A SIMPLIFIED MEANS OF PROVIDING FIRST ESTIMATES TO LAMINAR HEATING RATES ON ISOTHERMAL AXISYMMETRIC BLUNT BODIES

by

Michael Joseph Werle

Thesis submitted to the Graduate Faculty of the

Virginia Polytechnic Institute

in candidacy for the degree of

MASTER OF SCIENCE

in

Aerospace Engineering

August 1964

Blacksburg, Virginia

TABLE OF CONTENTS

		Page
I.	INTRODUCTION	8
II.	FORMULATION OF THE HEAT TRANSFER PARAMETER	11
	(a) Theoretical Considerations	11
	(b) A Quantitative Study of the Heat Transfer Parameter	14
III.	A SERIES EXPANSION OF THE HEAT TRANSFER PARAMETER ...	16
	(a) General Discussion	16
	(b) Higher Ordered Derivatives of ξ	16
	(c) Higher Ordered Derivatives of $\bar{u}$	18
	(d) Higher Ordered Derivatives of $\bar{q}$	19
	(e) Equation Summary	27
IV.	COMPUTATIONAL METHOD AND NUMERICAL EXAMPLES	30
	(a) General Computing Method	30
	(b) Heat Transfer Distribution on a Sphere	31
	(c) Heat Transfer Distribution on a Paraboloid of Revolution	33
	(d) Heat Transfer Distribution on a Hemisphere Cylinder	34
	(e) Heat Transfer Distribution on the Flat Nosed Cylinder of Reference 3	35
	(f) Heat Transfer Distribution on the Flat Nosed Cylinder of Reference 5	37
	(g) Heat Transfer Distribution on a Blunted Cone Cylinder	38
	(h) A Study of the Sensitivity of the Present Methods to Errors in the Input Constants	39

		Page
V.	CONCLUSIONS	43
VI.	ACKNOWLEDGMENTS	45
VII.	BIBLIOGRAPHY	46
VIII.	VITA	48

LIST OF FIGURES

Page

FIGURE 1. COORDINATE SYSTEMS 49

FIGURE 2. EMPIRICAL STUDY OF THE PARAMETER ξ 50

(a) Sphere 50

(b) Paraboloid of Revolution 51

(c) Hemisphere-Cylinder 52

(d) Flat-Nosed Cylinder 53

(e) Flat-Nosed Cylinder 54

(f) Blunted Cone Cylinder 55

FIGURE 3. INTERPOLATION CURVES FOR THE CONSTANTS

K_2, K_3, K_4 56

FIGURE 4. HEAT TRANSFER TO A SERIES OF BLUNT AXISYMMETRIC

BODIES 57

(a) Sphere - Method 1 57

(b) Sphere - Comparison of Methods 1 and 2 58

(c) Paraboloid of Revolution - Method 1 59

(d) Paraboloid of Revolution - Comparison of Methods 1 and 2 60

(e) Hemisphere-Cylinder - Method 1 61

(f) Flat-Nosed Cylinder - Method 1 62

(g) Flat-Nosed Cylinder - Method 1 63

(h) Blunted Cone Cylinder - Method 1 64

FIGURE 5. HEAT TRANSFER TO A PARABOLOID OF REVOLUTION EMPLOYING MODIFIED NEWTONIAN PRESSURE GRADIENTS 65

LIST OF SYMBOLS

C	mass concentration
F	diffusion function $\sum_i (Le_i - 1)\, h_i \frac{\partial C_i}{\partial h}$
H	total enthalpy
h	static enthalpy
K	see Eqs. (22) and (26)
K_1	constant of Eqs. (67) and (68), $K_1 = \frac{0.15}{1-t_w} \frac{\gamma-1}{\gamma}$
K_2, K_3, K_4	constants of Eq. (59) as determined from the data of Ref. 8
Le	Lewis number
M	Mach number
P, Q, $\mathcal{R}$, N	parameters in the correlation Eq. (47) as presented in Ref. 5
Pr	Prandtl number
p	static pressure
$\bar{p}$	nondimensional pressure, p/p_s
$\dot{q}_w$	heat transfer rate per unit area
$\bar{q}$	nondimensional heat transfer $\dot{q}_w/\dot{q}_{w_o}$
R	reference length
$\mathfrak{R}$	a nondimensional enthalpy function - see Eq. (37)
r	cross sectional radius of an axisymmetric body, see Fig. 1(a)
$\bar{r}$	nondimensional cross sectional radius, r/R

$\bar{\hbar}$	nondimensional normal distance from the longitudinal axis of a two-dimensional blunt body to the body surface - see Fig. 1(b)
s	surface distance from the stagnation point, see Fig. 1
$\bar{s}$	nondimensional surface distance, s/R
T	static temperature
t	enthalpy ratio, h/H, approximated as $t = \frac{T}{T_o}$
u	velocity of the fluid external to the boundary layer
u_∞	velocity of the fluid ahead of the bow shock
$\bar{u}$	nondimensional velocity, u/u_∞
α	recovery factor, here taken as $\alpha = 0.85$
β	pressure gradient parameter, see Eq. (31)
γ	specific heat ratio
η	$-\frac{1}{\gamma}$
θ	stagnation enthalpy profile, $\frac{H - H_e}{H_e - H_w}$, see Ref. 5
Γ	see Eq. (32)
μ	absolute viscosity
$\bar{\mu}$	nondimensional viscosity, μ/μ_{w_o}
ξ	three-dimensional heat transfer parameter, $\frac{\bar{r}}{\bar{p}}\frac{\bar{q}}{\bar{u}}\bar{u}_o$,
ξ_2	two-dimensional heat transfer parameter, $\frac{\bar{\hbar}}{\bar{p}}\frac{\bar{q}}{\bar{u}}\bar{u}_o$,
ρ	mass density of the fluid
$\bar{\rho}$	nondimensional mass density, ρ/ρ_s
$\bar{\omega}$	viscosity function, $\bar{\omega} = \frac{T_{w_o}}{T_w}\frac{\mu_w}{\mu_{w_o}}$

Superscripts

( )'	differentiation with respect to $\bar{s}$
iv,v, or vi	differentiation with respect to $\bar{s}$ for derivatives of order greater than three

Subscripts

aw	properties evaluated at adiabatic wall conditions
e	flow conditions external to the boundary layer
i	i^{th} species of a gas mixture
o	conditions evaluated at $\bar{s} = 0$
s	stagnation conditions aft of a normal shock
w	conditions evaluated at the wall

I. INTRODUCTION

While the subject of boundary layer heat transfer has been thoroughly exploited in the past two decades, recent developments in this field have prompted a critical review of the approximate methods now available for the study of this phenomenon. The developments referred to are the presentations of "exact" solutions to the boundary layer equations currently appearing in technical literature (for examples, see Refs. (1) and (2)).

One of the more thorough of these investigations is that developed by Davis and Flügge-Lotz (Ref. 2) at Stanford University. While said study is presently limited to the flow of a perfect gas about an axisymmetric body, these limitations are not at all essential, and the method is quite easily extended to the real gas regime. Since this method considers all second order boundary layer effects, it is, for all practical purposes, exact. The full Navier-Stokes equations are re-examined through an order-of-magnitude analysis, and an implicit finite difference scheme written for numerically integrating the resulting equations of motion. The method of computations is quite rapid and seems to enjoy a very wide range of application.

The point of interest here is that the prime difficulties surrounding this problem have been successfully overcome by Davis and Flügge-Lotz - resulting in a numerical scheme for the prediction of heating rates (among other quantities) for bodies of general interest.

If one now considers the more tried approximate schemes which are available for predicting laminar heating rates (Refs. (3), (4), and (5)), it is noted that, in general, these also require numerical analysis. Such a requirement stems from a somewhat universal approach to the approximate solution of Prandtl's boundary layer equations. Usually, one of the initial steps in any such analysis is to seek a transformation of the independent variables which will reduce the compressible equations of motion to the form of their incompressible counterparts. In general, this transformation is not of a one-to-one nature but requires differential relations between the two coordinate systems. Therefore, to return to the physical plane requires a reverse transformation which produces an integral relation for the heat transfer rate. In general, these integral relations cannot be expressed in terms of universal functions and thus the use of some numerical integration technique is required. It is seen, then, that while approximate schemes do provide explicit analytical expressions for the heating rate, they none the less imply the use of rather involved computational methods.

Comparing the "exact" and approximate solutions discussed above, one is tempted to conclude that for most applications little is to be gained in the use of any approximation when "exact" solutions are available. Such reasoning does of course subdue the difficulties normally associated with the numerical evaluation of nonlinear partial differential equations, but does serve to point up a very interesting fact. Namely, the engineer interested only in first estimates (or experimental data checks) logically must either relieve the approximate schemes of their integral relations or submit to an exact analysis of the problem at hand. The first alternative usually forces one to accept added approximations to the flow processes involved - which is somewhat analogous to the act of "patching a patch." The second alternative, although following logically, is much more involved than would be necessary for the purposes considered here.

In an effort to resolve this apparent dilemma then, the following study has been undertaken. The goal of this investigation has been to devise a simplified scheme for the evaluation of laminar boundary-layer heating rates. By use of the term "simplified" it is implied that the method devised should be free of any integral relations which would normally require the use of numerical integration techniques.

II. FORMULATION OF THE HEAT TRANSFER PARAMETER

(a) Theoretical Considerations:

As discussed in Section I, the basic goal of this investigation is to produce a heat transfer prediction scheme devoid of any integral relations. Obviously, a series expansion of the heating rate would serve this purpose and has, in fact, been considered by Lees (Ref. 4) for the case of a hemisphere cylinder.* In his analysis, Lees showed that through the use of isentropic gas relations and the modified Newtonian pressure coefficient, the basic heat transfer relation (see Figure 1(a) for clarification of nomenclature)

$$\bar{q} = \frac{\frac{1}{2}\,\bar{p}\,\bar{u}\,\bar{r}}{\left[\bar{u}_o{}' \int_o^{\bar{s}} \bar{p}\,\bar{u}\,\bar{r}^2\,d\bar{s}\right]^{1/2}} \tag{1}$$

may be written as

$$\bar{q} = 1 - \left[0.722 - \frac{0.667}{\gamma_\infty M_\infty^2}\right]\bar{s}^2 + \ldots \tag{2}$$

Since $\bar{q}$ usually experiences a very large variation in the nose region of a blunt body, one would expect that an excessive number of

* Another series representation - a Blasius type series expansion - shall be considered in detail in Section III (d).

terms of Eq. (2) would be required to yield consistently accurate estimates to $\bar{q}$ for other than small values of $\bar{s}$.

Thus while the use of a series expansion would, in principle, resolve the issue, its application would still be somewhat limited. Obviously a series representation would be of considerable value for a heat transfer parameter which did not experience the extreme variation of $\bar{q}$. A hint to the existence of such a parameter may be extracted from Eq. (1) by first noting that $\bar{p}$ and $\bar{r}^2$ are even in $\bar{s}$ while $\bar{u}$ may be considered as an odd function of $\bar{s}$. Thus one may write that

$$\bar{p} = 1 + \bar{p}_o'' \frac{\bar{s}^2}{2!} + \ldots \quad , \tag{3}$$

$$\bar{u} = \bar{u}_o' \, \bar{s} + \bar{u}_o''' \frac{\bar{s}^3}{3!} + \ldots \quad , \tag{4}$$

and

$$\bar{r}^2 = \bar{s}^2 + \bar{r}_o''' \frac{\bar{s}^4}{4!} + \ldots \quad , \tag{5}$$

so that to the first approximation

$$\int_o^{\bar{s}} \bar{p} \, \bar{u} \, \bar{r}^2 \, d\bar{s} = \bar{u}_o' \frac{\bar{r}^4}{4} \quad . \tag{6}$$

Thus Eq. (1) becomes

$$\bar{q} = \frac{\bar{p} \, \bar{u}}{\bar{r} \, \bar{u}_o'} \quad . \tag{7}$$

This result may be generalized to some extent by first defining

$$\xi \equiv \frac{\bar{q}\,\bar{r}}{\bar{p}\,\bar{u}}\,\bar{u}_o' \tag{8}$$

and then noting that according to Eq. (1)

$$\xi = \left[\frac{\dfrac{\bar{r}^4}{4}}{\displaystyle\int_o^{\bar{s}} \frac{\bar{p}\,\bar{u}\,\bar{r}^2}{\bar{u}_o'}\,d\bar{s}}\right]^{1/2} . \tag{9}$$

Again employing the concept of even and odd functions in $\bar{s}$, one may write that

$$\frac{\bar{r}^4}{4} = A_1 \frac{\bar{s}^4}{4!} + A_2 \frac{\bar{s}^6}{6!} + \ldots , \tag{10}$$

and

$$\frac{\bar{p}\,\bar{u}\,\bar{r}^2}{\bar{u}_o'} = B_1 \frac{\bar{s}^3}{3!} + B_2 \frac{\bar{s}^5}{5!} + \ldots , \tag{11}$$

then

$$\xi = \left[\frac{A_1 \dfrac{\bar{s}^4}{4!} + A_2 \dfrac{\bar{s}^6}{6!} + \ldots .}{B_1 \dfrac{\bar{s}^4}{4!} + B_2 \dfrac{\bar{s}^6}{6!} + \ldots .}\right]^{1/2} \tag{12}$$

Now, Eq. (12) is "exact" in the sense that no new approximations have been incorporated into Eq. (1). Thus, the similarity of the numerator and denominator of Eq. (12) would seem to imply the possibility that ξ, as determined from Lees' analysis, is only a weak function of $\bar{s}$ for the region of convergence of the series representation. The fact that the right-hand side of Eq. (12) is a radical would obviously strengthen this conclusion.*

Since the limited variation of ξ has, in truth, only been assumed - and that assumption based on a study of an approximate analysis - it is apparent that little more has been done thus far then to trace the evolution of a working hypothesis. An attempt to substantiate this hypothesis would next be in order.

(b) A Quantitative Study of the Heat Transfer Parameter.

Following naturally from the rather qualitative analysis of the preceding section, an effort is made here to give quantitative meaning to the fundamental hypothesis that ξ is only a weak function of $\bar{s}$.

---

* Note that for a two-dimensional blunt body, the factor $\bar{r}$ would not appear in Eq. (1), and subsequently not in ξ. This would, of course, force $\xi \rightarrow \infty$ as $\bar{s} \rightarrow o$. This problem may be resolved by introducing the two-dimensional counterpart of $\bar{r}$ into ξ to yield;

$$\xi_2 \equiv \frac{\bar{\hbar}\,\bar{q}}{\bar{p}\,\bar{u}}\,\bar{u}_o' = \left[\frac{\dfrac{\bar{\hbar}^2}{4}}{\displaystyle\int_o^{\bar{s}} \frac{\bar{p}\,\bar{u}}{\bar{u}_o'}\,d\bar{s}}\right]^{1/2}$$

which should provide the same weak dependence on $\bar{s}$ as noted for ξ.

After various attempts to accomplish this goal through an analytical approach proved futile, it became apparent that an empirical study would have to suffice.

Being aware that conclusions based on empirical results are only as general as the scope of the study conducted, an earnest effort has been made to cover a wide range of possible blunt, axisymmetric body configurations. For the sake of consistency, this study has been restricted to the case of isothermal surface temperatures.

Figure 2 presents the local variation of the parameter ξ for six shapes of general interest. When available (in the cited references of this figure) Lees' prediction of $\bar{q}$ has also been used to compute an ξ distribution. Quite obviously, from Figure 2, the presumed weak dependence of ξ on $\bar{s}$ seems well in line with the results obtained from Lees' method.

The experimental variation of ξ, while tending to be somewhat erratic, does generally display a relatively mild variation over the range of $\bar{s}$ values considered. It should be noted that for $\bar{s} < 1.5$, for all shapes considered, ξ does not vary appreciably from its first approximation; that is from $\xi = 1$. In view of this fact, then, a series expansion about the stagnation point for ξ should converge quite rapidly. Subsequently, only a minimum of terms of such a series would be required to yield good estimates to the true variation of ξ. Such a series representation is next considered in detail.

III. A SERIES EXPANSION OF THE HEAT TRANSFER PARAMETER

(a) General Discussion.

Here consideration is given to a Taylor series expansion of the form

$$\xi = \sum_{K=0}^{n} \frac{\bar{s}^K}{K!} \xi^{(K)}(0) + R_n \quad , \tag{13}$$

where $\xi^{(K)}$ is the K^{th}-ordered derivative of ξ and R_n is the remainder after n terms of the series.

It should be evident that since Eq. (13) requires continuity of ξ through its first n + 1 derivatives its use shall be considerably more restrictive than the simple requirement of axial symmetry and isothermal body surfaces. The fact that the continuity of ξ is quite easily violated may be shown through its relation to $\bar{r}$. Since the higher ordered derivatives of $\bar{r}$ are frequently discontinuous, it should be expected that the proposed series will have its radius of convergence reduced as more terms are added to Eq. (13). In turn, it is noted that aside from possible truncation errors, a single term expansion (ξ = 1) should be quite general and apply to a large number of shapes of practical interest. This statement seems well justified by the results of Figure 2 as discussed above.

(b) Higher Ordered Derivatives of ξ.

Again employing the concept of even and odd functions, as applied to the flow about blunt axisymmetric bodies, it is noted that $\bar{r}$ and $\bar{u}$

are in general odd functions of $\bar{s}$. Conversely, $\bar{p}$ and $\bar{q}$ may be considered as even in $\bar{s}$. It follows then that ξ is an even function of $\bar{s}$ so that Eq. (13) must be of the form

$$\xi = 1 + \frac{\bar{s}^2}{2} \xi_o'' + \frac{\bar{s}^4}{4!} \xi_o^{iv} \ldots + R_n \quad . \tag{14}$$

Now rewriting Eq. (8) as

$$\bar{p}\,\bar{u}\,\xi = \bar{u}_o'\,\bar{r}\,\bar{q} \quad , \tag{15}$$

and considering

$$(\bar{p}\,\bar{u}\,\xi)_o''' = \bar{u}_o'\,(\bar{r}\,\bar{q})_o'' \quad , \tag{16}$$

it follows that

$$\xi_o'' = \bar{q}_o'' + \frac{1}{3}\bar{r}_o''' - \frac{1}{3}\frac{\bar{u}_o'''}{\bar{u}_o'} - \bar{p}_o'' \quad . \tag{17}$$

In a similar manner one may compute

$$(\bar{p}\,\bar{u}\,\xi)_o^{v} = \bar{u}_o'\,(\bar{r}\,\bar{q})_o^{v} \tag{18}$$

to yield

$$\xi_o^{iv} = \bar{q}_o^{iv} + 2\,\bar{r}_o'''\,\bar{q}_o'' + \frac{1}{5}\bar{r}_o^{v} - \frac{1}{5}\frac{\bar{u}_o^{v}}{\bar{u}_o'} - 2\,\bar{p}_o''\,\frac{\bar{u}_o'''}{\bar{u}_o'}$$

$$- \bar{p}_o^{iv} - 2\,\xi_o''\left[\frac{\bar{u}_o'''}{\bar{u}_o} + 3\,\bar{p}_o''\right] \quad . \tag{19}$$

Convenient relations for the higher ordered derivatives of $\bar{q}$ and $\bar{u}$ are now considered.

(c) Higher Ordered Derivatives of $\bar{u}$.

Euler's equation for a steady inviscid flow field may be written as

$$\rho u \frac{du}{ds} = - \frac{dp}{ds} \tag{20}$$

and subsequently normalized to

$$\bar{\rho}\,\bar{u}\,\bar{u}' = - k\,\bar{p}' \quad , \tag{21}$$

where

$$K = \frac{\rho_s}{p_s} u_\infty^{\;2} \quad . \tag{22}$$

Since the isentropic forms of the gas relations are both familiar and convenient, they have been employed here. If need be, real gas effects may be accounted for through use of some effective specific heat ratio, γ.

Thus Eq. (21) may be written as

$$\bar{p}^{\eta}\,\bar{p}' = - K\,\bar{u}\,\bar{u}' \quad , \tag{23}$$

where

$$\eta = - \frac{1}{\gamma} \quad . \tag{24}$$

By considering

$$\lim_{\bar{s}\to o} \frac{\bar{p}^{\eta}\,\bar{p}'}{\bar{u}\,\bar{u}'} = \lim_{\bar{s}\to o} - K \quad , \tag{25}$$

it may be shown that

$$K = - \frac{\bar{p}_o''}{\bar{u}_o'^{\,2}} \quad . \tag{26}$$

Equation (23) now can be rewritten in a more convenient form as

$$\bar{p}^{\eta}\ \bar{p}' = \frac{\bar{p}_o''}{\bar{u}_o'^2}\ \bar{u}\ \bar{u}' \quad . \tag{27}$$

Considering the higher ordered derivatives of Eq. (27) allows one to write

$$\frac{\bar{u}_o'''}{\bar{u}_o'} = \frac{1}{4}\left[\frac{\bar{p}_o^{iv}}{\bar{p}_o''} + 3\ \eta\ \bar{p}_o'' \right] \tag{28}$$

and

$$\frac{\bar{u}_o^{v}}{\bar{u}_o'} = \frac{1}{6}\left[\frac{\bar{p}_o^{vi}}{\bar{p}_o''} + \frac{45}{4}\ \eta\ \bar{p}_o^{iv} + \frac{15}{8}\ \eta\ (5\eta\ -\ 8)(\bar{p}_o'')^2 - \frac{5}{8}\left(\frac{\bar{p}_o^{iv}}{\bar{p}_o''} \right)^2 \right] \quad . \tag{29}$$

(d) Higher Ordered Derivatives of $\bar{q}$.

For the computation of higher ordered derivatives of $\bar{q}$ two methods have been considered. The first method (designated as Method 1) is based on an accepted approximate solution, while the second method (Method 2) will be taken from an "exact" but restricted analysis. Method 2 only allows for the computation of second order derivative of ξ whereas Method 1 shares no such restriction.

Method 1:

To determine convenient relations for the higher order derivatives of $\bar{q}$ (in the stagnation region), it should be noted first that the stagnation point is a position at which true boundary layer similarity exists. Thus one might conclude that in a small region about $\bar{s} = 0$, local similarity results should very closely approximate the local variation of $\bar{q}$. This approximation is now assumed to be sufficiently accurate to provide good estimates of the derivatives of $\bar{q}$ at $\bar{s} = 0$.

One of the more thorough methods presently available in the literature, for computing $\bar{q}$ from local similarity concepts, is that presented by Beckwith and Cohen in Ref. 5.* This method combines local similarity results with the energy integral equation to approximate the dependence of the boundary layer parameters on the external flow field and the wall conditions.

From Beckwith's analysis one can write that

$$\bar{q} = \left[\frac{\left(\frac{\rho_w \mu_w}{\beta\, t_e} \right) \frac{du}{ds}}{(2\, \rho_{w_o}\, \mu_{w_o}) \left(\frac{du}{ds} \right)_o} \right]^{\frac{1}{2}} \frac{\frac{1 + F_w}{Pr_w}}{\left(\frac{1 + F_w}{Pr_w} \right)_0} \; \frac{t_{aw} - t_w}{(t_{aw} - t_w)_o} \; \frac{\theta_w{}'}{\theta_{w_o}{}'} , \tag{30}$$

where $\frac{\theta_w{}'}{\theta_{w_o}{}'}$ is a function of β and t_w, as given by the results of the similar solution analysis for $Pr = 1$ and $\frac{\rho\, \mu}{\rho_w\, \mu_w} = 1$.

---

* Lees' solution is a special case of the method discussed in Ref. 5.

The term β is determined by Beckwith and Cohen from an analysis combining local similarity with the energy integral equation to yield;

$$\beta = \frac{\frac{2}{u^2} \frac{1}{t_e} \frac{du}{ds}}{\rho_w \mu_w \bar{r}^2 \Gamma^2} \int_o^s \rho_w \mu_w u \bar{r}^2 \Gamma^2 ds \quad , \tag{31}$$

where

$$\Gamma = \frac{\frac{1 + F_w}{Pr_w}}{\left(\frac{1 + F_w}{Pr_w} \right)_o} \frac{1 - t_w}{(1 - t_w)_o} \frac{\theta_w'}{\theta_{w_o}'} \quad . \tag{32}$$

Rewriting Eq. (31) as

$$\left(\frac{\rho_w \mu_w}{\beta t_e} \frac{du}{ds} \right)^{1/2} = \frac{\rho_w \mu_w u \bar{r} \Gamma}{\left[2 \int_o^s \rho_w \mu_w u \bar{r}^2 \Gamma^2 ds \right]^{1/2}} \quad , \tag{33}$$

and noting that since

$$t_{aw_o} = \left(\frac{h_{aw}}{H_e} \right)_o = 1 \tag{34}$$

then

$$\frac{\frac{1 + F_w}{Pr_w}}{\left(\frac{1 + F_w}{Pr_w} \right)_o} \frac{t_{aw} - t_w}{(t_{aw} - t_w)_o} \frac{\theta_w'}{\theta_{w_o}'} = \Gamma \frac{t_{aw} - t_w}{1 - t_w} \quad , \tag{35}$$

so that Eq. (30) becomes

$$\bar{q} = \frac{\rho_w \mu_w u \bar{r} \Gamma^2 \Re}{\left[4 (\rho_w \mu_w)_o \left(\frac{du}{ds} \right)_o \int_o^s \rho_w \mu_w u \bar{r}^2 \Gamma^2 ds \right]^{1/2}} , \quad (36)$$

where

$$\Re \equiv \frac{t_{aw} - t_w}{1 - t_w} \quad (37)$$

Normalizing Eq. (36), then

$$\bar{q} = \frac{1}{2} \frac{1}{(\bar{u}_o{}')^{1/2}} \frac{\bar{\rho}_w \bar{\mu}_w \bar{u} \bar{r} \Gamma^2 \Re}{\left[\int_o^{\bar{s}} \bar{\rho}_w \bar{\mu}_w \bar{u} \bar{r}^2 \Gamma^2 d\bar{s} \right]^{1/2}} . \quad (38)$$

As was pointed out in Ref. 5, by considering wall temperatures less than 4400 ^{o}R, one is justified in the use of the thermally perfect gas relations in the vicinity of the wall. Accepting such a premise then;

$$\frac{\rho_w}{\rho_{w_o}} = \frac{p_w}{p_{w_o}} \frac{T_{w_o}}{T_w} \quad (39)$$

or

$$\bar{\rho}_w \bar{\mu}_w = \bar{p} \bar{\omega}_w , \quad (40)$$

where

$$\bar{\omega}_w \equiv \frac{T_{w_o}}{T_w} \frac{\mu_w}{\mu_{w_o}} . \quad (41)$$

For the case of an isothermal cold wall, Eq. (38) reduces to

$$\bar{q} = \frac{1}{2(\bar{u}_o{}')^{1/2}} \; \frac{\bar{p}\,\bar{u}\,\bar{r}\,\Gamma^2\,\Re}{\left[\int_o^{\bar{s}} \bar{u}\,\bar{p}\,\bar{r}^2\,\Gamma^2\,d\bar{s}\right]^{1/2}} \quad . \tag{42}$$

Computing $\bar{q}'$, and ridding the system of the integral by reintroducing Eq. (42), results in

$$\bar{u}\,\bar{p}\,\bar{r}\,\Gamma^2\,\Re^2\,\bar{q}' = \Re\,\bar{q}\left[\bar{u}\,\bar{p}\,\bar{r}\,\Gamma^2\,\Re\right]' - 2\,\bar{u}_o{}'\,\bar{q}^3\,\bar{r} \; . \tag{43}$$

Now taking the third derivative of Eq. (43), it may be shown that

$$\bar{q}_o'' = \frac{4}{3}\Gamma_o'' + \Re_o'' + \frac{2}{3}\bar{p}_o'' + \frac{2}{9}\frac{\bar{u}_o'''}{\bar{u}_o'} + \frac{1}{9}\bar{r}_o''' \quad . \tag{44}$$

In a similar manner, the fifth derivative of Eq. (43) may be computed to yield

$$\begin{aligned}
\bar{q}_o^{iv} = {} & \Re_o^{iv} + \frac{3}{2}\Re_o''\,\bar{q}_o'' + 15\,\Re_o''\Gamma_o'' + 3\,\Re_o''^2 + \frac{15}{2}\Re_o''\,\bar{p}_o'' \\
& + 3\,\bar{q}_o''\,\Gamma_o'' + \frac{3}{2}\bar{q}_o''\bar{p}_o'' + \frac{5}{2}\Re_o''\,\bar{r}_o''' + \frac{5}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\Re_o'' + \frac{1}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\bar{q}_o'' \\
& + \frac{3}{2}\Gamma_o^{iv} + \frac{9}{2}\Gamma_o''^2 + 9\,\Gamma_o''\,\bar{p}_o'' + \frac{3}{4}\bar{p}_o^{iv} + 3\,\bar{r}_o'''\,\Gamma_o'' + \frac{3}{2}\bar{r}_o'''\,\bar{p}_o'' \\
& + 3\,\frac{\bar{u}_o'''}{\bar{u}_o'}\Gamma_o'' + \frac{3}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\bar{p}_o'' + \frac{1}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\bar{r}_o''' + \frac{1}{10}\bar{r}_o^{v} \\
& + \frac{3}{20}\frac{\bar{u}_o^{v}}{\bar{u}_o'} - \frac{9}{2}\bar{q}_o''^2 - \bar{r}_o'''\,\bar{q}_o'' \quad .
\end{aligned} \tag{45}$$

In order to complete the analysis the higher ordered derivatives of Γ and $\Re$ must be evaluated.

For an isothermal wall, Γ simply becomes (see Eq. (32))

$$\Gamma = \frac{\theta_w'}{\theta_{w_o}'}$$

In Ref. 5, Beckwith and Cohen present a correlation for Eq. (46) which is valid over the entire range of β considered (0 to ∞) according to the relation

$$\frac{\theta_w'}{\theta_{w_o}'} = \frac{1 + P\,\beta^N}{Q + R\,\beta^N}\left[\frac{Q + R\beta^N}{1 + P\beta^N}\right]_o \frac{\theta_{w_{\beta=1}}'}{(\theta_{w_{\beta=1}}')_o} , \tag{47}$$

where P, Q, R and N are determined (by Beckwith and Cohen) as functions of t_w.

While Eq. (47) allows one to generate expressions for the higher ordered derivatives of Γ, the work involved may be greatly reduced by considering the case of $\Gamma = 1$. This essentially reduces Beckwith and Cohen's analysis to that of Lees' (Ref. 4); this is felt to be satisfactory for the exploratory study proposed here.

Accepting the implied restrictions, then

$$\Gamma_o'' = \Gamma_o^{iv} = 0 \quad . \tag{48}$$

Next considering the derivatives of $\Re$, it is noted (from Eq. (37)) that for an isothermal wall

$$\Re_o'' = \frac{t_{aw_o}''}{1 - t_{w_o}} \quad , \tag{49}$$

and

$$\Re_o^{iv} = \frac{t_{aw_o}^{iv}}{1 - t_{w_o}} \quad . \tag{50}$$

Since

$$t_{aw} = t_e + \alpha\,(1 - t_e) \quad , \tag{51}$$

and α is commonly considered a constant having a numerical value 0.85, Eqs. (49) and (50) become

$$\Re_o'' = \frac{1 - \alpha}{1 - t_{w_o}}\, t_{e_o}'' \quad , \tag{52}$$

and

$$\Re_o^{iv} = \frac{1 - \alpha}{1 - t_{w_o}}\, t_{e_o}^{iv} \tag{53}$$

respectively.

As in Section III(c), using the assumption of an effective specific heat ratio, one can write that

$$t_{e_o}'' = \frac{\gamma - 1}{\gamma}\, \bar{p}_o'' \quad , \tag{54}$$

and

$$t_{e_o}^{iv} = \frac{\gamma - 1}{\gamma}\, \bar{p}_o^{iv} - \frac{3}{\gamma}\left(\frac{\gamma - 1}{\gamma}\right) \bar{p}_o''^2 \quad , \tag{55}$$

thus

$$\Re_o'' = K_1\, \bar{p}_o'' \quad , \tag{56}$$

and

$$\Re_o^{iv} = K_1 \left[\bar{p}_o^{iv} - \frac{3}{\gamma} \bar{p}_o^{\prime\prime 2} \right] \quad , \tag{57}$$

where

$$K_1 = \frac{\gamma - 1}{\gamma} \; \frac{1 - \alpha}{1 - t_{w_o}} \quad ; \tag{58}$$

which completes the analysis for Method 1.

Method 2:

The second means of computing the higher ordered derivatives of $\bar{q}$ has been taken directly from the work of Davis and Flügge-Lotz (Ref. 8). In that paper, the authors develop an "exact" solution to the laminar boundary layer equations which includes second-order vorticity effects for blunt axisymmetric bodies. In so doing, a Blasius series representation of the flow parameters about $\bar{s} = 0$ is employed to obtain an "exact" solution to the equations of motion in the stagnation region.

Considering a perfect gas ($\gamma = 1.4$), a constant Prandtl number of 0.7, and a 1/2-power viscosity law, Davis and Flügge-Lotz are able to generalize the coefficients of the series representations so that they become only functions of the wall to stagnation temperature ratio at $\bar{s} = 0$. This dependence is not explicit and requires computer techniques to obtain the functional relationship. Values of these coefficients are presented in Ref. 8 for a range of t_{w_o} of 0.2 to 2.0.

Considering only first order effects, then, Ref. 8 allows one to write (for isothermal walls)

$$\bar{q}_o'' = K_2 \frac{\bar{p}_o^{iv}}{\bar{p}_o''} + K_3 \bar{r}_o''' + K_4 \bar{p}_o'' \quad , \tag{59}$$

where K_2, K_3, and K_4 are functions of t_{w_o} only.

Figure 3 presents these coefficients (K_2, K_3, and K_4) as determined from an interpolation of the data of Davis and Flügge-Lotz (Ref. 8). Also shown are values of the same coefficients as determined by Method 1; these were obtained by combining Eqs. (28), (44), (48), and (56), whereby

$$\bar{q}_o'' = \frac{1}{18} \frac{\bar{p}_o^{iv}}{p_o''} + \frac{1}{9} \bar{r}_o''' + \left[K_1 + \frac{2}{3} - \frac{1}{6\gamma} \right] \bar{p}_o'' \quad . \tag{60}$$

It would seem that the second order terms of Method 1 provide for a fair degree of accuracy for very cold walls but lose accuracy as t_w increase. This is to be expected since the restrictions employed in the derivation of Method 1 are essentially for the cold wall case.

(e) Equation Summary.

Thus far it has been shown that

$$\xi_o'' = \bar{q}_o'' + \frac{1}{3} \bar{r}_o''' - \frac{1}{3} \frac{\bar{u}_o'''}{\bar{u}_o'} - \bar{p}_o'' \quad , \tag{61}$$

$$\xi_o^{iv} = \bar{q}_o^{iv} + 2 \bar{r}_o''' \bar{q}_o'' + \frac{1}{5} \bar{r}_o^{v} - \frac{1}{5} \frac{\bar{u}_o^{v}}{\bar{u}_o'} - 2 \bar{p}_o'' \frac{\bar{u}_o'''}{\bar{u}_o'}$$

$$- \bar{p}_o^{iv} - 2 \xi_o'' \left[\frac{\bar{u}_o'''}{\bar{u}_o'} + 3 \bar{p}_o'' \right] , \tag{62}$$

$$\frac{\bar{u}_o'''}{\bar{u}_o'} = \frac{1}{4}\left[\frac{\bar{p}_o^{iv}}{\bar{p}_o''} + 3\,\eta\,\bar{p}_o''\right] \quad , \tag{63}$$

and

$$\frac{\bar{u}_o^{v}}{\bar{u}_o'} = \frac{1}{6}\left[\frac{\bar{p}_o^{vi}}{\bar{p}_o''} + \frac{45}{4}\eta\,\bar{p}_o^{iv} + \frac{15}{8}\eta(5\eta-8)\,\bar{p}_o''^{2} - \frac{5}{8}\left(\frac{\bar{p}_o^{iv}}{\bar{p}_o''}\right)^2\right] . \tag{64}$$

Also, according to Method 1:

$$\bar{q}_o'' = \frac{1}{18}\,\frac{\bar{p}_o^{iv}}{\bar{p}_o''} + \frac{1}{9}\bar{r}_o''' + \left[K_1 + \frac{2}{3} - \frac{1}{6\gamma}\right]\bar{p}_o'' \tag{65}$$

$$\begin{aligned}
\bar{q}_o^{iv} = {} & \Re_o^{iv} + \frac{3}{2}\Re_o''\,\bar{q}_o'' + 3\,\Re_o''^{2} + \frac{15}{2}\Re_o''\,\bar{p}_o'' \\
& + \frac{3}{2}\bar{q}_o''\,\bar{p}_o'' + \frac{5}{2}\Re_o''\,\bar{r}_o''' + \frac{5}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\Re_o'' \\
& + \frac{1}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\bar{q}_o'' + \frac{3}{4}\bar{p}_o^{iv} + \frac{3}{2}\bar{r}_o'''\,\bar{p}_o'' + \frac{3}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\bar{p}_o'' \\
& + \frac{1}{2}\frac{\bar{u}_o'''}{\bar{u}_o'}\bar{r}_o''' + \frac{1}{10}\bar{r}_o^{v} + \frac{3}{20}\frac{\bar{u}_o^{v}}{\bar{u}_o'} - \frac{9}{2}\bar{q}_o''^{2} \\
& - \bar{r}_o'''\,\bar{q}_o'' \quad ,
\end{aligned} \tag{66}$$

$$\Re_o'' = K_1\,\bar{p}_o'' \quad , \tag{67}$$

and

$$\Re_o^{iv} = K_1\left[\bar{p}_o^{iv} - \frac{3}{\gamma}\bar{p}_o''^{2}\right] . \tag{68}$$

Now, for Method 2;

$$\bar{q}_o'' = K_2 \frac{\bar{p}_o^{iv}}{\bar{p}_o''} + K_3 \bar{r}_o''' + K_4 \bar{p}_o'' \tag{69}$$

Combining Eqs. (61)-(68) it may be shown that for Method 1;

$$\xi_o'' = \bar{p}_o'' \left[K_1 - \frac{1}{3} + \frac{1}{12\gamma} \right] + \frac{4}{9} \bar{r}_o''' - \frac{1}{36} \frac{\bar{p}_o^{iv}}{\bar{p}_o''} \tag{70}$$

and

$$\xi_o^{iv} = \bar{p}_o^{iv} \left[\frac{5}{6} K_1 + \frac{5}{96} \frac{1}{\gamma} - \frac{5}{24} \right] + \bar{p}_o''^2 \left[1 - 2 K_1 - \frac{0.25+2.5 K_1}{\gamma} \right.$$

$$\left. - \frac{1}{64\gamma^2} \right] + \bar{r}_o''' \left[\bar{p}_o'' \left(\frac{8}{3} K_1 + \frac{1}{4\gamma} - 1 \right) - \frac{1}{12} \frac{\bar{p}_o^{iv}}{\bar{p}_o''} \right]$$

$$+ \frac{1}{18} \bar{r}_o'''^2 + \frac{3}{10} \bar{r}_o''' + \frac{7}{576} \left(\frac{\bar{p}_o^{iv}}{\bar{p}_o''} \right)^2 - \frac{1}{120} \frac{\bar{p}_o^{iv}}{\bar{p}_o''} . \tag{71}$$

Similarly, using Eq. (69) it may be shown that for Method 2;

$$\xi_o'' = \left[K_4 + \frac{1}{4\gamma} - 1 \right] \bar{p}_o'' + \left[\frac{1}{3} + K_3 \right] \bar{r}_o'''$$

$$+ \left[K_2 - \frac{1}{12} \right] \frac{\bar{p}_o^{iv}}{\bar{p}_o''} . \tag{72}$$

IV. COMPUTATIONAL METHOD AND NUMERICAL EXAMPLES

(a) General Computing Method.

To maintain the generality of the computational scheme, expressions for the higher ordered derivatives of $\bar{p}$ and $\bar{r}$ have not been considered. Evaluation of these terms is quite easily effected and shall be left to the specific examples considered.

Assuming that values of ξ have been obtained, it remains to present general relations for the heating rate, $\bar{q}$. One of the basic assumptions for the present method is that the pressure distribution over the body of interest is known. Thus, the final relations for $\bar{q}$ will be set down in terms of $\bar{p}$ and appropriate constants.

Making use of the definition of ξ (Eq. (8)) then

$$\bar{q} = \frac{\bar{p}\,\bar{u}\,\xi}{\bar{r}\,\bar{u}_o{}'} \quad . \tag{73}$$

Now, employing the perfect gas relations, the energy equation may be written as

$$u_\infty \bar{u} = \sqrt{2\,H_o}\left[1 - \bar{p}^{\frac{\gamma-1}{\gamma}}\right]^{1/2} \tag{74}$$

so that Eq. (73) becomes

$$\bar{q} = \frac{\xi\bar{p}}{\bar{r}}\left(1 - \bar{p}^{\frac{\gamma-1}{\gamma}}\right)^{1/2} \frac{\sqrt{2\,H_o}}{\bar{u}_o{}'\,u_\infty} \quad . \tag{75}$$

Approximating the term $\sqrt{2\,H_o}$ according to

$$\sqrt{2\,H_o} = \sqrt{\frac{2\gamma}{\gamma-1}\,\frac{p_s}{\rho_s}} \quad , \tag{76}$$

and noting that on combining Eqs. (22) and (26)

$$\sqrt{\frac{p_s}{\rho_s}} = \frac{u_\infty\,\bar{u}_o{}'}{\sqrt{-\,\bar{p}_o{}''}} \tag{77}$$

then

$$\sqrt{2\,H_o} = \frac{u_\infty\,\bar{u}_o{}'}{\sqrt{-\bar{p}_o{}''}}\,\sqrt{\frac{2\gamma}{\gamma-1}} \quad , \tag{78}$$

thus Eq. (75) becomes

$$\bar{q} = \xi\,\frac{\bar{p}}{\bar{r}}\left[1 - \bar{p}^{\frac{\gamma-1}{\gamma}}\right]^{1/2}\sqrt{\frac{-2\gamma}{(\gamma-1)\,\bar{p}_o{}''}} \quad . \tag{79}$$

It is seen that once $\bar{p}_o{}''$ has been determined, the heat transfer ratio can be readily evaluated from Eq. (79).

(b) Heat Transfer on a Sphere.

In Ref. 2, Davis and Flügge-Lotz apply their first and second-order analysis of the boundary layer equations to a sphere at a Mach number of 10 in a perfect gas. These results are shown in Figures 4(a) and 4(b) along with the results obtained from the present methods. It should be noted that the data employed in the present computations were the exact computer outputs obtained in Ref. 2; these have been graciously supplied by Dr. Davis. These results not only eliminate the uncertainties of experimental data, but their use also reduces the errors incurred when one extracts data from the usual graphical presentations of published works.

Also made available by Dr. Davis were highly accurate values of the higher ordered pressure derivatives at the stagnation point. As discussed in Sec. III(d), Davis and Flügge-Lotz (Ref. 2) employed a series representation to determine the flow parameters from their Blasius type series solution in the vicinity of the stagnation point. These results provide the following:

$$\bar{p}_o'' = -2.406 \quad , \qquad (80)$$

$$\frac{\bar{p}_o^{iv}}{\bar{p}_o''} = -6.3441 \quad , \qquad (81)$$

and

$$\frac{\bar{p}_o^{vi}}{\bar{p}_o''} = 61.6500 \qquad (82)$$

so that according to Method 1;

$$\xi = 1 + 0.06639\,\bar{s}^2 - 0.015629\,\bar{s}^4 \quad , \qquad (83)$$

and from Method 2;

$$\xi = 1 + 0.1138\,\bar{s}^2 \quad . \qquad (84)$$

As would be expected from a comparison of the coefficients of Eqs. (83) and (84), Figures 4(a) and 4(b) show very little difference in the results obtained from Methods 1 and 2. It is apparent that, for the case at hand, use of the simpler method - the first approximation - seems well justified for the entire range of $\bar{s}$. Reviewing the higher ordered approximations, the results of Method 2 seem to produce nearly exact correlation with the exact results of Ref. (2). for $\bar{s} < 1$. Note that a series expansion of $\bar{q}$ would yield

$$\bar{q} = 1 - 0.972\,\bar{s}^2 \quad , \tag{85}$$

which loses accuracy quite rapidly for $\bar{s} > 0.5$.

(c) <u>Heat Transfer Distribution on a Paraboloid of Revolution</u>.

Davis and Flügge-Lotz (Ref. 2) also presented data for a paraboloid of revolution at a Mach number of infinity in a perfect gas. Arbitrarily choosing the case of $t_w = 0.2$, the heat transfer distribution has again been obtained using the exact computer output of Ref. 2 as input to the present methods.

Figure 4(c) presents the variation of $\bar{q}$ as given by Method 1. It is noted that although the second and third approximations do correct for the failure of the first approximation to yield valid results, they seemingly do so only for $\bar{s} \leq 1.2$. Above $\bar{s} = 1.2$ the third approximation is seen to experience a divergent characteristic which is felt to be due to truncation errors. That is, the correction to the first approximation for $\bar{s}$ small becomes predominant as $\bar{s}$ takes on large values.

Comparing the higher approximations of Methods 1 and 2, Figure 4(d) shows the same results as noted above. That is, even though the second approximation of Method 2 does produce accurate estimates of $\bar{q}$ for $\bar{s} \leq 1$, it none the less shows a divergent characteristic for large $\bar{s}$.

(d) Heat Transfer Distribution on a Hemisphere Cylinder.

In Ref. 3, Kemp, Rose, and Detra present experimental data for a hemisphere cylinder obtained from a shock tube study. The test conditions corresponded to an equivalent flight velocity of 18,000 ft/sec at 70,000 feet of altitude. The wall to stagnation temperature ratio is stated as approximately 0.05. Since the pressure distribution taken in that study did not extend into the stagnation region, the authors imply the use of a modified Newtonian distribution of the form

$$\bar{p} = 1 - 0.8913 \sin^2 \bar{s} \tag{86}$$

for use in the stagnation region.

Thus from Eq. (86) one obtains

$$\bar{p}_o'' = - 1.7826 \quad , \tag{87}$$

$$\frac{\bar{p}_o^{iv}}{\bar{p}_o''} = - 4 \quad , \tag{88}$$

and

$$\frac{\bar{p}_o^{\,vi}}{\bar{p}_o^{\,''}} = 16 \quad . \tag{89}$$

Figure 4(e) presents the experimental results obtained in Ref. 3 along with the approximations of the present Method 1 when using the above data. As a comparison, Lees' method, as presented in Ref. 3 for this body, is also shown in the figure. Note that while any of the results of Method 1 would be satisfactory for first estimates, the higher ordered approximations are consistently in error over the entire range of $\bar{s}$. This same behaviour was noted for the higher ordered approximation of Method 2 (Method 2 is not shown since it produced results within 1 to 2 percent of the results of Method 1). This result seems due to the inability to extract accurate data from the figures of Ref. 3.

While Lees' method does definitely yield better results than the present method, it is also obvious that for this shape, little is lost by use of the first approximation instead.

(e) Heat Transfer Distribution on the Flat Nosed Cylinder of Reference 3.

Figure 4(f) presents the experimental data of Ref. 3 for a flat nosed cylinder with stagnation point conditions corresponding to a speed of 14,000 fps at 80,000 feet of altitude. The specific heat ratio has been taken as $\gamma = 1.195$ and $t_w \approx 0$.

As suggested by Solomon (Ref. 7), the pressure distribution on the forward portion of this body is approximated by

$$\bar{p} = 1 - 0.1849 \sin^2 \bar{s} \quad . \qquad (90)$$

Following the same procedure as in the previous sections, the heat transfer distribution, corresponding to the present Methods 1 and 2, have been computed and are also shown in the figure.* It is noted that the experimental heat transfer is 20 to 30 percent higher than that predicted (here) in the corner region. This same type of deficiency was pointed out in Ref. 3 for the local similarity results. However, there the discrepancy was more adverse than in the present case. In fact, in the corner region the present first approximation is almost equivalent to the local similarity heating rates.

It should be noted that the higher ordered approximations of the present methods are not valid for $\bar{s} > 0.75$, even though they appear to attain a peak value at about $\bar{s} = 0.83$. This limitation is due to the discontinuous nature of the higher order derivatives of $\bar{r}$ at the corner junction. Regardless of this fact, it is seen that the higher ordered approximations do continue to produce better results than the previously discussed approximate scheme, right up to the peak value. Thus, while $\bar{s} = 0.75$ is surely the rigorous limit of the present method, a practical limit for a shape of this type might be taken as the

---

* Again, since the results of Method 2 were essentially the same as those of Method 1 for the higher ordered approximation, a direct comparison is not shown.

position of the peak heating rate. Thereafter one would resort to the first approximation, which continues to be valid over the entire body.

(f) Heat Transfer Distribution on the Flat-Nosed Cylinder of Reference 5.

Here consideration is given to the flat-nosed cylinder depicted in Figure 4(g). Note the very small radius of curvature at the corner as compared to the case discussed in Section IV(d).

Once again only the results for Method 1 are shown since the higher ordered approximations of the two present methods were essentially the same.

In reviewing the results of the present method, as presented in Figure 4(g), one should note that the suggested velocity gradient of Ref. 5 has not been employed here. In Ref. 5, Beckwith and Cohen encountered difficulty in determining the stagnation point velocity gradient due to the erratic behaviour of the velocity distribution in the vicinity of $\bar{s} = 0$ (the velocity distribution having been determined from an experimental pressure distribution).*

---

* This difficulty is quite reasonable since, for $\gamma = 1.4$,

$$\bar{u} \propto \left[1 - \bar{p}^{0.29} \right]^{1/2}$$

and hence for $\bar{p} \approx 1$, $\bar{u}$ is quite sensitive to error.

The authors (Ref. 5) therefore made use of a velocity gradient determined experimentally for a similar body but at a different Mach number. However, it was found here that by plotting the pressure distribution, as given by Beckwith and Cohen, in the form - $\bar{p}$ versus $\sin^2 \bar{s}$ - a linear relationship was apparent for a reasonable region about $\bar{s} = 0$. From such a plot the slope has been determined, so that to a reasonable degree of accuracy

$$\bar{p} = 1 - 0.102 \sin^2 \bar{s} \quad , \qquad (90)$$

and hence

$$\bar{p}_o'' = - 0.204 \quad . \qquad (91)$$

The results obtained by using the derivative of Eq. (91) in the present Method 1 are also shown in Figure 4(g).* Note that while the second and third approximations are not valid for $\bar{s} > 0.91$, they do seem to predict the position and, very nearly, the value of the peak heating rate. This, of course, is a definite advantage over Lees' method and is surpassed only slightly by the results of Ref. 5.

(g) Heat Transfer Distribution on a Blunted-Cone-Cylinder.

A consideration of the heat transfer distribution over a blunted-cone-cylinder configuration provides a critical test of the present

* Figure 4(g) presents Lees method in lieu of Beckwith and Cohen's only for the sake of consistency. While the method of Ref. 5 is more accurate in the corner region, it is considerably more difficult to apply.

method. In the previous cases $\bar{s}$ has not attained values larger than 2, here values of $\bar{s}$ up to 15 must be considered.

Figure 4(h) presents a faired curve for the experimental heat transfer distribution, as obtained from Ref. 6. Also shown are the results of the present Method 1 (again Method 2 has not been shown so as to avoid redundancy). Note that the first approximation, valid over the entire body, does seem to adequately predict the experimental heat transfer except in the cone and corner region. At present there does not seem to be any way to correct for this deficiency; it is simply accepted as a limitation of the present methods. For "better than order-of-magnitude" results though, the first approximation would appear to be quite sufficient.

(h) A Study of the Sensitivity of the Present Methods to Errors in the Input Constants.

Quite obviously, from a study of Eqs. (70)-(72) and Eq. (79), the input parameters which may introduce errors are the pressure derivatives and the effective specific heat ratio, γ. Since close practical limits exist on the available values of γ, the investigation here will be directed toward the effect of variations in the stagnation point pressure derivatives.

A generally accepted means of expressing the pressure distribution in the forward region of blunt bodies is in the form of a modified Newtonian pressure coefficient. This method was employed in the previous examples when more accurate means were not available. It should be noted that use of the Newtonian approximation is usually only required to obtain estimates of the ratios, $\bar{p}_o^{iv}/\bar{p}_o''$ and $\bar{p}_o^{vi}/\bar{p}_o''$. The value of $\bar{p}_o''$ may often be obtained by simply plotting the pressure distribution in an appropriate manner (say $\bar{p}$ versus $\sin^2 \bar{s}$) so as to obtain a linear variation of $\bar{p}(\bar{s})$.

Since very accurate values of the pressure derivative ratios were available in the data of Ref. 2, an examination of the use of the Newtonian approximation has been made for the cases of a sphere and a paraboloid.*

For the case of the sphere previously considered, Davis and Flügge-Lotz (Ref. 2) had available the ratios

$$\frac{\bar{p}_o^{iv}}{\bar{p}_o''} = -6.3441 \quad , \tag{92}$$

and

$$\frac{\bar{p}_o^{vi}}{\bar{p}_o''} = 61.6500 \quad . \tag{93}$$

Equations (92) and (93) should be compared with the values obtained by the Newtonian approximations;

---

* The theoretical pressure distributions employed in Ref. 2 were made available to Davis and Flügge-Lotz by H. Lomax of the Ames Research Center of NASA.

$$\frac{\bar{p}_o^{iv}}{\bar{p}_o''} = -4 \quad , \tag{94}$$

and

$$\frac{\bar{p}_o^{vi}}{\bar{p}_o''} = 16 \quad . \tag{95}$$

Even though there is a large difference in these two results, the heat transfer distributions, generated from the respective ratios, differed so little that there was no significant change in the results of Figures 4(a) and 4(b).

While these results would seem to imply that the present methods are insensitive to errors in the stagnation point pressure derivatives, the following results obtained for the case of a paraboloid of revolution do not support this as a general conclusion.

From the data of Davis and Flügge-Lotz (Ref. 2), it was found that for the paraboloid previously considered,

$$\frac{\bar{p}_o^{iv}}{\bar{p}_o''} = -11.5075 \quad , \tag{96}$$

and

$$\frac{\bar{p}_o^{vi}}{\bar{p}_o''} = 259.4 \quad , \tag{97}$$

while from the Newtonian approximation

$$\frac{\bar{p}_o^{iv}}{\bar{p}_o''} = -16 \quad , \tag{98}$$

and

$$\frac{\bar{p}_o^{\,vi}}{\bar{p}_o^{\,''}} = 688 \quad . \tag{99}$$

Based on a percentage error these results are about as inaccurate as those presented for the sphere. Thus one would expect that this difference would have as little effect here as it did for the heat transfer distribution over a sphere. While for the second approximation this does prove to be generally true, Figure 5 shows that the third approximation (of Method 1) experiences a gross divergence from the true variation of $\bar{q}$ when employing the results given in Eqs. (98) and (99).

Since there seems to be no definite cause for this behaviour of the present method's third approximation, and since the second approximation (as seen in Figure 5) is as accurate as the third, it would appear that a restriction on the present method is required. Thus it is suggested that the third approximation be dropped completely unless accurate values of the pressure derivative ratios are available.

V. CONCLUSIONS

In view of the results obtained in articles IV (b) - (h), the following general conclusions have been formulated:

(1) The present scheme developed for predicting first estimates of the heat transfer distribution, on isothermal axisymmetric blunt bodies, yields satisfactory results so long as the series representation of ξ is not violated. Such a violation would be the extension of the method into regions not within the radius of convergence of the series employed to represents ξ.

(2) For order-of-magnitude purposes, the present first approximation yields excellent results for all cases considered. Since this approximation may be applied to a large number of shapes of general interest and is extremely simple to apply, it is felt to be of interest for future studies in this area.

(3) The sensitivity of the higher ordered approximations to the choice of input data has been demonstrated in article IV (h). These results indicate that the input accuracy becomes progressively more critical as the order of approximation is increased. To avoid this difficulty, it is suggested that one consider only approximations through the second order term unless very accurate values of the input constants are available. This rule of thumb

represents a compromise situation, but is one which is definitely applicable to the cases considered here.

(4) The choice of either Method 1 or 2 would seem to be irrelevant as indicated by the results obtained in Section IV. Since Method 2 is based on a more exacting study than that of Method 1, it is suggested that Method 2 be taken as generally more valid.

VI. ACKNOWLEDGMENTS

The author wishes to express his sincere gratitude to for his instructional guidance during the entire course of this study. display of endless patience and understanding has had a direct and significant bearing on the completion of this work.

The author also is indebted to for his continued encouragement and enlightening suggestions which have added greatly to this paper.

For his assistance in the securing and clarification of the experimental heat transfer data of Section IV (g), the author wishes to express his thanks to of ARO, Inc. of Tullahoma, Tennessee.

Also, for his enlightening comments and continued assistance, the author wishes to express his appreciation to of V.P.I. The work presented here has been influenced directly by the many stimulating discussions held with in the past few months.

To , the author wishes to give full credit and thanks for her extreme care in the preparing of this manuscript. Her patient and skillful handling of the somewhat complex notation and many arduous equations included here has proved to be welcomed enhancement to this final copy.

VII. BIBLIOGRAPHY

1. Clutter, D. W. and Smith, A. M. O.: "Solution of the General Boundary-Layer Equations for Compressible Laminar Flow, including Transverse Curvature," Douglas Aircraft Division, Report No. LB 31088, February 1963.

2. Davis, R. T. and Flügge-Lotz: "Laminar Compressible Flow Past Axisymmetric Blunt Bodies (Results of a Second-Order Theory)," Stanford University Division of Engineering Mechanics, Technical Report No. 143, December 1963.

3. Kemp, N. H., Rose, P. H., and Detra, R. W.: "Laminar Heat Transfer Around Blunt Bodies in Dissociated Air," Jour. Aero. Sciences, Vol. 26, No. 7, pp. 421-430, July 1959.

4. Lees, L.: "Laminar Heat Transfer Over Blunt-Nosed Bodies at Hypersonic Flight Speeds," Jet Propulsion, Vol. 26, No. 4, pp. 259-269, April 1956.

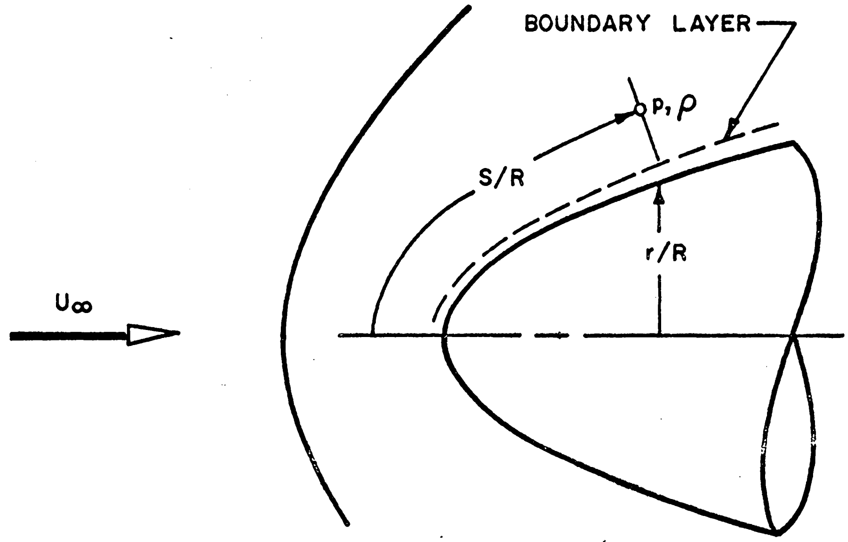
5. Beckwith, I. E. and Cohen, N. B.: "Application of Similar Solutions to Calculation of Laminar Heat Transfer on Bodies with Yaw and Large Pressure Gradient in High-Speed Flow," NASA TN-D-625, January 1961.

6. Knox, E. C.: Personal communication to ARO, Inc., Tullahoma, Tennessee.

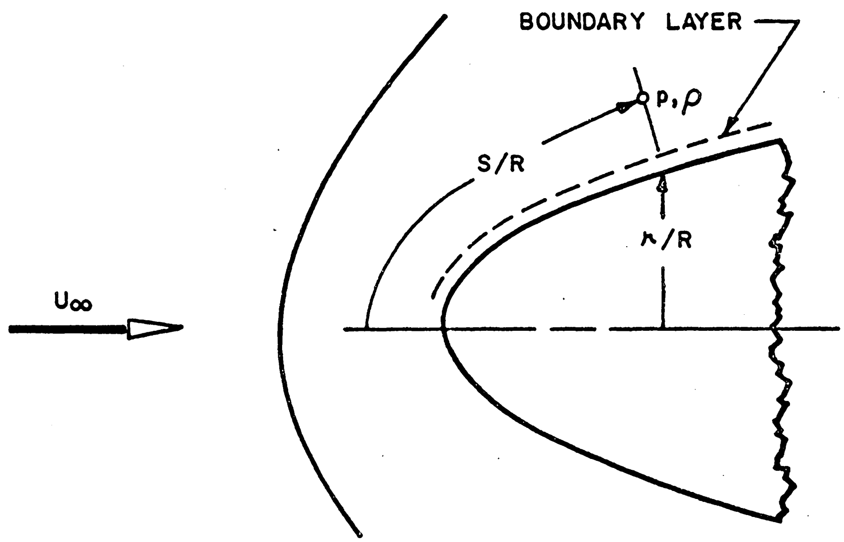
7. Solomon, J. M.: "The Calculation of Laminar Boundary Layers in Equilibrium Dissociated Air by an Extension of the Cohen and Reshotko Method," NOLTR 61-143, April 1962.

8. Davis, R. T. and Flügge-Lotz, I.: "The Laminar Compressible Boundary-Layer in the Stagnation Region of an Axisymmetric Blunt Body Including the Second-Order Effect of the Vorticity Interaction," Int. Jour. Heat and Mass Transfer, Vol. 7, pp. 341-370, Pergamon Press, 1964.

The vita has been removed from
the scanned document



(a) THREE DIMENSIONAL BODY



(b) TWO DIMENSIONAL BODY

FIGURE 1. COORDINATE SYSTEMS

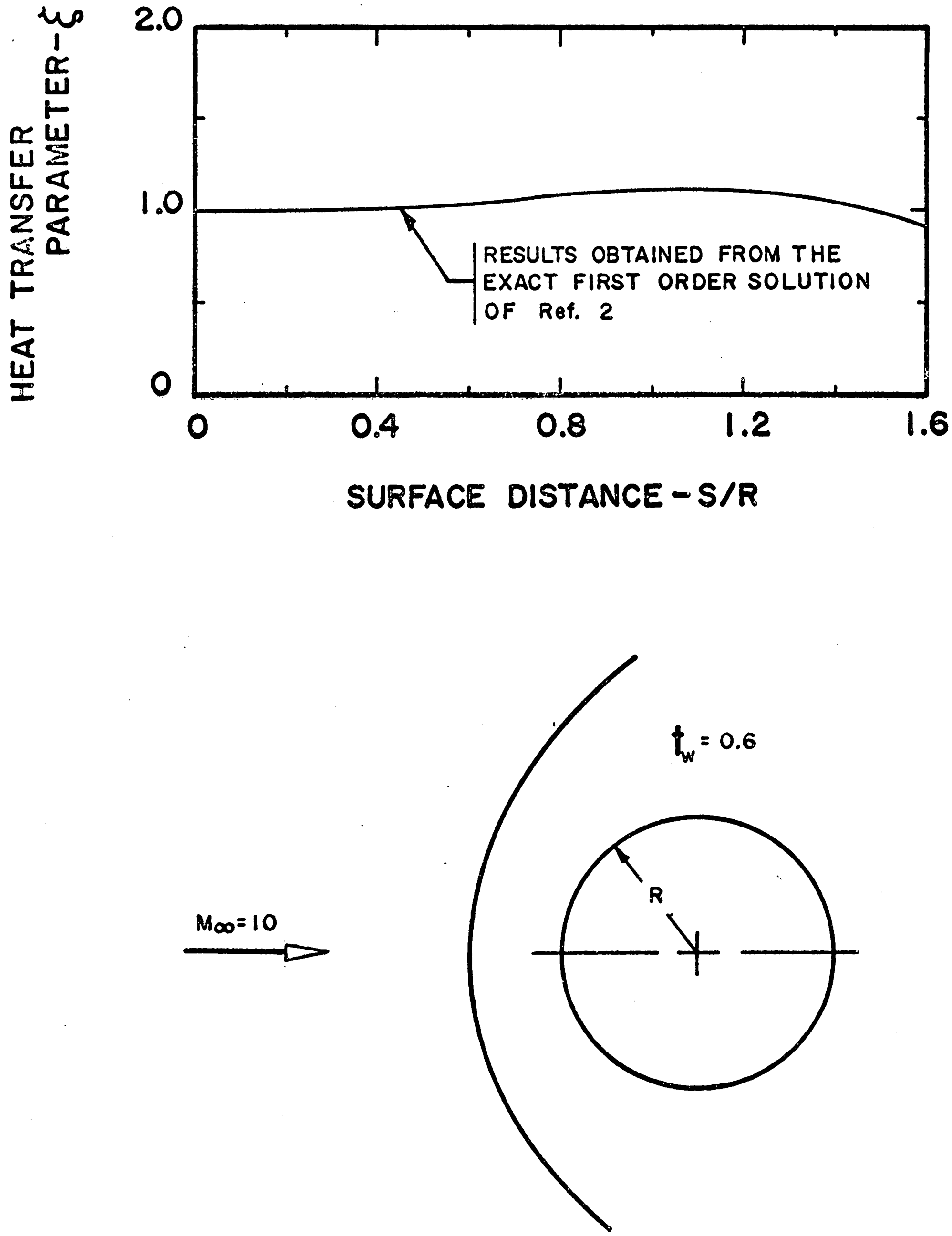
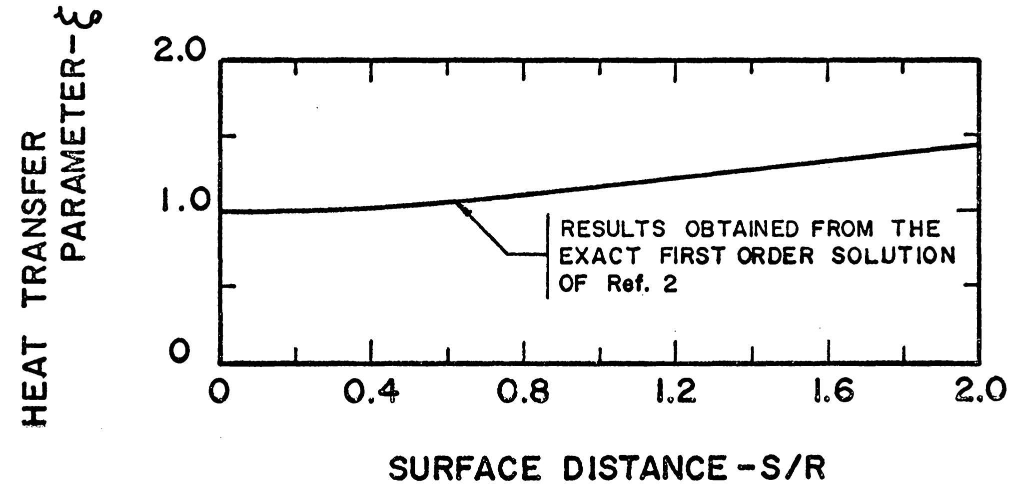


FIGURE 2. EMPERICAL STUDY OF THE HEAT TRANSFER PARAMETER-ξ

(a) SPHERE



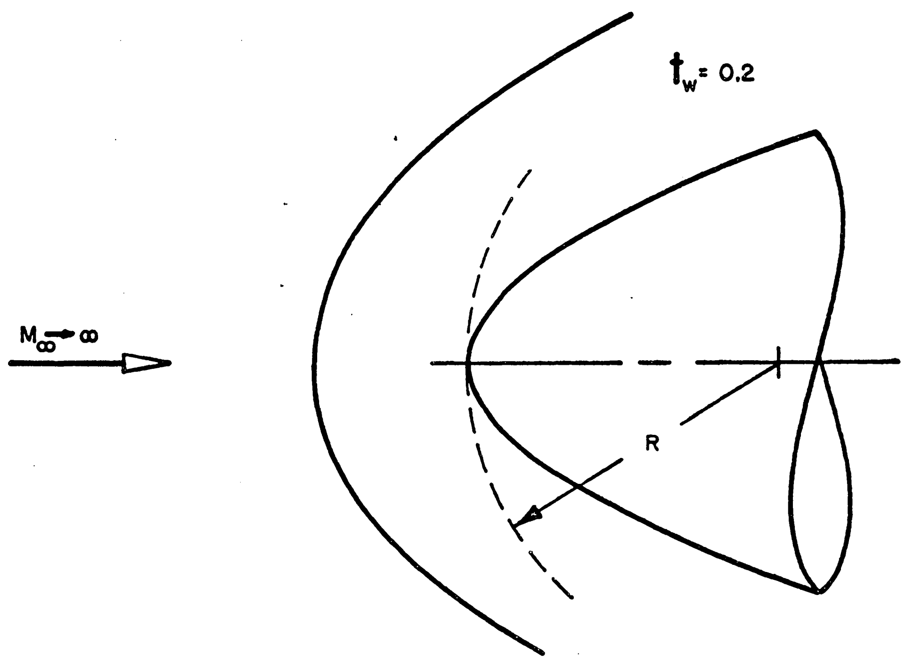
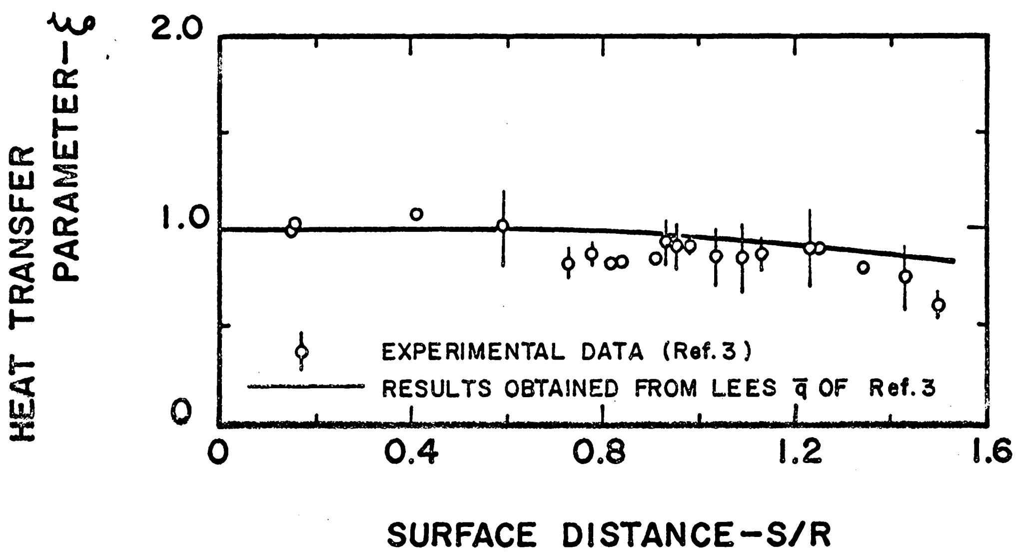


FIGURE 2. CONTINUED

(b) PARABOLOID OF REVOLUTION



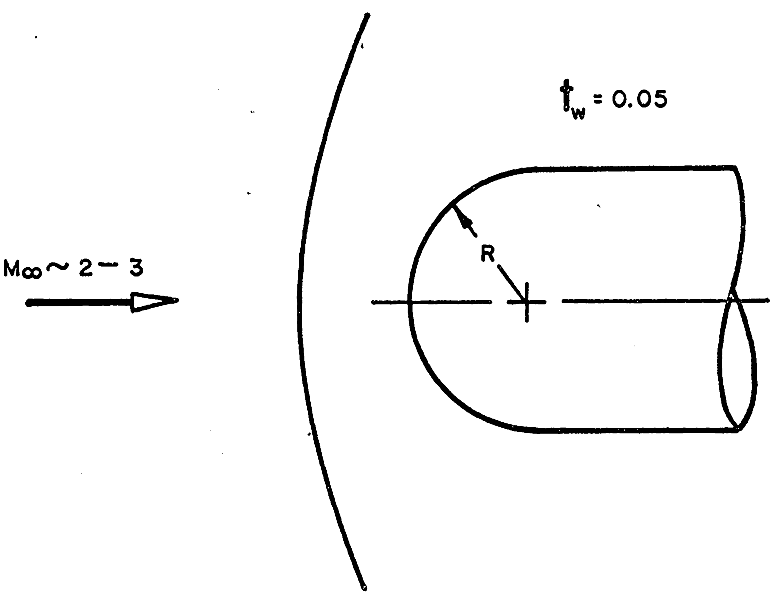
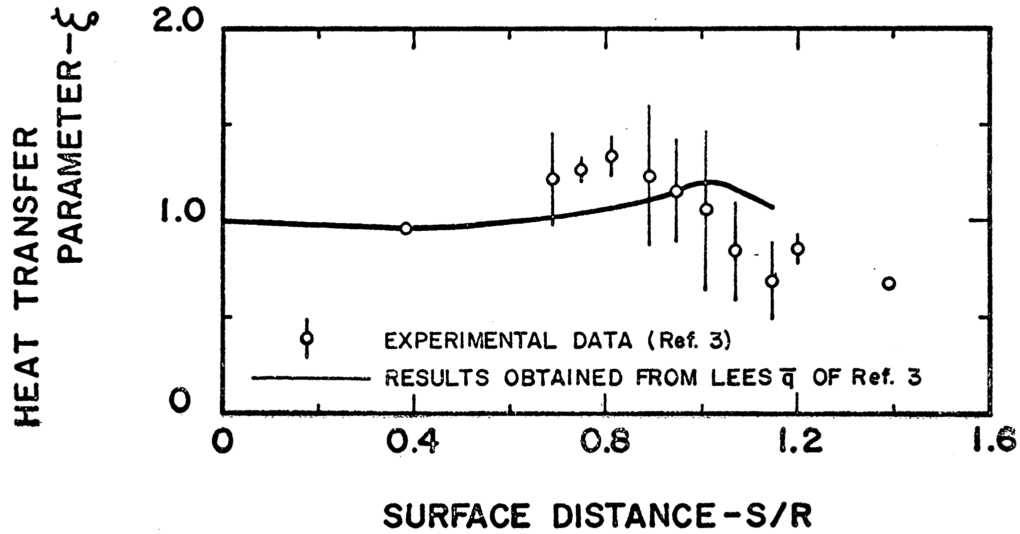


FIGURE 2. CONTINUED
(c) HEMISPHERE CYLINDER



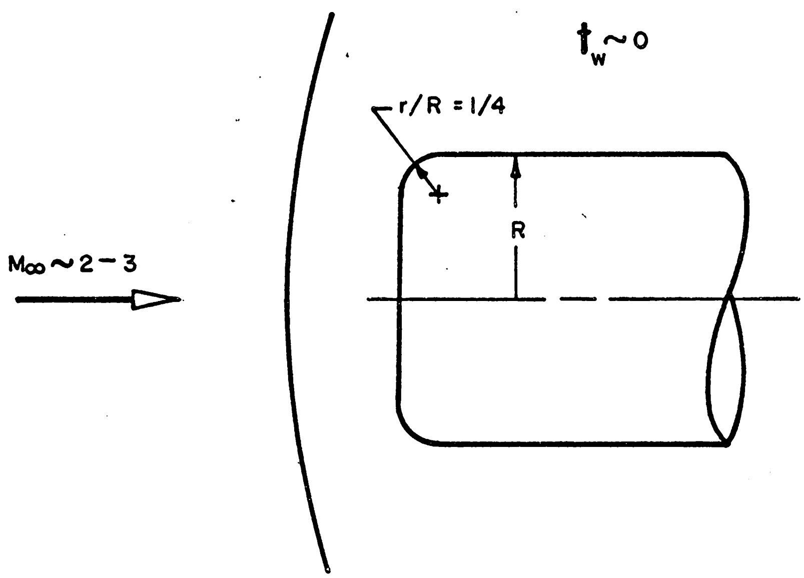
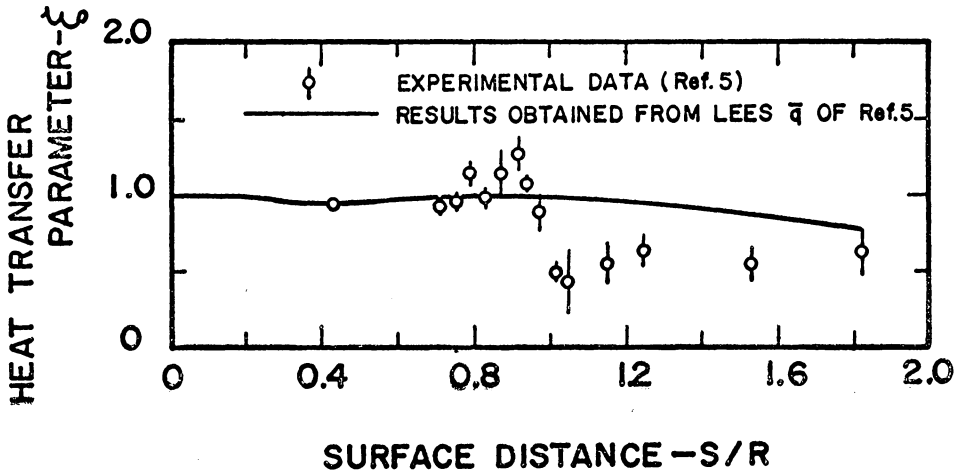


FIGURE 2. CONTINUED
(d) FLAT NOSED CYLINDER



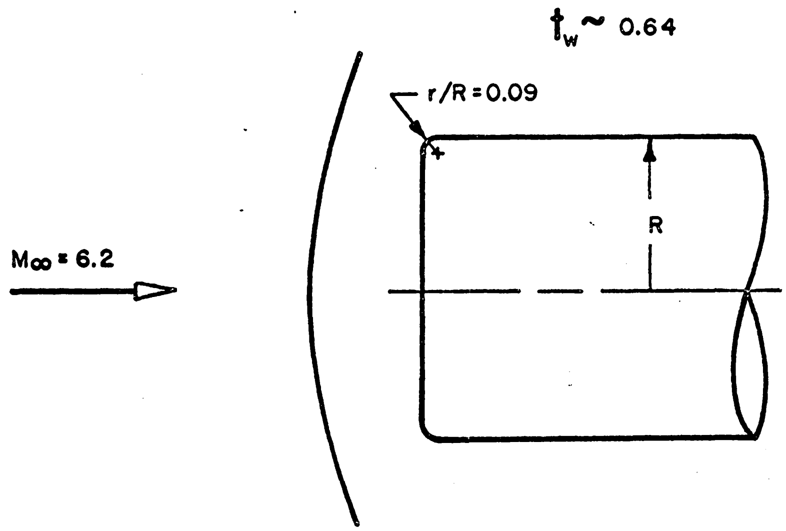
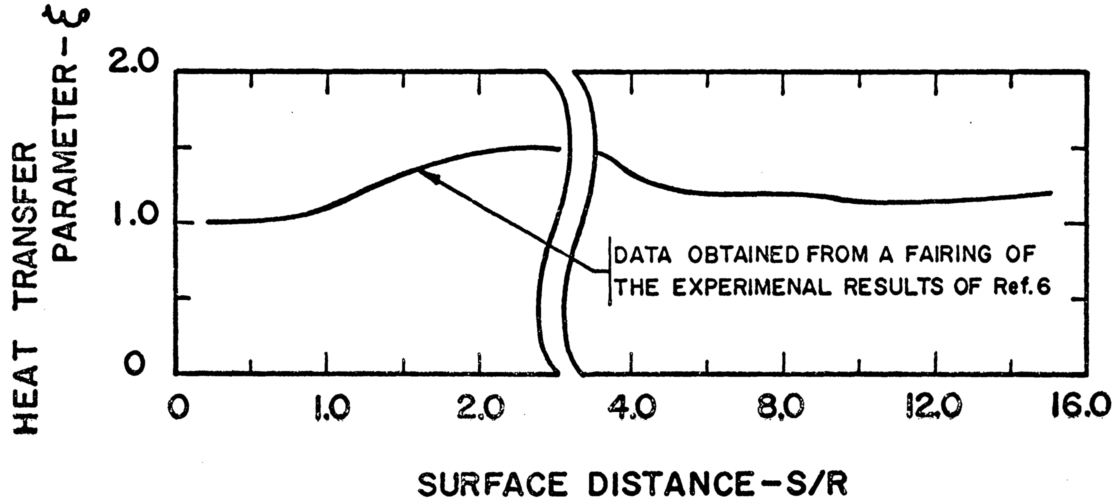


FIGURE 2. CONTINUED
(e) FLAT NOSED CYLINDER



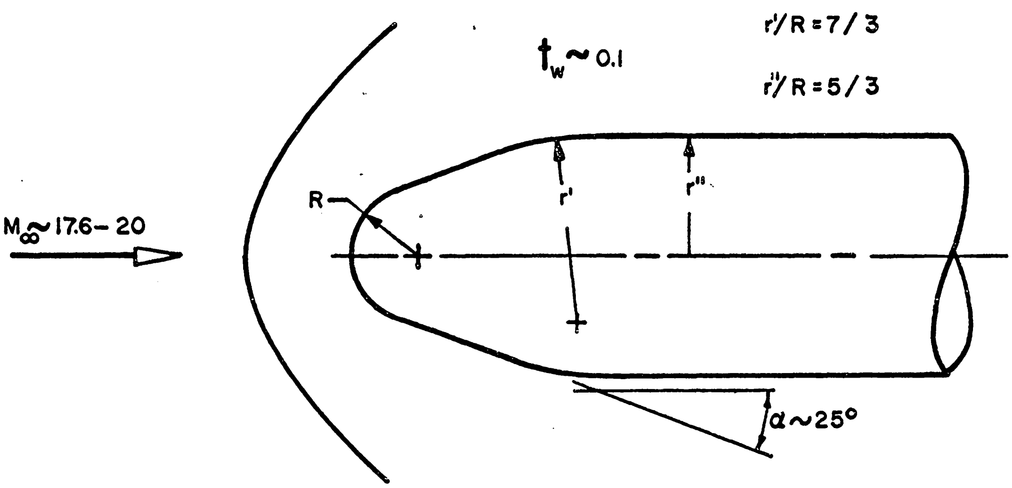


FIGURE 2. CONCLUDED
(f) BLUNTED CONE CYLINDER

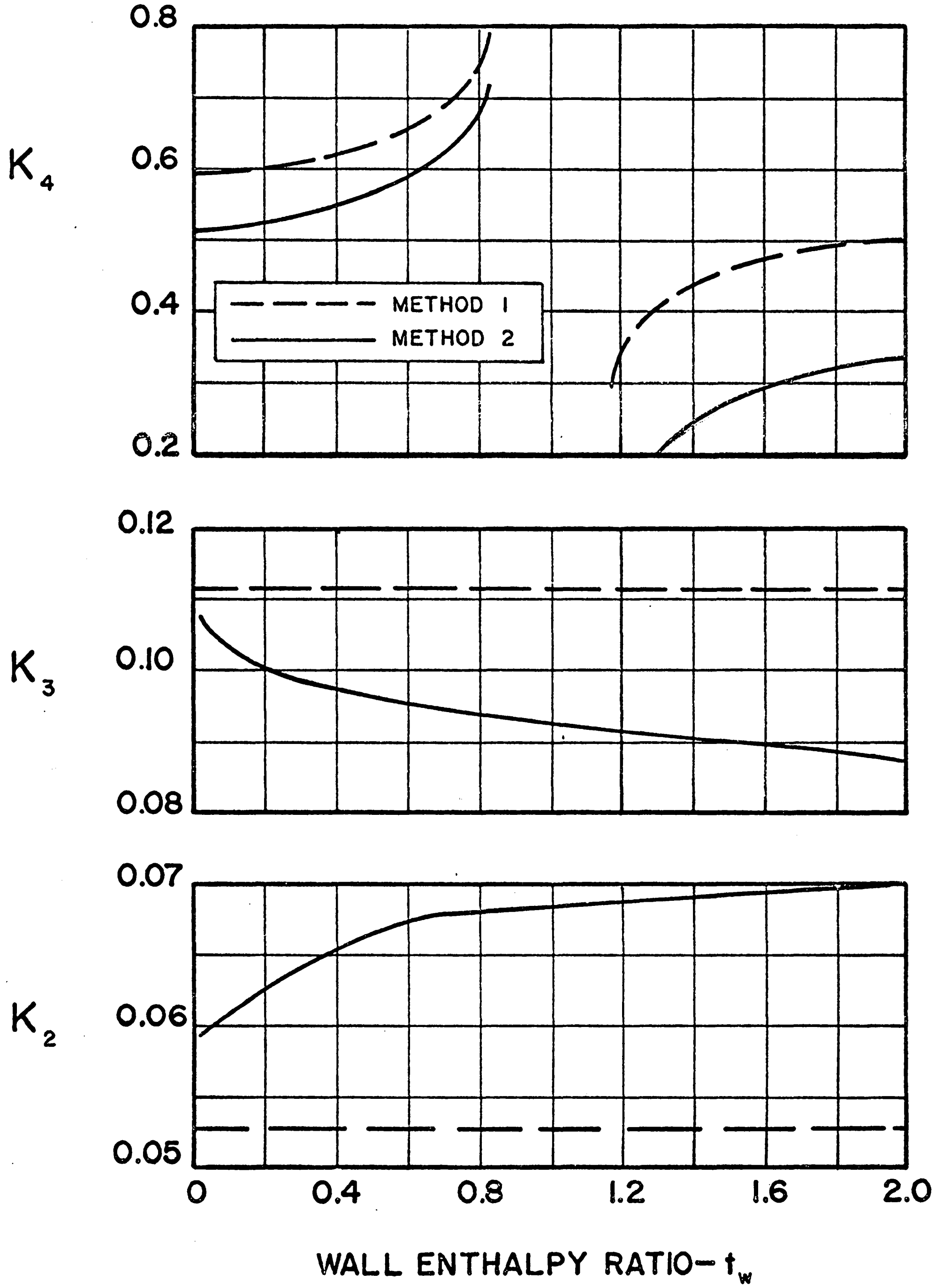


FIGURE 3. INTERPOLATION CURVES FOR THE CONSTANTS K_2, K_3, and K_4

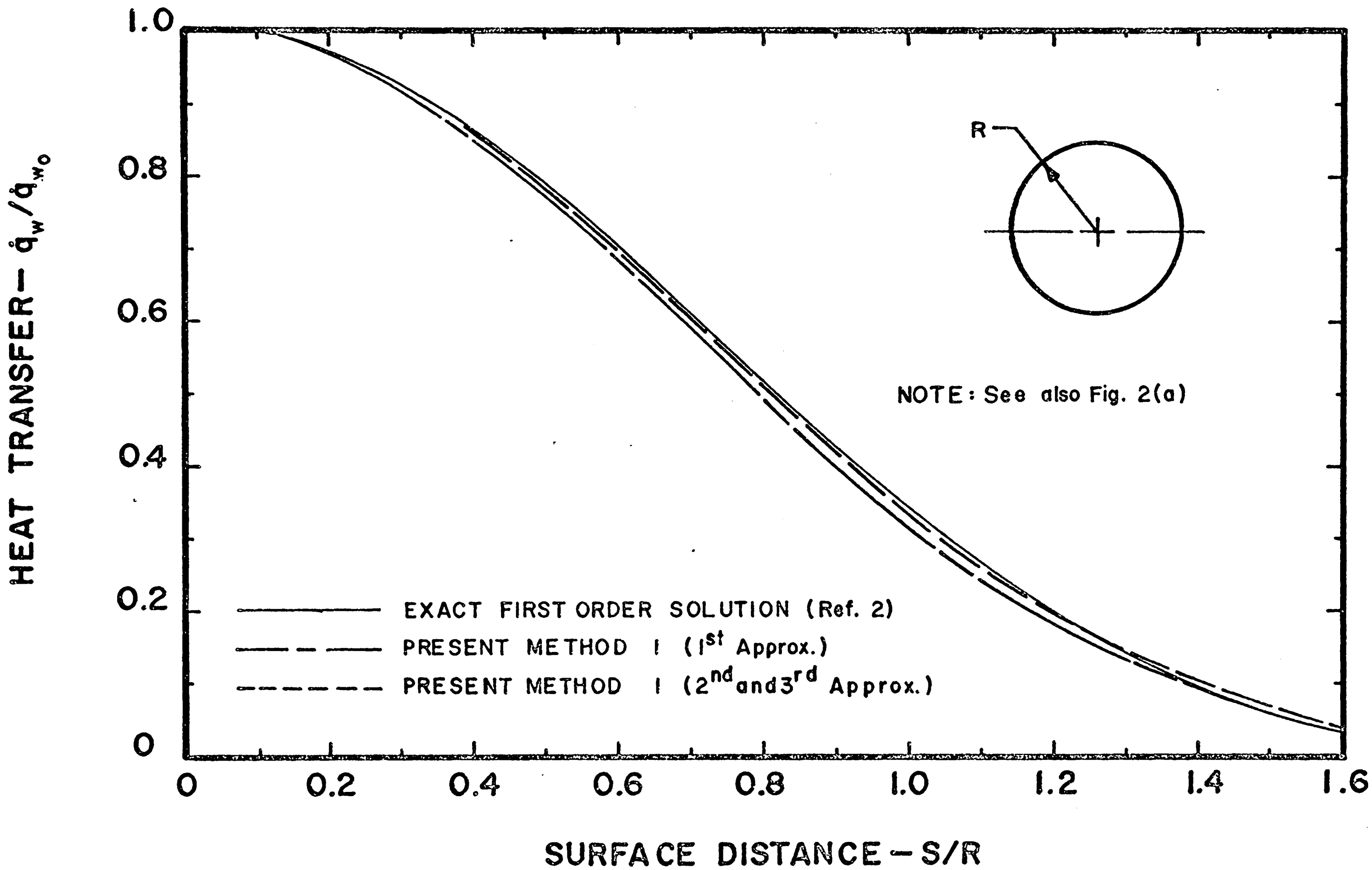


FIGURE 4. HEAT TRANSFER TO A SERIES OF BLUNT AXISYMMETRIC BODIES

(a) SPHERE — METHOD I

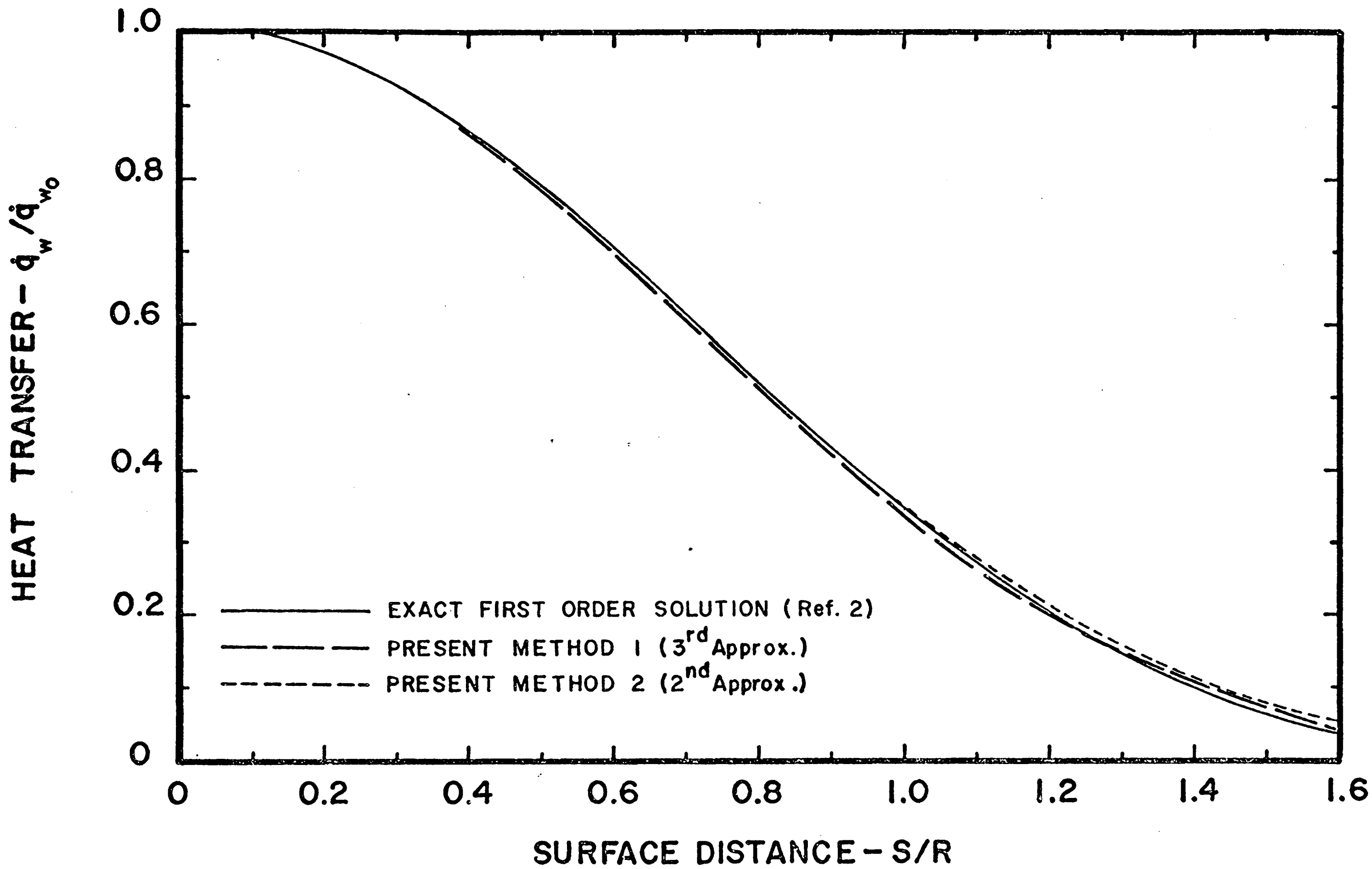


FIGURE 4. CONTINUED

(b) SPHERE-COMPARISON OF METHODS 1 and 2

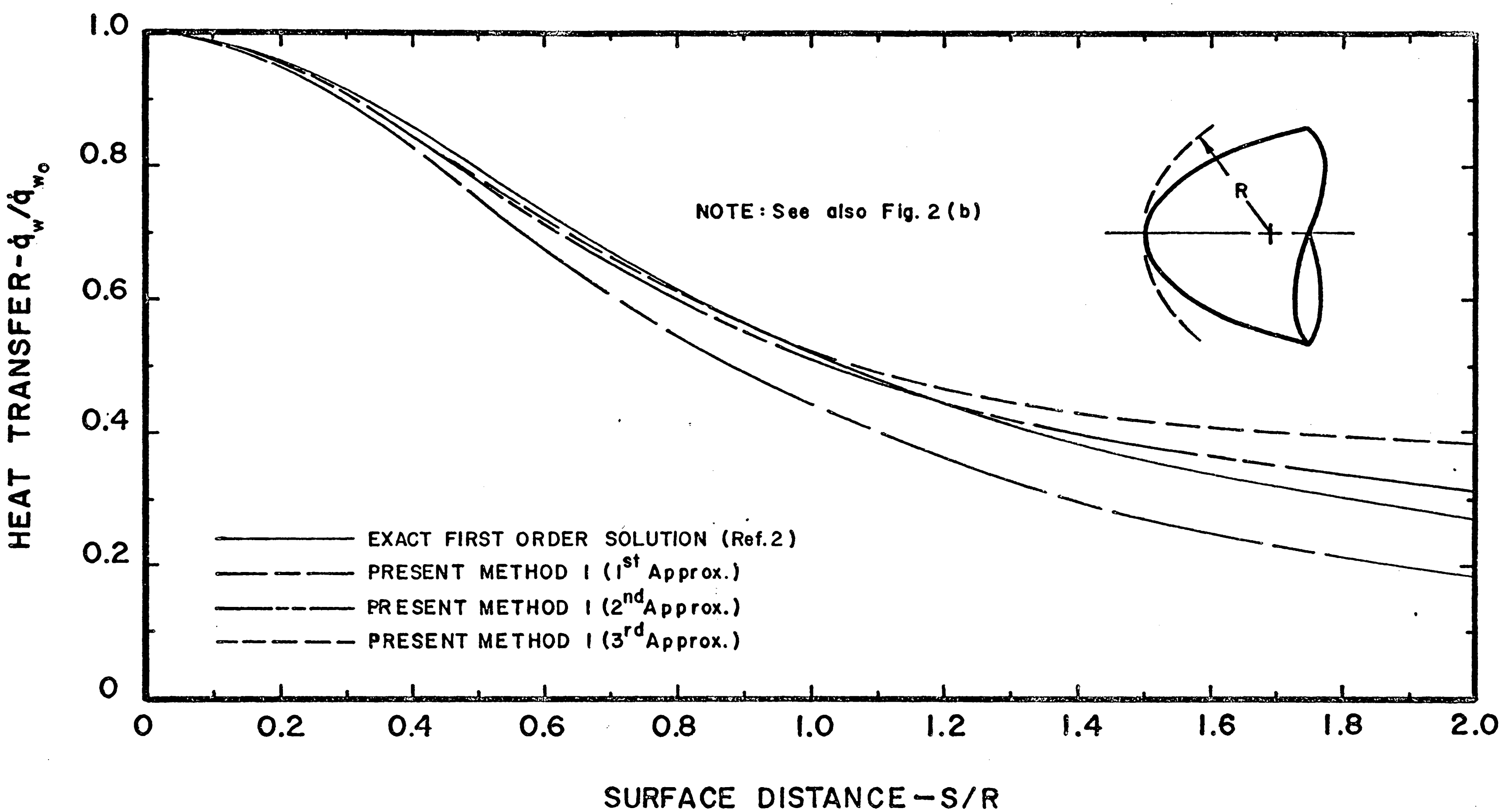


FIGURE 4. CONTINUED

(c) PARABOLOID OF REVOLUTION-METHOD I

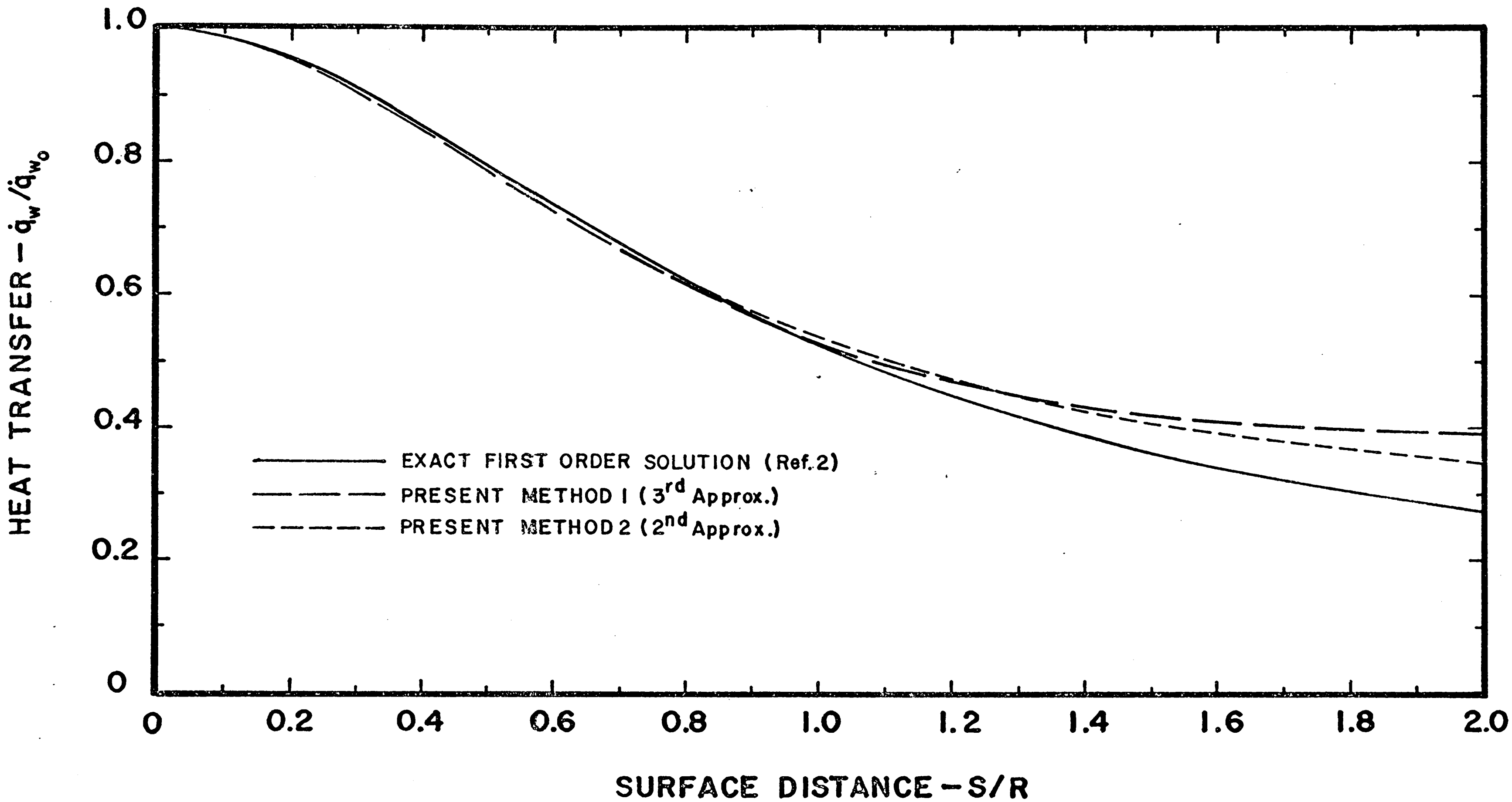


FIGURE 4. CONTINUED

(d) PARABOLOID OF REVOLUTION-COMPARISON OF METHOD 1 and 2

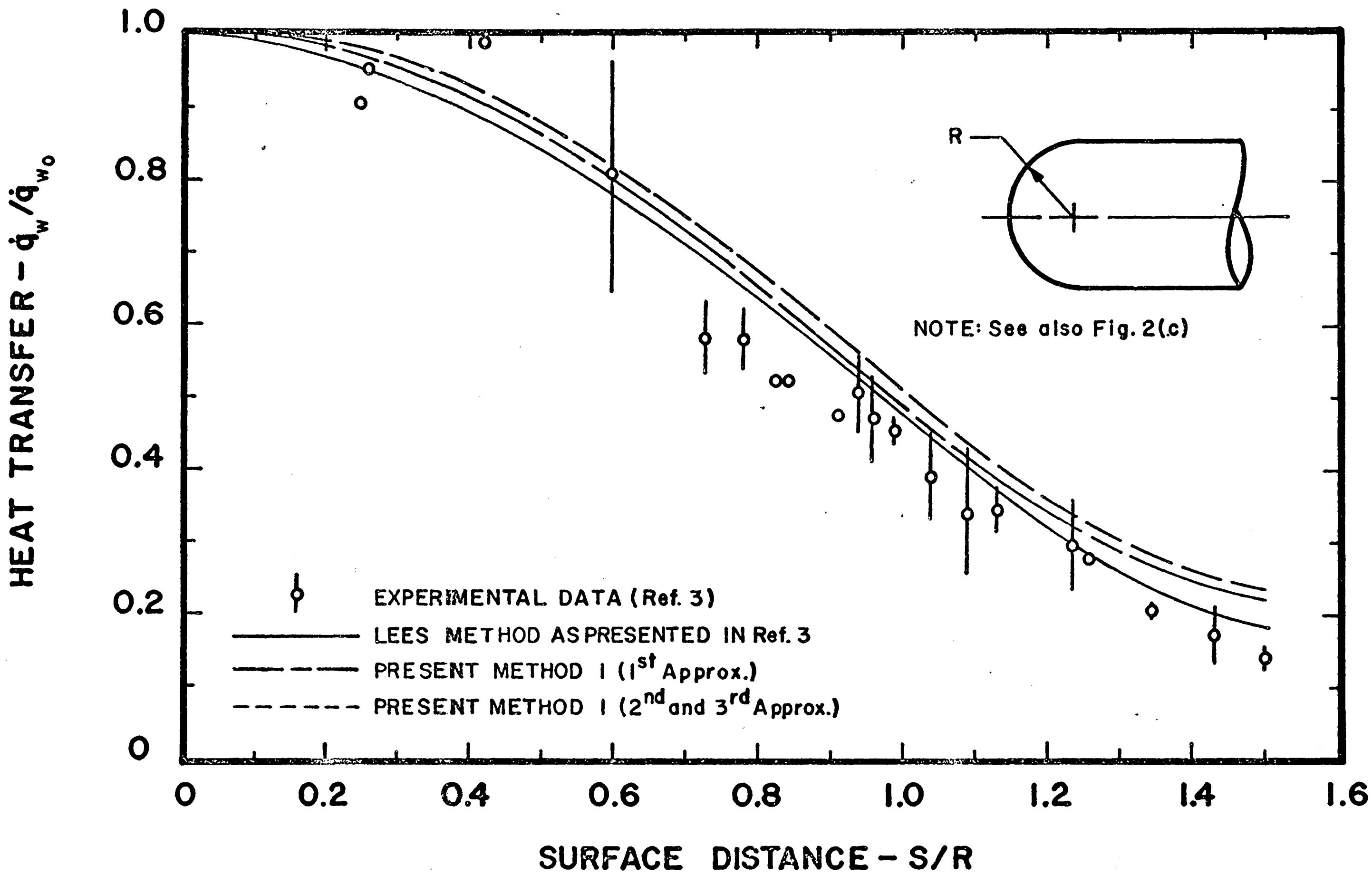


FIGURE 4. CONTINUED

(e) HEMISPHERE CYLINDER-METHOD I

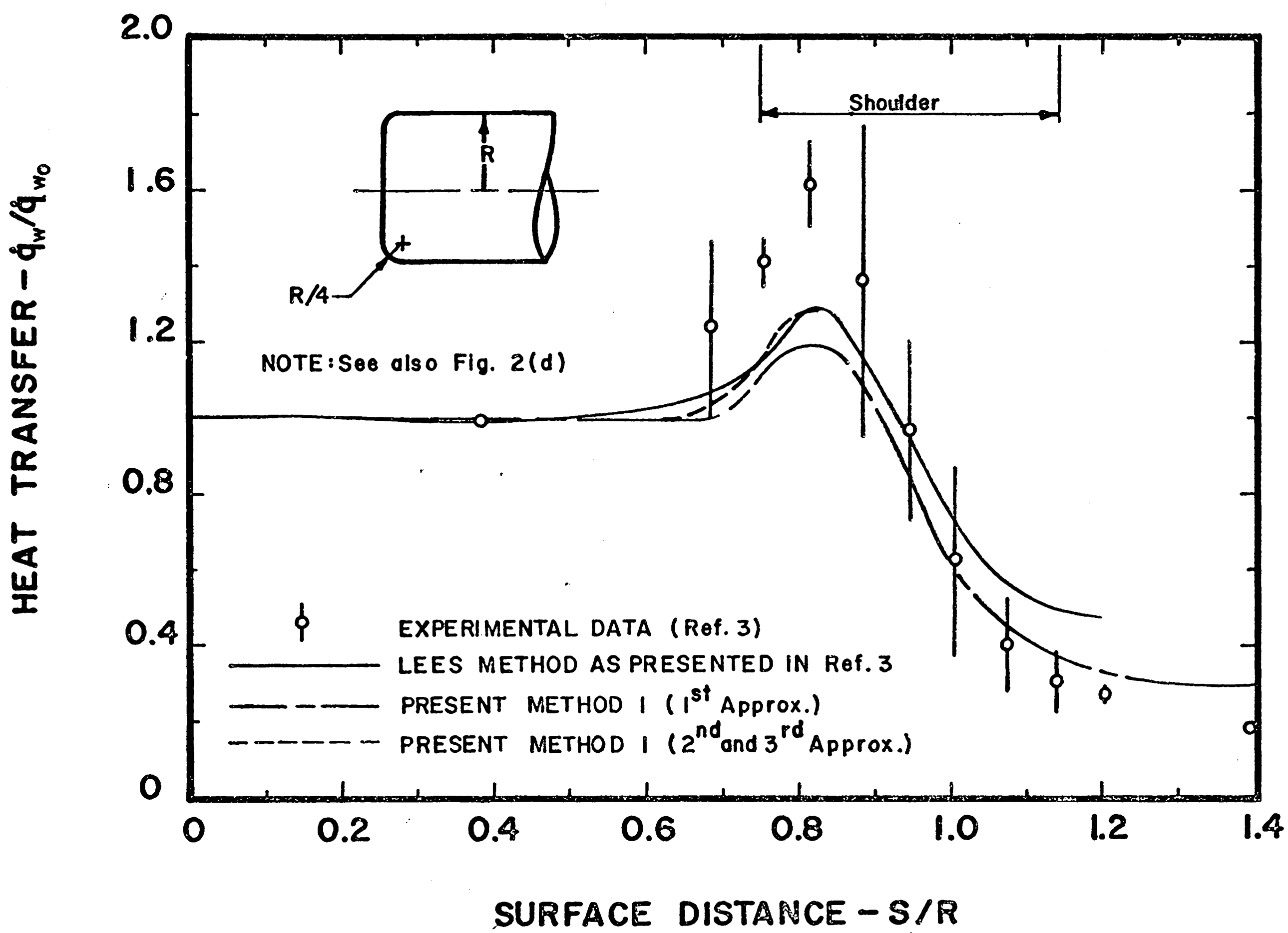


FIGURE 4. CONTINUED

(f) FLAT NOSED CYLINDER–METHOD I

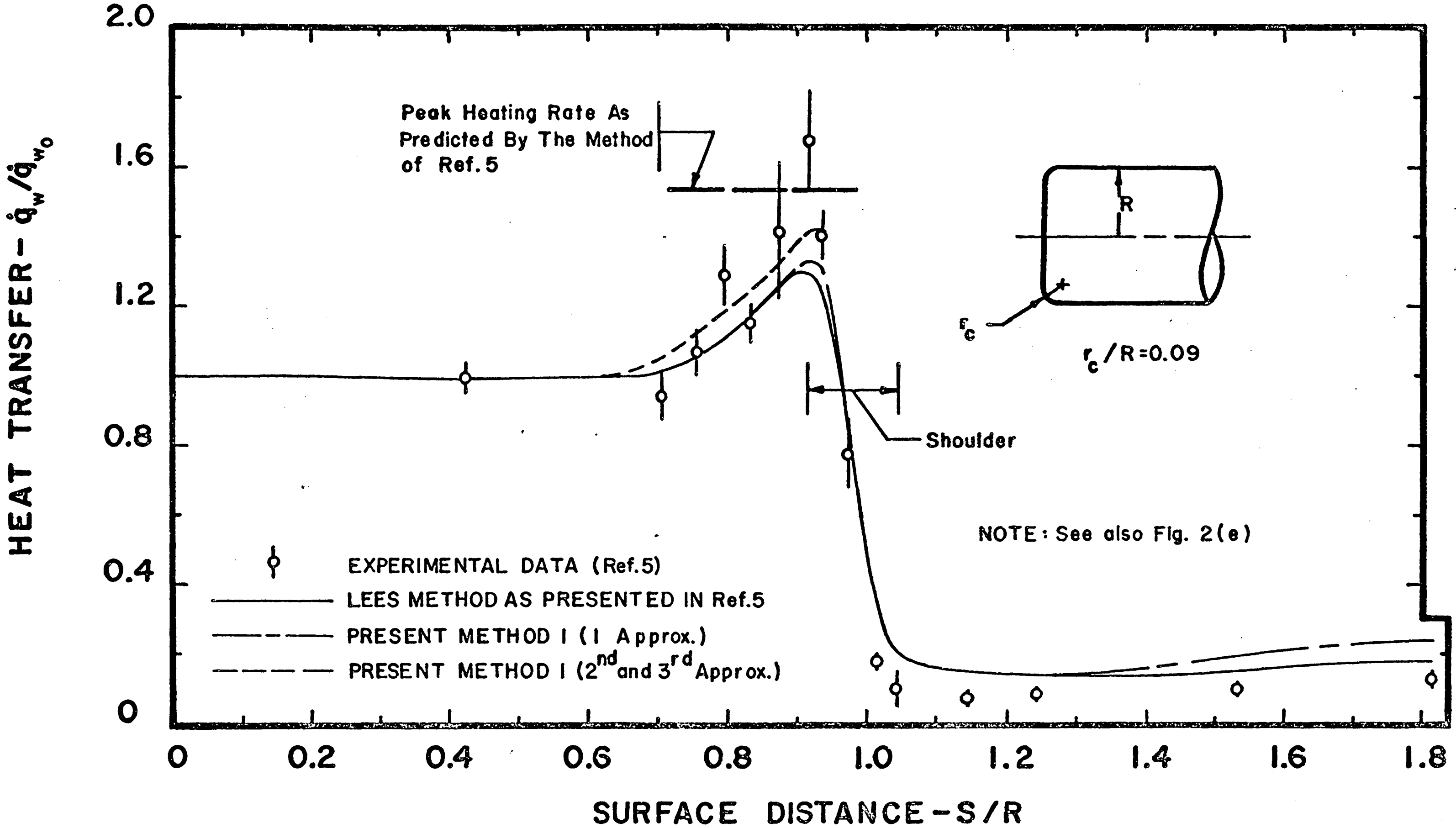


FIGURE 4. CONTINUED

(g) FLAT NOSE CYLINDER — METHOD I

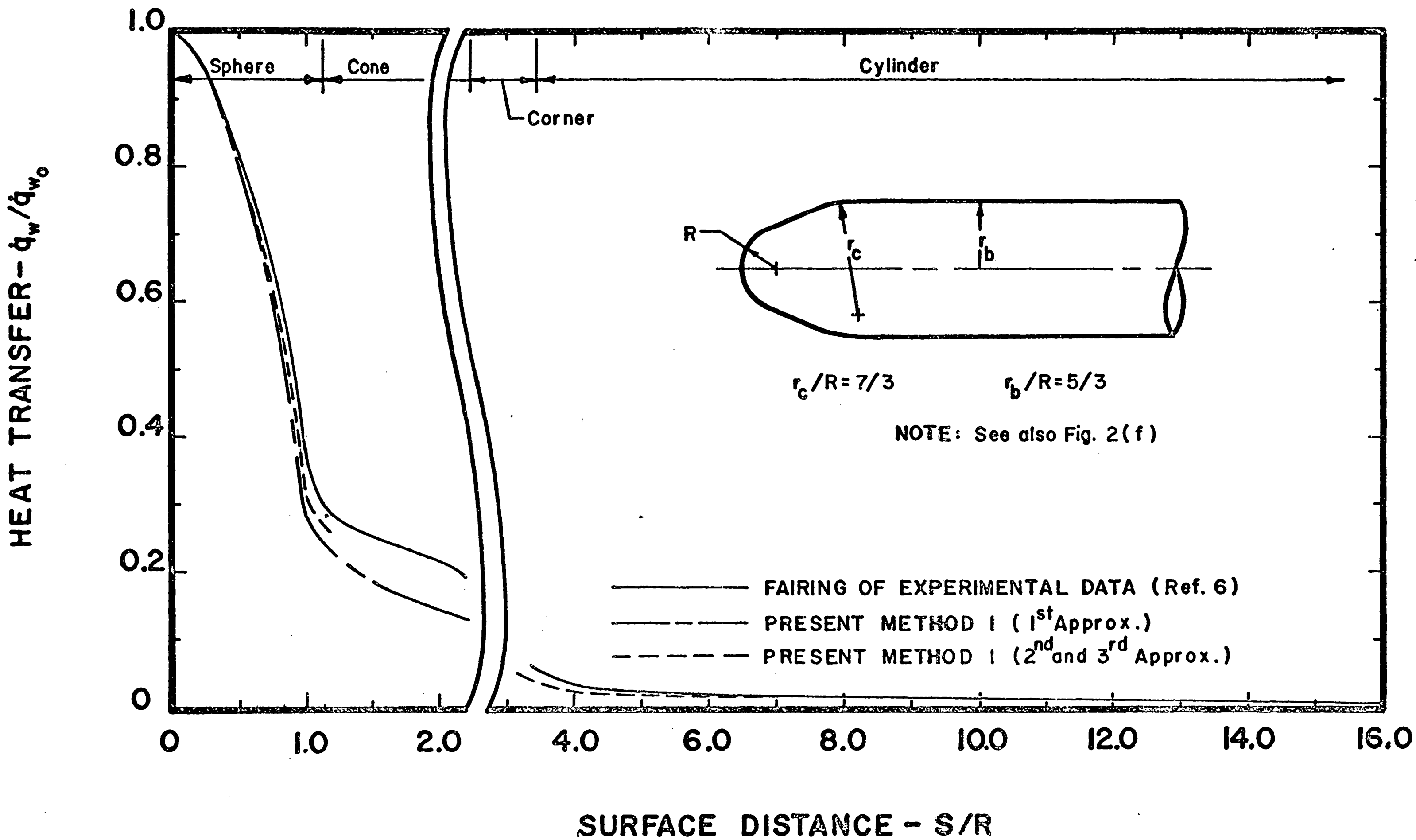


FIGURE 4. CONCLUDED

(h) BLUNTED CONE CYLINDER - METHOD I

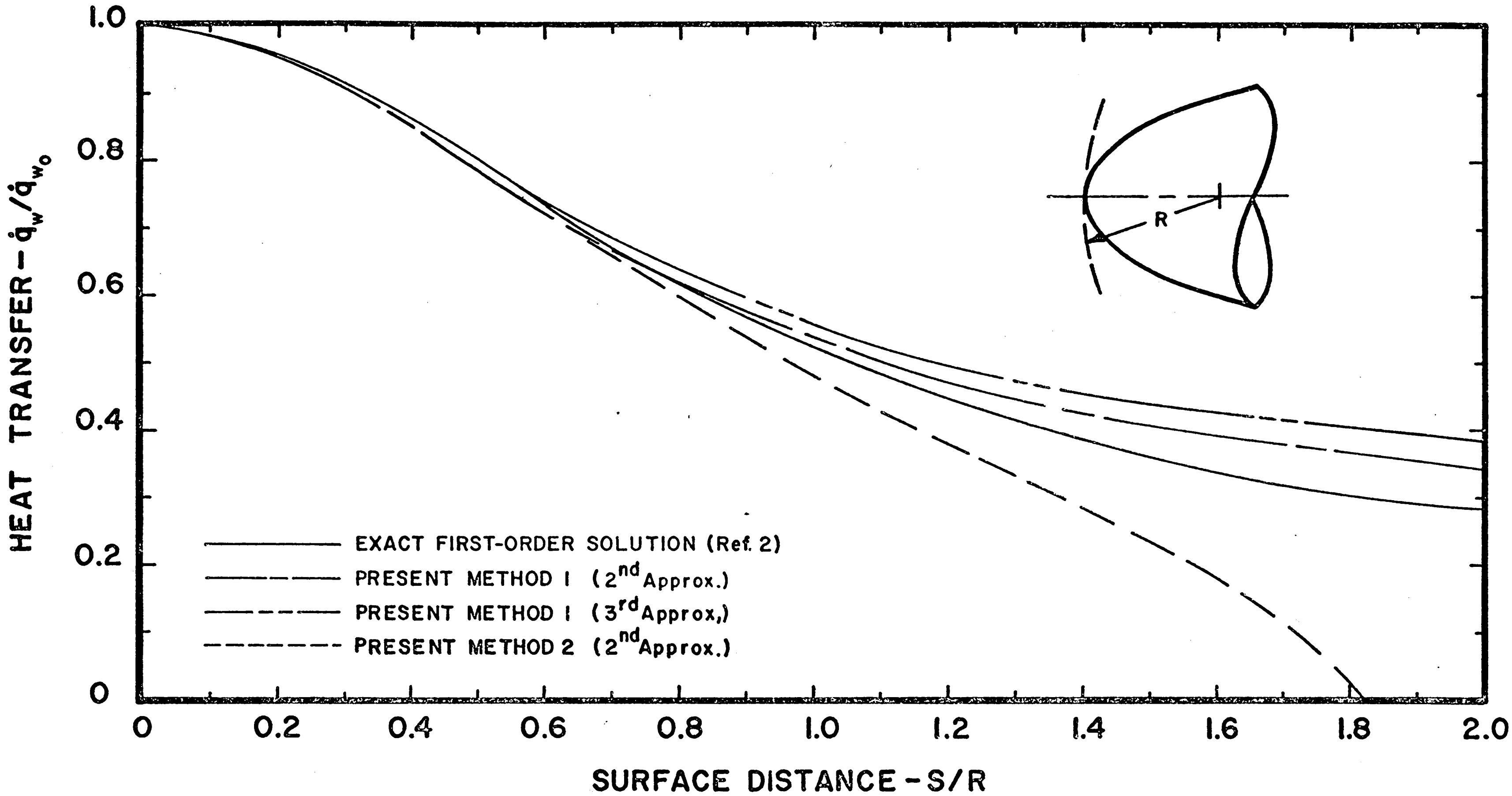


FIGURE 5. HEAT TRANSFER TO A PARABOLOID OF REVOLUTION EMPLOYING MODIFIED NEWTONIAN PRESSURE GRADIANTS

A SIMPLIFIED MEANS OF PROVIDING FIRST ESTIMATES TO LAMINAR HEATING RATES ON ISOTHERMAL AXISYMMETRIC BLUNT BODIES

Abstract

An approximate scheme for the rapid calculation of first estimates to the laminar heat transfer distribution over isothermal axisymmetric blunt bodies is developed. The method devised is free of any integral relations and reduces the required computing effort to a simple slide rule task. The simplicity of the method is due to the introduction of a new heat transfer parameter which is shown, from a semiempirical study, to undergo only moderate variation in regions where the heat transfer experiences order of magnitude changes. Based on these results, a series expansion for the parameter of interest is obtained through the fourth order term. Even though the perfect gas laws are employed in the series expansion, the resulting effect on the heat transfer ratio is felt to be small.

To substantiate the method, the heat transfer computed by the present scheme was compared with experimental, first-order exact, and Lees' approximate scheme for six body shapes of general interest. In all cases, fair to moderately good results were obtained. It is felt that any loss in accuracy is readily compensated for by the fact that the present method requires no numerical integration and therefore is extremely easy to apply.