

MATHEMATICAL EXPLORATIONS USING LOGO:
A TRAINING MODEL FOR PERSPECTIVE ELEMENTARY TEACHERS

by

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MATHEMATICAL EXPLORATIONS USING LOGO:
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There have been many studies that have shown that a substantial proportion of primary school teachers do not possess strong beliefs about mathematics and what and how it should be taught (eg. Dionne, 1986). Many have experienced failure in mathematics and as a result are anxious about learning and teaching it (Widmer & Chavez, 1982). To overcome the problem of poor elementary mathematics teachers we must focus on our teacher training programs and how they might be altered to better prepare students to enter the mathematics classroom. Our teacher training should provide students with opportunities for the individual's development of competence and confidence in teaching mathematics (Bishop & Nickson, 1983). Furthermore, prospective elementary school teachers should be equipped not only with essential mathematical knowledge, but cognitive knowledge as to how students learn mathematics and a working knowledge of mathematical procedures and materials (Howson, 1973). Finally, it is important that we work to create in prospective teachers an awareness of the change in priorities of mathematical topics in light of current and future technological possibilities. The revision of elementary mathematics education courses should to be considered in order to incorporate the need for student education in the use of computers and their potential for enhancing mathematical learning (Prichard, 1982).

The Concepts in Mathematics Series at Virginia Tech, (Math 1611, 1612, and 1613) incorporates a weekly lab which has been designed with the goal of providing opportunities for mathematical enrichment. The labs have been developed around the idea that students should be placed in situations where, by exploring and reflecting, they can identify learning processes which will enable them to describe the kinds of questions which would guide children to similar understandings. The labs provide a rich environment where students can further develop classroom mathematics topics and, in the process, "recreate" in mathematics; actively involving themselves in mathematical investigation and discovery. These labs have generally been carried out with traditional pen, paper, and manipulative activities. The purpose of this paper is to present a model for an alternative approach to the lab portion of these courses.

The alternative approach to investigating (and hopefully as a result discovering) mathematics utilizes the computer and the programming language Logo. The model introduces and uses the Logo language to develop "microworlds", ideal for illustrating important mathematical ideas visually and for making these abstract ideas concrete (Papert, 1980). The model develops the ideas that students learn best in an environment where they actively participate (Dienes, 1962) and that free investigation is needed to preserve spontaneity and creativity in mathematics (Biggs, 1973). Like Piaget, Brunner, and Dienes, I believe that students learn best when they are actively participating in their own mathematical environment and creating and adding to their own mathematical structures.

The interaction of student and computer can offer many opportunities for students to engage in activities such as investigating new ideas, formulating conjectures and original hypotheses, or proving original or existing theorems (Abelson & deSessa, 1981). This type of interaction can greatly change students' attitudes toward mathematics, their attitudes towards computers, and the way they approach and think about mathematics (Milner, 1972; Papert, 1980; Washburn, 1969).

OVERVIEW OF THE DESIGN PROCESS

In the fall of 1985, the Mathematics Education faculty at Virginia Tech established a need to integrate computers into their elementary mathematics program. Elementary teachers need to be knowledgeable about computers and how they can be integrated into the existing mathematics curriculum as well as how they can and are changing what we teach in mathematics (NCTM Report, 1985).

Many computer projects, at all levels, have sprung up as a result of the societal pressure placed upon educational institutions to prepare students for a technological world. It would be difficult to argue against the point that computers are affecting change in all areas of our society and that education must attempt to keep up with those changes. However, before integrating computers into our existing mathematics curriculum, we must ask several important questions. How do people learn mathematics? How can the computer enhance the learning of mathematics? What computer applications or languages best encourage the "understanding" of mathematics? We need to look to research for the answers to these questions. Research findings in the area of mathematics and computer learning theory will serve as a foundation for this design project.

As may be seen in Figure 1, a fundamental framework has been developed to ensure a systematic approach to designing the alternative labs. The framework will insure that the design process focuses both on

the needs of the target audience and the mathematical content to be taught.

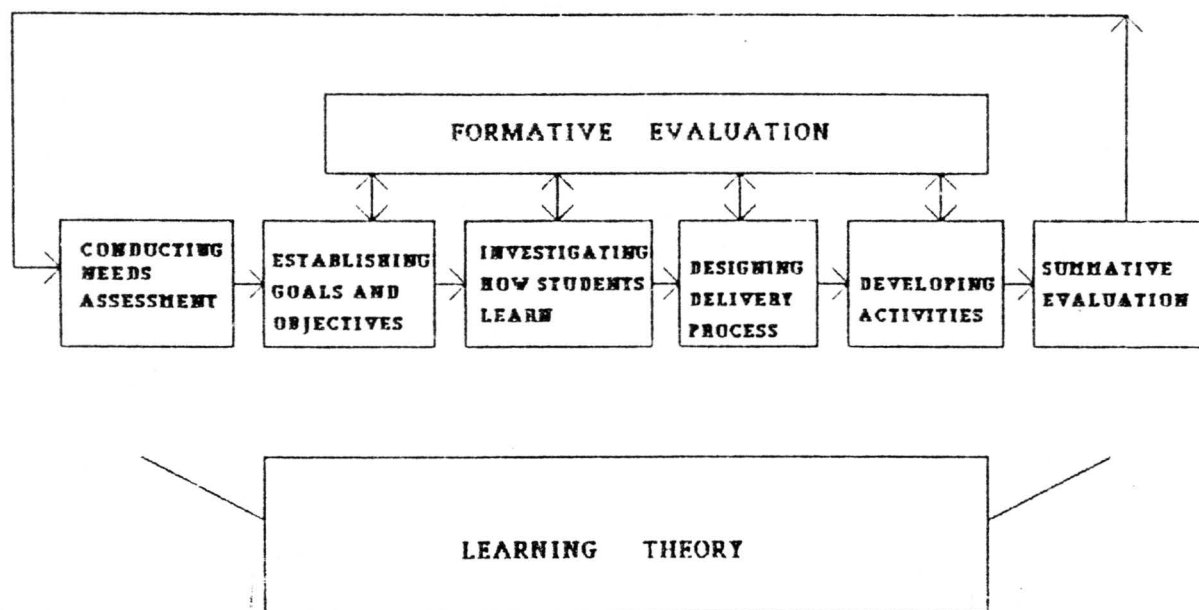


FIGURE 1

MODEL OF THE DESIGN PROCESS

In addition to addressing the need to integrate computers into the existing mathematics curriculum, a needs assessment was carried out to determine other areas of concern. Research indicates that the use of computers can both enhance mathematical understanding and develop positive attitudes toward mathematics (Prichard, 1982). Elementary school teachers need improvement in both of these areas. From the literature, five basic needs were ascertained. From those needs, project goals and objectives were established.

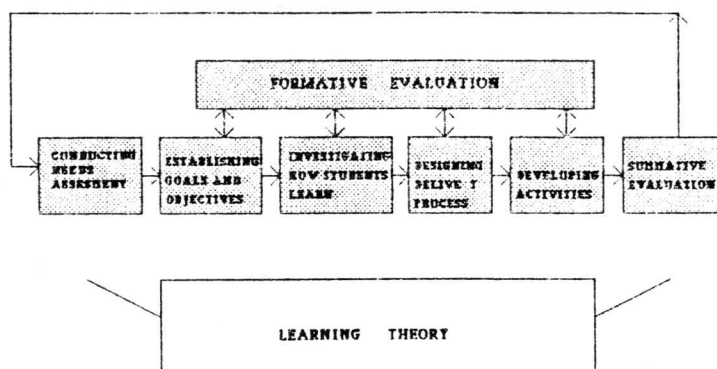
Before turning to the design of the delivery process, specific details on factors which influence how students learn (internal and external) were drawn from the literature. These details were considered throughout the design of the delivery process and the development of the computer activities. The design of the delivery process took on four phases: the method of delivery, grouping, feedback, and student evaluation.

At the next stage, current research helped determine which languages and/or software could best be used to facilitate mathematics understanding. After considering a number of alternatives, the decision was made to introduce computers through the use of the programming language Logo. Research projects using Logo were reviewed in order to determine guidelines for developing Logo activities. Topics were chosen from the course syllabus that could be effectively presented with the use of the computer and the Logo programming language. Finally, Logo procedures and activities were developed for each of the chosen topics.

As initial activities were developed, they were presented to a target audience and reviewed by a content specialist. The lab activities were then implemented during the fall and winter quarters. A formative evaluation process was carried out and feedback was provided on the effectiveness of each of the activities. This feedback was used to make revisions on existing lab activities and to develop new activities. The development and evaluation of additional labs and the revision of existing labs will continue throughout the spring and summer quarters. A

summative evaluation plan has been designed and will be implemented during the 1987-88 school year.

LEARNING THEORY



A recent study (Dionne, 1986) on the perception of mathematics learning among elementary school teachers shows a wide range of beliefs. Dionne places these perceptions of learning into three categories; traditionalist, formalist, and constructivist. Most learning theorists can be placed into either the traditionalist or the constructivist categories. Taking a look at these two points of view in light of the classic theories of learning will enable us to make appropriate decisions for developing effective mathematical learning environments.

LEARNING MATHEMATICS

Dionne (1986) defines the traditional perspective of mathematics, found among the majority of elementary school teachers, as a set of rules and computations which can be utilized by the user to gain power to solve problems. Students who are taught by teachers with this perception often feel that they have no power or control over their own mathematical

learning and are more likely to exhibit math anxiety (Wildmer & Chavez, 1982). The rules have little or no meaning, which results in students' lack of understanding.

Very early attempts to explain how we learn mathematics were made by E. L. Thorndike (1913). Dionne (1986) would most likely place Thorndike in the category of a traditionalist. He promoted the "Law of Effect" ("any act which is given a situation produces satisfaction becomes associated with that situation, so that when the situation recurs the action is more likely than before to recur also.") Rote learning of mathematics and continued practice will result in correct responses to given stimuli. The learner is able to develop mathematical rules to be used to determine correct responses.

Similar to Thorndike's behavioral approach to learning (Thorndike, 1913), Gagne (1962) defines a rule as a learned concept that governs one's actions. However, he goes further to say that rules are only the building blocks for the ultimate goal in learning, problem solving. The development of a set of rules enables the individual to respond to certain stimuli. Gagne (1962) proposes two major elements in the development of rules: discrimination and association. He describes discrimination as the ability to differentiate between variations in objects or groups of objects. By learning to discriminate, the individual is able to recognize new stimuli, develop a set of rules for responding to those stimuli, and finally, attain understanding of new concepts. Through the development of a set of rules and practice of these rules, the learner will be able to classify and

associate these new stimuli to certain responses, use the rules to develop more complex rules, and finally develop the capacity to solve problems.

Dienes, along with Piaget, and Brunner, would most certainly be placed in Dionne's third category as perceiving mathematics learning as constructive (Dionne, 1986). According to this perspective, new concepts are learned only if they are reconstructed by the learner, not simply transmitted by the teacher. Mathematics is something to "do", not something to simply look at.

Expanding on the traditionalist perspective that mathematics is a set of rules, the constructivist would view a simple rule as a means of constructing an infinite amount of information (Plumpton, 1972). Dienes (1961) describes this process as generalization; the discovery that a general rule extends beyond the first few cases. Dienes believes that the ability to generalize is essential to the learning of mathematics. Students gain an enormous amount of power when they are able to generalize a simple statement (Plumpton, 1972). Students can most effectively generalize from one case to another in a narrow field (Dienes, 1961). Therefore, we must provide students with a rich range of mathematical experiences.

Krutetskii (1985) stated two levels of generalization. At the first level, a person is able to see something general and known to him in what is particular and concrete. At the second level, a is able to defuse the general from particular cases in order to form a concept. Dienes (1961) sees the first level as an extension of an already formed class and the

second level as what he defines as abstraction; the awareness that a rule implies in a number of other situations. If students can first generalize within a narrow front, then abstraction to a wider front comes later (Dienes, 1961). Once again, a large number of rich mathematical activities must be provided for students in order for them to be able to abstract common elements from a wide range of different cases.

Piaget (1952) also develops in his theories this idea of abstraction as it specifically relates to mathematics. He defines abstraction as the forming of an idea separate from instances or material objects, an essential element in the learning of mathematics. However, children do not abstract automatically. Studies show that students can only abstract from what is concrete (Pulaski, 1980). In order to get to the abstract structure of mathematics, we must move students from a very concrete starting point by making sure that an appropriate number of concrete activities of varied experiences are provided (Piaget, 1952). This means that activities with symbols or numbers must be preceded by activities with objects and then symbols must be related to those objects. To accomplish this, we want to place in the hands of the students working tools which they can use to discover for themselves these relationships between abstract mathematics and concrete objects.

Dienes (1961), Piaget (1952), and Brunner(1960) propose that students need to be actively involved in the learning process. Piaget's theories propose that the extensive use of manipulatives will lead children to "think" about mathematics and to construct the operations that will help

them solve technical and practical problems. Piaget (1958) refers to this in his pre-operational and concrete operational stages of concept development. Once they understand what they are doing, the algorithms of arithmetic can be memorized as convenient shortcuts to solutions already worked out in concrete objects (Pulaski, 1980).

Children in the classroom suffer from "cognitive passivity" (Piaget, 1952). Children need to take a more active part in discovering facts and constructing relationships. Dienes (1962) refers to this as "play". He states that whenever an organism is placed in a new environment, its natural instincts are to actively explore and try to manipulate the environment in which it has been placed. Through mathematical exploration with concrete materials, students develop for themselves a set of rules which become the platform for extended play and discovery. In such an environment where mathematics is open-ended, students are more likely to become motivated and continue the learning process (Kersh, 1958). The learning environment needs to provide children with a variety of opportunities for "recreating" in mathematics; playing mathematical games, handling mathematical materials, and building concrete models of abstract mathematical concepts.

Bruner (1961) refers to this active participation as "enactive representation". He accepts Piaget's theories that children have different levels of development but extends this to say that individual mathematical concepts can also be developed at different levels. Children must first manipulate materials directly before being able to develop mental images

and later be able to manipulate those images with symbols. Piaget (1952) supports this theory of enactive representation when he states the importance of the development of sensory-motor structures. Actively grouping objects into different groups, (7 blocks into 1 and 6, 5 and 2, or, 4 and 3) allows for the constructions of numbers to develop along with the construction of logic and symbols. Many such studies have been performed where students were given a certain number of objects in a set and were told to recreate the set so that the new set had the same number of objects as the original set (eg. Piaget, 1952). It can be exciting for a group of students to discover the concept of "3ness" or "7ness." The concept of a number is no longer abstract but is made concrete through action upon objects. The act of discovering something new can be exciting at any age.

Theories of active learning have been the topic of much mathematical research in the area of "discovery learning" (Gagne & Brown, 1961; Kersch, 1958; Scandura, 1964; Suppes & Groen, 1967). Research findings strongly suggest that the presentation of mathematical concepts to elementary school children by techniques of "discovery learning", causes the learner to conceptually integrate the content into their own structures. Concepts can be retained more easily when "discovered" than if the concepts had been presented using an "expository learning" method (Brunner, 1961). Biggs (1973) describes discovery learning as a vital mathematical environment for investigation and problem solving involving three stages: free discovery, guided discovery, and practice. She emphasizes that the third state is essential for transfer of learned concepts to a broader

front. The concepts should be reinforced by varied experiences and practice.

Research supports this idea that discovery methods seem superior in terms of transfer of mathematical concepts (Suppes & Groen, 1967). However, researchers view the idea of transfer differently. Thorndike's (1913) theories stated that transfer could be accomplished only when it involved two activities which contained identical elements. Brunner (1961), however, believes that through discovery learning, massive general transfer can be achieved by appropriating learning and that learning properly under optimum conditions can lead one to "learn to learn." Principals "re-invented" by the learner and understood are more readily transferred than given or memorized principals (Haslerud & Myers, 1958; Katona, 1940). Ausubel (1961), however, claims that discovery learning has its merits but that the methods are not unique in their ability to generate self-confidence and continued motivation for learning.

LEARNING MATHEMATICS WITH COMPUTERS

Teaching consists of setting the stage for learners to develop their own mathematical structures. The use of multi-sensory materials can help to develop such an environment which will be stimulating and enjoyable (Krulik & Kaufman, 1963). School access to inexpensive computers has generated a new dimension to a traditional teaching process where the teacher is the transmitter and the student is the receiver. The

constructivist theory of learning mathematics, where students must actively participate in the learning environment, is effectively modeled through the use of the computer. The computer offer an instructional breakthrough as a result of its capacity to interact with students, allowing students to take on the role of teacher as they instruct the computer (Davies 1982). "Children teaching computers" turns the whole teaching concept around to something exciting, creative, and interactive (Raphael,1976).

The effects of computers on mathematical learning is dependent upon continued research, providing answers for how computers can best enhance mathematical instruction and learning. Research indicates that the use of the computer for discovery and investigation through computer programming can provide both opportunities for students to "think" about mathematics (Hatfield & Kieren, 1972; Milner 1972) and an environment where students can take control of their own learning environment (Billings, 1983). Students who explore mathematics through the use of a programming language are more likely to believe that learning mathematics is open-ended and not simply rote memorization (Jensen, 1986).

Theorists agree that mathematical learning must progress from what is informal to what is formal. Research indicates that a computer environment can provide opportunities for investigating mathematics at an informal level (Moser & Carpenter, 1983). The computer's graphic capabilities provide students with a tool for constructing their own mathematical models (Papert, 1980). Through programming the computer

with graphics, students can create visual representations of abstract ideas (Papert, 1980). By using the computer to draw shapes, manipulate them, and visualize arithmetic problems, we can operationalize Piaget's idea that students need to be actively involved in objects that represent numbers (Piaget, 1952). Students can use the computer to draw three red squares and four green squares or any other combination of 7 squares (2 and 5, 1 and 6) to discover the idea of "7ness". How exciting for all students to have their computer draw 7 objects and discover that their set of 7 flowers represents the same number as their friend's set of 7 houses. The computer can provide an active environment enabling the learner to bridge the gap between what is formal (abstract) and what is informal (concrete) (Papert, 1980).

Theorists also agree that generalization is essential to mathematical learning. Research in the area of computer programming indicates that mathematical generalization is effectively modeled through problemistic tasks requiring the use of computer programming (Pea & Kurland, 1984; Prichard, 1982). Programmers develop schemata for manipulating the computer to perform certain tasks. When faced with a new problem, the programmer must generalize the problems components in order to utilize the developed schema (Pea & Kurland, 1984).

Computers were first created as mathematical machines to help solve scientific and business problems. They are "problem solving tools" and, when used appropriately in an educational setting, can enable students to work at a higher level of sophistication and to approach the solution of a

problem in a number of ways (Abelson & deSessa, 1981). Real problems are complex and in order to develop "good" problem solving skills, we need a learning environment that provides us with adequate tools for performing algorithmic tasks such as computations, repetitions, and recursion (Lesh, 1982). Such an environment would allow students to concentrate on conceptualizing the problem. Time and energy spent on exhaustive computations would be limited and time spent on conceptual considerations would be increased (Richards & Wheeler, 1985).

Tolstoy (1939) wrote, "Some mathematician, I believe, has said that true pleasure lies not in the discovery of truth, but in the search for it." The computer can provide a new medium for searching for and discovering difficult mathematical ideas (Damarin, 1982). Abelson and diSessa (1981) promote the use of the computer as a medium for not only exploring mathematics and problem solving, but for reshaping what and how we teach mathematics. They agree with Piaget, Brunner, and others that students learn best in an environment which provides for exploration, investigation and discovery. They see the computer as a tool that can bring to learning the essential ingredient of surprise. A student, manipulating the computer, can write a procedure, carry it out, investigate the results, understand how it came about, and make appropriate changes to revise their solution. While writing computer programs, students are able to operationalize their ideas about mathematics. Research by Resnick and Ford (1981), indicates that while writing procedures, student errors reflect their lack of understanding. Furthermore, while revising those procedures,

students may be able to identify their misunderstandings and correct them (Prichard, 1982). Students who encounter the computer in such a way approach "discovery learning" as an open-ended adventure rather than the traditional discovery learning where the teacher already knows what the student will discover (Abelson and diSessa, 1981).

LEARNING MATHEMATICS WITH LOGO

Almost two decades ago, Seymour Papert and a group of his associates at Massachusetts Institute of Technology were faced with two important questions in the area of computers and education (Papert, 1980). First, is it possible to design computers so that learning to communicate with them can be a natural process? Second, can learning to communicate with a computer change the way we learn other things? Papert (1980) believed that the answer to both of these questions was yes. He and his associates developed a programming language called Logo. Papert's claims were, and still are, that Logo is a powerful tool for providing children with experiences to create their own structures for solving problems. He describes Logo as an environment where students can learn to "think" and to pull together all the fields of human knowledge (Papert, 1980). These claims and others have been widely accepted. As a result, administrators and teachers across the country have begun to respond by developing "Logo curriculums."

The statement has been made again and again that "a Logo world is a mathematics world" (eg. Papert, 1980; Goldstien, 1985). While using Logo, students will stumble across many different mathematical concepts and skills and use many mathematical processes implicitly if not explicitly (Goldstien, 1985). As a result, Logo research has been concentrated in the areas of problem solving and mathematics. Researchers have observed powerful learning taking place in Logo environments and have suggested that this learning can be transferred to other areas (Delclos, Kinzer, Littlefield, & Bransford, 1984; Rowe, 1985). High expectations have been placed on Logo which include development of problem solving skills and transfer of these skills to other areas, increased math understanding, reduction of math fears, development of important geometric concepts, and improved student attitudes towards mathematics and their view of themselves as mathematicians.

Papert has created a language that can be easily communicated with and which enables the student to program the computer, rather than the computer programming the student (Harvey, 1985). The use of the Logo language as an instructional model can provide students with new experiences in the study of mathematics (Abelson & deSissa, 1980; Hoyles & Sutherland, 1984). The power (the ability to focus on the problem) and ease of Logo lends itself to a greater depth of exploration (Harvey, 1985). The name Logo comes from the Greek word for thought (logos). Logo does just that, it creates an environment in which students are encouraged to

think (Papert, 1980). Emphasis is placed on "getting to know" how something works rather than simple skill success (Kellum-Scott, 1983).

While using Logo to teach the computer how to "think", students learn about how they themselves think (Hoyles & Sutherland, 1985). Investigations through Logo can provide students with a new insight to what they understand and what they do not understand. Students using the Logo programming language have been observed to more readily reflect upon their initial perception of a problem and make changes in their thinking processes in order to modify and/or correct the solution (Hoyles & Sutherland, 1985). Immediate and tangible output through the use of "turtle graphics" provides students with feedback concerning what they "thought" they were instructing the computer to do and what they actually instructed the computer to do (Billings, 1983). Furthermore, students exposed to Logo are able to more easily express mathematical ideas and apply the concepts that Logo has helped them to develop (Delclos et al., 1984). Student developed procedures can also provide valuable insight to teachers concerning how students perceive and solve certain tasks (Hoyles & Southerland, 1985).

Through Logo, students are not forced to approach problem solving in an abstract way but with concrete and powerful ways of thinking (Jensen, 1986). Logo is an aid to learning other things because it encourages the use of problem solving and mathematical strategies (Papert, 1980). The philosophy of Polya (1957), is that we should teach general methods of problem solving. Whenever we have a problem, we should go through a

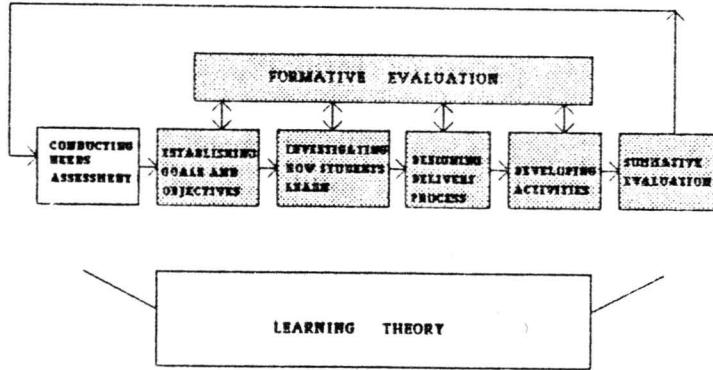
general list of heuristic questions such as, can this problem be divided into smaller parts, or have I seen and solved a similar problem? Positive results in favor of Logo were found when comparing the Logo problem solving environment to traditional "discovery learning" (Delclos et al., 1984). The procedural nature of Logo encourages students to approach problem solving activities by breaking the problem into parts and then using these parts as building blocks for deriving an appropriate solution (Hoyles & Sutherland, 1985). When Logo materials were developed to simulate real life problem solving, they became an effective instrument for enhancing mathematical topics (Rowe, 1985). With its structured language and use of procedures, Logo is an excellent device for staging and encouraging mathematical problem solving (Billings, 1983).

Much of the time and energy spent at the elementary school level is in memorizing rules and algorithms. Identical rules and algorithms are rememorized year after year with no additional depth of understanding. It is essential that our mathematics curricula turn from mathematics computation to mathematics understanding (NCTM Report, 1985). Logo provides an environment where students can more easily synthesize new concepts by re-inventing these concepts for themselves (Hoyles & Sutherland, 1985). A higher level of mathematical understanding can be reached as a result of learning through Logo activities (Milner, 1973). Studies show that Logo groups are far better in articulating their understanding of math concepts (Billstein, 1982). Logo provides students with a language for expressing themselves mathematically. Students who

are involved in Logo environments are more apt to express their mathematical understanding as having real life applications (Rowe, 1985)

Students who learned mathematics through a Logo environment have also been observed to be more motivated to continue their mathematical investigations (Billstein, 1982). This increase in motivation on the part of Logo students may be due to more positive perceptions of themselves as mathematicians (Billstein, 1982), that they tend to find mathematical learning more interesting (Rowe, 1984), or that they reflect increased mathematical understanding and improved attitudes (du Boulay, 1979).

Logo is uniquely characterized by its dynamic graphics capabilities provided by the "cybernetic turtle." Students are able to relate to the movement of the turtle just as they relate to the movement of their own body. This physical representation becomes a model for the student to build on (Papert, 1980). Existing inside a Logo "microworld", students are placed in an environment in which they have the opportunity to create these structures (du Boulay, 1979). In doing so, the student becomes an active learner rather than a passive learner. The computer becomes a modeling tool where the student has control and takes on a new image of himself as a mathematician.

NEEDS ASSESSMENT

As stated before, the need to improve the quality of primary school mathematics teaching lies in the hands of those instructing future teachers. Three problems exist: first, the difficulty of changing a student's attitude toward mathematics especially after a long history of failure (du Boulay, 1979) , second, providing the student with an "understanding" of mathematical concepts (Widmer & Chavez, 1982) and finally, creating an awareness of the change in priorities of mathematical topics in light of current and future technological possibilities (Prichard, 1984).

Many studies have shown that elementary teachers tend to have high anxiety in the areas of both mathematics and computers (eg. Widmer & Chavez, 1982). Widmer and Chavez studied math anxiety among elementary school mathematics teachers of all ages. One important finding was that teachers who described their mathematical training as "mathematics understanding" showed less anxiety than those teachers who described their math education as "mathematical computation." The need to improve

student attitudes towards mathematics goes hand-in-hand with the need for students to "understand" mathematics.

Mathematical understanding is emerging as a fertile area for curriculum reform in mathematics. A report given in 1985 by the National Conference of Teachers of Mathematics (NCTM) on the impact of computer technology, recommends that elementary instruction shift from an emphasis on computational skills to an emphasis on the meaning of arithmetic operations. The primary goal of mathematics education should be to ensure that students possess an intuitive understanding of what they are doing and why it is being done (von Glasersfeld, 1985). Many sophisticated mathematical topics such as randomness, scientific notation, variables, and transformation appear naturally when using computers. The use of computer based environments, sufficiently rich and yet flexible enough to provide for a variety of complex problems, can provide excellent opportunities for mathematical instruction and learning (Patterson & Smith, 1985). The NCTM report also recommends that future elementary teachers be exposed to the use of computers as problem solving tools and for instructional presentations. Furthermore, computer experience should be offered within the realm of mathematics content and not simply as a "computer science" course.

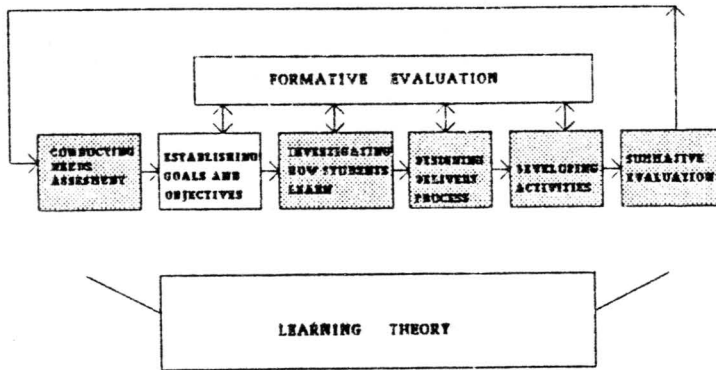
Active participation in the learning process through the use of computers can improve students' attitudes towards mathematics and mathematics learning (Milner, 1972; Papert, 1980). The exposure to computers in teacher education can help teachers to develop positive

attitudes towards mathematics and mathematics learning (Milner, 1972; Papert, 1980). The exposure to computers in teacher education can help teachers to develop positive attitudes toward mathematics and develop confidence in themselves as learners of mathematics (Prichard, 1982).

The literature suggests the following needs in the area of elementary mathematics:

- 1) The need to improve teacher attitudes about mathematics.
- 2) The need for mathematical understanding in areas where teachers have, at best, computational skills.
- 3) The need to understand the potential effects of technology on mathematics learning.
- 4) The need to improve teacher attitudes toward computers and their use in the mathematics classroom.
- 5) The need to create and participate in a mathematical learning environment which promotes students involvement as active learners.

GOALS AND OBJECTIVES



The following goals and objectives have been established after careful consideration of the determined needs.

GOALS

- 1) Students in elementary teacher education will take on a new image of themselves as mathematicians developing positive attitudes toward mathematics.
- 2) Students in elementary teacher education will use Logo to develop conceptual understanding of mathematical topics.
- 3) Students in elementary teacher education will use the computer and Logo to enhance understanding of mathematics. Students will be made aware of the potential effects of using Logo on mathematics learning.
- 4) Students in elementary teacher education will develop positive attitudes toward computers.

5) Students in elementary teacher education will experience a mathematical environment which allows for discovery and investigation in mathematics.

OBJECTIVES

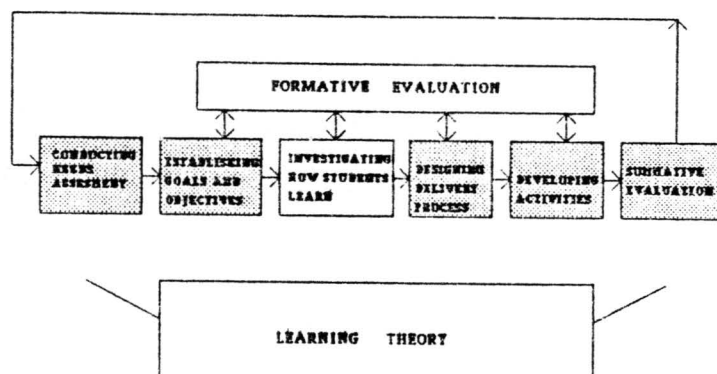
1) Student teachers will improve their attitudes toward learning and teaching mathematics.

2) Students will increase their understanding of mathematical concepts through the use of simple Logo procedures and the development of their own Logo procedures.

3) Students will see the instructional value of the computer and Logo as a tool for teaching mathematics.

4) Students will improve their attitudes toward computers and the uses of computers in teaching mathematics.

5) The student will act out the role of learner while participating in Logo activities. These activities will guide the learner through an investigative approach toward understanding various mathematical concepts.

INVESTIGATING HOW STUDENTS LEARN

The following ideas extracted from the literature on how students learn, were taken into consideration throughout the design of the delivery process and the development of Logo activities.

INTERNAL

1) The learner learns if the expected outcome is consistent with his view of the world and himself. He needs to have a positive attitude about mathematics and his ability to perform in this area (Wagner, 1983).

- a) We need to provide students with tasks at which they can be successful.
- b) We need to convince the learner that he can perform the tasks.
- c) We need to provide tasks that allow the student to develop at their own rate and to feel good about what they have discovered

no matter how much or how little.

2) The learner learns best if he can relate what he is learning to real life applications (Lesh, 1982).

- a) Use real life problems when available.
- b) If problems have to be contrived, at least make them interesting.

3) The learner learns when he can first recall needed prerequisite materials (Gagne & Briggs, 1974).

- a) Subject material should be sequenced.
- b) Prompts for student recall should be given.

4) The learner learns if he is able to generalize from a small field of knowledge to a larger field (Dienes, 1961).

EXTERNAL FACTORS

5) The learner learns when he actively participates (Bruner, 1960).

- a) Participate with the computer.
- b) Interact with peers.
- c) Asked to perform tasks which take action, i.e.,

manipulate, draw, position, etc.

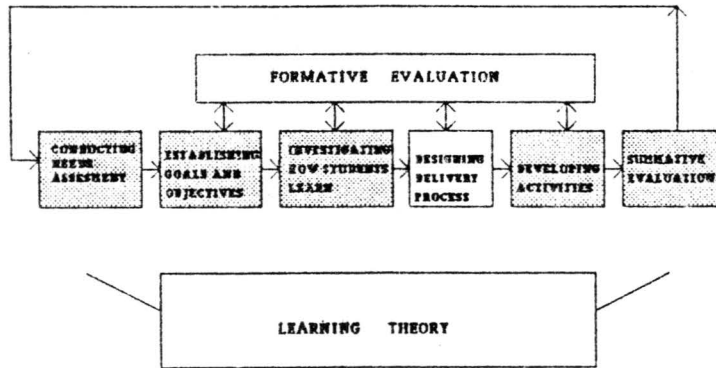
6) The learner learns when he is informed of his progress (Peeck, 1979).

a) Students need immediate feedback.

b) Students need to see when they are right and
when they are wrong.

7) The learner learns if given time to practice individual tasks (Biggs, 1973).

8) The learner learns if materials are presented schematically (Skemp, 1972).

DESIGN OF THE DELIVERY PROCESS

It has been suggested that students need time for free investigation, guided discovery, and practice (Biggs, 1973). These three ideas will be the major components of the weekly two hour labs. Computer lab space has been provided through the Department of Curriculum and Instruction. Our software contract permits the use of the Logo programming language with up to eight machines at a time. Eight IBM computers (with graphics capabilities) will be used to serve a maximum of 16 students per lab. Three to four math graduate students (preferably math education students) will be provided to act as lab assistants. One graduate student will assist with each scheduled lab. After considering these specifications, alternatives were listed and decisions made about each of the following four areas: method of delivery, grouping, feedback, and student evaluation. Decisions made about each of these areas are provided in the following pages.

METHOD OF DELIVERY

Each lab will be accompanied by a lab activity manual and lab activity disk. The activity disk will provide students with a number of pre-programmed Logo procedures simulating mathematical thinking. These activities will be referred to as "microworlds." The lab manual will provide the student with ideas for exploring these "microworlds", problems to be solved using the "microworlds, and ideas for expanding the "microworlds" to other areas.

Logo is ideal for allowing students to create their own learning structures and time is needed for this to occur (Papert, 1980) . Therefore, the delivery process will focus on student investigation and discovery by interaction with student, computer, and lab assistant. Student activities during lab time will be enhanced by: 1) involvement from lab assistants; 2) small group interaction with peers, and; 3) the use of the computer as a tool for investigation and as a means of immediate feedback.

The lab assistants will contribute greatly to the success of the project. Lab assistants should be confident in their knowledge of computers, elementary mathematics topics, and the programming language Logo. The main purpose of the lab assistants will be to give guidance to students throughout the investigative process. Lab assistants will meet weekly with course instructors and Logo specialist to prepare for the following week's lab as well as to formatively evaluate the previous lab(s).

It is evident from the research that the Logo language can be a powerful tool for enhancing mathematical concepts. Logo's turtle graphics deals with mathematical concepts in a beautiful and exciting way. However, many children are heavily exposed to Logo and its turtle graphics but have little concept of what it means in the real world. To maximize gains, lab assistants should emphasize the mathematical content of Logo activities and the connection between these concepts and the world around them (Delclos et al., 1984).

GROUPING

Three alternatives were considered when determining the best grouping strategy.

- Students working alone at the computer.
- Students working in pairs.
- Students working in groups of more than two.

It would seem that the ideal situation would be to have one computer per student. However, from personal experience and review of research (eg. Biggs, 1973), I have found that students seem to work best in groups of twos. With more than two people it is difficult for all students to participate with the actual implementation of the computer. They cannot get close enough to the computer to actively participate to the extent that they need to. With only one student to a computer, the brighter students

usually do fine but the average and/or below average students lack the benefit from peer involvement where they can share ideas, share computer skills, and work together toward solving problems.

Students seem to learn best when their attitudes are positive (Wagner, 1983). Grouping students by preference would improve attitudes. However, abilities of the two students should be kept in mind. When two students are working together at the computer, the faster learner will often dominate the slower learner and essentially do all the work.

FEEDBACK

It is important that students receive continuous feedback (Peeck, 1979). Feedback should be provided in the following three forms.

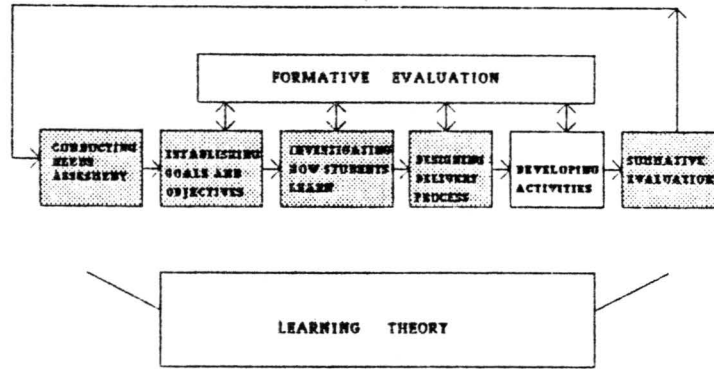
- From computer (immediate feedback)
- From lab instructor during lab and through comments about completed lab activities.
- From peers

STUDENT EVALUATION

Student evaluation will be carried out by individual lab assistants. The lab assistants should keep in mind that mathematics is open-ended. There may not be simply one correct answer, solution, or method of discovery.

Even though it is desirable for students to complete all lab activities, students should be encouraged to carry out investigations in individual areas of interest. Evaluation of students should include each of the following:

- Lab participation
- Successful completion of lab activities

DEVELOPMENT OF LOGO ACTIVITIES

Logo activities (disk and manual) will be provided as the primary source of instruction. Each lesson will provide the students with objectives, sample activities, Logo procedures, and problems for discovery. Logo activities will follow course topics. Activities will be designed to aid students in understanding a concept rather than simply providing opportunities for students to write procedures presenting ideas that they already understand.

Extensive work by Goldstein (1985) has been done using Logo to teach elementary geometry. Goldstein's studies provide a number of guidelines for implementing Logo as a tool for teaching mathematics. His studies support my personal belief that Logo should be used as a drawing board to provide students with an environment for investigating mathematics and to enhance their understanding of mathematical concepts. Students should be exposed to the Logo language, not for the purpose of learning a high level programming language, but for the purpose of communicating in a mathematical environment. Goldstein advocates providing students with

prepared Logo procedures which can be easily manipulated and expanded upon. The software (simple procedures) should set the stage for student exploration rather than controlling student discovery with set questions and responses. This is a very different approach to computerized instruction. There are no set questions, order, or responses, as found in traditional educational software. Students working with simple Logo procedures still have the full power of the language at their finger tips. Students can utilize this power to differing degrees depending on their varying levels of understanding.

A similar instructional model using Logo to teach elementary student teachers was implemented and evaluated by du Boulay (1979) at the University of Edinburgh. The results showed that using Logo's graphics capabilities to provide visual illustrations of mathematical ideas and rules leads to increased mathematical understanding as well as improved attitudes toward mathematics and computers. However, writing procedures to rework algorithms that students already knew was not advantageous and was described by most students as quite "frustrating." Writing a program to simulate an algorithm did in fact reinforce the steps taken to carry out the algorithm, but it took quite a long time and did not result in an understanding of why the algorithm worked.

A Logo "microworld" can provide a rich environment for students to conjecture, test those conjectures, refine them, and focus on mathematical generalizations (Jensen, 1986). Papert (1980) describes a "microworld" as a stage where mathematical thinking can hatch and develop with ease. It is

in this type of an environment that students can actively search for and discover exciting mathematical concepts.

Microworlds have been designed for the first two courses in the Virginia Tech elementary mathematics series (1611 and 1612). A brief description of each is given on the following pages. Each "microworld" was developed using the MIT Logo programming language for the Apple II series and revised to be used with the IBM Logo language. Sample lab activities can be found in Appendix A. Complete procedures are listed in Appendix B.

DESCRIPTION OF THE MICROWORLDS

Whole Numbers

The student is provided with two different models for exploring addition, subtraction, multiplication, and division of whole numbers. The first model provides the student with a number of procedures to produce a desired set of objects on the screen (eg. 4 red squares, 5 blue circles, or 6 black hats). Students are encouraged to write their own procedures to draw other objects on the computer's screen. The students can then manipulate the objects to discover properties of operations on whole numbers.

Example 1: After having the computer draw two sets of squares, one set containing 5 squares and one containing 3 squares, the student can then determine the total number of square on the screen (see Figure 2). The students might also compare the combined set to a set combining 3 squares with 5 squares. Is there any difference in the results?

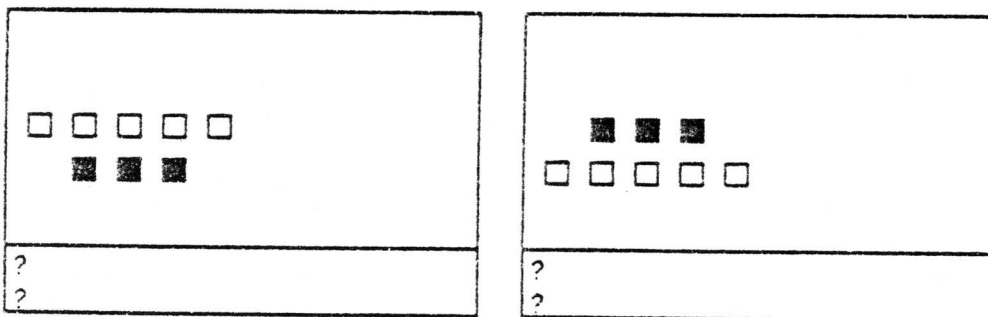


FIGURE 2

Logo Screen: Addition Model

Example 2: After having the computer draw a row of 7 hats, the student can then manipulate the computer to erase (take-away) 3 of the hats (see Figure 3). How many remain? What would happen if you tried to take 7 hats away from a set of 3 hats?

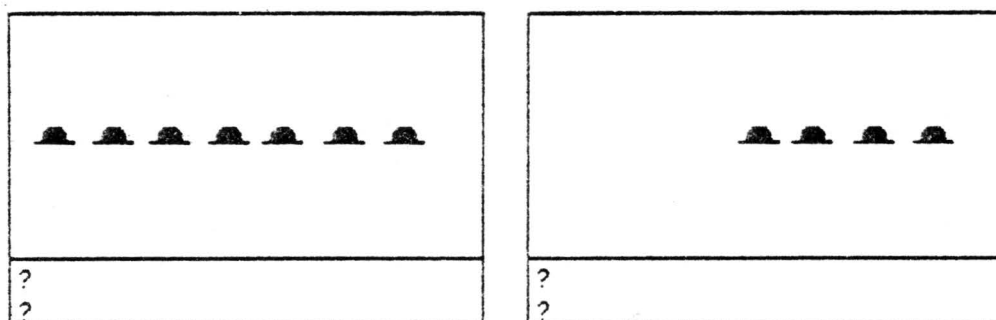


FIGURE 3

Logo Screen: Subtraction Model

Example 3: Draw two rows of hats. Let the top row contain 8 hats and the bottom row contain 3 hats. Keep adding one hat at a time to the bottom row (see Figure 4). How many do you need to add in order to get the same number on both rows?

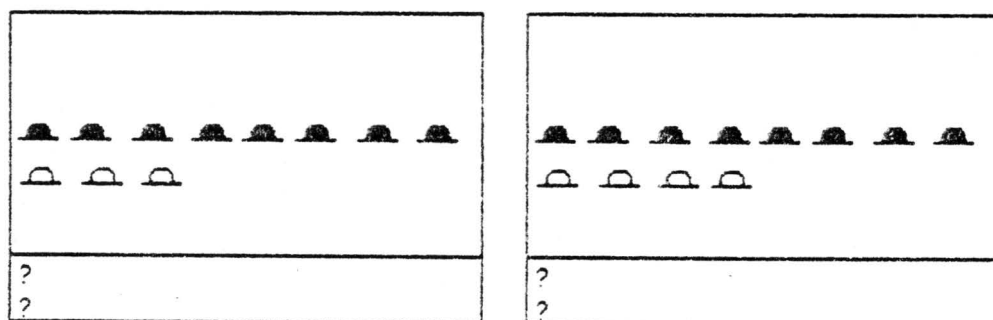


Figure 4

Logo Screen: Subtraction Model

Example 4: Students can look at multiplication and division as repeated addition and repeated subtraction respectively. The student can have the computer draw a set of 5 red circles, one time, two times, three times, and so on (see Figure 5). The reverse can be used to simulate division. The student can draw 15 circles on the screen and then erase 3 at a time until they are all gone. How many groups of 3 were erased? Were there any left over?

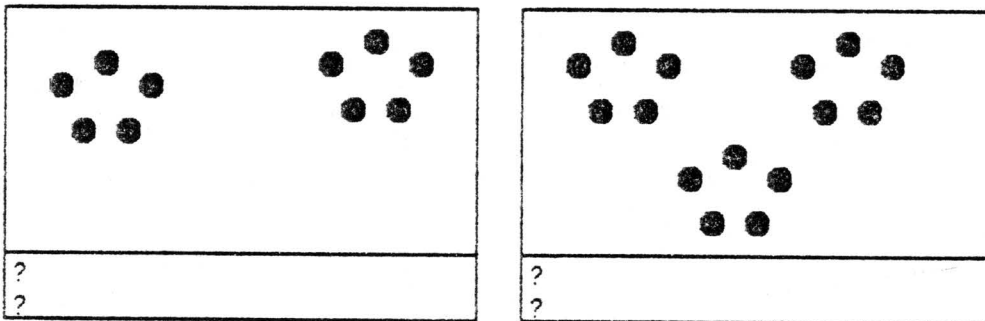


Figure 5

Logo Screen: Multiplication Model

The set of procedures discussed above, allows students to approach operations on whole numbers using manipulation of objects. A second model allows students to simulate addition, subtraction, multiplication and division using motion. Students are provided with a set of procedures that draw a number line and allow the students to move forward and backward on the number line. A forward motion on the number line means addition, a backward motion means subtraction, etc.

Example 5: After drawing the number line the student is asked to determine the position of the turtle after executing the commands F 5 (forward 5 units) and F 4 (forward 4 units) (see Figure 6). *Steps.*

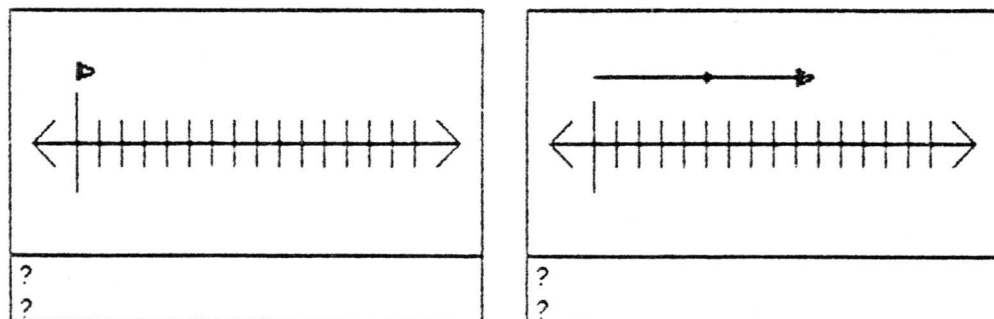


Figure 6

Logo Screen: Number Line Model

Number Theory

A number of procedures are provided for the purpose of exploring number theory topics such as prime and composite numbers, odd and even numbers, multiples and factors. One procedure draws a number of objects on the screen, depending on the given input, and groups the objects into the specified number of items.

Example 1: The student may use the procedures to draw 7 squares and group them by twos (see Figure 7). After experimenting with 7, the student will discover that when you try to group 7 objects by 2, 3, 4, 5, or, 6, there is always a remainder. Working with other groups of objects (eg. 4

squares, 8 squares, or 9 squares) students can discover properties of primes and composites.

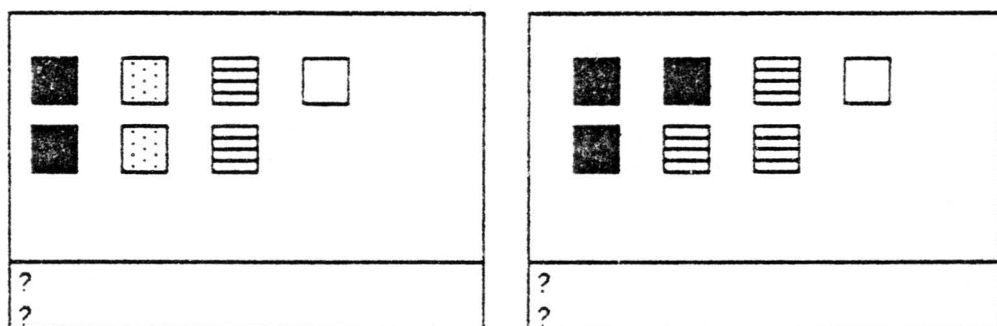


Figure 7

Logo Screen: Primes and Composites

Another group of procedures provide opportunities for discovering the difference between even and odd numbers.

Example 2: Given an input, the computer draws the indicated number of objects by stacking one on top of the other (see Figure 8). Odd numbers will always have one stack with just one square.

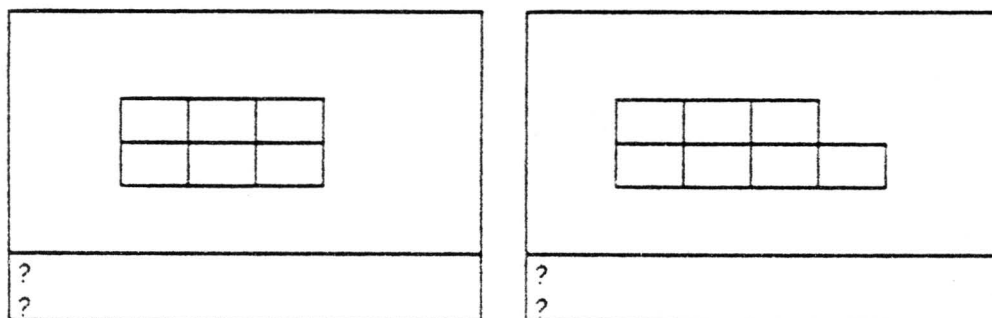


Figure 8

Logo Screen: Odd and Even Numbers

Example 3: Additional procedures allow students to draw a "train" of a certain length (see Figure 9). The students can then draw a number of "train cars" to equal the length of the original "train." Using these procedures, students can discover properties of factors and multiples.

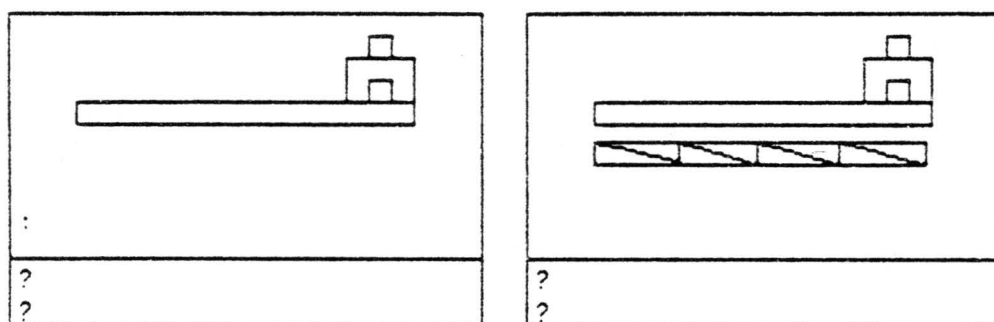


Figure 9

Logo Screen: Factors

Integers

Two different models for performing operations on integers are used. In the first, the student is provided with procedures that draw a number line on the screen going from -24 to 24. The student then uses the F (forward) and B (backward) commands to discover what happens when you move forward (-4) units or backward (-8) units. Does backward a negative number of units move you to the left or to the right? Using the idea of movement, the students can discover exciting properties for adding, subtracting, multiplying, and dividing positive and negative integers.

Example 1: Use the number line to find the sum of 7 and -12 (see Figure 10).

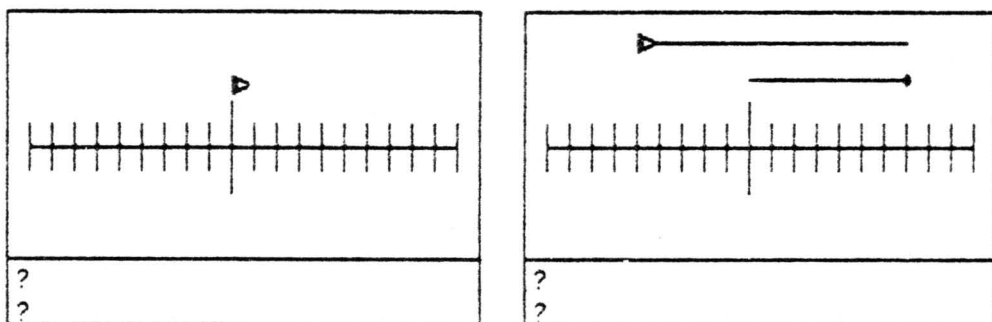


Figure 10

Logo Screen: Addition of Integers

A second set of procedures provides students with simple simulations of real life applications of negative and positive integers. The procedures draw items such as a thermometer, a time line with BC and AD, a bank account, or a hot air balloon. Problems are provided that students are to solve using these procedures.

Example 2: At 12:00 noon the temperature is 0 degrees celcius. The temperature rises 8 degrees over the next 2 hours. What is the temperature at 2:00 PM (see Figure 11)? What will be the temperature if it drops 12 degrees?

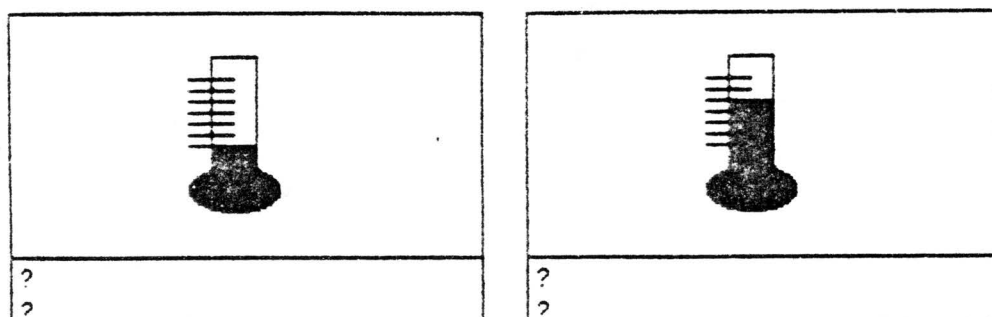


Figure 11

Logo Screen: Addition of Integers

Fractions

In this lab, students are provided with a set of procedures that simulate cutting a pie into different fractional parts. Students explore the terms denominator, numerator, half, fourth, third, etc. Students are then asked to write their own procedures for dividing a "chocolate bar" into different fractional parts (see Figure 12). Finally, students write procedures to move fractional units along the base of their candy bar. Moving along the bottom of the candy bar students can compare fractional parts, look at the sums and differences of fractional parts, and explore multiplication of fractions.

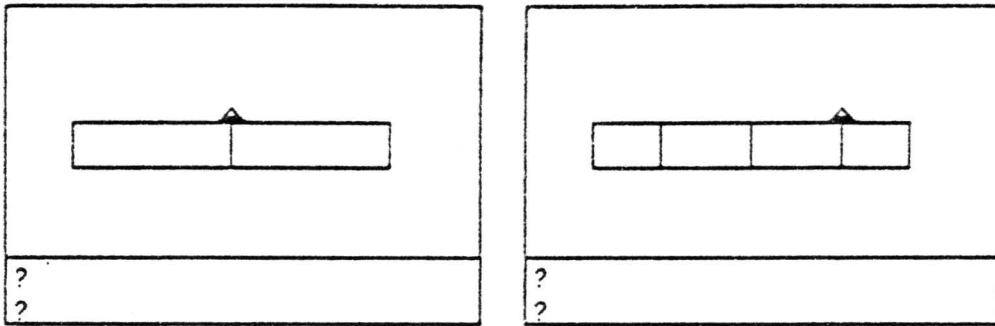


Figure 12

Logo Screen: Fractions

Probability

Two labs are dedicated to this topic. Students are provided with procedures for simulating the tossing of a coin, the throwing of a dice, or the throwing of a pair of die. Students are asked to perform certain experiments using the computer for exhaustive runs of the experiment. The students are also provided with procedures to help them graphically represent their experimental data (see Figure 13). Additional procedures are provided for different card games and dice games that simulate mathematical probability.

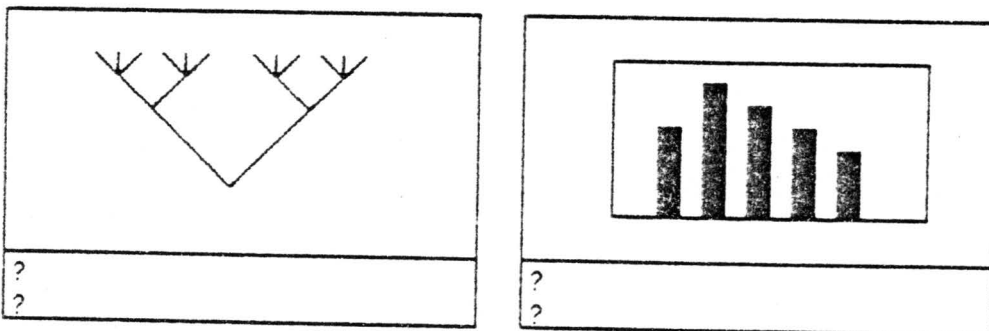
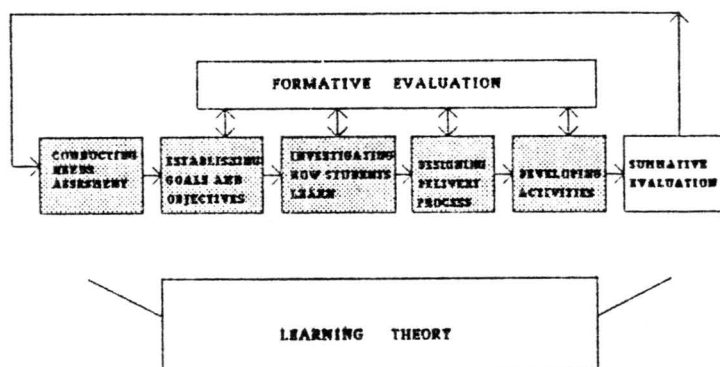


Figure 13

Logo Screen: Graphing Experiments

Counting (Permutations and Combinations)

These activities lead students through real life problems involving permutations and combinations. Computer procedures are provided and developed by the student to perform exhaustive operations. The student is provided with a simple procedure to find the factorial of a number. The students can then build on this procedure to develop tools for calculating permutations and combinations in order to solve complicated mathematical problems.

EVALUATION

This instructional unit offers a new approach to teaching and investigating elementary mathematics topics. The evaluation of the instructional model consists of a formative evaluation and a summative evaluation. The four components to be evaluated are Logo activities, attitudes toward learning and teaching mathematics, attitudes toward using computers to learn mathematics, and understanding of mathematical concepts.

At this time, Logo "microworlds" and lab activities have been developed to accompany mathematics topics which are taught in the first two quarters of the 1600 series. As these activities have been developed, they have been implemented in the weekly labs, evaluated, and revised. The formative evaluation for this stage of development is outlined below.

A. OBJECTIVE: Evaluate Logo activities with respect to:

- 1) Completion of tasks.
- 2) Ease of computer operation.
- 3) Clarity of activity materials.
- 4) Content.

Operational Outcome: Favorable participant and lab assistant attitudes toward the successfulness of lab activities and their implementation.

Data Collection Format:

- 1) Prior to implementation of each lab, a one to one evaluation was performed with a minimum of 2 people. Learners were chosen to represent a typical target audience. Learners were placed in a relaxed setting. The learners were guided and questioned on all areas of the instructional unit.
- 2) During initial implementation, weekly meetings were held with course instructors, Logo specialist, and lab assistants to discuss revisions of past labs and development and implementation of future labs.
- 3) Informal interviews were held with participants.

Criteria: Adequate responses and discussion.

Response: Feedback was used for revision of lab activities
and development of new activities.

B. OBJECTIVE: Measure student attitudes toward using the computer
to learn mathematics.

Operational Outcome: Improved participant attitudes toward
using computers for learning mathematics.

Data Collection Format:

- 1) Informal interviews were held with participants.
- 2) Students were asked to discuss (in small groups) and give feedback on strengths and limitations of Logo activities.

Criteria: Positive attitudes.

Response: Feedback was used to determine which mathematical topics had been improperly addressed or neglected. Students' attitudes toward programming mathematical algorithms they already knew were not favorable. However,

discovering "new" mathematics within existing Logo "microworlds" and expanding on those "microworlds" was favorable.

A final set of labs for the spring quarter will be developed, implemented, and formatively evaluated as described above. Formative evaluations of all labs will be used for further revisions which will be carried out during the summer of 1987. The completed and finalized activities will be implemented during the 1987-88 school year and formative and summative evaluations will be completed as described below.

A. OBJECTIVE: Measure student attitudes toward teaching and learning mathematics.

Operational Outcome: Improved participant attitudes toward learning mathematics.

Data Collection Format:

- 1) Weekly interviews with a minimum of 5 participants.
- 2) Mid-term and final questionnaires (free response) to all participants.

Criteria: Positive attitudes.

Alternative: In areas of non-improvement, reevaluate labs

in order to determine where areas are improperly addressed or neglected. If necessary, revise labs using alternative medium.

B. OBJECTIVE: Measure student attitudes toward using the computer to learn mathematics.

Operational Outcome: Improved participant attitudes toward using computers for learning mathematics.

Data Collection Format:

- 1) Weekly interviews with a minimum of 5 participants.
- 2) Mid-term and final questionnaires (free response) given to all participants.

Criteria: Positive attitudes.

Alternative: In areas of non-improvement, reevaluate labs in order to determine where areas are improperly addressed or neglected. If necessary, revise labs using alternative medium.

C. OBJECTIVE: Measure student understanding of mathematical concepts.

Operational Outcome: Improved scores on mathematics exam.

Data Collection Format: Pre and post examination to assess students' understanding of mathematical concepts. The exam is to be developed by course instructors.

Criteria: 70% of students (or better) obtaining improved mathematical understanding.

Alternative: Reevaluate how topics are being explored. Reevaluate which topics are being explored.

D. OBJECTIVE: Evaluate Logo activities with respect to:

- 1) Completion of tasks
- 2) Ease of computer operation
- 3) Clarity of activity materials
- 4) Content

Operational Outcome: Favorable participant and lab assistant attitudes toward the successfulness of lab activities and their implementation.

Data Collection Format:

- 1) Lab assistants will be asked to fill out weekly evaluations of the Logo labs.
- 2) Students participating in the labs will be asked to fill out mid-term and final evaluations of Logo labs.

Criteria: Adequate responses and discussion.

Alternative: Use feedback for continual revision of lab activities.

APPENDIX A
SAMPLE LOGO ACTIVITIES

LOGO AND WHOLE NUMBERS

GETTING STARTED

Load the Logo language disk.

Insert the Logo Activity disk into drive A and type LOAD

"WHOLE.NUMBERS

During this lab, we will explore the four operations of addition, subtraction, multiplication, and division of whole numbers. The WHOLE.NUMBER file will provide you with a number of models for addition, subtraction, multiplication, and division. You will be asked to create additional models on your own.

ADDITION

What does it mean to add 4 and 5?

We will begin by combining different groups of objects. Use the procedure MOVE to help you navigate about the screen. MOVE needs 2 inputs (an x coordinate and a y coordinate). Example: MOVE 30 40

Before starting, execute the procedure `SIZE`. This procedure will allow you to control the size of the figures that you are drawing.

Execute the procedure `SIZE` anytime during these activities if you want larger or smaller figures. `SIZE` needs one input. Example: `SIZE 30` will draw all figures with sides of length 30.

Use the procedure `PLACE.SQUARE` to place 4 squares anywhere on the screen.

`PLACE.SQUARE 4`

Now place 5 red squares anywhere on the screen. First, change the color.

`SETPALLET 0` These two commands together change the
`SETPC 2` color to RED.

How many squares are there on the screen? _____

Clear the screen.

Have the turtle discover the sum of the following:

3 brown squares and 7 green squares

Try these activities with figures other than squares. To do this, write a procedure to draw any figure with sides of length `:S`. Edit the `PLACE.SQUARE` procedure so that it places figures other than squares on the screen.

5 red items and 3 red items

5 red items and 2 brown items

Do not erase the last problem. Move the turtle above the row you just made.

Have the turtle add the following:

2 brown items and 5 red items

What did you discover about the last two problems?

Let's start again with a clean screen. Have the turtle place 7 red hats on the screen (PLACE.HAT 7). Once it has drawn the hats, place the turtle back in its starting position on the left side.

Draw over one of the hats in a different color (eg. green).

COVER.HAT 1

How many red hats? _____

How many green hats (or any other color)? _____

Draw over another hat in green.

Now how many red hats? _____

How many green hats? _____

Do we still have the same number of hats that we began with? _____

What can we say about the following three statements?

7 RED hats

6 RED hats AND 1 GREEN hat

5 RED hats AND 2 GREEN hats

Continue coloring in one hat at a time and fill in the following chart:

<u>RED hats</u>	<u>GREEN hats</u>
7	0
6	1
5	2
_____	_____
_____	_____
_____	_____
_____	_____
0	_____

Execute the procedure ADD. ADD needs 2 INPUTS.

Example: ADD 3 4

HOW COULD YOU IMPROVE THIS PROCEDURE? Carry out your improvements.

Write three addition story problems and describe how you might use Logo to help you find the solution.

WHAT ABOUT SUBTRACTION?

I have 9 candy bars and I give 4 of them away. How many do I have left?

Let the turtle help you discover the answer.

Have the turtle place 9 squares on the computer screen.

Now place the turtle back on top of the first square.

Type TAKE.AWAY.SQUARES 4

How many candy bars are left? _____

Try the following:

6 candy bars minus 4 candy bars

10 quarters minus 10 quarters

7 cookies minus 2 cookies

What would happen if we switched that last problem around and made it 2 cookies minus 7 cookies?

When we discover a problem that we can not work using the set of whole numbers, we say that that operation is not closed under the set of whole numbers.

Give two other subtraction problems that are impossible to work using the set of whole numbers.

1. _____

2. _____

ANOTHER MODEL FOR SUBTRACTION

Have the turtle draw 8 squares on the screen.

Now have the turtle place 5 triangles just above the set of squares.

Use the procedure `PLACE.TRI 5`

Which set of objects is larger? _____

How many more in the larger set? _____

Erase the screen and have the turtle redraw the 8 squares and 5 triangles but rather than placing one set above the other, place the triangles inside the squares. How many squares are left without triangles?

Explain how you could use Logo to simulate a third model of subtraction.

MULTIPLICATION

What does it mean to say 3 times 5?

Lets think of the first number as the number of repetitions and the second number as the quantity of items. For example, we have 3 baskets which each contain 5 eggs. We want to determine how many eggs we have.

Have the turtle move anywhere on the screen and draw 5 eggs.

EGGS 5

Place two more sets of 5 eggs anywhere on the screen.

How many eggs do we have in all? _____

Have the turtle solve this series of problems:

1 times 4 red eggs is _____

2 times 4 red eggs is _____

3 times 4 red eggs is _____

4 times 4 red eggs is _____

5 times 4 red eggs is _____

6 times 4 red eggs is _____

HOW ELSE MIGHT WE USE LOGO TO MODEL MULTIPLICATION?

DIVISION

Have the turtle draw 12 squares on the screen either in a straight row or in multiple rows.

What is 12 DIVIDED BY 3?

What does 12 DIVIDED BY 3 mean?

Let's say that we have 12 students that we want to divide into groups. We want to put 3 students into each group. How many groups will we have?

Take the 12 squares that you have drawn on the screen. Change the pen color and use the COVER.SQUARE procedure to group 3 of the squares together.

COVER.SQUARE 3

Continue covering three at a time until there are no longer 3 left to cover. How many different groups of 3 do you have? _____ Were there any squares left over? _____

Try the same process with 10 squares, covering 3 at a time.

How many groups did you have? _____

This is called the _____.

How many were left over? _____

This is called the _____.

Use the turtle to help you complete the following chart.

QUOTIENT	REMAINDER
6 DIVIDED BY 1	
6 DIVIDED BY 2	
6 DIVIDES BY 3	
6 DIVIDED BY 4	
6 DIVIDED BY 5	
6 DIVIDED BY 6	

List all of the numbers that you can divided 6 by and have no remainder.

These numbers are called FACTORS of 6.

What are the factors of 4? _____

Clear the screen and type in the following.

LOGS

Let this picture represent 3 logs. Suppose you have 9 frogs that you need to place on these three logs. You want each log to have the same number of frogs. How many frogs will be on each log? _____

Use one of the figures that you have created to represent a frog. Keep placing one frog on each log until you have placed all 9 frogs.

MORE WITH WHOLE NUMBERS

Clear the screen and type `NUMBER.LINE`.

Each slash mark on your number line represents 1 unit. One unit is 10 turtle steps.

The following procedures are available to help you add, subtract, multiply, and divide using the number line.

`TO F` Needs one input. `F 4` moves you forward 40 turtle steps.

`TO B` Needs one input. `B 4` moves you back 40 turtle steps.

`CLEAR` Clears the previous problem and gets you ready for the next problem without erasing the number line.

With the number line on the screen, determine what $1 + 3$ equals.

Type `F 1 F 3`

Where is the turtle located.

Type `CLEAR` before working another problem.

Type in each of the following sets of commands. Write an equation to represent each group of commands.

F 4 F 7 $4 + 7 = 11$

F 8 B 5

F 8 B 3 F 6

F 3 F 3 F 3

F 12 B 4 B 4 B 4

Use the number line to solve each of the following problems.

LOGO COMMANDS

Add 4 and 8 _____

Subtract 3 from 7 _____

Multiply 3 by 4 _____

Divide 10 by 2 _____

Write 8 story problems involving addition, subtraction, multiplication, and division of whole numbers. Use the number line to find the correct answer.

LOGO AND NUMBER THEORY

In today's lab, we will spend time exploring different topics in number theory. We will look at the following topics:

1. PRIMES AND COMPOSITES
2. FACTORS AND MULTIPLES OF A NUMBER
3. ODD AND EVEN NUMBERS

To get started, boot the Logo language disk and load the file NUMBER.THEORY from the activity disk.

Once again, begin by executing the SIZE procedure. You might want to begin with SIZE 25. Anytime during the lesson, you can execute the SIZE procedure to change the size of the objects you are drawing. This will help you if you need to draw many items on the screen.

Have the turtle place 8 squares (use the PLACE.SQUARE procedure) across the screen.

Place the turtle back at the first square and change the color.

Type: COVER.SQUARES 1

Continue grouping by 1 until all the 8 squares have been used.

How many times did you have to group? _____

Feel in the following statement.

8 can be divided into _____ groups of 1 with 0 left over.

Follow the same process to answer the following questions.

8 can be divided into _____ groups of 2 with _____ left over.

8 can be divided into _____ groups of 3 with _____ left over.

8 can be divided into _____ groups of 4 with _____ left over.

8 can be divided into _____ groups of 5 with _____ left over.

8 can be divided into _____ groups of 6 with _____ left over.

8 can be divided into _____ groups of 7 with _____ left over.

8 can be divided into _____ groups of 8 with _____ left over.

Which groupings from the previous page ended with 0 left over?

8 groups of 1, _____

All of these numbers are called **FACTORS** of 8.

List the factors of 8 in increasing order (smallest to largest).

1, _____, 8 (do not repeat factors)

How many different ways can you group 7 objects and have 0 left over?

List the factors of 7. _____

If we divide 10 by 5 and have no remainder, then we can say, "5 goes into 10 evenly", "5 divides 10", "10 is divisible by 5", or "5 is a factor of 10."

To determine if 5 is a factor of 10, type DIVIDE 10 5. Are there any squares that were not changed? _____ Does 5 DIVIDE 10 EVENLY? _____. Is 5 a factor of 10? _____

Why or why not? _____

Is 3 a factor of 10? _____ Why or why not?

Is 2 a factor of 10? _____ Why or why not?

List the factors of:

1 _____

2 _____

3 _____

4 _____

5 _____

6 _____

7 _____

8 _____

9 _____

10 _____

What number is a factor of every number in the list above?

What other characteristics do you find in common between all of the numbers or part of the numbers?

Which numbers only have two factors, 1 and itself?

These numbers are called PRIMES. They have 2 and only 2 factors.

Which numbers from the list above have more than 2 factors?

These numbers are called COMPOSITES. Composite numbers have more than 2 factors.

How many different factors does the number 1 have? _____

Is the number 1 PRIME? _____ COMPOSITE? _____

The number 1 is a special case. We say that it is neither prime nor composite.

FACTOR TRAIN

You have two procedures in this activity called TRAIN and CAR. TRAIN needs 1 input, the length of the TRAIN. Example: TRAIN 80. CAR needs 2 inputs, the length of each box car and the number of box cars. Example:
Car 40 2

Have the computer draw a TRAIN of length 160.

Have the turtle place 4 box cars of length 40 directly under the train. You should see that the entire train is the same length as the 4 box cars.

We can say that 4 times 40 equals 160. Four and 40 are FACTORS of 160.

Find 10 more factors of 160. To do this, find 5 different sets of box cars that are also the same length as the original train.

1. CAR ____

2.

3.

4.

5.

List 12 factors of 160.

Are there more? _____

Draw 3 more trains of different lengths. Determine as many sets of cars as possible that equal the length of the original train. Record your results.

EVEN or ODD

Type **BOXES 8**

Type **BOXES** using a number of different inputs. Record your inputs below the picture that best represents the picture that was drawn on your screen.

EVEN NUMBERS

ODD NUMBERS

Logo has a command called **REMAINDER**. Remainder needs 2 inputs, the dividend and the divisor, and outputs the remainder.

Type **REMAINDER 8 2**

Use this command to determine the remainder of any even number divided by 2.

Use the REMAINDER command to determine the remainder when any odd number is divided by 2.

WHAT OBSERVATIONS CAN YOU MAKE ABOUT EVEN NUMBERS AND ODD NUMBERS?

MULTIPLES

Clear the screen and place 3 squares anywhere?

How many squares are on the screen? 3 That was easy!

Now move the turtle to a new location and place three more square.

How many squares are there now? _____

Continue drawing 3 squares and counting the number of squares on the screen. Record your results below.

DRAWINGS	NUMBER OF SQUARES
0	0
1	3
2	6

3	9
4	_____
5	_____
6	_____
7	_____
8	_____
9	_____
10	_____

All of the numbers in the second column are called multiples of 3.

Can you keep drawing 3 more squares over and over again? _____

Do you get another multiple of 3 every time you draw 3 more squares?

How many multiples of 3 can you possibly get?

How might you describe all the numbers in the first column?

The set of whole number (numbers in the first column) are infinite. They go on forever.

If there is a number in the second column to represent every number in the first column then the multiples of 3 are also _____.

We represent an infinite set by listing a number of its members followed by 3 dots.

EXAMPLE: SET OF MULTIPLES OF 3

{ 0, 3, 6, 9, 12, ... }

Find the multiples of 6.

List all the multiples that 3 and 6 have in common.

Find the multiples of 8.

Find the multiples of 12.

List all of the multiples that 8 and 12 have in common.

24 is called the least common multiple of 8 and 12. Why do you think it is given that name?

Now that you have experimented with different number theory concepts, write your own definition for each of the following:

PRIME

FACTOR

MULTIPLE

COMPOSITE

LOGO AND INTEGERS

In this lab we will look at operations on the set of integers. We will use movement on the number line to look at addition, subtraction, and multiplication of negative and positive numbers. We will also look at some daily situations which involve calculations using negative and positive numbers.

Boot the Logo language disk and load INTEGERS from your activity disk.

TYPE: NUMBER.LINE

The following procedures will help you explore operations using negative and positive numbers.

TO F needs one input; F 1 moves you forward 10 turtle steps or 1 unit on the number line.

TO B needs one input; B 2 moves you back 20 turtle steps or 2 units on the number line.

CLEAR clears the most recent problem without erasing the number line.

We want to use this number line and the turtle to discover some things about negative and positive numbers.

If you typed F -5, what direction do you think the turtle would go in, left or right? _____

Try it.

What if you type B -5

Let's think of addition as forward movement and subtraction as backward movement. If we want to add a negative number we will use F followed by a negative input.

Place the turtle at the origin of the number line.

ADD 5 AND 6

F 5 F 6

WHERE IS THE TURTLE LOCATED?

Start again and ADD 4 and -3.

F 4 F -3

Experiment with adding a negative number to a positive number.

What have you discovered? Write a rule for adding a negative to a positive.

NOW ADD A NEGATIVE TO A NEGATIVE

ADD -6 AND -3

F -6 F -3

Experiment with adding two negatives. Make up problems of your own.

PROBLEM

LOGO COMMAND

$(-4) + (-5)$

F -4 F -5

Write a rule for what you have discovered about adding 2 negative numbers.

ADDING A POSITIVE TO A NEGATIVE

Remember: we are still adding so use FORWARD

LOGO COMMANDS

ADD -5 and 8

ADD -7 AND 3

ADD -3 AND 10 _____

What rule can you derive for adding a positive to a negative.

SUBTRACTION

We said at the beginning of this lab that we would think of addition as forward movement and subtraction as backward movement. If we want to subtract 5 from 8 we must first go forward 8 and then back 5.

Try this problem. Where did the turtle end up? _____

SUBTRACTING A POSITIVE FROM A POSITIVE

PROBLEM

LOGO COMMANDS

8 - 3

F 8 B 3

10 - 4

7 - 7

3 - 8

2 - 6

What happens when the second number is larger than the first. Using what you have discovered with the turtle, write a rule for this condition.

SUBTRACTING A NEGATIVE FROM A POSITIVE

$$4 - (-2)$$

F 4 B -2

Where is the turtle located? _____

Which direction did the turtle move when you typed

B -2 ? _____

What do you think it means to subtract a negative number?

Could you rewrite this problem using only forward movement?

Experiment with a few more problems.

PROBLEM

LOGO COMMAND

Write a rule for subtracting a negative from a positive.

SUBTRACTING A NEGATIVE FROM A NEGATIVE

$$-5 - (-8)$$

$$F -5 \quad B -8$$

Could you rewrite this problem using only forward movement?

Make up some more problems and experiment with the turtle.

What are your conclusions for subtracting a negative from a negative.

MULTIPLICATION

In multiplication, lets think of the first number as the number of repetitions. If the first number is negative then we will repeat the second

number going BACK. If the first number is positive, we will repeat the second number going FORWARD

4 times 5

4 TELLS US HOW MANY TIMES TO REPEAT

SINCE THE 4 IS POSITIVE WE WILL GO FORWARD 5, 4 TIMES

F 5 F 5 F 5 F 5

OR

REPEAT 4 [F 5]

MORE PROBLEMS:

3 times -2

REPEAT 3 [F -2]

WHAT DIRECTION DID THE TURTLE GO IN?

Try the following:

4 times -2

2 times -8

3 times -1

Was your final answer for these positive or negative? _____

TRY: -5 times 2

REMEMBER OUR RULE: The negative 5 tells us to move in a backward direction. The 5 tells us how many times to repeat.

REPEAT 5 [B 2]

Which direction did the turtle move in? _____

Try the following

-4 times 3

-2 times 1

-4 times 1

What did you discover about the statement: a negative times a positive is a _____.

Now we want to multiply a negative by a negative.

Remember our rules. Look at the first number. It tells us how many to repeat and in what direction. THE SECOND NUMBER IS SIMPLY THE QUANTITY.

-2 times -5

REPEAT 2 [B -5]

Which direction did the turtle move in? _____

Try the following:

LOGO COMMANDS

-3 times -2

-2 times -8

-5 times -2

-3 times -4

What can you conclude about multiplying a negative by a negative?

When do we need to use negative numbers? There are many times in real life that we deal with both negative and positive numbers. We state the temperature as 3 degrees celsius or -5 degrees celsius meaning 3 degrees above 0 or 5 degrees below 0. Banks use negative and positive numbers to represent withdrawals and deposits respectively. The stock market uses negative and positive numbers to indicate a loss or a gain in the market. There are many other examples in the real world for negative and positive numbers.

Clear the screen and type in THERMOMETER 8. The 8 represents the starting temperature, 8 degrees above 0.

What if the temperature drops 5 degrees? Type DROP 5 to determine the new temperature.

Now type RISE 7? What does the thermometer read now?

Write 3 story problems dealing with temperature and use the thermometer to find the answers.

Clear the screen and type BANK 80. The 80 represents the amount of money you have in the bank. Your money will be represented by a green bar across the screen.

What if you write a check for \$90? Your account should be overdrawn.

Type WITHDRAW 90

Watch what happens to your bank account. You better make a DEPOSIT quickly. Write 3 story problems dealing with your bank account.

Now type BALLOON.

You should have a picture of a hot air balloon on your screen. Right now, the balloon is stable at 1000 ft up. It is stable because it has equal amounts of hot air bags and sand bags, 5 each.

You have two procedures that you can use to help the balloon go up and down. You can change the number of hot air bags with the procedure HOT.AIR followed by one input. You can change the number of sand bags with the procedure SAND followed by one input.

Before typing in the following commands, determine if you think they will make the balloon go up or down.

HOT.AIR 3

SAND -2

HOT.AIR -5

SAND 3

DESCRIBE IN DETAIL, 4 DIFFERENT WAYS THAT YOU CAN MAKE THE
BALLOON MOVE. HOW DO THESE MODELS RELATE TO ADDITION AND
SUBTRACTION OF NEGATIVE AND POSITIVE NUMBERS.

PROBABILITY

Today's lab will provide you with the opportunity to discover the world of probability. You will need the following items before beginning:

Pencil

Paper

Lab Manual

Logo Language Disk

Storage Disk with Lab Activities

A Penny or Other Coin

A Pair of Dice

For today's lesson, load LAB.TWO into your computer's memory. Now you should be ready to begin.

WHAT IS PROBABILITY?

We use probability when we look at situations in which several things can happen and predict how likely each is to happen. When we toss a coin, there are two possible ways for it to land. It will show a head (H) or a tail (T). What is the probability that a head will come up when you toss a coin? There are two possible ways for a coin to land, each as likely as the other. Only one of these is a head. Therefore, we say:

THE PROBABILITY OF A HEAD IS $1/2$.

To shorten this we will write $\text{Pr}(\text{Head}) = 1/2$ or $\text{Pr}(H) = 1/2$.

Flip a coin 10 times and record the number of heads and the number of tails that occurred.

HEADS _____

TAILS _____

Is it possible to flip a coin 3 times and get exactly 3 heads? _____

What about 10 times and get exactly 10 heads? Yes, it is possible but not very probable. Why not?

It would seem even less probable to flip a coin 1000 times and get exactly 1000 heads.

Flip a coin 20 times and record the number of heads and tails that occurred.

HEADS _____

TAILS _____

Was it exactly 10 heads and 10 tails? _____ Why or why not?

About how many heads do you think you would get if you flipped a coin 1000 times _____, 10,000 times _____, or 1,000,000 times _____.

It would be silly to sit and flip a coin even 100 times. We can let the computer simulate this process for us. LAB.ONE contains a procedure called FLIP. FLIP needs one input to indicate how many times to FLIP the coin. For example: FLIP 100 will flip the coin 100 times and tell you how many heads and how many tails occurred. Use the procedure to complete the following chart:

NUMBER OF FLIPS	100	500	1000	1000000
-----------------	-----	-----	------	---------

NUMBER OF HEADS				
-----------------	--	--	--	--

NUMBER OF TAILS				
-----------------	--	--	--	--

Does the number of heads for 500 tosses seem to be 5 times the number of heads for 100 tosses?

How do you explain these results?

ROLLING A DIE

What are the possible outcomes when you roll a single die?

_____. How many outcomes did you list? _____

If we want to determine the probability of rolling a three, we must consider two questions. How many ways can I roll a three? 1 How many possible rolls? 6. Therefore, we say:

$$\Pr(3) = 1/6$$

What is the probability of rolling a number greater than 4? There are 2 numbers greater than 4 (5 and 6). Therefore we say:

$$\Pr(\text{number} > 4 \text{ on a die}) = 2/6 \text{ or } 1/3.$$

Mathematically, we define the probability of an event to be:

number of ways an event can happen

total number of possible events

Use the definition to solve the problems below.

1. Given a single die, what is the probability of rolling each of the following:

- a. a 1?
- b. a multiple of 4?
- c. a 1 or a 5?
- d. a multiple of 2?
- e. a number less than or equal to 3?

2. If you have a spinner with 1-10 equal divisions, what is the probability of spinning each of the following:

- a. a 6?

- b. a number less than 9?
- c. an even number?
- d. and even number greater than 7?
- e. a perfect square?

3. If you have a bag of marbles -- 3 blue, 4 red, 2 white, and 1 orange, what is the probability of drawing each of the following:

- a. a blue marble?
- b. an orange marble?
- c. a marble that is not white?
- d. a red, blue, white, or orange marble?
- e. a green marble?
- f. a marble that is not pink?

HOW BIG CAN A PROBABILITY BE?

What is the probability of rolling a one digit number when you roll a single die? You know that there are 6 possible outcomes; 1, 2, 3, 4, 5, and 6. All of these are 1 digit numbers. Therefore, $P(\text{rolling a one digit number}) = 6/6$ or 1.

Rolling a one digit number is certain to happen; its probability is 1. This is the largest probability an event can have.

Write four examples of events that are certain to happen.

- 1.
- 2.
- 3.
- 4.

What about the other extreme? How small can a probability be? The probability of shooting a hole-in-one in golf is $1/10,000$. That is pretty small but it can happen.

Can you think of a smaller probability? What is the probability of rolling a 2 digit number with a single die? How many 2 digit numbers are possible when you roll a single die? _____

$\text{Pr}(\text{rolling a 2 digit number with a single die}) = 0/6$ or 0 .

Write 4 examples of events that are impossible?

- 1.
- 2.
- 3.
- 4.

Probabilities vary from 0 to 1. They can be written as a fraction, decimal, or percent.

For example: $\text{Pr}(\text{head}) = 1/2 = .50 = 50\%$.

The probability of having two girls in a two-child family is $1/4$. Write this probability as a decimal and a percent.

If you roll a die 12 times, would you expect to get 2 ones, 2 twos, 2 threes and so on?

Since the probability for any given outcome is $1/6$, we would expect our outcomes to be fairly similar.

Roll a die 10 times and record your results below. Repeat the process for 20 rolls.

NOTE: IF YOU DO NOT HAVE A DIE, YOU CAN USE THE COMPUTER TO SIMULATE THIS. SIMPLY TYPE THE FOLLOWING IN IMMEDIATE MODE:

```
PRINT RANDOM 6 + 1
```

RANDOM 6 SELECTS A NUMBER FROM 0 TO 5. ADDING THE 1 CONVERTS THE NUMBERS FROM 1 TO 6.

NUMBER	TALLY	TOTAL	TALLY	TOTAL
--------	-------	-------	-------	-------

1

2

3

4

5

6

 10

20

The more times we roll a die, the more exact our results become. Let's roll the die 100, 500, and 1000 times and record your results.

You could do this very simply by typing: REPEAT 100 [PRINT RANDOM 6 + 1] However, the hundred numbers would soon scroll off your screen and you would have a difficult time tallying them.

This lab has a procedures called ROLL.DIE to help you simulate this activity and tally the results. ROLL.DIE needs one input. For example, to simulate rolling the die 100 times,

type ROLL.DIE 100

Record your results.

NUMBER	100 ROLLS	500 ROLLS	1000 ROLLS
--------	-----------	-----------	------------

1

2

3

4

5

6

What did you expect to happen in each case?

Were your expectations best met with 100, 500, or 1000 rolls of the die?

Why do you think this happened?

Make a bar chart to record your findings for 1000 rolls. This lab has a procedure GRAPH that will help you. Type GRAPH. The screen should clear and you should see the following at the bottom of your computer screen:

HOW MANY BARS?

Enter 6 since there were 6 different outcomes.

You will then be asked to enter the six totals from your chart. Use the PRINT SCREEN key on the number pad to print your graph.

EXPERIMENT

List the possible sums when you roll 2 dice:

What sum do you think will occur most frequently? _____

Roll two regular dice 20 times. Tally and total the sums below.

SUM 2 3 4 5 6 7 8 9 10 11 12

TALLY

TOTAL

Use the computer to simulate rolling the two dice 100, 500, and 1000 times.

Make sure to record your results below. The procedure `ROLL.TWO.DICE` will help you.

Type: `ROLL.TWO.DICE 100`

SUM 2 3 4 5 6 7 8 9 10 11 12

TOTAL

100

TOTAL

500

TOTAL

1000

Use the computer to make a bar graph to show the frequency of each sum (use the results after 1000 rolls). Print your graph.

What sum occurred most often after 1000 rolls? _____

What sum do you predict will occur most often after 5000 rolls?

_____ Why?

EXPERIMENT

List the possible differences that can occur when you roll 2 regular dice?

Which difference do you think will occur most often? _____

Roll two regular dice 20 times and record the differences below.

DIFFERENCE 0 1 2 3 4 5

TALLY

TOTAL

To have the computer simulate the difference of two dice, we need to change our procedure. Edit the procedure `ROLL.TWO.DICE` so that the line

```
MAKE "X :A + :B
```

becomes `MAKE "X ABS (:A - :B)`

`ABS` stands for absolute value. Why do we need to take the absolute value of the difference of `:A` and `:B`?

Once you have made the change, run the procedure `ROLL.TWO.DICE` three times using 100, 500, and 1000 as your inputs. Record your results and then use the computer to graph one of the results.

Was your prediction about the difference that would occur most often correct?

Think about rolling two regular dice. How many possible outcomes do you think there are? (not possible sums)

Suppose you have one red and one white die. List the possible outcomes from rolling the red die. _____

List the possible outcomes for rolling the white die. _____

When you roll your two dice, there are many ways they can land. Remember, a 4 and a 2 can appear two ways. You could roll a 4 on the red die and a 2 on the white die or a 4 on the white die and a 2 on the red die.

Find the probability of each of the following events if you are rolling two regular dice.

Pr(a sum of 6 or 8)

Pr(an odd sum)

Pr(a sum of 7 or 11)

Pr(a sum of 2, 3, or 12)

Pr(a sum that is a multiple of 4)

Pr a sum that is greater than 6)

The different possibilities of a two-child family are

BOY -- GIRL

GIRL -- BOY

BOY -- BOY

GIRL -- GIRL

There are four different outcomes. We say that the Pr(of having two girls) = $1/4$. What is the Pr(of having a one girl and one boy)?

List the different possibilities for a three child family.

Use your data from the three child family to find each of the following:

Pr(all boys)

Pr(two boys and one girl)

Pr(not all boys)

Carry out an experiment to see if your answers are correct.

Have the computer randomly choose a 1 (boy) or a 2 (girl) three times and print the results on the screen. The series 1 2 1 would represent a family with 2 boys and 1 girl. Run the experiment 100 times. If 23 times out of 100, there are 2 boys and 1 girl then the probability of this event is $23/100$ or approximately $1/4$. Determine the experimental probability for each of the questions above. Are your answers similar? Explain.

PROBABILITY

TO GET STARTED

Boot the Logo Language disk.

Load LAB.THREE from your storage disk.

A WELL KNOWN DICE GAME

In the game of Hazard, two regular dice are rolled. The basic rules are:

-- If the shooter rolls a sum of 7 or 11 on the first roll, he or she wins.

-- If the shooter rolls a sum of 2, 3, or 12 on the first roll, he or she loses.

-- If neither of these happen, the number rolled on the first roll is called the "point".

-- The player continues to roll until the "point" number or a 7 comes up. If a 7 comes up first, the player loses. If the "point" comes up before a seven, the player "makes the point" and wins.

The results that add up to 7 have been circled. Circle all the results that add up to 11.

6	*	*	*	*	*	*
5	*	*	*	*	*	*
4	*	*	*	*	*	*
3	*	*	*	*	*	*
2	*	*	*	*	*	*
1	*	*	*	*	*	*
	1	2	3	4	5	6

How many equally likely outcomes are there when rolling two dice?

How many of those outcomes give a sum of 7 or 11? _____

What is $\Pr(\text{rolling a 7 or 11})$? _____

$\Pr(\text{winning on the first roll})$? _____

Draw squares around the outcomes that give a sum of 2, 3, and 12.

What is $\Pr(2)$? _____ $\Pr(3)$? _____ $\Pr(12)$? _____ $\Pr(\text{losing on the first roll})$? _____

What is $\Pr(\text{for not winning and not losing on the first roll})$? _____

What is $1 - \Pr(\text{win on first roll or lose on first roll})$? _____

Explain why the above two statements are the same?

Roll a pair of dice 20 times. Keep track of how many times you win, lose, or neither win or lose on the first roll, according to the rules above.

Use the procedure HAZARD with one input to simulate the game for 100, 500, and 1000 rolls. Print your numerical results as well as a bar chart depicting the wins, loses, and neither.

What is the ratio of wins/loses for all three trials? The computer can easily act as a calculator. Example: type `PRINT 234/112` to find the ratio of 234 to 112.

Ratio of wins to loses:

100 rolls _____

500 rolls _____

1000 rolls _____

Explain why these ratios would be close to 2/1.

PROBABILITY WITH AND WITHOUT REPLACEMENT

Suppose you have a large bag of M & M candies. Select 10 pieces and place them in your hand: 2 tan, 3 brown, 2 yellow, 1 green, and 2 orange. Close your eyes and pick one piece of candy. Eat all of the pieces of that same color.

What is $\Pr(\text{of eating the yellow pieces first})?$

What is $\Pr(\text{of eating the green pieces first})?$

After eating all of one color, replace them with the same color and number of pieces. For example, if you ate 2 yellow, replace them with 2 yellow.

Now, what is $\Pr(\text{of eating yellow pieces second})?$

$\Pr(\text{of eating green pieces second})?$

If you happened to select the yellow pieces 10 times in a row and always replace them, what is $\Pr(\text{of eating yellow pieces on the eleventh trial})?$

Does the probability of eating a particular color change from trial to trial?

_____ Why is this so?

Now, let us assume that we do not have enough candies to replace the eaten pieces. Our first pick is yellow so we eat the 2 yellow pieces. How many pieces are left? _____

Now given that we ate the yellow first and that they were not replaced, what is:

$\Pr(\text{eating green second})?$

$\Pr(\text{eating yellow second})?$

$\Pr(\text{eating orange third if you ate green second})?$

$\Pr(\text{eating brown third if you ate orange second})?$

Explain how replacing the candies or not replacing the candies effects the probability on the next round of eating.

Problem: You are one of 25 members of a club. The club announces that it will have a drawing each week for 12 weeks to give a prize away to one of its members. What is the probability of you winning the third week if any member can win more than once? What is the probability of winning the third week if a member can only win once? How are these two questions different?

HOW MANY WAYS

Suppose you are trying to decide what to wear. You have several possible choices and it is hard to decide. You have three shirts, two sweaters, and 2 pair of jeans. How many outfits can you have? Using the cartesian product of multiplication, we can determine that there are $3 \times 2 \times 2$ or 12 different outfits to choose from. There is another way to look at this problem. Use a tree diagram. Counting along the branches in a tree diagram shows that there are 12 outfits for you to wear.

Logo can help you draw trees for all different combinations. Use the procedure TREE with one input. The input indicates the level of your tree

(example: the tree above has three levels; shirt, sweater, and jeans.) The procedure will then prompt you to enter the number of objects for each level.

Have the computer draw a tree diagram for each of the following situations. Print the diagram, label the levels, and tally the total number of possible events.

1. Number of meals if there are 3 soups, 4 entrees, and 2 desserts.
2. Number of gift boxes if there are 4 different wrapping papers, 2 different bows, and 2 different gift tags.

Now think about tossing two coins. Have the mputer draw a tree to represent the possible outcomes. What is $\text{Pr}(\text{Head on first coin})?$

_____ $\text{Pr}(\text{Head on second coin})?$ _____

List the paired outcomes for tossing two coins:

_____ How many ways are there for 2 coins to land? _____

What is $\text{Pr}(\text{HH})?$ _____

The tree diagram shows how to find the $\text{Pr}(\text{HH})$.

On the first coin you can get a head or a tail. On the second coin you can also get a head or a tail no matter what resulted on the first coin.

$\text{Pr}(\text{Head on first coin}) = 1/2$

$\text{Pr}(\text{Head on second coin}) = 1/2$

$\text{Pr}(\text{HH}) = \text{Pr}(\text{Head on first}) \times \text{Pr}(\text{Head on second}) = 1/2 \times 1/2$
 $= 1/4$

Find:

$\text{Pr}(\text{HT})?$ _____ $\text{Pr}(\text{TT})?$ _____ $\text{Pr}(\text{TH})?$ _____

Use the computer to draw a tree diagram to show the results of tossing:

3 coins

4 coins

Find: $\text{Pr}(\text{HHH})?$ _____ $\text{Pr}(\text{TTTT})?$ _____

NOTE: THE PROBABILITY OF EVENTS A AND B OCCURRING = $\text{PR}(A)$
 TIMES $\text{PR}(B)$

IF THE TWO EVENTS ARE INDEPENDENT. INDEPENDENT MEANS THAT
 ONE EVENT DOES NOT INFLUENCE THE OTHER EVENT.

MORE PROBABILITY

The goal of this activity is to write a Logo procedure or group of
 procedures to randomly draw a card from a deck of 52 regular playing cards.

We will use this procedure to develop some probability statements about our 52 card deck.

The Logo language, as most languages, has a built in primitive which randomly selects a number.

`RANDOM 6` randomly selects a number between 0 and 5

If you want to randomly assign to the variable "X" a number from 1 to 13, you could use the statements:

```
MAKE "X RANDOM 13  (SELECTS 0 TO 12)
MAKE "X :X + 1    (CONVERTS TO 1 TO 13)
```

Another nice feature of Logo is the use of lists. We can write a list of items and then select one at a time.

```
MAKE "SUITE [HEARTS CLUBS DIAMONDS SPADES]
```

To print the third item in the list above, we could type:

```
PRINT ITEM 3 :SUITE
```

To randomly choose an item, we could type:

```
MAKE "X RANDOM 4
MAKE "X X + 1
PRINT ITEM :X :SUITE
```

Write a procedure that will randomly select a card (example: 2 of HEARTS) from a full 52 card deck. Record your procedure below.

What is $Pr(\text{of drawing a 3 of diamonds from a full deck})$ _____

How many 3 of diamonds do you think you would draw if you picked from a full deck 100 times? _____ 1000 times? _____ 5000 times?

Use the procedure that you have written to randomly draw 1000 cards (replacing the card each time). Add a command to keep track of how many times the three of diamonds was drawn. What was your result?

Was the result what you expected?

Change your procedure to keep track of how many times a Jack was drawn.

What is $Pr(\text{of drawing any Jack from a full deck})$?

How many times do you think you would draw a Jack after 100 drawings from a full deck? 1000 drawings?

Run your procedure to simulate the drawing of 1000 cards. How many times was a Jack drawn?

Come up with 5 other probability statements that we can make about our deck of 52 cards?

- 1.
- 2.
- 3.
- 4.
- 5.

What if we do not return the card to the deck after each trial. Writing a procedure to simulate this is a little more complicated. You do not need to try. However, use what you have learned in this lab to answer each of the following:

1. PR(drawing a queen on the first draw)
2. PR(drawing a heart of the second draw if the first card was a King of diamonds.)
3. PR(drawing 4 Kings in a row)
4. PR(drawing a joker on the second draw if the first draw was not a joker)
5. PR(drawing a King, Queen, Jack, & Ten of hearts in that order)

SAVE YOUR WORK ON YOUR DISK

COMBINATIONS AND PERMUTATIONS

TO GET STARTED:

Load the Logo Language disk

Load COUNTING from your storage disk

What are combinations and permutations? Different arrangements of objects are called permutations. Permutations require that you place objects in a certain order (ex, A B C is a different arrangement from B A C). Combinations deal with arranging items where order does not matter (ex. a committee with John, Mary, and Sally is the same committee as Mary, Sally, and John.)

Suppose you have three different types of coins; a quarter, a nickel, and a dime. How many different ways can you line them up in a row?

You can think about the possibilities like this:

If you start with the quarter, the second one could be either a dime or a nickel. If the second is a dime then the third must be a nickel. Write all the possible arrangements.

How many possible arrangements did you find? _____

Let's look at this in a different way. Place three different coins in front of you. How many different choices do you have for picking a coin?

_____ There are three different choices. Now, take one coin away (any coin you choose). How many different choices do you have for the second selection? _____ Finally, take away a second coin. How many different coins do you have for the third selection? _____

First selection: 3 choices

Second selection: 2 choices

Third selection: 1 choice

Using the multiplication principal you discovered in LAB 3, you multiply and find $3 \times 2 \times 1 = 6$ possibilities. These different arrangements are called permutations. The order of the coins matter in determining the number of different arrangements.

How many permutations are there of 4 different coins? _____

List all of the permutations.

How many permutations for arranging the letters A B C D E? _____

How many permutations for arranging 26 different letters? This would take some calculating to find the answer. We would have to multiply

$$26 \times 25 \times 24 \times 23 \times \dots \times 1$$

The computer can help us find this result. Our procedure is called a recursive procedure because it multiplies over and over again by the next smaller number. List the procedure PERMUTATION and see if you can

understand how the procedure works. Run the procedure several times to determine the permutation of:

15 objects _____

26 objects _____

50 objects _____

In all of the problems above, you had to find the product of a list of numbers starting with a stated number and decreasing by 1 until you reached 1. For example; $5 \times 4 \times 3 \times 2 \times 1$. Since products like this come up often in mathematics, people have invented a special symbol for them.

The product of $5 \times 4 \times 3 \times 2 \times 1$ can be written as $5!$

The explanation point is called a FACTORIAL symbol and the expression $5!$ is read as "five factorial."

What would $1!$ mean. This is a special situation. It makes little sense to say, "Start with 1 and write smaller and smaller numbers until you get down to 1." If you tried that, you would just write 1 and be done.

Mathematicians have agreed that $1! = 1$. Zero causes even a worse problem.

What would $0!$ mean? For right now, we will accept what mathematicians have agreed upon and state that $0! = 1$.

Use the procedure FACTORIAL (this procedure is identical to PERMUTATION) to determine the following:

1. $4!$
2. $9!$
3. $10! - 8!$
4. $4! \times 5!$
5. $12! / 11!$
6. $7! + 6! + 5!$
7. A baseball team plays with 9 players. How many different batting lineups can the coach have?
8. How many different ways are there to introduce the 11 starting football players at a game?
9. Thirty-seven planes took off in the powder-puff derby. How many different finishes are there.

Eight swimmers are competing in a state wide competition. Only the top 4 receive medals. How many different ways can the medals be given out?

Any of the eight could place first. Any of the remaining 7 could place second. Any of the remaining 6 could place third and any of the remaining 5 could place fourth. Therefore, the number of possible ways that the medals could be given out is

8	X	7	X	6	X	5
first		second		third		fourth
place		place		place		place

What you have just done is to look at a permutation situation where you have chosen 4 things out of 8. Mathematicians sometimes abbreviate this with the notation:

$${}_8P_4$$

This is read as "the permutation of 8 things taken 4 at a time" or "the number of ways to choose 4 objects out of 8." To find the result, we start with 8 and multiply by 4 numbers each one less than the preceding one.

Suppose an artist wants to display 5 new paintings but there are only 3 places in the museum. How many different ways can the paintings be placed in the three places? _____

In general, if you are choosing r objects out of n , ${}_n P_r$ is the symbol you would use.

Write a new procedure for permutations that requires two inputs rather than 1. The first input will tell the number to begin with and the second input will determine how many numbers to multiply by. Make up three examples for testing your procedure. Record your results below.

Examples for testing procedure.

- 1.
- 2.
- 3.

PROBLEMS

1. In Michigan, a license plate "number" consists of 3 letters followed by 3 numbers. The vowels a, e, i, o, and u are NOT allowed. Each of the numbers may be any digit, for example, 222 or 000 may be used. How many different license plates are possible. This is a permutation problem which is a little different. When a letter is used, it is not "used up." It can be used again. Describe how you would solve this problem and use the computer to calculate your solution.

2. How many different 9 digit social security numbers can there be if the first digit can not be a zero?

3. Airplane identification numbers appear in several forms. All U.S. airplanes start with N. One form of the ID number has four numbers followed by a letter. The letters I, O, and Z may not be used. How many US airplane ID numbers of this form are possible.

COMBINATIONS

By now you should have a lot of experience with permutations - arranging things in order. There are also times when you need to use combinations, arranging in which order is not important. Suppose your teacher asks for a committee of two to plan a class party. It does not matter in which order the two are picked. How many committees of two can be selected from a class of 21 students? This is a combination problem. Finding answers to

combination problems can be quite easy or quite difficult. Lets suppose we just have a class of two students, (Jane and Bob). We can simply list all of the possibilities.

Committees- two students

Jane and Bob

List all the possibilities if the class has three students (you might call them student A, B, and C), four students, and five students.

Committees - three students

Committees - four students

Committees - five students

There is a special symbol for combinations. C_n^r is read this way: "The number of combinations of r objects selected from n " or "the number of combinations of n objects taken r at a time." In general C_n^r is read in this way: "The number of combinations of n objects taken r at a time."

A second way of solving combination problems is to use Pascal's Triangle. Blaise Pascal was a French mathematician and priest who studied probability. What you see below is only part of Pascal's triangle. It goes on and on, row after row, as far as you want to write the numbers.

Pascal's Triangle is full of patterns. See if you can discover the pattern and add three more rows to the triangle.

			1				
		1		1			
	1		2		1		
	1	3		3		1	
	1	4	6	4		1	
	1	5	10	10	5	1	
	1	6	15	20	15	6	1

Find each of the following combinations.

Look at the 6th row of Pascal's Triangle that contains 1 6 15 20 15 6 1.

This row represents combinations of 6 things taken 0 at a time, 1 at a time, 2 at a time, and all the way to 6 at a time.

Extend Pascal's triangle to 11 rows. Use the triangle to find the answers to each of the following. Write the mathematical notation for each problem.

- a) Combination of 8 objects taken 5 at a time.
- b) Combination of 4 objects taken 1 at a time.
- c) Combination of 10 objects taken 6 at a time.

- d) Combination of 5 objects taken 0 at a time.
- e) Combination of 7 objects taken 4 at a time divided by the combination of 5 objects taken 2 at a time.
- f) Combination of 7 objects taken 0 at a time plus combination of 7 objects taken 1 at a time plus combination of 7 objects taken 2 at a time plus combination of 7 objects taken 3 at a time plus combination of 7 objects taken 4 at a time plus combination of 7 objects taken 5 at a time plus combination of 7 objects taken 6 at a time plus combination of 7 objects taken 7 at a time.

A third way to solve combinations is with the use of factorials.

The combination of 6 things taken 2 at a time can be found using the following formula:

$$\frac{6!}{2!(6-2)!} \quad \text{or in general} \quad \frac{n!}{r!(n-r)!}$$

We can use our FACTORIAL procedure to solve this. Type in each of the following and record your results.

FACTORIAL 6

FACTORIAL 2

FACTORIAL 6-2

Now use the computer to calculate the final result.

Write a procedure called COMBINATION that will take 2 inputs :N and :R and find the combination of :N things taken :R at a time.

Test your procedure with the following examples and check the answer with Pascal's Triangle.

1. COMBINATION 8 3
2. COMBINATION 10 6
3. COMBINATION 4 4
4. COMBINATION 5 0

SAVE YOUR WORK ON YOUR DISK.

FRACTIONS

Fractions were invented long before decimals and have in the past received greater emphasis at the elementary level. However, with the use of handheld calculators and computers, the use of fractions has decreased. This is not to say that fractions will become non-existent. Concepts such as halves, thirds, and fourths are here to stay. This lab will provide us with the opportunity to explore fractions and develop different models for understanding fractions. To begin this lab, you will need the following:

Lab Manual, Logo Language Disk, & Activity Disk

Load FRACTIONS from your storage disk.

Execute the procedure PIE.

Now execute the procedure HALF.PIE.

You have probably known what "a half" is since you were quite young. However, write a general definition of "a half" that could apply to any item or group of items that is divided.

A HALF

We use $1/2$ to represent "one half." The 1 is called the numerator and the 2 is the denominator.

What does the 2 in this case represent? _____

Write a definition for denominator.

What does the 1 in this case represent? _____

Write a definition for numerator.

How many halves are there in the whole pie? _____ This can be written as "two halves" or $2/2$.

We can let the number 1 represent the whole pie since we have 1 pie.

Clear the screen and draw the PIE again.

Type HALF.PIE again

Now type FOURTH.PIE

This procedure divides the PIE into _____ equal parts. How many fourths are in the whole PIE? _____ in one half of the PIE?

Write two fractions that both represent half of the pie.
_____ These fractions are called equivalent fractions.

What is two-thirds? Use the example of a pie to draw a figure that represent two-thirds. Explain what the 2 and the 3 stand for.

In class we talked about the PART-WHOLE interpretation of fractions. On your disk, you have a procedures called PART.WHOLE. Use this procedure

to discover more about fractions. PART.WHOLE needs two inputs. Type in the following examples to determine what the procedure does and what each input is used for.

Examples: PART.WHOLE 8 3

PART.WHOLE 4 2

What does the first input determine?

What does the second input determine?

Try some more example on your own. Write a fraction that represents the results of your examples.

ANOTHER MODEL OF FRACTIONS

Write a simple Logo procedure to draw a rectangular bar on the screen similar to the one below. The bar can be of any length and any hieght. We will us our BAR to further investigate fractions and their properties. The length of the bar will be our major concern. The hieght will always stay the same. We will refer to our bar as a chocolate candy bar.

Record your procedure here: (Make sure the turtle is headed straight up when you finish any procedure.)

TO BAR

END

Have the turtle draw the chocolate bar

You are about to take a bite out of your candy bar when a friend asks you for a bite. Since you are a good friend, you decide to divide it evenly among the two of you.

Write a simple procedure to cut the candy bar in half. Execute both of the procedures to make sure they work together.

Record your procedure and draw a figure depicting a candy bar that has been cut in half.

PROCEDURE

FIGURE

What name would you give to each of the parts. _____

What mathematical notation would you use to symbolize the portion that you will give your friend? _____ Explain why this symbol was chosen. What does it mean.

Now you and your friend are ready to bite into your portion of the candy bar. But wait, two more friends are approaching. Now you must divide your candy bar into 4 equal parts. Write a procedures to do this.

Write other procedures to divide your candy bar into thirds, sixths, eighths, and tenths.

TO THIRD TO FOURTH TO SIXTH

END END END

TO EIGHTH TO TENTH

END END

On your computer screen, draw two different bars. Divide one of the bars into halves, and the other into thirds. (Note: Remember that SETXY can be used to help move you around the screen.)

What is bigger, $1/2$ or $1/3$? _____

Explain why this is true when we have already learned that 3 is greater than 2.

Clear the screen and draw 1 bar. Divide the bar into halves. If you now divide the bar into eighths, will the pieces be bigger or smaller? _____ . Try it. How many eighths are there in the whole bar? _____ How many halves are there in the whole bar? _____ Write

two equivalent fractions that represent the whole bar.

How many eighths are there in half of the bar? _____ Write two equivalent fractions that represent half of the bar.

Draw two new bars, divide one into 4 parts and one into 8 parts.

What is larger $3/8$ or $2/4$? _____

Write three inequalities and 3 equalities that would be true for these two candy bars.

MORE INVESTIGATION OF FRACTIONS

Now write a procedure that moves the turtle in fractional units of $1/2$, $1/4$, and $1/8$ along the base of the candy bar.

TO MOVE.HALF

FD 80 (or what ever is half the length of your candy bar)

END

TO MOVE.FOURTH

TO MOVE.EIGHTH

Now draw the bar and use your EIGHTH procedure to divide it into 8 parts. Place the turtle back at the left bottom corner of the candy bar facing to the right.

Type MOVE.HALF

How many eighths are in one half? _____

Now type MOVE.FOURTH MOVE.FOURTH

You should now be at the end of the bar. $1/2 + 1/4 + 1/4 = 1$ whole

List three other combinations that equal 1 whole.

We can easily write a procedure that will move any fractional part of the bar by using inputs. Write a procedure (on the next page) to do this. Your input should be in the form $1/2$, $1/3$, $1/7$.

TO MOVE :LENGTH

END

Draw the candy bar again and divide it into halves. Use the MOVE procedure with inputs to determine other fractions equivalent to $1/2$. For example: MOVE 100/200

Make a list of at least 5.

ADDING FRACTIONS

Clear the screen and draw a new bar. Divide the bar into 8 equal parts.

Try each of the following and record your results.

MOVE $1/4 + 1/4$

MOVE $5/8 + 2/8$

MOVE $3/4 - 2/4$

In all of these examples, the denominators are the same in the two fractions that are being added or subtracted. Write a rule for adding fractions that have the same denominator.

Now try examples where the denominators are not the same.

For example: MOVE $1/2 + 1/4$

Write 5 different examples, execute them, and record your results.

- 1.
- 2.
- 3.
- 4.
- 5.

Does your rule for adding numbers with the same denominator work here?

_____ Write an appropriate rule and describe how Logo might be used to model this rule.

MULTIPLYING AND DIVIDING FRACTIONS

Use the MOVE procedure to help you multiply fractions. Write down five multiplication examples, execute them and record your results.

EXAMPLE: MOVE $1/2 * 1/4$

- 1.
- 2.
- 3.
- 4.
- 5.

Write a rule for multiplying fractions.

Division with fractions can be confusing. What does $1/2$ divided by $1/4$ mean. You might remember from whole numbers that one model of division was repeated subtraction. $6 \div 2$ means how many 2's can I take away from 6.

How many $\frac{1}{4}$ can you take away from $\frac{1}{2}$.

Draw a candy bar and divide it into fourths.

Type MOVE $\frac{1}{2}$

Now type MOVE $-\frac{1}{4}$ until you have returned to the starting position.

How many $\frac{1}{4}$ could you take away? _____

$$\frac{1}{2} \quad \frac{1}{4} = 2$$

Use your Logo procedures to model each of the following.

1. $\frac{1}{2} \quad \frac{1}{8}$

2. $\frac{3}{4} \quad \frac{1}{8}$

3. $1 \quad \frac{2}{6}$

Explain how you might use your procedures to model each of the following examples.

1. $\frac{3}{8} \quad \frac{1}{4}$

2. $1 \quad \frac{3}{5}$

Save your work on your storage disk.

APPENDIX B
LGOG PROCEDURES

TO COVER.HAT :N
 PC 3
 PLACE.HAT :N
END

TO TAKE.AWAY :N
 PC 0
 PLACE.HAT :N
END

TO ADD :R :G
 MOVE -120 0 PC 3
 PLACE.SQUARE :R
 MOVE -120 50
 PLACE.SQUARE :G
END

TO COVER.TRI :N
 PC 3
 PLACE.TRI :N
END

TO COVER.SQUARE :N
 PC 3
 PLACE.SQUARE :N
END

TO PLACE.TRI :N
 REPEAT :N [TRIANGLE :S RT 90 PU FD :S+5 LT 90 PD]
END

TO PLACE.HAT :N
 REPEAT :N [HAT :S RT 90 PU FD 5 LT 90 PD]
END

```

TO PLACE.SQUARE :N
  REPEAT :N [ SQUARE :S RT 90 PU FD :S+5 LT 90 PD]
END

```

```

TO HAT
  RT 90 FD 5 LT 90
  REPEAT 2 [ FD :S/2 RT 90 FD (:S-10) RT 90]
  RT 90 FD (:S - 5) LT 90
END

```

```

TO TRIANGLE :S
  REPEAT 3 [ FD :S RT 120]
END

```

```

TO SQUARE
  REPEAT 4 [ FD :S RT 90]
END

```

```

TO EGGS :N
  PC 5
  HT REPEAT :N [ EGG PU RT 360/:N FD 45 PD]
  ST
END

```

```

TO EGG
  REPEAT 36 [ FD 2 RT 10]
END

```

```

TO NUMBER.LINE
  DRAW
  HT
  RT 90 REPEAT 28 [ FD 10 LT 90 FD 5 BK 10 FD 5 RT 90]
  MOVE -120 0
  SETH 0 FD 15 BK 30 PU FD 35 RT 90 PD
END

```

```

TO F :N
  FD :N * 10
  POINT
END

```

```

TO POINT
  LT 90 FD 4
  BK 8 FD 4
  PU FD 15
  RT 90
END

```

```

TO B:N
  BK:N * 10
  POINT
END

```

```

TO MOVE :X:Y
  PU SETXY :X:Y
  PD
END

```

```

TO CLEAR
  PU MOVE -120 16
  PC 0 PD HT
  SETH 90
  REPEAT 15 [ FD 500 LT 90 FD 1 RT 90 ]
  PC 1 PU MOVE -120 20
  SETH 90 ST
END

```

```

TO SIZE
  PRINT [ ENTER LENGTH FOR SIDES OF FIGURES ]
  MAKE "S FIRST REQUEST
END

```

NUMBER THEOREY

```

TO MOVE :X:Y
  PU
  SETXY :X:Y PD
END

```

```

TO X :S
  RT 45
  MAKE "M (:S*SQRT(2))
  FD :M BK :M/2 LT 90
  FD :M/2 BK :M
  RT 135 BK :S LT 90
END

```

```

TO SQUARE :S
  REPEAT 4 [ FD :S RT 90 ]
END

```

```

TO PLACE.SQUARE :N
  REPEAT :N [ SQUARE :S RT 90 PU FD :S + 5 LT 90 PD ]
END

```

```

TO PLACE.TRIANGLE :N
  REPEAT :N [ TRIANGLE :S RT 90 PU FD :S + 5 LT 90 PD ]
END

```

```

TO TRIANGLE :S
  REPEAT 3 [ FD :S RT 120 ]
END

```

```

TO COVER.SQUARE :N
  PC 3
  PLACE.SQUARE :N
END

```

```

TO SIZE
  PRINT [ENTER LENGTH FOR SIDES OF FIGURE]
  MAKE "S FIRST REQUEST
END

```

```

TO EVEN.ODD :A
  MAKE :Y (-20 * (:A/2))
  MOVE :Y 0
  REPEAT QUOTIENT :A 2 [ TWO.SQUARE ]
  REPEAT REMAINDER :A 2 [ SQUARE 20 ]
END

```

```

TO TWO.SQUARE
  SQUARE 20
  FD 20 SQUARE 20
  BK 20
  RT 90 FD 20 LT 90
END

```

```

TO DIVIDE :B :A
  PC (RANDOM 6+1)
  REPEAT QUOTIENT :B :A [PLACE.SQUARE :A PC (RANDOM (6+1))]
END

```

```

TO GROUP.BY :N
  MAKE "C (RANDOM 6 + 1)
  PC :C
  PLACE.X :N
  PRINT [CONTINUE...(Y/N)]
  MAKE "Q READCHARACTER
  IF :Q = "Y THEN GROUP.BY :N
END

```

```

TO TRAIN :L
  REPEAT 2 [ FD .25*:L RT 90 FD :L RT 90]
  FD .25*:L RT 90 FD :L- (.25*:L)
  LT 90 SQUARE .25*:L RT 90 FD 5 LT 90
  SQUARE .15*:L LT 90 FD 5
  RT 90 FD .25*:L
  REPEAT 2 [FD .2*:L RT 90 FD .05*:L RT 90]
END

```

```

TO CAR :A
  REPEAT 2 [ FD 20 RT 90 FD :A RT 90]
  LINES :A
  RT 90 FD :A LT 90
END

```

```

TO LINES :A
  REPEAT 4 [ REPEAT 2 [ FD 5 RT 90 FD :A RT 90] FD 5]
  BK 20
END

```

INTEGERS

TO NUMBER.LINE

HT HOME RT 90

REPEAT 24 [FD 10 LT 90 FD 5 BK 10 FD 5 RT 90]

PU HOME

PD FD 20 BK 40 FD 20

PU FD 30 SETH 0

PD ST

END

TO F :N

FD :N * 10

POINT

END

TO POINT

LT 90 FD 4

BK 8 FD 4

PU FD 15

RT 90

END

TO B :N

BK :N * 10

POINT

END

TO MOVE :X :Y

PU SETXY :X :Y

PD

END

TO CLEAR

PU MOVE -120 16

PC 0 PD HT

SETH 90

REPEAT 15 [FD 500 LT 90 FD 1 RT 90]

```

PC 1 PU MOVE -120 20
  SETH 90 ST
END

```

FRACTIONS

```

TO HALF.PIE
  PART.WHOLE 0 2
END

```

```

TO FOURTH.PIE
  PART.WHOLE 0 4
END

```

```

TO PIE
  HT REPEAT 36 [ FD 7 RT 10 ]
END

```

```

TO PART.WHOLE :X :Y
  PIE PU
  FD 3 RT 90 FD 40 LT 90 PD
  REPEAT :Y [ FD 40 BK 40 RT 360/:Y ]
  SHADE :X :Y
  PU LT 90 FD 40 RT 90 BK 3 PD
END

```

```

TO SHADE :X :Y
  REPEAT :X [ REPEAT 360/:Y [ FD 40 BK 40 RT 1 ] ]
END

```

PROBABILITY

```

TO FLIP :A
  MAKE "H 0
  MAKE "T 0
  REPEAT 2 [ PRINT [ ] ]
  PRINT [ PLEASE QAIT... ]
  REPEAT :A [ COIN ]

```

```

REPEAT 2 [PRINT [] ]
PRINT SE [NUMBER OF HEADS:] :H
PRINT SE [NUMBER OF TAIL:] :T
END

```

```

TO COIN
MAKE "X (RANDOM 2)
IF :X = 0 THEN MAKE "H :H+1
IF :X = 1 THEN MAKE "T :T+1
END

```

```

TO ROLL.DIE :N
MAKE "DICE [0 0 0 0 0 0]
MAKE "NUMBERS [ONE: TWO: THREE: FOUR: FIVE: SIX: ]
REPEAT 2 [PRINT [] ]
REPEAT :N [DIE]
REPEAT 2 [PRINT [] ]
PRINT (SE [OUTCOMES FOR] :N [ROLLS OF A DIE] )
PRINT [_____ ]
MAKE "X 1
REPEAT 6 [PRINT SE :NUMBER :X :DICE :X MAKE "X
:X+1]
PRINT [_____ ]
END

```

```

TO DIE
MAKE "X (RANDOM 6 +1)
MAKE "DICE :X :DICE :X + 1
END

```

```

TO GRAPH
DRAW
PRINT [HOW MANY BARS?]
MAKE "X FIRST REQUEST
PU HT SETXY -120 (-60)
PD REPEAT 2 [FD 180 RT 90 FD 240 RT 90]
SLASHES
PU MAKE "C (-1 * (:X/2*15 + 20))
SETXY :C (-60)
MAKE "N 1

```

```

REPEAT X [BAR MAKE "N :N+1]
END

```

```

TO BAR
  PRINT SE [ENTER DATA FOR BAR] :N
  MAKE "L FIRST REQUEST
  MAKE "W 15 PD
  REPEAT 15 [REPEAT 2 [ FD :L RT 90 FD :W RT 90]
    RT 90 FD 1 LT 90 MAKE "L :L-1 MAKE "W :W-1 ]
  PU RT 90 FD 13 LT 90 PD
END

```

```

TO SLASHES
  REPEAT 18 [ RT 90 FD 240 BK 240 LT 90 FD 10]
END

```

```

TO HAZARD :A
  MAKE "WIN 0
  MAKE "LOSE 0
  MAKE "NEITHER 0
  REPEAT 3 [PRINT [] ]
  PRINT [PLEASE WAIT...]
  REPEAT 2 [PRINT [] ]
  REPEAT :A [ROLL]
  PRINT [] PRINT []
  PRINT (SE [OUTCOMES OF] :A [GAMES OF HAZARD])
  PRINT

```

```

[ _____ ]
  PRINT SE [WINS:] :WIN
  PRINT SE [LOSES:] :LOSE
  PRINT SE [NEITHER:] :NEITHER
END

```

```

TO ROLL
  MAKE "C (RANDOM 6 + 1)
  MAKE "D (RANDOM 6 + 1)
  MAKE "E :C + :D
  IF OR :E=7 :E=11 THEN MAKE "WIN :WIN+1 STOP
  IF :E=12 THEN MAKE "LOSE :LOSE+1 STOP
  IF OR :E=2 :E=3 THEN MAKE "LOSE :LOSE +1 STOP
  MAKE "NEITHER :NEITHER + 1
END

```

```
TO TREE :L
  DRAW
  GET.DATA :L
  MAKE "TURN1 180/(:N1+1)
  MAKE "TURN2 180/(:N2+1)
  MAKE "TURN3 180/(:N3+1)
  PU BK 60 PD SETH -90
END
```

```
TO GET.DATA
  PRINT [NUMBER OF EVENTS FOR LEVEL 1]
  MAKE "N1 FIRST REQUEST
  IF :L = 1 THEN STOP
  PRINT [NUMBER OF EVENTS FOR LEVEL 2]
  MAKE "N2 FIRST REQUEST
  IF :L = 2 THEN STOP
  PRINT [NUMBER OF EVENTS FOR LEVEL 3]
  MAKE "N3 FIRST REQUEST
END
```

```
TO LEVEL1 :N1
  IF :N1 = 0 THEN STOP
  RT :TURN1
  FD 100
  MAKE "TEMP1 HEADING
  IF NOT (:L=1) THEN SETH -90 LEVEL2 :N2
  SETH :TEMP1 BK 100
  LEVEL1 :N1-1
END
```

```
TO LEVEL2 :N2
  IF :N2 = 0 THEN STOP
  RT :TURN2
  FD 45
  MAKE "TEMP2 HEADING
  IF NOT (:L=2) THEN SETH -90 LEVEL3 :N3
  SETH :TEMP2 BK 100
  LEVEL2 :N2-1
END
```

```
TO LEVEL3 :N3
  IF :N3 = 0 THEN STOP
  RT :TURN3
  FD 20 BK 20
  LEVEL3 :N3-1
END
```

PERMUTATIONS AND COMBINATIONS

```
TO FACTORIAL :X
  MAKE "FACT 1
  MAKE "C :X
  REPEAT :X [MAKE "FACT :FACT * :C MAKE "C :C+1]
  PRINT (SE [THE FACTORIAL OF] :X [IS] :FACT )
END
```

```
TO PERMUTATION :X
  MAKE "FACT 1
  MAKE "C :X
  REPEAT :X [MAKE "FACT :FACT * :C MAKE "C :C+1]
  PRINT (SE [THE PERMUTATIONS OF] :X [THINGS IS]
    :FACT )
END
```

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MATHEMATICS EXPLORATIONS USING LOGO:
A TRAINING MODEL FOR PERSPECTIVE ELEMENTARY TEACHERS

ABSTRACT

Research shows that many elementary school teachers dislike mathematics and have a fear of teaching the subject. To overcome this problem of poor mathematics teachers in the elementary schools we need to focus on our teacher training programs. This paper presents an alternative model for the lab portion of an undergraduate mathematics course for elementary school teachers and describes the design process used to develop that model. The model reflects the need to provide elementary school teachers with mathematical environments where they can actively investigate and construct their own mathematics. This model utilizes the computer and the programming language Logo to develop "microworlds", ideal for illustrating important mathematical concepts visually and for making abstract ideas concrete. Research indicates that the use of the computer in this manner as a tool for teaching and learning mathematics can not only enhance mathematical understanding but can improve the learnings attitudes about themselves as a mathematician.

Each lab provides the student with written lab materials as well as an activity disk with pre-developed Logo "microworlds". The "microworlds" set the stage for exploring elementary mathematics topics such as operations on whole numbers, number theory, fractions, and probability. The written materials provide student behavioral objectives as well as guided discovery activities to be used to explore and expand the developed "microworlds."

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