Essays on Signaling Games under Ambiguity

Min Suk Lee

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Adam P. Dominiak, Chair

Éric A. Bahel

Hans H. Haller

T. Nicolaus Tideman

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ABSTRACT

This dissertation studies two-person signaling games where the players are assumed to be Choquet expected utility maximizers à la Schmeidler (1989). The sender sends an ambiguous message to the receiver who updates his non-additive belief according to a f-Bayesian updating rule of Gilboa and Schmeidler (1993). When the types are unambiguous in the sense of Nehring (1999), the receiver's conditional preferences after updating on an ambiguous message are always of the subjective expected utility form. This property may serious limit the descriptive power of solution concepts under non-additive beliefs, and it is scrutinized with two extreme f-Bayesian updating rules, the Dempster-Shafer and the Bayes' rule.

In chapter 3, the Dempster-Shafer equilibrium proposed by Eichberger and Kelsey (2004) is reappraised. Under the assumption of unambiguous types, it is shown that the Dempster-Shafer equilibrium may give rise to a separating behavior that is never supported by perfect Bayesian equilibrium. However, it does not support any additional pooling equilibrium outcome. Since the Dempster-Shafer equilibrium may support implausible behaviors as exemplified in Ryan (2002a), a refinement based on coherent beliefs is suggested.

In chapter 4, a variant of perfect Bayesian equilibrium, the quasi perfect Bayesian equilibrium, is proposed, and its descriptive power is investigated. It is shown that the quasi perfect Bayesian equilibrium does not support any additional separating behavior compared to perfect Bayesian equilibrium. It may support additional pooling behavior only if the receiver perceives a correlation between the types and messages.

To my parents for their endless love, and

To my wife for her sacrifice.

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Chapter 1

Overview

Since the innovation of Savage (1954), the subjective expected utility (SEU) model has been foremost in describing human decision making. In particular, an SEU maximizer is modeled to use a unique probability distribution for evaluating the likelihoods of uncertain events. However, in real life, there are situations where quantifying uncertainty with a single probability is not apparent. Therefore, from the seminal article of Ellsberg (1961), the descriptive validity of SEU model (which proposes unique probability) has been challenged.

In Ellsberg's thought experiments, he considers the specific uncertainty in which some events provide their probabilistic likelihood of occurrence while the other events do not supply such information. For example, in the three-color ball experiment, an urn contains 30 red balls and 60 other balls that are either black or yellow. The probability of drawing a red ball is known to be 1/3, while the probability of drawing a black (or yellow) ball cannot be quantified with a probability. He distinguishes such uncertainty by naming it *ambiguity* and advocates that the choice behaviors that frequently arise under ambiguity are not describable with SEU model. In fact, it is

impossible with all models in which underlying beliefs are represented by a single probability.

Motivated by Ellsberg's experiments, generalizations of SEU model have been proposed in various directions, and many are grounded on the two pioneering works: the Choquet expected utility (CEU) model of Schmeidler (1989) and the maxmin expected utility (MEU) model of Gilboa and Schmeidler (1989). In both models, the demand for a single probability distribution is weakened. In MEU model, a decision maker evaluates the available choices over a set of probability measures, rather than a single probability. In CEU model, a decision maker's belief is represented by a capacity which is not-necessarily-additive measure.

Although such models involve single person decision making, they have been naturally applied to the models of multi-person decision problems; i.e., games where probabilities are universally used to represent the beliefs of players. Dow and Werlang (1994) is the first in applying CEU model to the class of strategic form games of two players. They replace the fundamental assumption that the players are SEU maximizers with one that the players are CEU maximizers. The beliefs of players are represented by capacities rather than probabilities. The Nash equilibrium concept is extended to "Nash equilibrium under uncertainty." And an equilibrium describes a set of beliefs of players under which every action in the support of each player is a best-response. A support reflects a belief of a player. It portrays the actions of the opponent which are believed to be played. In particular, Dow and Werlang provide a notion of support which guarantees its existence although it may not be unique.

Eichberger and Kelsey (2000) further extend the equilibrium under uncertainty to n-player strategic form games. Marinacci (2000) also studies a similar solution concept under capacities, "the equilibrium in beliefs," for strategic form games. But he

assumes a different notion of support.[1,2]

While there is literature investigating the applications of CEU model to various strategic form games, not many consider applications to dynamic games.[3] Eichberger and Kelsey (1999b) first explore education signaling games and Eichberger and Kelsey (2004) further explore two-person sequential games. Especially, they introduce a solution concept, the Dempster-Shafer equilibrium (DSE) which is a generalization of perfect Bayesian equilibrium (PBE). Its novelty lies on the fact that conditional beliefs may be well defined when the conditioning event is measured as zero, and hence the beliefs off-the-equilibrium-path may be uniquely defined without ad-hoc refinements. Despite its potential advantages, studies on the Dempster-Shafer equilibrium have been on hiatus, and it may be due to the note of Ryan (2002a).

In his note, Ryan provides one signaling game in which some DSEs support an implausible behavior. He points out that the receiver's beliefs in such DSEs violate

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[1] Notice that the existence theorem in Marinacci, and Dow and Werlang use the same support as they coincide in their proofs.

[2] A series of applications of MEU model to games is as follows. For strategic form games, Klibanoff (1996) and Lo (1996) extend the Nash equilibrium to the equilibrium in beliefs under Uncertainty. Lo (1999) further extends the equilibrium notion, *Multiple priors Nash equilibrium*, to the extensive form games. Kajii and Ui (2005) firstly practice MEU model into incomplete information games. A player may perceive a set of conditional probabilities about the opponents' private signals (thus maybe non-additive on types) and update the beliefs using the full-Bayesian updating rule by Jaffray (1992), and Fagin and Halpern (1991). Stauber (2011) also introduces the ambiguity into the incomplete information games based on Bewley (2002). Aryal and Stauber (2014) is applying MEU model to justify robustness equilibrium under ambiguous beliefs.

[3] For example, Haller (2000) studies *solvability* in CEU model and provides that two-person zero-sum games preserve the solvability with simple capacities. Eichberger, Kelsey, and Schipper (2009) generalize the Cournot and Bertrand competition model with pessimistic and optimistic in mind. They use a special class of capacities, the neo-additive capacity, to capture optimistic and pessimistic departures from the benchmark of the expected utility. Eichberger and Kelsey (2002) also consider pessimism and optimism for public good games and their experiment results are reported in Kelsey and le Roux (2015). Eichberger, Kelsey, and Schipper (2008) provide a view of how ambiguity may explain somewhat puzzling behaviors in experiments. Some literatures apply MEU model to auction and mechanism design: e.g., Salo and Weber (1995), Lo (1998), Bose, Ozdenoren, and Ozdenoren (2006), Chen, Katuk, and Ozdenoren (2007).

the belief persistence axiom and proposes to refine the DSEs. The belief persistence axiom is a requirement on belief change. Roughly speaking, it demands that a decision maker should not abandon any beliefs more than necessary in response to the new information.

In signaling games, the informed player, the sender, sends a message contingent on her private information, the type; and the uninformed player, the receiver, uses the message to make an inference about the sender's private information. An interesting outcome is separating equilibrium in which the sender fully reveals her private information that is learned by the receiver.

Hence, in signaling games, the receiver has to update his beliefs about the true type of the sender after receiving a message sent by the sender. According to the belief persistency, at a separating equilibrium, after observing a message, the receiver should never believe that a type has sent the message, if the type was deemed impossible in the ex-ante belief. If the receiver somehow becomes to believe that a type is the true type which was believed impossible, then his updated belief violates the belief persistence axiom.

The violation of belief persistency may not be a problem per se because it is already known that a departure from additive beliefs possibly violate dynamic consistency (see Lo, 1996; Dow and Werlang, 1994). However, after refining the equilibrium beliefs which violate the belief persistence postulate, the remaining ex-ante beliefs are all additive. That is, there is no room for admitting any non-additive belief at the outset. This can be a serious limitation of the Dempster-Shafer equilibrium as the main motivation of introducing ambiguity into games becomes negligible.

Motivated by Ryan's example, we reappraise the Dempster-Shafer equilibrium in signaling games via chapter 3. However, we aim to provide different perspectives on

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the solution concept. As Aumann (1985) points out, "[...] a solution concept should be judged more by what it does than by what it is." Thus, our investigation intends to gain sight on how much behaviors can be further explained by the Dempster-Shafer equilibrium compared to the perfect Bayesian equilibrium, which will be often referred as the descriptive power of DSE. Notably in Example 3.3.1, there exist DSEs which support separating equilibrium behavior that cannot be captured by any PBE. They, however, violate the belief persistency. To attain descriptive power, a solution concept should support such plausible behaviors while abandoning the implausible behaviors in Ryan's game. It is shown that this can be achieved by introducing the refinement notion of *coherent beliefs*.

Chapter 2 studies a restrict on CEU model under the assumption of unambiguous events of Nehring (1999) and the f-Bayesian updating rules of Gilboa and Schmeidler (1993). As ambiguity denotes the specific uncertainty where some events provide their probabilistic information, the distinction should be made between unambiguous and ambiguous events. To that end, the definition of Nehring (1999) is adopted. Under the Nehring's notion, the capacity value of every event can be decomposed as a linear sum of the capacity values of the intersection of the event and each unambiguous event. This property is called *additive-separability* across unambiguous events (see Definition 2.2.1).[4] It is noteworthy that this feature may be convenient for computational purposes especially when the marginal events of a coordinate of a product space are unambiguous.

Hence, we ask under which updating rules this additive separability is maintained. All f-Bayesian updating rules of Gilboa and Schmeidler (1993) preserve the additive separability, while the Full-Bayesian rule of Walley (1991) and Jaffray (1992) does

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[4]Due to Sarin and Wakker (1992) and Dominiak and Lefort (2011), the partition consisted of the unambiguous events by Nehring's notion satisfies the Sure-Thing-Principle.

not preserve the property. Every f-Bayesian updating rule maintains the Choquet preferences in conditional preferences for any conditioning event (see Gilboa and Schmeidler, 1993), and the Dempster-Shafer updating used in DSE and Bayes' rule belong to this class.

Moreover under any f-Bayesian updating rule, if the conditioning event is a marginal event that is ambiguous in a product space, then the conditional preferences not only preserve the additive-separability but also admit SEU representation; i.e., conditional capacities are additive.

In signaling games, product space is generally used by the convention of Harsanyi (1967-1968), and the probability distribution of the types of the sender is assumed to be a public knowledge to all players. Hence, the assumptions of unambiguous types and ambiguous messages imply that the conditional beliefs of the receiver must be additive after updating an ex-ante non-additive belief with f-Bayesian updating rule.

Chapter 3 mainly discusses the implication of this result to the solution concept of Dempster-Shafer equilibrium. The Dempster-Shafer updating rule of Dempster (1968) and Shafer (1976) is an extreme f-Bayesian updating rule. It is the *pessimistic* rule because the decision maker views that the best outcome is never realizable after observing an event. Hence, the decision maker always regards any conditioning event as "not-a-good-news." On the contrary, the Bayes' rule reflects *optimism* as the conditioning event is always viewed as "a-good-news." The decision maker perceives that the worst outcome is never realizable after observing an event.

A crucial feature of this pessimistic updating in signaling games is that the receiver may entirely neglect the signaling information conveyed by a message, and solely respect the unambiguous information. That is, the receiver's conditional belief can be the same as the probability distribution of the types after receiving an

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ambiguous message. In fact, as the receiver perceives more ambiguity on messages, his conditional beliefs resemble more closely the probability distribution of types. This feature is closely related to the violation of belief persistence axiom. At some separating DSEs, the receiver may ex-ante perceive that a type would never send a message. However, after observing the message that is ambiguous, the receiver may believe that the type has sent the message as his conditional belief may be the same with the prior information about the types. Thus, such DSEs naturally violate the belief persistency. However, if the belief persistency is exogenously enforced to be satisfied at any DSE, then the receiver must exhibit ex-ante additive beliefs at any separating equilibrium (see Proposition 3.3.2). Under the joint assumption of unambiguous types and belief persistence, all separating equilibrium precludes any ambiguity perception of the receiver.

This pessimism of the receiver, however, may offer a chance of separation to the sender. The possibility that the pessimistic receiver would never learn from a signal provides strong incentive to the sender to secure more safe option whenever possible. This behavior will never be captured by perfect Bayesian equilibrium as the learning of the receiver is a natural consequence of Bayes' updating. Further, such equilibrium is unattainable without the violation of belief persistency.

This fact provides a different perspective of DSE and motivates consideration of another refinement which is less stringent than the belief persistence axiom. We propose the notion of coherent beliefs which is directly related to the players' payoffs. It requires that if the receiver conditionally believes that a type could have sent the message observed, then there must exist a sender's belief which rationalizes the message to be optimal for the type. That is, the receiver's conditional beliefs must be coherent by being backed up with a sender's belief rationalizing her behavior. It is further shown that the Coherent DSEs, the DSEs which respect the coherency, suc-

cessfully capture the new separating behavior of DSE, while eliminating the anomaly raised by Ryan.

In chapter 4, we extend our study to the receiver who uses the Bayes' updating rule, the optimistic updating. An extended solution concept, quasi perfect Bayesian equilibrium (QPBE), is introduced and its existence is shown. It is an extension of perfect Bayesian equilibrium which admits non-additive beliefs while maintaining the Bayes' updating rule. Hence, the new solution concept requires the same consistency between beliefs and behavior as required by PBE, but only allows ex-ante non-additive beliefs - which is implied by the assumption of unambiguous types. Interestingly, under the assumption of unambiguous types, QPBE does not perform better than PBE in explaining additional behavior. Specifically, if the types of the sender are unambiguous, then any separating behavior of QPBE can be depicted by a PBE. That is, there exists a PBE which supports the same equilibrium behavior of QPBE. Further, if the types and messages are independent, in the sense of the receiver's ex-ante capacities are represented by an independent product capacity, then any pooling equilibrium behavior of QPBEs can be depicted by a PBE.[5]

However, once the receiver's belief admits a correlation between the types and messages, QPBE may capture a pooling behavior that is unattainable under PBE. It may be explained from the fact that the optimistic updating works exactly opposite to the pessimistic updating. The receiver may completely neglect the unambiguous information about the types of the sender, but solely respects his ex-ante belief. Hence, after receiving an ambiguous message, the receiver's conditional belief may reveal the opposite of the unambiguous information. It is further shown that such behaviors can be supported by PBE, if the probability distribution of types is not

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[5]By Nehring (1999), the product capacity which reveals that all marginal events of a coordinate are unambiguous, is unique.

uniquely specified, but is known to be a probability distribution.

We close up the chapter with a note that a small departure from the Bayesian paradigm may surprisingly open larger equilibrium behaviors that are not supported by perfect Bayesian equilibrium. This suggests that some experiments of signaling games may be understood in different ways. An observation which was regarded as supportive evidence of separating behavior might actually support pooling behavior. Likewise, an observation regarded as supportive evidence of pooling equilibrium might actually support separating behavior.[6]

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[6]Camerer and Karjalainen (1994), Ivanov (2011), Kelsey and le Roux (2015) consist of very small literatures on experiments of ambiguity in games.

Chapter 2

Choquet Preferences and f-Bayesian Updating

This chapter introduces the capacity model of Schmeidler (1989) and its extension to dynamic decision problems with f-Bayesian updating.

2.1   Static and Dynamic Choquet Preferences

Let S be a set of states of nature and X a set of consequences endowed with an weak order $\succsim$. The states in S are mutually exclusive and collectively exhaustive. An event E is a subset of S and its complementary event $S \setminus E$ is denoted by E^c. Σ is a σ-algebra of events of S. An act $g : S \to X$ is a mapping from states to consequences, and it is called Σ-measurable if for each $x \in X$ $\{s \mid g(s) \succ x\}, \{s \mid g(s) \succsim x\} \in \Sigma$. The set of all Σ-measurable acts is denoted by $\mathcal{A}$. An utility function u is a mapping $u : X \to \mathbb{R}$ such that $u(x) \geq u(y)$ if and only if $x \succsim y$ for all $x, y \in X$. For $g, h \in \mathcal{A}$,

the act gEh assigns the consequence $g(s)$ to $s \in E$, and $h(s)$ to $s \in E^c$. With abuse of notation, constant act, $g(s) = x$ for all $s \in S$, is simply denoted by x.

A capacity is a normalized and monotone set function defined as follows.

Definition 2.1.1

A function $\nu : \Sigma \to \mathbb{R}$ is a capacity if it satisfies:

$$
\begin{aligned}
(i) \quad & \nu(\emptyset) = 0, \quad \nu(S) = 1, \\
(ii) \quad & \nu(E) \leq \nu(F) \text{ for all } E \subseteq F \subseteq S.
\end{aligned}
$$

The capacity ν is a probability measure if it is additive;i.e., $\nu(E \cup F) = \nu(E) + \nu(F)$ for all disjoint $E, F \subseteq S$. It is called convex, if it further satisfies (iii) $\nu(E \cup F) + \nu(E \cap F) \geq \nu(E) + \nu(F)$ for all $E, F \subseteq S$. All the analyses hereafter are not restricted to convex capacities unless explicitly stated.

A real-valued function $k : S \to \mathbb{R}$ is Σ-measurable if for all $t \in \mathbb{R}$, $\{s \mid k(s) > t\}, \{s \mid k(s) \geq t\} \in \Sigma$.

Given a capacity $\nu : \Sigma \to \mathbb{R}$ and a function $k : S \to \mathbb{R}$ that is bounded, the *Choquet* integral of k with respect to ν by Choquet (1954) is given by

$$
\int k \, d\nu \quad := \quad \int_0^\infty \nu\left(\{s \in S \mid k(s) \geq t\}\right) dt + \int_{-\infty}^0 \left[\nu\left(\{s \in S \mid k(s) \geq t\}\right) - 1\right] dt,
$$

where on the right, we have Riemann integrals.

A preference relation $\succcurlyeq$ on $\mathcal{A}$ is said to admit a Choquet expected utility representation if there exists a capacity ν on Σ and a utility function $u : X \to \mathbb{R}$, such that

for any $g, h \in \mathcal{A}$,

$$g \succcurlyeq h \quad \Leftrightarrow \quad \int_S u\big(g(s)\big) \, d\nu(\{s\}) \geq \int_S u\big(h(s)\big) \, d\nu(\{s\}). \qquad (2.1)$$

Definition 2.1.2

Suppose S is finite with n states $s_1, ..., s_n$ and $u(g(s_1)) \geq \cdots \geq u(g(s_n))$. The Choquet integral of an act $g \in \mathcal{A}$ with respect to a capacity ν and a utility function u is defined by:

$$\int_S u\big(g(s)\big) \, d\nu(\{s\}) \;=\; \sum_{j=1}^{n-1} \Big[u\big(g(s_j)\big) - u\big(g(s_{j+1})\big) \Big] \, \nu\big(\{s_1, ..., s_j\}\big) + u\big(g(s_n)\big)$$

The maximization of Choquet integrals, as a decision criterion, has been axiomatized by Schmeidler (1989) - first version in 1982 - as an extension of Anscombe and Aumann (1963), and by Gilboa (1987) as an extension of Savage (1954). Wakker (1989), Sarin and Wakker (1992), Nakamura (1990), Chew and Karni (1994). Hereafter $\succcurlyeq$ represents a Choquet preference relation on $\mathcal{A}$.

In dynamic choice situations, a decision maker receives a new information in the form of an event $E \in \Sigma$, and maps the unconditional preference $\succcurlyeq$ into a conditional preference $\succcurlyeq_E$. For instance, two stages can be considered: the ex-ante and interim stage. At the ex-ante stage, the uncertainty is described by a set S of states of nature. At the interim stage, it is informed that an event $E \in \Sigma$ has been occurred. The arrival of new information is incorporated into the decision making process by updating the unconditional preferences $\succcurlyeq$.

Let $\succcurlyeq_E$ for some $E \in \Sigma$ be a conditional preference relation updated from fixed unconditional Choquet preference relation $\succcurlyeq$. Gilboa and Schmeidler (1993) identifies the class of updating rules which preserves Choquet preferences after updating.

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Assume that X contains a best $\bar{x}$ and worst $\underline{x}$ consequence.

Definition 2.1.3

An f-Bayesian updating rule is defined as follows: for any $g, h \in \mathcal{A}$ and any $E \in \Sigma$, there exists $D \in \Sigma$ such that

$$g \succcurlyeq_E h \quad \Leftrightarrow \quad gEf \succcurlyeq hEf, \tag{2.2}$$

where $f = \bar{x} \, D\underline{x} \in \mathcal{A}$.

Theorem 2.1.4 (Gilboa and Schmeidler (1993))

Let $\succcurlyeq_E$ be a conditional preference relation updated from a Choquet preference relation $\succcurlyeq$ on $\mathcal{A}$. Let $f \in \mathcal{A}$ and assume $|\Sigma| > 4$. The following statements are equivalent:

(i) $\succcurlyeq_E$ is a Choquet preference relation on $\mathcal{A}$, for all $E \in \Sigma$,

(ii) $\succcurlyeq_E$ is derived by an f-Bayesian updating rule.

Theorem 2.1.4 states that f-Bayesian updating rules is the class of updating rules which preserves Choquet preferences in conditional preference relation for all conditioning event in algebra. [1] Thus, a representation of conditional preference $\succcurlyeq_E$ is to evaluate acts by Choquet integrals with respect to the same unconditional utility index u and an updated capacity ν_E. The transformation of unconditional capacity to achieve ν_E is given by the following: for any $A, E \in \Sigma$, and fixed $D \in \Sigma$,

$$\nu_E^f(A) = \frac{\nu\big((A \cap E) \cup (D \cap E^c)\big) - \nu\big((D \cap E^c)\big)}{\nu(E \cup D) - \nu(D \cap E^c)}, \tag{2.3}$$

if the denominator is positive.

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[1]To the best of my knowledge, it is unknown whether there exist another rules which preserve Choquet preference relation with some conditioning events.

The class of f-Bayesian updating rules encloses two interesting cases. Firstly, if $D = S$, the conditional capacity ν_E coincides with the Dempster-Shafer rule by Dempster (1968) and Shafer (1976) which is:

$$\nu_E(A) = \frac{\nu\big((A \cap E) \cup E^c\big) - \nu(E^c)}{1 - \nu(E^c)}, \tag{2.4}$$

whenever $\nu(E^c) < 1$. Secondly, if $D = \emptyset$, the conditional capacity ν_E coincides with the Bayes' rule:

$$\nu_E(A) = \frac{\nu(A \cap E)}{\nu(E)}, \tag{2.5}$$

whenever $\nu(E) > 0$.

The Dempster-Shafer rule is interpreted as *pessimistic* rule because the decision maker views that the best consequence is in the complement of the event E and so impossible to occur anymore. Hence, the decision maker always regards any conditioning event as "not-a-good-news." On the contrary, the Bayes' rule reflects *optimism* as the conditioning event is viewed as "a-good-news." The perception that the worst outcome is in the complement of the conditioning event reflects that the worst could have happened otherwise.

Remark that the f-Bayesian updating rules respect the consequentialism, a fundamental property of conditional preferences introduced by Hammond (1988), but not the dynamic consistency (see Ghirardato, 2002).[2] Roughly, the dynamic consistency requires the decisions made at two different times to be consistent with one another. In consequentialism, however, the actions are evaluated purely on the basis of their consequences (Machina and Viscusi, 2014). For different choices with the identical feasible consequences, the revealed choice must generate the identical set of

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[2]These two properties of preferences can be maintained simultaneously if and only if the conditioning event is perceived as being unambiguous (see Dominiak, 2013).

consequences.[3]

Example 2.1.1

Let $S = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and $X = \{0, \ldots, 10\}$. $\mathcal{A}$ is the set of all mappings $g : S \to X$. Let $\succcurlyeq$ be the Choquet preference relation generated by $u(g) = g$ for all $g \in \mathcal{A}$ and ν given by:

$$\nu(\{\omega_1\}) = \nu(\{\omega_4\}) = \frac{1}{4}, \quad \nu(\{\omega_2\}) = \nu(\{\omega_3\}) = 0,$$

$$\nu(\{\omega_1, \omega_2\}) = \nu(\{\omega_3, \omega_4\}) = \frac{1}{2}, \quad \nu(\{\omega_1, \omega_3\}) = \nu(\{\omega_2, \omega_4\}) = \frac{1}{4},$$

$$\nu(\{\omega_1, \omega_4\}) = \frac{1}{2}, \quad \nu(\{\omega_2, \omega_3\}) = 0,$$

$$\nu(\{\omega_1, \omega_2, \omega_4\}) = \nu(\{\omega_1, \omega_3, \omega_4\}) = \frac{3}{4}, \quad \nu(\{\omega_1, \omega_2, \omega_3\}) = \nu(\{\omega_2, \omega_3, \omega_4\}) = \frac{1}{2}.$$

Consider two acts $g, h \in \mathcal{A}$ such that $g = (1, 1, 1, 1)$ and $h = (0, 1, 4, 1)$.[4] Then, $g \succ h$ as $CEU(g; \nu) = 1 > CEU(h; \nu) = \frac{1}{2}$.

Now, take $E = \{\omega_1, \omega_3\}$ as a conditioning event. Assume ν_E is revised by the Dempster-Shafer updating rule. Then, $\nu_E(\{\omega_1\}) = \frac{2}{3}$, $\nu_E(\{\omega_3\}) = \frac{1}{3}$, and we have $h \succ_E g$ as $CEU(h; \nu_E) = \frac{4}{3} > CEU(g; \nu_E) = 1$.

In a recent experimental study by Dominiak, Duersch, and Lefort (2012), however, provide an observational evidence that subjects behave more in line with consequentialism than dynamic consistency.

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[3]For more on the dynamic consistency and consequentialism, we guide to Machina and Viscusi (2014).

[4]Each component represents the mapped outcome from the states ω_is, $i = 1, 2, 3, 4$ in sequence; e.g., h returns 0 if the state is ω_1, and 1 for the state ω_2, and 4 for ω_3, and 1 for ω_4.

2.2 Unambiguous Events and f-Bayesian Updating

Many decision problems encompass the situation where the probabilities of some events are known while the others indicate no such an information. And thus the belief of a decision maker may reveal some events being unambiguous and others not. This distinction is made by following the definition of Nehring (1999).

Definition 2.2.1

For a capacity ν, an event $U \in \Sigma$ is revealed to be unambiguous if for any $A \in \Sigma$ the following is true:

$$\nu(A) = \nu(A \cap U) + \nu(A \cap U^c), \tag{2.6}$$

otherwise the event is ambiguous.

The capacity measure on each unambiguous event which satisfies Condition (2.6) does not variate for different acts being evaluated; i.e., the different rankings do not alter the measure attached. Since the capacity value of all events can be measured with two separated parts, the intersection with an unambiguous event and with its complement, a capacity satisfying Condition (2.6) is said to be "additively-separable" across the unambiguous events. It is noteworthy that Nehring's notion of unambiguous events has an important correspondence on Choquet preference relation. The following is restatement of Proposition 3.1 of Dominiak and Lefort (2011):

Let $U \in \Sigma$ be an unambiguous event satisfying Condition (2.6) for a given $\succcurlyeq$. Then $\succcurlyeq$ satisfies Savage's Sure-Thing Principle at the partition $\{U, U^c\}$. That is, for any $f, g, h, h' \in \mathcal{A}$, it holds true:

$$fUh \succcurlyeq gUh \iff fUh' \succcurlyeq gUh'. \tag{2.7}$$

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From Conditions (2.6) and (2.7), the Choquet integral of $g \in \mathcal{A}$ can be decomposed (Nehring, 1999, Theorem 1 and 3):

$$\int_S u\big(g(s)\big) \, d\nu(\{s\}) = \int_U u\big(g(s)\big) \, d\nu(\{s\}) + \int_{U^c} u\big(g(s)\big) \, d\nu(\{s\}). \qquad (2.8)$$

For conditional Choquet preferences, however, it is a question under what updating rules Condition (2.6) is maintained. The following proposition proves that all f-Bayesian updating rules preserve the condition.

Proposition 2.2.2

Let $\succeq$ be an unconditional Choquet preference relation with respect to a capacity ν, and $\mathcal{P} = \{U_1, ..., U_m\}$ be a partition of S consisting of unambiguous events. Take an event $E \in \Sigma$. If the conditional capacity ν_E is derived by a rule in the class of f-Bayesian updating, then the events in partition $\mathcal{P}$ remain unambiguous. That is, for any $A \in \Sigma$:

$$\nu_E(A) = \sum_{k=1}^{m} \nu_E(A \cap U_k). \qquad (2.9)$$

Remark: There is another updating rule, the Full-Bayesian updating rule, suggested by Walley (1991) and Jaffray (1992). Its definition is the following:

$$\nu_E(A) \;=\; \frac{\nu\,(A \cap E)}{1 - \big(\nu(A \cup E^c) - \nu(A \cap E)\big)}. \qquad (2.10)$$

Proposition 2.2.2 does not hold under the Full-Bayesian updating. The following example proves that the full-Bayesian updating rule does not maintain the additive separability.

Example 2.2.1

Let $S = \{s_1, s_2, s_3, s_4\}$, $\bar{\mathcal{P}} = \big\{\{s_1, s_2\}, \{s_3, s_4\}\big\}$ and label each set in the partition $\bar{\mathcal{P}}$

by U_1 and U_2. *Consider the capacity defined below:*

$$\nu(\{s_1\}) = \nu(\{s_4\}) = \frac{1}{3}, \quad \nu(\{s_2\}) = \nu(\{s_3\}) = 0,$$

$$\nu(\{s_1, s_2\}) = \nu(\{s_3, s_4\}) = \frac{1}{2}, \quad \nu(\{s_1, s_3\}) = \nu(\{s_2, s_4\}) = \frac{1}{3},$$

$$\nu(\{s_1, s_4\}) = \frac{2}{3}, \quad \nu(\{s_2, s_3\}) = 0,$$

$$\nu(\{s_1, s_2, s_4\}) = \nu(\{s_1, s_3, s_4\}) = \frac{5}{6}, \quad \nu(\{s_1, s_2, s_3\}) = \nu(\{s_2, s_3, s_4\}) = \frac{1}{2}.$$

Note that the events in partition $\bar{\mathcal{P}}$ are unambiguous. Letting $E = \{s_1, s_2, s_3\}$ and $A = \{s_1, s_3\}$, we have the following:

$$\nu_E(A) = \frac{\nu(\{s_1, s_3\})}{1 - \left[\nu(\{s_1, s_3, s_4\}) - \nu(\{s_1, s_3\})\right]} = \frac{2}{3},$$

$$\nu_E(\{s_1\}) = \frac{\nu(\{s_1\})}{1 - \left[\nu(\{s_1, s_4\}) - \nu(\{s_1\})\right]} = \frac{1}{2},$$

$$\nu_E(\{s_3\}) = \frac{\nu(\{s_3\})}{1 - \left[\nu(\{s_3, s_4\}) - \nu(\{s_3\})\right]} = 0.$$

Since $\nu_E(A) \neq \nu_E(\{s_1\}) + \nu_E(\{s_3\})$, the additive separability is not maintained under the Full-Bayesian updating.

The additive-separability for conditional capacities provides an interesting restriction on the conditional preferences. Consider the partition $\mathcal{P} = \{U_1, ..., U_m\}$, and construct a conditioning event E by selecting only one state from each unambiguous event in the partition; i.e., $E = \{s_1, \ldots, s_m \mid s_i \in U_i, i = 1, \ldots, m\}$. Then Proposition 2.2.2 immediately implies $\nu_E(\{s_1\}) + \cdots + \nu_E(\{s_m\}) = 1$ and $\nu_E(s) = 0$ for any $s \in E^c$. This implies ν_E only admits additive capacity when conditioning event comprises such conditions. This line of reasoning naturally fits to capacities

defined over n-fold Cartesian product. Let $\Omega = \mathcal{T} \times \mathcal{M}^1 \times \cdots \times \mathcal{M}^{n-1}$, an n-fold Cartesian product. Consider a capacity ν defined on 2^Ω. Suppose that the partition $\mathcal{P}_\mathcal{T} = \{\{t\} \times \mathcal{M} \mid t \in \mathcal{T}\}$ consists of unambiguous events while the partition $\mathcal{P}_\mathcal{M} = \{\mathcal{T} \times \{m_1\} \times \cdots \times \{m_{n-1}\} \mid m_i \in \mathcal{M}^i, \ i = 1, \ldots, n-1.\}$ consists of ambiguous ones. If the conditioning event E is an element of $\mathcal{P}_\mathcal{M}$, then the conditional preference $\succcurlyeq_E$ admits subjective expected utility representation.
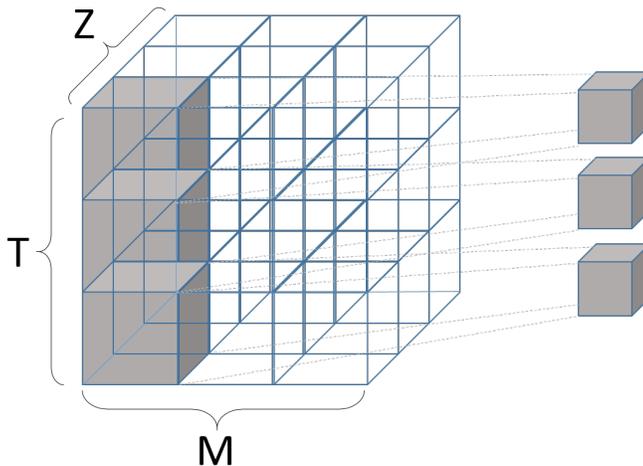
Figure 2.1: Conditional additivity for $\mathcal{T} \times \mathcal{M} \times \mathcal{Z}$.

Lemma 2.2.3

Let $\succcurlyeq$ be an unconditional Choquet preference relation with respect to a capacity ν on 2^Ω. Suppose that the events in partition $\mathcal{P}_\mathcal{T}$ are unambiguous and that for each $E \in \mathcal{P}_\mathcal{M}$, the conditional preference $\succcurlyeq_E$ is obtained by applying a f-Bayesian updating rule. Then, the conditional preference $\succcurlyeq_E$ admits SEU representation.

Figure 2.1 illustrates Proposition for 3-fold Cartesian product. When the conditioning event is the shaded vertical block, then each block can be separated due to Proposition 2.2.2 which means the conditional capacity is additive.

This result seems to be intuitive. In two-fold product space, where one coordinate is assumed to be perceived as being unambiguous while the others are not, a state can be categorized into two components: the unambiguous state and the other unambiguous states. After the uncertainty governing the ambiguous states are completely resolved, the true state now only depends upon the component that was ex-ante perceived unambiguous. The f-Bayesian fulfills this intuition by preserving the additive-separability and letting the decision maker behave as an expected utility maximizer with respect to an updated probability distribution on $\mathcal{T}$.

In the following sections, we apply the results to signaling games and explore how the equilibrium behaviors can be different compared to the perfect Bayesian equilibrium.

Chapter 3

Dempster-Shafer Equilibrium and Ambiguous Signals

3.1   Introduction

This chapter revisits the notion of the Dempster-Shafer Equilibrium (DSE) introduced by Eichberger and Kelsey (2004) as a solution concept for signaling games under ambiguity. Players are modeled via *Choquet* expected utility theory à la Schmeidler (1989). In Schmeidler's theory, beliefs are represented by capacities, a class of non-additive probabilities.

In the setup of Eichberger and Kelsey (2004), ambiguity is about the strategic behavior of the opponents but not about the types. So it is assumed that the types of the Sender are ex-ante perceived as unambiguous events. This is often taken for granted in incomplete information games due to the assumption that the probability distribution on the types is public information to all players. However, when the

definition of Nehring (1999) is applied for modeling unambiguous events, the assumption restricts the preferences of the players and also the equilibrium behaviors. The goal of this paper is to investigate how the assumption of unambiguous types limits the DSE behavior; in fact, it narrows down the DSE behavior almost to the same behavior in Perfect Bayesian Equilibrium (PBE).

Our main result states a restriction over the conditional preferences when the preferences are described with product space. Let us consider a Receiver's capacity defined over a Cartesian product of two finite sets, $\mathcal{T} \times \mathcal{M}$, types and messages. Each type is represented by a row-marginal event while each messages is represented by a column-marginal event. Assume that a probability distribution on the types is publicly known and thus the capacity reveals all the types to be unambiguous, while the messages are revealed to be ambiguous. Then it is shown that if the conditioning event is a message, which is perceived as being ambiguous, and the Dempster-Shafer updating rule is used to revise the ex-ante Choquet preference, then the derived conditional preference must exhibit the expected utility form. This result is free of the assumption that the capacity is convex or non-convex (ambiguity-averse or not).

Next result states that the constraint of the conditional preferences further restricts on the ex-ante preferences of the Receiver. Ryan (2002a) firstly provided an example in which DSE violates the belief persistence axiom: a rationality axiom over the belief change with new information. It requires the belief change to be smallest from the existing belief. He proposed the Robust Dempster-Shafer Equilibrium (RDSE) which are the DSEs survived from the process of eliminating the DSEs which violate the belief persistence axiom. In his example, the Receiver exhibited expected utility preference at the RDSE. We generalize his result to all class of signaling games under the assumption of unambiguous types: in any separating RDSE, the Receiver is ex-ante expected utility maximizer. Although the result is striking, we point out that

the violation of belief persistence axiom is not caused from the equilibrium definition itself, rather from the fact that the support is perceived as being ambiguous. If the support is perceived to be unambiguous to the players, then the belief persistence is maintained while the ambiguous support may (and may not) violate the belief persistence axiom.

We next investigate the impact of the assumption of unambiguous types over the equilibrium behavior at the limit of a sequence of DSEs, so called the Dempster-Shafer Equilibrium Limit (DSEL). In signaling games, the Receiver revises his belief by applying the Dempster-Shafer updating rule suggested by Dempster (1968) and Shafer (1976), and axiomatically justified by Gilboa and Schmeidler (1993). One attraction of the Dempster-Shafer rule is that it enables updating on the information sets that are off-the-equilibrium-paths (i.e., on the events of measure zero). Because of this property, the DSEL has been suggested as a novel refinement tool for the multiple PBE (Eichberger and Kelsey, 2004). The refinement idea is to consider the sequence of the DSEs which converges to PBE. Note that PBE often does not have well-defined beliefs off-the-equilibrium-paths as it is not well defined on probability-zero event. On the other hand, a conditional capacity derived by the Dempster-Shafer updating rule might be well-defined, even in the sequence of DSEs which converges to PBE. Thus, DSEL can provide a refinement of PBEs. However, this refinement becomes less powerful under the assumption of unambiguous types. With the Dempster-Shafer updating rule, the Receiver's conditional capacities in any sequence of DSEs must be additive. Therefore, if the sequence of DSEs converges to PBE, then DSEL must coincide with PBE. As a consequence, DSEL cannot be used as a refinement tool for PBE. The selection becomes possible only if the types are assumed to be ambiguous.

When the assumption of unambiguous types is relaxed, however, we can reexamine

the application of DSEL as a test-tool for PBE. Under the ambiguous types, some conditional capacities in DSEL can be non-additive although its unconditional capacity is additive; for example, the conditional capacity whose conditioning event is the action off-the-equilibrium path. Non-additive belief itself might not attract much attention, but it should if it directs the different behavior from the behavior of PBE as it may cause a deviation of the opponent's behavior. More specifically, we examine whether a PBE is immune against v ambiguity by exercising whether such a non-additive belief can be constructed.

3.2   Game Description

The class of signaling games studied here is described as follows. There are two players called "the Sender"(S) and "the Receiver"(R). In the ex-ante stage, Nature draws a type for the Sender from the set of types $\mathcal{T} = \{t_j\}_{j=1}^{J}$ according to a probability distribution p. The Sender learns her type and then chooses a message from $\mathcal{M} = \{m_k\}_{k=1}^{K}$, the set of messages. In the interim stage, the Receiver observes the message, but not the type, and selects a response from $\mathcal{R} = \{r_l\}_{l=1}^{L}$, the set of responses, and the game ends. Final payoffs are given by $u^i : \mathcal{T} \times \mathcal{M} \times \mathcal{R} \to \mathbb{R}$ for $i \in \{S, R\}$. We assume $J, K, N \geq 2$ and denote this class of signaling games by Γ.

The players hold beliefs about the opponent's behavior. For the Sender, a capacity ν^S defined on Σ^S, the set of all subsets of the mappings from $\mathcal{M}$ to $\mathcal{R}$, which reflects his beliefs about the Receiver's action(s) that will be chosen in response to each message $m \in \mathcal{M}$. Similarly, the Receiver's capacity ν^R is defined on Σ^R, the set of all subsets of $\mathcal{T} \times \mathcal{M}$, and it represents the Receiver's ex-ante joint belief about the Sender's strategic choice of messages and her possible types. For notational

convenience, let $T_j := \{t_j\} \times \mathcal{M}$ and $M_k := \mathcal{T} \times \{m_k\}$ denote the marginal events of the product space $\mathcal{T} \times \mathcal{M}$. We follow the previous notations: $\mathcal{P}_{\mathcal{T}} := \{T_j\}_{j=1}^J$ and $\mathcal{P}_{\mathcal{M}} := \{M_k\}_{k=1}^K$ denote the partitions; $\Sigma(\mathcal{P}_{\mathcal{T}})$ and $\Sigma(\mathcal{P}_{\mathcal{M}})$ are the algebras of events generated by the respective partitions.

In the interim stage, the Receiver observes the message sent by the Sender and revises his beliefs in accordance with the Dempster-Shafer updating rule. The Receiver's conditional beliefs, represented by the updated capacities $\{\nu_m^R\}_{m \in \mathcal{M}}$, reflect his inference about the Sender's types from messages.

In games under incomplete information, it is taken for granted that the probability distribution p over the types is publicly available to the players. To incorporate this probabilistic information into the Receiver's ex-ante beliefs, we follow the approach of Eichberger and Kelsey (2004), and assume that the Receiver's capacity agrees with the probability distribution p on $\mathcal{T}$.

Assumption 3.2.1

The Receiver's capacity ν^R agrees with p on $\mathcal{T}$. That is,

$$\nu^R(T_j) = p(T_j) \quad for \quad j = 1, \ldots, J. \tag{3.1}$$

However, a (non-convex) capacity which agrees with p on $\mathcal{T}$ does not necessarily reveal the events in partition $\mathcal{P}_{\mathcal{T}}$ to be unambiguous. For that to be true, the additive-separability condition of Definition 2.2.1 is further required.

Assumption 3.2.2

The Receiver's capacity ν^R reveals the types of the Sender to be unambiguous. That

is, for any $A \subseteq \mathcal{T} \times \mathcal{M}$,

$$\nu^R(A) = \sum_{j=1}^{J} \nu^R(A \cap T_j). \tag{3.2}$$

Note that the two Assumptions together assure that the perception of ambiguity solely stems from the messages to be sent by the Sender.

The equilibrium definition relies on a support notion. The support of a capacity reflects the actions of a player that are perceived possible by the opponent. There are various notions of support including Dow and Werlang (1994) and Marinacci (2000). The support notion of Dow and Werlang guarantees the existence for any capacity, but it may not be unique. The notion of Marinacci, however, provides the uniqueness whereas the existence is not guaranteed. Ryan (2002b) explores the support notions from an epistemological perspective. For a more detailed analysis about the support, see Eichberger and Kelsey (2014). In this study, the definition of Dow-Werlang is adopted to ensure non-emptiness. A Dow-Werlang (DW) support is the smallest event whose complement has the capacity value zero.

Definition 3.2.3
A DW-support of a capacity ν is an event $D \subset S$ such that $\nu(D^c) = 0$ and $\nu(F^c) > 0$ for any $F \subset D$.

The Dempster-Shafer equilibrium (DSE), suggested by Eichberger and Kelsey (2004), constitutes an equilibrium in beliefs.[1]

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[1]The DSE is an extension of the notion of "equilibrium in beliefs" in static games introduced by Dow and Werlang (1994); Eichberger and Kelsey (2000); and further studied by Marinacci (2000) and Haller (2000). The notion of equilibrium in beliefs assumes only pure strategies. There exist another approach in the literature which explicitly allows for mixed strategies, including Lo (1996), Klibanoff (1996), Bade (2011) and Lehrer (2012) for the static games; Kajii and Ui (2005) and Azrieli and Teper (2011) for dynamic games under incomplete information.

Definition 3.2.4

A Dempster-Shafer equilibrium (DSE) is a family of beliefs $[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}]$ for which there exist the associated supports satisfying

$$(i) \quad (t^*, m^*) \in supp(\nu^R) \quad \Rightarrow \quad m^* \in \underset{m \in \mathcal{M}}{\operatorname{argmax}} \int_{\mathcal{R}} u^S(m, r(m), t^*) d\nu^S,$$

$$(ii) \quad r^*(m) \in supp(\nu^S) \quad \Rightarrow \quad r^*(m) \in \underset{r \in \mathcal{R}}{\operatorname{argmax}} \int_{\mathcal{T}} u^R(m, r, t) d\nu_m^R \quad \forall m \in \mathcal{M},$$

$$(iii) \quad \nu_m^R \text{ is derived by the Dempster-Shafer updating rule.}$$

If the ex-ante capacities are additive, the definition of DSE coincides with the definition of perfect Bayesian equilibrium (PBE) in behavioral strategies.[2] The definition consists of three components. The first two conditions require the consistency of the actual behavior with the players' beliefs about the opponents' actions. That is, any action which belongs to the support of a player must be a best response of the opponent.[3] The last condition requires the Receiver to follow the Dempster-Shafer updating rule whenever possible. The existence of a DSE is guaranteed (see Eichberger and Kelsey, 2004, Proposition 2). Lemma 2.2.3 provides the first implication of the assumption of unambiguous types on the DSE notion. That is, the Receiver can only exhibit ambiguity-sensitive preferences in the ex-ante stage. After a message is observed, however, the Receiver no longer perceives ambiguity and uses a conditional probability distribution to make an inference about the type of the Sender.

In signaling games, two sorts of equilibria are of special interest: the separating and pooling equilibrium in pure strategies. Following the convention, in a separating

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[2]Note that the Dempster-Shafer rule coincides with Bayes rule if a capacity is additive.

[3]However, note that there is a subtle difference between two players. The condition (i) restricts the candidates in the Receiver's ex-ante beliefs.

DSE, each type of the Sender sends a different message; in a pooling DSE, all types of the Sender send the same message. Denote by $\psi_t(\nu^R) = \{m \in \mathcal{M} \mid (t, m) \in supp(\nu^R)\}$ the set of messages that are in the support of the Receiver's equilibrium capacity ν^R.

Definition 3.2.5

A DSE, $\left[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}\right]$, is called

(i) *separating if $\psi_t(\nu^R) \cap \psi_{t'}(\nu^R) = \emptyset$ for any $t, t' \in \mathcal{T}$, and*

(ii) *pooling if $\psi_t(\nu^R) = \psi_{t'}(\nu^R)$ for any $t, t' \in \mathcal{T}$.*

For given DSE, $\left[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}\right]$, let $\mathcal{B}(DSE) = \{supp(\nu^S), supp(\nu^R)\}$ be the equilibrium actions supported by the ex-ante equilibrium beliefs. Two equilibria, DSE^1 and DSE^2, are called *behaviorally equivalent* if they support the identical equilibrium actions; i.e., $\mathcal{B}(DSE^1) = \mathcal{B}(DSE^2)$.

3.3 Separating Equilibrium Behavior

The assumption of unambiguous types forces the Receiver's conditional preference to be an expected utility preference. If it is further assumed that the Sender is an SEU maximizer, the only ambiguity is about the messages. Given this "least" departure from the Bayesian framework, it is interesting to ask whether the DSE notion is flexible enough to accommodate behavior that differs from the PBE behavior. In this section, we address this question in the context of a separating DSE behavior.

As exemplified below, the ambiguity perceived at the ex-ante stage is sufficient for the existence of a separating behavior that is incompatible with the standard PBE notion.
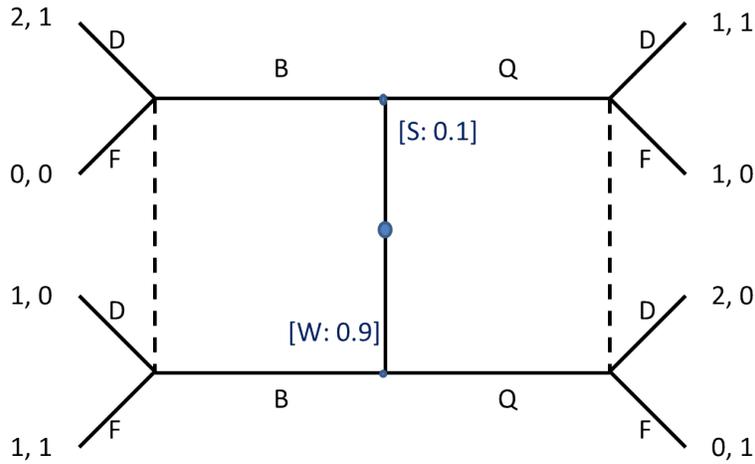
Figure 3.1: A variant of Beer and Quiche Game.

Example 3.3.1

Consider the game in Figure 4.2. It is a variant of Beer and Quiche game of Cho and Kreps (1987).[4] It can be verified that the game has neither pooling nor separating (pure-strategy) PBE. However, there exists the unique PBE in which the weak type mixes two messages while the strong type only sends message Q.[5] Notice that separating PBE does not exist because the Sender has an incentive to deviate once she anticipates that the Receiver will learn the Sender's true type from a message. For instance, the separation where the strong type sends Q, while the weak types sends B, cannot constitute a PBE. Once the Receiver learns that message Q has been sent by the strong type, he optimally responds by playing D. Knowing that D is played after Q, the weak type should profitably deviate by playing Q instead of B.

However, when the Receiver's beliefs are non-additive, the separating behavior - where

---

[4]In Figure 4.2, Q represents Quiche, B Beer, D Dismiss, and F Fight.

[5]The unique PBE is $\left[\left(Q, \left(\frac{8}{9}B + \frac{1}{9}Q\right)\right), \left(F, \left(\frac{1}{2}D + \frac{1}{2}F\right)\right)\right]$ where the strong type sends Q while the weak type mixes between B and Q; the Receiver responds with F after observing message B, and mixes between D and F after observing Q.

the strong type sends Q and weak B - can be explained by a DSE. The family of beliefs that constitute the separating DSE is defined in Table 3.1, 3.2, and 3.3. For given α_1 and α_2, the Receiver's conditional Choquet expected utility is

$$
\int_{\mathcal{T}} u^R(m, r, t)\ d\nu_m^R \;=\;
\begin{cases}
\frac{0.1}{1-\alpha_2} & \text{if } (m, r) = (Q, D), \\[2ex]
\frac{0.9-\alpha_2}{1-\alpha_2} & \text{if } (m, r) = (Q, F), \\[2ex]
\frac{0.1-\alpha_1}{1-\alpha_1} & \text{if } (m, r) = (B, D), \\[2ex]
\frac{0.9}{1-\alpha_1} & \text{if } (m, r) = (B, F).
\end{cases}
$$

Hence, it is optimal for the Receiver to respond with F regardless of the message he observes. Due to the pessimistic updating, the Receiver cannot infer the true type and this induces the Sender to separate. That is, by anticipating that the Receiver is incapable of learning the true type, the Sender will secure the certain payoff 1 rather than exposing herself to "strategic" uncertainty.

Table 3.1: Receiver's ex-ante capacity.

E	$\nu^R(E)$
$\{(t_s, Q)\}$	α_1
$\{(t_s, B)\}$	0
$\{(t_w, Q)\}$	0
$\{(t_w, B)\}$	α_2
$\{(t_s, B), (t_s, Q)\}$	0.1
$\{(t_w, B), (t_w, Q)\}$	0.9
$\{(t_s, Q), (t_w, Q)\}$	α_1
$\{(t_s, B), (t_w, B)\}$	α_2
$\{(t_s, B), (t_w, Q)\}$	0
$\{(t_s, Q), (t_w, B)\}$	$\alpha_1 + \alpha_2$
$\{(t_s, B), (t_s, Q), (t_w, B)\}$	$0.1 + \alpha_2$
$\{(t_s, B), (t_s, Q), (t_w, Q)\}$	0.1
$\{(t_w, B), (t_w, Q), (t_s, Q)\}$	$0.9 + \alpha_1$
$\{(t_w, B), (t_w, Q), (t_s, B)\}$	0.9

$$0 < \alpha_1 \leq 0.1 \ \text{and} \ 0 < \alpha_2 < 0.8$$
$$supp(\nu^R) = \{(t_s, Q), (t_w, B)\}$$

Table 3.2: Receiver's conditional capacity.

m	E	$\nu_m^R(E)$
$\{(t_s, Q), (t_w, Q)\}$	T_s	$\frac{0.1}{1-\alpha_2}$
	T_w	$\frac{0.9-\alpha_2}{1-\alpha_2}$
$\{(t_s, B), (t_w, B)\}$	T_s	$\frac{0.1-\alpha_1}{1-\alpha_1}$
	T_w	$\frac{0.9}{1-\alpha_1}$

$$supp(\nu_B^R) = \{T_s, T_w\}$$
$$supp(\nu_Q^R) = \{T_s, T_w\}$$

Table 3.3: Sender's capacity.

m	E	$\nu_m^S(E)$
Q	$\{D\}$	0
	$\{F\}$	1
B	$\{D\}$	0
	$\{F\}$	1

The driving force behind such DSE is the pessimistic attitude of the Dempster-Shafer updating rule. Notice that prior to observing a message, the support of the Receiver's capacity contains the event that the strong type sends Quiche but does not include the event that the weak type sends Quiche. However, after Quiche is observed, the Receiver's conditional capacity, which is a probability, attaches positive values to

both types. That is, the Receiver distrusts the message by taking into account that Quiche could also have been sent by the weak type. The pessimistic belief change hinders the Receiver from learning the true type, although the Sender reveals her private information by sending a different message for each type. This behavior is impossible under the Bayesian paradigm where, at any separating PBE, the Receiver learns the Sender's private information by ascribing probability one to the type who sent the message observed.

However, Ryan (2002a) questioned such DSE. He pointed out that the beliefs on-the-equilibrium-path may conflict with the so-called belief persistence axiom. Broadly speaking, the belief persistence postulate is a qualitative requirement for the updated beliefs to reflect the ex-ante beliefs as accurately as possible (see Battigalli and Bonanno, 1997).[6]

Definition 3.3.1

For a given capacity ν on Σ, a conditional capacity ν_E is said to respect the belief persistence if for all $E \in \Sigma$, it satisfies

$$supp(\nu) \cap E \neq \emptyset \implies supp(\nu_E) = supp(\nu) \cap E. \qquad (3.3)$$

In Example 3.3.1, the Receiver's conditional capacity assigns a strictly positive value to the weak type after observing Quiche, although the event that caused the week type to send Quiche was outside of the ex-ante support. To put it differently, the support of the Receiver's conditional capacity expands, relative to the ex-ante support, by adding the states that were deemed impossible at the ex-ante stage. To eliminate such irregularity, Ryan advocated to consider the DSEs that comply with the

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[6]The belief persistence axiom is one of the rationality principles postulated in the theory of belief change in the tradition of Alchourron, Gardenfors, and Makinson (1985). Stalnaker (1998) provides a concise exposition on the AGM theory.

principle of belief persistence, so called robust Dempster-Shafer equilibrium (RDSE).

However, if the belief persistence axiom is exogenously imposed on the beliefs on-the-equilibrium-path of a separating DSE, for all conditioning events in $\Sigma(\mathcal{P}_\mathcal{M})$, , then a remarkable result follows.

Proposition 3.3.2

Consider a signaling game in Γ with $|\mathcal{M}| = |\mathcal{T}| \geq 2$. Assume that the Receiver's capacity ν^R reveals the types to be unambiguous. At any separating DSE, provided it exists, if the Receiver's capacity respects the belief persistence axiom on $\Sigma(\mathcal{P}_\mathcal{M})$, then the Receiver must exhibit (ex-ante) SEU preferences.

Under the joint assumption of unambiguous types and belief persistence, the separating equilibrium precludes any ambiguity perception of the Receiver.

Remark: For the signaling games with two types and two messages, the imposition of the belief persistence axiom only for single message is enough to have Proposition 3.3.2. In general, however, satisfying the belief persistence axiom conditioning on single message is not enough. Interestingly though, to have the Receiver learn, without violating the belief persistency, at least three messages are required.

Ryan provided an example for intrinsically identical limitation. In his particular game, the Receiver's capacity is assumed to be an E-capacity. Since the E-capacity reveals the types of the Sender to be unambiguous, Ryan's result (2002a, Proposition 4.1) is a corollary of Proposition 3.3.2.

Corollary 3.3.3

In any separating DSE of Ryan's game which respects the belief persistence axiom, the Receiver displays (ex-ante) SEU preferences.

Hence, to model a separating equilibrium behavior that is due to the ambiguity

perception of the Receiver, one needs to relax either the belief persistence axiom or the assumption of unambiguous types (or both).[7] Let us assume for the rest of the section that the Receiver's capacity does not reveal the types to be unambiguous. When both the types and messages are ambiguous, then the Receiver's conditional beliefs may again violate the belief persistence axiom. Two questions rise. First, is there a way to respect the belief persistence under the pessimistic updating? Second, if the belief persistence is maintained, what behavior can be captured by the DSE notion?

Let us firstly address the former question. The main reason for the violation of the belief persistence axiom lies on the fact that the Dempster-Shafer updating rule assigns positive values to the events which were ex-ante valued zero. However, the events with the capacity value zero do not need to be unambiguous (i.e., such events are not necessarily Savage-null). An event E is said to be Savage-null if for any $f, g \in \mathcal{F}$, $f \sim g_E f$. Clearly, the Savage-null events are the unambiguous events with the capacity value zero.

Lemma 3.3.4

Let ν be a capacity on Σ. An event $E \in \Sigma$ is Savage-null if and only if E is unambiguous and $\nu(E) = 0$.

Indeed, the Receiver's equilibrium capacity ν^R (see Table 3.1) reveals its ex-ante support to be an ambiguous event. That is, the states outside of the support (e.g., the event that the weak type sends Quiche) are not Savage-null; they are payoff relevant. Since such events, although valued zero, contribute to the formation of unconditional preferences, they may also affect the formation of conditional preferences. However,

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[7]Of course, one could also assume that messages are unambiguous but types are not. However, if conditioning events are unambiguous then the Dempster-Shafer updating rule coincides with the Bayes rule (Proposition 5.2, Dominiak and Lefort, 2011). In this case, the pessimism inherent in the Dempster-Shafer updating rule vanishes.

an event that is ex-ante Savage-null remains so after the pessimistic belief change, thus making the event irrelevant for the conditional preferences.

Lemma 3.3.5

Let ν be a capacity on Σ and $supp(\nu)$ be a DW-support of ν. If $supp(\nu)$ is unambiguous, then for any $E \in \Sigma$ such that $supp(\nu) \cap E \neq \emptyset$, the following holds true:

$$\nu_E(A) = 0, \quad \forall\, A \subset \big[supp(\nu)\big]^c. \tag{3.4}$$

Lemma 3.3.5 suggests a natural way of respecting the belief persistence by assuming that a DW-support of the ex-ante capacity to be perceived unambiguous.[8] When support of a capacity is assumed to be unambiguous, two properties follow: (i) its DW-support is unique, and (ii) support of conditional capacity cannot expand.

Proposition 3.3.6

Let ν be a capacity on Σ and $supp(\nu)$ is a DW-support of ν. If the $supp(\nu)$ is an unambiguous event, then (i) it is unique, and (ii) for any $E \in \Sigma$, the following is true

$$supp(\nu) \cap E \neq \emptyset \implies supp(\nu_E) \subseteq supp(\nu) \cap E. \tag{3.5}$$

Based on this observation, the second question can be explored. If the support of the Receiver's ex-ante capacity is an unambiguous event, then the Receiver's equilibrium beliefs satisfy the belief persistence axiom at any separating DSE. However, if that is ture, any such separating DSE corresponds to a behaviorally equivalent PBE, albeit the fact that the Receiver may perceive the types and messages as being ambiguous.

Proposition 3.3.7

Consider a signaling game in Γ and assume that the support of the Receiver's capacity

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[8]The requirement for the support of a capacity to be an unambiguous event is equivalent to saying that the states outside of the support are Savage-null.

supp(ν^R) is an unambiguous event. Then, for any separating DSE, provided it exists, there is a separating PBE which is behaviorally equivalent to the DSE.

This result detects a serious limitation of the principle of belief persistence which forces the Receiver to learn the true type at any separating DSE. The ambiguity about messages and types, as well as the pessimistic belief change, plays no role. The strategic behavior will not differ from that of PBE. Consequently, deviations from PBE behavior, as discussed in Example 3.3.1, cannot be modeled via DSE unless the belief persistence axiom is abandoned.

3.4   Pooling Equilibrium Behavior

In this section, we examine the effect of the assumption of unambiguous types on strategic behavior in pooling DSE. At pooling DSE, the support of the Receiver's capacity (i.e., the message on which the Sender pools) has to be ambiguous. Otherwise, the conditional capacity off-the-equilibrium-path is not well defined, and the Dempster-Shafer rule loses its attractiveness.

We start by remarking that, in pooling DSE, the Receiver's beliefs never violate the belief persistence axiom. Recall that the belief persistence axiom requires the support of a conditional capacity to be equal to the intersection of the support of an unconditional capacity and a conditioning event; provided the intersection is non-empty. Consider a pooling DSE and suppose the Sender's types are unambiguous. If the message on-the-equilibrium-path is observed, the Receiver's conditional capacity coincides with the prior distribution on the Sender's types. If a message off-the-equilibrium-path is received, then this message cannot belong to the support of the Receiver's ex-ante capacity. This makes the belief persistence being vacuously

satisfied.

Proposition 3.4.1

Consider a signaling game in Γ, and assume that the Receiver's capacity ν^R reveals the Sender's types to be unambiguous. Then, in any pooling DSE, provided it exists, it is true that for any $E \in \Sigma(\mathcal{P}_{\mathcal{M}})$:

$$supp(\nu^R) \cap E \neq \emptyset \implies supp(\nu_E^R) = supp(\nu^R) \cap E.$$

Note that the coexistence of the assumption of unambiguous types and the belief persistence at pooling DSE does not restrict the Receiver's preferences.

However, the assumption of unambiguous types substantially constrains the pooling behavior supported by DSE. In short, any pooling DSE can be explained by a PBE that captures the identical behavior. Fix a pooling DSE. When the Sender's types are perceived unambiguous, the Receiver's conditional beliefs are additive regardless of whether the message observed is on or off-the-equilibrium path. Further, the conditional beliefs on-the-equilibrium-path coincide with the prior distribution over the types, and the beliefs off-the-equilibrium-path are well-defined although the conditioning event is measured zero. Thus, one can replace the Receiver's ex-ante capacity with the prior probability and define the conditional beliefs off-the-equilibrium-path, which are arbitrary under ex-ante additivity, as dictated by the Dempster-Shafer updating rule. Given that these beliefs support an optimal behavior at the pooling DSE, these beliefs must also support the same pooling behavior in PBE.

Proposition 3.4.2

Consider a pooling DSE, and assume that the Receiver's capacity ν^R reveals the Sender's types to be unambiguous. Then, for any pooling DSE, there exists a behaviorally equivalent pooling PBE.

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However, when types are perceived as ambiguous, then the Receiver's conditional beliefs do not need to be additive. We conclude this discussion with an example illustrating a pooling DSE behavior that cannot be explained under the regime of additive beliefs.
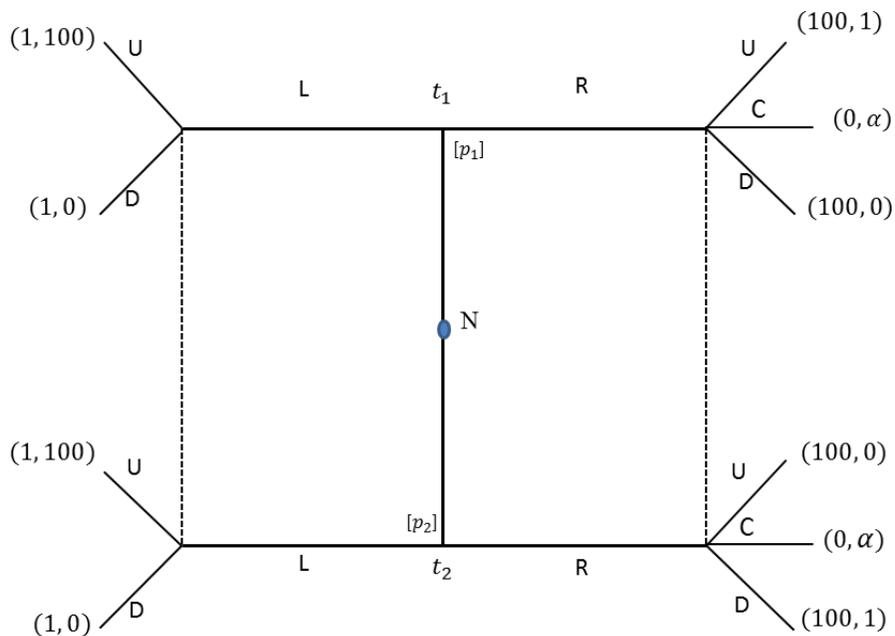
Figure 3.2: Pooling behavior under ambiguous types and messages.

Example 3.4.1

In the signaling game described by the Figure 3.2, for any $p \in (0,1)$, and $\alpha \in (0, \frac{1}{2})$, two pooling PBEs exist:

(i) Pooling on R; $\left[(R,R), (U,U), \mu(t_1 \mid L) = q \in [0,1], \mu(t_1 \mid R) = p_1 > p_2 \right]$,

(ii) Pooling on R; $\left[(R,R), (U,D), \mu(t_1 \mid L) = q \in [0,1], \mu(t_1 \mid R) = p_1 < p_2 \right]$.

For the Sender, if she pools on message L, she then always attains the payoff 1 *regardless of the Receiver's response. However, it is better for her to send message R as long as she believes that the Receiver will never play C after observing R since the payoff* 100 *will be guaranteed.*

For the Receiver, playing U after L is dominant action for any type of the Sender. Also, as long as the Receiver holds additive beliefs, playing C is never optimal after observing R. Thus, any equilibrium concept under the regime of additive beliefs of the Receiver will fail to explain the response C although it assures the payoff $\alpha \in (0, \frac{1}{2})$ to the Receiver. However, if the Sender believes that C will be the response to the message R, then she might want to consider sending L instead of R, which brings the safe payoff 1. *Tables 3.4 and 3.5 present capacities that support this behavior as pooling DSE. The types are no longer unambiguous, although the Receiver's capacity is additive on types. Note that the Receiver exhibits an extremely pessimistic attitude after observing message R, as he only takes into account the lowest outcome possible.*

$$\int_{\mathcal{T}} u^R(R, r, t) \ d\nu_R^R \ = \ \begin{cases} 0 & \text{if } r \in \{U, D\}, \\ \alpha \in (0, \frac{1}{2}) & \text{if } r = C. \end{cases}$$

Conclusively, the Receiver's pessimism leads him to play C and the Sender who has anticipated this will pool on L.

Table 3.4: Receiver's ex-ante beliefs.

E	$\nu^R(E)$
$\{(t_1, L)\}$	α_1
$\{(t_2, L)\}$	α_2
$\{(t_1, R)\}$	0
$\{(t_2, R)\}$	0
$\{(t_1, L), (t_1, R)\}$	p_1
$\{(t_2, L), (t_2, R)\}$	p_2
$\{(t_1, R), (t_2, R)\}$	0
$\{(t_1, L), (t_2, L)\}$	ρ
$\{(t_1, L), (t_2, R)\}$	p_1
$\{(t_1, R), (t_2, L)\}$	p_2
$\{(t_1, L), (t_2, L), (t_1, R)\}$	ρ
$\{(t_1, L), (t_2, L), (t_2, R)\}$	ρ
$\{(t_1, R), (t_2, L), (t_2, R)\}$	p_2
$\{(t_1, L), (t_1, R), (t_2, R)\}$	p_1

$$0 \le \alpha_1 < p_1, \ 0 \le \alpha_2 < p_2$$

$$\max\{p_1, p_2\} < \rho < 1$$

Table 3.5: Conditional beliefs.

m	E	$\nu_m^R(E)$
L	T_1	p_1
	T_2	p_2
R	T_1	0
	T_2	0

m	E	$\nu^S(E; m)$
L	$\{U\}$	1
	$\{D\}$	0
R	$\{C\}$	1
	$\{U, D\}$	0

3.5 Coherent Dempster-Shafer Equilibrium

The main rationale for Ryan's (2002a) refinement concept was his observation that the DSE might support an "implausible" behavior unless the Receiver's beliefs are restricted. However, if the equilibrium beliefs are restricted by the belief persistence axiom then, as discussed previously, deviations from PBE become unfeasible and the DSE notion looses it's descriptive power. In this section, an alternative refinement is suggested. Our approach is less stringent; while it successfully eliminates Ryan's "implausible" behavior, it is flexible enough to maintain the separating DSE of Example 3.3.1.

Let us briefly recall Ryan's argument. His game is depicted in Figure 3.3. Let m and m^o be two distinct messages. The message m is said to be strictly dominated by m^o for a type t^o, if the following holds true:

$$\min_{r \in \mathcal{R}} u^S(m^o, r(m^o), t^o) > \max_{r \in \mathcal{R}} u^S(m, r(m), t^o). \tag{3.6}$$

When all the messages $m \neq m^o$ are strictly dominated by m^o for the type t^o, then m^o is called the strictly dominant message for the type t^o.

Note that the message L and R are the strictly dominant messages for t_1 and t_2, respectively. Therefore, Ryan (2001, p. 170) argues that it *"seems reasonable that player 2 ought to choose U in any sensible analysis of this game."* However, since the Receiver (i.e., player 2) might perceive the Sender's messages as being ex-ante ambiguous, it is possible to construct a DSE in which the Sender plays her strictly dominant strategy while the Receiver always plays D giving him the pay-off of zero (for a collection of capacities supporting such DSE, see Appendix B). Ryan views such DSE as "troublesome" or "implausible."
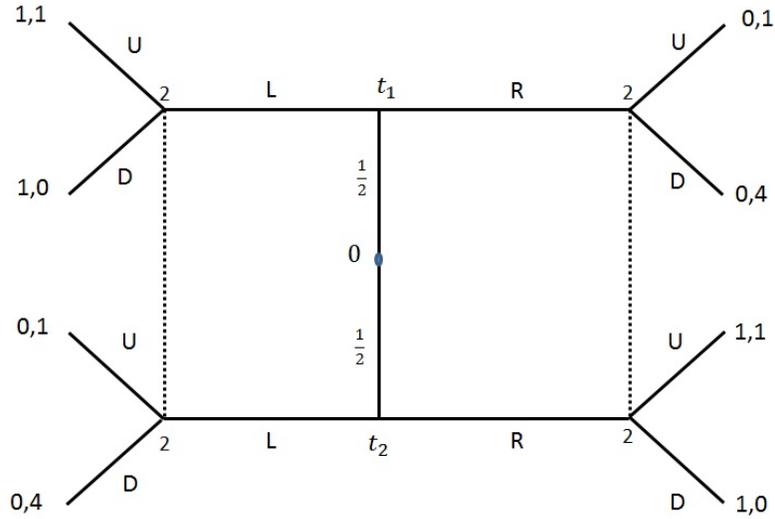
Figure 3.3: Signaling game of Ryan.

In the separating DSE above, it is again the pessimism of the updating rule that prevents the Receiver from learning the true type. At the interim stage, the Receiver regards both types of t_1 and t_2 as possible (i.e., both types are in the support of this conditional capacities). After observing L, the Receiver optimally responds with D by believing that t_2 could also have sent the message observed. However, for the Sender type t_2 sending L will never be rational. There is no additive belief over the Receiver's responses (after observing L) with respect to which sending L is optimal for t_2. The same argument applies to the Sender type t_1 and message R. Therefore, the Receiver's beliefs are inconsistent with the fact that the Sender - who by assumption is an SEU maximizer - never plays a strictly dominated strategy.

Motivated by this observation, we suggest a refinement of DSEs by demanding the Receiver's beliefs to be *coherent* with the assumption that the Sender is rational in the sense of never playing a strictly dominated message.[9]

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[9]Note that the Sender's (additive) beliefs are always coherent due to the condition (ii) of the

Definition 3.5.1

Fix a signaling game in Γ. A collection $\left\{\nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}\right\}$ of Receiver's beliefs is said to be coherent, if for each $m \in \mathcal{M}$ and $t \in \text{supp}(\nu_m^R)$, there exists a probability measure $\mu^S(m)$ on $\mathcal{R}$ so that

$$m \in B(t) = \underset{m' \in \mathcal{M}}{\arg\max} \int_{\mathcal{R}(m')} u^S(m', r(m'), t) \, \mathrm{d}\mu^S. \tag{3.7}$$

Definition 3.5.2

A Coherent Dempster-Shafer Equilibrium (CDSE) is a DSE $\left\{\nu^S, \nu^R, \{\nu^R\}_{m \in \mathcal{M}}\right\}$ in which the Receiver's beliefs are coherent.

It is important to remark that the separating DSE of Example (3.3.1) is utterly different. Although the Receiver believes that both types could have sent the message on-the-equilibrium-path (thus violating the belief persistence axiom), for each type off-the-equilibrium-path there exist an additive probability measure which rationalizes sending the message the Receiver factually observes. The separating DSE is coherent.

The notion of coherent beliefs constrains the Receiver's equilibrium beliefs as the belief persistence does. However, there is a substantial difference. The restriction imposed by the coherence notion is weaker and directly related to the player's payoff structure. Consider a separating DSE in which there is a type t^o with a strictly dominant message m^o on-the-equilibrium-path. If the Receiver's beliefs are coherent then the state (t^o, m^o) is perceived as being ex-ante unambiguous. Nonetheless, the massage m might still be ambiguous.

Proposition 3.5.3

Fix a signaling game in Γ. Assume that there exists (t^o, m^o) such that m^o is the

definition of DSE.

strictly dominant message for the Sender type t^o, and that the types are unambiguous. At any CDSE $\{\nu^S, \nu^R, \{\nu^R\}_{m \in \mathcal{M}}\}$, if ν^R is convex and $(t^o, m^o) \in supp(\nu^R)$, then $\{(t^o, m^o)\}$ is unambiguous.

Clearly, at any separating CDSE in which all the states in the support of the Receiver's ex-ante capacity consists of the pairs of strictly dominant messages and types, then the Receiver with convex capacity must have an ex-ante SEU preference.

Corollary 3.5.4

Let $\{\nu^S, \nu^R, \{\nu^R\}_{m \in \mathcal{M}}\}$ be a CDSE of a game in Γ. Assume that for every pair $(t, m) \in supp(\nu^R)$, m is the strictly dominant message for type t. If ν^R is convex, then the Receiver's ex-ante preference is SEU.

It is, thus, not surprising that in Ryan's game the restrictions imposed by the coherent beliefs are the same as those by the belief persistence axiom. Since each type has strictly dominant message, the Receiver has to be an SEU maximizer at any coherent DSE of Ryan's game; which in fact is PBE.

3.6 Dempster-Shafer Equilibrium Limit and Unambiguous Types

A shortcoming in signaling games with expected utility players is to define the beliefs off-the-equilibrium-paths. In Weak Perfect Bayesian Equilibrium, any additive belief which supports the equilibrium action being optimal is acceptable. The Perfect Bayesian Equilibrium further requires the equilibrium beliefs to be derived by the Bayes updating rule wherever possible. For the beliefs off-the-equilibrium-paths,

however, it does not provide a restriction as it is not regarded as the case of "wherever possible." To overcome this nuisance of Bayesianism, the sequential equilibrium further requires for the equilibrium belief system to have a sequence of beliefs at which the Bayes updating rule is well defined and the sequence converges to the equilibrium beliefs on and off; regardless of whether it is a "right" sequence.

A forté of the Dempster-Shafer updating rule is to provide well-defined conditionals as long as the measure of the complement of the conditioning event is strictly less than 1. Thus when players are allowed to have non-additive beliefs and they use the Dempster-Shafer updating, we may not encounter the plethora of off-the-equilibrium-beliefs although now we may encounter the plethora of on-the-equilibrium-beliefs. But if we consider a sequence of non-additive equilibrium beliefs which eventually converges to an additive PBE, then the limit of the conditional beliefs might be different with that of PBEs off-the-equilibrium-beliefs.

In the following, we demonstrate this advantage of DSEs in two perspectives: one to refine the beliefs of PBEs by assuming the Dempster-Shafer updating, and the other to test the equilibrium behavior of a PBE whether the equilibrium itself is mighty enough with a departure from additive-beliefs.

It begins with the definition of the Dempster-Shafer Equilibrium Limit (DSEL) introduced in Eichberger and Kelsey (2004). DSEL is defined to be the limit of the sequence of DSEs of which (ν^S, ν^R) converge to additive probabilities. To measure a distance from additive probabilities, a DSE is said to display the degree $\lambda = \max\{\lambda(\nu^S), \lambda(\nu^R)\}$ of strategic ambiguity such that:

$$\lambda(\nu^i) = 1 - \max_{E \in \Sigma^i}[\nu(E) + \nu(E^c)], \quad i \in \{S, R\}. \tag{3.8}$$

Definition 3.6.1

A tuple $\left[\bar{\nu}^S, \bar{\nu}^R, \{\bar{\nu}^R_m\}_{m\in\mathcal{M}}\right]$ is called Dempster-Shafer Equilibrium Limit if it is the limit of a sequence of the Dempster-Shafer Equilibria $\left\{\nu^S_n, \nu^R_n, \{\nu^R_{n,m}\}_{m\in\mathcal{M}}\right\}^{\infty}_{n=1}$ such that the degree of ambiguity $\lambda_n \to 0$ as $n \to \infty$. Consistent DSEL is a DSEL where all capacities are well-defined at every point of sequence.

Our focus lies on consistent DSEL to eliminate all sequences of DSEs in which the conditional beliefs are arbitrarily constructed due to the undefined denominator in the Dempster-Shafer rule.
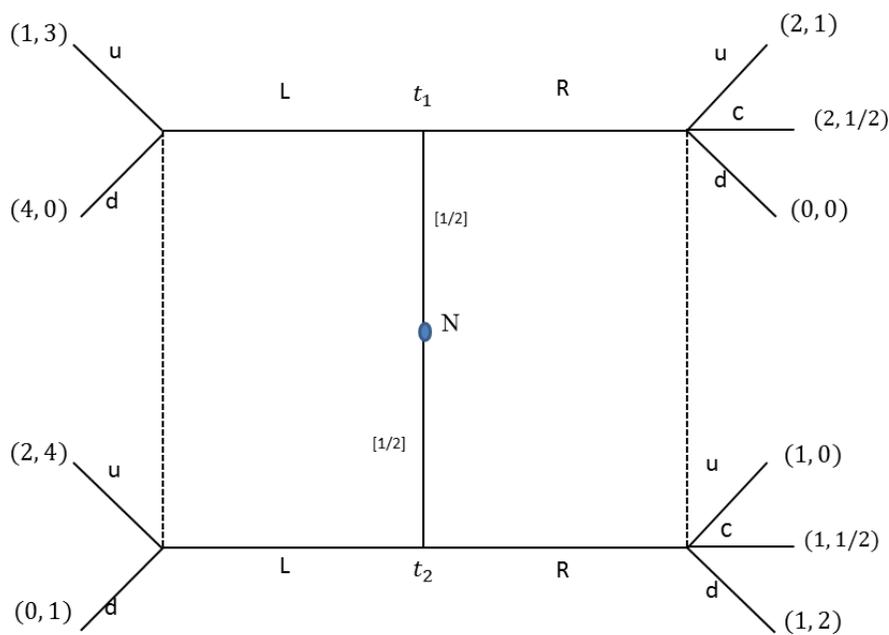

Figure 3.4: Signaling Game for DSEL.

Example 3.6.1

In the signaling game described by Figure 3.4, pooling and separating PBEs coexist:

(i) Pooling on L; $[(L, L), (u, d), \mu(t_1 \mid L) = \frac{1}{2}, \mu(t_1 \mid R) = q \leq \frac{2}{3}]$,

(ii) Separating; $[(R, L), (u, u), \mu(t_1 \mid L) = 0, \mu(t_1 \mid R) = 1]$.

We firstly explain how the ambiguity can play a role to refine some beliefs off-the-equilibrium-path; i.e., $q \leq \frac{2}{3}$ in pooling equilibria. Let us assume that the Receiver's non-additive belief is described by E-capacity. Then any DSE with the same pooling behavior, the conditional belief which is the belief after observing the message R, $\nu_R^R(T_1)$ is equal to $\frac{1}{2}$. Notice that the conditional belief is additive as E-capacity reveals the types to be unambiguous. Hence, if the Receiver's belief is represented by E-capacity, then only $q = \frac{1}{2}$ is uniquely determined for the belief off-the-equilibrium-path.

In general, however, there should be no "right" way to pin down a capacity which is believed to represent the Receiver's true belief. The proposition below states the limitation of this approach under the assumption of unambiguous types.

Proposition 3.6.2

For any game in Γ such that ν^R agrees with p on $\mathcal{T}$ and reveals the types to be unambiguous, the followings are true:

(i) For each PBE, there exists a consistent DSEL which converges to the PBE

(ii) Consistent DSEL is PBE.

To illustrate the result, consider the pooling equilibrium in the Example 3.6.1. The Receiver's capacity which reveals the types to be unambiguous should have the form

$$
\begin{array}{c|c|c|c}
 & L & R & \\
\hline
t_1 & \alpha_n & 0 & p(t_1) = \frac{1}{2} \\
t_2 & \beta_n & 0 & p(t_2) = \frac{1}{2}
\end{array}
$$

below. For each n, $\alpha_n, \beta_n \leq \frac{1}{2}, \alpha_n + \beta_n < 1$. And as $n \to \infty$, $(\alpha_n, \beta_n) \to (\frac{1}{2}, \frac{1}{2})$. Fix $q = \frac{1}{3}$, then the conditional belief after observing R should satisfy the following:

$$
\alpha_n, \beta_n \leq \frac{1}{2}, \qquad \alpha_n + \beta_n < 1,
$$

$$
\nu_R^R(T_1) = \frac{p_1 - \alpha_n}{1 - (\alpha_n + \beta_n)} = \frac{1}{3}, \quad \nu_R^R(T_2) = 1 - \nu_R^R(T_1)
$$

For $p_1 = \frac{1}{2}$, we attain the equation between α_n and β_n:

$$
\alpha_n = \frac{1}{4} + \frac{1}{2}\beta_n
$$

Thus, for any increasing sequence β_n converging to $\frac{1}{2}$, and the α_n coupled with such β_n will pick up the PBE of $q = \frac{1}{3}$.

The first result states that similar construction is possible for any additive belief especially for the one off-the-equilibrium paths. Thus if the types are assumed to be unambiguous, then the refinement for the beliefs off-the-equilibrium-paths is equivalent to selecting a form of capacities which might be believed to reflect the true belief of the Receiver. This selection is by no means better than any behavioral approach such as intuitive criterion of Cho and Kreps (1987). The second result confirms that the set of consistent DSEL is equivalent with the set of PBEs excluding the DSELs which allow arbitrary construction of the beliefs. Therefore, we conclude that maintaining the assumption of unambiguous types does not provide a stringent refinement tool.

The assumption of unambiguous types is once relaxed, however, DSEL allows non-

additive conditional beliefs. Thus the refinement of additive beliefs might be no longer available; but, it can eliminate a PBE which is less mighty against non-additive beliefs than the other(s). In Example 3.6.1, there is a response m of the Receiver which can be seen as reasonable to play; although it can never be optimal to play if the Receiver's capacity is additive. Also for the Sender with the type of t_2, playing R may be attractive to play. Note that both actions offer a constant payoff regardless of the opponent's response. We will be looking for a DSEL which supports those behaviors. If such a DSEL exists, then it immediately causes the deviation of one of players from the (pooling) PBE and hence we may conclude that the PBE is not immune against non-additive beliefs.

Definition 3.6.3

A PBE is not immune against non-additive beliefs if there exists a DSEL which converges to the PBE such that:

  *(i) There exists an action **a** to a player which is different with the action of the PBE,*

 *(ii) The action **a** must induce a deviation of the opponent to attain strictly higher payoff in the PBE.*

From the Example 3.6.1, consider the following sequence of DSEs for the Receiver; note that the Sender's belief can be simply assumed as the same with the PBE (i.e., $\nu_n^S(u; L) = 1, \nu_n^S(d; R) = 1$ for any n). The limit coincides with the pooling PBE as λ_n goes to 0.

The conditional capacities off-the-equilibrium-paths are: $\nu_R^R(T_1) = \gamma = \nu_R^R(T_2)$ and they are strictly non-additive. Consequently, the CEU payoff for each response of

Table 3.6: A sequence of DSEs.

E	$\nu_n^R(E)$
$\emptyset$	0
$\{(t_1, L)\}$	$(1 - \lambda_n)p_1$
$\{(t_2, L)\}$	$(1 - \lambda_n)p_2$
$\{(t_1, R)\}$	0
$\{(t_2, R)\}$	0
$\{(t_1, L), (t_1, R)\}$	p_1
$\{(t_2, L), (t_2, R)\}$	p_2
$\{(t_1, L), (t_2, L)\}$	$(1 - \lambda_n)$
$\{(t_1, R), (t_2, R)\}$	0
$\{(t_1, L), (t_2, L), (t_1, R)\}$	$1 - (1 - \gamma)\lambda_n$
$\{(t_1, L), (t_2, L), (t_2, R)\}$	$1 - (1 - \gamma)\lambda_n$
$\{(t_1, R), (t_2, L), (t_2, R)\}$	$1 - (1 - \lambda_n)p_1$
$\{(t_1, L), (t_1, R), (t_2, R)\}$	$1 - (1 - \lambda_n)p_2$
S	1

the Receiver is the below:

$$
CEU^R(r; \nu_R^R) \;=\; \begin{cases} \gamma & \text{if } r = u, \\ 2\gamma & \text{if } r = d, \\ \frac{1}{2} & \text{if } r = m. \end{cases}
$$

For $\gamma < \frac{1}{4}$, the Receiver will respond with m rather than d. Thus we have an action m of the Receiver which is different with the pooling PBE, and if the Receiver's response is m, then the Sender with type t_1 has an incentive to deviate from playing L to R to receive strictly better outcome. Hence, we conclude that the PBE pooling on L is not immune against non-additive beliefs.

3.7   Conclusion

A main motivation for introducing ambiguity into games is to explain strategic behavior that cannot be captured as an equilibrium within the Bayesian framework. Following Aumann (1985), "[...] a solution concept should be judged more by what it does than by what it is". In line with such motivation, we considered a "least" departure from the Bayesian setup by assuming that the ambiguity is perceived solely by the uninformed player, and examined the descriptive power of the DSE compared to the PBE notion.

The contribution of this chapter is twofold. On the one hand, our results unfolded the tension between the descriptive power of DSE concept and two assumptions imposed on the equilibrium beliefs: the unambiguous types and the belief persistence axiom. As a matter of fact, ambiguity cannot exist at all when both assumptions are simultaneously enforced. Taken individually, each assumption admits ambiguity but surprisingly limits the equilibrium behavior. The assumption of unambiguous types restricts the pooling DSE, and the belief persistence axiom the separating DSE, to be behaviorally equivalent with the PBE. As such, the results point out the limitations of the DSE notion under the two assumptions.

On the other hand, the results highlight the signaling behavior that is incompatible with the standard PBE, but with the DSE notion. Interestingly, the Sender's private information could be successfully revealed, not because the Receiver learns from the message observed, but because he distrusts it. This equilibrium behavior features the pessimism of the Dempster-Shafer rule, and will never be observed under the updating rules such as Bayes' rule which comply with the belief persistence axiom. We believe that the DSE concept provides a new perspective on the prevalent understanding of separating behavior in signaling games.

For pooling DSE behavior to be different from PBE, the Receiver has to perceive ambiguity both about the types and messages. Under the pessimistic belief change, ambiguity in the interim stage is only admissible when the types are ex-ante ambiguous. The Receiver may perceive the types being ambiguous either because he is not confident about the probabilistic information publicly available, or because such information is missing at all.

Gilboa and Schmeidler (1993) axiomatized the whole family of f-Bayesian updating rules for Choquet preferences. Besides the Dempster-Shafer rule, the family contains another extreme, the optimistic updating rule. One direction of future research would be to scrutinize the impact of the optimistic belief change on the strategic signaling under ambiguity and compare the behavioral differences between the two different approaches to signaling games.

Chapter 4

Signaling Games under Bayes' updating

4.1 Introduction

Bayes' rule is pervasive in and out of Economics to be often understood as a feature of rationality. The rule is simple, intuitive, and has an axiomatic foundation of the conditional Subjective Expected Utility (SEU) preferences (Ghirardato, 2002). Further, other non-Bayesian updating rules such as Dempster-Shafer by Dempster (1967) and Shafer (1976), and Full-Bayesian by Jaffray (1992) coincides with the Bayes' rule when the measure is additive. Exploring beyond the class of expected utility preferences, however, it makes the Bayes' rule be an updating rule among others. That being said, the Bayes' rule can be applied to more general preferences.

In this chapter, we maintain the Bayes' rule; but, allow the preferences of players to go beyond the expected utility class. Specifically, we assume that the players

are Choquet expected utility (CEU) maximizers à la Schmeidler (1989) and update their preferences by Bayes' rule. With CEU maximizing players, their (non-additive) beliefs are modeled via capacity, and each player's expected payoff is evaluated via the integral of Choquet (1954) with respect to own utility function and capacity (see Definition 2.1.2). Moreover, due to Gilboa and Schmeidler (1993), their conditional preferences preserve Choquet expected utility form.

They play a signaling game. In signaling game, the informed player, the sender, may successfully reveal her private information due to the *learning* of the uninformed player, the receiver, who infers the private information from the message sent by the opponent. Such behavior is explained by the solution concept, perfect Bayesian equilibrium, and learning, in general, is understood as a consequence of Bayes' updating. Such learning might be natural to the players with probabilistic beliefs. However, it is unclear to the players with non-additive beliefs. An important question of this study is whether such a *learning* would be still achievable by maintaining the Bayes' rule but abandoning the probabilistic beliefs.

A noteworthy assumption in this study is that the players not only share the probability distribution over the types of the sender but also regard the types to be unambiguous in the sense of Nehring (1999). As shown in Proposition 2.2.2 and Lemma 2.2.3, this assumption enforces the conditional preferences of the receiver to be an subjective expected utility (SEU). And hence, the ambiguity that the receiver may perceive stays only on the messages at the ex-ante stage.

As a solution concept, we technically use the same perfect Bayesian equilibrium (PBE) definition. However, the players' beliefs are allowed to be non-additive. To distinguish this point, it is named as quasi perfect Bayesian equilibrium (QPBE).[1]

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[1]The interpretation of PBE is suitably amended to equilibrium in beliefs. This tradition is based on the interpretation of mixed strategy equilibrium as *pure* strategy equilibria under equilibrium

With the notion of QPBE, we firstly state its existence. The analyses of its descriptive power ensue the existence statement. At separating QPBE, the receiver learns the type of the sender although the messages are perceived as ambiguous. Moreover, the separating equilibrium exists with all degree of ambiguity. Thus, the Bayes' rule assures the robustness of separating PBE against ambiguity. At pooling QPBE, however, equilibrium behavior can be much different from that of PBE. To have an introspection on the result, it is worthy to remark the behavioral characterization of the Bayes' rule under CEU by Gilboa and Schmeidler (1993). It is interpreted as an optimistic updating rule because the player regards the new information as "a-good-news." That is, the decision maker updates his preferences in a way that the worst outcome could have happened if this new event had not been occurred. In signaling games, this optimism is reflected that the receiver regards an ambiguous signal as a favorable one in the sense that his posterior weights more on what he had believed than on the public information; the probability distribution over the types. As shown in the sequel, a consequence of this optimism will be the posterior which is inconsistent with the public information. This optimistic belief change in turn brings different equilibrium behavior distinct from PBE.

In any QPBE, however, the public information no more provides a good description about the upcoming state of the world as it did in PBE. This suggests that the common prior assumption in signaling games may be entirely unrelated to the prediction of the outcome of a game although it is respected as unambiguous events in the ex-ante stage. Further, with different updating rules; e.g., the Dempster-Shafer rule

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beliefs. It has been explored in Harsanyi (1973) and Aumann (1987) among others. Briefly, Harsanyi viewed the mixed strategy equilibrium as pure strategy equilibria attained under the perturbed game in which each player is subjected to the small independent random perturbations onto the payoffs of opponent players. Aumann dropped the assumption of perturbation and explain a mixed strategy equilibrium as a special (and unnatural) point on which all players possess the same beliefs about what the opponents will play and about these beliefs are common knowledge.

of Dempster (1967) and Shafer (1976), the prediction of single signaling game can be much different from the classical prediction by PBE. We believe, this is a news to the experiments of signaling games.

The structure of this chapter is as follows: Section 4.2 introduces the new solution concept, quasi perfect Bayesian equilibrium, and state its existence. In Section 4.3, the descriptive power of QPBE compared to PBE is studied. Section 4.4 discusses the implications of the assumptions of common prior and updating in signaling game experiments.

4.2 Quasi Perfect Bayesian Equilibrium

For Choquet preference model, see § 2.1 and for the class of signaling games studied here, consult § 3.2.

As it was the case in the previous chapters, we follow the convention of Harsanyi (1967, 1968a,b) and manage the uncertain payoffs by introducing Nature who randomly assigns the type of the Sender according to a probability distribution. The probability distribution is public information, and hence we assume that the receiver's ex-ante capacity reveals the types to be unambiguous.

Assumption 4.2.1

The receiver's capacity ν^R reveals the probability distribution p on types of the sender to be unambiguous. That is,

$$\nu^R(T_j) = p(T_j) \ \ for \ \ j = 1, \ldots, J, \tag{4.1}$$

$$\nu^R(A) = \sum_{j=1}^{J} \nu^R(A \cap T_j), \quad \forall A \subseteq \mathcal{T} \times \mathcal{M}. \tag{4.2}$$

New solution concept is also in the same vein of equilibrium in beliefs. Thus, the equilibrium definition relies on the notion of support. There exist at least two distinct notions of support of a capacity: Dow and Werlang (1994) and Marinacci (2000). The support notion of Dow and Werlang, which is a smallest event whose complement has the capacity value zero, guarantees the existence for any capacity, but it may not be unique. The notion of Marinacci, however, provides the uniqueness whereas the existence is not guaranteed.

Definition 4.2.2

A DW-support of a capacity ν is an event $D \subset S$ such that $\nu(D^c) = 0$ and $\nu(F^c) > 0$ for all $F \subset D$,

A M-support of a capacity ν is the set $\{s \in S : \nu(\{s\}) > 0\}$.

The definition below extends the perfect Bayesian equilibrium in signaling games to non-additive capacities.

Definition 4.2.3

A Quasi Perfect Bayesian Equilibrium (QPBE) is a family of capacities $[\nu^S, \nu^R, \{\nu^R_m\}_{m \in \mathcal{M}}]$ for which there exist the associated supports such that:

$$(i) \quad (t^*, m^*) \in supp(\nu^R) \quad \Rightarrow \quad m^* \in \underset{m \in \mathcal{M}}{\operatorname{argmax}} \int_{\mathcal{R}} u^S(m, r(m), t^*) d\nu^S,$$

$$(ii) \quad r^*(m) \in supp(\nu^S) \quad \Rightarrow \quad r^*(m) \in \underset{r \in \mathcal{R}}{\operatorname{argmax}} \int_{\mathcal{T}} u^R(m, r, t) d\nu^R_m \quad \forall m \in \mathcal{M},$$

$$(iii) \quad \nu^R_m \text{ is derived by the Bayes' rule.}$$

Notice that if the ex-ante capacities are additive, the definition coincides with the definition of perfect Bayesian equilibrium in behavioral strategies. Also, if the condition (iii) is replaced with the Dempster-Shafer rule, then the definition coincides with the Dempster-Shafer Equilibrium of Eichberger and Kelsey (2014) (see Definition 3.2.4).

Recall that, under the unambiguous types, the conditional Choquet preferences admit SEU representation after an f-Bayesian updating (by Lemma 2.2.3). Thus, the following is an immediate corollary of Lemma 2.2.3.

Proposition 4.2.4

Consider a signaling game in Γ, and assume that the receiver's capacity ν^R reveals the types to be unambiguous. In any QPBE, the receiver exhibits conditional SEU preferences.

Two equilibria will be in our special interest: separating and pooling equilibrium. At separating equilibrium, each type of the sender sends different messages and the receiver responds optimally for each observation of the message. At pooling equilibrium, all types send the same message and the receiver optimally responds. To capture this, we require the ex-ante beliefs of the receiver satisfy the following definition at equilibrium. Denote by $\psi_t(\nu^R) = \{m \in \mathcal{M} \mid (t, m) \in supp(\nu^R)\}$ the set of messages that are in the support of the Receiver's equilibrium capacity ν^R.

Definition 4.2.5

A QPBE, $\left[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}\right]$, is called

(i) *separating if $\psi_t(\nu^R) \cap \psi_{t'}(\nu^R) = \emptyset$ for any $t, t' \in \mathcal{T}$, and*

(ii) *pooling if $\psi_t(\nu^R) = \psi_{t'}(\nu^R)$ for any $t, t' \in \mathcal{T}$.*

For given QPBE, $\left[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}\right]$, let $\mathcal{B}(QPBE) = \{supp(\nu^S), supp(\nu^R)\}$ be the equilibrium actions supported by the ex-ante equilibrium beliefs. Two equilibria, $QPBE^1$ and $QPBE^2$, are called *behaviorally equivalent* if they support the identical equilibrium actions; i.e., $\mathcal{B}(QPBE^1) = \mathcal{B}(QPBE^2)$.

Now is for the statement of the existence of QPBE.

Proposition 4.2.6

For any game in Γ, quasi perfect Bayesian equilibrium exists in which the receiver perceives the types to be unambiguous.

4.3 Equilibrium Behaviors

As implied by Proposition 4.2.4, the Receiver's beliefs are allowed to be non-additive only when he has not observed any message; i.e., at the ex-ante stage. To focus on the role of the Bayes' updating and to be simpler, the sender is assumed to have probabilistic beliefs.

Assumption 4.3.1

The sender's beliefs are additive.

For separating behaviors, QPBE cannot predict different behavior from that of PBE.

Proposition 4.3.2

Fix a game in Γ with $|\mathcal{M}| \geq |\mathcal{T}|$. Assume that the sender's beliefs are additive, and that the types are unambiguous. If there exists separating QPBE, then there exists separating PBE which supports the same equilibrium behavior;i.e., for all separating QPBEs and PBE,

$$\mathcal{B}(QPBE) = \mathcal{B}(PBE).$$

Note that Proposition 4.3.2 does not variate with different notions of support. As long as the receiver updates his beliefs with the Bayes' rule, the separating equilibrium will be achieved.

For pooling QPBE, there may not exist any PBE which explains the same behavior of the QPBE. This is mainly because the receiver who uses the Bayes' rule may

not respect the public prior information about the unambiguous distribution of the types. However, all deviations are the pooling behaviors within (different) probability priors. That is, a pooling QPBE can be explained by a PBE under the game with different prior.

Proposition 4.3.3

Fix a game in Γ and a probability distribution $p \in \Delta(\mathcal{T})$. Denote the set of equilibrium behaviors of pooling (Q)PBEs for given p by $\mathcal{B}((Q)PBE, p)$ under the M-support notion. Then the following is true:

$$\mathcal{B}(QPBE, p) = \bigcup_{q \in \Delta(\mathcal{T})} \mathcal{B}(PBE, q).$$

Notice that Proposition 4.3.3 almost holds with DW-support notion.[2]

To illustrate the point, consider the game in Figure 4.1. In the game, the sender has three types; t_1, t_2, t_3, and two messages; L, R. The receiver can respond with one of three actions; U, M, D. As the number of available messages are smaller than that of the types, full separation is infeasible for this game; actually, partial signaling is neither possible. Further notice that playing D is the strictly dominant response to the receiver after observing L. Thus, any sensible equilibrium outcome must be pooling on R: $[(R, R, R), (D, U); \mu^R(\cdot) = p(\cdot)]$.

However, there exists another pooling QPBE which explains the behavior that the sender pools on R, and the receiver responds with D after observing R. This QPBE

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[2]A difference between two support notions occurs when there exist some states whose ex-ante capacity values are equal to zero but the states are in DW-support. Hence, it has to be shown that there exists a probability $q \in \mathring{\Delta}(T)$ such that $\int u^R(r^*, m^*, t) dq \geq \int u^R(r, m^*, t) d\nu_m^R$ for all $r \in \mathcal{R}$. Notice that there must exist such q if $\int u^R(r^*, m^*, t) d\nu_m^R > \int u^R(r, m^*, t) d\nu_m^R$ for every $r \in \mathcal{R}$. However, it is unclear whether such q exists when $\int u^R(r^*, m^*, t) d\nu_m^R = \int u^R(r, m^*, t) d\nu_m^R$ for some $r \in \mathcal{R}$.
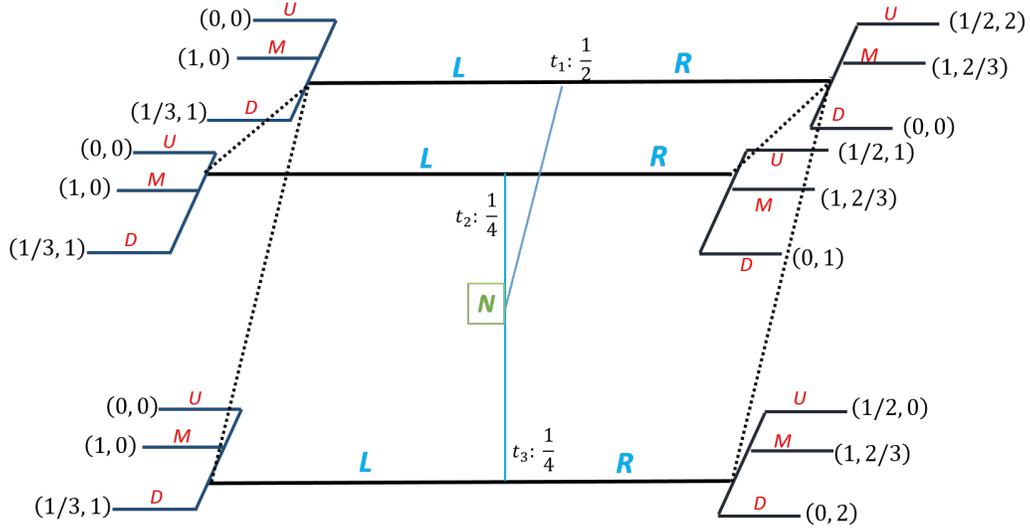
Figure 4.1: QPBE may be inconsistent with the public information.

occurs when the receiver ex-ante assigns values close to zero on the events that the types t_1 or t_2 would send the message R. That is, the receiver believes that 't_1 sends R' and 't_2 sends R' are close to be ambiguously impossible.

Although such equilibrium behavior is attainable, it is still unclear how such behavior can be explained from the given information. A clear note is that the absolute degree of perception of the receiver is less important than the relative degree of ambiguity for each type. Notice that the degree of ambiguity itself can be a small number, although the degree of ambiguity that the receiver perceives over the events of 't_1 or t_2 sends the message R' can be large enough to attain such QPBE.

From the following proposition, it is necessary that the receiver must view that the types and messages are somewhat correlated at all such QPBEs. If the types and messages are perceived as being *stochastically independent*, then QPBE in which the receiver's conditional beliefs disobey the public information is unattainable.

Definition 4.3.4

Let p be a probability measure on $2^{\mathcal{T}}$ and μ a capacity on $2^{\mathcal{M}}$. The capacity ν on $2^{\mathcal{T} \times \mathcal{M}}$ is called independent product of p and μ if it satisfies for every $T \subseteq \mathcal{T}$, $M \subseteq \mathcal{M}$,

$$\nu(T \times M) = p(T)\,\mu(M). \tag{4.3}$$

Proposition 4.3.5

Fix a game in Γ and a probability distribution $p \in \Delta(\mathcal{T})$. Assume that the types are unambiguous, and that the senders's beliefs are additive. Fix a pooling QPBE. If the receiver's capacity is an independent product capacity for some μ, $\mathcal{B}(QPBE, p) = \mathcal{B}(PBE, p)$.

Hence, to have a different pooling behavior, the receiver's capacity must be a form beyond the independent product capacity in the sense of Ghirardato (1997). Notice that by Nehring (1999), the product capacity which reveals the types to be unambiguous is unique and it is E-capacity of Eichberger and Kelsey (1999a).

Recall that the QPBE inconsistent with the public information was attainable even with small departure from the probabilistic belief. Looking at such QPBE from a different standpoint, we may ask how far we may go to attain a QPBE which can explain the same behavior of each PBE of interest. That is, we may ask how PBE is robust against the degree of ambiguity. Requiring the class of capacities for revealing the types to be unambiguous in the sense of Nehring (1999), we may find QPBE which is behaviorally equivalent to PBE for all degree of ambiguity.

Proposition 4.3.6

Fix a game in Γ. For each PBE, there exists QPBE that is behaviorally equivalent for every $\lambda \in (0, 1)$.
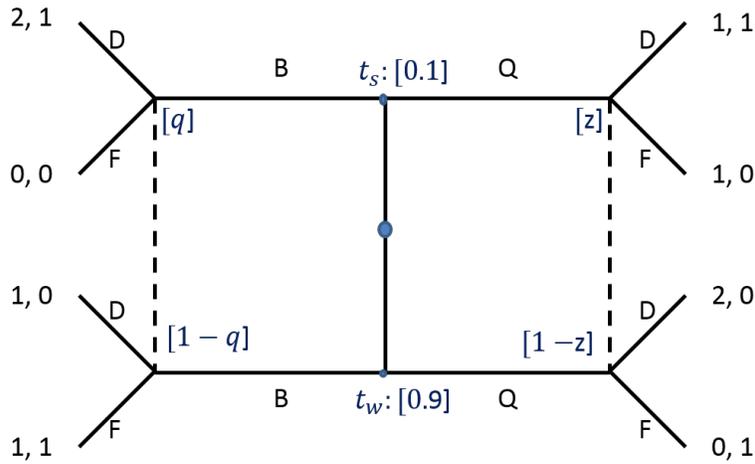
Figure 4.2: A variant of Beer and Quiche Game.

4.4 Note on Experiments

All the discussions hitherto provides a crucial introspection that a simple departure from the Bayesian paradigm can support different behaviors of a signaling game as equilibria. For example, in the signaling game of Example 3.3.1, there exists a separating DSE which is not captured by any PBE;moreover, $\mathcal{B}((C)DSE) \neq \mathcal{B}(PBE) \neq \mathcal{B}(QPBE)$. To illustrate the point, recall the variant of Beer and Quiche game introduced in a previous chapter.

In the game of Figure 4.1, the probability distribution over the strong and weak type is publicly known, and they are 10% for strong and 90% for weak type. If the strong type sends the message Quiche, then she attains the constant payoff 1 regardless of the receiver's responses. Likewise, the weak types may secure the same payoff by sending the message Beer. The receiver can achieve positive payoff only if he correctly inferred the type of the sender; he should fight against the weak type and dismiss against the strong type.

Recall that there is no (pure) separating nor pooling PBE. However, there exist two kinds of pooling quasi perfect Bayesian equilibria; namely, $[(Q,Q),(F,D),q \leq \frac{1}{2},z \geq \frac{1}{2}]$ and $[(B,B),(D,F),q \geq \frac{1}{2},z \leq \frac{1}{2}]$.[3] Note that the receiver's conditional beliefs do not respect the probability distribution over the types although each equilibrium is pooling.

Consider the QPBE at which the sender pools on B. At the ex-ante stage, it is feasible to construct the receiver's belief such that he is extremely ambiguous about the event of the weak type sends the message B. Specifically, assume that he attaches the measure $\frac{1}{20}$ on the event of $\{(t_w,B)\}$ and $\frac{2}{20}$ on $\{(t_s,B)\}$. The event of observing message Q is (unambiguously) null. Since he weights more on the event that the strong type sends B, the conditional beliefs, after observing B, will confirm what he has believed;i.e., assign strictly larger probability on the event of the message was sent by the strong type. Due to the assumption of unambiguous types, the receiver's conditional beliefs must be additive by Lemma 4.2.4. Thus, any pooling QPBE may be explained by PBE with different probability distribution on types which confirms Proposition 4.3.3.

Lastly, recall from the chapter 3 that there exist separating DSEs in which the strong type sends Q and the weak B, and the receiver responds with F regardless of the message observed.

To sum, we have a signaling game such that $\mathcal{B}((C)DSE) \neq \mathcal{B}(PBE) \neq \mathcal{B}(QPBE)$.

An implication of this can be critical to experiments in signaling games as they might have only considered the perfect Bayesian equilibrium as possible observations. In

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[3]The notation for the equilibrium consists of three components (Q,Q) describes the messages sent by the sender for each type; (F,D) describes the response of the receiver after observing B and Q respectively; and the conditional beliefs of the receiver that q represents $\nu_B^R(t_s)$ and z $\nu_Q^R(t_s)$. Note that there exist infinitely many ex-ante beliefs which can lead the conditional beliefs of q and z after Bayes' updating.
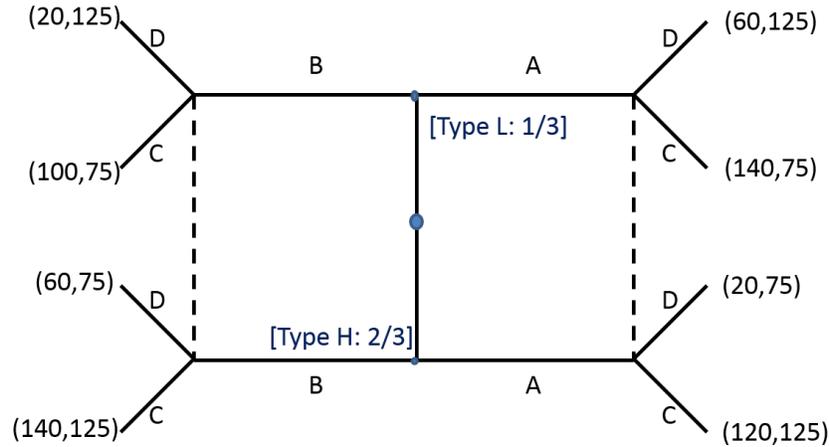
Figure 4.3: Signaling Game of Brandts and Holt (1992), "An Experimental Test of Equilibrium Dominance in Signaling Games," *The American Economic Review*, 82(5), 1350-1365, Used under the fair use, 2015.

below, we make a brief note on the experiment of Brandts and Holt (1992), a classical experiment literature of signaling games.[4]

In their experiments, the game depicted in Figure 4.3 was played and a result is reported in the Table 4.1.[5]

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[4]Although there exist many experiments in signaling games, the studies about the role of the common prior seem paid much attention to a possibility that the receiver may perceive different priors. Only early literatures were found to be possibly fault under the consideration of ambiguity. For example, the games in Goeree and Holt (2001) only contain PBE and no different behaviors can be explained with other notions. Also, the experiment of signaling games without common knowledge Drouvelis, Mller, and Possajennikov (2012) does not indicate different behaviors are feasible under non-additive beliefs of the receiver.

[5]Table 4.1 is slightly modified to explain the CDSE behavior of the game without changing the original result. In their paper, this table corresponds to Table 2. Also, the results from the treatments $3 - 5$ are not reported here as the original paper does not provide the information of which signal was sent under what types; only combination of signals and responses were reported. Part b reports when the roles is reversed and at Part c, the roles are reversed again. All procedures were announced before the experiment.

Table 4.1: An experiment result in Brandts and Holt.

Part	Signals given type(%)		Responses given signal(%)	
	B/H	A/L	C/B	C/A
a	100	79	98	26
b	100	42	100	0
c	100	25	98	40

There are two pooling PBEs: both types pool on A or B and the receiver responds optimally; i.e., $\left[(B,B),(D,C);\mu(L|A) \geq \frac{1}{2}, \mu(L|B) = \frac{1}{3}\right]$, and $\left[(A,A),(C,D);\mu(L|A) = \frac{1}{3}, \mu(L|B) \geq \frac{1}{2}\right]$.[6] Brandts and Holt mainly discuss the intuitive equilibrium - which is the equilibrium pooling on B - was hardly justified by the experimental observations.

Yet, there exist separating (coherent) DSEs in which the type L sends A and the type H, B; and the receiver responds with C for all messages. A course of argument in experiments is to compare the frequencies supporting different predictions made by distinct equilibria. For the game in Figure 4.3, the analysis assumed two different pooling equilibria which support two different observable behaviors. However, admitting a possibility of non-additive beliefs at the ex-ante stage, an observer is no more able to distinguish whether the sender is pooling or separating. Thus, all the analyses should be flawed. The question now should be an empirical one, whether the prediction of DSE or QPBE is more powerful than the existing equilibrium concept, PBE. We leave this question for a next research.

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[6]The first component of the receiver's response indicates the response after observing A and the second component indicates the response after observing B.

Table 4.2: Receiver's ex-ante capacity.

E	$\nu^R(E)$
$\{(t_L, A)\}$	α_1
$\{(t_L, B)\}$	0
$\{(t_H, A)\}$	0
$\{(t_H, B)\}$	α_2
$\{(t_L, B), (t_L, A)\}$	$\frac{1}{3}$
$\{(t_H, B), (t_H, A)\}$	$\frac{2}{3}$
$\{(t_L, A), (t_H, A)\}$	α_1
$\{(t_L, B), (t_H, B)\}$	α_2
$\{(t_L, B), (t_H, A)\}$	0
$\{(t_L, A), (t_H, B)\}$	$\alpha_1 + \alpha_2$
$\{(t_L, B), (t_L, A), (t_H, B)\}$	$\frac{1}{3} + \alpha_2$
$\{(t_L, B), (t_L, A), (t_H, A)\}$	$\frac{1}{3}$
$\{(t_H, B), (t_H, A), (t_L, A)\}$	$\frac{2}{3} + \alpha_1$
$\{(t_H, B), (t_H, A), (t_L, B)\}$	$\frac{2}{3}$

$$0 < \alpha_1 \le \tfrac{1}{3} \text{ and } 0 < \alpha_2 < \tfrac{1}{3}$$
$$supp(\nu^R) = \{(t_L, A), (t_H, B)\}$$

Table 4.3: Receiver's conditional capacity.

m	E	$\nu_m^R(E)$
$\{(t_L, A), (t_H, A)\}$	T_L	$\frac{1/3}{1-\alpha_2}$
	T_H	$\frac{2/3-\alpha_2}{1-\alpha_2}$
$\{(t_L, B), (t_H, B)\}$	T_L	$\frac{1/3-\alpha_1}{1-\alpha_1}$
	T_H	$\frac{2/3}{1-\alpha_1}$

$$supp(\nu_B^R) = \{T_L, T_H\}$$
$$supp(\nu_A^R) = \{T_L, T_H\}$$

Table 4.4: Sender's capacity.

m	E	$\nu_m^S(E)$
A	$\{D\}$	0
	$\{C\}$	1
B	$\{D\}$	0
	$\{C\}$	1

4.5 Conclusion

It is known that many decision behaviors of an economic agent are failed to be described by unique probability measure. Arguably, signaling games demand relatively

high level of cognitive efforts and hence the behaviors of the players may not be describable with unique probabilistic belief. This paper explores a direction of that possibility.

The receiver is assumed to perceive ambiguity on the messages, but not on the types. After observing an ambiguous message, he updates his ex-ante non-additive beliefs with the popular Bayes' rule. With the sender whose beliefs are additive, we show that ex-ante ambiguity on messages plays no role for the sender to separate. More precisely, any separating quasi perfect Bayesian equilibrium behavior can be supported by separating perfect Bayesian equilibrium. As long as the receiver uses the Bayes' rule to update his ex-ante beliefs and the payoff structure directs the separation, the ambiguity can play no role. This result can be different when the receiver uses the Dempster-Shafer updating. As the receiver revises his belief in a pessimistic manner, it was possible to neglect the message and follow the prior distribution on types; this is when the degree of ambiguity is 1 everywhere. However under the Bayes' rule, the receiver revises his belief in an optimistic manner. That is, he weights more on what he had believed rather than what the unambiguous information advises. As the separating equilibrium requires that the receiver should ex-ante believe that some types can never send a message, optimistic belief change will only confirm what he has known. Thus, ambiguity can play no role for a different separating behavior.

However, this optimistic belief change may completely ignore the unambiguous information; i.e., the probability distribution on the types. For a pooling quasi perfect Bayesian equilibrium, there may not exist any perfect Bayesian equilibrium supporting the identical behavior. The fuse of such behavior is the ex-ante ambiguous perception on the messages. Under the symmetric distribution on the types, the receiver may ex-post believe that a type is more likely to have sent the message

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observed because he ex-ante perceived more ambiguity on the event the other types send the message observed. Thus, he may neglect the symmetric prior information, but weights more how he believed instead.

This paper did not explore how much ambiguity perception *should* be allowed. From the stand point that the beliefs are purely subjective, any degree of ambiguity should be allowed. However, from the game theoretic perspective, there should be a reason why the receiver perceives more ambiguity on an event than another. For example, it would be puzzling to explain asymmetric ambiguity perception under a symmetric game as the payoff structure will not hint any reason for asymmetric beliefs.

Although the Bayes' rule is common and ubiquitous, a fact has been neglected that it can be still applied to an updating of non-additive beliefs. This paper is innovative in that regard. Moreover, for a signaling game, the equilibrium behaviors can be much different with different assumption on how the receiver updates his ex-ante beliefs. An implication to signaling games is that the previous analyses based on the solution concept of perfect Bayesian equilibrium may be seriously flawed as a small departure from Bayesian paradigm may lead an entirely opposite behavior.

Appendix A

Appendix: Proofs

A.1 Proofs for Chapter 2.

Proposition 2.2.2 Let $\succcurlyeq$ be an unconditional Choquet preference relation with respect to a capacity ν, and $\mathcal{P} = \left\{ U_1, ..., U_m \right\}$ be a partition of S consisting of unambiguous events. Take an event $E \in \Sigma$. If the conditional preferences $\succcurlyeq_E$ is derived by an h-Bayesian updating rule, then the events in partition $\mathcal{P}$ remain unambiguous after updating. That is, for any $A \in \Sigma$:

$$\nu_E(A) = \sum_{k=1}^{m} \nu_E(A \cap U_k). \tag{A.1}$$

Proof. Fix $U \in \mathcal{P}$ and $E \in \Sigma$. For any $f, g, k, k' \in \mathcal{A}$, the following is true

$$fUk \succsim_E gUk$$

$$\Longleftrightarrow \qquad (fUk)Eh \succsim (gUk)Eh \qquad\qquad (A.2)$$

$$\Longleftrightarrow \quad (fEh)U(kEh) \succsim (gEh)U(kEh)$$

$$\Longleftrightarrow \quad (fEh)U(k'Eh) \succsim (gEh)U(k'Eh) \qquad\qquad (A.3)$$

$$\Longleftrightarrow \qquad (fUk')Eh \succsim (gUk')Eh$$

$$\Longleftrightarrow \qquad fUk' \succsim_E gUk'$$

where (A.2) is followed from the Definition 2.1.3 and (A.3) from (2.7). Thus, we conclude

$$fUk \succsim_E gUk \quad \Longleftrightarrow \quad fUk' \succsim_E gUk'.$$

By Proposition 3.1 in Dominiak and Lefort (2011), $\succsim_E$ reveals every event in $\mathcal{P}$ to be unambiguous in the sense of (A.1).

$\square$

Lemma 2.2.3 Let $\succsim$ be an unconditional Choquet preference relation with respect to a capacity ν on 2^Ω. Suppose that the events in partition $\mathcal{P}_\mathcal{T}$ are unambiguous and that for each $E \in \mathcal{P}_\mathcal{M}$, the conditional preference $\succsim_E$ is obtained by applying a f-Bayesian updating rule. Then, the conditional preference $\succsim_E$ admits SEU representation.

Proof. Fix $D \in \Sigma$ and $E \in \mathcal{P}_\mathcal{M}$. Observe that $\nu_E(A) = 0$ for any $A \subseteq E^c$:

$$\big(\nu(D \cup E) - \nu(D \cap E^c)\big) \nu_E(A) = \nu\left((A \cap E) \cup (D \cap E^c \cap A)\right) - \nu\left(D \cap E^c \cap A\right)$$
$$= \nu\left(D \cap E^c \cap A\right) - \nu\left(D \cap E^c \cap A\right)$$
$$= 0$$

From the equation (2.2.3), we have:

$$\big(\nu(D \cup E) - \nu(D \cap E^c)\big) \nu_E(U_j) = \nu\left((U_j \cap E) \cup (D \cap E^c \cap U_j)\right) - \nu\left(D \cap E^c \cap U_j\right)$$

Then, $\sum_{j=1}^{n} \nu_E(U_j) = 1$ if and only if the following is true:

$$\nu(D \cup E) - \nu(D \cap E^c) = \sum_{j=1}^{n} \left[\nu\left((U_j \cap E) \cup (D \cap E^c \cap U_j)\right) - \nu\left(D \cap E^c \cap U_j\right) \right] \quad \text{(A.4)}$$

The following is true:

$$(U_j \cap E) \cup (D \cap E^c \cap U_j) = \left((U_j \cap E) \cup D\right) \bigcap \left(\underbrace{(U_j \cap E) \cup (E^c \cap U_j)}_{= U_j}\right)$$
$$= (U_j \cap E) \cup (D \cap U_j)$$
$$= U_j \cap (E \cup D)$$

Thus, RHS of the equation (A.14) can be rewritten as:

$$\sum_{j=1}^{n} \left[\nu\left(\underbrace{(U_j \cap E) \cup (D \cap E^c \cap U_j)}_{U_j \cap (E \cup D)}\right) - \nu\left(D \cap E^c \cap U_j\right) \right]$$

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$$= \sum_{j=1}^{n} \left[\nu \left(U_j \cap (E \cup D) \right) - \nu \left(D \cap E^c \cap U_j \right) \right]$$

$$= \nu(D \cup E) - \nu(D \cap E^c) \quad : \text{LHS of the equation (A.14)}$$

Therefore, $\sum_{j=1}^{n} \nu_E(U_j) = 1$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

A.2 Proofs for Chapter 3.

Proposition 3.3.2 Consider a signaling game in Γ with $|\mathcal{M}| = |\mathcal{T}| \geq 2$. Assume that the Receiver's capacity ν^R reveals the types to be unambiguous. At any separating DSE, provided it exists, if the Receiver's capacity respects the belief persistence axiom on $\Sigma(\mathcal{P}_\mathcal{M})$, then the Receiver must exhibit (ex-ante) SEU preferences.

Proof. Let $\left[\nu^S, \nu^R, \{\nu^R_m\}_{m \in \mathcal{M}} \right]$ be a separating DSE. By the definition of separating equilibrium, we may order types and messages so that, without loss of generality, $supp(\nu^R) = \{(t_1, m_1), \ldots, (t_J, m_J)\}$. That is, at separating equilibrium, the Sender sends m_j when her type is t_j with $j = 1, \ldots, J$. By the assumption of unambiguous types, let $p_j = \nu^R(T_j)$ denote the probability of type j revealed by ν^R. By the definition of DW-support, $\nu^R(\{s\}) = 0$ for any $s \notin supp(\nu^R)$. Thus, the equilibrium capacity ν^R on the states in $\mathcal{T} \times \mathcal{M}$ can be expressed as follows

$$\nu^R \big(\{(t_j, m_k)\} \big) = \begin{cases} 0 & \text{for } j \neq k, \\ \alpha_j \in [0, p_j] & \text{for } j = k, \text{ where } 0 \leq \sum \alpha_j \leq 1. \end{cases}$$

Notice that the definition of the DW-support and the assumption of unambiguous types suffice to conclude that if $\alpha_j = p_j$ for all $j = 1, \ldots, J$. Then, ν^R is additive

regardless of whether it is convex or not.

For any $E \in \Sigma(\mathcal{P}_{\mathcal{M}})$, suppose that the conditional capacity ν_E^R, derived from the equilibrium capacity ν^R, respects the belief persistence axiom. Fix a type $j = 1, \ldots, J$. Let $E = M_{-j}$ the messages sent by the types other than j. By the belief persistence axiom, we must have that $(t_j, m_j) \notin supp(\nu_E^R)$ and $\nu_E^R(T_j) = 0$. That is,

$$\nu_E^R(T_j) = \frac{\nu^R\left((T_j \cap M_{-j}) \cup M_j\right) - \nu^R(M_j)}{1 - \nu^R(M_j)} = 0. \tag{A.5}$$

Notice that $\nu^R(M_j) = \sum_{i=1}^{J} \nu^R(M_j \cap T_i) = \nu^R(M_j \cap T_j) < 1$. Hence, ν_E^R is well-defined. Thus, Equation (A.5) holds true if and only if

$$\nu^R\left((T_j \cap M_{-j}) \cup M_j\right) = \nu^R(M_j). \tag{A.6}$$

Since types are unambiguous, Equation (A.6) can be equivalently written as

$$\nu^R(T_j) + \sum_{i \neq j} \nu^R(T_i \cap M_j) = \sum_{i=1}^{J} \nu^R(T_i \cap M_j)$$
$$\nu^R(T_j) = \nu^R(T_j \cap M_j) = \alpha_j.$$

Thus, $\alpha_j = p_j$ for each $j = 1, \ldots, J$, and we conclude that ν^R must be additive at the separating DSE. Since the chosen equilibrium was arbitrary, the statement holds true for any separating DSE. $\qquad\square$

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Lemma 3.3.4 Let ν be a capacity on Σ. An event $E \in \Sigma$ is Savage-null if and only if E is unambiguous and $\nu(E) = 0$.

Proof. Call an event $E \in \Sigma$ as *dummy* if and only if $\nu(F \cup E) = \nu(F)$ for any $F \in \Sigma$. Due to Schmeidler (1989, p.586), E is Savage-null if and only if it is dummy. It remains to be shown that E is dummy if and only if E is unambiguous and $\nu(E) = 0$.

For sufficiency, assume E is dummy. Then $\nu(E) = 0$; otherwise $\nu(\emptyset \cup E) > 0$ which contradicts the assumption that E is dummy. Clearly, any $D \subset E$ is also dummy: $\nu(F \cup D) = \nu((F \cup D) \cup E) = \nu(F \cup E) = \nu(F)$ for any $F \in \Sigma$. Thus, for any $F \in \Sigma$, we have the following:

$$\begin{aligned}
\nu(F) &= \nu\big((F \cap E) \cup (F \cap E^c)\big) = \nu(F \cap E^c) \\
&= \nu(F \cap E) + \nu(F \cap E^c).
\end{aligned}$$

For necessity, assume E is unambiguous and $\nu(E) = 0$. Then, it is straight forward that for any $F \in \Sigma \setminus \emptyset$:

$$\begin{aligned}
\nu(F \cup E) &= \nu\big((F \cup E) \cap E\big) + \nu\big((F \cup E) \cap E^c\big) = \nu(F \cap E^c) \\
&= \nu(F).
\end{aligned}$$

For $F = \emptyset$, E is dummy because $\nu(E) = 0$. Therefore, we conclude that E is Savage-null if and only if E is unambiguous and $\nu(E) = 0$. $\qquad\square$

Lemma 3.3.5 Let ν be a capacity on Σ and $supp(\nu)$ its DW-support. Suppose that $supp(\nu)$ is revealed to be unambiguous. Then, for any $E \in \Sigma$ such that $supp(\nu) \cap E \neq$

$\emptyset$, the following is true

$$\nu_E(A) = 0, \quad \forall\, A \subset \big[supp(\nu)\big]^c. \tag{A.7}$$

Proof. Fix E and A, and denote $supp(\nu) = D$. Note that $\nu(A) = 0$ since $A \subseteq D^c$. By definition of unambiguous events, and since $A \cap D = \emptyset$, we have

$$
\begin{aligned}
\nu_E(A) &= \frac{\nu\big((A \cap E) \cup E^c\big) - \nu(E^c)}{1 - \nu(E^c)} \\
&= \frac{\nu\big(\big((A \cap E) \cup E^c\big) \cap D\big) - \nu(E^c \cap D)}{1 - \nu(E^c \cap D)} \\
&= 0.
\end{aligned}
$$

$\square$

Proposition 3.3.6 Let ν be a capacity on Σ and $supp(\nu)$ its DW-support. If the support is an unambiguous event, then (i) it is unique, and (ii) for any $E \in \Sigma$, the following is true

$$supp(\nu) \cap E \neq \emptyset \implies supp(\nu_E) \subseteq supp(\nu) \cap E. \tag{A.8}$$

Proof. To show uniqueness, assume there exist two distinct DW-supports D and G where D is unambiguous and G may be ambiguous with respect to ν. Then there exist at least one state $s \in S$ such that $s \in D$ and $s \notin G$. The latter implies

$$\nu(\{s\}) = 0 \text{ since } \{s\} \subset G^c. \tag{A.9}$$

Now, take $F = D \setminus \{s\}$. Since D is a DW-support, $F \subset D$ implies $\nu(F^c) > 0$. But $F^c = D^c \cup \{s\}$ and since D is an unambiguous event, $\nu(F^c) = \nu(D^c \cup \{s\}) =$

$\nu(D^c) + \nu(\{s\}) = \nu(\{s\}) > 0$ which contradicts $\nu(\{s\}) = 0$. Thus no such G can exist, and the unambiguous DW-support must be unique.

To show (A.8), denote $supp(\nu)$ by D and $supp(\nu_E)$ by D_E. Assume $D \cap E \neq \emptyset$ but $D_E \nsubseteq D \cap E$. Then there exists $\{\omega\} \in \Sigma$ such that $\omega \in D_E$ and $\omega \notin D \cap E$.

Note that $\nu_E(\{\omega\}) > 0$; otherwise $\nu_E\big((D_E \setminus \{\omega\})^c\big) = 0$, contradicting that D is the DW-support. Since $\omega \notin D \cap E$, it is either $\omega \notin D$ or $\omega \notin E$ (or both). However, $\nu_E(\{\omega\}) = 0$ trivially for the case of $\omega \notin E$, and by the Lemma 3.3.5 for the case of $\omega \notin D$. This contradicts that $\nu_E(\{\omega\}) > 0$ and thus we conclude that $supp(\nu_E) \subseteq supp(\nu) \cap E$. $\qquad\square$

Proposition 3.3.7 Consider a signaling game in Γ and assume that the support of the Receiver's capacity $supp(\nu^R)$ is an unambiguous event. Then, for any separating DSE, provided it exists, there is a separating PBE which is behaviorally equivalent to the DSE.

Proof. Consider a game in Γ with $|\mathcal{M}| = |\mathcal{T}|$.[1] Let $[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}]$ be a separating DSE. Then, Conditions $(i), (ii)$, and (iii) of Definition 3.2.4 are satisfied. By our assumption, ν^S is additive. Thus, it suffices to show that there exists an additive

---

[1]For simplicity, we assume $|\mathcal{M}| = |\mathcal{T}|$ in the proof. However, the result can be easily extended to the case of $|\mathcal{M}| \geq |\mathcal{T}|$.

capacity of the Receiver, denoted by π^R, such that

(i) $supp(\nu^R) = supp(\pi^R),$

(ii) $r^*(m) \in supp(\nu^S) \implies r^*(m) \in \underset{r \in \mathcal{R}}{\mathrm{argmax}} \int_{\mathcal{T}} u^R(m, r, t) d\pi_m^R \;\; \forall m \in \mathcal{M},$

(iii) π_m^R is derived by Bayes' rule whenever possible.

By the definition of separating equilibrium, we may write, without loss of generality, that $supp(\nu^R) = \{(t_1, m_1), \ldots, (t_j, m_j), \ldots, (t_J, m_J)\}$. Since ν^R is the equilibrium belief, the following is true:

(1) $\nu(\{s\}) > 0$ for any $s \in supp(\nu^R)$; otherwise, it contradicts the definition of DW-support.

(2) For any $j = 1, \ldots, J$, we have that $supp(\nu^R) \cap M_j = \{(t_j, m_j)\}$, and thereby $supp(\nu_{M_j}^R) = \{(t_j, m_j)\}$. This fact follows from Proposition 3.3.6 and the non-emptiness of the DW-support.

Proposition **??** implies that $\nu_{M_j}^R(supp(\nu_{M_j}^R)) = 1$. Thus, the family of conditional capacities $\{\nu_m^R\}_{m \in \mathcal{M}}$ satisfies: For any $j = 1, \ldots, J$ and $E \subset \mathcal{T} \times \mathcal{M}$,

$$
\nu_{M_j}^R(E) = \begin{cases} 1 & \text{if } (t_j, m_j) \in E, \\ 0 & \text{otherwise.} \end{cases}
$$

Now, set $\pi^R(\{(t_j, m_j)\}) = p_j > 0$ where p is a probability distribution on $\mathcal{T}$ with $p_j > 0$. Then, $supp(\nu^R) = supp(\pi^R)$, and $\{\nu_m^R\}_{m \in \mathcal{M}} = \{\pi_m^R\}_{m \in \mathcal{M}}$. Thus, $[\nu^S, \pi^R, \{\pi_m^R\}_{m \in \mathcal{M}}]$ is a PBE and it is behaviorally equivalent to the given DSE $[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}]$. Since the DSE was chosen arbitrary, we conclude that the result is true for any existent DSE. $\qquad\qquad\square$

Proposition 3.5.3 Fix a signaling game in Γ. Assume that there exists (t^o, m^o) such that m^o is the strictly dominant message for the Sender type t^o, and that the types are unambiguous. At any CDSE $[\nu^S, \nu^R, \{\nu^R\}_{m \in \mathcal{M}}]$, if ν^R is convex and $(t^o, m^o) \in supp(\nu^R)$, then $\{(t^o, m^o)\}$ is unambiguous.

Proof. Fix a DSE $[\nu^S, \nu^R, \{\nu^R\}_{m \in \mathcal{M}}]$ and assume, without loss of generality, that m_1 is the strictly dominant message for the Sender type t_1. By the assumption of unambiguous types, it suffices to show that, for all $E \subseteq T_1$:

$$\nu^R(E) = \nu^R(E \cap \{(t_1, m_1)\}) + \nu^R(E \cap \{(t_1, m_1)\}^c). \tag{A.10}$$

By the coherency (Definition 3.5.1), $\nu^R_{m_j}(T_1) = 0$ for all $j \neq 1$. Assuming $\nu^R(M_j^c) < 1$, it is equivalent to

$$\nu^R(T_1 \cap M_j^c) = \nu^R(T_1), \quad \text{for all } j \neq 1. \tag{A.11}$$

Consider distinct $j, i \neq 1$. By the convexity and monotonicity of ν^R, we get

$$
\begin{aligned}
\nu^R(T_1) \;&\geq\; \nu^R\big(\underbrace{[T_1 \cap M_j^c] \cup [T_1 \cap M_i^c]}_{=T_1}\big) + \nu^R\big(T_1 \cap (M_j \cup M_i)^c\big) \\
&\geq\; \nu^R(T_1 \cap M_j^c) + \nu^R(T_1 \cap M_i^c).
\end{aligned}
$$

By (A.11), the inequality holds true if and only if

$$\nu^R\big(T_1 \cap (M_j \cup M_i)^c\big) = \nu^R(T_1), \quad \text{for all } i \neq j, \text{ and } i, j \neq 1.$$

Now, consider another distinct $k, j, i \neq 1$. By the convexity and monotonicity,

$$\nu^R\big(T_1 \cap (M_k \cup M_j \cup M_i)^c\big) = \nu^R(T_1), \quad \text{for all } i \neq j \neq k, \text{ and } i, j, k \neq 1.$$

By continuing this argument for all messages other than m_1, we get

$$\nu^R\left(T_1 \cap \left(\bigcup_{k=2}^{l} M_k\right)^c\right) = \nu^R(T_1), \quad \text{for any } l \geq 2. \tag{A.12}$$

Thus, for any $E \in \Sigma^R$ such that $\{(t_1, m_1)\} \subseteq E \subseteq T_1$, we have

$$\nu^R(E) = \nu^R(T_1).$$

Now, we show that $\nu^R(E) = 0$ for all E such that $\{(t_1, m_1)\} \not\subseteq E$. Consider $A = T_1 \cap M_1$ and A^c. Note that $\nu^R(A) = \nu^R(T_1)$ by (A.12). Then, by the convexity and the assumption of unambiguous types,

$$\begin{aligned}
\nu^R(A \cup A^c) &\geq \nu^R(A) + \nu^R(A^c) \\
&= \nu^R(A) + \nu^R(A^c \cap T_1) + \nu^R(A^c \cap T_1^c) \\
&= \nu^R(T_1) + \nu^R(A^c \cap T_1) + \nu^R(T_1^c).
\end{aligned}$$

The inequality holds true if and only if $\nu^R(A^c \cap T_1) = \nu^R(T_1 \cap M_1^c) = 0$. By the monotonicity, $\nu^R(F) = 0$ for all $F \subseteq (T_1 \cap M_1^c)$. In sum, we have shown that, for any $E \subseteq T_1$:

$$\nu^R(E) = \begin{cases} p_1 & \text{if } (t_1, m_1) \in E, \\ 0 & \text{otherwise.} \end{cases} \tag{A.13}$$

Equation (A.13) implies (A.10). This completes the proof showing that the state $\{(t^o, m^o)\}$ is unambiguous.

$\square$

Proposition 3.4.2 Consider a pooling DSE, and assume that the Receiver's capacity ν^R reveals the Sender's types to be unambiguous. Then, for any pooling DSE, there exists a behaviorally equivalent pooling PBE.

Proof. Let $\left[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}\right]$ be a pooling DSE. It suffices to show that there exists π^R, an additive capacity of the Receiver such that

(i) $supp(\nu^R) = supp(\pi^R)$,

(ii) $r^*(m) \in \text{supp}(\nu^S) \quad \Rightarrow \quad r^*(m) \in \underset{r \in \mathcal{R}}{\text{argmax}} \int_{\mathcal{T}} u^R(m, r, t) d\pi_m^R \quad \forall m \in \mathcal{M},$

(iii) π_m^R is derived by Bayes' rule whenever possible.

Since the Sender's types are perceived unambiguous, let $\nu(T_j) = p_j$ be the probability distribution on $\mathcal{T}$ revealed by ν^R. Assume, without loss of generality, that m_1 is the message on-the-equilibrium path. By Definition 4.2.5 and DW-support, $supp(\nu^R) = \{(t_1, m_1), (t_2, m_1), \ldots, (t_J, m_1)\}$. By Lemma 2.2.3, ν_m^R is additive for any message $m \in \mathcal{M}$. Further, for the message on-the-equilibrium-path, it is true that $\nu_{M_1}^R(T_j) = p_j$ for all $j = 1, ..., J$ by the unambiguous types and the Dempster-Shafer updating.

Now, set $\{\pi_m^R\}_{m \in \mathcal{M}} = \{\nu_m^R\}_{m \in \mathcal{M}}$ and construct π^R such that for each $j = 1, \ldots, J$:

$$
\pi^R(\{(t_j, m_i)\}) = \begin{cases} \pi_{M_1}^R(T_j) & \text{for } i = 1, \\ 0 & \text{otherwise .} \end{cases}
$$

Thus, $supp(\pi^R) = supp(\nu^R)$, and the condition (ii) above must be satisfied as $\{\pi_m^R\}_{m \in \mathcal{M}} = \{\nu_m^R\}_{m \in \mathcal{M}}$. Furthermore, $\{\pi_m^R\}_{m \in \mathcal{M}}$ can be derived from π^R: apply the Bayes' rule on m_1, and arbitrarily define $\pi_m^R = \nu_m^R$ for $m \neq m_1$, because the Bayes' rule is not well defined.

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Therefore $[\nu^S, \pi^R, \{\pi_m^R\}_{m \in \mathcal{M}}]$ is a PBE and it is behaviorally equivalent to the given pooling DSE $[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}]$. Since the given DSE is arbitrarily chosen, we conclude that Proposition is true for any pooling DSE.

$\square$

A.3 Proofs for Chapter 4.

Proposition 4.2.6 For any game in Γ, quasi perfect Bayesian equilibrium exists in which the receiver perceives the types to be unambiguous.

Proof. The proof is by construction.

Step 1:

Let $\Delta_p(\mathcal{T} \times \mathcal{M}) = \{q \in \Delta(\mathcal{T} \times \mathcal{M}) \mid q(\{t\} \times \mathcal{M}) = p(\{t\} \times \mathcal{M})\}$ and define:

$$\nu_q^R(E) := \rho \sum_{t \in \mathcal{T}} p(t) \cdot \mathbb{1}_{\{\{t\} \times \mathcal{M} \subseteq E\}} + (1 - \rho)q(E).$$

For each $q \in \Delta_p(\mathcal{T} \times \mathcal{M})$ and $m \in \mathcal{M}$, the conditional capacity ν_m^R updated by the Bayes' rule is the following by computation.

$$\nu_{m,q}^R(T_j) = \frac{q(t_j, m)}{\sum_{t \in \mathcal{T}} q(t, m)}, \quad j = 1, \ldots, T. \tag{A.14}$$

Also for given m and q, consider the best response behavioral strategies which max-

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imizes the following.

$$\zeta(q,m) = \operatorname*{argmax}_{\sigma \in \Delta(\mathcal{R})} \sum_{r \in \mathcal{R}} \sigma(r) \cdot \int_{t \in \mathcal{T}} u^R(r,m,t) \mathrm{d}\nu^R_{m,q}$$

Note that $\nu^R_{m,q}$ is continuous for each q and also $\int u^R(r,m,t)\mathrm{d}\nu^R_{m,q}$ is continuous function of $\nu^R_{m,q}$. Thus $\int u^R(r,m,t)\mathrm{d}\nu^R_{m,q}$ is continuous at q. By the Maximum theorem of Debreu, $\zeta(q,m)$ is upper hemicontinuous at q (see Ok, 2007, p.306). Further note that $\Delta(\mathcal{R})$ is convex and $\sum_{r \in \mathcal{R}} \sigma(r) \cdot \int_{t \in \mathcal{T}} u^R(r,m,t)\mathrm{d}\nu^R_{m,q}$ is linear in σ. Thus $\zeta(q,m)$ is convex-valued.[2]

Step 2: Let $\gamma(m) \in \Delta(\mathcal{R})$ be an additive distribution and $\gamma = (\gamma(m))_{m \in \mathcal{M}}$ a vector of each distribution. consider $\nu^S(E;\gamma,m) = (1-\rho)\gamma(m)$. For each m, consider the best correspondence for the sender.

$$\eta(t,\gamma) = \operatorname*{argmax}_{\sigma \in \Delta(\mathcal{M})} \sum_{m \in \mathcal{M}} \sigma(m) \int_{r \in \mathcal{R}} u^S(m,r,t)\mathrm{d}\nu^S(\{r\};\gamma,m)$$

Note that ν^S is continuous in γ and and $\int_{r \in \mathcal{R}} u^S(m,r,t)\mathrm{d}\nu^S(\{r\};\gamma,m)$ in ν^S. Thus $\eta(t,\gamma)$ is upper hemicontinous. Since $\Delta(\mathcal{M})$ is convex and $\sum_{m \in \mathcal{M}} \sigma(m) \int_{r \in \mathcal{R}} u^S(m,r,t)\mathrm{d}\nu^S(\{r\};\gamma,m)$ is linear in σ, it is convex-valued. Let

$$\psi(\gamma) := \left\{ q \in \Delta_p(\mathcal{T} \times \mathcal{M}) \mid q(t,m) = \sum_{t \in \mathcal{T}} p(t)\, \sigma_t, \; \sigma_t \in \eta(t,\gamma) \right\}$$

Then $\psi(\gamma)$ is also upper hemicontinuous and convex-valued.

Step 3: Let $\chi(q,\gamma) = \left[\prod_{m \in \mathcal{M}} \zeta(q,m) \right] \times \psi(\gamma)$. Then it is a self-mapping $\chi : (q,\gamma) \mapsto X(q,\gamma)$ and each component of χ is upper hemicontinuous, convex valued.

---

[2]compact-valued follows from the finite set up.

By the Kakutani's fixed point theorem, there exist $(q^*, \gamma^*) \in \chi(q^*, \gamma^*)$.

Step 4: Now consider $\left[\nu^S(\gamma^*), \nu^R_{q^*}, \{\nu^R_{m,q^*}\}_{m \in \mathcal{M}}\right]$. Note that from the construction, $supp(\nu^R_{q^*}) = supp(q^*)$ and $supp(\nu^S(\gamma^*)) = supp(\gamma^*)$.

Now for any $(t', m') \in supp(\nu^R_{q^*})$, it satisfies $q^*(t', m') \in \psi(\gamma^*)$ and $\sigma_{t'} \in \eta(t', \gamma^*)$. Thus, $m' \in \text{argmax}_{m \in \mathcal{M}} \int_{r \in \mathcal{R}} u^S(m, r, t') \mathrm{d}\nu^S(\{r\}; \gamma^*, m)$. For any $r' \in supp(\nu^S(\gamma^*))$, $\gamma^*(m) > 0$ for each $m \in \mathcal{M}$ which in turn implies at q^*, $r'(m) \in \zeta(q^*, m)$ and $r' \in \text{argmax}_{r \in \mathcal{R}} \int_{t \in \mathcal{T}} u^R(r, m, t) \mathrm{d}\nu^R_{m,q^*}$ where ν^R_{m,q^*} is derived by the Bayes' rule.

Therefore, we conclude $\left[\nu^S(\gamma^*), \nu^R_{q^*}, \{\nu^R_{m,q^*}\}_{m \in \mathcal{M}}\right]$ is a QPBE distinct from PBE. $\square$

Proposition 4.3.2 Fix a game in Γ with $|\mathcal{M}| \geq |\mathcal{T}|$. Assume that the sender's beliefs are additive, and that the types are unambiguous. If there exists separating QPBE, then there exists separating PBE which supports the same equilibrium behavior;i.e., for all separating QPBEs and PBE,

$$\mathcal{B}(\text{QPBE}) = \mathcal{B}(\text{PBE}).$$

Proof. Fix a game in Γ and denote by $p \in \Delta(\mathcal{T})$, the probability distribution over the types. $\mathcal{B}(\text{PBE}) \subseteq \mathcal{B}(\text{QPBE})$ is clearly true. For $\mathcal{B}(\text{QPBE}) \subseteq \mathcal{B}(\text{PBE})$, take a QPBE $[\nu^S, \nu^R, \{\nu^R_m\}_{m \in \mathcal{M}}]$. Denote the set of the messages on-the-equilibrium-path by $\mathcal{M}^*$ and off-the-equilibrium-path by $\mathcal{M}_*$. Without loss of generality, assume that the sender of type j sends the message j on-the-equilibrium-path where $j = 1, \dots, l < T$. For the types $j' = l + 1, \dots, T$, we assume they do not send any message at the

equilibrium. Now construct π^R such that

$$\pi^R(E) = \begin{cases} p_i & \text{if } E = \{(t_i, m_i)\}, \ i = 1, \ldots, l, \\ 0 & \text{otherwise.} \end{cases}$$

Take $\pi^S = \nu^S$ and $\pi_m^R = \nu_m^R$ for $m \in \mathcal{M}_*$. Then $supp(\nu^R) = supp(\pi^R)$ and $\{\nu^R\}_{m \in \mathcal{M}} = \{\pi^R\}_{m \in \mathcal{M}}$. Thus $[\pi^S, \pi^R, \{\pi_m^R\}_{m \in \mathcal{M}}]$ constitutes a separating PBE which is behaviorally equivalent. $\qquad\qquad \square$

Proposition 4.3.3 Fix a game in Γ and a probability distribution $p \in \Delta(\mathcal{T})$. Denote the set of equilibrium behaviors of pooling (Q)PBEs for given p by $\mathcal{B}((Q)PBE, p)$. Then the following is true:

$$\mathcal{B}(QPBE, p) = \bigcup_{q \in \Delta(\mathcal{T})} \mathcal{B}(PBE, q).$$

Proof. Fix a game and $p \in \Delta(\mathcal{T})$. Show firstly $\mathcal{B}(\text{QPBE}, p) \subseteq \bigcup_{q \in \Delta(\mathcal{T})} \mathcal{B}(\text{PBE}, q)$. Take a QPBE $[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}]$ which is pooling equilibrium. Since ν^R reveals the types to be unambiguous, $\{\nu_m^R\}_{m \in \mathcal{M}}$ must be additive from the Lemma 4.2.4. Denote the messages on-the-equilibrium-path by m^*.

(i) If the notion of support is the M-support, then take $\pi_m^R = \nu_m^R$ for each $m \in \mathcal{M}$ and construct π^R such that:

$$\pi^R(E) = \begin{cases} \pi_{m^*}^R(T_i) & \text{for } E = \{(t_i, m^*)\}, \ i = 1, \ldots, T, \\ 0 & \text{otherwise.} \end{cases}$$

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Since $supp(\nu^R) = supp(\pi^R)$ and $\nu_m^R = \pi_m^R$, $[\nu^S, \pi^R, \{\pi_m^R\}_{m \in \mathcal{M}}]$ constitute a pooling PBE for $q = \pi_{m^*}^R$.

To show $\mathcal{B}(\text{PBE, q}) \subseteq \mathcal{B}(\text{QPBE, p})$, observe that the following Lemma is true.

Lemma A.3.1

For $q \neq p \in \Delta(\mathcal{T})$, there exists $\alpha \in (0, 1)^{|\mathcal{T}|}$ such that:

$$(i) \quad \alpha_i \leq p_i \quad i = 1, \ldots, T,$$

$$(ii) \quad \sum_{i=1}^{T} \alpha_i < 1,$$

$$(iii) \quad \frac{\alpha_i}{\sum_{i=1}^{T} \alpha_i} = q_i \quad i = 1, \ldots, T.$$

The Lemma can be simply shown by taking $\alpha_i = \min\{p_1, \ldots, p_T\} \cdot q_i$ for each $i = 1, \ldots, T$.

Now take PBE with $q \neq p$ and denote it by $[\pi^S, \pi^R, \{\pi_m^R\}_{m \in \mathcal{M}}]$.

We construct ν^R which reveals the types to be unambiguous, $supp(\nu^R) = supp(\pi^R)$, and $\{\nu_m^R\}_{m \in \mathcal{M}} = \{\pi_m^R\}_{m \in \mathcal{M}}$. By the Lemma A.3.1, for given $q \neq p$, there exist $\alpha \in (0, 1)^{|T|}$ such that ν^R is constructed below: for any $E \in \Sigma^R$,

$$\nu^R(E) = \sum_{i=1}^{T} \left[\mathbb{1}_{\left\{T_i \subseteq E\right\}} \cdot p_i + \mathbb{1}_{\left\{T_i \not\subseteq E, \ (t_i, m^*) \in E\right\}} \cdot \alpha_i \right]$$

Note that $supp(\nu^R) = supp(\pi^R)$ and by the condition (iii) of the Lemma A.3.1, $\nu_{m^*}^R = \pi_{m^*}^R$ by the Bayes' rule. For the messages off-the-equilibrium-path, the conditional capacity can be chosen arbitrarily as it is not well defined under the Bayes' rule. Taking $\nu_m^R = \pi_m^R$ for $m \neq m^*$, note that $[\pi^S, \nu^R, \{\nu_m^R\}_{m \in calM}]$ is QPBE which is behaviorally equivalent with PBE under q. Since q is chosen arbitrarily, $\mathcal{B}(\text{PBE, q}) \subseteq \mathcal{B}(\text{QPBE, p})$ is true for any $q \in \Delta(\mathcal{T})$.

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Therefore, we conclude $\mathcal{B}(QPBE, p) = \bigcup_{q \in \Delta(\mathcal{T})} \mathcal{B}(PBE, q)$. $\qquad \square$

Proposition 4.3.5 Fix a game in Γ and a probability distribution $p \in \Delta(\mathcal{T})$. Assume that the types are unambiguous, and that the senders's beliefs are additive. Fix a pooling QPBE. If the receiver's capacity is an independent product capacity for some μ, $\mathcal{B}(QPBE, p) = \mathcal{B}(PBE, p)$.

Proof. Denote $\mathcal{M}^*$ be the set of messages on-the-equilibrium-path. It is sufficient to show that $\nu_m^R(T_i) = p(T_i)$ for all $i = 1, ..., T$ and $m \in \mathcal{M}^*$. Fix $m \in \mathcal{M}^*$. Denote $M = \mathcal{T} \times \{m\}$. By the definition of independent product capacity and the Bayes' rule,

$$\nu_m^R(T_i) = \frac{p(T_i)\mu(M)}{\sum_{j=1}^{T} p(T_j)\mu(M)} = p(T_i).$$

$\qquad \square$

Proposition 4.3.6 Fix a game in Γ. For each PBE, there exists QPBE that is behaviorally equivalent for every $\lambda \in (0, 1)$.

Proof. Fix a PBE $\left[\pi^s, \pi^R, \{\pi^R\}_{m \in \mathcal{M}}\right]$, and $\lambda \in (0, 1)$. Consider $\left[\pi^s, \nu^R, \{\nu^R\}_{m \in \mathcal{M}}\right]$ such that $\nu^R(E) = (1 - \lambda)\pi^R(E)$ for all $E \in \Sigma$. Then, $supp(\pi^R) = supp(\nu^R)$ and $\{\nu^R\}_{m \in \mathcal{M}} = \{\pi^R\}_{m \in \mathcal{M}}$ after the Bayes' updating. Further notice that the degree of ambiguity of ν^R is equal to λ. Thus, $\left[\pi^s, \nu^R, \{\nu^R\}_{m \in \mathcal{M}}\right]$ is a QPBE by Definition 4.2.3 and it supports the same equilibrium behavior of the fixed PBE. As λ was chosen arbitrarily, the statement holds true. $\qquad \square$

Appendix B

Appendix: The Game of Ryan

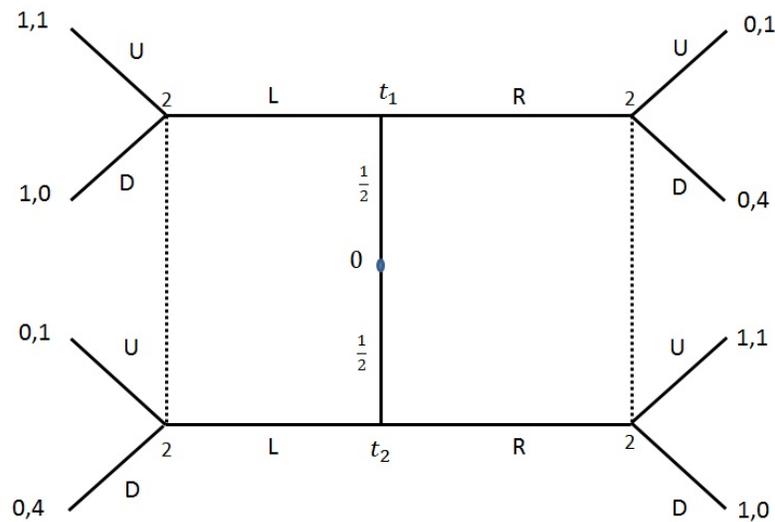This appendix briefly recalls the game of Ryan (2002a). Consider the signaling game

Figure B.1: Signaling game of Ryan.

in Figure B.1. For the Sender, sending L when his type is t_1 and R when t_2 is the

strictly dominant strategy. Therefore, Ryan argues that any reasonable equilibrium should capture the separating behavior.

Let $p(T_1) = p(T_2) = \frac{1}{2}$ and q be a probability distribution on $\mathcal{T} \times \mathcal{M}$ such that $q(\{t_1, L\}) = q(\{t_2, R\}) = \frac{1}{2}$. Let $\{\nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}\}$ be the family of Receiver's beliefs defined as follows.

$$\nu^R(E) = \rho q(E) + (1 - \rho) \sum_{j=1}^{2} w_{t_j}(E) \, p(T_j), \ 0 \le \rho < 1,$$

$$w_{t_j}(E) = \begin{cases} 1 & \text{if } \{t_j\} \times \mathcal{M} \subseteq E, \\ 0 & \text{otherwise,} \end{cases}$$

$$\nu_L^R(E) = \begin{cases} \frac{1}{2-\rho} & \text{if } E = T_1, \\ \frac{1-\rho}{2-\rho} & \text{if } E = T_2, \end{cases}$$

$$\nu_R^R(E) = \begin{cases} \frac{1-\rho}{2-\rho} & \text{if } E = T_1, \\ \frac{1}{2-\rho} & \text{if } E = T_2. \end{cases}$$

The associated supports are:

$$supp(\nu^R) = \{(t_1, L), (t_2, R)\},$$
$$supp(\nu_L^R) = \{(t_1, L), (t_2, L)\},$$
$$supp(\nu_R^R) = \{(t_1, R), (t_2, R)\}.$$

In fact, ν^R is an E-capacity which reveals the types to be unambiguous.

Consider $\rho < \frac{2}{3}$. Let ν^S be such that $\nu^S(D) = 1$, $\nu^S(U) = 0$ for all messages. The associated support is $supp(\nu^S) = \{D\}$. Then, $[\nu^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}]$ constitutes the DSE

where the Receiver plays D after observing any message, and the Sender t_1 plays L and t_2 sends R. In Ryan's paper, $\rho = \frac{1}{2}$.

Consider $\rho > \frac{2}{3}$. Let $\bar{\nu}^S$ be such that $\bar{\nu}^S(U) = 0$, $\bar{\nu}^S(D) = 1$ for all messages. The associated support is $supp(\bar{\nu}^S) = \{U\}$. Then, $[\bar{\nu}^S, \nu^R, \{\nu_m^R\}_{m \in \mathcal{M}}]$ constitutes the DSE in which the Receiver plays U after observing any message, and the Sender t_1 plays L and t_2 sends R. Thus, for each $\rho > \frac{2}{3}$, the DSE supports the identical behavior of the unique separating PBE. Nevertheless, the Receiver's beliefs violate the belief persistence axiom.

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