

**Piezoelectric Transformer Characterization
and
Application of Electronic Ballast**

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Dedicated to Jia-Bin (Robin) Chen
A deceased former graduate student/research assistant
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(ABSTRACT)

The characterization and modeling of piezoelectric transformers are studied and developed for use in electronic ballasts. By replacing conventional L-C resonant tanks with piezoelectric transformers, inductor-less piezoelectric transformer electronic ballasts have been developed for use in fluorescent lamps.

The piezoelectric transformer is a combination of piezoelectric actuators as the primary side and piezoelectric transducers as the secondary side, both of which work in longitudinal or transverse vibration mode. These actuators and transducers are both made of piezoelectric elements, which are composed of electrode plates and piezoelectric ceramic materials. Instead of the magnetic field coupling between the primary and secondary windings in a conventional magnetic core transformer, piezoelectric transformers transfer electrical energy via electro-mechanical coupling that occurs between the primary and secondary piezoelectric elements for isolation and step-up or step-down voltage conversion. Currently, there are three major types of piezoelectric transformers: Rosen, thickness vibration mode, and radial vibration mode, all three of which are used in DC/DC converters or in electronic ballasts for fluorescent lamps. Unlike the other two transformers, the characterization and modeling of the radial vibration mode piezoelectric transformer have not been studied and developed prior to this research work.

Based on the piezoelectric and wave equations, the physics-based equivalent circuit model of radial vibration mode piezoelectric transformers is derived and verified through characterization work.

Besides the major vibration mode, piezoelectric transformers have many spurious vibration modes in other frequency ranges. An improved multi-branch equivalent circuit is proposed, which more precisely characterizes radial vibration mode piezoelectric transformers to include other spurious vibration modes in wide frequency ranges, as compared with the characterizations achieved by prior circuits.

Since the equivalent circuit of piezoelectric transformers is identical to the conventional L-C resonant tank used in electronic ballasts for fluorescent lamps, piezoelectric transformers replace the conventional L-C resonant tank in order to reduce the amount and cost of electronic components for the electronic ballasts. With the inclusion of the radial vibration mode piezoelectric transformer, the design and implementation of inductor-less piezoelectric transformer electronic ballast applications have been completed.

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Nomenclature

A_v	Voltage gain
C	Capacitor of equivalent circuit model
C_m	Capacitor in the equivalent circuit model of the piezoelectric transformer
C_{m1}, C_{m2}	Capacitor inductor in the physics-based equivalent circuit of the piezoelectric ceramic material
C_{d1}	Input capacitor of equivalent circuit model
C_{d2}	Output capacitor of equivalent circuit model
D	Electric displacement
d	Piezoelectric constant
E	Electric field
f_r	Resonant frequency of piezoelectric transformer with output terminal shorted
f_s	Switching frequency
L	Inductor of equivalent circuit model
L_m	Inductor in the equivalent circuit model of the piezoelectric transformer
L_{m1}, L_{m2}	Inductor in the physics-based equivalent circuit of the piezoelectric ceramic material
N	Turns ratio
$N_1, N_2, \dots, N_n$	Turns ratio of equivalent circuit for #n vibration mode
n_1	Number of layers on the primary side
n_2	Number of layers on the secondary side
N_L	Frequency constant in the planar direction
N_T	Frequency constant in the thickness direction
N_P	Frequency constant in the radial direction
P	Polarization
P_o	Output power
Q_m	Mechanical quality factor
R	Resistor of equivalent circuit model
R_m	Resistor in the equivalent circuit model of the piezoelectric transformer

R_{m1}, R_{m2}	Resistor in the physics-based equivalent circuit of the piezoelectric ceramics
R_L	Load resistor
r	Radius
S	Strain
T	Stress
t	Thickness
t_1	Thickness of primary-side layer
t_2	Thickness of secondary-side layer
V_{in}	Input voltage
V_{out}	Output voltage
S^E	Elastic compliance
W	Width
Y_{in}	Input admittance
Z_{in}	Input impedance
Z_o	Output impedance
s^E	Elastic compliance at constant electric field
ψ	Turns ratio of the physics-based equivalent circuit
ϵ^T	Permittivity at constant stress
ϵ_o	Permittivity of free space
ℓ	Length
ℓ_1	Length of primary side of Rosen piezoelectric transformer
ℓ_2	Length of secondary side of Rosen piezoelectric transformer
ρ	Density
ω	Angular frequency ($2\pi f$) in rad/sec
ω_r	Resonant angular frequency ($2\pi f_r$) in rad/sec
λ	Wavelength
σ	Poisson's ratio

CHAPTER 1

INTRODUCTION

1.1 Background

Piezoelectric transformers, a combination of piezoelectric actuators and piezoelectric transducers, are electrical energy transmission devices that contain no conventional magnetic elements, and that function via the electro-mechanical coupling between the adjacent piezoelectric actuators and transducers. Due to their special characteristics, in the past few decades, piezoelectric transformers have been developed and used widely in many applications, such as DC/DC converters and electronic ballasts for fluorescent lamps. Both piezoelectric actuators and piezoelectric transducers are made of piezoelectric elements. A piezoelectric element, shown in Figure 1.1, is composed of two electrode plates and a piezoelectric ceramic material, such as barium titanate-based ceramics. Generally speaking, piezoelectric elements can work in either longitudinal mode or transverse mode with a corresponding resonant frequency. In the longitudinal mode, the direction of the mechanical stress, T , is parallel to the electric or polarization direction, P , with a corresponding resonant frequency, as shown in Figure 1.2. In the transverse mode, the direction of the mechanical stress, T , is perpendicular to the electric or polarization direction, P , with a corresponding resonant frequency, as shown in Figure 1.3. A piezoelectric element can work as either a piezoelectric actuator or a piezoelectric transducer.

Instead of the magnetic field coupling that occurs between the primary and secondary windings in a conventional magnetic core transformer, piezoelectric transformers transfer electrical energy via electro-mechanical coupling between the primary and secondary piezoelectric elements for step-up or step-down voltage conversion. At present, there are three main piezoelectric transformer categories: Rosen [A1, A2, A3], thickness vibration mode [A4] and radial vibration mode [A5], shown in Figures 1.4, 1.5 and 1.6, respectively.

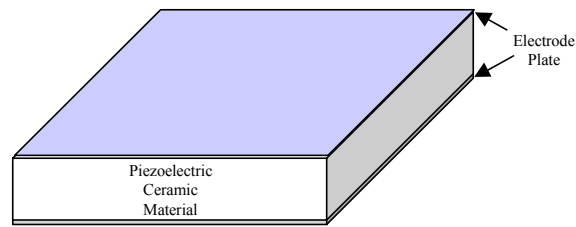


Figure 1.1. Piezoelectric element. This device is composed of two electrode plates and a piezoelectric ceramic material, such as barium titanate-based ceramics.

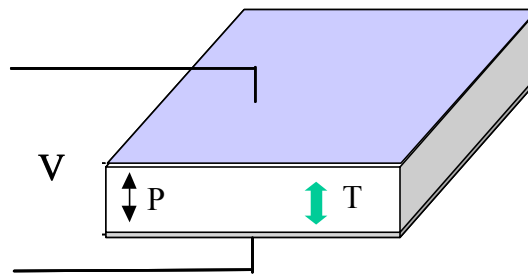


Figure 1.2. Longitudinal mode piezoelectric element. The direction of the operating stress, T , is parallel to the polarization direction, P , with a corresponding resonant frequency.

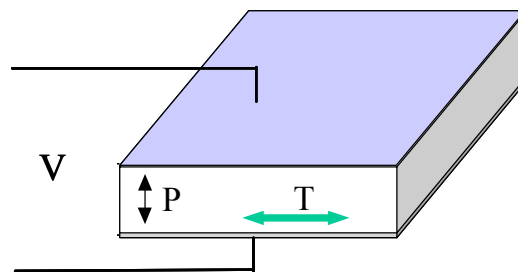


Figure 1.3. Transverse mode piezoelectric element. The direction of the operating stress, T , is perpendicular to the polarization direction, P , with a corresponding resonant frequency.

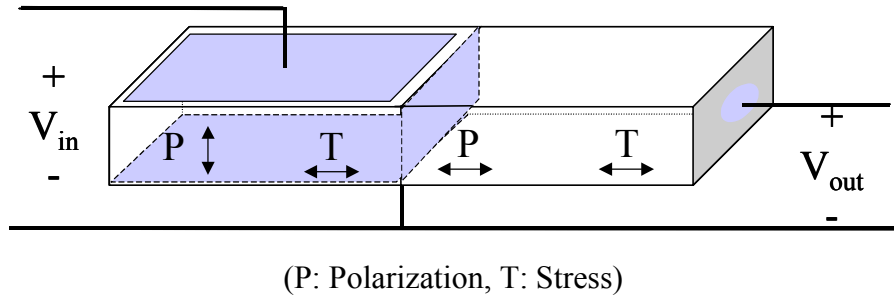


Figure 1.4. Rosen piezoelectric transformer. This piezoelectric transformer is a combination of a transverse mode piezoelectric actuator (primary side) and a longitudinal mode piezoelectric transducer (secondary side).

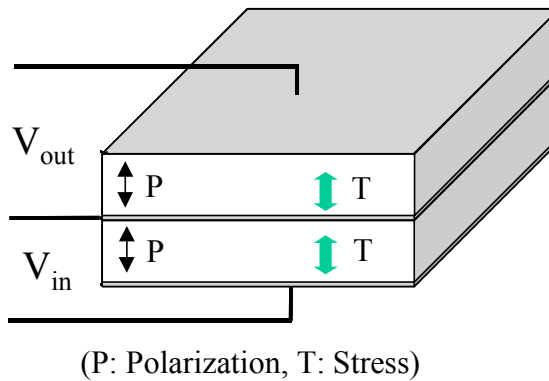


Figure 1.5. Thickness vibration piezoelectric transformer. This piezoelectric transformer is a combination of a longitudinal mode piezoelectric actuator (primary side) and a longitudinal mode piezoelectric transducer (secondary side).

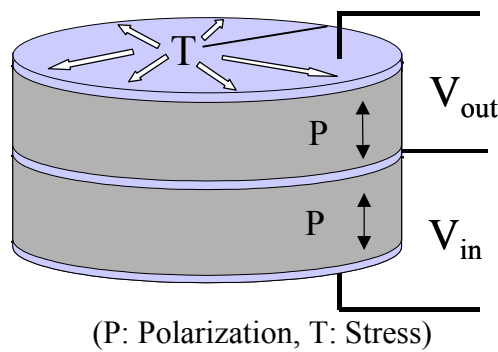


Figure 1.6. Radial vibration mode piezoelectric transformer. This piezoelectric transformer is a combination of a transverse mode piezoelectric actuator (primary side) and a transverse mode piezoelectric transducer (secondary side).

Invented by Dr. Rosen in the 1950s, the Rosen piezoelectric transformer, shown in Figure 1.4, is a combination of transverse mode piezoelectric actuators and longitudinal mode piezoelectric transducers. The characterization and modeling of Rosen piezoelectric transformers have been well studied and documented [A2, A3, A8]. Because of the inherent high voltage gain associated with the Rosen piezoelectric transformers, they are often referred to as high-voltage piezoelectric transformers. One application to which the Rosen piezoelectric transformer is well suited to drive high-voltage lamps, such as the cold cathode fluorescent lamps used as the backlight source for flat panel displays of notebook computers.

The thickness vibration mode piezoelectric transformer, developed by NEC of Japan in the 1990s, is a combination of longitudinal mode piezoelectric actuators and longitudinal mode piezoelectric transducers. Thickness vibration mode piezoelectric transformers, shown in Figure 1.5, have been studied and detailed characteristics and physics-based equivalent circuit models have been given [A8,A9]. The thickness vibration mode piezoelectric transformer is also known as the low-voltage piezoelectric transformer because of its inherent low voltage gain. Its present applications include DC/DC converter and adapter applications.

The radial vibration mode piezoelectric transformer, developed by FACE Electronics, USA in 1998, is a combination of piezoelectric actuators and transducers that both operate in the transverse mode. Although the radial vibration mode piezoelectric transformer, shown in Figure 1.6, has been invented and partially developed, its detailed characterization and modeling were not complete before this research work. This piezoelectric transformer can be utilized in such applications as DC/DC converters, adapters, and electronic ballasts for linear/compact fluorescent lamps [D8].

Because of their different vibration modes and mechanical structures, these three main piezoelectric transformer categories have different mechanical and electrical characteristics. These three transformers can be characterized by a single-branch

equivalent circuit model, shown in Figure 1.7, for the specific frequency bandwidths around the corresponding mechanical resonant frequencies [A2-A3, A6, A8-A13]. The mechanical dimensions and material parameters of piezoelectric transformers determine the parameters of the single-branch equivalent circuit model. This model is identical to a parallel-series resonant circuit, which has been widely applied to resonant converter, inverter or electronic ballast circuits. Figure 1.8 shows a typical conventional electronic ballast circuit with a complicated parallel-series resonant tank and a turn-off snubber capacitor, C_{d1} , for the switches, S_1 and S_2 .

Different mechanical structures associated with these three transformers result in different equivalent circuit parameters; therefore, they are suited to different applications.

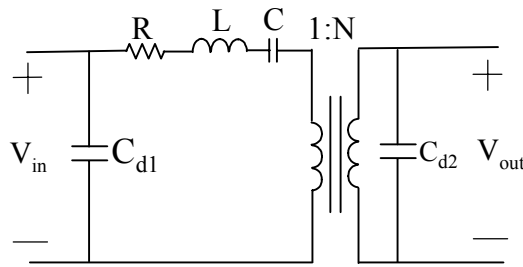


Figure 1.7. Equivalent circuit model for piezoelectric transformers. Components R , L and C are the equivalent mechanical components, analogous to electrical terms.

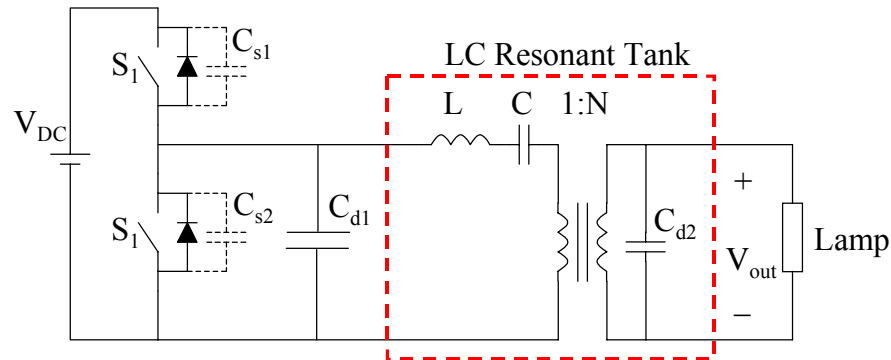


Figure 1.8. Typical conventional electronic ballast circuit. This ballast circuit has a complicated L-C resonant tank circuit with a turn-off snubber capacitor, C_{d1} , for the switches, S_1 and S_2 .

Many prior technologies [B1-B13] have tried to employ piezoelectric transformers in order to develop converter, inverter or electronic ballast. Because of the input capacitor of piezoelectric transformers, these previous topologies utilized one or more additional magnetic devices, such as an inductor, in order to achieve zero-voltage-switching (ZVS) condition. By using this prior approach, the specific characteristics of the piezoelectric transformers were not fully utilized, necessitating the extra expenses of additional magnetic devices. Ideally, piezoelectric transformers should be employed without any additional magnetic devices for the effective cost reduction of electronic components used in the electronic circuits.

Among the applications for piezoelectric transformers, the electronic ballast for fluorescent lamps is one of the most interesting areas of research for the lighting industry. Current estimates show that approximately 20% of the total electric energy consumption in the United States is for lighting. The most popular light sources are incandescent lamps and fluorescent lamps for residential and commercial uses. Residential and commercial fluorescent lamps include linear fluorescent lamps and compact fluorescent lamps (CFLs). The efficacy (the ratio of lumens or light flux to the consumed electrical power) and lifetime of lamps are the major considerations for choosing light sources, especially in industrial applications. High-efficacy lamps can save electrical energy, and hence directly affect the quantity of natural resources used by power plants. In particular, fossil fuel energy sources can affect levels of pollutants and atmospheric contaminants (such as carbon dioxide), which contribute to what is known as the greenhouse effect. Longer lamp life can conserve the natural resources for manufacturing lamps as well as the maintenance expense of replacing lamps in commercial use [D1, D2, D3].

With the continuing concern for efficient use of energy resources, the national Energy Policy Act (EPACT) provides a further incentive to encourage individuals, companies, organizations and institutions to purchase and use energy-efficient lighting products. This comprehensive bill affects virtually every aspect of U.S. energy resource allocation, including conservation, consumption, distribution and efficiency, and is

designed to dramatically cut the nation's energy consumption through better conservation and more competitive electricity-generation practices. This legislation naturally has a major bearing on the lighting industry [D3].

Generally speaking, the efficacy of incandescent lamps is 17.5 lumen/watt. However, the efficacy of fluorescent lamps ranges from 65 to 80 lumen/watt. It is quite evident that fluorescent lamps are much more efficient than incandescent lamps. Furthermore, the lifetime of incandescent lamps is 750 to 1,500 hours, while the lifetime of fluorescent lamps is 20,000 hours [D4].

A simple example can illustrate the savings potential of fluorescent-based over incandescent lighting. Based on 10 cents/kWh and a requirement of 1,400 continuous lumens over 20,000 hours, a 20W fluorescent lamp can provide the required illumination at a savings of \$120 in utility cost alone. This does not include the costs for replacing incandescent bulbs. The superiority of fluorescent lamps for saving energy and resources is obvious.

Ballast circuits for driving fluorescent lamps can be categorized as either magnetic or electronic. Because magnetic ballasts are bulky and emit an audible low-frequency humming noise, they are as not appealing as electronic ballasts. Electronic ballast circuit production is one of the highest volume of any electronic products in the world. The market for electronic ballasts is large and can be expected to enjoy a healthy rate of growth over the next several years. According to the valuable market survey from the premier industry research organization, the Darnell Group, Inc., the global market is expected to grow from about \$7.5 billion in 1999 to \$10.4 billion in 2004, a compound growth rate of 6.7 percent [D3].

The conventional electronic ballast circuit employs a complicated resonant tank circuit, as shown in Figure 1.8. This resonant tank circuit is composed of two capacitors and two magnetic components. In order to reduce the cost of these four components, most lighting companies worldwide have focused their efforts on finding cost-effective

component suppliers. Fortunately, the piezoelectric transformer is a potential alternative. A piezoelectric transformer is a electro-mechanical device that can replace the L-C resonant tanks of conventional electronic ballasts, thus providing a good method for reducing the cost and increasing the attraction to residential and commercial users in the worldwide lighting industry.

1.2 Motivation

Since the radial vibration mode piezoelectric transformer is more recently proposed and developed than either the Rosen or the thickness vibration mode piezoelectric transformer, its detailed characterization and modeling were not yet complete before this research work.

In order to provide a good reference for the design and application of radial vibration mode piezoelectric transformers, its physics-based equivalent circuit model for major vibration mode needs to be derived and verified.

Besides the major vibration mode, other spurious vibration modes need to be considered in order to design piezoelectric transformers so that they operate within a suitable frequency range for those application circuits with wide operation frequency ranges. Therefore, multi-branch equivalent circuit needs to be able to include these other spurious vibration modes.

To efficiently reduce the cost of electronic components for electronic ballasts, inductor-less piezoelectric transformer electronic ballast circuits for fluorescent lamps need to be developed by fully utilizing the intrinsic characteristics of piezoelectric transformers without requiring any additional magnetic devices, which is different from prior demonstrations. Furthermore, an inductor-less piezoelectric transformer electronic ballast circuit incorporating PFC function needs to be developed in order to meet input current harmonic regulations, such as the IEC-61000.

1.3 Objectives of Research and Method of Approach

Based on the motivations mentioned in the previous section, the research of this dissertation works toward the following goals:

- (1) Derivation of the physics-based equivalent circuit model for the major radial vibration mode of the newly invented radial vibration mode piezoelectric transformers, based on the piezoelectric and wave equations.
- (2) Characterization of the radial vibration mode piezoelectric transformers in order to verify the derived physics-based equivalent circuit model for the major radial vibration mode.
- (3) Proposal of an improved multi-branch equivalent lumped-parameter circuit model to more precisely describe the voltage gain characteristic of radial vibration mode piezoelectric transformers, which can include other spurious vibration modes in wide frequency ranges besides the major vibration mode frequency range.
- (4) Proposal of a circuit design technology to fully utilize the characteristics of piezoelectric transformers to design an electronic ballast circuit for driving linear fluorescent lamps. Without the use of any additional magnetic devices, this proposed electronic ballast circuit is still able to have its switches achieve ZVS in order to have very low switching losses.
- (5) Inclusion of PFC in the proposed inductor-less piezoelectric transformer electronic ballast circuit in order to meet the IEC-61000 input current harmonic regulation.

1.4 Dissertation Outline and Major Results

This dissertation is composed of five chapters, an appendix and references. The chapters are briefly described as follows.

Chapter 1 briefly reviews the development and application background of piezoelectric transformers. Then, this chapter provides the objectives and an outline of the research work in this dissertation.

Chapter 2 introduces the physics-based equivalent circuit model of piezoelectric elements as well as reviews the physics-based equivalent circuit models of the Rosen and the thickness vibration mode piezoelectric transformers. This chapter derives a physics-based equivalent circuit model of radial vibration mode piezoelectric transformers for major vibration mode in order to provide a good reference for the design and application of this piezoelectric transformer. Like the other two piezoelectric transformers, the derivation of this model is also based on the piezoelectric and wave equations.

Chapter 3 characterizes the equivalent circuit model for the major vibration mode of the radial vibration mode piezoelectric transformer in order to verify the physics-based equivalent circuit model derived in Chapter 2. Furthermore, this chapter presents an improved equivalent circuit model for more precisely describing the voltage gain characteristic of the radial vibration mode piezoelectric transformers, which can include other spurious vibration modes in wide frequency ranges. This improved model can effectively include the impact of the spurious vibration frequencies adjacent to the major radial vibration frequency for more accurate circuit prediction in prototype applications.

Based on the characteristics of the radial vibration mode piezoelectric transformer, Chapter 4 presents a cost-effective inductor-less piezoelectric transformer electronics ballast circuit. This circuit utilizes a radial vibration mode piezoelectric transformer that replaces the L-C resonant tank and turn-off snubber capacitor of conventional electronic

ballasts to ignite and sustain a linear fluorescent lamp. This topology can lead to a significant reduction in the number of components and total cost outlay in the construction of electronic ballasts. This chapter also presents the design and implementation of an inductor-less voltage source charge pump PFC (VS-CP-PFC) electronic ballast in order to meet input current harmonic regulations, such as the IEC-61000. This developed circuit also utilizes a radial vibration mode piezoelectric transformer that replaces the L-C resonant tank and turn-off snubber capacitor of conventional VS-CP-PFC electronic ballast and achieves PFC function. Finally, the experimental results of a developed prototype circuit are shown in order to verify the feasibility of the proposed technology.

Chapter 5 gives the conclusions of this dissertation, and proposes ideas for future work.

CHAPTER 2

MODELING OF PIEZOELECTRIC TRANSFORMERS

2.1 Introduction

The piezoelectric transformer is a combination of a piezoelectric actuator on the primary side and a piezoelectric transducer on the secondary side. Both the actuator and transducer are made of piezoelectric elements, and are composed of electrode plates and piezoelectric materials, such as barium titanate-based ceramics. With a corresponding resonant frequency, a piezoelectric element can work in either longitudinal vibration mode or transverse vibration mode to function as an actuator or a transducer.

Piezoelectric transformers can be categorized as one of three major types: Rosen [A1, A2, A3], thickness vibration mode [A4] or radial vibration mode [A5]. Because of their different vibration modes and mechanical structures, these three types of piezoelectric transformers have different mechanical and electrical characteristics. Based on the piezoelectric and wave equations, the physics-based equivalent circuit models of Rosen and thickness vibration mode piezoelectric transformers for their major vibration modes have been studied in prior works [A2, A3, A6, A7, A8, A9]. However, this kind of model for the major vibration mode of the newly invented radial vibration mode piezoelectric transformers had not yet been derived at the time of this work. With the good reference provided by physics-based equivalent circuit models, piezoelectric transformers can be actually designed rather than being manufactured by trial and error. Through use of the models, performance can actually be optimized because circuit networks can be analyzed and designed before implementation. In other words, the physics-based equivalent circuit model is a very important tool for the design and

analysis of piezoelectric transformers, and is also very useful for application circuit design.

This chapter first introduces the operational principles and physics-based equivalent circuit model of piezoelectric elements. After briefly reviewing the operational principles and existing physics-based equivalent circuit models for Rosen and thickness vibration mode piezoelectric transformers, this chapter will present the derivation of this type of model for the major vibration mode of the newly invented radial vibration mode piezoelectric transformers. Like the other two piezoelectric transformers, this derivation is also based on the piezoelectric and wave equations.

2.2 Physics-based Equivalent Circuit Model of a Piezoelectric Element

Piezoelectric transformers are composed of piezoelectric actuators on the primary side and piezoelectric transducers on the secondary side. These actuators and transducers are made of piezoelectric elements, as shown in Figure 2.1, which are composed of electrode plates and piezoelectric ceramic materials, such as barium titanate-based ceramics. Piezoelectric elements can work in longitudinal mode or transverse mode with a corresponding resonant frequency.

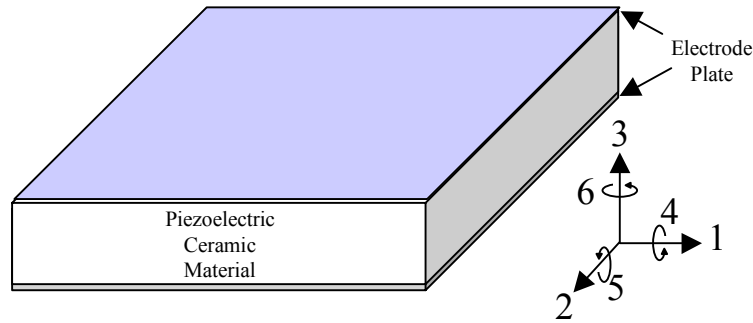


Figure 2.1. Piezoelectric element.

The behavior of piezoelectric elements can be described using the linear piezoelectric equation [A2, A6, A8], shown in Equations (2.1) and (2.2). Equation (2.1) describes that the mechanical strain, S , results from the applied mechanical stress, T , and the electric field, E , on a piezoelectric element. Without applying the mechanical stress, T , an applied electric field, E , on a piezoelectric element, results in the mechanical strain, S , in the piezoelectric element, which functions as an actuator. Equation (2.2) describes that the induced electric displacement results from the applied mechanical stress and applied electric field in a piezoelectric element. Without applying the electric field, E , an applied mechanical stress, T , on a piezoelectric element results in an electric displacement induced on the electric plates of the piezoelectric element, which functions as a transducer.

Piezoelectric Equations

- Piezoelectric Actuator:

$$S = s^E \cdot T + d_t \cdot E \quad (2.1)$$

- Piezoelectric Transducer:

$$D = d \cdot T + \epsilon^T \cdot E \quad (2.2)$$

where

S is mechanical strain,

T is mechanical stress,

E is electric field,

D is electric displacement,

d is piezoelectric constant,

s^E is elastic compliance at constant electric field, and

ϵ^T is permittivity at constant stress.

Since the electrode plates of the piezoelectric element are perpendicular to the direction of axis 3, the electric field, E, and electric placement, D, are in the direction of axis 3. Therefore, the non-zero components of the electric field, E, and electric placement, D, are E_3 and D_3 , respectively. Hence, the electric field, E, and electric placement, D, in Equations (2.1) and (2.2) can be re-written as the follows:

$$D = \begin{bmatrix} 0 \\ 0 \\ D_3 \end{bmatrix} \quad \text{and} \quad E = \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix}. \quad (2.3)$$

When the piezoelectric element operates in a different operation mode, such as longitudinal or transverse mode, with a corresponding operating frequency, the mechanical strain, S, and the mechanical stress, T, can occur in the direction of axes 1, 2, 3, 4, 5 or 6 [A2, A6, A8].

2.2.1 Longitudinal Mode Piezoelectric Element

When a piezoelectric element works in longitudinal mode, as shown in Figure 2.2, with its corresponding operating frequency related to the wavelength in the direction of operating stress, T , the direction of the operating stress, T , is parallel to the polarization direction, P . The polarization direction, P , is the same as that of the electrical field, E , and the electrical displacement, D , in Equations (2.1) and (2.2). Therefore, the non-zero components of the mechanical stress, T , and the mechanical strain, S , in Equations (2.1) and (2.2) are T_3 and S_3 , respectively. T_3 and S_3 are in parallel with the electric displacement component, D_3 , and the electrical field component, E_3 , in the direction of the axis 3. Hence, the mechanical stress, T , and mechanical stress, S , in Equations (2.1) and (2.2) can be re-written as follows [A2, A6, A8]:

$$T = \begin{bmatrix} 0 \\ 0 \\ T_3 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 0 \\ 0 \\ S_3 \end{bmatrix} \quad (2.4)$$

where T_3 is the component of mechanical stress in the direction of axis 3,
 D_3 is the component of electric displacement in the direction of axis 3, and
 E_3 is the component of electric field in the direction of the 3-axis.

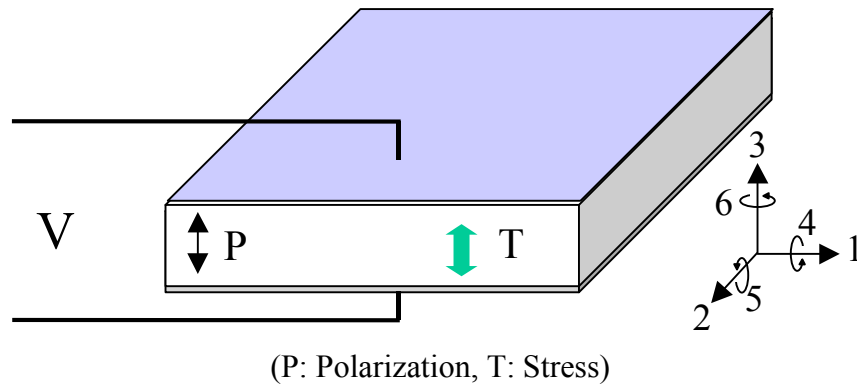


Figure 2.2. Longitudinal mode piezoelectric element.

Therefore, based on Equations (2.3) and (2.4), Equations (2.1) and (2.2) can be simplified into Equations (2.5) and (2.6) for the piezoelectric element operating in longitudinal mode as an actuator or a transducer, respectively [A2, A6, A8].

Piezoelectric Element in Longitudinal Mode

- Piezoelectric Actuator

$$\begin{bmatrix} 0 \\ 0 \\ S_3 \end{bmatrix} = \begin{bmatrix} S_{11}^E & S_{12}^E & S_{13}^E \\ S_{12}^E & S_{11}^E & S_{13}^E \\ S_{13}^E & S_{13}^E & S_{33}^E \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ T_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (2.5)$$

- Piezoelectric Transducer

$$\begin{bmatrix} 0 \\ 0 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{31} & d_{33} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ T_3 \end{bmatrix} + \begin{bmatrix} \epsilon_{11}^T & 0 & 0 \\ 0 & \epsilon_{22}^T & 0 \\ 0 & 0 & \epsilon_{33}^T \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (2.6)$$

2.2.2 Transverse Mode Piezoelectric Element

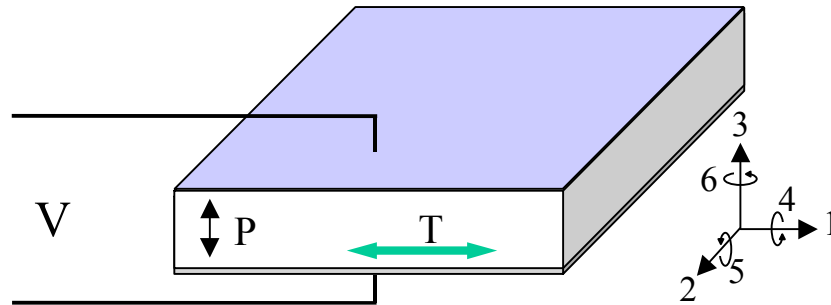
When a piezoelectric element works in transverse mode, as shown in Figure 2.3, with its corresponding operating frequency related to the wavelength in the direction of the operating stress, T , the direction of the operating stress, T , is perpendicular to the polarization direction, P . The polarization direction, P , is the same as that of the electrical field, E , and the electrical displacement, D , in Equations (2.1) and (2.2). Therefore, the non-zero components of the mechanical stress, T , and the mechanical strain, S , in Equations (2.1) and (2.2) are T_1 and S_1 , respectively. T_1 and S_1 are perpendicular to the electric displacement component, D_3 , and electrical field component, E_3 , in the direction of axis 3. Hence, the mechanical stress, T , and the mechanical strain, S , in Equations (2.1) and (2.2) can be re-written as follows [A2, A6, A8]:

$$T = \begin{bmatrix} T_1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} S_1 \\ 0 \\ 0 \end{bmatrix} \quad (2.7)$$

where T_1 is the component of mechanical stress in the direction of axis 1,

D_3 is the component of electric displacement in the direction of axis 3,

E_3 is the component of electric field in the direction of axis 3.



(P: Polarization, T: Stress)

Figure 2.3. Transverse mode piezoelectric element.

Therefore, based on Equations (2.3) and (2.4), Equations (2.1) and (2.2) can be simplified into Equations (2.8) and (2.9) for the piezoelectric element operating in transverse mode as an actuator or a transducer, respectively [A2, A6, A8].

Piezoelectric Element in Transverse Mode

- Piezoelectric Actuator

$$\begin{bmatrix} S_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} S_{11}^E & S_{12}^E & S_{13}^E \\ S_{12}^E & S_{11}^E & S_{13}^E \\ S_{13}^E & S_{13}^E & S_{33}^E \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (2.8)$$

- Piezoelectric Transducer

$$\begin{bmatrix} 0 \\ 0 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{31} & d_{33} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_{11}^T & 0 & 0 \\ 0 & \epsilon_{22}^T & 0 \\ 0 & 0 & \epsilon_{33}^T \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (2.9)$$

Based on the piezoelectric and wave equations, the physics-based equivalent circuit model for piezoelectric elements can be derived, as shown in Figure 2.4 [A2, A6, A8], where

V is the applied or induced voltage on the electrode plates,

C_{dm} is the capacitance between the electrode plates,

L_m is the equivalent mechanical mass analogy to electric inductor,

C_m is the equivalent mechanical compliance analogy to electric capacitor,

R_m is the equivalent mechanical resistance analogy to electric resistor,

E_m is the equivalent mechanical force analogy to electric field,

ψ is the equivalent mechanical force factor analogy to turns ratio.

These equivalent parameters in the physics-based equivalent circuit model are dependent on the dimensions, piezoelectric material coefficients and operation mode of the piezoelectric element. The physics-based equivalent circuit model can be derived based on this physics-based equivalent circuit model [A2, A6, A8].

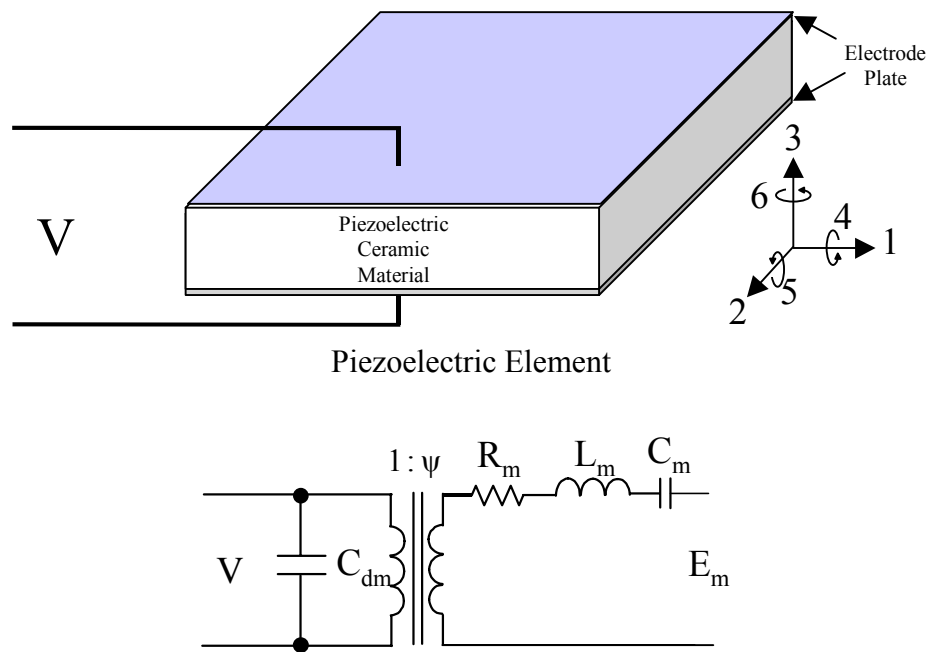


Figure 2.4. Physics-based equivalent circuit for piezoelectric element.

2.3 Physics-Based Equivalent Circuit Models for Piezoelectric Transformers

Since the piezoelectric transformer is the combination of two piezoelectric elements as the actuator and transducer, the physics-based equivalent circuit model of a piezoelectric transformer can be derived by connecting two physics-based equivalent circuit models of piezoelectric elements, as shown in Figure 2.5.

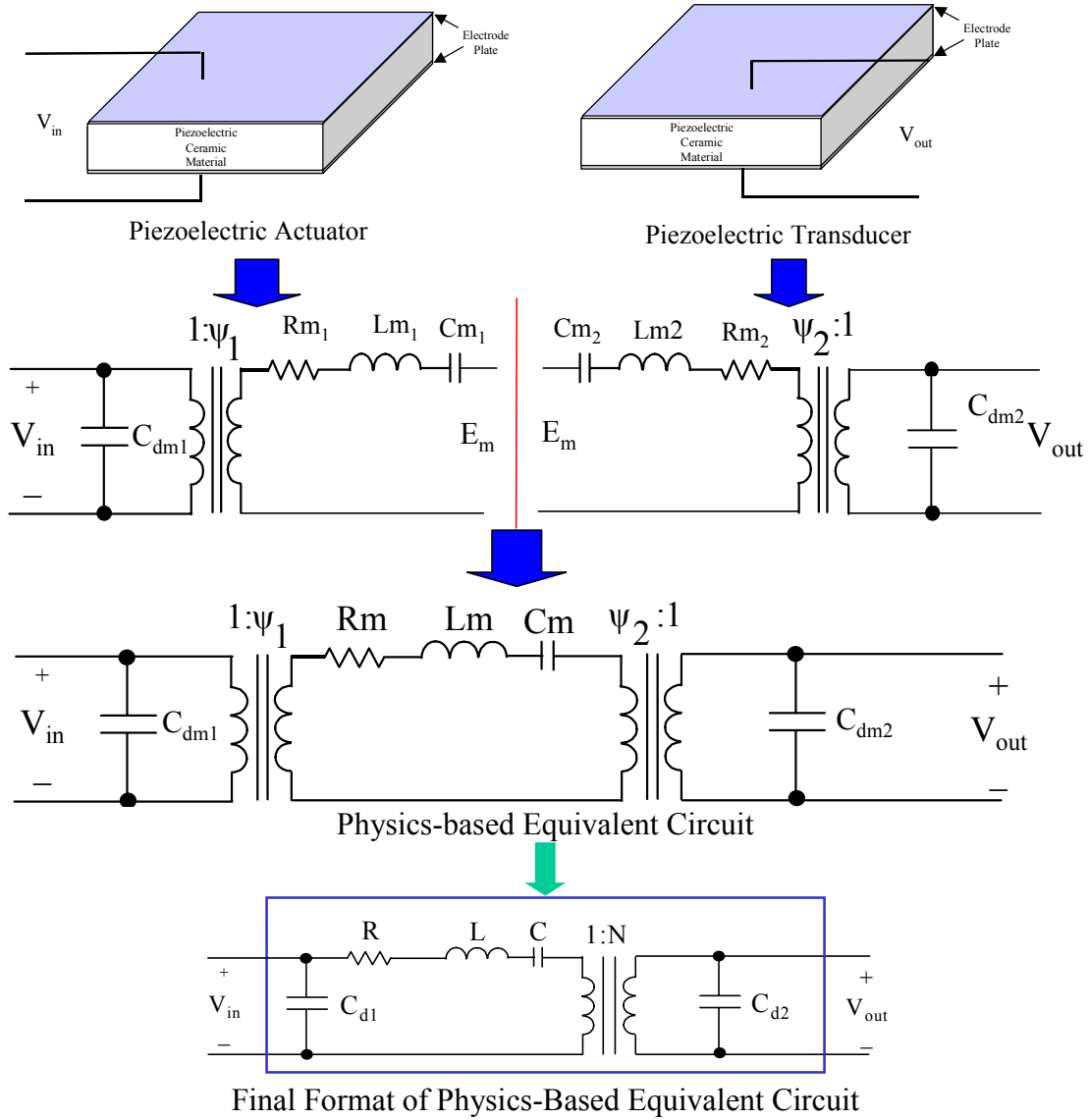


Figure 2.5. Derivation process of physics-based equivalent circuit model for piezoelectric transformers.

2.3.1 Rosen Piezoelectric Transformer

The Rosen piezoelectric transformer, shown in Figure 2.6, is the combination of two piezoelectric elements: a transverse mode piezoelectric actuator on the primary side and a longitudinal mode piezoelectric transducer on the secondary side. When an input voltage, V_{in} , is applied to the primary side, i.e., the transverse mode piezoelectric actuator, the material becomes polarized in the direction parallel to that of the material thickness. The greatest vibration strain occurs in the planar direction perpendicular to the polarization direction. The planar vibration of the transverse mode piezoelectric actuator transmits to the longitudinal mode piezoelectric transducer. With the transmitting vibration from the primary side, the longitudinal mode piezoelectric transducer induces an electric charge on the electrode plates of the piezoelectric transducer to generate the output voltage, V_{out} . The vibration direction of the secondary side is parallel to the direction of the induced polarization, P .

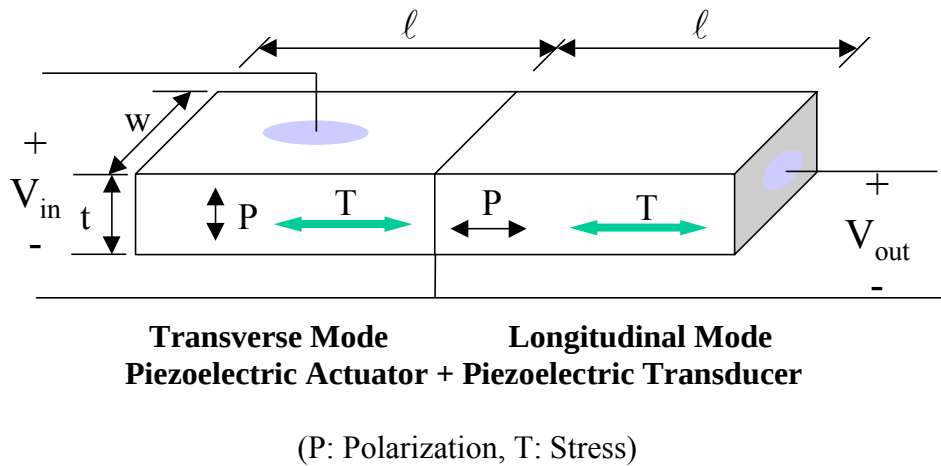


Figure 2.6. Rosen piezoelectric transformer.

Based on Equations (2.5) and (2.9), the parameters, R , L , C , N , C_{d1} and C_{d2} , of the equivalent circuit model for the Rosen piezoelectric transformer, shown in Figure 2.7, were derived and verified [A2, A3, A6, A8].

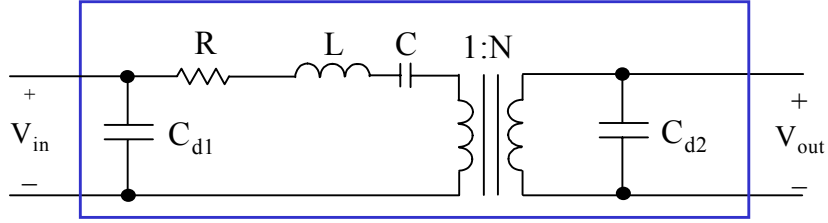


Figure 2.7. Physics-based equivalent circuit model of Rosen piezoelectric transformers.

Besides being dependent on piezoelectric material coefficients, the parameters of the physics-based equivalent circuit model are also dependent on the dimensions of the Rosen piezoelectric transformers, as shown in the following Equations (2.10) to (2.15) [A2, A3, A6, A8]:

$$C_{d1} \propto \frac{w \cdot \ell}{t} \quad (2.10)$$

$$C_{d2} \propto \frac{w \cdot t}{\ell} \quad (2.11)$$

$$R \propto \frac{t}{w} \quad (2.12)$$

$$L \propto \frac{t}{w} \cdot \ell \quad (2.13)$$

$$C \propto \frac{w}{t} \cdot \ell \quad (2.14)$$

$$N \propto \frac{\ell}{t} \quad (2.15)$$

In practical applications, the Rosen piezoelectric transformer can be made with a multi-layer structure in order to have a higher turns ratio, as shown in Figure 2.8.

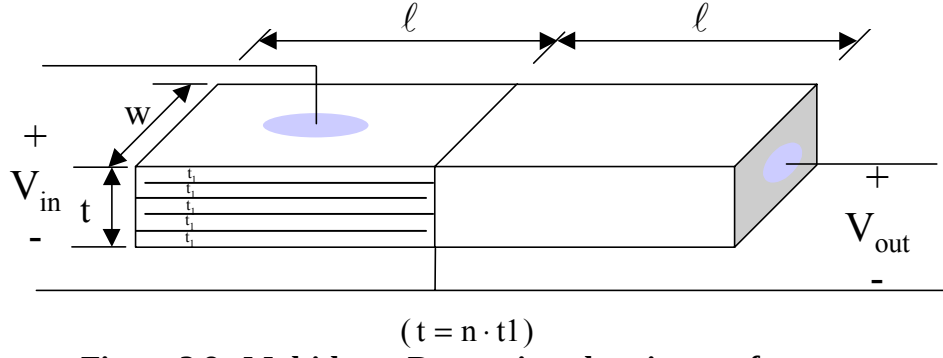


Figure 2.8. Multi-layer Rosen piezoelectric transformer.

Based on the equivalent circuit model of the single-layer Rosen piezoelectric transformer, the equivalent circuit model of the multi-layer Rosen piezoelectric transformer can be obtained. Like those of the single-layer transformer, the parameters of the physics-based equivalent circuit model are also dependent on the dimensions and piezoelectric material coefficients of Rosen piezoelectric transformers, as shown in the following Equations (2.16) to (2.21):

$$C_{d1} \propto \frac{w \cdot \ell}{t} \cdot n^2 \quad (2.16)$$

$$C_{d2} \propto \frac{w \cdot t}{\ell} \quad (2.17)$$

$$R \propto \frac{t}{w} \cdot \frac{1}{n^2} \quad (2.18)$$

$$L \propto \frac{t}{w} \cdot \ell \cdot \frac{1}{n^2} \quad (2.19)$$

$$C \propto \frac{w}{t} \cdot \ell \cdot n^2 \quad (2.20)$$

$$N \propto \frac{\ell}{t} \cdot n \quad (2.21)$$

where n is the layer number.

2.3.2 Thickness Vibration Mode Piezoelectric Transformers

The thickness vibration mode piezoelectric transformer, shown in Figure 2.9, is the combination of two piezoelectric elements: a longitudinal mode piezoelectric actuator and a longitudinal mode piezoelectric transducer. With the applied voltage, V_{in} , on the primary side, i.e., the piezoelectric actuator, the material becomes polarized in the direction parallel to that of the material thickness. The greatest vibration strain occurs in the thickness direction parallel to the polarization direction. The thickness vibration of the primary side, the piezoelectric actuator, transmits to the secondary side, the piezoelectric transducer. With the transmitting vibration from the primary side, the piezoelectric transducer induces an electric charge on the electrode plates of the piezoelectric transducer in order to generate the output voltage, V_{out} . The vibration direction of the secondary side, the transverse mode piezoelectric transducer, is also parallel to the direction of the induced polarization. The thickness vibration mode piezoelectric transformer is also known as a low-voltage piezoelectric transformer because of its inherent low voltage gain. Its present applications include use in converters and adapters.

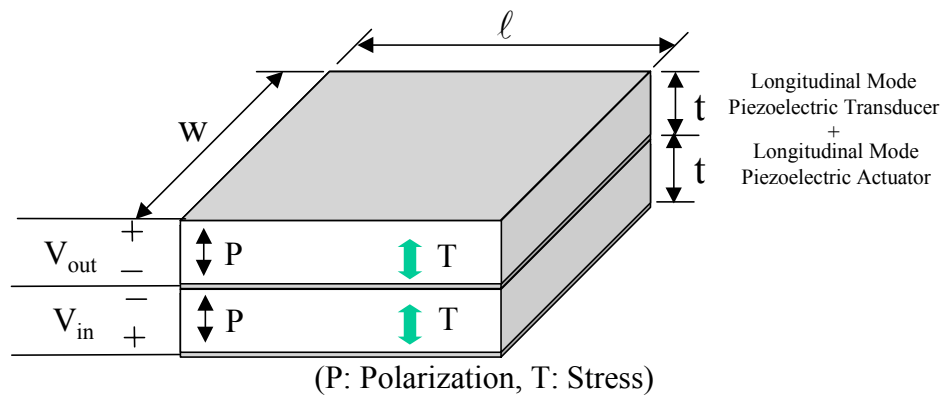


Figure 2.9. Thickness vibration mode piezoelectric transformer.

Based on Equations (2.5) and (2.6), the parameters, R , L , C , N , C_{d1} and C_{d2} , of the equivalent circuit model, as shown in Figure 2.10, for the major vibration mode of the thickness vibration mode piezoelectric transformer were derived and verified [A6, A8, A9].

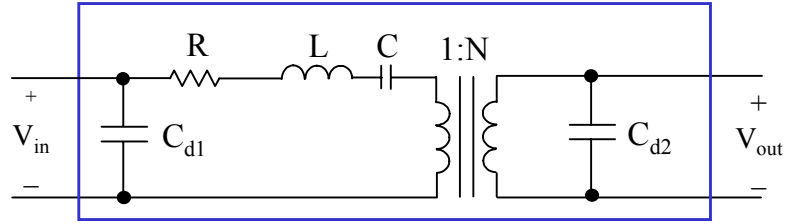


Figure 2.10. Equivalent circuit model of a thickness vibration mode piezoelectric transformer.

The parameters of the physics-based equivalent circuit model are dependent on the dimensions and piezoelectric material coefficients of thickness vibration mode piezoelectric transformers, as shown in the following Equations (2.22) to (2.27):

$$C_{d1} \propto \frac{\ell \cdot w}{t_1} \quad (2.22)$$

$$C_{d2} \propto \frac{\ell \cdot w}{t_2} \quad (2.23)$$

$$R \propto \frac{t_1^2}{\ell \cdot w} \quad (2.24)$$

$$L \propto \frac{(t_1 + t_2) \cdot t_1^2}{\ell \cdot w} \quad (2.25)$$

$$C \propto \frac{(t_1 + t_2) \cdot \ell \cdot W}{t_1^2} \quad (2.26)$$

$$N \propto \frac{t_1}{t_2} \quad (2.27)$$

Besides the single-layer structure, the thickness vibration mode piezoelectric transformers can be made in a multi-layer structure, as shown in Figure 2.11 [A9]. Equations (2.28) to (2.33) are the parameters of the equivalent circuit model for multi-layer thickness vibration mode piezoelectric transformers [A9].

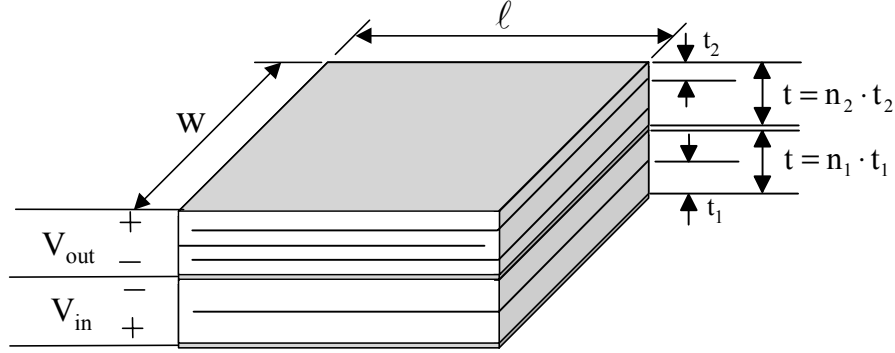


Figure 2.11. Multi-layer structure of thickness vibration mode piezoelectric transformer [A9].

$$C_{d1} \propto \frac{\ell \cdot w}{t} \cdot n_1 \quad (2.28)$$

$$C_{d2} \propto \frac{\ell \cdot w}{t} \cdot n_2 \quad (2.29)$$

$$R \propto \frac{t_1^2}{\ell \cdot w} \quad (2.30)$$

$$L \propto \frac{(t_1 + t_2) \cdot t_1^2}{\ell \cdot w} \quad (2.31)$$

$$C \propto \frac{(t_1 + t_2) \cdot \ell \cdot w}{t_1^2} \quad (2.32)$$

$$N \propto \frac{n_2}{n_1} \quad (2.33)$$

2.4 Derivation of Physics-Based Equivalent Circuit Model for Radial Vibration Mode Piezoelectric Transformers

The radial vibration mode piezoelectric transformer [A5], shown in Figure 2.12, is the combination of a transverse mode piezoelectric actuator and a transverse mode piezoelectric transducer. With the applied voltage, V_{in} , on the primary side, i.e., the piezoelectric actuator, the material becomes polarized in the direction parallel to that of the material thickness. In this case, the greatest vibration strain occurs in the planar direction perpendicular to the polarization direction. The planar vibration of the piezoelectric actuator transmits to the piezoelectric transducer. The vibration transmits from the primary side, inducing an electric charge on the electrode plates of the piezoelectric transducer in order to generate the output voltage, V_{out} . The vibration direction of the transverse mode piezoelectric transducer is perpendicular to the direction of the induced polarization.

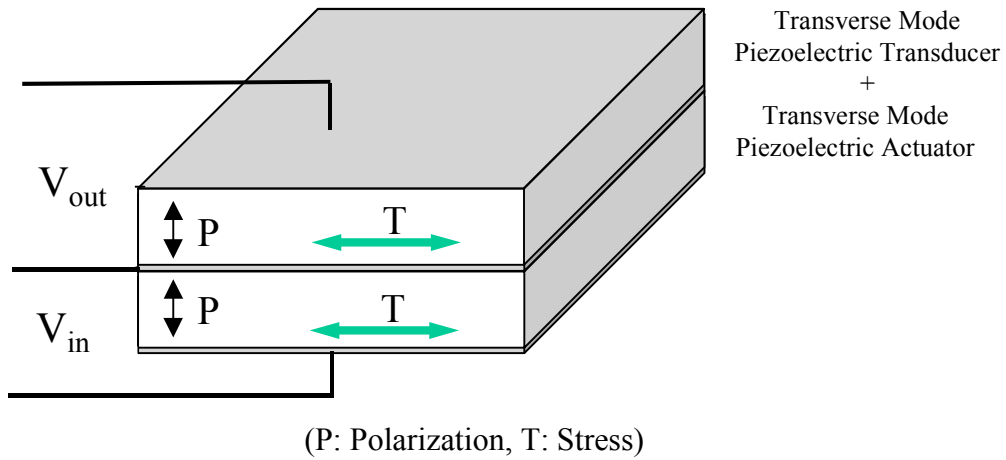


Figure 2.12. Radial vibration mode piezoelectric transformer [A5]. This piezoelectric transformer is a combination of a transverse mode piezoelectric actuator (primary side) and a transverse mode piezoelectric transducer (secondary side).

For the square-shaped radial vibration mode piezoelectric transformer, as shown in Figure 2.12, the distances from the center to the edges of the electrode plates are not the same. Therefore, its wavelengths of planar vibration are not the same, which causes additional vibration frequencies. In order to eliminate the vibration frequencies other than the fundamental vibration frequency, radial vibration mode piezoelectric transformers are made in a round shape in practical applications [D8], as shown in Figure 2.13. The round radial vibration mode piezoelectric transformer has the same distance, r , from its center to the edges of the electrode plates. Therefore, the fundamental vibration wavelength, λ , of the round radial vibration mode piezoelectric transformer is

$$\lambda = 2 \cdot r, \quad (2.34)$$

where r is the radius of the round-shaped radial vibration mode piezoelectric transformer.

This radial vibration mode piezoelectric transformer can be utilized in ballasts, adapters and converters.

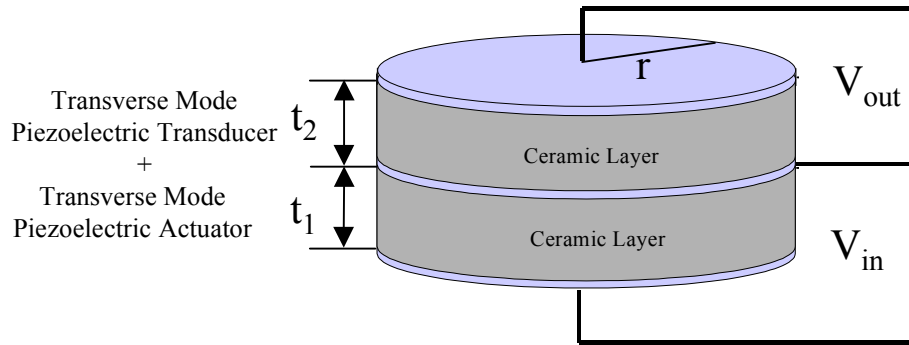


Figure 2.13. Round-shaped radial vibration mode piezoelectric transformer, where r is the radius, t_1 is the thickness of the primary side, and t_2 is the thickness of the secondary side. Because the distances, r , from the center to the edge of the electrode plates are the same, there are fewer vibration frequencies other than the fundamental frequency. These additional frequencies are problematic in the square-shaped radial vibration mode piezoelectric transformer.

This section will derive the equivalent circuit models for radial vibration mode piezoelectric transformers, including single-layer and multi-layer structures, based on the following assumptions:

- (1) *There are no mechanical losses within the interfaces of the piezoelectric ceramic layers (piezoelectric ceramic-to-glue, glue-to-copper);*
- (2) *The piezoelectric actuator and piezoelectric transducer vibrate identically to each other with no losses; and*
- (3) *Vibration modes other than radial vibration mode, such as thickness vibration mode and shear vibration mode, are not considered.*

2.4.1 Single-Layer Structure

As stated in previous chapters, piezoelectric transformers are a combination piezoelectric actuators and piezoelectric transducers. The behaviors of the piezoelectric actuators and transducers can be described by the following linear piezoelectric equations:

Piezoelectric Equations:

$$S = s^E \cdot T + d \cdot E \quad (2.35)$$

$$D = d \cdot T + \epsilon^T \cdot E \quad (2.36)$$

where

S is the mechanical strain,

T is the mechanical stress,

E is the electric field,

D is the electric displacement,

d is the piezoelectric constant,

s^E is the elastic compliance at constant electric field, and

ϵ^T is the permittivity at constant mechanical stress.

Equation (2.35) shows the behavior of piezoelectric actuators. The strain, S , of the piezoelectric actuator can be generated by either an applied electric field, E , on the electrode plates, or by an applied stress, T , on the piezoelectric actuator. Equation (2.36) shows the behavior of piezoelectric transducers. The electric displacement, D , of piezoelectric transducers can be generated on the electrode plates by either the applied electric field, E , or the applied stress, T .

Since the piezoelectric elements of radial vibration mode piezoelectric transformers work in transverse mode with a corresponding operating frequency, the mechanical stress, T_1 , is the only mechanical stress component. Therefore, Equations (2.35) and (2.36) can be simplified as shown in Equations (2.37) and (2.38), respectively:

Piezoelectric Actuator in Transverse Mode

$$\begin{bmatrix} S_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} S_{11}^E & S_{21}^E & S_{31}^E \\ S_{12}^E & S_{22}^E & S_{11}^E \\ S_{13}^E & S_{13}^E & S_{33}^E \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{31} & d_{33} \end{bmatrix}_t \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (2.37)$$

Piezoelectric Transducer in Transverse Mode

$$\begin{bmatrix} 0 \\ 0 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{31} & d_{33} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_{11}^T & 0 & 0 \\ 0 & \epsilon_{22}^T & 0 \\ 0 & 0 & \epsilon_{33}^T \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (2.38)$$

The simplified linear piezoelectric equations, (2.37) and (2.38), are in the Cartesian-coordinate system. However, the radial vibration mode piezoelectric transformer is made in a round shape, with its major vibration mode in the radial direction. Therefore, Equations (2.37) and (2.38) need to be converted into the cylindrical-coordinate system with the following transformation equations from (2.39) to (2.42). Equations (2.43) and (2.44) are the simplified linear piezoelectric equations in the cylindrical-coordinate system after coordinate transformation. The detailed derivation process is shown in the Appendix.

Mechanical Strain

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 1 & 1 & 0 \\ \sqrt{2} & \sqrt{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} S_r \\ S_\theta \\ S_z \end{bmatrix} \quad (2.39)$$

Mechanical Stress

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 1 & 1 & 0 \\ \sqrt{2} & \sqrt{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} T_r \\ T_\theta \\ T_z \end{bmatrix} \quad (2.40)$$

Electric Field

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 1 & 1 & 0 \\ \sqrt{2} & \sqrt{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} E_r \\ E_\theta \\ E_z \end{bmatrix} \quad (2.41)$$

Electric Displacement

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 1 & 1 & 0 \\ \sqrt{2} & \sqrt{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} D_r \\ D_\theta \\ D_z \end{bmatrix} \quad (2.42)$$

Piezoelectric Actuator in Transverse Mode

$$T_r = \frac{S_r - \sqrt{2} \cdot d_{31} \cdot E_z - \sqrt{2} \cdot S_{13}^E \cdot T_z}{(1 - \sigma) \cdot S_{11}^E} \quad (2.43)$$

Piezoelectric Transducer in Transverse Mode

$$D_z = \sqrt{2} \cdot d_{31} \cdot T_r + d_{33} \cdot T_z + \epsilon_{33}^T \cdot E_z \quad (2.44)$$

Since the radial vibration mode piezoelectric transformer is a combination of piezoelectric actuators and transducers, the single-layer radial vibration mode piezoelectric transformer sample can be separated into two piezoelectric elements, as shown in Figure 2.14.

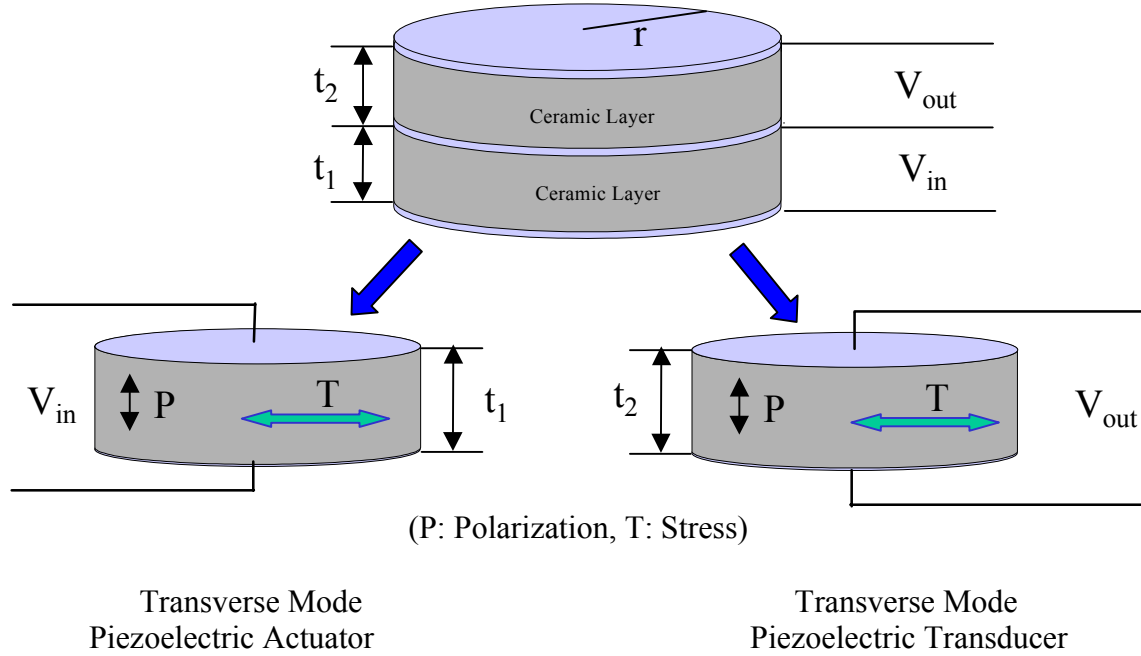


Figure 2.14. Radial vibration mode piezoelectric transformer.

One piezoelectric element works as an actuator, and the other works as a transducer. Referring to the derivation procedures of prior works [A2, A3, A8], the physics-based equivalent circuit, shown in Figure 2.15, of a round-shaped piezoelectric element can be derived from Equations (2.43) and (2.44). The parameters of the physics-based equivalent circuit for the transverse mode piezoelectric element are shown in Equations (2.45) to (2.49), which are dependent on the dimensions and the material coefficients of the piezoelectric element.

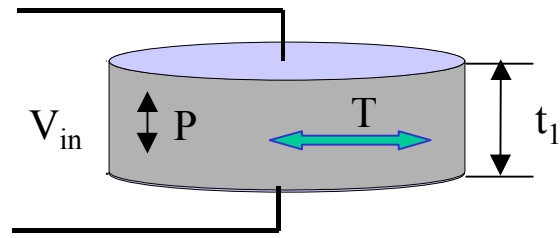
$$R_m = \frac{\pi^2 \cdot r \cdot t}{4 \cdot Q_m} \cdot \sqrt{\frac{2 \cdot \rho}{S_{11}^E \cdot (1 - \sigma)}} \quad (2.45)$$

$$L_m = \frac{\pi \cdot r^2 \cdot t \cdot \rho}{2} \quad (2.46)$$

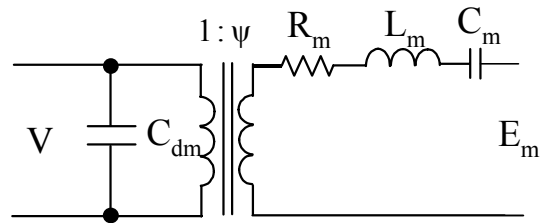
$$C_m = \frac{4 \cdot (1 - \sigma) \cdot S_{11}^E}{\pi^3 \cdot t} \quad (2.47)$$

$$\psi = \frac{2 \cdot \sqrt{2} \cdot \pi \cdot r \cdot d_{31}}{(1 - \sigma) \cdot S_{11}^E} \quad (2.48)$$

$$C_{dm} = \frac{\pi \cdot r^2}{t} \epsilon_{33}^T \left[1 - \frac{d_{31}^2}{(1 - \sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T} \right] \quad (2.49)$$



(a)



(b)

Figure 2.15. Physics-based equivalent circuit of transverse mode piezoelectric element. (a) transverse mode piezoelectric element. (b) physics-based equivalent circuit.

With the combination of the piezoelectric actuator and the piezoelectric transducer, their physics-based equivalent circuits can be linked so that they become a whole physics-based equivalent circuit of the radial vibration mode piezoelectric transformer, as shown in Figure 2.16.

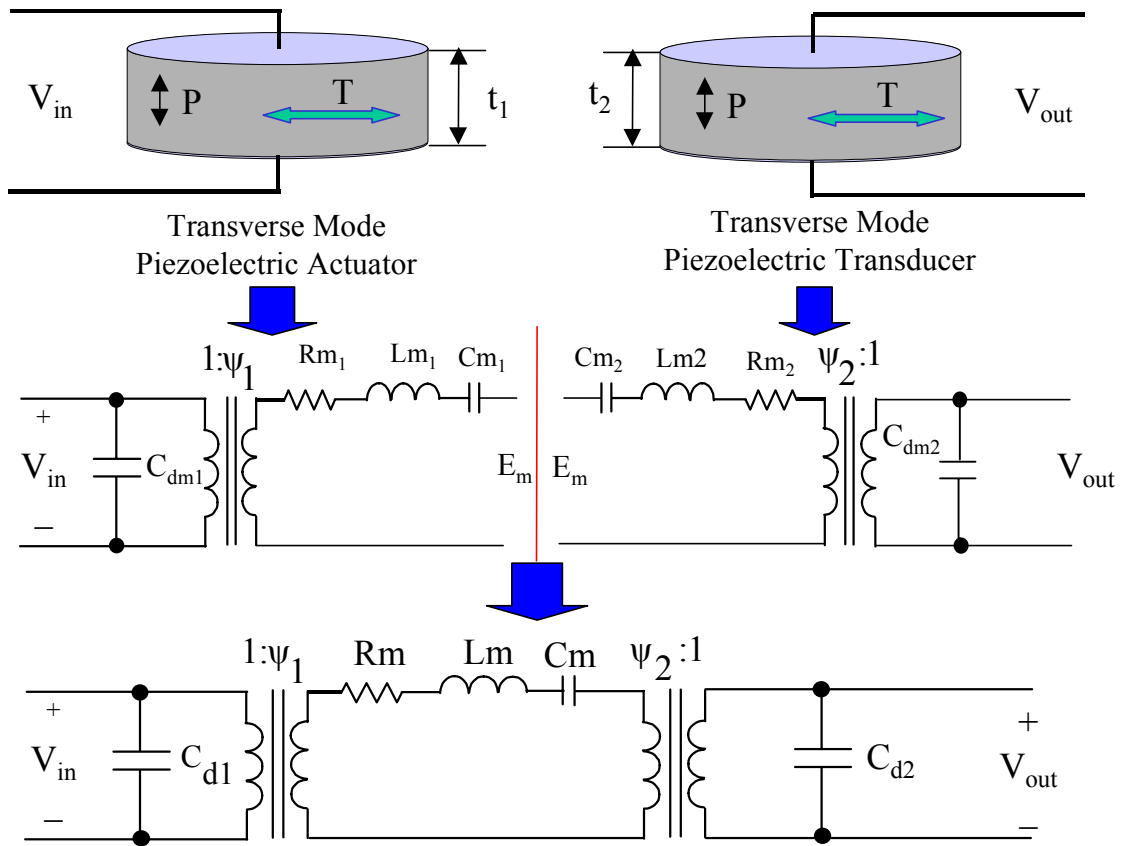


Figure 2.16. Derivation process of physics-based equivalent circuit for radial vibration mode piezoelectric transformers.

The parameters of the completed physics-based equivalent circuit for a single-layer radial vibration mode piezoelectric transformer, shown in Figure 2.16, can be derived, as shown in Equations (2.50) to (2.55), from Equations (2.45) to (2.49).

$$C_{d1} = C_{dm1} = \frac{\pi \cdot r^2}{t_1} \epsilon_{33}^T \left[1 - \frac{d_{31}^2}{(1-\sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T} \right] \quad (2.50)$$

$$C_{d2} = C_{dm2} = \frac{\pi \cdot r^2}{t_2} \epsilon_{33}^T \left[1 - \frac{d_{31}^2}{(1-\sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T} \right] \quad (2.51)$$

$$\psi_1 = \psi_2 = 2 \cdot \sqrt{2} \cdot \pi \cdot r \cdot \frac{d_{31}}{S_{11}^E \cdot (1-\sigma)} \quad (2.52)$$

$$R_m = R_{m1} + R_{m2} = \frac{\pi^2 \cdot r \cdot (t_1 + t_2)}{4 \cdot Q_m \cdot (1-\sigma)} \cdot \sqrt{\frac{2 \cdot \rho}{S_{11}^E \cdot (1-\sigma)}} \quad (2.53)$$

$$L_m = L_{m1} + L_{m2} = \frac{\pi \cdot r^2 \cdot (t_1 + t_2) \cdot \rho}{2} \quad (2.54)$$

$$C_m = \frac{C_{m1} \cdot C_{m2}}{C_{m1} + C_{m2}} = \frac{4 \cdot S_{11}^E \cdot (1-\sigma)}{\pi^3 \cdot (t_1 + t_2)} \quad (2.55)$$

The physics-based equivalent circuit, shown in Figure 2.16, can be converted into the final format of the physics-based equivalent circuit model, as shown in Figure 2.17, according to the following conversion Equations (2.56) to (2.59).

$$N = \frac{\psi_1}{\psi_2} \quad (2.56)$$

$$R = \frac{R_m}{2\psi_1} \quad (2.57)$$

$$L = \frac{L_m}{2\psi_1} \quad (2.58)$$

$$C = C_m \cdot \psi_1^2 \quad (2.59)$$

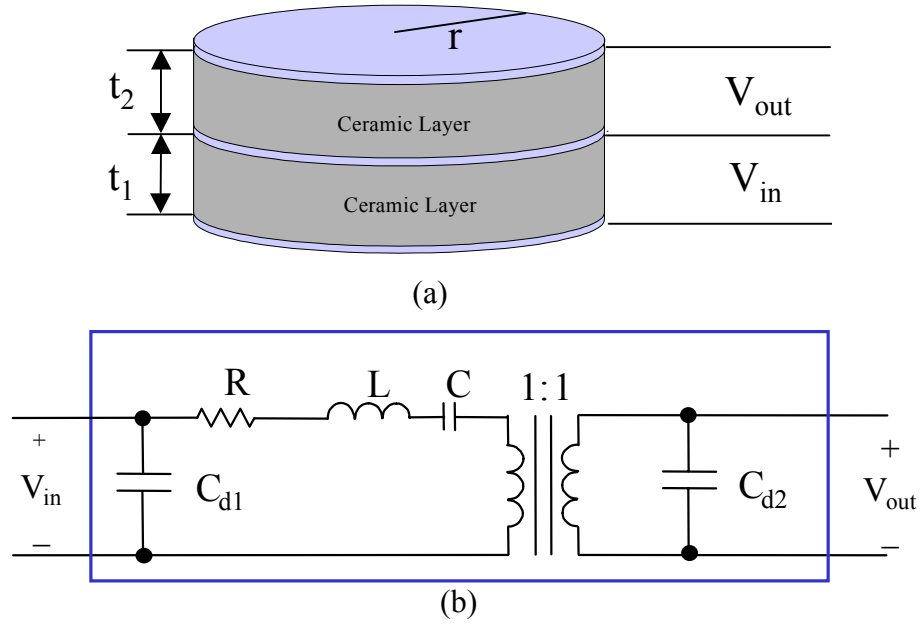


Figure 2.17. Physics-based equivalent circuit model for single-layer radial vibration mode piezoelectric transformers. (a) single-layer radial vibration mode piezoelectric transformer. (b) physics-based equivalent circuit model.

Therefore, the parameters of the physics-based equivalent circuit for single-layer radial vibration mode piezoelectric transformers can be obtained as shown in the following equations, from (2.60) to (2.65). These parameters are dependent on the dimension and piezoelectric material coefficients of the radial vibration mode piezoelectric transformers.

$$N = \frac{\Psi_1}{\Psi_2} = 1 \quad (2.60)$$

$$C_{d1} = \frac{\pi \cdot r^2 \cdot \epsilon_{33}^T \cdot [1 - \frac{d_{31}^2}{\epsilon_{33}^T \cdot S_{11}^E \cdot (1 - \sigma)}]}{t_1} \quad (2.61)$$

$$C_{d2} = \frac{\pi \cdot r^2 \cdot \epsilon_{33}^T \cdot [1 - \frac{d_{31}^2}{\epsilon_{33}^T \cdot S_{11}^E \cdot (1 - \sigma)}]}{t_2} \quad (2.62)$$

$$R = \frac{(t_1 + t_2)}{r} \cdot \frac{\sqrt{2 \cdot \rho \cdot S_{11}^E{}^3 \cdot (1 - \sigma)^3}}{32 \cdot Q_m \cdot d_{31}^2} \quad (2.63)$$

$$L = (t_1 + t_2) \cdot \frac{\rho \cdot S_{11}^E{}^2 \cdot (1 - \sigma)^2}{16 \cdot \pi \cdot d_{31}^2} \quad (2.64)$$

$$C = \frac{32 \cdot r^2}{(t_1 + t_2)} \cdot \frac{d_{31}^2}{\pi \cdot S_{11}^E \cdot (1 - \sigma)} \quad (2.65)$$

2.4.2 Multi-Layer Structure

Besides the single-layer structure, shown in Figure 2.17, the radial vibration mode piezoelectric transformer can also be made with a multi-layer structure in order to have electrical and mechanical characteristics different from those of the single-layer structure. Figure 2.18 shows the cross-section of a multi-layer radial vibration mode piezoelectric transformer sample, CK2. This sample has two layers on the primary side and one layer on the secondary side.

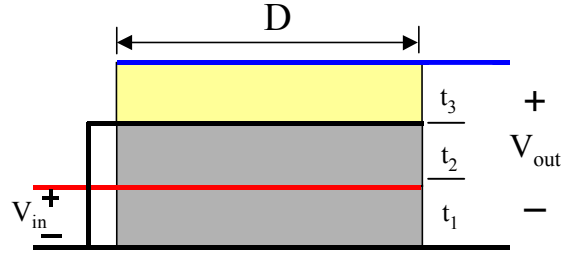


Figure 2.18. Cross-section of a multi-layer radial vibration mode piezoelectric transformer sample, CK2.

Based on the derivation principle of the physics-based equivalent circuit model for single-layer piezoelectric transformers given in prior works [A2, A3, A8], the physics-based equivalent circuit model for multi-layer piezoelectric transformers can be derived and obtained, as shown in Figure 2.19.

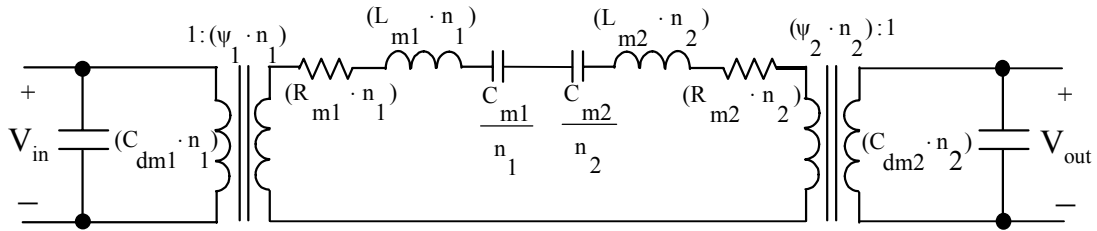


Figure 2.19. Physics-based equivalent circuit model of a multi-layer radial vibration mode piezoelectric transformer.

The parameters of the physics-based equivalent circuit model for multi-layer piezoelectric transformers can be calculated as shown in the following equations, from (2.66) to (2.74).

Resistors

$$R_{m1} = \frac{\pi^2 \cdot r \cdot t_1}{4 \cdot Q_m} \cdot \sqrt{\frac{2 \cdot \rho}{S_{11}^E \cdot (1 - \sigma)}} \quad (2.66)$$

$$R_{m2} = \frac{\pi^2 \cdot r \cdot t_2}{4 \cdot Q_m} \cdot \sqrt{\frac{2 \cdot \rho}{S_{11}^E \cdot (1 - \sigma)}} \quad (2.67)$$

Inductors

$$L_{m1} = \frac{\pi \cdot r^2 \cdot t_1 \cdot \rho}{2} \quad (2.68)$$

$$L_{m2} = \frac{\pi \cdot r^2 \cdot t_2 \cdot \rho}{2} \quad (2.69)$$

Capacitors

$$C_{m1} = \frac{4 \cdot (1 - \sigma) \cdot S_{11}^E}{\pi^3 \cdot t_1} \quad (2.70)$$

$$C_{m2} = \frac{4 \cdot (1 - \sigma) \cdot S_{11}^E}{\pi^3 \cdot t_2} \quad (2.71)$$

Input and Output Capacitors

$$C_{dm1} = \frac{\pi \cdot r^2}{t_1} \epsilon_{33}^T \left[1 - \frac{d_{31}^2}{(1 - \sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T} \right] \quad (2.72)$$

$$C_{dm2} = \frac{\pi \cdot r^2}{t_2} \epsilon_{33}^T \left[1 - \frac{d_{31}^2}{(1 - \sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T} \right] \quad (2.73)$$

Turns Ratio

$$\psi_1 = \psi_2 = 2 \cdot \sqrt{2} \cdot \pi \cdot r \cdot \frac{d_{31}}{S_{11}^E \cdot (1 - \sigma)} \quad (2.74)$$

Based on the conversion equations, Equations (2.75) to (2.79), the final format of the physics-based equivalent circuit model of multi-layer piezoelectric transformers can be derived and simplified, as shown in Figure 2.20, from the physics-based equivalent circuit model of multi-layer radial vibration mode piezoelectric transformers, as shown in Figure 2.19.

Inductor

$$L = \frac{(n_1 \cdot L_{m1} + n_2 \cdot L_{m2})}{(\psi \cdot n_1)^2} \quad (2.75)$$

Resistor

$$R = \frac{(n_1 \cdot R_{m1} + n_2 \cdot R_{m2})}{(\psi \cdot n_1)^2} \quad (2.76)$$

Capacitor

$$C = (\psi \cdot n_1)^2 \cdot \frac{C_{m1} \cdot C_{m2}}{C_{m1} \cdot N_2 + C_{m2} \cdot N_1} \quad (2.77)$$

Input Capacitor

$$C_{d1} = n_1 \cdot C_{dm1} \quad (2.78)$$

Output Capacitor

$$C_{d2} = n_2 \cdot C_{dm2} \quad (2.79)$$

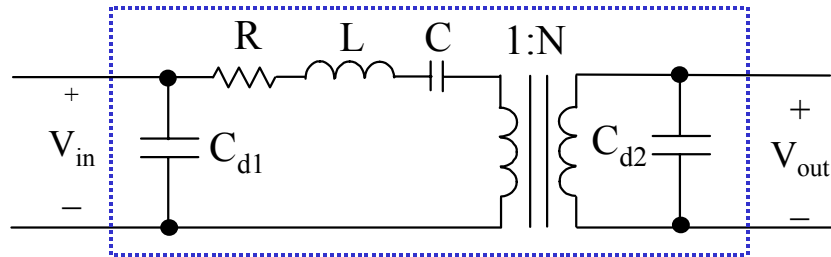


Figure 2.20. Physics-based equivalent circuit model for multi-layer radial vibration mode piezoelectric transformers.

The parameters of the physics-based equivalent circuit for multi-layer radial vibration mode piezoelectric transformers can be calculated as shown in the following equations, from (2.80) to (2.85).

Parameters of Physics-Based Equivalent Circuit Model

$$C_{d1} = \frac{n_1 \cdot \pi \cdot r^2 \cdot \epsilon_{33}^T \cdot \left[1 - \frac{d_{31}^2}{\epsilon_{33}^t \cdot S_{11}^E \cdot (1 - \sigma)} \right]}{t_1} \quad (2.80)$$

$$C_{d2} = \frac{n_2 \cdot \pi \cdot r^2 \cdot \epsilon_{33}^T \cdot \left[1 - \frac{d_{31}^2}{\epsilon_{33}^t \cdot S_{11}^E \cdot (1 - \sigma)} \right]}{t_2} \quad (2.81)$$

$$R = \frac{\sqrt{2 \cdot \rho \cdot S_{11}^E{}^3 \cdot (1 - \sigma)^3} \cdot (n_1 \cdot t_1 + n_2 \cdot t_2)}{32 \cdot Q_m \cdot d_{31}^2 \cdot n_1^2 \cdot r} \quad (2.82)$$

$$L = \frac{\rho \cdot S_{11}^E{}^2 \cdot (1 - \sigma)^2 \cdot (n_1 \cdot t_1 + n_2 \cdot t_2)}{16 \cdot \pi \cdot (n_1 \cdot d_{31})^2} \quad (2.83)$$

$$C = \frac{32 \cdot r^2 \cdot (d_{31} \cdot n_1)^2}{\pi \cdot S_{11}^E \cdot (n_1 \cdot t_1 + n_2 \cdot t_2) \cdot (1 - \sigma)} \quad (2.84)$$

$$N = \frac{n_1}{n_2} \quad (2.85)$$

2.5 Summary

This chapter has introduced the operational principles of piezoelectric elements and the three major piezoelectric transformers: Rosen, thickness vibration mode and radial vibration mode. After reviewing the existing equivalent circuit models of Rosen and thickness vibration mode piezoelectric transformers, this chapter also derived the physics-based equivalent circuit model for the major vibration mode of the newly invented radial vibration mode piezoelectric transformer.

The piezoelectric transformer is a combination of piezoelectric actuators on the primary side and piezoelectric transducers on the secondary side. Both the actuator and the transducer are made of piezoelectric elements, which are composed of electrode plates and piezoelectric materials, such as barium titanate-based ceramics. With a corresponding resonant frequency, piezoelectric elements can work either in longitudinal vibration mode or in transverse vibration mode. In the longitudinal mode, the direction of the mechanical stress is parallel to the electric or polarization direction at a corresponding resonant frequency. In the transverse mode, the direction of the mechanical stress is perpendicular to the electric or polarization direction at a corresponding resonant frequency.

The Rosen piezoelectric transformer has a transverse mode piezoelectric actuator on the primary side and a longitudinal mode piezoelectric transducer on the secondary side. The thickness vibration piezoelectric transformer has a longitudinal mode piezoelectric actuator on the primary side and a longitudinal mode piezoelectric transducer on the secondary side. The radial vibration mode piezoelectric transformer has a transverse mode piezoelectric actuator on the primary side and a transverse mode piezoelectric transducer on the secondary side. The applications of piezoelectric transformers include use in DC/DC converters and in electronic ballasts for fluorescent lamps.

Based on the piezoelectric and wave equations, the physics-based equivalent circuit model of the radial vibration mode piezoelectric transformer was derived in this chapter. The physics-based equivalent circuit model, comprising of an L-C resonant tank network, can provide a good reference for the design and application of piezoelectric transformers. In the next chapter, the derived physics-based equivalent circuit model for radial vibration mode piezoelectric transformers will be verified using a characterized equivalent circuit model.

CHAPTER 3

CHARACTERIZATION OF RADIAL VIBRATION MODE PIEZOELECTRIC TRANSFORMER

3.1 Introduction

The Y-parameter equivalent circuit model has been widely used to characterize the equivalent circuit model for Rosen and thickness vibration mode piezoelectric transformers for physics-based model verification and application of the circuit design [A8]. This chapter will use the Y-parameter equivalent of the circuit model to characterize samples of the radial vibration mode piezoelectric transformer in order to verify the physics-based equivalent circuit for the major vibration mode of radial vibration mode piezoelectric transformers, which was derived in Chapter 2.

Besides the major vibration mode, there are many spurious vibration modes that exist in the piezoelectric transformers. These spurious vibration modes need to be considered in some application circuits that operate in wide frequency ranges. Prior work [A8] proposed a multi-branch equivalent circuit to describe other spurious vibration modes adjacent to the major vibration mode. This chapter will show that this prior circuit model cannot very accurately describe the voltage gain characteristic for the radial vibration mode piezoelectric transformers; therefore, an improved multi-branch equivalent circuit model will be proposed in order to much more precisely match the measured voltage gain. This improved model is useful for detailed circuit analysis and design of application prototypes when considering spurious vibration mode effects in wider frequency ranges.

3.2 Measurement of Equivalent Circuit Model for Piezoelectric Transformers

The parameters of the physics-based equivalent circuit for piezoelectric transformers, shown in Figure 3.1, can be verified by using an HP-4194 impedance analyzer [A8].

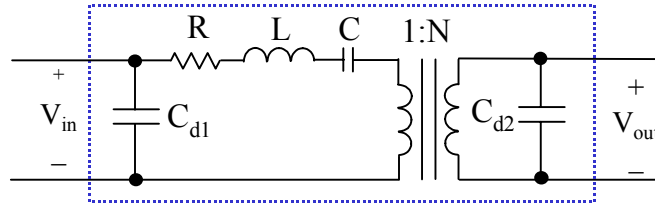


Figure 3.1. Physics-based equivalent circuit model for piezoelectric transformers.

3.2.1 Y-Parameter Equivalent Circuit Model

The HP-4194A impedance analyzer provides a Y-parameter equivalent circuit model, which is an admittance equivalent circuit, as shown in Figure 3.2. With the input or output terminal shorted, the physics-based equivalent circuit is identical to the Y-parameter equivalent circuit model shown in Figure 3.2. This Y-parameter equivalent circuit model has been used to measure and characterize the equivalent circuit model of piezoelectric transformers [A8].

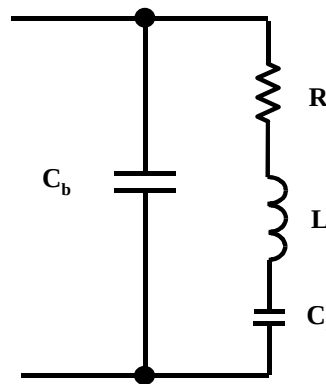


Figure 3.2. Y-parameter equivalent circuit model provided by the HP-4194A impedance analyzer. This Y-parameter equivalent circuit model is an admittance equivalent circuit and can be utilized to measure and characterize the equivalent circuit model of piezoelectric transformers [A8].

3.2.2 Measurement Procedure

Based on the Y-parameter equivalent circuit model provided by the HP-4194A impedance analyzer, the parameters of the equivalent circuit model for piezoelectric transformers around the resonant frequency range can be measured and characterized as described in the following outline [A8, A9],

Step 1. By shorting the output terminal, V_{out} , of the piezoelectric transformer, input admittance, Y_{in} , can be measured by the HP4194A impedance analyzer. This analyzer can then model this input admittance using the Y-parameter equivalent circuit. The Y-parameter equivalent circuit includes four parameters, R_1 , L_1 , C_{a1} and C_{b1} , as shown in Figure 3.3A. The Y-parameter equivalent circuit is identical to the equivalent circuit model of the piezoelectric transformer, but with one terminal shorted. The four parameters of the equivalent circuit model for piezoelectric transformers can be obtained as follows:

$$C_{d1}=C_{b1} \quad (3.1)$$

$$R=R_1 \quad (3.2)$$

$$L=L_2 \quad (3.3)$$

$$C=C_{a1} \quad (3.4)$$

Step 2. By shorting the output terminal, V_{in} , of the piezoelectric transformer, the output admittance, Y_{out} , can be measured by the HP-4194A impedance analyzer. The analyzer can then model the output admittance, Y_{out} , using the Y-parameter equivalent circuit. This Y-parameter equivalent circuit includes four parameters, R_2 , L_2 , C_{a2} and C_{b2} , as shown in Figure 3.3B. Two parameters of the equivalent circuit model, C_{d2} and N , can be obtained as follows:

$$C_{d2}=C_{b2} \quad (3.5)$$

$$N = \sqrt{\frac{L_2}{L_1}} \quad (3.6)$$

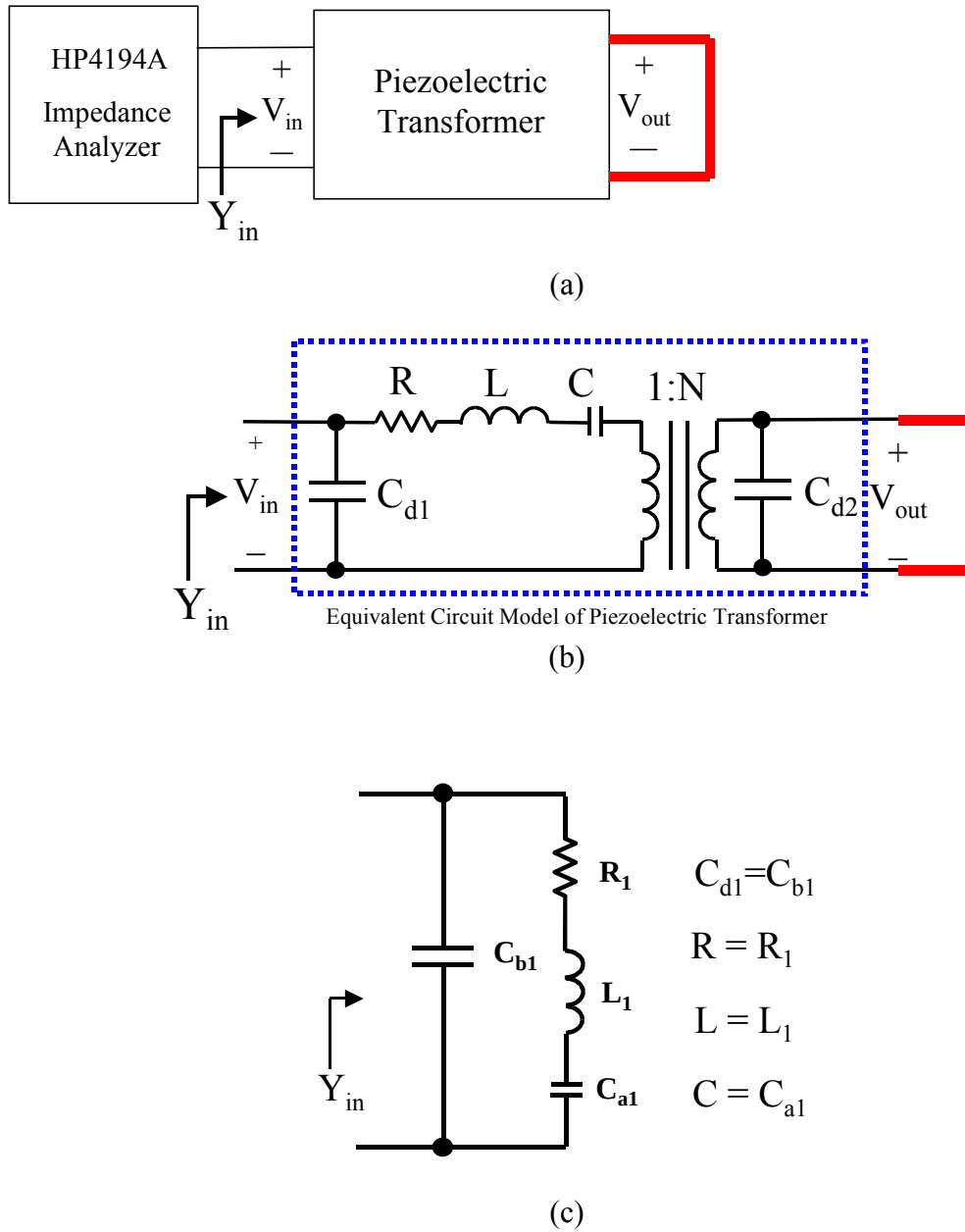


Figure 3.3A. Step 1 of measurement procedure for equivalent circuit model of piezoelectric transformers. (a) setup for measuring input admittance, Y_{in} . (b) equivalent circuit model with output terminal shorted. (c) Y-parameter equivalent circuit model provided by the HP-4194A impedance analyzer. With output terminal, V_{out} , shorted, the parameters, R , L , C and C_{d1} , can be measured and characterized using the Y-parameter equivalent circuit provided by the HP-4194A impedance analyzer.

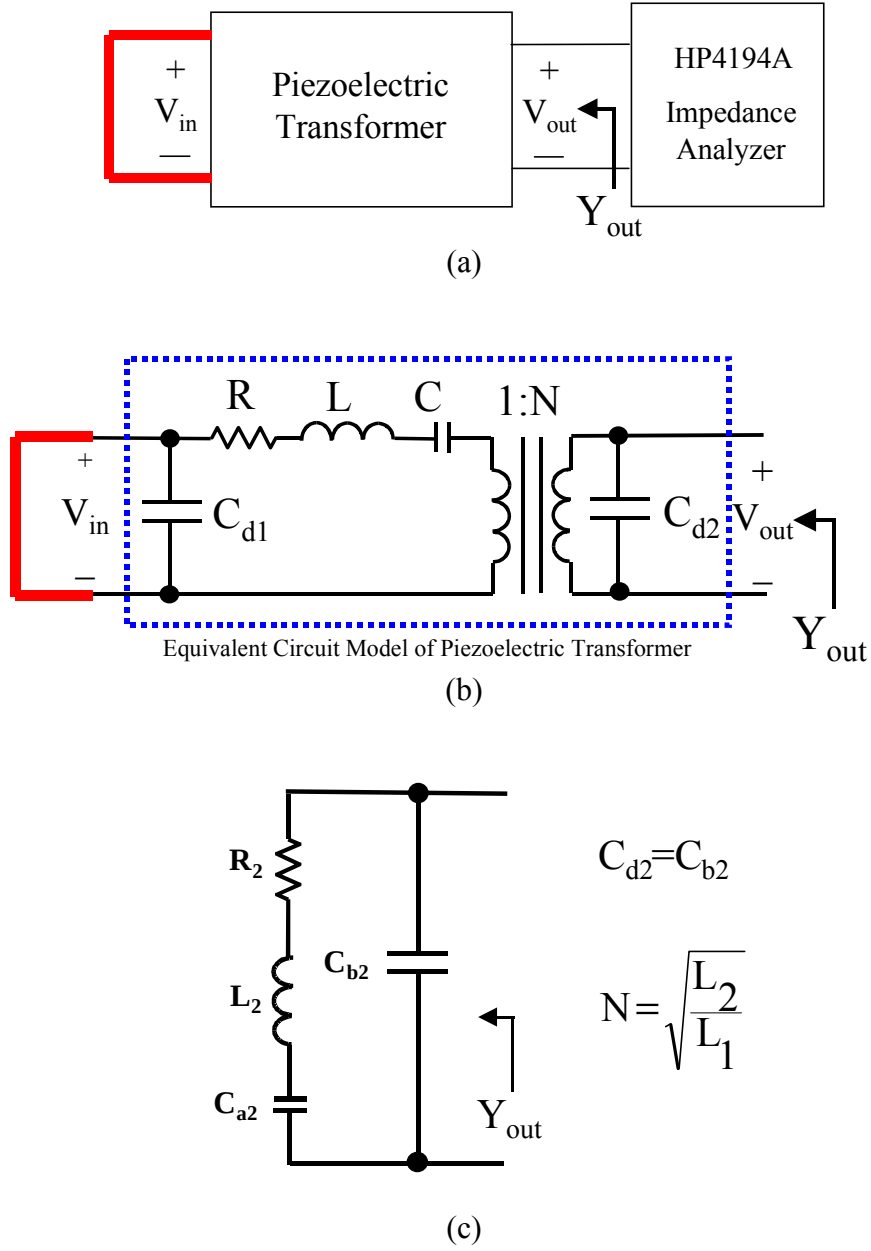


Figure 3.3B. Step 2 of measurement procedure for equivalent circuit model of piezoelectric transformers. (a) setup for measuring output admittance, Y_{out} . (b) equivalent circuit model with input terminal shorted. (c) Y-parameter equivalent circuit model provided by HP-4194A impedance analyzer. With the output terminal, V_{in} , shorted, the parameters, N and C_{d2} , can be measured and characterized using the Y-parameter equivalent circuit model provided by the HP-4194A impedance analyzer.

3.3 Measurement of Radial Vibration Mode Piezoelectric Transformer Samples

This section will use the characterization procedure described in the previous section to measure the parameters of the equivalent circuits for the single-layer sample, AS, and the multi-layer samples, CK2, CZ2, and CE1. These measured results will be compared with the parameters of the derived physics-based equivalent circuit model for radial vibration mode piezoelectric transformers.

3.3.1 Single-Layer Structure Samples

In order to verify the equivalent circuit model for the single-layer radial vibration mode piezoelectric transformer discussed in the previous chapter, ten identical samples, AS-1 to AS-10, were characterized, following the procedure outlined in Section 3.2. The dimensions of these nine samples are identical, as follows:

Diameter, $D = 825\text{mil} = 0.825\text{ inch} = 21\text{mm}$;

Thickness of primary side, $t_1 = 30\text{mil} = 0.030\text{ inch} = 0.76\text{mm}$; and

Thickness of secondary side, $t_2 = 90\text{mil} = 0.090\text{ inch} = 2.28\text{mm}$.

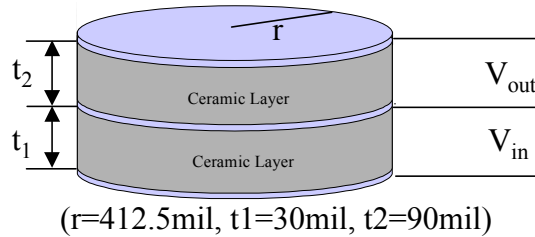


Figure 3.4. Single-layer radial vibration mode piezoelectric transformer sample, AS.

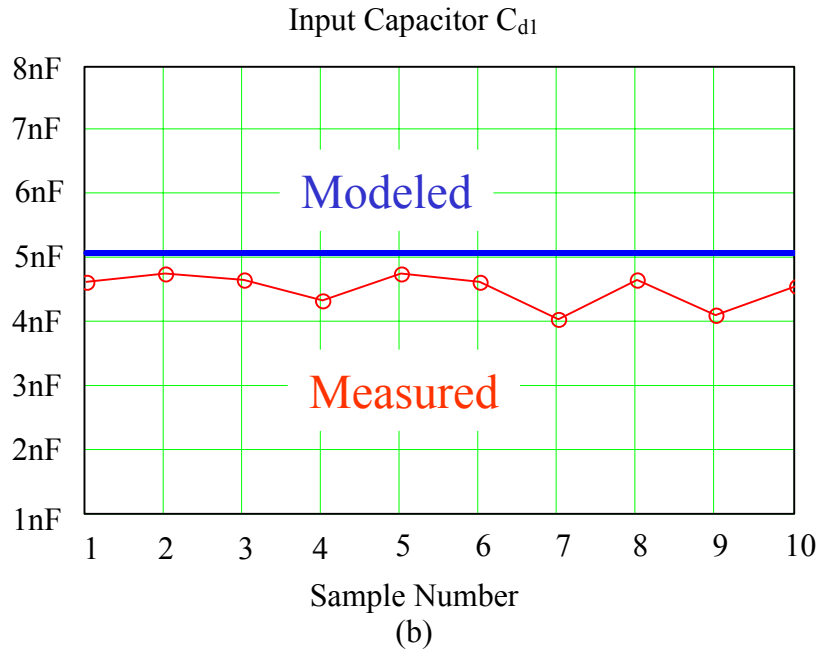
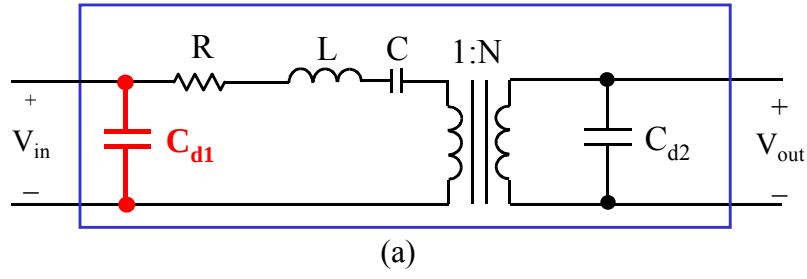
The coefficients of the piezoelectric material, APC 841 [D7], used for samples AS-1 to AS-10, are shown in Table 3.1. Figures 3.5, 3.6, 3.7, 3.8, 3.9 and 3.10 show the comparisons of the measured results and the calculated results for respective parameters, C_{d1} , C_{d2} , R , L , C and N , of the equivalent circuit model for the ten identical single-layer radial vibration mode samples. Figure 3.11 shows the comparison of the modeled and

measured results for the resonant frequency, f_0 . The differences between the measured and calculated results may result from the following factors:

- (1) *Tolerance of the piezoelectric ceramic coefficients,*
- (2) *Tolerance of the dimensions,*
- (3) *Additional mechanical losses due to the existence of the glue and copper layer.*

Table 3.1. Coefficients of piezoelectric ceramic material, APC 841 [D7].

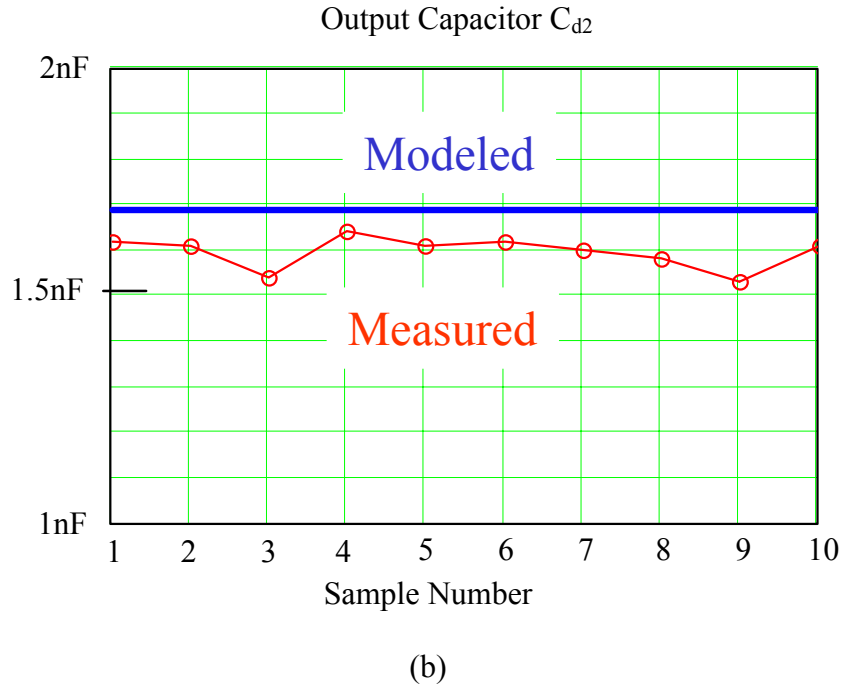
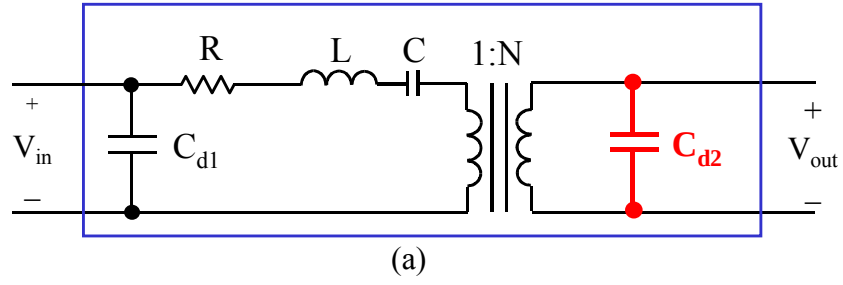
Coefficient	Value
Density	$\rho=7.6 \text{ (g/cc)}$
Poisson's Ratio	$\sigma=-0.32$
Relative Dielectric Constant	$\epsilon_{33}^T=1350\epsilon_0$
Mechanical Quality Factor	$Q_m=1400$
Coupling Coefficient	$k_{31}=0.33$
	$k_{33}=0.68$
Piezoelectric Coefficient	$d_{31}=-109 \cdot 10^{-12} \text{ (m/V)}$
	$d_{33}=275 \cdot 10^{-12} \text{ (m/V)}$
Young's Modulus	$Y_{11}^E=7.6 \cdot 10^{10} \text{ (N/m}^2\text{)}$
	$Y_{33}^E=6.3 \cdot 10^{10} \text{ (N/m}^2\text{)}$
Elastic Compliance	$S_{11}^E=11.7 \cdot 10^{-12} \text{ (m}^2\text{/N)}$
	$S_{33}^E=17.3 \cdot 10^{-12} \text{ (m}^2\text{/N)}$
Shear Frequency Constant	$N_L=1700 \text{ (m/s)}$
Thickness Frequency Constant	$N_T=2005 \text{ (m/s)}$
Radial Frequency Constant	$N_p=2055 \text{ (m/s)}$



$$C_{d1} = \frac{\pi \cdot r^2 \cdot \epsilon_{33}^T \cdot [1 - \frac{d_{31}^2}{\epsilon_{33}^T \cdot S_{11}^E \cdot (1 - \sigma)}]}{t_1} = 5.06 \text{ nF}$$

(c)

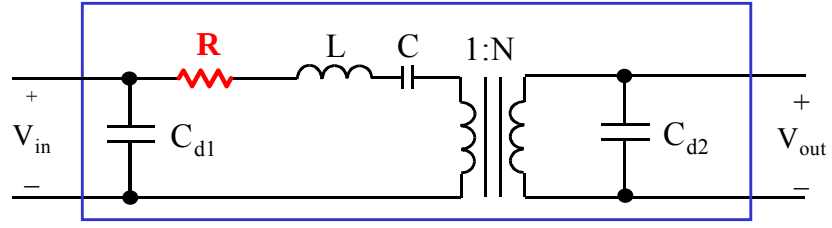
Figure 3.5. Comparison of modeled and measured results for input capacitor, C_{d1} . (a) physics-based equivalent circuit model. (b) modeled and measured results. (c) equation. The input capacitor, C_{d1} , is proportional to the area of the electrode plate and is inversely proportional the thickness of primary side, t_1 .



$$C_{d2} = \frac{\pi \cdot r^2 \cdot \epsilon_{33}^T \cdot \left[1 - \frac{d_{31}^2}{\epsilon_{33}^T \cdot S_{11}^E \cdot (1 - \sigma)}\right]}{t_2} = 1.69\text{nF}$$

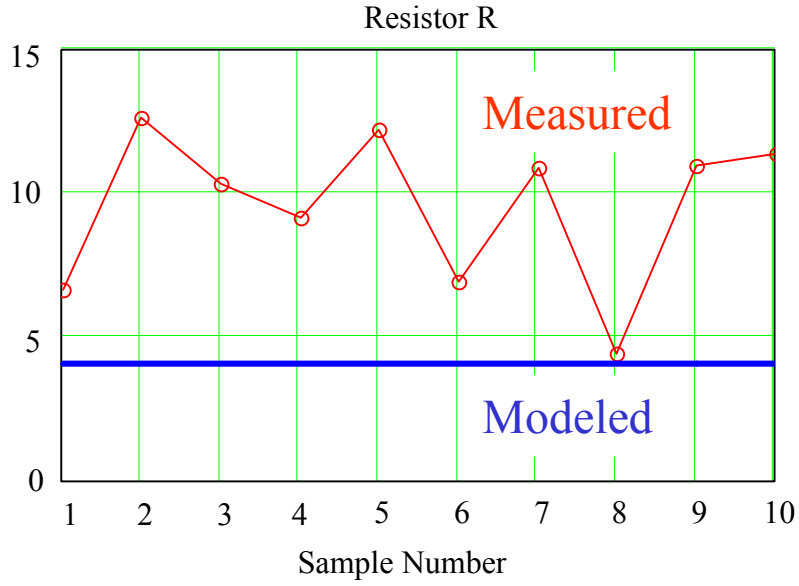
(c)

Figure 3.6. Comparison of modeled and measured results for output capacitor, C_{d2} . (a) physics-based equivalent circuit model. (b) modeled and measured results. (c) equation. The physics-based component of the output capacitor, C_{d2} , is proportional to the area of the electrode plate and is inversely proportional the thickness of secondary side, t_2 .



Physics-based equivalent circuit model

(a)

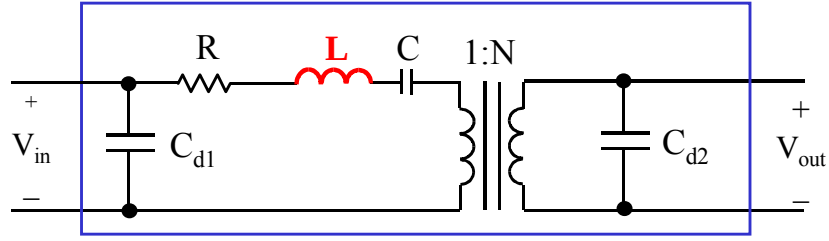


(b)

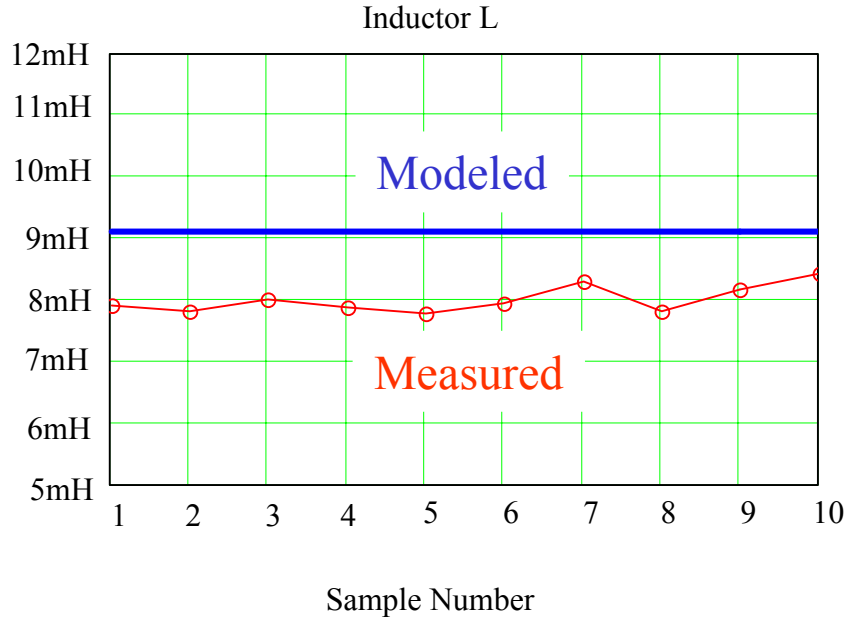
$$R = \frac{(t_1 + t_2)}{r} \cdot \frac{\sqrt{2 \cdot \rho \cdot S_{11}^E{}^3 \cdot (1 - \sigma)^3}}{32 \cdot Q_m \cdot d_{31}^2} = 4.04\Omega$$

(c)

Figure 3.7. Comparison of modeled and measured results for resistors, R. (a) physics-based equivalent circuit model. (b) modeled and measured results. (c) equation. The physics-based component of the resistor, R, is proportional to the total thickness, $(t_1 + t_2)$, and is inversely proportional to radius, r.



(a)

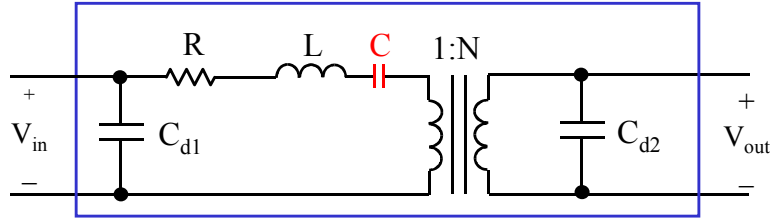


(b)

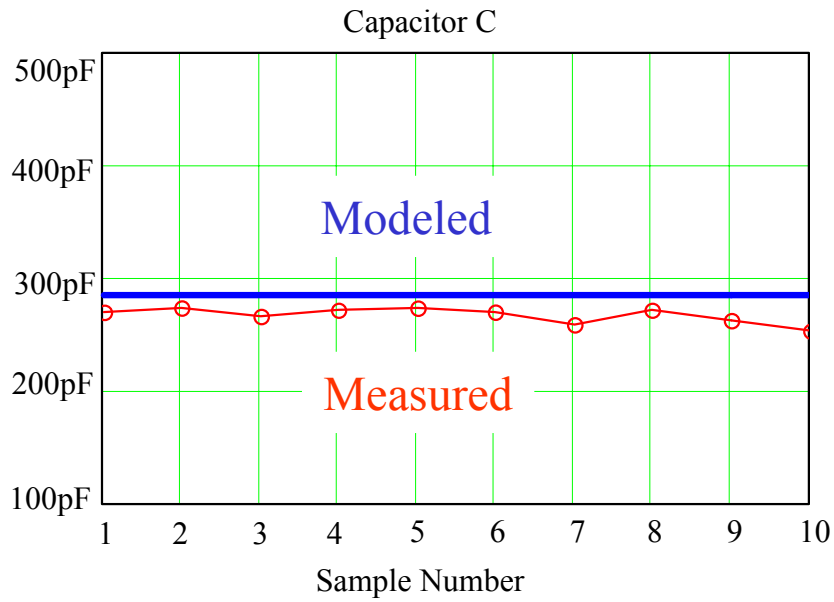
$$L = (t_1 + t_2) \cdot \frac{\rho \cdot S_{11}^E \cdot (1 - \sigma)^2}{16 \cdot \pi \cdot d_{31}^2} = 9.1\text{mH}$$

(c)

Figure 3.8. Comparison of modeled and modeled results for inductors, L. (a) physics-based equivalent circuit model. (b) modeled and measured results. (c) equation. The physics-based component of the inductor, L, is proportional to the total thickness, (t_1+t_2) .



(a)

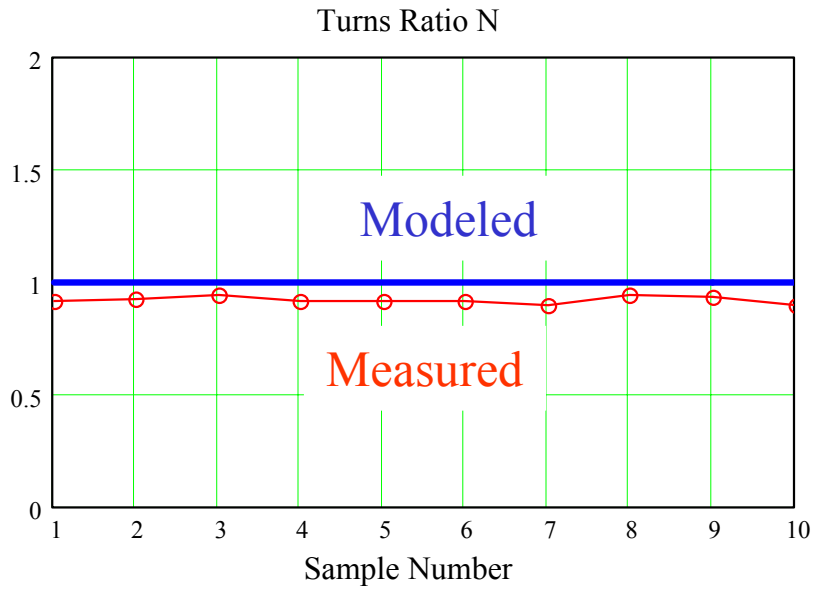
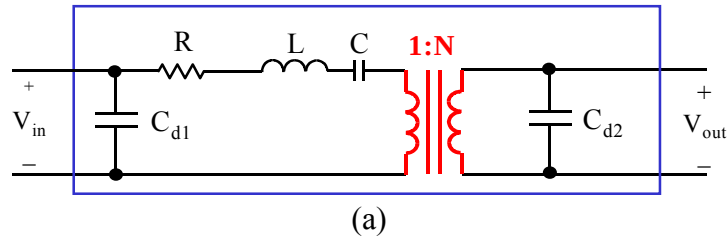


(b)

$$C = \frac{32 \cdot r^2}{(t_1 + t_2)} \cdot \frac{d_{31}^2}{\pi \cdot S_{11}^E \cdot (1 - \sigma)} = 284.4 \text{ nF}$$

(c)

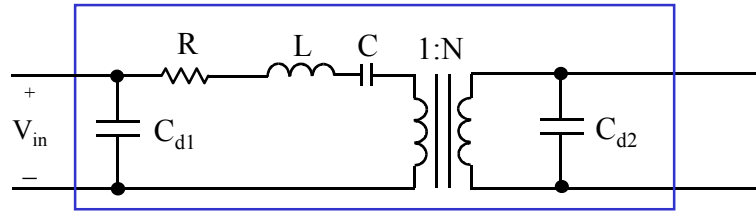
Figure 3.9. Comparison of modeled and measured results for capacitor, C. (a) physics-based equivalent circuit model. (b) modeled and measured results. (c) equation. The physics-based component of capacitor, C, is proportional to the square of radius, r, and is inversely proportional to the total thickness, $(t_1 + t_2)$.



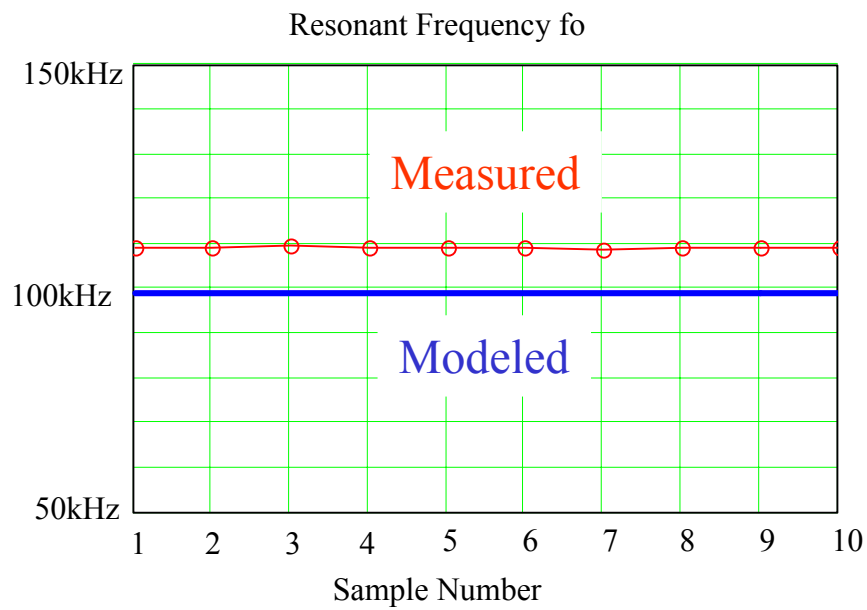
$$N=1$$

(c)

Figure 3.10. Comparison of modeled and measured results for turns ratio, N . (a) physics-based equivalent circuit model. (b) calculated and measured results. (c) equation.



(a)



(b)

$$f_o = \frac{1}{2\pi\sqrt{L \cdot C}} = 98.9\text{kHz}$$

(c)

Figure 3.11. Comparison of modeled and measured results for resonant frequency, f_o . (a) physics-based equivalent circuit model. (b) modeled and measured resonant frequency, f_o . (c) equation.

3.3.2 Multi-Layer Structure Samples

In order to verify the equivalent circuit model for the multi-layer radial vibration mode piezoelectric transformer derived in the previous chapter, three different multi-layer radial vibration mode piezoelectric transformer samples, CK2, CZ2 and CE1, will be characterized, following the procedure outlined in Section 3.2. Figure 3.12 shows the cross-sections of the multi-layer radial vibration mode piezoelectric transformer samples CK2, CZ1 and CE1.

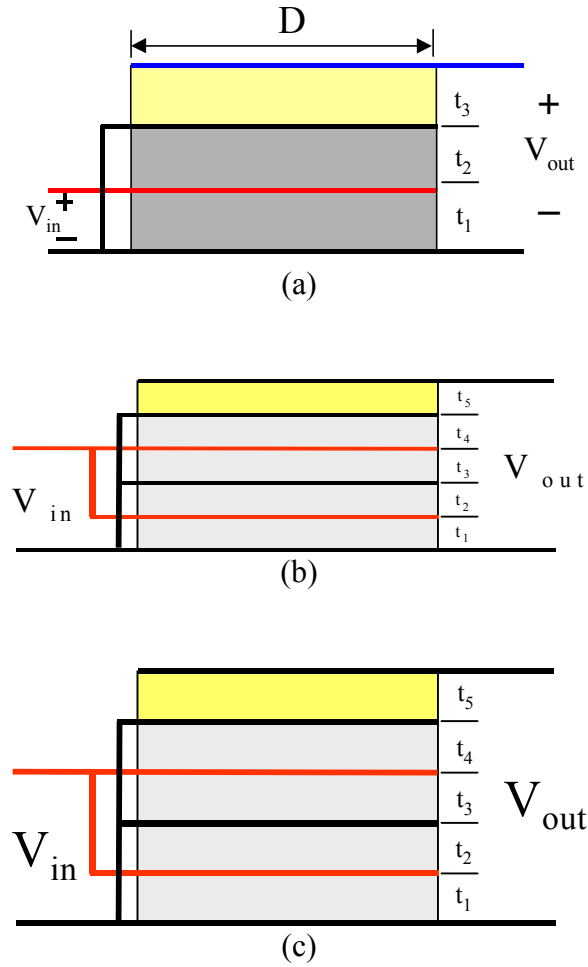


Figure 3.12. Cross-sections of multi-layer radial vibration mode piezoelectric transformer samples. (a) sample CK2 ($t_1=t_2=t_3=80\text{mils}$, $D=1180\text{mils}$, $n_1=2$, $n_2=1$). (b) sample CZ2 ($t_1=t_2=t_3=t_4=t_5=20\text{mils}$, $D=1180\text{mils}$, $n_1=4$, $n_2=1$). (c) sample CE1 ($t_1=t_2=t_3=t_4=t_5=80\text{mils}$, $D=1180\text{mils}$, $n_1=4$, $n_2=1$).

Table 3.2 shows the dimensions of the samples. Table 3.3 shows the coefficients of the piezoelectric ceramic material, PKI-802 [D9], used for the samples. Table 3.4 shows the modeled parameters, R , L , C , N , C_{d1} and C_{d2} , of the physics-based equivalent circuit models for these three samples, compared with the measured results of the characterized equivalent circuits. Referring to Table 3.4, the modeled resistor value, R , is much smaller than the measured result because the actual resistor value includes not only the piezoelectric vibration losses, but also the mechanical interface losses. The calculated turns ratio values of samples CK 2 and CZ2 very closely match their measured turns ratio values. However, the measured turns ratio of CE1 is 4.97, which is larger than the modeled value, 4, even though the diameter and layer number of sample CE1 are the same as those of CZ2 but with different layer thickness. This deviation can be caused by the interaction between radial vibration and thickness vibration modes in the five-layer sample CE1 when the mechanical vibration wavelengths of radial mode and thickness mode are very close. As stated in Section 2.4, the physics-based equivalent circuit model of radial vibration mode piezoelectric transformers was derived under the assumption of radial vibration mode only and without considering other vibration modes. Therefore, the derived physics-based equivalent circuit model agrees with the corresponding measured results when the total thickness is much less than the radius.

Table 3.2. Dimensions of multi-layer radial vibration mode piezoelectric transformer samples, CK2, CZ2, and CE1.

		Primary Side		Secondary Side	
Sample	Diameter (mil)	Layer Number	Layer Thickness (mil)	Layer Number	Layer Thickness (mil)
CK2	1,180	2	$t_1 = t_2 = 80$	1	$t_3 = 80$
CZ2	1,180	4	$t_1 = t_2 = t_3 = t_4 = 20$	1	$t_5 = 20$
CE1	1,180	4	$t_1 = t_2 = t_3 = t_4 = 80$	1	$t_5 = 80$

Table 3.3. Parameters of piezoelectric ceramic material, PKI-802 [D9].

Parameters	Value
Density	$\rho = 7.6 \text{ (g/cc)}$
Relative Dielectric Constant	$\epsilon_{33}^T = 1000\epsilon_0$
Poisson's Ratio	$\sigma = -0.32$
Dissipation Factor	0.004
Mechanical Quality Factor	$Q_m = 900$
Coupling Coefficient	$k_{31} = 0.30$
	$k_{33} = 0.61$
Piezoelectric Coefficient	$d_{31} = -100 \cdot 10^{-12} \text{ (m/V)}$
	$d_{33} = 220 \cdot 10^{-12} \text{ (m/V)}$
Elastic Compliance	$S_{11}^E = 10.4 \cdot 10^{-12} \text{ (m}^2\text{/N)}$
	$S_{33}^E = 13.5 \cdot 10^{-12} \text{ (m}^2\text{/N)}$
Shear Frequency Constant	$N_s = 1460 \text{ (m/s)}$
Thickness Frequency Constant	$N_t = 2100 \text{ (m/s)}$
Radial Frequency Constant	$N_p = 2360 \text{ (m/s)}$

Table 3.4. Comparison of measured and modeled results of equivalent circuit model for multi-layer radial vibration mode piezoelectric transformer samples, CK2, CZ2 and CE1.

		R	L	C	C _{d1}	C _{d2}	N	$f_s = \frac{1}{\sqrt{LC}}$ (kHz)
CK2	Calculated	2.21 Ω	4.3 mH	1093 pF	5.64 nF	2.82 nF	2	73.1
	Measured	6.42 Ω	4.79 mH	919 pF	5.41 nF	2.74 nF	2	75.9
CZ2	Calculated	0.23 Ω	450 μ H	10.5 nF	45 nF	11.28 nF	4	73.1
	Measured	2.94 Ω	686 μ H	6 nF	40 nF	11.2 nF	4.1	78.5
CE1	Calculated	0.92 Ω	1.8 mH	1.15 nF	11.28 nF	2.82 nF	4	73.1
	Measured	2.13 Ω	1.67 mH	2.85 nF	11.1 nF	2.75 nF	4.97	72.95

3.4 Improved Accuracy of Equivalent Circuit Model

Besides the major vibration mode, there are many other spurious vibration modes in piezoelectric transformers. Figure 3.13 shows the voltage gain comparison of the measured result and the single-branch model, as discussed in Section 3.3.2, for the radial vibration mode piezoelectric transformer sample CK2 with $R_L=600\Omega$. This comparison shows that the single-branch model can only match the major vibration mode within a specific frequency range of 60kHz to 90kHz. Above 90 kHz and below 60kHz, the modeled voltage gain curve deviates from that of the measured voltage gain. Obviously, this single-branch model may lead to the wrong voltage gain result in actual circuit designs. In order to model the true voltage gain of piezoelectric transformers for wider frequency ranges, it is necessary to have a more precise equivalent circuit model for circuit design.

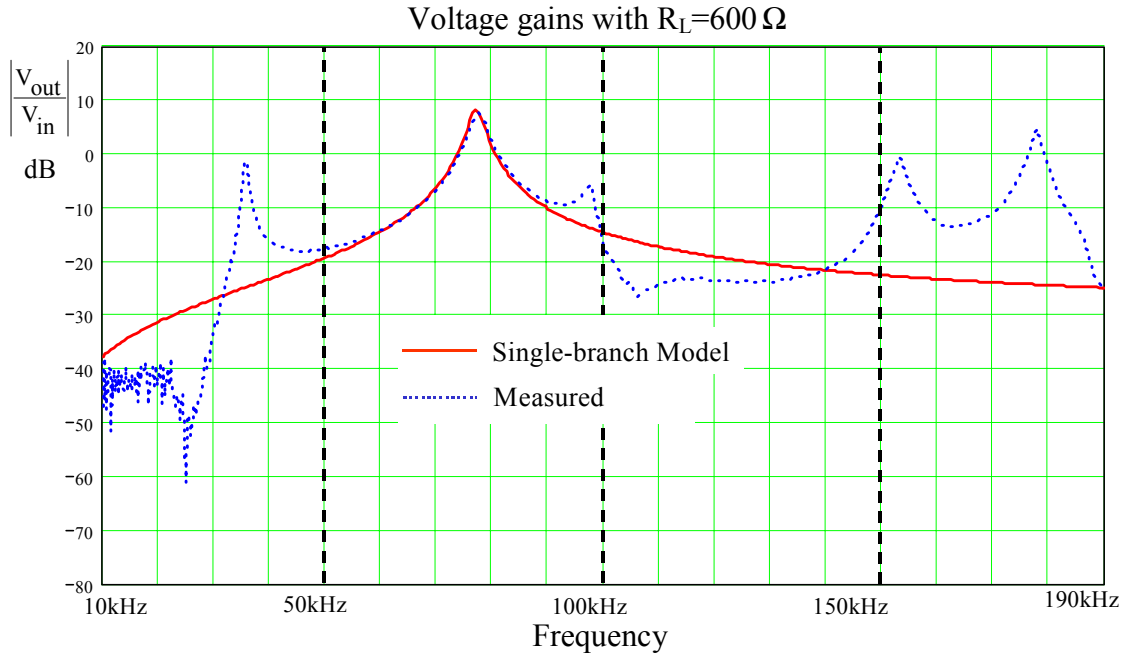


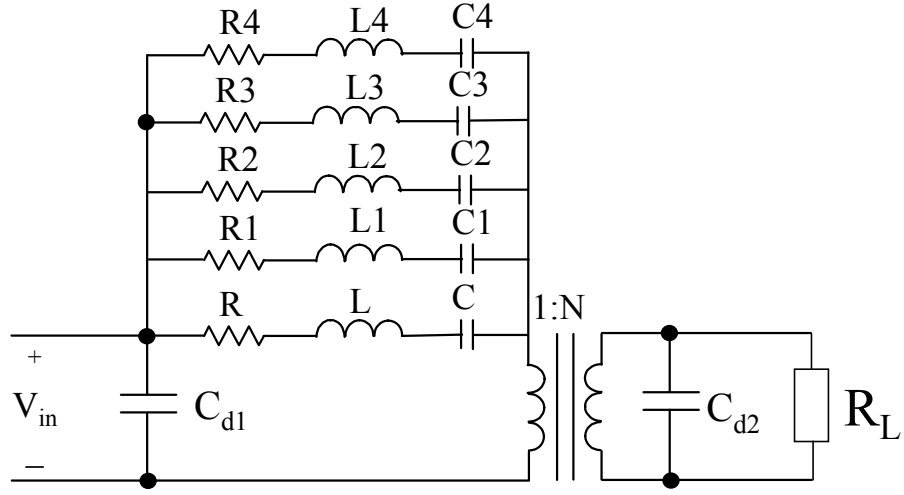
Figure 3.13. Voltage gains of measured result and single-branch equivalent circuit model. The single-branch model, discussed in Section 3.2.2, can only match the major vibration mode within a specific frequency range.

3.4.1 Prior Model

In order to describe the vibration mode at the major resonant frequency as well as other spurious vibration modes for piezoelectric transformers, prior work [A8] developed a multi-branch equivalent circuit model, as shown in Figure 3.14. This model has more R-L-C branches to describe other spurious vibration modes.

With the output terminal, V_{out} , shorted, the input admittance, Y_{in} , of sample CK2 can be measured, as shown in Figure 3.15. Referring to Figure 3.15, there are four spurious vibration modes, $fr1$, $fr2$, $fr3$ and $fr4$, in addition to the major radial vibration mode, fr , within the frequency range of 10kHz to 200kHz. With the frequency ranges corresponding to different vibration modes, as shown in Figure 3.16, each vibration mode can be characterized by a different Y-parameter equivalent circuit model, as shown in Table 3.5. Spurious vibration modes, $fr1$ and $fr3$, can result from spurious radial vibrations. Spurious vibration modes, $fr2$ and $fr4$, can result from spurious thickness vibrations. Table 3.5 shows that each vibration mode is represented by one corresponding R-L-C network branch with a different turns ratio. However, the multi-branch equivalent circuit based on the prior approach [A8], as shown in Figure 3.17, only uses the turns ratio of the major radial vibration mode while ignoring the other turns ratios of the spurious vibration modes. The parameters of this prior equivalent circuit model for sample CK2 are listed in Table 3.6.

Although the multi-branch equivalent circuit model based on the prior approach matches the measured input admittance curve Y_{in} , as shown in Figure 3.18, there is quite a deviation in the respective voltage gain curves, as shown in Figure 3.19. It is apparent that the prior multi-branch equivalent circuit model needs some modification in order to more precisely match the measured voltage gain curve of radial vibration mode piezoelectric transformers. Toward this goal, the next section will propose an improved multi-branch equivalent circuit model.



$$fr \Big|_{R_L=0} = \frac{1}{2\pi\sqrt{L \cdot C}}$$

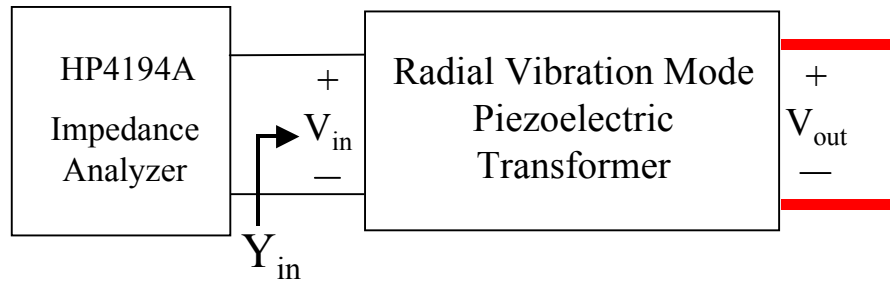
$$fr_1 \Big|_{R_L=0} = \frac{1}{2\pi\sqrt{L_1 \cdot C_1}}$$

$$fr_3 \Big|_{R_L=0} = \frac{1}{2\pi\sqrt{L_3 \cdot C_3}}$$

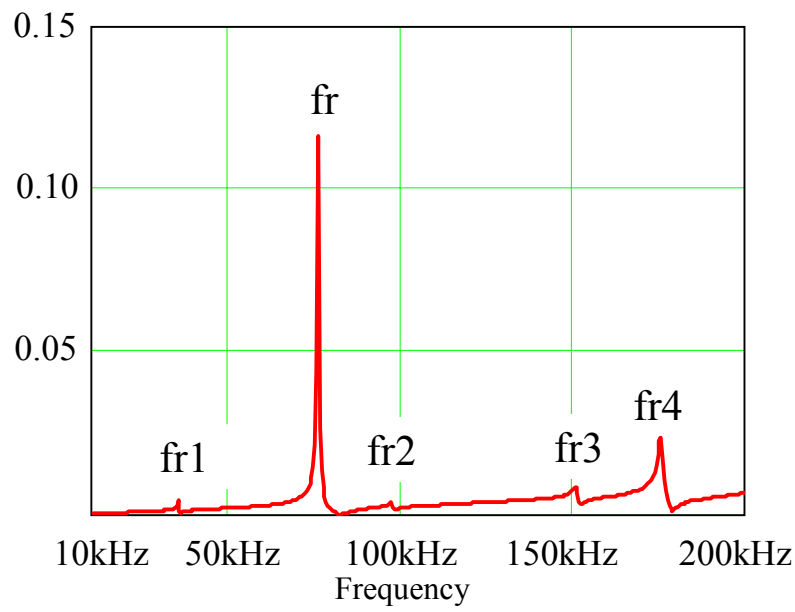
$$fr_2 \Big|_{R_L=0} = \frac{1}{2\pi\sqrt{L_2 \cdot C_2}}$$

$$fr_4 \Big|_{R_L=0} = \frac{1}{2\pi\sqrt{L_4 \cdot C_4}}$$

Figure 3.14. Multi-branch equivalent circuit model for piezoelectric transformers [A8]. This model describes the major resonant vibration mode at the major resonant frequency, fr , and other spurious resonant vibration modes at other spurious resonant frequencies, fr_1 , fr_2 , fr_3 and fr_4 .

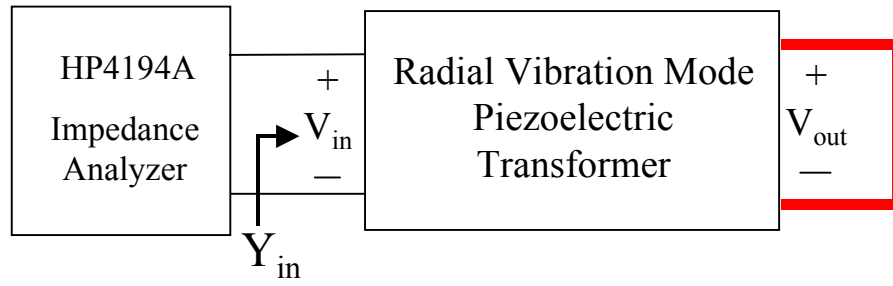


(a)

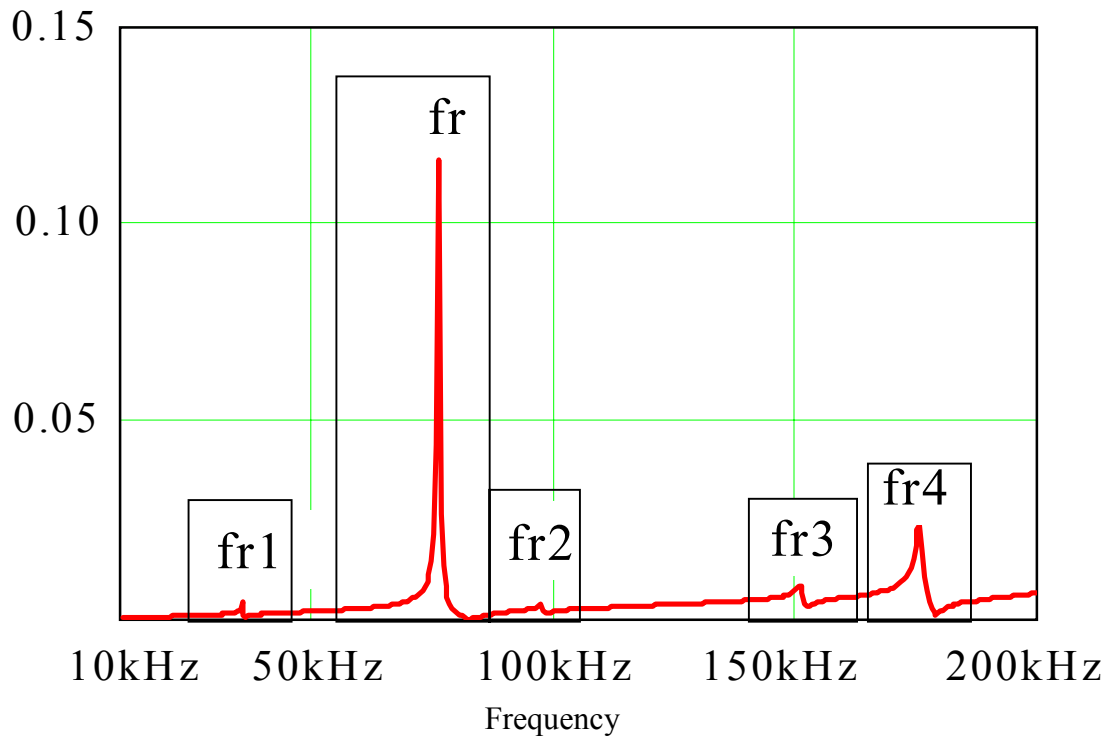


(b)

Figure 3.15. Measured input admittance of radial vibration mode piezoelectric transformer sample CK2. (a) measurement setup. (b) magnitude of input admittance Y_{in} (mho). Within the frequency range from 10kHz to 200kHz, there are four spurious vibration frequencies, $fr1$, $fr2$, $fr3$ and $fr4$, in addition to the major radial vibration mode resonant frequency, fr .



(a)



(b)

Figure 3.16. Frequency ranges corresponding to different vibration modes for characterizing Y-parameter equivalent circuit for sample CK2. (a) measurement setup. (b) magnitude of input admittance Y_{in} (mho).

Table 3.5. Measured parameters of Y-parameter equivalent circuits for radial vibration mode piezoelectric transformer sample CK2.

(10kHz to 200kHz)	Y-Parameter Equivalent Circuits		Turns Ratio
Resonant Frequency	Input Admittance Y_{in}	Output Admittance Y_{out}	
$fr = \frac{1}{\sqrt{L_{1s} \cdot C_{1s}}} = 75.87\text{kHz}$ $\approx \frac{N_p}{2 \cdot r} = 78.7\text{kHz}$ (Major Radial Vibration Mode)	$R_{1s} = 6.42\Omega$	$R_{2s} = 29.84\Omega$	$N = \sqrt{\frac{L_{2s}}{L_{1s}}}$ $= 2$
	$L_{1s} = 4.79\text{mH}$	$L_{2s} = 20.75\text{mH}$	
	$C_{1s} = 918.71\text{pF}$	$C_{2s} = 211.44\text{pF}$	
	$C_{b1s} = 5.41\text{nF}$	$C_{b2s} = 2.74\text{nF}$	
$fr1 = \frac{1}{\sqrt{L_{1s1} \cdot C_{1s1}}} = 35.9\text{kHz}$ $\approx \frac{N_p}{4 \cdot r} = 39.4\text{kHz}$ (Spurious Radial Vibration Mode)	$R_{1s1} = 103.93\Omega$	$R_{2s1} = 112.31\Omega$	$N1 = \sqrt{\frac{L_{2s1}}{L_{1s1}}}$ $= 1.04$
	$L_{1s1} = 112.36\text{mH}$	$L_{2s1} = 126.21\text{mH}$	
	$C_{1s1} = 181.215\text{pF}$	$C_{2s1} = 161.296\text{pF}$	
$fr2 = \frac{1}{\sqrt{L_{1s2} \cdot C_{1s2}}} = 97.5\text{kHz}$ $\approx \frac{N_t}{12 \cdot t} = 86.1\text{kHz}$ (Spurious Thickness Vibration Mode)	$R_{1s2} = 368.23\Omega$	$R_{2s2} = 365.9\Omega$	$N2 = \sqrt{\frac{L_{2s2}}{L_{1s2}}}$ $= 1.1$
	$L_{1s2} = 52.32\text{mH}$	$L_{2s2} = 66.07\text{mH}$	
	$C_{1s2} = 50.92\text{pF}$	$C_{2s2} = 40.27\text{pF}$	
$fr3 = \frac{1}{\sqrt{L_{1s3} \cdot C_{1s3}}} = 151.3\text{kHz}$ $\approx \frac{N_p}{r} = 157.5\text{kHz}$ (Spurious Radial Vibration Mode)	$R_{1s3} = 184.76\Omega$	$R_{2s3} = 218.2\Omega$	$N3 = \sqrt{\frac{L_{2s3}}{L_{1s3}}}$ $= 1.06$
	$L_{1s3} = 20.3536\text{mH}$	$L_{2s3} = 23.16\text{mH}$	
	$C_{1s3} = 54.3587\text{pF}$	$C_{2s3} = 47.73\text{pF}$	
$fr4 = \frac{1}{\sqrt{L_{1s4} \cdot C_{1s4}}} = 175.8\text{kHz}$ $\approx \frac{N_t}{6 \cdot t} = 172.2\text{kHz}$ (Spurious Thickness Vibration Mode)	$R_{1s4} = 42.75\Omega$	$R_{2s4} = 264.67\Omega$	$N4 = \sqrt{\frac{L_{2s4}}{L_{1s4}}}$ $= 2.4$
	$L_{1s4} = 4.81\text{mH}$	$L_{2s4} = 27.97\text{mH}$	
	$C_{1s4} = 170.25\text{pF}$	$C_{2s4} = 28.34\text{pF}$	

Note: r is the radius of sample CK2,
 t is the layer thickness of sample CK2,
 N_p is the radial frequency constant of the piezoelectric material, and
 N_t is the thickness frequency constant of the piezoelectric material.

Table 3.6. Characterized parameters of prior multi-branch equivalent circuit model for radial vibration mode piezoelectric transformer sample CK2.

Resonant Frequency	Parameter	Characterized Value
$f_r = \frac{1}{\sqrt{L_{1s} \cdot C_{1s}}} = 75.87\text{kHz}$ $\approx \frac{N_p}{2 \cdot r} = 78.7\text{kHz}$ (Major Radial Vibration Mode)	C _{d1}	5.41nF
	R	6.42Ω
	L	4.79mH
	C	918.71pF
	N	2
	C _{d2}	2.74nF
$f_{r1} = \frac{1}{\sqrt{L_{1s1} \cdot C_{1s1}}} = 35.9\text{kHz}$ $\approx \frac{N_p}{4 \cdot r} = 39.4\text{kHz}$ (Spurious Radial Vibration Mode)	R1	103.93Ω
	L1	112.36mH
	C1	181.2pF
$f_{r2} = \frac{1}{\sqrt{L_{1s2} \cdot C_{1s2}}} = 97.5\text{kHz}$ $\approx \frac{N_t}{12 \cdot t} = 86.1\text{kHz}$ (Spurious Thickness Vibration Mode)	R2	368.23Ω
	L2	52.32mH
	C2	50.92pF
$f_{r3} = \frac{1}{\sqrt{L_{1s3} \cdot C_{1s3}}} = 151.3\text{kHz}$ $\approx \frac{N_p}{r} = 157.5\text{kHz}$ (Spurious Radial Vibration Mode)	R3	184.76Ω
	L3	20.3536mH
	C3	54.3587pF
$f_{r4} = \frac{1}{\sqrt{L_{1s4} \cdot C_{1s4}}} = 175.8\text{kHz}$ $\approx \frac{N_t}{6 \cdot t} = 172.2\text{kHz}$ (Spurious Thickness Vibration Mode)	R4	42.75Ω
	L4	4.81mH
	C4	170.25pF

Note: r is the radius of sample CK2,

t is the layer thickness of sample CK2,

N_p is the radial frequency constant of the piezoelectric material, and

N_t is the thickness frequency constant of the piezoelectric material.

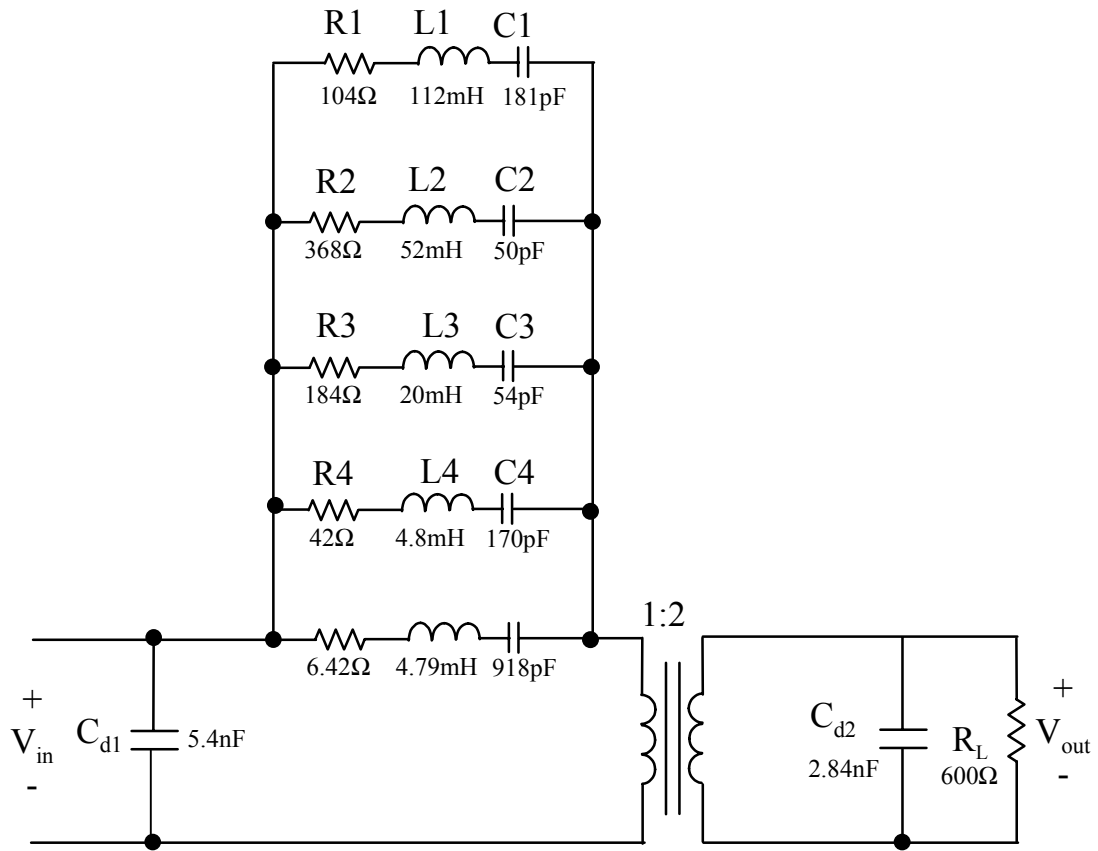
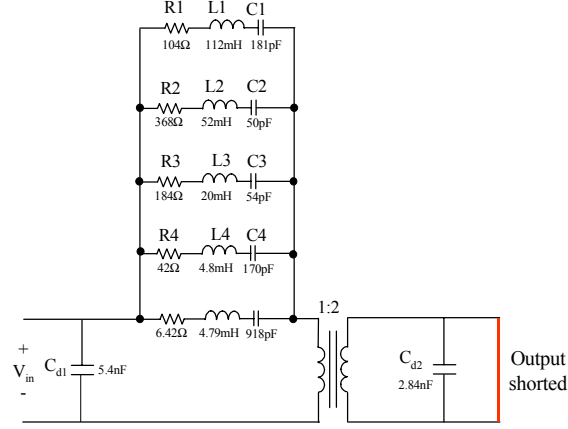
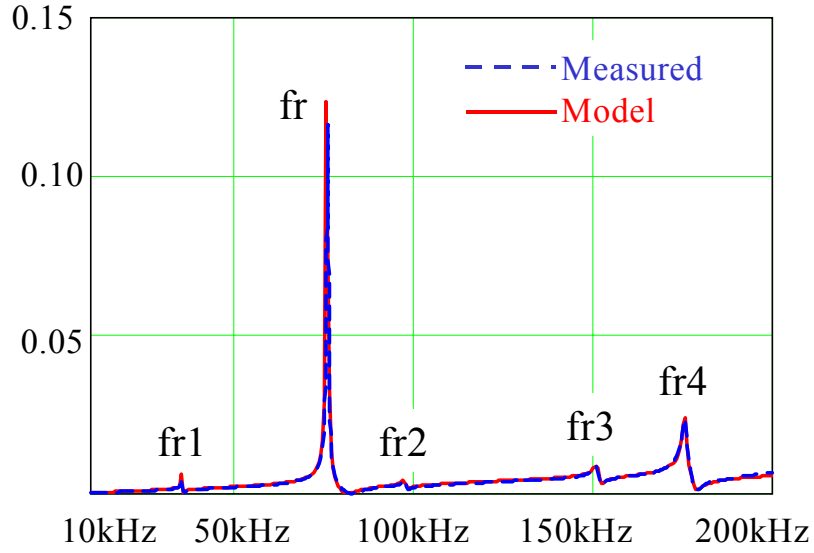


Figure 3.17. Multi-branch equivalent circuit model for the radial vibration mode piezoelectric transformer sample, CK2, based on prior approach [A8]. This multi-branch equivalent circuit is composed of five R-L-C branches. Each vibration mode is represented by one corresponding R-L-C branch.



(a)



(b)

Figure 3.18. Comparison of measured and prior model input admittance curves for sample CK2. (a) multi-branch equivalent circuit model based on prior approach [A8]. (b) magnitude of input admittance (mho). The prior multi-branch equivalent circuit model can very closely match the measured input admittance curve, Y_{in} .

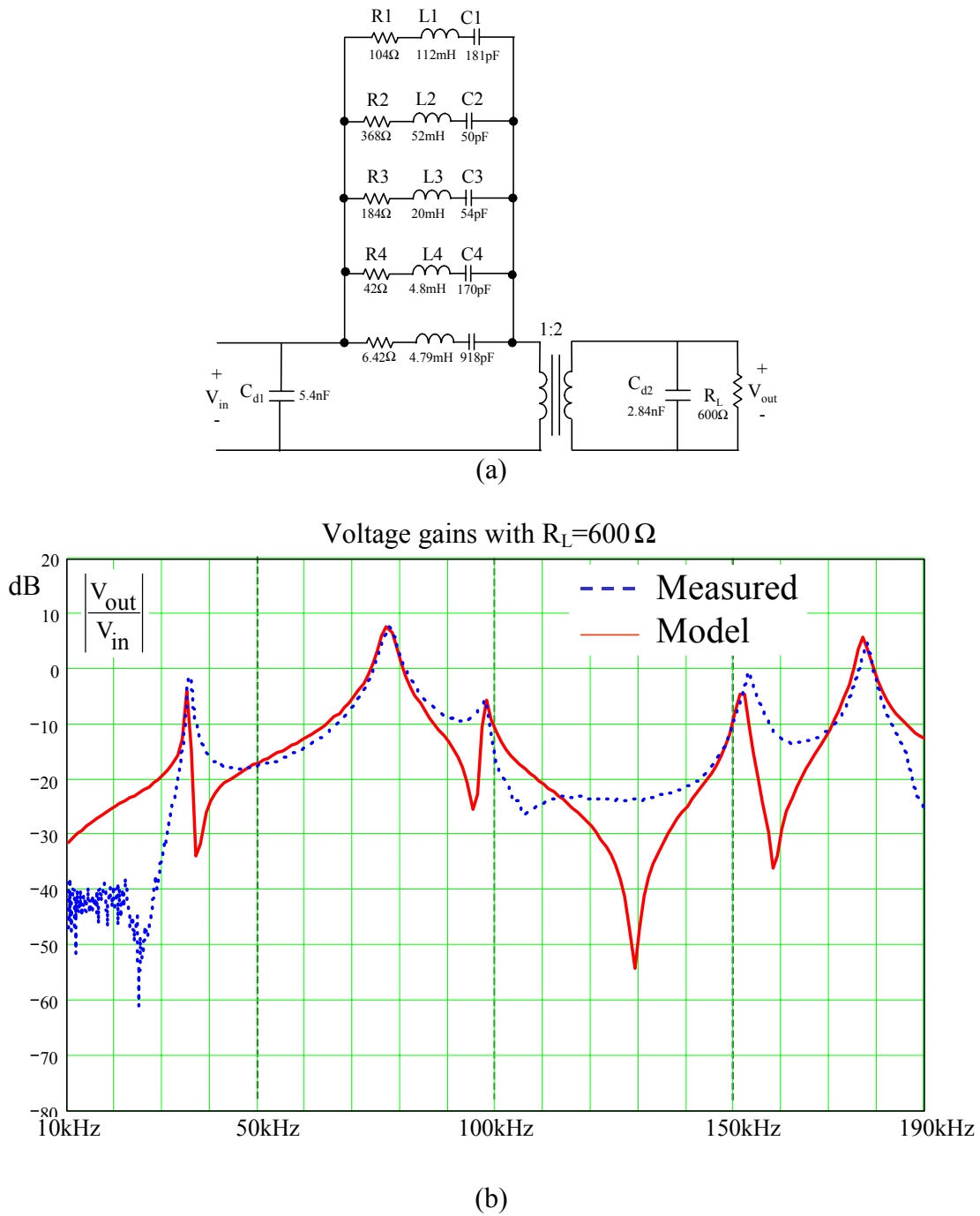


Figure 3.19. Comparison of measured voltage gain and modeled result for radial vibration mode piezoelectric transformer sample CK2 with $R_L = 600\Omega$. (a) prior multi-branch Equivalent circuit model [A8]. (b) measured and modeled results of voltage gains. The modeled voltage gain is based on a prior multi-branch equivalent circuit model [A8]. However, there is quite a deviation in the respective voltage gain curve.

3.4.2 Improved Model

Although the prior multi-branch equivalent circuit model based on the prior approach [A8] can very closely match the measured input admittance curve, Y_{in} , there is still quite a deviation in the respective voltage gain curves, as found in the last section. Since different vibration modes have different voltage gains and phases, as shown in Figure 3.20, the prior model cannot simply utilize one common ideal transformer to describe all the vibration modes within a specific frequency range. Figures 3.21 to 3.25 show the voltage gains and phases of different vibration modes, fr, fr1, fr2, fr3 and fr4, according to the parameters of the characterized Y-parameter equivalent circuits shown in Table 3.5. These figures show that the polarities of turns ratios, N, N1, N2, N3, and N4, for each single equivalent circuit network can affect the deviation of the phase curve. Figure 3.26 shows the voltage gain and phase curves of the multi-branch equivalent circuit of vibration modes, fr and fr2, with the correct polarities for their turns ratio, N and N2. Figure 3.27 shows the voltage gain and phase curves of the multi-branch equivalent circuit of vibration modes, fr, fr1, and fr2, with the correct polarities for their turns ratios, N and N2.

In order to more precisely describe the measured voltage gain, the prior multi-branch equivalent circuit model can be improved by accounting for the phase and branch gain ratio characteristics of all of the vibration modes, as shown in Figure 3.28. The parameters of the improved model for sample CK2 are shown in Table 3.7. This improved model can match the measured voltage gain curve much more accurately than the prior multi-branch equivalent circuit model, as shown in Figures 3.29 and 3.30. The improved circuit model uses more detailed networks to describe the voltage gains and phases of various spurious vibration modes for radial vibration mode piezoelectric transformers.

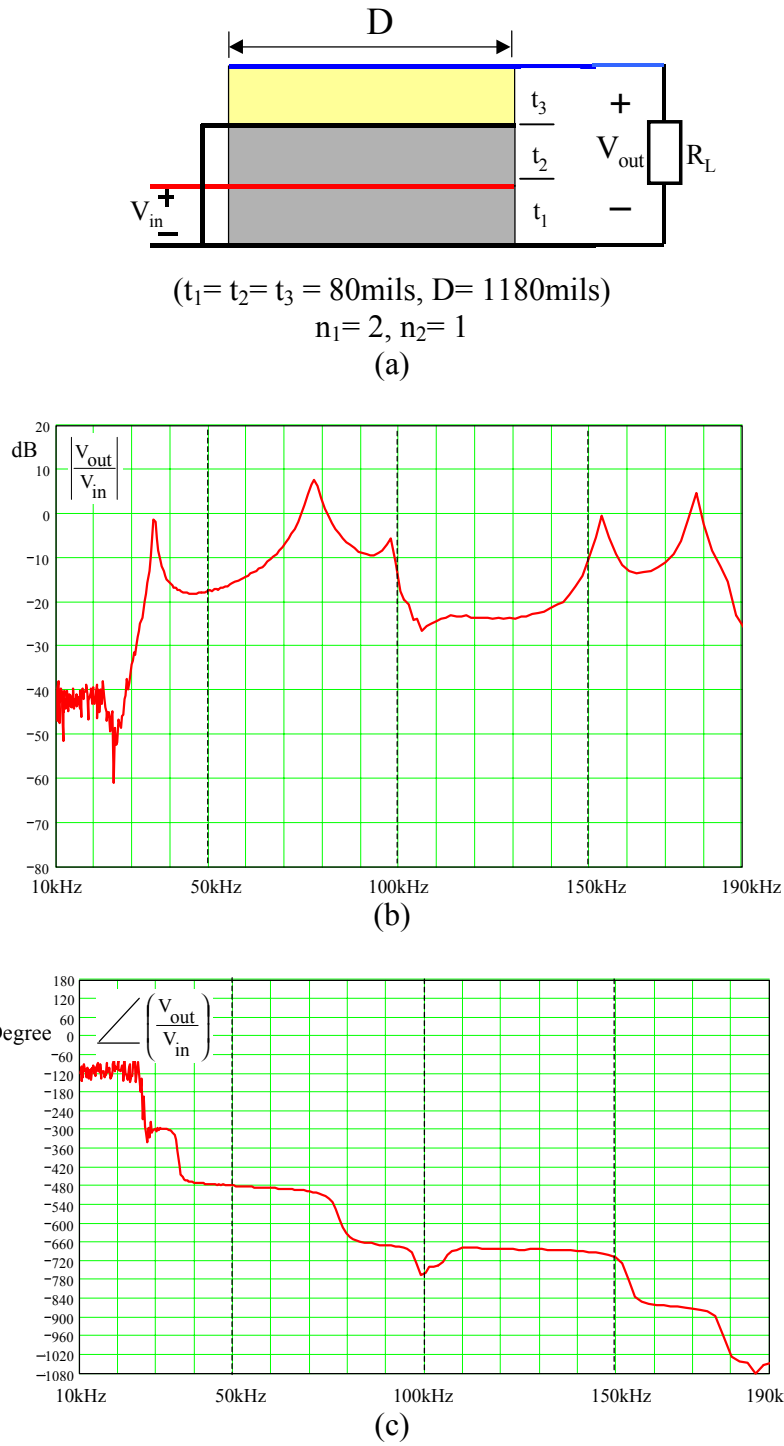


Figure 3.20. Measured voltage gain and phase of sample CK2. (a) sample CK2 with $R_L=600\Omega$. (b) voltage gain (dB). (c) phase (Degree).

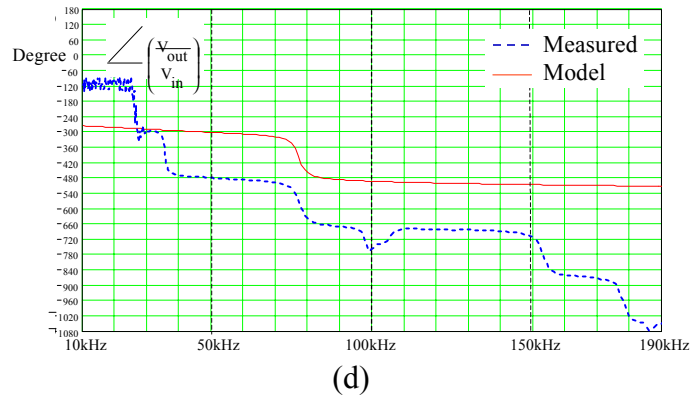
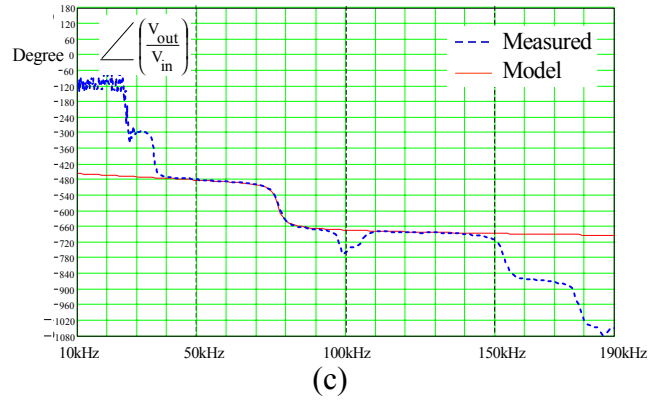
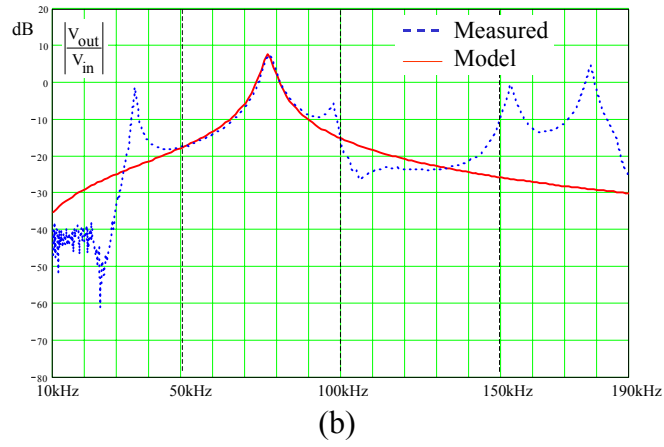
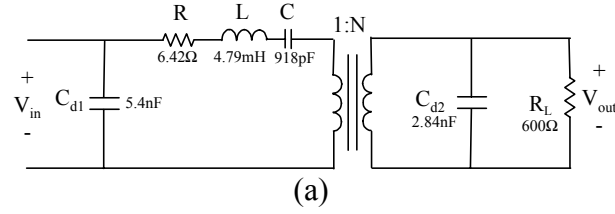


Figure 3.21. Voltage gain and phase of major vibration mode, fr. (a) equivalent circuit of major vibration mode, fr. (b) voltage gain ($N = +2$ or -2). (c) phase ($N = -2$). (d) phase ($N = +2$).

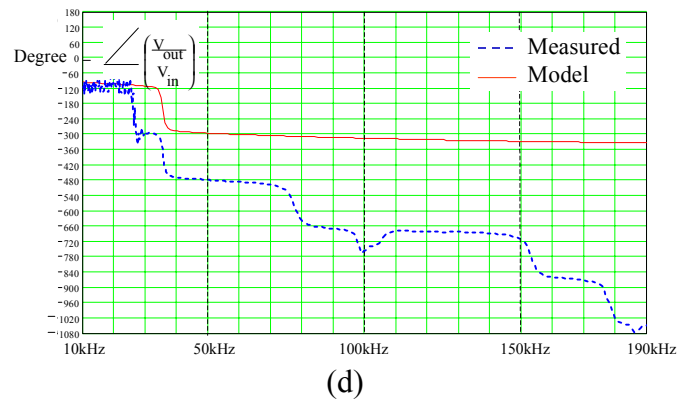
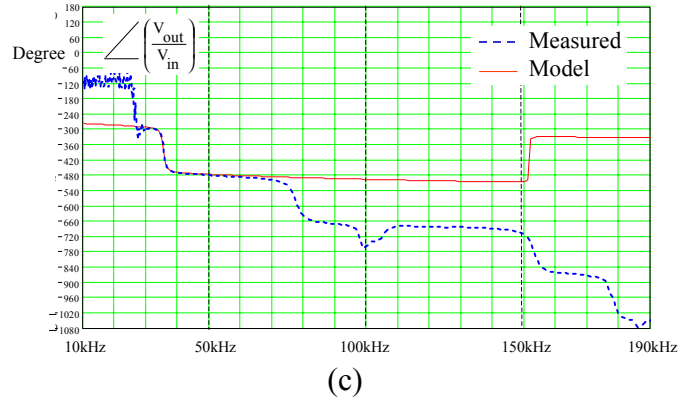
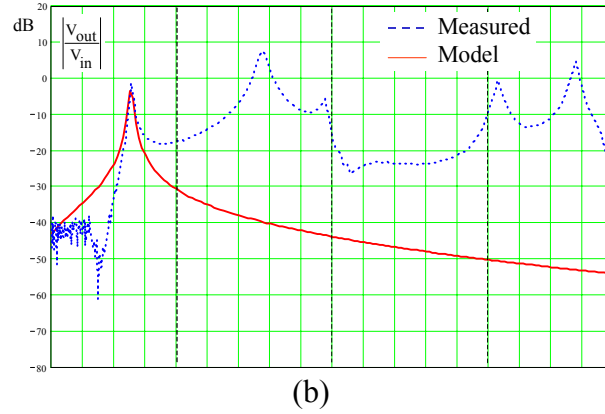
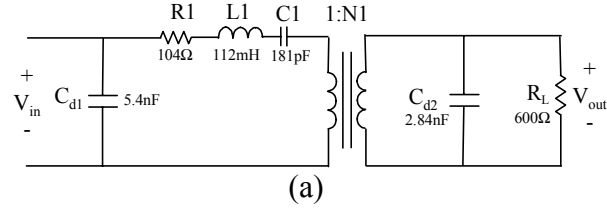
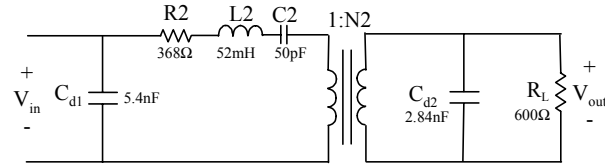
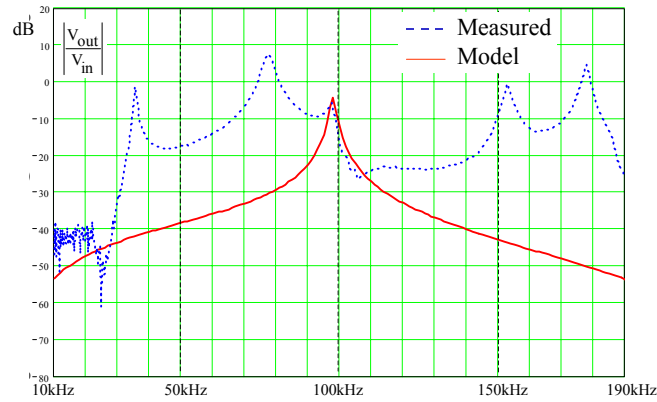


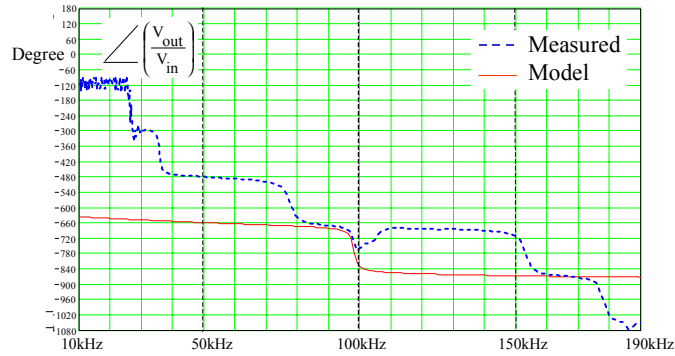
Figure 3.22. Voltage gain and phase of spurious vibration mode, fr1. (a) equivalent circuit of spurious vibration mode, fr1. (b) voltage gain ($N1 = +1.04$ or -1.04). (c) phase ($N1 = +1.04$). (d) phase ($N1 = -1.04$).



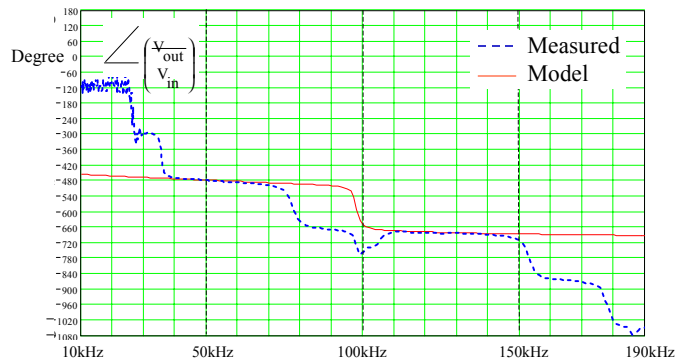
(a)



(b)



(c)



(d)

Figure 3.23. Voltage gain and phase of spurious vibration mode, fr2. (a) equivalent circuit of spurious vibration mode, fr2. (b) voltage gain ($N2= +1.1$ or -1.1). (c) phase ($N2= +1.1$). (d) phase ($N2= -1.1$) .

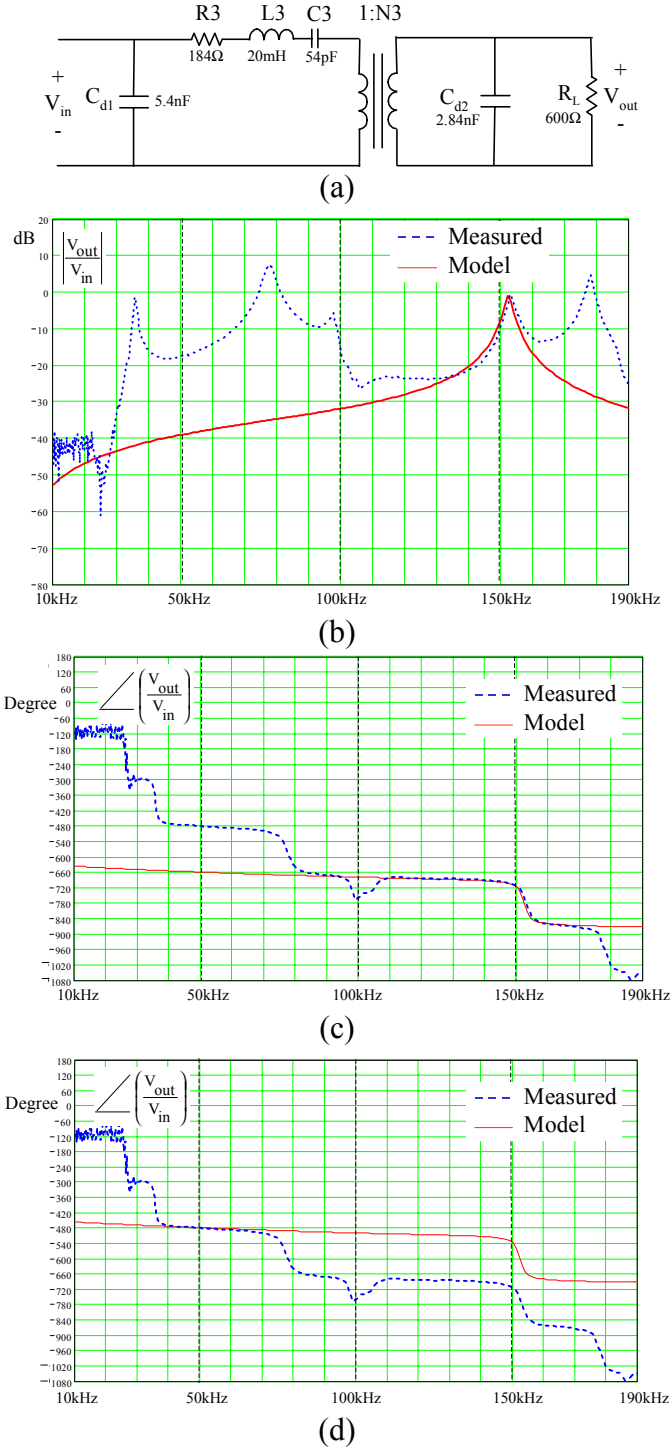


Figure 3.24. Voltage gain and phase of spurious vibration mode, fr3. (a) equivalent circuit of spurious vibration mode, fr3. (b) voltage gain ($N3 = +1.06$ or -1.06). (c) phase ($N3 = +1.06$). (d) phase ($N3 = -1.06$).

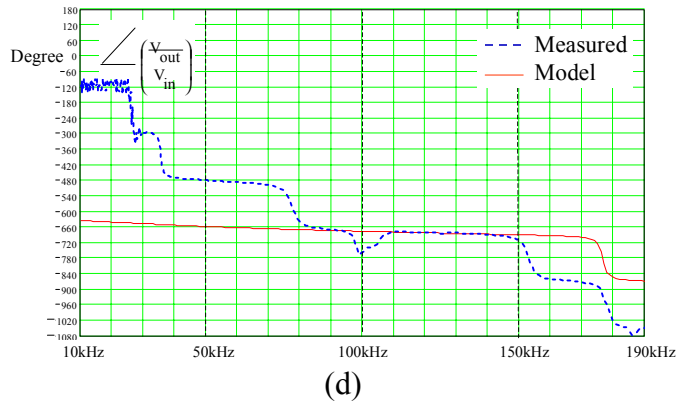
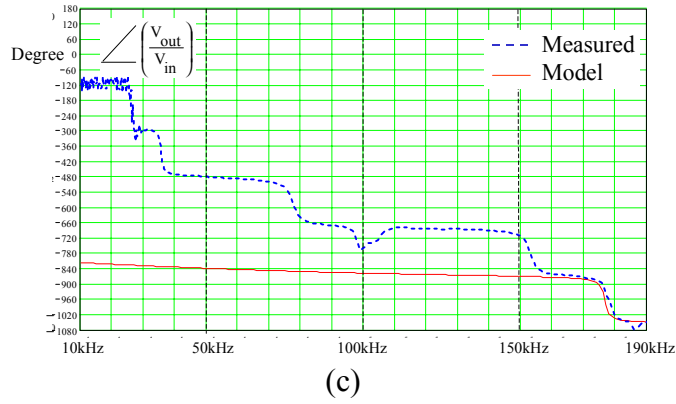
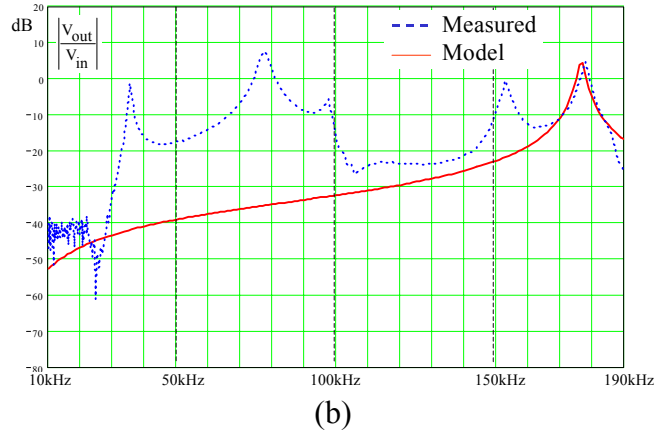
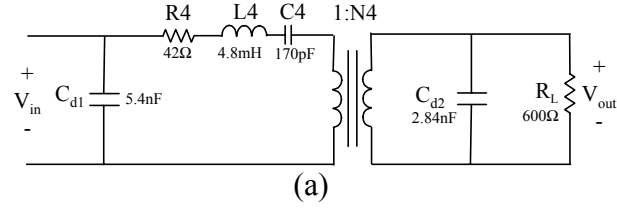
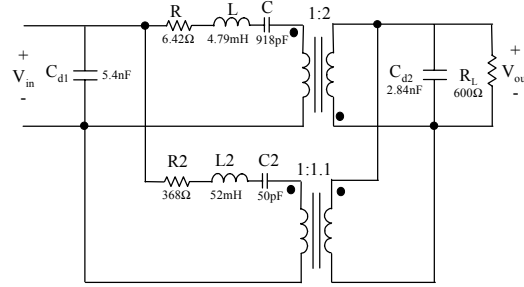
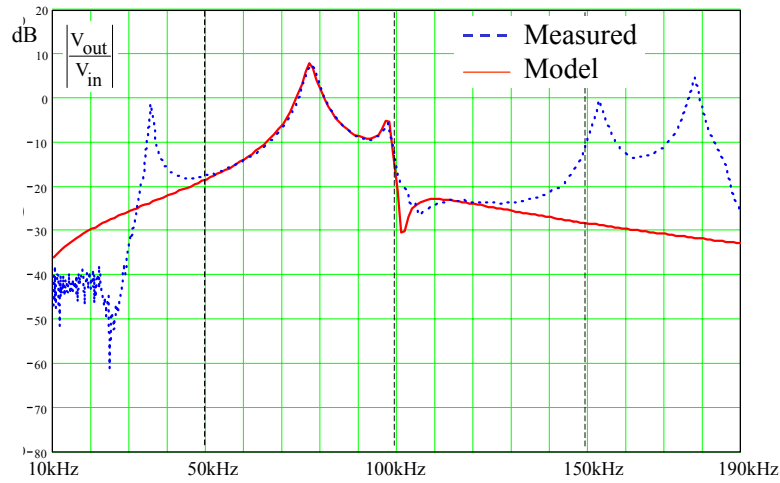


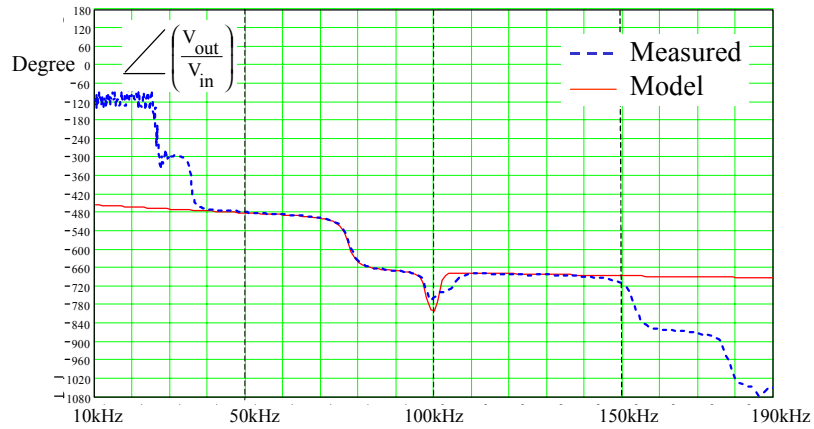
Figure 3.25. Voltage gain and phase of spurious vibration mode, fr4. (a) equivalent circuit of spurious vibration mode, fr4. (b) voltage gain ($N4 = +2.4$ or -2.4). (c) phase ($N4 = -2.4$) (d) phase ($N4 = +2.4$).



(a)

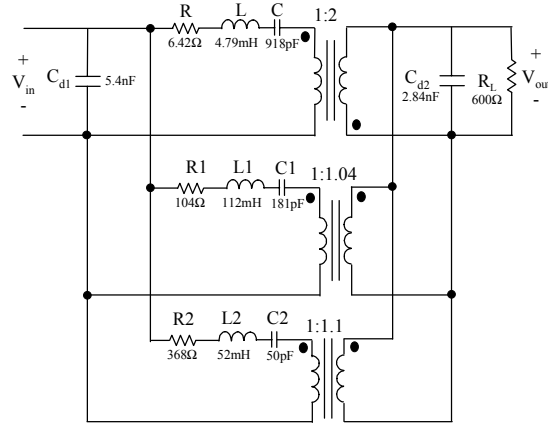


(b)

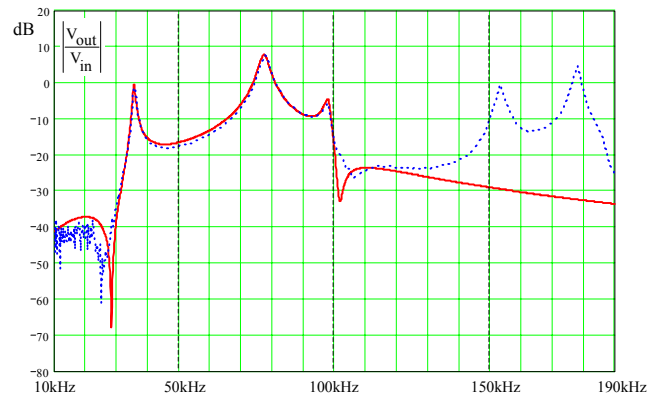


(c)

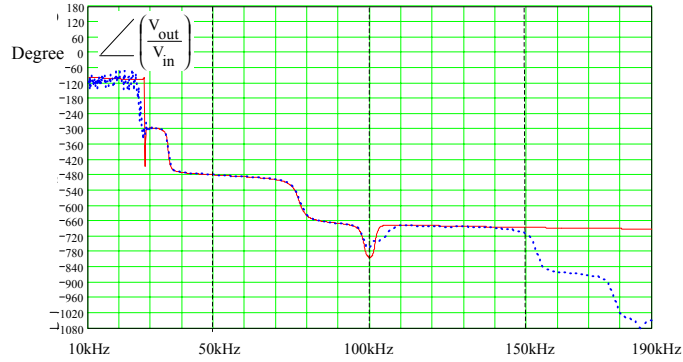
Figure 3.26. Voltage gain and phase of multi-branch equivalent circuit of vibration modes, fr and fr2. (a) multi-branch equivalent circuit of vibration modes, fr and fr2. (b) voltage Gain ($N = -2$ and $N2 = 1.1$). (c) phase ($N = -2$ and $N2 = 1.1$).



(a)



(b)



(c)

Figure 3.27. Voltage gain and phase of multi-branch equivalent circuits of major and spurious vibration modes, fr, fr1 and fr2. (a) multi-branch equivalent circuit of major and spurious vibration modes, fr, fr1 and fr2. (b) voltage gain ($N = -2$, $N1 = 1.04$ and $N2 = 1.1$). (c) phase ($N = -2$, $N1 = 1.04$ and $N2 = 1.1$).

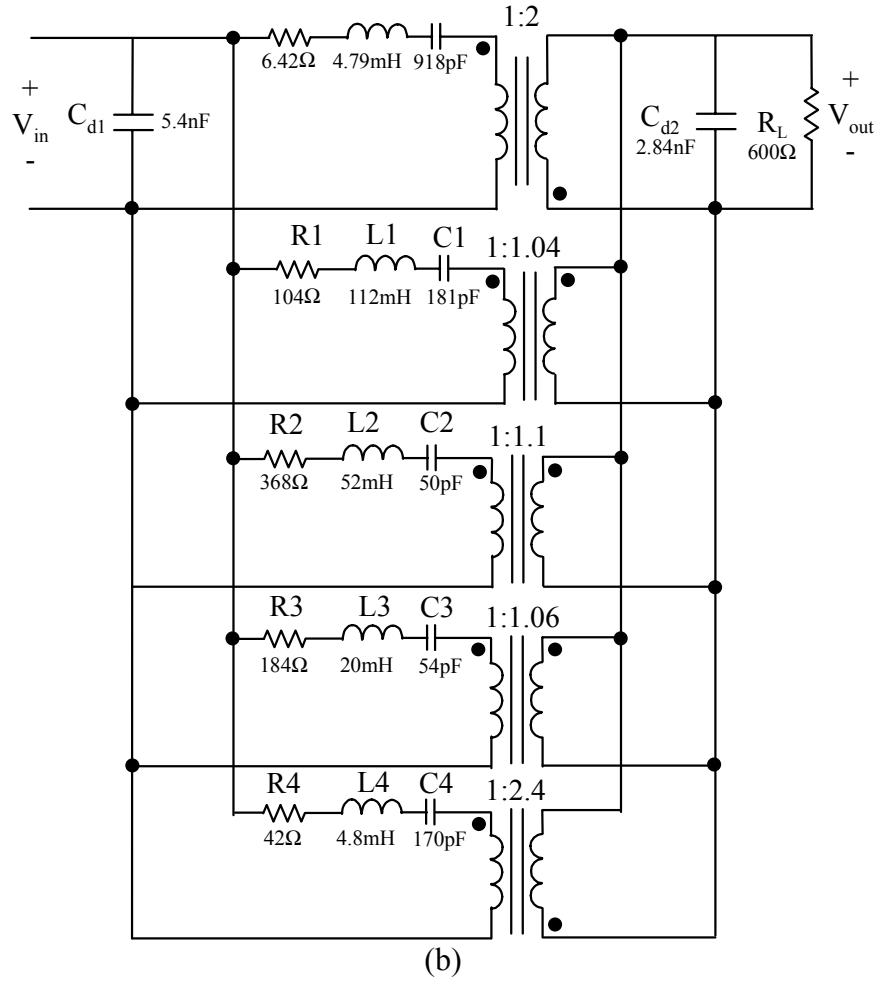
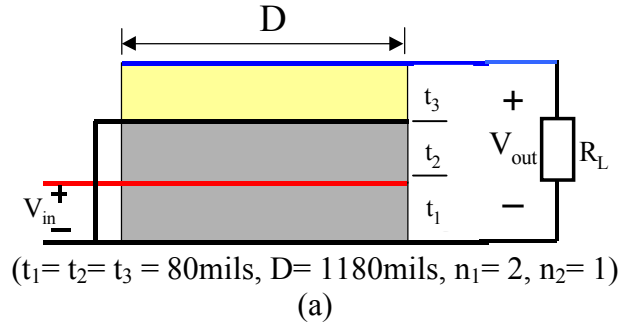


Figure 3.28. Improved multi-branch equivalent circuit model for radial vibration mode piezoelectric transformer sample CK2. (a) sample CK2 with $R_L=600\Omega$. (b) improved equivalent circuit model.

Table 3.7. Parameters of improved multi-branch equivalent circuit model for radial vibration mode piezoelectric transformer sample CK2.

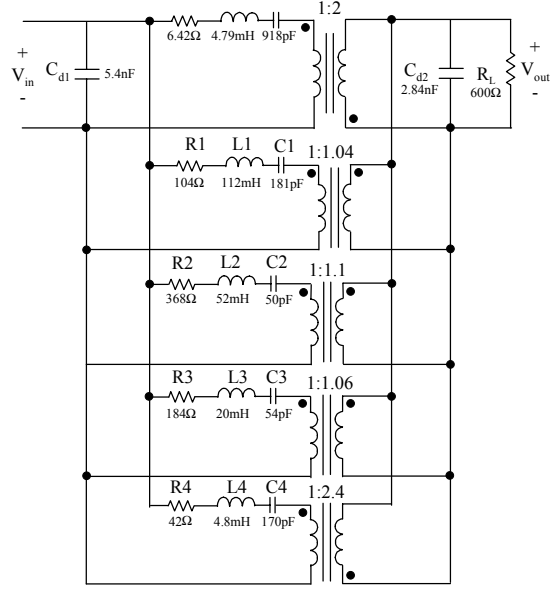
Resonant frequency	Parameter	Characterized value
$f_r = \frac{1}{\sqrt{L_{1s} \cdot C_{1s}}} = 75.87\text{kHz}$ $\approx \frac{N_p}{2 \cdot r} = 78.7\text{kHz}$ (Major radial Vibration Mode)	C_{d1}	5.41nF
	R	6.42Ω
	L	4.79mH
	C	918.71pF
	N	-2
	C_{d2}	2.742nF
$f_{r1} = \frac{1}{\sqrt{L_{1s1} \cdot C_{1s1}}} = 35.9\text{kHz}$ $\approx \frac{N_p}{4 \cdot r} = 39.4\text{kHz}$ (Spurious Radial Vibration Mode)	R1	103.93Ω
	L1	112.36mH
	C1	181.215pF
	N1	1.04
$f_{r2} = \frac{1}{\sqrt{L_{1s2} \cdot C_{1s2}}} = 97.5\text{kHz}$ $\approx \frac{N_t}{12 \cdot t} = 86.1\text{kHz}$ (Spurious Thickness Vibration Mode)	R2	368.23Ω
	L2	52.32mH
	C2	50.92pF
	N2	1.1
$f_{r3} = \frac{1}{\sqrt{L_{1s3} \cdot C_{1s3}}} = 151.3\text{kHz}$ $\approx \frac{N_p}{r} = 157.5\text{kHz}$ (Spurious Radial Vibration Mode)	R3	184.76Ω
	L3	20.3536mH
	C3	54.3587pF
	N3	1.06
$f_{r4} = \frac{1}{\sqrt{L_{1s4} \cdot C_{1s4}}} = 175.8\text{kHz}$ $\approx \frac{N_t}{6 \cdot t} = 172.2\text{kHz}$ (Spurious Thickness Vibration Mode)	R4	42.75Ω
	L4	4.81mH
	C4	170.25pF
	N4	-2.4

Note: r is the radius of sample CK2,

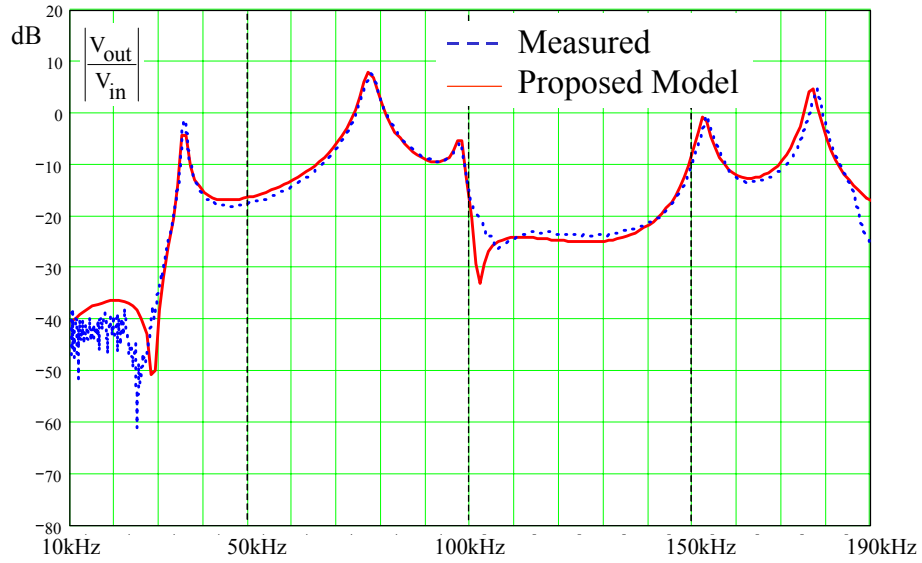
t is the layer thickness of sample CK2,

N_p is the radial frequency constant of the piezoelectric material, and

N_t is the thickness frequency constant of the piezoelectric material.

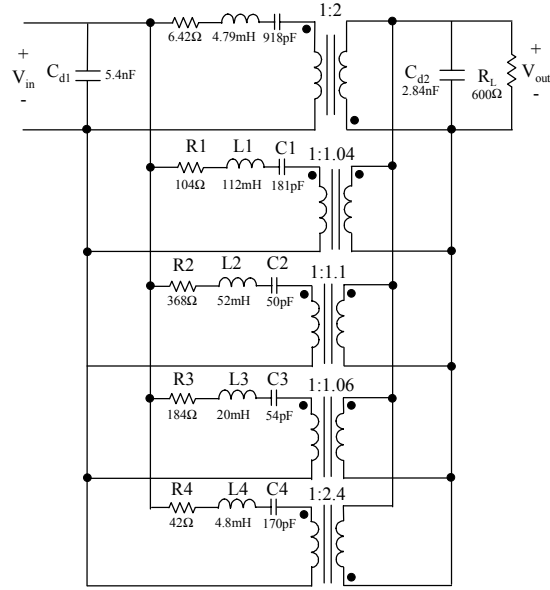


(a)

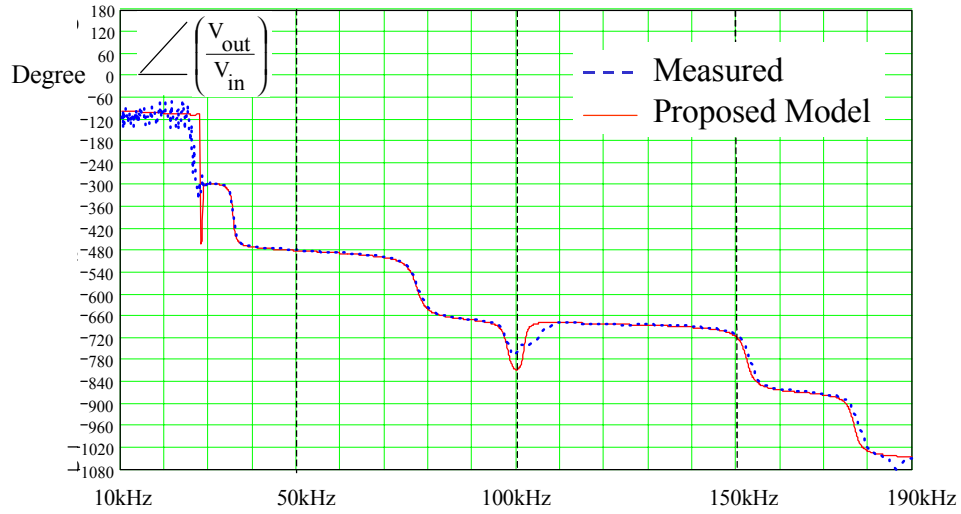


(b)

Figure 3.29. Comparison of voltage gains between the measured result and the improved multi-branch equivalent circuit model for radial vibration mode piezoelectric transformer sample, CK2. (a) improved multi-branch equivalent circuit model. (b) voltage gain curves of the measured result and the improved model. The improved model more accurately matches the measured voltage gain curve than the prior model.



(a)



(b)

Figure 3.30. Comparison of phase between measured result and improved multi-branch equivalent circuit model for radial vibration Mode piezoelectric transformer sample CK2. (a) improved multi-branch equivalent circuit model. (b) voltage phase curves of measured result and improved model. This improved model matches the measured phase curve with acceptable deviation.

3.5 Summary

By utilizing the Y-parameter equivalent circuit model provided by the HP4194A impedance analyzer, this chapter has characterized the samples of radial vibration mode piezoelectric transformer in order to verify the physics-based equivalent circuit model derived in Chapter 2 for the major vibration mode. The comparison results showed that the physics-based equivalent circuit model is in good agreement with the corresponding measured results of the piezoelectric transformer samples when the total thickness is much less than the radius.

Besides the major vibration mode, piezoelectric transformers have many spurious vibration modes in other frequency ranges. These vibration modes can be characterized with a multi-branch equivalent circuit model, composed of several single L-C resonant tanks, for the design and simulation of application circuits operating in wide frequency ranges rather than the major vibration mode. In order to more precisely characterize radial vibration mode piezoelectric transformers, this chapter has proposed an improved multi-branch equivalent circuit model to replace the prior circuit model. Although the prior circuit model can very precisely characterize the input admittance, it cannot characterize the voltage gain within an acceptable deviation. The improved multi-branch equivalent circuit model uses more detailed networks to describe the gain ratios and polarities of different spurious vibration modes for radial vibration mode piezoelectric transformers. Therefore, the proposed improved multi-branch equivalent circuit model can characterize the input admittance and voltage gain of radial vibration mode piezoelectric transformer much more precisely than the prior model. The comparison of the measured and modeled results showed that the improved model matches the measured phase curve much better than the prior multi-branch equivalent circuit model for radial vibration mode piezoelectric transformers.

CHAPTER 4

INDUCTOR-LESS PIEZOELECTRIC TRANSFORMER ELECTRONIC BALLAST

4.1 Introduction

Because of high efficacy and long life, fluorescent lamps are becoming more popular than incandescent lamps for residential and commercial uses. Ballast circuits for driving fluorescent lamps can be categorized as either magnetic or electronic. Since magnetic ballasts are bulky and emit an audible low-frequency humming noise, electronic ballasts are more appealing and are produced in greater numbers than almost any other electronic product of their type. However, the price of electronic ballasts is still not low enough to attract all residential and commercial users. In order to lower the price, most lighting companies worldwide have focused their efforts on finding cost-effective component suppliers. Fortunately, piezoelectric transformers provide a good alternative.

Figure 4.1 shows one typical electronic ballast circuit that uses an L-C resonant tank to generate high voltage for igniting and sustaining a linear fluorescent lamp. This L-C resonant tank is composed of resonant inductor L , DC-blocking capacitor C , a voltage step-up transformer, and high-voltage resonant capacitor C_{d2} . Input capacitor C_{d1} works as an additional turn-off snubber capacitor for half-bridge switches S_1 and S_2 . The cost of these five passive components constitutes the major cost of the conventional electronic ballast. However, the L-C resonant tank of the conventional electronic ballast shown in Figure 4.1 is almost identical to the equivalent circuit of the piezoelectric transformer shown in Figure 4.2, except for the addition of the resistor, R , in the latter.

Generally speaking, this resistance R is negligible as compared with the equivalent turn-on resistance of fluorescent lamps.

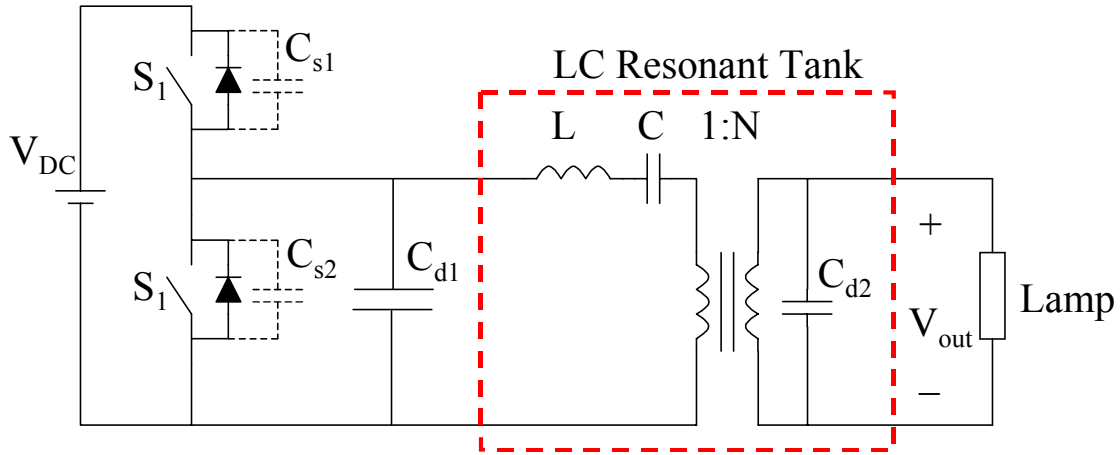


Figure 4.1. Typical conventional electronic ballast circuit. This circuit employs a typical L-C resonant tank circuit and an additional turn-off snubber capacitor C_{d1} for half-bridge switches, S_1 and S_2 .

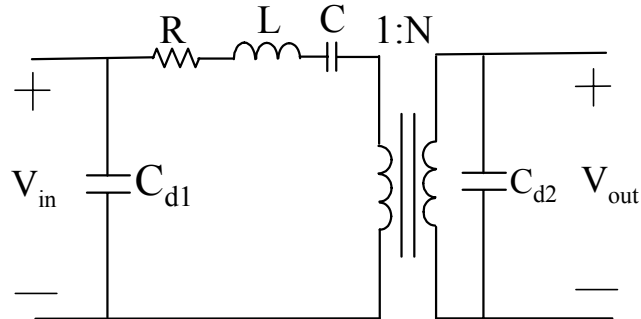


Figure 4.2. Equivalent circuit model of piezoelectric transformers. Components R , L and C are the equivalent physics-based components analogous to electrical terms. Compared with the equivalent turn-on resistance of fluorescent lamps, the resistance, R , in the equivalent circuit model can be ignored.

Many prior technologies [B1-B13] tried to utilize piezoelectric transformers to develop converter, inverter or electronic ballasts. Because of input capacitor C_{d1} , these previous topologies required additional magnetic device(s), such as an inductor, in order to enable their switches to work in ZVS condition. By using this prior approach, the specific characteristics of the piezoelectric transformers were not fully utilized, and thus extra expenses were incurred for the additional magnetic device(s). For example, Figure 4.3 shows a conventional piezoelectric transformer DC/DC converter utilizing a thickness vibration mode piezoelectric transformer [A9]. Because the input capacitor C_{d1} is equal to 2.6nF, this converter must use additional inductors, L_r and L_s , to have its switches, S_1 and S_2 , working in ZVS condition. Since the equivalent circuit of piezoelectric transformers has an inductor, this inductor may be utilized to have the half-bridge switches operate in ZVS condition in order to save the additional inductor, L_s . Furthermore, the input capacitor, C_{d1} , may be utilized as a turn-off snubber capacitor for the half-bridge switches, S_1 and S_2 . Therefore, this chapter will propose ZVS criteria to evaluate the equivalent circuit models of piezoelectric transformers for the required ZVS condition.

Based on the proposed ZVS criteria, this chapter will first present the design and implementation of a cost-effective inductor-less piezoelectric transformer electronic ballast employing a radial vibration mode piezoelectric transformer to drive a 4-foot 40-watt linear fluorescent lamp at $V_{DC}=280V$.

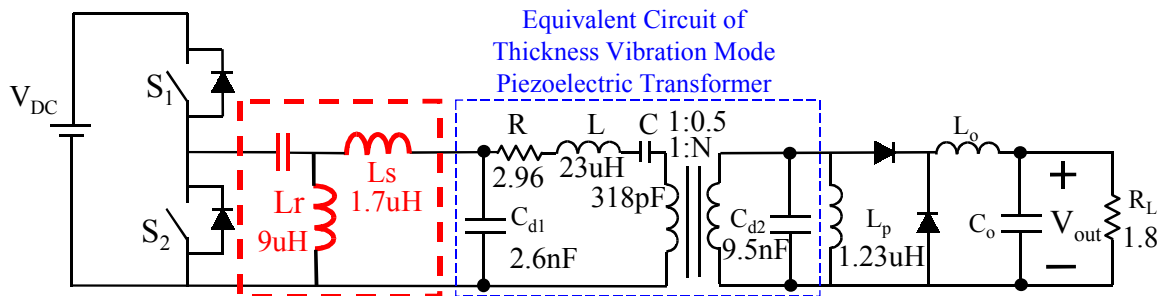


Figure 4.3. Conventional piezoelectric transformer DC/DC converter employing a thickness vibration mode piezoelectric transformer [A9].

This experimental lamp and its equivalent on-resistance of 600Ω requires an ignition voltage greater than $280V_{rms}$ and a sustain voltage greater than $109V_{rms}$. The radial vibration mode piezoelectric transformer serves as a piezoelectric transformer resonant tank that replaces the conventional passive L-C resonant tank. With its inherent piezoelectric resonant characteristics, the radial vibration mode piezoelectric transformer is able to both ignite and provide sustaining voltage to a linear fluorescent lamp. The experimental results of this inductor-less piezoelectric transformer electronic ballast circuit will be provided.

Incorporating the proposed ZVS criteria and the derived physics-based equivalent circuit model of radial vibration mode piezoelectric transformers, a suitable radial vibration mode piezoelectric transformer sample will be selected in order to implement inductor-less voltage source charge pump power factor correction (VS-CP-PFC) electronic ballast. The experimental results of this PFC inductor-less piezoelectric transformer electronic ballast circuit will be provided.

4.2 Voltage Gain

Since an electronic ballast should be able to provide sufficient output voltages, V_{out} , to ignite and sustain a fluorescent lamp, as shown in Figure 4.4, piezoelectric transformers must have sufficient voltage gain available for electronic ballast applications. If the DC voltage, V_{DC} , is 280V, then the fundamental component of the input voltage, V_{in} , applied to the piezoelectric transformer is 108Vrms. For a 4-foot 40-watt linear fluorescent lamp, its ignition voltage needs to be greater than 280Vrms and its sustain voltage needs to be greater than 109Vrms. Therefore, for the employed piezoelectric transformer sample to drive this linear fluorescent lamp, the required voltage gain to ignite the lamp needs to be greater than 2.6 and the required voltage gain to sustain lamp needs to be greater than 1.0.

A single-layer radial vibration mode piezoelectric transformer sample AF1 was first considered for this application. However, Figure 4.5 shows that sample AF1 does not have enough voltage gain with $R_{lamp}=600\Omega$ to sustain the lamp, even though it has enough voltage gain to ignite the lamp. The loaded quality factor Q_L of the L-C resonant tank at the corner frequency can be calculated as $Q_L=0.026$ according to the following equation [D10]:

$$Q_L = \frac{R_{Lamp}}{N} \cdot \sqrt{\frac{1}{L} \cdot \frac{C \cdot C_{d2}}{C + C_{d2} \cdot N^2}} \quad (4.1)$$

In order to have higher Q_L value and voltage gain with $R_{lamp}=600\Omega$, another sample CK2 with $Q_L=0.062$ can be considered, as shown in Figure 4.6, to have enough output voltage, V_{out} , to ignite and sustain the lamp.

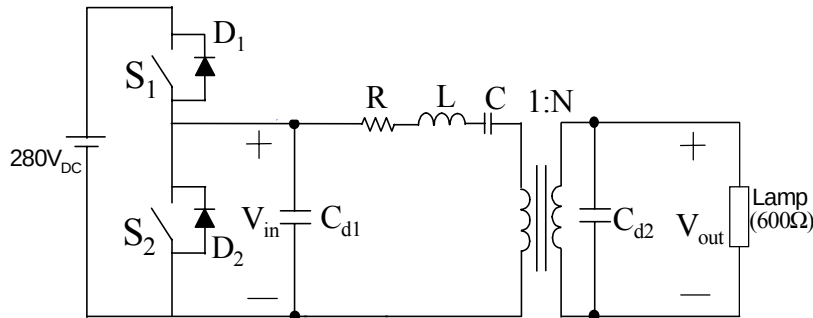
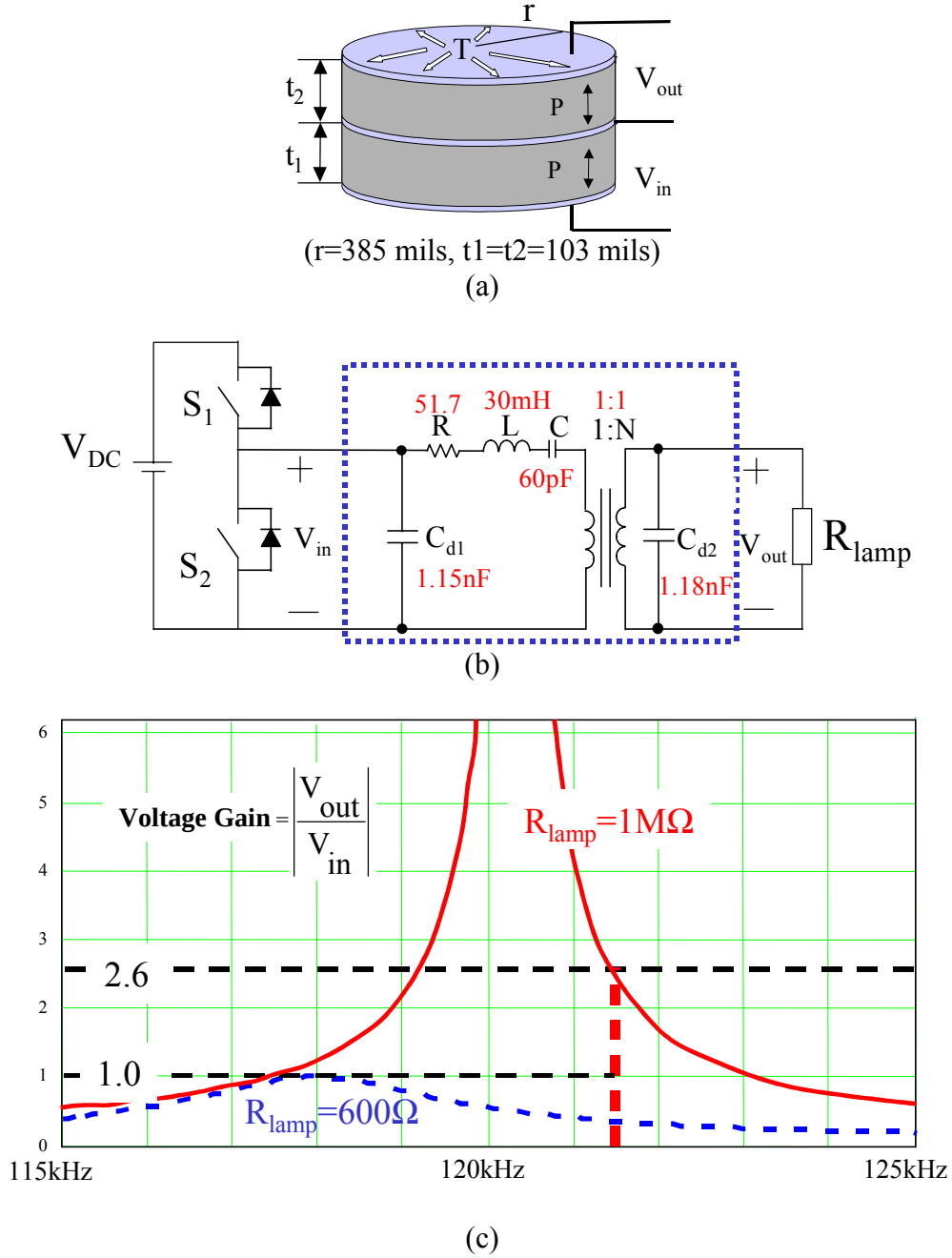


Figure 4.4. Electronic ballast.



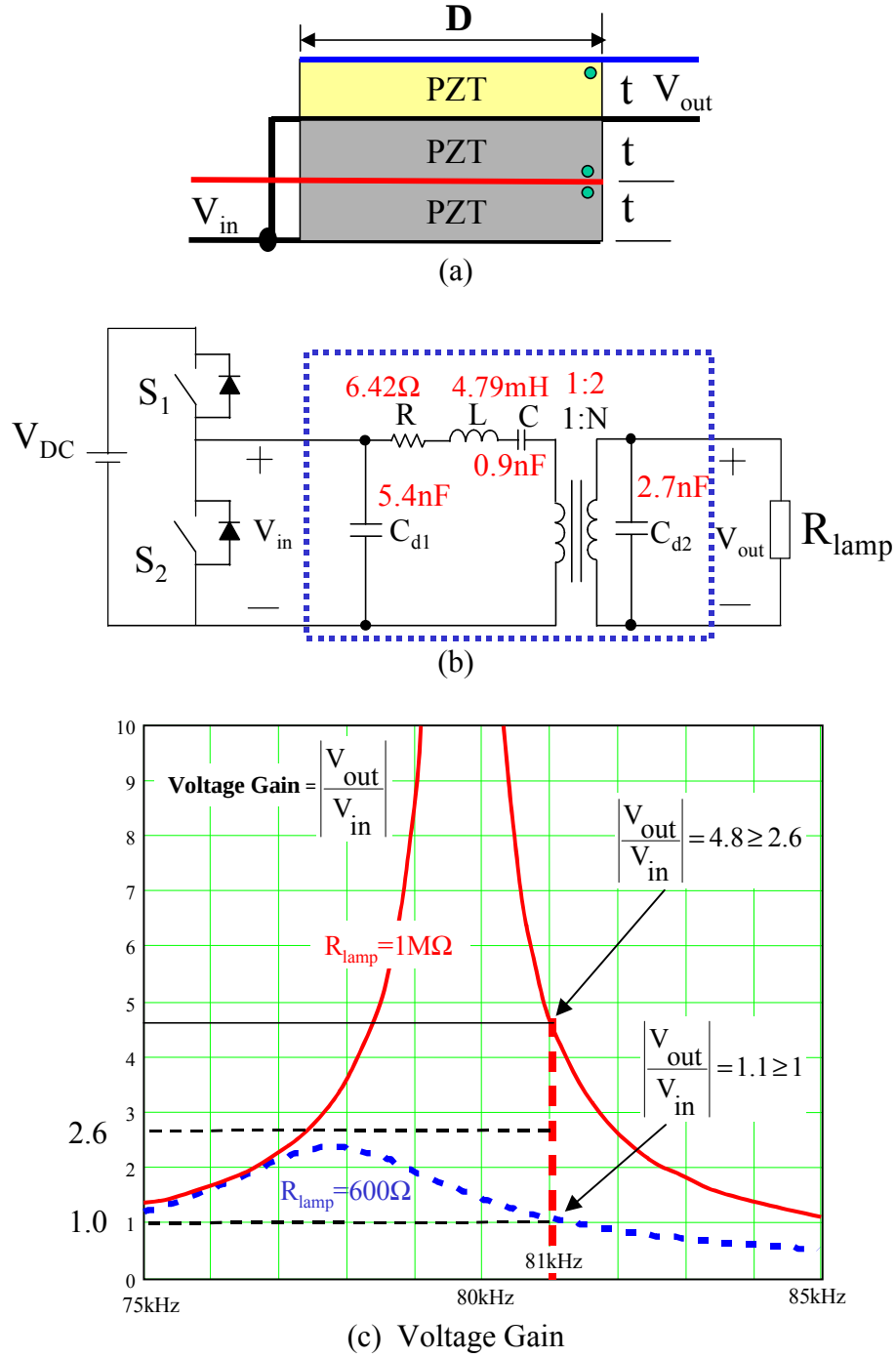


Figure 4.6. Voltage gains of piezoelectric transformer sample CK2 loaded with $R_L=1M\Omega$ and 600Ω . (a) single-layer radial vibration mode piezoelectric transformer sample CK2 ($D=1,180$ mils, $t=80$ mils). (b) half-bridge driver with equivalent circuit model of sample CK2. (c) voltage gain.

4.3 ZVS Condition

4.3.1 Operational Principle of ZVS

In the half-bridge amplifier driving an L-C resonant tank, shown in Figure 4.8, the input capacitor, C_{d1} , works as a turn-off snubber for the half-bridge switches, S_1 and S_2 , to reduce the turn-off switching losses. This input capacitor, C_{d1} , can be the parasitic capacitors of the half-bridge switches. In order to have minimal turn-on switching losses, the half-bridge switches need to be turned on at zero voltage. After the half-bridge switches, S_1 and S_2 , turn off, the instantaneous inductor current, i_L , must be sufficiently large to charge/discharge the input capacitor, C_{d1} , during the dead time, in order to reduce to zero the voltage across the half-bridge switches, S_1 and S_2 .

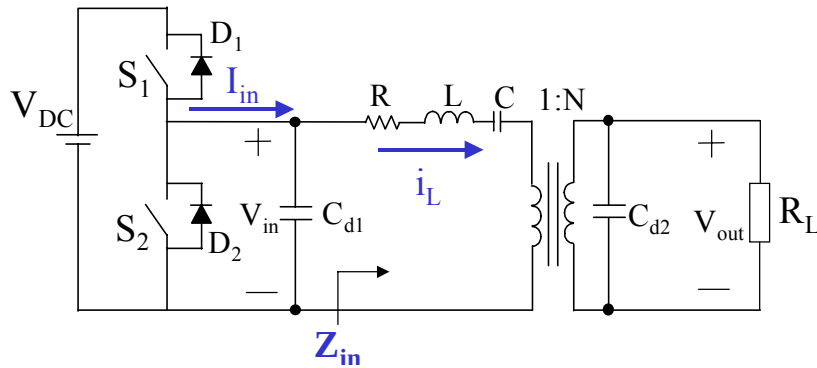
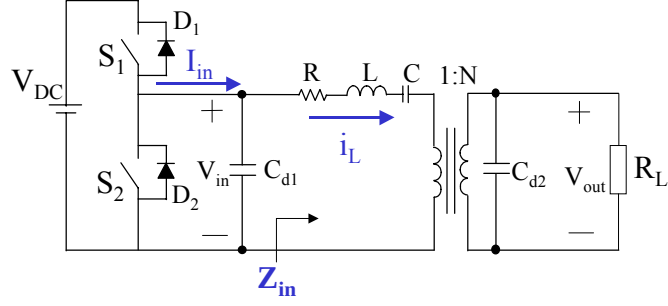


Figure 4.8. Half-bridge amplifier to drive an L-C resonant tank using turn-off snubber capacitor, C_{d1} .

Figure 4.9 shows the detailed switching timing diagram of the half-bridge amplifier for ZVS condition. After the bottom switch, S_2 , is turned off at t_0 , the input capacitor, C_{d1} , is charged by the inductor, L , during the period from t_0 to t_1 . When the voltage, V_{in} , across C_{d1} reaches the DC bus voltage, the diode, D_1 , starts to conduct at t_1 and the voltage across the upper switch, S_1 , becomes zero. The time periods, t_0 - t_2 and t_3 - t_5 , are the dead time between the switching of the half-bridge switches, S_1 and S_2 , when the inductor charges/discharges the input capacitor, C_{d1} .



(a)

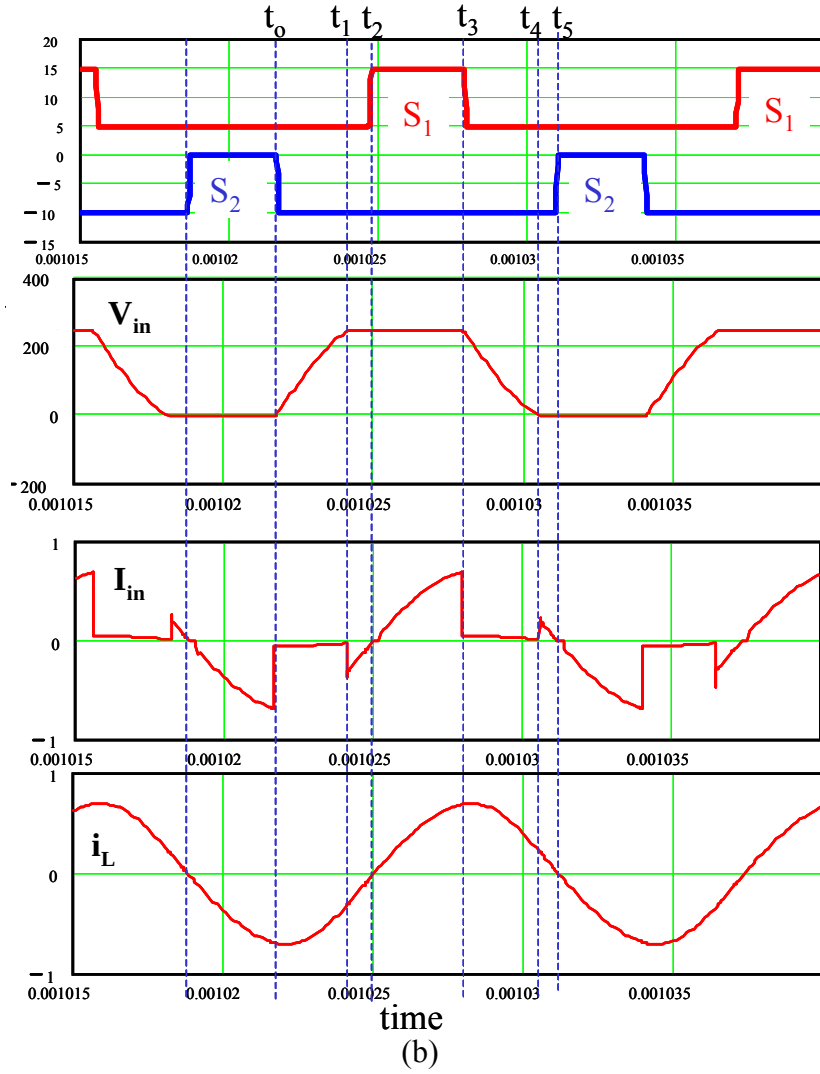


Figure 4.9. Switching timing diagram of half-bridge amplifier for ZVS condition. (a) half-bridge amplifier. (b) operational waveforms. The time periods, t_0 - t_2 and t_3 - t_5 , are the dead times between the switching of the half-bridge switches, S_1 and S_2 , for the inductor to charge/discharge the input capacitor, C_{d1} .

4.3.2 Criteria for ZVS Condition

In order for the half-bridge switches, S_1 and S_2 , to operate ZVS condition with minimal turn-on switching losses, the inductor current must be sufficiently large to charge or discharge the input capacitor, C_{d1} , within the dead time. Therefore, the charged/discharged input capacitor voltage and the dead time between the switching of the half-bridge switches, S_1 and S_2 , are two important criteria for determining ZVS condition. These two key criteria will be discussed in the following two required conditions.

Condition 1: Threshold Voltage of Input Capacitor to Achieve ZVS Condition

When the inductor, L , charges the input capacitor, C_{d1} , as shown in Figure 4.10, the electric charge of the input capacitor, C_{d1} , charged/discharged by the inductor, L , can be expressed as

$$dQ = C_{d1} \cdot dV = i_L \cdot dt \quad (4.2)$$

where dv is the voltage difference of the input capacitor, C_{d1} , after being charged/discharged by the inductor,

i_L is the magnitude of the instantaneous inductor current when the half-bridge switches are turned off, and

dt is the inductor charge/discharge time duration.

To achieve ZVS condition, the voltage difference of the input capacitor, C_{d1} , must be greater than or equal to the DC bus voltage; that is,

$$V_{C_{d1},pk} = \frac{Q}{C_{d1}} \geq V_{DC} \quad (4.3)$$

Referring to Figure 4.10 and excluding the input capacitor C_{d1} , $Z_{in}(f)$ is the input impedance of the L-C resonant tank and can be expressed as the following:

$$Z_{in}(fs) = R_{in}(fs) + jX_{in}(fs) \quad (4.4)$$

where fs is the switching frequency of the half-bridge switches, S_1 and S_2 .

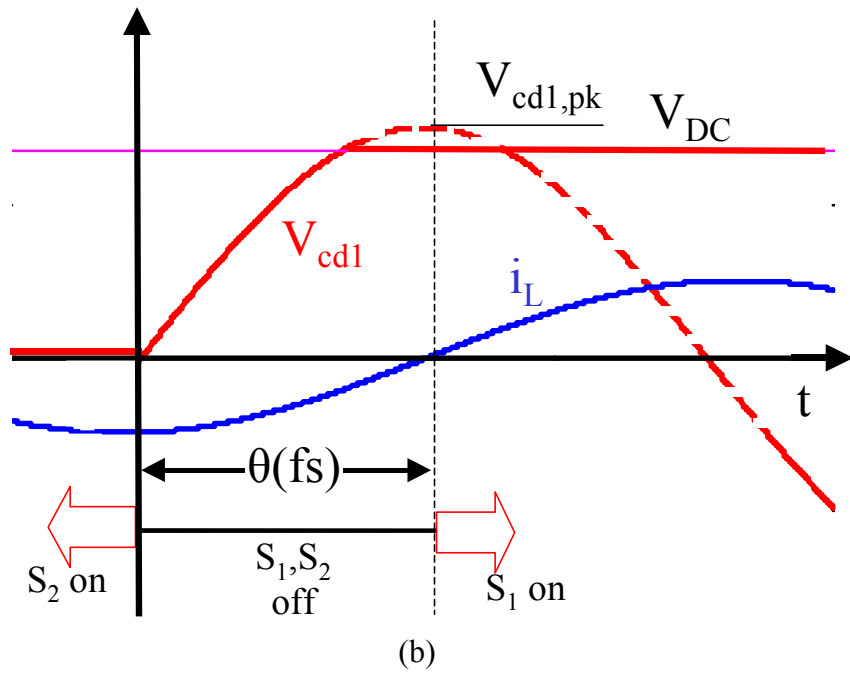
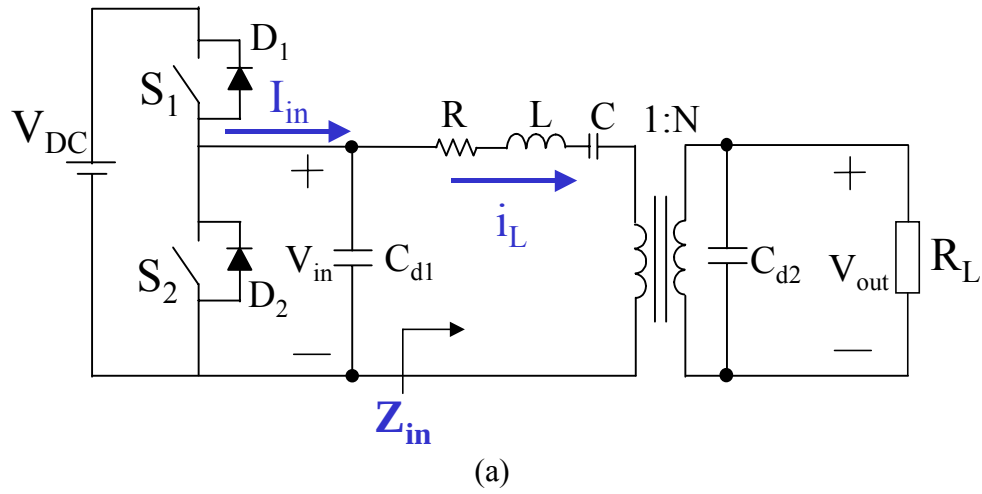


Figure 4.10. Phase relationship between inductor current, i_L , and input capacitor voltage, V_{cd1} . (a) half-bridge amplifier. (b) waveforms of inductor current and input capacitor voltage.

$$R_{in}(fs) = R + \frac{R_L}{N^2 \cdot \left[1 + \left(2 \cdot \pi \cdot fs \cdot C_{d2} \cdot R_L \right)^2 \right]} \quad (4.5)$$

$$X_{in}(fs) = 2 \cdot \pi \cdot fs \cdot L - \frac{1}{2 \cdot \pi \cdot fs \cdot C} - \frac{\left(2 \cdot \pi \cdot fs \cdot C_{d2} \cdot R_L \right)^2}{2 \cdot \pi \cdot f \cdot N^2 \cdot C_{d2} \cdot \left[1 + \left(2 \cdot \pi \cdot fs \cdot C_{d2} \cdot R_L \right)^2 \right]} \quad (4.6)$$

The phase difference between the input voltage, V_{Cd1} , and the inductor current, i_L , can be expressed as:

$$\theta(fs) = \tan^{-1} \left[\frac{X_{in}(fs)}{R_{in}(fs)} \right] \quad (4.7)$$

Therefore,

$$\cos(\theta(fs)) = \frac{R_{in}(fs)}{\sqrt{R_{in}(fs)^2 + X_{in}(fs)^2}} \quad (4.8)$$

$$\sin(\theta(fs)) = \frac{X_{in}(fs)}{\sqrt{R_{in}(fs)^2 + X_{in}(fs)^2}} \quad (4.9)$$

By integrating Equation (4.2), then

$$\int_0^{V_{Cd1,pk}} C_{d1} dV = \int_0^{\frac{\theta(f)}{2 \cdot \pi \cdot f}} i_L(t) dt \quad (4.10)$$

Since the piezoelectric transformer works as a band-pass filter, the input voltage, V_{in} , can be simplified and expressed as the fundamental component, as shown in Equation (4.11).

$$V_{in}(t) = V_{DC} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{\pi \cdot tr \cdot fs} \cdot \sin(2\pi \cdot fs \cdot t) \quad (4.11)$$

then

$$i_L(t) = \frac{-V_{in}(t)}{Z_{in}(fs)} = \frac{-V_{DC}}{|Z_{in}(fs)|} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{\pi \cdot tr \cdot fs} \cdot \sin(2\pi \cdot fs \cdot t + \theta(fs)) \quad (4.12)$$

From Equations (4.10) and (4.11), the peak input capacitor voltage, $V_{Cd1,pk}$, can be derived as shown in the following:

$$V_{Cd1,pk}(fs) = \frac{\frac{-V_{dc}}{|Z_{in}(fs)|} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{(\pi \cdot tr \cdot fs)}}{C_{d1}} \cdot \int_0^{\frac{\theta(fs)}{2 \cdot \pi \cdot fs}} \sin(2 \cdot \pi \cdot fs \cdot t + \theta(fs)) dt \quad (4.13)$$

After integration, Equation (4.13) can be derived as shown in Equation (4.14):

$$V_{Cd1,pk}(fs) = \frac{\frac{V_{DC}}{|Z_{in}(fs)|} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{\pi \cdot tr \cdot fs}}{C_{d1} \cdot 2 \cdot \pi \cdot fs} \cdot [\cos(2 \cdot \theta(fs)) - \cos(\theta(fs))] \quad (4.14)$$

Then,

$$V_{Cd1,pk}(fs) = \frac{\frac{V_{DC}}{|Z_{in}(fs)|} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{\pi \cdot tr \cdot fs}}{C_{d1} \cdot 2 \cdot \pi \cdot fs} \cdot [\cos(\theta(fs))^2 - \sin(\theta(fs))^2 - \cos(\theta(fs))] \quad (4.15)$$

Therefore,

$$V_{cd1,pk}(fs) = \frac{V_{DC} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{\pi \cdot tr \cdot fs}}{C_{d1} \cdot 2 \pi \cdot fs} \cdot \frac{R_{in}(fs)^2 - X_{in}(fs)^2 - R_{in}(f) \cdot \sqrt{R_{in}(fs)^2 + X_{in}(fs)^2}}{(R_{in}(fs)^2 + X_{in}(fs)^2) \cdot \sqrt{R_{in}(fs)^2 + X_{in}(fs)^2}} \quad (4.16)$$

To achieve ZVS condition, the peak input capacitor voltage, $V_{Cd1,pk}$, needs to be greater than or equal to the DC bus voltage, which means

$$V_{Cd1,pk}(fs) \geq V_{DC} \quad (4.17)$$

Equations (4.14) and (4.15) can be combined and derived as follows:

$$\left[\frac{V_{DC} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{\pi \cdot tr \cdot fs}}{C_{dl} \cdot 2\pi \cdot fs} \cdot \frac{R_{in}(fs)^2 - X_{in}(fs)^2 - R_{in}(fs) \cdot \sqrt{R_{in}(fs)^2 + X_{in}(fs)^2}}{(R_{in}(fs)^2 + X_{in}(fs)^2) \cdot \sqrt{R_{in}(fs)^2 + X_{in}(fs)^2}} \right] \geq V_{DC} \quad (4.18)$$

Figure 4.11(a) shows the calculated peak input capacitor voltage, $V_{Cd1,pk}$, compared with the DC bus voltage, according to Equation (4.16). The ZVS region is within the frequency range, in which the calculated $V_{Cd1,pk}$ is greater than the DC bus voltage. Furthermore, Equation (4.16) reveals that the larger the input capacitor, C_{dl} , the less input capacitor voltage, V_{Cd1} , can be charged by the inductor.

To find the maximum of $V_{Cd1,pk}(f)$ in Equation (4.16), let the derivative of $V_{Cd1,pk}(f)$ equal zero, as follows:

$$\frac{\partial V_{Cd1,pk}(fs)}{\partial fs} = \frac{\partial X_{in}(fs)}{\partial fs} \cdot \frac{\partial V_{Cd1,pk}(fs)}{\partial X_{in}(fs)} = 0 \quad (4.19)$$

which means either

$$\frac{\partial V_{Cd1,pk}(fs)}{\partial X_{in}(fs)} = 0 \quad \text{or} \quad (4.20A)$$

$$\frac{\partial X_{in}(fs)}{\partial fs} = 0 \quad (4.20B)$$

Since $\frac{\partial X_{in}(s)}{\partial fs} \neq 0$, then Equation (4.20A) is solved to obtain the following

Equation (4.21), which shows the relationship between $X_{in}(fs)$ and $R_{in}(fs)$:

$$X_{in}(fs) = \sqrt{7 - 2\sqrt{7}} \cdot R_{in}(fs) \quad (4.21)$$

According to Equation (4.7), the phase $\theta(f)$ between the input voltage, V_{in} , and the inductor current, i_L , can be calculated, as shown in the following Equation (4.22), when the maximal peak input capacitor voltage, $V_{Cd1,pk}$, is reached:

$$\theta(fs) = \tan^{-1} \left(\frac{X_{in}(fs)}{R_{in}(fs)} \right) = \tan^{-1} \left(\sqrt{7 - 2\sqrt{7}} \right) = 52.58^\circ \quad (4.22)$$

Figure 4.11(b) shows that the maximum input capacitor voltage, $V_{cd1,pk,max}$, is attained at the phase of the input impedance, Z_{in} , which is equal to 52.58° . According to Equations (4.16) and (4.22), the maximal peak charged/discharged voltage on the input capacitor, C_{d1} , is expressed as shown in the following:

$$V_{cd1,pk}(fs)_{max} = \frac{V_{DC} \cdot \frac{2}{\pi} \cdot \frac{\sin(\pi \cdot tr \cdot fs)}{\pi \cdot tr \cdot fs}}{C_{d1} \cdot 2 \cdot \pi \cdot fs} \cdot \frac{0.53}{R_{in}(fs)} = \frac{0.53 \cdot V_{DC} \cdot \sin(\pi \cdot tr \cdot fs)}{C_{d1} \cdot \pi^3 \cdot fs^2 \cdot tr \cdot R_{in}(fs)} \quad (4.23)$$

where $R_{in}(fs) = R + \frac{R_L}{N^2 \cdot \left[1 + \left(2 \cdot \pi \cdot fs \cdot C_{d2} \cdot R_L \right)^2 \right]}$

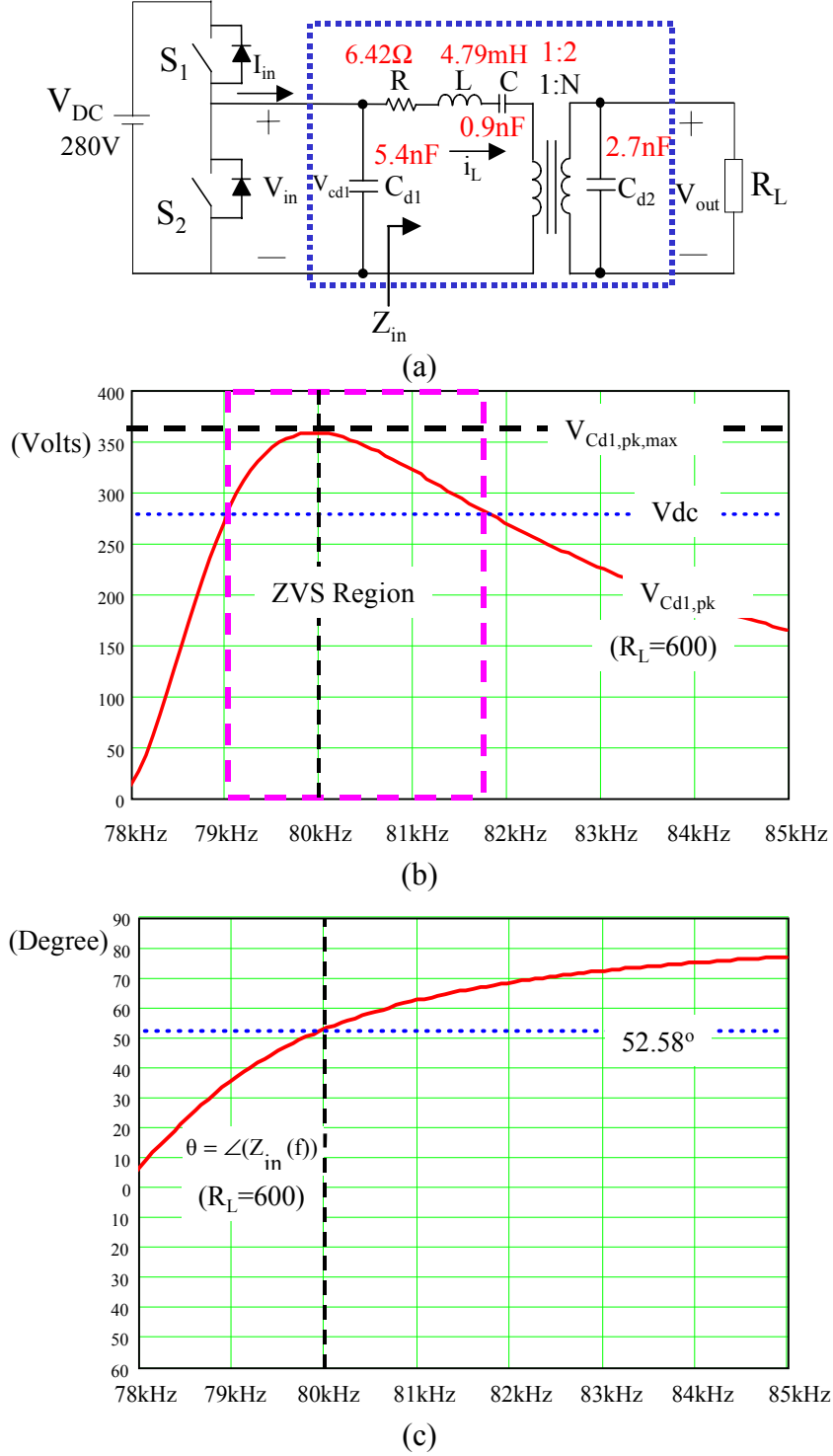


Figure 4.11. Calculated peak voltage on input capacitor, $C_{d1,pk}$, compared with DC bus voltage. (a) half-bridge amplifier with sample CK2 ($R_L=600\Omega$). (b) calculated peak voltage on input capacitor, C_{d1} , compared with DC bus voltage. (c) phase between input voltage V_{in} and inductor current i_L . To achieve ZVS condition, the charged input capacitor voltage, V_{Cd1} , must be greater than the DC bus voltage, V_{DC} .

Condition 2: Minimizing Dead Time for ZVS Condition

In order to provide sufficient time for the inductor to charge/discharge the input capacitor, C_{d1} , the dead time between the switching of the half-bridge switches, S_1 and S_2 , should be greater than one quarter of the resonant period for the serial resonating loop among C_{d1} , L , C , C_{d2} and R_L . With a given load, R_L , this serial resonant frequency, f_{serial} , of the serial resonating loop among C_{d1} , L , C , C_{d2} and R_L can be solved using Equation (4.24), as shown in Equation (4.25):

$$\omega_r \cdot L = \frac{1}{\omega_r} \cdot \left[\frac{1}{C} + \frac{1}{C_{d1}} + \frac{(\omega_r \cdot C_{d2} \cdot R_L)^2}{N^2 \cdot C_{d2} \cdot [1 + (\omega_r \cdot C_{d2} \cdot R_L)^2]} \right] \quad (4.24)$$

$$\text{where } \omega_r = 2\pi \cdot f_{\text{serial}}$$

then

$$f_{\text{serial}} = \frac{\sqrt{2 \cdot L \cdot C_{\text{eq}} \cdot [-L \cdot C_{\text{eq}} \cdot N^2 + N^2 \cdot C_{d2}^2 \cdot R_L^2 + C_{d2} \cdot R_L^2 \cdot C_{\text{eq}} + \sqrt{k}]}}{4\pi \cdot L \cdot C_{\text{eq}} \cdot N \cdot C_{d2} \cdot R_L} \quad (4.25)$$

where

$$\begin{aligned} k = & L^2 \cdot C_{\text{eq}}^2 \cdot N^4 + 2 \cdot L \cdot C_{\text{eq}} \cdot N^4 \cdot C_{d2}^2 \cdot R_L^2 - 2 \cdot L \cdot C_{\text{eq}}^2 \cdot N^2 \cdot C_{d2} \cdot R_L^2 \\ & + N^4 \cdot C_{d2}^4 \cdot R_L^4 + 2 \cdot C_{d2}^3 \cdot R_L^4 \cdot C_{\text{eq}} \cdot N^2 + C_{d2}^2 \cdot R_L^4 \cdot C_{\text{eq}}^2 \\ C_{\text{eq}} = & \frac{C \cdot C_{d1}}{C + C_{d1}} \end{aligned} \quad (4.26)$$

Therefore, the dead time for achieving ZVS condition can be calculated as the follows:

$$\begin{aligned} t_{\text{dead-time}} & \geq \frac{T_{\text{serial}}}{4} = \frac{1}{4 \cdot f_{\text{serial}}} \\ & = \frac{\pi \cdot L \cdot C_{\text{eq}} \cdot N \cdot C_{d2} \cdot R_L}{\sqrt{2 \cdot L \cdot C_{\text{eq}} \cdot [-L \cdot C_{\text{eq}} \cdot N^2 + N^2 \cdot C_{d2}^2 \cdot R_L^2 + C_{d2} \cdot R_L^2 \cdot C_{\text{eq}} + \sqrt{k}]}} \end{aligned} \quad (4.27)$$

where

$$k = L^2 \cdot C_{eq}^2 \cdot N^4 + 2 \cdot L \cdot C_{eq} \cdot N^4 \cdot C_{d2}^2 \cdot R_L^2 - 2 \cdot L \cdot C_{eq}^2 \cdot N^2 \cdot C_{d2} \cdot R_L^2$$

$$C_{eq} = \frac{C \cdot C_{d1}}{C + C_{d1}}$$

4.3.3 Minimizing Circulating Current for ZVS Condition

Figure 4.12 shows the waveforms of the input voltage and inductor current of the L-C resonant tank driven by a half-bridge amplifier. This inductor current, i_L , is dependent on the switching frequency, f_s , and the load resistance, R_L . The amplitude of the inductor current, i_{Lpk} , will be derived as follows in order to calculate the current stress on the half-bridge switches, S_1 and S_2 .

Since the piezoelectric transformer works as a band-pass filter, the magnitude of the input voltage, V_{in} , applied to the input terminal of the L-C resonant tank at switching frequency, f_s , can be simplified and expressed as the fundamental component, shown in the following:

$$V_{in, fund} \Big|_{f=f_s} = V_{DC} \cdot \frac{\sin(\pi \cdot t_r \cdot f)}{\pi \cdot t_r \cdot f} \cdot \frac{2}{\pi} \quad (4.28)$$

where V_{DC} is the DC bus voltage,

t_r is the rising time of input voltage, V_{in} , and

f_s is the switching frequency.

Therefore, the amplitude of the inductor current, i_{Lpk} , at switching frequency, f_s , can be expressed as shown in the following:

$$\left| i_{Lpk} \right|_{f=f_s} = \frac{V_{in, fundamental} \Big|_{f=f_s}}{\left| Z_{in} \right|_{f=f_s}} = V_{DC} \cdot \frac{\sin(\pi \cdot t_r \cdot f)}{\pi \cdot t_r \cdot f} \cdot \frac{2}{\pi} \cdot \frac{1}{\left| Z_{in} \right|_{f=f_s}} \quad (4.29)$$

where $\left| Z_{in} \right|_{f=f_s}$ is the magnitude of the input impedance excluding the input capacitor, C_{d1} .

If $tr \cdot f_s = \frac{t_r}{T_s} \approx \frac{1}{5}$, then the amplitude of the inductor current, i_{Lpk} , at switching frequency, f_s , can be plotted as shown in Figure 4.13.

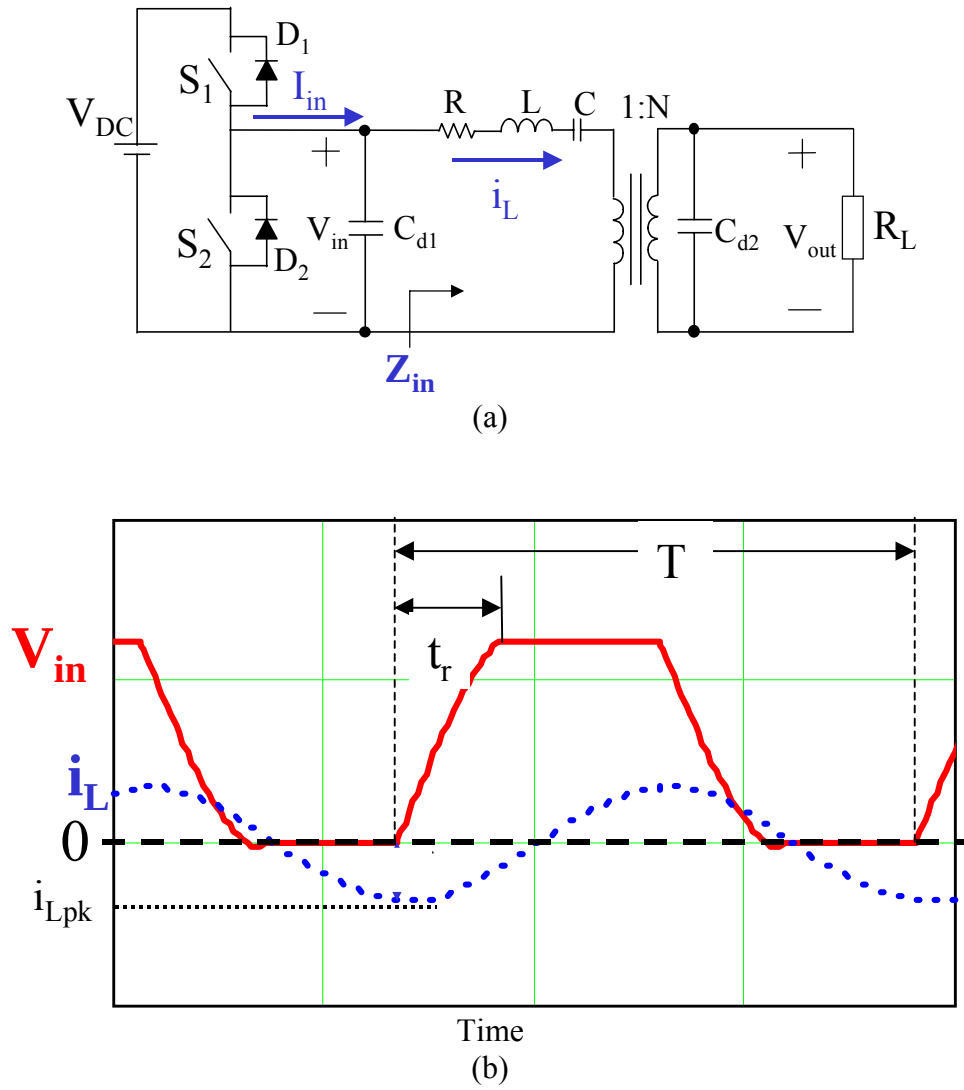


Figure 4.12. Input voltage waveform and inductor current waveforms of L-C resonant tank. (a) half-bridge amplifier. (b) waveforms of inductor current and input voltage.

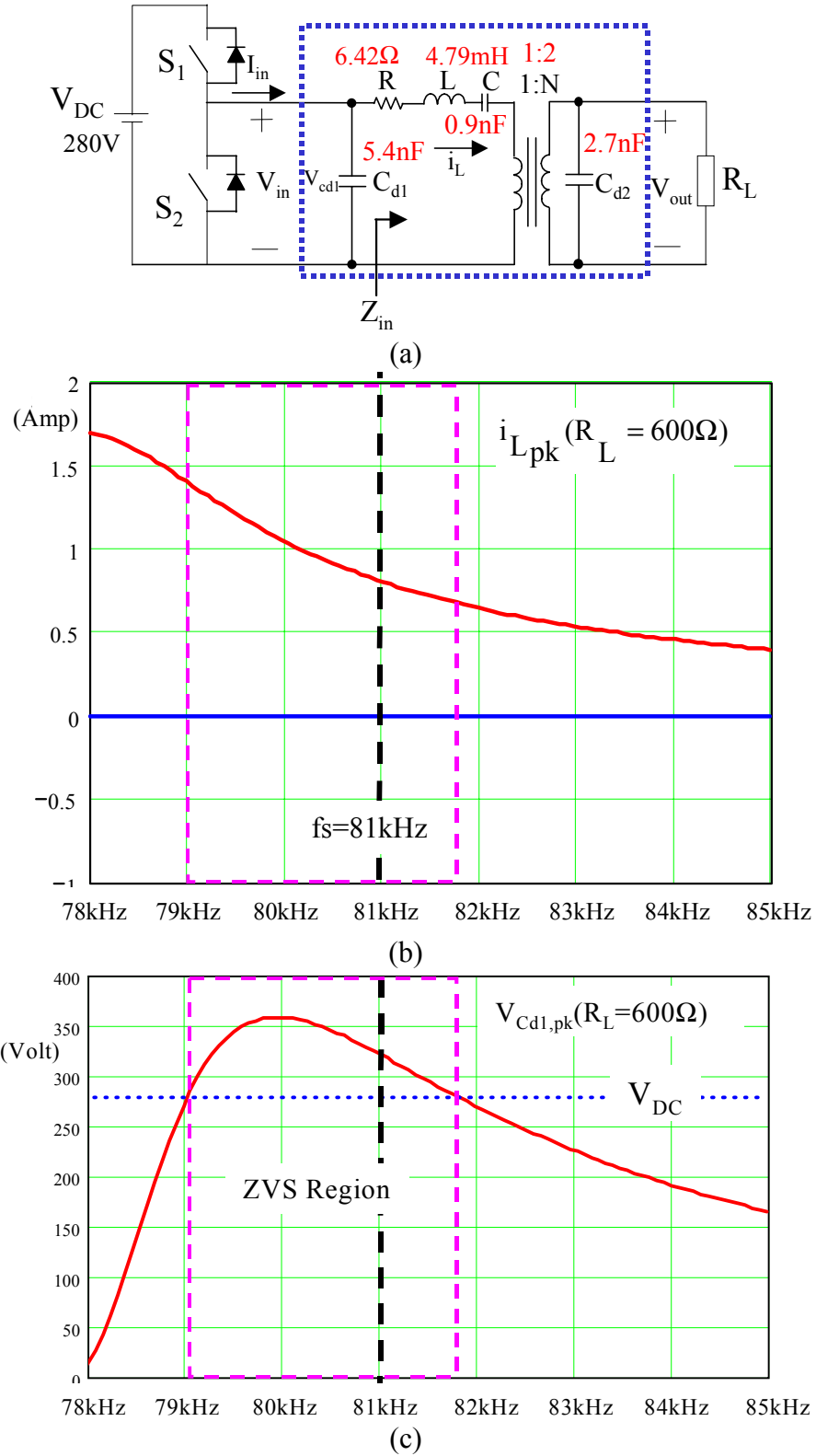


Figure 4.13. Amplitude of inductor current, i_{Lpk} , and peak input capacitor voltage, $V_{Cd1,pk}$. (a) half-bridge amplifier with sample CK2. (b) amplitude of inductor current. (c) magnitude of peak input capacitor voltage.

4.4 Matched Load for Optimal Efficiency

Besides the voltage gain and ZVS condition discussed in Sections 4.2 and 4.3, the efficiency achieved while utilizing a radial vibration mode piezoelectric transformer for electronic ballast applications remains to be evaluated in this section.

Since the loads of fluorescent lamps become resistive after ignition, fluorescent lamps can simply be modeled resistive. In order to calculate the efficiency of piezoelectric transformers, two dielectric loss components, R_{cd1} and R_{cd2} , for the input capacitor, C_{d1} , and the output capacitor, C_{d2} , respectively, need to be included in the equivalent circuit of a piezoelectric transformer with the resistive load, R_L , [A8] as shown in Figure 4.14.

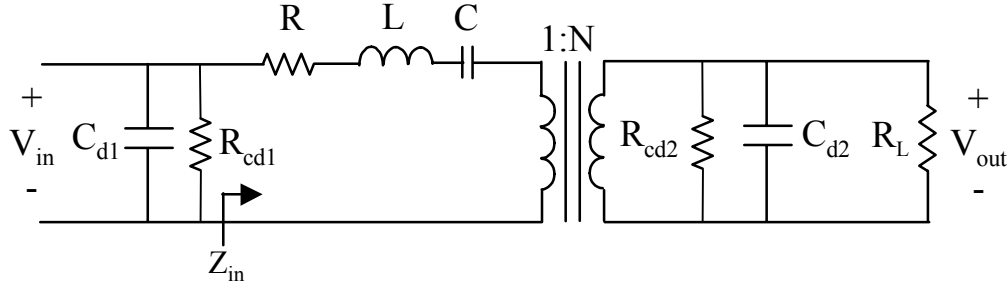


Figure 4.14. Piezoelectric transformer equivalent circuit model with dielectric loss components, R_{cd1} and R_{cd2} .

These two dielectric loss components, R_{cd1} and R_{cd2} , be calculated using the following Equations (4.30) and (4.31) [A8]:

$$R_{cd1} = \frac{1}{2 \cdot \pi \cdot f_s \cdot C_{d1} \cdot \tan \delta} \quad (4.30)$$

$$R_{cd2} = \frac{1}{2 \cdot \pi \cdot f_s \cdot C_{d2} \cdot \tan \delta} \quad (4.31)$$

where $\tan \delta$ is the dielectric tangent of the piezoelectric material, and f_s is the switching frequency of the half-bridge switches.

Referring to Figure 4.15, the piezoelectric transformer model can be converted to a series format, as shown in Figure 4.15(c) in order to conveniently calculate the efficiency. With the resistive load, R_L , the input power, output power and efficiency can be derived as shown in Equations (4.32A), (4.32B) and (4.32C), respectively:

$$P_{in} = \text{Re} \left[V_{in}^2 \cdot \left(\frac{1}{R_{cd1}} + \frac{1}{Z_{in}} \right) \right] = V_{in}^2 \cdot \left[\frac{1}{R_{cd1}} + \frac{R + R_2}{(R + R_2)^2 + (X_1 + X_2)^2} \right] \quad (4.32A)$$

$$P_{out} = V_{in}^2 \cdot \left| \frac{R_2 + jX_2}{(R + R_2) + j(X_1 + X_2)} \right|^2 \cdot \frac{1}{R_2} \quad (4.32B)$$

$$\text{Eff} = \frac{P_{out}}{P_{in}} = \frac{R_{cd1} \cdot (R_2^2 + X_2^2)}{(R_2^2 + X_2^2) + (X_1 + X_2)^2 + R_{cd1} \cdot (R + R_2)} \cdot \frac{1}{R_2} \quad (4.32C)$$

where

$$X_1 = \omega_s \cdot L - \frac{1}{\omega_s \cdot C} \quad (4.33)$$

$$X_2 = \frac{-1}{\omega_s \cdot C_{d2} \cdot N^2} \cdot \frac{q^2}{1 + q^2} \quad (4.34)$$

$$R_2 = \frac{R_L \cdot R_{cd2}}{R_L + R_{cd2}} \cdot \frac{1}{N^2 \cdot (1 + q^2)} \quad (4.35)$$

$$q = \omega_s \cdot C_{d2} \cdot \frac{R_L \cdot R_{cd2}}{R_L + R_{cd2}} \quad (4.36)$$

$$\omega_s = 2 \cdot \pi \cdot f_s$$

f_s is the switching frequency of the half-bridge switches.

To find the maximal efficiency in Equation (4.32C) with a given load R_L , assume its derivative, corresponding to X_1 , is equal to zero, as follows:

$$\frac{\partial}{\partial f_s} \text{Eff} = \frac{\partial X_1}{\partial f_s} \cdot \frac{\partial \text{Eff}}{\partial X_1} = 0 \quad (4.37)$$

which means,

$$\frac{\partial \text{Eff}}{\partial X_1} = 0, \text{ or} \quad (4.38A)$$

$$\frac{\partial X_1}{\partial f_s} = 0 \quad (4.38B)$$

Since $\frac{\partial X_1}{\partial f_s} \neq 0$, then solve Equation (4.38A) to obtain

$$X_1 = -X_2 \quad (4.39)$$

Equation (4.39) means that

$$\omega_s \cdot L - \frac{1}{\omega_s \cdot C} = \frac{1}{\omega_s \cdot C_{d2} \cdot N^2} \cdot \frac{q^2}{1+q^2} \quad (4.40)$$

$$\text{where } q = \omega_s \cdot C_{d2} \cdot \frac{R_L \cdot R_{cd2}}{R_L + R_{cd2}}$$

Equation (4.40) shows that the maximal efficiency is achieved at the resonant frequency with a specific resistive load, R_L . This resonant frequency, f_r , can be calculated by the following:

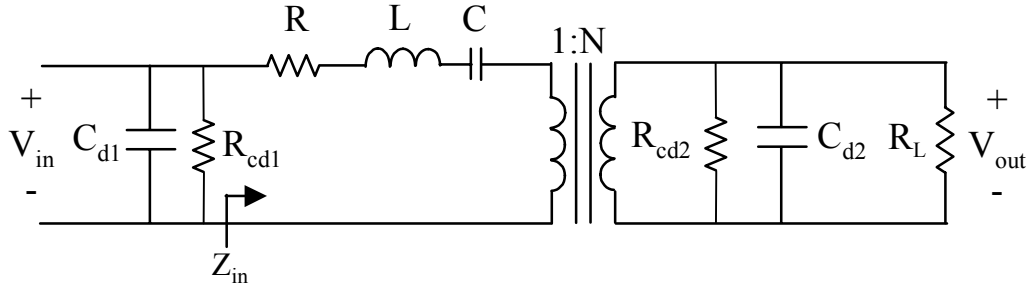
$$f_r = \frac{\sqrt{2 \cdot L \cdot C \cdot [-L \cdot C \cdot N^2 + N^2 \cdot C_{d2}^2 \cdot R_{eq}^2 + C_{d2} \cdot R_{eq}^2 \cdot C + \sqrt{k}]}}{4\pi \cdot L \cdot C \cdot N \cdot C_{d2} \cdot R_{eq}} \quad (4.41)$$

where

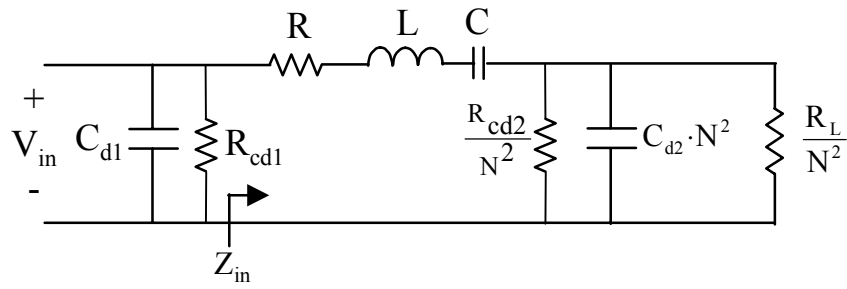
$$\begin{aligned} k = & L^2 \cdot C^2 \cdot N^4 + 2 \cdot L \cdot C \cdot N^4 \cdot C_{d2}^2 \cdot R_{eq}^2 - 2 \cdot L \cdot C^2 \cdot N^2 \cdot C_{d2} \cdot R_{eq}^2 \\ & + N^4 \cdot C_{d2}^4 \cdot R_{eq}^4 + 2 \cdot C_{d2}^3 \cdot R_{eq}^4 \cdot C \cdot N^2 + C_{d2}^2 \cdot R_{eq}^4 \cdot C^2 \end{aligned}$$

$$R_{eq} = \frac{R_L \cdot R_{Cd2}}{R_L + R_{Cd2}} \quad (4.42)$$

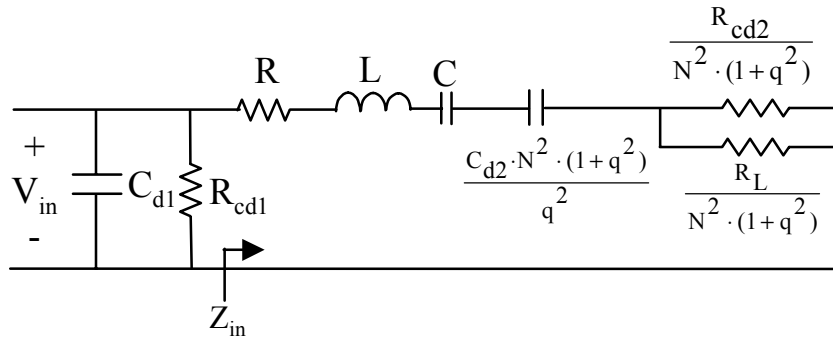
If $R_{cd2} \gg R_L$, then $R_{eq} \cong R_L$. According to Equations (4.30) and (4.31), the R_{cd1} and R_{cd2} of sample CK2 at 81kHz are 91k Ω and 182k Ω , respectively, both of which are much greater than $R_L=600\Omega$.



(a)



(b)



(c)

Figure 4.15. Optimal terminal of piezoelectric transformer with resistive load. (a) piezoelectric transformer model with dielectric loss components of C_{d1} and C_{d2} . (b) Reflecting R_L , R_{cd2} and C_{d2} to R-L-C branch. (c) converting the paralleled components to series format.

In Figure 4.14, if the values of dielectric loss components, R_{cd1} and R_{cd2} , are much greater than R and R_L , respectively, then these two dielectric loss components can be ignored and removed from the equivalent circuit, as shown in Figure 4.16. Prior work [A8] derived that the peak efficiency is attained when the load resistance equals the matched load resistance, $R_{L,max}$, at its corresponding resonant frequency, f_r , for the piezoelectric transformer model without the dielectric loss components, R_{cd1} and R_{cd2} . This matched load resistance, $R_{L,max}$, can be calculated as follows:

$$R_{L,match} = \frac{1}{2 \cdot \pi \cdot f_r \cdot C_{d2}} \quad (4.43)$$

where f_r is the corresponding resonant frequency with a given load R_L .

According to the parameters of CK2, shown in Table 4.1, the curves of efficiency vs. load resistance at the corresponding resonant frequencies can be plotted as shown in Figure 4.17 for the piezoelectric transformer models with and without the dielectric loss components, R_{cd1} and R_{cd2} . Figure 4.17 shows that the peak efficiency values of both models are achieved around the matched load resistance, $R_{L,match}$ ($=733\Omega$), at the corresponding resonant frequency.

Table 4.1. Measured parameters of equivalent circuit model for multi-layer radial vibration mode piezoelectric transformer sample CK2 ($t_1=t_2=t_3=80\text{mils}$, $D=1180\text{mils}$, $n1=2$, $n2=1$).

	R	L	C	C_{d1}	C_{d2}	N	$f_s = \frac{1}{\sqrt{LC}}$
Measured	6.42 Ω	4.79 mH	918.71 pF	5.41 nF	2.74 nF	2	75.86 kHz

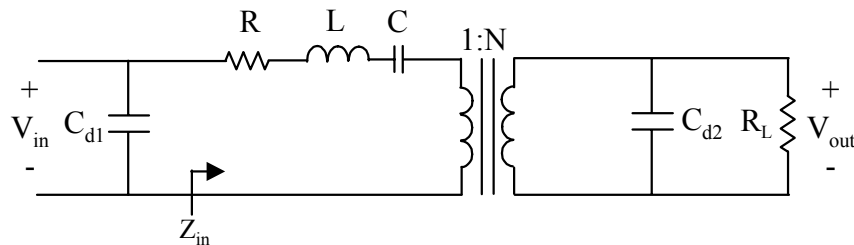


Figure 4.16. Piezoelectric transformer model without dielectric loss components.

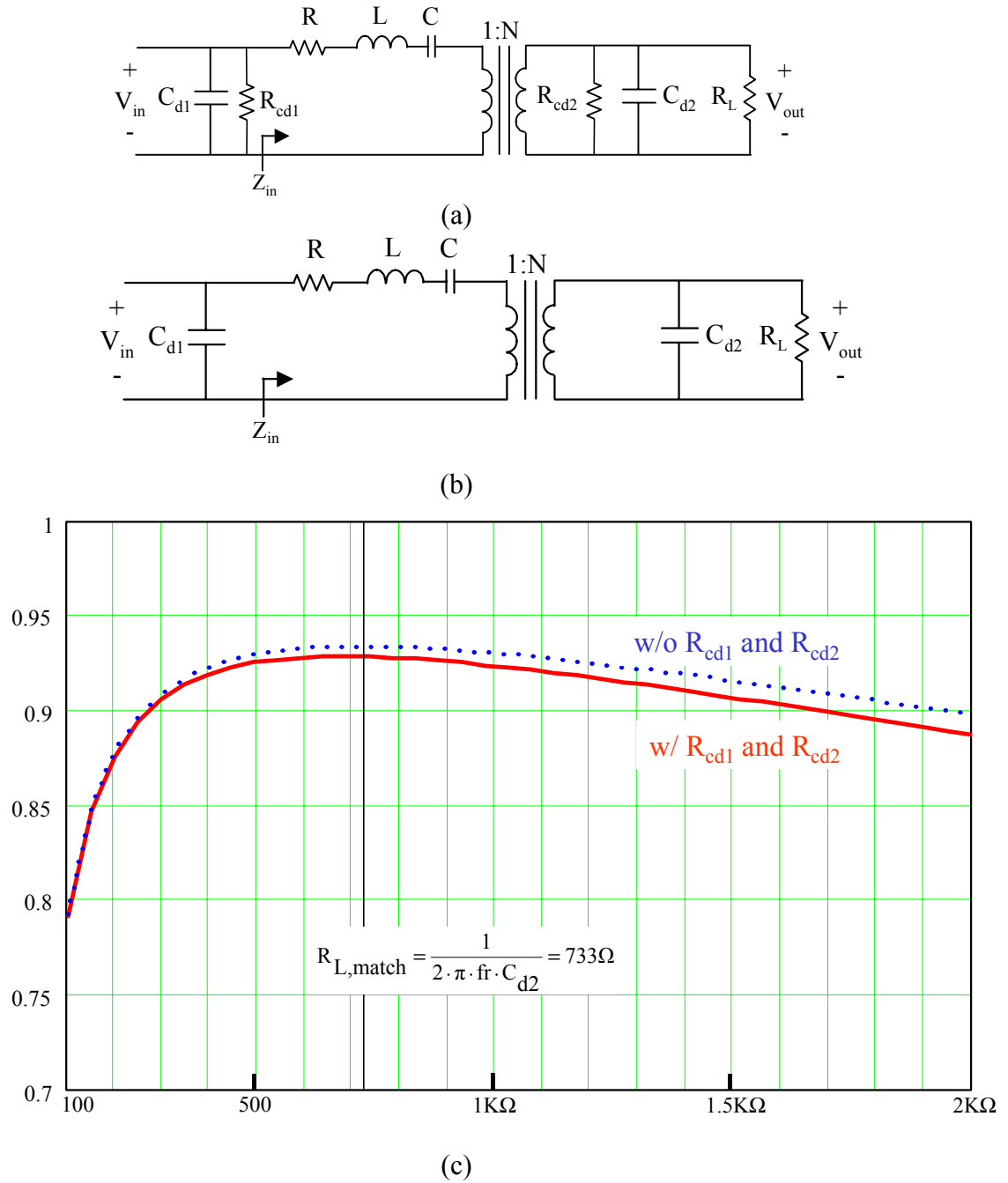


Figure 4.17. Efficiency vs. load resistance at corresponding resonant frequency for piezoelectric transformer sample CK2. (a) equivalent circuit model with dielectric loss components, R_{cd1} and R_{cd2} . (b) equivalent circuit model without dielectric loss components, R_{cd1} and R_{cd2} . (c) efficiency vs. load resistance at corresponding resonant frequency. When R_L is equal to $R_{L,match}$, the efficiency is maximal.

4.5 Inductor-less Electronic Ballast

Based on the ZVS criteria, proposed in Section 4.3, this section presents a cost-effective inductor-less piezoelectric transformer electronic ballast utilizing a radial vibration mode piezoelectric transformer to drive a 4-foot 40-watt linear fluorescent lamp at $V_{DC}=280V$. At steady state, the equivalent on-resistance of this lamp is equal to 600Ω . According to the calculated efficiency curves, shown in Figure 4.18, the efficiency with $R_L=600\Omega$ at the corresponding resonant frequency is still very high and is close to the efficiency peak. The radial vibration mode piezoelectric transformer serves as a piezoelectric transformer resonant tank that replaces the conventional passive L-C resonant tank. With its inherent piezoelectric resonant characteristics, the radial vibration mode piezoelectric transformer is able to both ignite and provide sustaining voltage to a linear fluorescent lamp. The operational principles, implementation and experimental results of the proposed inductor-less piezoelectric transformer electronic ballast circuit will be presented in this section.

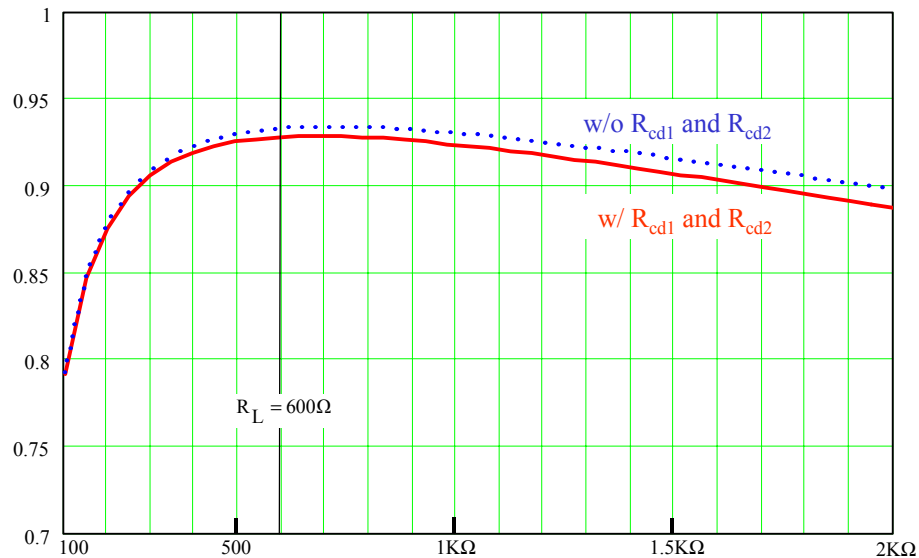


Figure 4.18. Efficiency vs. load resistance at corresponding resonant frequency for piezoelectric transformer sample CK2.

4.5.1 Implementation of Prototype Circuit

According to the evaluation of voltage gain, ZVS condition and efficiency discussed in Sections 4.2, 4.3 and 4.4, the radial vibration mode piezoelectric transformer sample CK2 can be used for the electronic ballast to drive a 4-foot 40-watt linear fluorescent lamp at $V_{DC}=280$. Figure 4.19 shows both the voltage gain and the peak input capacitor voltage, $V_{Cd1,pk}$, vs. frequency. Figure 4.20 shows both the amplitude of the inductor current, $i_{L,pk}$, and the peak input capacitor voltage, $V_{cd1,pk}$, vs. frequency. When the switching frequency is at 81kHz with $V_{DC}=280V$, the charged/discharged input capacitor voltage, V_{Cd1} , is greater than the DC bus voltage, V_{DC} , which means a ZVS condition can be achieved for the half-bridge switches, S_1 and S_2 . With $R_L=600\Omega$, the voltage gain is around 1(0dB), which is able to sustain the linear fluorescent lamp.

Figure 4.21 shows the PSpice simulation results of the prototype circuit employing the radial vibration mode piezoelectric transformer sample CK2. These results validate the ZVS condition at switching frequency $f_s=81kHz$. The schematic of the prototype circuit for the proposed inductor-less piezoelectric transformer electronic ballast is shown in Figure 4.22. This prototype circuit utilizes a voltage-controlled frequency oscillator (VCO), CD4046, to generate a constant-frequency pulse signal to the gate driver, L6384, which drives the half-bridge amplifier. The half-bridge switches, S_1 and S_2 , directly drives the radial vibration mode piezoelectric transformer (PT) without requiring any magnetic device. This prototype circuit also has a voltage doubler for universal-line applications. Figure 4.23 shows the completed prototype circuit for the proposed inductor-less piezoelectric transformer electronic ballast.

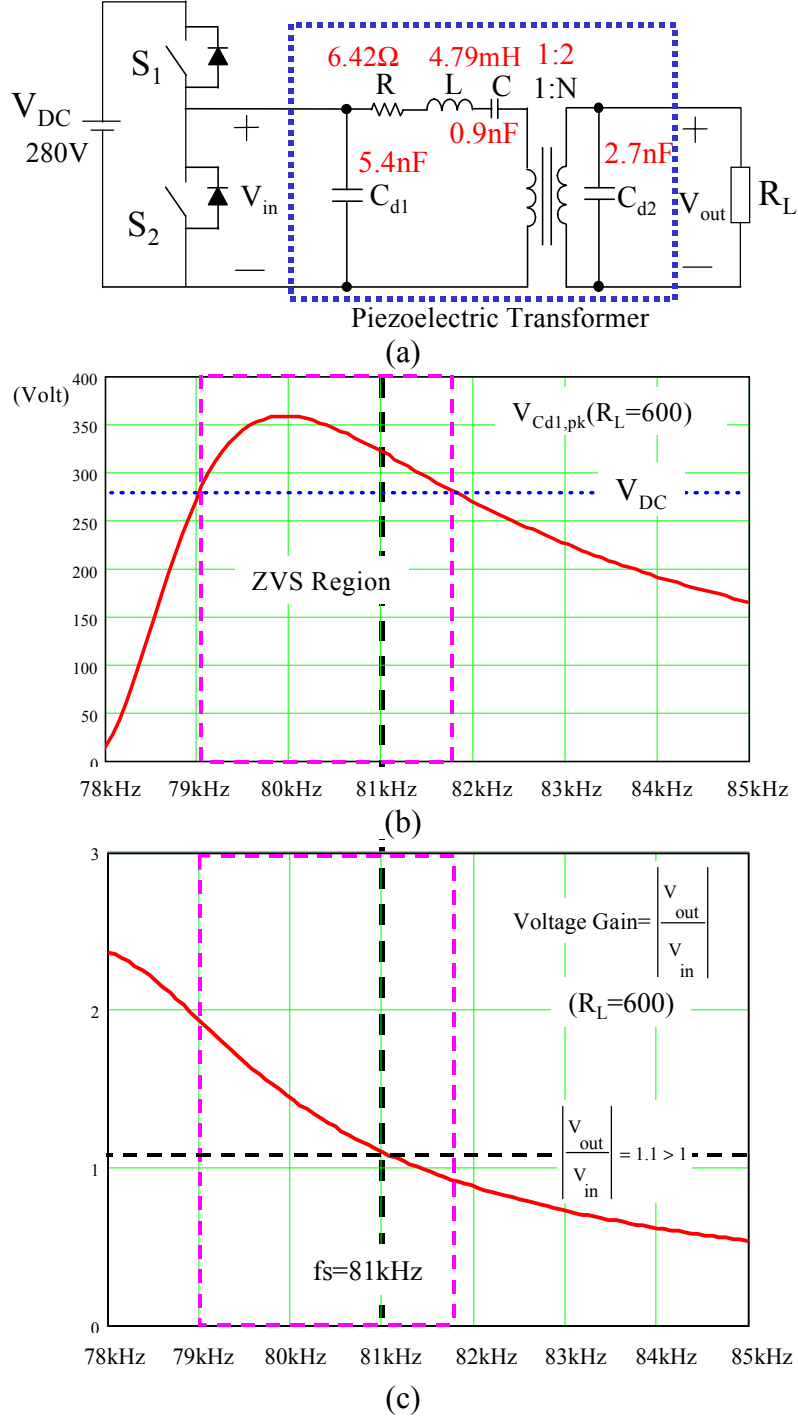


Figure 4.19. Peak input capacitor voltage, $V_{Cd1,pk}$, and voltage gain. (a) electronic ballast with piezoelectric transformer sample CK2. (b) magnitude of peak input capacitor voltage. (c) voltage gain. When the switching frequency is at 81kHz, the charged voltage, V_{Cd1} , on the input capacitor, C_{d1} , is greater than the DC bus voltage, which means the ZVS condition can be achieved for the half-bridge switches, S_1 and S_2 .

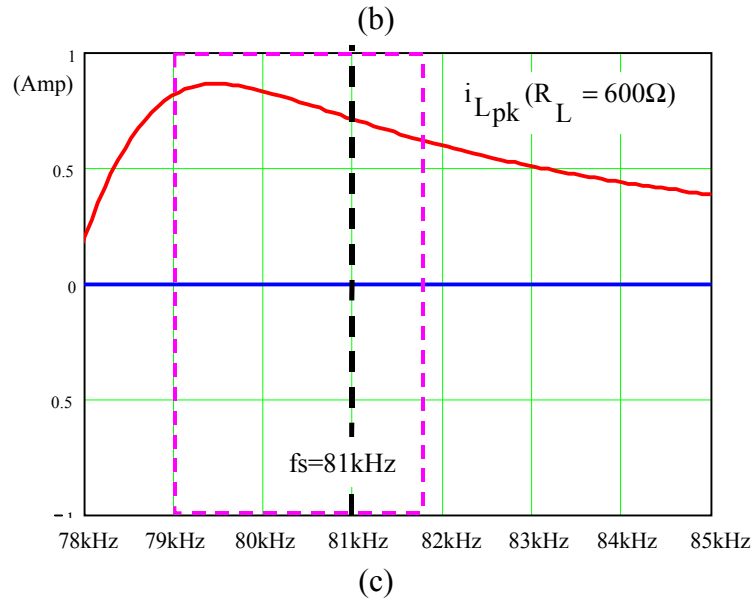
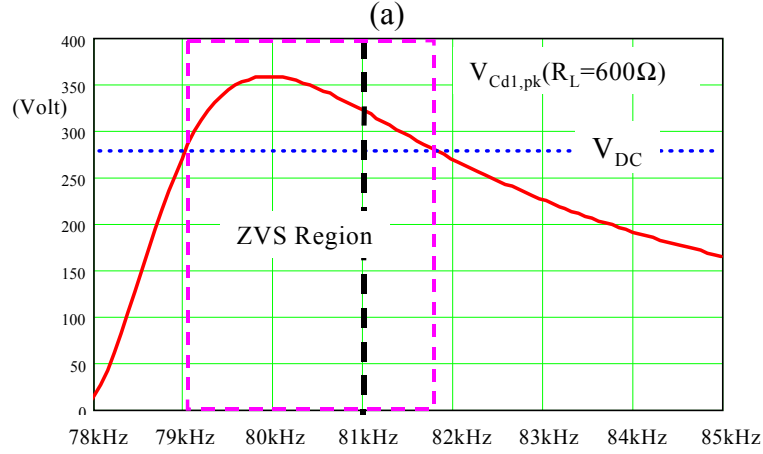
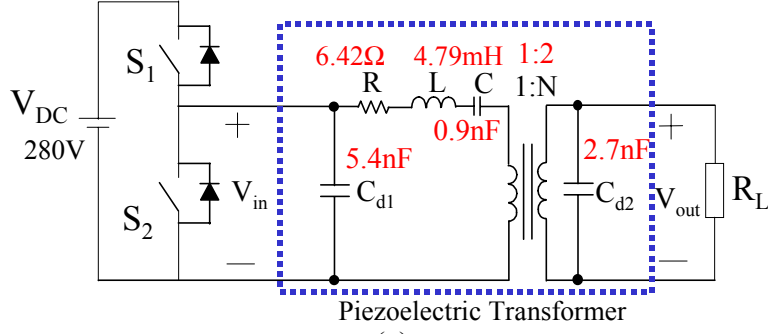


Figure 4.20. Peak input capacitor voltage, $V_{Cd1,pk}$, and amplitude of inductor current, $i_{L,pk}$. (a) electronic ballast with piezoelectric transformer sample CK2. (b) magnitude of the peak input capacitor voltage. (c) amplitude of inductor current.

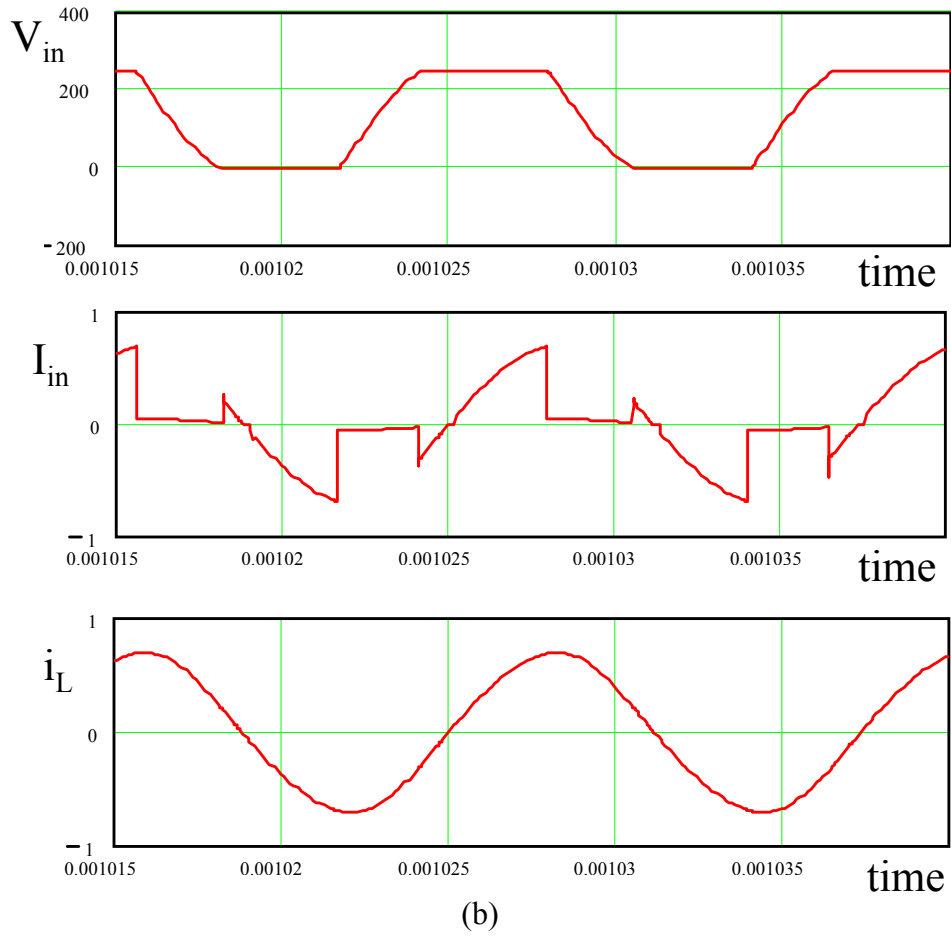
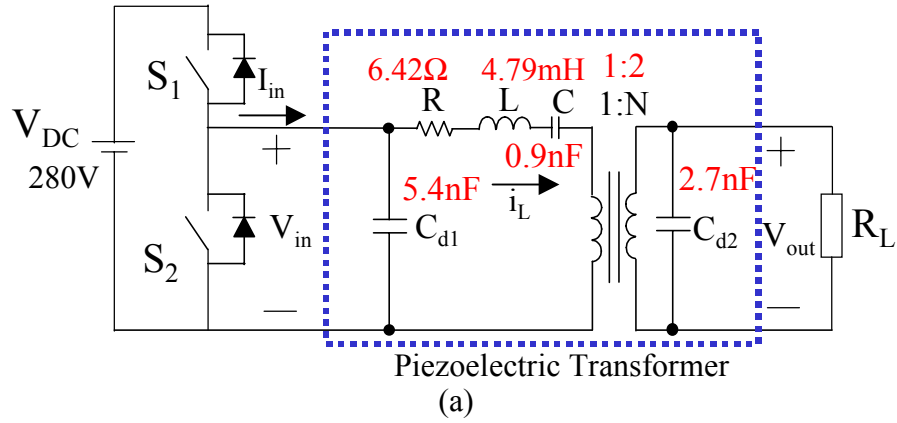


Figure 4.21. PSpice simulation results of inductor-less piezoelectric transformer ballast prototype circuit utilizing a radial vibration piezoelectric transformer sample, CK2. (a) electronic ballast with piezoelectric transformer sample CK2 ($R_L=600\Omega$). (b) simulated waveforms.

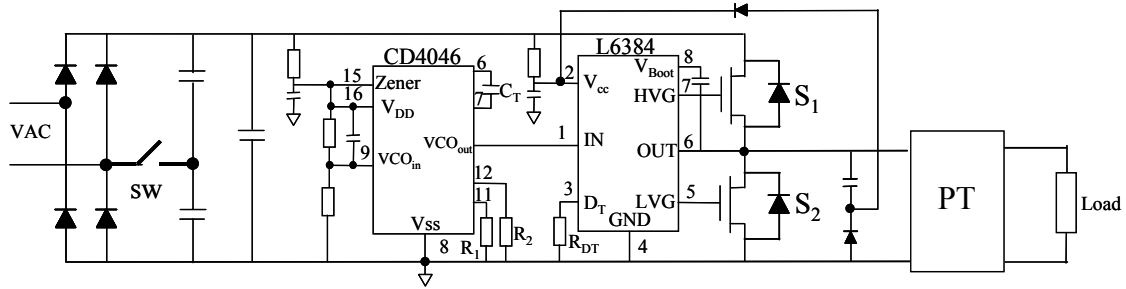
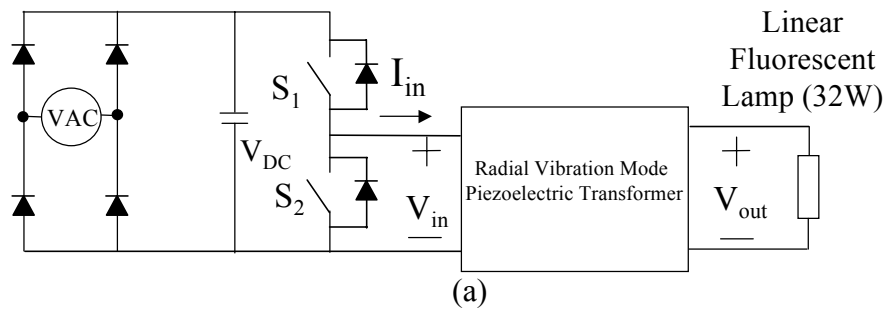
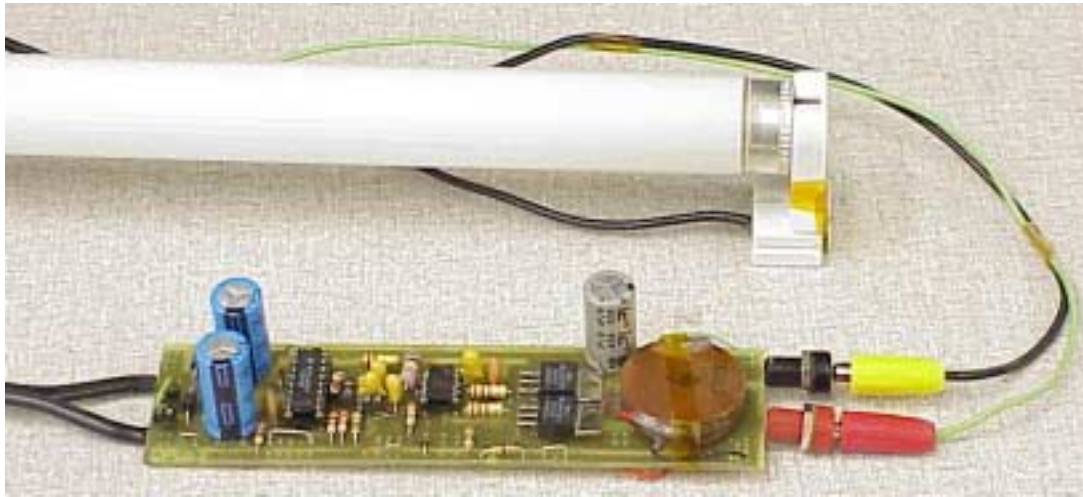


Figure 4.22. Detailed schematic of prototype circuit for proposed inductor-less piezoelectric transformer electronic ballast.



(a)



(b)

Figure 4.23. Photo of completed prototype circuit for inductor-less piezoelectric transformer ballast. (a) circuit block diagram. (b) photo of inductor-less piezoelectric transformer ballast. Without any magnetic device, the prototype circuit employs a radial vibration mode piezoelectric transformer sample CK2 as a piezoelectric transformer resonant tank to ignite and sustain a four-foot linear fluorescent lamp.

4.5.2 *Experimental Results of Prototype Circuit*

Without requiring any magnetic device, the prototype circuit utilizes a radial vibration mode piezoelectric transformer as a piezoelectric transformer resonant tank to ignite and sustain a linear fluorescent lamp. In Figure 4.24, this prototype circuit is shown driving a 4-foot 40-watt linear fluorescent lamp, with output power of 32 watts, without requiring any magnetic device. The efficiency of the whole prototype circuit was around 90%, including control circuit, half-bridge, and the radial vibration mode piezoelectric transformer. The experimental waveforms for the input voltage and current of the on-board radial vibration mode piezoelectric transformer are shown in Figure 4.25. The relationship shows the half-bridge switches, S_1 and S_2 , working in a ZVS condition. Figure 4.26 shows the input and output voltage waveforms. The input voltage waveform of the radial vibration mode piezoelectric transformer shows no voltage spikes and is characterized by slowly rising and falling slopes in order to create less dv/dt on the half-bridge switches, S_1 and S_2 . The parasitic input capacitor, C_{dl} , of the radial vibration mode piezoelectric transformer performs as a turn-off snubber for the half-bridge switches and yields this preferred switching characteristic.

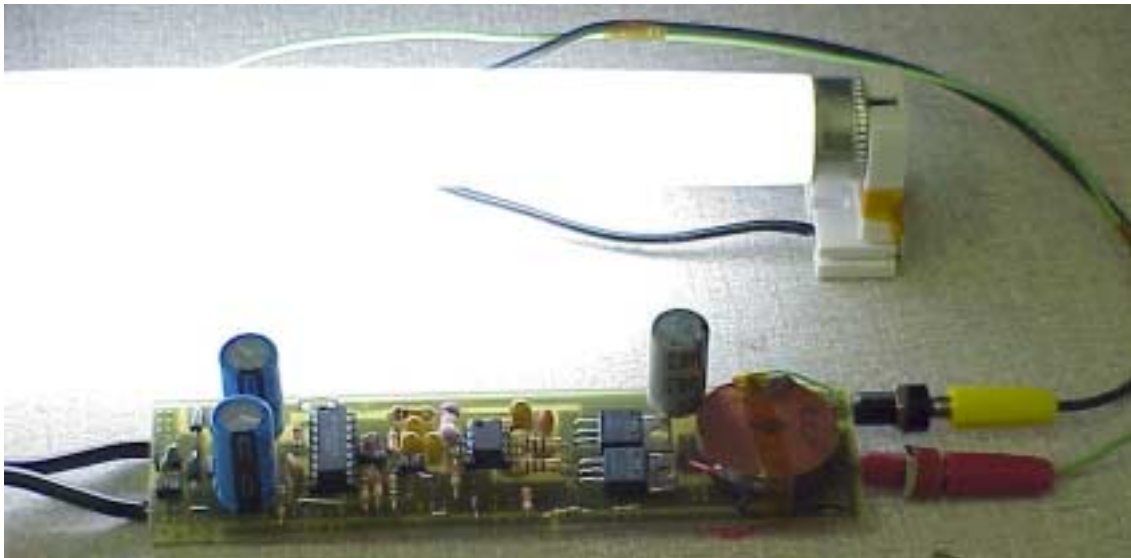
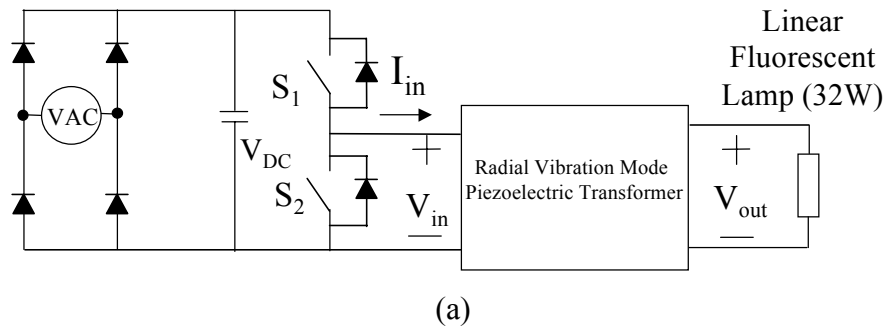


Figure 4.24. Photo of inductor-less piezoelectric transformer ballast driving a linear fluorescent lamp. (a) circuit block diagram. (b) after strike-on. This prototype circuit is driving a 4-foot 40-watt linear fluorescent lamp without requiring any magnetic device. The output power was 32 watts.

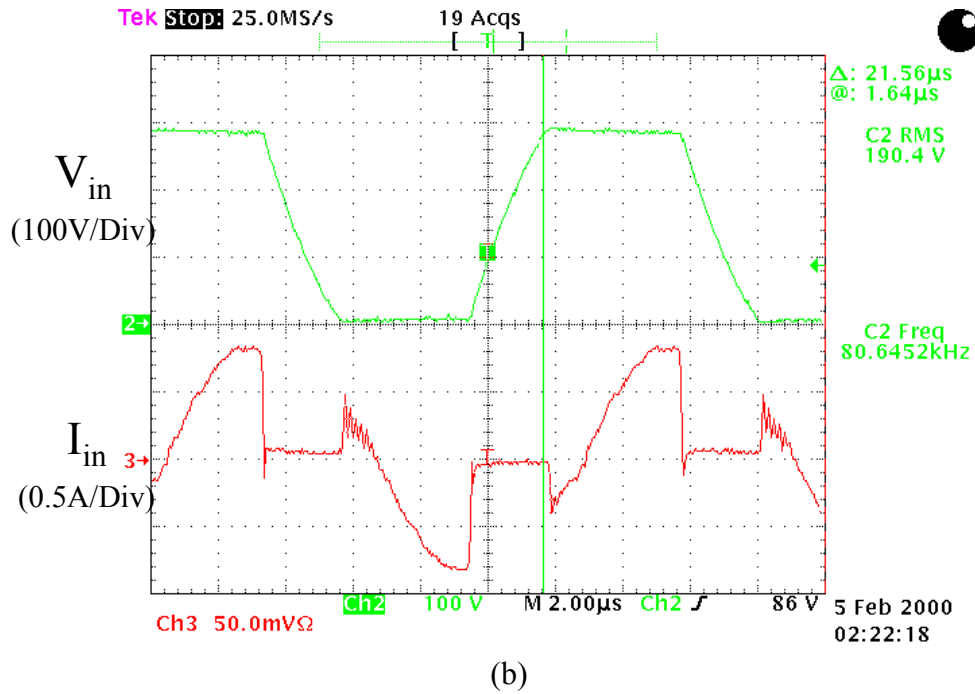
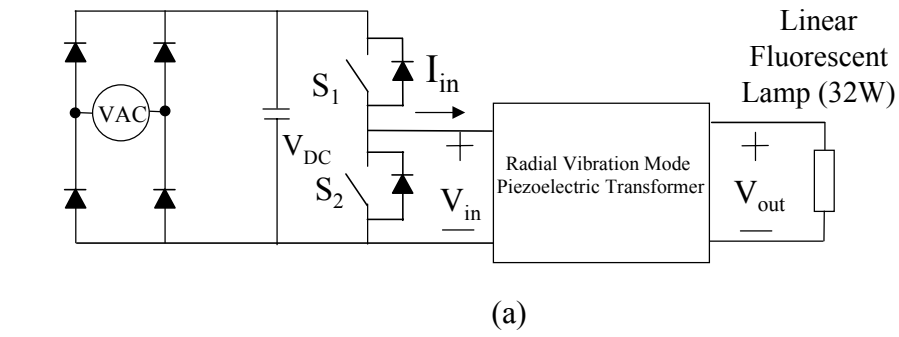


Figure 4.25. Experimental results of inductor-less piezoelectric transformer ballast prototype circuit. (a) circuit block diagram. (b) input voltage waveform (upper) and input current waveform (bottom). The input current and voltage waveforms of the radial vibration mode piezoelectric transformer show the switches, S_1 and S_2 , operate in a ZVS condition.

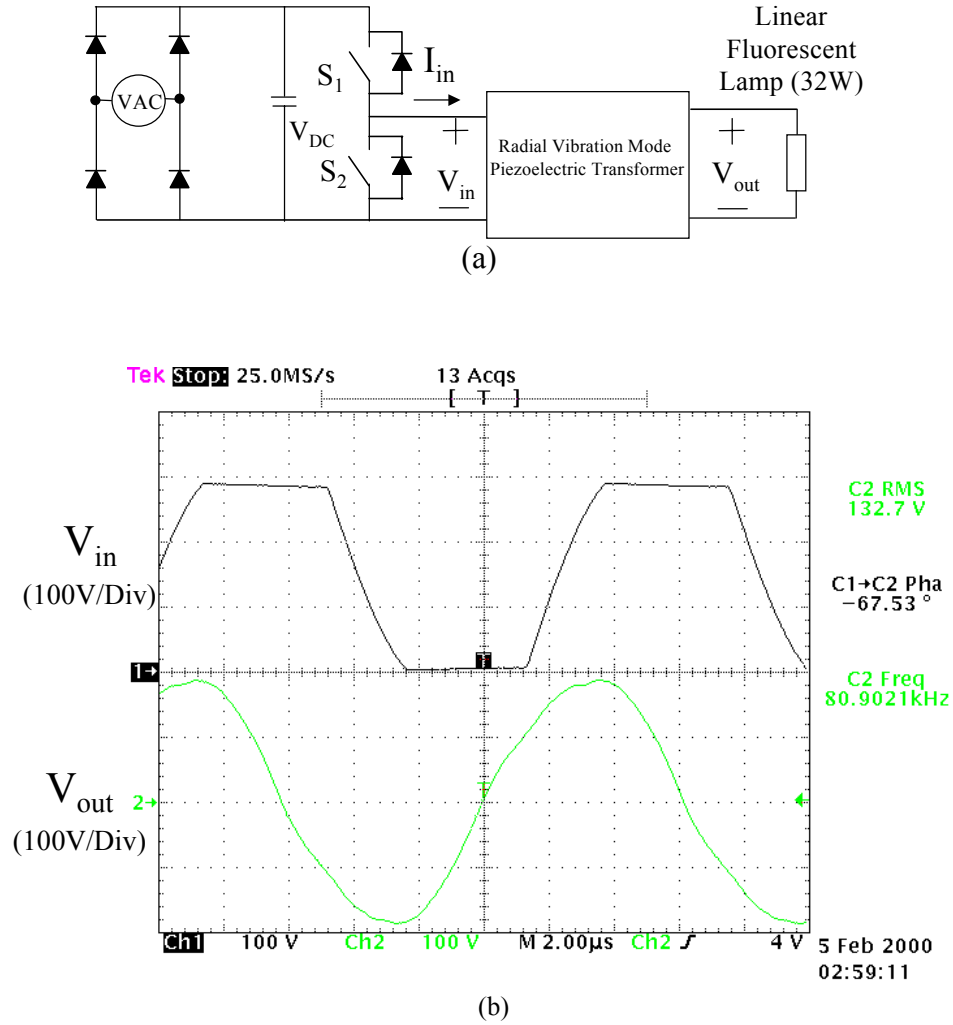


Figure 4.26. Input voltage waveform and output voltage waveform of prototype circuit utilizing a radial vibration mode piezoelectric transformer sample, CK2. (a) circuit block diagram. (b) input voltage waveform (upper) and output voltage waveform (bottom).

4.6 Inductor-less Electronic Ballast Incorporating PFC Function

Presently, there are many input power factor regulations concerning lighting products. The purpose of these regulations is to increase the operational efficiency of power generators for electric power plants, and hence directly reduce the quantity of natural resources used by electric power plants. In particular, fossil fuel energy sources can affect pollutants and atmospheric contaminants, such as carbon dioxide, which contribute to what is known as the greenhouse effect.

In order to meet power factor regulations, such as the IEC-61000, many charge pump power factor correction (PFC) schemes have been well developed and studied for use with electronic ballasts. These PFC schemes can be categorized into three major types: voltage source, current source, and voltage source plus current source [C1-C11]. However, all these have been implemented with conventional magnetic devices.

This section will present the design and implementation of an inductor-less PFC piezoelectric transformer ballast utilizing a radial vibration mode piezoelectric transformer. Considering its simplicity and performance, this chapter will focus on voltage source charge pump power factor correction (VS-CP-PFC) electronic ballast. This chapter will design and develop an inductor-less VS-CP-PFC electronic ballast utilizing a radial vibration mode piezoelectric transformer to drive a 40W white-light linear fluorescent lamp, with an equivalent on-resistance equal to 350Ω , at $V_{AC}=120V$ and output power at 39W. The radial vibration mode piezoelectric transformer sample, CK2, will first be evaluated for the application of the proposed inductor-less VS-CP-PFC electronic ballast. Based on the ZVS criteria described in Section 4.3 and the physics-based equivalent circuit model, derived in Chapter 2, a suitable radial vibration mode piezoelectric transformer sample will be selected for the prototype circuit.

4.6.1 Voltage Source Charge Pump Power Factor Correction Electronic Ballast

By adding two diodes, Dx and Dy, and charge pump capacitor, Cp, to the electronic ballast, as shown in Figure 4.27, the PFC function can be achieved by the VS-CP-PFC electronic ballast [C9, C10]. The charge pump capacitor, Cp, is in series with a high-frequency voltage source to pump energy from the AC line and discharge its stored energy to the bulk capacitor, CB, in order to reach unity input power factor [C1]. The minimal value of the charge pump capacitor, Cp, can be calculated by the following equation [C1]:

$$C_p = \frac{2 \cdot P_{out}}{\eta \cdot f_s \cdot |V_{in}|^2} \quad (4.44)$$

where Pout is the output power,

η is the efficiency,

fs is the switching frequency of the half-bridge switches, and

$|V_{in}|$ is the magnitude of the input AC voltage.

By replacing the L-C resonant tank, shown in Figure 4.28, with a piezoelectric transformer, the inductor-less VS-CP-PFC piezoelectric transformer electronic ballast is proposed, as shown in Figure 4.28.

This section will design and develop an inductor-less VS-CP-PFC electronic ballast employing a radial vibration mode piezoelectric transformer to drive a 40W white-light linear fluorescent lamp, with equivalent on-resistance equal to 350Ω , at $V_{AC}=120V$ and output power at 39W.

According to Equation (4.37), the value of the charge pump capacitor, Cp, needs to be greater than 33.4nF while the half-bridge switches operates at $f_s=81kHz$. However, the additional charge pump capacitor, Cp, paralleled with the input capacitor, Cd1, can cause the half-bridge switches to lose ZVS condition because the inductor must then charge/discharge the additional capacitor, Cp. Figure 4.29 shows that, if the additional

charge pump capacitor, C_p ($=33.4\text{nF}$) is paralleled with the input capacitor, C_{d1} , then the charged/discharged voltage is below the required DC bus voltage, $V_{DC}=170\text{V}$, which leads to the loss of ZVS condition for the half-bridge switches, S_1 and S_2 . Therefore, the radial vibration mode piezoelectric transformer sample, CK2, cannot be used for the proposed inductor-less VS-CP-PFC electronic ballast.

In order to assure that the half-bridge switches achieve ZVS condition, the next section will utilize the physics-based equivalent circuit model of the radial vibration mode piezoelectric transformer, developed in Chapter 2, to select a suitable radial vibration mode piezoelectric transformer sample for implementation of the inductor-less VS-CP-PFC electronic ballast.

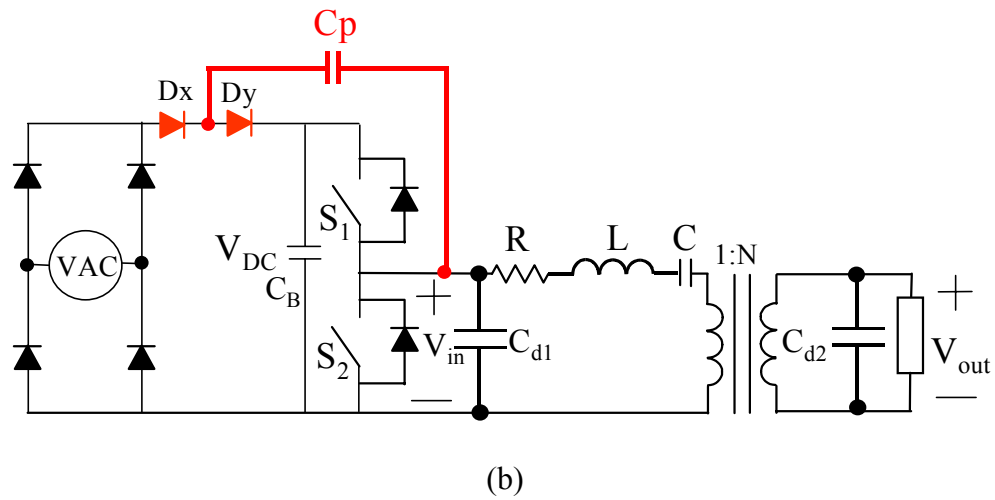
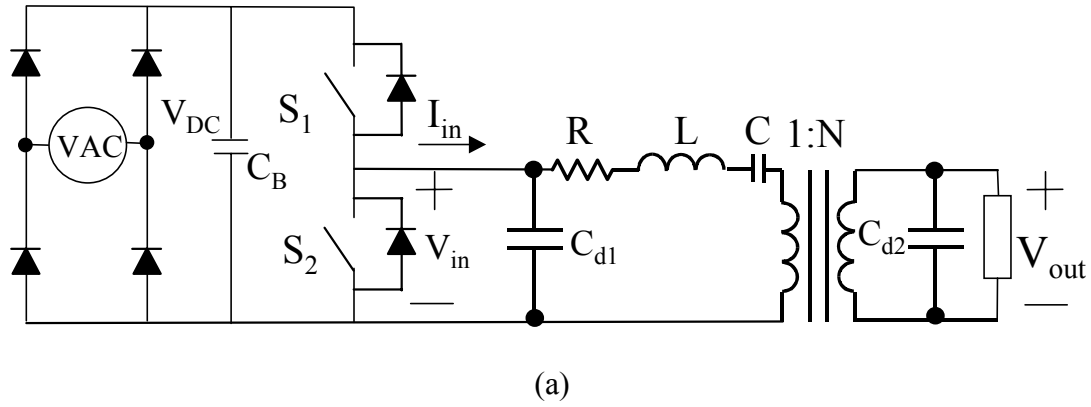


Figure 4.27. Voltage source charge pump power factor correction (VS-CP-PFC) electronic ballast [C9, C10]. (a) conventional electronic ballast. (b) VS-CP-PFC electronic ballast. With two additional diodes, Dx and Dy, and charge pump capacitor, Cp, the inductor-less electronic ballast can be modified to have power factor correction function.

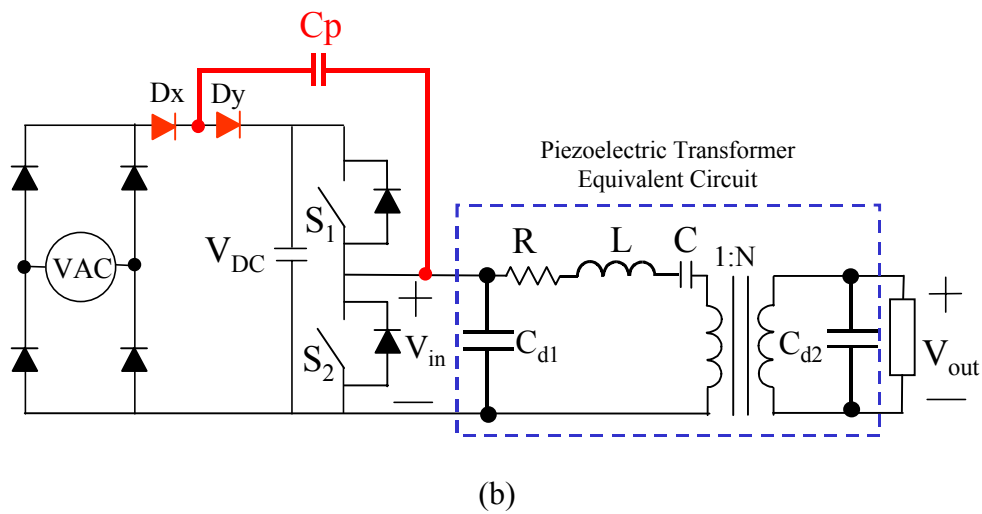
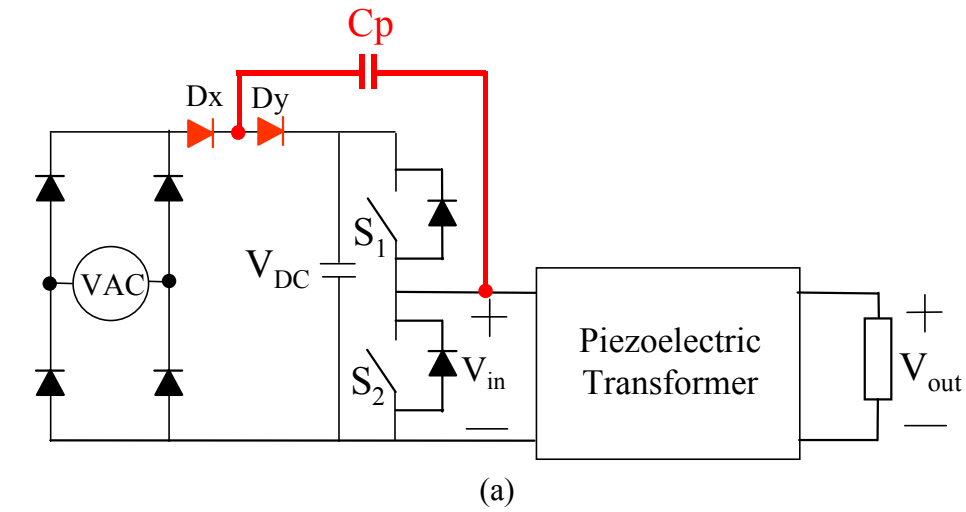


Figure 4.28. Inductor-less voltage source charge pump power factor correct (VS-CP-PFC) piezoelectric transformer electronic ballast. (a) VS-CP-PFC ballast utilizing a piezoelectric transformer. (b) VS-CP-PFC electronic ballast with an equivalent circuit of piezoelectric transformer. This circuit incorporates the concept of conventional VS-CP-PFC electronic ballast to implement inductor-less piezoelectric transformer electronic ballast function with power factor correction.

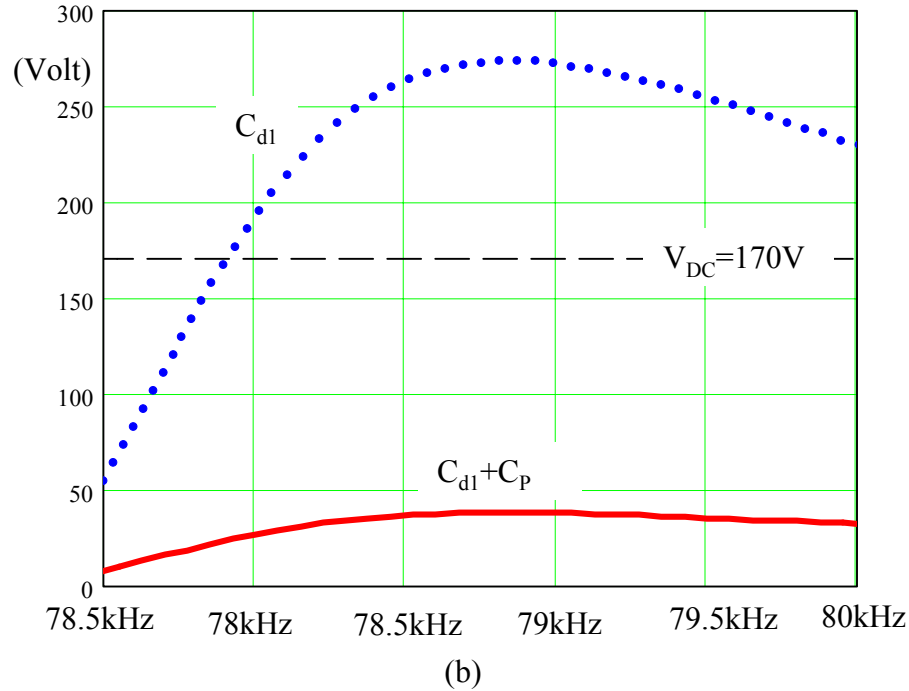
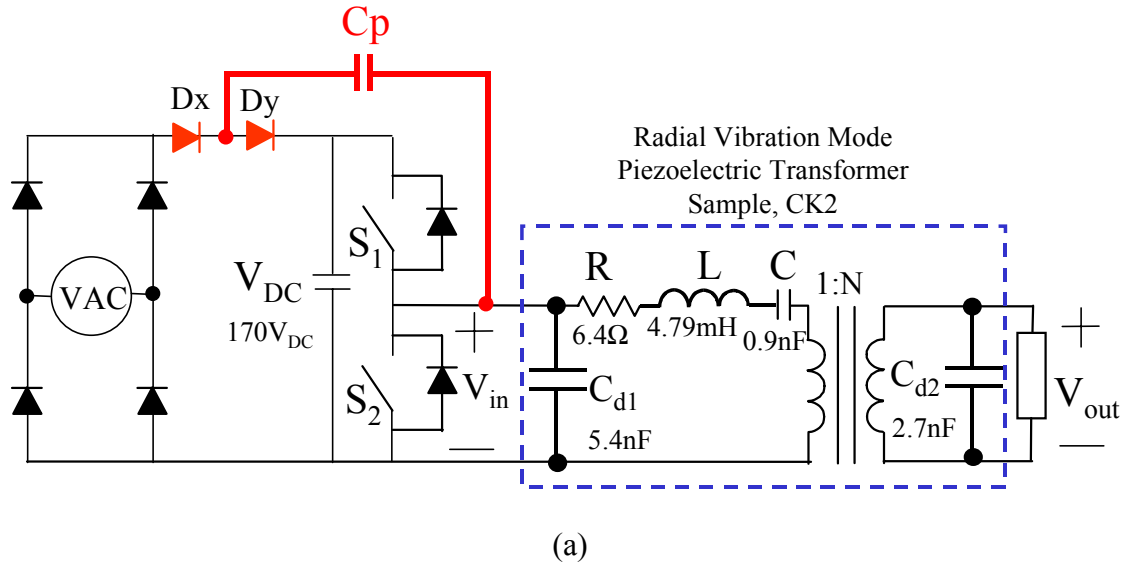


Figure 4.29. Charged/discharged voltage, V_{in} , of input capacitor, C_{d1} , paralleled with additional charge pump capacitor, $C_p=33.4\text{nF}$. (a) inductor-less VS-CP-PFC electronic ballast with radial vibration mode piezoelectric transformer sample CK2. (b) peak input capacitor voltage. With additional charge pump capacitor, C_p , paralleled with the input capacitor, C_{d1} , the charged/discharged voltage is far below the required DC bus voltage, $V_{DC}=170\text{V}$, which leads to the loss of ZVS condition for the half-bridge switches, S_1 and S_2 .

4.6.2 Design of Radial Vibration Mode Piezoelectric Transformer

According to the criteria for achieving ZVS condition, which was detailed in the previous chapter, the peak input capacitor voltage, $V_{Cd1,pk}$, needs to be greater than or equal to the DC bus voltage, V_{DC} , before the turn-on of the half-bridge switch, S_1/S_2 , as shown in Figure 4.30. However, with the additional charge-pump capacitor C_p , the actual input capacitor of the piezoelectric transformer becomes $C_{d1}+C_p$. Equation (4.23) expresses the maximal peak input capacitor voltage, $V_{Cd1,pk}$, and can be rewritten to include the additional charge-pump capacitor, C_p , as follows:

$$V_{Cd1,pk}^{(fs)}_{\max} = \frac{V_{DC} \cdot 0.53 \cdot \sin(\pi \cdot tr \cdot fs)}{\pi^3 \cdot fs^2 \cdot t_r} \cdot \frac{1}{R_{in}(fs) \cdot (C_{d1} + C_p)} \quad (4.45)$$

where t_r is rising time of the charged voltage of input capacitor, C_{d1} ,

fs is the switching frequency of the half-bridge switches S_1 and S_2 , and

$$R_{in}(fs) = R + \frac{R_L}{N^2 \cdot \left[1 + \left(2 \cdot \pi \cdot fs \cdot C_{d2} \cdot R_L \right)^2 \right]} \quad (4.46)$$

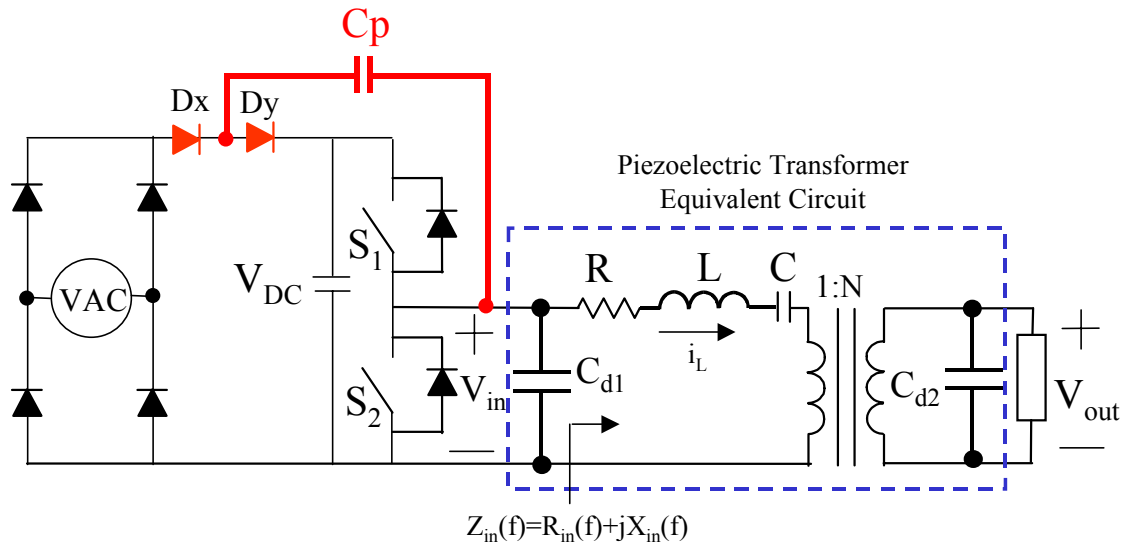


Figure 4.30. Half-bridge amplifier with equivalent circuit of piezoelectric transformer.

Referring to Chapter 2, the resistor, R , and the input capacitor, C_{d1} , of the physics-based equivalent circuit model for the radial vibration mode piezoelectric transformer can be expressed as:

$$R = \frac{\sqrt{2 \cdot \rho \cdot S_{11}^E \cdot (1 - \sigma)^3}}{32 \cdot Q_m \cdot d_{31}^2} \cdot \frac{(n_1 \cdot t_1 + n_2 \cdot t_2)}{n_1^2 \cdot r} \quad (4.47)$$

$$C_{d1} = \frac{n_1 \cdot \pi \cdot r^2 \cdot \epsilon_{33}^t \cdot \left[1 - \frac{d_{31}^2}{\epsilon_{33}^t \cdot S_{11}^E} \right]}{t_1} \quad (4.48)$$

When $R_L \approx \frac{1}{2 \cdot \pi \cdot f \cdot C_{d2}}$ for maximal efficiency, then

$$R_{in}(f) \approx R + \frac{R_L}{2 \cdot N^2} \quad (4.49)$$

Therefore, Equation (4.45) can be approximated as

$$V_{Cd1, pk}^{(fs)} \approx \frac{0.53 \cdot V_{DC} \cdot \sin(\pi \cdot tr \cdot fs)}{\pi^3 \cdot fs^2 \cdot t_r} \cdot \frac{1}{(C_{d1} + C_p) \cdot (2 \cdot R + \frac{R_L}{N^2})} \quad (4.50)$$

$$\text{where } N = \frac{n_1}{n_2},$$

n_1 is the layer number of the primary side, and

n_2 is the layer number of the secondary side.

Combining Equations (4.47), (4.48) and (4.50), the maximal voltage of the input capacitor, C_{d1} , and can be rewritten as follows:

$$V_{Cd1, pk}^{(fs)} \approx \frac{0.53 \cdot V_{DC} \cdot \sin(\pi \cdot tr \cdot f)}{\pi^3 \cdot f^2 \cdot t_r} \cdot \frac{1}{\left[\frac{n_1 \cdot \pi \cdot r^2 \cdot \epsilon_{33}^t \cdot \left(1 - \frac{d_{31}^2}{\epsilon_{33}^t \cdot S_{11}^E} \right)}{t_1} + C_p \right] \cdot \left[2 \cdot \frac{\sqrt{2 \cdot \rho \cdot S_{11}^E \cdot (1 - \sigma)^3}}{32 \cdot Q_m \cdot d_{31}^2} \cdot \frac{(n_1 \cdot t_1 + n_2 \cdot t_2)}{n_1^2 \cdot r} + \frac{R_L}{N^2} \right]} \quad (4.51)$$

Referring to Equation (4.51), there are many possible ways to increase the maximal peak input capacitor voltage, $V_{Cd1,pk}$, all of which are associated with the additional charge-pump capacitor. Examples of these methods include:

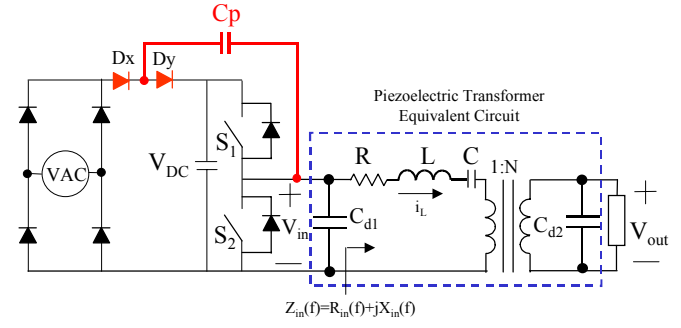
- (1) increasing $N = \frac{n_1}{n_2}$
- (2) decreasing the radius, r ,
- (3) increasing the layer thickness, t_1 , of the primary side, and
- (4) decreasing the layer thickness, t_2 , of the secondary side.

With the same dimensions and piezoelectric ceramic material as were used in the previous radial vibration mode piezoelectric transformer sample, CK2, a new sample, CE1, can be obtained by adding two more layers on the primary side. Therefore, Sample CE1 has four layers on the primary side and one layer on the secondary side, with the same diameter and layer thickness as sample CK2, i.e. $t_1=t_2=80\text{mils}$ and $\text{diameter}=1180\text{mils}$. Table 4.2 shows the measured parameters of the equivalent circuit model for five-layer radial vibration mode piezoelectric transformer samples, CK2 and CE1. The measured turns ratio, N , of sample CE1 is 4.97, which is much higher than that of sample CK1, in which $N=2$.

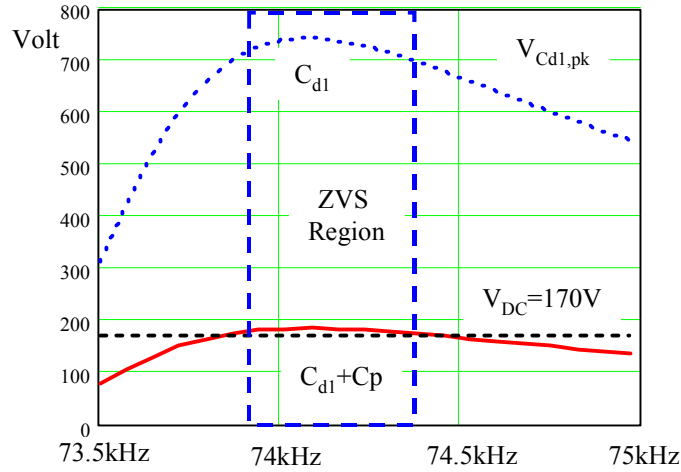
Table 4.2. Measured parameters of equivalent circuit model for five-layer radial vibration mode piezoelectric transformer samples, CK2 and CE1.

	R	L	C	C_{d1}	C_{d2}	N	$f_s = \frac{1}{\sqrt{LC}}$
CK2	6.42 Ω	4.79 mH	918.71 pF	5.41 nF	2.74 nF	2	75.86 kHz
CE1	2.13 Ω	1.67mH	2.85nF	11.1nF	2.75nF	4.97	72.95kHz

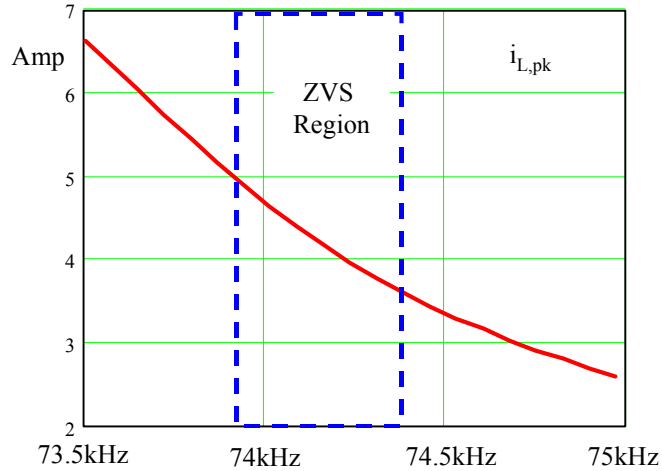
Figure 4.31 shows the new five-layer radial vibration mode piezoelectric transformer sample, CE1, has higher peak input capacitor voltage, $V_{Cd1,pk}$, than the required DC bus voltage, $V_{DC}=170V$, with an additional charge pump capacitor, C_p , which leads to ZVS conditions for the half-bridge switches, S_1 and S_2 .



(a)



(b)



(c)

Figure 4.31. Charged/discharged voltage, V_{in} , on input capacitor C_{d1} paralleled with additional charge pump capacitor, $C_p=33.4\text{nF}$. (a) VS-CP-PFC electronic ballast employing a radial vibration mode piezoelectric transformer sample CE1. (b) peak input capacitor voltage. (c) amplitude of inductor current.

4.6.3 Implementation and Experimental Results

Figure 4.32 shows the schematic of the developed inductor-less VS-CP-PFC piezoelectric transformer electronic ballast utilizing the five-layer radial vibration mode piezoelectric transformer sample, CE1. Figure 4.33 shows the experimental results of the inductor-less VS-CP-PFC electronic ballast circuit with $V_{in}=120V_{rms}$, including input current waveform, input voltage waveform, and input current harmonics. The measured current harmonics are below the required boundary. The measured power factor is 0.995 and the THD is 8.597%. However, the efficiency is only 72% due to the large circulating current flowing through the half-bridge switches and diodes, and thus reducing the efficiency of the circuit.

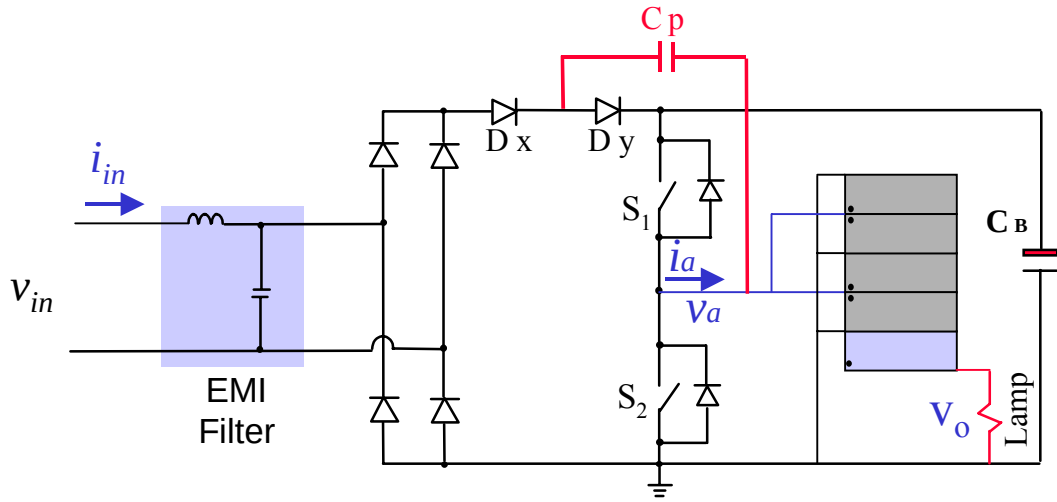


Figure 4.32. Schematic of inductor-less VS-CP-PFC piezoelectric transformer ballast utilizing five-layer radial vibration mode piezoelectric transformer sample, CE1, ($V_{in}=120V_{AC}$, $P_{out}=38.8W$, $C_p=33.4nF$). This five-layer sample has four layers at the primary side and one layer on the secondary side. The equivalent resistance of the lamp is 350Ω .

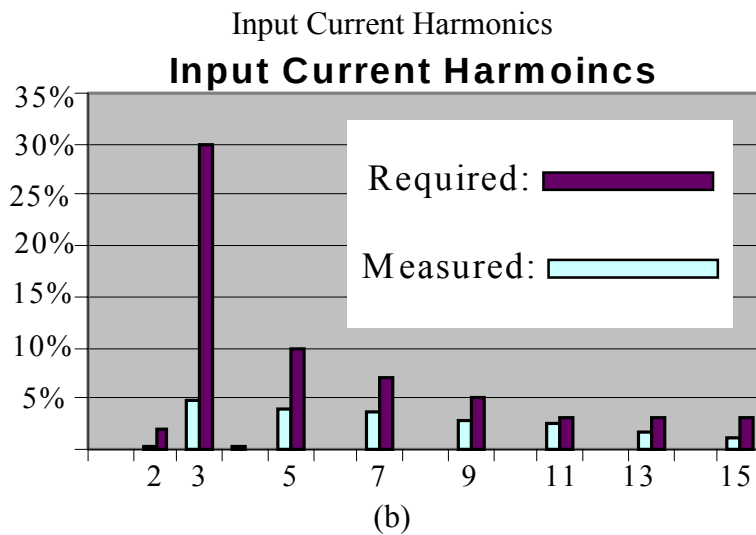
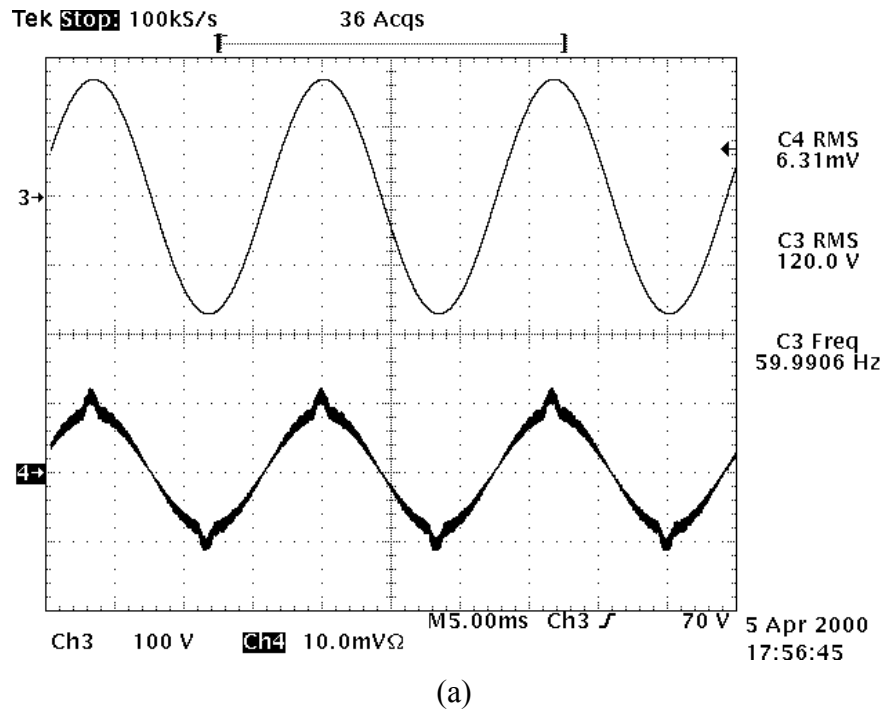


Figure 4.33. Experimental results of VS-CP-PFC piezoelectric transformer electronic ballast. (a) input voltage waveform (upper) and input current waveform (bottom). (b) measured and required input current harmonics. The measured input current harmonics is far below the requirement, IEC61000.

4.7 Summary

This chapter has proposed ZVS criteria to evaluate the equivalent circuit of piezoelectric transformers for ZVS condition in the electronic ballast application. The purpose of ZVS criteria is to analyze the charged/discharged voltage across the input capacitor of the piezoelectric transformer, as compared with the DC bus voltage. Based on the proposed ZVS criteria, this chapter has presented inductor-less piezoelectric transformer electronic ballast circuit utilizing suitable radial vibration mode piezoelectric transformer samples without requiring any magnetic devices.

The developed inductor-less ballast prototype was used to drive a 4-foot 40-watt linear fluorescent lamp with an output power of 32 watts. Since the switches of the proposed circuit work in a ZVS condition, their turn-on switching losses can be significantly reduced. In addition, the input capacitor of the radial vibration mode piezoelectric transformer can be a useful turn-off snubber for the half-bridge switches in order to reduce the turn-off voltage spike and to reduce the turn-off losses for the half-bridge switches of the proposed circuit. Through these innovative circuit techniques, the efficiency of the total proposed circuit can achieve a commendable level of around 90%. Furthermore, the low-profile design of the radial vibration mode piezoelectric transformer can minimize the total circuit packaging size of smaller lamp fixtures, translating into possible cost reduction as well.

Incorporating the proposed ZVS criteria and the derived physics-based equivalent circuit model of the radial vibration mode piezoelectric transformer, which was presented in Chapter 2, a suitable five-layer radial vibration mode piezoelectric transformer sample CE1 was selected for the implementation of an inductor-less VS-CP-PFC piezoelectric transformer electronic ballast. The measured current harmonics of a prototype using the 5-layer radial vibration mode piezoelectric transformer sample are less than the maximum specified by the IEC61000. The measured power factor is 0.995 and the THD is 8.597%. However, the efficiency of this PFC prototype circuit is only

72% due to the large circulating current flowing through the half-bridge switches and diodes, which subsequently reduces the efficiency of the entire circuit.

CHAPTER 5

CONCLUSION AND FUTURE WORK

Replacing conventional L-C resonant tanks with piezoelectric transformers reduces the component count and cost of DC/DC converters and electronic ballasts for fluorescent lamps. Furthermore, the low-profile design of piezoelectric transformers minimizes the total packaging size for the application circuit, possibly translating into additional cost reduction as well.

This dissertation has presented the characterization of the piezoelectric transformer and has described its use in electronic ballasts for fluorescent lamps. The characterization and model of the radial vibration mode piezoelectric transformer have also been revealed.

The piezoelectric transformer, an electromechanical device, is a combination of piezoelectric actuators on the primary side and piezoelectric transducers on the secondary side. Both the actuator and transducer are made of piezoelectric elements, which are composed of electrode plates and piezoelectric materials, such as barium titanate-based ceramics. A piezoelectric element can work either in longitudinal vibration mode or in transverse vibration mode at a corresponding resonant frequency. In the longitudinal mode, the direction of the mechanical stress is parallel to the electric or polarization direction at a corresponding resonant frequency. In the transverse mode, the direction of the mechanical stress is perpendicular to the electric or polarization direction at a corresponding resonant frequency.

Piezoelectric transformers can be categorized into three major types: Rosen, thickness vibration mode and radial vibration mode. The Rosen transformer is a combination of a transverse mode piezoelectric actuator on the primary side and a

longitudinal mode piezoelectric transducer on the secondary side. The thickness vibration piezoelectric transformer is a combination of a longitudinal mode piezoelectric actuator on the primary side and a longitudinal mode piezoelectric transducer on the secondary side. The radial vibration mode transformer is a combination of a transverse mode piezoelectric actuator on the primary side and a transverse mode piezoelectric transducer on the secondary side. The applications for piezoelectric transformers include use in DC/DC converters and in electronic ballasts for fluorescent lamps.

Based on the piezoelectric and wave equations, the physics-based equivalent circuit model for the major vibration mode of piezoelectric transformers can be derived. The physics-based equivalent circuit model, which is composed of a network of L-C resonant tanks, can provide a good reference for the design and application of these transformers. Therefore, a physics-based equivalent circuit model was derived for the newly invented radial vibration mode piezoelectric transformer for the major radial vibration mode.

The Y-parameter equivalent circuit model, provided by an HP4194A impedance analyzer, can be used to characterize the parameters of the equivalent circuit of piezoelectric transformers. Samples of radial vibration mode piezoelectric transformers were characterized to verify the derived physics-based equivalent circuit model. The comparison results showed that the circuit model is in good agreement with the corresponding measured results of the piezoelectric transformer samples when the total thickness is much less than the radius. Since this circuit model was derived for the major radial vibration mode, other spurious vibration modes, such as thickness vibration mode, were not considered. However, when the total thickness becomes equal to the radius, the resonant frequency of the thickness vibration mode approaches that of the major radial vibration mode, which causes the parameter deviation between the derived physics-based equivalent circuit model and the characterized equivalent circuit of radial vibration mode piezoelectric transformers.

Besides the major vibration mode, piezoelectric transformers have many spurious vibration modes in other frequency ranges. These vibration modes can be characterized with a multi-branch equivalent circuit model, composed of several single L-C resonant tanks, for the design and simulation of application circuits operating in wide frequency ranges rather than the major vibration mode. The prior multi-branch equivalent circuit model can very precisely characterize the input admittance, but cannot characterize the voltage gain with an acceptable deviation for the radial vibration mode piezoelectric transformer. This improved multi-branch equivalent circuit model uses more detailed networks to describe the voltage gain and phase for the different spurious vibration modes of the radial vibration mode piezoelectric transformer. Therefore, compared to the prior multi-branch equivalent circuit model, the proposed model can much more accurately characterize the input admittance and voltage gain of the radial vibration mode piezoelectric transformer.

In order to evaluate the equivalent circuit of piezoelectric transformers for achieving ZVS condition, two ZVS criteria were presented to analyze the charged/discharged voltage across the input capacitor of piezoelectric transformers. Based on these two ZVS criteria, a suitable radial vibration mode piezoelectric transformer sample can be evaluated and chosen for the implementation of inductor-less piezoelectric transformer electronic ballast circuit. Associating the proposed ZVS criteria and the derived physics-based equivalent circuit model of a radial vibration mode piezoelectric transformer, a suitable radial vibration mode piezoelectric transformer sample can be selected for the application of an inductor-less VS-CP-PFC piezoelectric transformer electronic ballast. Furthermore, the proposed ZVS criteria also can be used to evaluate and choose the suitable turn-off snubber capacitor for a general half-bridge inverter driving an L-C resonant tank.

The developed prototype of the inductor-less piezoelectric transformer was used to drive a four-foot 40-watt linear fluorescent lamp with an output power of 32 watts. Since the switches of the proposed circuit work in ZVS condition, their turn-on switching losses can be significantly reduced. In addition, the input capacitor of a radial

vibration mode piezoelectric transformer can be a useful turn-off snubber in order for the half-bridge switches to lower the turn-off voltage spike and to reduce the turn-off losses for the half-bridge switches of the proposed circuit. Through these innovative circuit techniques, the efficiency of the total proposed circuit can achieve a commendable level of around 90%.

By associating the proposed ZVS criteria and the derived physics-based equivalent circuit model of radial vibration mode piezoelectric transformer, a suitable five-layer radial vibration mode piezoelectric transformer sample, CE1, was selected for the implementation of an inductor-less VS-CP-PFC piezoelectric transformer electronic ballast. The measured current harmonics of a prototype using a 5-layer radial vibration mode piezoelectric transformer sample are less than the IEC-61000 requirement,. The measured power factor is 0.995 and the THD is 8.597%. However, the efficiency of this PFC prototype circuit is only 72% due to the large circulating current flowing through the half-bridge switches and the corresponding reduced efficiency of the circuit.

Recommended Future Work

The follows are some recommendations for future research work with the radial vibration mode piezoelectric transformers.

- **Advanced physics-based equivalent circuit:**
To include the interaction effects of spurious vibration modes, such as thickness vibration mode, for the physics-based equivalent circuit model of the radial vibration mode piezoelectric transformer.
- **More PFC applications:**
To incorporate other existing PFC schemes by utilizing the radial vibration mode piezoelectric transformers for electronic ballasts.
- **Packaging:**
To integrate radial vibration mode piezoelectric transformers with other electronic components on PCB board.

- Thermal analysis:

To investigate the thermal model and the power dissipation capability of radial vibration mode piezoelectric transformers.

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APPENDIX

Derivation of Physics-Based Equivalent Circuit Model for Radial Vibration Mode Piezoelectric Transformer

The radial vibration mode piezoelectric transformer, as shown in Figure A.1, is composed of two single-layer piezoelectric elements: a single-layer piezoelectric actuator and a single-layer piezoelectric transducer both of which operate in transverse mode. The behaviors of the actuator and transducer can be described as shown in the following linear piezoelectric Equations (A1) and (A2).

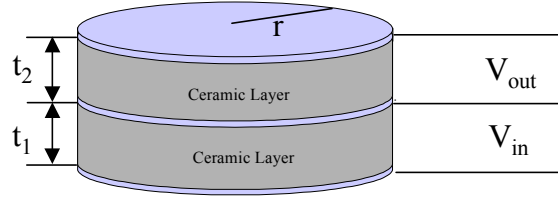


Figure A.1. Single-layer radial vibration mode piezoelectric transformer. This radial vibration mode piezoelectric transformer is composed of two single-layer piezoelectric elements: a single-layer piezoelectric actuator and a single-layer piezoelectric transducer, both of which operate in transverse mode.

Piezoelectric Equations

Piezoelectric Actuator

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E \\ s_{21}^E & s_{22}^E & s_{23}^E \\ s_{31}^E & s_{32}^E & s_{33}^E \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad \text{and} \quad (A1)$$

Piezoelectric Transducer

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{bmatrix} \epsilon_{11}^T & 0 & 0 \\ 0 & \epsilon_{22}^T & 0 \\ 0 & 0 & \epsilon_{33}^T \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (A2)$$

where

S is mechanical strain,

T is mechanical stress,

E is electric field,

D is electric displacement,

d is piezoelectric constant,

s^E is elastic compliance at constant electric field, and

ϵ^T is permittivity at constant stress.

Since both of the piezoelectric actuator and the piezoelectric transducer operate in transverse mode, the piezoelectric transformer Equations (A1) and (A2), can be simplified as shown Equations (A3) and (A4).

Piezoelectric Actuator in Transverse Mode

$$\begin{bmatrix} S_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E \\ s_{12}^E & s_{11}^E & s_{13}^E \\ s_{13}^E & s_{13}^E & s_{33}^E \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (A3)$$

Piezoelectric Transducer in Transverse Mode

$$\begin{bmatrix} 0 \\ 0 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{31} & d_{33} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_{11}^T & 0 & 0 \\ 0 & \epsilon_{22}^T & 0 \\ 0 & 0 & \epsilon_{33}^T \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \quad (A4)$$

The simplified linear piezoelectric Equations (A3) and (A4), are in the Cartesian-coordinate system. However, the radial vibration mode piezoelectric transformer is round, with its major vibration mode in the radial direction. Therefore, Equations (A3) and (A4) need to be converted into the cylindrical-coordinate system with the following transformation Equations (A5) to (A8). Following the derivation process, as shown in Equations (A9) to (A12), the linear piezoelectric equation for the radial vibration mode piezoelectric actuator can be simplified as shown in Equation (A13). Following the derivation process, as shown in Equations (A14) to (A15), the linear piezoelectric

equation for the radial vibration mode piezoelectric transducer can be simplified as shown in Equation (A16).

Transformation of Coordinates from Cartesian Coordinate System to Cylindrical Coordinate System

Mechanical Strain

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_r \\ S_\theta \\ S_z \end{bmatrix} \quad (\text{A5})$$

Mechanical Stress

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} T_r \\ T_\theta \\ T_z \end{bmatrix} \quad (\text{A6})$$

Electric Field

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} E_r \\ E_\theta \\ E_z \end{bmatrix} \quad (\text{A7})$$

Electric Displacement

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} D_r \\ D_\theta \\ D_z \end{bmatrix} \quad (\text{A8})$$

Radial Vibration Mode Piezoelectric Actuator

$$\begin{aligned}
 \begin{bmatrix} S_r \\ S_\theta \\ S_z \end{bmatrix} &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E \\ s_{12}^E & s_{11}^E & s_{13}^E \\ s_{13}^E & s_{13}^E & s_{33}^E \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} T_r \\ T_\theta \\ T_z \end{bmatrix} + \\
 &\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_r \\ E_\theta \\ E_z \end{bmatrix} \quad (A9)
 \end{aligned}$$

Then,

$$\begin{bmatrix} S_r \\ S_\theta \\ S_z \end{bmatrix} = \begin{bmatrix} (s_{11}^E + s_{12}^E) \cdot T_r + \sqrt{2} \cdot s_{13}^E \cdot T_z + \sqrt{2} \cdot d_{31} \cdot E_z \\ (s_{11}^E - s_{12}^E) \cdot T_\theta \\ \sqrt{2} \cdot s_{13}^E \cdot T_r + s_{33}^E \cdot T_z + d_{33} \cdot E_z \end{bmatrix} \quad (A10)$$

$$\Rightarrow S_r = (s_{11}^E + s_{12}^E) \cdot T_r + \sqrt{2} \cdot s_{13}^E \cdot T_z + \sqrt{2} \cdot d_{31} \cdot E_z \quad (A11)$$

$$= s_{11}^E \cdot (1 - \sigma) \cdot T_r + \sqrt{2} \cdot s_{13}^E \cdot T_z + \sqrt{2} \cdot d_{31} \cdot E_z \quad (A12)$$

where σ is the elastic compliance, and $\sigma = -\frac{s_{21}}{s_{11}}$.

$$\Rightarrow T_r = \frac{S_r - \sqrt{2} \cdot d_{31} \cdot E_z - \sqrt{2} \cdot s_{13}^E \cdot T_z}{(1 - \sigma) \cdot s_{11}^E} \quad (A13)$$

Radial Vibration Mode Piezoelectric Transducer

$$\begin{aligned}
 \begin{bmatrix} D_r \\ D_\theta \\ D_z \end{bmatrix} &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -1 & 1 & 0 \\ \sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{31} & d_{33} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} T_r \\ T_\theta \\ T_z \end{bmatrix} + \\
 &\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -1 & 1 & 0 \\ \sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{11}^T & 0 & 0 \\ 0 & \epsilon_{22}^T & 0 \\ 0 & 0 & \epsilon_{33}^T \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_r \\ E_\theta \\ E_z \end{bmatrix}
 \end{aligned}
 \tag{A14}$$

Then,

$$\begin{bmatrix} D_r \\ D_\theta \\ D_z \end{bmatrix} = \begin{bmatrix} \frac{E_r}{2} \cdot (\epsilon_{11}^T + \epsilon_{22}^T) - \frac{E_\theta}{2} \cdot (\epsilon_{11}^T - \epsilon_{22}^T) \\ \frac{E_r}{2} \cdot (\epsilon_{11}^T - \epsilon_{22}^T) + \frac{E_\theta}{2} \cdot (\epsilon_{11}^T + \epsilon_{22}^T) \\ \sqrt{2} \cdot T_r \cdot d_{31} + d_{33} \cdot T_z + \epsilon_{33}^T \cdot E_z \end{bmatrix}
 \tag{A15}$$

$$\Rightarrow D_z = \sqrt{2} \cdot d_{31} \cdot T_r + d_{33} \cdot T_z + \epsilon_{33}^T \cdot E_z
 \tag{A16}$$

Based on the simplified piezoelectric Equations (A13) and (A16), the current, I , flowing through the electrode plates of the radial piezoelectric element, shown in Figure A2, can be derived as shown in the following Equations (A17) to (A26).

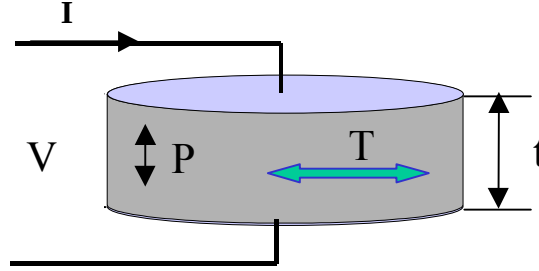


Figure A2. Transverse mode piezoelectric element.

$$I = \frac{d}{dt} Q = \frac{d}{dt} \int D \cdot dS = \frac{d}{dt} \int_0^r D_z \cdot 2 \cdot \pi \cdot r dr \quad (A17)$$

$$= 2 \cdot \pi \cdot \frac{d}{dt} \int_0^r (\sqrt{2} \cdot d_{31} T_r + d_{33} T_z + \epsilon_{33}^T \cdot E_z) \cdot r dr \quad (A18)$$

$$= 2 \cdot \pi \cdot \frac{d}{dt} \int_0^r [\sqrt{2} \cdot d_{31} \cdot (\frac{S_r - \sqrt{2} \cdot d_{31} \cdot E_z - \sqrt{2} \cdot S_{13}^E \cdot T_z}{(1-\sigma) \cdot S_{11}^E}) + d_{33} T_z + \epsilon_{33}^T \cdot E_z] \cdot r dr \quad (A19)$$

$$= \pi \cdot \frac{d}{dt} \int_0^r [2 \cdot \sqrt{2} \cdot (\frac{S_r d_{31} - \sqrt{2} \cdot d_{31}^2 \cdot E_z - \sqrt{2} \cdot d_{31} \cdot S_{13}^E \cdot T_z}{(1-\sigma) \cdot S_{11}^E}) + 2 \cdot d_{33} T_z + 2 \cdot \epsilon_{33}^T \cdot E_z] \cdot r dr \quad (A20)$$

$$= \pi \cdot \frac{d}{dt} \int_0^r [2 \cdot \frac{\sqrt{2} \cdot S_r d_{31}}{(1-\sigma) \cdot S_{11}^E} - \frac{2 \cdot d_{31}^2 \cdot E_z}{(1-\sigma) \cdot S_{11}^E} - \frac{2 \cdot d_{31} \cdot S_{13}^E \cdot T_z}{(1-\sigma) \cdot S_{11}^E} + 2 \cdot d_{33} T_z + 2 \cdot \epsilon_{33}^T \cdot E_z] \cdot r dr \quad (A21)$$

$$= \frac{2 \cdot \sqrt{2} \cdot \pi}{(1-\sigma) \cdot S_{11}^E} \cdot \frac{d}{dt} \int_0^r S_r \cdot d_{31} \cdot r dr + \pi \cdot \frac{d}{dt} \int_0^r [2 \cdot (d_{33} - \frac{d_{31} \cdot S_{13}^E}{(1-\sigma) \cdot S_{11}^E}) \cdot T_z \cdot r dr + j\omega [2\pi \cdot \frac{r^2}{2} \cdot \epsilon_{33}^T \cdot (1 - \frac{d_{31}^2}{(1-\sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T})] \cdot E_z \quad (A22)$$

$$= \frac{2 \cdot \sqrt{2} \cdot \pi \cdot d_{31}}{(1-\sigma) \cdot S_{11}^E} \cdot \frac{d}{dt} (S_r \cdot \frac{r^2}{2}) + \pi \cdot \frac{d}{dt} \int_0^r [2 \cdot (d_{33} - \frac{d_{31} \cdot S_{13}^E}{(1-\sigma) \cdot S_{11}^E}) \cdot T_z \cdot r dr + j\omega \cdot [\pi \cdot r^2 \cdot \epsilon_{33}^T \cdot (1 - \frac{d_{31}^2}{(1-\sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T})] \cdot E_z \quad (A23)$$

$$= \frac{2 \cdot \sqrt{2} \cdot \pi \cdot r \cdot d_{31}}{(1-\sigma) \cdot S_{11}} \cdot \frac{d}{dt} \cdot u_r + \pi \cdot \frac{d}{dt} \int_0^r 2 \cdot (d_{33} - \frac{d_{31} \cdot S_{13}}{(1-\sigma) \cdot S_{11}} \cdot E) \cdot T_z \cdot r dr +$$

$$j\omega \cdot [\pi \cdot r^2 \epsilon_{33}^T (1 - \frac{d_{31}^2}{(1-\sigma) \cdot S_{11} \cdot \epsilon_{33}})] \cdot \frac{V}{t} \quad (A24)$$

$$= \frac{2 \cdot \sqrt{2} \cdot \pi \cdot r \cdot d_{31}}{(1-\sigma) \cdot S_{11}} \cdot v + j\omega \cdot [\pi \cdot r^2 \epsilon_{33}^T (1 - \frac{d_{31}^2}{(1-\sigma) \cdot S_{11} \cdot \epsilon_{33}})] \cdot \frac{V}{t} \quad (A25)$$

$$= \psi \cdot v + j\omega \cdot C_{dm} \cdot V \quad (A26)$$

where

$$\psi = \frac{2 \cdot \sqrt{2} \cdot \pi \cdot r \cdot d_{31}}{(1-\sigma) \cdot S_{11}} \quad (A27)$$

$$C_{dm} = \frac{\pi \cdot r^2}{t} \epsilon_{33}^T (1 - \frac{d_{31}^2}{(1-\sigma) \cdot S_{11} \cdot \epsilon_{33}}) \quad (A28)$$

Based on the simplified piezoelectric Equations (A13) and (A16), the mechanical vibration force, F_r , of the radial pizeoelectric element can be derived as shown in the following Equations (A29) to (A36).

$$F_r = T_r \cdot A = T_r \cdot 2\pi \cdot r \cdot t \quad (A29)$$

Then,

$$F_r = \frac{S_r - \sqrt{2} \cdot d_{31} \cdot E_z - \sqrt{2} \cdot S_{13} \cdot E}{(1-\sigma) \cdot S_{11}} \cdot 2\pi \cdot r \cdot t \quad (A30)$$

$$= \frac{2 \cdot \pi \cdot r \cdot t}{(1-\sigma) \cdot S_{11}} \cdot (S_r - \sqrt{2} \cdot d_{31} \cdot E_z) \quad (A31)$$

$$= 2 \cdot \pi \cdot r \cdot t \cdot [\frac{1}{(1-\sigma) \cdot S_{11}} \cdot \frac{\partial u_r}{\partial r} - \frac{\sqrt{2} \cdot d_{31}}{(1-\sigma) \cdot S_{11}} \cdot E_z] \quad (A32)$$

$$= \frac{2 \cdot \pi \cdot r \cdot t}{(1 - \sigma) \cdot S_{11}} \cdot \beta (B1 \cdot \cos \beta \ell - B2 \cdot \sin \beta \ell) - \frac{\sqrt{2} \cdot d_{31}}{(1 - \sigma) \cdot S_{11}} \cdot 2 \cdot \pi \cdot r \cdot t \cdot E_z \quad (\text{A33})$$

$$= \frac{2 \cdot \pi \cdot r \cdot t}{(1 - \sigma) \cdot S_{11}} \cdot \beta \left(\frac{-1}{j\omega} \cdot \frac{v_2}{\sin \beta \ell} \cdot \cos \beta \ell \right) - \frac{\sqrt{2} \cdot d_{31}}{(1 - \sigma) \cdot S_{11}} \cdot 2 \cdot \pi \cdot r \cdot t \cdot E_z \quad (\text{A34})$$

$$= \frac{-1}{j\omega} \cdot \frac{2 \cdot \pi \cdot r \cdot t}{(1 - \sigma) \cdot S_{11}} \cdot \beta \frac{v_2}{\tan \beta \ell} - \frac{\sqrt{2} \cdot d_{31}}{(1 - \sigma) \cdot S_{11}} \cdot 2 \cdot \pi \cdot r \cdot t \cdot E_z \quad (\text{A35})$$

$$= \frac{Z_o}{j \tan \beta \ell} v - \frac{2 \cdot \sqrt{2} \cdot d_{31}}{(1 - \sigma) \cdot S_{11}} \cdot \frac{\pi \cdot r \cdot t}{t} V \quad (\text{A36})$$

where

$$Z_o = \frac{\beta \cdot 2\pi \cdot r \cdot t}{\omega \cdot (1 - \sigma) \cdot S_{11}} \approx \frac{\frac{\omega}{\sqrt{\frac{1}{2 \cdot \rho \cdot S_{11}^E \cdot (1 - \sigma)}}}}{\omega \cdot (1 - \sigma) \cdot S_{11}} \cdot 2\pi \cdot r \cdot t = 2 \cdot \pi \cdot r \cdot t \cdot \sqrt{\frac{2 \cdot \rho}{(1 - \sigma) \cdot S_{11}^E}} \quad (\text{A37})$$

Referring to the derivation process in previous works [A2, A3, A6, A7, A8], the five parameters of the physics-based equivalent circuit, shown in Figure A3, for the round-shaped transverse mode piezoelectric element can be derived as shown in Equations (A38), (A39) and (A42).

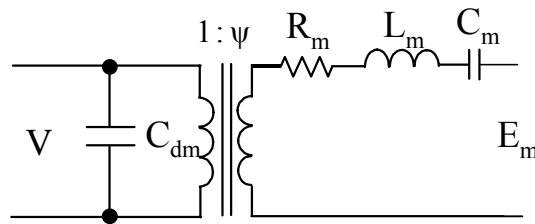


Figure A3. Physics-based equivalent circuit of transverse mode piezoelectric element.

$$R_m = \frac{\pi \cdot Z_o}{8 \cdot Q_m} = \frac{\pi^2 \cdot r \cdot t}{4 \cdot Q_m} \cdot \sqrt{\frac{2 \cdot \rho}{S_{11}^E \cdot (1 - \sigma)}} \quad (A38)$$

$$L_m = \frac{\pi \cdot Z_o}{8 \cdot \omega_o} = \frac{\pi \cdot r \cdot t}{8} \cdot \sqrt{\frac{2 \cdot \rho}{S_{11}^E \cdot (1 - \sigma)}} \cdot \frac{1}{f_o} = \frac{\pi \cdot r^2 \cdot t \cdot \rho}{2} \quad (A39)$$

where

$$\omega_o = 2 \cdot \pi \cdot f_o \quad (A40)$$

$$f_o = \frac{1}{2 \cdot r} \cdot \sqrt{\frac{1}{2 \cdot \rho \cdot S_{11}^E \cdot (1 - \sigma)}} \quad (A41)$$

$$C_m = \frac{1}{\omega_o^2 \cdot L_m} = \frac{4 \cdot (1 - \sigma) \cdot S_{11}^E}{\pi^3 \cdot t} \quad (A42)$$

From Equations (A27) and (A28), the turns ratio and input capacitor are as follows:

$$\psi = \frac{2 \cdot \sqrt{2} \cdot \pi \cdot r \cdot d_{31}}{(1 - \sigma) \cdot S_{11}^E} \text{ and} \quad (A43)$$

$$C_{dm} = \frac{\pi \cdot r^2}{t} \epsilon_{33}^T \left[1 - \frac{d_{31}^2}{(1 - \sigma) \cdot S_{11}^E \cdot \epsilon_{33}^T} \right] \quad (A44)$$

VITA

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In May 2000, he was employed by GE Lighting, Cleveland, Ohio, as an electronics design engineer working to design and develop cost-effective high-performance electronic ballasts for HID and fluorescent lamps.

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