Hyper-acceleration in Secondary Mathematics and Student Course Taking Patterns after Middle School Algebra

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ABSTRACT

The purpose of this study was to assess the impact of a school division policy on early algebra on students' course taking patterns in high school. Over the past two decades, there has been significant growth in the number of students taking Algebra 1 in middle school. Research about the advantages and drawbacks to completing Algebra 1 prior to high school have mixed conclusions, with some suggesting that students benefit from the opportunity to take more advanced mathematics and science courses in high school and others concluding that students are more likely to fail and need to repeat courses if they take Algebra 1 early (Stein et al., 2011). Most of the research has focused on students taking Algebra 1 in eighth grade. At the same time, there is an ever-growing group of students seeking to take Algebra 1 even earlier, as evidenced by expansive growth in the number of students accessing Advanced Placement Calculus prior to twelfth grade (College Board, 1997; College Board, 2017).

To assess the impact of early Algebra 1, the researcher considered transcript data for two cohorts of students in a large, suburban school district who took Algebra 1 in seventh or eighth grade. Statistical analysis was performed to assess whether students were likely to access the highest level mathematics courses available to them, whether they were staying in mathematics courses throughout all years of high school, and what patterns might emerge in mathematics and science course taking for students based on when they took Algebra 1. The findings indicated that students in this cohort who took Algebra 1 in eighth grade were more likely to complete the highest level mathematics courses available to them than those who took Algebra 1 in seventh

grade, but they also took, on average, fewer total mathematics and science courses. For all students taking middle school Algebra 1, there were sharp declines in students accessing honors-level mathematics coursework as they advanced through the mathematics sequence.

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GENERAL AUDIENCE ABSTRACT

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Chapter 1

Equity is a major theme in education research since 2000. Although it has not delivered all of the educational improvements that it promised, the No Child Left Behind (NCLB) Act (2002) "forced American schools to confront an issue that has long been ignored throughout much of American history" (Noguera, 2013, p. 62). In particular, NCLB contributed to our increased awareness of achievement gaps, i.e., that proficiency in academics differs based on student demographics, including race, ethnicity, socioeconomic status, primary language, and disability. Despite the increased awareness, differences in achievement and opportunities persist two decades after the passage of NCLB (Bromberg & Theokas, 2013; Cook-Harvey, Darling-Hammond, Lam, Mercer, & Roc, 2016). Furthermore, as schools focus attention on the lowest performing students in an effort to meet the accountability benchmarks of NCLB and successive legislation, it is a real concern that the progress of the top ten percent of students will lag (Loveless, et. al, 2008). Many researchers have studied the work done by states, school districts, and schools to close the achievement gap by bringing the lowest achieving students up to minimum proficiency levels (Dee & Jacob, 2011; Noguera, 2013, Cook-Harvey, et. al., 2016). However, fewer researchers study closing the gap at the upper end of the achievement spectrum, e.g., on increasing participation of disadvantaged students in advanced coursework. There is a need for more research about how to provide equity in disciplines such as STEM and in college preparatory courses like Advanced Placement and the impact such programs have on the students they are intended to serve and on the student population at large. Klugman (2013) studied programs in California designed to increase AP course enrollments in schools that serve a large proportion of minority or low-income students. He found that the programs met the goal of increasing enrollment in the targeted schools, but that the inequalities in opportunity between those schools and schools serving more advantaged students did not improve (Klugman, 2013). Specifically, the efforts to increase equity were "offset by the actions of advantaged families and schools, who are motivated to accumulate more opportunities for distinction, given intensified competition over admission to selective colleges" (Klugman, 2013, p. 26).

In our increasingly technology-centric world, the need for workers with skills in mathematics and related fields to support existing structures and to provide innovation is well-established. However, the STEM workforce does not reflect the demographics of the population. According to the National Science Board, the STEM labor force is aging, with a higher average age and a higher proportion of workers in their 60s than in the previous century (National Science Board, 2018). The number of women in STEM careers, while increasing, remains around 28% and is concentrated in areas such as life sciences and social sciences with persistent underrepresentation in engineering and computer science (National Science Board, 2018). Finally, the proportion of historically underrepresented racial and ethnic groups in the STEM workforce remains low, while Asian and foreign-born workers are represented at levels significantly higher than their presence in the population overall (National Science Board, 2018)

While we need to find ways to encourage more students, particularly students from underrepresented groups, to pursue STEM-related career pathways, it is not enough to simply encourage them to sign up for college majors in engineering and science. We also need to consider the factors that contribute to success in post-secondary education. Students' high school course history is the largest predictor of eventual bachelor's degree completion (Adelman, 2006). Looking closer, students who completed mathematics courses above the level of Algebra 2 were three times more likely to go on to earn a bachelor's degree (Adelman, 2006).

Study Background

The researcher became interested in student pathways to Algebra 1 after observing changes in the course selection patterns of high achieving students. Through work with high school mathematics teachers as a mathematics curriculum specialist, anecdotes accumulated about the increasing numbers of ninth grade students struggling in honors-level math courses. The researcher also observed that many students who were hyper-accelerated in mathematics, i.e. were more than a year ahead of grade level, were not continuing on a pathway to advanced mathematics courses throughout high school. Math curriculum supervisor colleagues from across the state shared similar experiences and concerns.

The number of students seeking accelerated pathways in mathematics in at least one school division has grown rapidly. Prior to 2005, the most common accelerated opportunity available to middle school students was Algebra 1 Honors for eighth graders. At the time, 5% or fewer seventh graders accessed that course each year, typically in gifted and talented (GT) centers. Beginning in 2005, a standard level or non-honors version of Algebra 1 became available to all eighth graders in every middle school. At the same time, the numbers of students taking Algebra 1 Honors in seventh grade began to rise. After a few years of rapid growth, the number of seventh graders in Algebra 1 Honors has leveled off, at about 13-15% every year (FCAG, 2012).

The issue of hyper-acceleration is not isolated to this school district. According to College Board data, the growth in students taking AP Calculus exams prior to twelfth grade far outpaces the growth in exams overall. For the period from 1997-2017, the total number of AP Calculus BC exams administered grew from 22,668 to 132, 514, an increase of 485%. In the same time interval, the number of students in grades 9-11 taking those exams grew from 4,192 to

44,024, an increase of 950%, or almost twice as much growth as the exam overall. The results for AP Calculus AB are even more dramatic – an increase of 183% (111,834 to 316,099) for exams overall compared to 611% (13,976 to 99,325) for 9th through 11th graders. In 2017, almost one-third of students taking the AP Calculus AB exam were not seniors (College Board, 1997; College Board, 2017).

Statement of the Problem

Students and their parents have a lot to consider when making choices about the courses that they will take in secondary school. They must take into account the opportunities at their school, potential career choices, the requirements for admission to desired colleges or other post-secondary programs, and an overall balance of coursework, extracurricular activities, and personal time. Schools and school systems have an obligation to provide information to help students make choices that will result in an educational program that provides an appropriate challenge to the student as well as preparation for the student's post-secondary goals. If a school or school system chooses to implement a policy that uses an objective measure for admission to particular courses or programs, then that measure should have some indicator of reliability. In other words, the policy should be set such that students who are admitted have a reasonable chance of success.

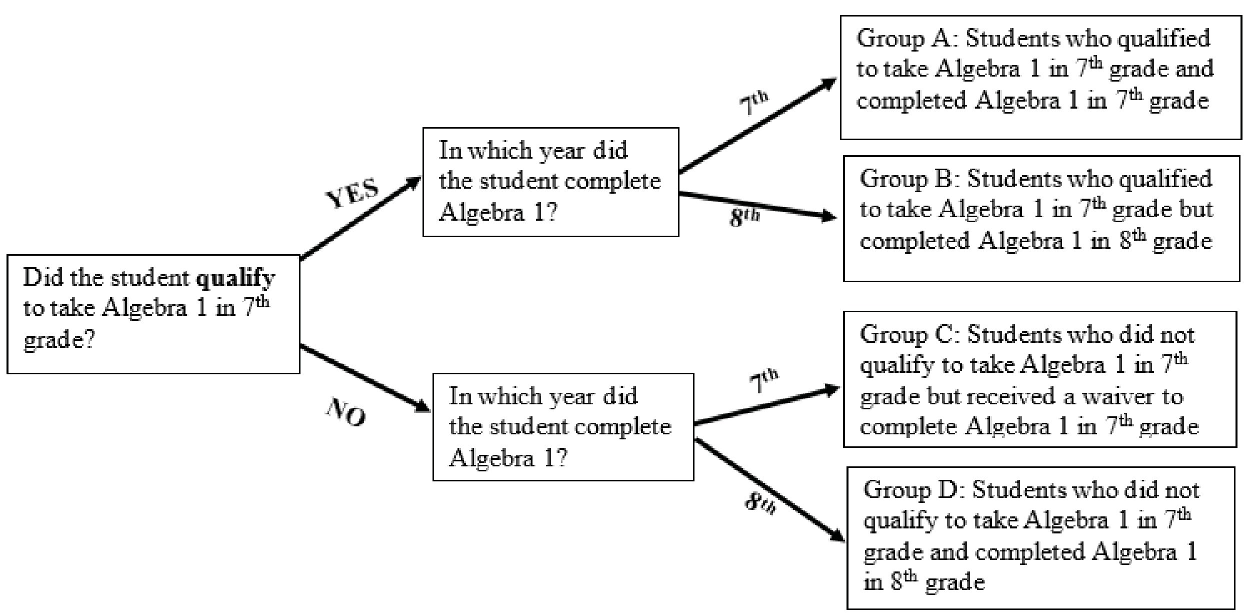
In the case of mathematics acceleration, students who meet the criteria to take Algebra 1 prior to eighth grade should be expected to have high mathematics aptitude as well as the interest to pursue mathematics courses at high levels throughout high school. Without that understanding, there is little point to students starting the high school mathematics sequence so early. However, what we observe is that some students meet the established criteria for hyper-acceleration and then do not follow through on a course of study which would reflect the aptitude and passion demonstrated in middle school. This study seeks to explore the degree to which a particular policy on mathematics hyper-acceleration accurately places students in course sequences that meet their goals and needs.

Theoretical and Conceptual Framework

Within the constraints of the policies and requirements for early algebra in the school division under consideration, most students participating in early algebra will fall into one of four groups: students who qualified for and took Algebra 1 in seventh grade, students who qualified for early algebra but chose to delay taking Algebra 1 until eighth grade, students who did not qualify for early algebra but still took it in seventh grade, which means that they were granted a waiver of one or both of the entry criteria, and finally students who did not qualify for early algebra and took Algebra 1 in eighth grade through the open enrollment policy. The group assignment is illustrated in Figure 1.

Figure 1

Assignment of students to groups based on early algebra



Across each of these groups there are similarities and differences in the achievement and aptitude levels of the students as measured by standardized tests, and on the experiences to which the students have access based on the entry criteria. Dividing students into these four groups allowed for comparison between students taking Algebra 1 in seventh vs. eighth grade, but also provided some insight into whether choices that the students made based on the placement criteria had any additional effect on their course taking choices later in high school.

Purpose of the Study

The purpose of this study was to identify the impact of a school division policy on early algebra on students' course taking patterns in high school. Through a quantitative analysis of transcript data, the patterns in mathematics and science course taking for students taking early algebra were investigated. Studying course-taking patterns allowed for some insight into whether students benefitted from the policy by taking the most advanced mathematics courses available to them, and how well the criteria set out by the policy identified students who were likely to benefit from the policy.

Rationale for the Study

An important question was whether this push for more advanced mathematics content earlier was beneficial to students, and if in fact it might have negative impacts. If the goal of hyper-acceleration in mathematics is to complete more advanced mathematics coursework in high school, then is that the actual outcome for most students? If the assumption is that students who take Algebra 1 prior to eighth grade have high interest and/or strong ability in mathematics, do their course taking patterns throughout high school reflect that? In other words, are they consistently choosing a sequence of challenging courses in mathematics and related disciplines, or do they at some point “hit a wall” and choose less challenging courses, or stop taking mathematics all together?

Research Questions

1. Are students more likely to take progressively more challenging courses each year and go on to take the highest level of mathematics available to them at the same rate based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?
2. What patterns emerge when comparing mathematics course pathways for students based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?
3. What are the differences in mathematics and science course-taking based on whether students met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

Definition of Terms

1. Acceleration: Enrolling in courses or learning specific content ahead of grade level (Van Tassel-Baska & Brown, 2015).
2. Early Algebra: This term is used two ways in the literature. One meaning is the extension of learning algebraic thinking into elementary curriculum as low as kindergarten. In this study, we will use the other definition in which the term refers to students completing a first course in algebra, typically called Algebra 1, before enrolling in ninth grade (Stein, et. al., 2011).

3. Hyper-acceleration: Enrolling in high school credit courses in middle school, two or more years before expected (Sheffield, 2017). For example, taking Algebra 1 in seventh grade or earlier.
4. Universal Algebra: Also referred to as Algebra for All, this is the practice of enrolling all eighth graders in a specific school or district in Algebra 1 (Stein, et. al., 2011).

Limitations

1. The assumption in this study was that students' progress towards higher mathematics and their choices of mathematics and science courses were based on achievement and passion for STEM. However, students make course choices based on many factors, including but not limited to strength of schedule, parent influence, peer influence, pressure for good grades, grade weighting for advanced coursework, extracurricular activities, and other responsibilities such as family obligations or afterschool jobs.
2. The researcher did not have control over students' mathematics learning. Variations in the instruction and level of rigor in mathematics classes across and within schools may have had an impact on student course choices.
3. The entry criteria for early algebra were set by the school division, and there may have been variation in the application of the policy across schools. For example, in some schools, all students who met the test score benchmarks for Algebra 1 in seventh grade were enrolled in Algebra 1 and were given the option to change to Mathematics 7 Honors, while in other schools, parents were notified of the qualification and asked to designate whether their child would enroll in Algebra 1 or Mathematics 7 Honors. There were also differences in how and when students were granted waivers to take Algebra 1 in seventh grade when they failed to meet one or both of the entry criteria.

Delimitations

1. The data came from a single school division with a particular early algebra policy. Other school divisions or individual schools have different policies and testing requirements, which means that the results of this study may not be generalizable to other school divisions where policies and criteria for mathematics acceleration may differ.
2. This study considered students who took Algebra 1 in seventh or eighth grade. However, a small number of students will have taken Algebra prior to seventh grade and a larger number will have taken Algebra 1 in ninth grade or later.
3. The group of students who took Algebra 1 in eighth grade but did not meet entry qualifications for early algebra were not a homogeneous group with regards to their prior mathematics experiences. Some of the students would have participated in the advanced mathematics sequence for at least part of elementary school, giving them access to the testing that leads to seventh grade algebra, while others would have been in the general mathematics sequence throughout elementary school, and their earliest opportunity for Algebra 1 would have been in eighth grade. Examining the differences within this group could be an opportunity for further investigation but was beyond the scope of this study.

Organization of the study

Chapter One has defined the problem and the background for the study, outlined the research questions, provided the terms used, and revealed limitations and delimitations of the study. Chapter Two presented the literature review related to the topic, while Chapter Three described the methodology. In Chapter Four, the analyzed data were presented, and in Chapter Five, the findings, implications, and recommendations for future research were discussed.

Chapter 2

This chapter presents a review of existing research on early algebra. First, there is an overview of trends in mathematics instruction, followed by a comparison of enrichment and acceleration for mathematically talented students and the implications of changes in mathematics standards and trends in mathematics instruction on policies and instruction for advanced students. Next there is a review of the history and purpose of early algebra, including the contrast between programs that identify certain students for accelerated mathematics programs with the implementation of universal algebra programs. The chapter concludes with a look at research in which the short- and long-term impacts of early algebra are investigated in the context of student achievement and participation in higher level mathematics.

Trends in Mathematics Instruction

Supported by research into detracking (Burris & Welner, 2005; Oakes, et. al., 1997) and growth mindset (Boaler, 2016), trends in mathematics education are slowly shifting. Mathematics education leaders agree that a student must know more than simply how to compute or solve an equation. Students should be able to make connections both within mathematics concepts and to other content, apply mathematics learning to new situations, and communicate their thinking using multiple models and strategies (NCTM, 2014). Experts promote practices that were formerly reserved for gifted students be used for all students and advocate for equitable access to advanced courses across the curriculum. Programs and policies designed to detrack high schools are growing in popularity, but the research is not yet available to assess the benefits and drawbacks of such programs. (Sawchuk, 2018; Peyser, Martinie, Schrick, & Fast, 2018).

Equity in Advanced Courses

Tracking was a pervasive trend in education, written about extensively in the 1990s (Ascher, 1992; Brewer, et. al., 1995; Oakes, et. al., 1997), followed by acknowledgments that many of the suggested detracking practices failed to reduce tracking and provide rigorous education for all students (Yonezawa, et. al., 2002). The move towards detracking is garnering renewed interest by educators as well as the general public with regards to mathematics instruction, as people recognize the need for students to be prepared for the increasingly technology-driven careers of the future (Burris & Welner, 2005). Schools have made a concerted effort to decrease the degree to which students become stuck in a cycle of remedial courses, but many educators regard mathematics as the most difficult subject to detrack (Rubin, 2006).

While much research has focused on access to algebra in middle school, tracking in mathematics begins much earlier in most students' educational experiences (Cogan, et al., 2001). While little research exists on the impact of prior mathematics education on participation in eighth grade algebra, particularly based on racial differences (Morton & Riegle-Crumb, 2019), existing research on tracking does confirm that students are unlikely to access advanced or accelerated coursework in middle or high school if they have not been exposed to advanced classes early on (Lucas, 1999; Oakes, 2005).

Mathematics Proficiency for All Students

Consider two descriptions of mathematics proficiency, one which describes all students and one which is intended for mathematically gifted students. These descriptions are remarkably similar. Mathematically proficient students "make sense of problems and persevere in solving them", "reason abstractly and quantitatively", "look for and express regularity in repeated reasoning", and "construct viable arguments and critique the reasoning of others" (CCSS, 2010).

But Gavin et. al. (2009) suggested that “mathematically talented students come to know and understand mathematics differently than other students”. They think like mathematicians, flexibly shifting among different problem-solving modes and making connections among concepts.

In general, recommendations for teaching mathematics to gifted students dating from the early 2000s (Renzulli, Leppien, &Hays, 2000; Tomlinson et al., 2002) resemble the recommendations now promoted for all students by NCTM (2014) and in the CCSS. Works focused on mathematically gifted students suggest that they should have access to a coherent and organized curriculum that is focused on solving problems that require critical and creative thinking and is built on essential concepts, skills, and principles. They should have the opportunity to develop their passions and strengths in a variety of ways (Gavin et al, 2009; NCTM, 2016). But NCTM presents a case for all students to have access to a mathematics curriculum that sounds just like that, one which “develops important mathematics along coherent learning progressions” and helps students to “view themselves as capable of using their growing mathematical understanding to make sense of new problems situated in the world around them” (NCTM, 2014, p. 1).

In seeking to improve mathematics education for all students, we must ensure that we do not restrict opportunities for mathematically talented students. “Appropriately challenging mathematics instruction should be provided for all students when they need it” (Spielhagen, 2006, p. 57). Researchers need to analyze the structure of mathematics instruction, particularly in the middle grades, and how to provide equitable identification protocols to ensure that students receive appropriate levels of challenge and rigor throughout their mathematics education (Bromberg & Theokas, 2014).

Mathematics Instruction for Gifted Students

Advocates for gifted students argue that federal accountability legislation and local and state policies and programs have created an environment in which gifted students are held back from reaching their full potential (Jolly & Makel, 2010). Acceleration in the form of offering algebra to students earlier than high school is only one approach. We must understand what opportunities can and should be offered to gifted students to challenge and engage them in mathematics.

Strategies for Teaching Gifted Students

There is limited research with mixed results on specific models for teaching mathematically promising students (Gavin et al, 2009). Most of the research addresses strategies to support students who are academically talented across disciplines. Specific strategies for instructing mathematically gifted students include ability grouping and various forms of acceleration, including grade skipping and early college entrance.

Ability grouping. A common practice for teaching advanced mathematics courses utilizes ability grouping, i.e. segregating gifted students in separate classes or fixed groups within a class. Studies on this practice return mixed results. Card and Giuliano (2014) found that participation in a separate gifted education program had little to no effect on standardized test achievement for students with high IQ scores who had always been placed in the program, but students who did not meet the IQ score cut off but were placed in the classes "to fill seats" showed significant gains. On the other hand, Hoffer (1992) found that ability grouping in mathematics provided only a slight benefit for students in the high performing groups, but a negative effect for the students placed in lower performing groups.

Studies on the social and psychological impact of ability grouping return mixed results, with no clear conclusion about the benefit to students. Some evidence suggests that ability grouping offers advantages for certain groups of students and detriments to others. The students themselves report having mixed feelings about participating in homogeneous and heterogeneous classes, citing the learning benefits of a more challenging atmosphere alongside concerns about the social and emotional impact of being with like-minded peers (Adams-Byers, et. al. 2004; Meijnen & Guldemond, 2002). Students are subject to a big-fish-small-pond effect in heterogeneous settings, as some students reported a preference for heterogeneous classes because they felt that they wouldn't need to work as hard to be considered successful in comparison with less-abled peers (Adams-Byers, et. al. 2004). Furthermore, when students placed in homogeneous classes were compared to those of equal ability in heterogeneous settings, significant differences arose in academic self-concept, particularly in regard to mathematics (Marsh, et. al., 2008). Kuriloff and Reichert (2003) found some benefit to ability grouping for students who are underrepresented in advanced courses, because they benefited from a peer group with similar backgrounds and goals. However, this study took place at an elite school, so the results are not generalizable to minority students in a more racially or socioeconomically homogeneous setting who might not choose to take the advanced courses based on peer perception or might not have access to advanced coursework in the first place.

Acceleration. The Stanley model of gifted education describes a process by which schools identify students based on testing and give them opportunities to participate in fast-paced courses in core academic subjects, especially mathematics and science (Van Tassel-Baska & Brown 2015). One of the longest running and most well-known implementations of the Stanley model is the Study of Mathematically Precocious Youth (SMPY) started at Johns Hopkins

University in 1971. Studies on students participating in programs associated with SMPY have shown overwhelmingly positive impacts on students.

The Stanley method is popular with students and parents, but less so with schools (Van Tassel-Baska and Brown 2015). A concern about the Stanley model is fast-paced instruction may result in shallow understanding by students, with an emphasis on breadth over depth. The evidence suggests that most teachers do not use best-practices in fast-paced instruction, including pre-testing, flexible grouping, student choice, and enrichment beyond textbook content (Lee & Olszewski-Kubilius, 2006). Teachers of the fast-paced classes use high level questioning and discussion strategies, but in a whole-class setting which relies on an assumption of greater auditory processing ability and longer attention span in gifted learners.

Grade skipping. Beginning in the latter half of the 20th century, researchers wrote substantially about acceleration in the form of whole-grade skipping (VanTassel-Baska, & Brown, 2015). In general, researchers viewed grade skipping as a way to ensure that students remained challenged and continued to learn, by not wasting time on things they already know. One longitudinal study of cohorts of students from the 1970s and 1980s suggested that for the top 1% of students based on mathematics ability, grade skipping led to higher achievement in STEM fields, including more advanced degrees, publications, and patents (Park, et al, 2013). In that case, the researchers suggest that the acceleration allowed students to complete schooling and training earlier and develop more productivity during the “prime” years of early adulthood compared to matched controls who did not experience grade skipping.

Advantages and disadvantages of mathematics acceleration. Some recent studies have shown benefits of mathematics acceleration for certain groups of students (Collins & Gan, 2013; Loveless, 2009), but other research suggests that students accelerated in early grades prove more likely to drop mathematics at the first opportunity and to experience lower achievement overall (Boaler, 2016). Sheffield (2017) summarized much of this work and suggested that acceleration of more than one year is not likely to be beneficial. Instead, "mathematics experiences that emphasize multi-faceted, complex problem solving are much more effective than simply going faster through classes that are based on memorized rules and algorithms." (Sheffield 2017, p. 22). Boaler, Sheffield, and other researchers in mathematics pedagogy agree that advanced mathematics students, like all students, should have the chance to struggle and make mistakes as they explore mathematical concepts in depth

Shifts in national and state standards for mathematics mean that schools teach more algebra content in middle grades mathematics courses (CCSS, 2010). Skipping those courses would result in students missing out on essential mathematics content that provides a foundation for success in high school and college courses. "Simply put, mathematics learning is not a race, and evidence suggests that students who speed through content without developing deep understanding are the very ones who tend to drop out of mathematics when they have the chance" (Boaler, 2016, p. 192).

Experts in gifted education resoundingly point to acceleration as the most effective way to challenge and educate academically talented students.

> It is about matching the level and complexity of the curriculum with the readiness and motivation of the child. Acceleration is about letting students soar. Acceleration is about respecting individual differences and the fact that some of these differences merit educational flexibility (Colangelo et. al., 2004, p. 1).

In other words, acceleration benefits students who are ready for it.

Early Algebra

An introductory course in algebra, commonly called Algebra 1, is traditionally considered to be the first course in high school mathematics. It is often the first mathematics course for which students earn a mathematics credit towards graduation, even if the course is taken in middle school. In a survey of graduation requirements, 32 states explicitly list Algebra 1 or make reference to mastery of algebraic concepts (Dounay, 2007). Algebra 1 was once considered a traditional high school course, but in many parts of the country students more commonly take it in middle school (National Center for Education Statistics, 2012).

Middle School Algebra Enrollment

Enrollment in Algebra 1 or similar courses in eighth grade or earlier has been increasing over the past several decades. According to the National Center for Education Statistics (2012), in 1986, 21% of 13-year-old students who took the National Assessment of Educational Progress (NAEP) reported that they were taking a course in algebra or higher. By 2011, that number had increased to 47%. However, that trend levelled off or may have reversed, with only 43% of eighth grade students in algebra or a more advanced course in 2015.

When disaggregating these national averages by district or student category, the actual numbers of students taking algebra courses in middle school varies widely. In North Dakota, New York, and Mississippi, 21% of eighth graders were taking Algebra 1 or higher, while 59% of eighth graders were enrolled in those courses in California, according to 2007 NAEP data (Loveless, 2008). Even within a state, the access to and enrollment in middle school algebra across different school divisions is not consistent. One study in Virginia reported that the percentage of eighth graders enrolled in Algebra 1 or a higher course ranged from 25% to 95% across multiple school divisions (Senechal, 2014). Furthermore, some racial groups and students with lower socioeconomic status continue to be underrepresented in early algebra (Stein et al., 2011).

Reasons for Early Algebra

Extensive research suggests that the mathematics courses that a student takes in high school have an impact on a “range of postsecondary outcomes”, including choice of major, degree attainment, and participation in STEM careers (Newton et al, 2011). These effects may be even more pronounced for women and students from underrepresented minority groups (Bozick & Ingels, 2007).

Numerous studies have made links between high school mathematics and science courses and postsecondary participation in STEM programs. The number of mathematics and science courses a student takes in high school significantly predicts their choice of STEM majors in college (Maltese & Tai, 2011; Simpson, 2001). In particular, participation in a calculus course in high school has been linked to both choice of STEM major and more importantly to degree completion (Tyson, 2011; Bressoud et al, 2013). In most schools, the pathway to calculus consists of five courses beginning with Algebra 1, which means that calculus completion in high school requires taking Algebra 1 in eighth grade or earlier (Bressoud et al, 2013).

Among minority students, higher mathematics grades and test scores are associated with whether students enter and complete STEM majors in college (Bonous-Hammarth, 2000; Cole & Espinoza, 2008). Although these studies do not directly attribute post-secondary success to early algebra, further analysis in this chapter will provide support to the theory that the mathematics courses that students take in middle school have a lasting effect on their grades, test scores, and course-taking patterns throughout high school.

Equity

Research on the underrepresentation of women and/or minorities in STEM fields often includes a reference to students' high school mathematics and science achievements as an underlying factor in their ultimate success in college and career (Simpson, 2001). Stein et. al. (2011) found significant evidence that Black and Hispanic students, students from lower socioeconomic groups, and students whose parents have fewer years of education enroll in eighth grade algebra less frequently than their more affluent white and Asian peers.

> Although some of these students from traditionally marginalized groups who do not take algebra in eighth grade may not be prepared for the course, some evidence suggests that prepared students from those groups are also barred from early algebra course taking (Stein et. al., 2011, p. 460).

Other factors found to contribute to student participation in middle school algebra include peer and parent encouragement and school type (i.e. rural, suburban, urban) (Stein et. al., 2011). While these factors are not strictly demographic, they are associated with social and economic differences and therefore contribute to the gap in enrollment among high-poverty and minority students.

Access and opportunity gaps begin early for some students. For minority children, teacher perception of their ability in mathematics is influenced by their participation in elementary gifted and talented programs. In particular, Black students who have been enrolled in gifted and talented programs are more likely to be viewed as capable or advanced (Callahan et. al., 2013), which would in turn make them more likely to pursue advanced coursework in middle and high school. When given the opportunity, however, Black and Latino students perform as well as their peers, passing algebra in eighth grade at a rate proportional to their participation. (Patrick et. al., 2020)

Participation in Early Algebra

Students' access to algebra depends on the opportunities and policies of the schools they attend. Some school districts tightly control access to early algebra and identify students based on criteria such as grades and standardized test scores. In contrast, some states and school divisions have implemented universal algebra programs to provide early algebra instruction for all students.

Algebra Readiness

If we define algebra readiness as having the skills and habits that will ensure success in a first-year algebra course, then we can consider how to measure those skills and habits in students. Some factors contributing to algebra readiness are explicitly taught in early grades and are easily measured with standardized assessments, including number sense, fluency with basic computation, and skills in problem solving and mathematical communication (Gojak, 2013). Identification of other factors is less objective and often depends on teacher observations or other more subjective measures. These skills and habits are mathematics self-concept and self-efficacy, work habits such as persistence in challenging tasks, and "a level of maturity that includes a readiness to … work with abstract models and representations, and to understand and make connections among mathematical structures" (Gojak, 2013, p. 1) The National Council of Teachers of Mathematics (NCTM), in a position paper on supporting students with mathematical promise, made the point that mathematical promise is fluid and can be developed, and that students may show stronger aptitude for particular topics. NCTM encouraged schools to look students who "demonstrate patterns of focused interest; are eager to try more difficult problems or extensions or to solve problems in different, creative ways; are particularly good at explaining complex concepts to others…; and/or are strongly interested in the material" (NCTM, 2016, p. 1).

For students who have participated in specialized instruction for gifted and talented students in elementary school and perceive themselves as (or are perceived as by adults) above average, taking algebra in middle school is the logical next step (Gojak, 2013). In theory, this specialized instruction allows for students to develop the depth of understanding of numerical concepts as well as the problem solving skills that will prepare them for success in algebra. But these programs are inconsistent and have a history of inequities. Admissions criteria favor students with educated or affluent parents whose parents pay for tutoring or test prep programs, or have the means to advocate for their children and influence the selection process (Tyre, 2016). Teacher influence in the identification process is equally problematic, with a prevalence of inconsistent understanding and professional development about the characteristics of gifted students (Siegle & Powell, 2004). In particular, teachers tended to favor mental computation and student interest over other characteristics in identifying students. Perhaps out of a fear of misidentifying students, teachers focused more on weaknesses than on strengths (Siegle & Powell, 2004).

To further complicate matters, current trends in mathematical standards have students learning algebraic concepts as early as the primary grades. The CCSS emphasizes representing arithmetic concepts in a variety of ways. As early as kindergarten, students are expected to work with addition and subtraction problems including unknown quantities, such as "what plus three makes five?" Furthermore, standards that were previously considered new content in Algebra 1 have been pushed down into middle school mathematics courses (VDOE, 2016). If students are learning about algebra throughout elementary and middle school, then does that change the concept of algebra readiness?

Mason (2008) argued that students have “powers”, including generalizing, conjecturing, and “dealing with not knowing”, but that these powers need to be explicitly connected to mathematics in order to prepare students for doing algebra. Some researchers argue that students need a strong foundation in arithmetic to be able to do algebra (Nathan & Koedinger, 2000), but children are able to think about algebraic concepts without numbers at an early age, including using algebraic symbols. Teachers and students often try to classify problems as either algebraic or arithmetic. In doing so they fail to recognize that very few problems that can be solved with algebra cannot also be solved with arithmetic. Students must be flexible and choose an appropriate solution method (Mason, 2008, p. 78-79).

Some advocates argue that early algebra should be open to all students due to the benefits algebra confers and the difficulty in developing criteria to select students. “The movement away from selective algebra policies and toward universal algebra can be characterized as trading in one type of error for another” (Stein, 2011, p. 484). When schools impose selection criteria, students who may have the potential for success are excluded, creating “false negatives”. These false negatives present a valid equity concern. On the other hand, schools and school systems that have attempted to have all students complete Algebra 1 in middle school have faced challenges in maintaining rigor and supporting struggling students (Stein, 2011).

Universal Algebra

Even with numbers of students taking algebra in middle school on the rise nationally, a persistent gap in the opportunity to take early algebra access remains based on race and socioeconomic status. Using data from the Early Childhood Longitudinal Study, Walston and

McCarroll (2010) found that among students in the top two quintiles for fifth grade mathematics achievement, only 35% of Black students took algebra by eighth grade, compared to 63% of White students, 68% of Hispanic students, and 94% of Asian students. Filer and Chang (2008) reported a significant correlation between SES and attending an algebra class in eighth grade.

To address the equity issue associated with opportunities for algebra, some states and school divisions have instituted universal algebra or "Algebra for All" policies. In these initiatives, officials expect that all or most students will be enrolled in a first course in algebra by eighth grade. Some notable universal algebra programs include the statewide initiative in California, early college high school programs in North Carolina, and single-district policies in Chicago and Long Island (Stein, 2011).

In general, overall achievement rates as measured by standardized tests do not change under universal algebra policies (Howard, et. al., 2015: Allensworth, et. al., 2009). The number of students taking advanced mathematics courses increases, particularly among low income and minority students (Stein, 2011). Of critical importance, the greatest gains were found in schools where structured programs to support struggling students accompanied the universal algebra policy. In particular, students who had access to extra time to learn, in the form of double-block classes or other structured mathematics learning time, benefitted from the increased opportunity to learn (Stein, 2011).

If the goal is for all students to be successful in algebra and subsequent mathematics courses, then "policy should focus on enhancing students' *opportunities to learn*" (Stein, 2011, p. 486). However, students vary in their interest and motivation, especially in middle school, and are not equally likely to benefit from such opportunities (Domina, 2014). In fact, there could be lasting negative effects if students are unsuccessful and need to retake the course (Howard, et. al., 2015; Clotfelter et al., 2015; Allensworth, et. al., 2009).

More students taking algebra means that more students passed it, but also that more students failed. Pass rates and average scores tended to drop slightly in the universal algebra programs (Stein, 2011; Domina, et. al., 2015). Domina (2014) expressed the concern that "placing students in academic environments for which they are academically and developmentally unprepared may have unintended negative consequences for student self-efficacy, motivation, and achievement." Furthermore, the research has focused on the results, i.e. the test scores and subsequent courses taken, but little is known about the instruction. In sharing their conclusions, researchers have raised questions about how students are grouped, the impacts of those groupings on teacher and peer interactions (Stein, 2011; Domina, et. al., 2015), what types of supports are available when students struggle, and the types of educational settings, curricula, and pedagogy to which students are exposed (Heckman, 2013).

Impacts of Early Algebra

The preponderance of the research on early algebra has focused on students who take their first course in algebra in eighth grade. Even with similar variables under consideration, individual studies have come to different conclusions about the benefits or drawbacks of early algebra. In a meta-analysis of studies on early algebra and universal algebra, Stein et al. (2011) found much support for the notion that early algebra is beneficial as a predictor of higher mathematics achievement and taking more advanced mathematics courses. The same report, however, cites a number of studies that provide evidence that these positive results may not be generalizable for all students and that in particular lower achieving students may in fact be harmed by starting algebra in middle school (Stein et al., 2011). More than half of the students who took algebra in eighth grade did not take an advanced mathematics course in high school, but no clear explanations exist for why that might be the case based on the reviewed research (Stein et al. 2011). In this section, research will be presented on the impacts of early algebra on mathematics achievement, course taking patterns, and post-secondary opportunities.

Long Term Impacts

Two recent studies have taken a closer look at the short and long term impacts of early algebra for students based on their prior mathematics achievement. Clotfelter et al. (2015) considered students from two North Carolina school districts that had implemented policies intended to increase the number of eighth graders enrolled in algebra courses. The researchers concluded that higher scoring middle school students were 10-20% more likely to enroll in a calculus course by twelfth grade if they took algebra in eighth grade, but that the majority of students scoring in the middle range in middle school will “fall off track at some point before they reach calculus” (Clotfelter et al., 2015, p. 182). The chance of low performing students taking calculus during high school is near zero no matter when they take algebra. In another study, Dougherty et al. (2017) found evidence of a “leaky pipeline”, noting that only one-seventh of students who took eighth grade algebra had completed precalculus by eleventh grade, even when the students had been selected for acceleration into algebra based on an objective measure predicting their success.

Researchers have also investigated the degree to which participation in early algebra has an impact on learning and achievement in mathematics. In general, taking advanced mathematics coursework, i.e. courses above Algebra 2, is associated with higher mathematics achievement (Ma, 2000; Bozick & Ingels, 2008; Wang & Goldschmidt, 2003; Smith, 1996; Newton, 2010; Spielhagen, 2006), so a logical conclusion would be that early algebra will be a good thing because it allows students more time in high school to take additional mathematics courses. However, the research does not support that this is happening. In Chicago, little change was reported in completion of higher level mathematics courses. The lowest-ability students were slightly more likely to take courses beyond Geometry than before the universal algebra mandate, but they did not take the full range of courses to which they had access (Allensworth, et. al., 2009),

Similar to the analysis of course-taking patterns, one must look more closely at these results. While a relationship exists between the number of mathematics courses taken and mathematics learning, "most of 12th-grade achievement is explained by background factors and previous learning" (Bozick & Ingels, 2008, p. 33). An even more troubling interpretation is students come to high school with persistent gaps based on their middle school experiences and that high school programs are not designed to close those gaps, leaving some students even further behind (Wang & Goldschmidt, 2003).

Short Term Impacts

The studies referenced above also examined short-term impacts of acceleration. Dougherty et al. (2017) concluded that no association exists between acceleration in middle school and improved middle school standardized test scores. Similarly, in a matched comparison of students who took and failed algebra to similar students who successfully completed a lower level course, researchers found no significant difference in their performance on an algebraic reasoning assessment (Howard, et. al., 2015). In other words, for average to low performing eighth graders, the learning advantages of Algebra 1 appear to be minimal.

The risk of failure increases as more students are identified for early algebra. For example, in a sample of students who had been selected for acceleration into Pre-algebra in seventh grade, only 54.2% passed Algebra 1 in eighth grade (Dougherty et al., 2017). The results are consistent in the study by Clotfelter et al. (2015), which found that among low to middle achieving students based on middle school test scores who were accelerated to algebra in eighth grade, the rate of repeating Algebra 1 was as high as 50%, compared to about 20% of students who repeated the first course in algebra after taking it in ninth grade or later. Similarly, in Chicago, failure rates and grades decreased for both low- and average-ability students, although standardized test scores were unaffected (Allensworth, Nomi, Montgomery, & Lee, V. E., 2009).

Struggling in or failing algebra can have a negative impact on students' attitude, identity, and interest related to mathematics. Even when controlling for achievement and demographic related factors, there can be a significant decrease in attitude about mathematics during Algebra 1 (McCoy, 2005). Howard et. al. (2015) found that, although students who failed Algebra 1 in eighth grade demonstrate similar mathematics proficiency in high school to students who had taken a lower-level mathematics class in eighth grade, the students who failed reported lower levels of mathematics interest, identity, and utility.

In summary, while the body of literature on early algebra suggests benefits for students, recent studies indicate that those benefits may not be universal. Prior research may have inaccurately implied causal relationships between early algebra and mathematics achievement for all students (Clotfelter et al., 2015). For the lowest performing students, starting algebra in middle school "introduces significant downside risks with little to no upside potential" (Clotfelter et al., 2015). Furthermore, most of these studies provide control by demographics or academic achievement but contain little to no information about factors such as the teacher quality, the effectiveness of instruction, school leadership, student motivation, or parent and peer influences.

Summary

This literature review has described the results of initiatives designed to increase algebra instruction in middle schools with the goal of providing access to STEM courses and careers for all students. In addition, the chapter addressed trends in mathematics instruction for gifted students as well as for all students, particularly in light of the introduction of the CCSS. The convergence of best practices for teaching gifted mathematics students and all mathematics students creates a challenge for educators to provide enrichment and acceleration opportunities to students who show promise. While selective and universal algebra programs for eighth graders have been widely covered, there is a gap in the research on students accessing Algebra 1 in earlier grades and the long-term impact on their achievement and course taking patterns.

Chapter 3

The purpose of this study was to identify the impact of a school division policy on early algebra on students' course taking patterns in high school. The study incorporated quantitative statistical analysis techniques to examine relationships among variables and differences among groups of participants, with regard to a school district policy that determines entry to early algebra based on test scores. Analyzing the expected and actual outcomes based on achievement on these entry criteria provided insight into whether the policy identified students for acceleration who ultimately benefited from it. If students who enroll in Algebra I in earlier grades also take more advanced or higher-level mathematics classes, then the acceleration would have the intended outcome. Comparing the differences in course taking patterns and overall completion of mathematics and science courses provides a measure of the benefit of early algebra.

Research Questions

1. Are students more likely to take progressively more challenging courses each year and go on to take the highest level of mathematics available to them at the same rate based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?
2. What patterns emerge when comparing mathematics course pathways for students based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?
3. What are the differences in mathematics and science course-taking based on whether students met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

Research Design

The basic design of this study was a transcript analysis. Both the explanatory and response variables were based on coursework taken by students. This is an example of causal-comparative, or *ex post facto* research, in which groups are determined by "something that happened in the past" and the groups are compared "on another variable in such a way that it makes possible drawing potential causal relationships between the two variables (Mertler, 2016, p. 12). In this study, the groups were determined by if the participant met the assessment criteria for early algebra and when the participant took Algebra 1. The response variables were whether the student completed the highest level of mathematics courses available to them and took mathematics all four years in high school and the total number of mathematics and science credits earned in high school.

The participants were sorted into four population groups based on Figure 1. One group is students who met the qualification criteria for early Algebra 1, i.e., completing seventh grade mathematics content in sixth grade, scoring in the 91st percentile or higher on the IAAT, and meeting the Pass Advanced benchmark on the Grade 7 Mathematics state assessment in sixth grade, and then successfully completed Algebra 1 Honors in seventh grade. The second group is students who met those criteria but chose to delay taking Algebra 1 until eighth grade. The third group is students who did not meet one or both of the assessment criteria for seventh grade algebra but were granted waivers to take Algebra 1 Honors in seventh grade. The final group is all remaining students who chose to take Algebra 1 in eighth grade. That final group contained students who took the qualifying exams for seventh grade algebra but did not meet the criteria and students who did not take the qualifying assessments. Students who completed Algebra 1 prior to seventh grade or after eighth grade are not included in the study.

To answer the first research question, a Chi-Square procedure was used to measure the association between the population variable and whether or not students accessed the highest mathematics course available to them. This course varies based on the year in which the student completed Algebra 1 and the school that they attend.

To address the second question, graphical and tabular displays were prepared, summarizing mathematics course choices across the years from seventh through twelfth grade for each group of students. The goal of those displays is to highlight similarities and differences in course choices among the four subgroups.

For the third question, a one-way ANOVA was performed, comparing the mean total number of mathematics and science courses taken in school. Mathematics courses include courses in computer science. The total number of mathematics and science courses is a proxy for the student's continuing passion and interest in STEM courses.

Ethics, IRB Approval, and Confidentiality

This research contains sensitive student information. To protect the identity of the division and any individual schools within the district, pseudonyms were used. The researcher completed CITI training for Social and Behavioral Research (Appendix A). This research proposal was submitted to and approved by the Internal Review Board (IRB) of Virginia Tech (Appendix B) and the Office of Research and Strategic Improvement of the participating school division (Appendix C).

The data were de-identified by the school division before receipt by the researcher, with only an arbitrary numerical identifier for each student. The individual school data were used only to identify the highest mathematics course available to each student and the published results do not include the identification of individual schools.

Data Collection

Transcript and test score data were obtained for students from a single, large, suburban school division. The district has had a clear policy on mathematical acceleration in place for over ten years. Because of its large size, there is adequate representation in each of the population groups to conduct the statistical analyses.

The students included in the study had been enrolled in schools within the same school division from seventh grade through high school graduation. This ensured that all of the participants had been subjected to the same policies and had equal access to higher level courses. Although specific course options vary by school, all schools in the division offered mathematics course sequences that allowed students who began high school level mathematics in middle school to matriculate to a higher level of mathematics each year until twelfth grade.

The information for each student included:

- Score on the Iowa Algebra Aptitude Test (IAAT);
- Scores on state assessment in Mathematics 7;
- High school from which they graduated;
- Mathematics courses, including computer science, taken in each year from seventh through twelfth grade;
- Science courses taken in each year from ninth through twelfth grade; and
- Demographic information (race, sex, socioeconomic status, English learner status).

The IAAT and state assessment scores were used to classify students by whether they met the qualifying criteria for taking Algebra 1 Honors in seventh grade. Their mathematics course history and school attended determines if they took the highest mathematics course available. Opportunities for advanced mathematics vary across high schools, especially as there is a difference in offerings at Advanced Placement (AP) schools and International Baccalaureate (IB) schools.

Procedures

The first step was to code the data in order to provide the proper input and output format for performing the statistical analysis. Using Microsoft Excel macros and conditional statements, variables were created to assign the students to the population groups based on IAAT and state assessment scores and when they completed Algebra 1. Other variables were the binary variables *completed the highest course available* and *took no mathematics course* and the total count of mathematics and science courses taken in high school. The statistical analysis was completed with the statistics software package JMP.

Chi-square Test

Chi-Square is the most common procedure for comparing two categorical variables. The null hypothesis was that the two variables were independent (Howell, 2017). In this case, the goal was to determine if there was an association between the population group variable, i.e., when and how a student took Algebra 1, and whether or not they completed the highest mathematics course available to them. In an ideal world, all students would feel successful in their courses and would maintain a course taking pathway that would progress to higher level content each year, and therefore they would achieve the highest level available. However, students choose to take courses that vary from that pathway for a variety of reasons. The Chi-

Square procedure assesses whether students in any one of the population groups are more likely to complete the highest mathematics courses than others.

One drawback of the Chi-square test is that it becomes less reliable when the expected frequency or count in any one category is small. This is because the test assumes that in repeated trials, the frequency in each category would be normally distributed. A conservative criterion for application of the Chi-square test is that all expected frequencies be at least 5 in a table with fewer than ten cells (Howell, 2017, p. 509). While there was a large variation in the size of each group, the smallest group contained over 200 participants, which far exceeded the minimum requirement.

One-way Analysis of Variance

The number of mathematics and science courses a student takes was a ratio variable. In other words, equal intervals represented equal differences and there was a true zero, i.e. a student could have no mathematics or science courses, and twelve mathematics or science courses represented twice as many courses as six. To compare multiple groups with a single independent variable and a quantitative output variable, the statistical technique was one-way Analysis of Variance (ANOVA) (Howell, 2017).

In ANOVA, the null hypothesis is that the mean value for each group defined by the independent variable is the same. In this case, the null hypothesis was that the total number of mathematics and science courses taken in high school would be the same for all four population groups. The alternative hypothesis was that any one group differed significantly. If the ANOVA reveals a significant difference among the means, then as a second step, Fisher's least significant difference (LSD) test allows for between-group comparisons to further clarify the differences among the group means (Howell, 2017).

The use of ANOVA requires three assumptions. First, that the values of the variables of interest in the populations from which the sample is drawn are normally distributed around the mean. However, the ANOVA procedure is considered robust and “even substantial departures from normality may… have remarkably little influence on the final result” (Howell, 2017, p. 406). The second assumption is that the observations are independent. While it can be acknowledged that there are relational factors that influence students’ course choices, ultimately each individual student makes their own decision and it is reasonable to assume that students were independent with regards to the response variable.

Finally, homogeneity of variance is assumed, i.e., that each population has the same variance in the population of interest. ANOVA is also robust in regard to this assumption (Howell, 2017). If the populations are normal, or at least symmetric or similar in shape, and the largest variance is less than five times the smallest, then the ANOVA should still be valid.

Another consideration in ANOVA is that of the size of the sample groups. Ideally, all of the sample groups would be the same size. In the case of this study, that was not realistic. The test statistic calculation took into account the unequal sizes. This assumption is not usually considered to be problematic unless it is accompanied by unequal variances.

Displays of Course-taking Patterns.

Graphical and tabular displays represented mathematics course pathways for students within each of the four groups. This pathway analysis consisted of grouping students by whether they continued to take progressively higher-level coursework, including whether they chose honors-level courses. The goal was to identify where there may have been a “leaky pipeline” (Dougherty, et. al., 2017) in students’ progress towards higher-level mathematics. In other words, are there patterns related to when students make choices to move into or out of honors-level courses, or to take less rigorous options?

Reliability

In general, school transcript records are considered to be a reliable source of data for educational research. However, most of the research on reliability of quantitative data for educational research is focused on the data collected through assessment instruments such as tests and surveys. In a study on the reliability of using medical records for research studies, Aaronson and Burman (1994) concluded that regarding objective measures, medical records are reliable. It is reasonable to extend that notion of reliability to educational transcripts. Furthermore, researchers should establish and follow clear and precise procedures for extracting and organizing the data to maintain the reliability. As a sole researcher, an effort was made to seek out an independent audit of the data coding and sorting process to ensure the reliability of the data to be analyzed.

Validity

Threats to validity in this study are related to the *ex post facto* nature of the investigation and to the types of data that were analyzed. Because there was no random selection of participants to the explanatory variable groups, there was a concern for selection bias. Students may have had differences that impacted their decision when to take algebra, including parent education level and involvement, prior school experiences, and peer influence. Those same factors may have also influenced their motivation to continue in a rigorous mathematics sequence and their chances of success in higher level courses. When participants share characteristics that may affect their response it is referred to as differential selection (Mertler, 2016).

Summary

Quantitative analysis of course-taking patterns gave some insight into whether students benefitted from hyper-acceleration, i.e., did they take the highest levels of mathematics courses available to them and did they pursue coursework that indicated an interest in STEM pathways. Examining graphical and tabular displays of summary quantitative data helped to illuminate whether there were patterns related to when students made decisions about continuing to pursue higher level mathematics. The results of these analyses may contribute to answering the question of whether or not hyper-acceleration in mathematics is beneficial for students.

Chapter 4

Introduction

The purpose of this study was to identify the impact of a school division policy on early algebra on students' course taking patterns in high school. Patterns in mathematics and science course taking for students taking early algebra were explored through a quantitative analysis of transcript data. Studying course-taking patterns allowed for some insight into whether students benefited from the policy by taking the most advanced mathematics courses available to them, and how well the criteria set out by the policy identified students who were likely to benefit from the policy.

There are many large-scale studies using student transcripts to explore course-taking patterns across states or internationally, such as the National Assessment of Educational Progress and the Programme for International Student Assessment. One difference in this study is that all of the students were enrolled in the same school division for middle and high school, and therefore were subject to the same placement policies and course opportunities. Furthermore, most of the studies related to taking algebra have focused on eighth graders, and the conclusions have been unclear about the benefits and drawbacks to Algebra 1 in middle school (Stein et al., 2011). In this study, course taking patterns for students were compared based on taking Algebra 1 in seventh or eighth grade, to seek insight into whether additional acceleration should be encouraged.

Research Questions

1. Are students more likely to take progressively more challenging courses each year and go on to take the highest level of mathematics available to them at the same rate based on

whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

2. What patterns emerge when comparing mathematics course pathways for students based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?
3. What are the differences in mathematics and science course-taking based on whether students met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

Data Sorting and Coding

Sorting and Classifying Subjects

For this study, transcript data were obtained for the graduating classes of 2017 and 2018 from a large, diverse, suburban school division. The data included demographic information, all mathematics and science courses taken from grades 7-12, scores on the IAAT (if taken), and scores and year taken for the Grade 7 Mathematics state assessment. After removing students who had not been enrolled continuously across the six years from grades 7-12, the data set included 18,700 student records. Further reducing the data to include only students who completed Algebra 1 in middle school left a total of 13,419 students, 2,511 who earned credit for Algebra 1 in seventh grade and 11,038 who earned credit for Algebra 1 in eighth grade.

The IAAT and state assessment scores were used to identify whether students had met the qualifications for taking Algebra 1 in seventh grade. Then students were assigned to groups as shown in Table 1.

Table 1

Assignment of students to groups

Group designation	Description	Number of students in group
A	Students qualified and completed Algebra 1 in seventh grade	1840
B	Students qualified for Algebra 1 in seventh grade but completed it in eighth grade	226
C	Students did not qualify for Algebra 1 in seventh grade but completed it in seventh grade	668
D	Students did not qualify for Algebra 1 in seventh grade and completed it in eighth grade	10659

Grouping of Courses

Students in the school division typically take Algebra 1, Geometry, and Algebra 2 (or the honors equivalents of those courses) as their first three high school mathematics credits. Beyond Algebra 2, students have options including AP and IB pathways as well as standard level electives. Each school has a different set of electives, so to simplify the analysis of individual pathways, courses are grouped as shown in Table 2.

Some students take more than one mathematics course in a school year, particularly in eleventh and/or twelfth grade. Students who took more than one course were then assigned to a pathway based on the highest priority course in that year. In general, when students took successive courses in the same year, such as Algebra 2 and Geometry, they were assigned to a pathway based on the higher-level course. If a student took an AP or IB course in conjunction with a standard level course, such as AP Statistics and Applied Calculus, they were assigned to the pathway based on the AP or IB course. Assigning students to pathways in this way eliminated double counting them and accounted for the most challenging pathways to be recognized in the analysis.

Course Progression

> **Research Question 1:** Are students more likely to take progressively more challenging courses each year and go on to take the highest level of mathematics available to them at the same rate based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

Data for Research Question 1

The first research question considered how students' progress through high school mathematics pathways. First consider whether students chose progressively more challenging courses each year and went on to take the highest level of mathematics available to them. For students who took Algebra 1 in seventh grade and followed the expected pathway through high school courses, the most advanced course available to them is Multivariable Calculus. For students who took Algebra 1 in eighth grade, there are several courses that could be considered the culmination of a progressively more challenging course sequence. For this study, students who completed AP Calculus AB, AP Calculus BC, AP Statistics, or a two-year International Baccalaureate sequence are all considered to have completed a highest course available to them.

Another measure to consider was how many students did not take a mathematics course at all in at least one year of high school. The graduation requirement for the school division under consideration is for students to take at least three years of mathematics at the high school level, beginning with Algebra 1. A student who successfully completes Algebra 1 in seventh grade could fulfill the mathematics requirement for graduation by the end of ninth grade. Thus, while these students would have completed their graduation requirements, reviewing the paths and additional courses that students who choose to accelerate two years in mathematics continued to take in mathematics throughout high school is relevant to this analysis.

Table 3 shows the summary by group of students who achieved the highest course available to them. It also shows the students who did not take a mathematics course in at least one year of high school.

Table 3

Students taking highest math and students taking no math, by group

	Total students	Completed highest math (count)	Completed highest math (percent of group)	No math at least one year (count)	No math at least one year (percent of group)
Group A	1840	559	30.4%	212	11.5%
Group B	226	164	72.6%	25	11.1%
Group C	668	146	21.9%	79	11.8%
Group D	10659	5446	51.1%	1527	14.3%

Highest Mathematics Course Completed

A chi-square test of independence was performed to examine the relation between when students took Algebra 1 and if they reached the highest mathematics course available to them. Table 4 is the contingency table for all four groups.

Table 4

Contingency table for Algebra 1 completion year and highest mathematics course completed (n=13,419)

Count Total % Col % Row %	Did not complete highest math course available	Completed highest math course available	Total
A	1281	559	1840
	9.56 %	4.17 %	13.74 %
	18.10 %	8.85 %	
	69.62 %	30.38 %	
B	61	165	226
	0.46 %	1.23 %	1.69 %
	0.86 %	2.61 %	
	26.99 %	73.01 %	
C	522	146	669
	3.90 %	1.09 %	4.99 %
	7.38 %	2.31 %	
	78.14 %	21.86 %	
D	5212	5447	10659
	38.92 %	40.67%	79.59 %
	73.66 %	86.23 %	
	48.90 %	51.10 %	
Total	7076	6317	13393
	52.83 %	47.17 %	

The relation between these variables was significant, X^2 (3, N = 13393) = 527.453, p < .0001. There is an association between when students take Algebra 1 and if they take the highest mathematics course available to them. Students in groups B and D, i.e., those who took Algebra 1 in eighth grade, were more than twice as likely to reach the highest mathematics courses available to them than students who took Algebra 1 in seventh grade. Students in group B, who met the qualifications for Algebra 1 in seventh grade but chose to wait until eighth grade to take it, were the most likely to go on to complete the highest level course available. Almost three-quarters of that group took an AP or IB mathematics course in twelfth grade. In contrast, only

21% of the students in group C, who did not qualify for seventh grade Algebra 1 but took it that year anyways, took the highest mathematics available to them.

Mathematics All Four Years

A chi-square test of independence was performed to examine the relationship between when a student took Algebra 1 and if they took mathematics all four years in high school. Table 5 is the contingency table.

Table 5

Contingency table for Algebra 1 completion year and no mathematics course (n=13,419)

Count Total % Col % Row %	Took math all four years	No math in at least one year	Total
A	1628 12.16 % 14.10 % 88.48 %	212 1.58 % 11.50 % 11.52 %	1840 13.74 %
B	201 1.50 % 1.74 % 88.94 %	25 0.19 % 1.36 % 11.83 %	226 1.69 %
C	589 4.40 % 5.10 % 85.67 %	79 0.59 % 4.29 % 11.83 %	668 4.99 %
D	9132 68.18 % 79.06 % 85.67 %	1527 11.40 % 82.85 % 14.33 %	10659 79.59 %
Total	11550 86.24 %	1843 13.76 %	13393

The relationship between these variables was significant, X^2 (3, N = 13419) = 15.13, p = .0017. Students who took Algebra 1 in eighth grade were slightly more likely to stop taking mathematics during high school than students who took Algebra 1 in seventh grade.

Summary

The data suggest a contradiction here. While students who benefited from hyper-acceleration and completed Algebra 1 in seventh grade were more likely to continue to take mathematics courses each year, they were less likely to access the highest mathematics course available to them, based on the Chi-squared results. This issue will be explored more in this section regarding course-taking patterns.

Course Taking Patterns

> **Research question 2:** What patterns emerge when comparing mathematics course pathways for students based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

Examining Course Pathways

While there are limited choices in mathematics courses for students at each school each year, there are actually over 1400 unique mathematics pathways that students in this sample may have followed. To simplify the examination and analysis of course pathways, courses with similar pre-requisites are grouped together and IB courses were grouped with non-IB courses that cover similar content**.** The course groupings are described in Table 3.

Table 6 displays the course progression for students who qualified for and completed Algebra 1 Honors in seventh grade (group A). The bold values represent students who continued to take honors courses each year. In this analysis, students are not tracked through pathways as individuals. Instead, the total number of students from the group in each course is reported for each grade level.

Table 6

Course Sequences, Group A. Algebra 1 Honors in seventh grade

	Grade Level					
Course	7th	8th	9th	10th	11th	12th
Algebra 1		5	2			
Algebra 1 Honors	**1842**	75				
Geometry			29	2		
Geometry Honors		**1739**	103			
Algebra 2			101	51	5	
Algebra 2 Honors		23	**1581**	86		
Precalculus or IB SL 1			3	512	124	12
Precalculus Honors or IB HL 1			23	**1131**	187	1
AP Calculus AB or IB SL 2				7	632	151
AP Calculus BC or IB HL 2				16	**688**	507
Multivariable Calculus					14	**544**
Other AP and IB courses				17	44	377
Other mathematics electives				8	85	84
No mathematics course				12	63	166

There is a decrease in students who continue in the honors pathway each year, with the largest decreases occurring between Algebra 2 Honors and Precalculus Honors (28.4 % decrease) and between Precalculus Honors and AP Calculus BC (39.2 % decrease).

Students who did not qualify for seventh grade algebra but were still able to complete Algebra 1 Honors that year (group C) exhibit similar patterns, as shown in Table 7. The bold values represent students who continued to take honors courses each year. Similar to group A, there is a decrease in students who continue in the honors pathway each year. There is a decrease of 35.7 % between Algebra 2 Honors and Precalculus Honors and a decrease of 49.0 % between

Precalculus Honors and AP Calculus BC. While this group is smaller, by percentage these drops in students are somewhat larger than the drops in the previous group.

Table 7

Course Sequences, Group C

	Grade Level					
Courses	7th	8th	9th	10th	11th	12th
Mathematics 8		2				
Algebra 1	4	5	2			
Algebra 1 Honors	**665**	46				
Geometry			19	2		
Geometry Honors		**613**	46			1
Algebra 2			58	35	4	
Algebra 2 Honors		3	**540**	39		
Precalculus or IB SL 1			2	222	91	12
Precalculus Honors or IB HL 1			2	**347**	59	2
AP Calculus AB or IB SL 2					244	88
AP Calculus BC or IB HL 2				4	**177**	156
Multivariable Calculus					4	**142**
Other AP and IB courses				14	29	169
Other mathematics electives				4	41	36
No mathematics course				2	20	63

Table 8displays the course progression for students who qualified for seventh grade algebra but did not complete Algebra 1 until eighth grade (group B). The bold values again represent students who continued to take honors courses each year. Similar to groups A and C, there is a decrease in students who continue in the honors pathway each year. There is a decrease of 40.9 % between Algebra 2 Honors and Precalculus Honors and a decrease of 64.8 % between Precalculus Honors and AP Calculus BC.

Table 8

Course Sequences, Group B

	Grade Level					
Course	7th	8th	9th	10th	11th	12th
Mathematics 7	2					
Mathematics 7 Honors	**224**					
Algebra 1		36	2			
Algebra 1 Honors		**190**	1			
Geometry			48	3		
Geometry Honors			**174**			
Algebra 2				66	4	1
Algebra 2 Honors			1	**154**		
Precalculus or IB SL 1				1	117	3
Precalculus Honors or IB HL 1					**91**	
AP Calculus AB or IB SL 2						102
AP Calculus BC or IB HL 2					1	**32**
Multivariable Calculus						1
Other AP and IB courses						27
Other mathematics electives					8	39
No mathematics course				2	5	21

For students who did not qualify for Algebra 1 in seventh grade and then took it in eighth grade (group D), it is not as clear what might constitute an expected pathway. More than half of students in this school division take Algebra 1 by eighth grade, so it is reasonable to assume that group D is more diverse than the other groups in terms of their interest and abilities in mathematics and in STEM associated courses and career pathways. However, the same pattern that emerges in groups A, B, and C holds true with group D when considering the distribution of students in honors and standard level courses in each year. Table 9 displays the course pathways for Group D. In seventh grade, about 67% of the students in the group were in Mathematics 7 Honors. In eighth and ninth grades, the proportion of students in honors-level Algebra 1 and Geometry was around 50%, but then in tenth grade only 42.5% of the students were in Algebra 2 Honors. Those percentages account for students who followed the traditional course sequence, and do not include students who repeated a course or accelerated by taking a summer course.

Table 9

Course Sequences, Group D

	Grade Level					
Courses	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}	12^{th}
Mathematics 7	3540					
Mathematics 7 Honors	7142					
Algebra 1		5357	953	29	1	
Algebra 1 Honors		5322	204	2		
Geometry			4696	1031	79	6
Geometry Honors		3	4678	143		
Algebra 2			21	5297	1190	153
Algebra 2 Honors			130	3919	79	1
Precalculus or IB SL 1				76	5506	613
Precalculus Honors or IB HL 1				116	1989	31
AP Calculus AB or IB SL 2					82	2840
AP Calculus BC or IB HL 2				1	70	714
Multivariable Calculus					1	54
Other AP and IB courses				3	313	1744
Other mathematics electives				2	1117	3179
No mathematics course				63	255	1347

Graphical Representations of Twelfth Grade Course Completion

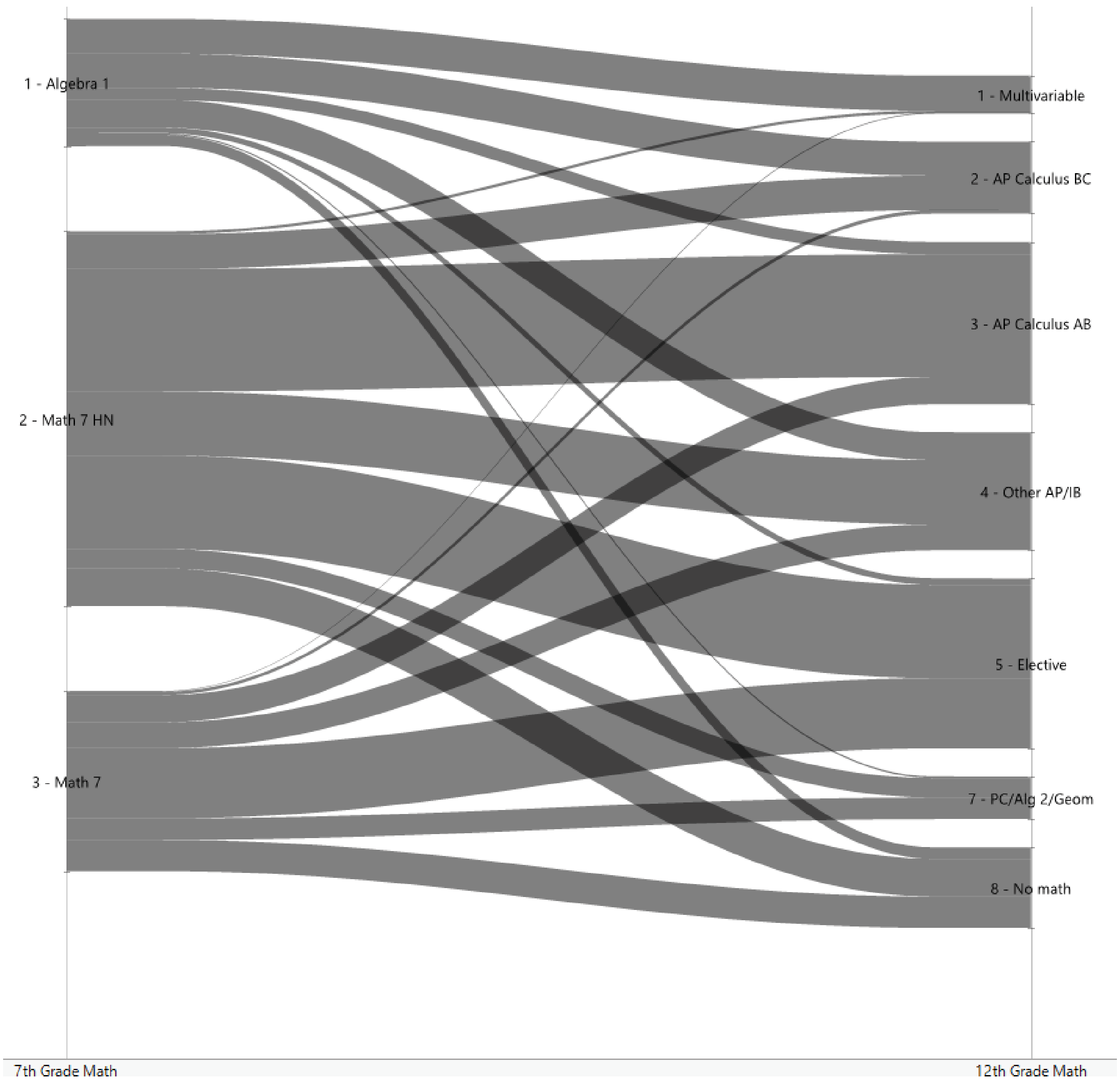
Due to the wide variation in options for students, it is challenging to represent all possible course choices in a single graphical display. A Sankey diagram is a type of aggregate visual display (Oran, et. al, 2019), which means that its intent is to represent characteristics of cohorts of students. It is a subcategory of flow diagrams, which represents progress from point to point with the width of each flow based on its magnitude. Sankey diagrams have been widely used in fields such as energy and medicine but have only recently been used in educational settings (Oran, et. al., 2019).

Figure 2 shows a Sankey diagram, or flow diagram, to illustrate the pathways from seventh grade mathematics to twelfth grade mathematics for students who took Algebra 1 in middle school. At each grade level, honors and standard level courses have been combined. AP

Calculus includes AP Calculus AB, AP Calculus BC, IB Mathematics SL2, and IB Mathematics HL 2. Other mathematics electives include any other courses above Algebra 2 and Precalculus, including other AP and IB courses.

Figure 2

Flow diagram of mathematics from seventh to twelfth grades



This diagram revealed that in the group of students who completed Algebra in seventh grade, only about half of the students completed the highest mathematics courses available to any high school student in the district, i.e. Multivariable Calculus or AP Calculus BC. However, the majority of students in that group ended high school with an AP or IB course, and only a small percentage of them took lower level courses or no mathematics at all in twelfth grade. More than half of the students who were in Mathematics 7 Honors in seventh grade, thus completing Algebra 1 in eighth grade, also completed high school with an AP or IB course, with a similar proportion having no mathematics course in twelfth grade. For students who took standard level Mathematics 7 and then Algebra 1 in eighth grade, a much smaller proportion of students took AP or IB courses in twelfth grade, while the proportion not taking mathematics that year appears to be slightly higher than in the other groups.

Mathematics Pathways Across Secondary School

The Sankey diagram reveals general patterns of change over time, but with so many pathways available, such a display becomes very busy when including course choices for the intervening grade levels. Because there are so many options in any given year for students, and some students move in and out of honors pathways, an alternate way to look at the data is by classifying students based on how their course choice in any given year compares to the course they completed in the previous year. This approach is similar to the analysis of students' progress and mobility through university programs carried out by Oran et. al (2019). In that study, they created a data visualization approach based on Sankey diagrams that "summarize the progress of students through a curriculum in an easily understandable way" (Oran et al, 2019).

To create this data visualization, it was necessary to classify the types of transitions in mathematics coursework that students make each year. Many students choose to follow an expected pathway through a high school mathematics sequence, either at the standard or honors level. The district open enrollment policy allows students to transition to an honors course at any time; for example a student may choose to take Algebra 2 Honors after taking standard level Geometry the previous year. A student may also choose to “drop out” of the honors track or may take a summer course to accelerate through the curriculum. A separate category is defined for students who took AP Calculus AB followed by AP Calculus BC. The course transitions are defined for this analysis in Table 10.

Table 10

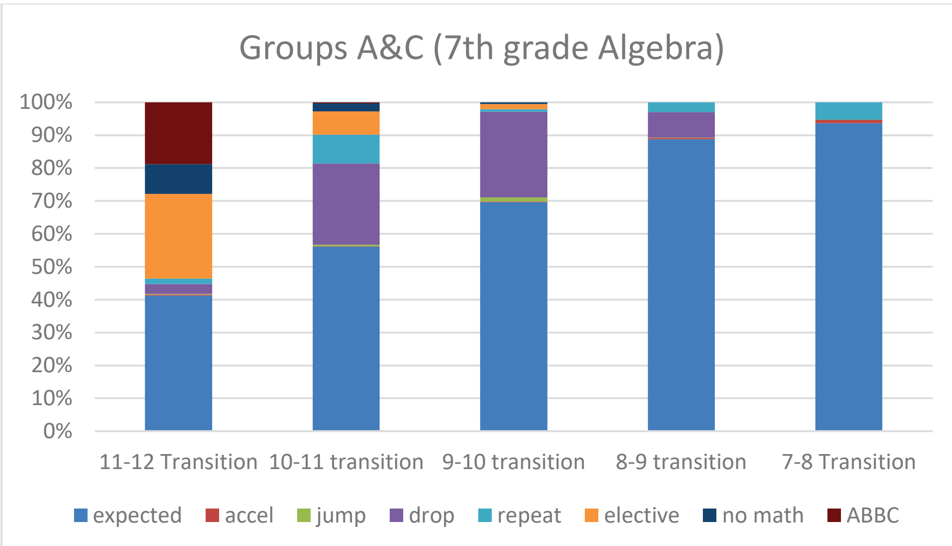
Course transitions for pathways analysis

Course Transition	Description
Expected	Student progressed to the next course in the pathway
Drop	Student progressed to the next level but moved from honors to standard
Jump	Student progressed to the next level and moved from standard to honors
Accelerate	Student moved to a course two levels above the previous year
Repeat	Student took the same course as the previous year
No math	Student took no mathematics course
ABBC	Student took AP Calculus BC after taking AP Calculus AB the previous year

Figure 3 represents the year-to-year transitions for the group of students who took Algebra 1 Honors in seventh grade. In eighth and ninth grades, there were a small number of students who repeated a course, but most students took the expected course the following year. Starting in the transition from ninth to tenth grade, there was a noticeable decline in students continuing in their expected pathway, with over 20% of students in each of those grades

Figure 3

Course transitions for seventh grade Algebra 1

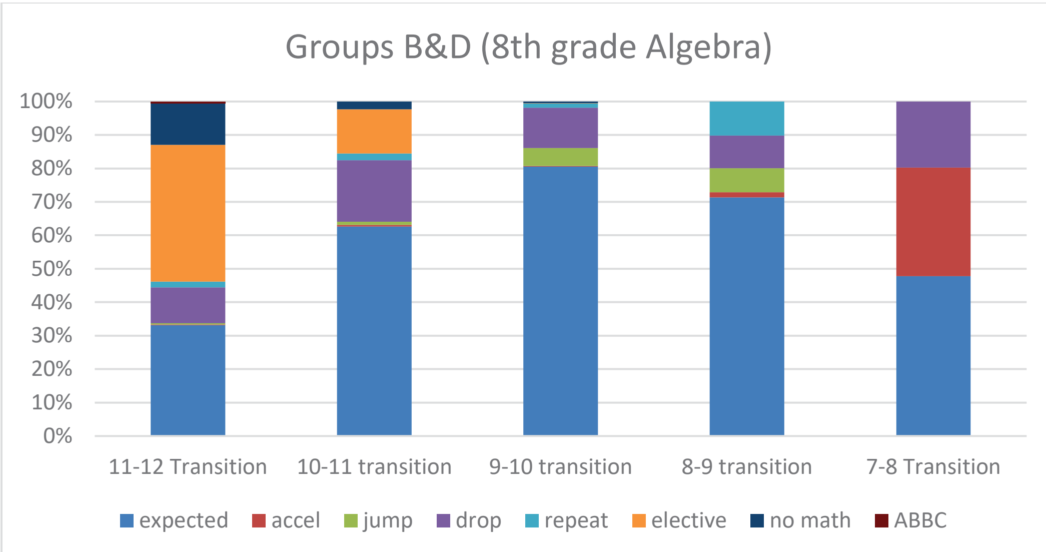


dropping from an honors level course to a standard level course in the next grade level. Also notable in this group is that about 20% of students took AP Calculus AB in eleventh grade and then AP Calculus BC in twelfth grade.

The group of students who completed Algebra 1 in eighth grade exhibit a different pattern in course transitions, as shown in Figure 4. The seventh to eighth grade transition represents some students accelerating from Mathematics 7 to Algebra 1 and some students taking standard level Algebra 1 after taking honors level in the previous year. From eighth to ninth grade, about 10% of students repeated Algebra 1. About the same number of students chose to move up from standard to honors level as chose to move in the opposite direction. In the transitions to tenth and eleventh grade, a higher proportion of students stayed in their expected pathway and a smaller proportion of students chose to move from honors to standard level courses compared to the seventh grade algebra students depicted in Figure 3.

Figure 4

Course transitions for eighth grade Algebra 1



Total Mathematics and Science Credits

Research Question 3: What are the differences in mathematics and science course-taking patterns based on whether students met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

Data for Research Question 3

The third research question considers whether there is a relationship between students accessing hyper-acceleration in mathematics and their demonstrated interest in STEM fields as measured by their course choices in high school. For this analysis, all mathematics and science courses, including computer science, taken in grades nine through twelve were totaled for each student. The minimum course total was five, which reflects the minimum graduation requirement of three years of mathematics and three years of science, but only 67 students in the sample took the minimum number of required courses. Six students had the maximum course total of fourteen. All of those students took multiple mathematics and science courses in a single year and took at least one computer science course.

Data Analysis for Research Question 3

A one-way ANOVA was performed to compare the total number of mathematics and science credits in high school across all four groups. The one-way ANOVA revealed that there was a statistically significant difference in the total number of mathematics and science credits between at least two groups, $F(3, 13392) = 254.32$, $p < 0.0001$. The means are displayed in Table 11.

Table 11

Means for one-way ANOVA on total math and science credits by group

Group	Number	Mean	Std Error	Lower 95%	Upper 95%
A	1840	8.96793	0.02524	8.9185	9.0174
B	226	8.42920	0.07203	8.2880	8.5704
C	668	8.87126	0.04190	8.7891	8.9534
D	10659	8.28014	0.01049	8.2596	8.3007

Eighth grade algebra students (groups B and D) went on to complete an average of 8.3 mathematics and science courses in high school, while seventh grade algebra students (groups A and C) took around 8.9 mathematics and science courses in high school. It should be noted that the averages for all groups are above the number of mathematics and science credits required for graduation.

Table 12 shows the pairwise comparisons of each group. Tukey's HSD Test for multiple comparisons found that the mean value of total mathematics and science courses was significantly different when comparing the students who took Algebra 1 in seventh grade (groups

A and C) and the students who took Algebra 1 in eighth grade (groups B and D), but there was no statistically significant difference between the groups within each grade level. Students who take Algebra 1 in seventh grade earn on average one-half to three-quarters more mathematics and science credits in high school than students who take Algebra 1 in eighth grade, regardless of whether the students met the qualifications for seventh grade algebra.

Table 12

Ordered Differences Report

Group Comparison	Difference	Std Err Diff	Lower CL	Upper CL	p-Value
A-D	0.7725058	0.0261592	0.705294	0.8397180	<.0001*
C-D	0.6539809	0.0413239	0.547805	0.7601567	<.0001*
A-B	0.6136702	0.0730803	0.425901	0.8014394	<.0001*
C-B	0.4951453	0.0797753	0.290174	0.7001162	<.0001*
B-D	0.1588356	0.0696973	-0.020242	0.3379127	0.1030
A-C	0.1185249	0.0468047	-0.001733	0.2387828	0.0551

Data Analysis Summary

The data provide evidence of differences in course-taking patterns depending on which year in middle school that students complete Algebra 1. Students who took Algebra 1 in seventh grade completed on average more mathematics and science courses in high school than students who took Algebra 1 in eighth grade. This result is consistent with the finding that students who take Algebra 1 in eighth grade are more likely to not be enrolled in a mathematics class in at least one year in high school.

On the other hand, the data suggest that students who take Algebra 1 in seventh grade are less likely to reach the highest course available to them than those who take Algebra 1 in eighth grade. Further investigation into the course-taking of each group of students revealed that a significant proportion of students who completed Algebra 1 in seventh grade take two years of AP Calculus in high school.

In the following chapter the results will be discussed, including additional context, implications of the finding, how the findings relate to previous literature, and recommendations for further research.

Chapter 5

The purpose of this study was to identify the impact of a school division policy on early algebra on students' course taking patterns in high school. Patterns in mathematics and science course taking for students taking early algebra were explored through a quantitative analysis of transcript data. This chapter presents the significant findings and implications based on the analysis of the impact of hyper-acceleration in mathematics on students' course taking patterns in high school. This chapter also includes recommendations for further research and the researcher's personal reflections.

Patterns in mathematics and science course taking for students taking early algebra were explored through a quantitative analysis of transcript data. Studying course-taking patterns allows for some insight into whether students benefit from the policy by taking the most advanced mathematics courses available to them, and how well the criteria set out by the policy identify students who are likely to benefit from the policy. The research questions were:

1. Are students more likely to take progressively more challenging courses each year and go on to take the highest level of mathematics available to them at the same rate based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?
2. What patterns emerge when comparing mathematics course pathways for students based on whether they met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?
3. What are the differences in mathematics and science course-taking based on whether students met or did not meet the criteria for hyper-acceleration and whether they took algebra in seventh or eighth grade?

Findings

The analysis of the data was presented in Chapter 4, and the following findings have been identified.

Finding 1

Students who completed Algebra 1 in seventh grade went on to take more mathematics and science courses in high school than those who completed Algebra 1 in eighth grade. The results of the one-way ANOVA indicated a significant difference in total mathematics and science credits when comparing groups of students based on whether they took Algebra 1 in seventh or eighth grade. The results are displayed in Tables 9 and 10. On average, students who completed Algebra 1 in seventh grade took 0.5 to 0.75 more credits in mathematics and science, including computer science, in their four years in high school.

This research study does not include the reasons that students choose to take Algebra 1 in seventh grade, two years ahead of what is considered grade-level, but it is reasonable to believe that students choosing this option have aptitude and/or interest in mathematics and related fields. This finding would follow from such an assumption. Although there is little research on students taking high school mathematics before eighth grade, at least one study (Domina, 2014) found that students who benefited the most from early algebra were those who were already demonstrating high levels of achievement in mathematics.

Finding 2

Students who completed Algebra 1 in eighth grade were more likely to have at least one year in high school with no mathematics course than students who completed Algebra 1 in seventh grade. In the Chi-squared analysis, students who completed Algebra 1 in eighth grade (group D) were slightly more likely to not continue taking mathematics courses in each year in high school. Among the students who qualified for and/or completed Algebra 1 in seventh grade, about 11% of students did not take mathematics in at least one year, while about 14% of the students who completed Algebra 1 in eighth grade did not take mathematics in at least one year. The results of the Chi-squared analysis are reported in Table 5.

The research on long-term achievement of students who take Algebra 1 in middle school is not consistent regarding the benefits or drawbacks to students. Stein et al. (2011) found much support for the notion that early algebra is beneficial as a predictor of higher mathematics achievement and taking more advanced mathematics courses. The same report, however, cites a number of studies that provide evidence that these positive results may not be generalizable for all students and that in particular lower achieving students may in fact be harmed by starting algebra in middle school (Stein et al., 2011).

Because the students in this study who completed Algebra 1 in seventh grade had to qualify for that opportunity through mathematics assessment scores, they are a more homogeneous group in terms of their prior mathematics understanding and high achievement. The findings that these students are more likely to take mathematics throughout high school and that they take overall more mathematics and science courses is consistent with the results of the research by Stein et al (2011), Domina (2014), and Clotfelter et al (2015) showing that higher achieving students benefit from the opportunity for early algebra.

Due to the school division's open enrollment policy, the students who took Algebra 1 in eighth grade represent a more heterogeneous group, including many students who would not have participated in honors or advanced courses prior to eighth grade or students with consistently lower achievement in mathematics courses prior to Algebra 1 as measured by grades and standardized test scores. Although this study does not include the reasons that students choose not to take mathematics in a particular year, this finding is consistent with Stein's research that not all students will demonstrate high mathematics achievement and take advanced mathematics courses.

Finding 3:

Students who took Algebra 1 in 8th grade were more likely to reach their highest potential mathematics course than students who took Algebra 1 in 7th grade. One way to measure the benefit gained by students taking Algebra 1 in seventh grade is whether they go on to take the highest level of mathematics available to them, which in the school division being studied is Multivariable Calculus. For students who take Algebra 1 in eighth grade, the highest mathematics courses available without any acceleration are AP Calculus AB and BC, AP Statistics, and IB mathematics courses. The Chi-squared analysis results are displayed in Table 4.

Students in group B, who met the qualifications for Algebra 1 in seventh grade but chose to wait until eighth grade to take it, were the most likely to go on to complete the highest-level course available. Almost three-quarters of that group took an AP or IB mathematics course in twelfth grade. About half of the remaining students who took Algebra 1 in eighth grade went on to take an AP or IB course. In contrast, less than a third of students who took Algebra 1 in seventh grade took a course beyond AP Calculus. In particular, 21% of the students in group C, who did not qualify for seventh grade Algebra 1 but took it that year anyways, took the highest mathematics available to them.

This finding seems to be a contradiction to the previous findings. Students who complete Algebra 1 in seventh grade take more courses in mathematics and science overall and are more likely to take mathematics each year in high school, yet they are significantly less likely to take the highest-level mathematics course available to them. One factor that connects these findings is that if students in this group repeat a course or take both AP Calculus courses, they appear to be taking more mathematics that students who take progressively more challenging courses each year. In the metanalysis of middle school algebra studies, Stein et al (2011) reported that more than half of the students who took algebra in eighth grade did not take an advanced mathematics course in high school, but no clear explanations exist for why that might be the case based on the reviewed research.

Finding 4

Some students who took Algebra 1 in seventh grade took multiple introductory calculus courses in high school. This finding provides one possible explanation for the contradiction described in finding 3 above. About 20% of the students who complete Algebra 1 in seventh grade take both AP Calculus AB and AP Calculus BC. The College Board, who provides curriculum and assessment for these courses, does not explicitly prohibit nor endorse this pathway (College Board, 2022). However, the two courses are designed to be standalone courses in introductory calculus. The content of AP Calculus AB is aligned to a first semester college introductory course in calculus, while AP Calculus BC covers the content of the first and second semesters (College Board, 2020). In other words, students who take AP Calculus AB followed by AP Calculus BC encounter a repeat of a significant portion of the mathematics content, although it may be presented in a different way. This course pathway effectively slows down the previous acceleration.

Finding 5

The largest drops from honors to standard level pathways occurred between Algebra 2 Honors and Precalculus Honors and between Precalculus Honors and AP Calculus, regardless of when a student took Algebra 1. Dougherty et al. (2017) found evidence of a “leaky pipeline”, noting that only one-seventh of students who took eighth grade algebra had completed precalculus by eleventh grade, even when the students had been selected for acceleration into algebra based on an objective measure predicting their success. The results for this school division are displayed in Tables 6 through 9 and are not as dire. Over 70% of the students who took Algebra 1 in eighth grade had completed a course in precalculus or higher by the end of eleventh grade, and about 90% of the seventh grade Algebra 1 students completed Precalculus or higher by the end of tenth grade.

The leaky pipeline in this case would refer to the loss of students from the honors pathway. Students who qualify for and choose Algebra 1 in seventh grade have participated in advanced mathematics instruction and have usually also been identified for other gifted services in the school division. Their expected pathway in mathematics is to take honors courses each year. From Table 6, there is a decrease in the number of students in the honors level courses in each year, but the largest decreases occur from ninth to tenth and tenth to eleventh grade. In ninth grade there were 1581 students in Algebra 2 Honors but only 1131 were in Precalulus Honors in tenth grade, a decrease of 28.4%. A further decrease of 39.4% occurs in the next year, with 688 students in AP Calculus BC. Table 9 displays the course data for students who took Algebra 1 in eighth grade. Not all of those students start out in Algebra 1 Honors, but there is still a similar pattern of attrition from the honors level courses, with 3919 students in Algebra 2 Honors in tenth grade, 1989 students in Precalculus Honors in eleventh grade (an almost 50% decrease), and only 714 students in AP Calculus BC in twelfth grade.

Implications of Findings

Analysis of the findings leads to several implications for schools and the school division to consider. These implications are based on findings from the school division in this study but may have impact on further study or action on a broader scale.

Implication 1

Schools and school divisions should offer rigorous mathematics courses other than calculus. Based on findings 1 and 2, students should have access to courses that are relevant to their own interests or career plans but also rigorous. In a 2016 survey of job requirements, Handel (2016) reported that only about 5% of all workers require calculus in their work. As calculus is considered a university-level mathematics course, all of those careers also require a college degree. On the other hand, data analyst is one of the fastest growing career clusters and in a world of *big data*, all citizens require data literacy (Boaler, 2016).

Students in the school division being studied have access to AP Statistics, but with the exception of the International Baccalaureate program at several schools, no other mathematics electives are offered at honors, advanced, or dual enrollment level. In response to business and industry needs, some states, including the one in which this study was implemented, have begun to diversify offerings in advanced mathematics, including data science and mathematical modeling (Anderson and Burden, 2022). Schools should develop and make available course offerings, including advanced and dual enrollment courses, to “reflect the way mathematics is used in many careers” (Anderson and Burden, 2022).

Implication 2

Schools and school divisions should develop and share communication about diverse mathematics pathways, and counsel students to pursue coursework aligned to their future plans. Related to findings 1-4 and considering new course offerings as described in implication 1, teachers, counselors, students, and parents should have clear information about the impact of their mathematics course choices in middle and high school. Teachers and counselors should communicate with students and parents as early as upper elementary school about course pathways and implications of course choices in middle school.

Calculus should not necessarily be the only goal of a high school mathematics program. Only about half of students who take introductory calculus in high school will go on to study further calculus-intensive courses in college (Bressoud, 2021). Yet calculus remains the ultimate goal for a large proportion of college-bound students, regardless of their intended majors or careers. "The belief that AP Calculus looks good on college applications, and the concomitant, though often undeclared, encouragement of mathematics teachers and counselors, plays a prominent motivational role in students taking AP Calculus" (Rosenstein and Ahluwalia, 2017). The perception is not in line with a changing reality. Anderson and Burdman (2022) found that an increasing number of colleges and universities, including competitive ones, "have made clear that non-traditional courses such as data science are not only accepted for admission but are considered advanced math courses".

All students in this study had access to AP Statistics. However, of the 13,419 students in the sample, only 2,429 students took AP Statistics. Furthermore, over half of those students took it in addition to AP Calculus, instead of as their primary culminating mathematics course. This suggests that the school division could do more to promote rigorous alternatives to calculus, even as other opportunities are still being developed as recommended in implication 1.

For students who do have a goal to take calculus in high school, it should be clear that they don't have to take Algebra 1 in seventh grade to achieve that. Regarding finding 4, many students are entering the "race to Calculus" (Galanti, et. al., 2021) early, only to repeat content across multiple years in different calculus courses. Taking the time to build strong conceptual understanding and participate in meaningful algebraic reasoning in prealgebra courses would benefit many students who would still have the opportunity to take AP Calculus in twelfth grade (Bressoud, 2021; Galanti, et. al., 2021).

Implication 3

Schools should provide intervention supports to students in honors courses. Based on finding 5, there is a "leaky pipeline" in the pathways to calculus, regardless of when students take Algebra 1. Significant proportions of students leave the honors pathway between Algebra 2 and Precalculus and between Precalculus and AP Calculus. It is reasonable to infer that some students are struggling in those courses, and potentially earning grades that they deem unsatisfactory. If students have conceptual gaps or need more time to make sense of increasingly complex mathematical ideas, it may seem easier to "drop" to a standard level course than to engage in the extra work it would take to be successful in an honors class.

Schools should provide extra support and scaffolds to students who struggle in honors courses. In the review of literature about early algebra, one common theme was that among the inconsistencies in outcomes for students, researchers raised questions about what types of supports are available when students struggle, and the types of educational settings, curricula, and pedagogy to which students are exposed (Heckman, 2013). Galanti et. al. (2021) expressed concern that hyper-acceleration may lead to a loss of opportunities for students to engage in meaningful algebraic reasoning.

Intervention structures and supports in high schools are usually reserved for students who are behind grade-level or at risk of not meeting state assessment benchmarks and/or not graduating on time. Providing the same kinds of intervention supports would allow more students to stay in the honors pipeline and reach the highest levels of mathematics courses. Additionally, providing extra supports could make honors courses more accessible for more students, including students from groups that are historically underrepresented in advanced mathematics and other STEM fields. In other words, schools "should focus on enhancing students' *opportunities to learn*" (Stein, 2011, p. 486) even in honors and advanced coursework.

Recommendations for Future Research

This study examined course taking patterns for students in a single, large, suburban school division. The purpose of this study was to identify the impact of a school division policy on early algebra on students' course taking patterns in high school. The data revealed significant differences in attainment of higher-level courses for students associated when they took Algebra 1. There are also patterns of students leaving honors level courses after ninth and tenth grade. Future studies could investigate why students make the course choices that they do. One approach might be to survey students about their attitudes towards mathematics and their plans for future coursework prior to high school and then again close to graduation.

Another area for future studies would be the type of instruction that students receive. Hyper-accelerated students are earning two and sometimes three mathematics credits in middle school, and it would be interesting to investigate whether the instruction in those courses is at the same level of depth and rigor as the same content taught in high school courses. Additionally, a study of the instruction in honors and other advanced courses in high schools, particularly how teachers differentiate and support students who struggle in those courses, could provide some insight into the course choices and overall course attainment for students.

Summary

This chapter provided a summary and discussion of the findings, implications, and recommendations for further research. There are impacts on students who participate in hyper-acceleration as evidenced by differences in course taking patterns and course attainment for students taking Algebra 1 in seventh and eighth grades. Due to the lack of clear predictive methods to select students with a higher chance of being successful in a hyper-accelerated mathematics pathway (Stein, et. al., 2011), schools and school divisions should consider structures that provide access to rigorous and appropriate mathematics at any level. To that end, schools should expand the opportunities for students to engage in a variety of rigorous advanced mathematics coursework and should provide clear communication and guidance to students and parents about mathematics pathways, including and not including calculus. Schools should review the instruction, differentiation, and supports for students in honors and advanced courses to ensure that all students have an opportunity to learn and reach their mathematics potential.

Personal Reflection

The process of sorting, formatting, and analyzing real, and therefore messy, data was challenging at times but ultimately rewarding. To put into practice skills and procedures learned only in a classroom is fulfilling, and I will apply my learning to my current job and future professional pursuits.

The biggest challenge for me in this study was to maintain objectivity and resist the urge to ask *why* for every result. The data analysis was done in aggregate, looking for patterns and statistical significance in the behavior of groups of students. Each data point, however, represents an actual student who makes choices every day in how to engage in mathematics learning and has multiple influences in how they select mathematics courses each year.

At the heart of this study are two questions about the impact of school policy around participation in advanced mathematics (and in a larger sense gifted programs in general). One is that of equity, i.e., whether schools and school divisions are providing real opportunities for all students to access appropriately rigorous content. The second is whether school policies that use specific assessment scores as the sole measure of allowing students to hyper-accelerate in mathematics may in fact do more harm than good, setting students up for unnecessary challenges and stress and denying them the opportunity to develop rich conceptual knowledge in mathematics. I believe the findings in this study are steps towards answering these questions but are not in themselves conclusive.

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Appendix A

CITI training for Social and Behavioral Research

CITI PROGRAM

Completion Date	22-Sep-2021
Expiration Date	21-Sep-2024
Record ID	45250196

This is to certify that:

Jennifer Allard

Not valid for renewal of certification through CME.

Has completed the following CITI Program course:

Social & Behavioral Research
(Curriculum Group)
Social & Behavioral Research
(Course Learner Group)
1 - Basic Course
(Stage)

Under requirements set by:

Virginia Polytechnic Institute & State University (Virginia Tech)

CITI
Collaborative Institutional Training Initiative

Verify at www.citiprogram.org/verify/?w42bfbe70-b07e-489d-b845-6d32ad3a7bae-45250196

Appendix B

Virginia Tech IRB Approval

VIRGINIA TECH.

Division of Scholarly Integrity and Research Compliance
Institutional Review Board
North End Center, Suite 4120 (MC 0497)
300 Turner Street NW
Blacksburg, Virginia 24061
540/231-3732
irb@vt.edu
http://www.research.vt.edu/sirc/hrpp

MEMORANDUM

DATE: October 18, 2021

TO: Carol S Cash, Jennifer Evans Allard

FROM: Virginia Tech Institutional Review Board (FWA00000572)

PROTOCOL TITLE: How does taking Algebra 1 by 8th grade effect students high school science course taking patterns?

IRB NUMBER: **18-1069**

Effective October 18, 2021, the Virginia Tech Human Research Protection Program (HRPP) determined that this protocol meets the criteria for exemption from IRB review under 45 CFR 46.101 (b) category(ies) 4.

Ongoing IRB review and approval by this organization is not required. This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are made and there are questions about whether these activities impact the exempt determination, please submit an amendment to the HRPP for a determination.

This exempt determination does not apply to any collaborating institution(s). The Virginia Tech HRPP and IRB cannot provide an exemption that overrides the jurisdiction of a local IRB or other institutional mechanism for determining exemptions.

All investigators (listed above) are required to comply with the researcher requirements outlined at:

https://secure.research.vt.edu/external/irb/responsibilities.htm

(Please review responsibilities before beginning your research.)

PROTOCOL INFORMATION:

Determined As: **Exempt, under 45 CFR 46.101(b) category(ies) 4**
Protocol Determination Date: **December 3, 2018**

ASSOCIATED FUNDING:

The table on the following page indicates whether grant proposals are related to this protocol, and which of the listed proposals, if any, have been compared to this protocol, if required.

Invent the Future

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
An equal opportunity, affirmative action institution

Appendix C

School Division Research Agreement

[redacted]

Office of Research and Strategic Improvement
[redacted]

Data Agreement

Part I

This agreement is entered into by, and between, [redacted], the Office of Research and Strategic Improvement (ORSI), and William Glenn (Researcher).

This Data Agreement certifies that data, as defined in Part II of this document, are to be released from the Office of Research and Strategic Improvement to the Researcher for an established fee.

The parties do hereby agree to as follows:

1. The Researcher shall comply with all federal and state statutes, as well as [redacted] policies and regulations related to the use of student demographic, behavioral, socio-emotional, and achievement data.
2. The Researcher agrees to pay the cost of the data provision, as established in Part II of this agreement, prior to [redacted] releasing data to the Researcher.
3. The Researcher's use of data is restricted to the activities and purposes defined in the approved research proposal (see Part III). Any other or additional use of the data may result in the immediate termination of the agreement.
4. All data provided by FCPS pursuant to this agreement is the sole property of [redacted] and may not be copied or reproduced in any form or manner.
5. A breach of any of the provisions of the agreement will terminate this agreement. If [redacted] terminates the agreement, all data previously provided by FCPS, including any copies of the data, regardless of form, will be destroyed or returned to [redacted] immediately. No further data will be released to, nor agreements entered into with, the Researcher and collaborators for a period of time to be determined by [redacted]
6. The terms of this agreement shall commence as of the Effective Date and shall continue for so long as the Researcher retains the data, unless sooner terminated as set forth in this agreement.
7. This document constitutes the agreement in its entirety, and there shall be no deviation from the terms unless expressly agreed to, and executed by the parties, by way of written amendment.

Part II

This part of the agreement certifies that the following data is to be released to the Researcher by FCPS.

Data Description: [redacted] will provide the data listed below to the Researcher, following the terms set forth in this agreement.

The following deidentified data will be provided for seniors who had been enrolled in [redacted] since Grade 6 for SY 2017-18, SY 2016-17, SY 2015-16, and SY 2014-15:

- Pseudo ID (assigned by ORSI);
- Demographic data (race/ethnicity, gender, special ed status, ELP level, and FRM status);
- Math course taking history for Grades 7-12 (high school credit bearing courses only);
- Math 7 SOL scores (scaled scores and proficiency level);

Application ID #20020

- Iowa Algebra Aptitude Test scores for Grade 6.

If a student is enrolled in more than one math or science course per year, each course will be provided. Student data will be provided in one row with the most recent courses listed first. If the number of seniors who have been enrolled in [redacted] since Grade 6 exceeds 20,000 with fewer than 4 years of data, then less than 4 years of data will be provided.

Data Format:

The data will be provided in up to four separate Excel files, one per graduating class.

Timeline for Providing Data:

The data files will be provided within three weeks of signature of this data agreement.

Cost: It is estimated that the request detailed within this agreement will require 8 hours of staff time at a cost of $50 per hour. The estimated total cost of the request is $400. The Researcher will receive an invoice for the actual cost of the data provision for the data in each section, not to exceed a total of $400.

Part III

This part of the agreement certifies the following limited purposes and activities for which the Researcher may use the data sets.

The Researcher will follow the guidelines and requirements set forth in this agreement and in the signed Acknowledgment of Researcher Responsibilities dated August 30, 2019, for use in the study titled *Hyperacceleration in Secondary Mathematics* (Research Application ID #20020). The use of data is limited to the purposes laid out in the approved study as well as this Data Agreement. More specifically, the Researcher may analyze the provided data to answer the research questions laid out in the proposal. The Researcher may not use this data for other purposes nor share it with any other researchers for their use other than as specified in this agreement.

FCPS reserves the right to review documents and presentations that result from this study for technical adequacy.

The terms and conditions of the Data Agreement shall not be amended without mutual written consent of both parties. No duties or obligations may be assigned without prior written agreement by the other party. The parties shall not assign or subcontract the whole or any part of the Data Agreement without the other party's written consent. This agreement shall become effective upon the date of the last authorized signatory hereto.

Signatures

Entered into and agreed to by the following and effective, upon whichever signatory date is the latter.

[redacted]

12/19/19
Date

Director, Office of Research and Strategic Improvement
Chief Operating Officer's Office
[redacted]

FOR THE RESEARCHER:

William J. Glenn

William Glenn, Ph.D.
Professor
Virginia Tech

12/14/2019
Date

Application ID #20020