

**FEASIBILITY OF HELICALLY STIFFENED CONSTRUCTION
FOR A FORMULA RACING CAR STRUCTURAL SHELL**

by

Carlos Alejandro Font

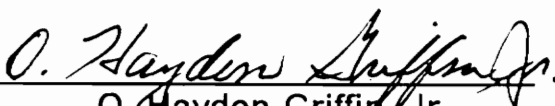
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Feasibility of Helically Stiffened Construction for a Formula Racing Car Structural Shell

by

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ABSTRACT

The feasibility of replacing currently widely used sandwich construction with helically stiffened construction for a Formula racing car structural shell is studied. The torsional deformation behavior of circular and square cross-section shells is analyzed as an approximation to the real car structure. Shells with different sandwich and helically stiffened configurations are analyzed with finite elements. For closed square and circular cross-section shells, the highest torsional stiffness is obtained with helical stiffening. For circular cross-section shells with the cockpit cutout and reinforcements usually present in real racing car structural shells, $\pm 45^\circ$ helically stiffened shells are 200% stiffer in torsion than sandwich shells. For square cross-section shells, the torsional stiffness improvement obtained with the helical stiffening is only 27%. The cross-sectional shape of the shell, cockpit opening, and different type of reinforcements (present in a real car structure) affects the selection of the best stiffening for torsional stiffness. The role of the terms of the stiffness matrix of the helically stiffened configuration in the torsional behavior of the shells is studied. The 0/90 waffle stiffening is more efficient than the helical stiffening for the square cross-section shells with the cutout and reinforcements. In the case of circular cross-section shells, the 0/90 waffle stiffening yields approximately the same results than the helical stiffening. The skin-stiffener configuration for maximum torsional stiffness depends on the cross-sectional shape of the shell. The advantages of the $\pm 45^\circ$ helical stiffening over the sandwich construction depend on the cross-sectional shape of the shell and on the way the cutout region is reinforced.

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Nomenclature

- E = Young's modulus
 ν_{12} = Poisson ratio in the 1-2 plane
 G_{ij} = shear modulus in the i-j plane
 X_t = tensile strength in the 1-direction
 X_c = compressive strength in the 1-direction
 Y_t = tensile strength in the 2-direction
 Y_c = compressive strength in the 2-direction
 S_{ij} = shear strength in the i-j plane
 ρ = density
 N_i = resultant force in the i-direction
 M_i = moment resultant in the i-direction
 A_{ij} = extensional stiffnesses
 B_{ij} = bending-extension coupling stiffnesses
 C_{ij} = bending-extension coupling stiffnesses for the sandwich laminates
 D_{ij} = bending stiffnesses
 ϵ_i = strain in the i-direction
 γ_{xy} = shear strain in the x-y plane
 κ_x = curvature in the x-direction
 κ_y = curvature in the y-direction
 κ_{xy} = twisting curvature in the x-y plane
 t_c = thickness of the core
 t_f = thickness of the facings
 Q_i = shear force resultant
 $K_{\alpha 3}$ = transverse shear stiffness
 P_1 = stiffener load in 1-direction (Figure 2.2)
 P_2 = stiffener load in 2-direction (Figure 2.2)
 b_s = width of the stiffener
 a = spacing between stiffeners
 h_s = height of the stiffeners
 t = thickness of the skin
 z = distance from the skin midplane to a skin layer midplane
 $\bar{Q}_{ij}$ = transformed reduced stiffnesses (Equations (26))

$\bar{z}$ = distance from the centroid of the stiffener to the middle surface of the skin
 A_s = stiffener cross-sectional area
 S = first moment of the area of the stiffeners
 J = polar moment of inertia of the stiffeners
 I = moment of inertia of the stiffeners
 $\bar{D}_{ij}$ = resultant bending stiffnesses of the stiffened constructions
 $\bar{A}_{ij}$ = resultant extensional stiffnesses of the stiffened constructions
 $\bar{B}_{ij}$ = resultant bending-extension coupling stiffnesses of the stiffened constructions
 L = size of the grid unit cell
 $\bar{t}$ = distance from skin middle surface to stiffener cross section most distant point
 ϕ = twist angle of the stiffener
 c_1, c_2 = torsion coefficients for rectangular bars (Equation (40))
 P_e = equivalent load in the stiffener
 N_{cr} = critical buckling load
 a_{cr} = critical stiffener spacing
 K_θ = torsional stiffness
 R = radius of the mid-plane of the shells
 u_θ = circumferential displacement
 T = torque
 N = number of nodes at the loaded end of the shell
 P = equivalent nodal load
 u_r = radial displacement
 D_r = internal diameter of the bulkheads

SUBSCRIPTS:

c = core or compressive
 s = stiffener
 $i = 1, 2, 3$
 $j = 1, 2, 3$
 $\alpha = 1, 2$
 t = tensile
 f = facings
 e = equivalent
 cr = critical

Chapter 1

Introduction

1.1 BACKGROUND

The purpose of this study is to investigate a new stiffening concept for the structural shell of a Formula 3 racing car using composite materials. The reason for the use of composite materials is based on the most important attributes of a successful racing car. These attributes are high stiffness, especially torsional, in order to enhance the handling of the vehicle and low weight in order to have a high power-to-weight ratio. By improving the stiffness and the power-to-weight ratio, we will rapidly improve the overall behavior of the car because cars are very susceptible to these two variables.

Other reasons that make composite materials a good choice are that they are easier and less expensive to shape than metals for the short fabrication runs typical of racing cars. The reason that makes composite materials easier and less expensive is that the tooling needed to make short runs of composite parts is easier to make, quicker, and less expensive than the tooling needed for metal part fabrication. Thus, composite materials appear to be a very good choice.

A typical modern Formula racing car is composed of various parts which we will group in three categories:

- A) Structural Shell
- B) Aerodynamic Shell
- C) Aerodynamic Attachments

A schematic drawing of a Formula racing car is shown in Figure 1.1. In the upper part of this figure, we see a longitudinal section of the car, and in the lower part we see a top view of the car. The locations of the parts that will be named in the following definition of the categories labeled A, B, and C are shown in Figure 1.1.

A) **Structural Shell:** This part is the real structure of the car. The structural shell supports the loads from the front wheels, from the rear wheels through the engine (which is attached to the rear part of the structural shell), and from the Aerodynamic Shell and other secondary parts such as the sidepods where the radiators are housed, front and rear wings, etc. Also, the structural shell houses the cockpit and usually contains the fuel tank, which is located between the engine and the back of the driver.

B) **Aerodynamic Shell:** This part is a very light shell that covers the structural shell and sometimes the engine also. The purpose of this shell is to direct the air flow around the structural shell, engine, and other parts that do not have a good aerodynamic shape.

C) **Aerodynamic Attachments:** Under this category are front and rear wings, covers of the sidepods, etc.

Type of Structure: The type of structure used in racing car structural shells has changed over the years. The designers started with a bridge-type structure, then came the space tubular frames, and presently the most-used structure is the thin-walled shell. Some examples of these structures are shown in Figure 1.2 [1]. The structure of the twin-tube Lister Jaguar is shown in Figure 1.2a. The name "twin-tube" comes from the two tubes that form the bottom part of the structural shell. Two examples of space tubular frames, the multi-tube Cooper Monaco and the Lotus 19, are shown in Figures 1.2b and 1.2c, respectively. The name "multi-tube" is given because all the members of the structure are tubes. The structure of the Lotus 25/33 is shown Figure 1.2d. In this case, we see that the tube structure is replaced by a shell-type structure which is formed by two box-type beams, one at each side of the structure. The thin-walled shell has a higher torsional stiffness than the other concepts.

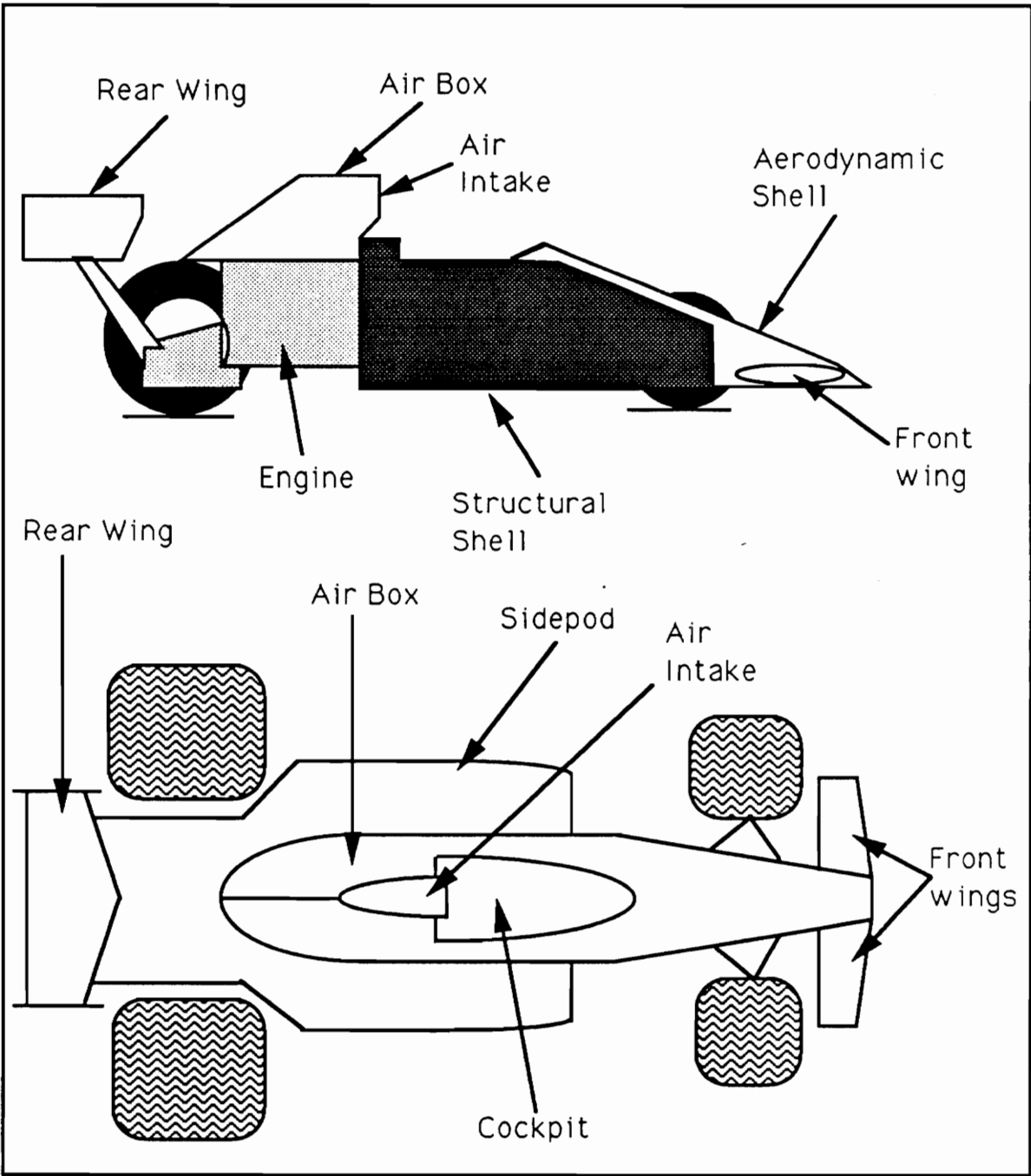


Figure 1.1 Formula Racing Car

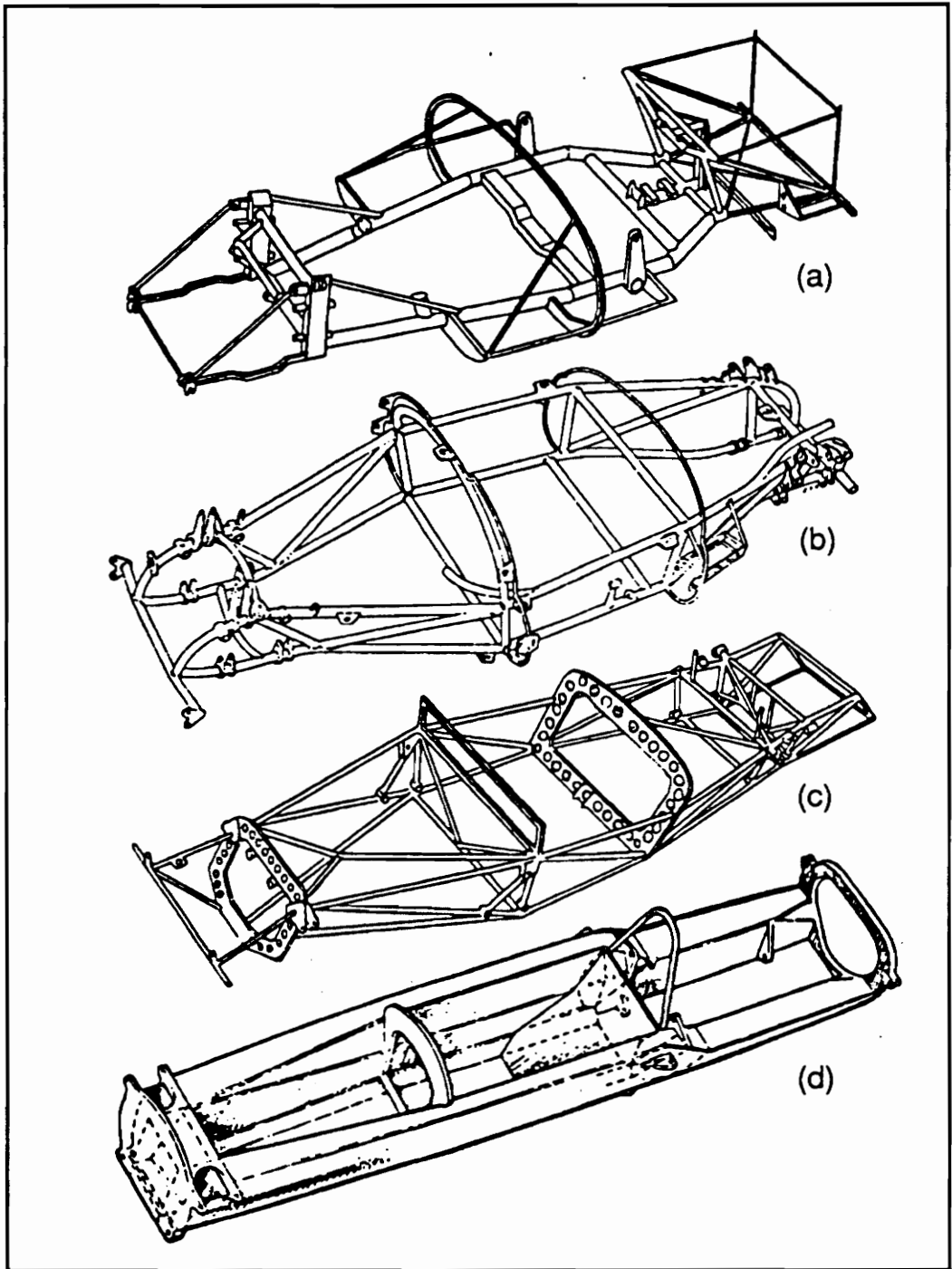


Figure 1.2 Evolution of the Racing Car Structures [2]

The thin-walled shell is stiffer than the other concepts because of the presence of the box beams like the ones used in the Lotus 25/33, which give the structure an enhanced torsional stiffness. In the other case of the multi-tube structures, the bending stiffness of the members alone governs the stiffness of the whole structure. An example of a modern structural shell is shown in Figure 1.3. If we compare this structure with the structure of the Lotus 25/33, we see that presently instead of using the box beams to give torsional stiffness to the structure a closed shell with an opening for the cockpit is used. The shape shown in Figure 1.3 corresponds to a contemporary Formula 3 racing car. All the data used in this thesis, such as the shape, the loads that act on the structure, and the size of the structure, were provided by Mr. Oreste Berta who is a racing car builder [2]. These data correspond to a Formula 3 racing car.

1.2 STEPS IN THE DESIGN OF THE RACING CAR

Definition of the Loads: The main loads that act on the structural shell are:

Front Suspension Loads: The front suspension loads are applied at each of the points where the suspension frames are fastened to the structural shell on the two front bulkheads. The magnitude of these loads is 7840 N, and they can act in both directions, in and out, relative to the structural shell. See loads labeled 1 in Figure 1.3.

Rear Suspension and Engine Loads: The rear suspension and engine loads are applied at the points where the engine is bolted to the back panel of the structure, and their magnitude is 19600 N at each point. They can act in both senses in the direction of the longitudinal axis of the structural shell. See loads labeled 2 in Figure 1.3.

The loads just described are the main forces that act on the structure. Of course there are others, but they are all negligible compared with these previous two, which are the loads that appear when the car hits one of the bumps that surround the curves of the racing tracks. Therefore, the loads described before are extreme loads that are not present all the time, but are frequent during a race.

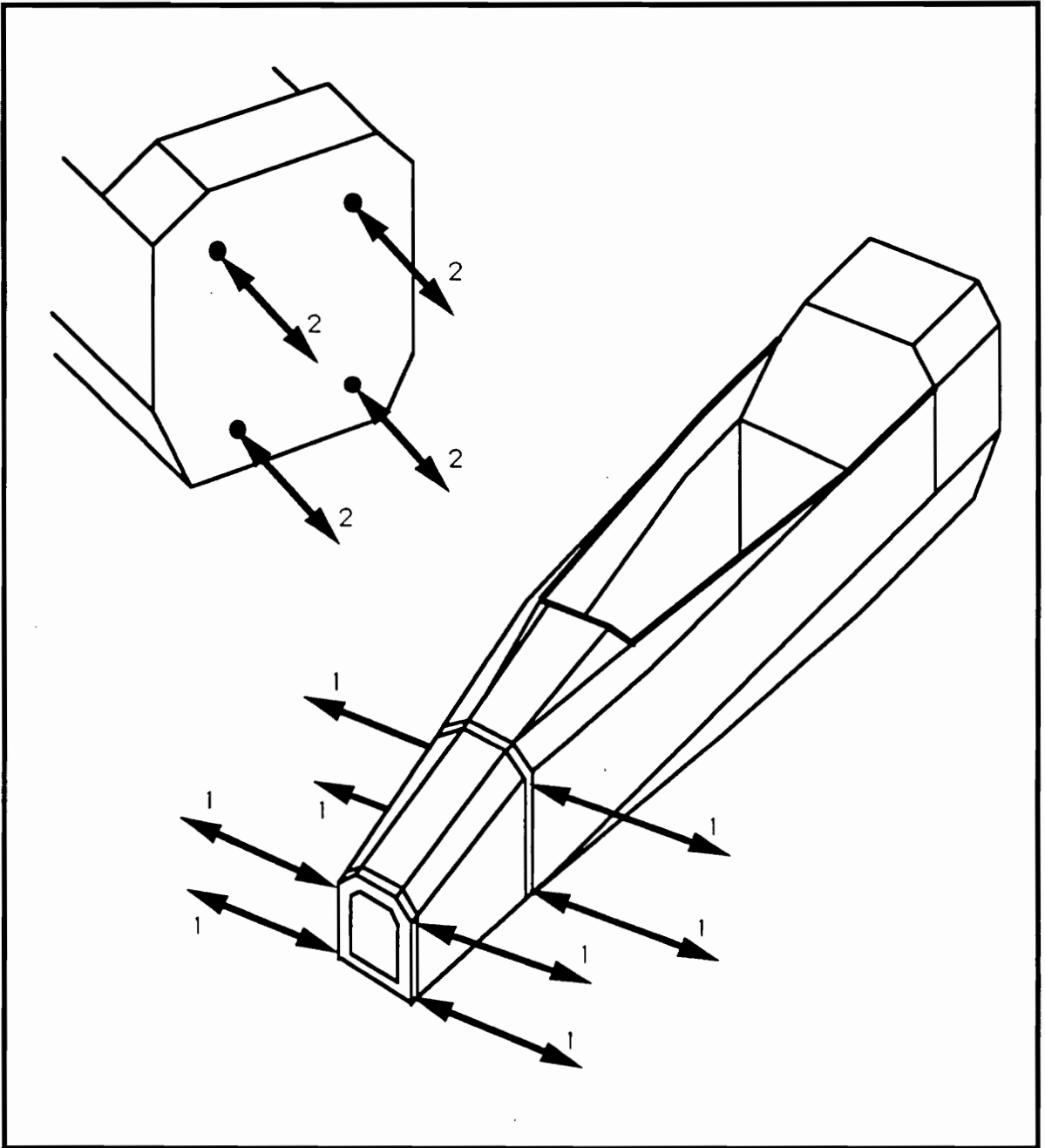


Figure 1.3 Shape of and Loads on the Structural Shell

Design Drivers: Design drivers are those design variables that, when changed, affect the behavior of the design more than other design variables. In order to determine the design drivers, first we must discuss the attributes that enhance the performance of a racing car and the target values of those attributes for the design. Then, the structural shell structure must be designed to meet these values. A racing car must have good handling and a high power-to-weight ratio. The reason for the first attribute is obvious, and the reason for the second one is that increasing the power-to-weight ratio will enhance the responsiveness of the vehicle.

1) To have good handling characteristics, the car must be stiff because if the vehicle starts bending or twisting excessively during its operation, it becomes unstable. Hence, the design criterion for vehicle structures is normally based on stiffness, rather than strength, and in the analysis deflections are more important than stresses. For ordinary street cars, the mid-span deflection (between the front and rear wheels) should not exceed 1.3 mm and the torsional stiffness should be bigger than 6000 Nm/degree of twist between front and rear axles [1]. Values of torsional stiffness ranging from 6500 to 7500 Nm/degree are acceptable.

A racing car is subjected to more demanding situations than a street car in ordinary daily use. Thus, one objective of the design is to obtain as high torsional stiffness as possible within the weight requirements. For a racing car, the values for mid-span bending should be lower and the torsional stiffness higher than those listed in the previous paragraph for a street car because better levels of handling are needed. For actual Formula 3 racing cars, usual values of torsional stiffness are: 9200 Nm/degree for aluminum structural shell structures and 11000 to 14000 Nm/degree for composite structural shells [3]. The value of 9200 Nm/degree is the best value of torsional stiffness obtained for the last aluminum cars that competed in the 1989 South American Formula 3 Tournament. In the 1990 season, there were very few of these cars left. The aluminum cars could not perform successfully any more against cars with composite structural shells which had developed higher values of torsional stiffness than the aluminum cars.

2) The power-to-weight ratio of the car governs the reaction of the car to rapid

accelerations, so this ratio is very important in a racing car. Although the regulations for the Formula 3 racing car category have a minimum weight for the car, each gram saved can be distributed to other convenient places. The only way we can change the power-to-weight ratio is to lower the weight of the structural shell as much as possible, taking into account the stiffness issue. The weight of the structural shell of a typical Formula 3 racing car is around 25 kg [3]. From the considerations stated before, the design drivers are :

- 1) Torsional stiffness
- 2) Bending Stiffnesses
- 3) Weight

Structural Analysis: For the analysis of this structure, we use the finite element analysis program ABAQUS version 4.8, which has the kind of elements that we need for composite materials such as 3D shell elements [4]. This program runs on the Virginia Tech mainframe computer. For finite element mesh generation and to plot deformed shapes, we use a preprocessor and postprocessor program called PATRAN [5]. Also, we use a Fortran program that calculates the stresses in each layer of the laminated structure from the forces and moments in each element.

Shape of the Structure: The shape of the structural shell that was provided to us [2] is shown in Figure 1.3. There are some troublesome regions that must be analyzed with special care. The first region is the opening of the cockpit. This opening will decrease the torsional stiffness of the structural shell structure, so we must study ways of stiffening this region without increasing the weight too much.

Other troublesome parts are the two front bulkheads where the suspension are attached. The problem here is that the forces that act between the suspensions and the bulkheads are out of the plane of the surface of the structural shell, so they act in a weak direction of laminated composite materials. The same problem occurs in the back panel where the engine is bolted to the structure. The front bulkheads and back panel regions are shown in Figure 1.4. To give an idea of the size of the structural shell of a Formula 3 racing car, the different projections and dimensions of the structural shell structure are shown in Figure 1.5.

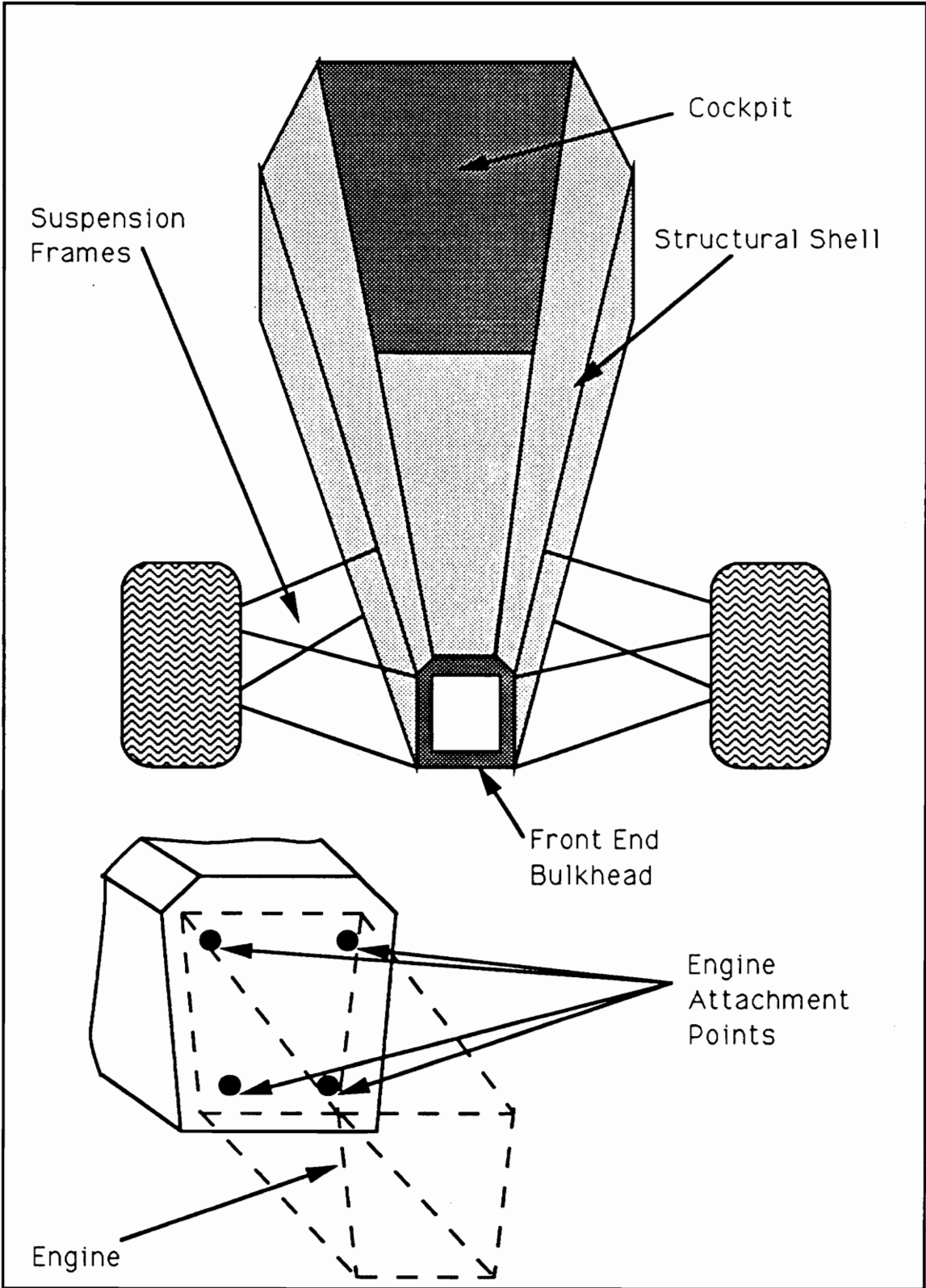


Figure 1.4 Front Bulkheads and Back Panel Areas

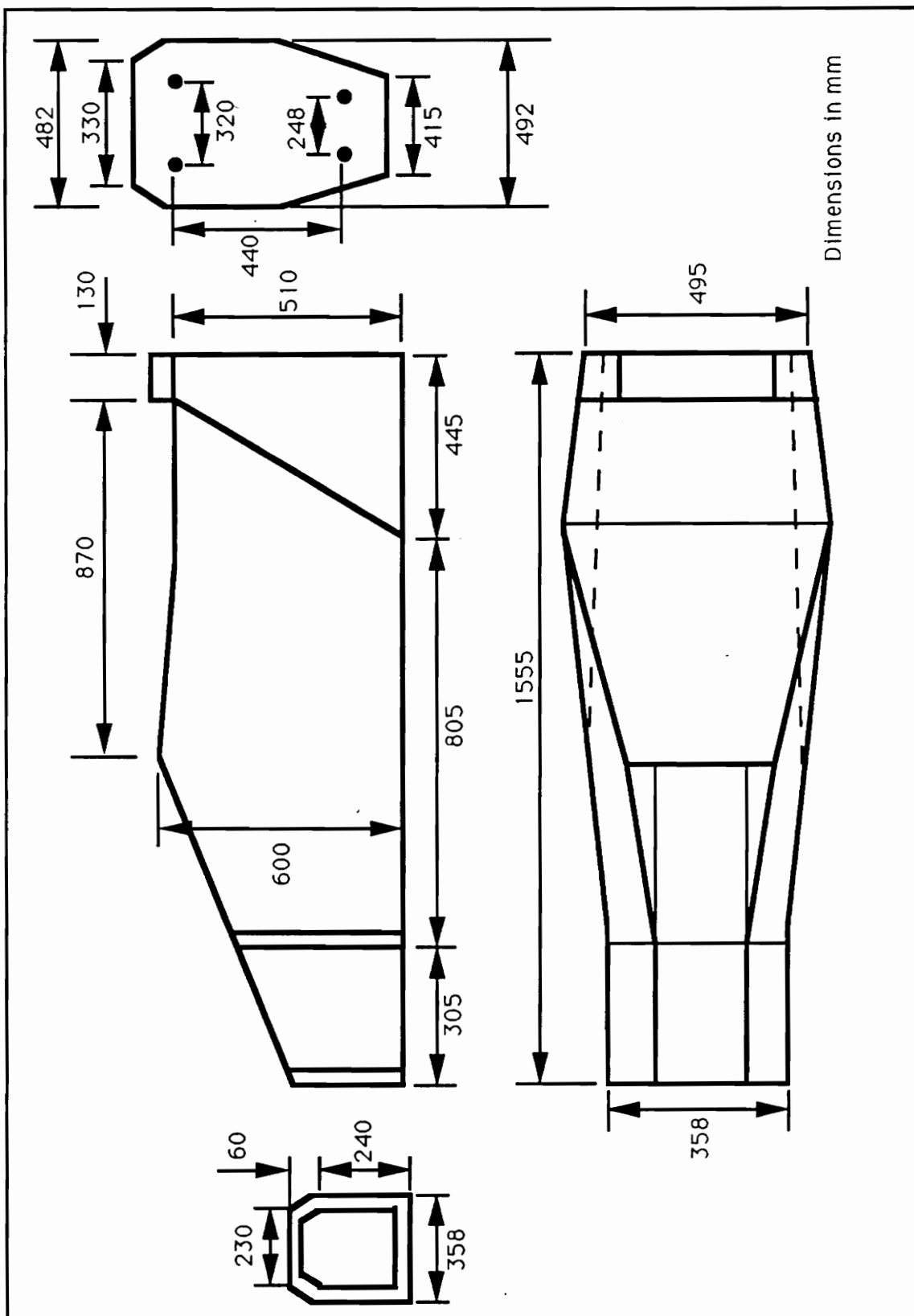


Figure 1.5 Principal Dimensions of the Structural Shell

1.3 OBJECTIVES

At present, the composite structural shells of all the Formula racing cars are built using sandwich construction. In this thesis, we investigate the possibility of replacing the sandwich construction by a $\pm 45^\circ$ helically stiffened construction. Sketches of the sandwich and helically stiffened constructions are shown in Figure 1.6. The motivation for this study is to see if stiffened construction has higher values of torsional stiffness than sandwich construction without increasing the weight of the structure.

To perform this study, we compare the behavior in torsion of a sandwich baseline configuration with different stiffened configurations. We analyze the behavior of circular and square cross-section cylindrical shells to approximate the behavior of a real structure. In Chapter 2, we develop the mathematical expressions that govern the mechanical behavior of both configurations, and we analyze circular cylindrical shells in torsion. We study the behavior of closed shells, shells with a cutout simulating the opening of the cockpit, and shells with a cutout and different types of reinforcements around the cutout and inside the shell such as bulkheads, etc. Also, we study the effect of the internal or external location of the stiffeners with respect to the skin of the shells. In Chapter 3, we move one more step toward the real shape of a structural shell of a Formula racing car by analyzing the behavior of square cross-section cylindrical shells with rounded corners. Again, we analyze the cases of closed shells, shells with a cockpit-size cutout, and shells with the cutout and reinforcements. Throughout Chapters 2 and 3, we explain the reasons for the differences in behavior between the two shell constructions. In this thesis, we do not analyze a real structure because we could not get data for a real design. Because it was too complex to try to design a real structure, we try to make as close a comparison as possible. In Chapter 4, we perform a stiffness analysis in which we study the impact that each term of the stiffness matrix has on the overall torsional stiffness of the structure. In Chapter 5, we address the problem of the manufacturing of a stiffened structure, and we propose guidelines for building the structure. In this Chapter, we discuss the different ways of building helically structures and how to solve the problem of the stiffener lay-up operation. Finally, the conclusions of this work are in Chapter 6.

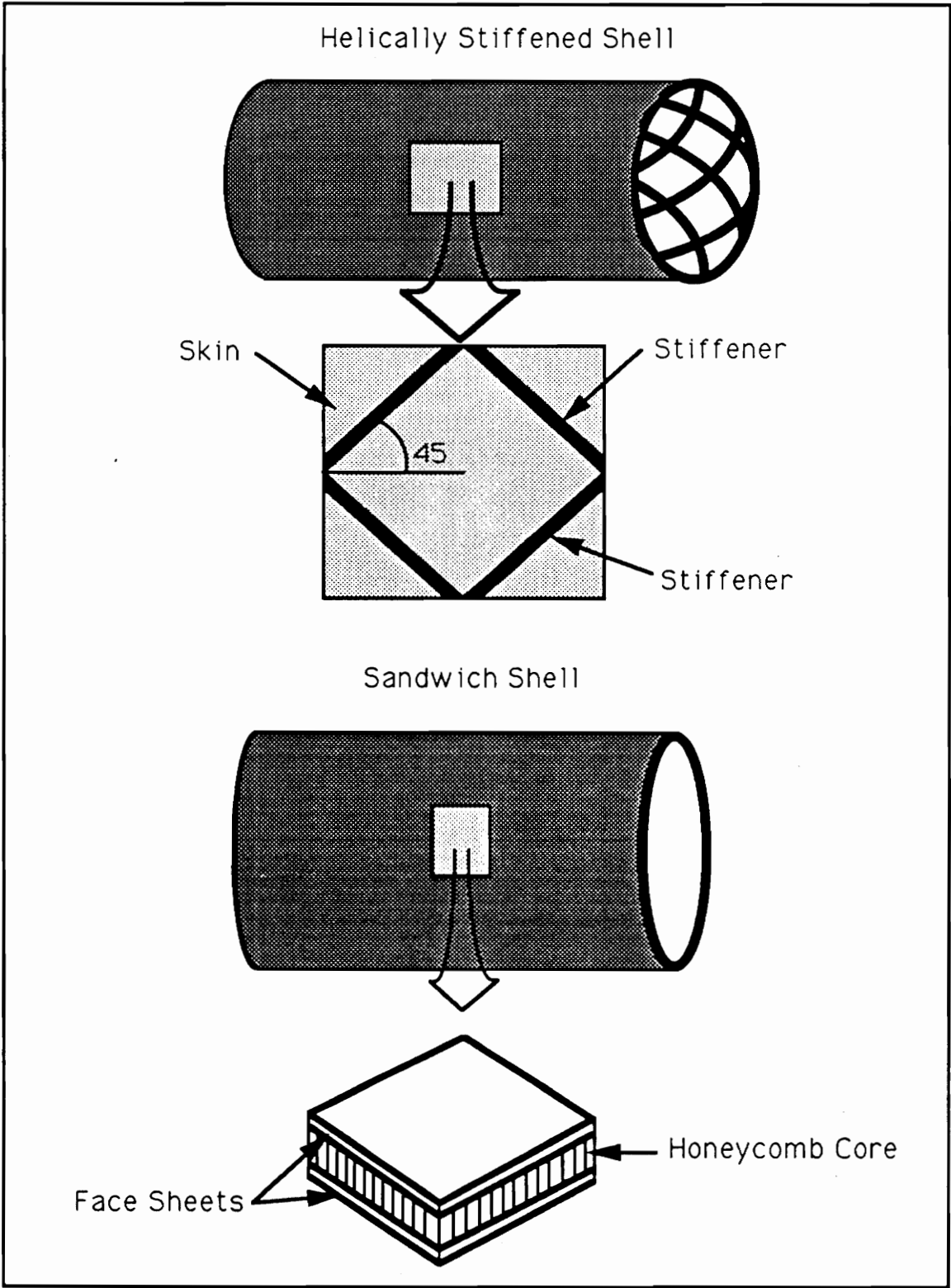


Figure 1.6 Helically Stiffened Shell and Sandwich Shell

Chapter 2

Selection of the Best Structure for Torsional Stiffness of Circular Cylindrical Shells

2.1 INTRODUCTION

This part of the thesis is concerned with the selection of the best stiffening concept to enhance the torsional stiffness of the racing car structural shell. The selection will be made between two different types of construction, sandwich construction and $\pm 45^\circ$ helically stiffened construction. Sketches of the $\pm 45^\circ$ helically stiffened shell and of a typical sandwich construction shell are shown in Figure 1.6.

We analyze two circular cylindrical shells with the same external dimensions and weight. The first shell is made of sandwich construction, and the other is a $\pm 45^\circ$ helically stiffened shell. The external dimensions of these shells are chosen as close as possible to the most important dimensions of the structural shell of the racing car, and both shells are subjected to the same loads as the racing car.

To compare both types of construction, we first perform the analysis of closed cylindrical shells i.e., without the effect of the cockpit and the shell tapering seen in Figure 1.3. To study the effect of the cockpit opening, we first analyze shells with a small cutout and then with a cockpit-size cutout. We also study the effect of the presence of the bulkheads and of the reinforcement of the longitudinal edges of the cutout as shown in Figure 2.1. In this study, we will compare the behavior of the sandwich baseline skin configuration with the behavior of two different stiffener-skin configurations, one with the same bending stiffness and one with the same weight as the sandwich skin configuration. In this way, we can analyze the bending stiffness importance, especially in the case of the shells with cutouts.

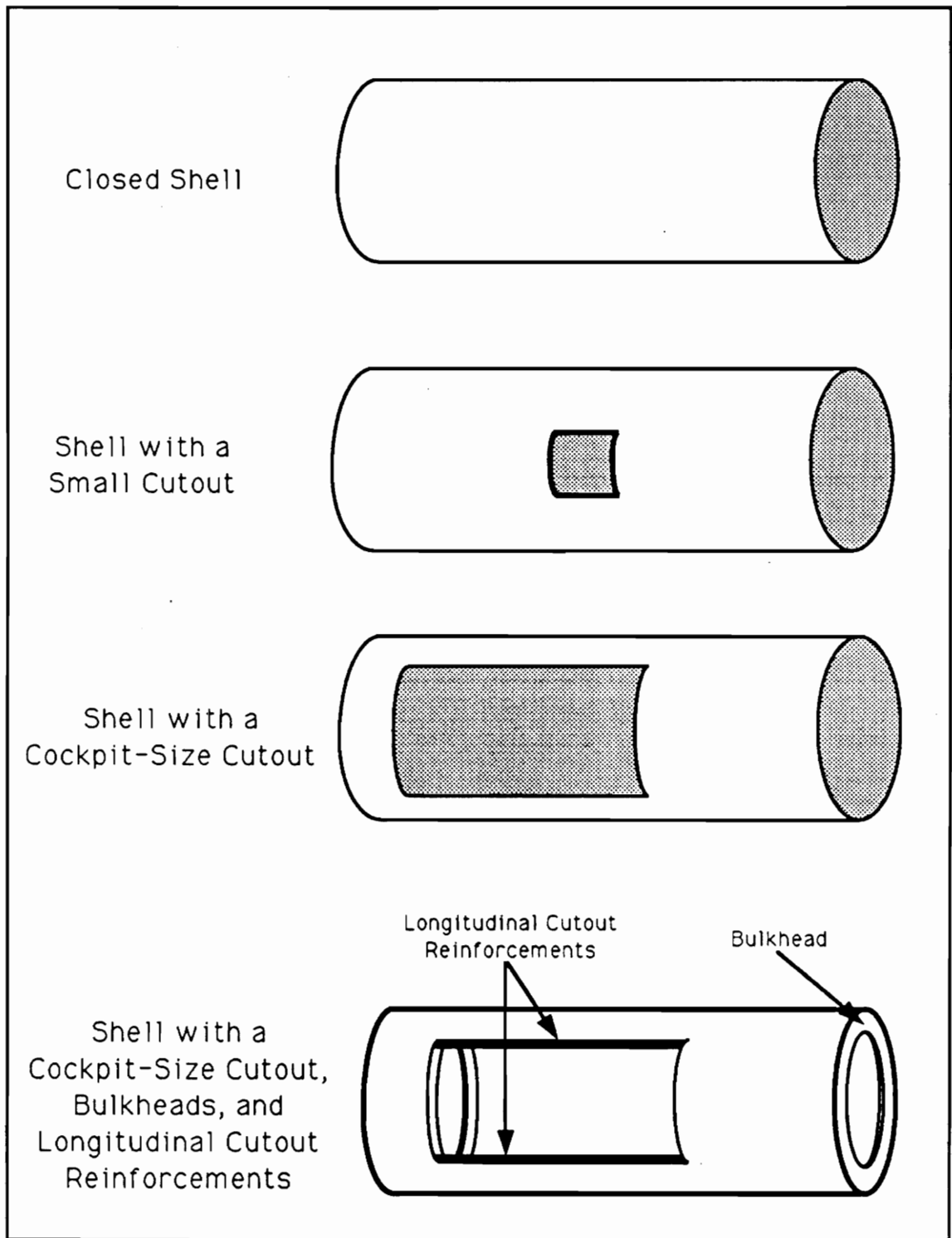


Figure 2.1 Circular Cylindrical Shells

2.2 MATERIALS

We use the AS-3501 graphite-epoxy unidirectional tape [6] for the sandwich and helically stiffened constructions. In the cases where a graphite-epoxy fabric is needed, T300-Fiberite 934 [7] is used. The sandwich shell has core material of Nomex honeycomb HRH10/F50 - 4.5 (5) [8], which has a special cell shape called Flexcore that makes this honeycomb appropriate to use in regions of compound curvature. The properties of these graphite-epoxy and honeycomb materials are listed in Tables 2.1 and 2.2, respectively, where ρ stands for the density.

2.3 DERIVATION OF THE STIFFNESS MATRICES AND STRESSES FOR THE SANDWICH AND $\pm 45^\circ$ HELICALLY STIFFENED CONSTRUCTIONS

In this section, we derive the stiffness matrices for the sandwich and the $\pm 45^\circ$ helically stiffened constructions. With these expressions, we calculate the values of the stiffness matrix terms that are used as input for the ABAQUS finite element analysis program. Also in this section, we develop the expressions to perform the stress and failure calculations for both constructions, and, for the stiffened construction, we address the problem of skin and stiffener buckling.

2.3.1 Stiffness Matrix for the Sandwich Construction

For a general sandwich laminate, the constitutive relations given by Whitney [9] are:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ C_{ij} & D_{ij} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (1)$$

Table 2.1 Properties of the AS-3501 and T300-Fiberite 934 Materials

Property	AS-3501	T300-Fiberite 934
E_1 kN/mm ²	127	66
E_2 kN/mm ²	11	66
ν_{12}	.25	.04
G_{12} kN/mm ²	4.48	4.1
X_t N/mm ²	1170	375
X_c N/mm ²	1100	279
Y_t N/mm ²	40	368
Y_c N/mm ²	170	278
S_{12} N/mm ²	50	46
ρ kg/mm ³	1.5×10^{-6}	1.5×10^{-6}
Ply thickness mm	.127	.175

Table 2.2 Honeycomb Properties

HRH10/F50 Nomex Honeycomb	
$G_{13} = .05$ kN/mm ²	$S_{13} = 1.55$ N/mm ²
$G_{23} = .048$ kN/mm ²	$S_{23} = 0.965$ N/mm ²
$\rho = 7.21 \times 10^{-8}$ kg/mm ³	

where the B_{ij} and the C_{ij} are equal only in the case in which the stacking sequence for the sandwich laminate is symmetric. The strains and curvatures of the midplane of the laminate are ϵ and κ . For a symmetric sandwich laminate in which both skins have the same stacking sequence and materials:

$$B_{ij} = C_{ij} = 0 \quad (2)$$

$$A_{ij} = 2 A_{ij}^f \quad (3)$$

$$D_{ij} = .5 t_c^2 \left[\left(\frac{t_c + t_f}{t_c} \right) A_{ij}^f \right] \quad (4)$$

where superscript and subscript f stands for facing and subscript c stands for core.

By integrating the shear stress resultants through the thickness of the core, we get the transverse shear relations

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \int_{t_c} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz \quad (5)$$

Where $\tau_{xz} = G_{13} \gamma_{xz}$ and $\tau_{yz} = G_{23} \gamma_{yz}$. Performing the integral and knowing that the stresses are constant through the thickness of the core, we get

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = t_c \begin{bmatrix} G_{13} & 0 \\ 0 & G_{23} \end{bmatrix}^c \begin{bmatrix} \gamma_{xz}^c \\ \gamma_{yz}^c \end{bmatrix} \quad (6)$$

where superscript and subscript c stands for core, so the transverse shear stiffnesses are:

$$K_{\alpha 3} = t_c G_{\alpha 3} \quad \alpha = 1, 2 \quad (7)$$

2.3.2 Stresses for the Sandwich Construction

In the theory given by Whitney [9], all in-plane stresses are carried by the facings, and the transverse shear stresses are carried only by the core. Thus, the stresses for the k^{th} layer of the i^{th} facing are:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{ij}^k \end{bmatrix} \begin{bmatrix} \epsilon_x^k \\ \epsilon_y^k \\ \gamma_{xy}^k \end{bmatrix} \quad i = 1, 2 \quad (8)$$

For the core:

$$\begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} G_{13} & 0 \\ 0 & G_{23} \end{bmatrix}^c \begin{bmatrix} \gamma_{xz}^c \\ \gamma_{yz}^c \end{bmatrix} \quad (9)$$

Thus, in order to get the stresses in each layer, we first multiply the forces and moments obtained for each element by the inverse of the stiffness matrix of that element to get the mid-plane deformations and curvatures for that element. Finally, we can obtain the values of the stresses for each layer using Equations (8) and (9).

2.3.3 Stiffness Matrix for the $\pm 45^\circ$ Helically Stiffened Construction

For a stiffened shell with an arbitrary skin stacking sequence and stiffeners making angles of $\pm 45^\circ$ with the axis of the shell i.e., helical stiffeners, the constitutive relations are:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} \bar{A}_{ij} & \bar{B}_{ij} \\ \bar{B}_{ij} & \bar{D}_{ij} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (10)$$

where the $\bar{A}\bar{B}\bar{D}$ matrix is the stiffness matrix of the helically stiffened construction with the effect of the stiffeners averaged over the spacing of the stiffeners. Next, we derive the expression for each term of the stiffness matrix.

Stress-Strain Relations for the Stiffeners: Here, we use the approach derived by Meyer [10]. The stress-strain relations for the stiffeners are found by a rotation of the engineering strains from the x-y shell coordinates to the $\bar{x}$ - $\bar{y}$ stiffener coordinates shown in Figure 2.2.

$$\begin{aligned}\epsilon_{\bar{x}} &= \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta \\ \epsilon_{\bar{y}} &= \epsilon_x \sin^2\theta + \epsilon_y \cos^2\theta - \gamma_{xy} \sin\theta \cos\theta\end{aligned}\tag{11}$$

For $\theta = 45^\circ$

$$\begin{aligned}\epsilon_{\bar{x}} &= .5 \epsilon_x + .5 \epsilon_y + .5 \gamma_{xy} \\ \epsilon_{\bar{y}} &= .5 \epsilon_x + .5 \epsilon_y - .5 \gamma_{xy}\end{aligned}\tag{12}$$

From the unidirectional Hooke's law, the stresses in stiffeners 1 and 2 shown in Figure 2.2 are

$$\sigma_{\bar{x}} = E \epsilon_{\bar{x}} \qquad \sigma_{\bar{y}} = E \epsilon_{\bar{y}}\tag{13}$$

Now, we distribute these stresses over the spacing a between two parallel stiffeners, i.e., we average or "smear" the stiffener effects over their spacing. We do this by replacing the stiffeners by a fictitious layer that has the same behavior as the stiffeners. Thus, the load acting on this fictitious layer must be equal to the load acting in the stiffeners and must be applied at the same point as in the stiffeners. Thus, the thickness of this layer must be the same as the stiffener height h_s . Then

$$\sigma_{\bar{x},s} b_s h_s = \sigma_{\bar{x}} a h_s \quad \Rightarrow \quad \sigma_{\bar{x}} = \sigma_{\bar{x},s} \frac{b_s}{a}\tag{14}$$

where $\sigma_{\bar{x},s}$ is the stress in the stiffener, $\sigma_{\bar{x}}$ is the stress in the fictitious layer, b_s is the stiffener width, and a is the stiffener spacing as shown in Figures 2.2 and 2.3.

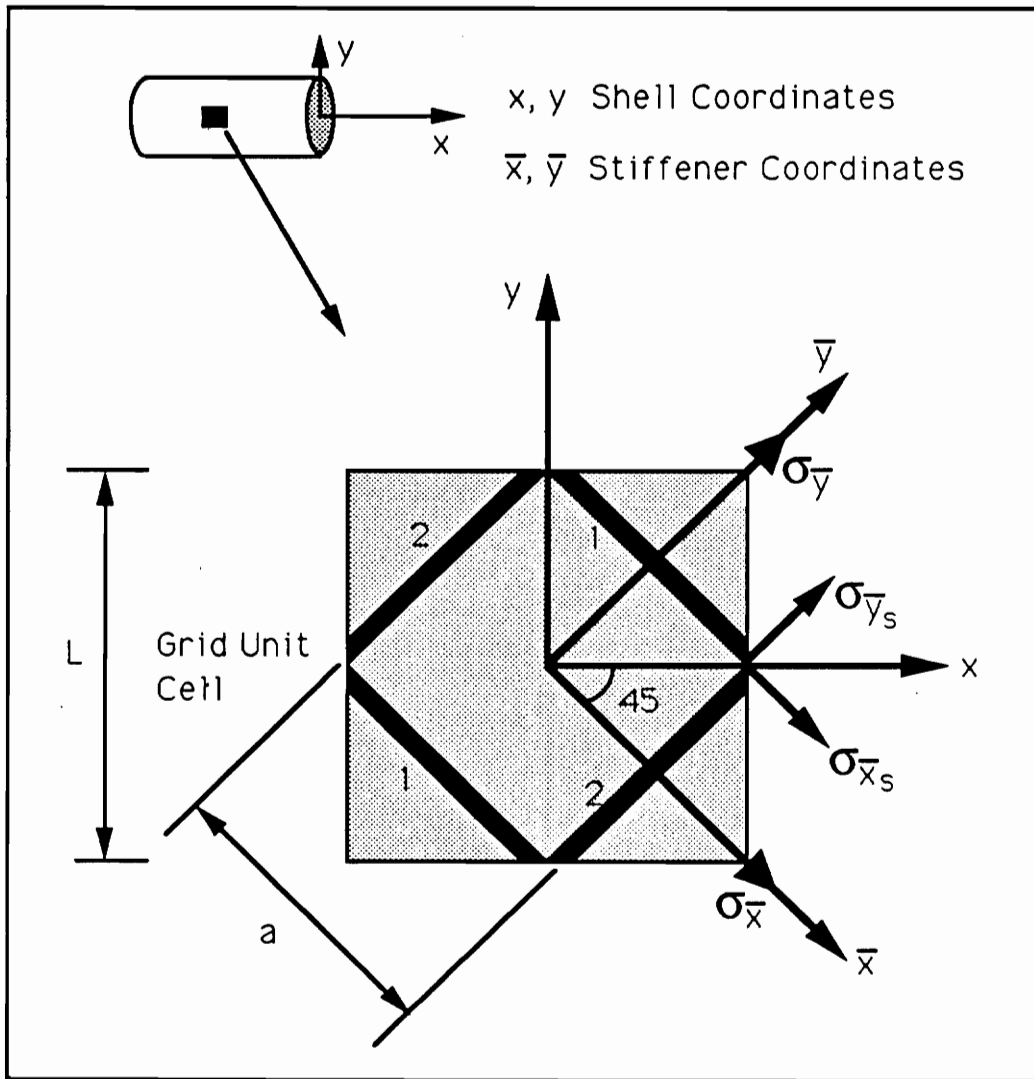


Figure 2.2 Shell and Stiffener Coordinate Systems

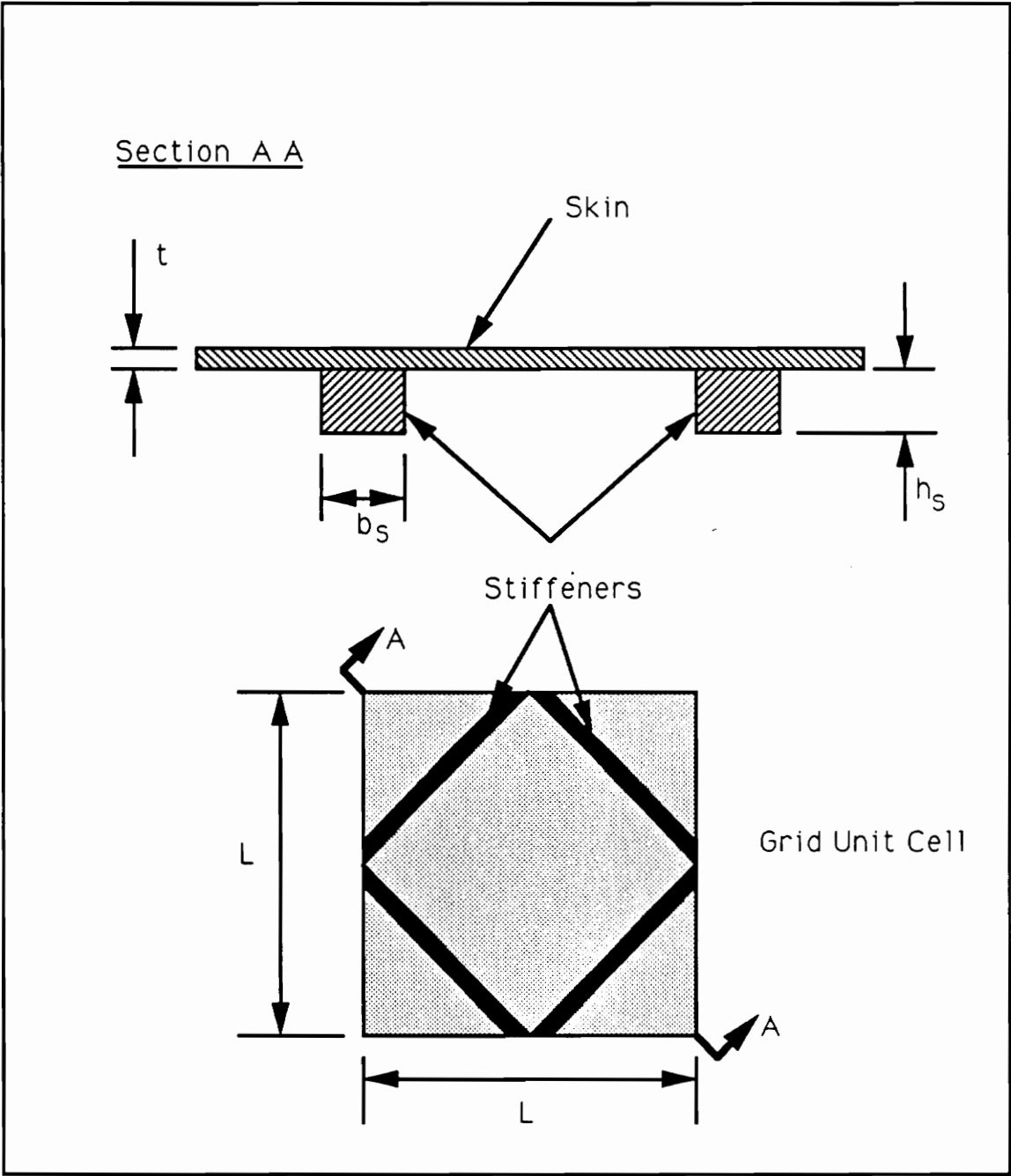


Figure 2.3 Grid Unit Cell

Now, from Equations (13) and (14) the stresses in the smeared stiffeners in stiffener coordinates are

$$\sigma_{\bar{x}} = \frac{b_s E}{a} \epsilon_{\bar{x}} \quad \sigma_{\bar{y}} = \frac{b_s E}{a} \epsilon_{\bar{y}} \quad (15)$$

We find the stresses in the smeared stiffeners in shell coordinates by transforming the stresses:

$$\begin{aligned} \sigma_x &= \frac{\sigma_{\bar{x}} + \sigma_{\bar{y}}}{2} + \frac{\sigma_{\bar{x}} - \sigma_{\bar{y}}}{2} \cos 2\theta - \tau_{\bar{x}\bar{y}} \sin 2\theta \\ \sigma_y &= \frac{\sigma_{\bar{x}} + \sigma_{\bar{y}}}{2} - \frac{\sigma_{\bar{x}} - \sigma_{\bar{y}}}{2} \cos 2\theta - \tau_{\bar{x}\bar{y}} \sin 2\theta \\ \tau_{xy} &= \frac{\sigma_{\bar{x}} - \sigma_{\bar{y}}}{2} \sin 2\theta + \tau_{\bar{x}\bar{y}} \cos 2\theta \end{aligned} \quad (16)$$

For $\theta=45^\circ$,

$$\begin{aligned} \sigma_x &= \sigma_y = \frac{b_s E}{2a} (\epsilon_{\bar{x}} + \epsilon_{\bar{y}}) \\ \tau_{xy} &= \frac{b_s E}{2a} (\epsilon_{\bar{x}} - \epsilon_{\bar{y}}) \end{aligned} \quad (17)$$

From Equations (12), in shell coordinates,

$$\begin{aligned} \sigma_x &= \sigma_y = \frac{b_s E}{2a} (\epsilon_x + \epsilon_y) \\ \tau_{xy} &= \frac{b_s E}{2a} \gamma_{xy} \end{aligned} \quad (18)$$

From the Love-Kirchhoff hypothesis,

$$\begin{aligned} \epsilon_x &= \epsilon_x^* + Z \kappa_x \\ \epsilon_y &= \epsilon_y^* + Z \kappa_y \\ \gamma_{xy} &= \gamma_{xy}^* + Z \kappa_{xy} \end{aligned} \quad (19)$$

Substituting Equations (19) in Equations (17) and (18), we get expressions for the stresses of the smeared stiffeners in shell coordinates:

$$\begin{aligned}\sigma_x = \sigma_y &= \frac{b_s E}{2a} [(\epsilon_x + \epsilon_y) + z (\kappa_x + \kappa_y)] \\ \tau_{xy} &= \frac{b_s E}{2a} (\gamma_{xy} + z \kappa_{xy})\end{aligned}\tag{20}$$

Force Resultants: We know that the force resultants of the stiffened element have 2 components, one coming from the integration of the skin stresses through the thickness of the skin, and the other from the integration of the stiffener stresses over the cross-sectional area of the stiffener:

$$\{N\} = \int_t \{\sigma\} dz + \frac{1}{a} \int_{A_s} \{\sigma\} dA\tag{21}$$

where t is the thickness of the skin and A_s is the cross-sectional area of the stiffener. For a laminated skin, we must integrate the stresses in each lamina over its own thickness and then add them as done by Jones [11]:

$$\{N\} = \sum_{k=1}^n [\bar{Q}]_k \int_{t_k} \left[\{\epsilon\} + z \{\kappa\} \right] dz + \frac{1}{a} \int_{A_s} \{\sigma\} dA\tag{22}$$

Also, we know that

$$\begin{aligned}A_{ij} &= \sum_{k=1}^n [\bar{Q}]_k (z_k - z_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^n [\bar{Q}]_k (z_k^2 - z_{k-1}^2) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^n [\bar{Q}]_k (z_k^3 - z_{k-1}^3)\end{aligned}\tag{23}$$

Finally, performing the integration in Equation (21), we get the expressions for the force resultants,

$$\begin{aligned}N_x &= (A_{11} + \frac{EA_s}{2a})\epsilon_x + (A_{12} + \frac{EA_s}{2a})\epsilon_y \\ &+ (B_{11} + \frac{EA_s}{2a})\kappa_x + (B_{12} + \frac{EA_s}{2a})\kappa_y\end{aligned}\tag{24}$$

$$\begin{aligned}
N_y &= (A_{12} + \frac{EA_s}{2a}) \epsilon_x + (A_{22} + \frac{EA_s}{2a}) \epsilon_y \\
&\quad + (B_{12} + \bar{z} \frac{EA_s}{2a}) \kappa_x + (B_{22} + \bar{z} \frac{EA_s}{2a}) \kappa_y \\
N_{xy} &= (A_{66} + \frac{EA_s}{2a}) \gamma_{xy} + (B_{66} + \bar{z} \frac{EA_s}{2a}) \kappa_{xy}
\end{aligned} \tag{24}$$

where $\bar{z}$ is the distance from the centroid of the stiffener to the middle surface of the skin, i.e., the stiffener eccentricity. In matrix form,

$$\{N\} = [\bar{A}_{ij}] \{\epsilon\} + [\bar{B}_{ij}] \{\kappa\} \tag{25}$$

so the expressions for the $\bar{A}_{ij}$ and $\bar{B}_{ij}$ for the helically stiffened shell are from Equations (24):

$$\begin{aligned}
\bar{A}_{11} &= \bar{A}_{22} = (A_{11} + \frac{EA_s}{2a}) = (A_{22} + \frac{EA_s}{2a}) \\
\bar{A}_{12} &= (A_{12} + \frac{EA_s}{2a}) & \bar{A}_{66} &= (A_{66} + \frac{EA_s}{2a}) \\
\bar{B}_{11} &= \bar{B}_{22} = (B_{11} + \bar{z} \frac{EA_s}{2a}) = (B_{22} + \bar{z} \frac{EA_s}{2a}) \\
\bar{B}_{12} &= (B_{12} + \bar{z} \frac{EA_s}{2a}) & \bar{B}_{66} &= (B_{66} + \bar{z} \frac{EA_s}{2a})
\end{aligned} \tag{26}$$

Moment Resultants: For the moment resultants, we proceed in the same way that we did for the force resultants, so

$$\{M\} = \int \{\sigma\} z \, dz \tag{27}$$

Here we also have two contributions - one from the skin and one from the stiffeners. The contribution of the skin is the integration of the skin stresses times $z \, dz$ through the thickness of the skin. Because the skin of our shell is laminated, we proceed for the integration of the skin stresses as we did for the force resultants.

For the contribution of the smeared stiffeners, apart from the stresses given by

Equations (20), we must take into account the torsional stiffness of the stiffener. For $\pm 45^\circ$ stiffeners, if we bend the stiffened element in the x direction, for example, then apart from the bending, the stiffener will twist as well. If instead of bending the stiffened element we apply a twisting moment M_{xy} , the stiffeners will bend without any twisting.

Meyer [10] and Reddy, Valisetty, and Rehfield [12] did not take into account the contribution of the stiffener torsional stiffness. However, they still got good correlation between the analytical and experimental results because, in the cases analyzed in those papers, the contribution of the torsional stiffness of the stiffeners is small. In our study, we take the torsional stiffness of the stiffeners into account because Jones and Card showed that it has a significant contribution to the stiffened shell buckling resistance [13].

In order to find the contribution of the twisting stiffness of the stiffeners, we must find how much the stiffeners twist when the stiffened element bends. Thus, we must find the twisting curvature κ_{xy} in stiffener coordinates in terms of the curvatures in shell coordinates. If we rotate our coordinate system an angle θ , we have

$$\kappa_{\bar{x}\bar{y}} = -2 \cos\theta \sin\theta \kappa_x + 2 \cos\theta \sin\theta \kappa_y + (\cos^2\theta - \sin^2\theta) \kappa_{xy} \quad (28)$$

where $\bar{x}$, $\bar{y}$ are stiffener coordinates. For our case, $\theta = \pm 45^\circ$, so

$$\begin{aligned} \kappa_{\bar{x}\bar{y}} &= (-\kappa_x + \kappa_y) & \text{for } \theta = 45^\circ \\ \kappa_{\bar{x}\bar{y}} &= (\kappa_x - \kappa_y) & \text{for } \theta = -45^\circ \end{aligned} \quad (29)$$

The signs of these curvatures depend on which bending moment we are applying (M_x or M_y).

The torsional stiffness of the smeared stiffeners in stiffener coordinates in the manner of Block, Card, and Mikulas [14] is $GJ/2a$, so, expanding Equation (27), we have:

$$\begin{aligned}
M_x &= \sum_{k=1}^n \int_{t_k} \sigma_x^k z \, dz + \frac{1}{a} \int_{A_s} \sigma_x z \, dA + \frac{GJ}{2a} (\kappa_x - \kappa_y) \\
M_y &= \sum_{k=1}^n \int_{t_k} \sigma_y^k z \, dz + \frac{1}{a} \int_{A_s} \sigma_y z \, dA + \frac{GJ}{2a} (-\kappa_x + \kappa_y) \\
M_{xy} &= \sum_{k=1}^n \int_{t_k} \tau_{xy}^k z \, dz + \frac{1}{a} \int_{A_s} \tau_{xy} z \, dA
\end{aligned} \tag{30}$$

where superscript and subscript k stand for k^{th} layer of the skin and n is the number of layers. Performing these integrations and keeping in mind Equations (28), we get:

$$\begin{aligned}
M_x &= (D_{11} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a} + \frac{GJ}{2a}) \kappa_x + (D_{12} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a} - \frac{GJ}{2a}) \kappa_y \\
&\quad + D_{16} \kappa_{xy} + (B_{11} + \bar{z} \frac{EA_s}{2a}) \epsilon_x + (B_{12} + \bar{z} \frac{EA_s}{2a}) \epsilon_y \\
M_y &= (D_{12} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a} - \frac{GJ}{2a}) \kappa_x + (D_{22} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a} + \frac{GJ}{2a}) \kappa_y \\
&\quad + D_{26} \kappa_{xy} + (B_{12} + \bar{z} \frac{EA_s}{2a}) \epsilon_x + (B_{22} + \bar{z} \frac{EA_s}{2a}) \epsilon_y \\
M_{xy} &= D_{16} \kappa_x + D_{26} \kappa_y + (D_{66} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a}) \kappa_{xy} + (B_{66} + \bar{z} \frac{EA_s}{2a}) \gamma_{xy}
\end{aligned} \tag{31}$$

where I is the centroidal moment of inertia of the stiffener and J is the polar moment of inertia of the stiffener. From Equations (31), we can identify the expressions for the bending stiffnesses $\bar{D}_{ij}$ and for the bending-extension coupling stiffnesses $\bar{B}_{ij}$ (the same as in Equations (26)) of the $\pm 45^\circ$ helically stiffened construction:

$$\begin{aligned}
\bar{D}_{11} &= (D_{11} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a} + \frac{GJ}{2a}) \\
\bar{D}_{22} &= (D_{22} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a} + \frac{GJ}{2a}) \\
\bar{D}_{12} &= (D_{12} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a} - \frac{GJ}{2a})
\end{aligned} \tag{32}$$

$$\bar{D}_{66} = (D_{66} + \bar{z}^2 \frac{EA_s}{2a} + \frac{EI}{2a}) \quad (32)$$

$$\bar{D}_{16} = D_{16} \quad \bar{D}_{26} = D_{26}$$

Because the skin is thin, we can neglect transverse shear deformations due to the assumption that the normals to the middle surface remain straight and normal under deformation. This assumption is good if the laminate is thin as in our case. On the other hand, the stiffeners are not thin, so the transverse shear deformations are important, and we will take them into account. We integrate the transverse shear stresses through the thickness of the stiffeners h_s to obtain the transverse shear stress resultants in the stiffeners [12]

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \int_{h_s} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz \quad (33)$$

Performing the integration and knowing that the shear stresses are constant through the thickness of the stiffeners, we get

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} K_{13} & 0 \\ 0 & K_{23} \end{bmatrix}^c \begin{bmatrix} \gamma_{xz}^c \\ \gamma_{yz}^c \end{bmatrix} \quad (34)$$

where

$$K_{13} = K_{23} = \frac{2 G_{13} A_s \cos 45^\circ}{L} \quad (35)$$

where K_{13} and K_{23} are the transverse shear stiffnesses of the $\pm 45^\circ$ helically stiffened construction [12].

2.3.4 Stresses for the $\pm 45^\circ$ Helically Stiffened Construction

From ABAQUS, we get the force and moment resultants, and, as for the sandwich shell, we multiply them by the inverse of the stiffness matrix to get the skin mid-plane strains and curvatures. To get the stiffener longitudinal strain and

curvature and the stiffener twist angle, we rotate the strains and curvatures of the skin mid-plane from the shell coordinates x - y to the stiffener coordinates $\bar{x}$ - $\bar{y}$ as in Figure 2.2.

A) **Skin Stresses:** Because the skin can be regarded as a usual laminate, we obtain the skin stresses as is done by Jones [11], so for the k^{th} layer of the skin:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{ij}^k \end{bmatrix} \begin{bmatrix} \epsilon_x^k \\ \epsilon_y^k \\ \gamma_{xy}^k \end{bmatrix} \quad (36)$$

where Q_{ij}^k are the reduced stiffnesses of the k^{th} layer.

B) **Stiffener Stresses:**

Bending stress: We calculate the bending stress at the most distant point of the stiffener from the mid-plane of the skin.

$$\begin{aligned} \text{Stiffener 1:} \quad \sigma_{max} &= E_1 (\epsilon_x + \bar{t} \kappa_x) \\ \text{Stiffener 2:} \quad \sigma_{max} &= E_1 (\epsilon_y + \bar{t} \kappa_y) \end{aligned} \quad (37)$$

where $\bar{t} = t/2 + h_s$ and stiffeners 1 and 2 are shown in Figure 2.2.

Shear stress due to transverse shear forces:

$$\begin{aligned} \tau_{xz} &= G_{13} \gamma_{\bar{x}z} \\ \tau_{yz} &= G_{23} \gamma_{\bar{y}z} \end{aligned} \quad (38)$$

In our case G_{13} is equal to G_{12} , the in-plane shear modulus, because the stiffeners are made with unidirectional tape with the fiber axis parallel to the longitudinal axis of each stiffener ($\bar{x}$ for stiffener 1, and $\bar{y}$ for stiffener 2 in Figure 2.2).

Shear stress due to twist of the stiffeners: The twist angle of the stiffeners is $\phi = \kappa_{xy} L/\sqrt{2}$, where $L/\sqrt{2}$ is the stiffener length. Then, using the approach used by Timoshenko [15] for the twist of non-circular prismatic members, we get

$$\tau_{yz_{max}} = \phi c_2 G_{13} b_s \sqrt{2} / c_1 \quad (39)$$

where t is the thickness of the skin, h_s is the height of the stiffeners, b_s is the width of the stiffeners, and c_1 and c_2 are coefficients that depend on h_s/b_s .

A FORTRAN program was developed to post-process the force and moment resultants obtained from ABAQUS to get the stresses in the skin and stiffeners. In this program, the Tsai-Hill failure criterion is used to evaluate whether there are any ply failures in the skin layers. To predict failure in the stiffeners, we check if the values of the different stresses in the stiffeners reach the values of the respective strengths because the stiffeners are stressed only in one direction.

To perform this failure analysis, we need the values of the shear strengths in $\bar{x}$ - z and in $\bar{y}$ - z coordinates. We assume that the shear strength $S_{\bar{x}z}$ will be equal to the in-plane shear strength S_{xy} . We can make this assumption because the stiffeners are made with unidirectional tape with the axis of the fibers coinciding with the longitudinal axis of each stiffener ($\bar{x}$ for stiffener 1 and $\bar{y}$ for stiffener 2 as shown in Figure 2.2).

The value of the strength S_{yz} should be measured, but this effort is beyond the scope of this work, so we will assume that:

$$S_{yz} = .7 \times S_{yz} \quad (40)$$

The value of .7 is the relative value found for a similar graphite-epoxy material (T300-5208) [16]. This approximation leads to $S_{yz} = 35 \text{ N/mm}^2$.

2.3.5 Skin and Stiffener Buckling

Up to this point, we can predict the failure of the structure due to skin or stiffener

material failure, but we must also ensure that there will not be any local buckling before the material failure. Two types of local buckling can occur for this type of structure: skin buckling between stiffeners and stiffener buckling (also known as rib crippling).

Skin Buckling:

To perform this buckling analysis, we assume the skin to be a simply supported square plate with in-plane dimensions equal to a , which is the spacing between stiffeners. This support assumption is conservative enough to ensure that there will be no buckling because in reality the skin is more supported than we are assuming here. The loads acting on the plate are the equivalent loads in stiffener coordinates. The actual N_{xy} loading and the equivalent loads are shown in stiffener coordinates in Figures 2.4a and b. The plate with the loads is shown in Figure 2.4c. The values of the loads are obtained in the following way: the shear force resultant acting on the structure as a function of the torque T applied is

$$N_{xy} = \frac{2 T}{\pi D^2} \quad (41)$$

The equivalent load P_e in stiffener coordinates is:

$$P_e = \frac{N_{xy} L}{\cos 45^\circ} \quad (42)$$

By distributing P_e over the spacing between stiffeners, we get

$$-N_x = N_y = \frac{P_e}{a} = \frac{N_{xy} L}{\cos 45^\circ a} \quad (43)$$

but $L = \sqrt{2} a$, so

$$-N_x = N_y = 2 N_{xy} \quad (44)$$

We use a safety factor of 1.5, so the final values of N_x and N_y , which are the critical loading, are:

$$-N_x = N_y = 3 N_{xy} \quad (45)$$

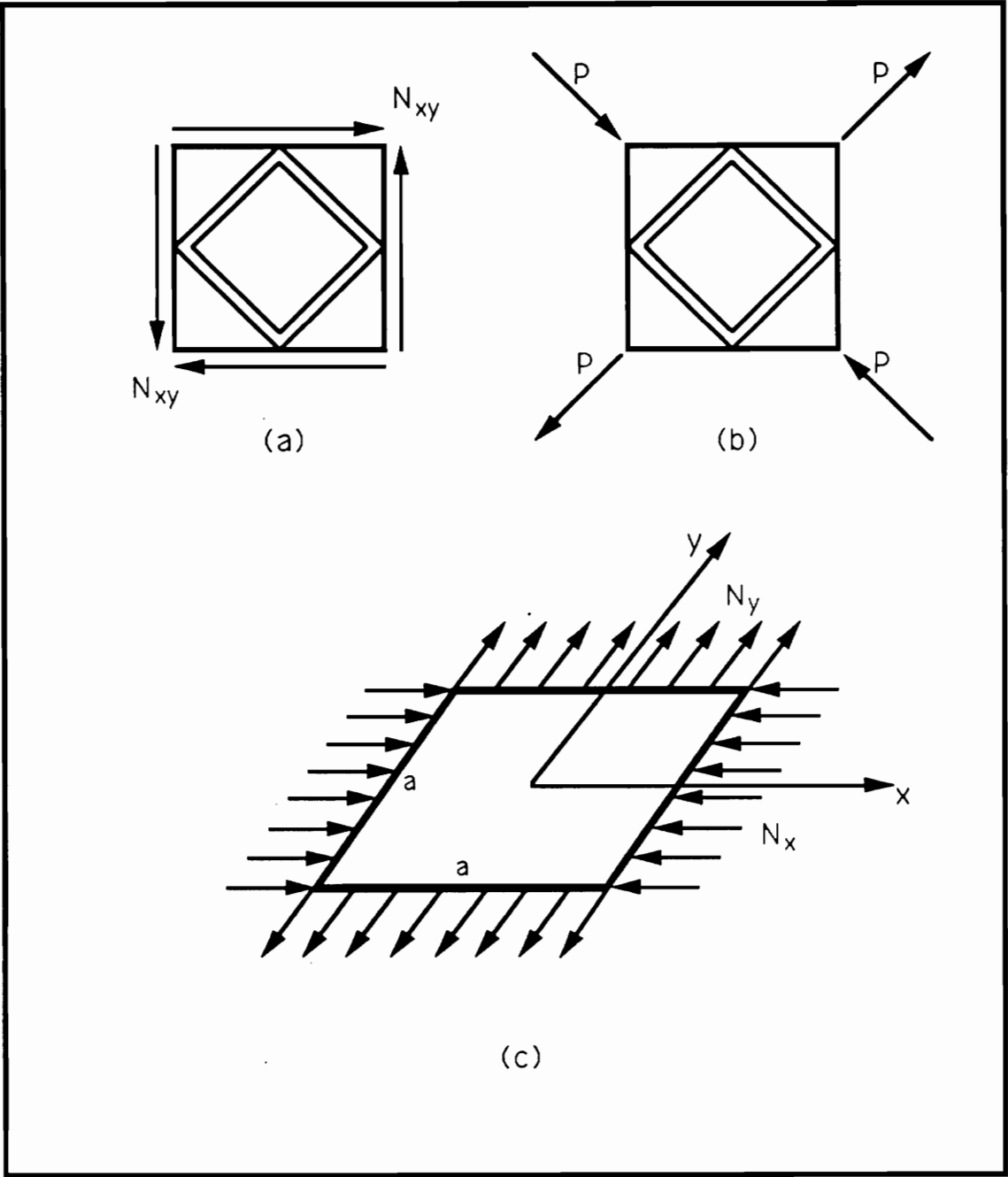


Figure 2.4 Equivalent Loading for the Skin Buckling Problem

For the buckling load prediction, we use the following expression which is for a simply supported symmetric cross-ply laminated rectangular plate under N_x and N_y loading [9]

$$N_{cr} = \frac{-\pi^2 [D_{11} (\frac{m}{a})^2 + 2 (D_{12} + 2 D_{66}) (\frac{n}{b})^2 + D_{22} (\frac{n}{b})^4 (\frac{a}{m})^2]}{1 + \alpha (\frac{an}{bm})^2} \quad (46)$$

where

$$\alpha = \frac{N_y}{N_x}$$

and the buckling deflection w that satisfies the boundary conditions is of the form

$$w = A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a} \quad (47)$$

where m and n are the number of buckle half waves in the x - and y - directions, respectively. For our case, $N_y = -N_x$, so $\alpha = -1$. For a square plate, $a = b$, so

$$N_{cr} = \frac{-\pi^2 [D_{11} (\frac{m}{a})^2 + 2 (D_{12} + 2 D_{66}) (\frac{n}{b})^2 + D_{22} \frac{n^4}{a^2 m^2}]}{1 - (\frac{n}{m})^2} \quad (48)$$

From Equation (48), we notice that the buckling load for $m=1$ and $n=1$ is infinite. Also, substituting in Equation (48) different values of m and n , we see that the lowest buckling load occurs for $m=2$ and $n=1$.

Multiply Equation (48) by D_{22}/D_{22} and group terms to obtain

$$N_{cr} = \frac{\pi^2 D_{22}}{1 - (\frac{n}{m})^2} [K_1 (\frac{m}{a})^2 + K_2 (\frac{n}{a})^2 + \frac{n^4}{a^2 m^2}] \quad (49)$$

where

$$K_1 = \frac{D_{11}}{D_{22}} \quad K_2 = \frac{2 (D_{12} + D_{66})}{D_{22}} \quad (50)$$

From Equation (49), setting $N_{cr} = N_x$, we get the maximum safe spacing between stiffeners for a given laminate. Thus, solving Equation (49) for a , with

m=2 and n = 1, we obtain

$$a = \sqrt{\frac{\pi^2 D_{22}}{3 N_{cr}} [16 K_1 + 4 K_2 + 1]} \quad (51)$$

The value of a obtained from Equation (51) is the critical value, so we must choose values of a lower than this critical a. The final value of a is chosen so that we do not complicate the manufacturing step.

Stiffener Buckling:

To predict stiffener buckling, we use the expression by Rehfield, Deo, and Renieri [17]. This expression comes from assuming the stiffeners to be orthotropic plate elements. The stiffener element support is assumed to be clamped at the nodes where the stiffener intersects two perpendicular adjacent stiffeners and simple support at the rib-skin attachment. Thus, for an orthotropic plate clamped at two opposite edges and simply supported and free at the other two, the critical value of the stress resultant can be estimated from [17]

$$N_{cr} = b_s^3 \left[\frac{4 \pi^2 E_1}{12 a^2 \left(1 - \nu_{12} \frac{E_2}{E_1} \right)} + \frac{G_{12}}{h_s^3} \right] \quad (52)$$

where a is the spacing between stiffeners or the length of the stiffener element and E_1 , E_2 , G_{12} , and ν_{12} are the properties of the stiffener material in principal material coordinates.

We compare the value of the N_{cr} obtained from Equation (52) with the value of N that is applied on the stiffener. If N is smaller than N_{cr} , then the stiffeners do not buckle. From Equations (44) and (45), the expression for P_e is

$$P_e = 2 N_{xy} a \quad (53)$$

To obtain the value of the load applied on the stiffener, we use the following

expression:

$$N = \frac{P_e}{h_s} \quad (54)$$

where P_e is the equivalent load in stiffener coordinates obtained in Equation (53). To prevent stiffener buckling, N must be smaller than N_{cr} .

2.4 DEFORMATION BEHAVIOR OF CLOSED SANDWICH AND $\pm 45^\circ$ HELICALLY STIFFENED CIRCULAR CYLINDRICAL SHELLS

Here, we consider two closed circular cylindrical shells with the mean external dimensions of the actual racing car structural shell. First, we analyze a sandwich shell, and then we perform the same analysis for a $\pm 45^\circ$ helically stiffened shell. For the length of the shell L , we use the total length of the structural shell, $L = 1600$ mm. For the external diameter of the shell, the biggest dimension of the back plate of the structural shell is used, $D = 570$ mm. Both shells are subjected to a torque at one end whereas the other end is clamped. The shell with its dimensions, loads, and boundary conditions is shown in Figure 2.5.

2.4.1 Finite Element Analysis

For the analysis of these shells, we use the ABAQUS finite element program [4], and the PATRAN [5] pre- and post-processor. From the shell element library of ABAQUS, we use element S8R, which is an 8-noded, reduced-integration, doubly curved shell element, with six degrees of freedom per node (3 displacements and 3 rotations).

The input to the ABAQUS program is the loads, the boundary conditions, and the stiffness matrix of the shell section, including the values of the transverse shear stiffnesses. As output from ABAQUS, we get five force resultants per unit length, three in-plane and two transverse, three moment resultants at the centroid of each element, and the nodal displacements and rotations. The force and moment resultants per unit length (hereafter called simply forces and moments) are shown in Figure 2.6.

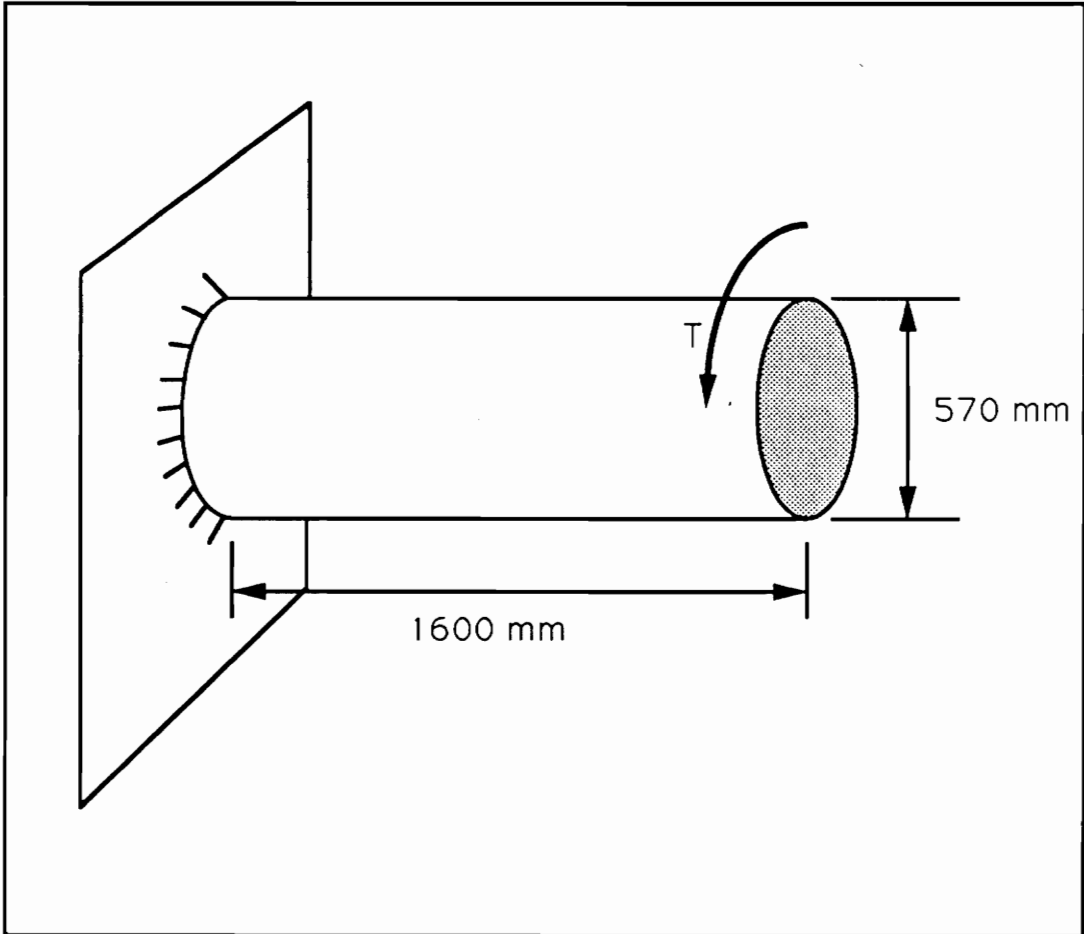


Figure 2.5 Circular Cylindrical Shell Subjected to Torsion

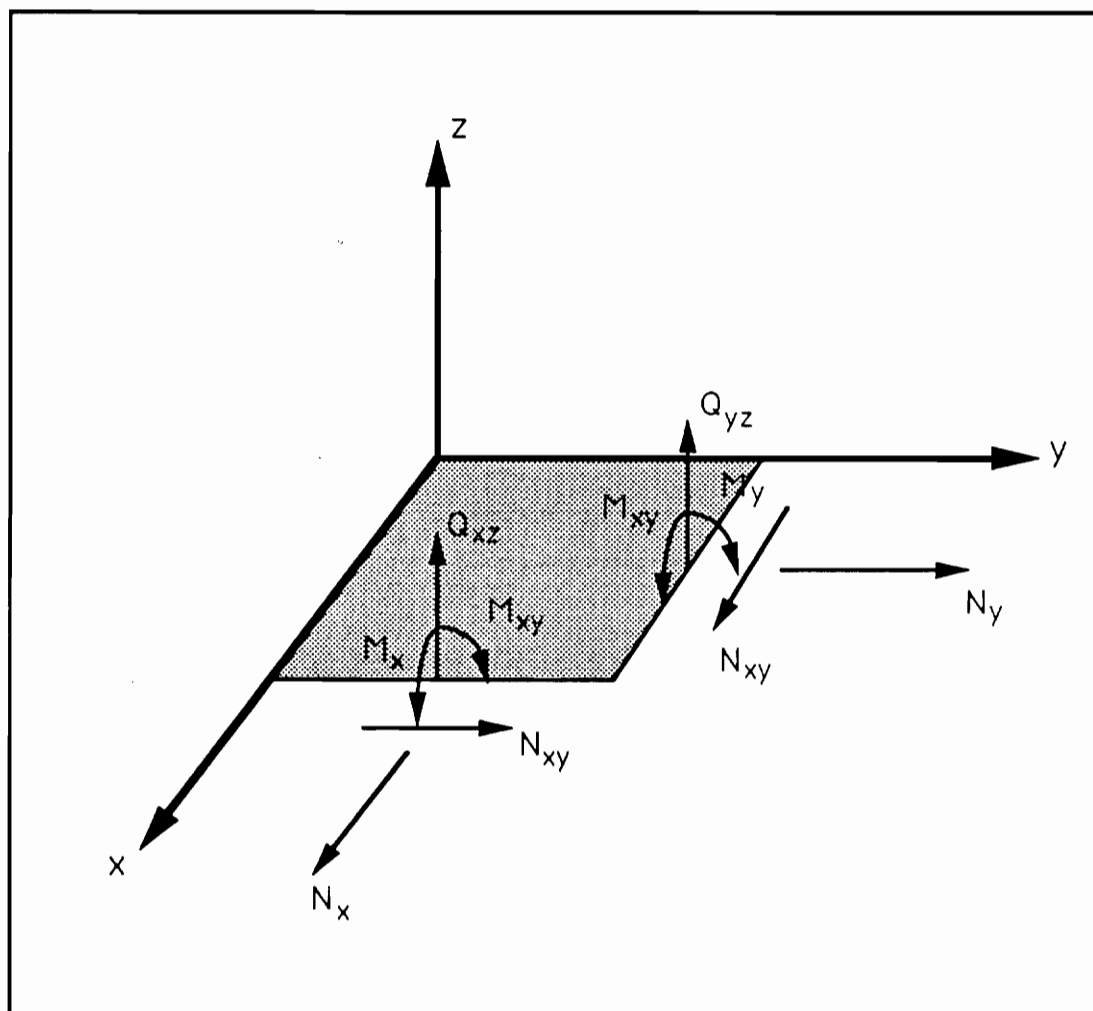


Figure 2.6 Forces and Moments per Unit Length.

In order to get the values of stress in each layer of the skins, in the core, or stiffeners, Fortran programs were created. We multiply the inverse of the stiffness matrix by the forces and moments to get the values of strains and curvatures. Then, we multiply the reduced stiffness matrix $[Q_{ij}]$ of each layer by the corresponding strains of that layer to get the stresses for each element. We also compare the values of stress with the Tsai-Hill failure criterion [18] in order to be able to predict failure.

In the deformed shapes of the closed shell meshes generated with PATRAN, we found a radial deformation at the loaded end of the shells that should not exist. The loaded end, instead of remaining circular, was distorted as in the case of a noncircular shell. This error is due to geometrical inaccuracies in the mesh generated by PATRAN. This effect is reduced by generating the surface to be meshed in several pieces or patches. When PATRAN generates circular arcs, it is actually generating a parametric curve instead of a circle. This parametric curve is exact in the radius only in the beginning and at the end of the curve. Thus, the bigger the number of patches, the better the accuracy in the mesh generated. The minimum number of patches used in our meshes was never lower than 8 which was enough to get negligible radial deformations.

2.4.2 Loads and Boundary Conditions

In this Chapter, we are concerned only with the torsional response of the shells. We use the maximum torque that the loads of the suspension members transmit to the structure through the two front bulkheads. These loads are shown in Figure 2.7 where the front part of the structural shell with its loads is sketched. The torque, $T = 6637 \text{ kNmm}$, is applied to one end of the shell, whereas the other end is clamped as shown in Figure 2.5.

To model the torque at one end, we put an equivalent concentrated load at each node of that end. These equivalent loads are tangent to the surface of the shell, and the collective effect of these loads should be the same as the torque.

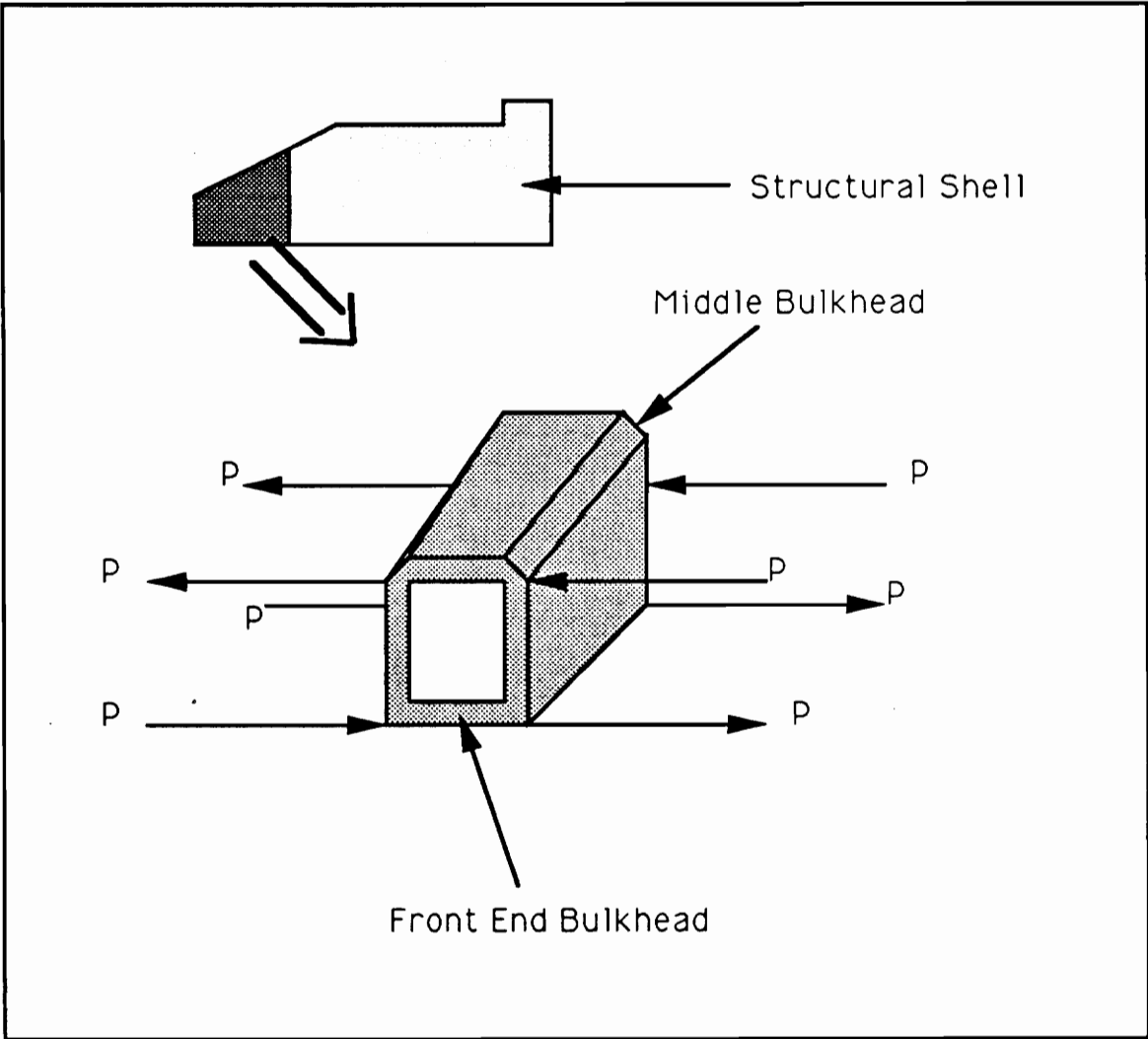


Figure 2.7 Loads on the Front and Middle Bulkheads

The equivalent loading for a shell end with 4 nodes in the loaded end is shown in Figure 2.8. The way in which we model the torque is the same as in Figure 2.8 with the difference that we always have more than four nodes. The sketch in Figure 2.8 is just a simple example.

To calculate the value of the equivalent load that we apply at each node of the loaded end, we use:

$$T = N P R$$

$$\Rightarrow P = \frac{T}{N R} \quad (55)$$

where P is the equivalent load, T is the applied torque, R is the radius of the mid-plane of the shells, and N is the number of nodes at the loaded end of the shells.

The result in which we are interested is the torsional stiffness of the shells. We calculate this torsional stiffness as:

$$K_{\theta} = \frac{T}{\theta} \quad (56)$$

where θ is the angle rotated by the loaded end of the shells around their longitudinal axis. From ABAQUS, we get the circumferential displacement u_{θ} , so θ is:

$$\theta = \sin^{-1} \frac{u_{\theta}}{R} \quad (57)$$

2.4.3 Cross Section and Lay-Up of the Sandwich and $\pm 45^{\circ}$ Helically Stiffened Shells

As the baseline skin configuration for the comparison of the torsional behavior between the sandwich and stiffened shells, we use the approximate sandwich lay-up used in the construction of an actual Formula 3 racing car structural shell which was provided to us by Mr. Oreste Berta [19]. This sandwich lay-up is $[[CF]_3/+45/-45/HC]_s$, where CF stands for graphite-epoxy fabric and HC stands for honeycomb core.

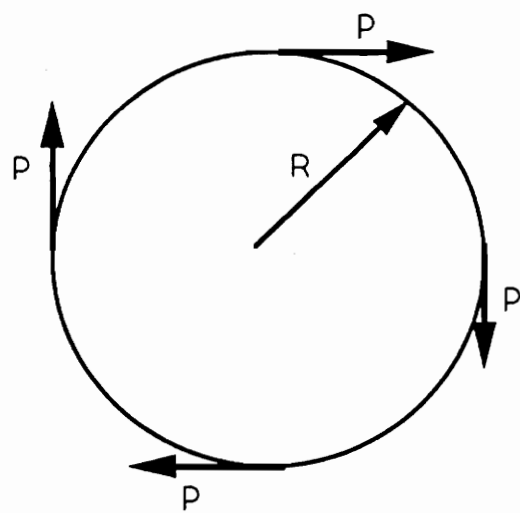


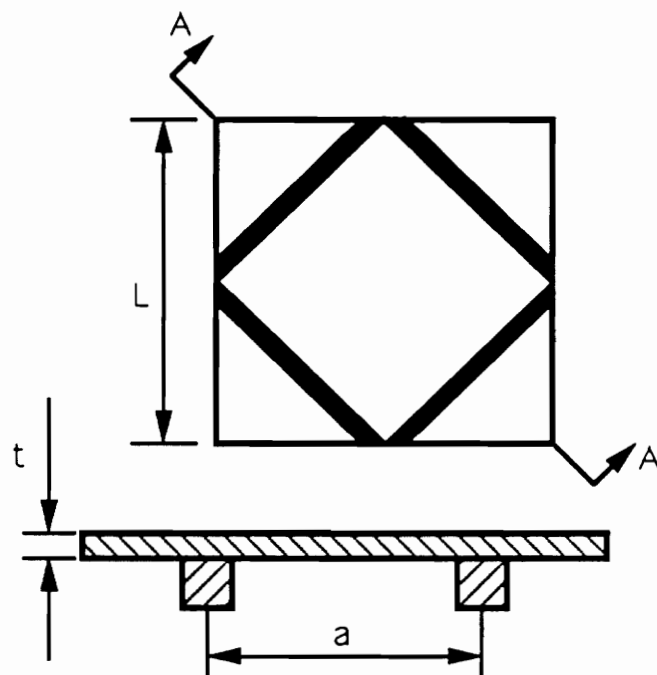
Figure 2.8 Equivalent Loads P for Torque T

For the stiffened construction, we try to find a stiffener-skin configuration that would have approximately the same bending stiffness as the sandwich lay-up. We analyze three different $\pm 45^\circ$ symmetric skin lay-ups using Equations (51) to take into account the problem of local skin buckling, and we look for the skin lay-up that has values of a and L that are convenient for manufacturing (not so small that would complicate the lay-up of the stiffeners). For the moment, we are not concerned with the stiffeners.

The skin lay-ups analyzed are $[\pm 45]_s$, $[+45/-45/+45/-45]_s$, and $[45/-45/45/-45/45/-45]_s$. For the $[\pm 45]_s$ lay-up, the value obtained for a is impractical because it is too small to manufacture. The $[45/-45/45/-45]_s$ skin lay-up has a maximum permissible spacing a of 83.5mm. This value of spacing is too small because we do not have enough freedom in modifying it to decrease or increase the number of stiffeners used around the shell. For this lay-up, the minimum permissible number of stiffener unit cells around the shell is 16. In the case of the $[45/-45/45/-45/45/-45]_s$ lay-up, the minimum permissible number of stiffener unit cells around the shell is 8. Having a low number of stiffener unit cells around the shell is desirable because of the lower number of stiffeners and consequently less effort in manufacturing. The values of a , L , and t for which there is no buckling for the last two lay-ups are shown in Table 2.3. To select the final stiffener-skin configuration for the shell, we choose a permissible value of L that divides the perimeter of the shell in an integer number of stiffener unit cells. Also, we look for the stiffener aspect ratio that yields approximately the same bending stiffnesses obtained for the sandwich lay-up. To verify that there is no stiffener buckling, we use Equation (53).

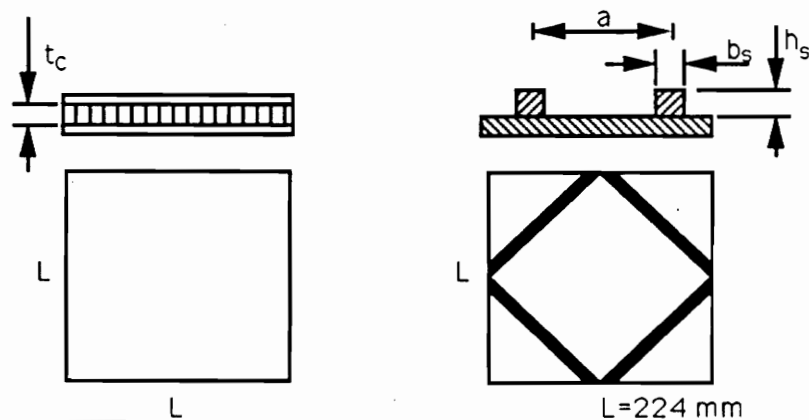
After changing the values of a , h_s , and b_s , we obtained the same bending stiffness as in the sandwich skin configuration with the stiffener-skin configuration shown in Table 2.4. This final stiffener-skin configuration has a stiffener aspect ratio t_s/b_s of 2.5, and its value of L yields 8 unit cells around the shell. The geometry, weight, and stiffness matrix values are given in Table 2.4. The weights shown in this table are those of the flat panels shown on the top of the table which have equal in-plane dimensions. The equal-bending-stiffness stiffener-skin configuration is about 30% heavier than the sandwich skin configuration.

Table 2.3 Values of a , L , and t for Two Skin Layups



	$[45/-45/45/-45]_s$	$[45/-45/45/-45/45/-45]_s$
a (mm)	83.3	162.3
t (mm)	1	1.5
L (mm)	118.1	226.3

Table 2.4 Final Skin and Stiffener Configurations
for Circular Cylindrical Shells



Configuration	Sandwich	$\pm 45^\circ$ Stiffened
Lay Up	$[[CF]_3/+45/-45]_s$ $t_c = 15 \text{ mm}$	Skin: $[\pm 45]_{3s}$ Stiffeners: $h_s = 17 \text{ mm}$ $b_s = 6.8 \text{ mm}$ $a = 158 \text{ mm}$
Weight (kg)	.17	.225
$A_{11} = A_{22}$ (kN/mm)	90.1	108.2
A_{12} (kN/mm)	18.82	94.5
A_{66} (kN/mm)	21.22	97.12
B_{1j}^* (kN)	0	429.4
$D_{11} = D_{22}$ (kN mm)	5326	5230
D_{12} (kN mm)	1114	4979
D_{66} (kN mm)	1256	5103.6
$D_{16} = D_{26}$ (kN mm)	0	2.15

* $B_{1j} = B_{11} = B_{22} = B_{12} = B_{66}$

2.4.4 Closed Sandwich Shell Analysis

In this section, we perform the finite element analysis of a closed sandwich circular cylindrical shell with the skin configuration described at the top of Table 2.4. The transverse shear stiffnesses are obtained from Equation (6). We use the ABAQUS S8R element to model the middle surface of the shell that has a radius of $R=276.7$ mm. This value is obtained by subtracting one half of the thickness of the shell wall from the external radius of the shell. To study the convergence of the finite element solution, we use three different meshes: 8 around by 10 along the shell, 16 by 15, and 24 by 22. The shell is subjected to a torque at one end whereas the other end is clamped. The way in which the torque and the boundary conditions are modeled is explained in Section 2.4.2.

A) 8 by 10 mesh: In this case, 16 nodes exist at each end of the shell (because each element has 3 nodes per edge), so we apply 16 tangent concentrated loads P , one at each node of the loaded end of the shell. These loads are applied in the same way that we did in Figure 2.8. Thus, by using Equation (55) with $N=16$, we get the equivalent load to be $P= 1.499$ kN.

B) 16 by 15 mesh: For this case, 32 nodes exist at each end, so in the same way as we did for the previous mesh for $N=32$, the value of the equivalent load is $P=.75$ kN.

C) 24 by 22 mesh: For this case, 48 nodes exist at each end, so for $N=48$ the value of the equivalent load is $P=.5$ kN.

D) 36 by 33 mesh: For this case, 72 nodes exist at each end, so for $N=72$ the value of the equivalent load is $P=.333$ kN.

The results for the four meshes are shown in Table 2.5. There, we see that convergence is achieved for u_θ and θ . The decrease in their value between the values obtained for the C) and D) meshes is much smaller than the decrease obtained between the A) and B) and between the B) and C) meshes.

Table 2.5 Results for Torsion of a Closed Sandwich Shell

	Mesh			
Behavior	8 x 10	16 x 15	24 x 22	36 x 33
U_{θ} (mm)	1.299	1.137	1.039	1.037
θ (deg)	.269	.236	.218	.217
K_{θ} (Nm/deg)	24674	28179	30503	30562

Because of the very small difference among the values obtained for the last two meshes, there is no practical need to use the most-refined mesh because the 24 by 22 mesh gives the same results as the most-refined mesh. In the 45° view plots of the deformed shape of the first three meshes, we see that there is no bending deformation and that the wall of the shells deform in shear as shown in Figures 2.9, 2.10 and 2.11. The values of radial displacement obtained for these meshes are negligible. During the postprocessing of the element forces and moments (to get the values of stress in each ply), no failure diagnostics were generated.

2.4.5 Closed $\pm 45^\circ$ Helically Stiffened Shell Analysis

In this section, we perform the finite element analysis of a closed $\pm 45^\circ$ helically stiffened circular cylindrical shell with the stiffener-skin configuration shown at the top of Table 2.4. The transverse shear stiffnesses are obtained from Equation (36). To study the convergence of the finite element solution, we use the same meshes that we used for the sandwich case. In this case, the middle surface of the shell skin has a radius of 284.2 mm. This value is obtained by subtracting one half of the thickness of the shell skin from the external radius of the shell. The loading of these shells is the same as in the sandwich case. The way in which the torque and the boundary conditions are modeled was explained in Section 2.4.2. We will also analyze the difference in behavior between an externally stiffened and an internally stiffened shell. The difference between these two shells is that the $\bar{B}_{ij}$ terms of the stiffness matrix have different sign. For the externally stiffened shell, the $\bar{B}_{ij}$ are positive, whereas for the internally stiffened shell the $\bar{B}_{ij}$ terms are negative.

The equivalent loads P are found in the same way as for the sandwich shell. The values of P for the three meshes that we use are:

- 8 by 10 element mesh: $P=1.46$ kN
- 16 by 15 elements mesh: $P=.73$ kN
- 24 by 22 elements mesh: $P=.49$ kN

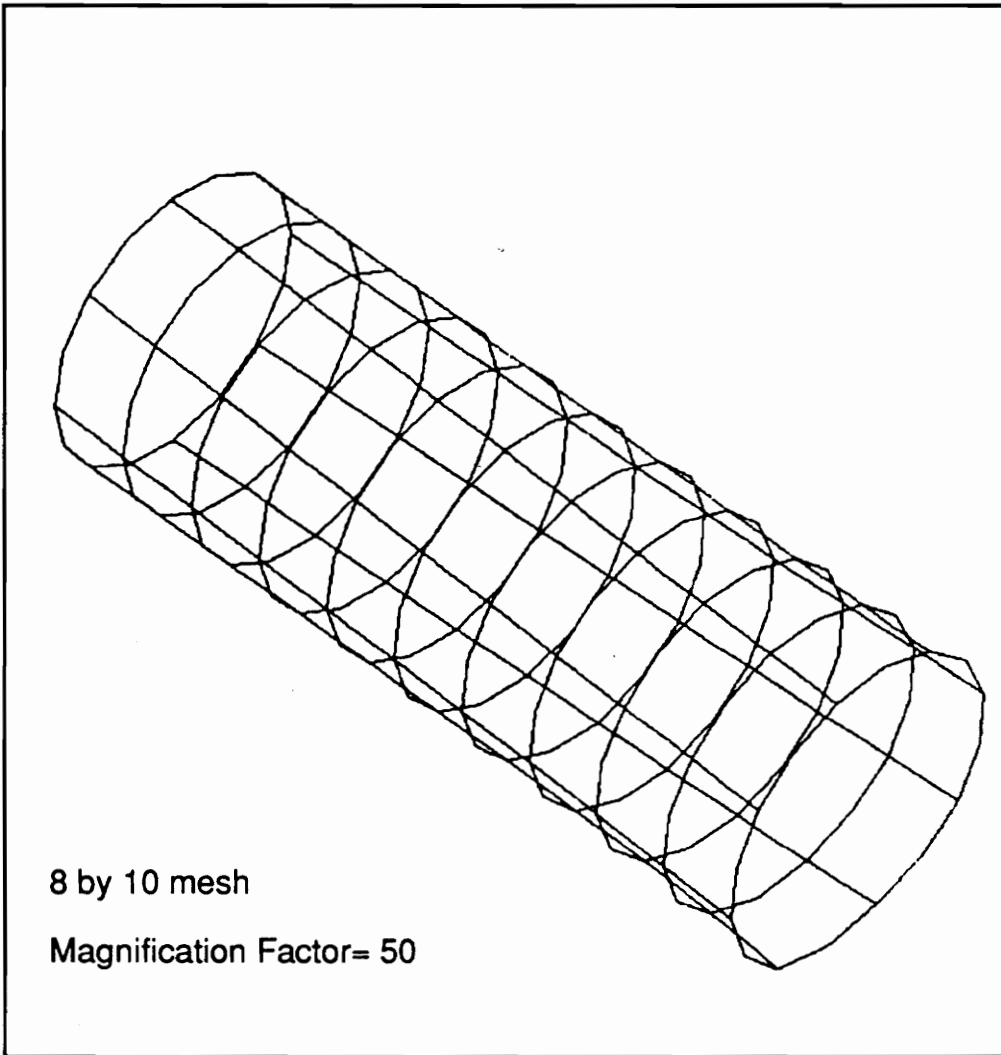


Figure 2.9 Deformed Shape for the Closed Sandwich Shell

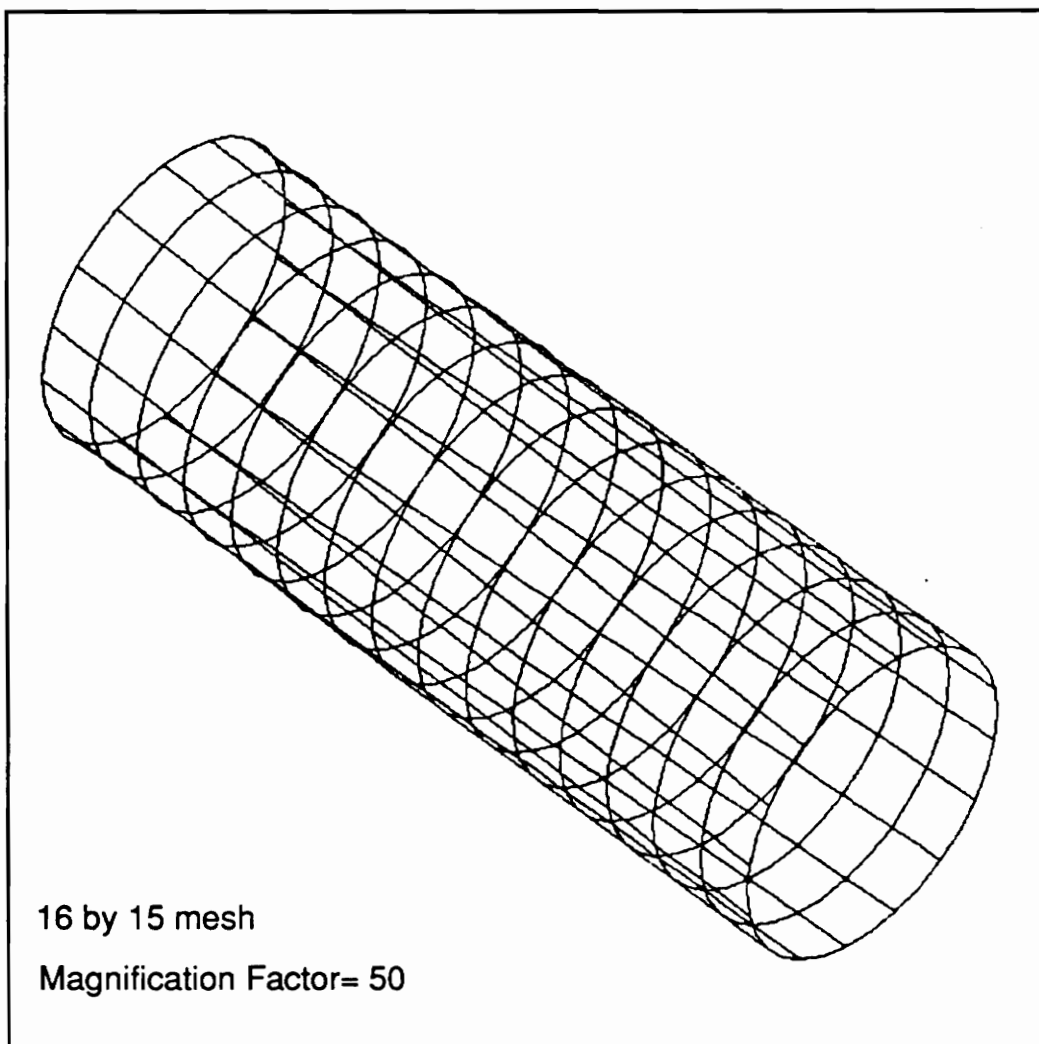


Figure 2.10 Deformed Shape of the Closed Sandwich Shell

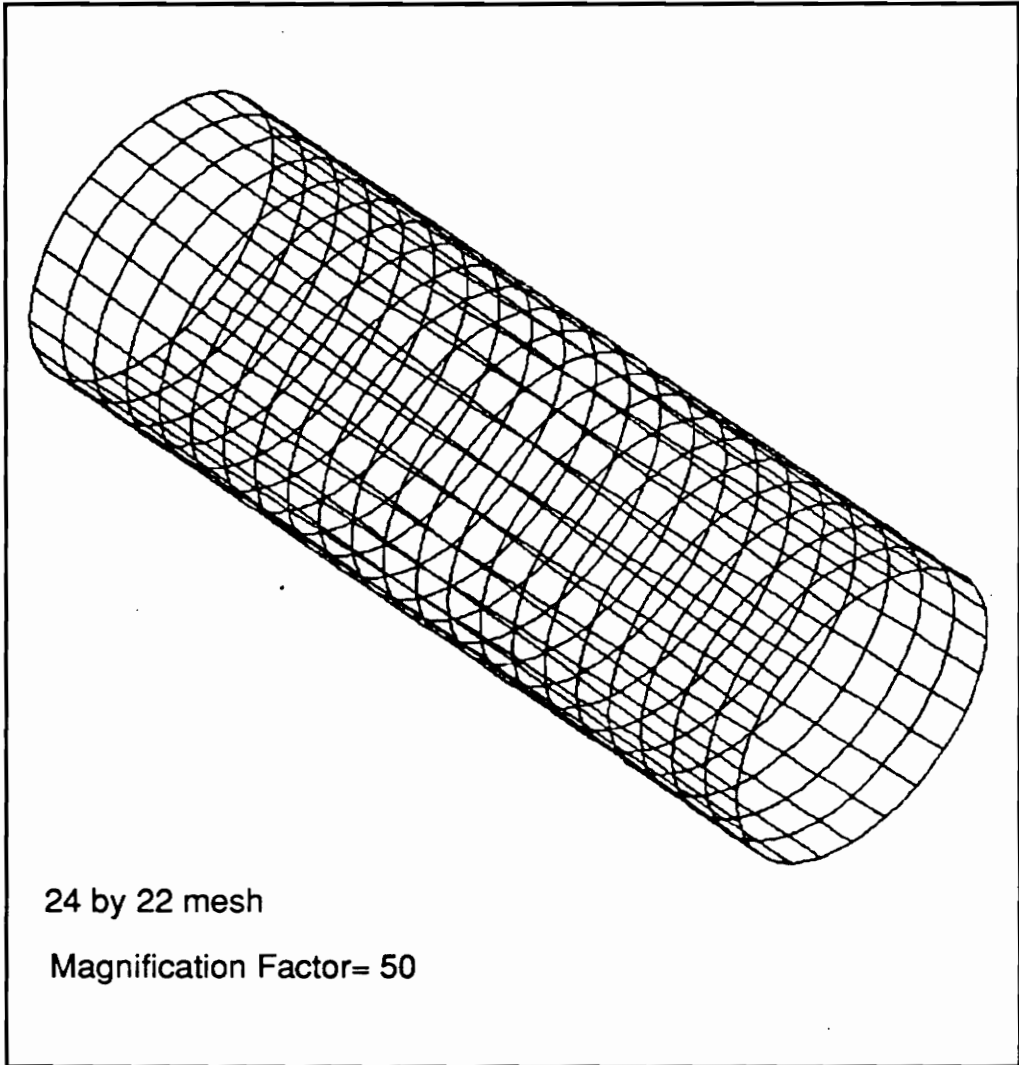


Figure 2.11 Deformed Shape for the Closed Sandwich Shell

As happened with the sandwich shells, the results for torsion of externally and internally stiffened shells converge, but we need only use the 24x22 mesh to achieve convergence. There is a 9% increase in torsional stiffness for the externally stiffened shell over the value obtained for the internally stiffened shell as shown in Tables 2.6 and 2.7. This increase is due to the fact that when we put the stiffeners outside, the terms of the bending-stretching coupling stiffness matrix $\bar{B}_{ij}$ are positive, whereas in the case of an internally stiffened shell they are negative. Finally, if we compare the torsional stiffness of the closed stiffened shell with the torsional stiffness of the closed sandwich shell, we see that the stiffened shell is 5.1 times stiffer than the sandwich shell.

If we look at the deformed shapes obtained for these shells (Figures 2.12 and 2.13) we see that the pattern of deformation is the same as for the sandwich shell (Figures 2.9, 2.10, and 2.11). We see that there is no bending and that the walls of the shells are deformed in shear. Thus, the improvement in the torsional stiffness of the stiffened shell over the sandwich shell is due to the fact that the in-plane shear stiffness A_{66} of the stiffened shell is nearly 5 times that of the sandwich shell (see Table 2.4).

2.5 DEFORMATION BEHAVIOR OF SANDWICH AND $\pm 45^\circ$ HELICALLY STIFFENED CIRCULAR CYLINDRICAL SHELLS WITH A SMALL CUTOUT

In this section, we move one more step toward the real structure of a Formula car. Up to this point, we have been concerned only with the torsion of closed circular cylindrical shells. Here we analyze a sandwich shell and a $\pm 45^\circ$ stiffened shell with a small cutout to try to begin to understand what happens in the region of the cockpit of the car.

The geometry and the skin and stiffener configurations of the shells are the same as those used for the closed shells except for the cutout. The location and size of the cutout are shown in Figure 2.14. We apply to these shells the same loading and boundary conditions that we used for the closed shells.

Table 2.6 Results for Torsion of
a Closed Externally Stiffened Shell

	Mesh		
Behavior	8 x 10	16 x 15	24 x 22
U_{θ} (mm)	.2129	.2112	.2103
θ (mm)	.043	.043	.042
K_{θ} (Nm/deg)	154613.4	155876	156617.6

Table 2.7 Results for Torsion of
a Closed Internally Stiffened Shell

	Mesh		
Behavior	8 x 10	16 x 15	24 x 22
U_{θ} (mm)	.2326	.2311	.2301
θ (mm)	.047	.047	.046
K_{θ} (Nm/deg)	141534.9	142453.6	143072.6

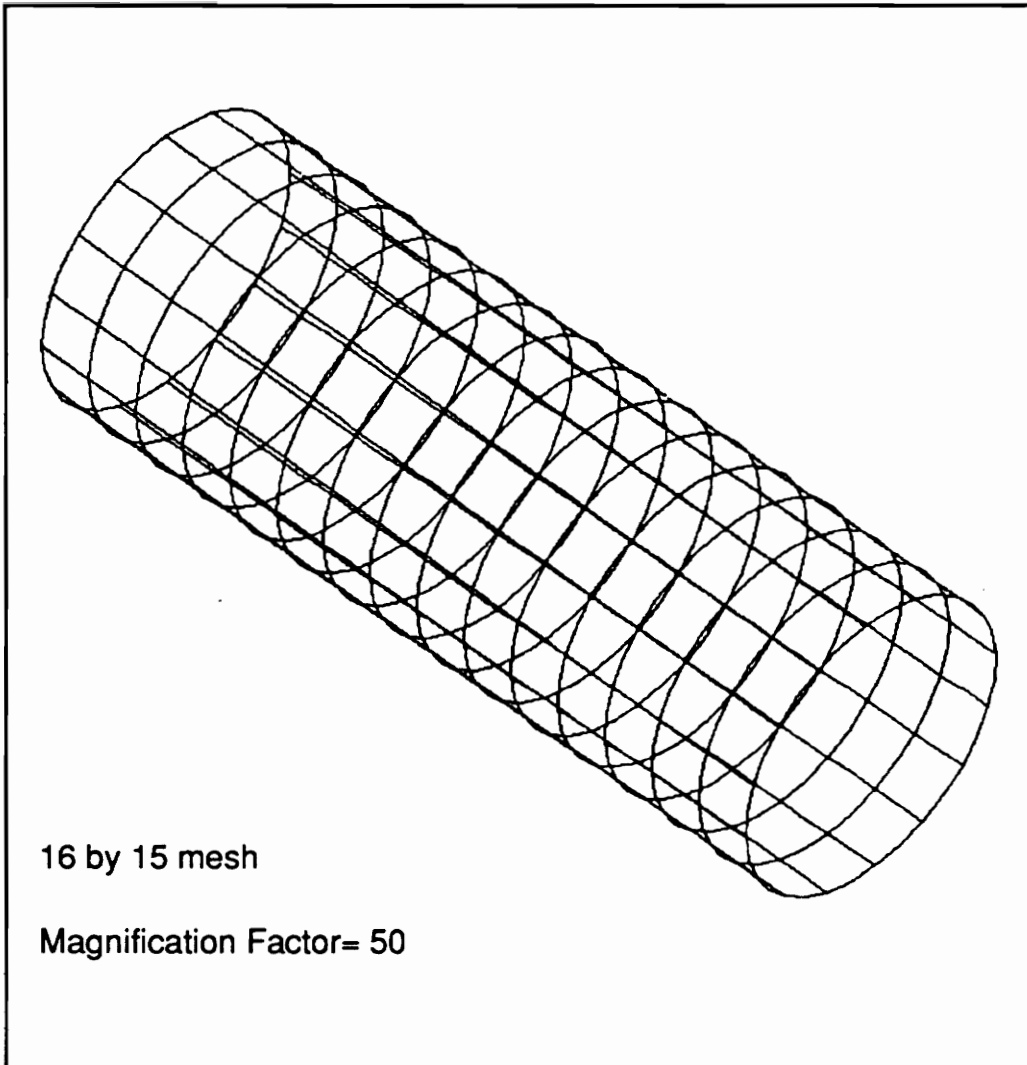


Figure 2.12 Deformed Shape for
the Closed Externally Stiffened Shell

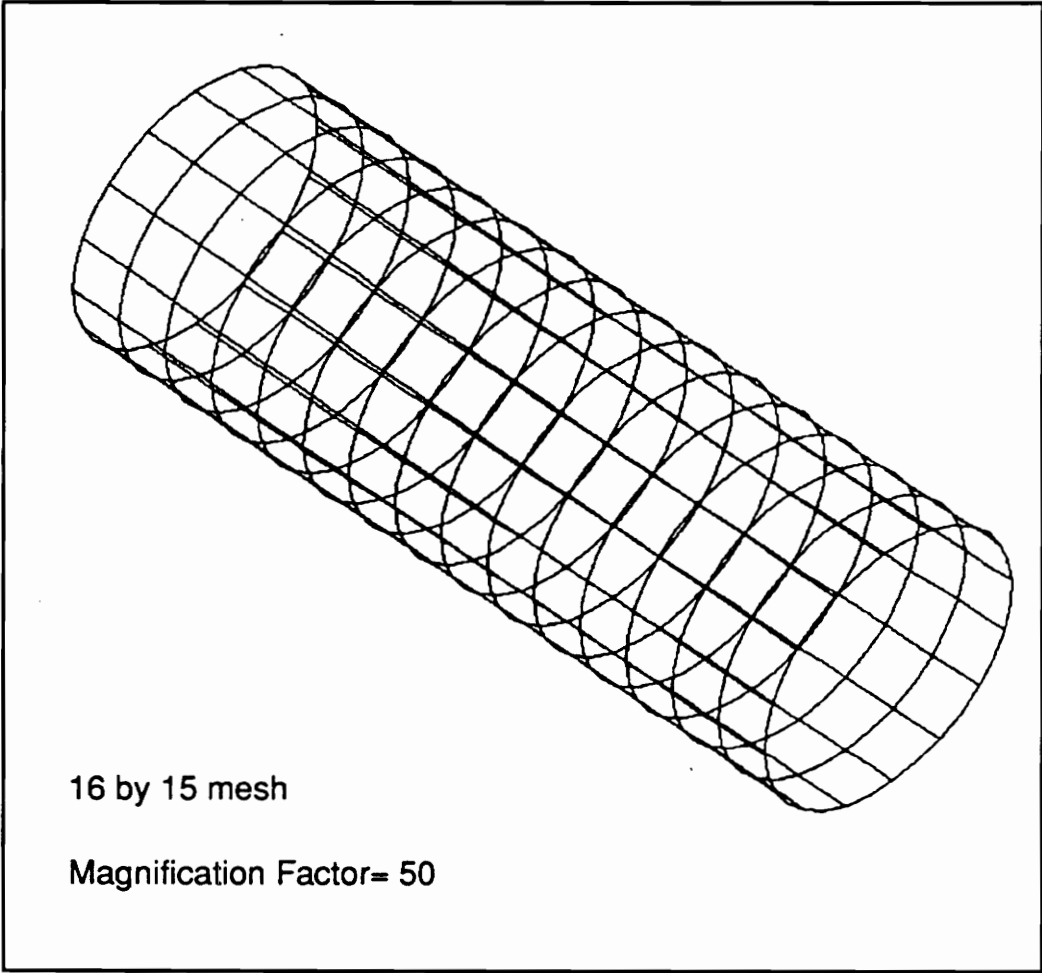


Figure 2.13 Deformed Shape for
the Closed Internally Stiffened Shell

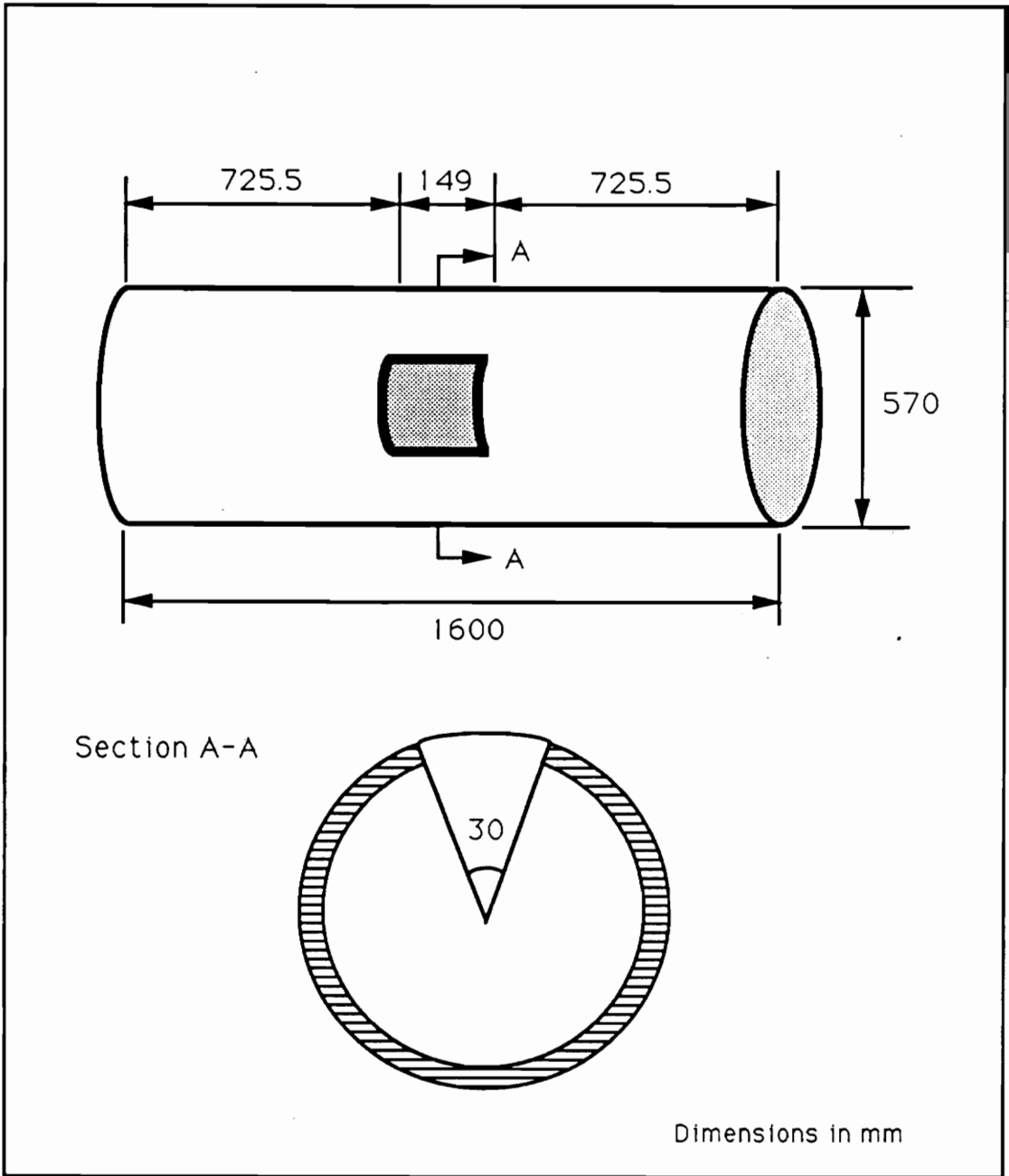


Figure 2.14 Circular Cylindrical Shell with a Small Cutout

2.5.1 Analysis of a Sandwich Shell with a Small Cutout

In this section, we analyze the deformation behavior of sandwich shell with a small cutout. This shell is subjected, as all the cases analyzed before, to a torque of 6637 kNmm. For this analysis, we used a mesh of 24 elements around the shell by 23 along the shell. In this way, the longitudinal and circumferential edges of the cutout have three and two elements, respectively, without complicating the generation of the mesh.

Apart from shear deformation, local bending appears in the vicinity of the hole because of the presence of the cutout. The two corners of the hole that are in compression sink into the shell, whereas the other two corners have a positive radial displacements, as shown in Figure 2.15. Thus, some local bending occurs around the hole. The circumferential displacement u_θ at the loaded end of this shell is 1.088 mm. The value of the torsional stiffness K_θ for this shell is 29460 Nm/deg. The maximum radial displacement obtained for this shell is .314 mm. The decrease in torsional stiffness from the value obtained for the closed shell is 4.5%.

2.5.2 Analysis of a $\pm 45^\circ$ Helically Stiffened Shell with a Small Cutout

We compare the deformation behaviors of two stiffened shells, one internally stiffened and another externally stiffened, with a small cutout. Both shells are subjected, as always, to the same torque of 6637 kNmm. For this analysis, we use for both shells the same mesh used for the sandwich shell.

The circumferential displacement u_θ at the loaded end is .269 mm for the internally stiffened shell and .246 mm for the externally stiffened shell. The value of the torsional stiffness K_θ for these shells is 122,400 Nm/deg for the internally stiffened shell and 133,800 Nm/deg for the externally stiffened shell. Again, as happened for the closed shells, the externally stiffened shell is stiffer by 9.3% than the internally stiffened shell. The shells with the small cutout are 14% less stiff than the closed shells.

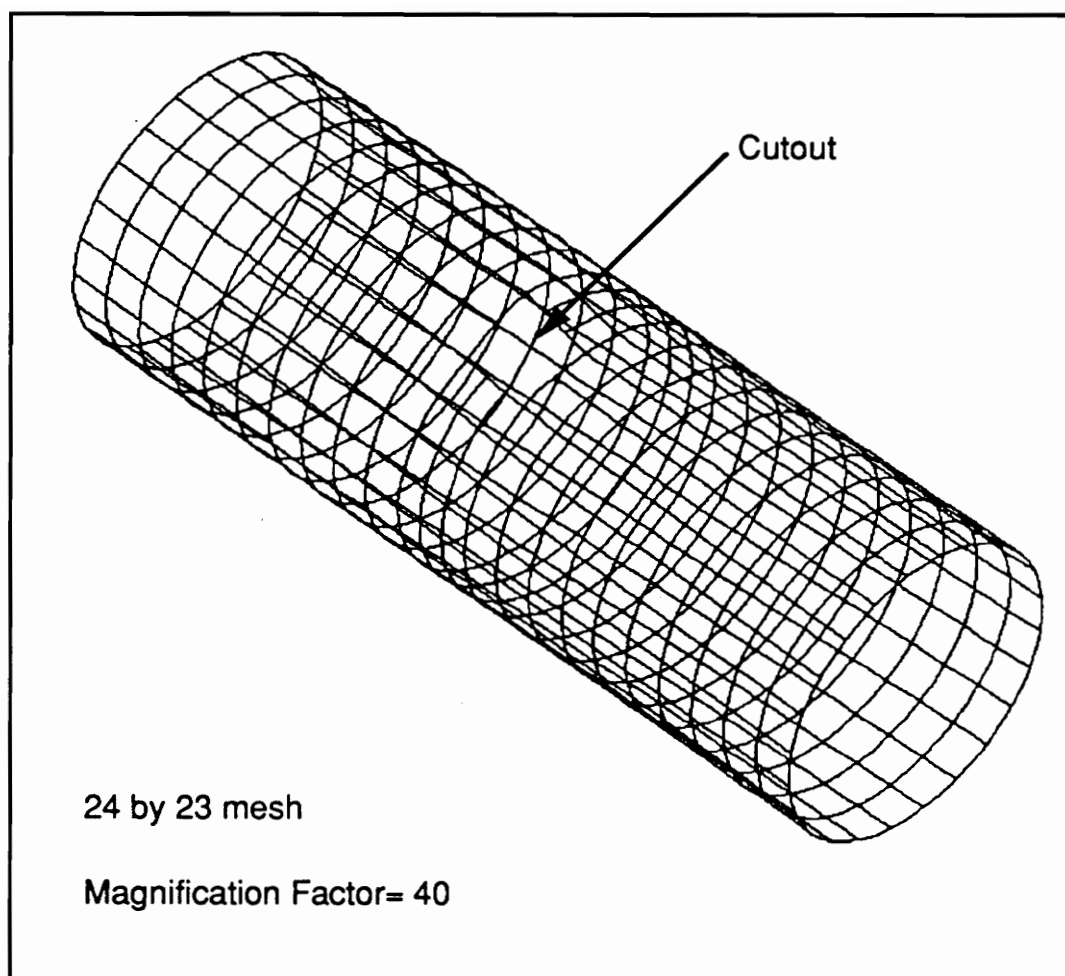


Figure 2.15 Deformed Shape for a Sandwich Shell with a Small Cutout

For this case, the radial displacements obtained for the internally stiffened shell are slightly smaller than for the externally stiffened shell. The maximum radial displacement is .183 mm for the internally stiffened shell and .192 mm for the externally stiffened shell. Both shells deform in the same way as we see in the 45° view of the deformed shapes of the internally and externally stiffened shells shown in Figures 2.16 and 2.17, respectively. Apart from the shear deformation of the hole, the two corners that are in compression sink into the shell, whereas the other two corners have positive radial displacements. Thus, some local bending occurs around the hole. This local bending could be the responsible for the decrease in torsional stiffness of these structures.

2.5.3 Comparison Between Results for the Sandwich and Stiffened Shells

From the results obtained for the sandwich and stiffened shells, we see that the stiffened shells are affected more than the sandwich by the presence of the small cutout. The stiffened shells decreased their torsional stiffness by 14% whereas the torsional stiffness of the sandwich shell decreased only 4.7%. This difference can be seen in Figure 2.18 where the circumferential displacement u_θ is plotted for all the nodes along a line on one side of the shell as indicated in the shell side-view drawing.

For the stiffened shells, we see that in the region of the cutout and in its surroundings there is a change in the slope as shown in Figure 2.18. It appears as if in this zone there is an increase in the torsional stiffness of the shells which can not be possible. On the contrary, we believe that this decrease in the slope means that in addition to the circumferential displacement now there is bending. Thus, this part of the shell has bending deformation along with the shear deformation. If we now look at the curves for the sandwich shell, we see the slope effect also decreases but is not as big as it is for the stiffened shells. Thus, we could say that, although there is also bending in the sandwich case, it is present in less amount than in the stiffened case. The reason for having less bending in the sandwich shell could be due to the fact that the bending stiffnesses D_{11} and D_{22} for the sandwich skin configuration are bigger than for the stiffener-skin configurations.

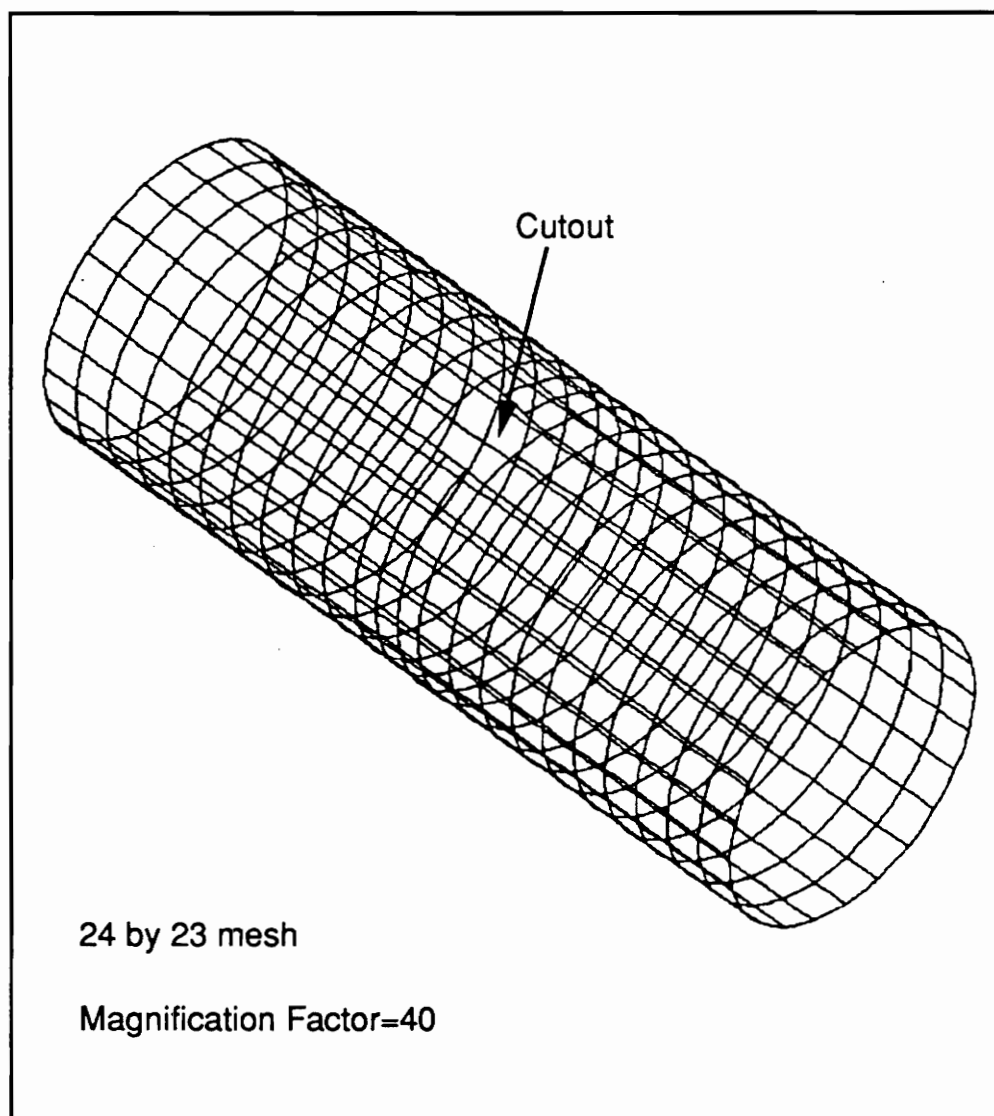


Figure 2.16 Deformed Shape for
the Internally Stiffened Shell with a Small Cutout

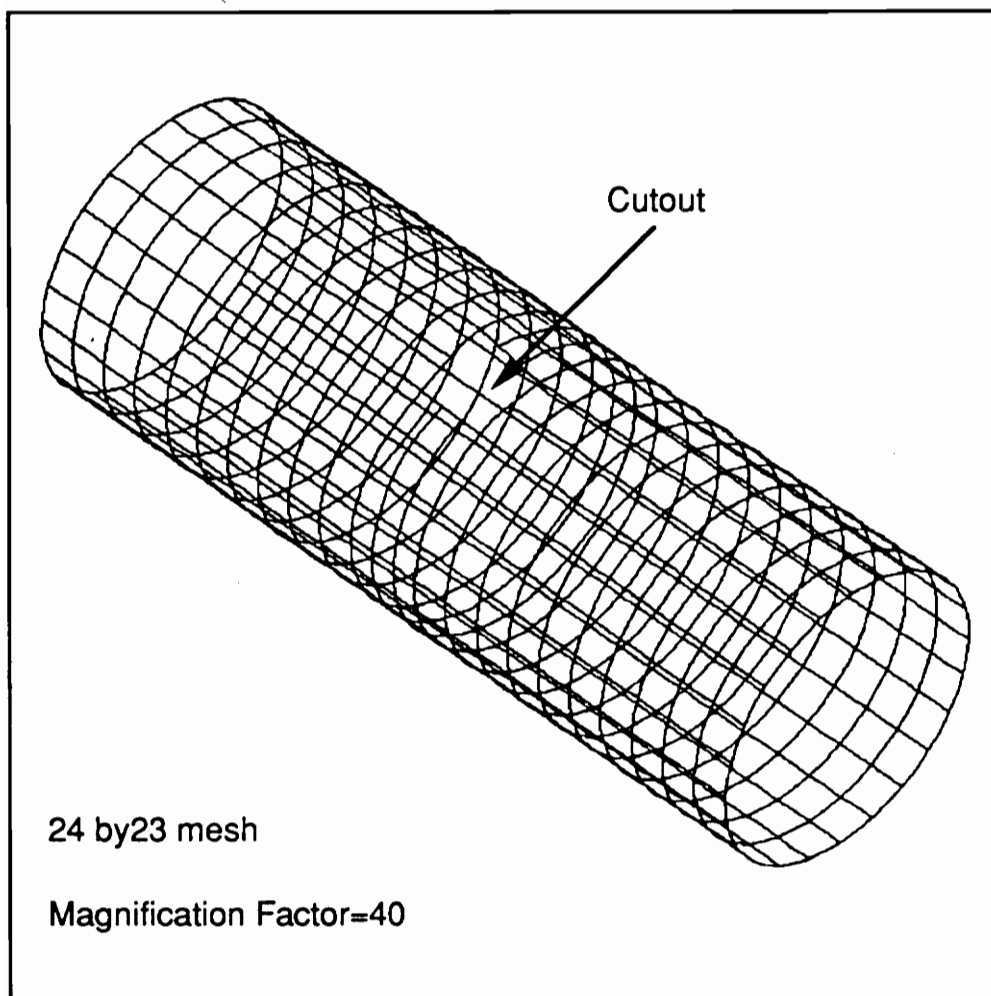


Figure 2.17 Deformed Shape for
the Externally Stiffened Shell with a Small Cutout

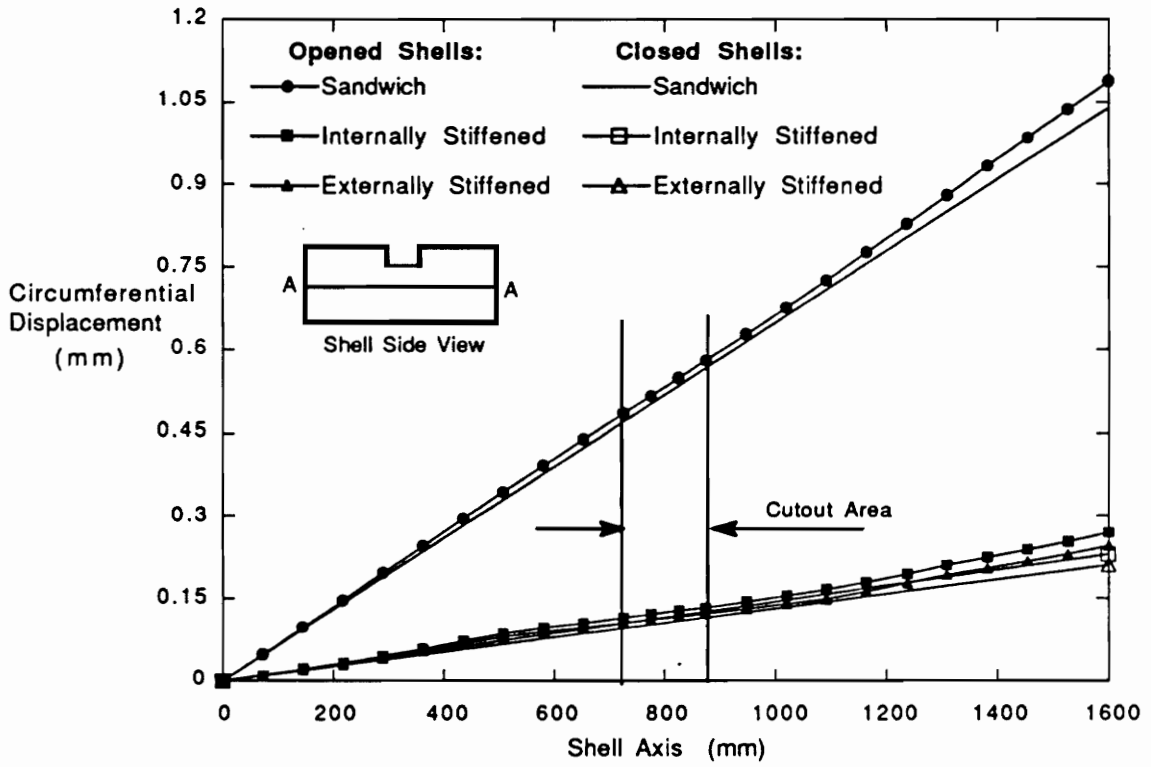


Figure 2.18 Circumferential Displacement Along Line A-A for Shells with a Small Cutout

2.6 DEFORMATION BEHAVIOR OF SANDWICH AND $\pm 45^\circ$ HELICALLY STIFFENED CIRCULAR CYLINDRICAL SHELLS WITH A REAL-COCKPIT-SIZE CUTOUT

We analyze the case of a shell with a cutout of approximately the same size as the opening for the cockpit in Formula racing cars as shown in Figure 2.19. Also, in this section, we will investigate some ways of stiffening the vicinity of the hole in order to improve the overall torsional stiffness of the structure. The geometry of the shells is the same as that used in the previous sections, except for the size of the cutout. We apply to these shells the same loading and boundary conditions that we used for the closed shells. For all cases, we use a mesh of 24 elements around and 22 elements along the shell. We will measure the circumferential displacements at the nodes located along the line A-A shown in Figure 2.20. We measure the circumferential displacement at these locations because that is where the displacements are measured in the real cars.

2.6.1 Analysis of a Sandwich Shell with a Cockpit-Size Cutout

We analyze the deformation behavior of a sandwich shell with a cockpit-size cutout. The results for the circumferential displacement u_θ at the loaded end node located on one side of the shell as indicated in Figure 2.20 is 2.959 mm. The value of the torsional stiffness K_θ for this shell is 10850 Nm/deg. The maximum radial displacement obtained for this shell is 5.3 mm. The decrease in torsional stiffness from the closed sandwich shell is 65%.

In the 45° view of the deformed shape of this shell, shown in Figure 2.21, we see that again the corners of the hole that are in compression tend to sink into the shell, whereas the other two corners which are in tension have positive radial displacement. Also, we see extensive bending around the hole.

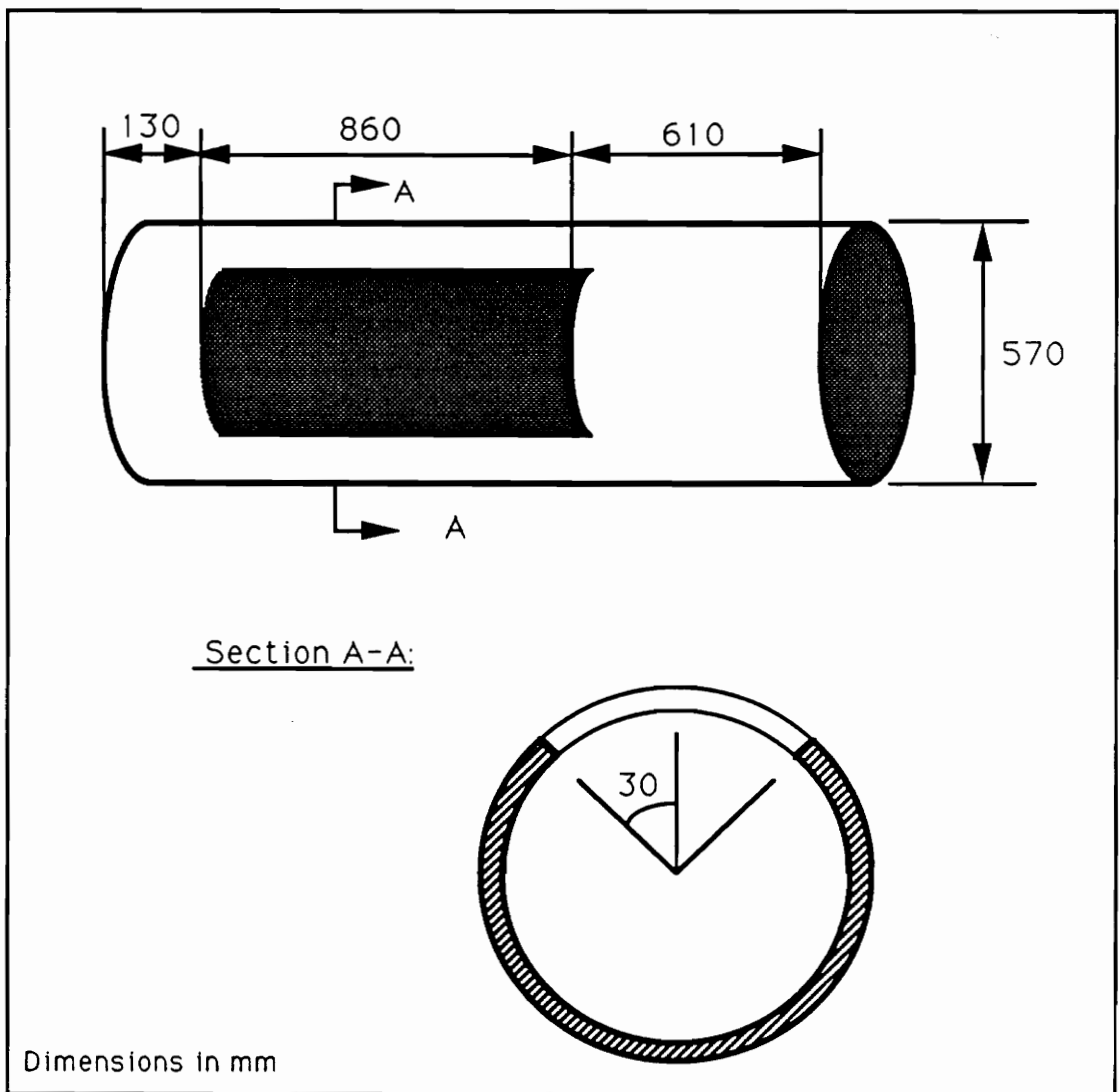


Figure 2.19 Circular Cylindrical Shell with a Cockpit-Size Cutout

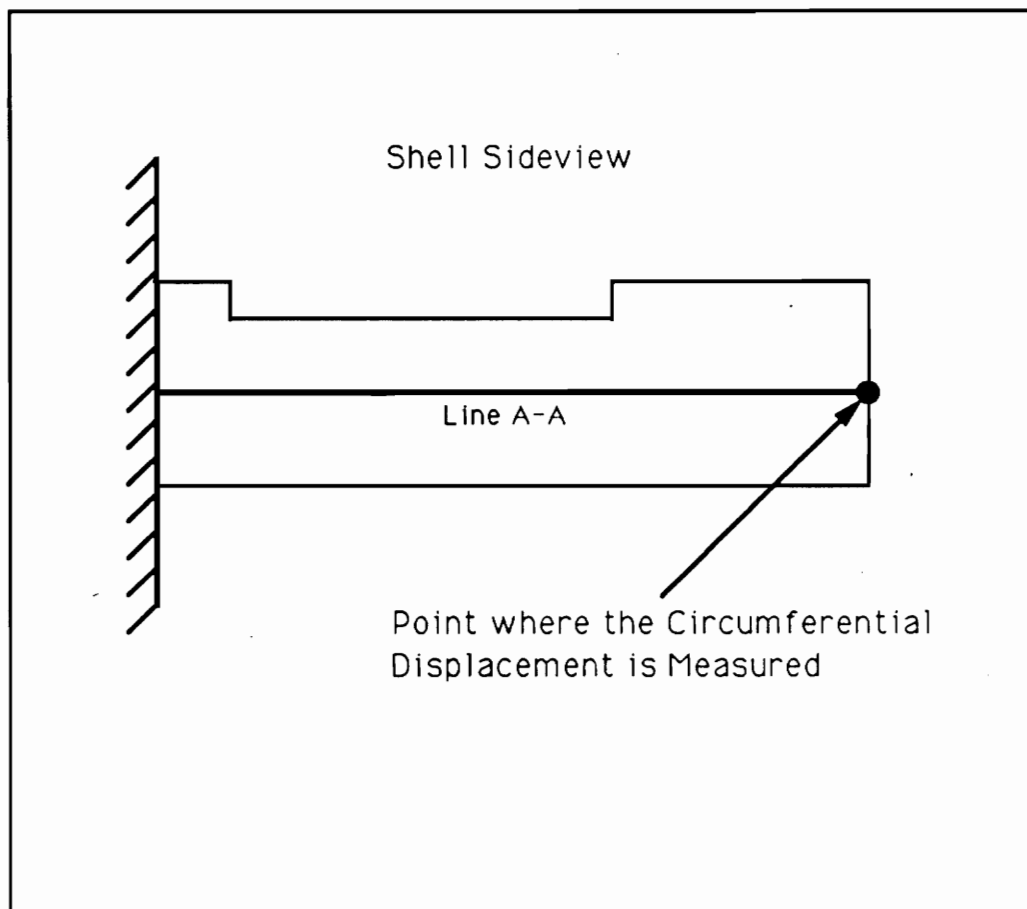


Figure 2.20 Point where the Circumferential Displacement is Measured

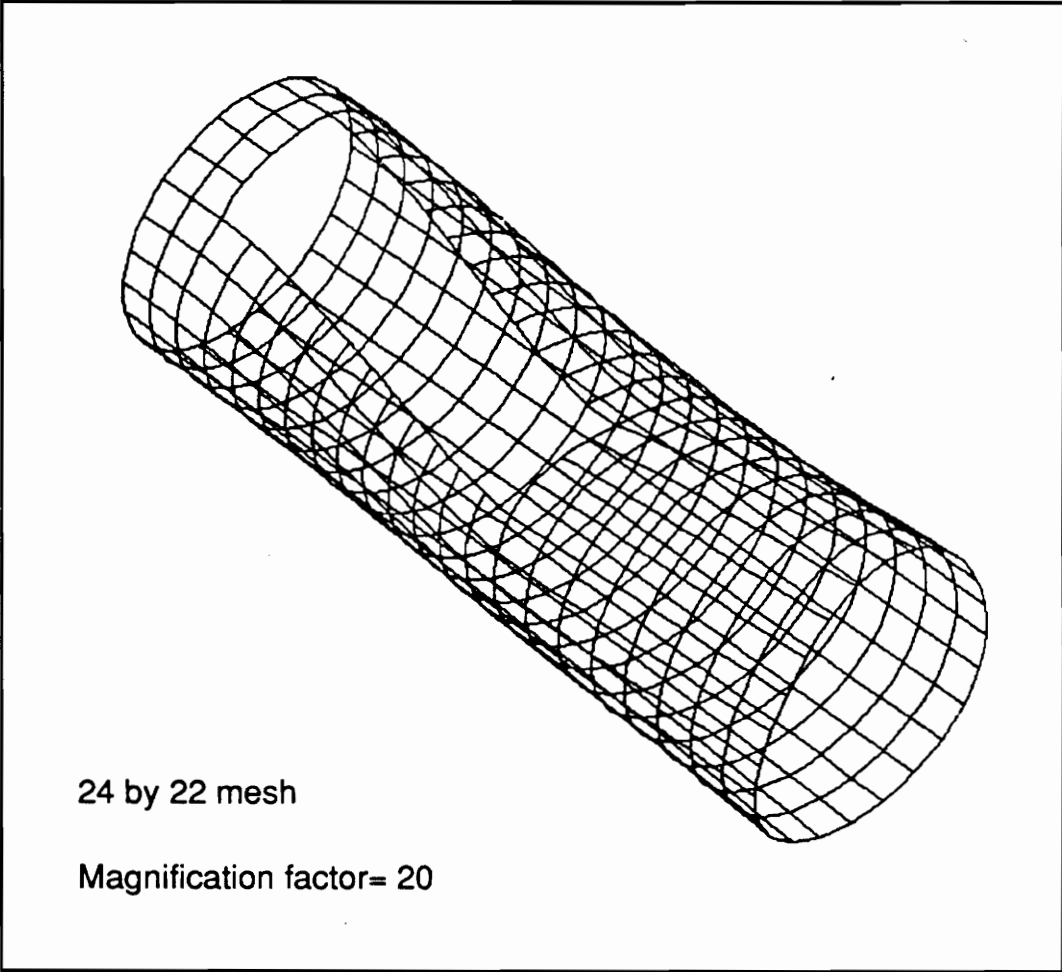


Figure 2.21 Deformed Shape for the Sandwich Shell with a Cockpit-Size Cutout

2.6.2 Analysis of a $\pm 45^\circ$ Helically Stiffened Shell with a Cockpit-Size Cutout

In this section, we compare the deformation behaviors of three stiffened shells, an internally stiffened, an externally stiffened, and a shell with the $\bar{B}_{ij}$ terms equal to zero with a cutout of the size of the opening for the cockpit. We analyze the case with the $\bar{B}_{ij}$ terms equal to zero in order to see how the $\bar{B}_{ij}$ coupling terms affect the deformation behavior for the shell.

The results for the circumferential displacement, maximum radial displacement, angle of twist, torsional stiffness, and stiffness decrease from the case of the closed shells for both stiffened shells and for the shell with no coupling are shown in Table 2.8. The value of u_θ is measured at the loaded end node indicated in Figure 2.20. The torsional stiffness obtained for the externally stiffened shell is 19% bigger than for the internally stiffened shell. Also, both shells are affected in the same amount by the hole because their respective stiffness are decreased by 92% compared to the values for the closed shells.

The circumferential displacement u_θ is plotted for all the nodes along a line on one side of the shells as indicated in the shell side-view drawing in Figure 2.22. Both stiffened shells have more bending and torsional deformations than the uncoupled fictitious case, as we see comparing the deformed shapes of the internally stiffened, externally stiffened, and of the shell with no coupling (Figures 2.23, 2.24, and 2.25, respectively). Thus, the coupling does not make the shells any stiffer. On the contrary, we get a lower stiffness when the bending-extension coupling is present. Also, we see that the externally stiffened shell is less affected than the internally stiffened shell. The externally stiffened shell has a torsional deformation behavior closer to the uncoupled shell than the internally stiffened shell, as shown in Figure 2.22.

Table 2.8 Results for the Stiffened Shells with a Cockpit-Size Cutout

Behavior	Internally Stiffened Shell	Externally Stiffened Shell	Shell without Coupling
U_{θ} (mm)	3.17	2.65	2.49
θ (deg)	.639	.535	.502
K_{θ} (Nm/deg)	10,400	12,400	13,200
ΔK_{θ} (%)	-92.7	-92	—
U_r (mm)	6.74	6.38	5.22

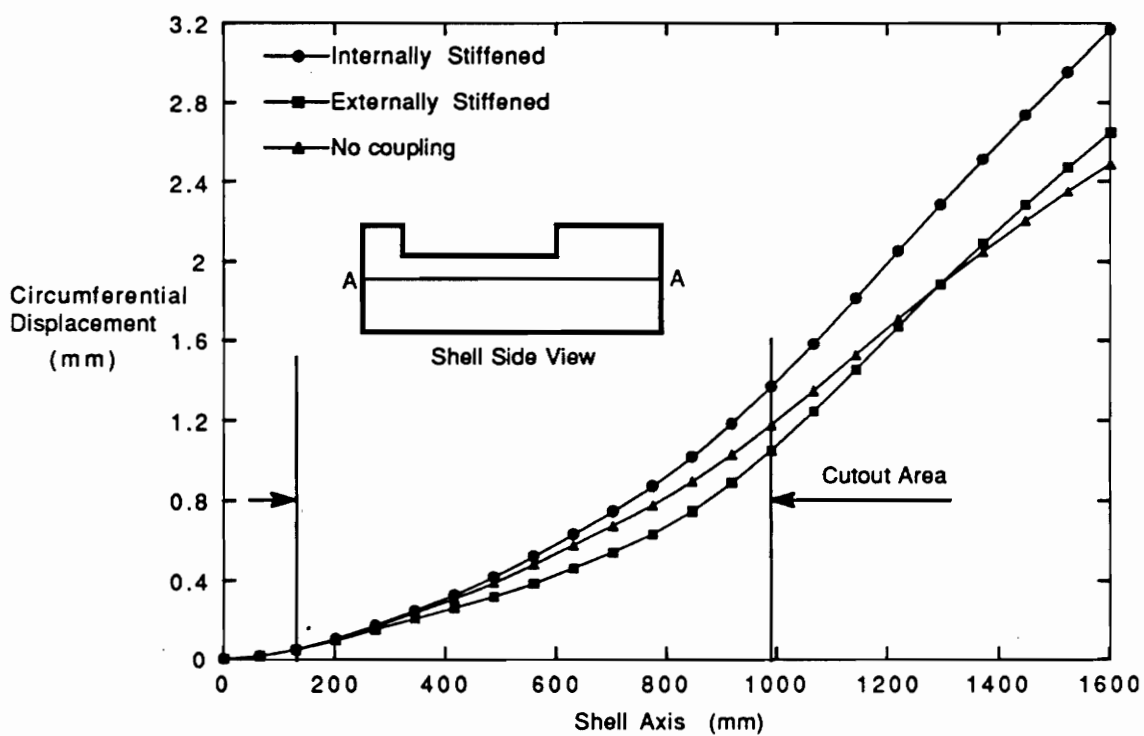


Figure 2.22 Circumferential Displacement Along Line A-A for Stiffened Shells with a Cockpit-Size Cutout

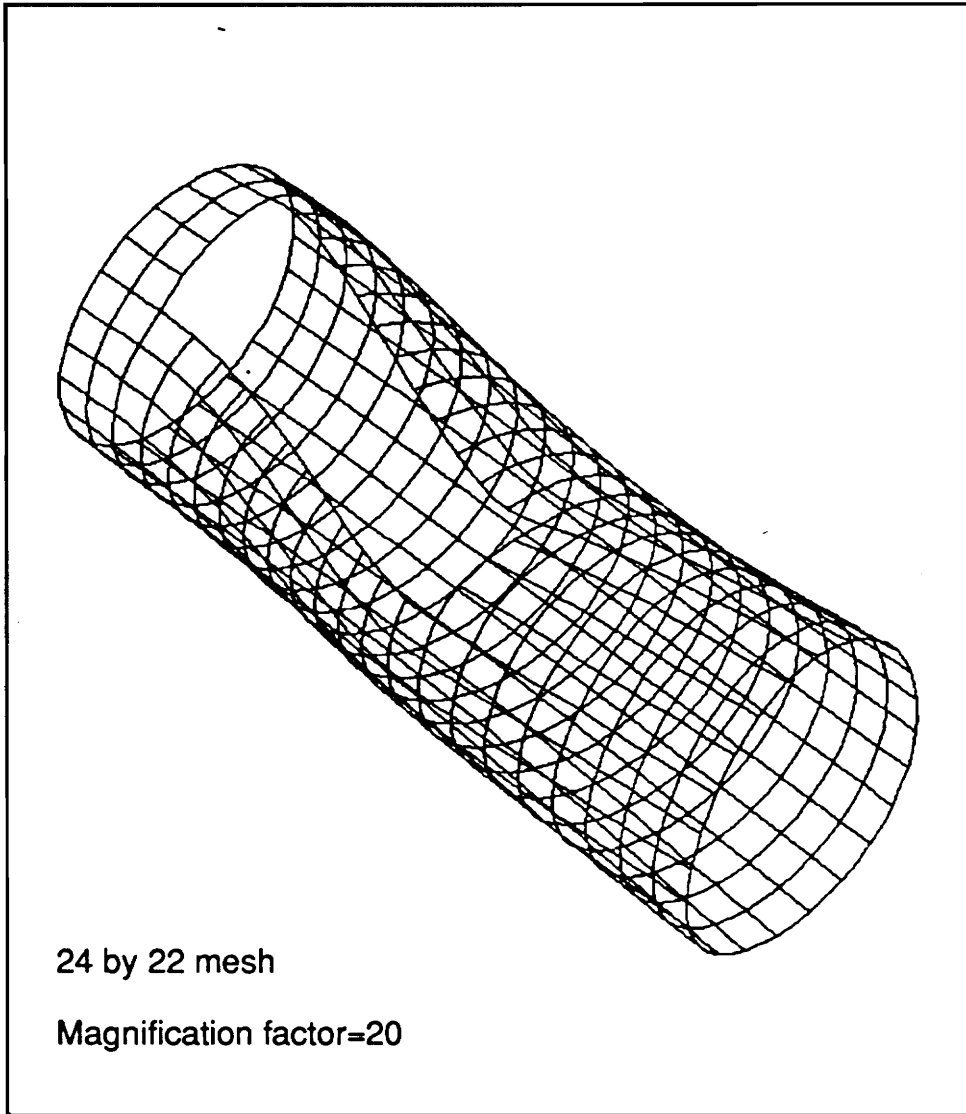


Figure 2.23 Deformed Shape for
the Internally Stiffened Shell with
a Cockpit-Size Cutout

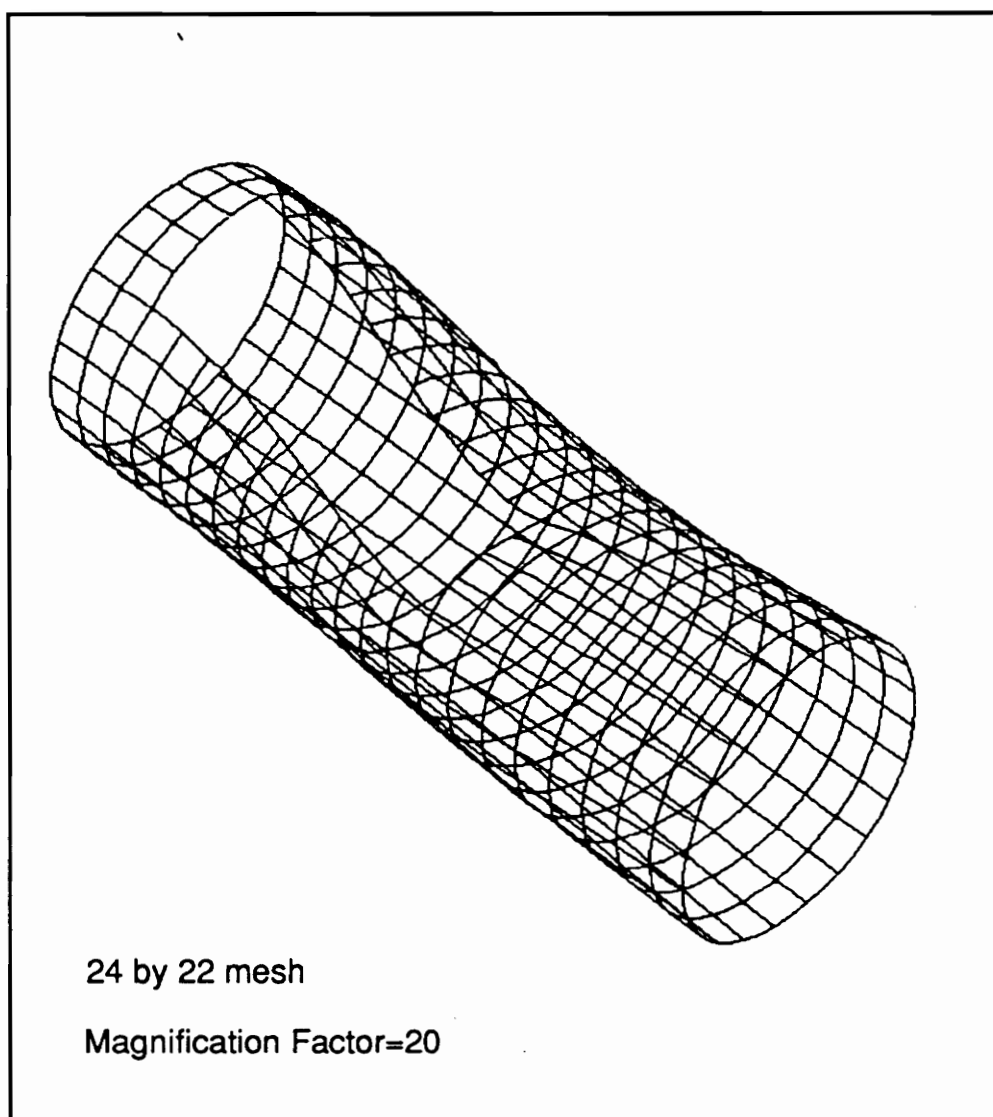


Figure 2.24 Deformed Shape for
the Externally Stiffened Shell with
a Cockpit-Size Cutout

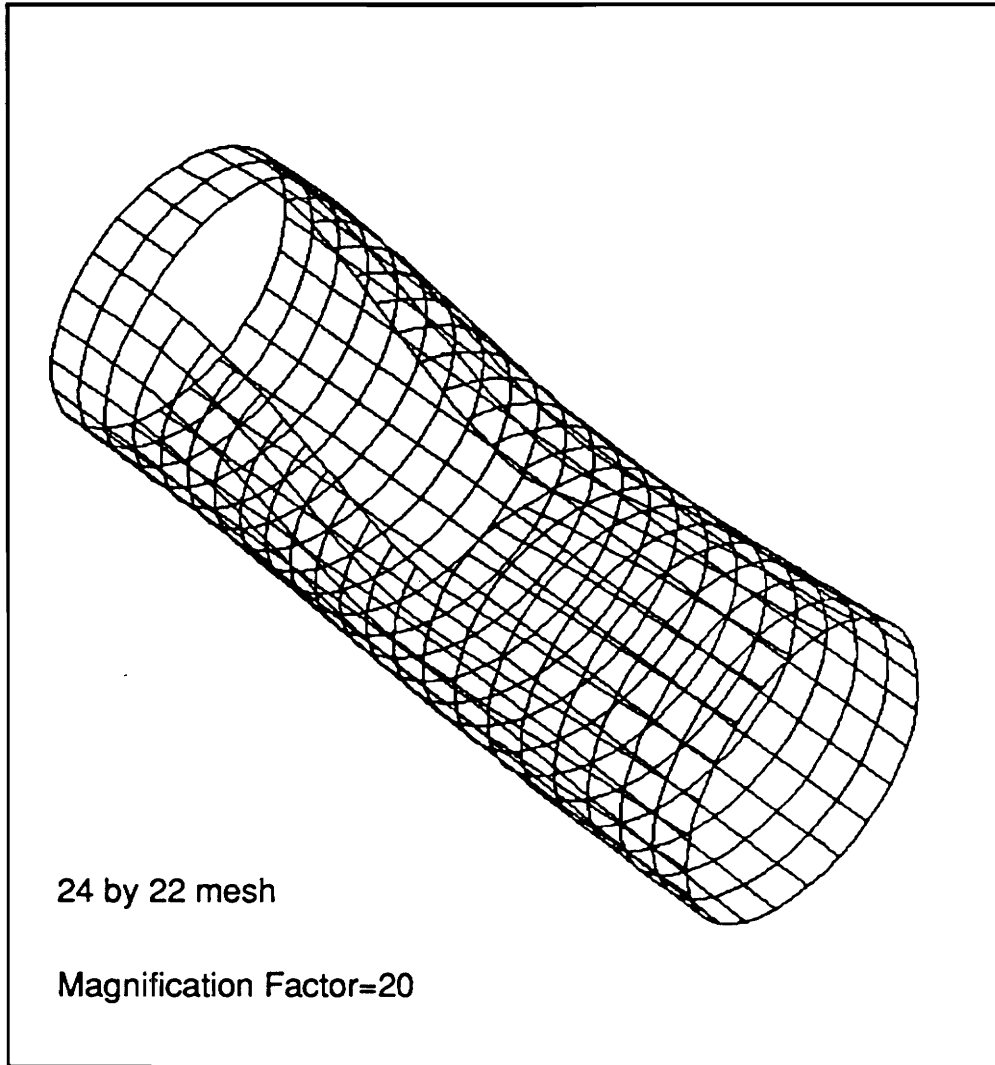


Figure 2.25 Deformed Shape for
the Stiffened Shell with a Cockpit-Size Cutout
with no Coupling

2.6.3 Comparison Between Results for the Sandwich and Stiffened Shells

The torsional stiffness for the shells with the big hole diminished by 64% for the sandwich shell and around 92% for the stiffened shells compared with the values obtained for the closed shells. Also, the gap between the torsional stiffness for the stiffened and sandwich shells diminished from around five times for the closed shells to only 1.14 times for the case of the externally stiffened shell. This decrease can be seen in Figure 2.26 where the circumferential displacement u_θ for the open and closed shells is plotted for all the nodes along line A-A as indicated in the shell side-view drawing. The internally stiffened shell has a torsional stiffness even lower than that of the sandwich shell.

This decrease in the torsional stiffness for the stiffened shells is caused by the fact that now in the region of the hole other stiffnesses apart from the in-plane shear stiffness gain importance in governing the torsional behavior of the shell. Thus, for the open stiffened shells, possessing a bigger value of A_{66} than the sandwich shell is not the same advantage as it was for the closed shells.

In the following sections of this Chapter, we will try to improve the torsional stiffness of both shells. From the results obtained in the last two sections, it seems that the way to improve the torsional stiffness of the stiffened shell is to make the region of the hole deform in in-plane shear instead of bending. Thus, we will try as solutions putting some deep ring stiffeners to diminish the radial displacement of the corners of the hole. These ring stiffeners will be called bulkheads from now on because they simulate the presence of the bulkheads in the real cars. Then, we will add longitudinal stiffeners on the longitudinal edges of the hole to diminish the bending deformations.

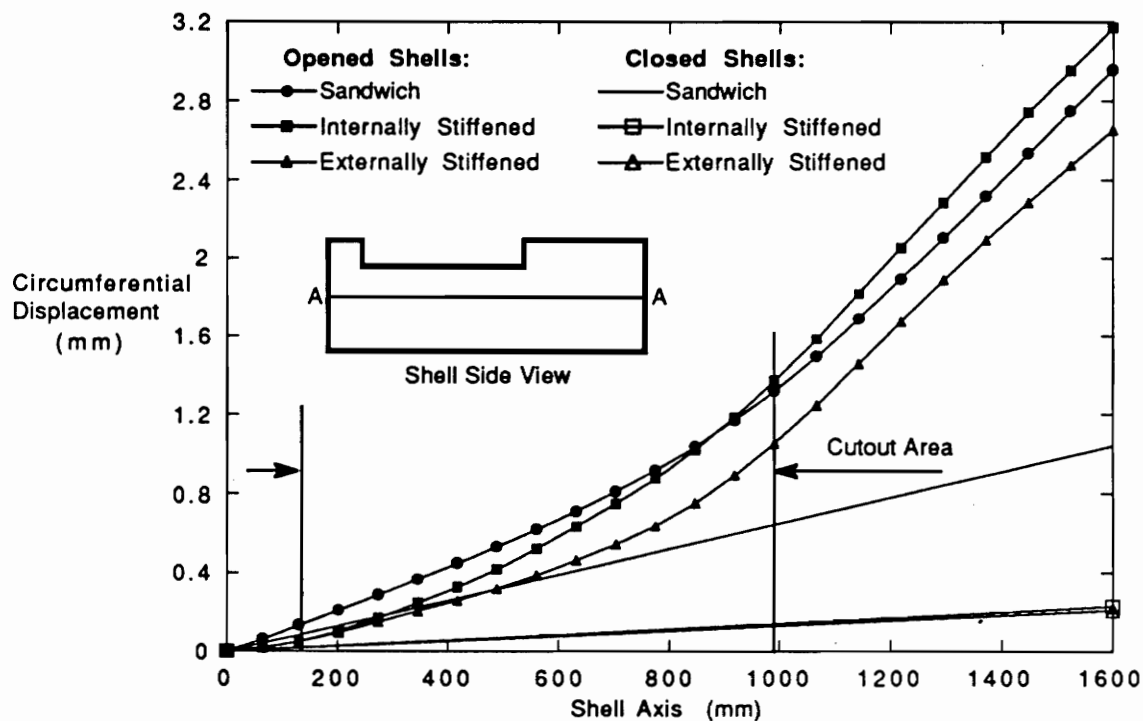


Figure 2.26 Circumferential Displacement Along Line A-A for Shells with a Cockpit-Size Cutout

2.6.4 Deformation Behavior of Sandwich and $\pm 45^\circ$ Helically Stiffened Circular Cylindrical Shells with a Real-Cockpit-Size Cutout and Bulkheads

The front suspension members are attached to two bulkheads as shown in Figures 1.3 and 1.4 which are different drawings of the structural shell of a Formula racing car. These bulkheads, or ring-type stiffeners, receive all the loads coming from the front suspension members. Also, we see that another ring is located in the front of the cockpit opening, and a wall separates the fuel tank from the back of the driver. These bulkheads are deep ring-type stiffeners that are usually made of sandwich construction. Also, the wall separating the back of the driver from the fuel tank is a sandwich wall. In this section, we study the behavior of our shells with the big cockpit-size opening with ring stiffeners in the positions shown in Figure 2.27. Both the sandwich and the stiffened shells have the same external dimensions and internal diameter of the bulkheads or ring stiffeners of 453 mm. These stiffeners are made with sandwich construction, and their lay-up is the same for stiffened and sandwich shells. The lay-up of these sandwich bulkheads is the same as used for the sandwich shells shown in Table 2.4.

For the finite element modeling of these bulkheads, we use the same type of shell elements that we use for modeling of the shells, i.e., the ABAQUS S8R element. The meshes used are the same as those used for the cases without bulkheads where the bulkheads are modeled with a row of 24 elements around the shell.

2.6.4.1 Analysis of a Sandwich Shell with a Cockpit-Size Cutout and Bulkheads

We analyze the deformation behavior of a sandwich shell with a cockpit-size cutout and bulkheads. The circumferential displacement u_θ at the loaded end node located on one side of the shell as indicated in Figure 2.20 is 2.4 mm. The value of the torsional stiffness K_θ for this shell is 13,400 Nm/deg. The radial displacement varies along the shell, and the maximum value obtained is 3.4 mm. By adding the bulkheads to the shell, we make the shell 23.7% stiffer than the shell without bulkheads. The shell deforms as the shell without bulkheads, but the deformations are smaller (see Figures 2.28 and 2.21).

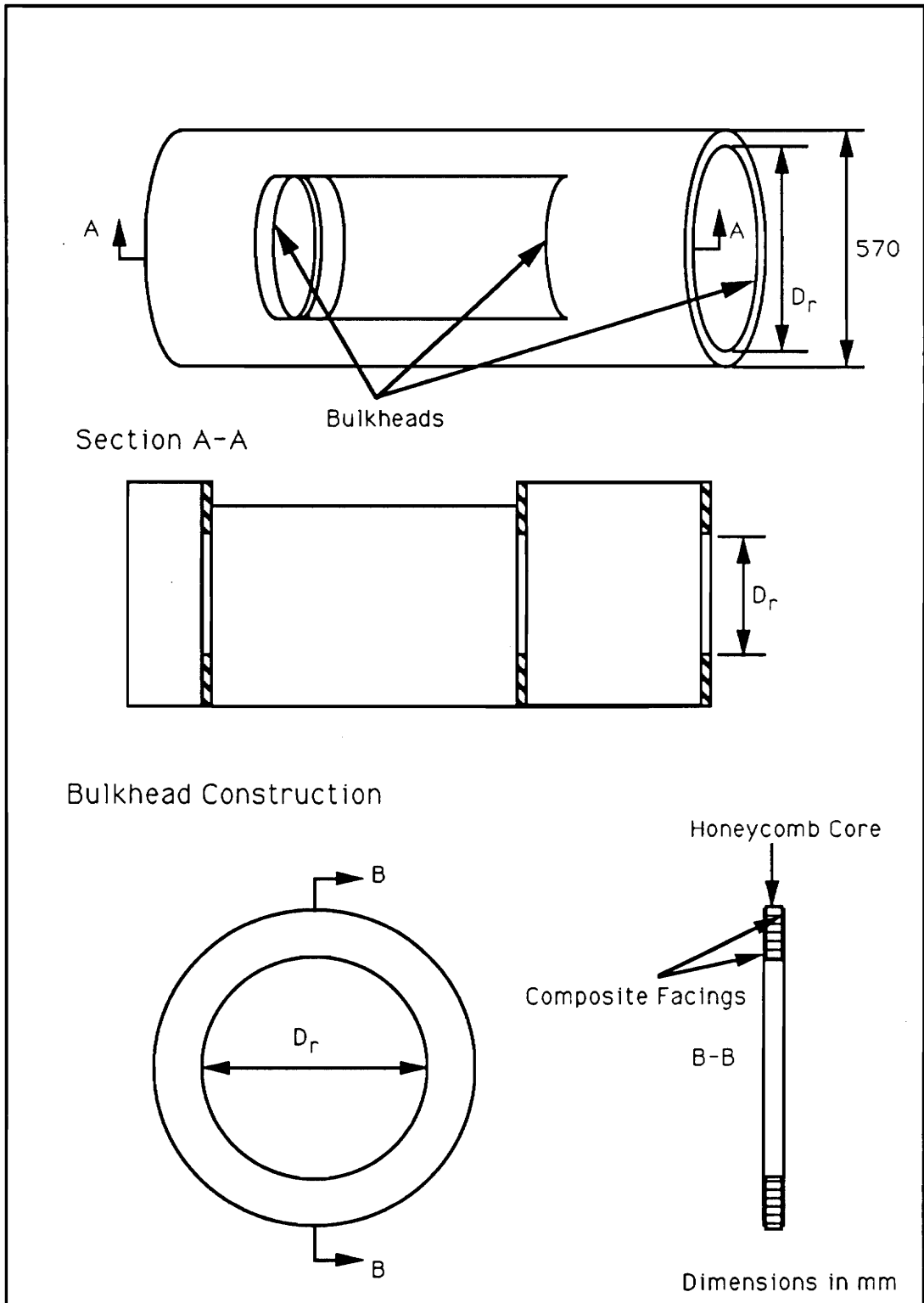


Figure 2.27 Bulkhead Locations and Construction

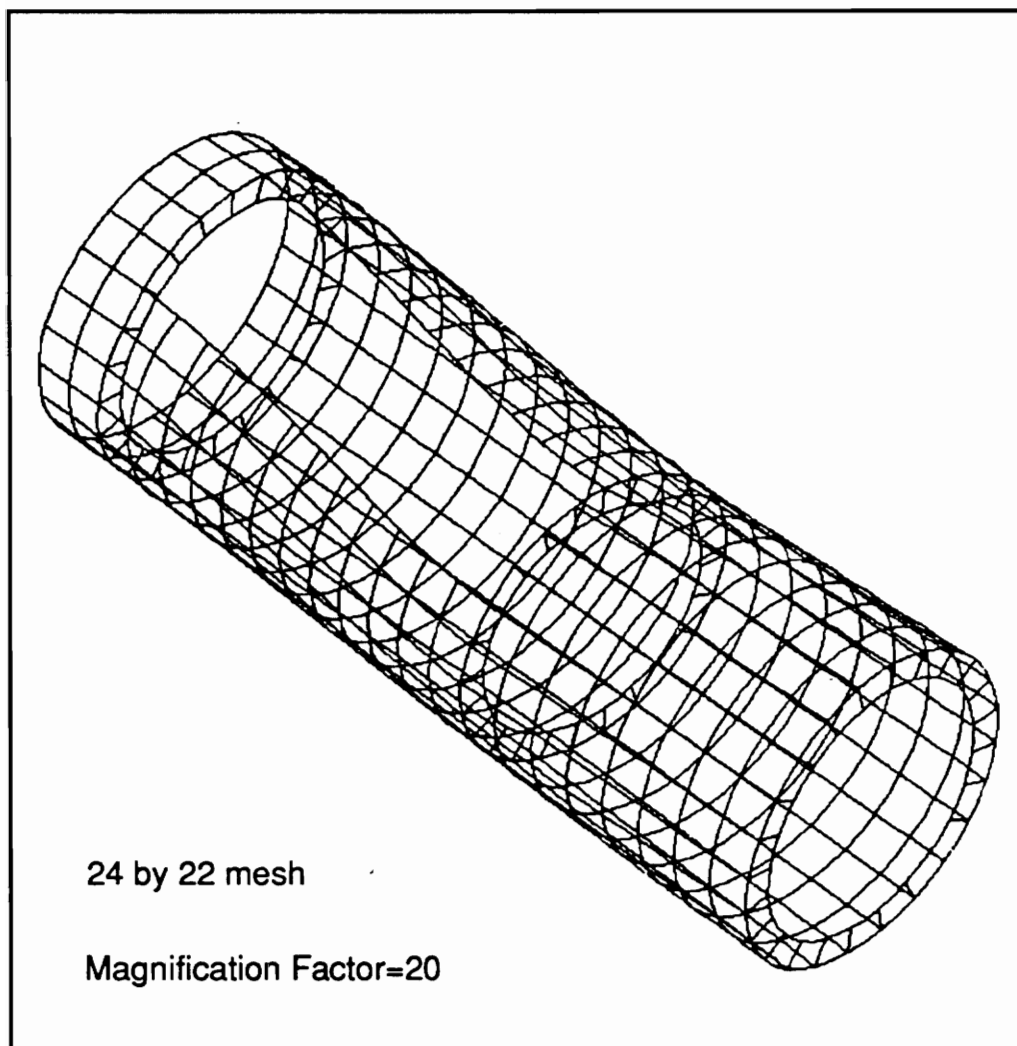


Figure 2.28 Deformed Shape for the Sandwich Shell with a Cockpit-Size Cutout and Bulkheads

2.6.4.2 Analysis of a $\pm 45^\circ$ Helically Stiffened Shell with a Cockpit-Size Cutout and Bulkheads

We analyze the deformation behavior of externally and internally stiffened shells with a big cutout and bulkheads. The circumferential displacement of the loaded end for these two shells, the torsional stiffness, and the maximum radial displacement are shown in Table 2.9. The externally stiffened shell is 18.3% stiffer in torsion than the internally stiffened shell. Also, comparing results with the unreinforced case (Table 2.8), we see that the improvement in stiffness is around 45% for both stiffened shells. The pattern of deformation is the same as for the unreinforced cases, but the deformations are much smaller, as shown in Figures 2.29 and 2.30. The maximum radial displacements for the reinforced cases are around 50% smaller than for the unreinforced cases. This decrease in radial displacement gives us a measure of the lower bending deformation that we obtain when we use bulkheads.

2.6.4.3 Comparison between the Sandwich and Stiffened Shell Results

To compare the torsion behavior of the sandwich and stiffened shells, we plot the circumferential displacement u_θ for all the nodes along a line on one side of the shells as indicated in the shell side view in Figure 2.31. The curve obtained for the sandwich shell is much straighter than for the stiffened shells. This difference between the curves could be due to the fact that the front portion of the stiffened shells now has the approximately the same torsional stiffness as the fixed-free closed shell whereas the open portion has less torsional stiffness. On the other hand, the torsional stiffness of the fixed-free closed sandwich shell is five times smaller than the torsional stiffness of the stiffened shells, and, thus, the behavior of the sandwich shell is smoother.

Table 2.9 Results for Stiffened Shells with
a Cockpit-Size Cutout and Bulkheads

Behavior	Internally Stiffened Shell	Externally Stiffened Shell
U_{θ} (mm)	2.17	1.83
K_{θ} (Nm/deg)	15200	18000
U_r (mm)	4.54	4.1

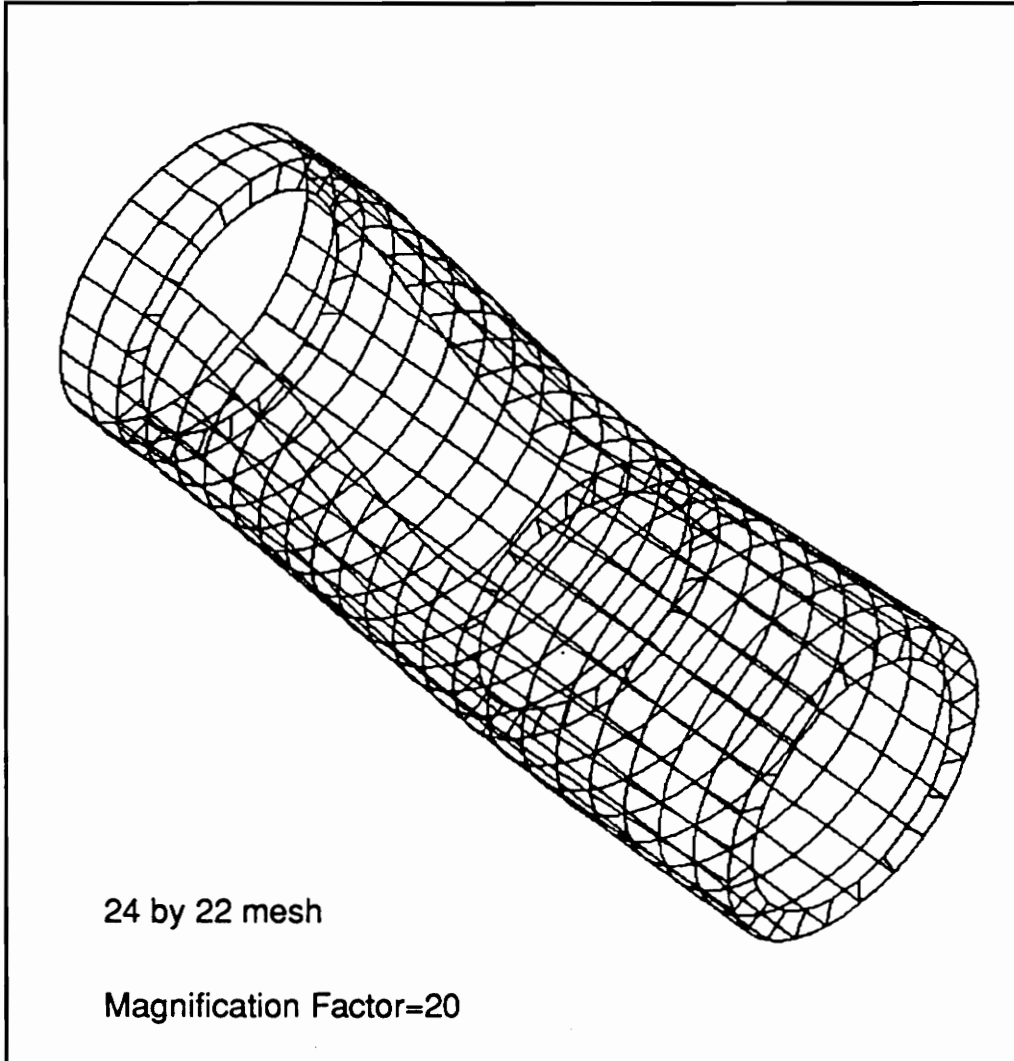


Figure 2.29 Deformed Shape for
the Internally Stiffened Shell with
a Cockpit-Size Cutout and Bulkheads

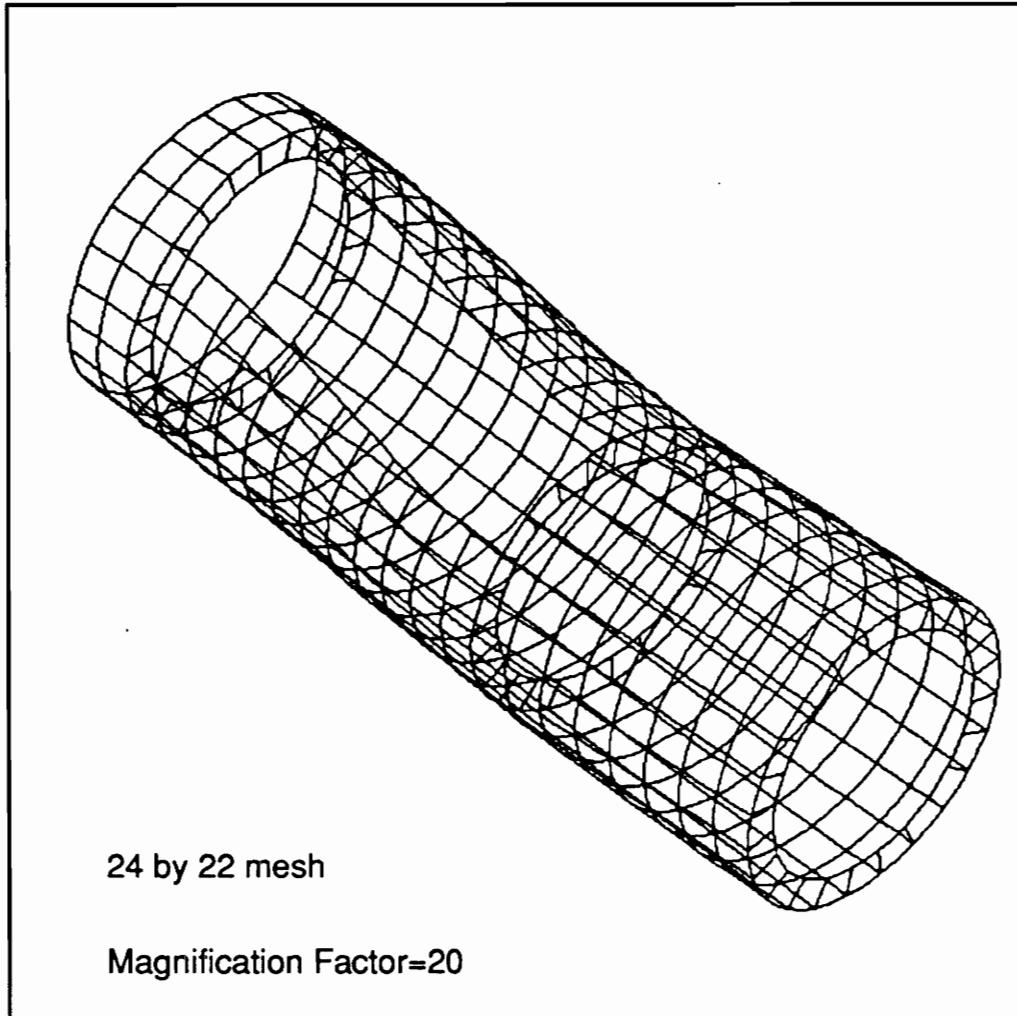


Figure 2.30 Deformed Shape for
the Externally Stiffened Shell with
a Cockpit-Size Cutout and Bulkheads

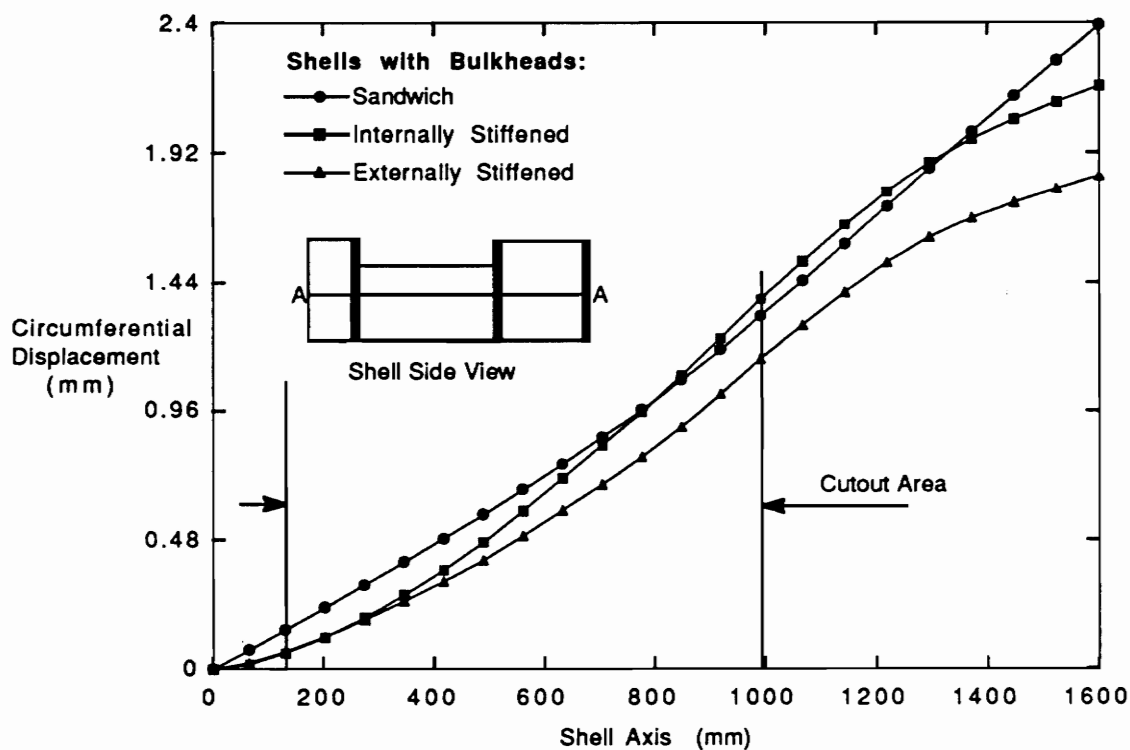


Figure 2.31 Circumferential Displacement Along Line A-A for Shells with a Cockpit-Size Cutout and Bulkheads

The externally stiffened shell is 34% stiffer than the sandwich shell, whereas the internally stiffened shell is only 13% stiffer than the sandwich shell. The gap in torsional stiffness between the sandwich and the externally stiffened shells has increased from 14% in the unreinforced case to 34% in the reinforced case.

The bulkheads have helped the stiffened shells more than the sandwich shell because the torsional stiffness of the stiffened shells has increased by 45% and only by 23% for the sandwich shell. This difference in stiffness increase could be due to the fact that, with the bulkheads, the walls of the shells are working more in shear than they did in the unreinforced case. The stiffener-skin configuration has a much bigger in-plane shear stiffness than the sandwich skin configuration. Also, the bulkheads make the shells work more in shear than in bending. Thus, the stiffened shell takes more advantage of the bulkhead presence than the sandwich shell.

2.6.5 Deformation Behavior of Sandwich and ±45° Helically Stiffened Circular Cylindrical Shells with a Cockpit-Size Cutout, Bulkheads, and Stiffening of the Longitudinal Edges of the Cutout

In this section, we analyse the behavior of shells with a cockpit-size cutout and bulkheads as in the previous section, but now we also stiffen the longitudinal edges of the cutout. The location and shape of these stiffeners, which from now on will be called longitudinal reinforcements, are shown in Figure 2.32. The longitudinal reinforcements are square cross-section stiffeners (25 mm by 25 mm) that run between the bulkheads located at the front and rear edges of the cutout. The middle plane of these reinforcements is located at the midplane of the skin of the shell. The longitudinal reinforcements are made of unidirectional AS-3501 graphite-epoxy tape with the fiber axis coinciding with the longitudinal axis of the reinforcements.

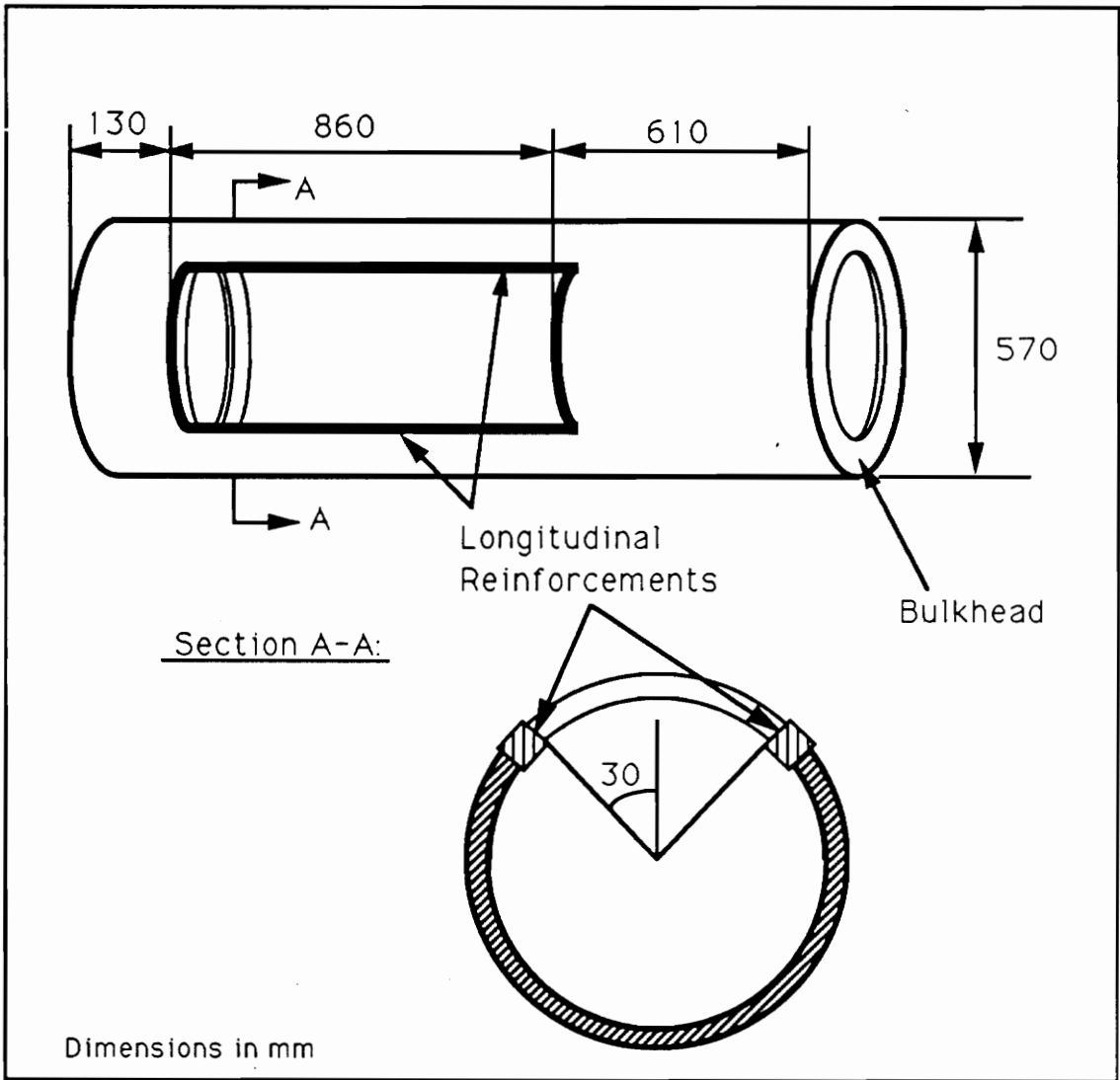


Figure 2.32 Circular Cylindrical Shell with a Cockpit-Size-Cutout, Bulkheads, and Longitudinal Reinforcements

For the finite element modeling of these reinforcements, 12 ABAQUS B32 beam elements per reinforcement are used. The ABAQUS B32 beam element has three nodes, quadratic interpolation, and six degrees of freedom per node. As output for this element, we get the axial force, two shear forces, two bending moments, and the twisting moment about the beam axis. To model the shell and bulkheads, we used the same 24 by 22 element mesh used for the shells with bulkheads only.

2.6.5.1 Analysis of a Sandwich Shell with a Cockpit-Size Cutout with Bulkheads and Longitudinal Reinforcements

We analyze the deformation behavior of a sandwich shell with a cockpit-size cutout, bulkheads, and longitudinal reinforcements. The circumferential displacement u_θ at the loaded end node as shown in Figure 2.20 is 1.9 mm. The value of the torsional stiffness K_θ for this shell is 17,240 Nm/deg which is 29% bigger than the stiffness obtained for the shell with bulkheads only. The maximum radial displacement obtained for this shell is 2.2 mm which is 34% smaller than the displacement obtained for the shell with bulkheads only.

The bending in the longitudinal edges of the cutout has decreased greatly with the presence of the reinforcements (Figure 2.33) compared to the unreinforced shell (Figure 2.21). Also, the radial deformation on the bulkhead located at the front edge of the cutout has diminished considerably with the addition of the longitudinal stiffeners (see Figure 2.28).

2.6.5.2 Analysis of a $\pm 45^\circ$ Helically Stiffened Shell with a Cockpit-Size Cutout, Bulkheads and Longitudinal Reinforcements

We analyze the deformation behavior of a sandwich shell with a cockpit-size cutout, bulkheads, and longitudinal reinforcements. The externally stiffened shell is 14% stiffer than the internally stiffened shell. The results obtained for these two shells are shown along with the results obtained for the sandwich shell in Table 2.10.

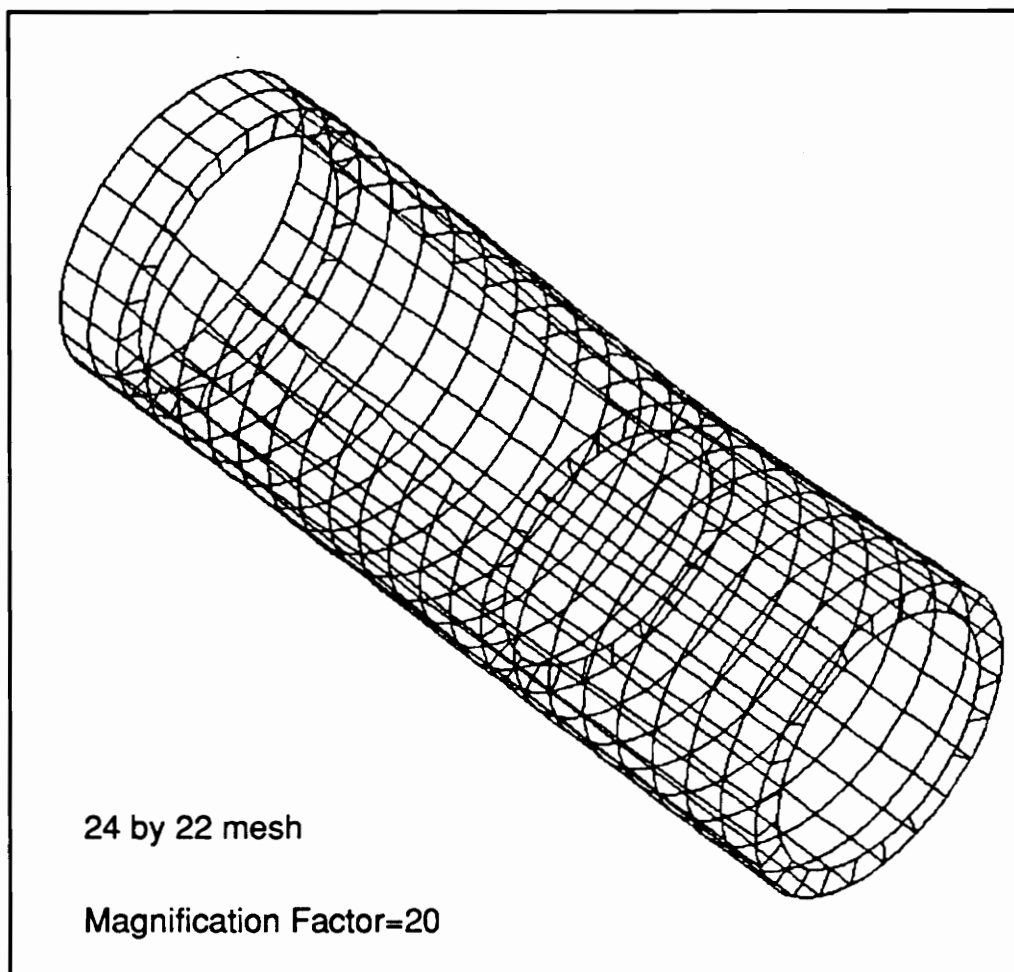


Figure 2.33 Deformed Shape for the Sandwich Shell with a Cockpit-Size Cutout with Bulkheads and Longitudinal Reinforcements

Table 2.10 Results for the Torsion Problem for Shells with a Cockpit-Size-Cutout, Bulkheads, and Longitudinal Reinforcements

Behavior	Sandwich shell	Internally Stiffened Shell	Externally Stiffened Shell
U_{θ} (mm)	1.9	.9	.8
θ (deg)	.384	.180	.158
K_{θ} (Nm/deg)	17,240	36,820	42,050
U_r (mm)	2.228	2.147	1.901

There is considerable improvement in torsional stiffness from the shells with the bulkheads only (Table 2.9). The internally stiffened shell is 143% stiffer and the externally stiffened shell is 134% stiffer than the shells with bulkheads only. There is a reduction in the radial displacements of 53% when we add the longitudinal reinforcements. The improvement in torsional stiffness and the decrease of the radial displacement is clear in the 45° view of the deformed shapes of the internally and externally stiffened shells shown in Figures 2.34 and 2.35, respectively.

2.6.5.3 Comparison of Results for the Sandwich and Stiffened Shells

With the addition of longitudinal reinforcements along the cutout edges, we can increase the torsional stiffness for the sandwich shell by 29% and for the stiffened shells by around 140%. The gain in torsional stiffness obtained by adding longitudinal reinforcements to the shells with bulkheads is bigger for the stiffened shells than for the sandwich shells. The reason for this difference in improvement is that, when we increase the gross-body bending stiffness of the cutout region, the $\bar{A}_{66}$ and the $\bar{D}_{66}$ terms of the stiffness matrix gain influence on the deformation behavior of the shell. For the stiffened shell, $\bar{A}_{66}$ is 4.6 times bigger and $\bar{D}_{66}$ is four times bigger than the values for the sandwich shell. Thus, whenever we decrease the local bending, the increase in torsional stiffness obtained with stiffened shell is bigger than the increase obtained with the sandwich shell.

The improvement in torsional stiffness for the stiffened shells is clearly bigger than for the sandwich shells as shown in Figure 2.36 for the shells with and without longitudinal reinforcements. Also, although the shape of the curves obtained in Figure 2.36 for the shells with and without longitudinal reinforcements is the same, the curves are smoother for the shells with longitudinal reinforcements.

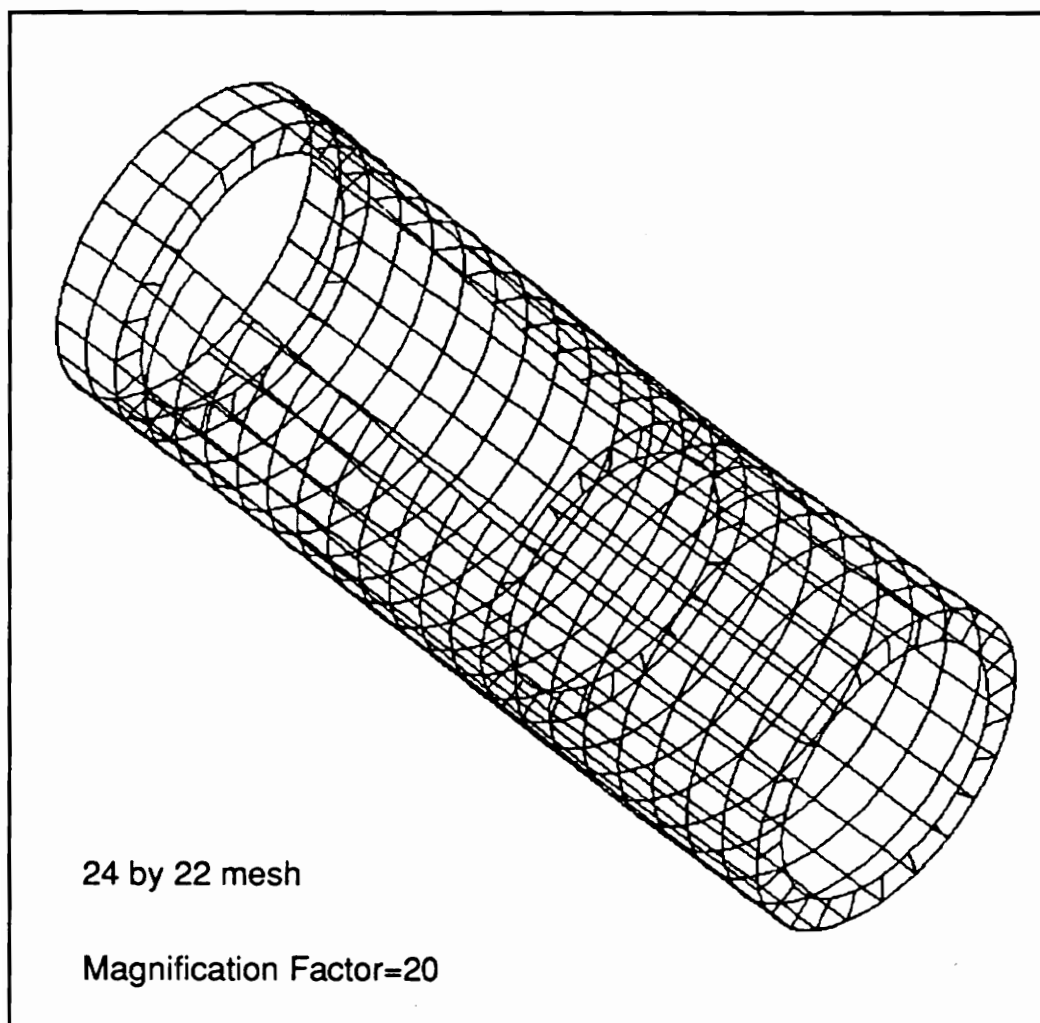


Figure 2.34 Deformed Shape for
the Internally Stiffened Shell with
a Cockpit-Size Cutout, Bulkheads
and Longitudinal Reinforcements

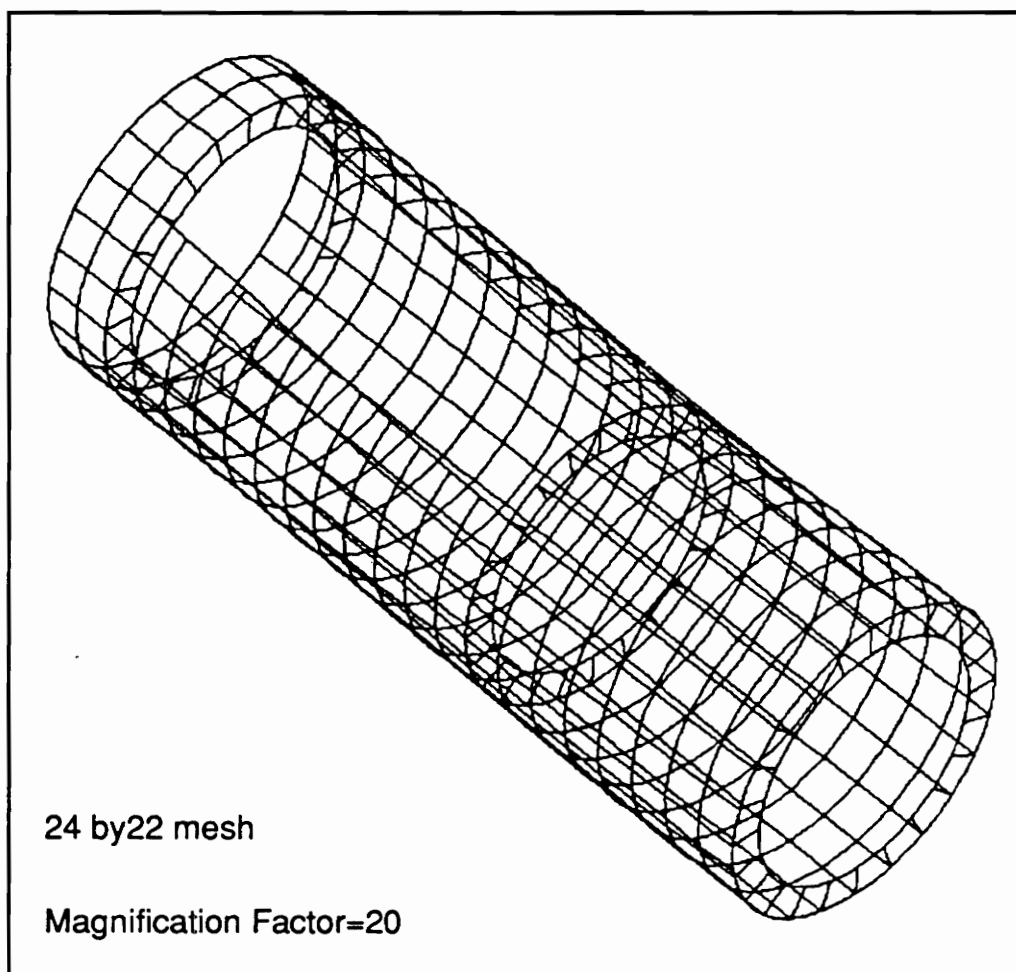


Figure 2.35 Deformed Shape for
the Externally Stiffened Shell with
a Cockpit-Size Cutout, Bulkheads
and Longitudinal Reinforcements

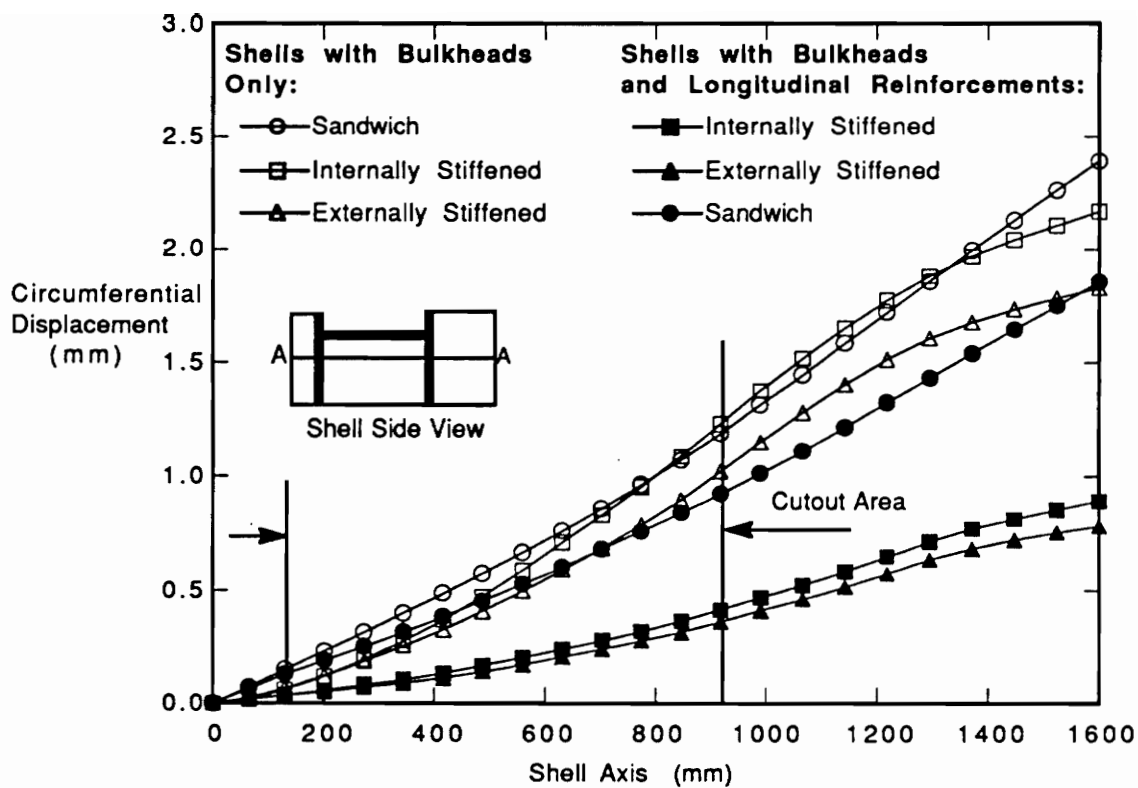


Figure 2.36 Circumferential Displacement Along Line A-A for Shells with a Cockpit-Size Cutout with Bulkheads and Longitudinal Reinforcements

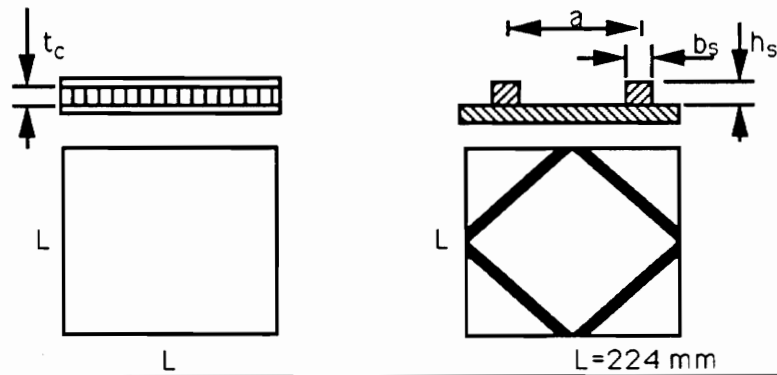
Although the torsional stiffness of the stiffened shell is clearly bigger than the torsional stiffness of the sandwich shell, the stiffened shell is heavier. The difference in weight is 30%, but this difference exists only in the skin of the structural shell because the bulkheads and reinforcements are the same for both configurations. Thus, the total weight increase for the stiffened shell is less than 30% because we must consider the weight of other parts of the structural shell that is the same for both shell. Among these parts are included the bulkheads, longitudinal reinforcements, fuel tank, etc.

2.6.6 Analysis of $\pm 45^\circ$ Helically Stiffened Shell with a Cockpit-Size Cutout with Bulkheads, Longitudinal Reinforcements, and the Same Weight as the Equivalent Sandwich Shell

Up to this point, we have studied the behavior of a stiffener-skin configuration that is 30% heavier and has the same bending stiffness as the sandwich skin configuration used as a baseline configuration. In this section, we analyze the behavior of a stiffener-skin configuration that has the same weight as the sandwich skin configuration studied in the previous sections. We perform this analysis for the externally stiffened shell that has proved to be stiffer than the internally stiffened shell.

The only way of lowering the weight of the stiffener-skin configuration without getting local skin buckling is to remove material from the stiffeners while keeping the same skin lay-up. The smaller size of the stiffener cross section of the equal-weight stiffener-skin configuration will yield much smaller bending stiffnesses and also smaller in-plane stiffnesses than for the equal-bending-stiffness stiffener-skin configuration. The decrease of the in-plane stiffnesses is smaller than the decrease in the bending stiffnesses. In order to achieve the same weight as the sandwich skin configuration, the new cross-sectional area of the stiffeners is 58 mm^2 and the stiffener aspect ratio is three. The values of the stiffness matrix terms for the baseline sandwich skin configuration and the stiffener-skin configurations are listed in Table 2.11. We see that although all the stiffnesses have diminished, the A_{66} stiffness of the equal-weight stiffener-skin configuration is still more than three times bigger than A_{66} for the baseline sandwich skin configuration.

Table 2.11 Sandwich and Stiffened Equal-Weight Skin and Stiffener Configurations for Circular Cylindrical Shells



Configuration	Sandwich	$\pm 45^\circ$ Stiffened Equal-Weight
Lay-Up	$[[CF]_3/+45/-45]_S$ $t_c = 15 \text{ mm}$	Skin: $[\pm 45]_{3S}$ Stiffeners: $h_s = 13.2 \text{ mm}$ $b_s = 4.4 \text{ mm}$ $a = 158 \text{ mm}$
Weight (kg)	.17	.17
$A_{11} = A_{22}$ (kN/mm)	90.1	85.1
A_{12} (kN/mm)	18.82	71.4
A_{66} (kN/mm)	21.22	74
B_{ij}^* (kN)	0	171.4
$D_{11} = D_{22}$ (kN mm)	5326	1650.5
D_{12} (kN mm)	1114	1570.8
D_{66} (kN mm)	1256	1609.8
$D_{16} = D_{26}$ (kN mm)	0	2.15

* $B_{1j} = B_{11} = B_{22} = B_{12} = B_{66}$

Although we have decreased the cross-sectional area of the stiffeners we still get a substantial improvement in torsional stiffness over the sandwich shell, as shown in Table 2.12. The circumferential displacement u_θ at the loaded end for the equal-weight stiffener-skin configuration is 1mm. The value of the torsional stiffness K_θ for this shell is 33,000 Nm/deg. The equal-weight stiffened shell is 22% less stiff than the externally stiffened equal-bending-stiffness shell and around two times stiffer than the baseline sandwich shell.

The deformation behavior of the equal-weight stiffened shell is similar to that of the equal-bending-stiffness shells. The curve circumferential displacement of the nodes along line A-A obtained for the equal-weight stiffened shell has the same shape as the curves obtained for the equal-bending-stiffness stiffened shells, as shown in Figure 2.37. The same conclusion can be drawn if we compare the deformed shapes of the equal-weight and equal-bending-stiffness shells shown in Figures 2.38 and 2.35, respectively.

2.7 SUMMARY

In this Chapter, we studied the torsional behavior of circular cross-section shells with a cutout simulating the the opening for the cockpit of a Formula racing car. If the edges of the cutout are stiffened, the bending stiffness of the structural shell skin is not as important as we thought at the start of this study when we considered the stiffener-skin configuration with equal bending stiffness. Also, we conclude that the $\pm 45^\circ$ helically stiffened construction appears to be a very good alternative for the construction of a formula racing car structural shell. Using the $\pm 45^\circ$ stiffening, we obtained an improvement in torsional stiffness of around 200% over the sandwich shell without adding any weight to the structure.

Because real racing car structural shells usually have non-circular cross sections, in the next Chapter we will study the torsional behavior of square cross-section cylindrical shells. We will perform the same analysis for these shells that we did for the circular cylindrical shells.

Table 2.12 Results for the Torsion Problem for Shells with a Cockpit-Size-Cutout, Bulkheads, and Longitudinal Reinforcements

Behavior	Shell			
	Sandwich Baseline	Equal D_{ij} Internally Stiffened	Equal D_{ij} Externally Stiffened	Equal-Weight Externally Stiffened
U_θ (mm)	1.9	.9	.8	1
θ (deg)	.384	.180	.158	.202
K_θ (Nm/deg)	17,240	36,820	42,050	33,000
U_r (mm)	2.228	2.147	1.901	2.311
$\frac{K_{\theta \text{stiffened}}}{K_{\theta \text{sandwich}}}$	—	2.14	2.44	1.91

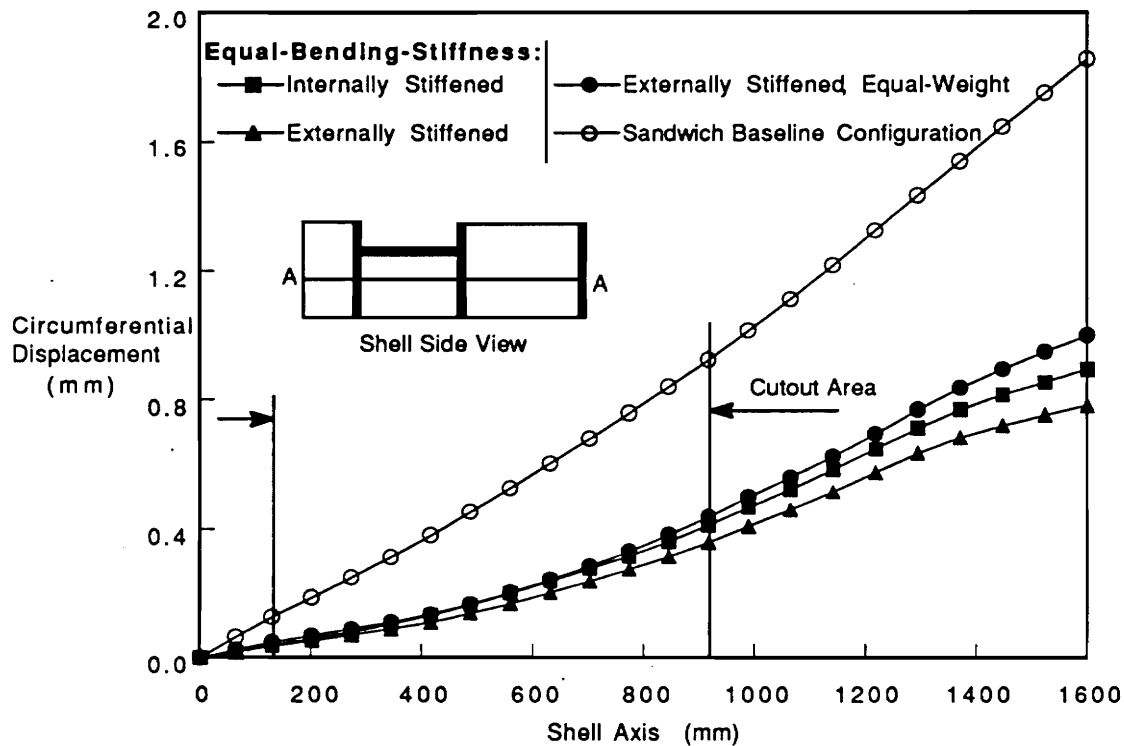


Figure 2.37 Circumferential Displacement Along Line A-A for Shells with Equal-Bending-Stiffness and Equal-Weight Configurations

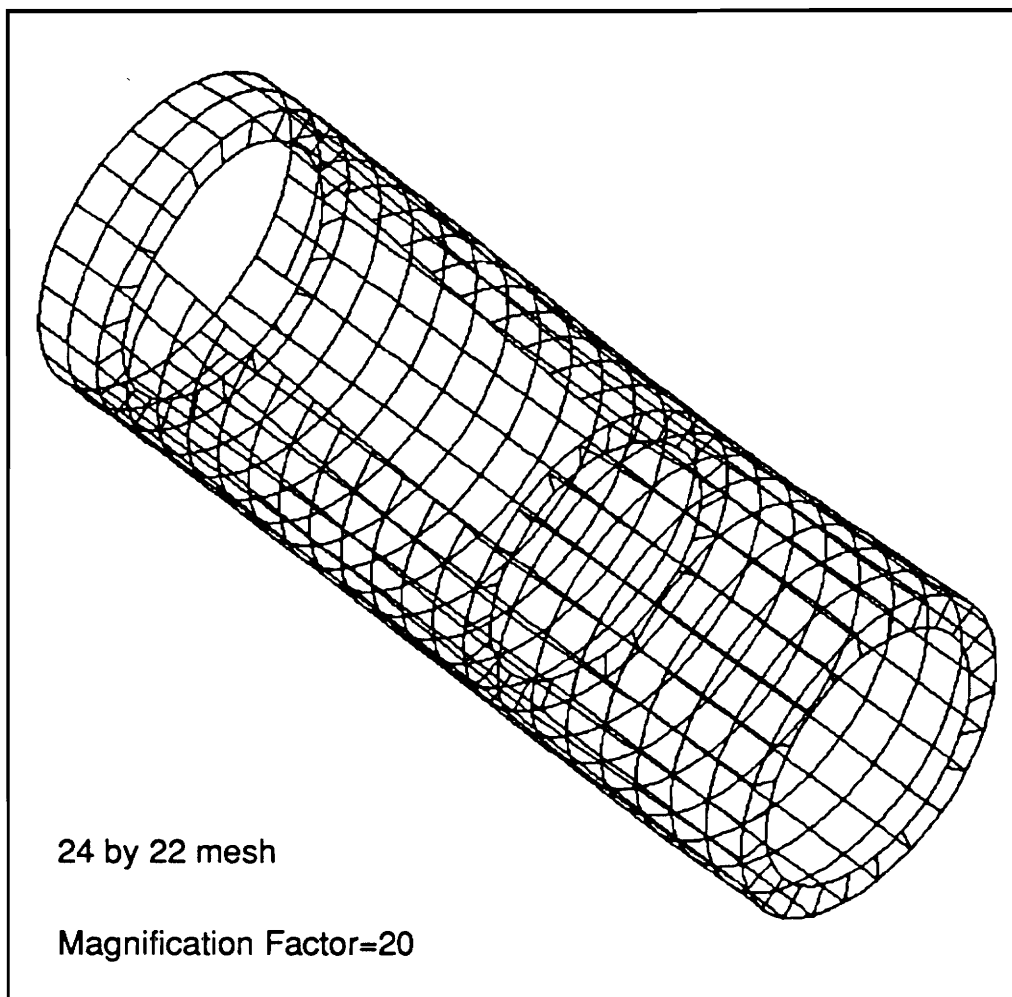


Figure 2.38 Deformed Shape for the
Equal-Weight Externally Stiffened Shell with
a Cockpit-Size, Bulkheads,
and Longitudinal Reinforcements

Chapter 3

Selection of the Best Structure for Torsional Stiffness of Square Cross-Section Cylindrical Shells with Rounded Corners

3.1 INTRODUCTION

In this Chapter, we study the torsional behavior of square cross-section cylindrical shells to try to make a better approximation to a real racing car structural shell than with the circular cross-section cylindrical shells. The conclusion from Chapter 2 was that the $\pm 45^\circ$ helically stiffening concept has a bigger torsional stiffness than the sandwich construction for shells with a circular cross section. For the circular cylindrical shell with bulkheads and longitudinal reinforcements using the equal-weight stiffener-skin configuration, we obtained nearly twice the torsional stiffness of the sandwich baseline skin configuration. In this Chapter, we will see if we obtain similar results for the square cross-section shells.

Real racing car structural shells are generally non-circular cross-section cylindrical shells (see Figure 1.3). Usually, the structural shell of a racing car has a rectangular or hexagonal cross section, and the portion of the shell in front of the cockpit is tapered. The reason for a rectangular cross section is that in this way the designers minimize the frontal cross-sectional area of the car improving the aerodynamic penetration. This aerodynamic improvement enables the car to perform better in the long straight parts of the race tracks where the speed is high and the aerodynamic drag is very important. We use rounded corners to avoid stress concentration in the corners and because of manufacturing reasons as will be explained later in this thesis.

In this Chapter, we will investigate the performance of square cross-section cylindrical shells with rounded corners under torsion to see if the helically stiffened concept is still better than the sandwich construction. In the case of circular cross-section shells, it was sufficient to stiffen the edges of the cutout to make the equal-weight stiffened shell (with lower bending stiffness) be nearly 2 times stiffer in torsion than the sandwich shell. If we compare the values of the equal-weight, helically stiffened shell with the equal-bending-stiffness (over weight) stiffened shell, we see that there is only a 21% difference in torsional stiffness in favor of the heavier shell. This small difference could be due to the fact that because the wall of a circular cylindrical shell is curved, it has an added bending stiffness in the direction of the longitudinal axis of the shell provided by the curvature of the wall. In the case of a square cross-section shell, this curvature does not exist, so that added stiffness is not present.

In this Chapter, we will perform the same analyses for the square cross-section cylindrical shells that we did for the circular cylindrical shells. The dimensions of the square cross-section cylindrical shell are shown in Figure 3.1. These dimensions were chosen so that the shells will weigh the same as their circular cross-section counterparts. Thus, the square cross-section cylindrical shell has the same length and perimeter as the circular cylindrical shells.

3.2 FINITE ELEMENT ANALYSIS

For the analysis of these shells, we use the ABAQUS Finite Element Program [4] and the PATRAN [5] pre- and post-processor, as we did for the circular cross-section cylindrical shells. To model the shells, we use the ABAQUS S8R element, which is an 8-noded, reduced-integration, doubly curved shell element with 6 degrees of freedom per node (3 displacements and 3 rotations). To model stiffeners other than the bulkheads and the helical stiffeners, we use the ABAQUS B32 beam element. The ABAQUS B32 element is a 3-noded, quadratic interpolation beam element with 6 degrees of freedom per node (3 displacement and 3 rotations).

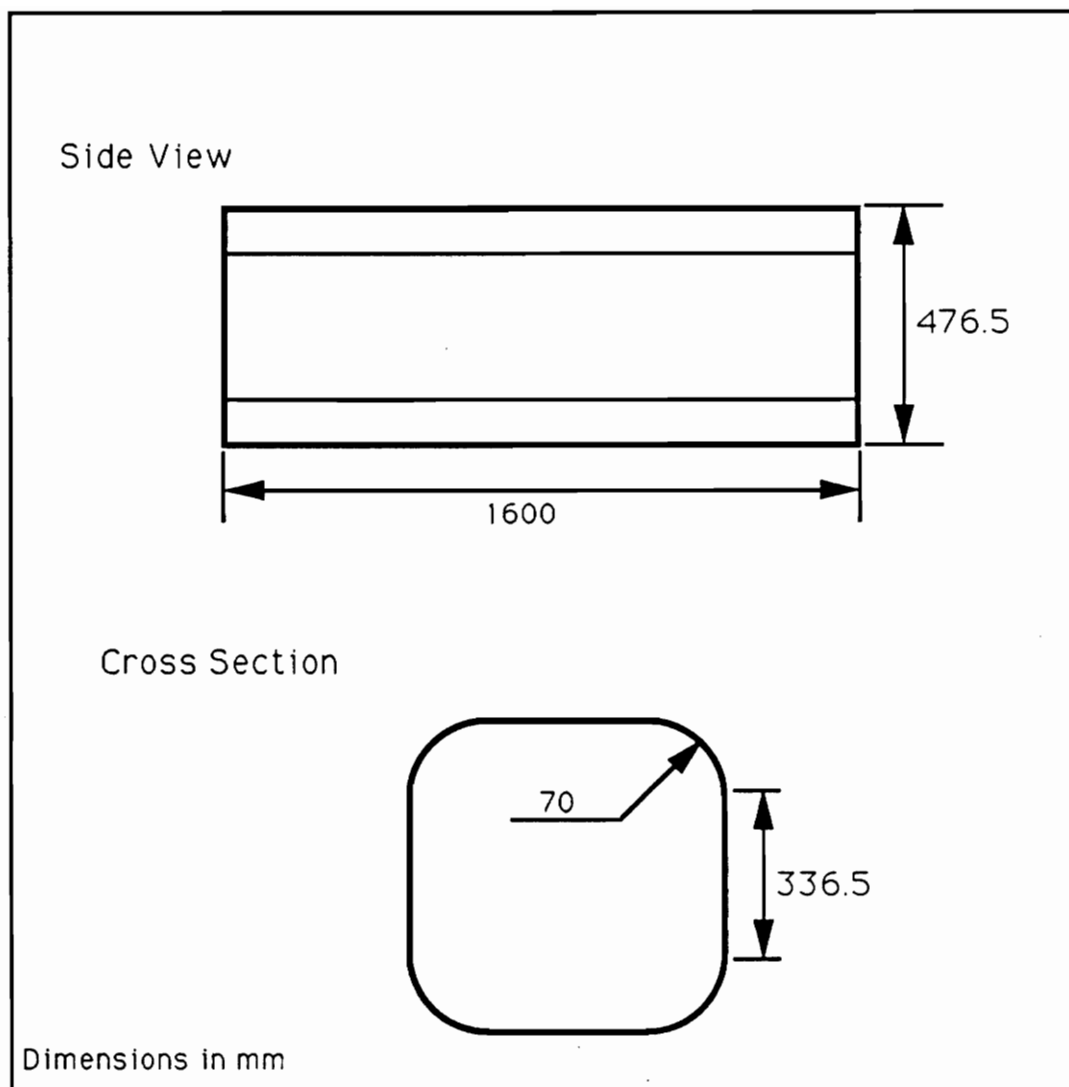


Figure 3.1 Shape and Dimensions of the Square Cross-Section Cylindrical Shell with Rounded Corners

In order to get the values of stresses in the skin, stiffeners, etc., we use the same procedure used for the the circular cross-section cylindrical shells (see Section 2.3.4). For failure prediction, we use the Tsai-Hill failure criterion [18] as was done before for the circular cross-section shells.

3.3 LOADS AND BOUNDARY CONDITIONS

The torque applied is the same used in Chapter 2 for the circular cross-section cylindrical shells, i.e., $T=6637\text{kNmm}$. This value of torque is the maximum torque that the loads of the front suspension members can transmit to the structure through the two front bulkheads. This torque is applied to one end of the shell, whereas the other end is clamped as shown in Figure 3.2. To model the torque at one end, we put four equivalent concentrated loads P , one at each middle node of each side of the end as shown in Figure 3.3. The collective effect of these four loads is the same as the torque. To calculate the value of these loads, we use Equation (55) with $N=4$ and $R=476.5\text{mm}$. The value of the equivalent load P for the square cross-section cylindrical shells is 6.964kN .

The boundary conditions at the clamped end are modeled in the same way as for the circular cylindrical shells. We achieve the clamped condition by setting all the degrees of freedom of the nodes at that end of the shell to zero.

We calculate the torsional stiffness of the square cross-section cylindrical shell in the same way as we did for the circular cylindrical shells. We use Equations (56) and (57) of the previous Chapter. In Equation (57), we set the value of R to the distance from the node where we get the value of u_θ to the centroid of the cross section of the shell.

3.4 SKIN AND STIFFENER BUCKLING

As we did for the circular cylindrical shells, we must ensure that there will not be any local buckling before material failure. We perform these buckling analysis in the same way as we did in Section 2.3.5. For prediction of stiffener buckling, we use Equation (53) [17].

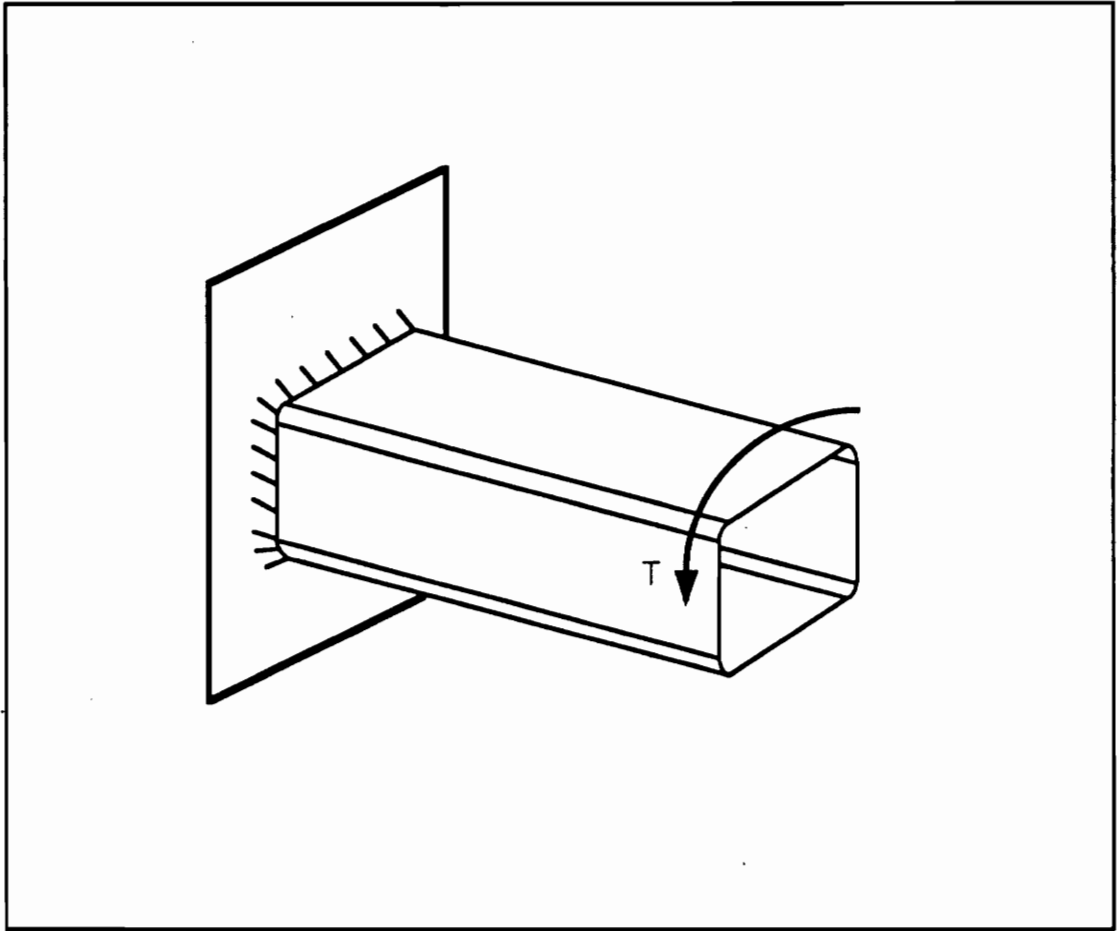


Figure 3.2 Loading and Support of
the Square Cross-Section Cylindrical Shells
with Rounded Corners

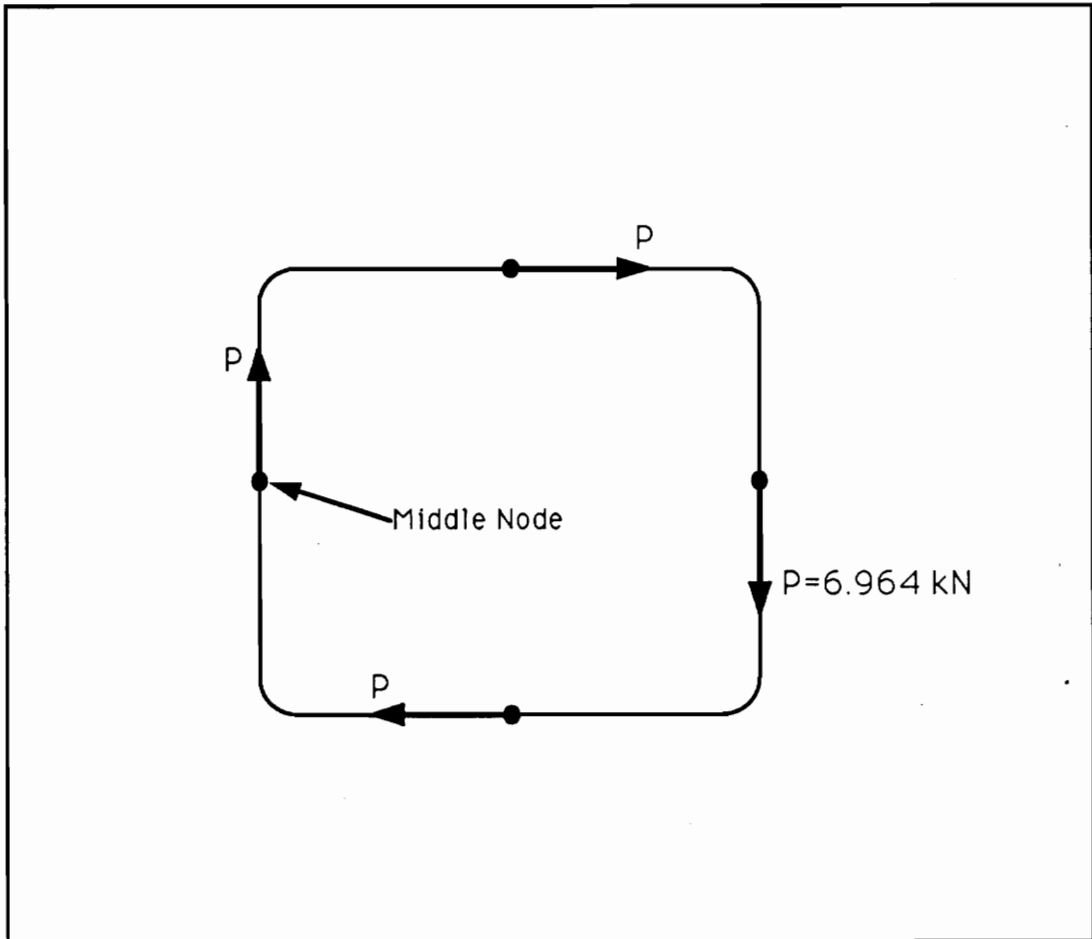


Figure 3.3 Location of the Equivalent Loads at the Loaded end of the Shell

3.5 DEFORMATION BEHAVIOR OF CLOSED SANDWICH AND $\pm 45^\circ$ HELICALLY STIFFENED SQUARE CROSS-SECTION CYLINDRICAL SHELLS

We analyze the behavior of a sandwich shell and of three stiffened shells. For the four shells, we use the same mesh of 36 elements around and 27 elements along the shell. Of the 36 elements around the shell, we used 3 elements to model each corner and 6 elements to model each flat face. The loads and boundary conditions used are those explained in Section 3.3.

For the sandwich construction shell, we use the same sandwich lay-up of the circular cross-section cylindrical shells as shown in Table 3.1. Three stiffened shells are analyzed. The first two are an externally and an internally stiffened shell with the equal-bending-stiffness stiffener-skin configuration used in Chapter 2 as shown in Table 3.1. We analyze these two cases to see if the effect of the internal or external location of the stiffeners is the same as in the circular cylindrical shells. Finally, we analyze an externally stiffened shell with the same equal-weight stiffener-skin configuration that we used in Chapter 2 as shown in Table 3.1.

Throughout all the analysis in this Chapter, we plot the circumferential displacements along the longitudinal axis of the shells for all nodes along the midline A-A of one of the flat faces of the shells as shown in Figure 3.4. We use these nodes and not the mid-nodes of one of the corners because this is the place at which the circumferential displacements are measured in real cars. Thus, the results for the flat faces will be more useful than the results at the corners.

3.5.1 Analysis of a Closed Sandwich Shell

We perform the finite element analysis of a closed sandwich shell with the skin configuration shown in Table 3.1. In the curves of circumferential displacement along line A-A obtained for all the shells, there is a sudden change of slope near the loaded end of the shells as shown in Figure 3.5. This effect is due to the way in which the torque is modeled.

Table 3.1 Baseline Sandwich, Equal-Bending-Stiffness, and Equal-Weight Skin and Stiffener Configurations for Square Cross-Section Cylindrical Shells



	Sandwich Baseline Configuration	$\pm 45^\circ$ Stiffened Equal-Bending- Stiffness Configuration	$\pm 45^\circ$ Stiffened Equal-Weight Configuration
Lay-Up	$[[CF]_3/+45/-45]_s$ $t_c = 15 \text{ mm}$	Skin: $[\pm 45]_{3S}$ Stiffeners: $h_s = 17 \text{ mm}$ $b_s = 6.8 \text{ mm}$ $a = 158.3 \text{ mm}$	Skin: $[\pm 45]_{3S}$ Stiffeners: $h_s = 13.2 \text{ mm}$ $b_s = 4.4 \text{ mm}$ $a = 158 \text{ mm}$
Weight (kg)	.17	.225	.17
$A_{11} = A_{22}$ (kN/mm)	90.1	108.2	85.1
A_{12} (kN/mm)	18.82	94.5	71.4
A_{66} (kN/mm)	21.22	97.12	74
B_{1j}^* (kN)	0	429.4	171.4
$D_{11} = D_{22}$ (kN mm)	5326	5230	1650.5
D_{12} (kN mm)	1114	4979	1570.8
D_{66} (kN mm)	1256	5103.6	1609.8
$D_{16} = D_{26}$ (kN mm)	0	2.15	2.15

* $B_{1j} = B_{11} = B_{22} = B_{12} = B_{66}$

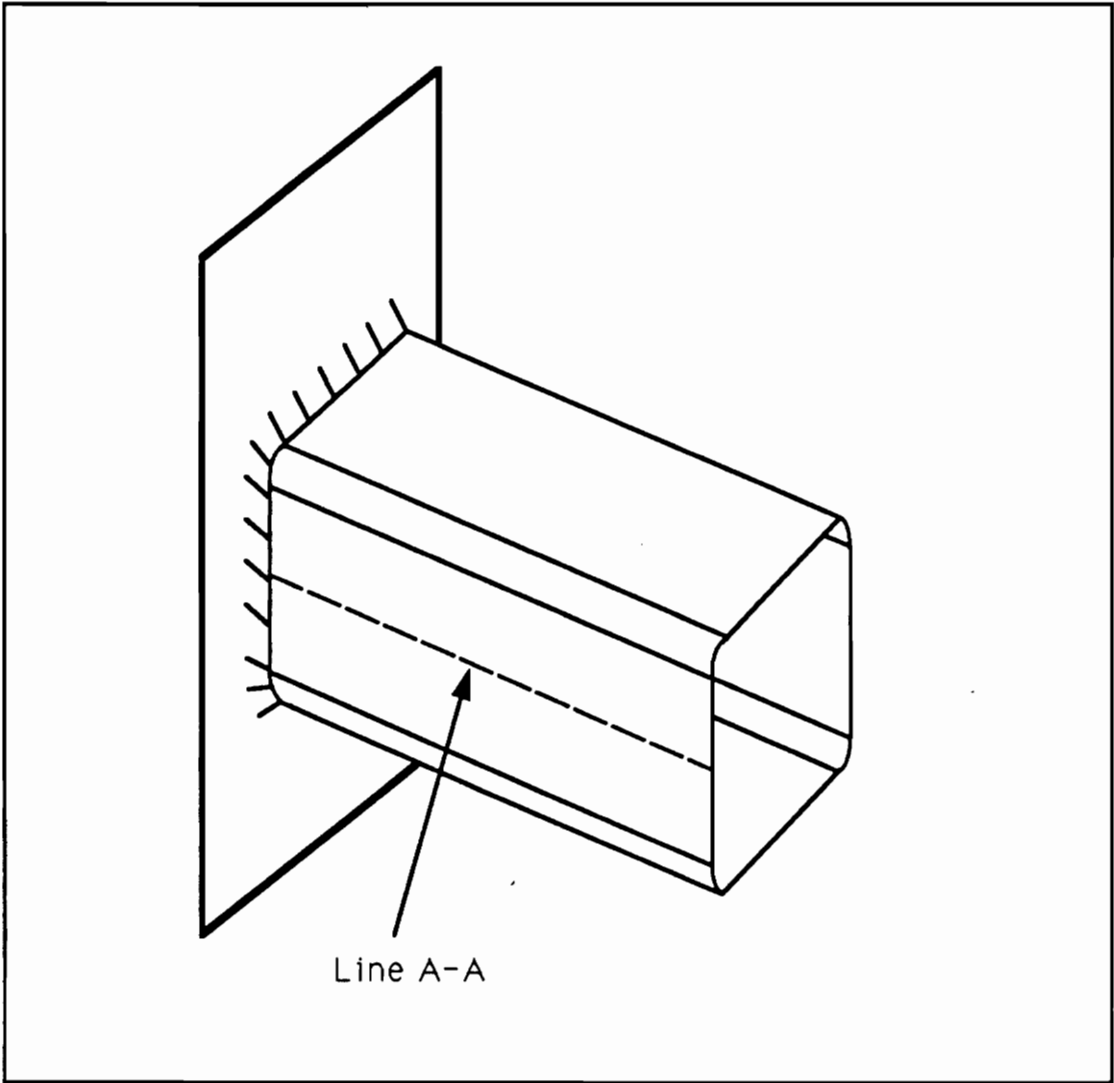


Figure 3.4 Line where the Circumferential Displacement is Measured

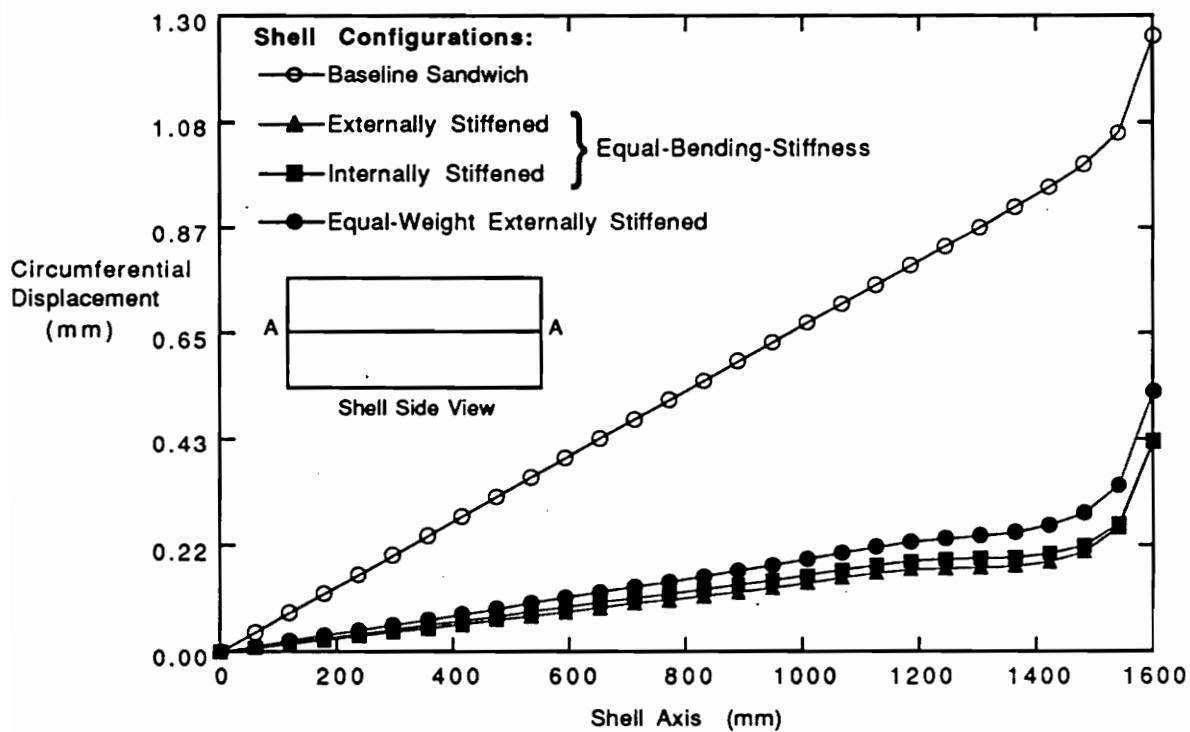


Figure 3.5 Circumferential Displacement Along Line A-A for Closed Square Cross-Section Cylindrical Shells

We used only four forces to model the torque, and one of the forces is applied at the node where we measure the displacement. To calculate the values of torsional stiffness and circumferential displacement at the loaded end, we neglect the change in the slope of the circumferential displacement curves.

The torsional stiffness of the sandwich square cross-section cylindrical shell is smaller than the torsional stiffness of the sandwich circular cross-section cylindrical shell. The circumferential displacement u_θ at the loaded end (neglecting the change in curvature at the end) for the sandwich shell is 1.07 mm. The value of the torsional stiffness K_θ for this shell is 25,800 Nm/deg. The maximum radial displacement obtained for this shell is .23 mm. The square cross-section shell is 16% less stiff than the circular cross-section shell. This difference is due to the fact that the circular cross section is better for torsion than the square section. The effect of having only one force per side of the shell to model the torque can be seen in the 45° view of the deformed shape of the sandwich shell shown in Figure 3.6. There is a localized deformation at the middle node of the flat sides. A better but more complicated way of modeling the torque is to distribute the load over all the nodes of the loaded end.

3.5.2 Analysis of Closed $\pm 45^\circ$ Helically Stiffened Shells

We analyze three shells, (1) an equal-bending-stiffness internally stiffened shell, (2) an equal-bending-stiffness externally stiffened shell, and (3) an equal-weight externally stiffened shell. We analyze the internally stiffened shell only for the equal-bending-stiffness configuration to see if we get a lower torsional stiffness than for the externally stiffened shells.

Before calculating the torsional stiffness of the stiffened shells, we must check for local buckling. We perform the buckling predictions for the skin and stiffeners using Equations (46), (51), and (53). The value of N_{xy} acting on these shells is equal to the summation of the four forces applied to the shell divided by the perimeter of the shell, so $N_{xy}=1.47 \times 10^{-2}$ kN/mm. The values of critical stiffener spacing a_{cr} and critical load N_{cr} obtained for these shells were bigger than the values used so there is no local buckling.

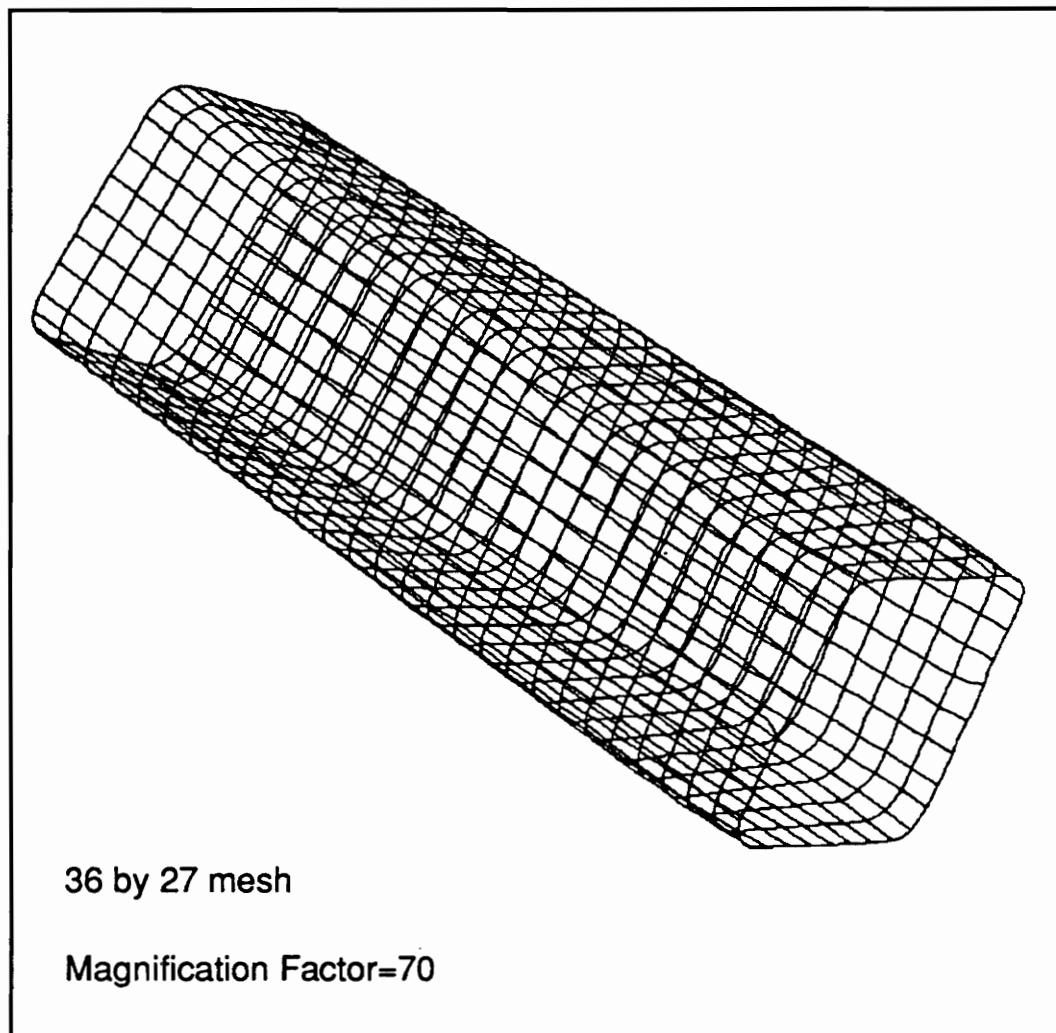


Figure 3.6 Deformed Shape of the Closed Sandwich Shell

The curves for circumferential displacement along the shell axis for the sandwich and stiffened shells have the same sudden slope increase near the loaded end as shown in Figure 3.5. This sudden slope increase is due to the way in which the loads are applied to the shells. Thus, the torsional stiffness and the circumferential displacement at the loaded end are calculated as was done for the sandwich shell.

As happened for the circular shells, the internally stiffened shell has a lower torsional stiffness than the corresponding externally stiffened shell as shown in Table 3.2. The difference in torsional stiffness between the square internally and externally stiffened shells is 18%, twice the difference obtained for the circular shells. The maximum radial displacement is lowest for the internally stiffened shell. This difference in the radial displacement is clear in the plots of the deformed shape as in Figures 3.7, 3.8, and 3.9, where we see that the deformation in the loaded end is higher for the externally stiffened shells than for the internally stiffened.

For the two externally stiffened shells, there is a 36% difference in torsional stiffness in favor of the equal-bending-stiffness shell, but there is also an increase in weight of 32%. Also, the equal-bending-stiffness and the equal-weight shells are five and 3.6 times stiffer than the sandwich shell, respectively. Thus, for a racing car structural shell where the weight is very important, the equal-weight stiffener-skin configuration appears again as a very good option because it has more than three times the torsional stiffness obtained with the baseline sandwich skin configuration and the same weight.

The difference in torsional stiffness between the sandwich and equal-bending-stiffness externally stiffened shells is around 5 times for both the square and circular cross-section shells (see Tables 2.5, 2.6, 2.7, and 3.2). The absolute values of torsional stiffness obtained for the circular cross-section shells are 20% bigger for the sandwich shell, 25% bigger for the externally stiffened shell, and 34% bigger for the internally stiffened shell than for the square cross-section shells. Thus, we see that the best shape for maximizing torsional stiffness is the circular cross section.

Table 3.2 Results for the Torsion Problem for
Closed Square Cross-Section Cylindrical Shells

Shell Configurations		U_θ (mm)	θ (deg)	K_θ (Nm/deg)	$U_{r \max}$ (mm)
Baseline Sandwich		1.07	.26	25,800	.23
Equal Bending Stiffness	Externally Stiffened	.22	.05	125,500	.26
	Internally Stiffened	.26	.06	106,150	.1
Equal-Weight Externally Stiffened		.3	.07	92,000	.5

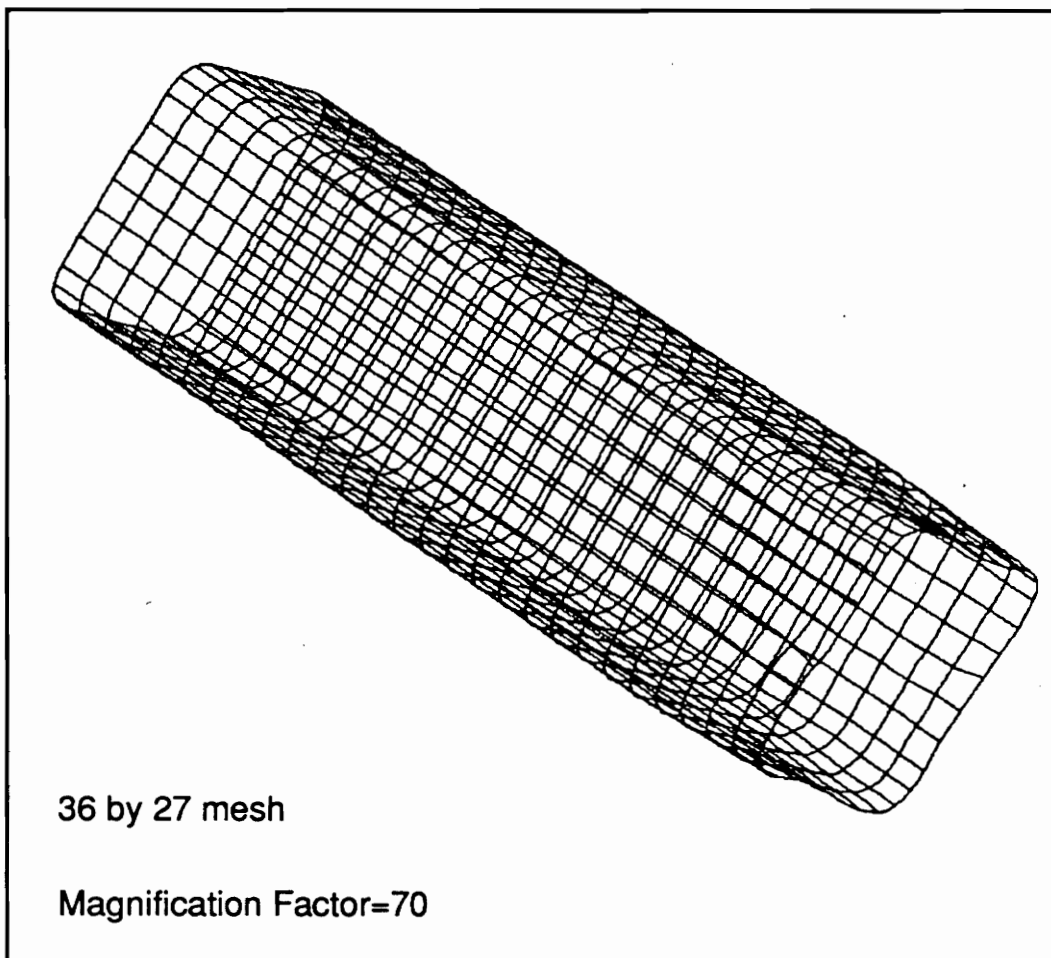


Figure 3.7 Deformed Shape of
the Equal-Bending-Stiffness Externally Stiffened
Closed Shell

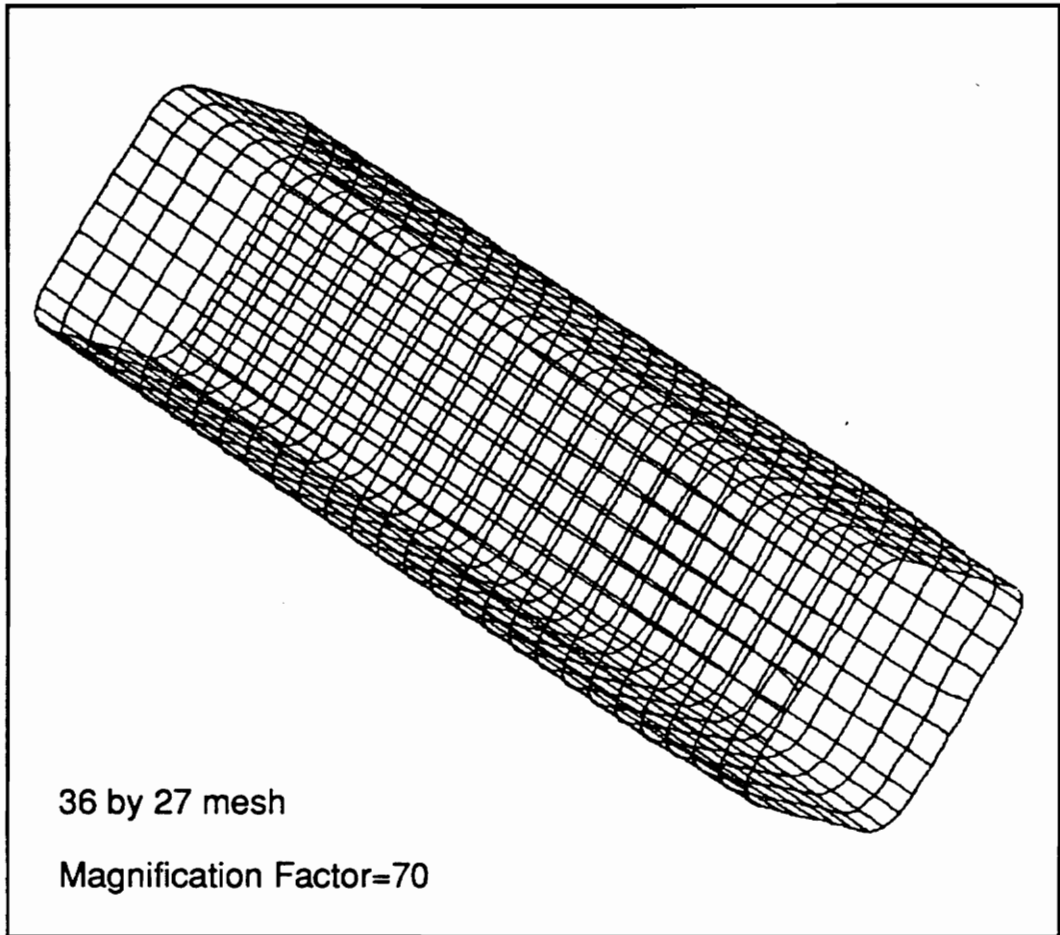


Figure 3.8 Deformed Shape for
the Equal-Bending-Stiffness Internally Stiffened
Closed Shell

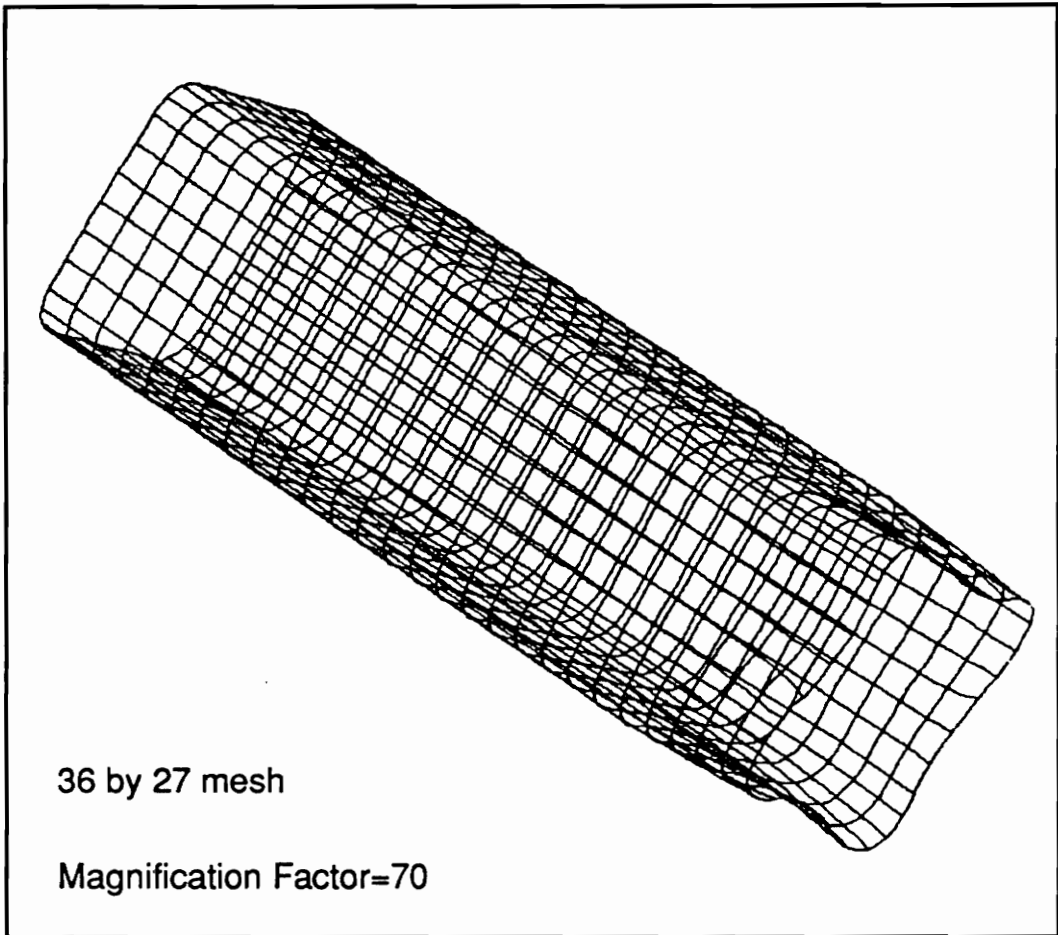


Figure 3.9 Deformed Shape for
the Equal-Weight Externally Stiffened
Closed Shell

3.6 DEFORMATION BEHAVIOR OF SANDWICH AND $\pm 45^\circ$ HELICALLY STIFFENED SQUARE CROSS-SECTION CYLINDRICAL SHELLS WITH A COCKPIT-SIZE CUTOUT

We analyze the behavior of a sandwich shell and of two stiffened shells with a cockpit-size cutout. The case of the shells with the cutout is obviously more representative than the closed shells of the behavior of a real racing car structural shell, which has an opening for the driver to enter the cockpit. The location and size of the cutout is shown in Figure 3.10. For all the shells, we use the mesh of 36 elements around and 27 elements along the shell used for the closed shells. Of the 36 elements around the shell, we used 3 elements to model each corner and 6 elements to model each flat face. The loads and boundary conditions used are those explained in Section 3.3.

For the sandwich shell, we use the sandwich skin configuration shown in Table 3.1 which is the same as for the circular cross-section cylindrical shells. For the stiffened shells, we analyze two different shells. The first shell is an externally stiffened shell with the equal-bending-stiffness stiffener-skin configuration used in Chapter 2 as shown in Table 3.1. Finally, we analyze an externally stiffened shell with the same equal-weight stiffener-skin configuration used in Chapter 2 as shown in Table 3.1. In this Section, we do not analyze an internally stiffened shell because in Section 3.5.2 the use of this type of shell is shown to yield lower values of torsional stiffness than the externally stiffened shells.

3.6.1 Analysis of a Sandwich Shell with a Cockpit-Size Cutout

We perform the finite element analysis of a sandwich square cross-section cylindrical shell with a cockpit-size cutout. For this shell, we use the baseline sandwich skin configuration shown in Table 3.1. The circumferential displacement u_θ at the loaded end for the sandwich shell is 16.1 mm. The values of torsional stiffness k_θ and maximum radial displacement for this shell are 1700 Nm/deg and 30 mm, respectively. The torsional stiffness obtained for the open shell is 15 times smaller than for the closed shell. Also, the maximum radial displacement for the open shell is 130 times bigger than for the closed shell.

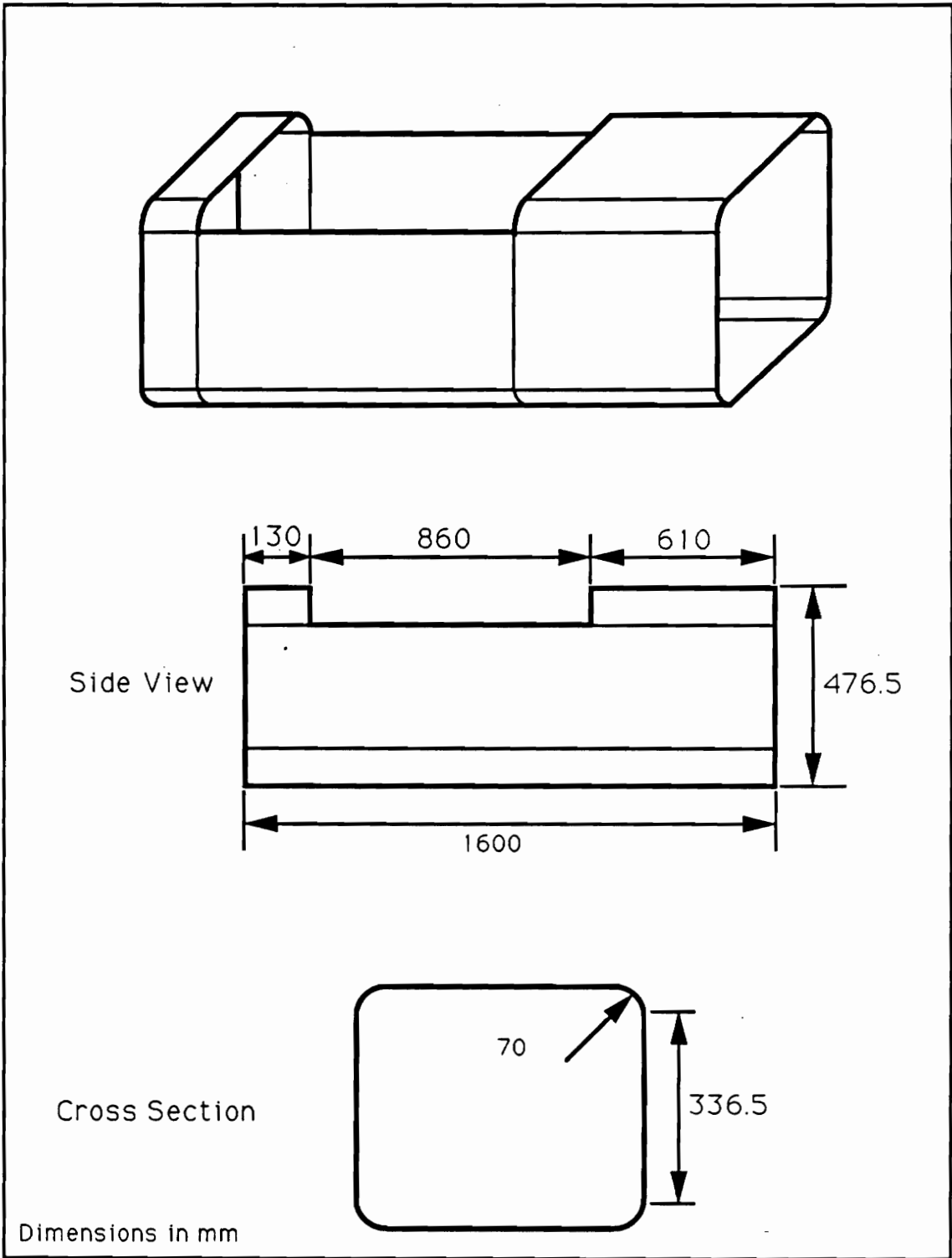


Figure 3.10 Square Cross-Section Cylindrical Shell with a Cockpit-Size Cutout

The decrease in torsional stiffness for the circular cross-section shells is not as big as for the square cross-section shells. For the open circular cross-section shells, the torsional stiffness is 3 times smaller than for the closed shells whereas in the case of an open square cross-section shell the torsional stiffness is 15 times smaller than for the closed shell. The bigger decrease in torsional stiffness for the square cross-section shell is due to the fact that for the square cross-section shell the effect of local bending in the vicinity of the cutout is bigger than for the circular cross-section shell. This difference can be seen if we compare the deformed shape of the square cross-section shell shown in Figure 3.11 with those of the circular cross-section shell shown in Figure 2.21. In the case of the square cross-section shell, the walls on the sides of the cutout are flat whereas in the circular cross-section shell these walls are curved. This curvature increases the bending stiffness of the walls in the longitudinal direction. Thus, the walls of the circular cross-section shells carry the load partly in bending and partly in shear, whereas the walls of the square cross-section shells carry the load mainly in bending. Also, in the square cross-section shells, the presence of warping is more evident than in the circular cross-section shells where warping does not exist unless there is a cutout. Warping is typical in the torsion of non-circular cross sections.

3.6.2 Analysis of $\pm 45^\circ$ Helically Stiffened Shells with a Cockpit-Size Cutout

In this Section, we analyze the behavior of two externally stiffened shells with $\pm 45^\circ$ helical stiffeners, one with the equal-bending-stiffness stiffener-skin configuration and the other with the equal-weight stiffener-skin configuration. These configurations are shown in Table 3.1.

The results for the circumferential displacement u_θ , maximum radial displacement, angle rotated at the loaded end θ , and torsional stiffness K_θ for the stiffened and sandwich shells are shown in Table 3.3. The torsional stiffness for the equal-bending-stiffness shell is 47 times smaller than for the closed shell. For the equal-weight open shell, the torsional stiffness is 58 times smaller than for the closed shell. The torsional stiffness obtained for the equal-bending-stiffness shell is 55% bigger than for the equal-weight shell. This difference is 70% bigger than the difference obtained for closed shells.

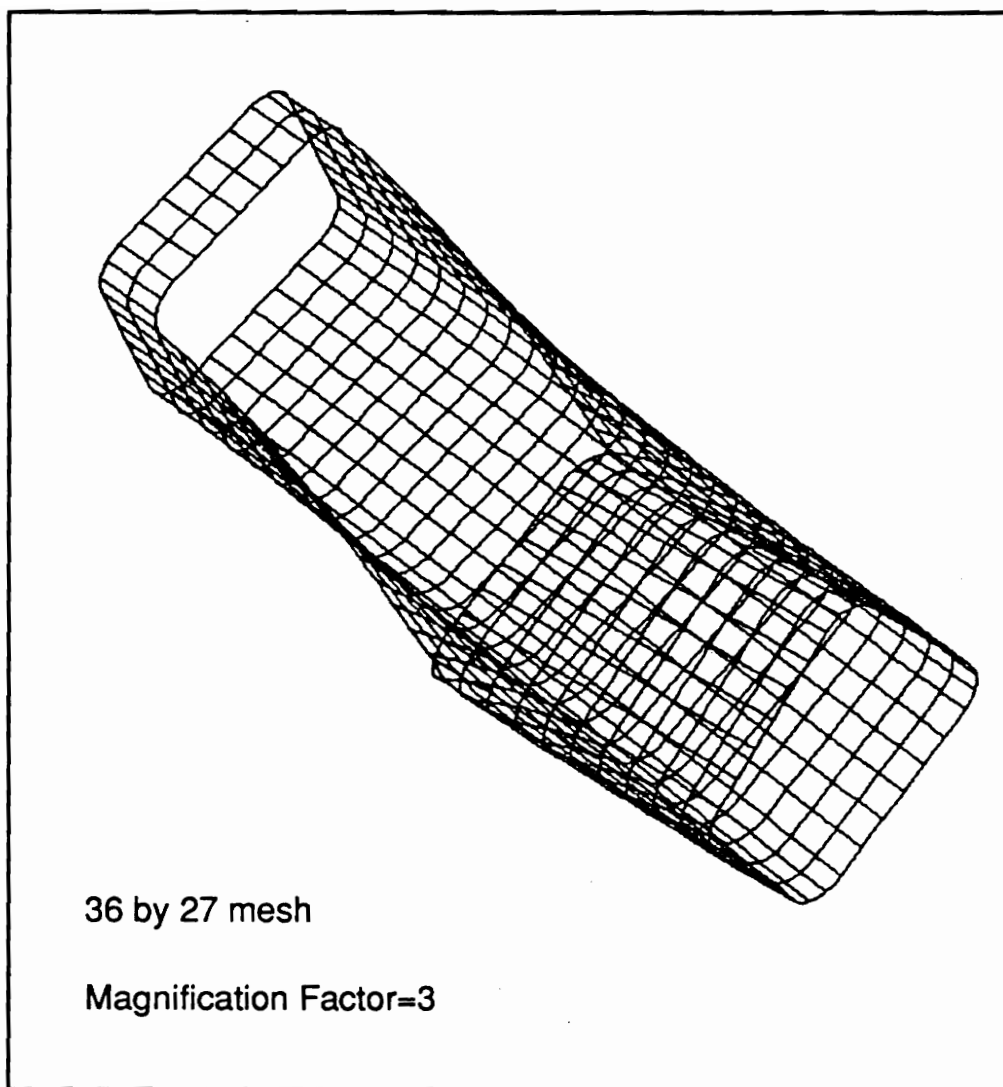


Figure 3.11 Deformed Shape of the Sandwich Shell
with a Cockpit-Size Cutout

Table 3.3 Results for the Torsion Problem for Square Cylindrical Shells with a Cockpit-Size Cutout

Shell	u_{θ} (mm)	θ (deg)	K_{θ} (Nm/deg)	$u_{r \max}$ (mm)
Baseline Sandwich	16.1	3.87	1,700	30
Equal-Bending Stiffness Externally Stiffened	20	4.8	1,380	35
Equal-Weight Externally Stiffened	31	7.5	890	67

The torsional stiffness of the sandwich shell is 25% higher than for the equal-bending-stiffness shell and 94% higher than for the equal-weight shell. We recall from Chapter 2 that the open circular cylindrical stiffened shells were 14% stiffer than the sandwich shells. From these results, we can draw the conclusion that the local bending in the vicinity of the cutout and warping of the square cross-section of the shell affects the stiffened shells more than the sandwich shell. The effect of the shape of the cross section of the shell is evident if we compare the deformed shape of the sandwich shell shown in Figure 3.11 with the deformed shape of the stiffened shells shown in Figures 3.12 and 3.13. In these three Figures, we see that the effect of the warping is bigger for the stiffened shells than for the sandwich shells. In the following Sections, we will try to reinforce these shells in the same way as we did with the circular cross-section cylindrical shells to see if we can diminish the adverse effect of the shape of the cross section on the torsional stiffness of the shells, especially for the stiffened shells.

3.6.3 Deformation Behavior of Sandwich and $\pm 45^\circ$ Helically Stiffened Square Cross-Section Cylindrical Shells with a Cockpit-Size Cutout and Bulkheads

In this section, we reinforce the shells by adding three ring stiffeners called bulkheads. These bulkheads are located at the loaded end and at the front and back edges of the cutout as shown in Figure 3.14. In real cars, these bulkheads are used to reinforce the places where the front suspension members are attached to the structural shell. The bulkheads are deep stiffeners that are usually built using sandwich construction and are modeled as described in Section 2.6.4.

3.6.3.1 Analysis of a Sandwich Shell with a Cockpit-Size Cutout and Bulkheads

The presence of the bulkheads increases the torsional stiffness of the sandwich shell. The circumferential displacement obtained at the loaded end for this shell is 10.43 mm. The value of the torsional stiffness K_θ for this shell is 2650 Nm/deg. The maximum radial displacement $u_{r_{max}}$ obtained for the sandwich shell is 17 mm. The presence of these bulkheads increase the torsional stiffness of the shell by 55%. This 55% improvement is bigger than the 24% improvement obtained

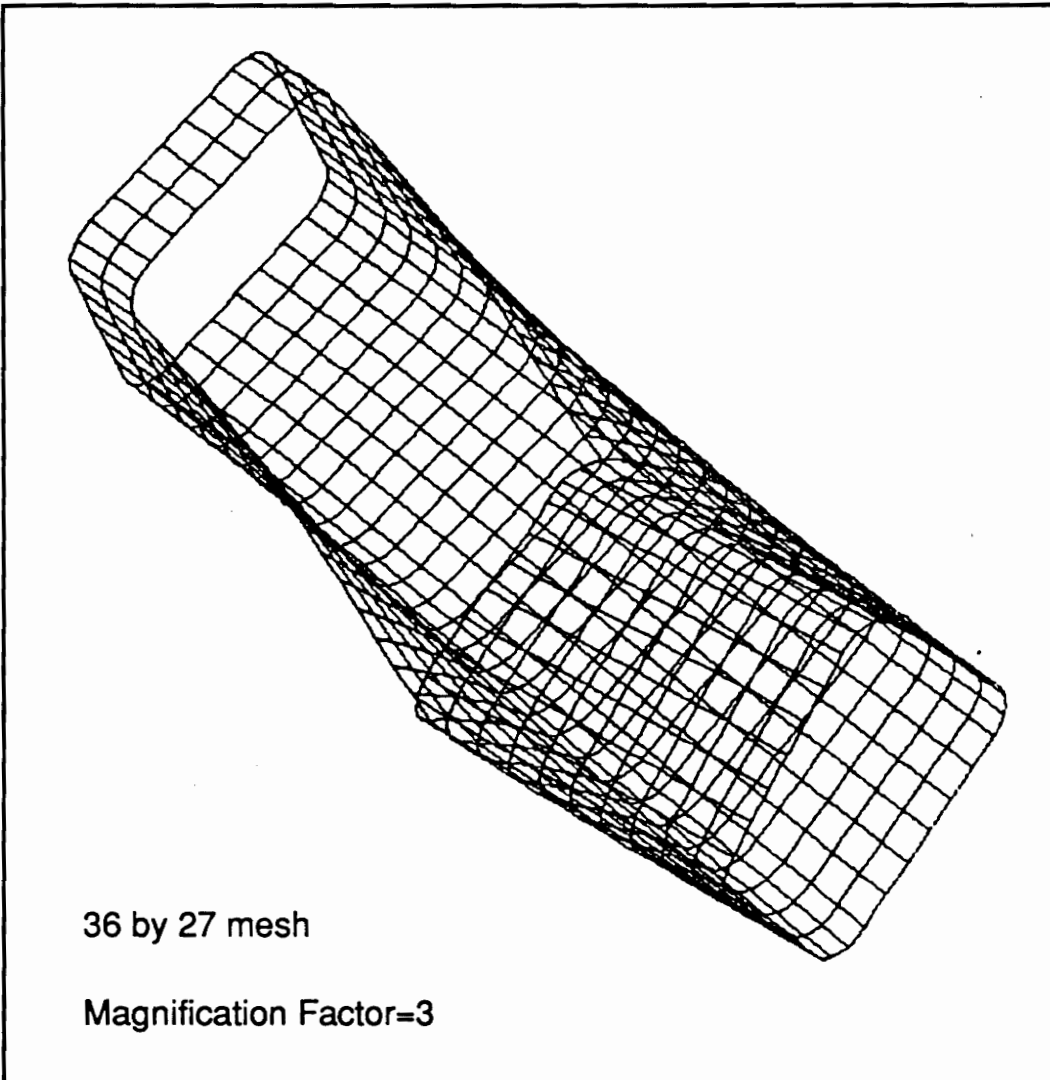


Figure 3.12 Deformed Shape for
the Equal-Bending-Stiffness Shell
with a Cockpit-Size Cutout

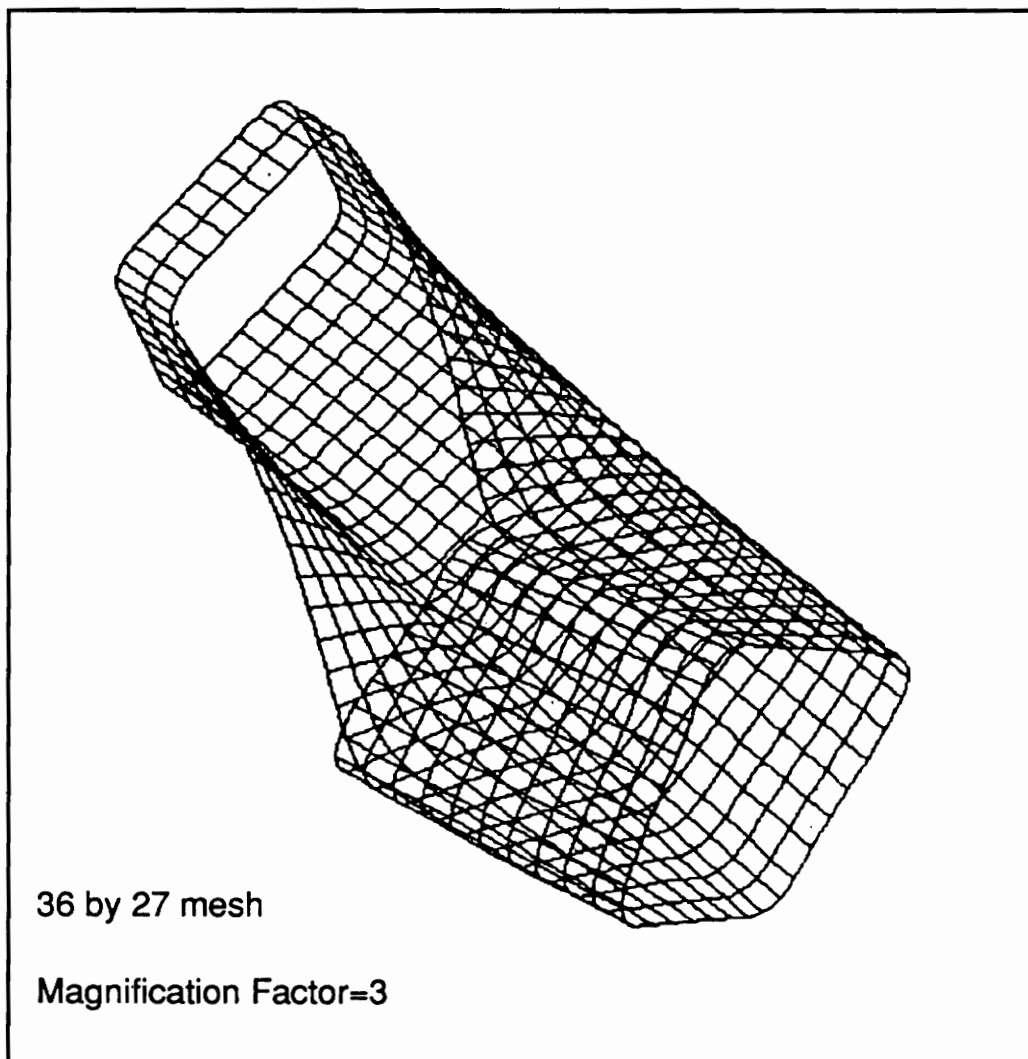


Figure 3.13 Deformed Shape for
the Equal-Weight Externally Stiffened Shell
with a Cockpit-Size Cutout

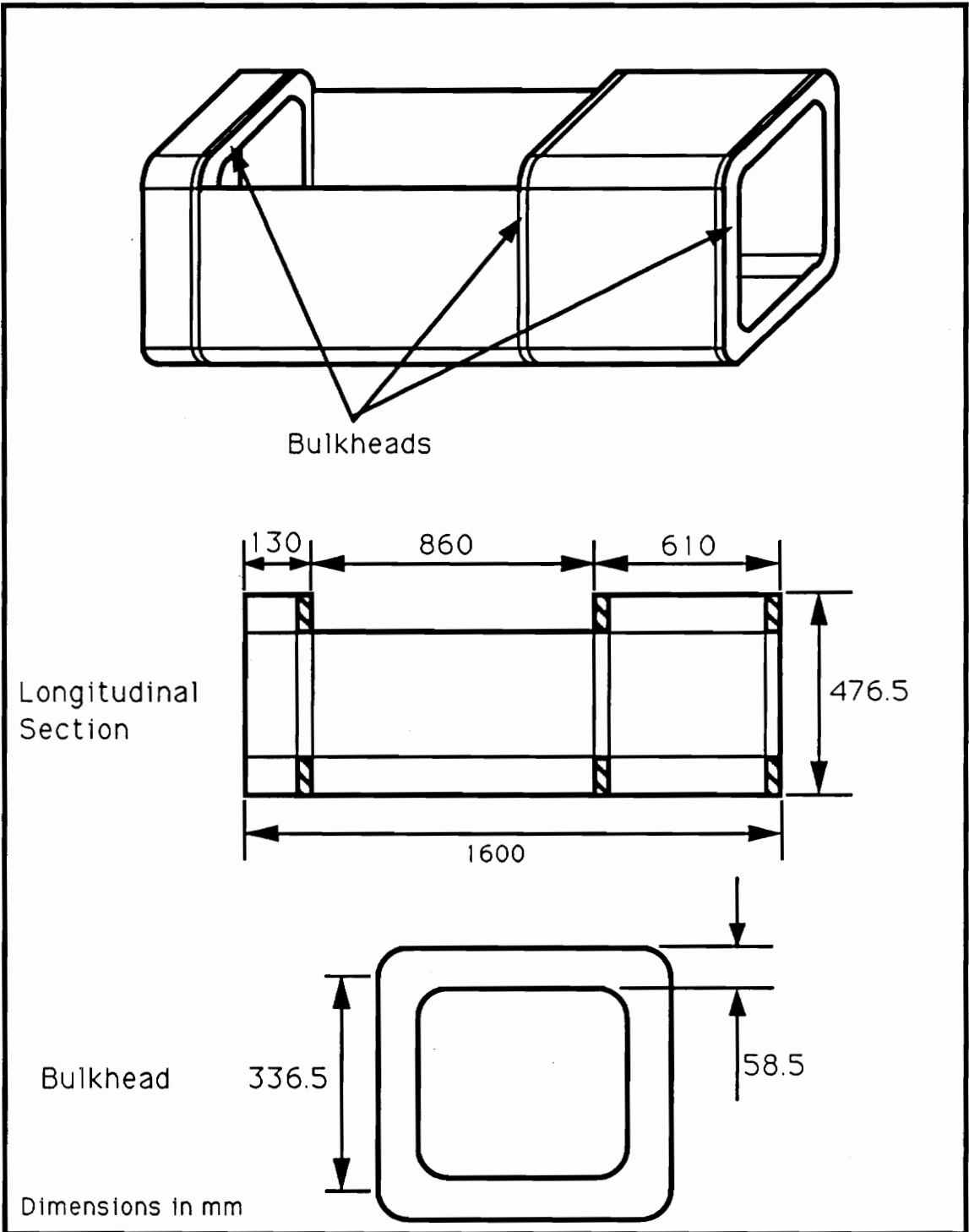


Figure 3.14 Square Cross-Section Cylindrical Shell with a Cockpit-Size Cutout and Bulkheads

for the circular cross-section shell. Thus, the presence of the bulkheads is more favorable for the square than for the circular cross-section shell. The 55% torsional stiffness improvement obtained with the bulkheads is clear if we compare the deformed shape of the sandwich shell with bulkheads with the deformed shape of the sandwich shell without bulkheads, shown in Figures 3.15 and 3.11, respectively.

3.6.3.2 Analysis of $\pm 45^\circ$ Helically Stiffened Shells with a Cockpit-Size Cutout and Bulkheads

We analyze the torsional behavior of two externally stiffened shells with a cockpit-size cutout and bulkheads, one with the equal-bending-stiffness stiffener-skin configuration and the other one with the equal-weight stiffener-skin configuration. These two configurations are shown in Table 3.1.

The presence of the bulkheads improves the value of torsional stiffness of the unreinforced shell by 73% for the equal-bending-stiffness and by 106% for the equal-weight shells. The value of torsional stiffness obtained for the equal-bending-stiffness shell is 2400 Nm/deg and for the equal-weight shell is 1800 Nm/deg. The improvement in torsional stiffness for the stiffened shells is bigger than the improvement obtained for the sandwich shell (55%). This difference in improvement is evident if we compare the deformed shapes of the stiffened shells, shown in Figures 3.16 and 3.17, with the deformed shape of the sandwich shell in Figure 3.15. Again, we find that the equal-bending-stiffness shell is 31% stiffer than the equal-weight shell, as shown in Table 3.4.

The improvement due to the bulkheads is bigger for the square than for the circular cross-section shells. For the equal-bending-stiffness square cross-section shell, we obtain a 73% improvement whereas for the equivalent circular cross-section shell the improvement is only 45%. Unlike the circular cross-section shells, with the presence of the bulkheads, the sandwich shell is still 11% stiffer than the stiffest stiffened shell. The circular cross-section shells are 5 and 7.5 times stiffer than the sandwich and equal-bending-stiffness square cross-section shells, respectively.

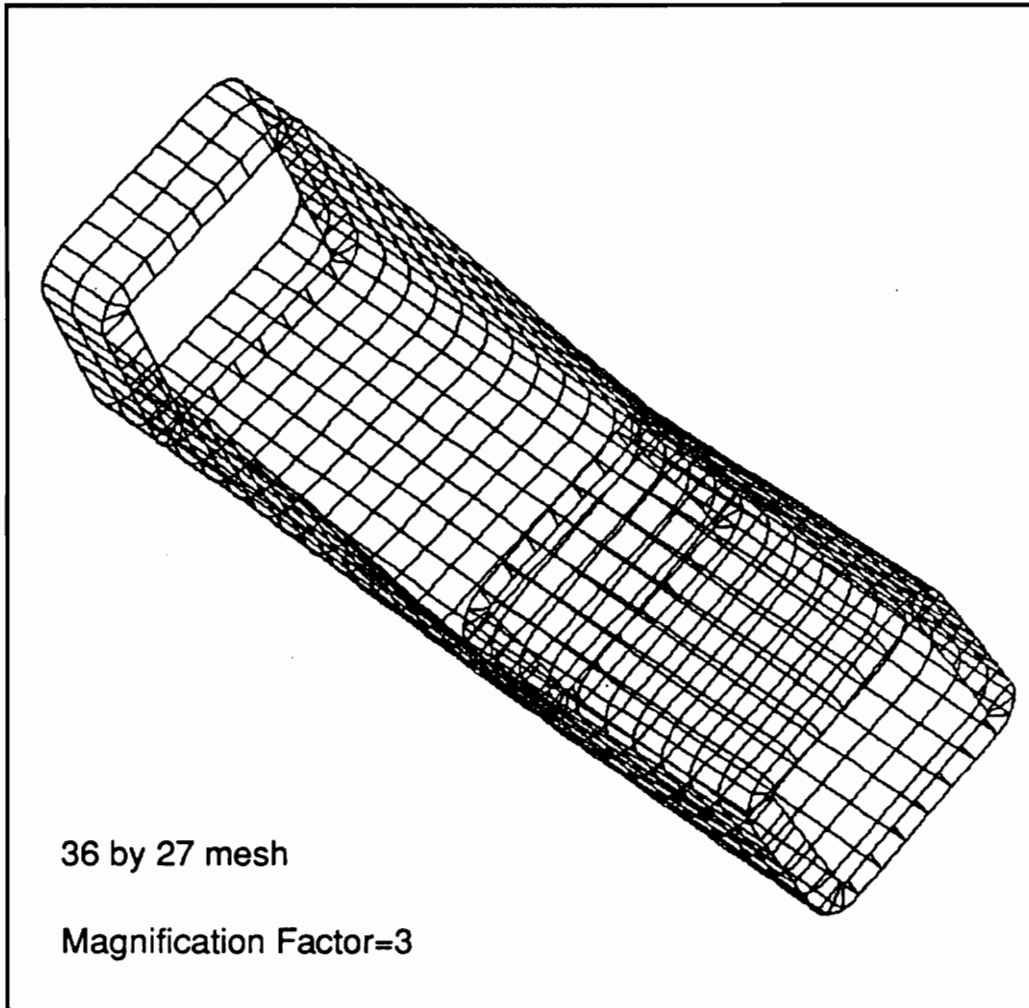


Figure 3.15 Deformed Shape for the Sandwich Shell
with a Cockpit-Size Cutout and Bulkheads

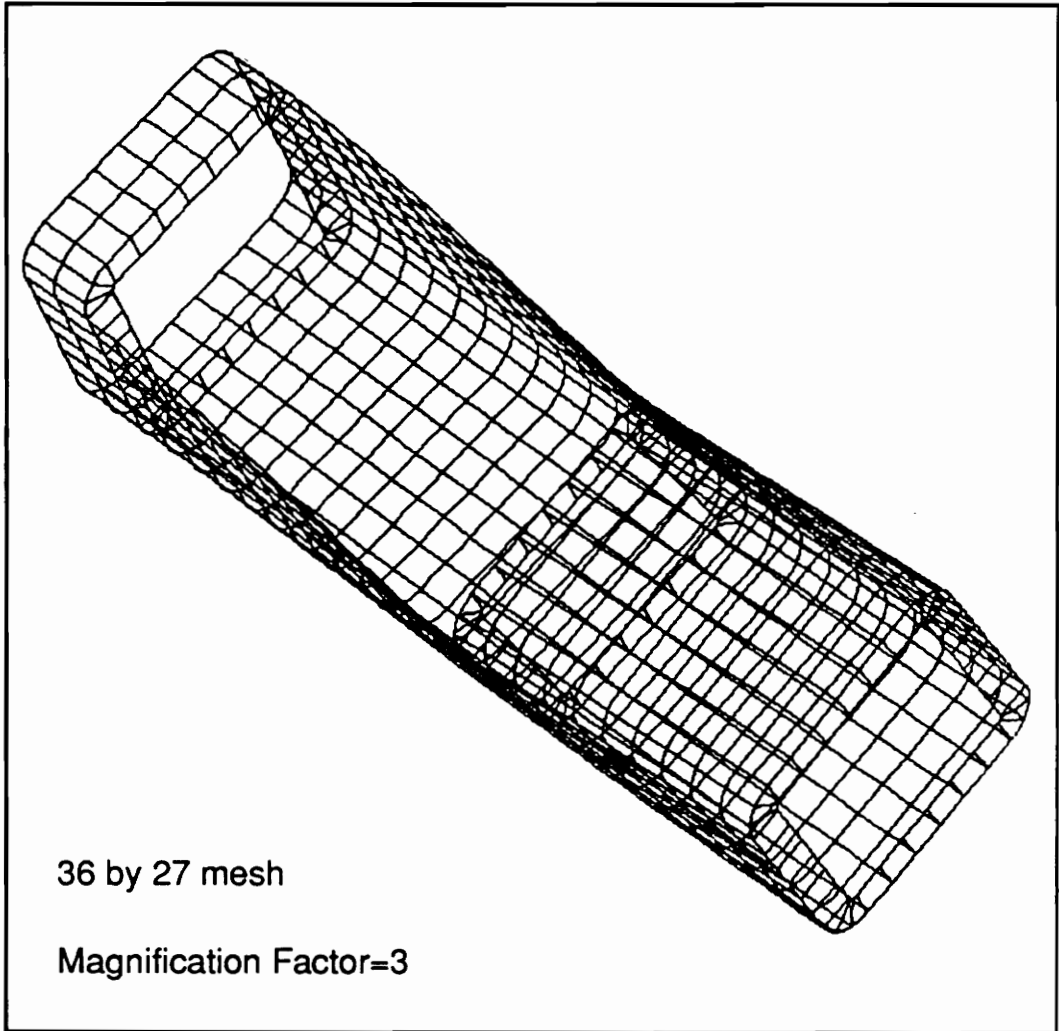


Figure 3.16 Deformed Shape for
the Equal-Bending-Stiffness Shell
with a Cockpit-Size Cutout and Bulkheads

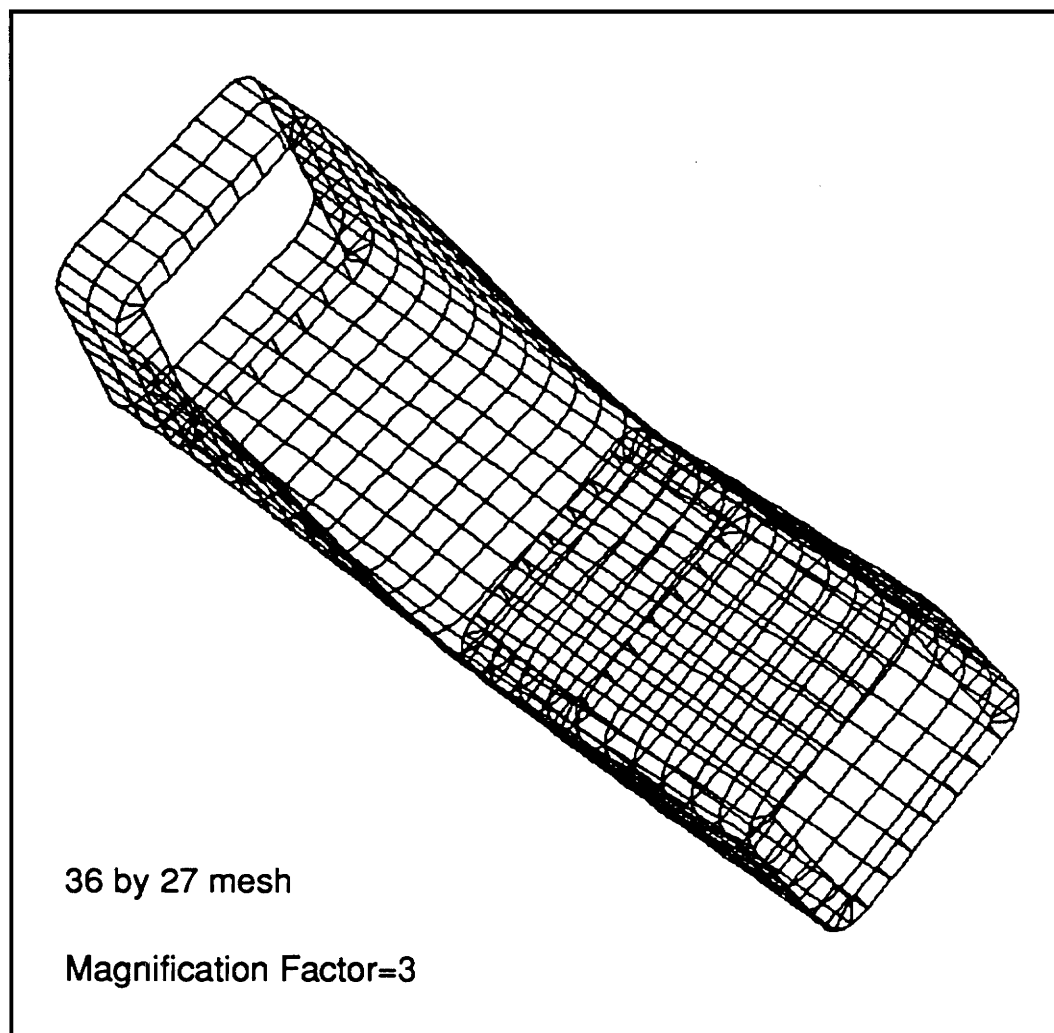


Figure 3.17 Deformed Shape for
the Equal-Weight Shell
with a Cockpit-Size Cutout and Bulkheads

Table 3.4 Results for the Torsion Problem for
Square Cross-Section Cylindrical Shells
with a Cockpit-Size Cutout and Bulkheads

Shell	u_{θ} (mm)	θ (deg)	K_{θ} (Nm/deg)	$u_{r \max}$ (mm)
Baseline Sandwich	10.43	2.5	2650	17
Equal-Bending Stiffness Externally Stiffened	11.56	2.8	2400	20.5
Equal-Weight Externally Stiffened	15.1	3.6	1800	27

3.6.4 Deformation Behavior of Sandwich and $\pm 45^\circ$ Helically Stiffened Square Cross-Section Cylindrical Shells with a Cockpit-Size Cutout, Bulkheads, and Stiffening of the Longitudinal Edges of the Cutout

In this Section, we reinforce the shells with bulkheads in Section 3.6.3 by adding a stiffener along the longitudinal edges of the cutout. The longitudinal reinforcements of the cutout are square cross-section stiffeners (25mm by 25mm) that run along the longitudinal edges of the cutout between the bulkheads located at the front and rear edges of the cutout. The centroidal line of these stiffeners coincides with the edge of the middle-plane of the shell skin. The locations of the longitudinal reinforcements and bulkheads are shown in Figure 3.18. The reinforcements of the longitudinal edges of the cutout are made with unidirectional AS-3501 graphite-epoxy tape whose fibers are aligned with the longitudinal axis of the stiffeners.

For the finite element modeling of the longitudinal reinforcements, 15 ABAQUS B32 elements are used. The ABAQUS B32 is a beam element with three nodes, quadratic interpolation, and six degrees of freedom per node. The meshes used for the shells are the same as for the case of shells with bulkheads only.

3.6.4.1 Analysis of a Sandwich Shell with a Cockpit-Size Cutout, Bulkheads, and Longitudinal Reinforcements

In this Section we analyze the shell of Section 3.6.3.1 with the addition of the longitudinal reinforcements. The circumferential displacement obtained at the loaded end is 7.86 mm and the torsional stiffness K_θ for this shell is 3510 Nm/deg. The maximum radial displacement $u_{r_{max}}$ obtained is 13.6 mm. The addition of these reinforcements increase the torsional stiffness by 33%. The improvement obtained for the square cross-section shell is practically the same as that obtained for the circular cross-section shell (29%).

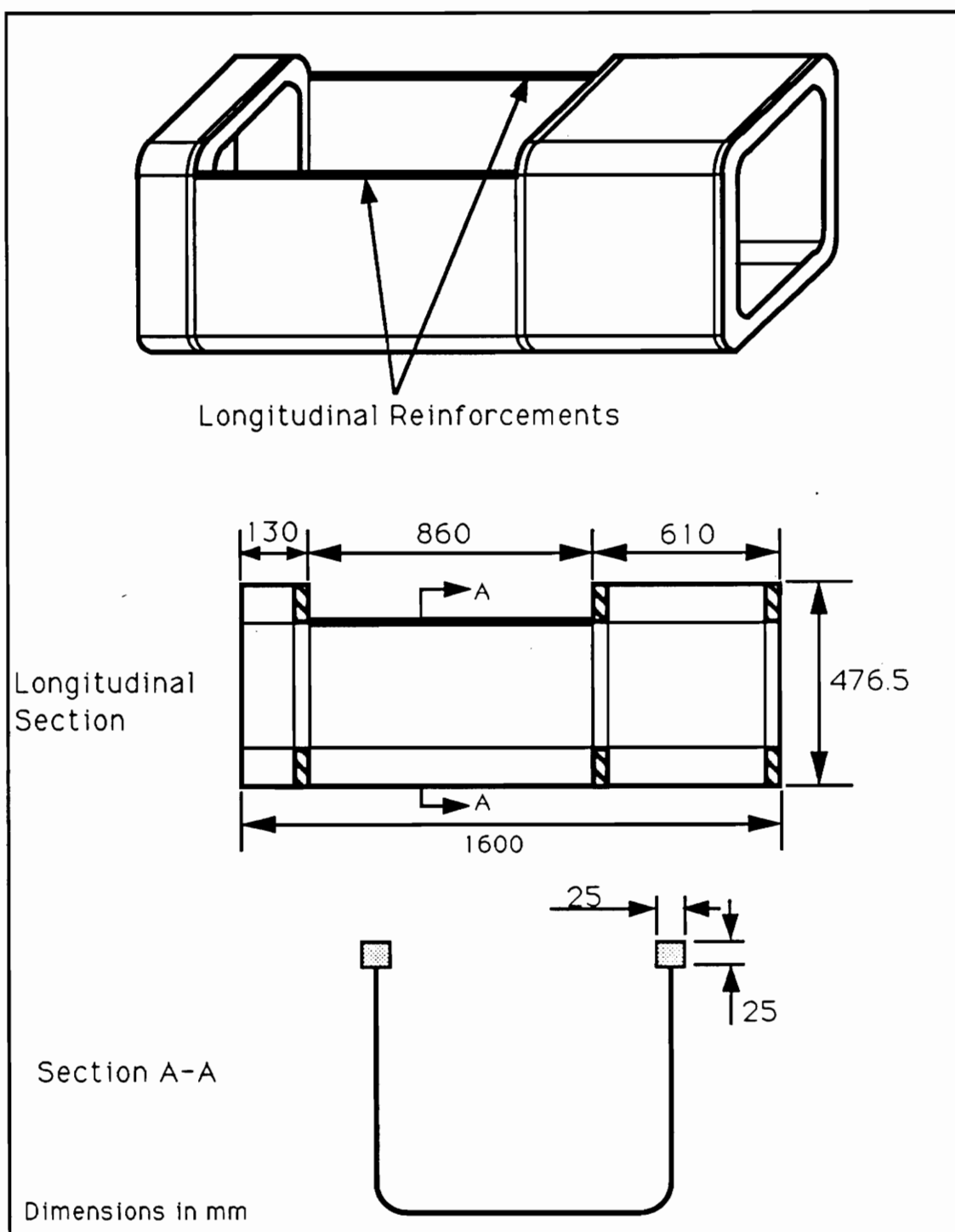


Figure 3.18 Square Cross-Section Cylindrical Shell with a Cockpit-Size Cutout, Bulkheads, and Longitudinal Reinforcements

The improvement obtained with the longitudinal reinforcements is clear if we compare the 45° view of the deformed shapes of the sandwich shell with and without longitudinal reinforcements, shown in Figures 3.19 and 3.15 respectively. The effect of warping that was so important in the case with no bulkheads and reinforcements (see Figure 3.11) tends to disappear as we add bulkheads and longitudinal reinforcements.

3.6.4.2 Analysis of $\pm 45^\circ$ Helically Stiffened Shells with a Cockpit-Size Cutout, Bulkheads, and Longitudinal Reinforcements

We analyze the torsional behavior of two externally stiffened shells with a cockpit-size cutout, bulkheads, and longitudinal reinforcements, one with the equal-bending-stiffness stiffener-skin configuration and the other one with the equal-weight stiffener-skin configuration. These two configurations are shown in Table 3.1.

The presence of the longitudinal reinforcements has increased the value of stiffness by around 57% for the equal-bending-stiffness and equal-weight shells. The value of torsional stiffness obtained for the equal-bending-stiffness shell is 3700 Nm/deg and 2880 Nm/deg for the equal-weight shell. As in the case of the shells with bulkheads only, the 57% torsional stiffness improvement obtained for the stiffened shells is bigger than for the sandwich shell (33%). As in the case of the sandwich shell, the effect of the warping that was so important in the shells without bulkheads and longitudinal reinforcements tends to disappear as we add bulkheads and longitudinal reinforcements. This improvement is evident in the deformed shapes of the stiffened shells with and without longitudinal reinforcements, shown in Figures 3.20, 3.21, 3.16, and 3.17, respectively. With the longitudinal reinforcements, the equal-bending-stiffness shell is stiffer than the equal-weight shell, as shown in Table 3.5. The difference in torsional stiffness between both shells is 29%, approximately the same difference as in the case with bulkheads only.

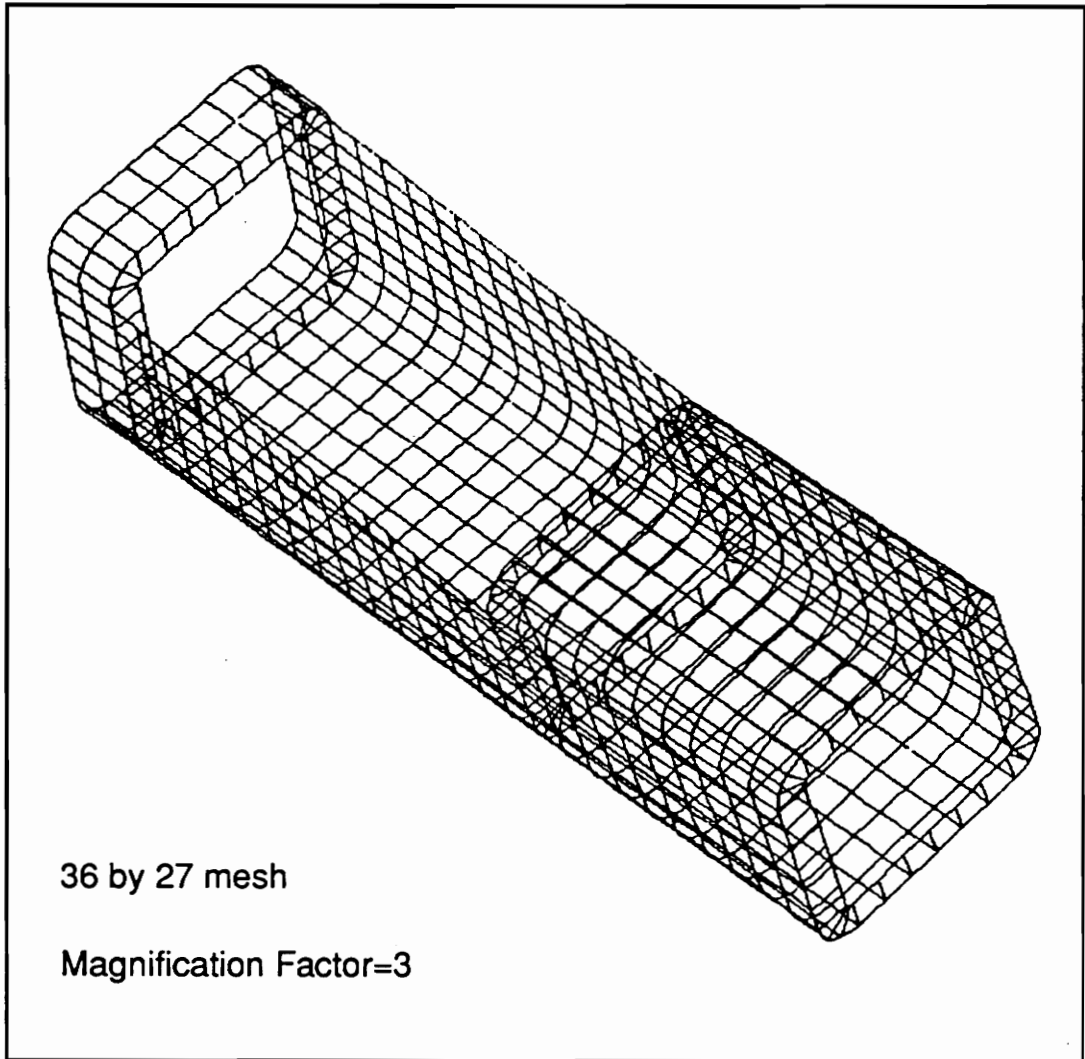


Figure 3.19 Deformed Shape for the Sandwich Shell
with a Cockpit-Size Cutout, Bulkheads,
and Longitudinal Reinforcements

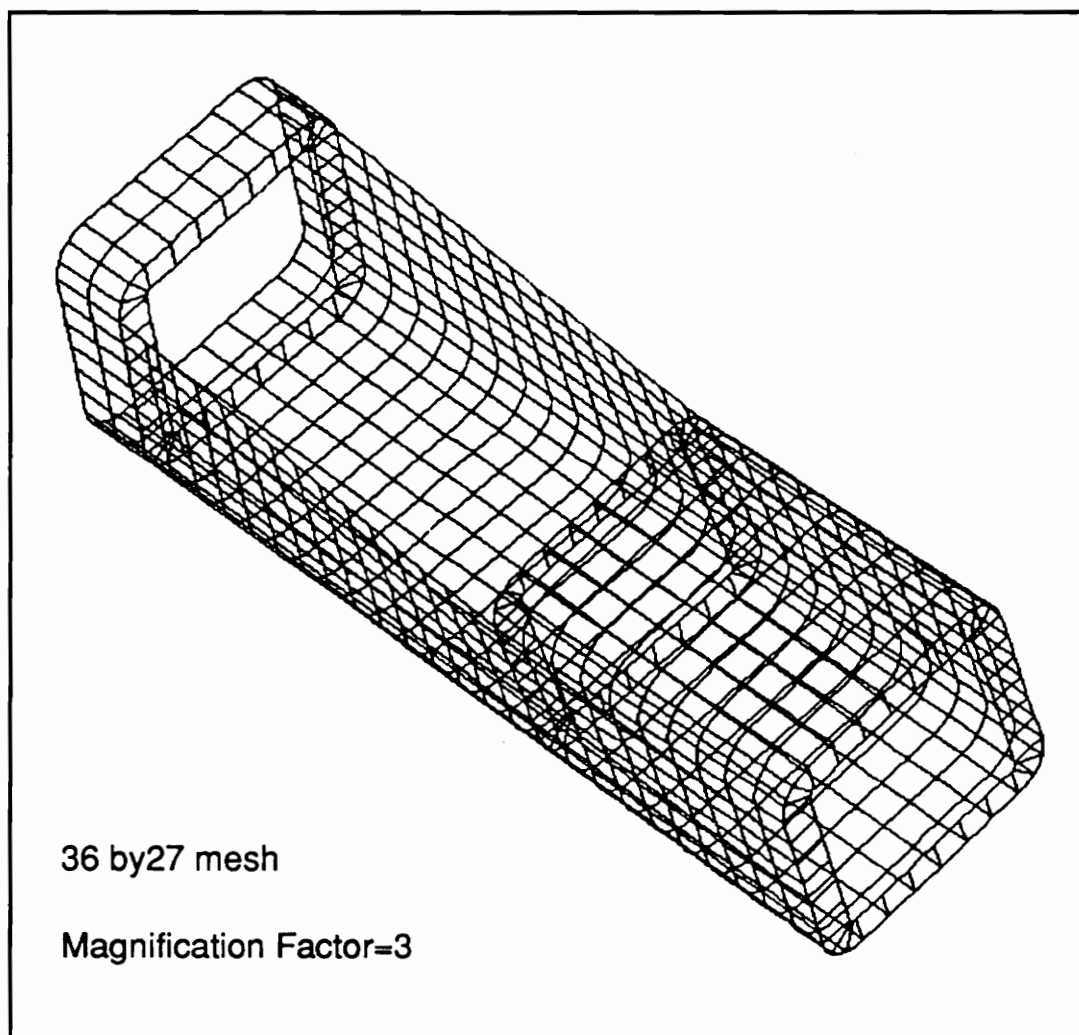


Figure 3.20 Deformed Shape for
the Equal-Bending-Stiffness Shell
with a Cockpit-Size Cutout, Bulkheads,
and Longitudinal Reinforcements

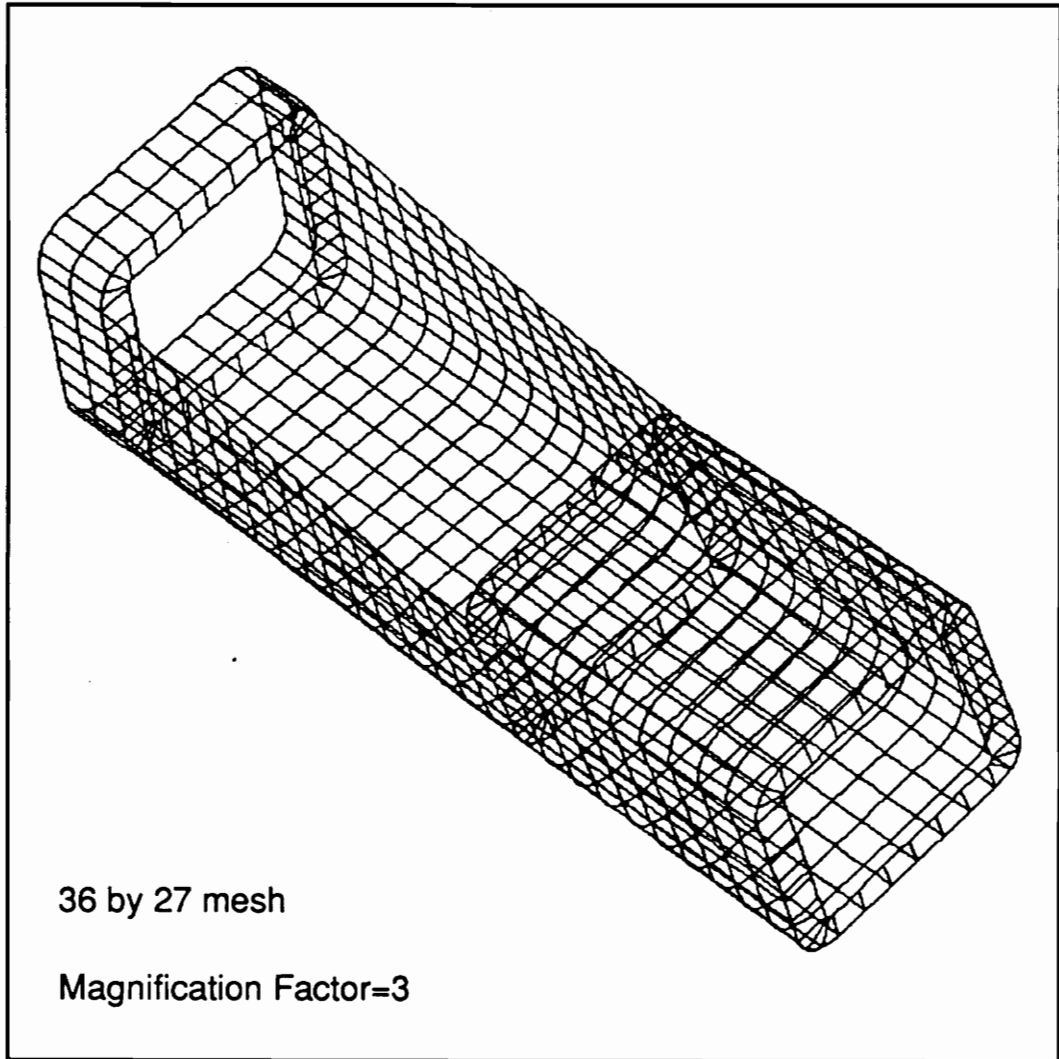


Figure 3.21 Deformed Shape for
the Equal-Weight Shell
with a Cockpit-Size Cutout, Bulkheads,
and Longitudinal Reinforcements

Table 3.5 Results for the Torsion Problem for Square Cross-Section Cylindrical Shells with a Cockpit-Size Cutout, Bulkheads, and Longitudinal Reinforcements

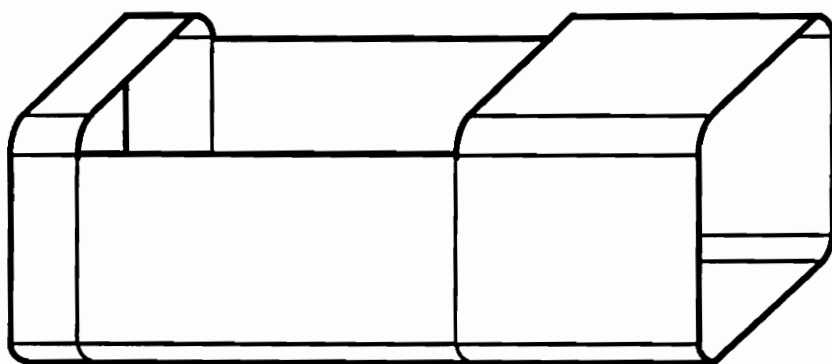
Shell Configurations	u_{θ} (mm)	θ (deg)	K_{θ} (Nm/deg)	$U_{r \max}$ (mm)
Baseline Sandwich	7.86	1.9	3510	14
Equal-Bending Stiffness Externally Stiffened	7.41	1.8	3700	15
Equal-Weight Externally Stiffened	9.58	2.3	2880	20

The improvement obtained with the addition of the longitudinal reinforcements for the circular cross-section shells is bigger than for the square cross-section shells. For the equal-bending-stiffness square cross-section shell, we obtain an improvement of 56% whereas, for the equal-bending-stiffness circular cross-section shell, the improvement is 134%. This big difference could be due to the fact that when we put the longitudinal stiffeners on the circular cross-section shell we diminish the bending deformation in the region of the cutout. Thus, this part of the shell will deform more in shear with a consequent increase in torsional stiffness. In the case of the square cross-section shell, although we diminish the bending deformation in the cutout region, still the walls of the cutout will bend, instead of shear, because they are flat.

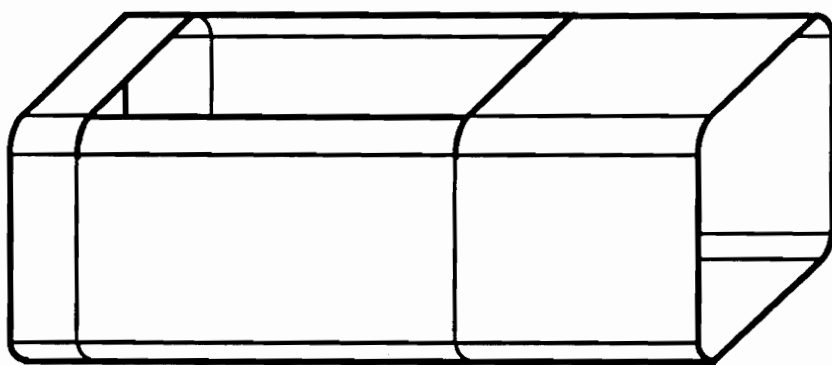
In the present case, the presence of the longitudinal reinforcements makes the equal-bending-stiffness stiffened shell slightly stiffer (only 6%) than the sandwich shell. Still the equal-weight stiffened shell is 22% less stiff than the sandwich shell. The circular cross-section sandwich shell is five times stiffer than the square cross-section sandwich shell. In the case of stiffened shells, the circular cross-section shells are more than eleven times stiffer than the square cross-section shells for both stiffened shells. These comparisons, confirm the explanation in the previous paragraph for the fact that longitudinal reinforcements are more useful when used on circular than on square cross-section shells.

3.6.5 Deformation Behavior of Sandwich and $\pm 45^\circ$ Helically Stiffened Square Cross-Section Cylindrical Shells with a Cockpit-Size Cutout with Curved Edges, Bulkheads, and Stiffening of the Longitudinal Edges of the Cutout

In this Section, we diminish the width of the cutout so that the walls of the shell in the vicinity of the longitudinal edges of the cutout now are curved instead of flat as in the shells studied up to this point. The shape of the shells with curved and flat walls at the cutout sides are shown in Figure 3.22. These curved edges provide walls at the cutout sides with an increased bending stiffness. These shells have bulkheads and longitudinal reinforcements. We study the effect of increasing the bending stiffness of the walls at the cutout sides.



Shell with Flat Cutout Longitudinal Edges



Shell with Curved Cutout Longitudinal Edges

Figure 3.22 Square Cross-Section Cylindrical Shells
with Flat and Curved Longitudinal Cutout Edges

The locations of the bulkheads is the same as in the shells with flat walls at the cutout sides. For the bulkheads, we use the same finite element modeling as in Section 3.6.3. The longitudinal reinforcements of the cutout are, as in Section 3.6.4, square cross-section stiffeners (25mm by 25mm) that run along the longitudinal edges of the cutout between the bulkheads located at the front and rear edges of the cutout. The centroidal line of these stiffeners coincides with the edge of the middle plane of the shell skin.

3.6.5.1 Analysis of a Sandwich Shell with a Cockpit-Size Cutout with Curved Edges, Bulkheads, and Longitudinal Reinforcements

The addition of the curved edges of the cutout to the sandwich shell yields a considerable improvement in torsional stiffness. The value of the circumferential displacement at the loaded end for this shell is 2.94 mm which yields a value of torsional stiffness of 9400 Nm/deg. The improvement in torsional stiffness achieved by having curved cutout longitudinal edges is 167%. Still, the circular cross-section shell is 83% stiffer than the square cross-section shell. Thus, the circular cross-section shell again is more efficient in torsion than the square cross-section shell.

3.6.5.2 Analysis of $\pm 45^\circ$ Helically Stiffened Shells with a Cockpit-Size Cutout with Curved Edges, Bulkheads, and Longitudinal Reinforcements

We analyze the torsional behavior of two externally stiffened shells with a cockpit-size cutout with curved edges, bulkheads, and longitudinal reinforcements: (1) equal-bending-stiffness shell, and (2) equal-weight shell. These two configurations are shown in Table 3.1.

The presence of the curved longitudinal edges of the cutout improves the value of stiffness of the shell with flat cutout edges by 300% and 310% for the equal-bending-stiffness and equal-weight shells, respectively. The value of torsional

stiffness obtained for the equal-bending-stiffness shell is 14800 Nm/deg whereas the equal-weight shell yields a torsional stiffness of 11900 Nm/deg. As in all the previous cases, the improvement in torsional stiffness for the stiffened shells is bigger than the improvement obtained for the sandwich shell (167%). Again, the equal-bending-stiffness shell is stiffer than the equal-weight shell, as shown in Table 3.6. The difference in torsional stiffness between both shells is 24%, which is slightly smaller than in the case with flat cutout edges. This time the equal-weight and equal-bending-stiffness shells are around 27% and 57% stiffer than the sandwich shell, respectively. Still, the circular cross-section shells are 185% and 177% stiffer than the square cross-section equal-bending-stiffness and equal-weight shells.

3.7 SUMMARY

In this Chapter, we analyzed the torsional behavior of square cross-section shells. From this analysis we conclude that the $\pm 45^\circ$ helical stiffening for square cross-section shells is not as efficient in torsional stiffness as for the circular cross-section shells. By using helical stiffening in a square cross-section shell, we were able to improve the torsional stiffness by only 26% whereas with a circular cross-section shell we can nearly double the torsional stiffness of the sandwich shell. In the next Chapter, we will determine which stiffnesses govern the torsional stiffness of circular and square cross-section shells with a cockpit-size cutout to see if there are better ways of increasing the torsional stiffness than with the $\pm 45^\circ$ helical stiffening.

Table 3.6 Results for the Torsion Problem for Square Cross-Section Cylindrical Shells with a Cockpit-Size Cutout, Bulkheads, Longitudinal Reinforcements, and Curved Longitudinal Cutout Edges

Shell	u_{θ} (mm)	θ (deg)	K_{θ} (Nm/deg)	$u_{r \max}$ (mm)
Baseline Sandwich	2.94	.71	9400	4.3
Equal-Bending Stiffness Externally Stiffened	1.87	.45	14800	3.8
Equal-Weight Externally Stiffened	2.32	.56	11900	4.8

Chapter 4

Further Considerations on the Torsional Stiffness of Helically Stiffened Circular and Square Cross-Section Shells with Cutouts

4.1 INTRODUCTION

In Chapters 2 and 3, we found that the torsional behavior of the $\pm 45^\circ$ helical stiffening is completely different for circular and square cross-section cylindrical shells. For the circular cross-section shells with a cockpit-size cutout, bulkheads, and longitudinal reinforcements using the helical stiffening, nearly twice the torsional stiffness of the sandwich shell is obtained without adding any weight to the structure. On the other hand, in Chapter 3, the use of the helical stiffening in square cross-section shells with a cockpit-size cutout, bulkheads, longitudinal reinforcements, and curved walls at the sides of the cutout increases the torsional stiffness over the value obtained for the sandwich shell only by 27% for an equal-weight stiffened shell. Furthermore, if the walls at the sides of the cutout are flat instead of curved, the sandwich shell is stiffer than the stiffened shell by 22%. In this case, the only way to get a bigger torsional stiffness than with the sandwich shell is to use the equal-bending-stiffness stiffener-skin configuration which is 30% heavier than the sandwich skin configuration. The torsional stiffness increase obtained over the torsional stiffness of the sandwich shell is only 6%. Moreover, the stiffest circular cross-section shell is 177% stiffer than the stiffest square cross-section shell (with equal weight) .

The values obtained for the square cross-section shells make the $\pm 45^\circ$ helical stiffening not as attractive as it was after the study of the circular cross-section shells where we doubled the performance of the sandwich shells. In this Chapter, we will study the reasons for the big decrease in torsional stiffness by investigating

which terms of the stiffness matrix dominate the behavior of the different stiffened shells. Once these stiffnesses are identified, we will try to see what improvements can be made to the $\pm 45^\circ$ helical stiffening concept.

4.2 TORSIONAL STIFFNESS OF SANDWICH AND $\pm 45^\circ$ HELICALLY STIFFENED CLOSED CIRCULAR CYLINDRICAL SHELLS

In this Section, we derive the expression for the torsional stiffness of a sandwich or $\pm 45^\circ$ helically stiffened closed circular cross-section cylindrical shell. We use the same approach used by Timoshenko [15] for circular cross-section tubes of isotropic material under torsion. First, we will develop the expression for the torsional stiffness for the helically stiffened shell. For the sandwich shell, we use the same expression of the stiffened shell, but we set width of the stiffeners b_s to zero and take the core as a layer of the skin with nearly zero in-plane properties.

Consider a circular tube of length L and external radius R_o with one end clamped and loaded with a torque at the other end as shown in Figure 4.1a. Assume that circular cross sections of the tube remain circular during torsion and that their diameters and the distance between them do not change. If we isolate the element ABCD in Figure 4.1a, we see that side BD rotates with respect to side AC through an angle $d\theta$ in Figure 4.1b, where θ is the angle rotated by any cross section with respect to the clamped end. Thus, the element is in a state of pure shear, and the magnitude of the shearing strain γ is found from the triangle BAB' as

$$\gamma = \frac{\overline{BB'}}{\overline{AB'}} \quad (58)$$

Because $\overline{BB'}$ is a small arc of radius R_o corresponding to the rotated angle $d\theta$, $\overline{BB'} = R_o d\theta$. Also, $\overline{AB'} = dx$, so

$$\gamma = R_o \frac{d\theta}{dx} \quad (59)$$

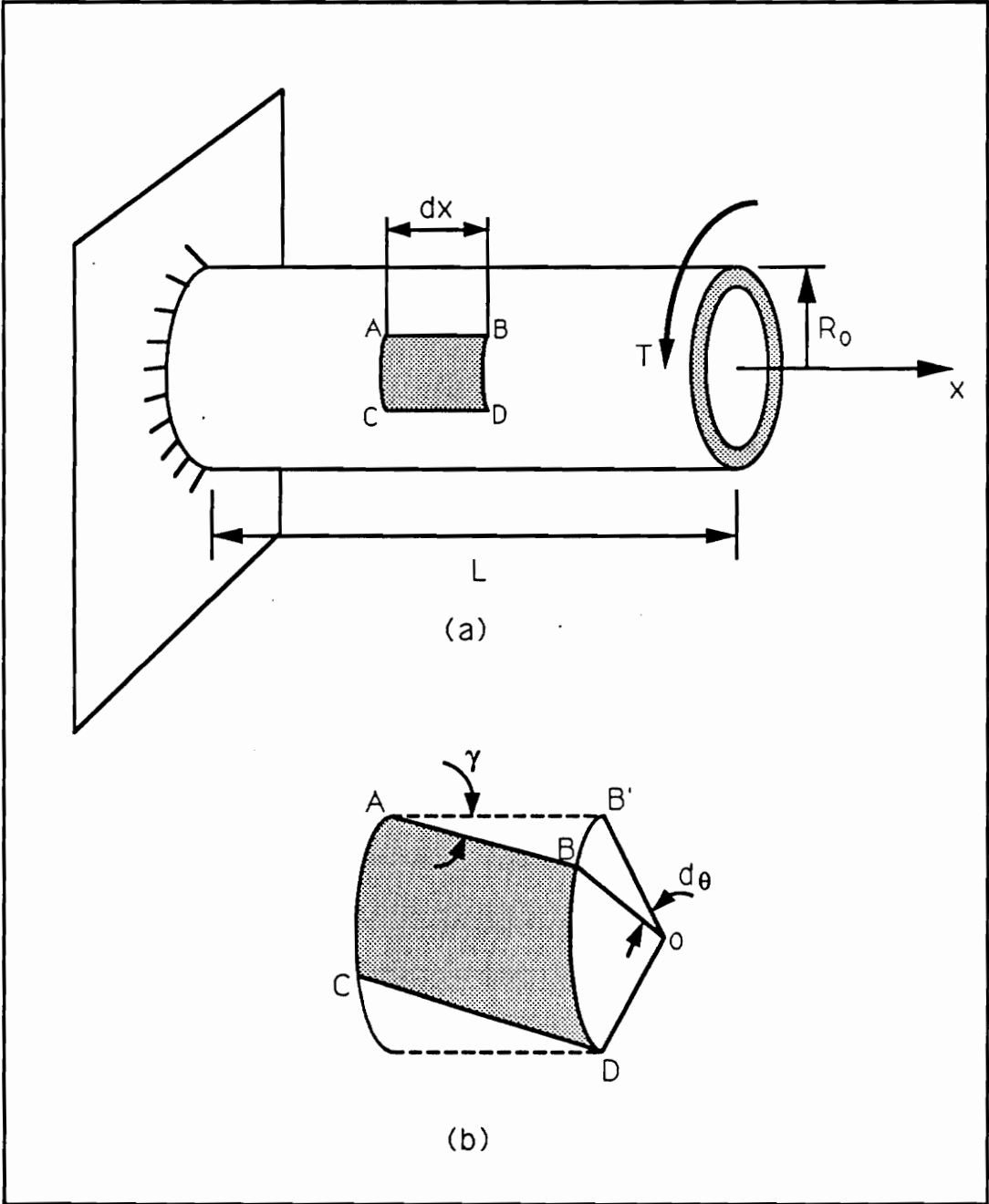


Figure 4.1 Circular Cylindrical Shell Under Torsion

For a tube twisted by a torque, the angle rotated θ is proportional to the length of the tube and the angle of twist per unit length $\frac{d\theta}{dx}$ is constant, so

$$\text{if } \frac{d\theta}{dx} = \phi \quad \Rightarrow \quad \gamma = R_o \phi \quad (60)$$

Now we look for a relationship between the applied torque and the stresses it produces in the skin and in the smeared stiffener grid. We apply the equations of equilibrium to the portion of the tube between the cross section m-n and the loaded end of the tube shown in Figure 4.2. The shearing stresses distributed over the cross section of the skin and of the smeared stiffener grid are statically equivalent to a couple equal and opposite to the applied torque T . For each element of area dA , the shearing force is τdA . The moment of this force about the axis of the tube is $\tau r dA$, so

$$dT = \tau_{xy} r dA \quad (61)$$

whereupon

$$T = \int_A \tau_{xy} r dA \quad (62)$$

In the case of a stiffened shell, the integral shown in Equation (62) is composed of two integrals, one over the area of the cross section of the skin of the shell and the other one over the area of the cross section of the smeared stiffeners, so

$$T = \int_{A_{\text{skin}}} \tau_{xy} r dA + \int_{A_{\text{stiff}}} \tau_{xy} r dA \quad (63)$$

where A_{skin} is the cross-sectional area of the skin of the shell and A_{stiff} is the cross-sectional area of the smeared stiffeners. Recall from Section 2.3.3 that we idealized the smeared stiffeners as a fictitious layer with the same properties of the actual stiffeners and with a thickness equal to the height of the stiffeners. Thus, the differential areas of the smeared stiffeners and of the skin are the cross-sectional area of cylinders with a thickness of differential radius of the stiffeners and of the skin.

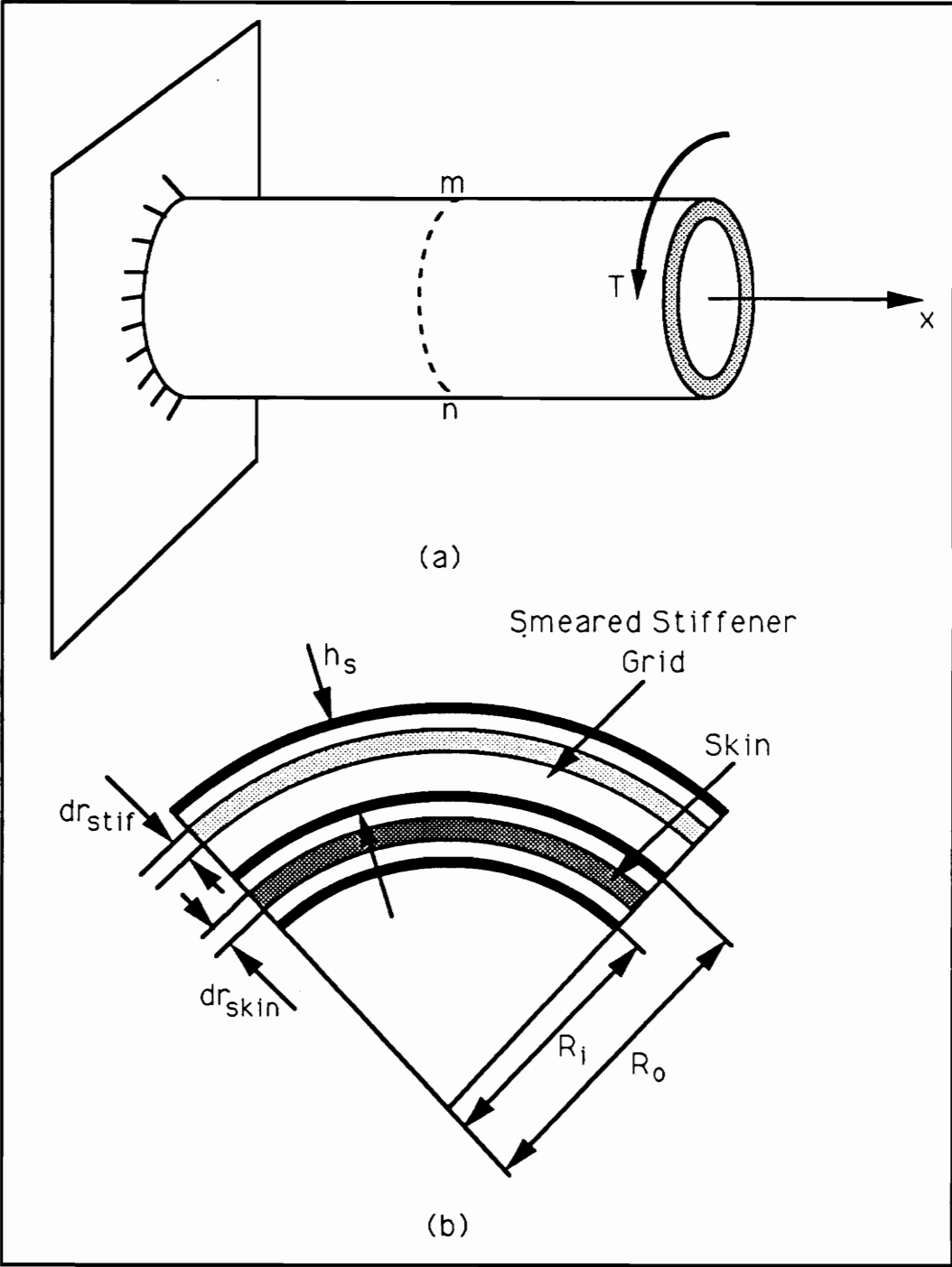


Figure 4.2 Helically Stiffened Circular Cylindrical Shell

The differential areas for the skin and for the smeared stiffeners are shown in Figure 4.2b and their expressions are

$$\begin{aligned} dA_{\text{skin}} &= 2\pi r dr & R_i &\leq r \leq R_o \\ dA_{\text{stiff}} &= 2\pi r dr & R_o &\leq r \leq R_o + h_s \end{aligned} \quad (64)$$

From the constitutive relationship for a lamina, the shearing stress for a skin lamina is $\tau = \bar{Q}_{66}\gamma$ and $\gamma = \phi r$. Thus, the shearing stress in a layer of the skin is $\tau = \bar{Q}_{66}r\phi$. In the same way as for the smeared stiffeners from Section 2.3.3, we know that the shearing stress is $\tau = \frac{b_s E}{2a} \phi r$. Substituting these expressions for the shearing stresses and the expressions of the differentials of area from Equations (64) in Equation (62) and in the case of the skin integrating for each layer, we get

$$T = 2\pi\phi \sum_{k=1}^n \bar{Q}_{66}^k \int_{r_{k-1}}^{r_k} r^3 dr + \frac{\pi b_s E \phi}{2a} \int_{R_o}^{R_o + h_s} r^3 dr \quad (65)$$

Performing the integration in Equation (64), we get

$$T = \phi \sum_{k=1}^n \bar{Q}_{66}^k \frac{\pi (r_k^4 - r_{k-1}^4)}{2} + \frac{b_s E \phi}{2a} \frac{\pi}{2} \left((R_o + h_s)^4 - R_o^4 \right) \quad (66)$$

where:

- $\frac{\pi (r_k^4 - r_{k-1}^4)}{2}$ = polar moment of inertia of each layer of the skin
- $\frac{\pi}{2} \left((R_o + h_s)^4 - R_o^4 \right)$ = polar moment of inertia of the smeared stiffeners

Finally, we know that the angle rotated at the loaded end $\theta = \phi L$ and that the torsional stiffness $K_\theta = T/\theta$. Thus, K_θ with the angle θ in degrees is

$$K_\theta = \frac{1}{180L} \left\{ \sum_{k=1}^n \bar{Q}_{66}^k \frac{\pi (r_k^4 - r_{k-1}^4)}{2} + \frac{b_s E}{2a} \frac{\pi}{2} \left((R_o + h_s)^4 - R_o^4 \right) \right\} \quad (67)$$

With this expression, we can calculate the torsional stiffness of sandwich or $\pm 45^\circ$

helically stiffened circular cylindrical shells. In the case of a sandwich shell, b_s would be zero and the core is treated as another layer of the skin but with nearly zero properties. We compared the results from Equation (67) for the closed circular cylindrical shells with the results obtained from the finite element analysis. The values of torsional stiffness given by Equation (67) were within 2% for the stiffened shell and within 0.1% for the sandwich shell.

One of the assumptions made in the derivation of Equation (67) was that the shell carried only in-plane shear forces. Looking at the values of the forces and moments obtained from ABAQUS for each element of the finite element meshes of the closed circular cross-section shells in Chapter 2, we see that this assumption is incorrect. From the results of the finite element analysis of the closed circular cross-section shells, we see that the shells carry not only in-plane shear forces but also twisting moments. Thus, why does Equation (67) give accurate results? The reason for the accuracy of Equation (67) can be found in the constitutive equations. For the case where the only applied forces and moments are N_{xy} and M_{xy} , the constitutive equations are

$$N_{xy} = \gamma_{xy} \bar{A}_{66} + \kappa_{xy} \bar{B}_{66} \quad (68)$$

$$M_{xy} = \gamma_{xy} \bar{B}_{66} + \kappa_{xy} \bar{D}_{66}$$

To be able to see how N_{xy} and M_{xy} affect the deformation of the shells, we invert Equations (68) to obtain expressions for γ_{xy} and κ_{xy} ,

$$\gamma_{xy} = N_{xy} \left[\frac{\bar{D}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] - M_{xy} \left[\frac{\bar{B}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] \quad (69-a)$$

$$\kappa_{xy} = -N_{xy} \left[\frac{\bar{B}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] + M_{xy} \left[\frac{\bar{A}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] \quad (69-b)$$

If we replace in Equations (69) the values of the stiffnesses of the sandwich skin and stiffener-skin configurations, we obtain,

- Equal-bending-stiffness stiffener-skin configuration:

$$\gamma_{xy} = .164 N_{xy} - .0014 M_{xy} \quad (70)$$

$$\kappa_{xy} = -.0014 N_{xy} + .0003 M_{xy}$$

- Equal-weight stiffener-skin configuration:

$$\gamma_{xy} = .02 N_{xy} - .002 M_{xy} \quad (71)$$

$$\kappa_{xy} = -.002 N_{xy} + .0008 M_{xy}$$

- Sandwich baseline skin configuration:

$$\gamma_{xy} = .05 N_{xy} \quad (72)$$

$$\kappa_{xy} = .0008 M_{xy}$$

Because the absolute values of N_{xy} and M_{xy} are of the same order, we see from Equations (70), (71), and (72) that the contribution of M_{xy} is negligible compared with the contribution of N_{xy} . Thus, the assumption made in the derivation of Equation (67) is correct.

Also, from Equations (69) we can see why the internally stiffened shells are always less stiff than the externally stiffened shells. In Chapter 2 the terms of the stiffness matrix of an internally stiffened shell are the same as for an externally stiffened shell except for the $\bar{B}_{ij}$ terms which are negative. This change of sign causes the second term and the first term of Equations (69-a) and (69-b), respectively, to be added instead of subtracted. Thus, because γ_{xy} and κ_{xy} are bigger for the internally stiffened shell than for the externally stiffened shell, the internally stiffened shell is less stiff than the externally stiffened shell.

Going back to Equation (67), we see that the torsional stiffness of the closed circular cylindrical shell depends on the ability of the wall of the shell to carry shear load. In the first term of Equation (67), $\bar{Q}_{66}$ is the reduced shear stiffness of each layer of the skin. In the case of an isotropic material, $\bar{Q}_{66}$ is the shear

modulus G . In the second term of Equation (66), we see that, instead of G , we have E , but if we recognize that in smeared fictitious layer the Poisson's ratio ν is zero, then $G = E/2$ which is what we have in Equation (67). Therefore, we see that obviously the closed circular cross-section stiffened shells are stiffer than the closed circular cross-section sandwich shells because of their increased shear stiffness. We can confirm this observation by comparing the in-plane shear stiffnesses of the sandwich skin configuration with that of the stiffener-skin configurations shown in Table 3.1.

4.3 TORSIONAL STIFFNESS OF CLOSED SQUARE CROSS-SECTION CYLINDRICAL SHELLS

In Section 4.2, we saw that the only forces acting on the wall of a closed circular cylindrical shell under torsion are in-plane shear loads. In closed square cross-section cylindrical shells with one end clamped, the walls of the shell carry shear loads as before. However, due to the shape of the cross section, there will also be in-plane normal loads and bending moments, especially near the clamped end where the shear forces due to torsion are minimum. Thus, in this Section, we investigate whether the stiffer shell would be the one with the biggest in-plane shear stiffness. We investigate the importance of the $\bar{A}_{66}$ term by evaluating the impact that other terms of the stiffness matrix have on the overall torsional stiffness of the shell.

To derive an expression for the torsional stiffness of a closed square cross-section shell is a complicated task due to the different type of forces acting in the walls of the shell along the axis of the shell that we can expect for these shells. Thus, we artificially change the values of the terms of the stiffness matrix of the stiffened shell one by one to the value that each term has in the sandwich skin configuration. After each change, we performed a finite element analysis using the fictitious stiffness matrix to find the change in the circumferential displacement at the loaded end of the shell. The stiffness terms that we change are $\bar{A}_{12}$, $\bar{A}_{66}$, $\bar{D}_{12}$, $\bar{D}_{66}$, and the $\bar{B}_{ij}$ terms which for the sandwich skin configuration are zero. All the terms are changed to the value that they would

have in the sandwich skin configuration (see Table 3.1). Of course, each time we artificially change the value of one term of the stiffness matrix, we obtain another fictitious stiffness matrix.

To see the effect of changing only one term of the stiffness matrix, we analyzed what happens with the constitutive equations when we make the change. As happened with the circular cross-section shells, for a square cross-section closed cylindrical shell under torsion, the only forces acting in the points of the longitudinal middle-line of the shell flat faces N_{xy} and M_{xy} . To confirm the presence of N_{xy} and M_{xy} only, we checked the values of the forces and moments of all the elements along the middle-line of one of the flat faces of the shell given by ABAQUS. All the forces but N_{xy} and M_{xy} pass through zero at the longitudinal middle-line of the faces of the square cross-section shells. Thus, the constitutive equations for the points of the longitudinal middle-line of each face of the shell are also given by Equation (68). With Equations (69) we can see what happens when we change the value of one term of the stiffness matrix (Equations (69) are the inverse of Equations (68)): In Equations (69), we see that the term $[\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2]$ can change its sign or even turn zero depending on the values of the stiffnesses involved. This explains the artificial nature of arbitrarily changing only one term of the stiffness matrix. When the $[\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2]$ changes sign, the direct terms of Equations (69) become negative and the coupling terms become positive which is completely unrealistic. When the $[\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2]$ term is zero, the terms of Equations (69) are indeterminate. Thus, from the approach of changing the values of the stiffness matrix, we can only find which terms govern the deformation behavior. It is not realistic to analyze the effect on the deformation behavior of the reduction or increase of each term because we are then analyzing a stiffer skin configuration that does not exist and that has an artificial (unrealistic) stiffness matrix that will yield unrealistic results. Of course, we can use the results given by this approach as guidelines for how to start attacking the problem.

The terms of the stiffness matrix that have an important impact on the deformation of the closed square cross-section shell are $\bar{A}_{66}$, $\bar{D}_{66}$, and the bending-extension coupling terms $\bar{B}_{ij}$ as shown in Table 4.1. In Table 4.1, we list:

Table 4.1 Results for
Closed Square-Cross-Section Cylindrical Shells

Skin Configuration	Circumferential Displacement u_θ (mm)	Δu_θ (%)
Real Skin Configurations	Sandwich = 1.260 Stiffened = .428	—
No Coupling $B_{ij}=0$	.3892	-9
Equal A_{12}	.4151	-3
Equal A_{66}	1.024	139
Equal D_{12}	.4349	1.6
Equal D_{66}	.3685	-14

(1) The values of u_θ obtained for the sandwich and equal-bending-stiffness shells with the "real stiffener-skin configurations" used in Chapters 2 and 3; (2) The values of u_θ obtained for the different fictitious stiffness matrices with one of the terms changed to the sandwich skin configuration value; and (3) The increase or decrease that the change produced in u_θ from the value obtained for the "real" stiffened shell.

By setting the coupling terms $\bar{B}_{ij}$ to zero, we see in Table 4.1 that the displacement decreases 9%. This result is realistic because we can make all the $B_{ij}=0$ by having the midlines of the stiffeners aligned with the middle surface of the shell skin. Terms $\bar{A}_{12}$ and $\bar{D}_{12}$ do not have an impact of more than 3% which confirms the validity of Equations (68). Also, $\bar{A}_{66}$ and $\bar{D}_{66}$ are important because of the large change obtained in the circumferential displacement.

From the values obtained, we see that when we decrease the in-plane shear stiffness $\bar{A}_{66}$, we increase the circumferential displacement by 139%. On the other hand, if we decrease $\bar{D}_{66}$, we diminish the circumferential displacement by 14% which means that, by diminishing one of the stiffnesses of the shell, we stiffen the shell. To explain this strange behavior, we go back to Equations (69) and we see what happens when we change $\bar{D}_{66}$. Using the values of the stiffnesses of the sandwich skin and stiffener-skin configurations we have for the real stiffener-skin configuration $[\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2] = 311,300$ whereas for the fictitious equal $\bar{D}_{66}$ skin configuration $[\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2] = -62,400$. Thus, Equations (69) for the real and fictitious skin configurations are,

- Real Stiffener-Skin Configuration:

$$\gamma_{xy} = .164 N_{xy} - .0014 M_{xy}$$

$$\kappa_{xy} = -.0014 N_{xy} + .0003 M_{xy}$$

- Equal D_{66} Fictitious Skin Configuration:

$$\gamma_{xy} = -.02 N_{xy} + .007 M_{xy}$$

$$\kappa_{xy} = .007 N_{xy} - .0016 M_{xy}$$

We see the reduction of $\bar{D}_{66}$ produces a change in the signs of Equations (69). Thus, the only conclusion that we can draw from the analysis of the equal $\bar{D}_{66}$ fictitious skin configuration is that $\bar{D}_{66}$ has an important role in the deformation behavior of the shell. We also studied what happens when we have equal $\bar{D}_{66}$ and no coupling ($\bar{B}_{ij}=0$). For this case, we obtained a value of circumferential displacement of 0.39 mm that is the same as the value obtained for the shell with no coupling ($\bar{B}_{ij}=0$). Thus, if there is no coupling in the square cross-section shell, the torsional deformation depends only on the value of $\bar{A}_{66}$. On the other hand, when the $\bar{B}_{ij}$ terms are different from zero, we have coupling between in-plane shear and twisting. Another case that we could analyze is when we have an internally stiffened shell for which the $\bar{B}_{ij}$ terms are negative. Thus, in Equations (69) we see that the negative sign of the $\bar{B}_{ij}$ increases the value of γ_{xy} and κ_{xy} . Thus, the torsional stiffness of the internally stiffened shell is lower than for the externally stiffened shell.

From all that was said in the foregoing, we conclude that the best way of building a square cross-section shell for torsional resistance is to use $\pm 45^\circ$ helically stiffened construction but without coupling. To make the $\bar{B}_{ij}$ coupling terms zero, the midplane of the stiffener grid must coincide with the midplane of the skin of the shell (the eccentricity of the stiffeners relative to the skin midplane must be zero). By using the $\pm 45^\circ$ helical stiffening without coupling, we get the biggest possible value of $\bar{A}_{66}$, and we do not have the bad effect of the coupling. The only problem is that the construction of a shell without coupling is more complicated and costly than the construction of an externally stiffened shell. The added difficulty is that in the case of no coupling we have stiffeners on both sides of the skin, and this complicates the construction of the mold and the lay-up of the stiffeners.

4.4 TORSIONAL STIFFNESS OF CIRCULAR AND SQUARE CROSS-SECTION CYLINDRICAL SHELLS WITH A COCKPIT-SIZE CUTOUT

In Chapter 3, we found that the torsional stiffness of square cross-section sandwich shells with a cockpit-size cutout is 93% higher than the torsional stiffness of the

equal-weight stiffened shell. In Chapter 2, the equal-bending-stiffness circular cross-section stiffened shell was only 11% stiffer than the sandwich shell. In this case, the torsional stiffness difference between the sandwich and stiffened shells decreased considerably due to the presence of the cutout. Thus, in this Section, we perform the same analysis as done for the closed square cross-section shell in Section 4.3. We find the terms of the stiffness matrix that govern the torsional stiffness of the shell and determine possible changes to improve the stiffening of the shells.

The terms of the stiffness matrix with an important contribution to the torsional stiffness for both shells are $\bar{A}_{12}$, $\bar{A}_{66}$, $\bar{D}_{66}$, and the bending-extension coupling terms $\bar{B}_{ij}$ as shown in Table 4.2. The condition of no coupling ($\bar{B}_{ij}=0$) has the effect of diminishing the value of the circumferential displacement for both shells, but the decrease in u_θ is different. For the circular cross-section shell, there is a decrease in u_θ of only 6%, whereas for the square cross-section shell we have a 15% decrease. As happened in Sections 4.2 and 4.3 for closed shells, a decrease in $\bar{A}_{66}$ produces an important increase in u_θ for both shells. For the square cross-section shell, we even get a negative value of circumferential displacement. The reason for this strange behavior could be that when we change $\bar{A}_{66}$ we have the same problem that we had with the closed shells when we changed $\bar{D}_{66}$ (see Section 4.3).

In the case of shells with a cutout, the $\bar{A}_{12}$ term has a very important effect in the torsional stiffness. For the square and circular cross-section open shells, a decrease in $\bar{A}_{12}$ increases the torsional stiffness of the shells by approximately 50%. Due to the very complicated distribution of forces along the open shells, it is very difficult to try to explain the reason for this behavior.

For the circular and square cross-section open shells, a decrease in $\bar{D}_{66}$ produces an important decrease (around 600%) in torsional stiffness. A possible explanation for this behavior is that during the torsional deformation of the open shells the walls of the cutout are twisted. Thus, when we decrease $\bar{D}_{66}$, we decrease the overall stiffness of the shell. For the closed square cross-section shell, we analyzed the case of no coupling and equal $\bar{D}_{66}$. Decrease of $\bar{D}_{66}$ decreased

Table 4.2 Results for Circular and Square-Cross-Section Cylindrical Shells with a Cockpit-Size Cutout

Skin Configuration	Circumferential Displacement u_θ (mm)	Δu_θ (%)
Circular Cylindrical Shell with a Cockpit-Size Cutout		
Real Skin Configurations	Sandwich = 2.959 Stiffened = 2.653	—
No Coupling $B_{ij} = 0$	2.492	-6
Equal A_{12}	1.306	-50
Equal A_{66}	4.707	77
Equal D_{12}	2.721	2.6
Equal D_{66}	17.41	556
Square Cylindrical Shell with a Cockpit-Size Cutout		
Real Skin Configurations	Sandwich = 16.1 Stiffened = 20	—
No Coupling $B_{ij} = 0$	17.03	-15
Equal A_{12}	10.12	-49
Equal A_{66}	-326.4	
Equal D_{12}	.1938	-3
Equal D_{66}	149.9	645

the torsional stiffness of the circular and square cross-section shells by 43% and 63%, respectively. Thus, the influence of $\bar{D}_{66}$ is bigger for the square cross-section shell than for the circular cross-section shell. Also, we see that the presence of the bending-extension coupling accentuates the influence of the twist stiffness.

Finally, we can draw the following three conclusions: (1) For square and circular cross-section open shells, bending-extension coupling diminishes the torsional stiffness of the shell, and its effect is bigger for the square cross-section shells than in the circular cross-section shells; (2) The $\bar{A}_{12}$ term is inversely proportional to the torsional stiffness of the shells, and its effect is the same for both shells; and (3) The decrease of $\bar{A}_{66}$ and $\bar{D}_{66}$ decreases the stiffness of the shells. From the conclusions of the preceding paragraph, in order to increase the torsional stiffness of an open shell, we should try to increase $\bar{A}_{66}$ and $\bar{D}_{66}$ while decreasing $\bar{A}_{12}$. It is not possible to increase $\bar{A}_{66}$ and $\bar{D}_{66}$ and to decrease $\bar{A}_{12}$ at the same time. Also, because all the skin layers and the stiffeners are oriented at $\pm 45^\circ$, we have already maximized $\bar{A}_{66}$ and $\bar{D}_{66}$. Thus, now we investigate what happens when we decrease $\bar{A}_{12}$. One way of decreasing $\bar{A}_{12}$ is to change the angle of the stiffeners. For example, using longitudinal and circumferential stiffeners instead of helical stiffeners. In this case, the stiffeners do not contribute to the $\bar{A}_{12}$, $\bar{D}_{12}$, $\bar{A}_{66}$, $\bar{D}_{66}$, $\bar{B}_{12}$, and $\bar{B}_{66}$ terms of the stiffness matrix. In the next Section, we will investigate whether we can increase the torsional stiffness of the circular and square cross-section shells with a cockpit-size cutout by replacing the helical stiffeners with longitudinal and circumferential stiffeners (0/90 waffle stiffening).

4.5 ANALYSIS OF CIRCULAR AND SQUARE CROSS-SECTION 0/90 WAFFLE SHELLS

In this Section, we investigate the use of longitudinal and circumferential stiffeners (0/90 waffle stiffening) for the circular and square cross-section shells with a cockpit-size cutout with and without reinforcements and we compare results with the $\pm 45^\circ$ helically stiffened shells ($\pm 45^\circ$ waffle shells). We perform this study because in Section 4.4 we saw that the decrease of $\bar{A}_{12}$ increases the

torsional stiffness of the open shells. We can not decrease $\bar{A}_{12}$ alone, so we must decrease the $\bar{A}_{12}$ and $\bar{D}_{12}$ terms together. With the change in the angle of the stiffening to 0° and 90° , we diminish the $\bar{A}_{12}$ and $\bar{D}_{12}$ terms of the stiffness matrix but we also diminish $\bar{A}_{66}$ and $\bar{D}_{66}$. The decrease of $\bar{A}_{12}$, $\bar{D}_{12}$, $\bar{A}_{66}$, and $\bar{D}_{66}$ occurs because in the case of the axial and circumferential stiffeners, the stiffeners do not contribute to the 1-2 and 6-6 terms. In the case of $0/90$ waffle shells, the $\bar{A}_{12}$, $\bar{D}_{12}$, $\bar{A}_{66}$, $\bar{D}_{66}$, $\bar{B}_{12}$, and $\bar{B}_{66}$ terms of the stiffness matrix are those of the skin alone except $\bar{D}_{66}$ which has the contribution of the torsional stiffness of the stiffeners. The $\bar{A}_{11}$, $\bar{D}_{11}$, $\bar{A}_{22}$, $\bar{D}_{22}$, $\bar{B}_{11}$, and $\bar{B}_{22}$ terms have contributions from the stiffeners and from the skin. The expressions for the terms of the $0/90$ waffle shell are [20],

$$\begin{aligned}\bar{A}_{11} &= A_{11} + \frac{E A_s}{b} & \bar{A}_{22} &= A_{22} + \frac{E A_s}{a} \\ \bar{A}_{12} &= A_{12} & \bar{A}_{66} &= A_{66} \\ \bar{B}_{11} &= \frac{E \bar{z} A_s}{b} & \bar{B}_{22} &= \frac{E \bar{z} A_s}{a}\end{aligned}\tag{68}$$

$$\bar{B}_{12} = \bar{B}_{66} = \bar{B}_{16} = \bar{B}_{26} = \bar{A}_{16} = \bar{A}_{26} = 0$$

$$\begin{aligned}\bar{D}_{11} &= D_{11} + \frac{E I}{b} & \bar{D}_{22} &= D_{22} + \frac{E I}{a} \\ \bar{D}_{12} &= D_{12} & \bar{D}_{66} &= D_{66} + \frac{G J}{a} \\ \bar{D}_{16} &= D_{16} & \bar{D}_{26} &= D_{26}\end{aligned}\tag{68}$$

where a and b are the stiffener spacings in the longitudinal and circumferential directions, respectively, and $\bar{z}$ is the distance from the centroid of the stiffener to the middle surface of the skin.

4.5.1 Analysis of Circular and Square Cross-Section 0/90 Waffle Cylindrical Shells with a Cockpit-Size Cutout

In this Section, we analyze circular and square cross-section shells with a cockpit-size cutout with 0/90 waffle stiffening. The dimensions and shape of these shells are the same that we used in Chapters 2 and 3 and are shown in Figures 2.32 and 3.18. We compare the torsional behavior of three different 0/90 waffle stiffener-skin configurations that we will call A, B, and C, respectively. All stiffener-skin configurations have the same weight as the sandwich baseline skin configuration. Configurations A and B have the same $[\pm 45]_{3s}$ skin stacking sequence that we used for the helically stiffened shells and the same stiffener cross section. Stiffener-skin configuration A has a longitudinal stiffener spacing a bigger than the circumferential spacing b whereas in stiffener-skin configuration B the spacing b is bigger than a . This difference causes the A_{11} , D_{11} , and B_{11} terms in stiffener-skin configuration A to be smaller than the A_{22} , D_{22} , and B_{22} whereas in stiffener-skin configuration B the opposite happens. Stiffener-skin configuration C has a $[0/90]_{3s}$ skin stacking sequence, and the stiffener cross-section and spacings are the same used in configuration A. The reason for changing the skin stacking sequence is to lower $\bar{A}_{12}$ as much as possible because in Section 4.4 we saw that decreasing $\bar{A}_{12}$ produces a torsional stiffness increase. The values of the terms of the stiffness matrix and stiffener spacings for the A, B, and C 0/90 waffle stiffener-skin configurations are listed in Table 4.3

The results for the torsion problem for the circular cylindrical shells with stiffener-skin configurations A, B, and C are listed along with the results for the $\pm 45^\circ$ waffle and sandwich shells in Table 4.4. The biggest value of torsional stiffness was obtained with the A stiffener-skin configuration which is 8% stiffer than configuration B and 195% stiffer than configuration C. With the $[0/90]_{3s}$ skin stacking sequence of stiffener-skin configuration C, we get a lower torsional stiffness than with configuration A. This decrease is due to the fact that when we decrease the $\bar{A}_{12}$ and $\bar{D}_{12}$ terms of the stiffness matrix, we also decrease the $\bar{A}_{66}$ and $\bar{D}_{66}$ terms so now the shear and twisting stiffnesses are too small.

The values of torsional stiffness for the 0/90 waffle shells are bigger than for the

Table 4.3 Stiffnesses and Stiffener-Skin Configuration for Three 0/90 Waffle Shells

	A	B	C
Skin	$[\pm 45]_{3S}$	$[\pm 45]_{3S}$	$[0/90]_{3S}$
h_s (mm)	18	18	18
b_s (mm)	4	4	4
a (mm)	199	179	199
b (mm)	178	200	178
A_{11} (kN/mm)	107.8	113	151.7
A_{22} (kN/mm)	113.2	107.5	157.1
A_{12} (kN/mm)	48.14	48.14	4.2
A_{66} (kN/mm)	50.76	50.76	6.8
B_{11} (kN)	448.6	498.7	448.6
B_{22} (kN)	501.5	446.3	501.5
D_{11} (kN mm)	5631.2	6259.1	5635.4
D_{22} (kN mm)	6294.2	5603	6307
D_{12} (kN mm)	9.32	9.32	.82
D_{66} (kN mm)	151.4	150.7	143
$D_{\alpha 6}$ (kN mm)	2.15	2.15	0

Table 4.4 Circular Cross-Section Cylindrical Shells
with a Cockpit-Size Cutout

Shell		$\frac{\text{Weight}_{\text{stiffened}}}{\text{Weight}_{\text{sandwich}}}$	U_{θ} (mm)	K_{θ} (Nm/deg)
Sandwich		1	2.96	10,800
± 45 Waffle	Equal Dii	1.32	2.65	12,400
	Equal Weight	1	3.67	9,000
0/90 Waffle	A	1	2.236	14,760
	B	1	2.358	14,000
	C	1	4.689	7,000

$\pm 45^\circ$ waffle and sandwich shells except for Stiffener-skin configuration C. Configuration A is 37% stiffer than the sandwich skin configuration, 20% stiffer than the equal-bending-stiffness stiffener-skin configuration, and 65% stiffer than the equal-weight stiffener-skin configuration. Recall from Chapter 2 that the equal-bending-stiffness stiffener-skin configuration is 30% heavier than the sandwich skin configuration so even adding 30% in weight the $\pm 45^\circ$ waffle shells are less stiff than the 0/90 waffle shells with a big cutout.

The results for the torsion problem for the square cross-section cylindrical shells with stiffener-skin configurations A, B, and C are listed along with the results for the $\pm 45^\circ$ waffle and sandwich skin configurations in Table 4.5. The biggest value of torsional stiffness was obtained with the C configuration which is 8% stiffer than configuration A and 16% stiffer than configuration B. Because configuration C has the smallest values of $\bar{A}_{66}$ and $\bar{D}_{66}$ of the three configurations, we can conclude that for square cross-section shells the influence of $\bar{A}_{66}$ and $\bar{D}_{66}$ on the torsional deformation of the shells is smaller than for the circular cross-section shells. Also, the torsional stiffness of the square cross-section shell is more influenced by $\bar{A}_{12}$ than by $\bar{A}_{66}$ and $\bar{D}_{66}$. The reason for this effect is that, in the cutout region, the normal in-plane forces and bending moments are more important for square cross-section shells than for the circular cross-section shells.

In the case of square cross-section shells, the 0/90 waffle shells are stiffer than the $\pm 45^\circ$ waffle shells but less stiff than the sandwich shell. The sandwich shell is 7% stiffer than the shell with stiffener-skin configuration C, 22% stiffer than the shell with configuration B, and 17% stiffer than the shell with configuration A. On the other hand, the shell with configuration C is 80% stiffer than the equal-weight $\pm 45^\circ$ waffle configuration. Again, even if we add 30% weight to the $\pm 45^\circ$ waffle shell, we still get more stiffness with the 0/90 waffle stiffening as happened before for the circular shells.

From the analysis of the square and circular cross-section shells with a cockpit-size cutout using the 0/90 waffle stiffening, the torsional problem is governed by the presence of the cutout. The main loads in the region of the cutout are not in-plane shear loads alone as happened for the closed shells. We see that, because of

Table 4.5 Square Cross-Section Cylindrical Shells
with a Cockpit-Size Cutout

Shell		$\frac{\text{Weight}_{\text{stiffened}}}{\text{Weight}_{\text{sandwich}}}$	U_{θ} (mm)	K_{θ} (Nm/deg)
Sandwich		1	16.1	1712
± 45 Waffle	Equal Dii	1.32	20	1378
	Equal Weight	1	31.2	887.7
0/90 Waffle	A	1	18.93	1460
	B	1	19.57	1400
	C	1	17.29	1600

the presence of the cutout, in-plane normal forces and bending moments appear and their importance is also influenced by the shape of the cross section of the shell. The influence of the shell cross-sectional shape is clear if we recall that the best results for the circular and square cross-section shells were obtained using different stiffener-skin configurations (configurations A and C, respectively).

The main difference between stiffener-skin configurations A and C is in the terms of the in-plane stiffness matrix and in the $\bar{D}_{12}$ term. All the other terms including $\bar{B}_{11}$ and $\bar{B}_{22}$ are similar for both configurations. The terms $\bar{A}_{12}$, $\bar{D}_{12}$, and $\bar{A}_{66}$ are much smaller for configuration C than for configuration A. Thus, configuration C should be stiffer than configuration A where the in-plane normal forces are more important than the in-plane shear forces. Conversely, configuration A should be stiffer where the in-plane shear forces are more important than other in-plane forces because of its bigger in-plane shear stiffness.

From the observations of this Section, we can conclude that the selection of the best stiffening concept for torsional stiffness of shells with a cutout is influenced by the shell cross-sectional shape and by the size of the cutout. In the next Section we will study the behavior of orthogonally stiffened shells when we add bulkheads and longitudinal stiffening of the edges of the cutout. We will see if we get an improvement in torsional stiffness by using 0/90 waffle stiffening.

4.5.2 Analysis of Circular and Square Cross-Section 0/90 Waffle Cylindrical Shells with a Cockpit-Size Cutout, Bulkheads, and Longitudinal Reinforcements

In this Section, we analyze circular and square cross-section shells with a cockpit-size cutout, bulkheads, and longitudinal reinforcements with 0/90 waffle stiffening. The dimensions and shape of these shells are the same that used in Chapters 2 and 3 as shown in Figures 2.32 and 3.18. The stiffener-skin configurations of these shells are the same configurations used for unreinforced shells in Section 4.5.1.

As with the unreinforced shells in Section 4.5.1, stiffener-skin configuration A is the stiffest among the orthogonally stiffened circular cross-section shells whereas configuration C is the least stiff, as shown in Table 4.6. Configuration A is 3%

Table 4.6 Circular Cross-Section Cylindrical Shells
with a Cockpit-Size Cutout, Bulkheads,
and Longitudinal Reinforcements

Shell		$\frac{\text{Weight}_{\text{stiffened}}}{\text{Weight}_{\text{sandwich}}}$	U_{θ} (mm)	K_{θ} (Nm/deg)
Sandwich		1	1.858	17,768
± 45 Waffle	Equal Dii	1.32	.783	42,163
	Equal Weight	1	1	33,014
0/90 Waffle	A	1	1.019	32,400
	B	1	1.03	31,440
	C	1	3.986	8280

stiffer than configuration B and 290% stiffer than configuration C. Configurations A and B are approximately two times stiffer than the sandwich skin configuration and as stiff as the $\pm 45^\circ$ waffle equal-weight stiffener-skin configuration. Thus, for the circular cross-section shells, we obtain roughly the same stiffness using $\pm 45^\circ$ or 0/90 waffle stiffening.

Results for the square cross-section shell with the 0/90 waffle stiffening are shown along with results obtained for the sandwich and helically stiffened shells in Table 4.7. As for unreinforced shells, a bigger torsional stiffness results for 0/90 waffle stiffening than for $\pm 45^\circ$ waffle stiffening. In this case, the shell with stiffener-skin configuration A yields the best results and, unlike for unreinforced shells configuration C is the worst. The low value of stiffness obtained for configuration C could be due to the fact that when we add the reinforcements the importance of the shear loads must increase. Thus, because the configuration C has a very low value of A_{66} , the overall stiffness of the shell decreases.

In this study, decreasing the values of $\bar{A}_{12}$, $\bar{D}_{12}$, $\bar{A}_{66}$, and $\bar{D}_{66}$ leads to two different results. For the reinforced circular cross-section shells, we obtained the same value of stiffness with 0/90 and $\pm 45^\circ$ waffle stiffenings. On the other hand, for the reinforced 0/90 waffle square cross-section shell, the torsional stiffness improves 42% over the equal-weight $\pm 45^\circ$ waffle shell and 16% over the sandwich shell. Again, as happened for the unreinforced shells, the shape of the shell cross section has a great influence in behavior of both stiffening concepts.

For circular cross-section shells, the bulkheads and longitudinal reinforcements of the cutout edges increase the influence of the in-plane shear force in the vicinity of the cutout. Thus, when we replace $\pm 45^\circ$ waffle stiffening with 0/90 waffle stiffening, the benefit of decreasing $\bar{A}_{12}$ cancels with the negative effect of decreasing $\bar{A}_{66}$ and $\bar{D}_{66}$. Also, for stiffener-skin configuration C we decrease so much $\bar{A}_{66}$ and $\bar{D}_{66}$ that the change of stiffening is counterproductive.

For square cross-section shells, the bulkheads and longitudinal reinforcements also increase the importance of the in-plane shear force, but the in-plane normal and bending loads are still very important because of the cross-sectional shape of the

Table 4.7 Square Cross-Section Cylindrical Shells
with a Cockpit-Size Cutout, Bulkheads,
and Longitudinal Reinforcements

Shell		$\frac{\text{Weight}_{\text{stiffened}}}{\text{Weight}_{\text{sandwich}}}$	U_{θ} (mm)	K_{θ} (Nm/deg)
Sandwich		1	7.86	3511
± 45 Waffle	Equal D _{II}	1.32	7.41	3722
	Equal Weight	1	9.58	2880
0/90 Waffle	A	1	6.778	4080
	B	1	6.885	4010
	C	1	10.95	2520

increase the importance of the in-plane shear force, but the in-plane normal and bending loads are still very important because of the cross-sectional shape of the shell. Thus, in the reinforced shells we obtain the best results with the stiffener-skin configuration A instead with configuration C as we did for the unreinforced shells.

4.6 SUMMARY

From the results obtained in this Chapter we can extract the following conclusions. The square shaped cross sections are not very efficient in torsion especially if the walls at the sides of the cutout are flat. Also, the selection of the best stiffening concept is greatly affected by the cross-sectional shape of the shells and by the size of the cutout. To be able to choose the best stiffening concept, we must try different skin stacking sequences and stiffener angles to try to obtain the best balance among the $\bar{A}_{12}$, $\bar{A}_{66}$, and $\bar{D}_{66}$ terms of the stiffness matrix for each cross-sectional shape. With the circular cross-section shells we obtained the best results for torsional stiffness. Using the circular cross-section $\pm 45^\circ$ waffle shell, we obtain twice the torsional stiffness of the sandwich shell. Thus, when designing the real racing car structural shell, we must try to use a cross-sectional shape as close as possible to the circular shape

Chapter 5

Tentative Manufacturing Procedure for a Racing Car Stiffened Structural Shell

5.1 INTRODUCTION

In this Chapter, we explain the usual procedures used in the manufacturing of $\pm 45^\circ$ waffle and 0/90 waffle shells and the problems that we might encounter associated with the shape of the structure. Also, a tentative procedure for construction of the racing car structural shell is described. All the manufacturing processes addressed here are tentative because they should be the subject of tests and analysis that are beyond the scope of this thesis.

5.2 MANUFACTURING OF $\pm 45^\circ$ AND 0/90 WAFFLE SHELLS

The $\pm 45^\circ$ and 0/90 waffle constructions are structures formed by a composite laminated skin and by a grid of stiffeners uniformly distributed over the skin. The stiffeners are made by laying up several very narrow layers of unidirectional tape, with the fibers aligned with the longitudinal axis of the stiffener, until the desired stiffener height is achieved. The three most common types of stiffening concepts are 0/90 waffle stiffening (stiffeners at $0^\circ/90^\circ$), isogrid (stiffeners at $90^\circ/\pm 60^\circ$), and the $\pm 45^\circ$ waffle stiffening (stiffeners at $\pm 45^\circ$) used in this thesis. These three concepts are shown in Figure 5.1 for circular cylindrical shells.

The $\pm 45^\circ$ waffle stiffening has two sets of 45° helical stiffeners that go around the shell in opposite directions as shown in Figure 5.1a. A $\pm 45^\circ$ waffle or helically stiffened shell and the detail of the grid unit cell is shown in Figures 1.6 and 2.3.

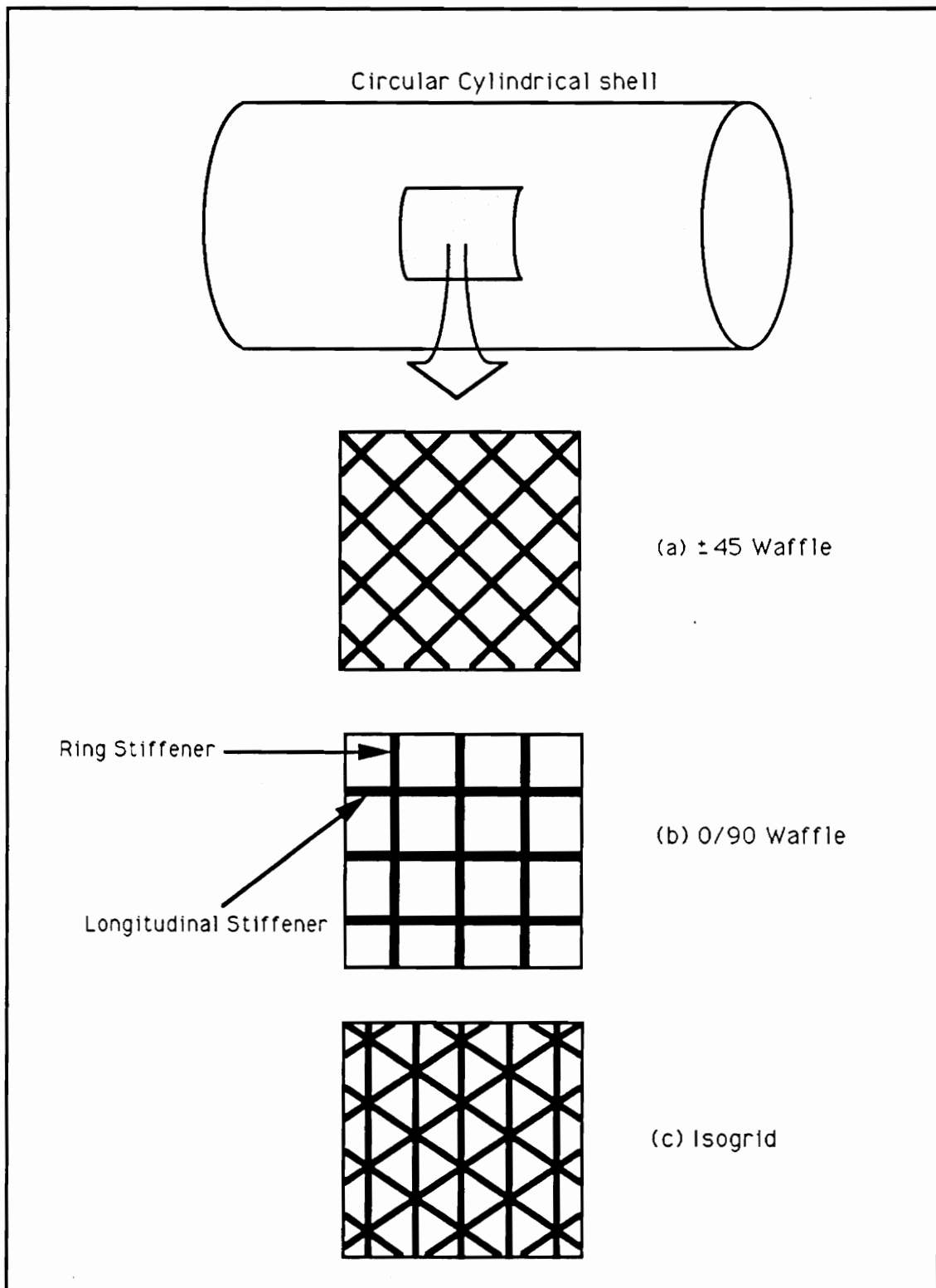


Figure 5.1 Types of Grid-Stiffened Constructions

The 0/90 waffle stiffening concept is composed of two sets of stiffeners, one of longitudinal stiffeners and the other of ring stiffeners as shown in Figure 5.1b. This array is very common when the structure is loaded axially or by internal or external pressure and, as we saw in Chapter 4, could be the best way of building the racing car structural shell depending on the cross-sectional shape of the structure.

Finally, the isogrid stiffening concept is formed by stiffeners that make equilateral triangular cells as shown in Figure 5.1c. All the angles of the triangular cell are equal (60°). This concept was developed by Dr. Robert R. Meyer of McDonnell Douglas Astronautics Company-West in 1964 under NASA contract [21]. The name isogrid refers to the fact that the triangular array of stiffeners (without the skin) behaves in a gross sense as an isotropic material. This isotropic behavior disappears when a skin is added because of the bending-extension coupling that now exists. "Unlike the other grid stiffening concepts, isogrid is a stable configuration that can be used with or without attached skin" [17].

5.2.1 Manufacturing of $\pm 45^\circ$ Waffle Shells

The main problem in the construction of $\pm 45^\circ$ waffle structures is the laying up operation for the stiffeners. One way of doing this operation after having the skin laminated would be to wind one layer of unidirectional tape for one set of stiffeners (for example, the set that goes around the shell clockwise) and then one layer of tape for the other set of stiffeners. Then, we repeat this procedure until the desired stiffener height is achieved. The problem with this procedure is that at the nodes where the two stiffeners intersect, the stiffener height is twice the height of other parts in the stiffeners as shown in Figure 5.2.

This material build up is problematic from the manufacturing and strength points of view. In the node region where two stiffeners intersect, the different layers are not perfectly bonded together, so during the curing process part of the resin of each layer will have to fill the gaps between layers. This process is not controllable, so there is the risk of generation of voids in the region of the intersection of two stiffeners. Also, the region of the intersection of two stiffeners

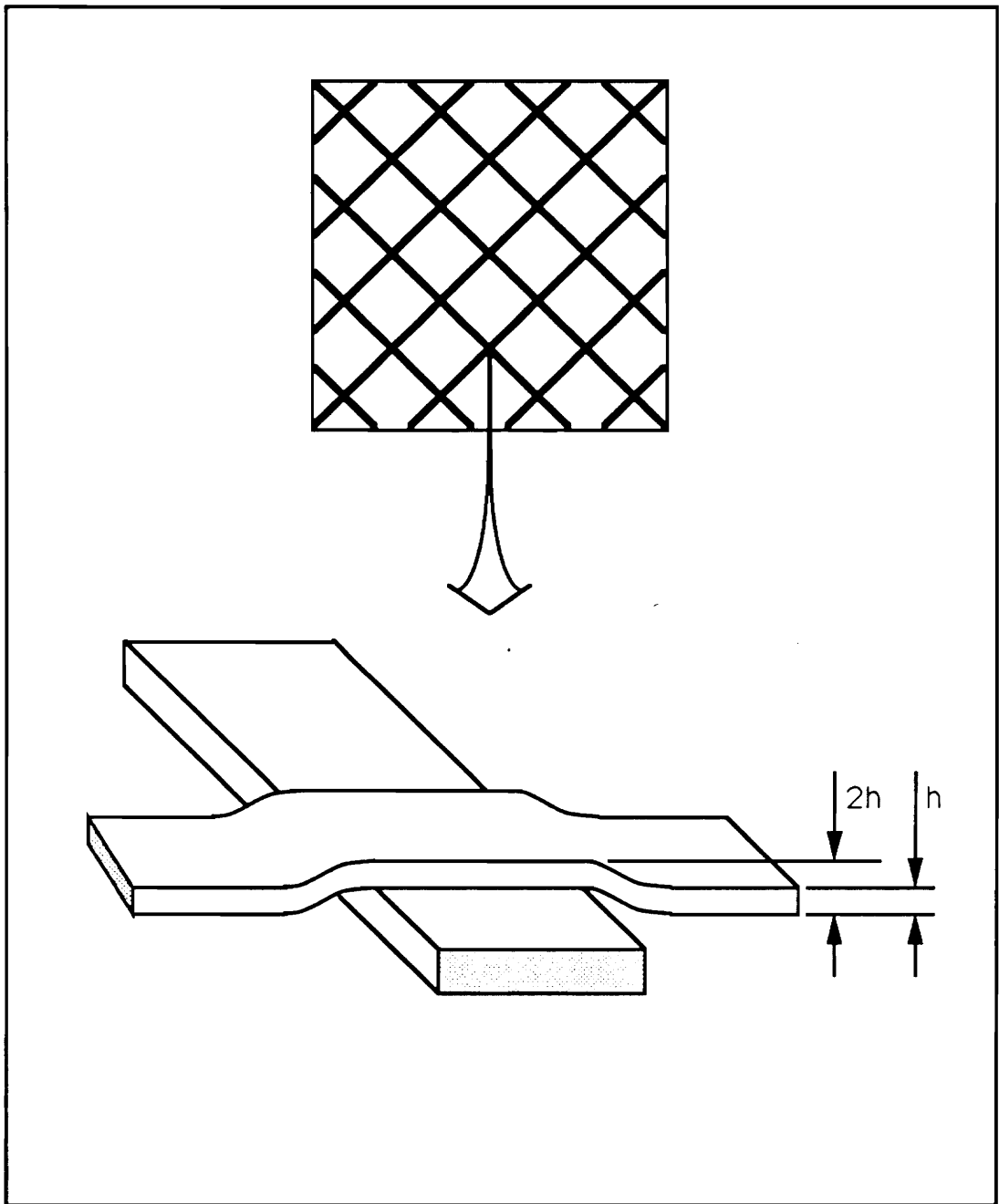


Figure 5.2 Material Build up at the Intersection of Two Stiffeners

is weaker than the other parts of the stiffeners because the fibers are not straight. Thus, when tensile or compressive loads are applied, the fibers will tend to straighten or to separate (buckle), respectively.

A way of laying up the stiffeners without material build up in the intersections is as follows. First, we lay up the first layer for the clockwise stiffeners. After this, we lay up the first layer for the counterclockwise stiffeners, but only in the space between two clockwise stiffeners as shown in Figure 5.3. Thus, we must cut the tape to avoid going over the clockwise stiffener layer. Then, we lay up the second layer for the counterclockwise stiffeners without cutting the tape this time. After this operation, we lay up the second layer for the clockwise stiffeners, but this time cutting the tape and filling only the space between two counterclockwise stiffeners. We repeat these operations until the desired stiffener height is achieved.

This last stiffener lay up procedure is much more complicated and labor intensive than the procedure that generated the material build up explained before, but we avoid the problems of the material build up. The number of operations necessary to achieve the height of the stiffeners is regulated by the thickness of the unidirectional tape (.127 mm for our AS-3501 material), but this number can be diminished by sticking together a number of tapes before laying a layer. There are automatic ways of performing this procedure, and one was developed by McDonnell Douglas Astronautics Company in St. Louis. They use a syntactic resin layer to fill the space between two stiffeners instead of cutting the tape [12]. Of course, this syntactic resin layer does not have the mechanical properties of the graphite-epoxy tape.

Another procedure to manufacture uniformly stiffened structures involves weaving. In this procedure, the stiffener grid is constructed separately of continuous unidirectional fibers by the use of weaving. After the stiffener grid is fabricated, it can be co-cured with the skin of the structure being made or bonded to it at a later time in a separate operation [17].

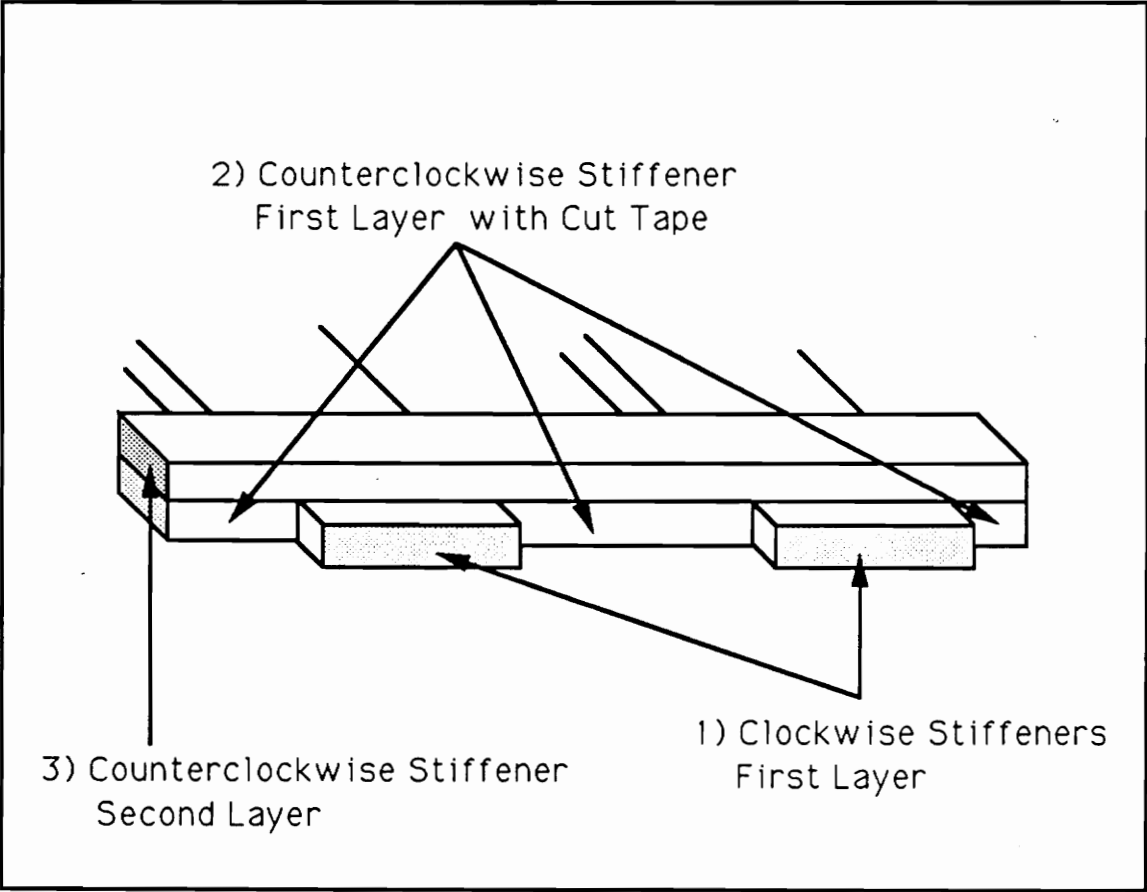


Figure 5.3 Stiffener Layup Procedure to Avoid Material Build Up

Apart from the problem of cutting and laying up involved in the manufacturing of the stiffener grid using the procedure that avoids the material build up, there are problems in maintaining the stiffener angle and spacing constant if the shell does not have a constant cross section. When the structures are conical, pyramidal or have some kind of tapering, there are problems in maintaining the angle and spacing of the stiffeners constant because after we lay a stiffener layer around one of the corners of a pyramid, for example, it is very difficult to maintain in the same angle we had before the corner. Real racing car structural shells always have tapering in the front part of the structural shell (see Figure 1.3), so we are going to have the problems just stated. The difficulty in keeping the same stiffener angle increases with stiffener width. However, the stiffener width that we are using is around 6 mm, so it should not be too difficult to maintain the angle of the stiffeners constant. The problem of stiffener spacing change is unavoidable. This change in spacing between stiffeners brings the additional complication of how to model the tapered part of the structure with finite elements because the terms of the stiffness matrix are functions of the stiffener spacing.

5.2.2 Manufacturing of 0/90 Waffle Shells

The procedure to lay up the stiffeners for the 0/90 waffle construction is the same as the procedure used for the $\pm 45^\circ$ waffle construction. The difference is that in this case we have stiffeners parallel and perpendicular to the longitudinal axis of the shell. The problem of the conservation of the angle of the stiffeners when the shell is tapered does not exist in this type of construction. On the other hand, we still have the problem of the change in size of the unit cell. Thus, the 0/90 waffle stiffening appears easier to manufacture than the helical stiffening.

With the 0/90 waffle stiffening it would be easier, for example, to use foam or honeycomb to increase the cross-sectional area of the stiffeners to obtain an increased stiffener bending stiffness than with the helical stiffening. The difference is that with $\pm 45^\circ$ waffle stiffening we bend and twist, at the same time, the layers of tape to lay them up along and around the shell. For the 0/90 waffle stiffening, we only bend the layers of tape when we lay them up. If we were going to lay up sandwich helical stiffeners, we would have to bend and twist the core

material and then hold it in place until we lay up the layers that go on top of it. With the 0/90 waffle stiffening, we only bend the core material so the process is easier.

5.3 CONSTRUCTION PROCEDURE OF THE FORMULA RACING CAR STIFFENED STRUCTURAL SHELL

In this Section, we explain a tentative procedure for construction of the structural shell of a Formula racing car. This procedure is based on the usual way in which these structures are constructed using sandwich construction as explained by B. P. O'Rourke from Willams Grand Prix Engineering Limited which builds Formula 1 racing cars [22].

In order to minimize the problems associated with the stiffener lay-up operation that were stated in Section 5.2, we must avoid the presence of sharp corners, and the cross-sectional shape should be as close as possible to a circular shape. Of course, the specification of the final shape of the structural shell should be subjected to a study of tradeoffs between maximizing stiffness and minimizing the problems of manufacturing without hurting the aerodynamics of the car. In the design of a real car, this tradeoff study must be done with input from the structural, aerodynamics, and manufacturing engineers.

If we ask the structural, aerodynamics, and manufacturing engineers what would be the best shape for the structural shell, we would obtain three different answers. Perhaps the structural and manufacturing engineers would agree that the best solution would be to have a circular cylindrical structural shell whereas the aerodynamics engineer would like to have a cone with the smallest frontal cross section possible. Here is where the tradeoff study must take place to arrive at the best possible solution. We take the shape shown in Figures 1.3 and 1.5 and make some changes to avoid too many manufacturing problems.

In this tentative manufacturing procedure, the skin of the structural shell is laminated in two parts. One part is the top part of the shell skin which includes the front bulkhead, the back plate, and the opening for the cockpit as shown in

Figure 5.4. The front bulkhead and the back panel are made of sandwich construction. The other part is the floor panel. The two intermediate bulkheads and the wall that divides the fuel tank from the cockpit are made of sandwich construction in separate molds. All these parts are cured separately, and then they are bonded together with some adhesive. Because we are going to laminate the stiffeners on the outer side of the skin of the structural shell, this surface must be as smooth as possible. To obtain a smooth outer surface, both parts of the skin must be molded using female molds. Using female molds, we laminate the outer surface of the skin against the mold and therefore we will get a smooth surface.

In this stage, we have the whole skin of the structural shell with bulkheads, etc. in one piece. Next comes the lay up of the stiffeners with the procedure explained in Section 5.2 using the skin as a mandrel. Finally, once the stiffener lay-up procedure is completed, the whole structure is placed in the autoclave for the stiffeners to be cured.

5.4 SUMMARY

The process for manufacturing a stiffened structural shell is much more complicated than manufacturing a sandwich structural shell. Of course, the cost of the stiffened structure is higher than the cost of the sandwich structure, mainly because of the complicated manufacturing procedure. Thus, the issue of manufacturing cost must be taken into account when studying the possibility of making a stiffened structural shell.

The importance of the manufacturing cost depends on what Racing Formula you are making the car for. In the case of Formula 1 cars the costs have much less importance than in Formula 3 cars. Thus, the cost issue could be a disqualifying factor for the stiffened construction in some Racing Formulas where the cost of the cars must be within certain limits so that the manufacturers can sell them. In the case of Formula 1, usually each racing team manufactures its own cars and the budget that a Formula 1 team has permits them to expend a lot more in the development of a car than a Formula 3 racing team, for example.

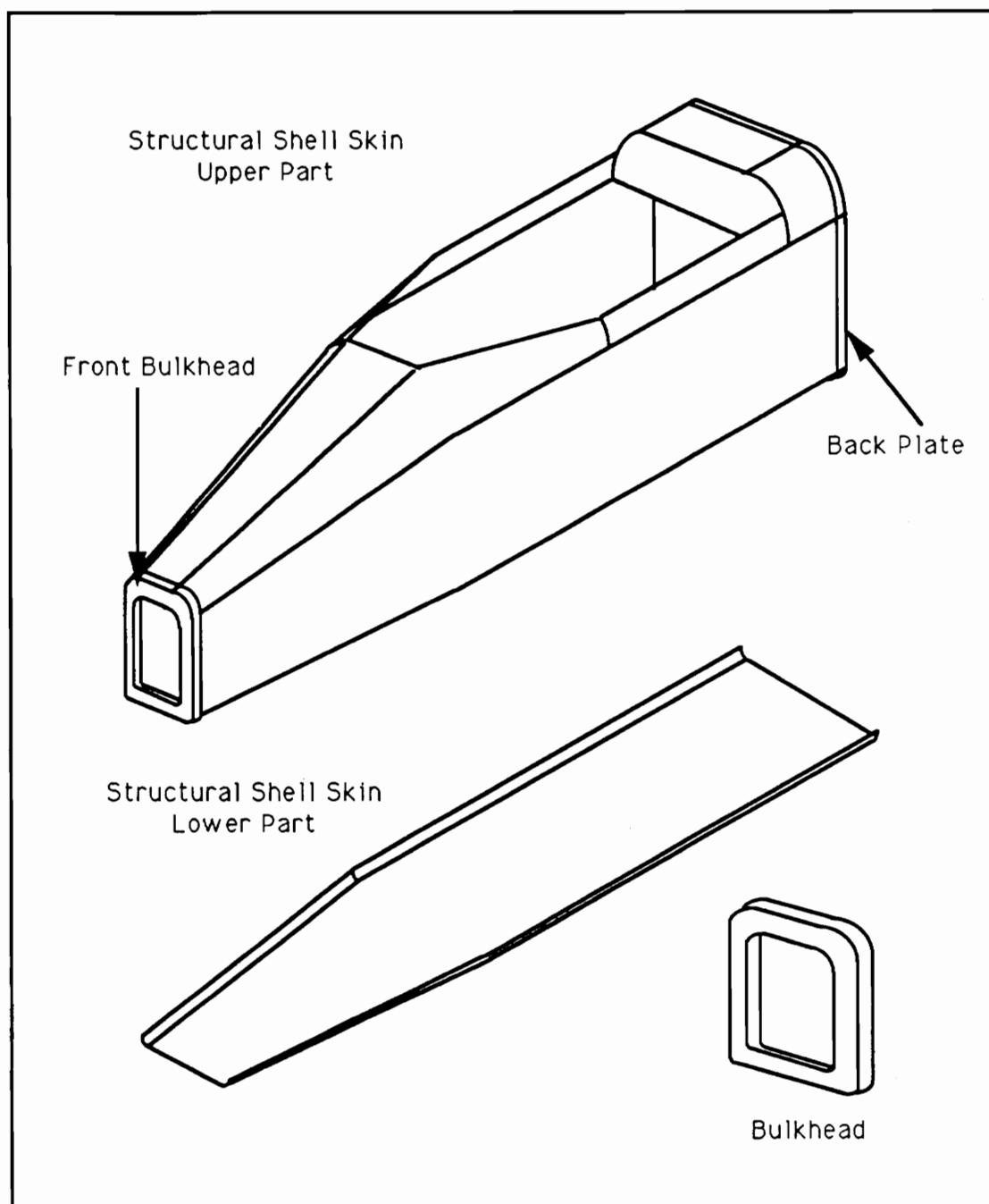


Figure 5.4 Parts of the Structural Shell

Chapter 6

Conclusions

6.1 INTRODUCTION

When we started this work, we intuitively thought that the best way to maximize the torsional stiffness of a shell structure such as the structural shell of a racing car was to use $\pm 45^\circ$ helical stiffening instead of sandwich construction. The fact that led us to this conclusion was that because presumably the main forces acting in the shell are in-plane shear loads the solution was to increase as much as possible the in-plane shear stiffness. These in-plane shear loads in a sandwich shell are taken only by the facings of the sandwich and not by the core which does not contribute to the in-plane stiffnesses. Thus, the best way of stiffening the structure would be to replace the core of the sandwich construction by helical stiffeners at $\pm 45^\circ$ with the longitudinal axis of the shell. These stiffeners would increase in-plane shear stiffness and, at the same time, they would contribute to the bending stiffness.

In the analyses performed in Chapters 2, 3, and 4, we see that the conclusion of the previous paragraph is true only for closed shells and still holds, but to a lesser extent, for circular cross-section shells with a cockpit-size cutout. In Chapter 3, we saw that, for square cross-section shells with a cockpit-size cutout, helical stiffening yields lower values of torsional stiffness than sandwich construction. The square cross-section sandwich shell with a cockpit-size cutout is more than 30% stiffer than the helically stiffened shell with a cockpit-size cutout. Thus, the usefulness of $\pm 45^\circ$ helical stiffening depends on the cross-sectional shape of the shell and on the size of the cutout. In the following Sections, we discuss the conclusions for the square and circular cross-section shells with and without cutouts and reinforcements shown in Figure 6.1.

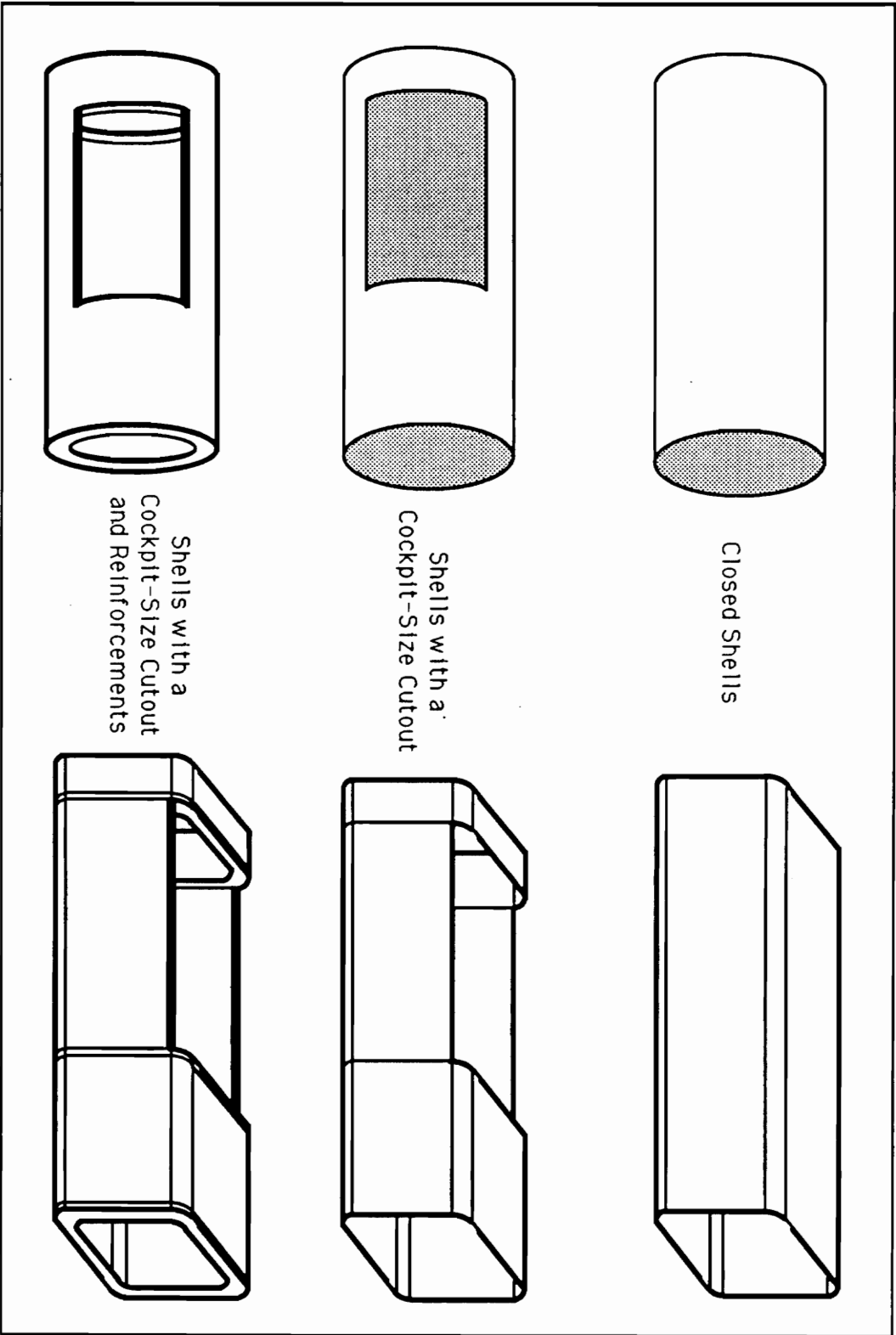


Figure 6.1 Circular and Square Cross-Section Cylindrical Shells

6.2 CIRCULAR AND SQUARE CROSS-SECTION CLOSED SHELLS

In Chapters 2 and 3, we found that for closed circular and square cross-section cylindrical shells, helical stiffening increases the torsional stiffness without regard to the cross-sectional shape. Circular and square cross-section helically stiffened shells are 5 and 3.7 times stiffer, respectively, than sandwich shells.

6.2.1 Closed Circular Cross-Section Shells

In Section 4.2, we obtained the following expression for the torsional stiffness of closed circular cross-section sandwich or helically stiffened shells,

$$K_{\theta} = \frac{1}{180} L \left\{ \sum_{k=1}^n \bar{Q}_{66}^k \frac{\pi (r_k^4 - r_{k-1}^4)}{2} + \frac{b_s E}{2 a} \frac{\pi}{2} \left((R_o - h_s)^4 - R_o^4 \right) \right\} \quad (73)$$

in which we treated the core of the sandwich shells as another layer with very low properties. The torsional stiffness of the shell depends on the value of $\bar{Q}_{66}$ of the layers of the skin, on the in-plane shear stiffness of the smeared stiffener layer, and the polar moment of inertia of each skin layer and of the smeared stiffener layer. The results obtained with this expression were very accurate compared with the results obtained in Chapter 2 from the ABAQUS finite element analysis program.

This expression for the torsional stiffness was derived under the assumption that the shells carry only in-plane shear force. From the results obtained in Chapter 2 from ABAQUS for closed circular cross-section shells, we know that this assumption is incorrect because the shells carry not only N_{xy} but also M_{xy} . In Section 4.2, we analyzed the contribution of N_{xy} and M_{xy} to the shearing strain γ_{xy} and to the twisting curvature κ_{xy} . We performed this study by inverting the constitutive equations, and we obtained the following expressions for γ_{xy} and κ_{xy} .

$$\gamma_{xy} = N_{xy} \left[\frac{\bar{D}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] - M_{xy} \left[\frac{\bar{B}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] \quad (74-a)$$

$$\kappa_{xy} = -N_{xy} \left[\frac{\bar{B}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] + M_{xy} \left[\frac{\bar{A}_{66}}{\bar{D}_{66} \bar{A}_{66} - \bar{B}_{66}^2} \right] \quad (74-b)$$

Substituting in the last two Equations the stiffness matrix terms values of the sandwich and stiffened skin configurations, we saw that in all cases the contribution of M_{xy} is negligible compared with the contribution of N_{xy} . Thus, we can assume with good approximation that the shells carry only in-plane shear forces.

From Equations (73), (74-a), and (74-b), we see the the terms of the stiffness matrix that govern the torsional stiffness of the shell are $\bar{A}_{66}$, $\bar{B}_{66}$, and $\bar{D}_{66}$. Also, from Equations (74) we can explain the fact that the externally stiffened closed shells were stiffer than the internally stiffened closed shells. If both shells have exactly the same skin and geometry and spacing of the stiffeners, the only difference between both shells is in the sign of the $\bar{B}_{ij}$ terms of the stiffness matrix. We know that externally stiffened shells have positive $\bar{B}_{ij}$ terms whereas internally stiffened shells have negative $\bar{B}_{ij}$ terms. The negative sign of the $\bar{B}_{ij}$ terms of the internally stiffened shell change the sign of the second and first term in Equations (74-a) and (74-b). Thus, internally stiffened shells are less stiff than externally stiffened shells because the values of γ_{xy} and κ_{xy} for internally stiffened shells are bigger than for the externally stiffened shells.

In Chapter 2, we saw that stiffened shells were stiffer than sandwich shells. The reason that makes stiffened shells stiffer than sandwich shells is clear in Equation (73). The torsional stiffness of a sandwich shell is given by the first term of Equation (73), where the core is treated as another layer with very low properties. If we replace the weight of the core of the sandwich shell with $\pm 45^\circ$ stiffeners uniformly distributed over the shell and we join the facings of the sandwich construction to form the skin of the stiffened shell, then the second term of Equation (73) is no longer zero. The contribution of the stiffeners is always bigger than the contribution of the core layer that has nearly zero properties. Thus, we can conclude that if a helically stiffened closed circular cylindrical shell has a skin equal to the summation of the two facings of the sandwich shell, then the helically stiffened shell is always stiffer than the sandwich shell. Also, the best stiffening

concept for closed circular cross-section shells is the $\pm 45^\circ$ helical stiffening because with this stiffener orientation we get the biggest stiffener contribution to the $\bar{A}_{66}$ term of the stiffness matrix.

6.2.2 Closed Square Cross-Section Shells

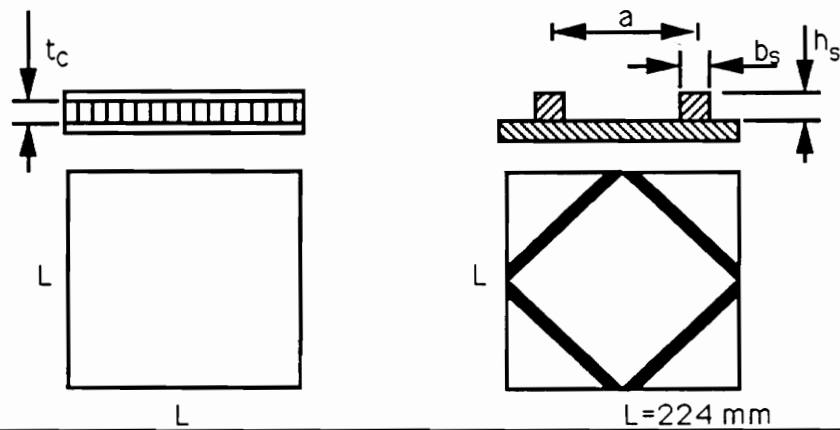
In Section 3.1, we analyzed closed square cross-section sandwich and $\pm 45^\circ$ helically stiffened shells. The stiffened shell was 3.7 times stiffer than the sandwich shell. Thus, as for the circular cross-section shells, $\pm 45^\circ$ helical stiffened construction has higher torsional stiffness than sandwich construction.

Because of the lack of an expression for the torsional stiffness of closed square cross-section shells, we changed the value of each of the stiffness terms to determine the effect of each one on the deformation behavior. Each stiffness matrix term is changed to the value that it has in the sandwich skin configuration stiffness matrix. The stiffness matrices of the stiffened and sandwich skin configurations used in this analysis are shown in Table 6.1.

The terms with a significant effect on the torsional stiffness of the shell are $\bar{A}_{66}$, $\bar{D}_{66}$, and the bending-extension coupling terms ($\bar{B}_{ij}$), as shown in Table 6.2. The absence of bending-extension coupling ($\bar{B}_{ij}=0$) increases the shell stiffness by 9%. When we decrease $\bar{A}_{66}$, the torsional stiffness decreases significantly. When we decrease $\bar{D}_{66}$, we get a 14% increase in torsional stiffness. To explain why a decrease in $\bar{D}_{66}$ produces a torsional stiffness increase, we analyse the cases of two shells with no coupling and equal $\bar{A}_{66}$ and $\bar{D}_{66}$ terms, respectively. The results obtained with the decrease of $\bar{A}_{66}$ and $\bar{D}_{66}$ were a 200% and a 1% decrease in the torsional stiffness of the shell, respectively. Thus, in the case of $\bar{B}_{ij}=0$, we see that $\bar{A}_{66}$ governs the torsional stiffness of the shell whereas there is practically no contribution of $\bar{D}_{66}$. On the other hand, if there is coupling, both the $\bar{A}_{66}$ and $\bar{D}_{66}$ terms affect the value of torsional stiffness.

The reason for the difference between the coupled and uncoupled cases is that for a square shell, apart from the in-plane shear deformation, the shell walls twist. Thus, if $\bar{B}_{ij} \neq 0$, then the in-plane shear and twist deformations are coupled, and

Table 6.1 Sandwich and Stiffened Skin Configurations



Configuration	Sandwich	$\pm 45^\circ$ Stiffened
Lay Up	$[[CF]_3/+45/-45]_S$ $t_c = 15 \text{ mm}$	Skin: $[\pm 45]_{3S}$ Stiffeners: $h_s = 17 \text{ mm}$ $b_s = 6.8 \text{ mm}$ $a = 158 \text{ mm}$
Weight (kg)	.17	.225
$A_{11} = A_{22}$ (kN/mm)	90.1	108.2
A_{12} (kN/mm)	18.82	94.5
A_{66} (kN/mm)	21.22	97.12
B_{1j}^* (kN)	0	429.4
$D_{11} = D_{22}$ (kN mm)	5326	5230
D_{12} (kN mm)	1114	4979
D_{66} (kN mm)	1256	5103.6
$D_{16} = D_{26}$ (kN mm)	0	2.15

* $B_{1j} = B_{11} = B_{22} = B_{12} = B_{66}$

Table 6.2 Results for
Closed Square-Cross-Section Cylindrical Shells

Skin Configuration	Circumferential Displacement u_θ (mm)	Δu_θ (%)
Real Skin Configurations	Sandwich = 1.260 Stiffened = .428	—
No Coupling $B_{ij}=0$	.3892	-9
Equal A_{12}	.4151	-3
Equal A_{66}	1.024	139
Equal D_{12}	.4349	1.6
Equal D_{66}	.3685	-14

both $\bar{A}_{66}$ and $\bar{D}_{66}$ govern the torsional stiffness of the shell. The effect of the $\bar{B}_{ij}$ terms can be seen if we write the constitutive equations for one of the walls of the square cross-section shell

$$\begin{aligned} N_{xy} &= \bar{A}_{66} \gamma_{xy} + \bar{B}_{66} \kappa_{xy} \\ M_{xy} &= \bar{B}_{66} \gamma_{xy} + \bar{D}_{66} \kappa_{xy} \end{aligned} \quad (75)$$

If the $\bar{B}_{ij}$ terms are zero (no coupling), the twist and in-plane shear deformations are decoupled. Thus,

$$N_{xy} = \bar{A}_{66} \gamma_{xy} \quad M_{xy} = \bar{D}_{66} \kappa_{xy} \quad (76)$$

In the case where $\bar{B}_{ij} \neq 0$, we invert Equations (75) to obtain the expressions for γ_{xy} and κ_{xy} as a function of the stiffness terms and the applied N_{xy} and M_{xy} :

$$\begin{aligned} \gamma_{xy} &= N_{xy} \left(\frac{\bar{D}_{66}}{\bar{D}_{66}\bar{A}_{66} - \bar{B}_{66}^2} \right) - M_{xy} \left(\frac{\bar{B}_{66}}{\bar{D}_{66}\bar{A}_{66} - \bar{B}_{66}^2} \right) \\ \kappa_{xy} &= -N_{xy} \left(\frac{\bar{B}_{66}}{\bar{D}_{66}\bar{A}_{66} - \bar{B}_{66}^2} \right) + M_{xy} \left(\frac{\bar{A}_{66}}{\bar{D}_{66}\bar{A}_{66} - \bar{B}_{66}^2} \right) \end{aligned} \quad (76)$$

In Equations (76), the sign of the terms in parentheses depends on the value of $\bar{D}_{66}$. The value of these terms for the real stiffened skin and fictitious configurations with $\bar{D}_{66}$ equal to that of the sandwich skin configuration are:

- Real Skin Configuration:

$$\gamma_{xy} = .164 N_{xy} - .004 M_{xy} \quad (77)$$

$$\kappa_{xy} = -.0014 N_{xy} + .0003 M_{xy}$$

- Equal $\bar{D}_{66}$ Fictitious Configuration:

$$\gamma_{xy} = -.02 N_{xy} + .007 M_{xy} \quad (78)$$

$$\kappa_{xy} = .007 N_{xy} - .0016 M_{xy}$$

If the value of $\bar{D}_{66}$ is decreased to the value it has in the sandwich skin configuration, the sign of the terms in parentheses in Equation (76) is the opposite of that of the real skin configuration. This change of sign is the reason for the increase in torsional stiffness when we diminish the value of $\bar{D}_{66}$ without diminishing $\bar{A}_{66}$ and $\bar{D}_{66}$. Thus, all we can say is that in the case of square cross-section shells the torsional behavior always depends on $\bar{A}_{66}$. If the $\bar{B}_{ij}$ terms are different from zero, then the $\bar{D}_{66}$ term also affects the torsional behavior of the shell.

Finally, we conclude that the best stiffening concept for torsional stiffness of closed square cross-section shells is the one that increases more the value of the $\bar{A}_{66}$ term of the stiffness matrix. Thus, as for closed circular cross-section shells, $\pm 45^\circ$ helical stiffening is the best concept for torsional stiffness of closed square cross-section shells.

6.3 CIRCULAR AND SQUARE CROSS-SECTION SHELLS WITH A COCKPIT-SIZE CUTOUT

The presence of the cutout introduces a new complication in the understanding of the torsional behavior of the shells. The type of forces that act in the walls of the shells are different. Thus, the analysis of these shells is much more complicated than the analysis of the closed shells. The circular and square cross-section shells with a cockpit-size cutout are shown in Figure 6.1

In the case of shells with a cockpit-size cutout, the forces acting in the walls of the shell are not in-plane shear forces alone as in the case of closed circular cross-section shells. In the vicinity of the cutout, we also have in-plane normal forces and bending moments. The distribution of forces around the cutout changes with the stiffness matrix of the walls of the shell. When we change from a sandwich to a stiffened shell, the distribution of forces changes according to the changes in the stiffness matrix. Also, the torsional stiffness of the whole shell depends on the stiffness of the surroundings of the cutout of the shell. The stiffness of this part depends on the shape of the walls surrounding the cutout and on the way in which

we stiffen it.

In the analysis of the open shells performed in Chapters 2 and 3, we compare the behavior of the following three different sandwich and stiffener skin configurations:

- **Sandwich Baseline Skin Configuration.**

- **Equal-Bending-Stiffness Stiffened Skin Configuration:** $\pm 45^\circ$ helically stiffened skin configuration with the same values of $\bar{D}_{11}$ and $\bar{D}_{22}$ as the sandwich skin configuration and with an excess in weight of 30% over the sandwich skin configuration.

- **Equal-Weight Stiffened Skin Configuration:** $\pm 45^\circ$ helically stiffened skin configuration with the same weight as the sandwich skin configuration.

We use the equal-bending-stiffness and equal-weight skin configurations to study the effect that the bending stiffnesses have on the behavior of the shells. The lay-up and the stiffness matrix terms of these skin configurations are shown in Table 6.3.

In Chapter 2 and 4, for circular cross-section shells, we obtained a 14% higher torsional stiffness with the equal-bending-stiffness stiffened shell (30% heavier than the sandwich shell) than with the sandwich shell. On the other hand, the equal-weight shell was 19% less stiff than the sandwich shell. In the case of the square cross-section shells (Chapters 3 and 4), not even the equal-bending-stiffness shell could exceed the stiffness of the sandwich shell. The sandwich shell was 24% and 92% stiffer than the equal-bending-stiffness and equal-weight stiffened shells.

In Chapter 4, the same analysis done for the closed square cross-section shells was performed for circular and square cross-section shells. For both cross-sectional shapes, not only the $\bar{A}_{66}$ and $\bar{D}_{66}$ terms of the stiffness matrix affect the torsional stiffness of the shell, but also the $\bar{A}_{12}$ term. Also, the presence of bending-extension coupling decreases the torsional stiffness of the shells. This coupling affects more the square than the circular cross-section shells. These results are

Table 6.3 Baseline Sandwich, Equal-Bending-Stiffness, and Equal-Weight Skin Configurations



Configuration	Sandwich Baseline Configuration	$\pm 45^\circ$ Stiffened Equal-Bending- Stiffness Configuration	$\pm 45^\circ$ Stiffened Equal-Weight Configuration
Lay Up	$[[CF]_3/+45/-45]_s$ $t_c = 15 \text{ mm}$	Skin: $[\pm 45]_{3S}$ Stiffeners: $h_s = 17 \text{ mm}$ $b_s = 6.8 \text{ mm}$ $a = 158.3 \text{ mm}$	Skin: $[\pm 45]_{3S}$ Stiffeners: $h_s = 13.2 \text{ mm}$ $b_s = 4.4 \text{ mm}$ $a = 158 \text{ mm}$
Weight (kg)	.17	.225	.17
$A_{11} = A_{22}$ (kN/mm)	90.1	108.2	85.1
A_{12} (kN/mm)	18.82	94.5	71.4
A_{66} (kN/mm)	21.22	97.12	74
B_{1j}^* (kN)	0	429.4	171.4
$D_{11} = D_{22}$ (kN mm)	5326	5230	1650.5
D_{12} (kN mm)	1114	4979	1570.8
D_{66} (kN mm)	1256	5103.6	1609.8
$D_{16} = D_{26}$ (kN mm)	0	2.15	2.15

* $B_{1j} = B_{11} = B_{22} = B_{12} = B_{66}$

shown in Table 4.2. Lowering the $\bar{A}_{12}$ and increasing $\bar{A}_{66}$ and $\bar{D}_{66}$ improves the torsional stiffness of the shell.

It is imposible to increase $\bar{A}_{66}$ and $\bar{D}_{66}$ and decrease the $\bar{A}_{12}$ term of the stiffness matrix at the same time. Thus, in Chapter 4 we replaced the $\pm 45^\circ$ helical stiffening with a 0/90 waffle stiffening where there is no contribution of the stiffeners to the $\bar{A}_{12}$ term of the stiffness matrix. We used three different 0/90 waffle skin configurations. Configurations A and B have the same $[\pm 45]_{3s}$ skin stacking sequence and different spacings a and b. Configuration C has a $[0/90]_{3s}$ skin stacking sequence and the same spacing as configuration C. These three stiffened skin configurations are shown in Table 4.3, where the values of the stiffness matrix terms and stiffener orientation, geometry, and spacing are listed. These three stiffened skin configurations have the same weight as the sandwich baseline skin configuration.

For both the circular and square cross-section stiffened shells, we obtained the highest torsional stiffness with one of the 0/90 waffle shells as shown in Table 6.4. The highest torsional stiffness for the circular cross-section stiffened shell is obtained with 0/90 waffle configuration A. The use of 0/90 waffle configuration A for the circular cross-section shells yields an increase in torsional stiffness of 19% over the equal-bending-stiffness shell (30% heavier than the A 0/90 waffle configuration). For the square cross-section stiffened shell, the highest torsional stiffness is obtained with 0/90 waffle configuration C but still the sandwich shell is slightly stiffer than C 0/90 waffle configuration shell.

From the fact that the best skin configurations for the circular and square cross-section shells are different, we conclude that the distribution of forces in the two shells are completely different. 0/90 waffle Configuration A has higher $\bar{A}_{12}$, $\bar{A}_{66}$ and $\bar{D}_{12}$ terms and lower $\bar{A}_{11}$ and $\bar{A}_{22}$ terms than 0/90 waffle configuration C. All the other terms are practically equal for both configurations. Thus, the effect of decreasing $\bar{A}_{12}$ is more important than that of increasing $\bar{A}_{66}$ for the square cross-section shell whereas for the circular cross-section shell an intermediate solution seems better to increase the torsional stiffness of the shells. From these results, we can conclude that the governing factor of the torsional stiffness is the

Table 6.4 Circular and Square Cross-Section Cylindrical Shells with a Cockpit-Size Cutout

Shell		$\frac{\text{Weight}_{\text{stiffened}}}{\text{Weight}_{\text{sandwich}}}$	U_{θ} (mm)	K_{θ} (Nm/deg)	
C I R C U L A R	Sandwich		1	2.96	10,800
	+ 45 Waffle	Equal Dii	1.32	2.65	12,400
		Equal Weight	1	3.66	9,027
	0/90 Waffle	A	1	2.236	14,760
		B	1	2.358	14,000
		C	1	4.689	7,000
	S Q U A R E	Sandwich		1	16.1
+ 45 Waffle		Equal Dii	1.32	20	1378
		Equal Weight	1	31.2	887.7
0/90 Waffle		A	1	18.93	1460
		B	1	19.57	1400
		C	1	17.29	1600

balance between $\bar{A}_{12}$ and $\bar{A}_{66}$. Thus, to get the skin configuration with the largest torsional stiffness, we must study several different skin configurations changing the orientation of the stiffeners and the skin lay-up until we get the best balance between $\bar{A}_{12}$ and $\bar{A}_{66}$. Also, the best solution depends on the cross-sectional shape of the shell we are trying to stiffen.

6.4 CIRCULAR AND SQUARE CROSS-SECTION SHELLS WITH A COCKPIT-SIZE CUTOUT, BULKHEADS, AND STIFFENING OF THE LONGITUDINAL EDGES OF THE CUTOUT

In Chapters 2 and 3, we analyzed the behavior of circular and square cross-section shells with bulkheads and longitudinal reinforcements as shown in Figure 6.1. The results obtained for both shells are completely different from each other. In the case of circular cross-section shells, the equal-weight helically stiffened shell is nearly two times stiffer than the sandwich shell. On the other hand, the square cross-section sandwich shell is 22% stiffer than the square cross-section equal-weight helically stiffened shell.

In Chapter 4, we compared the torsional behavior of the sandwich skin configuration, the two $\pm 45^\circ$ helically stiffened skin configurations, and the A, B, and C 0/90 waffle configurations as we did for the unreinforced shells. The highest torsional stiffness for circular cross-section shells is obtained with helical stiffening, although the difference with the A and B 0/90 waffle configurations is negligible. For square cross-section shells, the highest torsional stiffness is obtained with the A 0/90 waffle configuration instead of the C 0/90 waffle as occurred with the unreinforced shells as shown in Table 6.5.

The best results for circular and square cross-section shells are obtained with the helically and with the A 0/90 waffle configurations, respectively. Thus, the importance of $\bar{A}_{66}$ relative to $\bar{A}_{12}$ has increased. For both shells, we changed to skin configurations with an increased value of $\bar{A}_{66}$ compared with the skin configurations used to get the best results for the unreinforced shells. The reason for this change is that, because the reinforcements allow less bending deformation in the region of the cutout, the in-plane shear forces in the vicinity of the cutout

Table 6.5 Circular and Square Cross-Section Cylindrical Shells with a Cockpit-Size Cutout, Bulkheads, and Longitudinal Reinforcements

Shell		$\frac{\text{Weight}_{\text{stiffened}}}{\text{Weight}_{\text{sandwich}}}$	U_{θ} (mm)	K_{θ} (Nm/deg)
C I R C U L A R	Sandwich	1	1.858	17,768
	+ 45 - Waffle	Equal Dii	1.32	42,163
		Equal Weight	1	33,014
	0/90 Waffle	A	1	32,400
		B	1	31,440
		C	1	8280
S Q U A R E	Sandwich	1	7.86	3511
	+ 45 - Waffle	Equal Dii	1.32	3722
		Equal Weight	1	2880
	0/90 Waffle	A	1	4080
		B	1	4010
		C	1	2520

have increased. Thus, a further increase of the stiffening of the longitudinal edges of the cutout will decrease even more the bending deformation of the edges of the cutout and increase the benefits of using helical stiffening. This change from 0/90 waffle to $\pm 45^\circ$ helical stiffening is slower for square cross-section shells than for circular cross-section shells. The bending stiffness of the longitudinal edges of the cutout necessary to benefit from the change to the helical stiffening is much higher for the square than for the circular cross-section shell. This effect can be seen in Section 3.6.5.1 where we analyze the torsional behavior of square cross-section shells with a cockpit-size cutout, bulkheads, longitudinal reinforcements, and curved longitudinal cutout edges. These shells are equal to the square cross-section shells used before except that, near the cutout, the walls at each side of the cutout are curved instead of flat as shown in Figure 6.2. With the addition of this curved cutout edges, the torsional stiffness of the equal-weight skin configuration helically stiffened shell is 27% stiffer than the sandwich shell. The reason for this increase in the torsional stiffness of the helically stiffened shell is that with the rounded longitudinal edges of the cutout we have increased drastically the bending stiffness of the longitudinal edges of the cutout. Thus, we take more advantage of the increased value of $\bar{A}_{66}$ of the helically stiffened skin configuration, as explained in the previous paragraph. In the case of circular cross-section shells, it is easier to make the helical stiffening useful than in the case of square cross-section shells because the walls of the cutout are curved so they already have an increased bending stiffness.

6.5 CONCLUDING REMARKS

The choice of the best stiffening concept for torsional stiffness of shells depends on whether the shell has an opening or not, on the size of the opening, and on the cross-sectional shape of the shell. For closed shells, the best stiffening concept for torsional resistance is by far $\pm 45^\circ$ helical stiffening. For shells with a cockpit-size cutout, the cross-sectional shape of the shell plays a much more important role than in the case of the closed shells, and the presence of the cutout complicates the distribution of forces in the shell. Thus, it is impossible to use intuition to determine which is the best way of stiffening the shell.

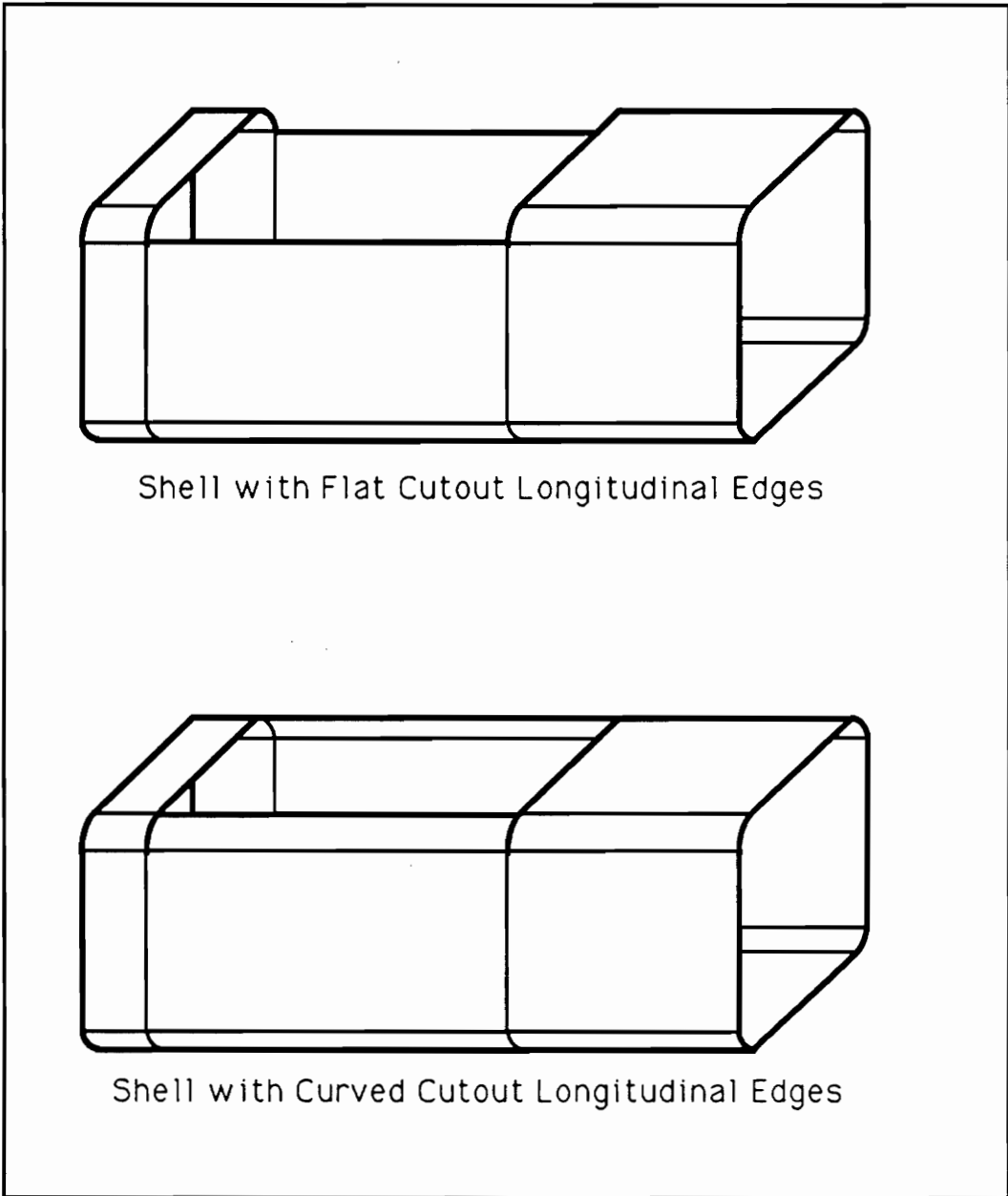


Figure 6.2 Square Cross-Section Cylindrical Shells
with Flat and Curved Longitudinal Cutout Edges

It is impossible to determine the best stiffening concept for torsional resistance of shells with a big cutout without going through an analysis like the present. The cutout affects the distribution of forces in the shell in different ways for different cross-sectional shapes. Also, we see that the way in which the cutout edges are reinforced influences drastically the choice of stiffening concept. Thus, to improve the torsional stiffness of shells with a cutout and different cross-sectional shapes, we must analyze them separately. Most likely the best solution is going to be different for each shell.

Although more research is needed to ensure feasibility of using the $\pm 45^\circ$ helical stiffening for a racing car structural shell, helical stiffening appears a very attractive solution. In order to determine if $\pm 45^\circ$ stiffening is the best solution, first the cross section and shape of the shell must be defined. Also, all the other loads present in the car must be added to the torsional loads in the analysis. From what we have seen in this thesis, the advantages of using the $\pm 45^\circ$ helical stiffening increase as the cross-sectional shape of the shell tends toward circular. Thus, the cross-sectional shape structural shell should be as close as possible to a circle. Other problems that must be studied are the effect of having the front part of the structural shell tapered and the crashworthiness of a stiffened structural shell.

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Vita

Carlos Alejandro Font was born on September 8, 1960 in Buenos Aires, Argentina. After graduating from Colegio Nacional San Isidro in December 1978, he attended Universidad Tecnologica Nacional and graduated in December 1987 with a Bachelor's degree in Mechanical Engineering. During his undergraduate studies, he also worked at the Pilot Plant of the Technological Research Center for the Plastic Industry (C.I.T.I.P.) in the National Institute of Industrial Technology (I.N.T.I), Argentina (1983-1988). In August 1988, he enrolled in the Engineering Science and Mechanics graduate program at Virginia Polytechnic Institute & State University to pursue a Master's degree.

A handwritten signature in black ink, appearing to read 'Carlos Font', with a long horizontal line extending to the right.