development of these equations can be found in Mabie and Ocvirk (4) pp. 131-2.) The equations for the outside radii are as follows

$$R'_{o1} = C' - R_2 - e_2 + \frac{k}{P} \qquad (28)$$

$$R'_{o2} = C' - R_1 - e_1 + \frac{k}{P} \qquad (29)$$

and the equation for the depth of cut is

$$h_t = R'_{o1} + R'_{o2} - C' + c \qquad (30)$$

where c is the clearance.

The equations for the dedendum circle radii are as follows

$$R'_{d1} = R'_{o1} - h_t \qquad (31)$$

$$R'_{d2} = R'_{o2} - h_t \qquad (32)$$

The derivation for the length of action and contact ratio will be developed for a pair of nonstandard gears in mesh. In Figure 5, E_1 and E_2 are the points of tangency of the line of action and base circles. Point A is the beginning of contact, defined as the point at which the addendum circle of the driven gear intersects the line of action. Point B is the end of contact, defined as the point at which the addendum circle of the driver intersects the line of action. Z' is the nonstandard length of action and is expressed as

$$Z' = AB = E_1B + E_2A - E_1E_2$$

but $$(E_1B)^2 + (R_{b1})^2 = (R'_{o1})^2$$

$$(E_2A)^2 + (R_{b2})^2 = (R'_{o2})^2$$

$$E_1E_2 = R'_1 \sin \phi' + R'_2 \sin \phi' = C' \sin \phi'$$

Therefore

$$Z' = \sqrt{(R'_{o1})^2 - (R_{b1})^2} + \sqrt{(R'_{o2})^2 - (R_{b2})^2} - C' \sin \phi' \qquad (33)$$

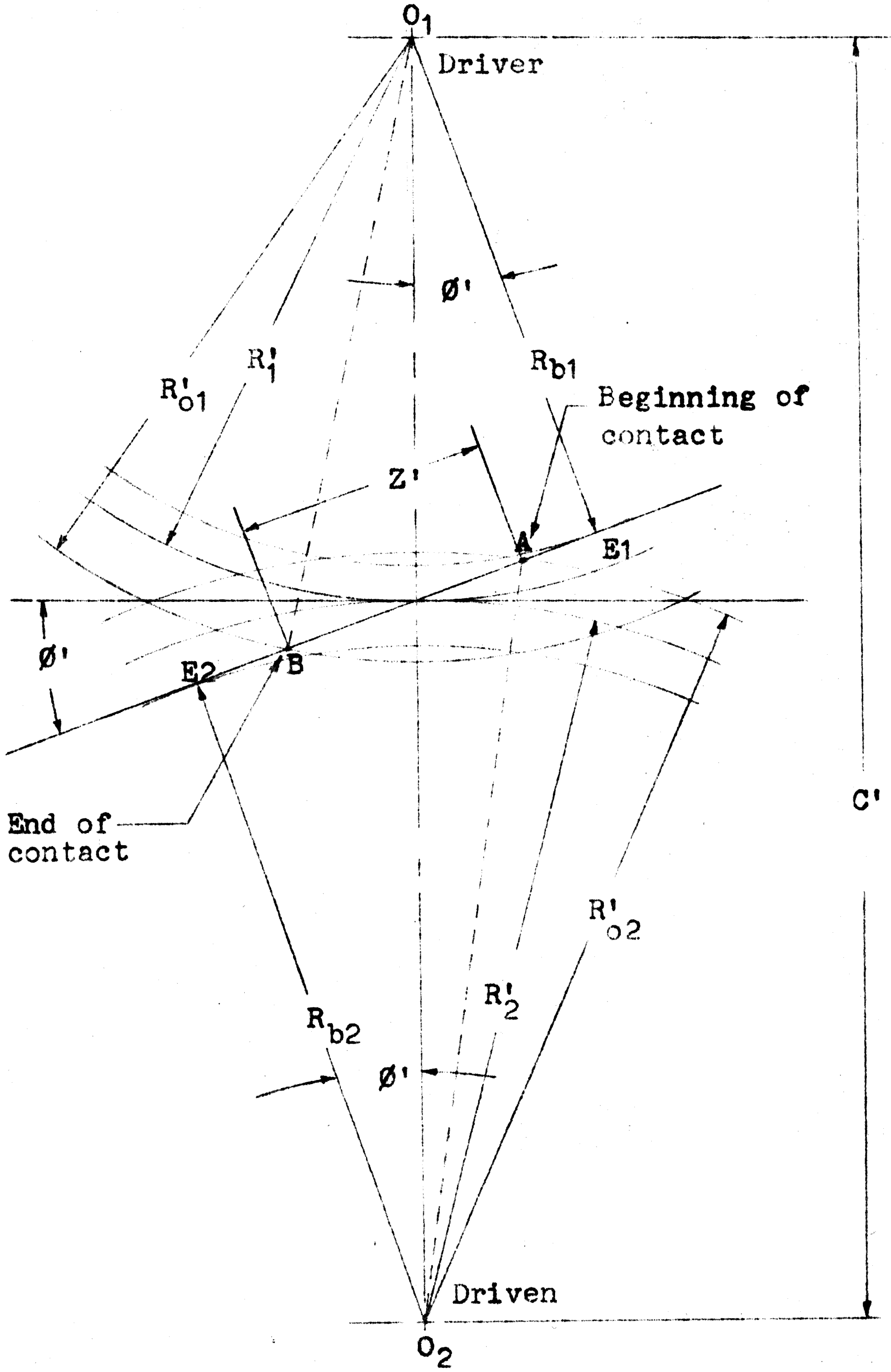

FIGURE 5. LENGTH OF ACTION

From the definition of base pitch given by Equation (1), it is now possible to define the contact ratio for nonstandard gears as

$$m_p = \frac{Z'}{p_b} \tag{34}$$

IV. HOB OFFSET CALCULATION BASED ON APPROXIMATE EQUAL STRENGTH OF GEAR AND PINION TEETH

A. Theory and Method

Consider a spur gear tooth as shown in Figure 6 for which the following assumptions are made.

1. The tooth material is homogeneous and isotropic.
2. The tooth is rigidly supported at the dedendum circle.
3. Transverse planes before bending remain transverse planes after bending; no warping takes place.
4. The load is gradually applied in the x-y plane such that no twisting of the tooth occurs.
5. The component of the tooth load parallel to the tooth axis, F_v, will be neglected.

The elementary bending stress equation for a straight beam having a constant, prismatic, cross section is

$$s = \frac{Mc}{I}$$

It will be assumed that the tooth is a cantilever beam and that the above equation holds. The usual bending stress formula for gear teeth is

$$s = \frac{F_b}{byp} \tag{35}$$

In Equation (35), y is the Lewis factor which can be found in tables (1), b is the face width, p is the circular pitch, and F_b is the bending load. These tables, however, only apply to standard gears. Values of the Lewis factor

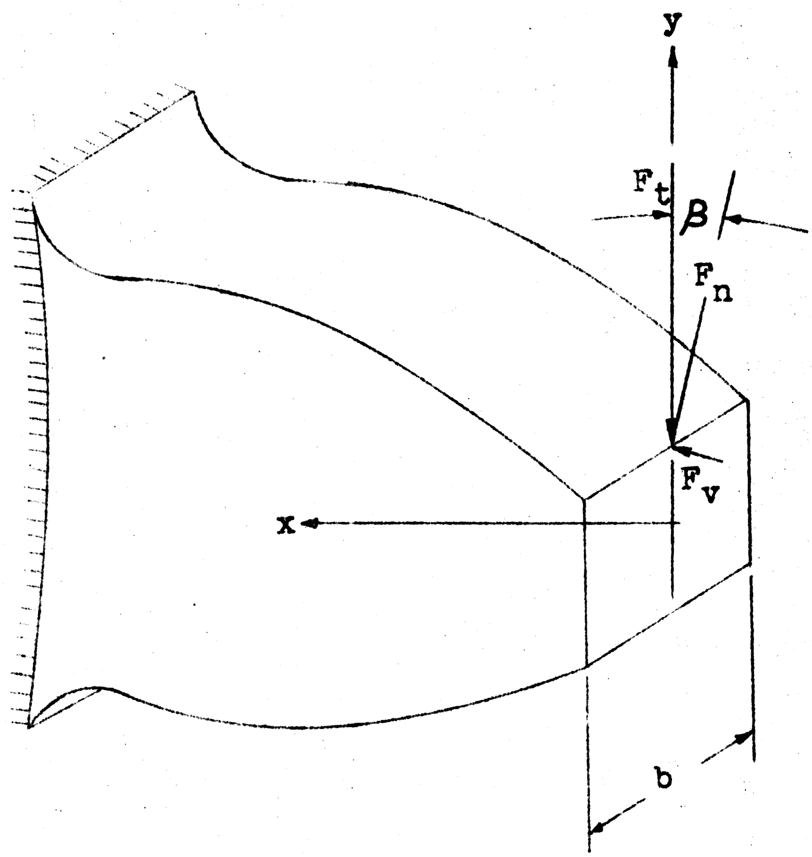

FIGURE 6. SPUR GEAR TOOTH

for nonstandard gears are not readily available. The only place mention is made of nonstandard Lewis factors is in Kinsman's thesis (3), but these apply only if the load is applied a distance 1/P from the tooth tip. The Lewis factor is a purely geometric property depending on the size and shape of the tooth.

The actual angle of load application, β, at the tooth tip will be developed first and then the stress equations will be derived. It will be assumed that the entire load is carried by a single pair of teeth and that the load is acting through the corner or most unfavorable point on the tooth as shown in Figures 7 and 8.

Figure 7 shows the case of two nonstandard gears in mesh at the particular phase when the gear tip is in contact with the flank of the pinion. It is necessary to determine the angle β_{A2}, since, in the stress equation, the bending moment caused by the tangential load at the tip of the gear tooth is wanted.

The angle AO_2E2 as shown in Figure 7 is

$$\phi'_{A2} = \cos^{-1}(R_{b2}/R'_{o2})$$

The involute of this angle is

$$inv(\phi'_{A2}) = \tan(\phi'_{A2}) - \phi'_{A2} \qquad (36)$$

The angle between the gear tooth centerline and R'_{o2} at point A is

$$\alpha_{A2} = \frac{t_{A2}/2}{R_{A2}}$$

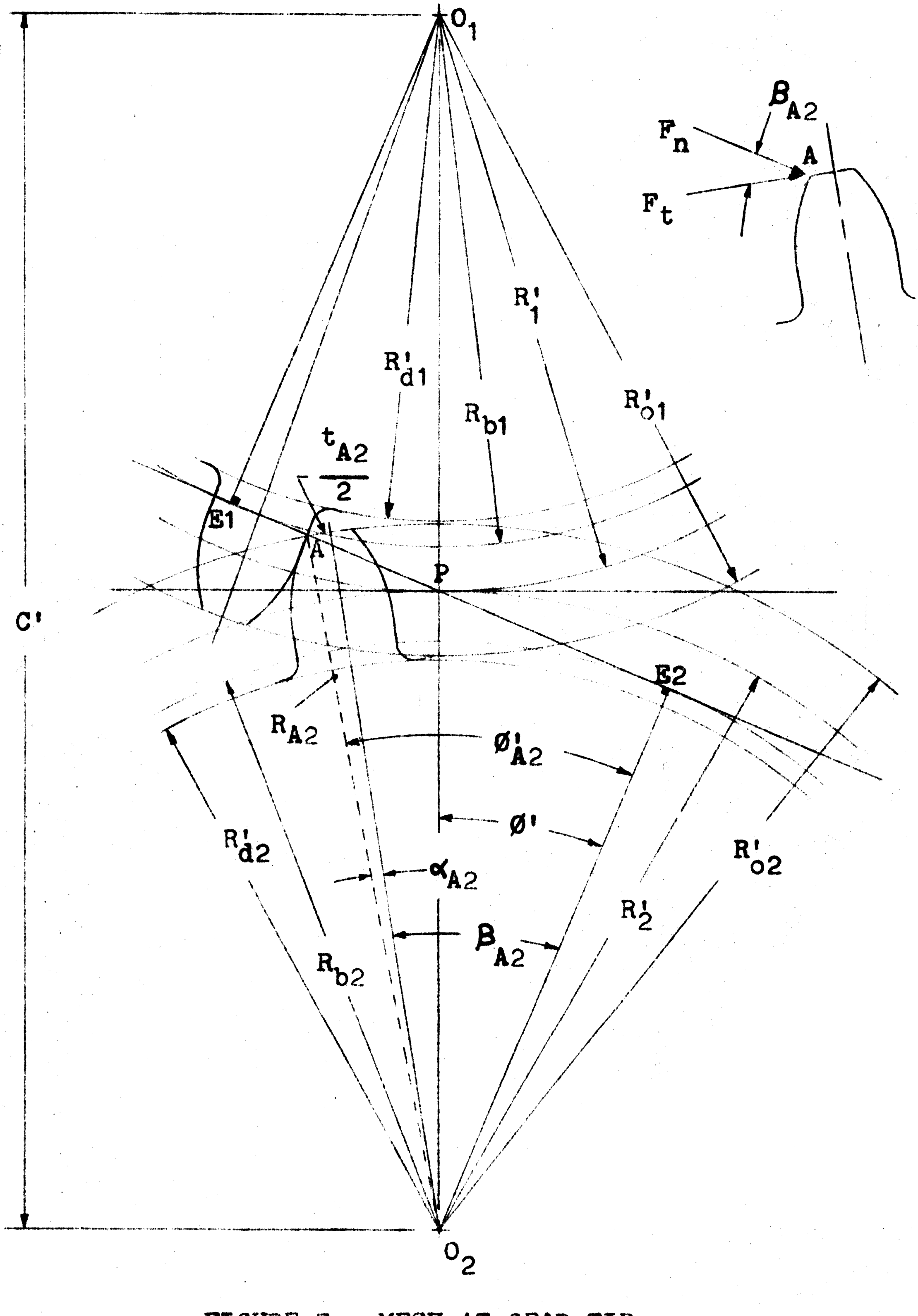

FIGURE 7. MESH AT GEAR TIP

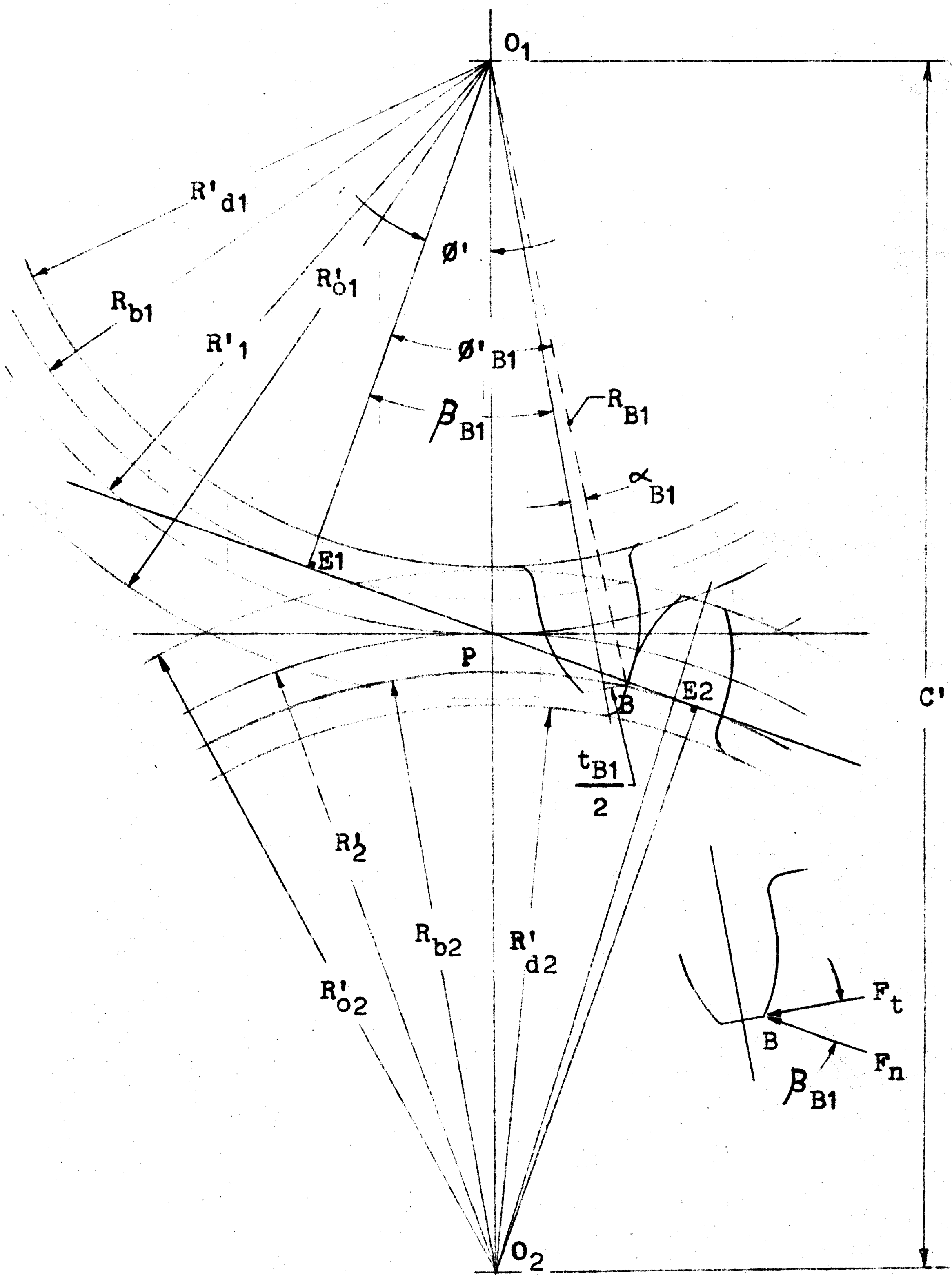

FIGURE 8

MESH AT PINION TIP

but $$t_{A2} = 2R_{A2}\left(\frac{t_2}{2R_2} + \text{inv } \phi - \text{inv } \phi'_{A2}\right)$$

Therefore $$\alpha_{A2} = \frac{t_2}{2R_2} + \text{inv } \phi - \text{inv } \phi'_{A2} \qquad (37)$$

where $$t_2 = 2e_2 \tan \phi + p/2$$

is the thickness of the gear tooth on its cutting pitch circle, R_2 the standard pitch radius of the gear, ϕ the cutting pressure angle, and e_2 the hob offset of the gear.

The angle between F_n and F_t for the gear at point A is

$$\beta_{A2} = \phi'_{A2} - \alpha_{A2} \qquad (38)$$

Figure 8 shows the case of two nonstandard gears in mesh at the particular phase when the pinion tip is in contact with the flank of the gear. For this case it is necessary to find the angle β_{B1} in order to calculate the bending moment caused by the tangential load at the tip of the pinion tooth.

The angle BO_1E_1 as shown in Figure 8 is

$$\phi'_{B1} = \cos^{-1}(R_{b1}/R'_{o1})$$

The involute of this angle is

$$\text{inv}(\phi'_{B1}) = \tan(\phi'_{B1}) - \phi'_{B1} \qquad (39)$$

The angle between the pinion tooth centerline and R'_{o1} at point B is

$$\alpha_{B1} = \frac{t_{B1}/2}{R_{B1}}$$

but $$t_{B1} = 2R_{B1}\left(\frac{t_1}{2R_1} + \text{inv } \phi - \text{inv } \phi'_{B1}\right)$$

Therefore $$\alpha_{B1} = \frac{t_1}{2R_1} + \text{inv } \phi - \text{inv } \phi'_{B1} \tag{40}$$

where $t_1 = 2e_1 \tan \phi + p/2$
is the thickness of the pinion tooth on its cutting pitch circle, R_1 the standard pitch radius of the pinion, ϕ the cutting pressure angle, and e_1 the hob offset of the pinion. The angle between F_n and F_t for the pinion at point B is

$$\beta_{B1} = \phi'_{B1} - \alpha_{B1} \tag{41}$$

The derivation for the length and thickness of the cantilever beam used to approximate the gear can be developed along with the bending stress equation. Two cases must be considered. Case 1 will involve Figure 9 which shows the base circle radius as less than or equal to the dedendum circle radius. For this condition the entire tooth has an involute profile. Case 2 involves Figure 10 which shows the base circle as being greater than the dedendum circle radius. For this condition the tooth profile is involute from the addendum circle to the base circle, and from the base circle to dedendum circle the profile is that of a radial straight line drawn from the gear center.

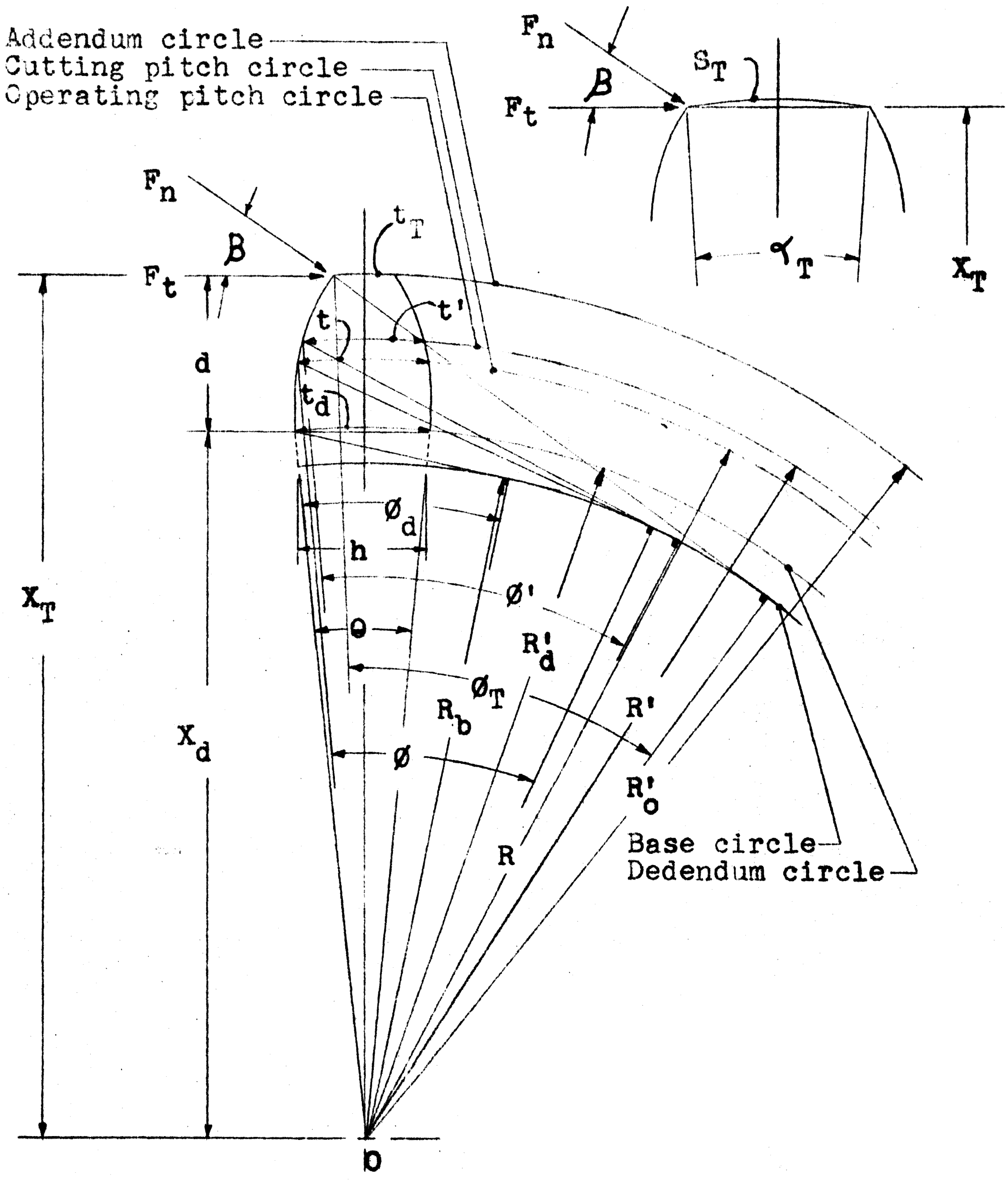

FIGURE 9

TOOTH STRESS MODEL $R_b \leq R'_d$

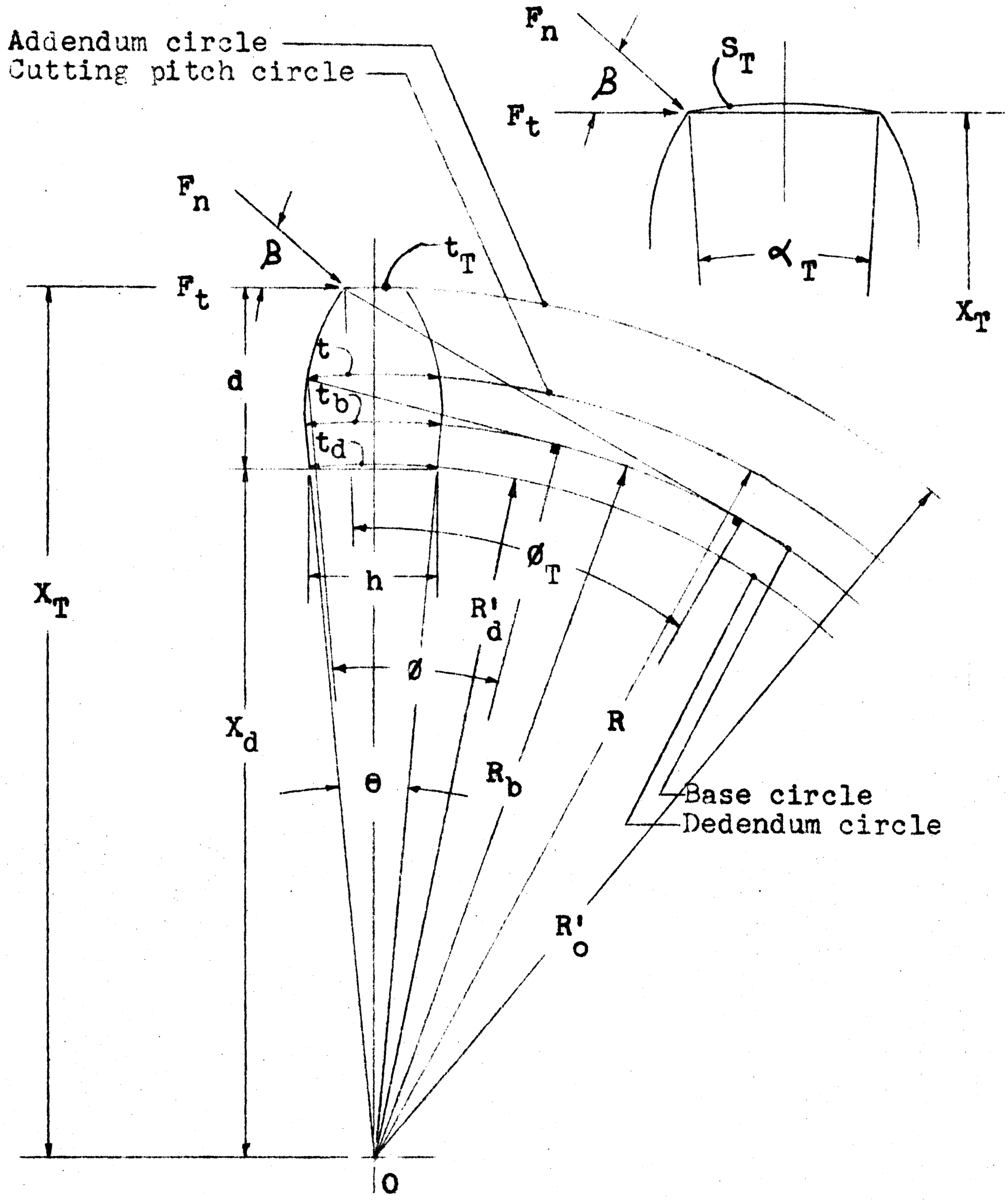

FIGURE 10

TOOTH STRESS MODEL $R_b > R'_d$

CASE 1

For $R_b \leq R'_d$ **Figure 9**

By Equation (19)

$$t = 2e \tan \phi + p/2$$

Since the involute profile goes to the dedendum circle

$$R \cos \phi = R'_d \cos \phi_d$$

Therefore $\phi_d = \cos^{-1}(\frac{R}{R'_d} \cos \phi)$ (42)

and $\quad inv(\phi_d) = \tan(\phi_d) - \phi_d$ (43)

By Equation (17)

$$t_d = 2R'_d(\frac{t}{2R} + inv\ \phi - inv\ \phi_d) \quad (44)$$

From trigonometry

$$\theta = t_d/R'_d$$

or $\quad \theta = 2(\frac{t}{2R} + inv\ \phi - inv\ \phi_d)$ (45)

Also $\quad h = 2R'_d \sin(\theta/2)$ (46)

$$X_d = R'_d \cos(\theta/2) \quad (47)$$

The tooth thickness at the tip of the tooth is

$$t_T = 2R'_o(\frac{t'}{2R'} + inv\ \phi' - inv\ \phi_T) \quad (48)$$

which is equal to the arc length S_T. The base circle radius is given by

$$R_b = R' \cos \phi' = R'_o \cos \phi_T$$

Therefore $\emptyset_T = \cos^{-1}(\frac{R'}{R'_o} \cos \emptyset')$ (49)

Let $C1 = \emptyset_T$ (50)

$C2 = \text{inv } \emptyset_T = \tan C1 - C1$ (51)

By Equation (17)

$t' = 2R'(\frac{t}{2R} + \text{inv } \emptyset - \text{inv } \emptyset')$ (52)

Substituting Equations (51 and 52) into Equation (48) gives

$$t_T = 2R'_o(\frac{t}{2R} + \text{inv } \emptyset - C2)$$

Then the arc angle α_T is

$$\alpha_T = \frac{S_T}{R'_o} = \frac{t_T}{R'_o}$$

which on substitution of t_T is

$$\alpha_T = 2(\frac{t}{2R} + \text{inv } \emptyset - C2)$$

Defining $C4 = R'_o$ (53)

and $C5 = \text{inv } \emptyset$ (54)

Therefore $X_T = R'_o \cos \alpha_T/2$

or $X_T = C4 \cos(\frac{t}{2R} + C5 - C2)$ (55)

where t is given by Equation (19)

The length of the cantilever beam is

$d = X_T - X_d$ (56)

As mentioned before, the bending stress equation is

$$s = \frac{Mc}{I}$$

where $M = (F_n \cos\beta)\, d$

$c = \frac{1}{2}h$

$I = \frac{1}{12}bh^3$

and substituting into the stress equation

$$s = \frac{6dF_n \cos\beta}{bh^2} \qquad (57)$$

where the angle β is defined by either Equation (38) or by Equation (41).

CASE 2

For $R_b > R_d'$ — Figure 10

From Equation (17)

$$t_b = 2R_b\left(\frac{t}{2R} + \text{inv}\,\phi - \text{inv}\,\phi_b\right)$$

but $\phi_b = 0$

Therefore $t_b = 2R_b\left(\frac{t}{2R} + \text{inv}\,\phi\right)$ (58)

By trigonometry

$$\theta = \frac{t_b}{R_b}$$

or $\theta = 2\left(\frac{t}{2R} + \text{inv}\,\phi\right)$ (59)

where t is given by Equation (19).

Also $h = 2R_d' \sin(\theta/2)$ (60)

$$X_d = R'_d \cos(\theta/2) \tag{61}$$

$$d = X_T - X_d \tag{62}$$

The bending stress equation is

$$s = \frac{Mc}{I}$$

where $M = (F_n \cos\beta)\, d$

$$c = \tfrac{1}{2}h$$

$$I = \frac{1}{12}bh^3$$

and substituting in the above equation

$$s = \frac{6dF_n \cos\beta}{bh^2} \tag{57}$$

which is the same result obtained in case 1 with "d" and "h" defined differently. The angle β is defined by either Equation (38) or by Equation (41) and X_T by Equation (55).

Comparing Equation (35) with Equation (57) shows that

$$F_b = F_n \cos\beta = F_t$$

and $$y_p = \frac{h^2}{6d}$$

This shows that Equation (57) includes the Lewis factor and also considers the angle of load application which Equation (35) neglects. Note: The face width b does not change.

Equation (57) was modified to

$$\frac{sb}{F_n} = \frac{6d \cos\beta}{h^2} \qquad (63)$$

so that the load and face width would not have to be considered in the calculations. A computer program was written so that a secondary relationship involving e_1 and e_2 could be determined on the basis of approximately equal tooth bending stress with the load applied at the tip.

The program which was written based all calculations on the following assumptions: pressure angle ϕ = 20 degrees, diametral pitch $P = 1$, full depth gears ($k = 1$), and coarse pitch (clearance $c = 0.250/P$ for $P = 1$ to 19.99). Values for the change in center distance ΔC were used as input. These values varied as follows: ΔC = 0.050 inches to 1.600 inches in steps of 0.025 inches and ΔC = -0.050 inches to -0.999 inches in steps of -0.025 inches. N_1 was held constant. N_2 varied from $N_2 = 18$ to $N_2 = 130$ teeth in steps of one so that a least square analysis could be applied to the results: $e_1/(e_1 + e_2)$ versus $N_2/(N_1 + N_2)$.

Figure 11 shows a very simple flow diagram which shows the primary logic of the program. A complete listing of the program can be found in Appendix A. The program is written in double precision and required approximately seven hours to calculate the results for all of the values of ΔC used.

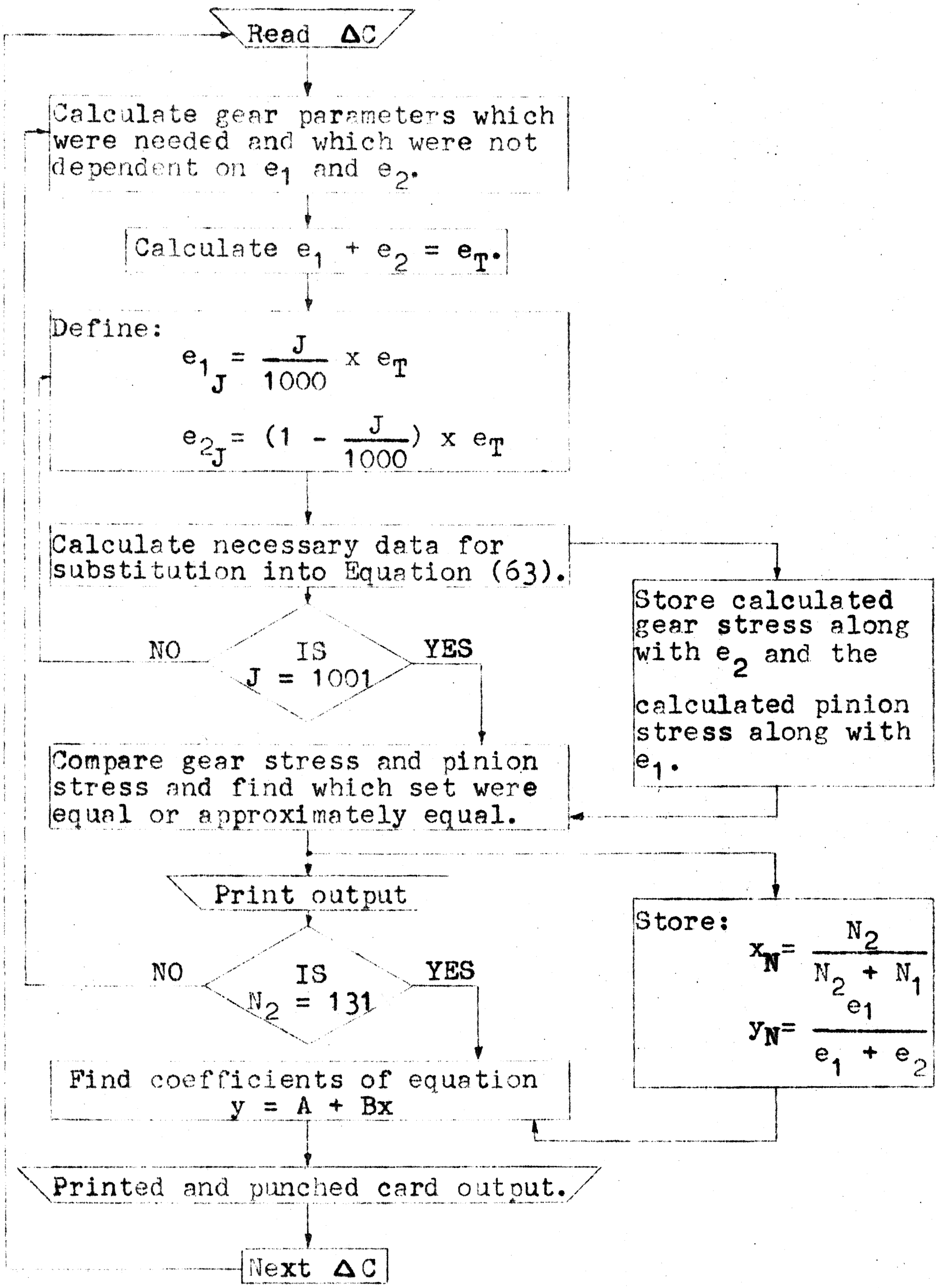

FIGURE 11. FLOW DIAGRAM

B. Data and Results

A sample set of data is shown in Tables 1, 2, and 3 which has been taken from the computer printout. Figures 12 through 24 show the plots of hob offset ratio for the cases where the hob is withdrawn a distance ($+\Delta C$) from the gear blank and Figures 25 through 32 show the plots of hob offset ratio for the case where the hob is pushed a distance ($-\Delta C$) into the gear blank.

These plots are only valid for a cutting pressure angle of $\phi = 20$ degrees, full depth gears ($k = 1$), and coarse pitch. Although these plots were plotted for data based on a diametral pitch of one and for $N_1 = 18$ teeth, they are good for any pitch, as the stress is independent of the pitch. When N_1 takes on other values, a very slight error (less than 4%) is introduced. This was determined by running the main program with other values of N_1 and comparing the results of the ratio $e_1/(e_1 + e_2)$ to $N_2/(N_1 + N_2)$.

Figure 18 shows the plot for $\Delta C = 0.875$ which corresponds to the data in Tables 1, 2, and 3. It is seen that as N_2 increases, so does the value of e_1. In other words, as the ratio of $N_2/(N_1 + N_2)$ increases so does the value of the ratio $e_1/(e_1 + e_2)$. A review of Equation (26) will show that this is the same trend that this relationship suggests. If Equation (26) were plotted, it would be seen that this equation is a straight line with a slope of 45°. A review

TABLE 1. EQUAL STRESS, N_2 = 18 TO 55

N_1 = 18 $\phi = 20^{\circ}$ P = 1 ΔC = 0.875

N_2	e_1	e_2	m_p	$\frac{sb}{F_n}$	$\frac{e_1}{e_1 + e_2}$
18	.507	.507	1.23	2.654	.500
19	.514	.496	1.24	2.636	.509
20	.523	.485	1.24	2.619	.519
21	.530	.474	1.25	2.602	.528
22	.538	.464	1.26	2.585	.537
23	.545	.453	1.27	2.568	.546
24	.553	.443	1.27	2.552	.555
25	.560	.433	1.28	2.535	.564
26	.568	.423	1.29	2.518	.573
27	.576	.413	1.29	2.502	.582
28	.583	.404	1.30	2.486	.591
29	.591	.394	1.30	2.471	.600
30	.596	.386	1.30	2.458	.607
31	.602	.379	1.31	2.446	.614
32	.608	.371	1.31	2.435	.621
33	.612	.364	1.32	2.425	.627
34	.617	.358	1.32	2.415	.633
35	.622	.351	1.32	2.406	.639
36	.626	.346	1.33	2.398	.644
37	.630	.341	1.33	2.390	.649
38	.633	.335	1.33	2.383	.654
39	.637	.330	1.33	2.375	.659
40	.641	.324	1.34	2.368	.664
41	.644	.320	1.34	2.362	.668
42	.648	.315	1.34	2.355	.673
43	.651	.311	1.34	2.349	.677
44	.654	.306	1.35	2.344	.681
45	.657	.302	1.35	2.338	.685
46	.659	.299	1.35	2.333	.688
47	.662	.295	1.35	2.328	.692
48	.665	.290	1.35	2.323	.696
49	.667	.287	1.36	2.319	.699
50	.669	.284	1.36	2.314	.702
51	.672	.280	1.36	2.309	.706
52	.674	.277	1.36	2.305	.709
53	.677	.274	1.36	2.301	.712
54	.679	.271	1.36	2.297	.715
55	.681	.267	1.36	2.294	.718

A = 0.05670   B = 0.87774

TABLE 2. EQUAL STRESS, N_2 = 56 TO 93

N_1 = 18 $\emptyset$ = 20°   P = 1 ΔC = 0.875

N_2	e_1	e_2	m_p	$\frac{sb}{F_n}$	$\frac{e_1}{e_1 + e_2}$
56	.682	.265	1.37	2.290	.720
57	.684	.262	1.37	2.287	.723
58	.687	.259	1.37	2.283	.726
59	.688	.257	1.37	2.280	.720
60	.690	.254	1.37	2.277	.731
61	.691	.252	1.37	2.274	.733
62	.694	.249	1.37	2.270	.736
63	.695	.247	1.37	2.268	.738
64	.696	.245	1.37	2.265	.740
65	.697	.243	1.38	2.262	.742
66	.700	.240	1.38	2.259	.745
67	.701	.237	1.38	2.257	.747
68	.702	.235	1.38	2.254	.749
69	.704	.233	1.38	2.252	.751
70	.705	.231	1.38	2.249	.753
71	.707	.229	1.38	2.247	.755
72	.708	.227	1.38	2.244	.757
73	.709	.225	1.38	2.242	.759
74	.711	.223	1.38	2.240	.761
75	.711	.222	1.38	2.238	.762
76	.713	.220	1.38	2.236	.764
77	.714	.218	1.38	2.234	.766
78	.716	.216	1.38	2.232	.768
79	.716	.215	1.39	2.231	.769
80	.717	.213	1.39	2.228	.771
81	.719	.211	1.39	2.226	.773
82	.719	.210	1.39	2.225	.774
83	.721	.208	1.39	2.223	.776
84	.722	.206	1.39	2.221	.778
85	.723	.205	1.39	2.220	.779
86	.724	.203	1.39	2.218	.781
87	.725	.202	1.39	2.217	.782
88	.726	.200	1.39	2.215	.784
89	.727	.199	1.39	2.214	.785
90	.728	.197	1.39	2.212	.787
91	.729	.196	1.39	2.211	.788
92	.731	.194	1.39	2.209	.790
93	.731	.193	1.39	2.208	.791

A = 0.05670   B = 0.87774

TABLE 3. EQUAL STRESS, N_2 = 94 TO 130

N_1 = 18 $\emptyset = 20^{\circ}$ P = 1 ΔC = 0.875

N_2	e_1	e_2	m_p	$\frac{sb}{F_n}$	$\frac{e_1}{e_1 + e_2}$
94	.732	.192	1.39	2.207	.792
95	.733	.190	1.39	2.205	.794
96	.734	.189	1.39	2.204	.795
97	.735	.188	1.39	2.203	.796
98	.736	.186	1.39	2.201	.798
99	.737	.185	1.39	2.200	.799
100	.737	.184	1.39	2.199	.800
101	.738	.183	1.39	2.198	.801
102	.739	.181	1.39	2.196	.803
103	.740	.180	1.40	2.195	.804
104	.741	.179	1.40	2.194	.805
105	.741	.178	1.40	2.193	.806
106	.742	.177	1.40	2.192	.807
107	.743	.176	1.40	2.191	.808
108	.744	.175	1.40	2.189	.810
109	.745	.174	1.40	2.188	.811
110	.746	.173	1.40	2.187	.812
111	.746	.172	1.40	2.186	.813
112	.747	.171	1.40	2.185	.814
113	.748	.170	1.40	2.184	.815
114	.748	.169	1.40	2.184	.816
115	.749	.168	1.40	2.183	.817
116	.750	.167	1.40	2.182	.818
117	.750	.166	1.40	2.181	.819
118	.751	.165	1.40	2.180	.820
119	.752	.164	1.40	2.179	.821
120	.752	.163	1.40	2.178	.822
121	.753	.162	1.40	2.177	.823
122	.754	.161	1.40	2.176	.824
123	.754	.160	1.40	2.175	.825
124	.755	.159	1.40	2.174	.826
125	.755	.159	1.40	2.174	.826
126	.755	.158	1.40	2.173	.827
127	.756	.157	1.40	2.172	.828
128	.757	.156	1.40	2.172	.829
129	.758	.155	1.40	2.171	.830
130	.758	.154	1.40	2.170	.831

A = 0.05670 B = 0.87774

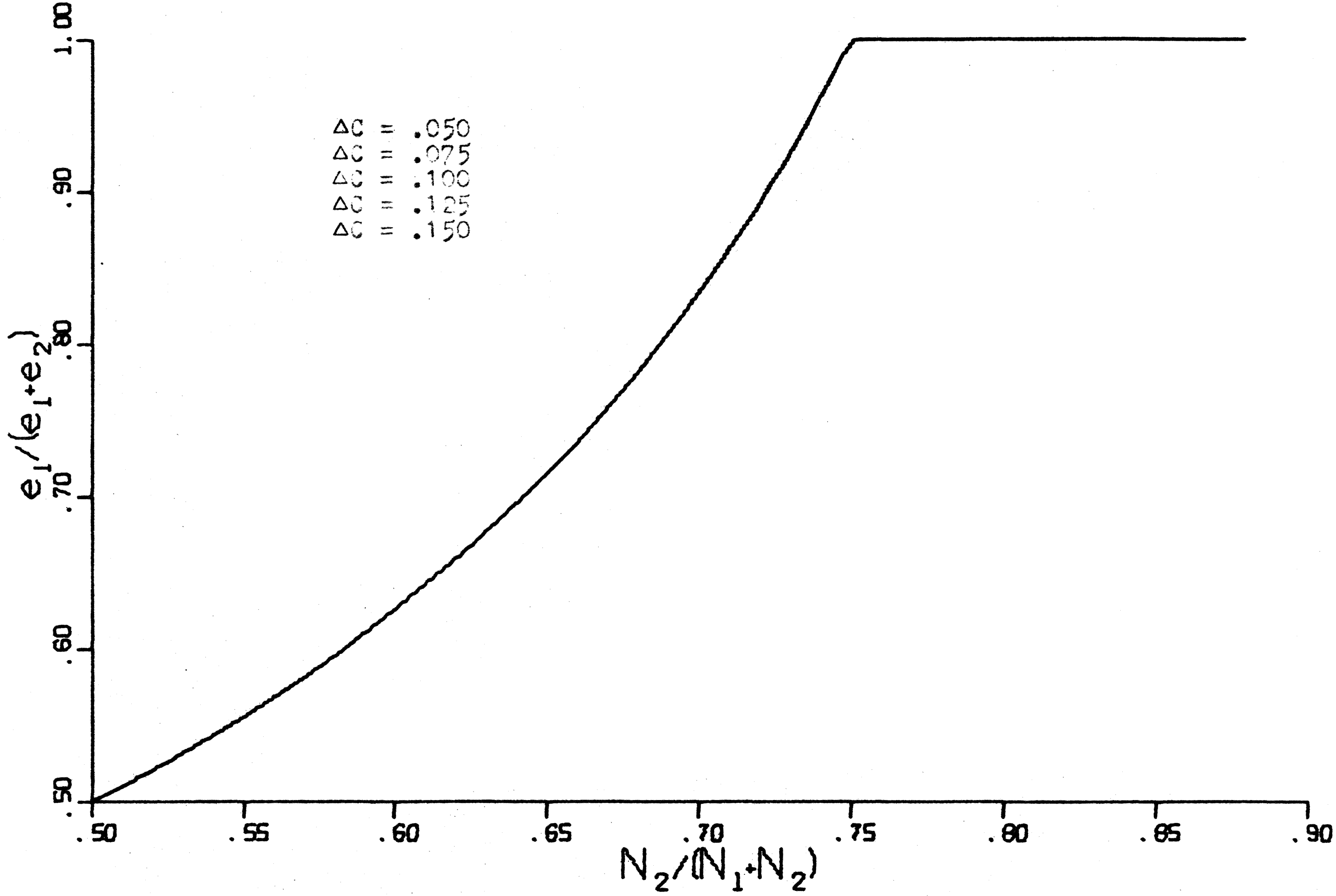

FIGURE 12 HOB OFFSET RATIO

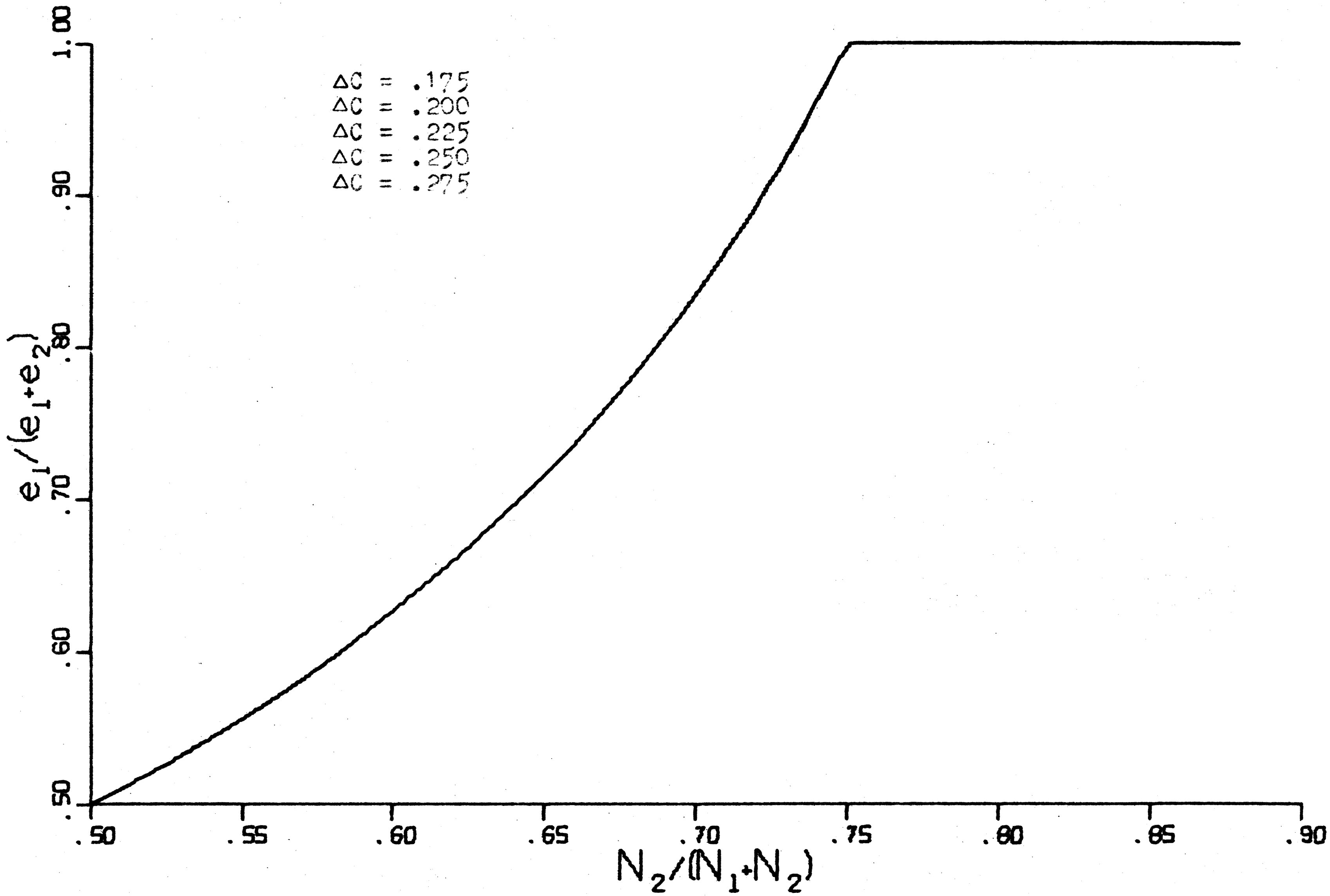

FIGURE 13 HOB OFFSET RATIO

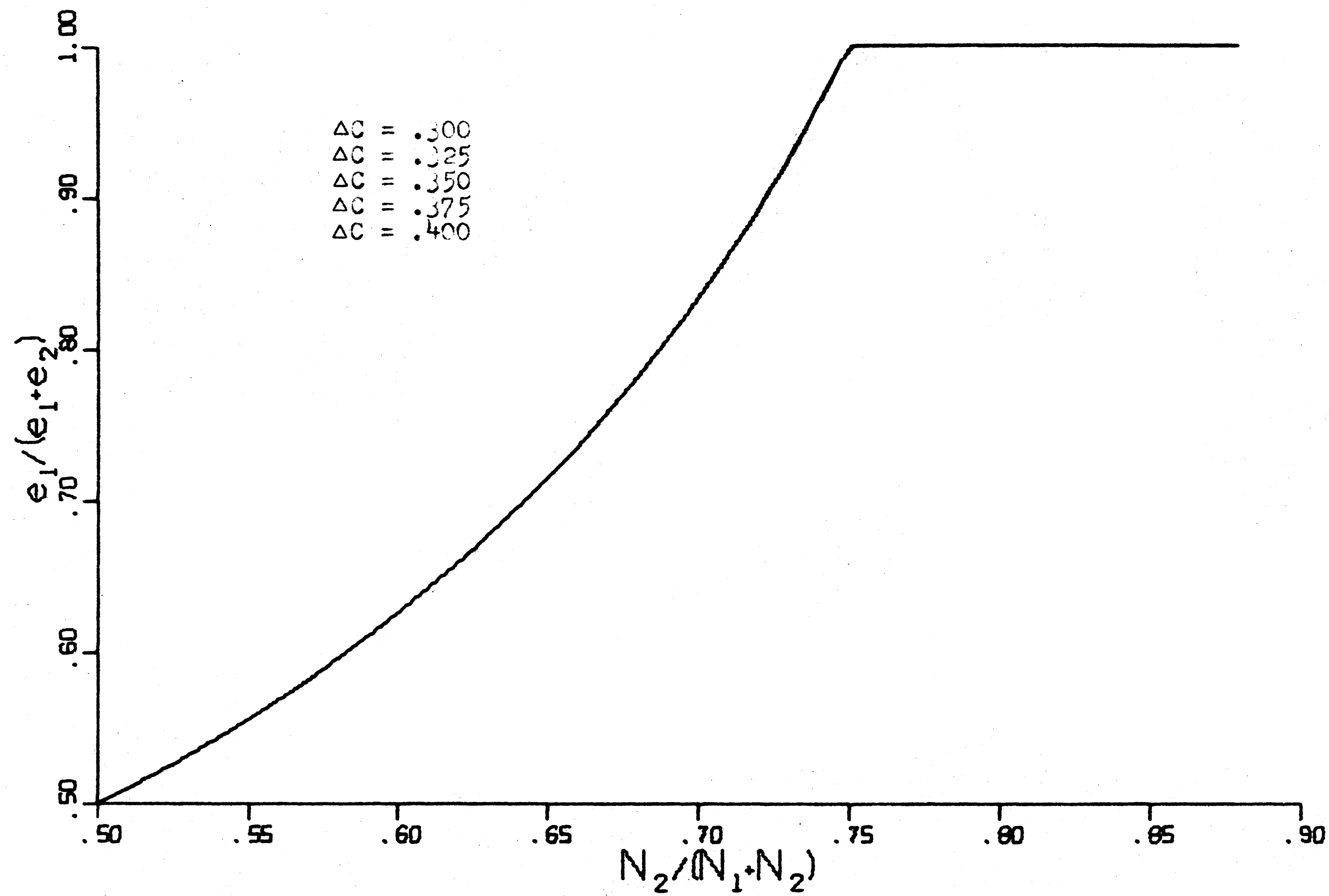

FIGURE 14 HOB OFFSET RATIO

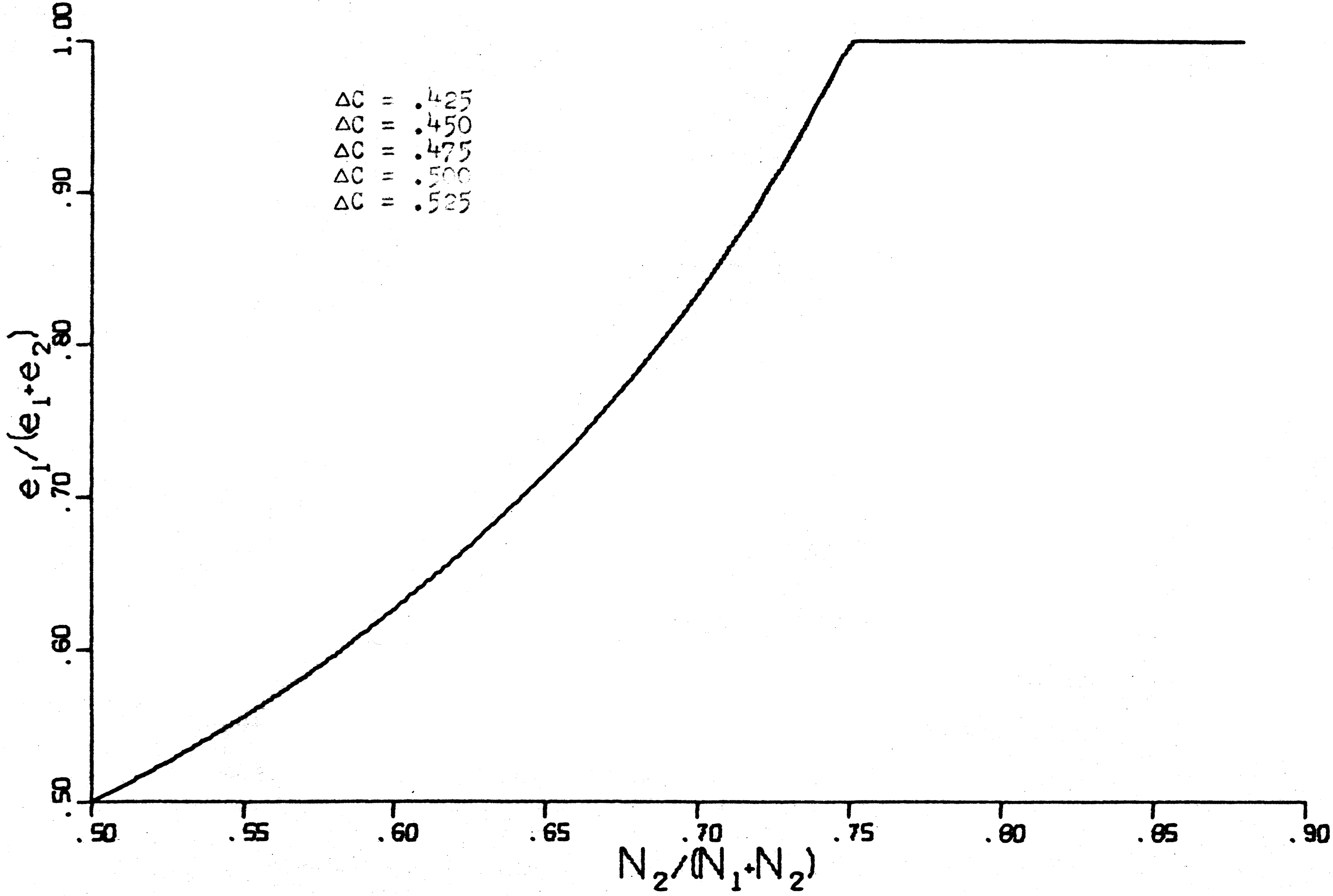

FIGURE 15 HOB OFFSET RATIO

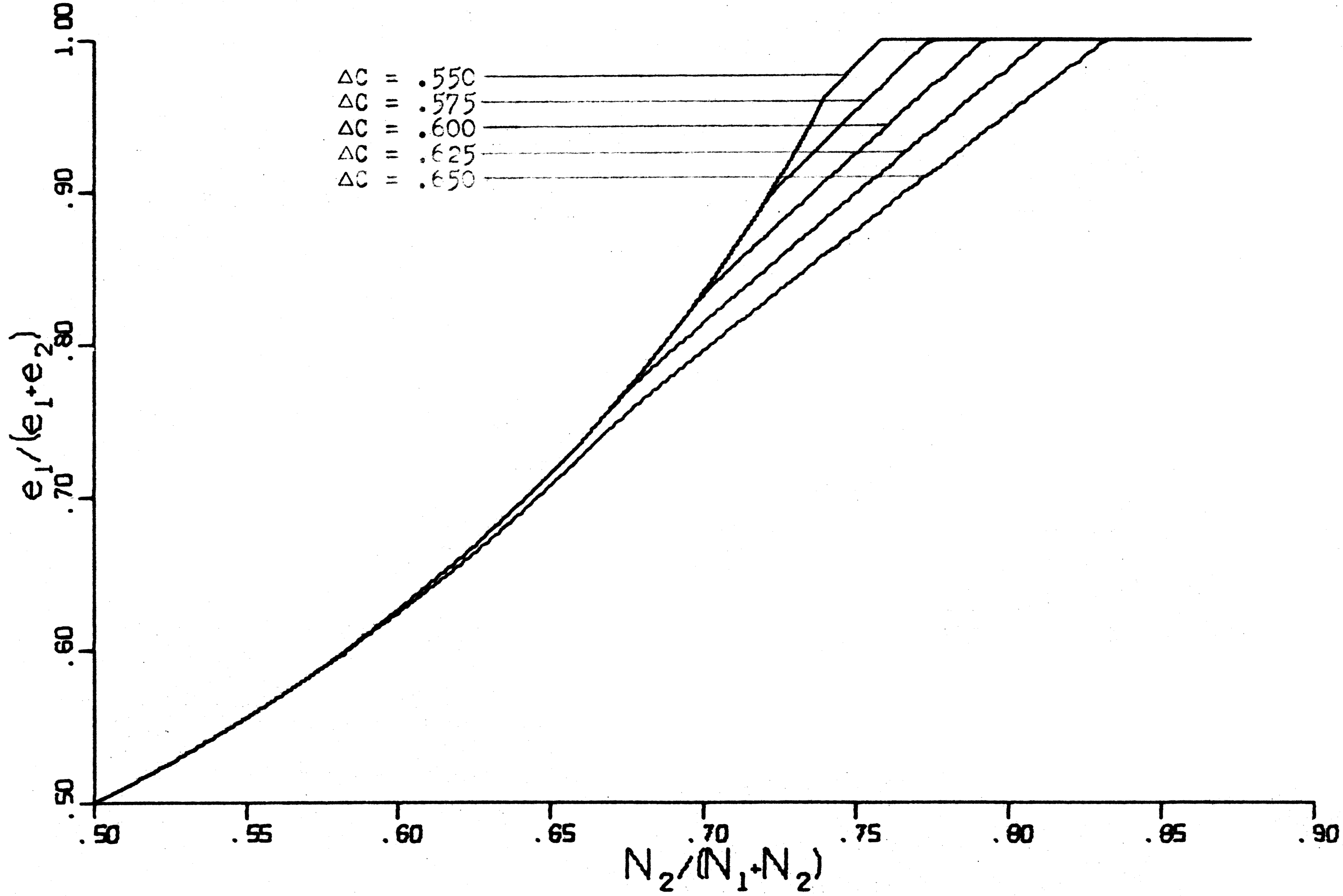

FIGURE 16 HOB OFFSET RATIO

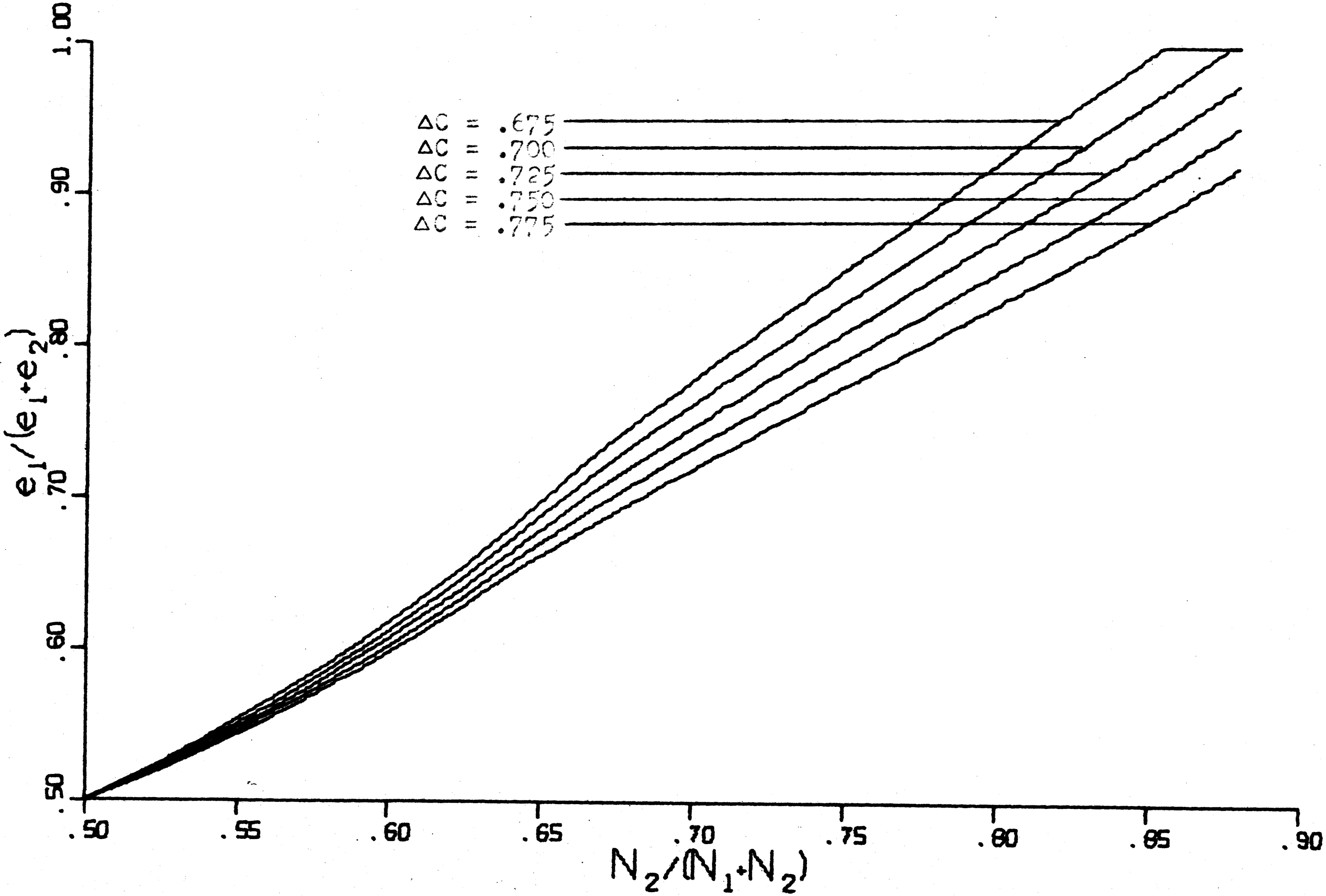

FIGURE 17 HOB OFFSET RATIO

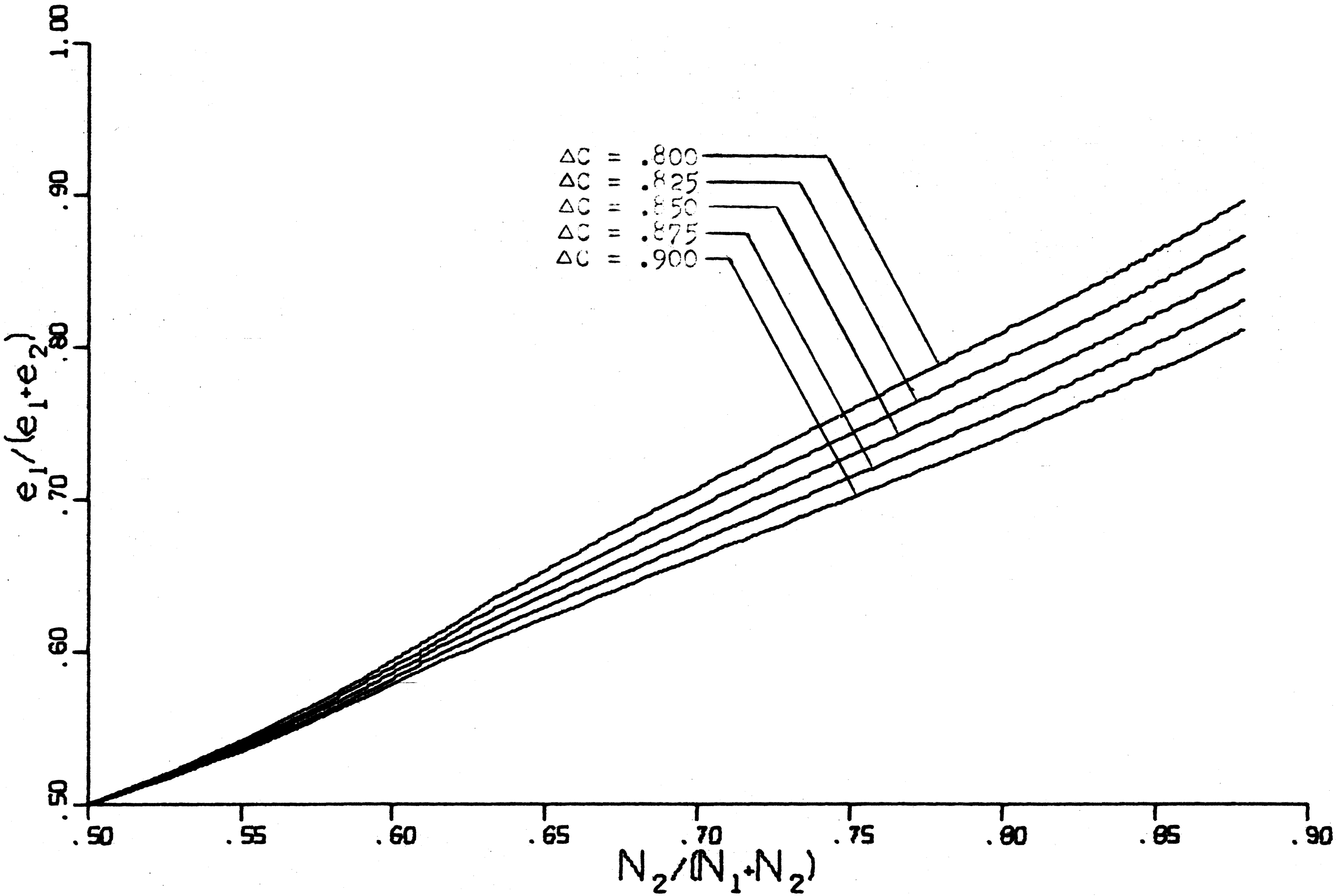

FIGURE 18 HOB OFFSET RATIO

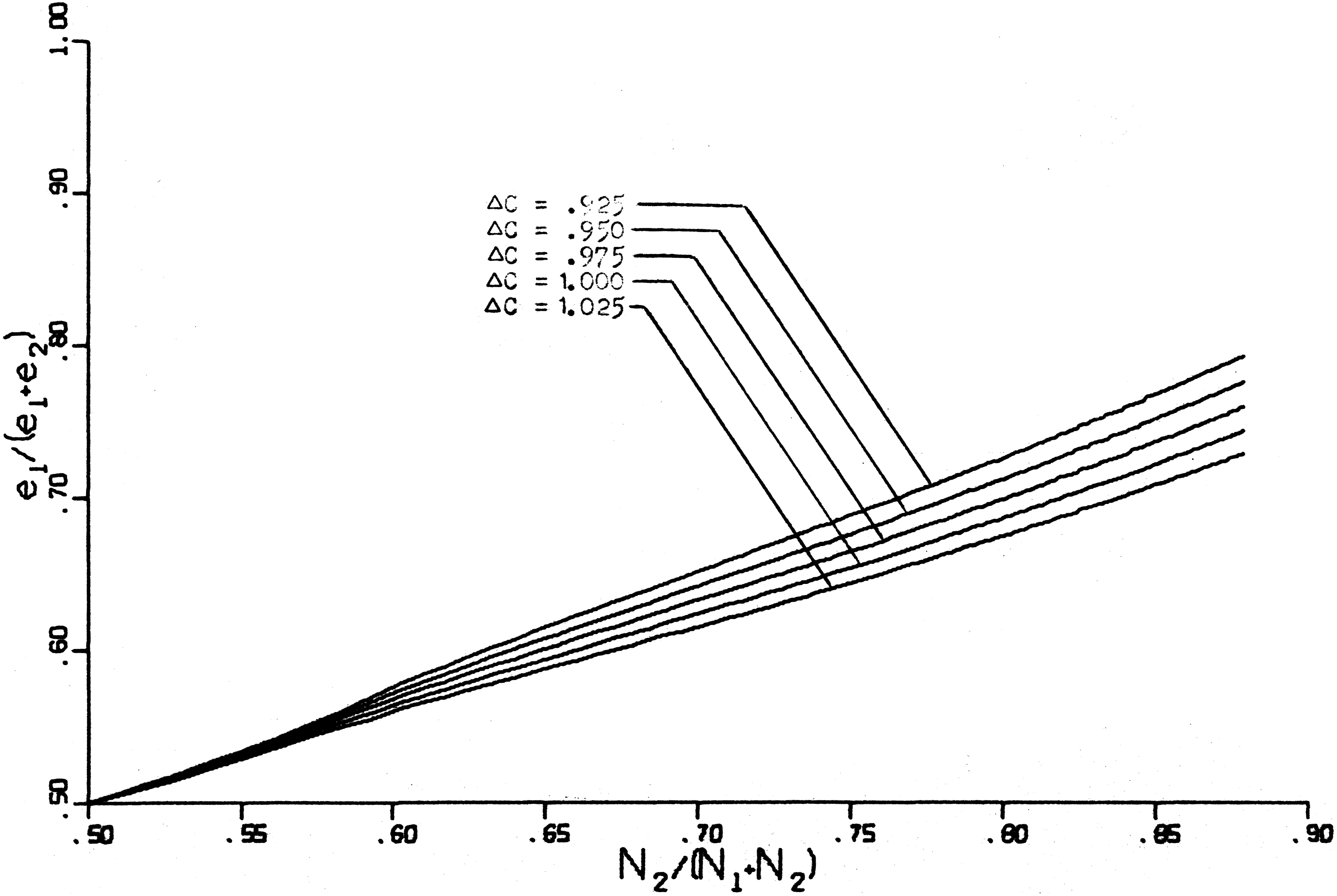

FIGURE 19 HOB OFFSET RATIO

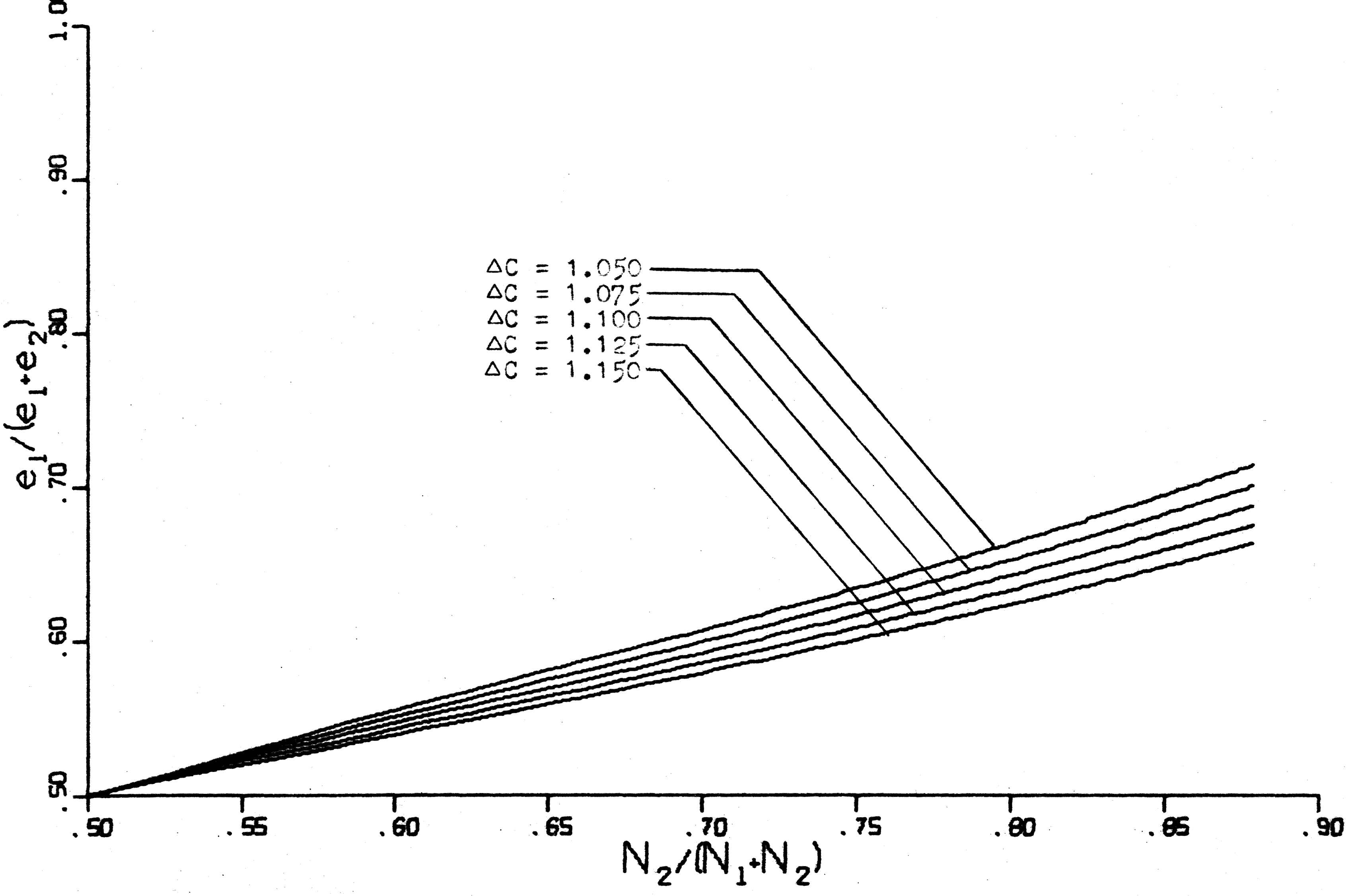

FIGURE 20 HOB OFFSET RATIO

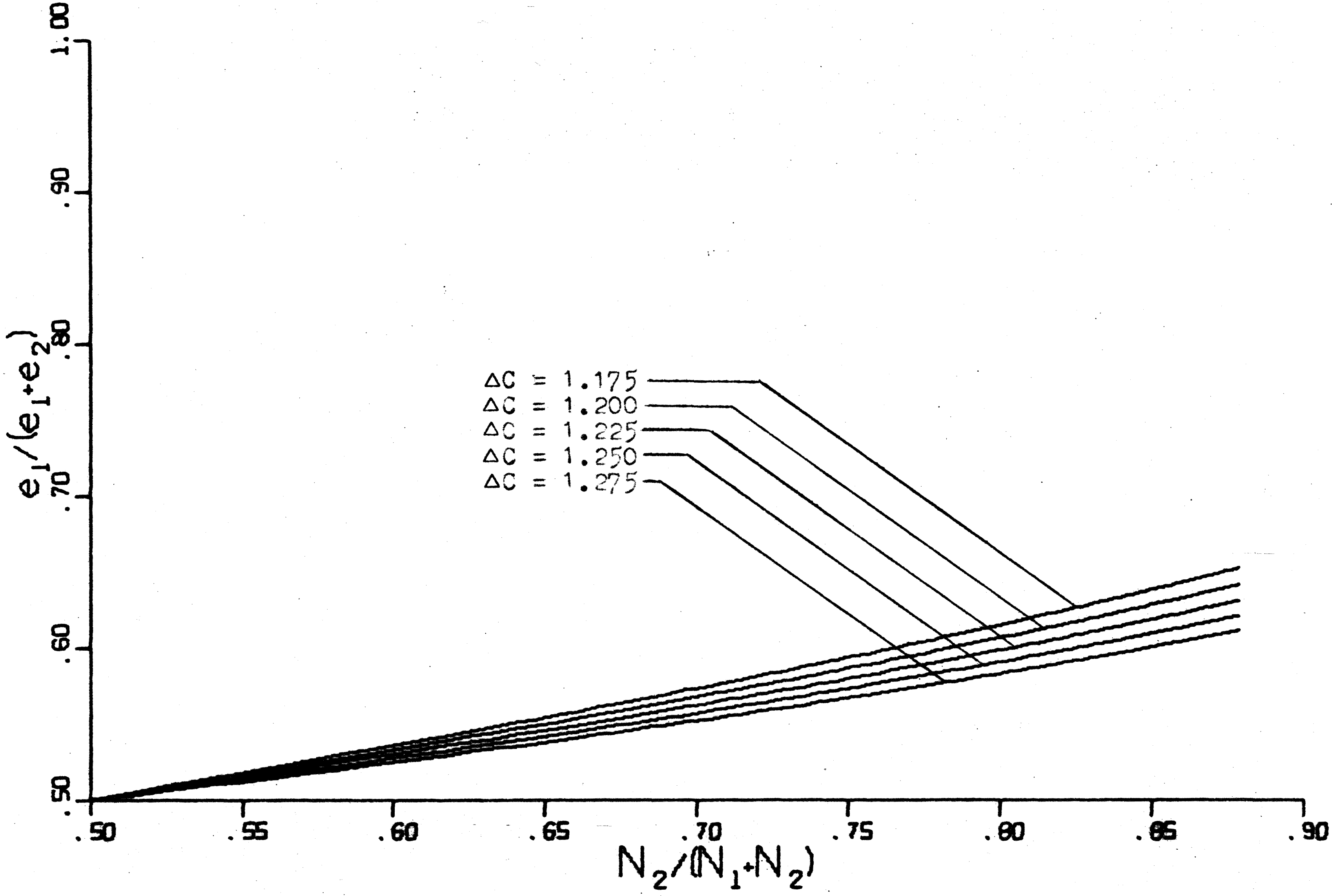

FIGURE 21 HOB OFFSET RATIO

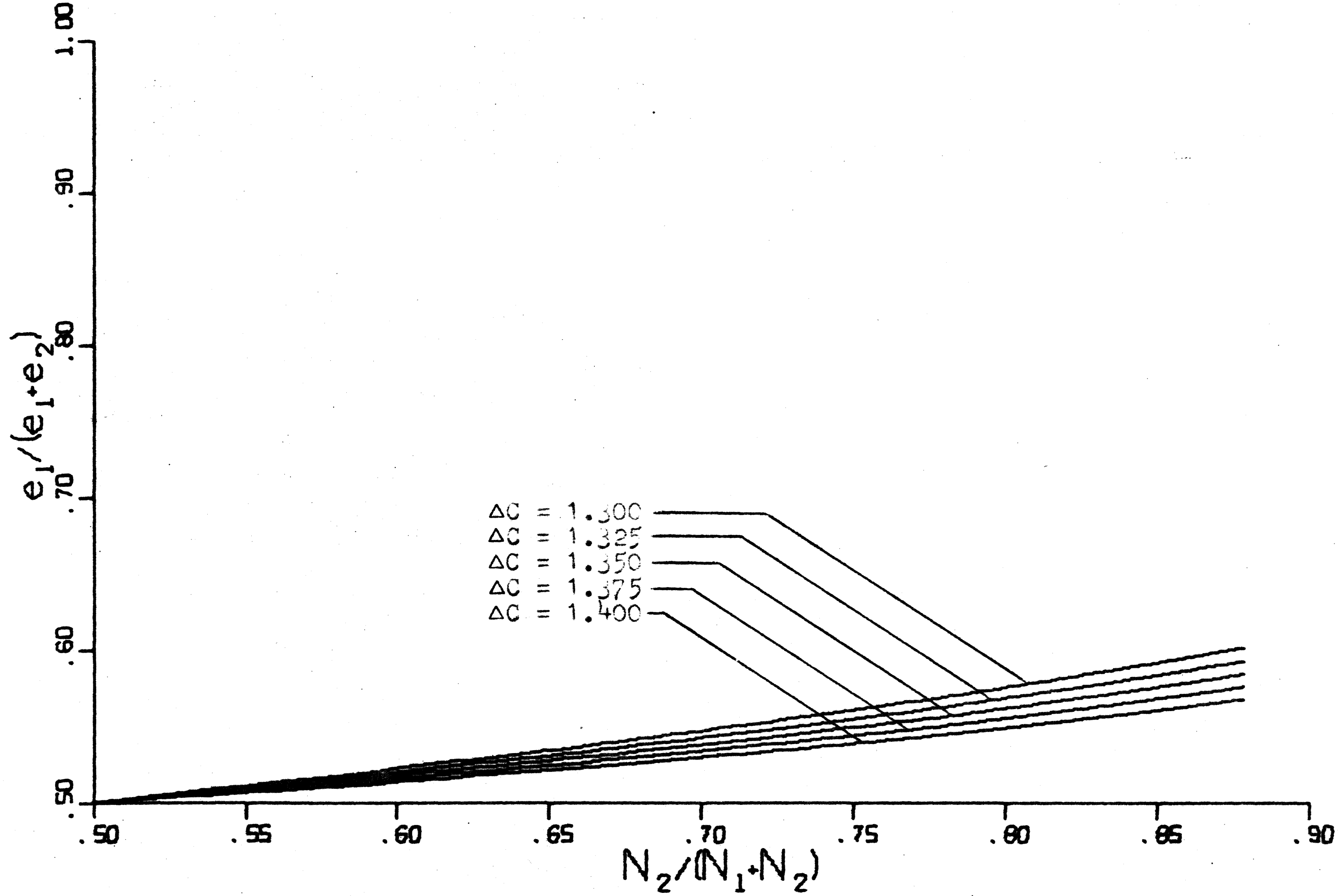

FIGURE 22 HOB OFFSET RATIO

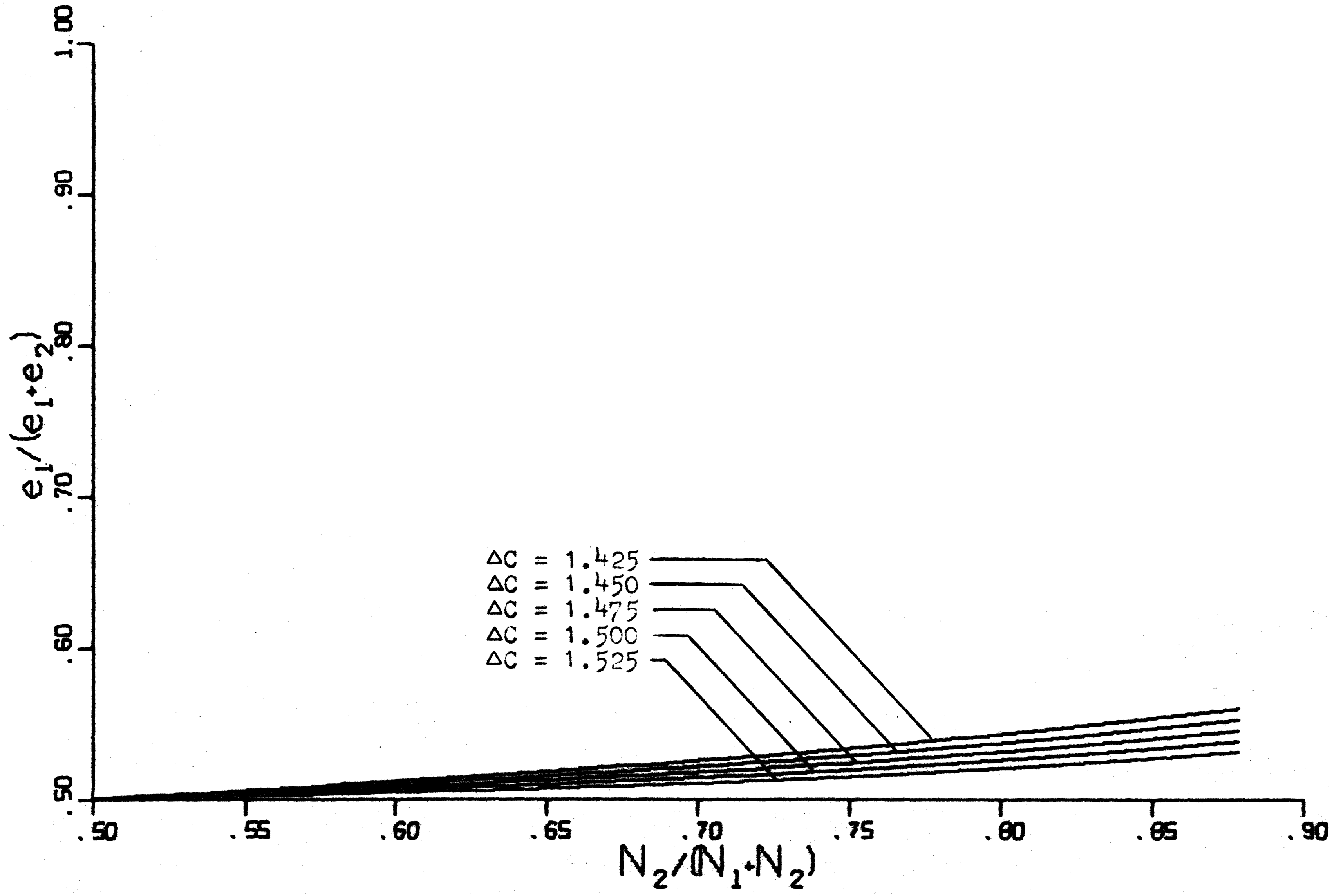

FIGURE 23 HOB OFFSET RATIO

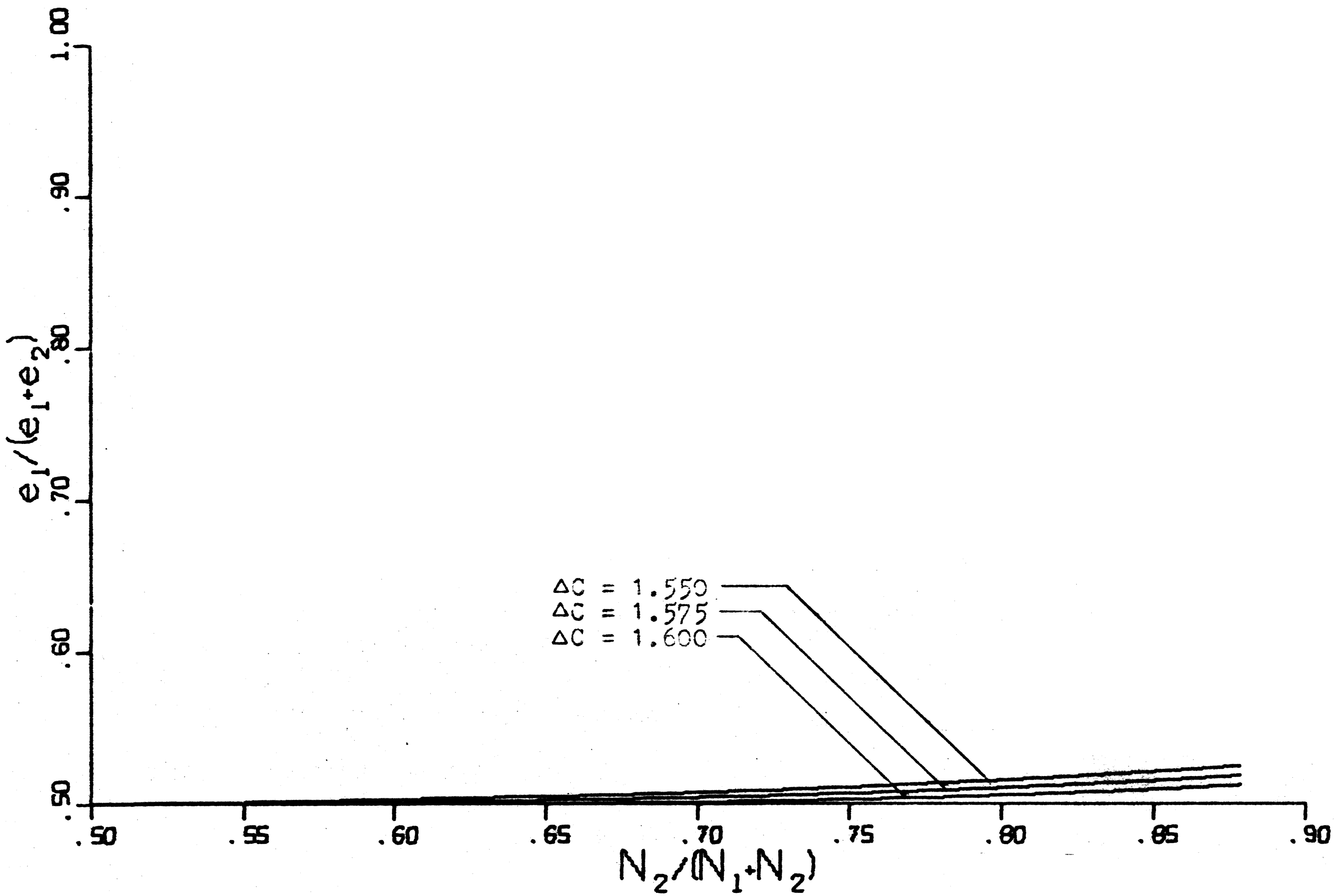

FIGURE 24 HOB OFFSET RATIO

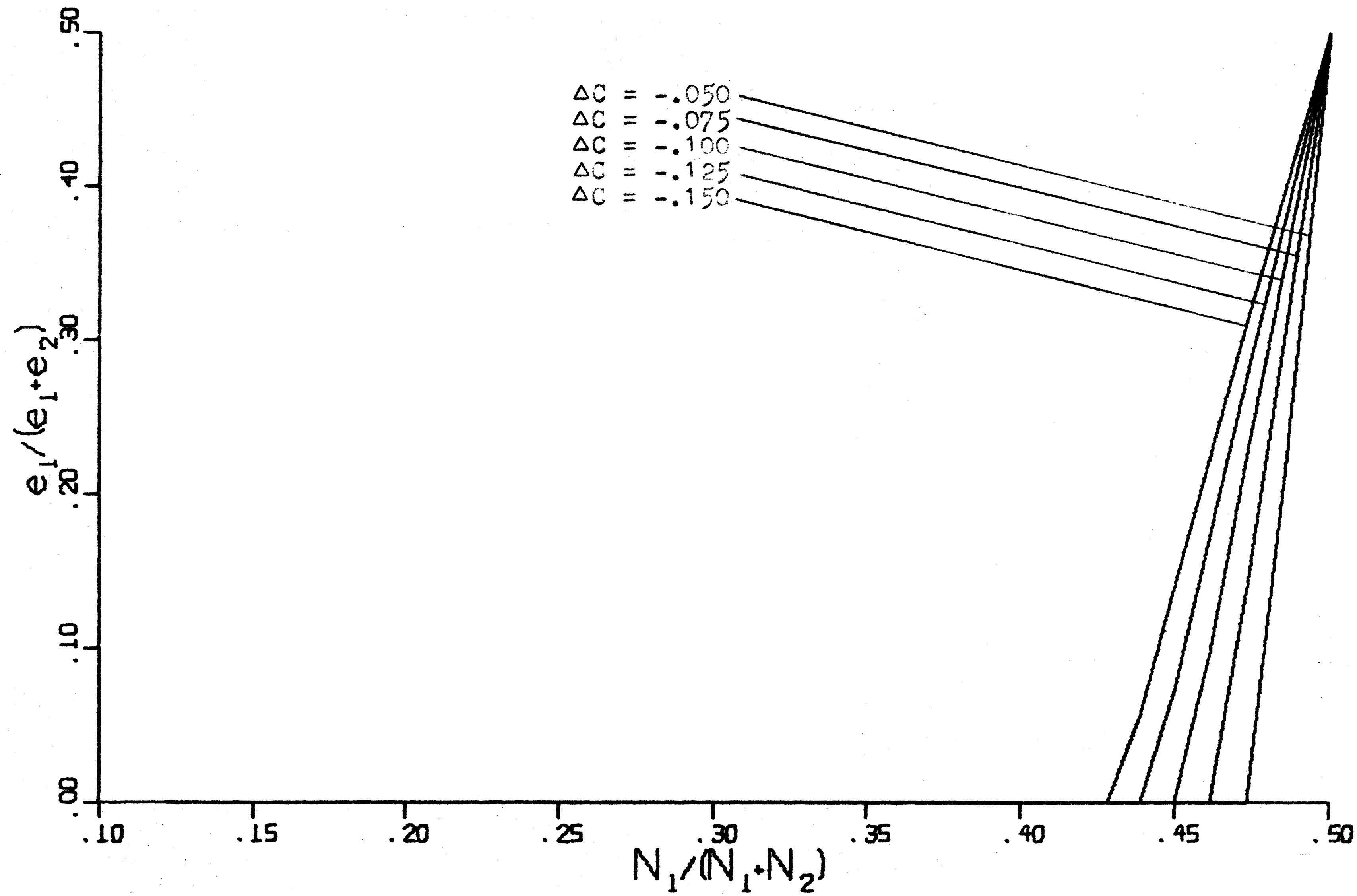

FIGURE 25 HOB OFFSET RATIO

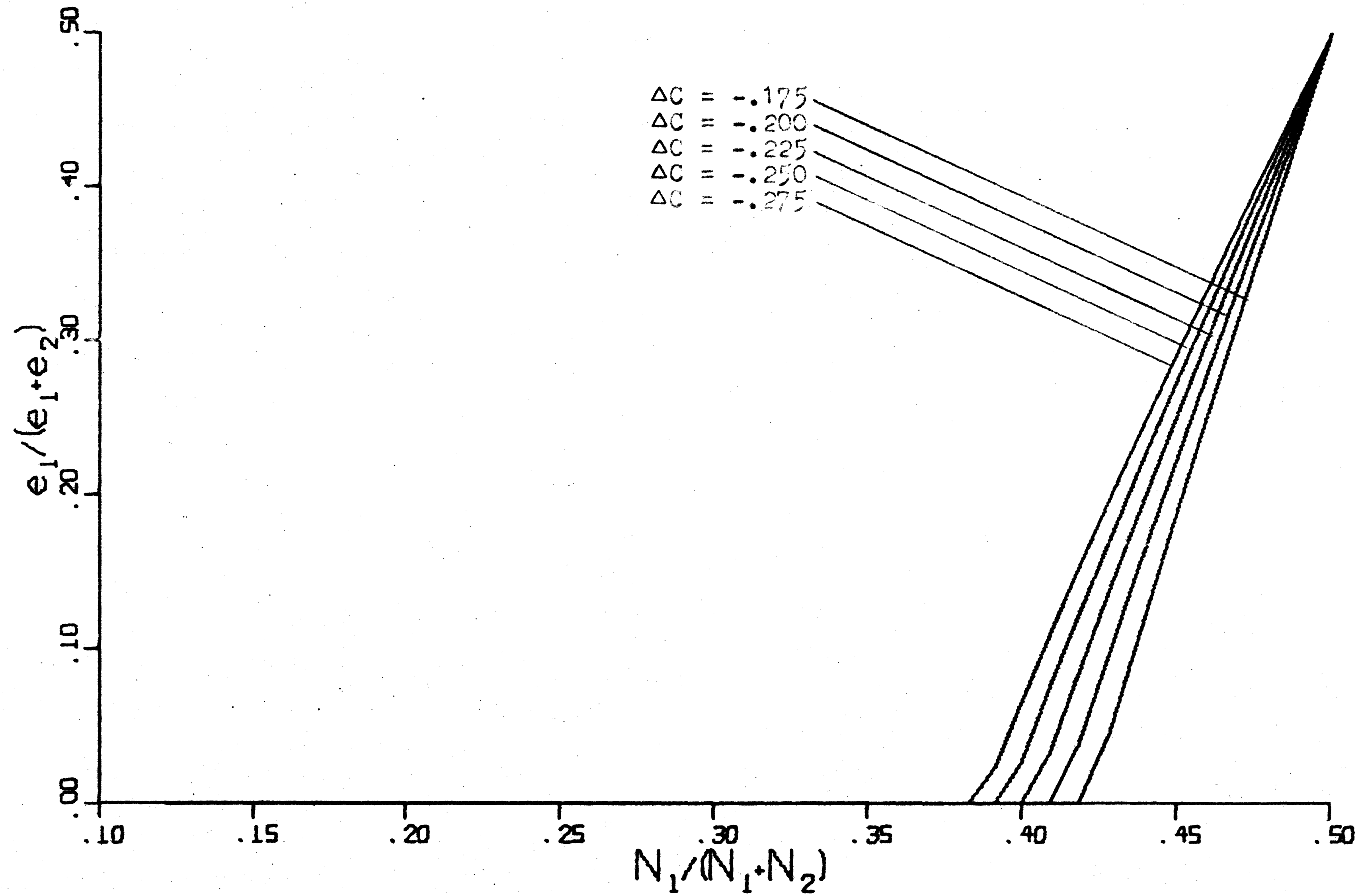

FIGURE 26 HOB OFFSET RATIO

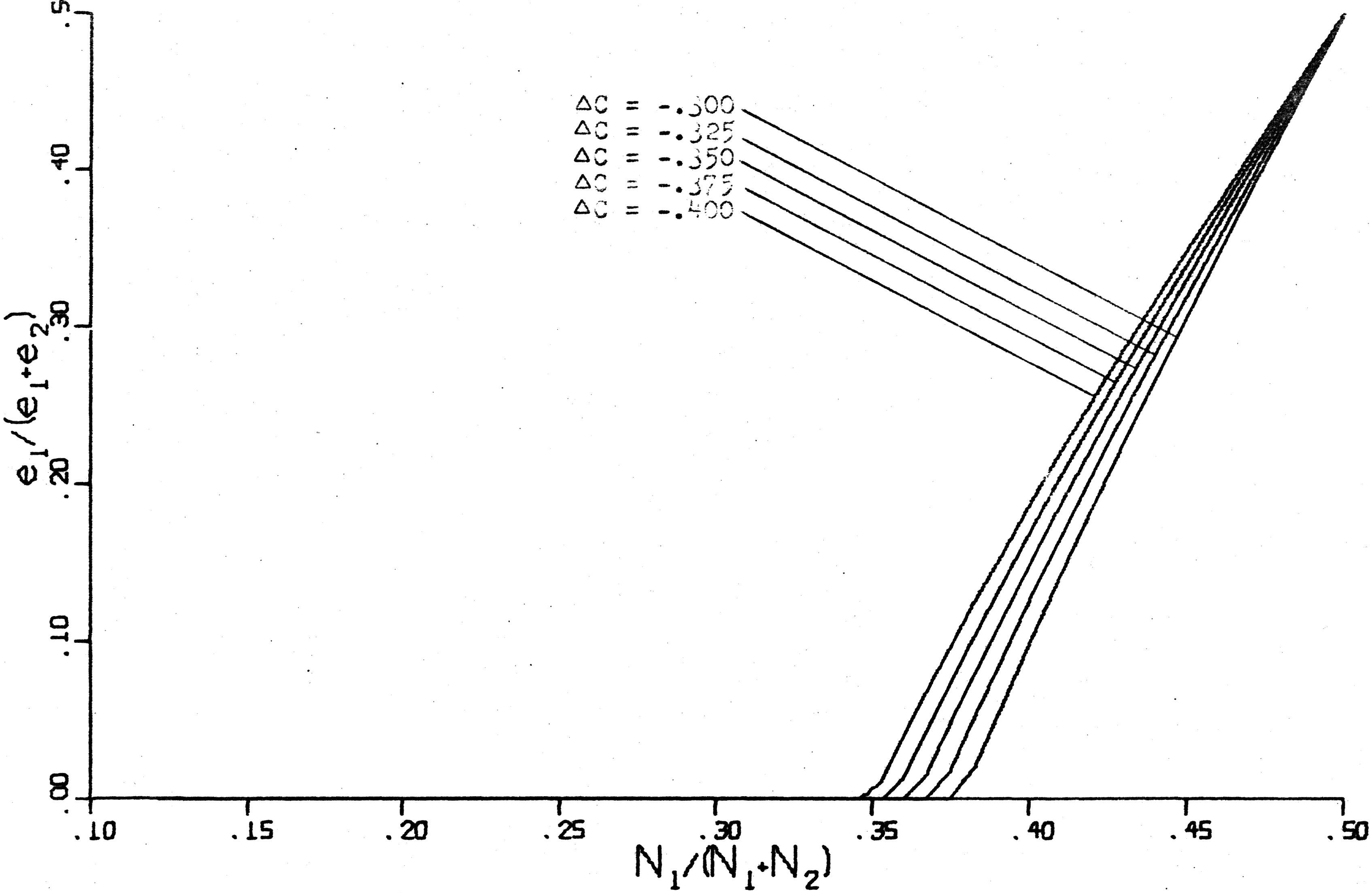

FIGURE 27 HOB OFFSET RATIO

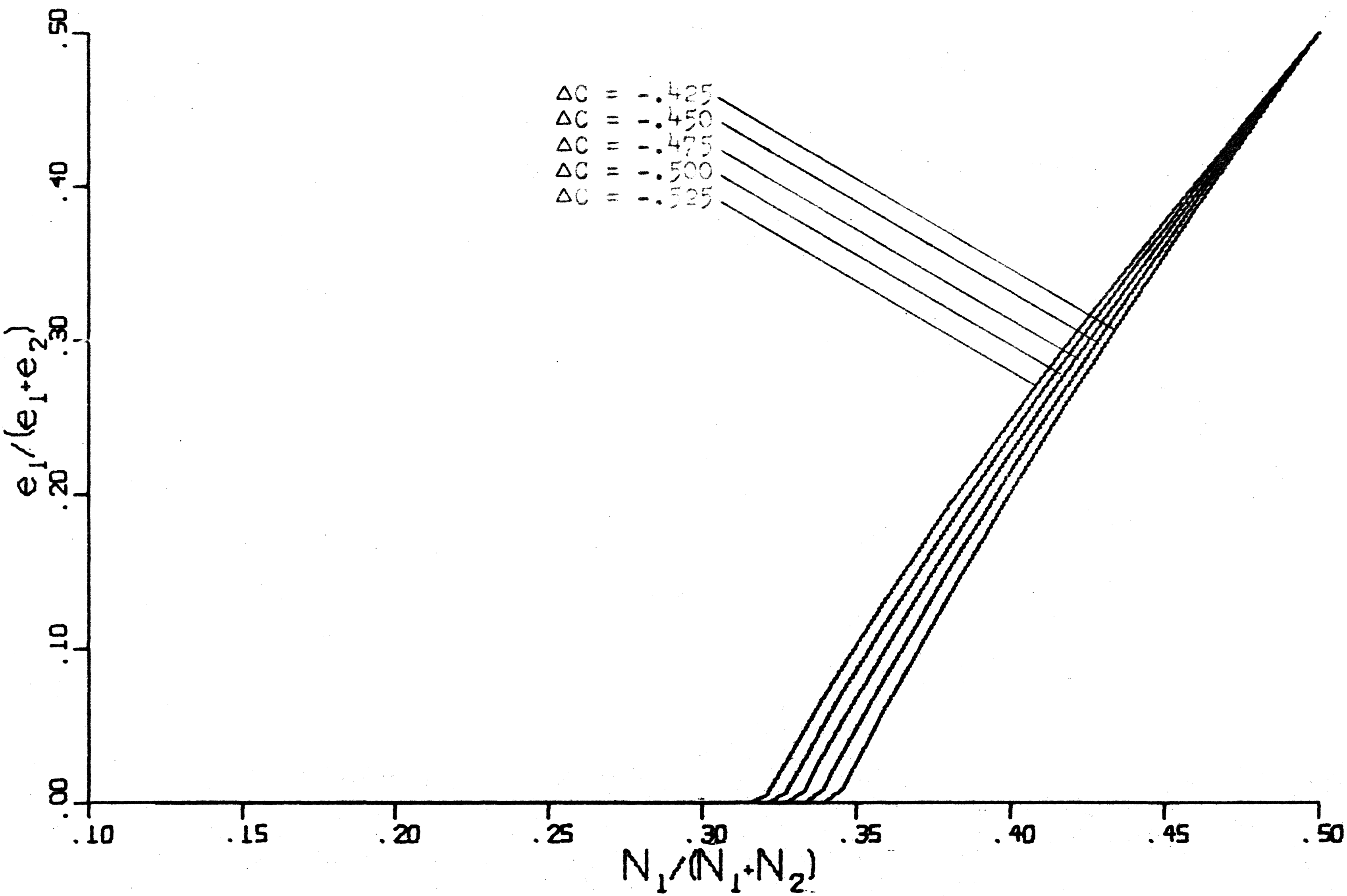

FIGURE 28 HOB OFFSET RATIO

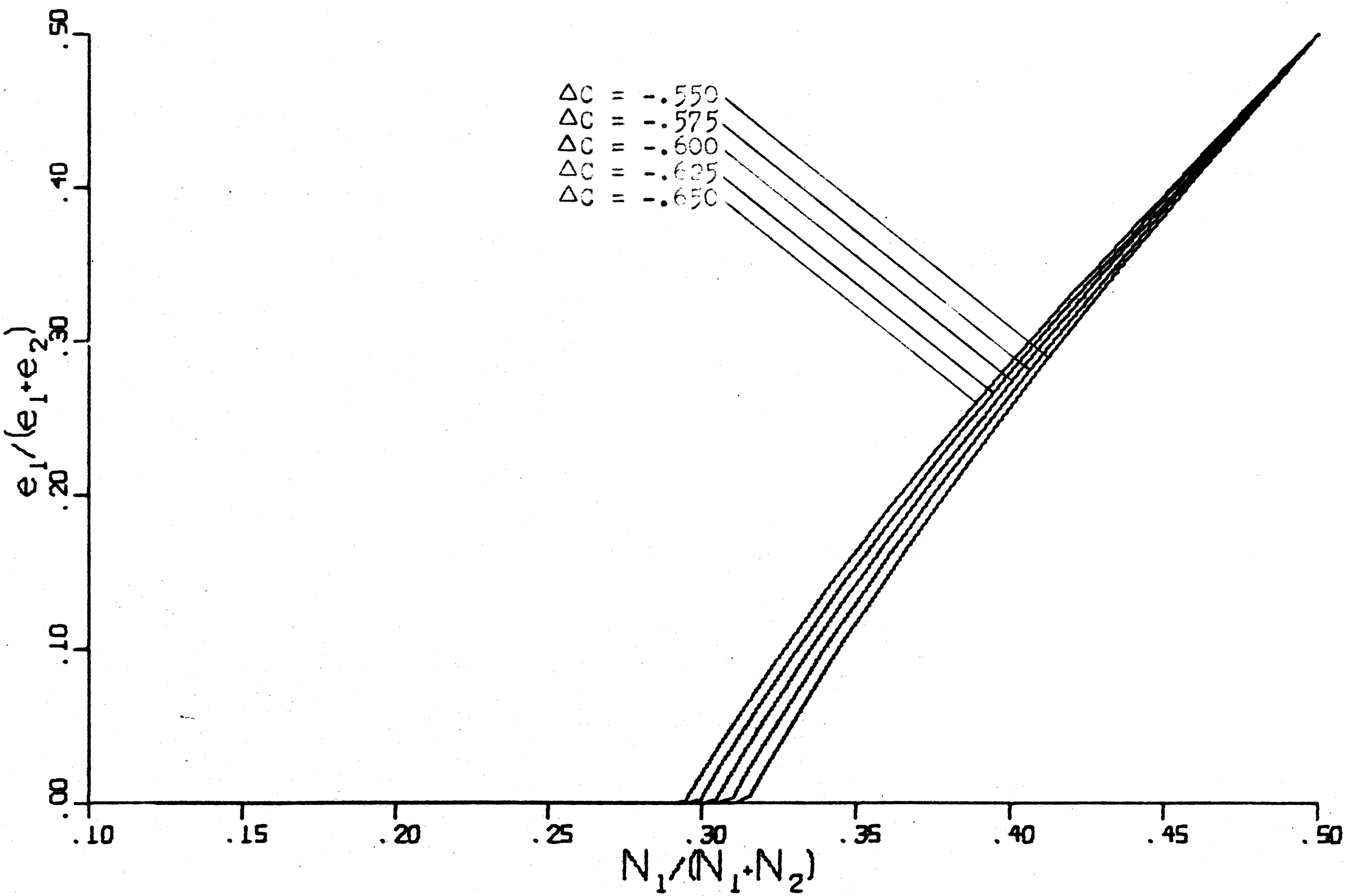

FIGURE.29 HOB OFFSET RATIO

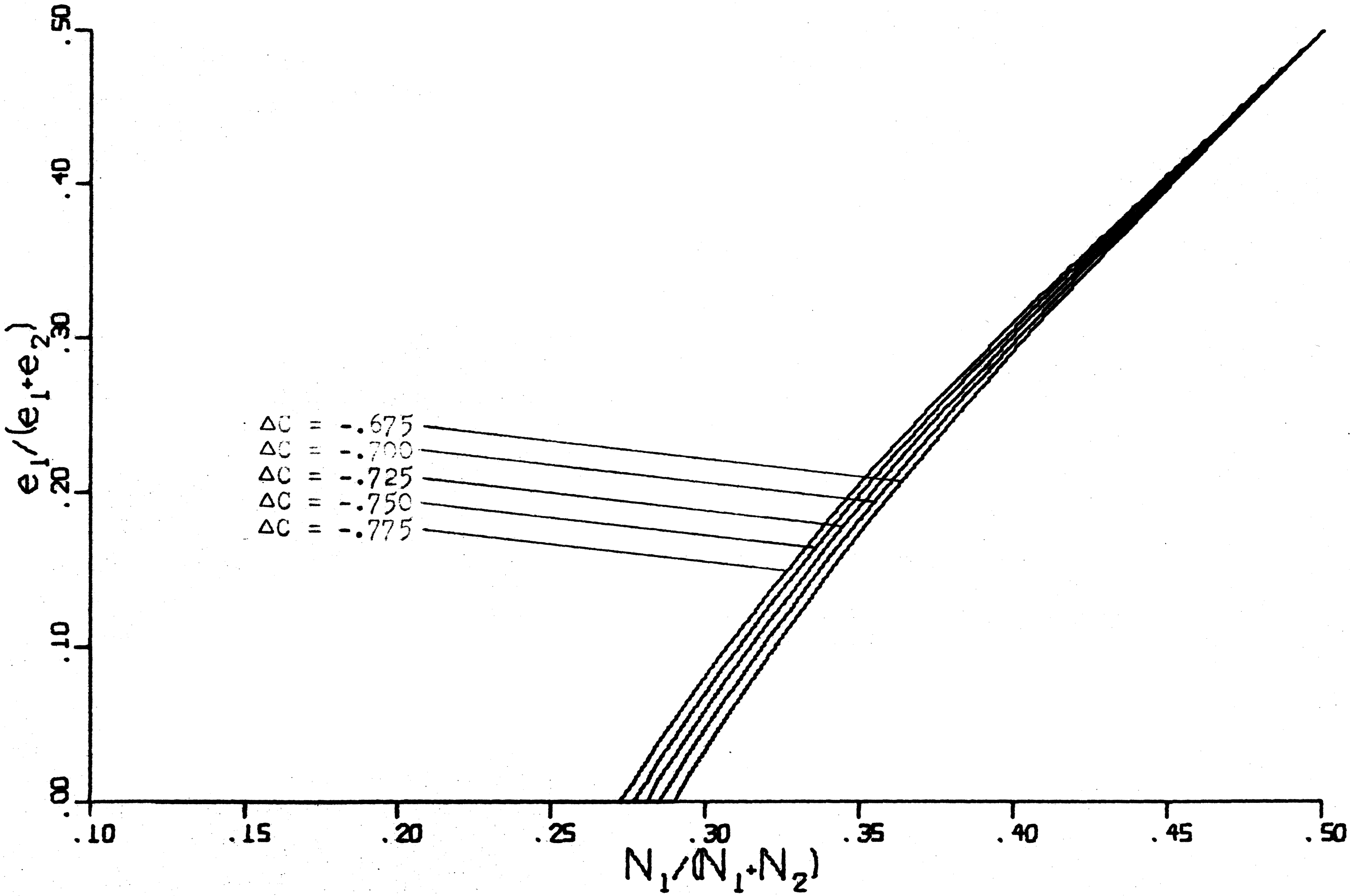

FIGURE 30 HOB OFFSET RATIO

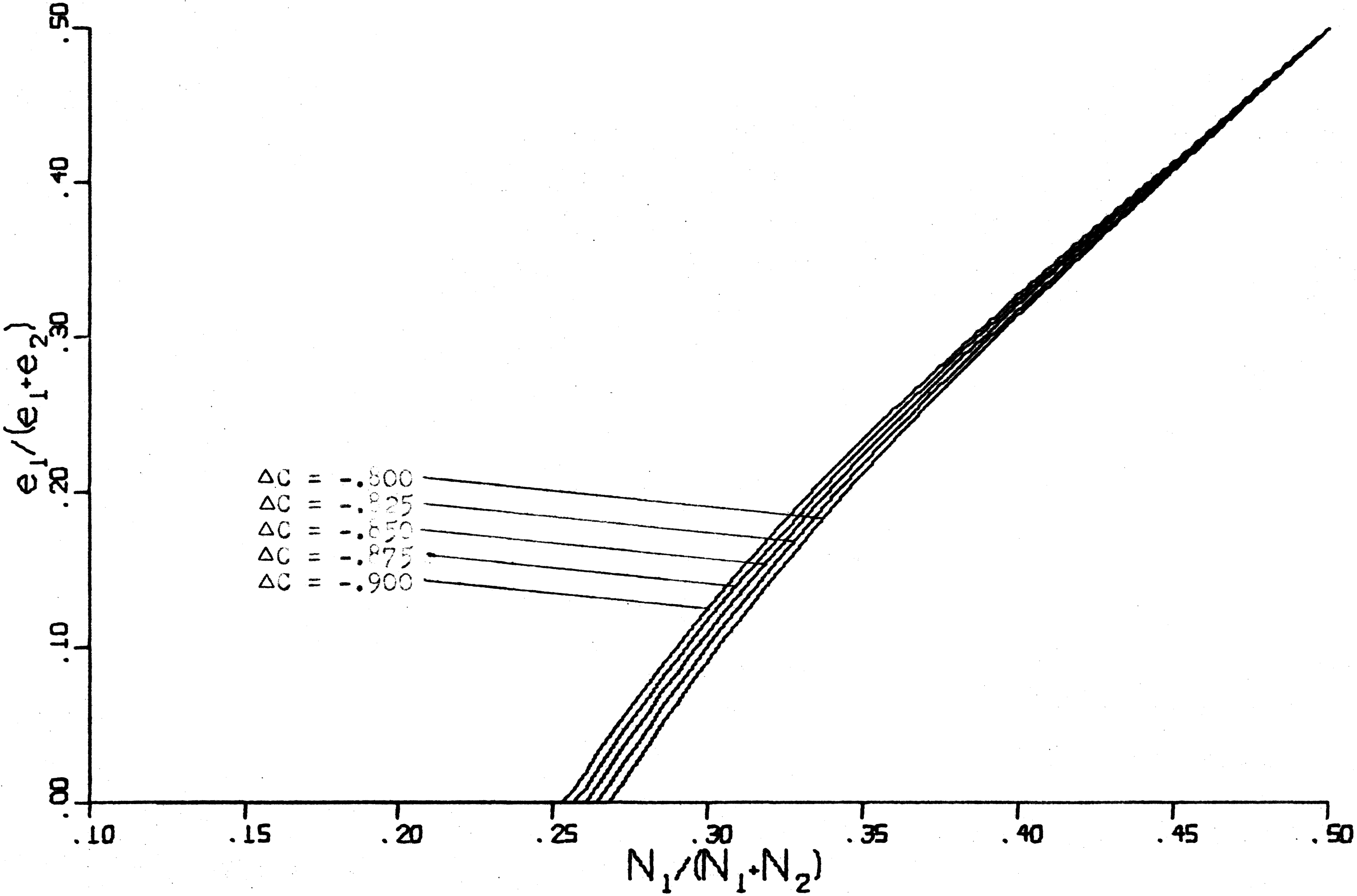

FIGURE 31 HOB OFFSET RATIO

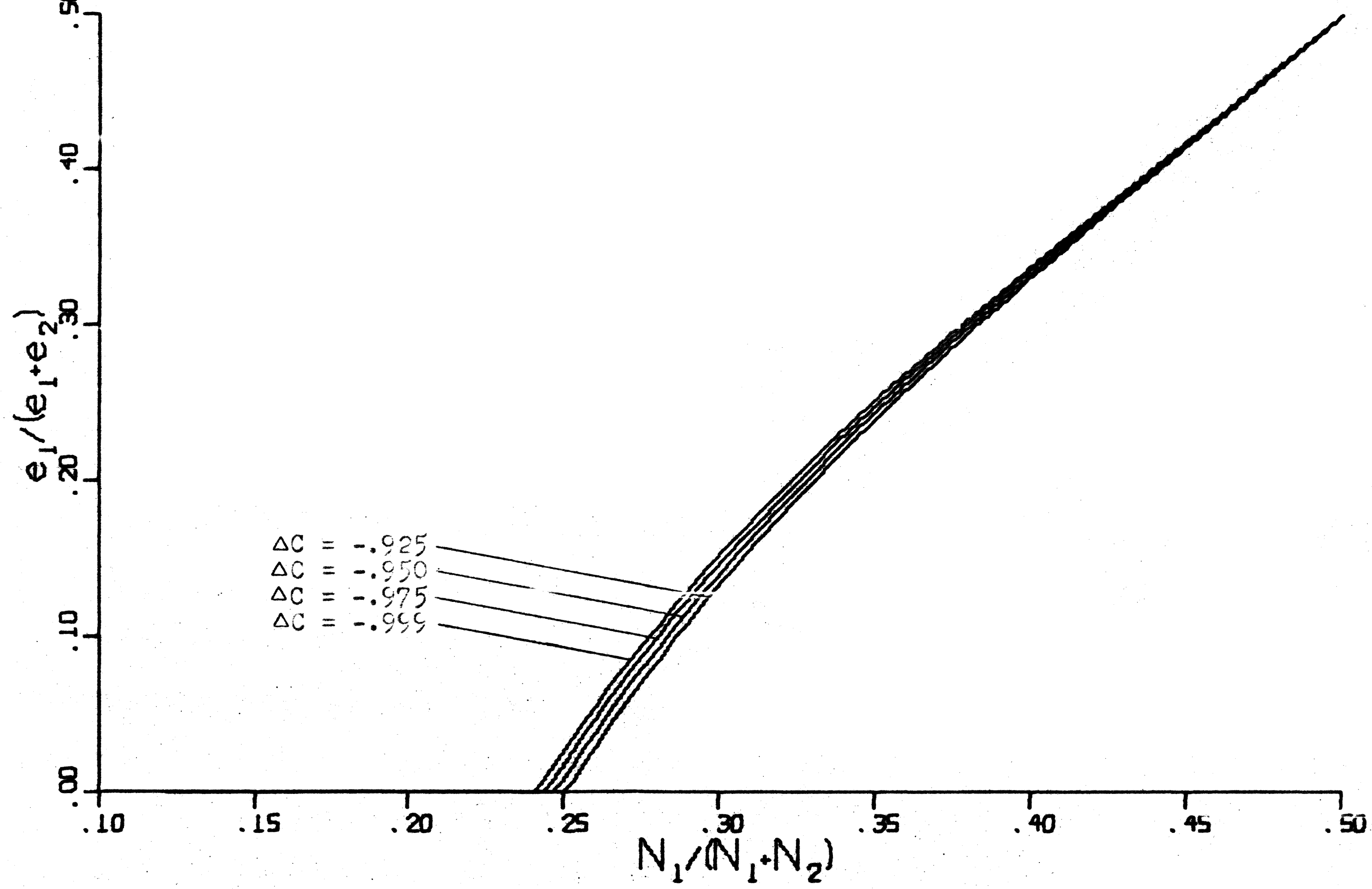

FIGURE 32 HOB OFFSET RATIO

of Figures (12) to (24) shows that the slopes are not 45° and that these curves are dependent on the value of ΔC used. In some cases it can be seen that the curve becomes a straight horizontal line with the ratio $e_1/(e_1 + e_2) = 1$ which indicates that the hob should only be pulled out on the pinion blank to increase the thickness of the teeth and therefore, to increase their strength.

A review of Figures 25 through 32 for values of $(-\Delta C)$ show that as the ratio of $N_1/(N_1 + N_2)$ decreases (increasing N_2) the value of $e_1/(e_1 + e_2)$ decreases (decreasing e_1). A review of Equation (27) will show that this is the same trend as this relationship suggests. If Equation (27) were plotted, a straight line with a slope of 45° would be obtained. Figures 25 through 32 do not show this to be the case as they are dependent on the value of $-\Delta C$. It can also be seen that the ratio $e_1/(e_1 + e_2)$ is zero over a good portion of the ratio $N_1/(N_1 + N_2)$ indicating that the hob should be pushed into the gear blank to thin the teeth and decrease their strength, thus making the pinion and gear more nearly equal on a strength basis.

As mentioned previously, an attempt was made at developing a series of equations so that it would be possible to solve explicitly for either the hob offset of the pinion or the hob offset of the gear. Tables 4 and 5 show the values of the coefficients A and B for the linear equation

$$y = A + Bx$$

TABLE 4. EQUATION COEFFICIENTS ΔC = 0.50 TO 0.850

$$\frac{e_1}{e_1 + e_2} = A + B \frac{N_2}{N_1 + N_2}$$

ΔC	A	B
0.050	USE GRAPH	
0.075		
0.100		
0.125		
0.150		
0.175		
0.200		
0.225		
0.250		
0.275		
0.300		
0.325		
0.350		
0.375		
0.400		
0.425		
0.450		
0.475		
0.500		
0.525		
0.550		
0.575		
0.600		
0.625		
0.650		
0.675		
0.700	USE GRAPH	
0.725	-.15711	1.28839
0.750	-.11453	1.20738
0.775	-.07463	1.13142
0.800	-.03784	1.06101
0.825	-.00436	0.99613
0.850	0.02751	0.93470

TABLE 5. EQUATION COEFFICIENTS ΔC = 0.875 TO 1.600

$$\frac{e_1}{e_1 + e_2} = A + B \frac{N_2}{N_1 + N_2}$$

ΔC	A	B
0.875	0.05670	0.87774
0.900	0.08442	0.82366
0.925	0.10974	0.77382
0.950	0.13304	0.72733
0.975	0.15537	0.68304
1.000	0.17579	0.64173
1.025	0.19458	0.60353
1.050	0.21235	0.56720
1.075	0.22871	0.53332
1.100	0.24486	0.50029
1.125	0.25988	0.46923
1.150	0.27403	0.43987
1.175	0.28814	0.41103
1.200	0.30260	0.38229
1.225	0.31601	0.35521
1.250	0.32921	0.32888
1.275	0.34170	0.30376
1.300	0.35399	0.27934
1.325	0.36566	0.25590
1.350	0.37738	0.23283
1.375	0.38857	0.21068
1.400	0.39953	0.18904
1.425	0.40988	0.16853
1.450	0.42003	0.14847
1.475	0.42981	0.12909
1.500	0.43945	0.11012
1.525	0.44866	0.09189
1.550	0.45765	0.07411
1.575	0.46613	0.05718
1.600	0.47518	0.03969

corresponding to the range of values of $+\Delta C$ studied. These coefficients were only obtained for positive values of ΔC. It is recommended that the plots of $e_1/(e_1 + e_2)$ vs. $N_2/(N_1 + N_2)$ be used if available rather than the above equation as the curves are not exactly linear.

A review of Tables 1, 2, and 3 shows that the contact ratio varies from a low of 1.23 to a high of 1.40. These values of contact ratio are on the low side. A contact ratio of 1.40 is generally recommended as a practical minimum with 1.20 for extreme cases. With modern manufacturing methods, however, the author feels that the degree of machining accuracy required for these low contact ratios can be easily obtained to secure quiet running.

The computer program shown in Appendix A can be easily modified so that it can be set up on a time sharing computer terminal. The program can be changed so that N_1, N_2, P and C' or ΔC can be read in and the values of e_1 and e_2 determined. A program of this type would be a considerable aid to the designer of nonstandard gears. The program can also be easily modified to handle fine pitch gears without a great deal of trouble.

C. Applications

Two example problems will be worked to demonstrate the use of the Hob Offset Ratio charts. Example 1 will cover the case in which the hob is pulled out of the gear blank and Example 2 will cover the case in which the hob is fed into the gear blank.

EXAMPLE 1

Given: A pinion and gear of 20 and 30 teeth, respectively, are to be cut by a 5-pitch, 20 degree full-depth hob to operate on a center distance of 5.25 inches with no backlash.

Required: The value of e_1 and e_2 to give teeth of the proper thickness.

Solution:

Standard center distance $C = \frac{N_1 + N_2}{2P} = \frac{20 + 30}{2 \times 5}$

$$C = 5.00 \text{ in.}$$

Operating pressure angle $\cos \emptyset' = \frac{C}{C'} \cos \emptyset = \frac{5.0}{5.25} \cos 20^\circ$

$$\emptyset' = 26.50^\circ$$

Change in center distance $\nabla C = C' - C = 5.25 - 5.0$

$$\nabla C = +0.250 \text{ in.}$$

In order to determine which chart to use, ∇C must be multiplied by the diametral pitch P since the charts are based on P = 1.

$$\Delta C = \nabla C \times P = 0.250 \times 5$$

$$\Delta C = 1.25 \text{ in.}$$

This value of $\Delta C = 1.25$ indicates that Figure 21 should be used with

$$\frac{N_2}{N_1 + N_2} = \frac{30}{20 + 30} = 0.60$$

Therefore from Figure 21

$$\frac{e_1}{e_1 + e_2} = 0.543$$

Calculating the value of $e_1 + e_2$ from Equation (24)

$$e_1 + e_2 = \frac{(N_1 + N_2)\ (\text{inv}\ \phi' - \text{inv}\ \phi)}{2P\ \tan\ \phi} \tag{24}$$

$$= \frac{(20 + 30)\ (\text{inv}\ 26.5 - \text{inv}\ 20)}{2 \times 5 \times \tan\ 20}$$

$$e_1 + e_2 = 0.29073 \text{ in.}$$

Combining these results

$$e_1 = 0.543\ (e_1 + e_2) = 0.543 \times 0.29073$$

$$e_1 = 0.15787 \text{ in.}$$

and $e_2 = (e_1 + e_2) - e_1 = 0.29073 - 0.15787$

$$e_2 = 0.13286 \text{ in.}$$

If the necessary equations are solved, the following results will be obtained:

$$R_{b1} > R_{d1}$$

$$R_{b2} > R_{d2}$$

$$X_{T1} = 2.316 \text{ in.}$$

$$X_{T2} = 3.291 \text{ in.}$$

$$X_{d1} = 1.894 \text{ in.}$$

$X_{d2} = 2.874$ in.

$d_1 = 0.422$ in.

$d_2 = 0.418$ in.

$h_1 = 0.458$ in.

$h_2 = 0.462$ in.

$\beta_1 = 34.32^\circ$

$\beta_2 = 31.71^\circ$

$$\frac{s_1 b}{F_n} = 9.959$$

$$\frac{s_2 b}{F_n} = 9.991$$

These results show that the pinion and gear are approximately equal in strength. In fact, for this particular problem the pinion is stronger than the gear.

EXAMPLE 2

Given: A pinion and gear of 35 and 44 teeth, respectively, are to be cut by a 10 diametral pitch, 20 degree full depth hob to operate on a center distance of 3.90 inches with no backlash.

Required: The value of e_1 and e_2 to give teeth of proper thickness.

Solution:

Standard center distance $C = \frac{N_1 + N_2}{2P} = \frac{35 + 44}{2 \times 10}$

$$C = 3.95 \text{ in.}$$

Operating pressure angle $\cos \emptyset' = \frac{C}{C'} \cos \emptyset = \frac{3.95}{3.90} \cos 20^\circ$

$$\emptyset' = 17.87^\circ$$

Change in center distance $\nabla C = C' - C = 3.90 - 3.95$

$$\nabla C = -0.05 \text{ in.}$$

In order to determine which chart to use, ∇C must be multiplied by the diametral pitch P since the charts are based on P = 1.

$$\Delta C = \nabla C \times P = -0.05 \times 10$$

$$\Delta C = -0.50 \text{ in.}$$

This value of $\Delta C = -0.50$ indicates that Figure 28 should be used with

$$\frac{N_1}{N_1 + N_2} = \frac{35}{44 + 35} = 0.443$$

Therefore from Figure 28

$$\frac{e_1}{e_1 + e_2} = 0.36$$

Calculating the value of $e_1 + e_2$ from Equation (24)

$$e_1 + e_2 = \frac{(N_1 + N_2)(\text{inv } \emptyset' - \text{inv } \emptyset)}{2P \tan \emptyset} \qquad (24)$$

$$= \frac{(35 + 44)\ (\text{inv } 17.87 - \text{inv } 20)}{2 \times 10 \times \tan 20}$$

$e_1 + e_2 = -0.047496$ in.

Combining these results

$e_1 = 0.36(e_1 + e_2) = 0.36 \times -0.047496$

$e_1 = -0.017098$ in.

and $e_2 = (e_1 + e_2) - e_1 = -0.047496+$

$e_2 = -0.030397$ in.

If the necessary equations are solved, the following results will be obtained:

$R_{b1} > R_{d1}$

$R_{b2} > R_{d2}$

$X_{T1} = 1.830$ in.

$X_{T2} = 2.267$ in.

$X_{d1} = 1.605$ in.

$X_{d2} = 2.042$ in.

$d_1 = 0.225$ in.

$d_2 = 0.224$ in.

$h_1 = 0.181$ in.

$h_2 = 0.186$ in.

$$\beta_1 = 24.78^\circ$$

$$\beta_2 = 23.57^\circ$$

$$\frac{s_1 b}{F_n} = 37.458$$

$$\frac{s_2 b}{F_n} = 35.530$$

These results show that the pinion and gear are approximately equal in strength. For this particular problem the stress in the pinion is 5.42 percent greater than the stress in the gear.

V. OTHER APPROACHES

The primary purpose of this chapter is to give a summary of two other attempts made during the course of this investigation at developing a method for determining the hob offset.

The first attempt involved determining a relationship relating e_1 and e_2 based on maximum contact ratio m_p. Equation (34) gives the formula for the contact ratio for nonstandard gears. A computer program was written so that a secondary relationship involving e_1 and e_2 could be determined on the basis of maximum constant ratio corresponding to a pair of gears in mesh at a given C'.

A sample set of data is shown in Table 6 for two values of N_1 and varying values of N_2 with the change in center distance $\Delta C = 0.875$. The following conclusion can be drawn by comparing the results of Table 6 to Equation (26): Equation (26) indicates that for a given N_1, e_1 should increase as N_2 increases. When the selection of e_1 is based on maximum contact ratio, e_1 decreases and becomes zero for increasing values of N_2 with a given N_1. Equation (26) suggests favoring the pinion, whereas the maximum contact ratio method suggests favoring the gear. For gears that transmit power, selecting e_1 and e_2 so as to favor the gear would be a mistake since the strength of the stronger gear tooth would be further increased over that of the pinion tooth. It was for this reason that the contact method approach was abandoned as not suitable.

TABLE 6. CONTACT RATIO

P = 1 $\phi = 20^{\circ}$ $\Delta C = 0.875$

$N_1 = 18$

N_2	C'	ϕ'	e_1	e_2	$e_1 + e_2$	m_p
18	18.875	26.35	.507	.507	1.014	1.226
20	19.875	26.06	.433	.574	1.007	1.245
30	24.875	24.95	.147	.835	.982	1.329
35	27.375	24.54	.039	.934	.973	1.363
40	29.875	24.19	0	.965	.965	1.394
130	74.875	21.76	0	.912	.912	1.603

$N_1 = 25$

N_2	C'	ϕ'	e_1	e_2	$e_1 + e_2$	m_p
25	25.875	24.78	.489	.489	.978	1.343
30	28.375	24.40	.359	.611	.970	1.376
40	33.375	23.79	.153	.803	.956	1.432
50	38.375	23.33	.009	.937	.946	1.479
55	40.875	23.14	0	.942	.942	1.499
130	78.375	21.69	0	.911	.911	1.641

The second attempt involved relating e_1 and e_2 based on equal deflection of the pinion tooth and of the gear tooth at the operating pitch point. Equation (86) gives the formula for the deflection of an approximated gear tooth (as shown in Figure 33) due to bending and shear as developed by Timoshenko and Baud (6).

$$\delta = \frac{4F_n \cos\beta}{Eb} \left\{ 3\left(\frac{L}{H_o}\right)^3 \left[\frac{1}{2}\left(3 - \frac{a}{L}\right)\left(\frac{a}{L} - 1\right) + \ln\left(\frac{1}{a/L}\right)\right] + \frac{(1+\mu)(L-a)}{H_o(1 + a/L)} \right\} \qquad (86)$$

The derivation of the necessary equations needed to solve Equation (86) are given in Appendix B. Appendix B also contains a flow chart for the computer program that was written so that a secondary relationship involving e_1 and e_2 could be determined on the basis of approximately equal deflection of the pinion tooth and of the gear tooth at the operating pitch point.

A typical set of data is shown in Tables 7, 8, and 9 which has been taken from the computer program printout for a change in center distance $\Delta C = 0.875$. At first, it appeared that the deflection method was the best method for arriving at a secondary equation so that e_1 and e_2 could be solved. After further thought and analysis of the data, however, it was decided that this method had the following disadvantages: For a positive value of ΔC, Equation (26)

TABLE 7. EQUAL DEFLECTION, N_2 = 18 TO 55

N_1 = 18 $\emptyset = 20^o$ $\mu = 0.303$ P = 1 $\Delta C = 0.875$

N_2	e_1	e_2	m_p	$\frac{SEb}{F_n}$	$\frac{e_1}{e_1 + e_2}$
18	.507	.507	1.23	1.787	.50
19	.495	.515	1.24	1.791	.49
20	.484	.523	1.24	1.795	.481
21	.473	.531	1.25	1.799	.471
22	.463	.539	1.26	1.805	.462
23	.452	.546	1.27	1.810	.453
24	.443	.553	1.28	1.815	.445
25	.434	.559	1.29	1.820	.437
26	.425	.566	1.29	1.825	.429
27	.416	.573	1.30	1.830	.421
28	.408	.578	1.31	1.835	.414
29	.401	.584	1.31	1.840	.407
30	.393	.589	1.32	1.845	.400
31	.385	.595	1.33	1.849	.393
32	.378	.601	1.33	1.854	.386
33	.371	.606	1.34	1.849	.380
34	.365	.610	1.34	1.854	.374
35	.358	.615	1.35	1.859	.368
36	.352	.620	1.36	1.872	.362
37	.346	.624	1.36	1.876	.357
38	.341	.628	1.37	1.881	.352
39	.335	.632	1.37	1.884	.346
40	.329	.636	1.38	1.889	.341
41	.324	.640	1.38	1.893	.336
42	.320	.643	1.38	1.897	.332
43	.314	.647	1.39	1.901	.327
44	.309	.651	1.39	1.904	.322
45	.305	.654	1.40	1.908	.318
46	.301	.657	1.40	1.912	.314
47	.296	.660	1.41	1.915	.310
48	.292	.663	1.41	1.918	.306
49	.288	.666	1.41	1.922	.302
50	.284	.669	1.42	1.925	.298
51	.280	.672	1.42	1.928	.294
52	.277	.674	1.43	1.931	.291
53	.273	.677	1.43	1.935	.287
54	.269	.679	1.43	1.938	.284
55	.265	.683	1.44	1.941	.280

A = 0.74593 B = -0.24468 C = -0.49436

TABLE 8. EQUAL DEFLECTION, N_2 = 56 TO 93

N_1 = 18 $\emptyset$ = 20° μ = 0.303 P = 1 ΔC = 0.875

N_2	e_1	e_2	m_p	$\frac{\delta E b}{F_n}$	$\frac{e_1}{e_1 + e_2}$
56	.262	.685	1.44	1.944	.277
57	.259	.687	1.44	1.946	.274
58	.256	.689	1.44	1.949	.271
59	.252	.692	1.45	1.952	.267
60	.250	.694	1.45	1.955	.265
61	.247	.696	1.45	1.957	.262
62	.244	.698	1.46	1.960	.259
63	.241	.700	1.46	1.962	.256
64	.238	.703	1.46	1.965	.253
65	.236	.704	1.47	1.967	.251
66	.233	.706	1.47	1.970	.248
67	.230	.709	1.47	1.972	.245
68	.228	.710	1.47	1.974	.243
69	.225	.712	1.48	1.976	.240
70	.223	.714	1.48	1.979	.238
71	.221	.715	1.48	1.981	.236
72	.218	.717	1.48	1.983	.233
73	.216	.719	1.49	1.985	.231
74	.214	.720	1.49	1.987	.229
75	.212	.721	1.49	1.989	.227
76	.201	.723	1.49	1.991	.225
77	.208	.724	1.49	1.993	.223
78	.206	.726	1.50	1.995	.221
79	.204	.727	1.50	1.997	.219
80	.202	.729	1.50	1.999	.217
81	.200	.730	1.50	2.000	.215
82	.198	.731	1.51	2.002	.213
83	.196	.733	1.51	2.004	.211
84	.194	.734	1.51	2.006	.209
85	.192	.736	1.51	2.007	.207
86	.191	.736	1.51	2.009	.206
87	.189	.738	1.51	2.011	.204
88	.187	.739	1.52	2.012	.202
89	.185	.741	1.52	2.014	.200
90	.184	.741	1.52	2.016	.199
91	.182	.743	1.52	2.017	.197
92	.181	.744	1.52	2.019	.196
93	.179	.745	1.53	2.020	.194

A = 0.74593 B = -0.24468 C = -0.49436

TABLE 9. EQUAL DEFLECTION, N_2 = 94 TO 130

N_1 = 18 $\varnothing$ = 20° μ = 0.303 F = 1 ΔC = 0.875

N_2	e_1	e_2	m_p	$\frac{\delta Eb}{F_n}$	$\frac{e_1}{e_1 + e_2}$
94	.178	.746	1.53	2.022	.193
95	.176	.747	1.53	2.023	.191
96	.175	.748	1.53	2.025	.190
97	.173	.749	1.53	2.026	.188
98	.172	.750	1.53	2.028	.187
99	.170	.751	1.53	2.029	.185
100	.169	.752	1.54	2.030	.184
101	.168	.753	1.54	2.032	.183
102	.167	.754	1.54	2.033	.181
103	.166	.755	1.54	2.034	.180
104	.164	.755	1.54	2.036	.179
105	.163	.757	1.54	2.037	.177
106	.162	.758	1.54	2.038	.176
107	.161	.758	1.55	2.039	.175
108	.160	.760	1.55	2.041	.174
109	.158	.760	1.55	2.042	.172
110	.157	.761	1.55	2.043	.171
111	.156	.762	1.55	2.044	.170
112	.155	.762	1.55	2.045	.169
113	.154	.763	1.55	2.046	.168
114	.153	.764	1.56	2.047	.167
115	.152	.764	1.56	2.049	.166
116	.150	.766	1.56	2.050	.164
117	.149	.767	1.56	2.051	.163
118	.148	.767	1.56	2.052	.162
119	.147	.768	1.56	2.053	.161
120	.146	.769	1.56	2.054	.160
121	.145	.769	1.56	2.055	.159
122	.144	.770	1.57	2.056	.158
123	.143	.771	1.57	2.057	.157
124	.142	.772	1.57	2.058	.156
125	.142	.773	1.57	2.059	.155
126	.141	.773	1.57	2.060	.154
127	.140	.774	1.57	2.061	.153
128	.139	.774	1.57	2.062	.152
129	.139	.774	1.57	2.063	.152
130	.138	.775	1.57	2.064	.151

A = 0.74593 B = -0.24468 C = -0.49436

indicates that as N_2 increases, e_1 should increase as should the ratio $e_1/(e_1 + e_2)$. An examination of the data in Tables 7, 8, and 9 shows that just the opposite is true when the relationship between e_1 and e_2 is based on equal deflection.

At first, it was felt that perhaps the guideline given by Equation (26) was in error. The reasoning behind the guideline given by Equation (26) was to favor the pinion as far as strength is concerned by increasing the thickness of the pinion tooth. This led to the question "Does equal deflection of the teeth at the operating pitch point necessarily mean that the gears are of equal strength?" To determine the answer, a simple stress analysis was made of gears hypothetically cut using the hob offsets e_1 and e_2 indicated in Tables 7, 8, and 9. The formula used for this purpose is the one used to calculate the bending stress of the tooth.

$$s = \frac{F_b}{byp} \qquad (35)$$

Kinsman (3) has developed a series of charts that give the Lewis factor as a function of the values of e_1 and e_2 with the load applied a distance $1/P$ from the end of the tooth. It was assumed that the distance $1/P$ placed the load approximately at the operating pitch point. With this assumption, and knowing the values of e_1 and e_2, the Lewis factor for the pinion and gear were found from Kinsman's charts. The stresses of the pinion and the gear were calculated for a number of hypothetical gear meshes. It was found that the stress in the pinion was 30 to 80 percent greater than that in the gear, indicating that the pinion was weaker.

This method is not considered suitable for power gearing because of the great difference in strength caused by the fact that the thickness of the gear tooth is increased instead of that of the pinion tooth. This method may, however, have considerable application in the precision gear field where power transmission is not the main concern and loads are low.

VI. CONCLUSIONS

The following conclusions concerning the method for determining the hob offset of the pinion and of the gear can be reached as a result of this investigation:

a. The equation needed so that e_1 and e_2 can be determined explicitly should be based on approximately equal bending tooth stresses of the pinion and of the gear with the load applied at the tip.

b. The separation of $e_1 + e_2$ into values of e_1 and e_2 should not be based on maximum contact ratio or on equal tooth deflections at the operating pitch point for power gearing applications.

c. Figures 12 through 34 can be used as an efficient method for obtaining the equation needed to solve for the hob offset of the pinion e_1 and the hob offset of the gear e_2.

VII. RECOMMENDATIONS

The following recommendations are made with respect to further work in determining the hob offset of the pinion and of the gear required for the gears to operate properly at the required center distance.

a. Develop a method of finding e_1 and e_2 based on equal stress for fine pitch gears that will not result in large volumes of output due to the fact that diametral pitch must be included.

b. Develop a chart or table giving the Lewis factor for nonstandard gears as a function of e_1 and e_2 with the load applied at the tip.

c. Run an investigation to determine if finding e_1 and e_2 based on equal deflections is a suitable method when the gear set is for precision gearing application.

d. Expand the plots given in Chapter IV to include the stress factor and contact ratio as a function of the velocity ratio.

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APPENDIX A

Computer Program for Determining Hob Offset Based on Equal Strength

```
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 INVOC,INVOO,K,N1,N2,L,L1,
   51 READ(5,52) DC
   52 FORMAT (D7.5)
      XYZ=0.0D0
      IF(DC.EQ.XYZ) GO TO 9
      RTD=57.2957795130824
      PI=3.14159265358979
      SX=0.0D0
      SY=0.0D0
      SXY=0.0D0
      SXSQ=0.0D0
      GEAR DATA
      N1=18.0D0
      N2=18.0D0
      EN1=19.0D0
      EN2=131.0D0
      P=1.0D0
      DELC=DC/P
      K=1.0D0
      CLEARANCE FOR COARSE PITCH GEARS
      CL=0.25D0/P
      CUTTING PRESSURE ANGLE RADIANS
      OC=20.0D0/RTD
      INVOC=DTAN(OC)-OC
      C5=INVOC
      BEAM DATA
      POI=0.303D0
      FN=1.0D0
      B=1.0D0
      CUTTING PRESSURE ANGLE DEGREES
      A=OC*RTD
      LP=P
      WRITE (6,29)
   29 FORMAT (1H1,1X//////////,6X,63HBASED ON APPROXIMATELY EQUAL BEAM T
     1OOTH STRESS AT BEAM BASE FOR)
      WRITE (6,30)
   30 FORMAT (8X,57HPINION AND GEAR WITH LOAD APPLIED AT THE TIP OF THE
     1TOOTH/////)
      WRITE (6,31) LP,A
   31 FORMAT (22X,4HP = ,I1,18X,4HO = ,F5.2//)
      WRITE (6,32) POI,DELC
   32 FORMAT (22X,6HPOI = ,F5.3,14X,4HC = ,F5.3)
    7 LN1=N1
      LK=1001
      IJ=1
  700 CONTINUE
      M=1
      WRITE (6,34) LN1
   34 FORMAT (1H1,43X,5HN1 = ,I3//)
      WRITE (6,35)
```

```
 35 FORMAT (11X,2HN2,6X,2HC',7X,2HO',7X,2HE1,8X,2HE2,7X,5HE1+E2,7X,2HM
   1P,3X,13HAVSTRESS*B/FN/)
C    STD CENTER DISTANCE
400 C=(N1+N2)/(2.0D0*P)
C    NONSTANDARD CENTER DISTANCE
     CN=C+DELC
C    OPERATING PRESSURE ANGLE RADIANS
     OO=DARCOS((C/CN)*DCOS(OC))
     INVOO=DTAN(OO)-OO
C    OPERATING PRESSURE ANGLE DEGREES
     F=OO*RTD
C    CUTTING PITCH RADIUS
     R1=N1/(2.0D0*P)
     R2=N2/(2.0D0*P)
C    STANDARD CIRCULAR PITCH
     CP=PI/P
C    OPERATING PITCH RADIUS
     R1O=(N1/(N1+N2))*CN
     R2O=(N2/(N1+N2))*CN
C    BASE CIRCLE RADIUS
     RB1=R1*DCOS(OC)
     RB2=R2*DCOS(OC)
C    BASE PITCH
     PB=2.0D0*PI*RB1/N1
C    VALUE OF E1+E2
     ET=((N1+N2)*(INVOO-INVOC))/(2.0D0*P*DTAN(OC))
     DIMENSION STSA2(1001),STSB1(1001),AMP(1001),E1(1001),E2(1001)
     DIMENSION ZZ(1001),Y(1001),X(1001),XY(1001),XSQ(1001)
     DIMENSION AX(1001),AY(1001)
     DO 10 I=1,LK
     J=I-1
     W=J/1000.0D0
     E1(I)=W*ET
     E2(I)=(1.0D0-W)*ET
C    NONSTD OUTSIDE RADIUS
     RO1N=CN-R2-E2(I)+K/P
     RO2N=CN-R1-E1(I)+K/P
C    DEPTH OF CUT
     HT=RO1N+RO2N-CN+CL
C    NONSTD DEDENDUM CIRCLE RADIUS
     RD1N=RO1N-HT
     RD2N=RO2N-HT
C    NONSTD LENGTH OF ACTION
     ZN=DSQRT((RO1N**2)-RB1**2)+DSQRT((RO2N**2)-RB2**2)-CN*DSIN(OO)
C    THICKNESS OF TOOTH ON CUTTING PITCH CIRCLE
     T1=2.0D0*E1(I)*DTAN(OC)+CP/2.0D0
     T2=2.0D0*E2(I)*DTAN(OC)+CP/2.0D0
C    POINT B ON GEAR NO 1
     PHB1=DARCOS(RB1/RO1N)
     VPHB1=DTAN(PHB1)-PHB1
     ALB1=(T1/(2.0D0*R1))+INVOC-VPHB1
```

```
      BTB1=PHB1-ALB1
      POINT A ON GEAR NO 2
      PHA2=DARCOS(RB2/RO2N)
      A2INV=DTAN(PHA2)-PHA2
      ALA2=(T2/(2.0D0*R2))+INVOC-A2INV
      BTA2=PHA2-ALA2
      COORDINATES XT AND YT OF TOOTH TIP T
      C11=DARCOS((R10/RO1N)*DCOS(OO))
      C12=DARCOS((R20/RO2N)*DCOS(OO))
      C21=DTAN(C11)-C11
      C22=DTAN(C12)-C12
      C41=RO1N
      C42=RO2N
      YT1=C41*DSIN((T1/(2.0D0*R1))+C5-C21)
      YT2=C42*DSIN((T2/(2.0D0*R2))+C5-C22)
      XT1=C41*DCOS((T1/(2.0D0*R1))+C5-C21)
      XT2=C42*DCOS((T2/(2.0D0*R2))+C5-C22)
      IF(RB1.GT.RD1N) GO TO 699
      IF(RB1.LE.RD1N) GO TO 701
      FOR BASE RADIUS GREATER THAN DEDENDUM RADIUS
  699 THETA1=2.0D0*((T1/(2.0D0*R1))+INVOC)
      H1=2.0D0*RD1N*DSIN(THETA1/2.0D0)
      XD1=RD1N*DCOS(THETA1/2.0D0)
      D1=XT1-XD1
      GO TO 702
      FOR BASE RADIUS LESS THAN OR EQUAL TO DEDENDUM RADIUS
  701 OD1=DARCOS((R1/RD1N)*DCOS(OC))
      ODINV1=DTAN(OD1)-OD1
      THETA1=2.0D0*((T1/(2.0D0*R1))+INVOC-ODINV1)
      H1=2.0D0*RD1N*DSIN(THETA1/2.0D0)
      XD1=RD1N*DCOS(THETA1/2.0D0)
      D1=XT1-XD1
  702 CONTINUE
      IF(RB2.GT.RD2N) GO TO 703
      IF(RB2.LE.RD2N) GO TO 704
      FOR BASE RADIUS GREATER THAN DEDENDUM RADIUS
  703 THETA2=2.0D0*((T2/(2.0D0*R2))+INVOC)
      H2=2.0D0*RD2N*DSIN(THETA2/2.0D0)
      XD2=RD2N*DCOS(THETA2/2.0D0)
      D2=XT2-XD2
      GO TO 705
      FOR BASE RADIUS LESS THAN OR EQUAL TO DEDENDUM RADIUS
  704 OD2=DARCOS((R2/RD2N)*DCOS(OC))
      ODINV2=DTAN(OD2)-OD2
      THETA2=2.0D0*((T2/(2.0D0*R2))+INVOC-ODINV2)
      H2=2.0D0*RD2N*DSIN(THETA2/2.0D0)
      XD2=RD2N*DCOS(THETA2/2.0D0)
      D2=XT2-XD2
  705 CONTINUE
      STRESS AT BEAM TOOTH BASE WITH LOAD AT TOOTH TIP
      STSB1(I)=(6.0D0*FN*DCOS(BTB1)*D1)/(B*H1**2)
```

```
      STSA2(I)=(6.0D0*FN*DCOS(BTA2)*D2)/(B*H2**2)
      ZZ(I)=DABS(STSA2(I)-STSB1(I))
      CONTACT RATIO
      AMP(I)=ZN/PB
   10 CONTINUE
      ZX=ZZ(1)
      DO 99 I=1,LK
      IF(ZX-ZZ(I)) 99,97,97
   97 II=I
      ZX=ZZ(I)
   99 CONTINUE
      AV=(STSA2(II)+STSB1(II))/2.0D0

      LN2=N2
      KK=II-1
      AX(IJ)=N2/(N1+N2)
      AY(IJ)=KK/1000.0D0
      WRITE (6,36) LN2,CN,F,E1(II),E2(II),ET,AMP(II),AV,AY(IJ)
   36 FORMAT (10X,I3,3X,F7.3,3X,F6.3,3X,F7.5,3X,F7.5,3X,F7.5,3X,F7.5,3X,
     1F8.5,10X,F6.4)
      IF(IJ.NE.5) GO TO 500
      PQR=0.50D0
      IF(AY(5).LT.PQR) GO TO 9
  500 N2=N2+1.0D0
      IF(N2.EQ.EN2) GO TO 8
      IJ=IJ+1
      M=M+1
      LK=KK+15
      IF(LK.GT.1001) GO TO 313
      GO TO 314
  313 LK=1001
  314 CONTINUE
      IF(M.EQ.51) GO TO 700
      GO TO 400
    8 CONTINUE
      IJ=1
      DO 15 I=1,113
      X(IJ)=AX(I)
      Y(IJ)=AY(I)
      XY(IJ)=Y(IJ)*X(IJ)
      XSQ(IJ)=X(IJ)**2
      SX=SX+X(IJ)
      SY=SY+Y(IJ)
      SXY=SXY+XY(IJ)
      SXSQ=SXSQ+XSQ(IJ)
      IJ=IJ+1
   15 CONTINUE
      AN=113.0D0
      BM=(SXY-(SX*SY)/AN)/(SXSQ-(SX**2)/AN)
      YBAR=SY/AN
      XBAR=SX/AN
```

```
      AI=YBAR-BM*XBAR
      WRITE(6,54) AI,BM,DELC,AI,BM
   54 FORMAT (1H1,10X,4HA = ,F9.5//,11X,4HB = ,F9.5//,11X,5H C = ,F9.5//
     1,11X,27HE1/(E1+E2)=A+B*(N2/(N1+N2))//,11X,11HE1/(E1+E2)=,F9.5,1H+,
     2F9.5,11H*N2/(N1+N2))
      WRITE(7,875) AI,BM,DELC
  875 FORMAT (3F8.5)
      WRITE(7,876) (AX(IJ),IJ=1,113)
  876 FORMAT (10F7.4)
      WRITE(7,877) (AY(IJ),IJ=1,113)
  877 FORMAT (10F6.3)
      N1=N1+1.0D0
      IF(N1.EQ.EN1) GO TO 51
      N2=N1
      GO TO 7
    9 STOP
      END
```

APPENDIX B

Hob Offset Based on Equal Deflection

In applying the results of Furrow's (8) investigation to nonstandard gears, it was decided to use Furrow's method of approximating the gear tooth by constructing a plane surface through the tooth tip and the operating pitch point as shown in Figure 33. The dimensions of the beam can then be derived in terms of known or assumed gear parameters. The thickness on the cutting pitch circle is given by

$$t = 2e \tan \phi + p/2 \qquad (19)$$

and the thickness on the operating pitch circle is

$$t' = 2R'\left(\frac{t}{2R} + \text{inv } \phi - \text{inv } \phi'\right)$$

which is equal to the arc length S_p.

The arc angle α_p is

$$\alpha_p = \frac{S_p}{R'} = \frac{t'}{R'}$$

Upon substitution of t'

$$\alpha_p = 2\left(\frac{t}{2R} + \text{inv } \phi - \text{inv } \phi'\right)$$

The cord length C_p is

$$C_p = 2R' \sin (\alpha_p/2)$$

or

$$C_p = 2R' \sin\left(\frac{t}{2R} + \text{inv } \phi - \text{inv } \phi'\right)$$

The y coordinate of the operating pitch point is then

$$Y_p = C_p/2$$

or

$$Y_p = R' \sin\left(\frac{t}{2R} + \text{inv } \phi - \text{inv } \phi'\right) \qquad (64)$$

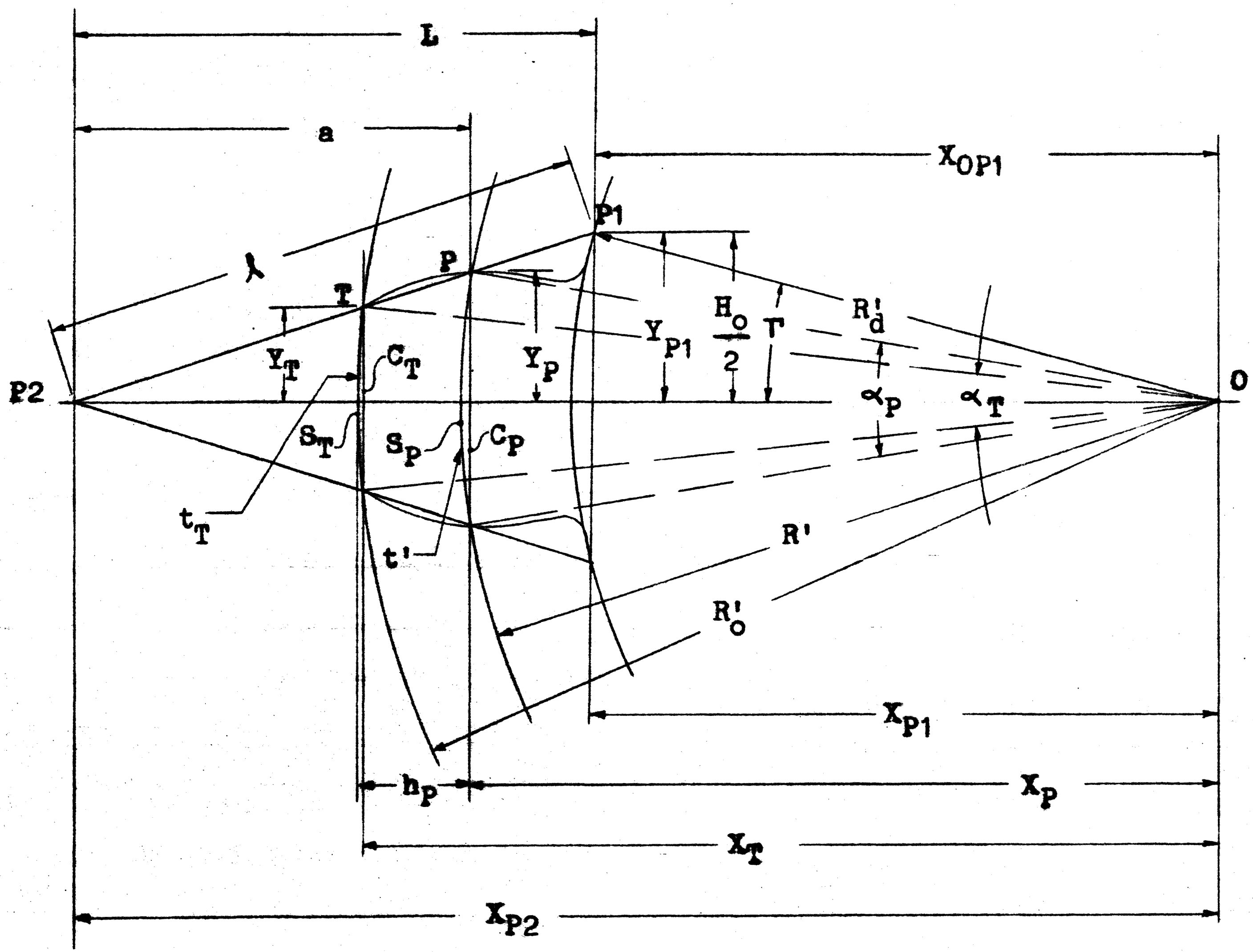

FIGURE 33 TAPER BEAM MODEL

The x coordinate of the operating pitch point is

$$X_P = R' \cos(\alpha_P/2)$$

or $$X_P = R' \cos\left(\frac{t}{2R} + \text{inv}\,\phi - \text{inv}\,\phi'\right) \quad (65)$$

where t is given by Equation (19).

The tooth thickness at the tip of the tooth is

$$t_T = 2R_o'\left(\frac{t'}{2R'} + \text{inv}\,\phi' - \text{inv}\,\phi_T\right) \quad (48)$$

which is equal to the arc length S_T. The base circle radius is given by

$$R_b = R' \cos\phi' = R_o' \cos\phi_T$$

Therefore, $$\phi_T = \cos^{-1}\left(\frac{R'}{R_o'} \cos\phi'\right) \quad (49)$$

Let $$C1 = \phi_T \quad (50)$$

$$C2 = \text{inv}\,\phi_T = \tan C1 - C1 \quad (51)$$

By Equation (17)

$$t' = 2R'\left(\frac{t}{2R} + \text{inv}\,\phi - \text{inv}\,\phi'\right) \quad (52)$$

Substituting Equations (51 and 52) into Equation (48)

$$t_T = 2R_o'\left(\frac{t}{2R} + \text{inv}\,\phi - C2\right)$$

Then the arc angle α_T is

$$\alpha_T = \frac{S_T}{R_o'} = \frac{t_T}{R_o'}$$

which on substitution of t_T is

$$\alpha_T = 2\left(\frac{t}{2R} + \text{inv}\,\phi - C2\right)$$

The cord length at the tooth tip is given by

$$C_T = 2R_o' \sin (\alpha_T/2)$$

or $$C_T = 2R_o' \sin (\frac{t}{2R} + \text{inv}\ \phi - C2)$$

Defining $C4 = R_o'$ (66)

and $C5 = \text{inv}\ \phi$ (67)

C_T can be expressed as

$$C_T = 2C4 \sin (\frac{t}{2R} + C5 - C2)$$

The y coordinate of the tooth tip is then

$$Y_T = \tfrac{1}{2}\ C_T$$

or $$Y_T = C4 \sin (\frac{t}{2R} + C5 - C2) \qquad (68)$$

The x coordinate for the line ℓ can now be developed. From analytic geometry

$$\frac{Y - Y_T}{X_T - X} = \frac{Y_P - Y_T}{X_T - X_P} \qquad (69)$$

Defining the slope of line ℓ as m;

$$m = \frac{Y_P - Y_T}{X_T - X_P}$$

which on substitution into Equation (69) gives

$$Y = -mX + Y_T + mX_T$$

Let $b = Y_T + mX_T$

Therefore, $Y = -mX + b$ (70)

Letting $X = X_{P2}$ and $Y = Y_{P2}$ Equation (70) becomes

$$Y_{P2} = -mX_{P2} + b$$

But $$Y_{P2} = 0 \tag{71}$$

and $$X_{P2} = \frac{b}{m} \tag{72}$$

Letting $X = X_{P1}$ and $Y = Y_{P1}$ Equation (70) then becomes

$$Y_{P1} = -mX_{P1} + b \tag{a}$$

From Figure 33

$$X_{P1}^2 + Y_{P1}^2 = R_d'^2 \tag{b}$$

Squaring Equation (a) and substituting into Equation (b) and solving for X_{P1}, the following results:

$$X_{P1} = \frac{2mb + \sqrt{(-2bm)^2 - 4(1 + m^2)\ (b^2 - R_d'^2)}}{2(1 + m^2)} \tag{73}$$

The y coordinate of point P1 is now given by Equation (a)

$$Y_{P1} = -mX_{P1} + b \tag{74}$$

Knowing the x coordinates of P2 and P1, the beam length can be defined as

$$L = X_{P2} - X_{P1} \tag{75}$$

From Figure 33 it is seen that

$$Y_{P1} = \tfrac{1}{2} H_o$$

or $$H_o = 2Y_{P1} \tag{76}$$

The angle $\top$ is defined as

$$\top = \sin^{-1}\left(\frac{H_o}{2R'_d}\right) \tag{77}$$

Therefore, the x distance from the origin O to the point P1 is

$$X_{OP1} = R'_d \cos(\top) \tag{78}$$

X_{OP1} is equal to X_{P1} and is shown here because it was used in the computer program.

Figure 34 shows two nonstandard gears in mesh at the instant when the teeth are in contact with each other at the operating pitch point.

The angle between O_1P and the pinion tooth centerline is

$$\gamma_{P1} = \frac{t'_1/2}{R'_1}$$

where $t'_1 = 2R'_1\left(\frac{t_1}{2R_1} + \text{inv}\,\phi - \text{inv}\,\phi'\right)$

and $t_1 = 2e_1 \tan\phi + p/2$

$$\text{Therefore } \gamma_{P1} = \frac{t_1}{2R_1} + \text{inv}\,\phi - \text{inv}\,\phi' \tag{79}$$

The distance along the pinion tooth centerline of the tooth to a point on the centerline corresponding to the operating pitch point is (see distance h_p, Figure 33)

$$h_{P1} = R'_{o1} - R'_1 \cos(\gamma_{P1}) \tag{80}$$

The angle between the normal and tangential load for the pinion is

$$\beta_{P1} = \phi' - \gamma_{P1} \tag{81}$$

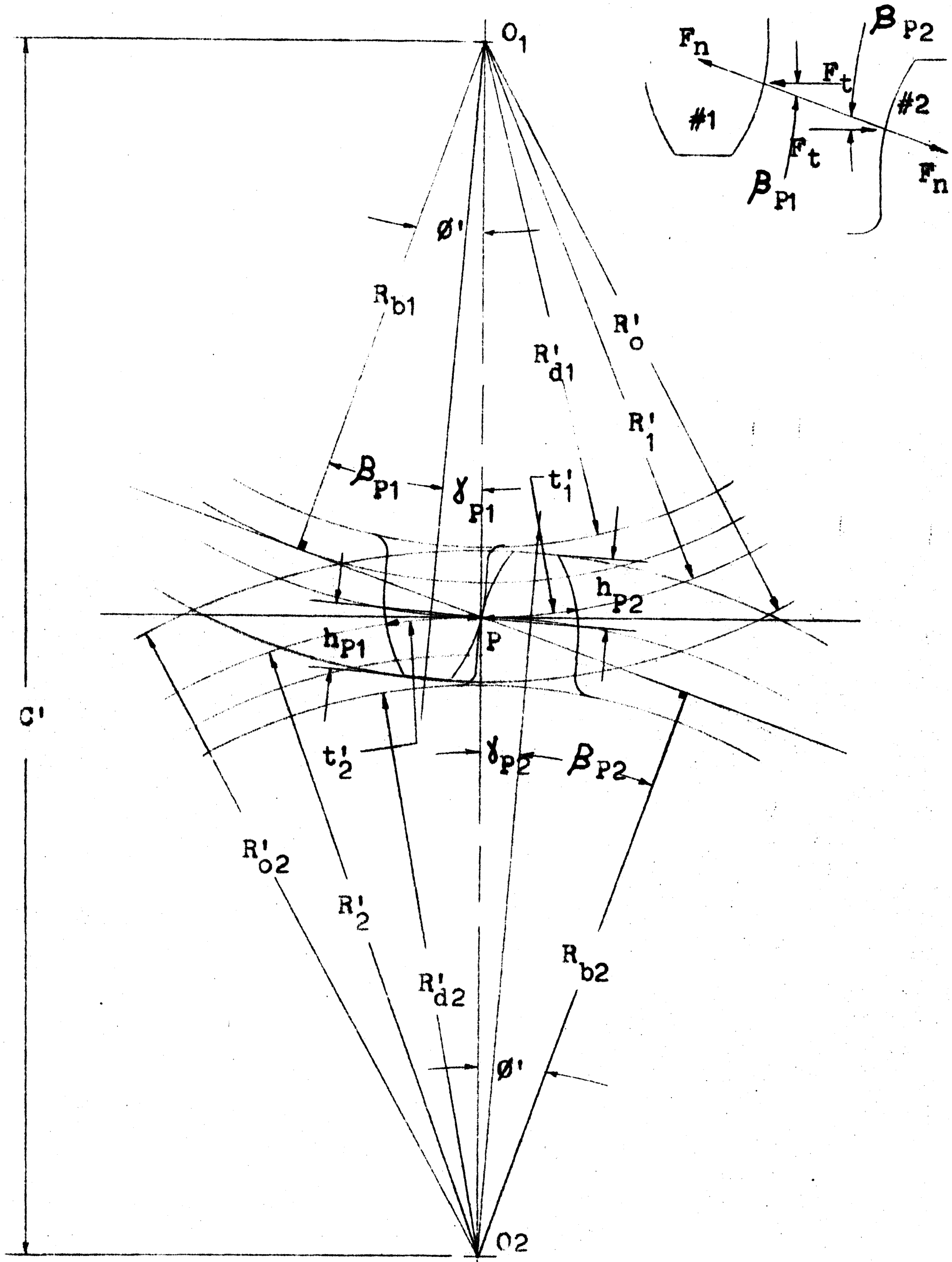

FIGURE 34

MESH AT OPERATING PITCH POINT

The angle between O_2P and the tooth centerline of the gear is

$$\gamma_{P2} = \frac{t_2'/2}{R_2'}$$

where $t_2' = 2R_2'\left(\frac{t_2}{2R_2} + \text{inv}\,\phi - \text{inv}\,\phi'\right)$

and $t_2 = 2e_2 \tan\phi + p/2$

Therefore $\gamma_{P2} = \frac{t_2}{2R_2} + \text{inv}\,\phi - \text{inv}\,\phi'$ (82)

The distance along the gear tooth centerline of the tooth to a point on the centerline corresponding to the operating pitch point is (see distance h_p, Figure 33)

$$h_{P2} = R_{o2}' - R_2' \cos(\gamma_{P2}) \qquad (83)$$

The angle between the normal and tangential load is

$$\beta_{P2} = \phi' - \gamma_{P2} \qquad (84)$$

The distance "a" shown in Figure 33 can now be defined as

$$a = L + X_{OP1} - R_o' + h_P \qquad (85)$$

where h_p is given by either Equation (80) or Equation (83).

Timoshenko and Baud's (6) equation for the deflection of an approximated gear tooth (as shown in Figure 33) due to bending and shear can be shown to be

$$\delta = \frac{4F_n \cos\beta}{Eb}\left\{3\left(\frac{L}{H_o}\right)^3\left[\frac{1}{2}\left(3 - \frac{a}{L}\right)\left(\frac{a}{L} - 1\right) + \ln\left(\frac{1}{a/L}\right)\right] + \frac{(1+\mu)(L-a)}{H_o(1 + a/L)}\right\} \qquad (86)$$

where b is the face width and μ is Poisson's ratio.

A computer program was written which would determine the value of e_1 and e_2 to give equal or nearly equal tooth deflections measured at the operating pitch circle. The program was written for coarse pitch gears with a cutting pressure angle ϕ of 20 degrees. A diametral pitch P of 1 was assumed since the deflection is independent of the pitch. Poisson's ratio was taken as 0.303.

Figure 35 shows a flow chart giving the primary logic of the program. The program is also listed.

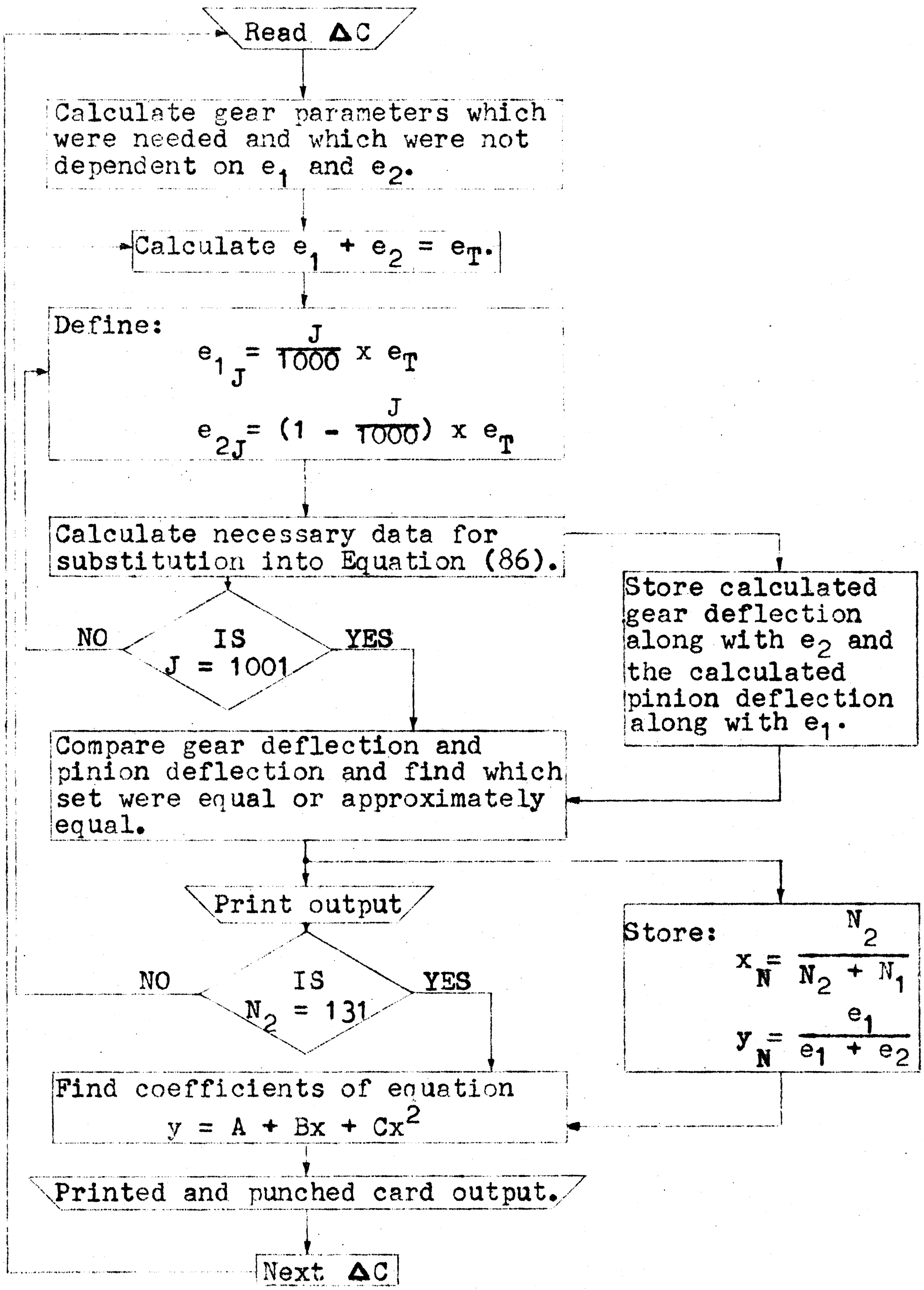

FIGURE 35. FLOW DIAGRAM

```
      IMPLICIT REAL*8(A-H,O-Z)
      REAL INVOC,K,N1,N2,INVOO,L,L1,L2
      EQUATION FOR BENDING AND SHEAR DEFLECTION
      FNCT(BTPT,HO,L,APT)=((4.0D0*FN*DCOS(BTPT))/(E*B))*((3.0D0*((L/HO)*
     1*3))*(0.50D0*(3.0D0-APT/L)*((APT/L)-1.0D0)+DLOG(1.0D0/(APT/L)))+((
     2(1.0D0+POI)*(L-APT))/(HO*(1.0D0+APT/L))))
   51 READ(5,52) DC
   52 FORMAT (D7.5)
      XYZ=0.0D0
      IF(DC.EQ.XYZ) GO TO 9
      RTD=57.2957795130824
      PI=3.14159265358979
      SX=0.0D0
      SY=0.0D0
      SXY=0.0D0
      SXSQ=0.0D0
      GEAR DATA
      N1=18.0D0
      N2=18.0D0
      EN1=19.0D0
      EN2=131.0D0
      P=1.0D0
      DELC=DC/P
      K=1.0D0
      CLEARANCE FOR COARSE PITCH GEARS
      CL=0.250D0/P
      CUTTING PRESSURE ANGLE RADIANS
      OC=20.0D0/RTD
      INVOC=DTAN(OC)-OC
      C5=INVOC
      BEAM DATA
      POI=0.303D0
      FN=1.0D0
      B=1.0D0
      E=1.0D0
      CUTTING PRESSURE ANGLE DEGREES
      A=OC*RTD
      LP=P
      WRITE (6,29)
   29 FORMAT (1H1,1X//////////,11X,53HBASED ON APPROXIMATELY EQUAL DEFLE
     1CTION OF THE PINION)
      WRITE (6,30)
   30 FORMAT (22X,31HAND THE GEAR AT THE PITCH POINT/////)
      WRITE (6,31) LP,A
   31 FORMAT (22X,4HP = ,I1,18X,4HO = ,F5.2//)
      WRITE (6,32) POI,DELC
   32 FORMAT (22X,6HPOI = ,F5.3,14X,4HC = ,F5.3)
    7 LN1=N1
      LK=1001
      IJ=1
  700 CONTINUE
```

```
      M=1
      WRITE (6,34) LN1
   34 FORMAT (1H1,43X,5HN1 = ,I3//)
      WRITE (6,35)
   35 FORMAT (11X,2HN2,6X,2HC',7X,2HO',7X,2HE1,8X,2HE2,7X,5HE1+E2,7X,2HM
     1P,3X,13HE*B*AVDBSP/FN/)
      STD CENTER DISTANCE
  400 C=(N1+N2)/(2.0D0*P)
      NONSTANDARD CENTER DISTANCE
      CN=C+DELC
      OPERATING PRESSURE ANGLE RADIANS
      OO=DARCOS((C/CN)*DCOS(OC))
      INVOO=DTAN(OO)-OO
      OPERATING PRESSURE ANGLE DEGREES
      F=OO*RTD
      CUTTING PITCH RADIUS
      R1=N1/(2.0D0*P)
      R2=N2/(2.0D0*P)
      STANDARD CIRCULAR PITCH
      CP=PI/P
      OPERATING PITCH RADIUS
      R1O=(N1/(N1+N2))*CN
      R2O=(N2/(N1+N2))*CN
      BASE CIRCLE RADIUS
      RB1=R1*DCOS(OC)
      RB2=R2*DCOS(OC)
      BASE PITCH
      PB=2.0D0*PI*RB1/N1
      VALUE OF E1+E2
      ET=((N1+N2)*(INVOO-INVOC))/(2.0D0*P*DTAN(OC))
      DIMENSION DBSP1(1001),DBSP2(1001),AMP(1001),E1(1001),E2(1001)
      DIMENSION ZZ(1001),Y(1001),X(1001),XY(1001),XSQ(1001)
      DIMENSION AX(1001),AY(1001)
      DO 10 I=1,LK
      J=I-1
      W=J/1000.0D0
      E1(I)=W*ET
      E2(I)=(1.0D0-W)*ET
      NONSTD OUTSIDE RADIUS
      RO1N=CN-R2-E2(I)+K/P
      RO2N=CN-R1-E1(I)+K/P
      DEPTH OF CUT
      HT=RO1N+RO2N-CN+CL
      NONSTD DEDENDUM CIRCLE RADIUS
      RD1N=RO1N-HT
      RD2N=RO2N-HT
      NONSTD LENGTH OF ACTION
      ZN=DSQRT((RO1N**2)-RB1**2)+DSQRT((RO2N**2)-RB2**2)-CN*DSIN(OO)
      PHB1=DARCOS(RB1/RO1N)
      DEL1=90.0D0/RTD-PHB1
      PHA2=DARCOS(RB2/RO2N)
```

OMIT !

```
THICKNESS OF TOOTH ON CUTTING PITCH CIRCLE
 T1=2.0D0*E1(I)*DTAN(OC)+CP/2.0D0
 T2=2.0D0*E2(I)*DTAN(OC)+CP/2.0D0
 POINT P ON GEAR NO 1
 ALP1=T1/(2.0D0*R1)+INVOC-INVOO
 HP1=R01N-R10*DCOS(ALP1)
 BTP1=OO-ALP1
 POINT P ON GEAR NO 2
ALP2=T2/(2.0D0*R2)+INVOC-INVOO
HP2=R02N-R20*DCOS(ALP2)
BTP2=OO-ALP2
COORDINATES XP AND YP OF PITCH POINT P
YP1=R10*DSIN(T1/(2.0D0*R1)+INVOC-INVOO)
YP2=R20*DSIN(T2/(2.0D0*R2)+INVOC-INVOO)
XP1=R10*DCOS(T1/(2.0D0*R1)+INVOC-INVOO)
XP2=R20*DCOS(T2/(2.0D0*R2)+INVOC-INVOO)
COORDINATES XT AND YT OF TOOTH TIP T
C11=DARCOS((R10/R01N)*DCOS(OO))
C12=DARCOS((R20/R02N)*DCOS(OO))
C21=DTAN(C11)-C11
C22=DTAN(C12)-C12
C41=R01N
C42=R02N
YT1=C41*DSIN(T1/(2.0D0*R1)+C5-C21)
YT2=C42*DSIN(T2/(2.0D0*R2)+C5-C22)
XT1=C41*DCOS(T1/(2.0D0*R1)+C5-C21)
XT2=C42*DCOS(T2/(2.0D0*R2)+C5-C22)
COORDINATES XP2 AND YP2 OF POINT P2
SLP1=(YP1-YT1)/(XP1-XT1)
SLP2=(YP2-YT2)/(XP2-XT2)
B1=YT1-SLP1*XT1
B2=YT2-SLP2*XT2
XP21=-B1/SLP1
XP22=-B2/SLP2
COORDINATES XP1 AND YP1 OF POINT P1
XP11=((-2.0D0*SLP1*B1)+DSQRT(((2.0D0*SLP1*B1)**2)-4.0D0*(1.0D0+SLP
11**2)*((B1**2)-RD1N**2))))/(2.0D0*(1.0D0+SLP1**2))
XP12=((-2.0D0*SLP2*B2)+DSQRT(((2.0D0*SLP2*B2)**2)-4.0D0*(1.0D0+SLP
12**2)*((B2**2)-RD2N**2))))/(2.0D0*(1.0D0+SLP2**2))
YP11=SLP1*XP11+B1
YP12=SLP2*XP12+B2
TAPER BEAM LENGTH LENGTH L
L1=XP21-XP11
L2=XP22-XP12
VALUE OF HO
HO1=2.0D0*YP11
HO2=2.0D0*YP12
ANGLE TAU
TAU1=DARSIN(HO1/(2.0D0*RD1N))
TAU2=DARSIN(HO2/(2.0D0*RD2N))
DISTANCE FROM GEAR CENTER TO POINT P1
```

```
      XOP11=RD1N*DCOS(TAU1)
      XOP12=RD2N*DCOS(TAU2)
C     DISTANCE 'A' TO POINT P AND B MEASURED FROM APEX
      AP1=L1+XOP11-R01N+HP1
      AP2=L2+XOP12-R02N+HP2
C     BENDING AND SHEAR DEFLECTION
      DBSP1(I)=FNCT(BTP1,H01,L1,AP1)
      DBSP2(I)=FNCT(BTP2,H02,L2,AP2)
      ZZ(I)=DABS(DBSP1(I)-DBSP2(I))
C     CONTACT RATIO
      AMP(I)=ZN/PB
   10 CONTINUE
      ZX=ZZ(1)
      DO 99 I=1,LK
      IF(ZX-ZZ(I)) 99,97,97
   97 II=I
      ZX=ZZ(I)
   99 CONTINUE
      AV=(DBSP1(II)+DBSP2(II))/2.0D0
      LN2=N2
      KK=II-1
      AX(IJ)=N2/(N1+N2)
      AY(IJ)=KK/1000.0D0
      WRITE (6,36) LN2,CN,F,E1(II),E2(II),ET,AMP(II),AV,AY(IJ)
   36 FORMAT (10X,I3,3X,F7.3,3X,F6.3,3X,F7.5,3X,F7.5,3X,F7.5,3X,F7.5,3X,
     1F8.5,10X,F6.4)
      N2=N2+1.0D0
      IF(N2.EQ.EN2) GO TO 8
      IJ=IJ+1
      M=M+1
      LK=KK+15
      IF(M.EQ.51) GO TO 700
      GO TO 400
    8 CONTINUE
      IJ=1
      DO 15 I=2,113
      X(IJ)=AX(I)
      Y(IJ)=(AY(I)-AY(1))/(AX(I)-AX(1))
      XY(IJ)=Y(IJ)*X(IJ)
      XSQ(IJ)=X(IJ)**2
      SX=SX+X(IJ)
      SY=SY+Y(IJ)
      SXY=SXY+XY(IJ)
      SXSQ=SXSQ+XSQ(IJ)
      IJ=IJ+1
   15 CONTINUE
      AN=112.0D0
      C=(SXY-(SX*SY)/AN)/(SXSQ-(SX**2)/AN)
      YBAR=SY/AN
      XBAR=SX/AN
      WA=YBAR-C*XBAR
```

```
      G=WA-C*AX(1)
      H=AY(1)-G*AX(1)-(C*AX(1)**2)
      WRITE(6,54) H,G,C,H,G,C
   54 FORMAT (1H1,10X,4HA = ,F9.5//,11X,4HB = ,F9.5//,11X,4HC = ,F9.5//,
     111X,43HE1/(E1+E2)=A+B*N2/(N1+N2)+C*(N2/(N1+N2)**2)//,11X,11HE1/(E1
     2+E2)=,F8.5,1X,F8.5,11H*N2/(N1+N2),F8.5,15H*(N2/(N1+N2)**2)
      WRITE(7,875) H,G,C,DELC
  875 FORMAT (4F8.5)
      WRITE(7,876) (AX(IJ),IJ=1,113)
  876 FORMAT (10F7.4)
      WRITE(7,877) (AY(IJ),IJ=1,113)
  877 FORMAT (10F6.3)
      N1=N1+1.0D0
      IF(N1.EQ.EN1) GO TO 51
      N2=N1
      GO TO 7
    9 STOP
      END
```

A SIMPLIFIED METHOD FOR DETERMINING HOB OFFSET VALUES IN THE DESIGN OF NONSTANDARD SPUR GEARS

(Edward James Walsh II)

Abstract

A typical problem which a mechanical designer might encounter concerns the design of a set of spur gears for a predetermined center distance which is not considered standard. The design of these gears can be accomplished by determining the value that a standard cutter can be moved in or withdrawn from its normal cutting position such that when the pinion and gear are cut, they will operate at the required center distance when meshed together. The design problem, however, is trying to establish what values of cutter offset should be used to cut the pinion and the gear so that the standard cutter may be used.

Several authors have shown the derivation for a mathematically correct relationship which will give the <u>sum</u> of the hob offsets for the pinion and the gear. The major problem is that there is no second mathematical expression whereby it is possible to solve explicitly for either the hob offset of the pinion or the hob offset of the gear.

Accordingly, the object of this investigation was to develop a method for determining the hob offset of the pinion and of the gear so that the gears would operate properly at the required predetermined center distance.

The following result can be reached from this investigation: The equation needed so that the hob offsets, e_1 and e_2, can be determined explicitly should be based on approximately equal bending tooth stresses of the pinion and of the gear with the load applied at the tip of the tooth.