THE TERM STRUCTURE OF ZERO-COUPON AND COUPON BONDS: A COMPARATIVE ANALYSIS

by

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ABSTRACT

The behavior of the term structure of zero-coupon bonds parallels the behavior of coupon bonds. Previous studies have demonstrated that there are synchronous changes among different maturities of coupon bonds. Results of statistical and sign tests are used to measure the degree of synchronicity, the relative amplitude of change and the amount of sensitivity among different paired maturities for zero-coupon bonds. A comparison is then made to the results of coupon bonds to verify that the behavior of the term structure of zero-coupon bonds are consistent with observations in previous studies, including a study of high-grade corporate bonds from 1901-1954 by Dr. David I. Meiselman. Generally, all maturities move in the same direction with short term rates more synchronous than longer term rates. Also, there is a general tendency for relative volatility to vary inversely with maturity, therefore, short term maturities have a greater degree of volatility than do long term maturities. The results are generally consistent with Meiselman's earlier findings.

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1. INTRODUCTION

1.1 Thesis Objectives

The purpose of this study is to evaluate the term structure of interest rates for zero-coupon bonds, comparing findings to previous studies conducted on corporate and government coupon bonds. Coupon bonds, as a whole, impose an implicit problem when one is trying to analyze their yield curves because the stated maturity of a coupon bond is not representative of the true weighted maturity, or duration, of the bond. A result of this study will be an ability to quantify the amount of systematic difference in estimation of the yield curves for coupon bonds this coupon/duration effect has. Chapter 1 of this thesis provides the reader with an introduction to the relevance of term structure, highlights previous studies of term structure of interest rates for various types of coupon bonds, and provides definitions of key terms relevant to the study of term structure. Chapter 2 describes the data used in this study and the methods of testing. Chapter 3 summarizes the results obtained from the tests outlined in Chapter 2; and Chapter 4 describes the implications of the results in comparison to previous studies' results.

1.2 Background

The term structure of interest rates plays an integral part in understanding and estimating interest rate movements within financial markets. "The term structure of risk-free interest rates is fundamental to every aspect of the theory and practice of investment and corporate finance".[1] Given the term structure, any certain future cash flow pattern can be represented as a present value. Policymakers, money managers, and individual investors often look to the term structure of interest rates for clues about the market's expectation regarding future interest rates. If individuals have a different forecast of market outlook this may cause them to speculate against the market through the trading of bonds. This trading moves bond prices and thus leads to a melting of private information with public information embodied in the term structure.[2]

In The Term Structure of Interest Rates (1962), David Meiselman analyzed yield curves from 1901-1954 for annual high-grade corporate bond data. Meiselman found that the movements of interest rates were systematically synchronous for all maturities and that the amplitude of change was inversely related to maturity. Therefore, short term maturities had a greater relative change in rates compared to long term maturities. An important conclusion of Meiselman was that the yield curve as a whole is a family of rates each member of which is systematically related to each other.[3]

Voss (1990) conducted the same tests as Meiselman, however, using monthly Treasury bond data from 1955-1989 . Voss found that there are synchronous changes in all maturities and that generally, all maturities move in the same direction with a high degree of correlation[4], thus confirming Meiselmans' results

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[1]Thomas Coleman, Lawrence Fisher, Roger Ibbotson, U.S. Historical Treasury Yield Curves, 1993 Edition, Ibbotson Associates, 1993.

[2]Peter A. Abken, "Innovations in Modeling the Term Structure of Interest Rates", Financial Derivatives New Instruments and Their Uses, Federal Reserve Bank of Atlanta, December 1993.

[3]David Meiselman, The Term Structure of Interest Rates, Englewood Cliffs: Prentice Hall, 1962.

Sultan (1995) continued Voss's study by carrying out the same tests but with some modifications: (1) extending the time covered from 1980-1994 and (2) using daily Treasury data. Sultan found that there was a high level of synchronization among component parts of the Treasury issue yield curve and that volatility varied inversely with maturity[5], thus confirming Meiselman's and Voss's results.

1.3 Definitions

Coupon Bond

A coupon bond is a security that obligates the issuer to make specified payments to the holder on specified intervals for the life of the bond and then, in addition, to pay the bond's par value at the bond's maturity date. The coupon rate of the bond is the coupon payment divided by the bond's par value. The cash flows from a coupon bond consist of coupon payments until the maturity date plus the final payment of par value, which can be expressed as:

$$\text{Bond Value} = \sum_{t=1}^{T} \frac{\text{Coupon}}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^t}$$

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[4]Maj-Lis A. Voss, The Term Structure Of Interest Rates: U.S. Government Bonds, 1955-1989, Blacksburg: Virginia Polytechnic Institute and State University, 1990.

[5]Michael Sultan, An Analysis Of The Term Structure Of Interest Rates, January 1980 - March, 1994, Blacksburg: Virginia Polytechnic Institute and State University, 1995.

T is the maturity date in years, r is the interest rate per year, and each coupon payment is discounted based on the time until it will be paid[6]. This assumes the same rate of interest (or discount) for all the elements in the payment stream.

Duration

Duration is a weighted average term-to-maturity of the security's cash flows[7]. The weight applied to each time to payment should be a proportion of the present value of the payment divided by the bond price[8]. The weight, W_t, associated with the cash flow made at time t(CF_t) is:

$$W_t = \frac{CF_t/(1+y)^t}{\text{Bond Price}}$$

y = the bond's yield to maturity

The numerator on the right-hand side of the equation is the present value of the cash flow occurring at time *t*. The denominator is the value of all the payments forthcoming from the bond. Using the above values to calculate the weighted average of the times until receipt of each bond's payments we obtain Macualay's formula for duration, denoted D:

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[6]Zvi Bodie, Alex Kane, Alan J. Marcus, Essentials Of Investments, Second Edition, Irwin, 1995.

[7]Frank Fabozzi and T. Dessa Fabozzi, The Handbook Of Fixed Income Securities, Fourth Edition, Irwin, 1995.

[8]Zvi Bodie, Alex Kane, Alan J. Marcus, Essentials Of Investments, Second Edition, Irwin, 1995.

$$D = \sum_{t=1}^{T} t \times W_t$$

The greater the duration of a bond, the greater its percentage price volatility per basis point change in interest rates. A zero-coupon bond will have a larger duration than a coupon bond of the same duration, because with other factors held constant, the lower the coupon rate the greater the duration of the bond. A zero-coupon's return is solely dependent upon price appreciation, therefore, it is understandable that a zero-coupon bond would have a greater duration and thus greater price volatility than a coupon bond.[9]

Term Structure

The term structure of interest rates is defined as the relationship between interest rates (or spot rates) on bonds of different maturities.[10] Stating the term structure of interest rates is equivalent to specifying the form and parameters of the function that shows the relationship between the present value and the time until payment.[11] This relationship applies directly to single-payment notes such as zero-coupon bonds. The shape of the

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[9]Frank Fabozzi and T. Dessa Fabozzi, The Handbook Of Fixed Income Securities, Fourth Edition, Irwin, 1995.

[10]Peter A. Abken, "Innovations in Modeling the Term Structure of Interest Rates", Financial Derivatives New Instruments and Their Uses, Federal Reserve Bank of Atlanta, December 1993.

[11]Thomas Coleman, Lawrence Fisher, Roger Ibbotson, U.S. Historical Treasury Yield Curves, 1993 Edition, Ibbotson Associates, 1993.

term structure appears to have an impact on future real economic activity, including consumption and investment, and it also contains useful forecasts of future inflation.[12]

Yield Curve

A yield curve is a graphical depiction of the relationship between yield to maturity, the measure of the average rate of return that will be earned on a bond if held to maturity, and the term to maturity. Typically the yield to maturity is graphed on the vertical axis (y-axis) and the term (time) to maturity is graphed on the horizontal axis (x-axis). There are four general shapes of yield curves that have been observed in the United States over time: (a) an upward sloping yield curve, yield rises steadily as maturity increases; (b) a downward sloping or inverted yield curve, yields decline as maturity increases; (c) a humped yield curve, yields first increase as maturities ? then decline as maturity increases; and (d) a flat yield curve, yields stay the same as maturity increases.

Zero-Coupon Bond

A zero-coupon bond makes only one payment, the par value at maturity date and thus has a zero coupon rate. Before maturity, zero-coupon bonds sell at a discount from par, because of the time value of money. The final return on a zero-coupon bond is derived solely from the price appreciation at the time of maturity.[13] US Treasury bills are

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[12]Frank Fabozzi and T. Dessa Fabozzi, The Handbook Of Fixed Income Securities, Fourth Edition, Irwin, 1995.

[13]Peter A. Abken, "Innovations in Modeling the Term Structure of Interest Rates", Financial Derivatives New Instruments and Their Uses, Federal Reserve Bank of Atlanta, December 1993.

examples of short-term zero-coupon instruments. Longer-term zero-coupon bonds may be created synthetically. Investment banking firms buy coupon-paying Treasury bonds and notes and sell rights to single payments backed by the bonds. The coupons are stripped by the investment banks and so are called Treasury strips. The single payments are zero-coupon bonds collateralized by the coupon or principle payments of the original Treasury securities and therefore are virtually free of default risk. The yield to maturity for a zero-coupon bond is:

$$B(t) = 1/\,[1 + R(n)]^{n}$$

$R(n)$, is the yield to maturity of an n period zero-coupon bond, and $B(t)$, is the bond price at time t.

2. DATA

2.1 Description

This study examines U.S. Treasury obligations of 10 different maturities; ranging from six months to 30 years. The observation period begins April 30, 1991 and ends August 16, 1991. The data consists of daily bid yields on coupon interest of U.S. Treasury Strips; obtained from the Wall Street Journal. The bid yields on coupon interest are used instead of the bid yields on note and bond principle because some Treasury notes and bonds have a call feature and the yields on these instruments are calculated to the earliest call date, therefore, there are sequential time periods of no quoted bid yields on principle. For example, from 2010 to 2015 there are no bid yields on bond principle listed for Treasury bonds maturing during these years because all Treasury bonds issued to mature during 2010 to 2015 were issued with a call feature.

Richard Anderson of the Federal Reserve Bank of St. Louis converted my daily bid yields to prices using equations from the Standard Securities Calculation Methods (Security Industry Association, 1993). Next, the Federal Reserve Board's Mathmatica program was used to fit yield curves to the price data. The computer program fits an interpolated curve using cubic spines with endogenously chosen knots. The knots are selected according to a prediction error criterion, generalized cross-validation. The yield curve cannot be written as an equation as it exits only as an interpolating (not interpolated) function. The points that were selected on the curve are 180, 360, 720, 1080, 1800, 2520, 3600, 4900, 7200, and 10800 days (based on a 360 day year). The

Federal Reserve Board uses the same program to fit yield curves to coupon bonds which they publish daily. The Sultan and Voss studies used this data. The estimated zero-coupon yield data will be referred to as the Anderson-Leies zero-coupon yield data.

The object of this study is to specifically compare the results of tests on the zero-coupon data set with results obtained by Sultan using coupon bonds for the same time frame and generally compare the results on the zero-coupon data set with previous studies by Voss and Meiselman.

Sultan's data set consisted of daily observations on Treasury note yields of 10 maturities for the period from January 2, 1980 to March 31, 1994.

2.2 Overview of Analyses

Comparison of Averages

Monthly averages were calculated and a comparison was made to Sultan's data and also to the zero-coupon data that is contained in the Historical U.S Treasury Yield Curves. Data was compared to Sultan's to identify trends and to specifically compare his data on the one year Treasury Bill (a pure zero-coupon bond) to the Anderson-Leies, (synthetically created) zero-coupon bond. Data was compared to the estimated zero-coupon yield data in the Historical U.S. Treasury Yield Curves for two reasons: (1) as a verification of my yield curve data set and (2) to compare the results of two different estimation processes, the one used by the authors of the Historical U.S Treasury Yield Curves and the one used in this study.

Sign Test

The sign test involves comparing changes in the yield of maturity A with maturity B. The change is recorded as either "+", "-", or "0" for positive, negative or zero changes, respectively. These signs are compared daily for all combinations of all pairs of, and results are counted. The results are then organized into four matrices, three of which will contain the total number of individual simultaneous matches, i.e. simultaneous positive matches, simultaneous negative matches, and simultaneous zero matches, out of the total number of either positive, negative, or zero movements in each sample. The fourth matrix will contain the total number of same direction matches out of the total number of observations. The matrices will be presented as totals and as percentages and then will be compared to the same type of matrices from Sultan's data set.

Synchronization

To test for synchronizational movements of the term structure, i.e. where two or more rates move in the same direction, the sums of the entries of the sign test matrices are examined. These sums show how often rates move together (in the same direction), move in opposing directions, or do not move at all. First an examination will be made of the total number of same directional movements among rates and then an examination will be made of whether the movements seem to follow a particular pattern, positive negative or zero. These results will be compared to Sultan's results.

Regression Correlation

Regression analysis is performed in order to produce coefficients estimating functional relationships among Treasure yields. Prior to running the regressions it is necessary to take the first differences of the daily yields. Regressions are performed, first Y on X (the independent variable on the dependent variable), and then X on Y (the dependent variable on the independent variable). Correlation and regression coefficients are obtained from the regression results. Correlation coefficients indicate the "goodness of fit" of the data and regression coefficients indicate relative volatility among rates. These correlation and regression coefficients are organized into separate matrices so that the relationships can be easily identified. A comparison will be made between the zero-coupon results and the coupon bond results found by Sultan.

3. RESULTS AND DISCUSSION

The statistical and analytical results of the tests described in Chapter 2 are presented in this chapter. There are four sections, Section 3.1 compares the end of month and monthly averages of the estimated zero-coupon yields from the Historical U.S. Treasury Yield Curves, my zero-coupon yield data, Sultan's coupon yield data, and the 1year Treasury Bill yield data. These comparisons are made in order to independently compare and verify the Federal Reserve Board's method used in this study to estimate the zero-coupon yield curves. Sections 3.2 and 3.3 examine the synchronization of the direction of change of the alternative pairs of maturities. This analysis highlights how often rates on specific maturities move with the majority of the other rates in the yield curve as well as focusing on any dominant single directional movements. The last section, Section 3.4 examines the regression results. The correlation coefficients and regression coefficients highlighted in Section 3.4 describe the volatility and sensitivity of and among the different issues of the zero-coupon yield curve.

3.1 Comparison Results

Table 3.1-1 shows the end of month yield data and the monthly averages of the end of month yield data for the estimated zero-coupon yields from the Historical U.S. Treasury Yield Curves (hereafter referred to as the Ibbotson zero-coupon yield data), for the Anderson zero-coupon yield data, and for Sultan's coupon yield data. A comparison is made between my zero-coupon data and the estimated zero-coupon data, and between the Anderson zero-coupon data and Sultan's coupon data. The Ibbotson zero-coupon data is derived from the calculated forward rates of coupon bonds and then a "discount

Table 3.1-1

Comparison Of Month-End Yield Data, Anderson-Leies and Ibbotson Zero-Coupon and Coupon Curve Data

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	20YEAR	30YEAR
A. Anderson-Leies Zero-Coupon Yield Curve Data									
April	5.83	6.15	6.72	7.14	7.64	7.97	8.16	8.34	7.94
May	5.82	6.1	6.62	7.07	7.66	7.98	8.2	8.38	7.96
June	6.01	6.46	6.96	7.4	7.97	8.26	8.46	8.65	8.23
July	5.91	6.19	6.74	7.21	7.83	8.14	8.32	8.54	8.13
B. Ibbotson Yield Curve Data									
April	5.86	6.29	6.88	7.32	7.78	8.04	8.33	8.46	8.23
May	5.86	6.33	6.81	7.31	7.8	8.07	8.38	8.55	8.33
June	5.95	6.48	6.93	7.52	8.04	8.28	8.54	8.71	8.56
July	5.89	6.27	6.8	7.36	7.88	8.16	8.46	8.63	8.45
C. Coupon Yield Curve Data									
April	5.81	6.06	6.8	7.15	7.63	7.88	8.02		8.2
May	5.93	6.16	6.68	7.1	7.69	7.92	8.06		8.26
June	5.93	6.32	6.9	7.33	7.9	8.14	8.24		8.42
July	5.91	6.19	6.81	7.21	7.77	8.03	8.2		8.36

**Note that the Coupon Yield Curve Data did not contain 20year maturity data.

factor" is applied to these forward rates to obtain the estimated zero-coupon yield, spot rate, data (the Ibbotson study did not present enough detail to pursuit precise replication). Therefore, by comparing the Anderson-Leies zero-coupon data with Sultan's data and the Anderson-Leies zero-coupon data with the Ibbotson zero-coupon data we will be able to observe the amount of systematic difference resulting from the coupon effect on the yield data.

In comparing the end of month data, we can see that there is, generally, the least difference between the zero-coupon rates and the coupon rates for the longer term maturities, 10 and 30 years. The figures under the two column headings: 10year and 30year of Table 3.1-2 show the difference between the Anderson-Leies zero-coupon data, the Ibbotson yield data and the coupon yield data. Comparing the figures under the two column headings of 10year and 30year for subtable A. Anderson-Leies Zero-Coupon Yield Data Minus Ibbotson Yield Data and subtable C. Anderson-Leies Zero-Coupon Yield Data Minus Coupon Yield Data shows the differences among these series, between the Anderson-Leies zero-coupon yield data and the Ibbotson yield data or between the Anderson-Leies zero-coupon yield data and the coupon yield data.

When comparing the figures within the table, only their absolute amounts were considered. In every instance but one (June 10year) the figures in subtable C. Anderson-Leies Zero-Coupon Yield Data Minus Coupon Yield Data and under the column headings of 10year and 30year of Table 3.1-2, Comparison of Month-End Yield Data, are less than the numbers in subtable A. Anderson-Leies Zero-Coupon Yield Data Minus Ibbotson Yield Data. The fact that the values are less in subtable C than the figures in subtable A signifies that for long term maturities there is less divergence

Table 3.1-2

Comparison Of Month-End Yield Data, Anderson-Leies and Ibbotson Zero-Coupon and Coupon Curve Data

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	20YEAR	30YEAR
A. Anderson-Leies Zero-Coupon Yield Data Minus Ibbotson Yield Data									
April	-0.03	-0.14	-0.16	-0.18	-0.14	-0.07	-0.17	-0.12	-0.29
May	-0.04	-0.23	-0.19	-0.24	-0.14	-0.09	-0.18	-0.17	-0.37
June	0.06	-0.02	0.03	-0.12	-0.07	-0.02	-0.08	-0.06	-0.33
July	0.02	-0.08	-0.06	-0.15	-0.05	-0.02	-0.14	-0.09	-0.32
B. Coupon Yield Data Minus Ibbotson Yield Data									
April	-0.05	-0.23	-0.08	-0.17	-0.15	-0.16	-0.31		-0.03
May	0.07	-0.17	-0.13	-0.21	-0.11	-0.15	-0.32		-0.07
June	-0.02	-0.16	-0.03	-0.19	-0.14	-0.14	-0.3		-0.14
July	0.02	-0.08	0.01	-0.15	-0.11	-0.13	-0.26		-0.09
C. Anderson-Leies Zero-Coupon Yield Data Minus Coupon Yield Data									
April	0.02	0.09	-0.08	-0.01	0.01	0.09	0.14		-0.26
May	-0.11	-0.06	-0.06	-0.03	-0.03	0.06	0.14		-0.3
June	0.08	0.14	0.06	0.07	0.07	0.12	0.22		-0.19
July	0	0	-0.07	0	0.06	0.11	0.12		-0.23

**Note that the Coupon Yield Curve Data did not contain 20year maturity data.

between the Anderson-Leies zero-coupon yield data and coupon yield data in comparison to between Ibbotson zero-coupon yield data and the Anderson-Leies zero-coupon yield data. In turn, this implies that the amount of systematic difference due to the coupon is greater for the Ibbotson (estimated) zero-coupon yield data for the long term maturities.

The coupon effect, however, cannot be observed when examining the shortest term maturity yield data, the first 3 columns of Table 3.1-2. The Treasury Bill is already a zero-coupon instrument and the duration difference between a one or two year zero and a one or two year coupon issue is so small that it is not likely to be evident in the data. In principal, the coupon effect, itself, increases with maturity, thus, even intermediate term issues may not show a discernible or distinct coupon effect either, as is shown in columns 4 through 6 of Table 3.1-2.

Subtables A. Anderson-Leies Zero-Coupon Yield Data Minus Ibbotson Yield Data and C. Anderson-Leies Zero-Coupon Yield Data Minus Coupon Yield data show that nether subtable has a predominantly greater difference for either of the shortest term or intermediate maturity lengths. For example, comparing the absolute numbers under the 6month column heading for subtables A and C, subtable A has a greater difference for April (3 compared to 2) and for July (2 compared to 0) but subtable B has a greater difference for May (11 compared to 4) and June (8 compared to 6). Continuing across to the 2year column heading, subtable A has greater differences for April and May but subtable C has greater differences for June and July. Continuing to the 5year column heading, subtable A has greater differences for April and May but subtable C has greater differences for July. Again, continuing across to the 7year column heading, subtable A

Table 3.1-3

Comparison of Monthly Averages Yield Data, Anderson-Leies Zero-Coupon and Coupon Curve Data

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	20YEAR	30YEAR
A. Anderson-Leies Zero-Coupon Yield Curve Data									
May	5.9	6.19	6.72	7.15	7.72	8.03	8.23	8.42	7.99
June	6.12	6.4	6.94	7.38	7.93	8.22	8.42	8.6	8.18
July	5.99	6.29	6.86	7.33	7.93	8.22	8.41	8.61	8.18
B. Coupon Yield Curve Data									
May	5.85	6.13	6.78	7.12	7.7	7.934	8.07		8.26
June	6	6.35	6.94	7.38	7.93	8.16	8.27		8.46
July	5.95	6.31	6.92	7.37	7.91	8.15	8.27		8.45
C. 1Year T-Bill Yield Data									
May		6.19							
June		6.42							
July		6.38							
D. Anderson-Leies Zero-Coupon Yield Data Minus 1Year T-Bill Yield Data									
May		0							
June		-0.02							
July		-0.09							

**Note that the Coupon Yield Curve Data did not contain 20year maturity data.

has greater differences for May but subtable C has greater differences for April, June and July. Under some short term and intermediate maturity columns subtable A has the larger number of greater differences and under other maturities subtable C has the larger number of greater differences. Therefore, there is no implication here that the amount of difference between each (the Ibbotson yield data and the coupon yield data) and the Anderson-Leies zero-coupon yield data is predominantly greater for either of the bodies of data, as was the case with the long term maturities.

A comparison between the 1 year Treasury Bill, which is a pure discount bond, and my 1 year zero-coupon bond is seen in Table 3.1-3, subtable D. Anderson-Leies Zero-Coupon Yield Data Minus 1Year T-Bill Yield Data. This comparison is an independent verification to see how "good" or close the Federal Reserve Board's estimation method of the (Anderson-Leies) zero-coupon yields are to a true zero-coupon Treasury Bill yield. Subtable D, which shows the difference between the 1 year T-bill yield data and the 1 year Anderson-Leies zero-coupon yield data, indicates some clear results: for the monthly averages from May 1991 to July 1991 the average amount of difference between the 1year Anderson-Leies zero-coupon yield data and the 1year T-bill was 3.67 basis points and the largest difference between the two yields was only 9 basis points. The small differences suggest that the Federal Reserve Board's estimation method is implicitly a good estimate of a true zero-coupon yield curve.

3.2 Synchronization

Next, I examine whether changes in different maturities systematically move in the same direction, opposite direction, or no direction at all. Yield curve behavior, with respect to direction of change classified by maturity, can easily be examined by evaluating the matrices containing the total counts of the simultaneous positive, negative, or no-change changes in yields. These counts are not only indicative of synchronicity among rates but also how often yields move in the same direction. Tables 3.2-1 and 3.2-2 confirm the hypothesis that rates, generally, tend to move in the same direction. Examination of these tables indicates that there is high degree of sychronicity among all the various maturities for the Anderson-Leies zero-coupon yields and the coupon yields, with the level or percentage of synchronicity never falling below 50%.

Table 3.2-1 shows the levels of sychronicity for the Anderson-Leies zero-coupon yields for various maturities, and Table 3.2-2 shows the levels of synchronicity of the yields on coupon issues for various maturities. In comparing the results of the Anderson-Leies zero-coupon data set with that of the coupon issue data set we can see that, generally, the short term maturities of the Anderson-Leies zero-coupon rates are more synchronous than the coupon issue rates. For example, in Table 3.2-1 the level of synchronicity of the Anderson-Leies zero-coupon 1year/6month pair, with the 6months on the horizontal axis and the 1year on the vertical axis, is 93%. The level of synchronicity of the 6month/2year pair is 77%. Moving down the first column on the left-hand side of Table 3.2-1, the 6month/5year pair has a 67% level of synchronicity, the 6month/10year pair has a 56% level of synchronicity and the 6month/30year pair has a 59% level of synchronicity. Even though there is a slight increase between the levels of synchronicity for the 6month/10year pair and the 6month/30year pair, it is easy to see that the levels of synchronicity are lower as the maturity increases for one of the members of the pair.

Table 3.2-1

Synchronization Of Interest Rate Changes
Anderson-Leies Zero-Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	75/75									
1YEAR	70/75	75/75								
2YEAR	58/75	59/75	75/75							
3YEAR	47/75	48/75	60/75	75/75						
5YEAR	50/75	50/75	57/75	56/75	75/75					
7YEAR	44/75	42/75	49/75	48/75	63/75	75/75				
10YEAR	42/75	39/75	49/75	46/75	58/75	67/75	75/75			
13YEAR	43/75	40/75	47/75	50/75	55/75	57/75	65/75	75/75		
20YEAR	44/75	42/75	49/75	49/75	58/75	63/75	67/75	63/75	75/75	
30YEAR	44/75	44/75	49/75	45/75	55/75	57/75	61/75	59/75	63/75	75/75

The data represent the total simultaneous same direction matches for various maturities out of the total number of observations in the sample.

Synchronization Of Interest Rate Changes
Anderson-Leies Zero-Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	100%									
1YEAR	93%	100%								
2YEAR	77%	79%	100%							
3YEAR	63%	64%	80%	100%						
5YEAR	67%	67%	76%	75%	100%					
7YEAR	59%	56%	65%	64%	84%	100%				
10YEAR	56%	52%	65%	61%	77%	89%	100%			
13YEAR	57%	53%	63%	67%	73%	76%	87%	100%		
20YEAR	59%	56%	65%	65%	77%	84%	89%	84%	100%	
30YEAR	59%	59%	65%	60%	73%	76%	81%	79%	84%	100%

The data represent the % of the figures from the above table.

Table 3.2-2

Synchronization Of Interest Rate Changes
Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	75/75							
1YEAR	53/75	75/75						
2YEAR	48/75	57/75	75/75					
3YEAR	43/75	48/75	49/75	75/75				
5YEAR	40/75	50/75	58/75	55/75	75/75			
7YEAR	44/75	23/75	48/75	49/75	52/75	75/75		
10YEAR	43/75	44/75	43/75	47/75	52/75	60/75	75/75	
30YEAR	41/75	39/75	40/75	46/75	49/75	54/75	59/75	75/75

The data represents the total simultaneous same direction matches for various maturities out of the total number of observations in the sample.

Synchronization Of Interest Rate Changes
Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	100%							
1YEAR	71%	100%						
2YEAR	64%	76%	100%					
3YEAR	57%	64%	65%	100%				
5YEAR	53%	67%	77%	73%	100%			
7YEAR	59%	31%	64%	65%	69%	100%		
10YEAR	57%	59%	57%	63%	69%	80%	100%	
30YEAR	55%	52%	53%	61%	65%	72%	79%	100%

The data represent the % of the figures from the above table.

Comparing the above Anderson-Leies zero-coupon levels of synchronicity with the coupon yield levels in Table 3.2-2 we see the level of synchronicity for the coupon 1year/6month pair is 71%, a -22% difference from the zero-coupon level. The level of synchronicity among the coupon 6month/2year pair is 64%, a -13% difference from the zero-coupon level of synchronicity. The level of sychronicity among the coupon 6month/5year pair is 53%, a -14% difference from the zero-coupon level of synchronicity. The level of sychronicity among the 6month/10year pair is 57%, a +1% difference from the zero-coupon level of synchronicity; and the level of sychronicity among the 6month/30year pair is 55%, a -4% difference from the zero-coupon level of synchronicity. The predominant negative sign of the differences signifies that the Anderson-Leies zero-coupon level of synchronicity is greater than the coupon level of synchronicity.

Table 3.2-1 also examines the other relationships between all the other yield-maturity combinations of the Anderson-Leies zero-coupon rates. For example, the synchronization, changes in the same direction, between the 2year Anderson-Leies zero-coupon rate, on the horizontal axis, and all the other paired rates, on the vertical axis, is as follows: the pair of 2year/3year has a 80% level of synchronicity; the 2year/5year pair has a 76% level of synchronicity; the 2year/10year pair has a 65% level of synchronicity; and the 2year/30year pair has a 65% level of sychronicity.

Moving across the horizontal axis of Table 3.2-1 to the column heading 5year, the relationship between the Anderson-Leies zero-coupon 5year rate and all the other paired Anderson-Leies zero-coupon rates, on the vertical axis is examined. The 5year/7year pair has a level of synchronicity of 89%; the 5year/10year pair has a level of synchronicity of 77%; and the 5year/30year pair has a level of synchronicity of 73%. Table 3.2-3 shows the results from comparing the Anderson-Leies zero-coupon levels of synchronicity with the coupon levels of synchronicity. The numbers (0, 15, 8, 8) under the horizontal column heading 5year, in Table 3.2-3, represent the difference between the Anderson-Leies zero-coupon levels of synchronicity and the coupon levels of synchronicity for related pairs of maturities. The positive sign of these numbers indicates that the levels of synchronicity for the Anderson-Leies zero-coupon pairs are higher than the levels for the coupon pairs.

Continuing across the horizontal axis of Table 3.2-1 to the 10year column heading, the relationship between the Anderson-Leies zero-coupon 10year rate with all the other paired Anderson-Leies zero-coupon rates is examined. The 10year/13year pair has a 84% level of synchronicity; the 10year/20year pair has a level of synchronicity of 89%; and the 10year/30year pair has a 81% level of synchronicity. The numbers (0, 2) under the column heading 10year, in Table 3.2-3, represent the difference between the Anderson-Leies zero-coupon levels of synchronicity and coupon levels of synchronicity for the related pairs of maturities (notice that the coupon data set do not contain a 13year maturity nor a 20year maturity, hence differences can only be taken between the 10year/10year and 10year/30year combinations). Again the positive sign of the "2" indicates that the levels of synchronicity for the Anderson-Leies zero-coupon pairs are higher than the levels of synchronicity for the coupon pairs.

Table 3.2-3 shows the above observations of the differences between the levels of synchronicity for the Anderson-Leies zero-coupon and coupon yields. Table 3.2-3 also emphasizes the more volatile relationship between the intermediate paired rates. For

Table 3.2-3

Synchronization Of Interest Rate Changes
Anderson-Leies Zero-Coupon Minus Coupon Yields; Daily Figures, Classified By Maturity

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	0							
1YEAR	22	0						
2YEAR	13	3	0					
3YEAR	6	0	15	0				
5YEAR	14	0	-1	2	0			
7YEAR	0	-1	1	-1	15	0		
10YEAR	-1	-4	8	-2	8	9	0	
30YEAR	4	7	12	-1	8	4	2	0

This data represent the difference between the percentage levels for zero-coupon yields minus the percentage levels for coupon yields.

example, the difference between the Anderson-Leies zero-coupon and coupon level of synchronicity for the 7year/3year combination is -1, thus signifying that the coupon level of synchronicity is 1 percentage point higher than the Anderson-Leies zero-coupon level of synchronicity. The next combination, the 7year/5year, has a difference of 15, thus signifying that the Anderson-Leies zero-coupon level of synchronicity is 15 percentage points higher than the coupon level of synchronicity. This movement from -1 for one combination to 15 for the next combination characterizes the volatile pattern that takes place among the intermediate rates.

3.3 Sign Tests and Individual Synchronization

I next examined whether synchronization is affected by the direction of change in yields. In order to evaluate the degree of synchronization among the different maturities for the Anderson-Leies zero-coupon yields and the coupon yields, first differences of the daily yield data for both my set of data were taken as well as the overlapping time period of Sultan's data set. After the first differences had been calculated, a sign test was performed for all of the combinations of pairs of the 10 separate maturities. Each first difference was converted from it's numerical value into a sign; either a "+", "-", or "0" for a positive, negative, or zero change. It is important to distinguish all of the same direction movements among the different pairs of maturities. For example, if the 1 year rate increased, how many of the other rates increased and if the 10 year rate decreased, how many of the other rates also decreased?

Table 3.3-1 shows the number and the percentage of simultaneous positive matches (i.e. where both rates increased) for the Anderson-Leies zero-coupon yield data set. Tables

Table 3.3-1

Synchronization Of Interest Rate Changes When Specific Maturity Rates Increased
Anderson-Leies Zero-Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	34/34									
1YEAR	31/32	32/32								
2YEAR	23/27	23/27	27/27							
3YEAR	22/33	21/33	25/33	33/33						
5YEAR	21/32	20/32	21/32	25/32	32/32					
7YEAR	19/33	17/33	18/33	21/33	26/33	33/33				
10YEAR	21/39	19/39	21/39	22/39	27/39	33/39	39/39			
13YEAR	21/35	18/35	19/35	24/25	25/35	28/35	33/35	35/35		
20YEAR	21/40	20/40	20/40	24/40	27/40	30/40	35/40	32/40	40/40	
30YEAR	20/36	20/36	20/36	22/36	25/36	26/36	32/36	31/36	33/36	36/36

The data show the total number of simultaneous positive matches out of the total number of positive movements for the various pairs of maturites, i.e. 6month/1year pair - 31/32, the 31 is the number of simultaneous positive matches there were out of a total number,32, of positive, or increasing, movements within the entire time period.

Synchronization Of Interest Rate Changes When Specific Maturity Rates Increased
Anderson-Leies Zero-Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	100%									
1YEAR	97%	100%								
2YEAR	85%	85%	100%							
3YEAR	67%	64%	76%	100%						
5YEAR	66%	63%	66%	78%	100%					
7YEAR	58%	52%	55%	64%	79%	100%				
10YEAR	54%	49%	54%	56%	69%	85%	100%			
13YEAR	60%	51%	54%	96%	71%	80%	94%	100%		
20YEAR	53%	50%	50%	60%	68%	75%	88%	80%	100%	
30YEAR	56%	56%	56%	61%	69%	72%	89%	86%	92%	100%

The data represent the % of the figures from the above table.

Table 3.3-2

Synchronization Of Interest Rate Changes When Specific Maturity Rates Increased
Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	28/28							
1YEAR	23/32	32/32						
2YEAR	18/25	23/25	25/25					
3YEAR	16/25	19/25	18/25	25/25				
5YEAR	15/29	20/29	23/29	23/29	29/29			
7YEAR	19/36	20/36	22/36	21/36	23/36	36/36		
10YEAR	19/32	22/32	19/32	20/32	25/32	29/32	32/32	
30YEAR	17/31	19/31	16/31	19/31	23/31	27/31	27/31	31/31

The data represent the total number of simultaneous positive matches out of the total number of positive movements for the various maturities, i.e. 6month/1year pair 23/32, 23 represents the number of total simultaneous positive matches out of the total number, 32, of positive, or increasing, movements within the entire time period.

Synchronization Of Interest Rate Changes When Specific Maturity Rates Increased
Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	100%							
1YEAR	72%	100%						
2YEAR	72%	92%	100%					
3YEAR	64%	76%	72%	100%				
5YEAR	52%	69%	79%	79%	100%			
7YEAR	53%	56%	61%	58%	64%	100%		
10YEAR	59%	69%	59%	63%	78%	91%	100%	
30YEAR	55%	61%	52%	61%	74%	87%	87%	100%

The data represent the % of the figures from the above table.

Table 3.3-3

Synchronization Of Interest Rate Changes When Specific Maturity Rate Decreased
Anderson-Leies Zero-Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	40/40									
1YEAR	39/41	41/41								
2YEAR	35/46	36/46	46/46							
3YEAR	25/37	27/37	35/37	37/37						
5YEAR	29/42	30/42	36/42	31/42	42/42					
7YEAR	25/40	25/40	31/40	27/40	36/40	40/40				
10YEAR	21/34	21/34	28/34	24/34	31/34	34/34	34/34			
13YEAR	22/35	22/35	28/35	26/35	30/35	32/35	32/35	35/35		
20YEAR	23/34	22/34	29/34	25/34	31/34	33/34	32/34	31/34	34/34	
30YEAR	24/34	24/34	29/34	23/34	30/34	31/34	29/34	28/34	30/34	34/34

The data represent the total number of simultaneous negative matches out of the total number of negative movements for the various maturites, i.e. the 6month/1year pair 39/41, the 39 represents the total number of simultaneous negative matches out of the total number,41, of negative, or decreasing, movements within the entire time period.

Synchronization Of Interest Rate Changes When Specific Maturity Rate Decreased
Anderson-Leies Zero-Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	100%									
1YEAR	95%	100%								
2YEAR	76%	78%	100%							
3YEAR	68%	73%	95%	100%						
5YEAR	69%	71%	86%	74%	100%					
7YEAR	63%	63%	78%	68%	90%	100%				
10YEAR	62%	62%	82%	71%	91%	100%	100%			
13YEAR	63%	63%	80%	74%	86%	91%	91%	100%		
20YEAR	68%	65%	85%	74%	91%	97%	94%	91%	100%	
30YEAR	71%	71%	85%	68%	88%	91%	85%	82%	88%	100%

The data represent the % of the figures in the above table.

Table 3.3-4

Synchronization Of Interest Rate Changes When Specific Maturity Rate Decreased
Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	39/39							
1YEAR	30/36	36/36						
2YEAR	29/38	33/38	38/38					
3YEAR	27/38	29/38	29/38	38/38				
5YEAR	25/36	29/36	32/36	31/36	36/36			
7YEAR	25/31	23/31	26/31	28/31	28/31	31/31		
10YEAR	24/31	22/31	24/31	27/31	27/31	29/31	31/31	
30YEAR	22/29	20/29	24/29	27/29	26/29	27/29	28/29	29/29

The data represent the total number of simultaneous negative matches out of the total number of negative movements, i.e 6month/1year pair, 30/36, 30 represents the total number of simultaneous negative matches out of the total number, 36, of negative, or decreasing, movements withing the entire time period

Synchronization Of Interest Rate Changes When Specific Maturity Rate Decreased
Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	100%							
1YEAR	83%	100%						
2YEAR	76%	87%	100%					
3YEAR	71%	76%	76%	100%				
5YEAR	69%	81%	89%	86%	100%			
7YEAR	81%	74%	84%	90%	90%	100%		
10YEAR	77%	71%	77%	87%	87%	94%	100%	
30YEAR	76%	69%	83%	93%	90%	93%	97%	100%

The data represent the % of the figures of the above table.

Table 3.3-5

Synchronization Of Interest Rate Changes When There Was No Change In Specific Maturity Rates
Anderson-Leies Zero-Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH										
1YEAR	0									
2YEAR	0	0								
3YEAR	0	0	0							
5YEAR	0	0	0	0						
7YEAR	0	0	0	0	0					
10YEAR	0	0	0	0	0	1				
13YEAR	0	0	0	0	0	1	2			
20YEAR	0	0	0	0	0	0	1	0		
30YEAR	0	1	0	0	0	0	1	1	2	

The data represent the simultaneous zero matches for various maturities.

Table 3.3-6

Synchronization Of Interest Rate Changes When There Was No Change In Specific Maturity Rates
Coupon Yields; Daily Figures, Classified By Maturity, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH								
1YEAR	0							
2YEAR	1	1						
3YEAR	0	0	2					
5YEAR	0	1	3	1				
7YEAR	0	0	0	0	1			
10YEAR	0	0	0	0	0	2		
30YEAR	2	0	0	0	0	0	3	

The data represent the simultaneous zero matches for various maturities

3.3-2 also identifies the number of positive matches but for Sultan's coupon data set. Tables 3.3-3, 3.3-4, 3.3-5, and 3.3-6 show the number of simultaneous negative and zero matches, respectively for the Anderson-Leies zero-coupon and coupon data sets. Comparing the results in the three tables for the Anderson-Leies zero-coupon yield data, Tables 3.3-1, 3.3-3, and 3.3-5, we can see that simultaneous negative changes dominate the pattern for the 2year, 3year, 5year, and 7year maturities listed on the vertical axis of Table 3.3-3. The simultaneous negative changes dominate the pattern such that the percentage levels of synchronization are higher for the simultaneous negative matches than for either the simultaneous positive matches or the simultaneous zero matches. The fact that simultaneous negative matches tend to dominate the pattern for some of the short term and some of the intermediate rates does not indicate that there is not a high degree of synchronicity among the other rates. It merely indicates that there is a pattern of simultaneous negative matches for those maturities.

3.4 Regression Results

Next, I evaluate the relative sensitivities of changes in yields of different maturities. For example, if the 5year rate increases by a given amount, how much is the change in the 1year rate, the 10year rate, and so forth. The corresponding correlation coefficients show how dependable the regression relationship is.

Tables 3.4-1, 3.4-2 and 3.4-3 contain the regression results from running regressions of the first differences for all the pairs of the 10 maturities for the Anderson-Leies zero-coupon data set as well as the coupon data set. A total of 100 alternative combinations

Table 3.4-1

Table 3.4-1A
Regression Matrix - Correlation Coefficient
Anderson-Leies Zero-Coupon Yield Curve Data, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	1									
1YEAR	0.745	1								
2YEAR	0.538	0.818	1							
3YEAR	0.425	0.655	0.927	1						
5YEAR	0.397	0.532	0.679	0.788	1					
7YEAR	0.23	0.319	0.448	0.539	0.706	1				
10YEAR	0.232	0.305	0.412	0.527	0.778	0.769	1			
13YEAR	0.17	0.279	0.426	0.543	0.733	0.81	0.888	1		
20YEAR	0.204	0.311	0.426	5.37	0.787	0.794	0.937	0.939	1	
30YEAR	0.249	0.328	0.387	0.471	0.72	0.701	0.825	0.823	0.881	1

Table 3.4-1B
Regression Matrix - Regression Coefficient
Anderson-Leies Zero-Coupon Yield Curve Data, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6MONTH	1	0.64	0.48	0.394	0.311	0.225	0.23	0.199	0.224	0.257
1YEAR	1.164	1	0.798	0.66	0.487	0.358	0.355	0.345	0.372	0.399
2YEAR	1.122	1.025	1	0.89	0.623	0.48	0.468	0.483	0.494	0.491
3YEAR	1.078	0.993	1.042	1	0.726	0.57	0.572	0.59	0.6	0.586
5YEAR	1.273	1.094	1.091	1.085	1	0.798	0.85	0.838	0.888	0.886
7YEAR	1.021	0.892	0.933	0.946	0.885	1	0.89	0.929	0.94	0.92
10YEAR	1.011	0.859	0.882	0.922	0.916	0.864	1	0.958	1.006	0.984
13YEAR	0.85	0.809	0.881	0.92	0.874	0.872	0.927	1	0.99	0.967
20YEAR	0.914	0.835	0.862	0.895	0.886	0.845	0.932	0.948	1	0.979
30YEAR	0.966	0.823	0.788	0.804	0.813	0.761	0.838	0.851	0.9	1

Table 3.4-2

Table 3.4-2A
Regression Matrix - Correlation Coefficient
Coupon Yield Curve Data, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	1							
1YEAR	0.555	1						
2YEAR	0.5	0.681	1					
3YEAR	0.376	0.623	0.758	1				
5YEAR	0.391	0.563	0.734	0.805	1			
7YEAR	0.34	0.479	0.646	0.722	0.907	1		
10YEAR	0.316	0.436	0.565	0.677	0.871	0.911	1	
30YEAR	0.267	0.356	0.492	0.61	0.828	0.882	0.931	1

Table 3.4-2B
Regression Matrix - Regression Coefficient
Coupon Yield Curve Data, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	1	1.046	1.133	0.943	0.906	0.779	0.757	0.725
1YEAR	0.53	1	0.942	0.865	0.773	0.659	0.633	0.596
2YEAR	0.441	0.723	1	0.836	0.774	0.67	0.631	0.613
3YEAR	0.398	0.72	0.907	1	0.844	0.738	0.72	0.712
5YEAR	0.432	0.727	0.948	0.953	1	0.879	0.868	0.881
7YEAR	0.436	0.727	0.963	0.977	1.032	1	0.961	0.985
10YEAR	0.417	0.688	0.894	0.94	1.003	0.947	1	1.004
30YEAR	0.369	0.598	0.801	0.857	0.939	0.895	0.927	1

Table 3.4-3

Regression Matrix - Correlation Coefficient
Anderson-Leies Zero-Coupon Yield Data Minus Coupon Yield Data, April 30, 1991 - August 16, 1991

	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
6MONTH	0							
1YEAR	0.19	0						
2YEAR	0.038	0.137	0					
3YEAR	0.049	0.032	0.169	0				
5YEAR	0.006	-0.031	-0.055	-0.017	0			
7YEAR	-0.11	-0.16	-0.198	-0.183	-0.201	0		
10YEAR	-0.084	-0.131	-0.153	-0.15	-0.093	-0.142	0	
30YEAR	-0.018	-0.028	-0.105	-0.139	-0.108	-0.181	-0.106	0

The data represent the zero-coupon correlation coefficient minus the coupon correlation coefficient.

of maturities was regressed so that the relationship of the independent variable on the dependent variable and also the dependent variable on the independent variable could be addressed separately. The resulting regression coefficients and correlation coefficients are shown in matrix form. The correlation coefficient tells us how good the "fit" is between the two maturities in the combination and the regression coefficient describes the sensitivity of the systematic relation between the dependent variable and the independent variable.

It is important to note the comparison of the correlation coefficients for the Anderson-Leies zero-coupon yield curve data and the coupon yield curve data for the combination 6month/1year. When examining Tables 3.4-1, 3.4-2 and 3.4-3, the 6month yield is the dependent variable and the 1year yield is the independent variable for this combination. The correlation coefficient for the Anderson-Leies zero-coupon yield data in Table 3.4-1 is .745 and the correlation coefficient for the coupon yield data in Table 3.4-2 is .555. This .19 difference is consistent with the hypothesis that the coupon causes variation both to the degree of sychronicity described previously and as well as the level of the correlation coefficient.

However, it is very interesting that as maturity increases this difference reduces and then disappears. In fact, for longer term maturities the coupon yield data have higher correlation coefficients than do the Anderson-Leies zero-coupon yield data. This effect can be seen in Table 3.4-3, which highlights the differences between the correlation coefficient for the Anderson-Leies zero-coupon yield data and the correlation coefficient for the coupon yield data. For example, the results for the 6month, 1year and 2year column headings and 6month, 1year, 2year and 3year row headings of Table 3.4-3 indicate that the difference amounts are positive, thus signifying that the Anderson-Leies zero-coupon correlation coefficient is greater than the coupon correlation coefficient. However, as maturity increases, along both the horizontal and vertical axes of Table 3.4-3, notice that the difference amounts become negative numbers, thus signifying that the coupon correlation coefficients are greater than the Anderson-Leies zero-coupon correlation coefficients.

The fact that the differences between the correlation coefficients declines as maturity increase can be explained by the different duration properties of zero-coupon and coupon bonds. As described in chapter 2, a zero-coupon bond's duration is equal to it's maturity. A coupon bond's duration, however, is always less than it's maturity, due to the effect the inclusion of coupons have on measuring the effective maturity (or duration) of the bond. Consequently, it could be more accurate to compare a 20year zero-coupon bond with a 30year coupon bond, in order to compare instruments with equal duration. The different role that duration plays for zero-coupon and coupon bonds explains why we see a decline in the differences in correlation coefficients as maturity increases.

The regression coefficients examine the relationship between the dependent variable and the independent variable for each combination. Comparing the regression coefficient results obtained in Tables 3.4-1B and 3.4-2B, shows a greater difference between the Anderson-Leies zero-coupon yield data and the coupon yield data. The Anderson-Leies zero-coupon yield data suggests a monotonic decreasing relationship, (for example, the relationship between the 1year rate and the 20year rate is nearly flat - .372), while the coupon yield data suggests a monotonic increasing relationship, (for example, the numbers increase as you move to the right in Table 3.4-2B).

Furthermore, the higher regression coefficients under the column headings 6month, 1year and 2year of Table 3.4-1B in comparison to the regression coefficients under the same column headings of Table 3.4-2B show that there is higher volatility among the short term rates in comparison to the long term rates for the Anderson-Leies zero-coupon yield curves; this is consistent with the results Meiselman found when studying high-grade corporate bonds. Table 3.4-1B indicates that if there is a 100 basis point change in the 6month rate, on the vertical axis, then the 10year rate, on the horizontal axis, will change by 22.4 basis points and if there is a 100 basis point change in the 30year rate, on the vertical axis, then the 6month rate, on the horizontal axis, will move 96.6 basis points. This indicates that the short term rates are more volatile. As with Sultan's earlier findings, the regression coefficients generally vary inversely with maturity. This inverse relationship confirms that there is systematically greater volatility among shorter term maturities.

4. SUMMARY AND IMPLICATIONS

4.1 Summary of Results

The purpose of this paper was to estimate and evaluate a yield curve derived from zero-coupon Treasury bond issues. Several methods were used to validate the accuracy of the estimated yield curves. The methods of validation included comparing the Anderson-Leies zero-coupon yield data to Ibbotson estimated zero-coupon yield data, comparing the 1year Anderson-Leies zero-coupon yield data to the 1year Treasury Bill, and lastly comparing the Anderson-Leies zero-coupon yield data to coupon yield data.

The results, highlighted in detail in chapter 3, showed that the estimation process used to obtain the Anderson-Leies zero-coupon yield data in this study is consistent with it being a valid and accurate methodology. For example, the 1year Anderson-Leies zero-coupon yield data was notably similar to the 1year T-bill yield data, thus signifying that the Federal Reserve Board's estimation method is implicitly a good estimate, at least of the 1year zero rate. Further analysis of the comparative results showed that there are larger differences for the long term maturities between the yields of the Anderson-Leies zero-coupon data and the Ibbotson estimated zero-coupon yield data than between the Anderson-Leies zero-coupon and the coupon yield data. This larger difference is puzzling and raises questions about the Ibbotson estimates. It is notable that the Anderson-Leies estimates of the one year zero-coupon yields are closer to the 1year Treasury Bill yields than the Ibbotson estimates. This fact serves as independent evidence for the reliability of the Anderson-Leies estimates, at least at the short end of the yield curve.

The behavior of the Anderson-Leies zero-coupon yield curve, with respect to synchronization, is similar to the behavior of yield curves previously studied by Meiselman, Voss and Sultan; where the majority of rates tend to move in the same direction. Examination of the movements of the Anderson-Leies zero-coupon yield curve over the period of the study shows that the rates move together in a synchronized and generally regular manner. The short term rates of the Anderson-Leies zero-coupon yield curve tend to exhibit the highest degree of synchronization. The greatest divergence in amounts of synchronicity between the Anderson-Leies zero-coupon and coupon yields occurs when comparing the shortest maturity pair of this study, the 6month/1year pair. The amount of divergence decreases as the maturity increases for at least one of the issues in the pair. However, the Anderson-Leies adjacent zero-coupon paired issues, consistently, have a greater level of synchronization than the adjacent coupon paired issues.

The results of the regression analysis showed that when the yield curve changes the shorter the maturity the greater the change, and the longer the maturity the less the change in the rate. In other words, volatility is generally inversely related to maturity; which is consistent with the Meiselman, Voss and Sultan results.

4.2 Implications of Results

There are several implications of the results obtained in this study. First and foremost, the method of estimation used by Anderson of zero-coupon yield curves appears to be a valid estimation method. The implications of the accuracy of this method can be seen in the comparison of the Anderson-Leies zero-coupon yield curves to the coupon yield curves. The comparison shows, in general, a higher degree of synchronicity for the entire family of Anderson-Leies zero-coupon rates than for coupon issues, especially among adjacent issues; and a higher degree of volatility and sensitivity among the short term issues of the Anderson-Leies zeros than of the coupon issues.

Previous findings have shown a synchronous characteristic of coupon yield curves, where both the short term and long term rates generally move in the same direction. Thus, if the Federal Reserve increased the Federal Funds Rate, and thereby other short term rates, then long term rates would also increase and vice versa. The Anderson-Leies zero-coupon yield curve data support these norms drawn from the history of coupon yield curves. Thus, it is inconsistent with established empirical norms that a change in short term rates, by Federal policies or other factors, would dependably cause opposite directional changes in the longer end of the yield curve. Yield curves generally do not rotate.

The graphical depiction of the Anderson-Leies yield data, see Appendix B, shows a humped yield curve, monotonically increasing until approximately the 20year maturity and then declining. The fact that the yield curve is humped affects some of the comparisons between the Anderson-Leies zero-coupon and coupon issues. For example, because the coupon bond has a shorter duration, the 20year or 30year coupon issue is actually closest to the highest peak of the Anderson-Leies zero-coupon yield curve. The fact that the yield curve is humped also suggests that the forward rates would be falling even more sharply for longer than the 20year maturity.

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Anderson-Leies Zero-Coupon Yield Data

Date	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
30-Apr-91	5.8285	6.1477	6.7155	7.1354	7.6442	7.965	8.1548	8.3074	8.335	7.9383
1-May-91	6.0195	6.2786	6.7699	7.1846	7.6952	7.9497	8.1528	8.2863	8.3274	7.9564
2-May-91	5.8087	6.1174	6.6743	7.0987	7.6127	7.9084	8.099	8.2508	8.2867	7.8909
3-May-91	5.9245	6.2072	6.7293	7.1501	7.6813	7.9834	8.181	8.3464	8.3652	7.9724
6-May-91	5.8889	6.1852	6.7252	7.1485	7.6752	7.9941	8.1877	8.3526	8.374	7.9765
7-May-91	5.9544	6.2278	6.7293	7.1356	7.6832	7.996	8.1949	8.3447	8.3848	7.9924
8-May-91	5.8918	6.199	6.7467	7.1655	7.6926	8	8.1998	8.3496	8.3841	7.9783
9-May-91	5.9012	6.2047	6.7491	7.1695	7.6986	7.9865	8.1834	8.3503	8.3843	7.9817
10-May-91	5.9576	6.2612	6.8093	7.2358	7.7658	8.0917	8.294	8.4496	8.4824	8.0767
13-May-91	5.965	6.2383	6.7482	7.174	7.7396	8.0363	8.24	8.3979	8.4344	8.0327
14-May-91	5.942	6.2209	6.7444	7.1899	7.7956	8.1066	8.3182	8.4801	8.5217	8.065
15-May-91	5.9574	6.2305	6.7427	7.1785	7.7752	8.0865	8.3037	8.4679	8.5097	8.0509
16-May-91	5.9498	6.2253	6.7416	7.1788	7.7707	8.0776	8.2937	8.4581	8.4999	8.0402
17-May-91	5.9853	6.2595	6.7723	7.2034	7.7694	8.0612	8.248	8.404	8.4363	7.9762
20-May-91	5.8933	6.1947	6.7493	7.2037	7.7758	8.0642	8.2667	8.423	8.4601	7.9865
21-May-91	5.8294	6.1702	6.7368	7.166	7.7304	8.0217	8.2077	8.3775	8.3998	7.91
22-May-91	5.8521	6.1471	6.6862	7.1304	7.7178	8.0291	8.2302	8.3943	8.4208	7.9249
23-May-91	5.8812	6.1624	6.6885	7.1341	7.7352	8.0584	8.2732	8.4347	8.4696	8.0464
24-May-91	5.8836	6.1622	6.688	7.1388	7.7431	8.0698	8.2808	8.4452	8.4727	8.0467
28-May-91	5.8266	6.1148	6.6513	7.101	7.7029	8.0324	8.2445	8.4117	8.4402	8.014
29-May-91	5.8259	6.1132	6.6488	7.1008	7.702	8.0359	8.2484	8.419	8.4491	8.0168
30-May-91	5.82	6.0977	6.6196	7.065	7.6629	7.9855	8.1966	8.3543	8.3828	7.9557
3-Jun-91	5.91073	6.18908	6.70963	7.14725	7.71986	8.04094	8.24934	8.40457	8.43313	8.00894
4-Jun-91	6.01724	6.28494	6.79099	7.22662	7.81399	8.1182	8.31427	8.47071	8.49925	8.07809
5-Jun-91	6.03615	6.30263	6.80574	7.2347	7.80927	8.11236	8.31303	8.46126	8.50049	8.08594
6-Jun-91	6.00539	6.29586	6.83741	7.28659	7.85866	8.15925	8.35578	8.51206	8.54736	8.12332
9-Jun-91	6.04272	6.33872	6.8888	7.34135	7.9024	8.19059	8.39105	8.52528	8.5658	8.15035
10-Jun-91	6.24949	6.52549	7.03913	7.45698	7.97446	8.25633	8.44241	8.59034	8.62559	8.20752
11-Jun-91	6.194	6.47833	7.00609	7.43745	7.96692	8.24994	8.44498	8.59075	8.62284	8.20625
12-Jun-91	6.16693	6.46011	7.00122	7.43818	7.96586	8.24291	8.43243	8.58252	8.61321	8.19567
13-Jun-91	6.20416	6.50836	7.06187	7.49955	8.02619	8.31168	8.50198	8.66191	8.68961	8.27129
14-Jun-91	6.16992	6.48906	7.05808	7.49129	8.0059	8.29414	8.47848	8.63151	8.66071	8.24064

Appendix A
(cont)

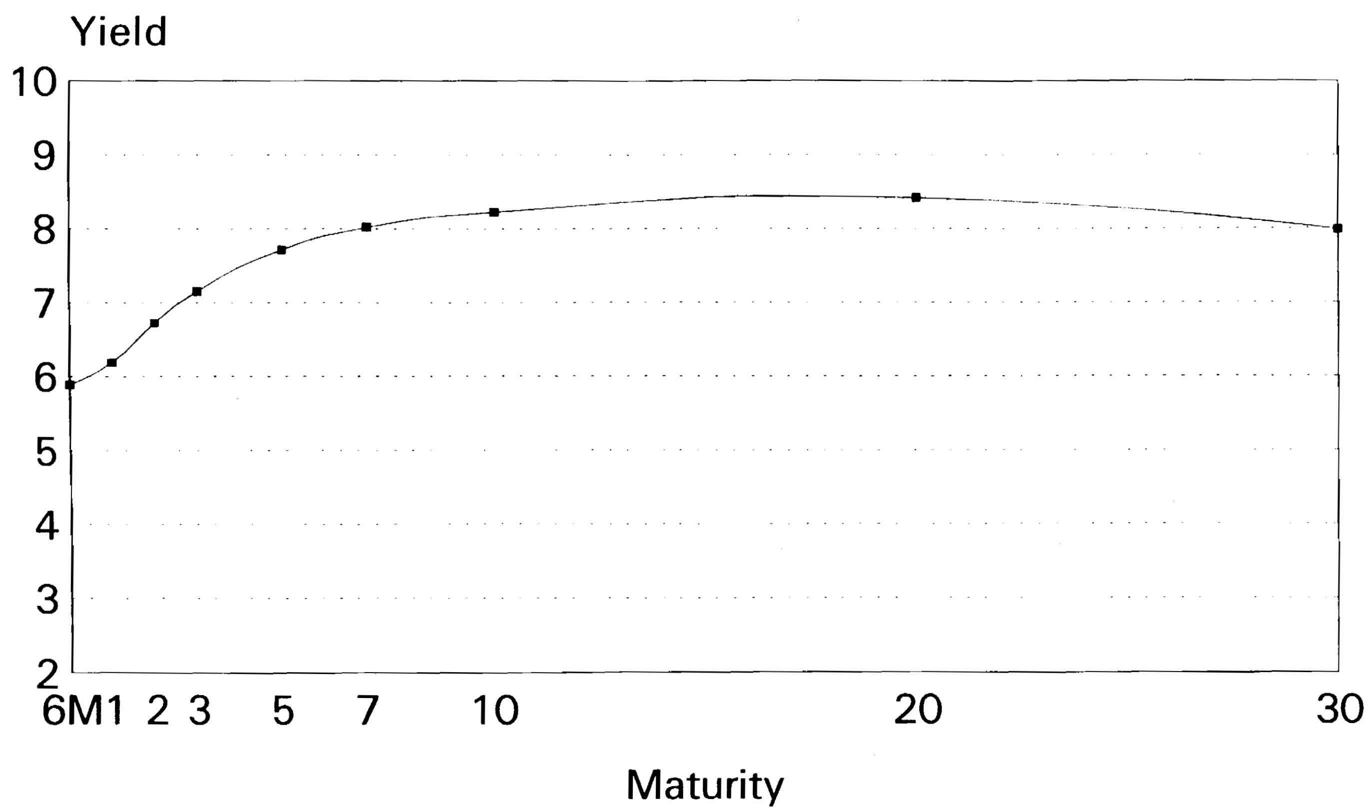
Date	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
17-Jun-91	6.19228	6.49027	7.03087	7.45559	7.96638	8.24375	8.43269	8.58913	8.61532	8.19398
18-Jun-91	6.16235	6.46527	7.01547	7.4487	7.96872	8.25072	8.44786	8.60228	8.63203	8.21235
19-Jun-91	6.17526	6.46729	7.00624	7.44537	7.98035	8.26861	8.46778	8.62137	8.65118	8.23154
20-Jun-91	6.14205	6.43119	6.96983	7.41682	7.97645	8.27858	8.47689	8.63381	8.6605	8.23813
21-Jun-91	6.12441	6.40055	6.92113	7.36242	7.92495	8.23394	8.43657	8.59345	8.62938	8.20457
24-Jun-91	6.19665	6.47334	6.9936	7.43125	7.97469	8.26943	8.47358	8.6291	8.66061	8.23229
25-Jun-91	6.14115	6.43356	6.97817	7.43039	7.99867	8.28156	8.48226	8.62745	8.66789	8.25077
26-Jun-91	6.15103	6.43415	6.96806	7.42319	8.0005	8.28207	8.48274	8.62768	8.6679	8.25073
27-Jun-91	6.13988	6.43001	6.97267	7.42941	8.00704	8.28913	8.48497	8.6259	8.66783	8.25043
28-Jun-91	6.19076	6.45461	6.95719	7.39759	7.974	8.25733	8.45755	8.60671	8.64865	8.2313
1-Jul-91	6.05022	6.3324	6.86803	7.32923	7.92633	8.20519	8.39599	8.53466	8.58464	8.16618
2-Jul-91	6.0217	6.3236	6.8905	7.3647	7.9423	8.2111	8.4064	8.5348	8.5911	8.1577
3-Jul-91	6.0448	6.3478	6.9109	7.3784	7.951	8.2146	8.4086	8.541	8.5924	8.1466
5-Jul-91	6.057	6.3508	6.9026	7.3635	7.9347	8.2049	8.3909	8.5238	8.5746	8.1303
8-Jul-91	6.1112	6.4137	6.9804	7.4474	8.0201	8.2924	8.4634	8.5921	8.6429	8.1978
9-Jul-91	6.0382	6.3443	6.9218	7.4071	8.0047	8.2887	8.4668	8.5984	8.6499	8.2053
10-Jul-91	6.0236	6.3337	6.9187	7.4105	8.0223	8.3075	8.4818	8.6212	8.6784	8.2333
11-Jul-91	5.9807	6.2983	6.8959	7.3947	8.0047	8.2937	8.4683	8.6088	8.6696	8.2213
12-Jul-91	5.9694	6.274	6.8526	7.3447	7.957	8.2399	8.4146	8.5611	8.6219	8.1751
15-Jul-91	5.9665	6.2635	6.8267	7.3035	7.9122	8.1954	8.3809	8.5263	8.5855	8.1708
16-Jul-91	5.9423	6.2423	6.8097	7.2917	7.9094	8.1929	8.386	8.5324	8.5925	8.1769
17-Jul-91	5.9395	6.2388	6.8034	7.2813	7.9065	8.1999	8.3923	8.5411	8.5932	8.1899
18-Jul-91	5.9582	6.2662	6.8426	7.3207	7.9345	8.2315	8.4231	8.5714	8.6313	8.2258
19-Jul-91	5.9361	6.2508	6.8402	7.3282	7.9425	8.2315	8.4229	8.5712	8.6411	8.2245
22-Jul-91	5.9699	6.2708	6.8357	7.3074	7.9185	8.2164	8.4147	8.5643	8.6341	8.2189
23-Jul-91	5.9427	6.2516	6.8294	7.3077	7.9115	8.2096	8.398	8.5418	8.6122	8.1907
24-Jul-91	5.9514	6.2686	6.8596	7.3424	7.9318	8.2293	8.4207	8.5706	8.6409	8.2171
25-Jul-91	5.916	6.2263	6.8038	7.275	7.8602	8.1548	8.3484	8.4863	8.555	8.1336
26-Jul-91	5.9826	6.2572	6.7793	7.2271	7.8277	8.1291	8.3173	8.4515	8.5272	8.118
29-Jul-91	5.9575	6.2405	6.7796	7.2408	7.848	8.1497	8.3272	8.4771	8.5386	8.129
30-Jul-91	5.9048	6.1925	6.7394	7.2082	7.8329	8.1432	8.323	8.4734	8.5363	8.1286
1-Aug-91	5.9203	6.2057	6.7475	7.2107	7.8307	8.1394	8.3273	8.4794	8.537	8.1298
2-Aug-91	5.8893	6.1758	6.716	7.1756	7.7921	8.114	8.2943	8.4405	8.4981	8.0837

Appendix A
(concl)

Date	6MONTH	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	13YEAR	20YEAR	30YEAR
6-Aug-91	5.8177	6.0896	6.6069	7.0513	7.6573	7.982	8.25	8.3554	8.4057	7.9824
7-Aug-91	5.8408	6.1442	6.6933	7.1208	7.6433	7.964	8.1815	8.342	8.3837	7.9626
8-Aug-91	5.6477	5.9235	6.4507	6.9105	7.5527	7.8927	8.1108	8.2778	8.3264	7.9172
9-Aug-91	5.542	5.847	6.4263	6.9131	7.5366	7.8693	8.1059	8.2791	8.3256	7.9076
12-Aug-91	5.5315	5.8342	6.4108	6.901	7.5518	7.8992	8.1498	8.3288	8.3829	7.9722
13-Aug-91	5.6144	5.8907	6.422	6.8889	7.5456	7.9058	8.1726	8.3336	8.415	7.9982
14-Aug-91	5.5205	5.8132	6.374	6.8583	7.5158	7.8831	8.1496	8.3317	8.3917	8.0263
15-Aug-91	5.5118	5.8039	6.3614	6.8363	7.4772	7.849	8.1111	8.2929	8.3534	7.9873
16-Aug-91	7.0702	7.1272	7.2408	7.3533	7.5699	7.7659	7.9984	8.1487	8.0234	7.0329

Zero-Coupon Yield Curve

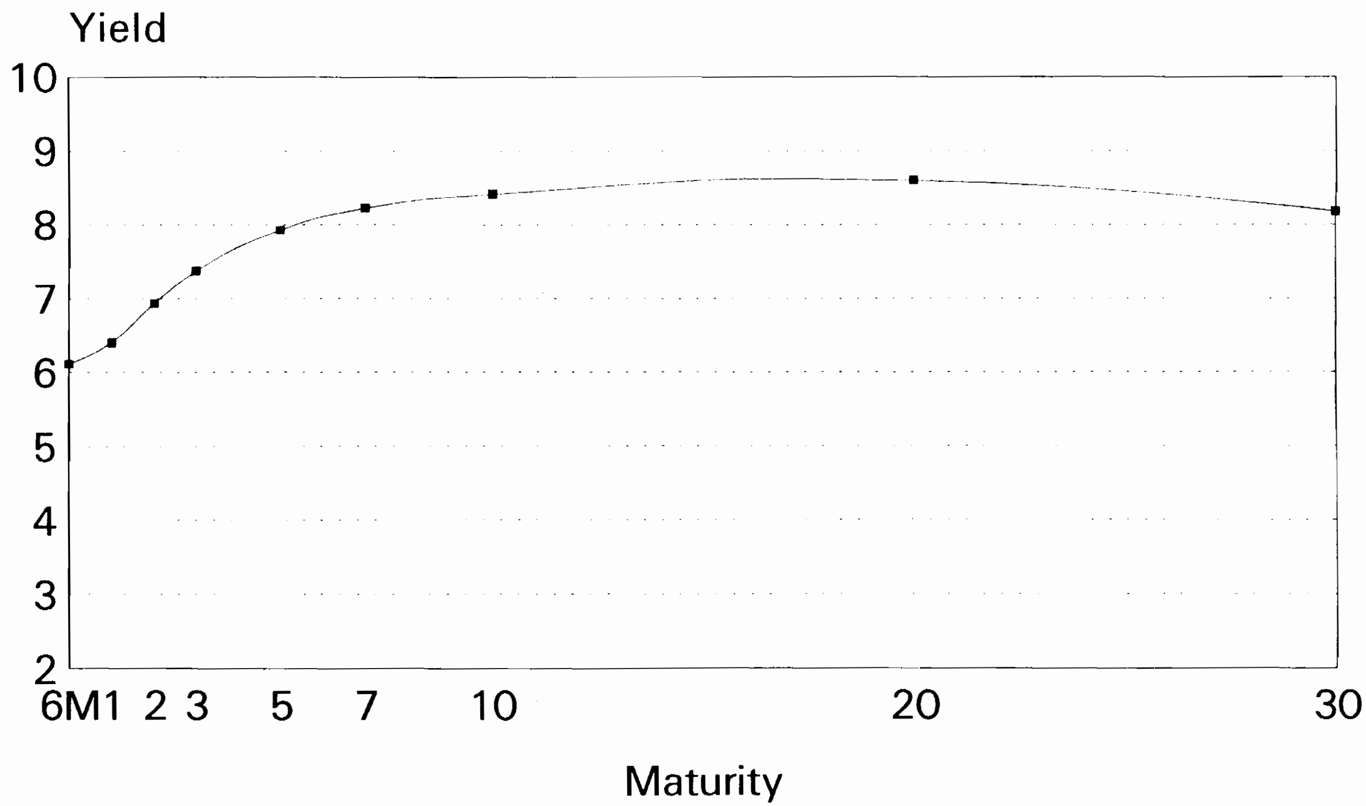
May 1991



Data Represents Monthly Averages

Zero-Coupon Yield Curve

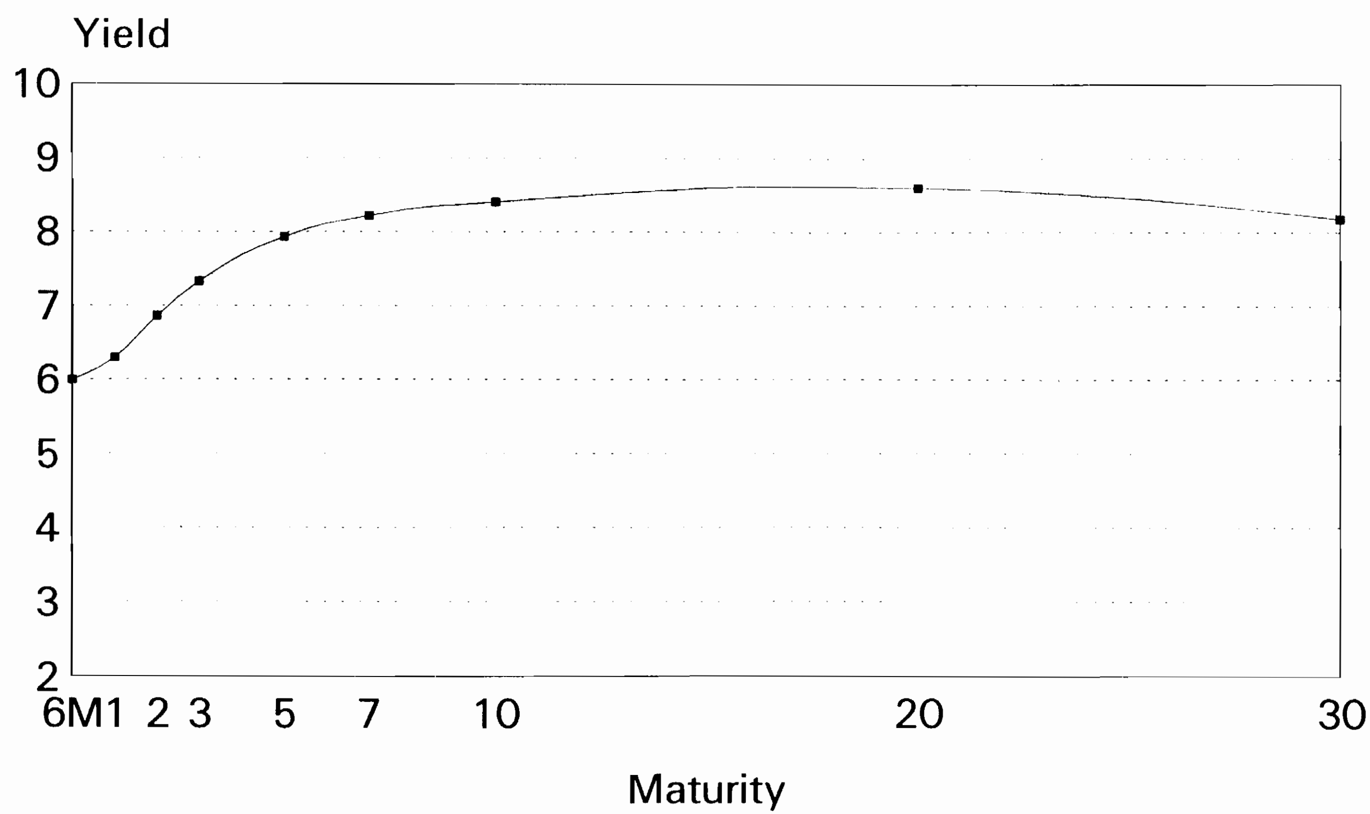
June 1991



Data Represents Monthly Averages

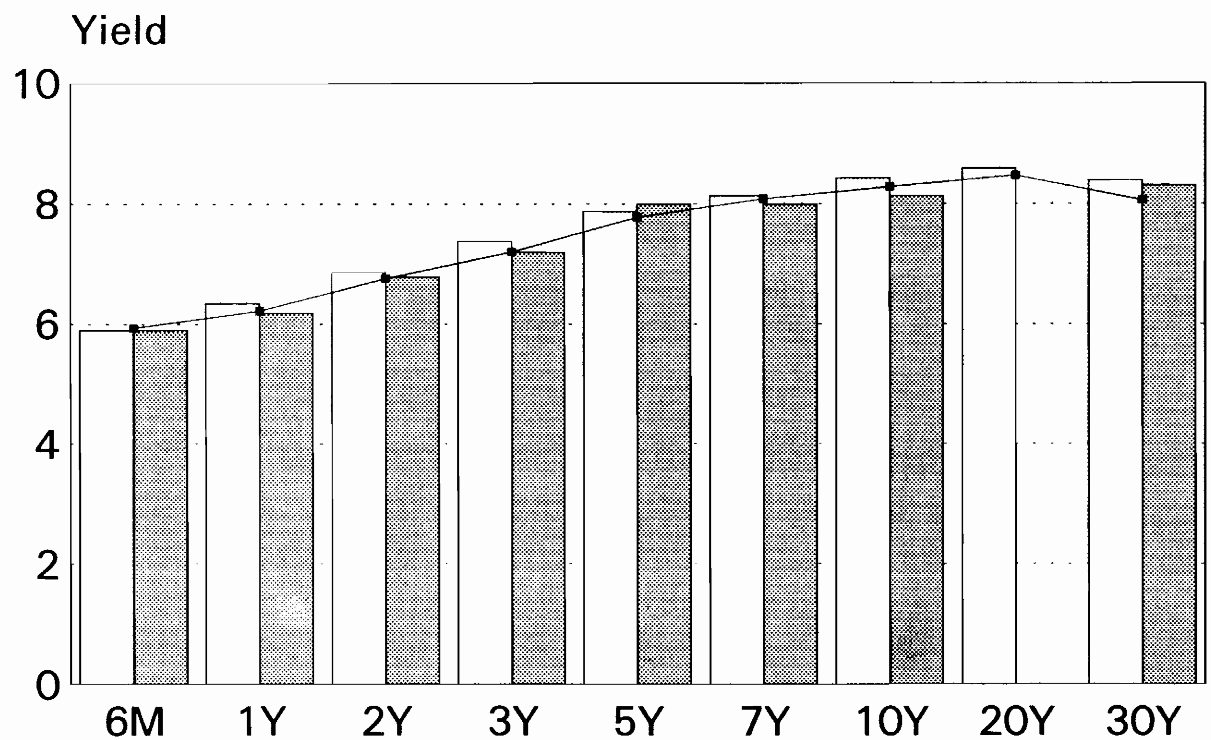
Zero-Coupon Yield Curve

July 1991



Data Represents Monthly Averages

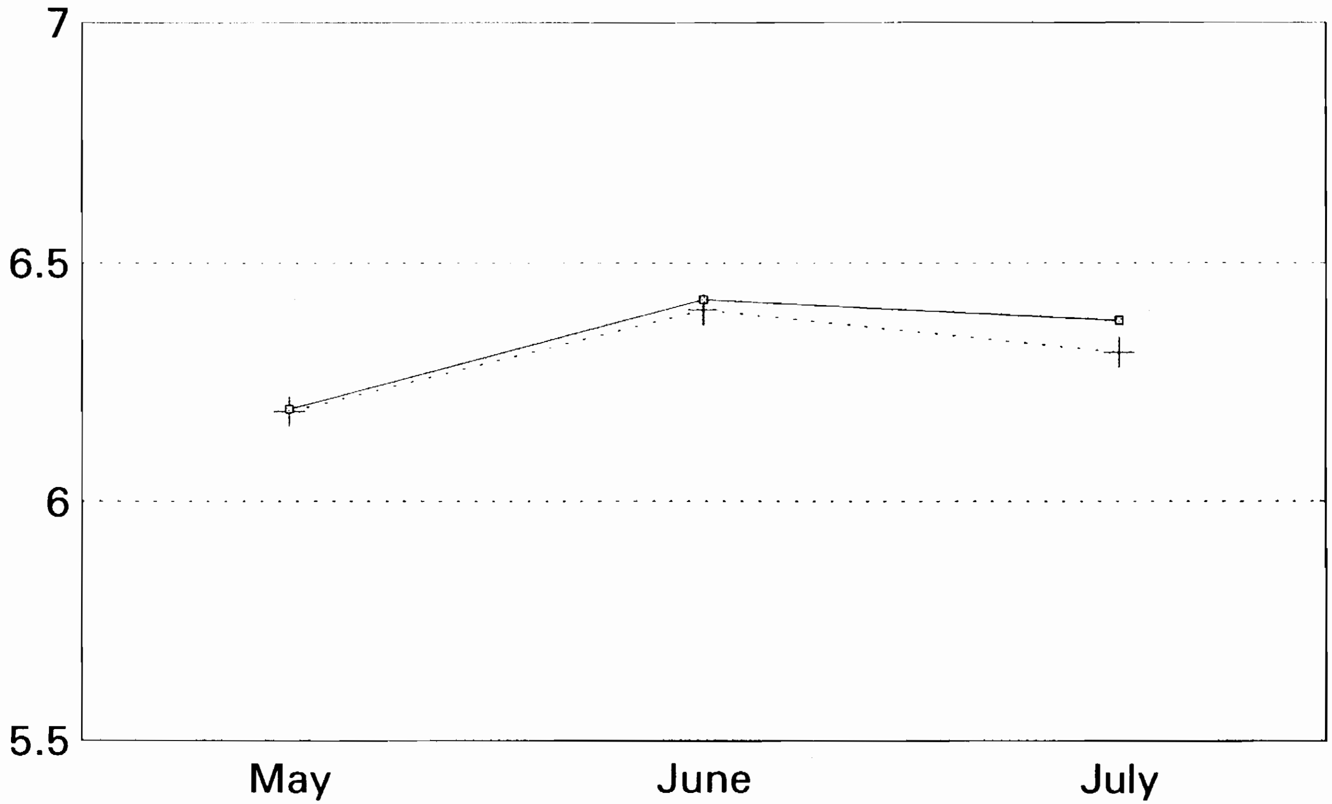
YIELD COMPARISON FOR MONTHLY AVERAGES



	6M	1Y	2Y	3Y	5Y	7Y	10Y	20Y	30Y
ZCYC	5.936	6.223	6.758	7.202	7.778	8.088	8.283	8.476	8.063
EST ZCYC	5.89	6.343	6.855	7.378	7.875	8.138	8.428	8.588	8.393
CYC	5.895	6.183	6.789	7.198	7.998	7.993	8.13		8.31

COMPARISON OF 1 YEAR T-BILL AND THE 1 YEAR ZERO-COUPON

Monthly Averages From May 1991 - July 1991



	May	June	July
1Y TB	6.194	6.424	6.38
1T ZC	6.189	6.402	6.313

VITA

Caroline Marie Leies was born August 19, 1969 in Alexandria Virginia. She moved to Florida in 1973, where she lived until 1987. Ms. Leies began her undergraduate college career at Rutgers University, in New Brunswick, New Jersey. Ms. Leies finished her undergraduate studies at Marymount University, in Arlington, Virginia, graduating with a B.A. degree in Economics.

Upon graduation of college, Ms. Leies was employed as a stock trader for a regional brokerage firm, Wheat First Butcher and Singer in Bethesda, Maryland. After a year, Ms. Leies left Wheat First Butcher and Singer and went to work for Booz•Allen & Hamilton, as a Financial Consultant. In 1994, Ms. Leies decided to return to academic life full-time to pursue graduate studies after being awarded an assistantship with the Graduate Economics Department at the Northern Virginia Graduate Center of Virginia Polytechnic Institute and State University. Subsequent to finishing her Masters in Economics degree, Ms. Leies has accepted a position as a Senior Consultant with Price Waterhouse LLP.

Caroline Marie Leies