TWO-DIMENSIONAL SPATIAL FREQUENCY CONTENT AND CONFUSIONS AMONG DOT MATRIX CHARACTERS

by

Michael Edward Maddox

Dissertation submitted to the Graduate Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

in

Industrial Engineering and Operations Research

APPROVED:

H.L. Snyder, Chairman

S.P. Almeida

A.M. Prestrude

W.W. Wierwille

R.C. Williges

March, 1979 Blacksburg, Virginia

ACKNOWLEDGEMENTS

This research was supported by the U.S. Army Research Office, Grant Number DAAG29-77-G-0067.

Many people contributed to the conduct of the research reported in this dissertation. Dr. Harry L. Snyder gave general direction to the research, in addition to frequent and detailed advice on specific problems when they were encountered.

At one time or another, most of the laboratory personnel were consulted for help on this project. Specifically,

worked many long hours on the hardware modifications to the display which was used in this research. provided a tremendous

amount of software support during all phases of this undertaking.

I would like to express my appreciation to each of my committee members for their helpful comments concerning both the research and the preparation of this dissertation.

Finally, I thank my wife, , who has put up with a lot during the past few years to enable me to reach this point.

TABLE OF CONTENTS

			Page
INTRODUCTION		,	1
Spatial Frequency Analysis Mode	1		2
Dot Matrix Displays		· · · · ·	4
Purpose		1	5
BACKGROUND		· · ·	8
Orientation and Frequency Sensi	tivity		9
Existence of Frequency Analysis	Mechanism	a se Xe a se	11
Experimental Tests of the SFA M	odel		14
One-Dimensional vs. Two-Dimensi	onal Analysis	3	19
Shortcomings of Past Research		en an de Carlos de La composition de la Composition de la composition de la comp	24
METHOD			27
Phase I	н. А		27
Subjects			32
Apparatus		4	32
Experimental design			33
Procedure			56
Analysis of the data		n an	58
Phase II			58
Apparatus			59
Procedure			59
Analysis of the data			62

iii

		Page
RESULTS		68
Phase I	× .	68
Number of errors		68
Confusion matrices		74
Phase II	•	74
DISCUSSION	• • •	103
Phase I		103
Number of errors		103
Confusion matrices		105
Phase, II		106
SUMMARY AND CONCLUSIONS		112
Font Selection		112
Intercharacter Similarity		112
General	•	113
APPLICATIONS		115
REFERENCES		117
VITA		120

iv

LIST OF FIGURES

Figure	Title	Page
1	Sine-wave, Square-wave, and Sawtooth Gratings	13
2	Concepts of MTF and MTFA	18
3	Three Fonts used in Maddox, et al. (1977) Study	21
4	Errors versus Trials for the Three Fonts shown in Figure 3	22
5	Schematic of Activities in Research Program	28
6	Matrix Size / Character Size Configurations	31
7	Experimental design for Phase I	34
8	Huddleston Font in 5 X 7 Matrix	36
~ 9	Huddleston Font in 7 X 9 Matrix	37
10	Huddleston Font in 9 X 11 Matrix	38
11	Huddleston Font in 7 X 9 (=5X7) Matrix	39
12	Huddleston Font in 9 X 11 (=5X7) Matrix	40
13	Maximum Dot Font in 5 X 7 Matrix	41
14	Maximum Dot Font in 7 X 9 Matrix	42
15	Maximum Dot Font in 9 X 11 Matrix	43
16	Maximum Dot Font in 7 X 9 (=5X7) Matrix	44
17	Maximum Dot Font in 9 X 11 (=5X7) Matrix	45
18	Maximum Angle Font in 5 X 7 Matrix	46
19	Maximum Angle Font in 7 X 9 Matrix	47
20	Maximum Angle Font in 9 X 11 Matrix	48
21	Maximum Angle Font in 7 X 9 (=5X7) Matrix	49

v

Figure	Title	Page
22	Maximum Angle Font in 9 X 11 (=5X7) Matrix	50
23	Lincoln/Mitre Font in 5 X 7 Matrix	51
24	Lincoln/Mitre Font in 7 X 9 Matrix	52
25	Lincoln/Mitre Font in 9 X 11 Matrix	53
26	Lincoln/Mitre Font in 7 X 9 (=5X7) Matrix	54
27	Lincoln/Mitre Font in 9 X 11 (=5X7) Matrix	55
28	Effect of Font upon Number of Errors	70
29	Effect of Character/Matrix Size upon Number of Errors	71
30	Effect of Character/Matrix Size by Font Interaction upon Number of Errors	73
31	Confusion Matrix for 5 X 7 Huddleston Font	75
32	Confusion Matrix for 7 X 9 Huddleston Font	76
33	Confusion Matrix for 9 X 11 Huddleston Font	77
34	Confusion Matrix for 7 X 9 (=5X7) Huddleston Font	78
35	Confusion Matrix for 9 X 11 (=5X7) Huddleston Font	79
36	Confusion Matrix for 5 X 7 Maximum Dot Font	80
37	Confusion Matrix for 7 X 9 Maximum Dot Font	81
38	Confusion Matrix for 9 X 11 Maximum Dot Font	82
39	Confusion Matrix for 7 X 9 (=5X7) Maximum Dot Font	83
40	Confusion Matrix for 9 X 11 (=5X7) Maximum Dot Font	. 84
41	Confusion Matrix for 5 X 7 Maximum Angle Font	85
42	Confusion Matrix for 7 X 9 Maximum Angle Font	86

vi.

Figure		•	Title	Page
43	Confusion Matrix	for	9 X 11 Maximum Angle Font	87
44	Confusion Matrix Font	for	7 X 9 (=5X7) Maximum Angle	88
45	Confusion Matrix Angle Font	for	9 X 11 (=5X7) Maximum	89
46	Confusion Matrix	for	5 X 7 Lincoln/Mitre Font	90
47	Confusion Matrix	for	7 X 9 Lincoln/Mitre Font	91
48	Confusion Matrix	for	9 X 11 Lincoln/Mitre Font	92
49	Confusion Matrix Font	for	7 X 9 (=5X7) Lincoln/Mitre	93
50	Confusion Matrix Font	for	9 X 11 (=5X7) Lincoln/Mitre	94

vii

LIST OF TABLES

Table	Title	Page
1	Summary of Analysis of Variance for Correct Responses	69
2	Mean Values of Similarity Measures	95
3	Correlations Between Variables for Full Data Set	97
4	Correlations Between Variables for Confusions ≥ 8	s 99
5	Correlations Between Variables for 2-D R \geq .9	90 101
6	Correlations Between Variables for Phi \geq .60	102

INTRODUCTION

The capabilities of the human visual system have been studied for quite some time. Many psychophysical techniques have been developed over the years to enable precise and repeatable measurements of sensory and perceptual capabilities. These methods generally rely on observable outputs which are related in some way to sensory inputs. Through the use of such techniques, the capabilities of the human visual system have been measured accurately by many researchers. Much is known, for instance, about the ability of the human visual system to resolve detail under many environmental conditions (e.g., McCormick, 1970). The study of color perception has resulted in many published reports concerning, among other things, the efficacy of various color coding schemes (Haeusing, 1976).

As more and more data concerning visual performance became available, certain generalizations were formulated relating stimulus characteristics to observer performance. Two of the more familiar generalizations or "laws" relate stimulus size and perceived brightness (Ricco's and Piper's Laws) and stimulus duration and perceived brightness (Block's Law). Such generalizations have found enough widespread applicability that they can be found in many human engineering handbooks (e.g., VanCott and Kinkade, 1972). While these "laws" can be very useful in describing the functioning of the visual system for limited ranges of input, they do not explain the underlying

mechanism(s) through which the visual stimulus is perceived.

The attempt to formulate the underlying mechanisms which describe the function of a system is known as modeling. An ongoing research problem in the field of vision is the formulation of a viable model of the human visual system. Ideally a visual system model should account for known visual phenomena as well as predict hitherto unobserved properties which can be experimentally verified. Of course, models of the visual system have been postulated by various researchers. Most of these models tend to be over-specific. That is, the models attempt to account for a relatively narrow range of visual phenomena, e.g., Mach bands. Unfortunately, these models generally produce spurious outputs when they attempt to account for a more general range of visual capabilities. Some models have been suggested which take a more general approach to the visual system. Specifically, Almagor (1977) has postulated a visual system model which accounts for a wide range of phenomena.

This proliferation of visual models serves to underscore the fact that, despite the numerous studies which have measured the various properties of the human visual system, few definitive data exist which would support the predictions of one model over others. This is particularly true for predictions of observer performance in practical visual tasks.

Spatial Frequency Analysis Model

An extremely large body of research has been devoted to the

discovery of the visual system's spatial processing mechanism. That is, how does one perceive and differentiate the world around us within our field of view? While the real visual world is made up of many complex and interacting shapes and details which most persons can sort out with very little difficulty, there is precious little explanation of this process.

One of the most tested and most popular theories of spatial perception is generally referred to as the spatial frequency analysis model of the visual system. The term "spatial frequency" is the spatial analog to the better known concept of temporal frequency. In the time domain, a periodic signal which oscillates ten times each second is referred to as a 10 Hz (cyc/sec) signal. In the space domain, a pattern of high and low luminance (brightness) which oscillates between the high and low extremes ten times in a distance of one millimeter is referred to as a 10 cyc/mm spatial frequency distribution.

There are mathematical techniques which transform a visual scene (spatial domain) into the discrete spatial frequencies (frequency domain) which, when added together in the proper proportions, can be combined to reproduce that visual scene. This is known as spatial frequency decomposition since the purely spatial information contained in the original scene is decomposed into the components which exist in the frequency domain.

The Spatial Frequency Analysis (SFA) model of the human visual system assumes that some mechanism exists within the visual system

which decomposes the external scene into those spatial frequencies most prevalent in it. The sensitivity of the visual system to the scene then is postulated to be dependent upon the relative magnitudes of those component spatial frequencies. For example, suppose the visual system is more sensitive to a particular spatial frequency, say 10 cyc/mm, than to other spatial frequencies. The SFA model of the visual system predicts that an observer will be more sensitive (have a lower threshold) to scenes in which the magnitude of the 10 cyc/mm spatial frequency is greater.

The SFA model is sometimes referred to as a multiple-channel model. This terminology is the result of the visual mechanism postulated by the model. It is assumed that spatial frequency decomposition is accomplished in the visual system by the presence of cells which are sensitive to very narrow ranges of spatial frequencies. The magnitude of cell excitation is proportional to the relative magnitude of the spatial frequency to which that cell is sensitive. All the cells which are sensitive to a particular spatial frequency are considered to form a "channel". This concept implies that the information in one channel has little or no effect on the performance of another channel. The sets of frequency-sensitive cells form multiple channels through which spatial frequency decomposition is accomplished.

Dot Matrix Displays

In recent years a display known as the computer-generated dot

matrix display has come into widespread use. This display type is distinguished from an ordinary stroke display by its use of small dots to make up the alphanumeric characters. The form of these alphanumerics lends itself to the study of visual spatial perception, in that stationary specific spatial relationships are maintained within the dot matrix structure. That is, the dots are generally consistent in size and shape and are regularly spaced within the matrix.

The regularity of spacing is known as periodicity. It should be fairly obvious from the preceeding discussion of spatial frequency analysis that the more periodic the intensity distribution then the easier it becomes to analyze the distribution in the frequency domain. The periodic nature of dot matrix displays makes them very useful in the study of spatial frequency mechanisms in the human visual system. The rationale for using dot matrix displays in this research is the relatively high degree of periodicity exhibited in dot matrix intensity distributions. If the visual system is sensitive to spatial frequency information, then the spatial frequency analysis mechanism should manifest itself in the use of dot matrix displays.

Purpose

The purpose of the research is twofold. The basic purpose is to ascertain the efficacy of the two-dimensional spatial frequency analysis model of the human visual system in accounting for observable phenomena. This research was done with a dot matrix display.

The task utilized was tachistoscopic identification of dot matrix alphanumerics written in various fonts. Font is a descriptive term used to identify alphanumerics with certain distinctive characteristics. Most common fonts, such as gothic, pica, and elite, are generally associated with printed alphanumerics. However, there are also fonts designed specifically for dot matrix characters. The utility of any particular font can be determined by presenting single characters to observers for very short periods of time. This technique is known as tachistoscopic presentation.

By using tachistoscopic presentation as the task in this research, the utility of various dot matrix fonts was determined. This determination is the secondary objective of the research, and is, in itself, an important, worthwhile area of study.

The primary purpose of the research was fulfilled by analyzing the various alphanumerics using two-dimensional spatial frequency analysis. The results of these analyses were compared to the observer performance data obtained from the tachistoscopic identification task. These comparisons determine whether the spatial frequency content of displayed information can be reliably related to observer performance. For example, if the letters 'A' and 'P' have very similar spatial frequency components, this technique will show whether or not this similarity is manifested in the confusion of these characters by observers.

In addition, an attempt was made to determine whether more simple calculational procedures can be utilized to predict the relative

confusion among various alphanumerics. If a more simple method can be utilized, then much calculational complexity can be circumvented in the design process.

BACKGROUND

Some basic psychophysical capabilities have been described in detail (e.g., Cornsweet, 1970; Graham, 1965), and many models have been proferred to account for the phenomena exhibited by the human visual system. For instance, fairly early in visual psychophysical experimentation a chemical theory was put forth to explain the observed change in Critical Flicker Frequency (CFF) with stimulus intensity (Jahn, 1946). Most models which have been suggested suffer a common failing with Jahn's (1946) theory. That is, most visual models attempt to account for certain isolated phenomena at the expense of the more general functioning of the visual system.

There are theories concerning the functioning of the visual system which attempt to account for a wide range of observable phenomena. Of the more general theories, those dealing with spatial frequency analysis are currently the most popular and widespread. There are two major aspects to most theories which attribute some type of spatial frequency analysis mechnism to the visual system. The first aspect is the analysis mechnism itself. Although some dispute may arise as to the exact mechanism of analysis and its level of sophistication, *some* mechanism is assumed to exist in any SFA model. The second aspect of such models is the concept of "channels" in the visual system. SFA models are often referred to as multi-channel models, since most of these models assume that there exists some

method of ascertaining the relative magnitude of many different spatial frequency bands. This independent assessment requires the presence of different channels in the visual system.

To understand the reasons for the acceptance of such theories it is helpful to look at some of the known visual characteristics which would support the logical conclusions of SFA models. Both components of SFA theory, i.e., analysis mechanism and channels, have been the subject of many investigations. The concept of individual channels has been demonstrated in several studies. The general results of these studies will be described in the following section.

Orientation and Frequency Sensitivity

The first studies of independent channels in the visual system were performed with animals, using the general method of single cell recording. This methodology consists of inserting electrodes into single cells in the visual cortex or retinal ganglia of anesthetized animals. The animals were then presented with visual stimuli in which the orientation was varied. It was discovered that individual cells respond to only a narrow range of orientations. The cell excitation is maximum for a specific orientation and drops rapidly to noise levels outside a small band of orientations centered on that specific value. Different cells respond to different orientations so that, on a macro level, orientation sensitivity appears to be a continuous function. At the cell level, however, definite discrete channels have been found to exist. Much of the animal work on this

subject is summarized by Pantle (1974).

In the human visual system, single cell recording has certain ethical and practical limitations. However, the use of threshold reporting techniques has been widely used in human experimentation. While discrete channels for orientation cannot be proved to exist in humans, much work exists which confirms that the human visual system is more sensitive to vertical and horizontal patterns than to oblique figures (e.g., Campbell, Kulikowski, and Levinson, 1966). This orientation selectivity has implications which will be pointed out in the discussion of two-dimensional frequency analysis.

In addition to orientation sensitivity, it has been shown that certain animal visual systems contain cells which are sensitive to a very narrow range of spatial frequencies. The method generally used in these studies is single cell recording, as in the orientation studies. The stimuli in the frequency studies, however, are intensity distributions of different spatial frequencies. As in the orientation studies, the cell excitation is maximum at some specific spatial frequency and drops off on either side of this frequency. In addition, it has been found that this frequency sensitivity becomes more and more pronounced as cells are selected from higher and higher in the visual system (Maffei and Fiorentini, 1973).

The sensitivity of the human visual system to various spatial frequencies has been studied using a variety of psychophysical techniques. One such technique is known as simultaneous masking. Simultaneous masking refers to the increased contrast threshold (decreased

sensitivity) of one spatial frequency due to the presence of other spatial frequencies in the visual field. A very exhaustive simultaneous masking study was done by Pantle (1974) in an attempt to demonstrate the existence of multiple channels in the human visual system. In essence, this method consists of simultaneously presenting two spatial frequencies (target and background, respectively). This particular study showed that the masking effects are very dependent upon the relationship between the target and background frequencies. The masking is most pronounced when both target and background have the same spatial frequency and becomes less so on either side of the target frequency.

When all the information from these studies is considered together, there emerges a fairly persuasive argument for the existence of multiple channels in the human visual system. These channels behave as though they are sensitive to relatively narrow ranges of orientation and spatial frequency.

Existence of Frequency Analysis Mechanism

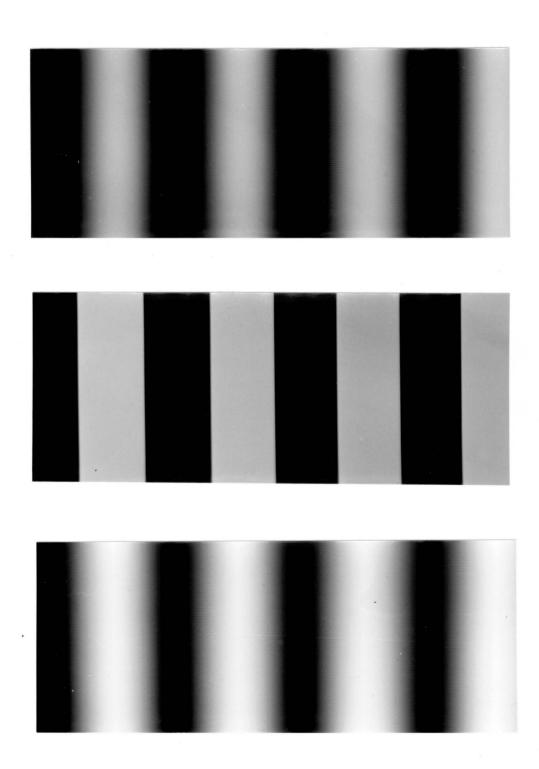
The existence of multiple channels in the human visual system is a necessary but not sufficient condition for the acceptance of SFA models. The most compelling support for these theories has come from studies which have demonstrated that the human visual system does indeed behave as though some form of spatial frequency snalysis mechanism does exist. One of the most quoted studies which demonstrated the analysis effect is the experiment of Campbell and Robson (1968).

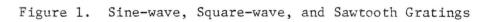
In this study, the visual stimuli were one dimensional grating patterns. These patterns appear as a series of alternating light and dark stripes. The intensity distributions perpendicular to the stripes were either square wave, rectangular wave, or sawtooth. Examples of some commonly used gratings are shown in Figure 1.

The names of the intensity distributions refer to the sharpness of the transition from dark to light. For instance, a square wave distribution is characterized by a saturated black area of a specific width followed by a sharp transition to a white area of the save width. This pattern is repeated for a number of cycles. A sawtooth distribution, on the other hand, is characterized by a saturated black region which is gradually desaturated until a white region is reached. This gives the appearance of a relatively smooth transition from dark to light.

The researchers showed that the sensitivity of observers was proportional to the amplitude of the fundamental spatial frequency component of the wave form. In addition, the threshold was shown to be dependent on the sum of the magnitudes of all spatial frequency components within the observer's range of sensitivity. This phenomenon is known as threshold summation. The relative values of observer thresholds were very close to the relative magnitudes predicted by spatial frequency analysis (Fourier analysis).

Work done by others, also summarized by Pantle (1974), has shown that under certain conditions the sensitivity of observers to complex gratings (gratings of more than one frequency) is a function of the





sensitivity to each separate frequency component. The agreement between the measured observer thresholds and predictions from spatial frequency analysis has provided strong support for the existence of some mechanism within the visual system whereby some form of spatial frequency analysis is performed.

Experimental Tests of the SFA Model

Given a theoretical structure of the human visual system, it remains to be shown that any practical significance can be attributed to the postulated model. Several studies exist in which the underlying assumptions of the SFA model have been put to some degree of testing. The basis for one series of experiments is the concept of the Modulation Transfer Function (MTF), which has had increasing popularity in the field of optics. Optical systems, that is, systems of lenses, are quantitatively described in terms of their MTF. Basically, given the MTF of a system and a description of some spatial frequency distribution to be input to that system, the output spatial frequency distribution of the system can be completely specified. The attractive feature of the MTF is that the output can be determined without actually performing separate measurements for each input frequency. The derivation of the MTF for a physical system is not usually a complicated matter. Attempts to derive a valid MTF for the human visual system have met with rather limited success. This is thought to be due to the nature of the performance measures used in these situations (Lowry and DePalma, 1961) and to the nonlinearities in the

visual system (Cornsweet, 1970).

The Modulation Transfer Function is the ratio of output to input modulation plotted versus spatial frequency. Modulation is a term which describes the luminance contrast of a display at a certain spatial frequency and is usually given by

$$M = \frac{L_{max} - L_{min}}{L_{max} + L_{min}} , \qquad (1)$$

where L_{max} = maximum display luminance, and L_{min} = minimum display luminance (Cornsweet, 1970).

As described previously, the spatial frequency of an object or a displayed pattern is analogous to its detail and is the inverse of its size. For any periodic pattern, such as a sine or square wave, the spatial frequency can be given in terms of lines/mm, cycles/mm, or cycles/degree of subtended angle at the system input or output. To be completely accurate, the term MTF should only be used when the system under study is presented with sinusoidal intensity distributions as input. A function describing the throughput characteristics of a system for other than sine-wave distributions is simply called a "describing function".

The MTF forms the basis of a variety of tests of the SFA model. By itself, the MTF is inadequate to explain the performance of observers on display-oriented tasks. One reason for this shortcoming is the fact that the MTF is concerned solely with the operating characteristics of the display while not accounting for the characteristics of the observer. To account for the effects of the human visual system it is necessary to understand not only how well the display can reproduce a range of spatial frequencies, but also how sensitive an observer is to this spatial frequency range.

The modulation versus spatial frequency (MTF) curve is empirically determined for any given display. There are data available, however, which define the threshold detectability curve of human observers to nearly flat-field sine- and square-wave intensity distributions for various ranges of spatial frequencies (Campbell and Robson, 1967; DePalma and Lowry, 1962; Keesee, 1976). These data take the form of sets of measurements of how much modulation is required by observers so that gratings of certain spatial frequencies are barely visible.

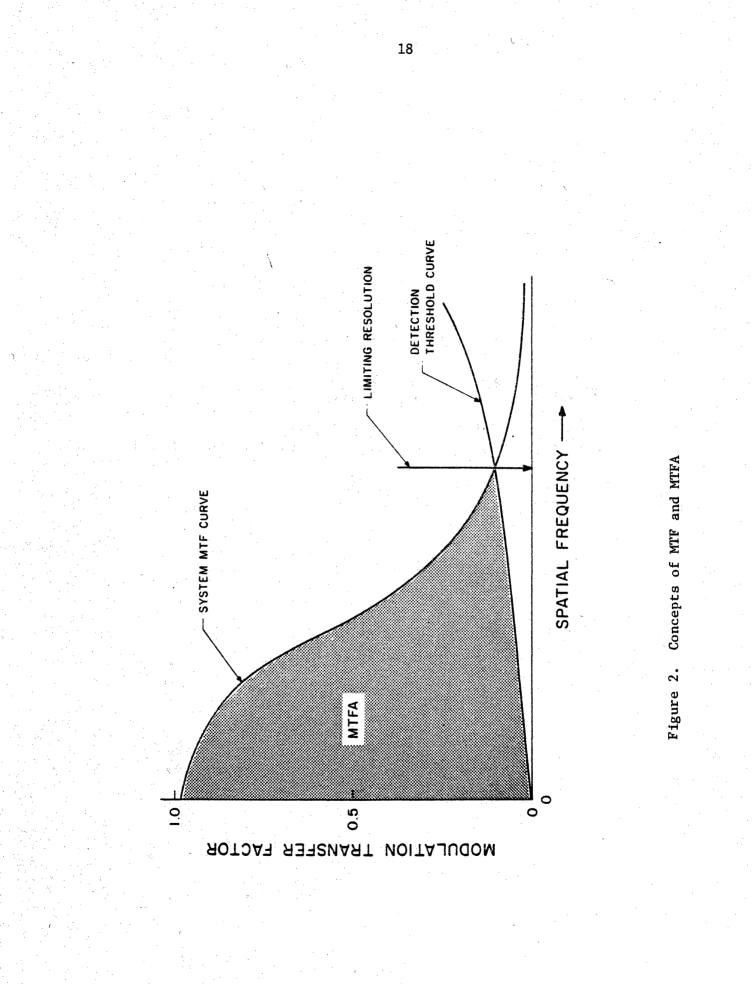
One very well known performance predictor which includes the effects of the human visual system's spatial frequency sensitivity is the Modulation Transfer Function Area (MTFA). The MTFA was developed by Charman and Olin (1965) for photographic systems and has since been refined and expanded for human performance prediction in CRT display situations (Snyder, 1973). The MTFA is a measure of the difference between the display's ability to modulate signals of various spatial frequencies and the observer's threshold modulation requirements at those frequencies.

Specifically, the MTFA is the integrated difference between the display MTF (or describing function) and the observer's detectability threshold from zero to some limiting (crossover) spatial frequency. This limiting spatial frequency is the point beyond which an observer

requires greater modulation than the system is capable of producing. The concepts of MTF and MTFA are illustrated in Figure 2.

The utility of the MTFA as a measure of dot matrix display quality was demonstrated by Albert (1975). In this study, observers were required to read anagrams using 7 X 9 dot matrix characters. A spatial frequency analysis was then performed on these characters. The MTFA for the characters was then computed using the threshold data of Campbell and Robson (1967). These MTFA calculations were then correlated with the performance data of the observers. The resultant prediction equation actually used a weighted log-log MTFA measure, but the correlation between predicted and observed performance was quite high (r = 0.82). This study demonstrated that observer performance on a practical task can be adequately predicted on the basis of the spatial frequency content of the simulated dot matrix display.

Other studies have attempted to derive empirical metrics based on the spatial frequency content of the displayed information and then to relate those equations to observer performance on visual tasks. One such study by Maddox (1977) used multiple regression techniques to correlate spatial frequency terms with performance on several tasks. In this study, 5 X 7 dot matrix characters were displayed using a variety of intracharacter parameters such as dot shape, dot size, and dot spacing. In addition, two levels of ambient illuminance were also used. Observer performance was measured on one reading task and two search tasks. The characters were then subjected to spatial frequency analysis in both horizontal and vertical directions. A variety of



terms based on spatial frequency content was calculated. These terms included MTFA, spatial frequency range, fundamental spatial frequency, etc.

When the measured performance data and the spatial frequency terms were subjected to multiple regression analysis, the resulting equations accounted for a significantly large proportion of observed variance. A separate equation was derived relating performance to spatial frequency terms for each task. One interesting outcome of this study was the observation that the type of spatial frequency information included in the prediction equations changed from task to task. As in the Albert (1975) study, this experiment clearly demonstrated the utility of using spatial frequency information to predict observer performance on visual tasks.

The previously cited studies tend to support the general SFA model. However, some methodological simplifications tend to limit the generality of the conclusions drawn. These limitations will be discussed in the following section.

One-Dimensional vs. Two-Dimensional Analysis

Most visual information exists in at least two dimensions. On flat displays the information is presented in both horizontal and vertical dimensions. Since both dimensions are present simultaneously in the visual field, there exists the possibility of some type of interaction between vertical and horizontal intensity gradients. The studies cited thus far have employed spatial frequency analysis procedures which proceed in one dimension at a time. That is, a horizontal scan of a row of dots (for dot matrix characters) is done and then analyzed. Likewise, a vertical scan is then taken and analyzed. These scans are done independently, thus precluding any interaction effects in the analysis.

It is known, however, that certain display parameters affect observer performance measurably while having no effect on the onedimensional spatial frequency content of the display. One example of such an effect is the change in performance due to differences in font. The font of a set of alphanumerics is simply the style of the letters and numerals or all the characteristics which make one set of alphanumerics distinguishable from other sets. It has been demonstrated that character legibility can differ widely among different fonts. A study by Maddox, Burnett, and Gutmann (1977) demonstrated that identification errors among characters is highly dependent on the font of these characters. The three fonts that were used in this study are shown in Figure 3 and the associated errors are depicted graphically in Figure 4. Despite these statistically significant differences, the one-dimensional spatial frequency distributions remain identical for the different fonts. This is due to the microscopic nature of the single dimensional scans which require only a few dots in a row (or column) to be analyzed.

There is no compelling reason to believe that the human visual system functions in one dimension only or in one dimension at a time. Much theoretical development concerning SFA models of the visual system

ABCDEFGHIJKLMNOPQRSTUVWXYZØ:23456789 ABCDEFGHIJKLMNOPQRSTUVWXYZØI23456789 RECDEFGHIJKLMNOPORSTUVWXYZ0123456789 HIT THE RETURN KEY WHEN FINISHED

Figure 3. Three Fonts used in Maddox, et al. (1977) Study

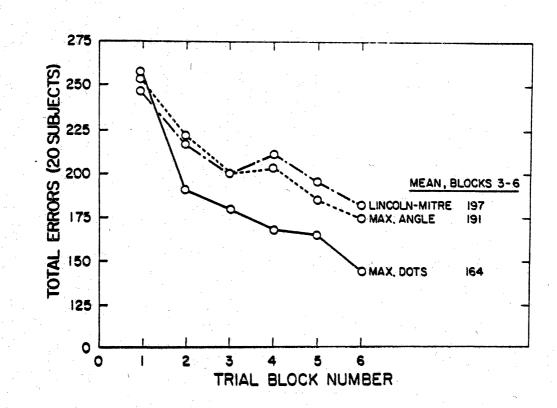


Figure 4. Errors versus Trials for the Three Fonts shown in Figure 3

assumes a two-dimensional analysis within the visual system, even three dimensions if time-varying intensity distributions are considered (Schnitzler, 1976a; 1976b). The primary difference between a onedimensional and a two-dimensional transform is the allowance in the two-dimensional case for interactions among the horizontal and vertical frequency components.

Several researchers have demonstrated that the effects of twodimensional spatial frequency interaction are manifested in observer performance (Burton, 1976; Kelly, 1976; Kelly and Magnuski, 1975). One of the more vivid examples of such interaction is the observer task employed by Kelly (1976). In this task, observers' thresholds to two-dimensional gratings were measured. The intensity distributions of the gratings were manipulated so that the spatial frequency components, as calculated by two-dimensional analysis, exhibited higher magnitudes along different axes. For example, some gratings were designed so that the highest magnitude frequency components were oriented vertically and horizontally. In other gratings, the frequency components were oriented at acute angles to the coordinate axes.

The results of this study showed that observers were more sensitive to gratings which exhibited a horizontal and vertical orientation of frequency components. The observers were less sensitive to gratings which contained more oblique components. This result is in agreement with work cited earlier in which observers were more sensitive to horizontal and vertical patterns than to oblique figures. The important point of Kelly's (1976) research is that this orientation difference

becomes explicit only when the gratings are subjected to two-dimensional spatial frequency analysis. There is no such difference when one-dimensional analyses are used.

This particular study illustrates quite well that certain performance data can only be explained in terms of two-dimensional spatial frequency analysis. Such analyses include the interaction of multi-dimensional spatial frequency information by allowing all points in the visual scene to contribute to the frequency spectrum of the scene. By contrast, the single dimensional analysis allows only limited portions of the visual stimulus to be included in the spectrum. Most studies have not used two-dimensional transforms due to the complexity of the calculations involved. In the last 15 years, however, algorithms have been formulated which greatly facilitate the computation of two-dimensional frequency spectra of visual stimuli (Brigham, 1974; Cooley and Tukey, 1965), thus making such analyses feasible in the typical laboratory environment.

Shortcomings of Past Research

The extent to which two-dimensional spatial frequency analysis by the human visual system contributes to observer performance has yet to be established. This is due to a pragmatic tradeoff between task complexity and computational difficulty. Experiments which have employed relatively realistic (complex) stimuli have generally used only single-dimensional spatial frequency analysis. The resulting one-dimensional transformation cannot account for certain aspects of

visual performance. On the other hand, experiments which have employed more complicated two-dimensional computations have been constrained to use extremely simple visual stimuli (checkerboards, Bessel's functions, etc.). These simple patterns can be analyzed rather easily, since they can be expressed in closed functional form and evaluated analytically. Thus, little practical design utility has come from this two-dimensional research.

The analytical evaluation of intensity distributions bypasses a rather important procedural aspect of visual display research. In order to simplify the computation of two-dimensional spectra, very few studies have photometrically sampled the visual stimuli used in the experimentation. In fact, it is much easier to perform frequency transformations upon functional representations of visual stimuli than to transform sampled data arrays containing actual photometric data. That is, if an intensity distribution can be represented as a mathematical function, then a transformation can be computed analytically, without having to scan the distribution photometrically. Of course, the possibility exists that the results of these simplified experiments are quite valid. However, the elementary stimuli employed in the experiments plus the simplified and abstracted computational procedures limit the generality of the results.

A relatively simple but realistic task which lends itself to spatial frequency analysis could provide some indication of the use of spatial frequency interactions by the visual system. This research employed just such a task and various forms of analysis to relate the

two-dimensional frequency spectra of visual stimuli to observer performance with such stimuli. This research was the next logical link in the experimental attempts to ascertain the utility of the spatial frequency analysis model of the human visual system. METHOD

This research was done in two distinct phases. A general overview of the experimental methodology is shown schematically in Figure 5. This overview shows that the research is divided into Phase I, which required that experimental performance data be gathered using human observers and Phase II, which required the digitization and analysis of the visual stimuli used in Phase I.

The Phase I research required that human subjects view a dot matrix display and make certain decisions based on the displayed information. In Phase II, the displayed information was scanned, digitized, decomposed into component spatial frequencies, and subjected to analyses designed to measure similarity of spatial frequency content. The findings from each phase were then analyzed together to determine how well the performance data could be accounted for on the basis of the analyses performed. Each phase of the research will be described separately.

Phase I

The first phase of the research used human observers to obtain performance data. The subjects viewed single characters on a dot matrix display. The characters were chosen from each of four different fonts. Three of these fonts (Maximum Dot, Maximum Angle, and Lincoln/Mitre) are described in the introduction and have been used in research cited in that section. The fourth font is known as the

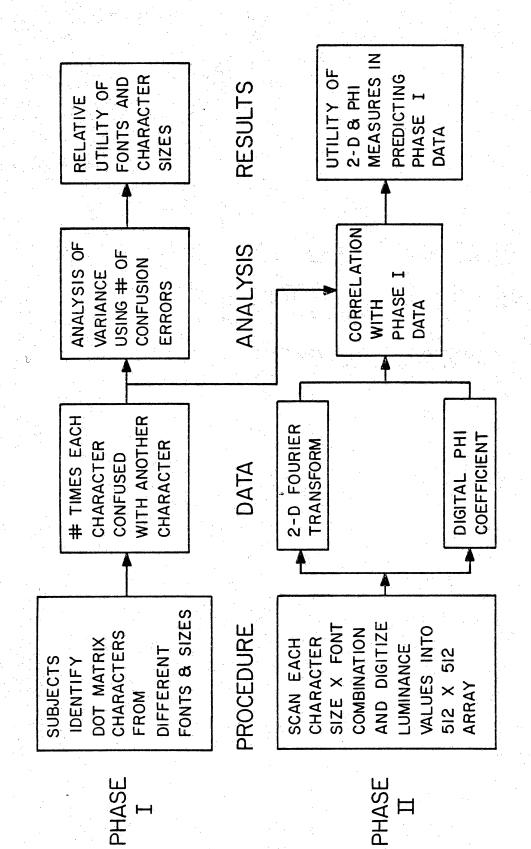


Figure 5. Schematic of Activities in Research Program

Huddleston font and has been designed for high visibility in extermely high ambient light environments (Huddleston, 1971). The Lincoln/ Mitre and Huddleston fonts have been studied by several researchers. The Maximum Dot and Maximum Angle fonts were designed by Maddox, *et al.* (1977) using the most and least dots, respectively, to construct individual characters.

In addition to the font, two other character variables were used in Phase I. These variables are matrix size and character size (subtended visual angle). Matrix size refers to the number of dots available with which to construct the dot matrix characters. A 5 X 7 matrix, for example, is a matrix with a maximum of 5 horizontal and 7 vertical (35 total) dots. The three standard dot matrix sizes were used in the first phase of this research. These three sizes are 5 X 7, 7 X 9, and 9 X 11. If the dot size and interdot spacing remain constant, the dot matrix character will become larger and larger as more dots are used to construct each character. If left unaccounted for, this character size effect will confound any result due to the matrix size used.

To remove much of this confounding, the relative character size was also varied in Phase I. This was done by making each character the same absolute size, no matter what matrix size was used. The same character height is maintained by keeping constant the dot size to dot spacing ratio for each matrix size. However, as the number of dots is increased, the absolute size of the dots is reduced. The various matrix size/character size combinations which were used are shown

conceptually in Figure 6. The use of all five sizes allowed any effect due to matrix size to be statistically separated from effects due to character size (for the 5 X 7 character size).

For sizes one through three, the same dot size and dot spacing are maintained for all matrix sizes (5 X 7, 7 X 9, and 9 X 11). For sizes four and five, the dot size and spacing are manipulated so that the dot-to-dot spacing ratio is held constant and equal to that of the 5 X 7 matrix (size one). The overall character height is fixed for sizes four and five so that the 7 X 9 and 9 X 11 matrices occupy approximately the same area as the 5 X 7 matrix.

It should be noted that this is an incomplete factorial design, in that all character sizes (visual subtense) are not completely crossed with all matrix sizes (number of dots). The selection of the 5 X 7 matrix size as the one within which the expanded (7 X 9 and 9 X 11) matrix sizes are confined is a purely pragmatic decision. The goal of most display design is to pack as many characters as possible on the display while minimizing the number of electrical connections to the device. Toward this end, the designer would undoubtedly opt to retain the smallest overall character size and limit the number of available dots within the character matrix. With this constraint, a complete factorial was deemed unnecessary. The results must also be viewed in this light, since all character size/matrix size combinations were not tested.

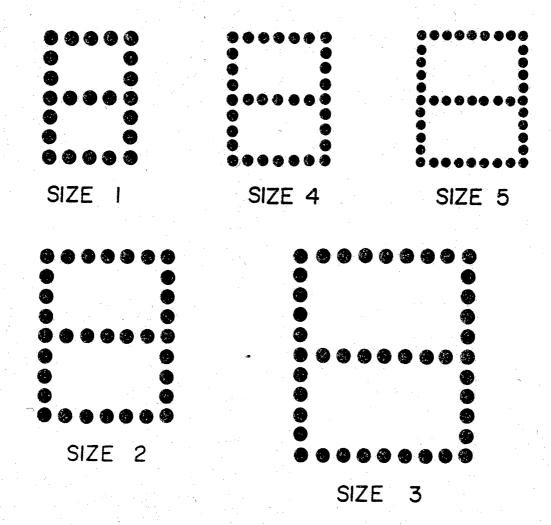


Figure 6. Matrix Size / Character Size Configurations

Subjects. Forty subjects, 20 male and 20 female, were used in this study. All subjects were screened for normal acuity, at least 20/25 corrected, and absence of gross visual defects using a Bausch and Lomb Orthorater. Each subject served a total of approximately four hours and was paid for his/her participation.

Apparatus. The display used in this study was a Tektronix 4014-1 computer graphics terminal. This terminal is equipped with an Enhanced Graphics Module which allows 3072 X 4096 separate points to be displayed on the face of the CRT. These points are slightly irregular due to phosphor blooming. Their size, determined by microphotometric measurement, is .508 mm high by .394 mm wide with center-to-center spacing of .091 mm in either direction. Thus, adjacent points (or minipoints) have substantial overlap. The maximum luminance of these minipoints is approximately 21 cd/m^2 . Larger illuminated areas are obtained by simply illuminating many of these small minipoints. In this research, the illuminated dots had a luminance of 21 cd/m^2 , against a background of approximately 3 cd/m^2 .

In order to increase the data transmission capabilities of the display, a major modification to the character generation circuitry was made. Two special programmable read-only memories (PROMs) were implemented as the alternate character set feature of the 4014-1. By programming the PROMs and selecting the alternate character set from software, individual dots in dot matrix characters were designed to be any shape and size, and then written using only a single character

write command. This proved to be much faster than an earlier method of drawing each dot of the dot matrix character by illuminating a certain sequence of minipoints on the face of the display from software. The older method required much more complicated software and necessitated sending up to 100 bytes per dot for each dot in a character. The new method required only 6 bytes per dot.

The computer system used in this study was a Digital Equipment Corporation (DEC) PDP 11/55 with a dual disk operating system and dual magnetic tape transports. A DEC Laboratory Peripheral System (LPS-11) was employed to supply the external time base used to accomplish all timing delays for generating the dot matrix characters. In addition, an ASCII keyboard was connected to the computer system through the intra-lab connection system. This keyboard served as the subjects' response apparatus such that all data were entered into the computer via the keyboard.

The only other major piece of equipment was a combination forehead rest and keyboard table which was located within a curtained cubicle inside the experimentation room. The Tektronix display was also located within this cubicle. The forehead rest was used to keep the plane of the subjects' eyes approximately 102 cm from the surface of the display.

Experimental design. The basic experimental design for this study is shown in Figure 7. Four character fonts were used in this study and have been described previously. The five character size/ matrix size combinations included the standard matrix sizes (5 X 7,

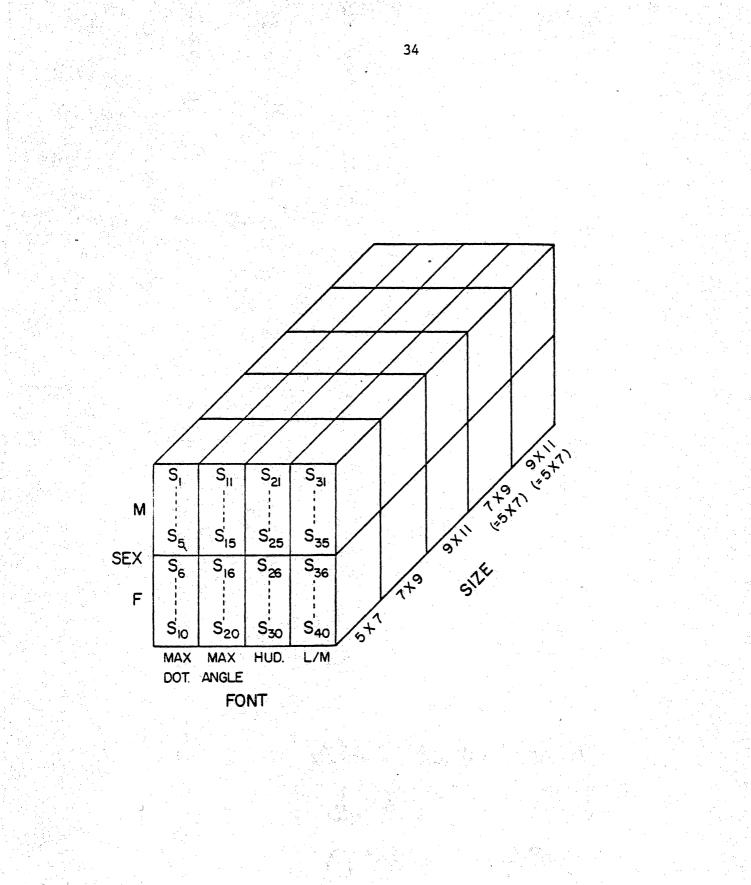


Figure 7. Experimental design for Phase I

7 X 9, and 9 X 11), allowing the character size to expand as more dots are added to the matrix. The 5 X 7, 7 X 9, and 9 X 11 matrices were 14.4, 18.7, and 22.9 mm high, respectively. At the 102 cm viewing distance, they subtended vertical angles of 48.5, 63.0, and 77.2 arcminutes. The remaining two levels were obtained by designing a 7 X 9 and 9 X 11 matrix size character set which remained the same size as the 5 X 7 characters. The various levels of font and matrix/character size are shown in Figures 8 thru 27.

A learning effect has been found to exist in this type of study, i.e., tachistoscopic presentation of single alphanumerics (Maddox, *et al.*, 1977). To make certain that a plateau was reached before experimental trials were begun, each subject was given a series of practice trials on his/her first day of participation. From previous experimentation it had been shown that this response plateau occurred after 10-20 passes through the set of alphanumerics. A pass consists of all 36 characters being presented once. Therefore, subjects were run through the practice alphanumerics 12 complete passes. Since each subject saw only one font, the size for practice was completely counterbalanced across subjects within each font/sex cell.

The order of size presentations was randomized. Once the orderings were obtained, one male and one female subject was run under each ordering. The runs were conducted over two days to minimize fatigue, and the orderings were constrained so that the first size seen on the second day was the same as the practice size seen on the first day. All of these precautions served to make any significant ordering,

PECDEFGHIJK NOTORXUN Sistestow Siste

Figure 8. Huddleston Font in 5 X 7 Matrix

Figure 9. Huddleston Font in 7 X 9 Matrix

1¹11 ۲ • ····· •••• ۳.

Figure 10. Huddleston Font in 9 X 11 Matrix

• Ţ.,

Figure 11. Huddleston Font in 7 X 9 (=5X7) Matrix

_____ ____ F**---ACYM

Figure 12. Huddleston Font in 9 X 11 (=5X7) Matrix

....

Figure 13. Maximum Dot Font in 5 X 7 Matrix

••••• ____ •. ••••

Figure 14. Maximum Dot Font in 7 X 9 Matrix

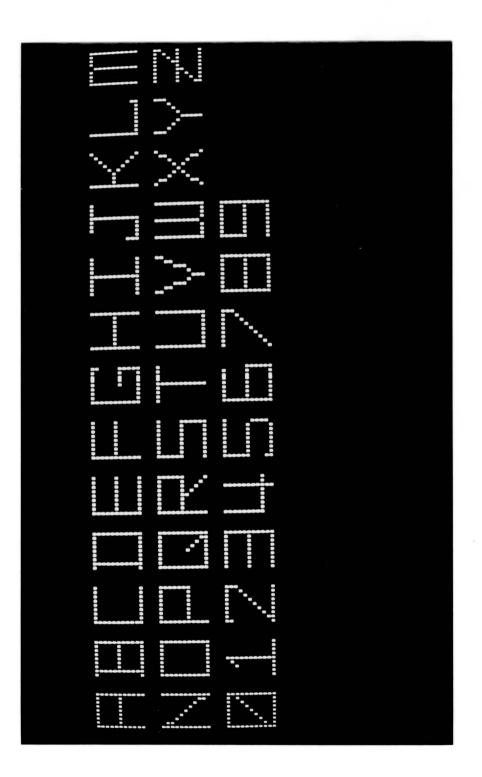


Figure 15. Maximum Dot Font in 9 X 11 Matrix

¢ -

Figure 16. Maximum Dot Font in 7 X 9 (=5X7) Matrix

...... ••••

Figure 17. Maximum Dot Font in 9 X 11 (=5X7) Matrix

ABCDEFGHIJKLM Noporstunuxyz Ø:23456789

Figure 18. Maximum Angle Font in 5 X 7 Matrix

Figure 19. Maximum Angle Font in 7 X 9 Matrix

. •••• ŀ. •••• 1 · · · •••• ••••••

Figure 20. Maximum Angle Font in 9 X 11 Matrix

ABCDEFGHIJKLM NoporstuvuxyZ Ø123456709

Figure 21. Maximum Angle Font in 7 X 9 (=5X7) Matrix

SCDEFGHI UK LM 200RSTUVKUX 23455709]O---Z⊗ ന്റ

Figure 22. Maximum Angle Font in 9 X 11 (=5X7) Matrix

Figure 23. Lincoln/Mitre Font in 5 X 7 Matrix

aoro

Figure 24. Lincoln/Mitre Font in 7 X 9 Matrix

. . . ····· **"--**-----· · · · ·

Figure 25. Lincoln/Mitre Font in 9 X 11 Matrix

ЕГСНТ ЈКЦМ 23ТUVWXYZ 456789 J Œ Ţ ABCD6 NOPQF Ø1231

Figure 26. Lincoln/Mitre Font in 7 X 9 (=5X7) Matrix

6 H I J K L M 6 7 0 9 X Y Z 6 7 8 9

Figure 27. Lincoln/Mitre Font in 9 X 11 (=5X7) Matrix

learning, or fatigue effects highly improbable.

Procedure. Subjects were seated comfortably behind the forehead rest in front of the Tektronix display. The subject was then informed that the entire set of alphanumerics which would be seen on the experimental trials would be displayed simultaneously and would remain visible until the subject was familiar with them. It was emphasized that any similarities or differences among the characters should be noted.

The experimenter then explained the presentation and response entry procedure and answered any questions posed by the subject. The subject was also told of the intercom link between the computer room and the experimental room. For the first day of trials, the subject was told that the first few trials were practice trials. After this, the experimenter retired to the computer room, initiated the program, and asked the subject, via the intercom, if he/she was prepared to begin.

The program for Day One included a brief review of instructions and a series of graphic instruction pages reviewing the method of presentation and response. The actual study contained one practice and two experimental sizes on Day One and three experimental sizes on Day Two. Thus, three sizes were shown on each day. The procedure for each size was nearly identical, as follows.

For each size, the entire set of 36 characters was displayed on the CRT. This included the letters A through Z and the numerals 0 through 9. The subject had as long as he/she desired to study this character set. In practice, no subject took longer than three or four minutes for this phase. After the familiarization phase, a shore review of instructions was given if the subject was on Day One of the trials.

The experimental trials always consisted of the same sequence of events. First, a fixation box was drawn in the middle of the screen. A short time later, a single character was placed in the middle of the box. Each character was constrained so that the average time to write an entire character of any size from any font was 35 ms (± 0.5 ms). After the character was fully written, the program delayed 10 ms and then overlaid the character with a full matrix of dots. The full matrix remained on the screen until the subject responded with the keyboard.

After the character was overlaid, a prompting message appeared in the lower left-hand corner of the display. Following this message, the subject typed in the character which he/she saw, or thought he/she saw, on the preceding trial. When this response was entered, the screen was erased and the next trial was begun. The subject was forced to make a response, by guessing if necessary, on each trial.

The experimental trials were blocked. Each block contained two presentations of the 36 characters in the set, or 72 total trials per block. Characters were chosen randomly from two complete sets of alphanumerics to fill the 72 trials for each block. An experimentercontrolled rest break was initiated after every two blocks, or 144 trials. A total of 4 blocks (288 trials) was given for each experi-

mental size. The practice size was given for a total of six blocks (432 trials) on the first day to assure that the subjects reached a performance plateau. On the second day, the same size as the subject was shown in practice was presented for four blocks of experimental trials.

Day One and Day Two procedures were essentially the same. On Day Two, no extensive instruction period was required and all sizes were run for four blocks of trials.

Analysis of the data. All character presentations and subject responses were stored on disk. An analysis program compared responses with presentations, tagged errors, tabulated statistics, and formatted the output for each subject. From these data sheets, confusion matrices were constructed and an analysis of variance (ANOVA) was performed on the numbers of errors.

Phase II

The overall procedural purpose of this phase of the research was to digitize the visual stimuli which were presented to observers in Phase I. The digitization of a continuous intensity distribution requires some form of sampling mechanism which allows the luminance of the display to be measured at certain discrete locations. The usual method of measuring luminance is to use a photometer to convert the luminous flux of the display into a scaled voltage. This voltage is proportional to the radiant flux weighted by the spectral sensitivity of the human visual system.

Apparatus. The character digitization process required both a photometer system and a mechanism with which to move the photometer sampling head over the display surface. All photometric measurements in this research were made with a Gamma Scientific Model 2400 Digital Photometer. In this particular research, the output of the photomultiplier tube was fed directly through an amplifier into the laboratory computer system. The reasons for this are explained in the subsequent procedures section.

Movement of the photometer sampling head was required in both horizontal and vertical directions. To accomplish this, an Aerotech 260D X-Y stage was used to sample the required area of the display. The X-Y stage was controlled by the laboratory computer system through a special interface designed and built in the Human Factors Laboratory. The X-Y stage is capable of moving in increments as small as 2.54 X 10^{-3} cm (.001 in.) in either axis with a maximum travel of 15.24 cm (6 in.).

Procedure. The digitization of a continuous image requires some basic tradeoffs between resolution and size of the digital array containing the image. The analysis program used in this research has a maximum limit of 512 X 512 points which it can analyze. It was decided to scan the visual stimuli at this density to retain the resolution available on the Tektronix display. It should be recalled that the smallest character size resulted in a vertical character height of more than 13 mm. (This is the linear dimension of a 512 X 512 scan at

.025 mm resolution.) The largest character had a height of 22.9 mm. Therefore, all scans were taken at .051 mm resolution. Physically this is accomplished by scanning an area 26.01 by 26.01 mm and taking digital samples at every other step. Thus, all displayed characters were totally contained within the 512 X 512 matrix scan, which covered an area of 26.01 mm squared on the display.

The photometer eyepiece used for these scans was a circular spot which was 450 microns in diameter. The microscope objective was 2.5 to 1 magnification. Since the sampling aperture was located in the image plane, the 450 micron spot actually sampled a 180 micron diameter circle in the object (display) plane. This can be compared to the step size of the X-Y stage by noting that the step size was approximately 51 microns.

Scans of every character from every font at all matrix sizes would have required 720 total scans (36 characters X 4 fonts X 5 sizes). Each scan stores approximately 260,000 words on magnetic tape. Thus, if all stimulus combinations were scanned, a total of four weeks (40 hours/week) and 40 magnetic tapes for data would have been required. Instead, it was decided that both pragmatic and experimental requirements could be satisfied by scanning all characters and sizes from two of the four total fonts. The fonts chosen were the Huddleston and the Maximum Angle, with the choices based on the errors recorded during Phase I in which the Huddleston font produced the fewest errors and the Maximum Angle font produced the most errors. In all, a total of 360 scans (512 X 512 points each) were made and the digitized points stored on 20 magnetic tapes.

The procedure for scanning each character was the same for all characters. Basically, a single character at a given matrix/character size from a given font was displayed in the center of the Tektronix display. The X-Y stage was moved so that the photometer scanning head sampling aperture was located at the lower left corner of the area to be scanned. This required positioning the sampling head so that the character to be digitized was located in the center of a 26.01 X 26.01 mm field.

After the X-Y stage was positioned to its starting point, the data acquisition program was started. This program is unique in that it operates synchronously with the X-Y stage. This is accomplished by starting the stage in motion with a specific number of steps to travel, 1024 in this case. The X-Y stage stepper motor output is used as an external input to the LPS-11 Schmitt trigger. The program then samples and digitizes the output from the photomultiplier tube every second pulse of the stepper motor. Thus the program operates as quickly or as slowly as necessary to keep up with the X-Y stage stepping motor. When the required number of steps have been completed in one direction (Y), the X position is incremented two steps, and the program directs the stage to reverse direction and start again. This process continues until all 512 columns have been scanned.

It was noted earlier that the photometer was not calibrated to a standard light source for these scans. The reason for this is that

the digital photometer is actually a series of amplifiers which integrate the output from the photomultiplier (PM) tube. The PM tube is the device in which the light energy is actually converted to electrical energy. Unfortunately, the integrating amplifiers introduce a time delay due to the capacitors used to integrate the voltage from the PM tube. As the sampling head is moved rapidly over the surface of the display, the effect of the integrating capacitance is to smooth the edges of the intensity distributions on the display. For example, a sharp transition from very bright to very dark on the display appears to be a sloping transition due to the time constant of the capacitor. Therefore, a small DC amplifier with a gain of approximately 1000 was built in the Human Factors Laboratory. This amplifier was used to take the signal directly from the PM tube and feed it into the input of the LPS-11. The resulting voltage is, thus, in relative luminance values, having been corrected for the photopic visual sensitivity function by the photopic filter matched to the PM tube. Because the same high voltage was applied to the PM tube for all scans, the PM tube output per unit display luminance is linear and constant, but not directly known. This is irrelevant because subsequent analyses disregard the DC component of the luminance signal.

Analysis of the data. There are many calculational techniques available to quantify the similarity between functions. In essence, the array of luminance values associated with the dot matrix characters represents a discrete two-dimensional function. If the spatial

frequency analysis model of the visual system has observable consequences in the task used in this research, then the similarity of spatial frequency content between characters should correlate with observed behavior. To determine the spatial frequency content of the displayed characters, a two-dimensional Fourier transform was performed on the luminance arrays obtained by the X-Y scans.

The multi-dimensional Fourier transform is described in detail elsewhere (Bracewell, 1965). The two-dimensional transform decomposes a two-dimensional spatial intensity distribution into the corresponding spatial frequency distribution in two-dimensional frequency space. The advantage of a direct transform is the explicit representation of spatial frequencies present in the original scene. The disadvantages of the transform are (1) the computational complexity of actually calculating the transform and (2) the necessity of calculating some similarity measure between the transforms in addition to calculating the transforms themselves. The computational difficulty was overcome by utilizing a pre-existing 2-D transform program obtained from the University of Arizona. This program is designed for use with a 512 X 512-point luminance array and can be used with smaller arrays down to 32 X 32 points. The main advantage of using the University of Arizona program was its compatability with the present computer operating sys-The program is based upon the procedures described by Cooley and tem. Tukey (1965).

Each digitized file on magnetic tape was analyzed with the 2-D Fourier transform program. The procedure for doing this analysis was

identical for each file (character). The file was read from tape, formatted to be compatible with the fast Fourier transform (FFT) program, and written to disk. The FFT program then performed the required transform and returned the coefficients to disk by overwriting the locations where the data had resided. A computer program then calculated the magnitude of the real and imaginary parts of each coefficient and added both plus and minus frequency components together for each frequency. The file was truncated to 64 X 64 frequencies so that a manageable statistical procedure could be used to correlate frequency spectra. This truncation did not alter the final results, since the modulations for frequencies above this highest spatial frequency (about 45 cyc/deg) value were well below observer thresholds.

The information contained in this array of 64 X 64 two-dimensional coefficients differs fundamentally from the information contained in two one-dimensional scans. To understand this difference, it is necessary to look at the sources of specific sections of the two-dimensional coefficient array. This array is square with 64 rows of 64 columns (coefficients) per row. The top row of coefficients contains information of the power contributed by the presence of vertical columns of dots in the dot-matrix character. This information is analogous to that contained in a one-dimensional horizontal scan across the character. Likewise, the left hand column of the 2-D array represents the power contributed by the presence of horizontal rows of dots in the dot-matrix character. Such information is also contained in a one-dimensional vertical scan of the character.

The top row and left-most column of the 2-D array contain only 128 of the 4096 coefficients. The remainder of the coefficients contain information unique to the 2-D scan and resultant transform. All other coefficients in the 2-D array represent the power in the spatial frequency domain contributed by the presence of particular vertical and horizontal spatial components. Such information is simply not available from a one-dimensional scan. Probably the most instructive analog is that of a one- vs. two-way analysis of variance. The interaction effect which can be extracted from the two-way analysis is not available in either one-way analysis. In much the same manner, the interaction terms present in a two-dimensional transform are not present in the one-dimensional transform. (Such interaction terms are, of course, critical to the visual system (Kelly, 1976), as indicated previously.)

After truncation, the files were eventually written out to another magnetic tape for transportation to the IBM 370. This tape contained the analyzed and truncated coefficients for all 360 digitized characters. This amounted to all characters in all sizes for both Huddleston and Maximum Angle fonts.

Similarity of spatial frequency spectra among all 36 characters within a given size and font was calculated using a simple productmoment correlation between corresponding Fourier coefficients. Thus, each such correlation was of the 4096 coefficients for each character with the corresponding 4096 coefficients for all other characters, for a total of 630 correlations/size, font combination, or 6300 correla-

55 °

tions in all. The correlations were calculated with the Statistical Analysis System (SAS) implemented on the University IBM 370 computer system (Barr, Goodnight, Sall, and Helwig, 1976).

In addition to the spatial frequency spectra similarity between characters, another type of similarity measure was calculated. This measure, known as the Phi coefficient, is a well known nonparametric technique which determines the degree of correlation between two dichotomous variables (Ghiselli, 1964).

The basis for using the Phi coefficient is the observation that only a limited number of dots is available for character formation in any given matrix size. For instance, in a 5 X 7 matrix only 35 possible locations exist in which to place dots. If the presence of a dot is denoted by a "1" and the absence of a dot by a "0", then each character can be represented by a two-dimensional array of binary values. The functional form of the Phi coefficient is given by

Phi =
$$\frac{P_{c} - P_{a}P_{b}}{(P_{a}Q_{a}P_{b}Q_{b})^{\frac{1}{2}}}$$
, (2)

where P_a = proportion of dots "on" in symbol a, P_b = proportion of dots "on" in symbol b, P_c = proportion of dots "on" in common, Q_a = 1.0 - P_a , and Q_b = 1.0 - P_b .

The similarity between any two characters can be evaluated by computing the Phi correlation coefficient between the two binary arrays

corresponding to the dot distributions in the characters. A total of 6300 Phi coefficients was thus calculated.

Therefore, after these calculations, two measures of similarity between every possible pair of characters within a given font and matrix/character size existed. These were (1) the product-moment correlation coefficient between the 2-D Fourier coefficients and (2) the nonparametric Phi coefficient. The final step in the data analysis was to relate these objective measures of similarity to the performance data gathered in Phase I. From the confusion matrices generated in Phase I, the confusions between each pair of characters was calculated and punched onto IEM cards. Also on each card was the value for each of the two similarity measures described above. For a given font and character/matrix size, these values were intercorrelated using the SAS calculational package. Both the Pearson productmoment and Spearman rank-order coefficients were calculated. The results of all data analyses are presented in the next section.

RESULTS

Phase I

Number of errors. The mean numbers of identification errors per subject per experimental condition were evaluated by an analysis of variance, which is summarized in Table 1. Individual comparisons were made by the Newman-Keuls technique for all meaningful significant effects. From Table 1, it can be seen that the Font and Character/Matrix Size main effects and their interaction were all statistically significant (p < .05).

The Font main effect is illustrated in Figure 28, which indicates that there is no overall significant difference between the Huddleston and Lincoln/Mitre fonts (p > .05), and that each of these was superior to both the Maximum Angle and Maximum Dot fonts (p < .01). Further, the Maximum Dot font was found to be superior to the Maximum Angle font (p < .05).

The Character/Matrix Size main effect is illustrated in Figure 29, which shows several interesting results. First, the 5 X 7 matrix size produced more errors than any of the other sizes (p < .01). The 7 X 9 matrix size yielded the next largest error total, and was in turn inferior to all three remaining matrix/character sizes (p < .01).

The next poorest size was the 7 X 9 matrix size reduced in character size to be equal to the 5 X 7; it, in turn, was inferior to both the 9 X 11 and the reduced 9 X 11 size. The next poorest was the

			· · · · · · · · · · · · · · · · · · ·	
Source	df	MS	F	р
Font (F)	3	5473.58	2.97	0.046
Character/Matrix Size (C)	4	3087.61	21.50	0.0001
Sex (S)	1	1866.60	1.01	0.32
FXC	12	308.26	2.15	0.018
FXS	3	577.10	0.31	0.82
CXS	4	116.37	0.81	0.52
FXCXS	12	43.98	0.31	0.99
Subjects within Font, Sex (Ss/F,S)	32	1843.06		
C X Ss/F,S	128	143.60		
Total	199			
				~

Table 1. Summary of Analysis of Variance for Correct Responses

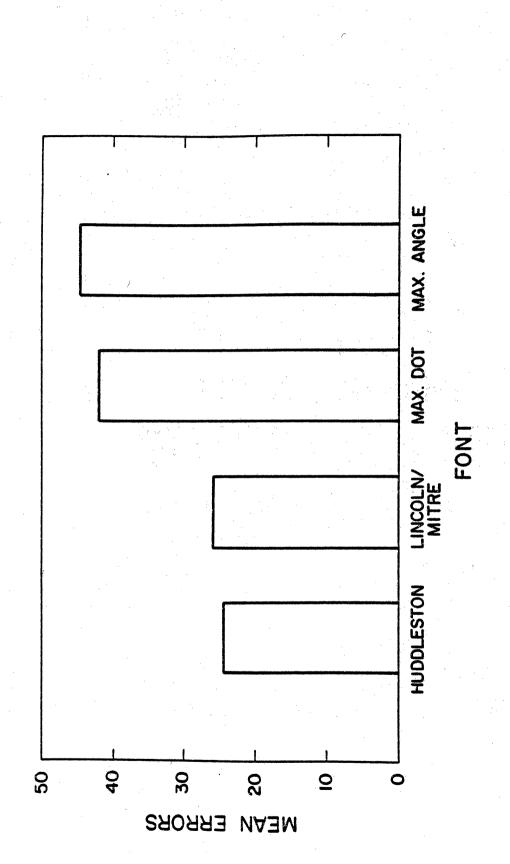
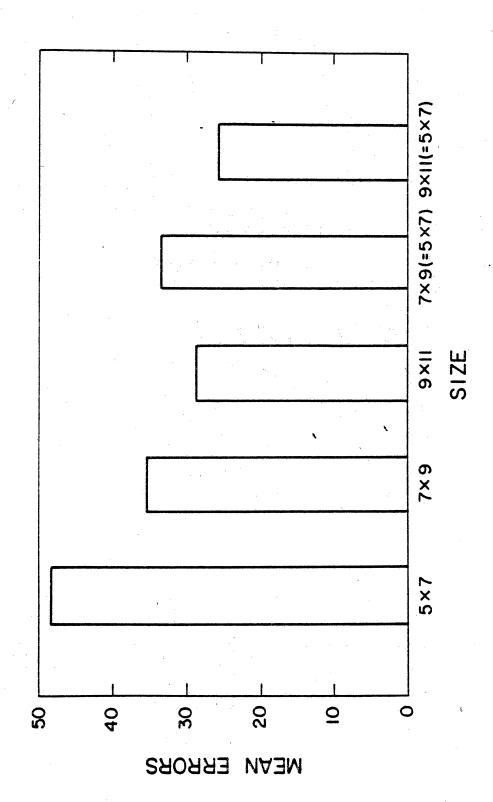


Figure 28. Effect of Font upon Number of Errors





9 X 11 size, which was inferior to the reduced 9 X 11 (p < .01). In summary, the larger the matrix size within the character size of the 5 X 7 matrix, the fewer the recognition errors.

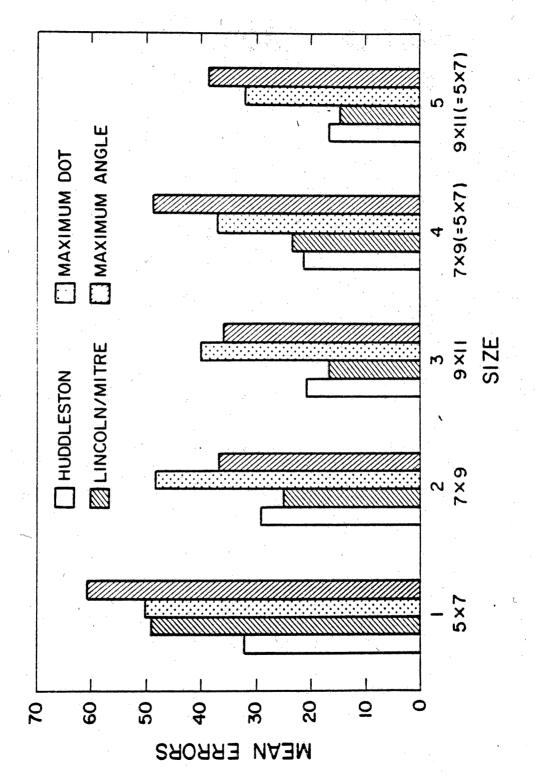
The Font X Character/Matrix Size interaction is shown in Figure 30. For the 5 X 7 size, the Huddleston font is superior to the other three (p < .01), while the Lincoln/Mitre and Maximum Dot fonts are essentially equivalent (p > .05). In this matrix size, the Maximum Angle font produced more errors than any of the other three fonts (p < .01).

For the 7 X 9 font, the Lincoln/Mitre font was superior to the other three (p < .05 for Huddleston; p < .01 for other comparisons). All comparisons among the Huddleston, Maximum Angle, and Maximum Dot error rates are statistically significant (p < .01).

Similarly, all comparisons among fonts for the 9 X ll matrix size are statistically significant (Huddleston vs. Lincoln/Mitre, p < .05; Maximum Angle vs. Maximum Dot, p < .05; all remaining comparisons, p < .01). For this matrix size, the Lincoln/Mitre is best and the Huddleston next best.

The Lincoln/Mitre and Huddleston fonts are essentially equivalent (p > .05) for the reduced 7 X 9 size, while all other comparisons are significant (p < .01), with the Maximum Angle the poorest.

Similarly, for the reduced 9 X 11 size, the Lincoln/Mitre and Huddleston fonts are nondifferent (p > .05), with the Maximum Angle font again the poorest (p < .01) and the Maximum Dot font next poor (p < .01).



Effect of Character/Matrix Size by Font Interaction upon Number of Errors Figure 30.

These interaction effects are not all possible such effects. As noted previously, character size and matrix size were not completely crossed. Therefore, these results must be interpreted within the scope of the conditions which were included in the design. As pointed out earlier, it is doubtful that the treatment levels which were not included would be of any interest to the display designer.

Confusion matrices. The confusion matrices constructed from the Phase I performance data are illustrated in Figures 31-50. Much can be learned from such matrices about the nature of the confusions among certain characters within a given font. For example, in the Maximum Dot font, major confusions were between 5 and S, 2 and 7, and Y and V. The Y-V confusion also existed with considerable frequency for the Huddleston and Lincoln/Mitre fonts, while 4-1 confusions were frequent for the Huddleston and Z-2 for the Lincoln/Mitre. Of interest is the fact that there were no predominant confusions for the Maximum Angle font; rather, the errors were distributed throughout the confusion matrices.

Phase II

The correlation between objective similarity and performance was calculated in several steps. First, the average values for both similarity measures for every matrix size and font combination analyzed is shown in Table 2. As indicated in the Phase II Procedure section, each of these mean values is based upon 630 correlations, or all possible intercorrelations among the 36 characters per font/size

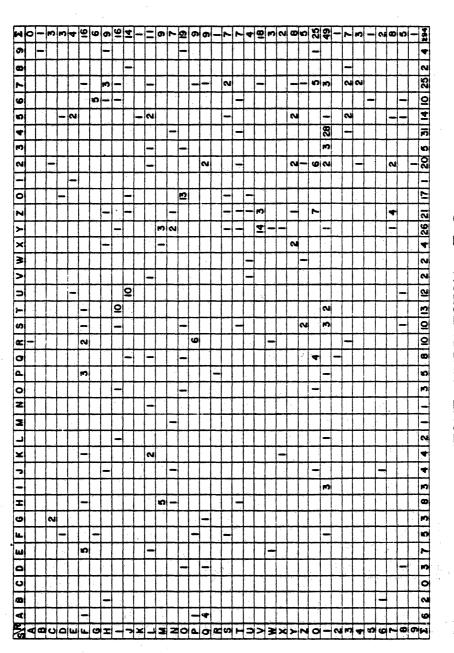
Figure 31. Confusion Matrix for 5 X 7 Huddleston Font

X7
S
7
Q
Ĕ
S
Щ
DLE
Q
4
2
1
•••
Ę
Z
ō
Ŀ

_	_						_						_										_													_	
	-	+	10	0	4	~	2	6	9	a	-	6	8	R	2	G	Ģ	4	2	2	9	R	10	-		T	z	7	-	=	17	2	0	ø	0	-	317
									Γ																							Γ					0
											-		Γ			-			-															-	Π		2
						-	Γ			Γ	Γ	Γ	Γ		Γ			-		-				-	-	9	2	-		~		_	1				E
0		1			-		-			Γ	T	N													-		-	-				-					2
5								-	-	t	1		1	-					2								- i		\vdash	37							-
4	1						, i		-	F	1			-	-					-						-		80	-	-					H		2
3										1			F		-				-															-			2
2			·	Ĺ	-						1		F		2		2		1							2	5	•			3						27
	÷		-					┢			1		N	-		H							-								Η						0
0			-		-			┢			\vdash	-	-		5		-		Ð				-					-								-+	=
Z	-		نبعه				-	-		\vdash	1	-	-			Η			10	-									-					2		+-	6
Y	Η	Η	-		-		-	2	-	-	\vdash	\vdash	-	-	-				Ĥ			33				Η		5			Η	-				- 1	30
X	-	-	-	\vdash	-	Η	\vdash	2	-	┝	E	+	10	m		-	-					677	\vdash		2	Η	-				Η	-	\vdash	Η			14
W I >	Η		-	\vdash		\square	-	E	÷		F	-	2		-	Η				-	Η	2	-	Η		\vdash			\vdash		\vdash	-		Η		day 1	13
<u>v</u>		-					-	-	-	┝	┢	-	-	-	-	\square	-		_		9			-		-	÷			-						-+	8
-	-		-			\vdash	┝─	┝╌	-	8	┝	F	-		-	\vdash	-		-	-	-	2	_	-	-	\vdash	2				-				-		5
D		-	-		-		-	-	8	8	-	-	-	3 1	-	\square	-			_	_				-	-	54	5	-		\vdash	-					13
-	•		-		-		-	-	-	┝┯	┝	-		-		\vdash			-		-				-		-	-		-						-+	
S		_	-	-					-	┝	-	-	-	1			_	_			_							-	_		\square	F				+	N
×			-				-		-	-	+-	-	-		-	2	1			_	_	_			_		-	-	÷.	2		- 1			-	-+	
0		_	-				-		-	-	-		Ļ	-	-	\square			_			_					3		-				-				4
•		_			-								_					2			_		_					-						-		-+	9
0			-		4.1					-		ļ.,	Ļ			_	-				ţ.						6			_						-+	7
Z		-							L				-							_								-				•				-+	0
X							.,									-				-		÷					·									_	2
_										-					Ľ													2								_	2
×														2														-									n
2									-											1								-								_	2
-													ſ							-								4									0
I													=	ŋ													-										5
9			m						-																		2										8
-					-		-		-				Γ			3				-						÷		-									0
						3			-		Γ	-				Π										Π		2				—				1	0
2									-	F	Γ		ļ-		2					2								-		÷.,				h		-	8
5												-	-		.,.				-							Η			-						H		N
8		-		Η	-					F	\vdash					H				-							2					-	ŗ.	H		÷i	5
4				Η	-	Η	-	-	-	-	1	-				Η			-							H				h	H	-	┢	Η		+	_
_	_			٥			-			-	×		-		0		-		-		-			X	-	Ŀ.		-	<u> </u>	-	ببا			-	8		-

Figure 32. Confusion Matrix for 7 X 9 Huddleston Font

FONT : HUDDLESTON 7x9



FONT : HUDDLESTON 9×11

Confusion Matrix for 9 X 11 Huddleston Font

Figure 33.

Figure 34. Confusion Matrix for 7 X 9 (=5X7) Huddleston Font

M	0	-	N	-	4	2	8	m	~	=	a	8	+	0	2	9	1	a	2	3	2	56	0	2	80	m	17	9	2	ر م	¢4	-	-	**	4	0	217
0																																			Π		0
-																											-										-
N	-		-			-								2	+					-						-	-	~				-	Γ				0
6	÷		-				2													2								-			-						~
5	-																																				0
-			-			_			-																			8		-							2
m				Γ	-	-																				ſ			2								4
~												2							-	-						-	3	0			-						2
-																				-									ľ					-			~
0							2								1					-	-																2
N						ļ				-		1																-						-			6
স						-														-	-	23					-			-							28
×																						-			4	-	-	-									8
≥								-		-		ľ																									2
>														·										-	-						- 1					-	2
7										8		· ·										2			2			-									2
Н						-			3																												4
3						1			-						ŀ					-								-		1.1							-
æ		-			-	10						·				9												N				•					2
9																											5										5
4						2			-																									•			3
0							2						1														17										6
Z													2											-	-			-									10
Z							•																					2									2
-								-	-																	,											2
¥						-				-		2	-															-								•	0
7						-		-																									-	ŀ			m
-		·		Ľ									÷							10								2									2
Ξ																									,							L	L		\square		0
9					-					Ĺ						-								5				-					Ľ	-			•
4					-		2					-				2												-					L				~
w			-	-		=)						2																									~
þ																			Ĺ							ŀ	-						Ĺ	L			_
ų		·																	-				•							·							_
6													-			-											-								n		9
◄														ŀ				1.1				÷															0
5	4	•	ပ	0	w	4	0	I	-	5	×	1	2	z	0	4	a	œ	S	-	Э	>	3	×	>	N	0	-	2	1	*	5	6	~	8	Ø	57

FONT: HUDDLESTON 7x9 (=5x7)

Figure 35. Confusion Matrix for 9 X 11 (=5X7) Huddleston Font

	9 x11 (=5 x7)
	UDDLESTON
	FONT : HUDE

10	1			0	10	~	C		2	0	<u>ı</u> –	0	-	-	8	10	-	0	m	0	2	2	0	0	m	2	9	8	0	Ò	0	0	2	2	0	à	8
	t	-				F	F		T	Ť	T	1	T	f		T		F																			0
0	t	-			-	F	\vdash	+	t	F	t	┢	F	-	\vdash	┢				-	h																_
+	t	۲				N		+		t	t	t	ŀ			ŀ				-							3						1				
0	t	-		\square	2			+-	\mathbf{h}	t	t	\vdash	÷		·	\vdash		-				-		-	-									1			4
0	$^{+}$	-	-		-	\vdash	┢	+	+-	F	+	┢╌	-	İ.	-	-				┝				-		H		•		-						-	-
e	t	-	-		┢	\vdash	┢	+		t	+	İ.		-	-	┢												2	[0
5	t	-	-			t	F	+	1	-	>	1-	F	F		F										_	-	-							H		~
N	t	-		÷		╞	┢	+	ŕ	F	t	N	\vdash	F		┢				-							2	10									0
1	t	-			ŀ	┢	-	+	+	┢─	+	-	-	÷				-											-							-	_
ᆂ	t	-			-			1	+		+	F	-		1	-	÷			-	ŕ				ŀ									1		_	5
1	t					1	F	+	t	ŀ	\uparrow	-	h	-	-	-		-	2		-				-	Η	~	-	·			H					đ
	t	-		H	÷	-	-	+	1-	1_	-	\vdash	-	ß				-		4		5						-									2
<	t	-	-	Η	-	\vdash	F	1	E		\uparrow	1	<u> </u>	-	-	-				ļ.	Η	3	-	-		-	-	2				-		- i			5
-	t				-	1	-	\uparrow	\vdash	1-	+	-	-			-		 .		ŀ						-		-	·					-			~
>	t	-		-	-		ŀ	┢	-	-	-	-	-		┝╍	┢╴				┢	H					÷		-		-							2
5	t	-			-	ŀ	\vdash	┢	h	¢.		-	-	┢		-		-	•	<u> </u>		n	-					-				-				-	6
+	t	-				ŀ	┢┈	+-	8	-	-	F		-	┝╌	r						÷		·				m						•	-	-	2
1	t					_		+	F		T	\vdash	-	\vdash		-					h						ŀ.	÷						÷	H	•	ŝ
2	t			Η		-	-	+	\vdash	h	┢		-	-	-	+	-										_					·		-			Q
5	t	-				-	┢	t	\vdash	┢╴	+	\vdash	-	┢─		-							_						-		·		-				_
	t					ŀ	-	1	h	F	F	ŀ	┢╴		-	-		-		-							-				-		÷		-		0
5	t	-	_	1		\vdash	-	╈	\vdash	F	1.	\vdash	┢╴			-				-			-				-										_
z	t					┢	-	\uparrow	1	h	\uparrow	\vdash	-		-	\vdash									÷						-			-			2
S I	t	-					-	+	┢	F	+	┢	-	_	-	-				-				-		ŕ		2							-		5
1	t			-		\vdash	1	\mathbf{t}	┢	┢	\mathbf{t}	+		-	\vdash	┢	-	-		+-			-					-									_
<	t				†		ŀ	t	t	t	\uparrow		-	\vdash	-					-		Η	-			H			٦					Η	-		0
2	t			\square	1	ľ	F	t	1	t			-	ŀ				Ľ		-	-		-			H		-						2	Ϋ́		2
+	t	٦	-			† †	-	t	h	t	\uparrow				ŀ					-			-					2	Η		-						50
z	t	•			\vdash	\vdash	٣	t	t	t	+	F		-	-	-			ŀ	-			Η		-			-									_
5	t	-	-	Η	1		F	t	1	t	t		-			\vdash				-		Η					\vdash								H	H	-
	Ť	٦				F	F	1	t	ŀ	ľ	F	t	F		-		-		1		Η														÷	_
	t					-	F	t.	t	t	t	Ń	1		-			-		\vdash															÷		4
5	t		-		F	\vdash	1	ŀ	┢	 	+	t	\vdash	ŀ	~	\vdash				-					\vdash	H			Η						H	Η	10
5	t	-			-	+	-	t÷	÷	┢	+	\vdash	-	ŀ	-				-			Η				-		-						\vdash	-		_
	t	-	-		-			ŀ	1	t	+		\vdash	1		-		-	ŀ	-	-	H	Η		H				$\left \right $			-				H	2
4	t	-		-	-		\vdash	+	+	f	+	\vdash	ŀ	ŀ	-	-	-	-		-	-				ŀ					-	-						_
		8	0	0	L.	14	e	+-	i-		×		5	5	0	0	0	0	5		5	>			2	~	0	-	0	-	-	10	6		-	0	ų

79

V

Figure 36. Confusion Matrix for 5 X 7 Maximum Dot Font

1
5×7
L)
<u> </u>
DOT
Ų
\cap
ليسبعا
_
>
-
<u> </u>
<
<
×
خبرت
MUMIXAM
~
~
•••
-
-
FONT
$\overline{}$
\mathbf{O}
1.
_

															- 1																						
X	2	8	2	2	17	5	4	2	=	-	0	8	8	=	5	8	ଷ	2	47	9	17	52	7	2	5	13	57	2	S	0	2	5	3	30	37	=	610
8					Γ		Γ	Γ		Γ								Γ		·												-			4		10
8					-			F	Γ			-			2			Γ		-		-		·			-	-						- i			80
3		-		-									-		-		F			2							2		2						-	-	9
0					5	-	m					8			-		4		_		-		-					_		-		5			5	П	-
0	-	-		-			┢		1	-							-	-	46	-	-		-							-			~		_		4 50 44 15
t		-	v				┝		\vdash	<u> </u>	\square		Η		-		-		4			H									-		-				4
0					_			1		-		2						-									-								_		· · · · ·
2			-		-			F	\square	┢																3	21			÷				24	4	H	63 5
							-	÷		┢		-						┢─			-								÷	-					-	-+	0
5									-	\vdash					3		5	-	-									_						·	-		=
3			-	\vdash			ŀ-	-	\vdash	-	H		Η			-		-									17		3	÷		l		9			
7				÷			-	-	-	-	\vdash		\vdash				-	-	\vdash	4		0	2	-	-				-					-		\vdash	27 10 58 27
×								-	-	-	Η		l	6			-	-		-		-					5	-	Η						-	\vdash	50
^ ≫	-							+	+	\vdash	Η	9	2	10	-		2	-	H		0	F	Η	Η	F				-	i-	-	\vdash		-	-	\vdash	ž
2				-	-	-		-	\vdash	H	$\left \cdot \right $		\vdash		-			-	\vdash		5	\vdash	-		4				Η	Η	-		-	-	-		1 2
5	-			\square	-		┝─	÷	┝	-	\square	2		2				┝		2	-										-				-		8
-								-	Q	-	H		2	-							-	-			-	-			-					-		1	4
	-	_	_	-	-		-	-	Ē					-			-	-	-				-	-	÷			- 1		-		8			0		8
2	2						\vdash	-	-	-	-	_	-		-							-	-	-		-		-	-							H	4
7	-					• •	-	-	-		Н				-	_		-			-	•	-			-	-		-		-	-	-	-	-	\vdash	2
2	-	_					-		-			-	-					-			_				-	-	-									<u> </u>	_
2	_	-		-			-	-	\vdash			_					6	-	\square	_	_		-	-	-	-	1 1	,	-	-	_		-		-	H	2
2	_				-		-		-			_				-	-									-			_			-				\vdash	3
Z	_	_	_		-	-	-		-	-	$\left \right $,						-	_	L.								•		-	-		-	\vdash	2
Z	_				-	_		Ľ	-	-	-				-			÷					2			_					_	·	-				0
긕		-		H					-	-	$\left \right $			-	÷	-							\square			Ц	_	·	-				-		_		_
Ľ	_		_				-	ļ	-	-					-			-				\vdash	-			Ц	.]	-	\vdash		_	μ		Ц		\vdash	0
2				\vdash			-		Ļ.,			-				-	ļ	-		2			\vdash				_		-			Ŀ	_			F	4
-							-	-	L			-						ļ.,	Ļ.,	5								7	_			ļ		_			0
Ξ			-	L-		\square	Ļ	Ļ	-	ľ	Ļ		-	-				<u> </u>	H			_	Ļ		Ц					ŀ		μ				\vdash	4
٥		_	5	μ			-		-	\vdash	μ				-	-	0	-					\square		L	\square	_		Н			μ	Ξ		-	\vdash	3 24
1								L	-	-	Ļ.		ļ.,	-	L			-											÷					_		Ĺ	
2			2			-	ŀ	Ŀ	L									_									-						Ļ.,			Н	4
		17					-	-	Ļ		Ц	-			8		11)	L		2							2						Ŀ	÷		Ц	34
2								ŀ.	ļ			-	Ľ			·	ŀ	L			(Ľ	-				ļ.,		Ļ				à
8								L																											1	\square	5
۷						2		-	-							10		~									-								-		2
2	◄	a	Ü	٥	u	L.	0	II	-	7	¥	3	X	z	0	م	0	æ	S	-)	>	3	×	≻	N	0	-	à	Ð	4	-	9	2	0	9	S

Figure 37. Confusion Matrix for 7 X 9 Maximum Dot Font

FONT: MAXIMUM DOT 7x9

														•				-														2		÷.,		s'	
×	2	2	2	4	Ŧ	2	5	m	N	0	0	2	Б	0	2	2	E	0	3	11	8	8	5	-	ŝ	_	55	6	27	0	2	4	20	27	35	=	462
2		-					-										-	1									1	~						Π	~		4
R			-											1	2		í -		Ť								8						Π	Π		Π	11
-											-				17		-		÷	2								Ś	2						10	Π	13
0			10		7	2	m		1			2			-																	-		Π	~	-	ខ
0			Γ				-								-				ES	1								-				1	4	Π	·	-	3
4		·										-														1	2										-
0	ŀ	I			2											·	-																		2	2	
N			-		-							-			2		n	·		2	-					-	23	2					-	4	5		57
-																																					0
C															8		4																				2
N		-									Ċ				-									-			*		25					5	-		56
Σ																	-			2	-	2					1										R
×							-								-		-					-			1		Ð								N		2
≥								-				2	2				3				12	-						-			_				-		53
>																	4				2		ļ		4		2				-						2
				_					-			-		·									1											Ц	•		10
-							Ĺ		Ξ						-													-						Ľ			Ē
n		-	-									લ					-															4	~		-		8
z	2					3		-	L	_						8	1					•												Ц			2
2							L								10					_							-							Ц			=
-						2																									-			Ц		-+	4
0				2					L_,							_	9							_			-			·				Ц	2		Ξ
Z							_	-			Ĺ		-																		. •			Ц		H	2
Z							ļ		ĺ.														2											\square		<u>⊢</u>	2
ت			-	-			Ļ	· ·	Ļ	L		-												Ц		Ĺ							Ц	Ц			2
×							ļ.,	Ļ.	ļ	ļ																			Ŀ	·			Ц	μ		i-	0
2	L		L_				_	L	-	L.		_			-						2				_							_	$\left \cdot \right $	μ		┝─┥	2
-	Ц					•			-	L.,		Ľ			Ļ.,		-	_		4								2					μ	μ		Ľ.	9
I	Ц							-		-		Ļ								_		-		2					Ľ		-	-		\vdash		H	-
9	Ц		2		2	-	\vdash	-		-		-	Ļ			_	5							Ц			_		\vdash			-	13	\vdash		H	23
-	Ĥ	_	-	L	μ		-	-	-	-	-	-	-	-	ŀ	-				_				μ					\vdash	Ц		-	H	$\left \cdot \right $		H	-
1		-	~			5	-	-	-	-	-	 			-						_		-	μ				-				-	Н	Н	2	++	=
0		2	-			-	-		-	-		-	\vdash		2		-							μ			2	-	\vdash			-	\vdash	\vdash	3		0
5					~			-		ļ.,					-														Ŀ			-	μ	H		·	10
8	μ		-		-	L	-	-	-	-	-	-	-	ļ	-			-		_				\square	-				\vdash			-	μ	μ	8		2
AN	μ			-	-	-	-	-	-	-	-	-		_	-	2	-	_			_		-				_				-	-					4
2	×	80	C	0	<u>س</u>	L	6	I	1-	17	×		X	Z	0		0		5	F	Э	>	3	×	≻	2	0		N	1	4		9		9	5	M

Figure 38. Confusion Matrix for 9 X 11 Maximum Dot Font

FONT: MAXIMUM DOT 9 × 11

_										_															_									'ı			
w	0	15	-	-	~	-	2		D	-	- -	-1-	0	0	2	4	Q	0	47	Q	ø	46	0	m	4	0	3	~	2	0	0	10	g	3	2	2	ş
9		-	Γ	Γ	Γ		Γ	Γ	Т	T	T	Τ	Τ	Γ												Γ									3		4
8		2		Γ	Γ	Γ	Γ	Γ	Τ	Т	T	Τ	Г	Γ	~	Γ	-				-		·													m	0
2	Ż	Γ		Γ		Γ	T	Г	T	T	T	T	T	1	N			Γ				Π					N										4
9		-	F	F	m		m		T	T	T	×	1	T					~																5		8
5		h	F	┢	ľ	-	2		T	t	t	t	T	T	F				45		-							-								_	40
-	-			F	1		T	t	T	T	t	1-	t	1			-	-						·								-	-				~
2		-		Γ	÷		Γ	T	T	T	T	T	Γ	Γ			-																-		_	-	ø
2				F	-			T	1-		T	1-	Ţ	Γ											u		2							26	2		525
-		1	-	T			1	T.	T	T	1	T	T	T				-	ľ.																		0
0				T				T	ţ	T	T	Ť	T		2		10																		2		4
N				Γ				Γ	T	Т	T	T	T				-								-		2		21					3	-		30
7		-	F	F				T	T	t	1-	+	T							-		4			-												44
×								T	T	T	T	1-						-				2					~										5
≥							F	T	T	t	Ť	~	1				-				16				1												24
>							\vdash	t	T	t	t	T	T				-							3	2								i.				9
5								T	T	T	t	\uparrow			-				-																		-
				1	-		-	t	G	1	Ť	1	Ľ					-					-			Ť		-									~
S					Γ		F	t	+	t	t	\dagger	T	t			Ť															2				5	2
æ				ŀ	-		F	t	1	t	1	+	T	1	-	4	-	-								÷										1	4
0				F			F	T	t	t	Ť	\uparrow	F		6											÷					-				2		Ξ
۵.				÷	-	F		ſ	T	t	T	+	T	T																							0
0				-				f	1	T	t	\uparrow	t	†-			1										2							Ì		1	0
z					Ē			T	T	t	T	\uparrow	T					-								-									-	1	-
Z			-			-	\vdash	T	\uparrow	T	t	$^{+}$	T	1			-	-							÷												0
1					ŕ			t	T	t	\dagger	╧		T	_								-														_
¥	-				-	F		ŀ	╈	┢	\dagger	\uparrow	┢	┢╴									-														0
5								t	6	t	\dagger	t	-	F											÷											-	2
_	-		ŀ			1	†-	t	\uparrow	-	+	+		\vdash						5			-					-		-				-			2
I			-	-			\vdash	t	\dagger	t	t	\uparrow	t					-	Η			Η							H	Η					_		
9	-		3	F		-	\vdash	t	t	t	\dagger	+	t	\vdash			2		Η			Η					_						4		2		B
	-		h	-	-		┢	t	\dagger	t	\dagger	t	t	h		-	:					Η		÷							-	Η	-		-	<u>م</u> ب	10
w	-	Ť	-	┢		┢	┢	t	t	t	\dagger	1-	\mathbf{T}	\vdash		-				-	_	Η	-			Η	-		H								6
0		0		-	-		\vdash	┢	+	t	+	+	F	1	~	-				-		Η				Η	_									-	2
5			-	-		-	+	t	+	+	+		+	\vdash			-		Η		H	Η	-		-			-	\square				Η				2
-		\vdash	-	\vdash	\vdash	┝	ŀ	┢	+	+	+	+	┢	-	\vdash		H		Η		-	$\left \cdot \right $	-		-	Η			Η	-		\vdash		-	-	-	4
<		•	-	┢	\vdash	-	┝	┢	+	+	t	+	†-	-	-							Η	-	•		H		-	H	H		$\left \right $	Η		5		8
-	_	Ø	0	5		-	-	II	+	t	1	+-	3	-	0			-	-		n	7		X			0	_	2			-	9		_	5	

Figure 39. Confusion Matrix for 7 X 9 (=5X7) Maximum Dot Font

5 7 1 5 13 2 11 5 7 1 5 13 2 18 6 0 1 1 1 1 1 2 8 9 2 10 6 0 17 1 7 19 0 8 0 5 の記ら聞の語とも記のこうちがうほのつ ŋ ~ ø ~ R S Ó مام

U V W X Y Z O

JKLMNOPQRST

SMABCDEFGH

FONT: MAXIMUM DOT 7x9 (=5x7)

Figure 40. Confusion Matrix for 9 X 11 (=5X7) Maximum Dot Font

WN=ONNO-NO 2 -1 10 9 2 12 1 4 1 5 1 0 2 11 2 3 10 7 7 1 4 5 11 47 41 6 0 33 1 1 56 8 9 3 2 6 8 ~ B -2 ø N 49 5 2 3 4 4 m I 0 m 2 2 ~ XYZ 44 2 - 9 2 2 3 2 > 2 > S T 5 s α N . 0 B 0 2 a L M N O r) 2 4 ¥ 2 4 I G 9 2 . w 2 m 4 0 ~ v 8 S 2 m ۷ 2 R A B U O W L 005 ¥ ZOLORWH 0 Nm 400 σI Z > NX XX

FONT : MAXIMUM DOT 9 × 11 (= 5 × 7)

Figure 41. Confusion Matrix for 5 X 7 Maximum Angle Font

W	m	ō	4	-	20	=	4	~	25	m	_	N	4	4	=	2	2	0	12	5	4	2	9	5	4	9	7	8	6	40	<u>ر</u>	o	2	5	5	φ	8
6	-	2	F		-	=	-	-	100	2	-	r)	-	=	1	F	•	Ŭ	n	~	2	2	-		-		9	9				F		-	-	2	
0			-		-		-	┝	-			-	-	·		L		_		-	-			_		-	-	-	·		-	-	-	\vdash		Η	4
~	N		-					┝	┢	+	-	5	-	-	2	\vdash	2		9	8		2	-		5	3	4	8			+	-	2	-	-	9	
9	_	2			-		2	-	\vdash	-		-		-	2		-		5					-	-	\vdash	8	4		-	-	-	-	-	10	F	39 74
5		_	┝			2		-	-	-			-	-	-			-	2		-			-	-		2	2			-	-	\vdash			Η	Ξ
4								4				-			-			-			÷		-		-			~				-	Η	\vdash		Η	5
3			-		-			ŀ	\vdash	i.			-			\square	-	-	0	_	-			-			-	4				-	\vdash	2	L.	H	କ୍ଷ
2			-		-	-		-	-			_			-	_		÷	_	_						=	_	2		2		-	H	Η	-	~	29
								<u> </u>	2			2	-	-						N												-		\square	-	Η	9
0			-							-	-				6		-		-				-							-				\square	-	Π	Ξ
N		Π			-	Η	-		-	4		9	i				ŝ		5	_	_	2	-	-			m	-	9		-		2	m		2	4
7								-	-	-		-			-		2		÷	2	m	Q		2			_	۲ŋ		-			H	M		Ċ	2
×	4	-	-			Η		-		_		-	2	-							-		-		4	Η	2	5				1	2	Π	Η	-	28 27
3						Π		-		-	-	-	_	-	-				÷.		2		7					-	-				Π				2
>		-				Π				4		-			-					-	Ŧ	Π			m	ŀ	2						Π		Π	Π	26
5										~						Π						2		-			-						Π				Ξ
F	-								Q			m			-	Π			2									9								Π	31
S												2					-			2							9	-		-							13
œ						2		÷								2			-								-										Ξ
σ															0												Q										9
٩						-						-							-			8															=
0									·								2										2										24
z						-		-					ŝ					·					2				-	2							-		5
Σ														2											-				1	÷							4
_																																					0
×												-					-							-	-								\square				4
2												2		2					-		-						Ю	5							_		15
_		1																		10		-						2				<u> </u>	Ŀ			Ц	51
Ξ	4	-					2					_	4	5									-					-					Ľ			Ц	6
ဗ	-	-	n									-		-		-	5										5	-			_		-	Ш		Ц	18
u.																-											-						Ц				2
ш						Ð					Ĺ	2	-		-	Ц			-	-								R							_		28 12
0		=	-		-										n	4	8									Ŀ	-	2			; ,					Ц	28
ပ				-												Ù			-	1							2										4
8																-				-	-							:	1	21	e,			Ц	-		4
4																	2			·							-	1	ц.					Ш		Ŀ	0
S/F	٩	8	ပ	0	ų	L.	9	I	-	ר	¥	1	Σ	Z	0	٩	0	æ	S	-	D	>	¥	×	7	2	0	-	2	3	4	5	9	2	8	თ	ĩ

FONT: MAXIMUM ANGLE 5x7

Figure 42. Confusion Matrix for 7 X 9 Maximum Angle Font

w	0	m	4	_	N	m	2	~	3	2	0	2	9	n	2	S	53	0	5	8	2	8	2	M	6	4	9	20	d	n	_	4	1	2	5		273
9							İ I				Ì						-	- •					·	ŀ					÷.	_					Ť		3
8									ŕ													i i	-											÷	1	t	0
7						m		2		-	- 1	S	-			÷			-	3					_	2	Ð	2			-				T	4	26
9						-	-					2			-					j.	÷	-			-		S	1				÷	, i	4	~	1	ର
5						1		-		-			• •)							-		_						-							T	-	ø
4				-		-			-			2								-								13	•	-		1			T	1	ର
3	·														-					-						1	-	5								4	5
Ń									-	-		2		-					-							+	5				-			-		1	
-				- 1 -							7	2																						-			n
0								·							~		6																	_			0
Z			-			-	-	-		~					4		m				-	-	-				ဖ	2	·					4		e	28
۲									-	-		-	-							1	ιn)	3		2			-	ñ								1	2
×						Ç		-	-	-		2			-		2					1			4		5	4								2	Ŋ
×			Ľ	·						-		-					• .				-					÷									_	·	n
>										ი	_				-		-				2				-		2	17									62
Э						_			-	2					-		ġ					-	Ĺ					-					•		1		4
۲						Ŀ			II			R)													ณ			ი	, ,					2		-	58
S		-		-	-		·		ŝ	-		2		۰.	M		· .	,		4							5	4					-	_	-	_	27
æ						-	1.									4			-											_	_		_			4	0
Ø						-		·	Ĺ						10												2								_	1	Ξ
٩			ļ																÷.				L.		·					_				_	_		
0			Ļ			-		·								·	3										-						-	_	\rightarrow		9
Z			ļ										4											<u> </u>					_				_		·	+	4
Σ	•			-	•	_								-	-					•			-					4			÷			_	4	-	2
Ľ				-	_	L							-	-									_	Ļ				-		_				_	\rightarrow	+	
¥			-				-	2				-					_								4	2		2	_	4		_		.	-+	+	9
ר						-					_				1					2	4		_		•	_	_			-		_		_	+	-	8 7
	-						-			-		2		e. F			Ľ			4	1.	а. 1			_	_		2 2		-		-	-	-	+	-	0
H		Ļ	2			F	$\left \cdot \right $			_	-			\square								-		F	-				-					-	-+		9
9				-	-	┝					Н			·								_	ļ	ļ.,				-	•	-		-		\neg	+	+	
<u>ب</u>	Ļ.		-	-		4										-	Η						····				-	-				Н			+	+	
E	-			-	-	3	$\left \right $				-		-	-				- - -		-	ι.		L.	2	1	-				_		-	_	-	+	-+-	2
c o	-			-		-	$\left \right $		\vdash	-	÷	<u> </u>		-				.*.		<u>5.</u>		2		- 		-	2						-	-	<u></u>	-+	4
BC			-	ŀ		÷						_				-	-	-							-								-		2	-+-	2
A			-	-	_	-			\square		-			-		-	-				-		ľ		-	1.			<u></u>				<u>.</u>				2
5,8	-	8	0	0	ju i	L	9	I			×		3	Z	0	a	C	œ	G	1	n N	>	3	×	۲	N	0	-	2	10		ŝ	j j	2	80	÷	-
Ś						-		-				-			-	-				Ľ.	Ļ				L.			L.,	-			-	1		_	-1	

FONT: MAXIMUM ANGLE 7x9

Figure 43. Confusion Matrix for 9 X 11 Maximum Angle Font

11×6
NGLE
AN
MUM
MXIN
2
FONT
The second

1	-	-	=	-	N	0	m	2	26	36	2	8	2	-	1	N	ß	0	8	2	18	5	m	2	60	2	\$	50	0	0	2	~	m	=	5	-	3
ת			Γ	1			Γ			Γ								:									-									Π	2
8			T	T	Γ	Γ	Γ	Γ.		-				Γ	-																	Π	-			Π	2
~			T	Γ	Γ	Γ		Γ	1	2		4							2								5	m					-			-	9
9			1-	-	Γ	-		-		Γ						-				2							2				-			~	-		2
ŝ	-		T	T		F				-				F	2				_						-						-				·		9
4			t	T		-			-	F	-	2							-	2		-						60			-	-				Π	8
N	-	-	T	1-	-				-	m		2					Ю	ŕ	-		-				•	-	-	N						4	2		33
2		F	1-	T				-		_		_		Γ	-											-	-										2
			Γ		1				2			-								2												\square					5
0			T	T		Γ				-	Π				m					1															1		m
N	Γ		Γ	Ť			2	Γ	-	9	F	-	-		S		2				~				2		5	m				Π		-			R
>		1	T	t		-		Γ	-		Π	_			-					2	2	=		2			-	-				Ċ,		n	-	Π	26
×			T	1	1					-	Π	2	-						-		-	-	2				0		Π			Π					6 0
3			Γ	T			-		-	-							-					2			-		-					Π	Π		1	Π	2
>			T	T		,		1.		2				-		-					2				ŧ		-	-								\square	33
5			T	1		F				0																						\square					6
-			T	t					0			80													-		2	5	ĺ								44
S	-		1-		-	-	-		-			n					2			-							4	-				-					2
æ	-		T	T	†-	-		T.								8	-		-						-		_						\square				2
0	F	F	T	T	F			1.							1				-						-		2								-		2
٥.	F		t	\uparrow					1	-																					÷					1.1	0
0	F		t	T	T	T	T	T		-							Ť					-				6					-	Π					0
z	F		T	\uparrow	F			T		-																				·							0
Σ			T	T	F		-	1-																													0
_			t	T			F	ţ-						-																		Π				Π	0
¥	Г	Γ	V	•	Γ	Γ	Γ	Γ			Π		Γ	Γ						ř														-	Ϊ		3
5	Γ		T	T	1			Γ	-	Γ	-		-								.0	·						-	Π				Π			Π	4
-	Γ	Γ	T	T	Γ	Γ	Γ	Γ	<u> </u>	Γ		4		Γ			·		:	2			1					4				Ť					0
I	Γ	1	T	T	Γ	Γ	Γ	Γ	Γ	Γ	Π		Γ	Γ	Γ	Γ																			-	Π	0
σ	Γ	Γ	6			Γ	Γ	Γ	Γ	Γ			Γ	—			n										-	-									2
L.	Γ	-	T	T	Γ	Γ	-	1.	Γ	ſ		-	Γ	Ĩ	Γ	ŀ		Γ					-				-									Π	Ю
ω	Г	 	†-	T	T	9			Γ	Γ					-												-					Π	П			Π	6
0	Γ		1-	t		Γ		ľ	Γ	Γ	Γ	-		Γ	 	Γ			-		-			-								Π				Π	n
ç	Γ	T	t	t	1	T	1		T	T			-	1		-		-											Π			Π	Π			Π	-
8		T	T	T		-	-	T	Γ	T			-	\vdash	F	10	H	-		-												Π	Π		Π	Π	10
4	F		t	t	F	ŀ	T	t	T		T			F																		Π	Π	Π		П	0
ž	◄	æ	i.	10	4	L	0	II	-	5	×	-	5	Z	0	a	0	æ	S	-	n	>	3	×	7	N	0	-	2	m	4	5	9	2	8	6	S

Figure 44. Confusion Matrix for 7 X 9 (=5X7) Maximum Angle Font

(=5×7
7×9
ANGLE
MAXIMUM
FONT: M4

)

												5		1																							
M	2	2	9	4	m	თ	õ	7	8	5	2	2	2	4	8	4	31	2	0	2	8	ର୍ଷ	4	9	6	9	50	3	ŋ	8	3	2	*	19	18	5	34
9							Γ			1																			-		-						N
Ð						•			1		1								-	-																	ณ
					-	-	-			2	1	m					_			-					2	2	11	-					3			2	3
0		-				-						-	·				-		÷					-	-		l	2		-		-			9	-	Q
0									N				7		-					-								N			÷		-	-	-	÷	9
4	-							2	-	2		5	-				-			N							ļ	1	-		-			4			2
n	-	-		_				2	-	2		4		۰.	~		-							-	-	-	2	5	-		2			-			5
N				-								-			-		-		-	÷.,						2	4	-									2
_							-	-			Γ	-								-																	2
5	•										1		-		2		2										,						2			-	2
V			-	-		-	-		-	2	-	-		2	9		20	-	1	-	4	S	÷				11	9					-	10	-		8
~						·			2	-	Γ		n		-					-	-	4		Ð			÷	ŝ						2			र्द
×						-			FD)	2		-		-	2	-	2	·	1	2	3	4	n		2		4	ŝ						-			8
3	1		_					-	Γ	_	Γ								-		-							Ì	t. L					-		- J	ø
>					-					σ	ſ										4			·			1	-							-		27
5										80					-			÷				4							2								2
-						-			9		-	3				÷	۰.		÷		-	-			-			80		2				m			8
n						-			-	4	ŀ	2			-		30					۰.					B	3		-							ð
æ			•						Γ	Γ	Γ	-				2	-														v	e e					4
0											Γ				5				·			-					9				P				2		Z
1	1																					. a.N					•				:						0
0																	0										2										=
z											Γ	-	2			·					+							-							2		5
Z						·					Γ			-				•																			-
_																						•	• •														0
¥												m	ŀ						·											ĺ		-					4
7								2							·				ľ	. *	n											1.1					5
_																				-								-	•								2
I									1	Ĺ	ŀ																								-		-
c	1	1	2								ŀ				-		-		ŀ																-		13
			-										-							2	,																+
ų			-			m			-	-		4					-				ŀ				-			30							~		17
0	·		2			-			-																		2				Ľ						9
<u>ن</u>								Γ	Γ		Γ	Γ	Γ								1	ŀ			-					-							-
D				-					-	2						-	Ĺ			-				-	-	-		-		R					2		3
4					-		Γ			-		-			2		2					-					-							-	n		2
ξ	A	8	S	0	ш	L.	6	II	-	-	×		3	z	0	٩	O	æ	S	F	Э	>	3	×	1>	2	0	-	2	m	-	5	9	1	8	6	ŝ

Figure 45. Confusion Matrix for 9 X 11 (=5X7) Maximum Angle Font

																										1											
Σ	0	-	6	-	2	9		m	25	35	-	28	-	0	9	4	9	-	4	8	3	9	-	0	2	m	43	57	-	-	6	Ň	ŝ	8	-	-	372
6																		·		-				-													2
8					ľ	Γ		Γ							·								÷										-				-
2					1	-						-							_	-				_	_	3	9	4			2		2	П	÷	_	24
6					-	\vdash	-	-	-	ŀ		-	-	-			-									-	N	2							-		6
5	-		T		-			<u> </u>	-	-		_	÷	-		-	-		-		_				-			2		-	2		2				6
4				-				-	-	·	-	_												-				2		-	-					-	8
3		-	+	-	-	+-		\vdash								÷				-				-	_	$\left \right $		4			1	_		_	-		õ
2			1				-	-		-		3			-					2							S			1		÷	-	_			2
1			 	-		\vdash			-			_											÷		-	-				÷						1	2
0			L		-	\vdash			-	-		i			9	-										-				_							2
Z			~		-	1	-		-	m		2			2	· .			-		2	_					n	9		-				2		-	26
Υ	-		-			÷-		-	01	S		-	<u> </u>			_						2	-	4	-	H		9		-			_	2			38 2
×	-	-	-		-	 .		-	<u> </u>	4		5	-		2		n				5			-	n	$\left \right $	=	9		-	_			_	÷	\vdash	503
N)					Ļ.	-	-	-	ŀ				-				_		-		2	-				$\left \right $			_	-	-					-	=
V V			-				-	H		n			-	-			_			_	17			_		$\left \cdot \right $	_			-			-				34
1 0	-	-			-	-		-		8	-		-		- 1					-÷	-			-	-	$\left \right $	_		·	-		-		Η			6
	-	-	-			<u> </u>		-	5	-		r n	-	-							\vdash	-	-	-	2	Η		5				_	-			-	25
ST	-					-		-			-				÷	-				IO				\dashv	-	$\left \cdot \right $	10	-		-	-	•					16 2
R	_					F		\vdash	Ē		-	F	-			4			2			-		_		$\left \right $		H					\vdash	Η			8
Q R	_					-		-	ŀ		-		-	-	2	-		\vdash	-	-				-		H	4	Η	-					Η		ŀ	9 9
-	_				-	<u> </u>		-	Ļ		-		-	-				_	\vdash							$\left \cdot \right $	_	-	-	-				Η	-	-	0
9	-	-		-	-			-	-	-	-			+					\vdash					-	-	Н	2	$\left \right $		-	-			Η		-	4
0	-		·							<u>·</u>						-	-		-	-	-			-	_	H			\square	-				H		-	m
Z	-		\vdash			-	-	-	ļ		\vdash	-	F			-			-					-		Η		$\left \right $		-			-	Н		·	0
Σ.				_		-		H		-	\vdash		ļ								-			-	-	\vdash)	Η		-					_		0
	_					-	-	$\left \right $	-		$\left \right $	H	-	-	_			5.	ŀ					-	-	\mathbb{H}				-		·	-	Η			8
¥	_		R	_	-			\vdash	F			F	-		_	_						_			-	Н	_			_		-	-	μ		ŀ	8
2		-				-				-		-				_			-	0	ю	•		_		$\left \right $		6 2		_			-	μ			16 B
	_				-	-		μ						-					-	Ξ				_		\mathbb{H}		-					-	ŀ		. 	
I					<u> </u>			-	-		$\left \right $	-			_	-				_	-					$\left \right $		-		-			-	\vdash			2
9	-		2		-				 			-	-	-				\vdash								$\left \right $		\vdash	$\left \right $	_		-	Ļ	-		Ŀ	m
LL				-	-	Ľ	_	$\left \right $	-	Ŀ	\square			-				-		.,	-	_				μ				_		<u> </u>				-	4
ш		-	 						-	-	-	a,	-		-	•					-																8
9					_	-		Ц	-	-		-		-			´																-	_	_		2
<u>0</u>											Ц	•													_			ļ		_		L		L		•	0
8						2							Ĺ							2	Ĺ						-	-					_	L		-	9
۷													ļ													Ц	_						<u> </u>	L		Ļ	-
5	۲	9	0	0	W	٤.,	G	Ξ	-	-	¥	_	Z	z	0	٩	Ø	æ	S	F) >	>	X	×	>		0	-	2	1	4	S	90	1	8	5	W

FONT: MAXIMUM ANGLE 9 x 11 (=5x7)

Figure 46. Confusion Matrix for 5 X 7 Lincoln/Mitre Font

	÷		
-			
		Ĺ	>
		Ľ	2
		Ľ)
		1	1
		ō	Ē
			-
		Ξ	ŝ
		~	
		_	>
		~	Ī
		Ć	5
			2
		2	Ξ
		_	٦
		·	•
		F	_
		4	₹
			2

3	0	=	ю	-	9	2	R	4	36	E)	0	9	4	15	9	0	8	-	23	4	-	39	7	٤	9	8	30	8	29	2	9	ę	m	2	45		
6						Γ											5										9		3								2
8																	-		2						1		ν.	-					-	:			9
~		-	-						4								-		1	-			2			4	6	2	m				_			2	2
9					-				-					÷					2					÷				_				5			~		n
5										Ì									2				ļ				ł	_					-		4		0
4								~	-	•		-	•	ŀ		-	-		2									2									=
m										-		1							4					·		-			-		-				24		55
2			÷						-								4		2				-			2	3	-		-						4	8
						-			5			-						-							i-				-								8
0			-			-									2		4		2		, i					ì			2							2	2
2	-	3							n						·				2				-				9		9					9	m	N	46
٢			-			 			-		•	1		2					-	2		27	X	-				S		•					N		8
×	-					Γ								4					-			-	-		+		-	F)							m		9
R	5														-							0															9
>	-		_											2							-		8	_	4			-									8
5						F				-			-									-									2					-	3
L		3				T			5								-											2				·					କ୍ଷ
S			-			Γ			-								,										~	3				-			-		9
æ			-		-											6	5											-						-			52
0		-				Γ	-								9				-				-							Π							~
٩						2		-										-									4	2									6
0																								-													0
N	·					1			-	-			ŝ		÷																						5
W								-						E)									-	-													9
-																																					0
¥			-									-												m				-								_	2
ſ	-								2					2														2		ŝ	-				-		4
-															•				-									12									Ē
H										ŀ			-	-																-							F)
0			-																																-		2
L.				-	4			-								-																					2
ш		-							-			-																									m
٥		2							4						-																					-	8
v							2																	-													ŝ
8	-					Ī																				ŀ					-				m		5
A								1								•																Γ					0
SR	A	8	U	0	ω	L	0	I	-	7	¥	-	X	Z	0	۵	o	æ	S	ĺ.	Э	>	3	×	×	N	0	-	2	m	4	8	6	2	8	6	3

Figure 47. Confusion Matrix for 7 X 9 Lincoln/Mitre Font

5
0,
×
7x9
-
TRE
~
E
L
~
2
_
~
~
Z
Р
20
ICOL
NCOL
INCOLN
INCOL
LINCOL
FONT : LINCOL

Σ	3	0	8	-	0	3	2	0	4	80	-	~	2	2	117	4	60	Ņ	0	9	20	29	2	-	m	9	8	8	39	0	F)	4	-	-	4	2	3
9										Γ						Γ	ŀ				ŕ						2	-	2						1		g
8							Γ			ŀ			ľ			Γ								•								-		-			2
2	-							Γ	-	-		-									1	2	·			-	11	m	3						2		26
9						Γ.			2		Γ					Γ											3	m				-			1		2
3						T.	T	Γ	-	2	Γ	-				1											1				-						9
4						1				-											.5		-					S									2
3						Γ	Γ	Γ	-	Γ																		-									2
2						Γ				-		-				Γ	-									5		-						-		_	2
_						Γ	Γ		-							Γ	Ĩ.					-							-								m
0							Γ	Γ		Γ	Γ			1	m					-								-									5
Z							Γ			ſ			Γ			1						-			-		1	-	29					2			40
λ								ľ	2	-	Γ	-				Γ				4		18		-				2		.,			<u>, -</u>	~			8
×	2					ţ` ,	Γ	1		ľ	Γ		1			Γ					-	2						-									5
N		•				Γ	-				Γ					Γ	Γ					3	÷														ю
>			-			Γ	Γ			Γ	Γ							·			-							2									4
0								-		-								·				-						-			-						Á.
-						F			m																		Ì	4					-				0
s						Γ				-										-	·		1					4				-		1			8
ž						ŀ	Γ			ŀ	Γ					n	9					1						1	-			-					Ē
0						-	m	1																			-										4
4						1	Ĩ.							·													-		-							-	m
0				÷		-	2		Γ								_																				m
Z										ŀ		-	-			Γ																					2
X										Γ				-																							_
L.						,	, I		Γ		1												-					-									-
×							-											÷											2								2
7						-				[ľ												~									2
_				Π																					•			9									9
H							-		-			×.													•			17)									5
9			9			-			-					1														-				·					8
			_				-						-												Ĺ			-									4
μ				-		2			-	ŀ	-	~					Ē	-																			2
0				Π					-		-										-	Π			_			·				·					17
5							1		Γ	-	-																										_
8				Π						Γ	-			-		-						Ť			-						-						3
A				Π					<u> </u>																			-									-
5	A	8	J	a	w	u.	9	I	-	5	¥	1	X	Z	0	٩	0	æ	S	÷	0	Χ	W	×	γ	Z	a	_	2	10	4	5	9	7	8	Ø	N

Figure 48. Confusion Matrix for 9 X 11 Lincoln/Mitre Font

	_	_
-	-	-
1	× J	٢
(7	١
`	'	1
l		_
ſ	7	_
į.	-	_
5	*	-
2		>
-		
	2	-
•	2	j
-		
		こうい
		こうシント

																																				~	
2	0	0	-	0	4	-	-	-	1	2	-	0	0	0	10	9	+	-	3	P)	-	9	9	0	0	4	36	36	9	I	0	0	9	2	2	-	171
6			Γ		-	T		Γ	T		Ť						-				•		·						-								2
8			T	T	1	T	Ì	T	T	Ť	T	t-							ŝ								1	-					·				4
7			Γ			Γ	1		1	Γ		Γ			1											2	3	-					4		-	T	=
9			F	t		T	T.	Γ	T	-	1																3										-
2		i.		\square		F	t	t	t	†-	1	\vdash											, i				1									1	-
+				1		1	1.	F	T	F	T															Π	4	5		-			-			1	11
3				Γ		Γ			T	-											. ;										·				-		2
2						Γ	Γ	Γ	Γ	-																	*									T	3
_						Γ		Γ	N		1									2																	2
0						T		Γ	Γ	ľ.	T				-		1																				2
N				1		Γ	Γ	Γ	-										-			1					4	2	2					~			21
γ				Γ	1	Γ	Γ	Γ	m	-										2		4	9				3	9									25
×			Γ	Γ		Γ		Γ	Γ	-												-				-									·		4
3						Γ		F	T								1					2				Π	-						-				9
>			-						Γ	Γ	Γ										1															T	2
5			-			Γ	-	Γ	Γ	ø																											6
F						Γ.		Γ	4													-						0								T	5
S						1		Γ	Γ	-																	2										ŝ
æ					-	-	Γ	-	1							10																				1	8
0	1					Γ	Γ		Γ	Γ					4																						4
a		-				Γ			T	F							-										2	-									4
0			•			Γ		1	Γ	Γ	,					-																					-
z						Γ		1	Γ	Γ																					·					ľ	0
Σ								Γ	Γ																											Τ	0
						1	1	Γ	T	Γ																				·							0
¥								Γ	[ŀ																1										-
5						Γ	ľ	Γ	ſ	Γ	-															Π	2									1	n
-																												7									2
I									Γ											-																	-
9		·			-			[-										2
-					2		-	Ē	Γ	ł			·												. 1												m
w						Γ	Γ	Γ	-	T	Γ							-					·				-	-									4
0						Γ			Γ	Γ	Γ							·									-	-									2
0							[ľ	Γ	Γ																										0
							Γ		2																		2	÷									4
4				Ì		1		1	ŀ	Γ	Γ									÷						-										T	-
ş	A	æ	U	0	i.	-	0	II	-	5	×	1	X	Z	0	٥.	0	æ	ŝ	-	n	>	A	X	λ	N	0	_	2	m	4	5	9	2	8	9	ŝ

Confusion Matrix for 7 X 9 (=5X7) Lincoln/Mitre Font Figure 49.

r i	_	, - ,	-		.	-			.			- 			_	-			-	12		6						6				_					67
W	0	0	4	-	2	-	-	-	E	~	C	-	0	-	4	T	o	0	m	2	-	ន	87)	2	100	4	2	49	¥	-	0	2		8	3	2	232
6			_					Ŀ	L		Ŀ							L	L								-										2
8							Ľ	L									2		Ľ	-							-									-	10
~									-										-							-	2	2	4		•		-		2		15
6	. *		j																								3										3
5											1									-																	1
4																			Ì				-					2					Π				=
3										-			·				-						Ϊ.					-					\square				Ð
2						-						-		-								-				3								~			8
-					Γ					Γ	÷		•				Γ	—		2							2						Π				4
0						Γ	Γ	Γ	ſ		Γ				-		Γ	Γ					-					-					Π			-	m
Z					Γ	1		Γ	-																			4	37					-			43
7				-	Γ		Γ	Γ		Γ	Γ			-	Γ		Γ	Γ		10	-	2		-	Γ	Π	-	5					Π	2			8
×				 -				Γ	-	-									-			5	-		2			-					П		-		E
3									Γ						1							2											П				2
2					F		-	ļ												-													Π			-	_
5			-				-		-	3																									÷		5
F								F	2								F											10						2			4
S					-			F		-	-						-					Ť			-			10					Π				5
æ				-				┝			-					0	9				-		-									_		·			21
a	Ì							F					-		10	-												-					H				4
a	-					÷		┢					-						i			-										_	Н				
0			-					-	-	$\left \cdot \right $					ŀ						-			H		- i							H		-		1
z	-	-,					-	-		·	-			-	\vdash	-	-						-			Η			-		-		H				0
Σ			÷						\vdash						ŀ	-				÷			-										Н	-	-		1
	-	-	-	-									-				-		$\left - \right $					-		-					-		Η				
Z	-	-	-				-	-	-	Η	ŀ	Η		. .	-	+	-	-			\vdash	Η	-	Η	-	Η	- 1			-	-		Η	Η	-		3
5	_	-	-	-		H	-		┝─	\vdash	-				-	-	-	\vdash			Η	Н		Η	-	Η	-	0	-	-	-		Η	Η			12
-	-	-	-	-	-	-		-	-	Η		-			-	-	-	÷	\vdash	-	Η	Η	-	\vdash		\vdash	\mathbb{H}	2		-			Η	-			3.1
Ξ	_	-	-					-	-			$\left \cdot \right $			-	-	-				-	-	-	Η	-	Η		-		-	-		Η	\vdash		-	2
5			4	-		Н	-	-	-	F	-	Н				-	-	-	\vdash		-	-		Н	\vdash	\vdash	-		H				\vdash	\vdash	-		6
-+	-	-	-	-			\vdash	-		Η	-		-	-	-	2	ŀ	$\left \cdot \right $	\vdash				-	\vdash		Η	-			-			Η		-		4 6
-	-	-	_		\vdash		$\left \right $		-	$\left \right $	-		_			-		-		-	-	-	-	Η	ŀ	Н		-	-	_	-	-	\vdash	\vdash	-	_	
		-	-				-	-	-	Н	ļ		-	·	Ļ	-	-	-	\vdash	-		Η		Н	-	-	-			_			\vdash	\vdash		-	2
	-		-	_			F		F	Ц	ļ			: 	-	-	-		-	L,	-		-	Н	\vdash							-	\vdash	$\left \right $		_	2
2	_	4		_	_			-	<u> </u>	$\left \cdot \right $						Ļ		-		L		-	-		-	$\left \cdot \right $							Н				2
	_	_	_	4					-	L,	Ļ	-		_	-	-		-		-	-			Ц	-				-	_			$\left - \right $				2
	_	_	_		-	_					L				ļ	ŀ																					-
2	۲	-	С	9	Ē	4	0	I	-	7	¥	-	2	2	0	٩	0	æ	S	-	2	>	3	×	1	N	a	-	2	10	4	10	9	Z	8	თ	w

FONT : LINCOLN / MITRE 7x9 (=5x7)

Confusion Matrix for 9 X 11 (=5X7) Lincoln/Mitre Font Figure 50.

ষ্ঠ 0+ 1 7 0 4 7 5 2 55 0500000 125 0 25 0 N 8 1 2 2 2 2 2 3 4 5 6 7 24 20 1 0 3 1 2 2 7 2 21 5 5 4 4 15 2 6 3 7 3 2 5 0 3 - 2 4 X M V 2 2 2 m ____ 3 2 14 GHIIJKLMNOPQRST ~ 'n N 4 2 8 0 3 2 50 3 3 SVA B C D E F 2 5 2 ABUDUL olr ORSIN NO 500 000

FONT: LINCOLN / MITRE 9 × II (=5 × 7)

Font/Size	2-D Product-Moment Correlation	Phi Coefficient		
Maximum Angle				
5 X 7	.814	.060		
7 X 9	.775	.077		
9 X 11	.751	.106		
7 X 9 (=5X7)	.764	.077		
9 X 11 (=5X7)	.682	.106		
Huddleston				
5 X 7	.820	.089		
7 X 9	.812	.144		
9 X 11	.801	.184		
7 X 9 (=5X7)	.807	.144		
9 X 11 (=5X7)	.751	.184		

Table 2. Mean Values of Similarity Measures

combination.

Two general trends can be seen in these data. As the number of dots per character increases, the correlation between the 2-D Fourier coefficients generally decreases. These correlations, of course, indicate the extent to which luminous power is contained in the same spatial frequencies for all 36 characters in the font/size combination. This is to be expected because the characters can be made more and more distinct as more dots become available from which to construct each character. Next, note that the average Phi coefficients remain very close to zero no matter what matrix size is used. Additionally, the Phi coefficients for the last two matrix sizes in each font are equal to their counterparts higher in the list. For example, in either font the Phi coefficient for the 7 X 9 matrix size is equal to the coefficient for the compressed 7 X 9 (=5X7) matrix size. This is necessarily so because the compressed size is still a 7 X 9 matrix with the dot size and spacing altered. All dot locations remain unchanged between the two sizes and, therefore, the Phi coefficient is also the same.

Secondly, intercorrelations between the number of confusions among all pairs of characters and the 2-D Fourier coefficient correlations for those same pairs of characters were calculated, as were the intercorrelations between the confusions and Phi coefficients. These calculations are based upon the entire data set, i.e., the performance and similarity measures were correlated for all 630 character pairs for each font/size combination. The results are shown in Table 3.

Font/Size	Confusions/2-D Pearson Spearman		· · · · · ·	Confusions/Phi Pearson Spearman	
				tana Tanang tanang	
Maximum Angle					
5 X 7	0. 173 ^{***}	0.097*	0.328***	0.197***	
7 X 9	0.096*	0.040	0.247***	0.149	
9 X 11	0.078*	0.003	0.192***	0.064	
7 X 9 (=5X7)	0.048	0.012	0.176***	0.086*	
9 X 11 (=5X7)	0.029	-0.038	0.164	0.062	
Huddleston		· · · · · · · · · · · · · · · · · · ·			
5 X 7	***	*** 0.145	0.239***	0.213 ***	
7 X 9	0.078*	0.052	0.184***	0.227	
9 X 11	0.107	0.098	0.187	0.175	
7 X 9 (=5X7)	0.111	0.132	0.151	0.161	
9 X 11 (=5X7)	0.056	0.043	0.165	0.124	

Table 3. Correlations Between Variables for Full Data Set

5

*** p < .005

Both Pearson and Spearman coefficients are included in this table along with the significance levels attained. While most of these correlations are fairly low, a general trend should be noted for both the Pearson and Spearman correlations between confusions and Phi values. As the characters become more and more distinct (i.e., as the matrix size increases), the correlation between objective similarity and performance decreases. This effect is evident regardless of the character size. That is, whether the character is restrained to be the same size or allowed to grow as more dots are available, the correlations decrease. The high significance levels are the result of the large number of data points (N = 630) contained in each full data set.

An examination of the confusion matrices will reveal that only certain character pairs were confused very much. By far, most entries in the matrices are very low or zero. Attempts to correlate such performance with objective similarity measures could mask any real relationship. Therefore, three more correlations were calculated with selected data points.

First, the data were sorted so that only those character pairs which subjects confused eight or more times were retained. This number is arbitrary and represents 10 percent of the total number of confusions possible for any given character pair. These correlations are shown in Table 4. Note that while some of these correlations appear to be quite high, none are, in fact, significantly different from zero (p > .05).

<u>98</u>°

Font/Size		ons/2 - D Spearman		ons/Phi Spearman	
Maximum Angle					a ana ana amin'ny faritr'o dia mandritry any faritr'i dia amin'ny faritr'i dia amin'ny faritr'i dia amin'ny far
5 X 7	0.403	0.282	0.271	0.278	18
7 X 9	0.299	0.111	0.067	-0.068	10
9 X 11	0.078	0.003	0.192	0.064	11
7 X 9 (=5X7)	-0.027	-0.023	-0.108	-0.060	12
9 X 11 (=5X7)	0.443	0.018	0.427	0.036	7
Huddleston			· · · · · · · · · · · · · · · · · · ·		
5 X 7	0.033	0.208	-0.288	0.117	10
7 X 9	-0.712	-0.738	-0.814	-0.527	5
9 X 11	-0.547	-0.500	-0.814	-0.500	3
7 X 9 (=5X7)	-0.740	-0.718	-0.782	-0.564	5
9 X 11 (=5X7)	0.219	0.500	0.097	0.500	3

Table 4. Correlations Between Variables for Confusions ≥ 8

All correlations p > .05

The next data sort was done to retain only those pairs of characters with very high correlations between 2-D Fourier coefficients $(r \ge .90)$. This number is also arbitrary and is intended to be an indication of very high similarity in the 2-D spatial frequency domain. The rationale behind this sort is that high similarity 2-D Fourier coefficients should lead, a priori, to large numbers of confusions. Thus, this is another means of selecting a data subset which has a large number of possible confusions and therefore permits greater opportunity to uncover the relationship between confusions and spatial frequency information. These correlations are listed in Table 5. Note that the correlations between Phi coefficients and performance are generally greater and more often significant than are the correlations between 2-D coefficients and performance.

The third and final data sort retained only those pairs of characters with high positive Phi coefficients (Phi \geq .60). This number is also arbitrary and represents a relatively high value of obtained Phi coefficients. The rationale is the same as that used in the previous data sort. These correlations are listed in Table 6. Note that the product-moment correlations between Phi coefficients and performance are generally high and statistically significant. There appears to be no trend in the magnitude of correlations either between fonts or among sizes within a given font.

Font/Size		ons/2-D Spearman			
Maximum Angle	Transie wagan and a series			-	
5 X 7	0.273*	0.357**	0. 488	0.557***	70
7 X 9	0.276*	0.364**	0. 446	0.490***	61
9 X 11	0.278	0.313*	0.391*	0.283	41
7 X 9 (=5X7)	0.086	0.212	0.451*	0.470*	29
9 X 11 (=5X7)	-0.554	-0.800	0.545	0.800	4
Huddleston					
5 X 7	0.092	0.222	***	0.603***	55
7 X 9	0.171	0.105	0.439***	0.463***	84
9 X 11	0.191	0.224*	0.361**	0.245*	77
7 X 9 (=5X7)	0.275*	0.170	0.458***	0.468***	66
9 X 11 (=5X7)	-0.070	-0.042	0.443	0.665**	18

Table 5. Correlations Between Variables for 2-D $R \ge .90$

* p < .05 ** p < .005 *** p < .0005

Font/Size	Confusio Pearson	ons/2 - D Spearman	Confusio Pearson	ns/Phi Spearman	# Cells Included
Maximum Angle					
5 X 7	-0.154	-0.153	0.795***	0.541*	24
7 X 9	0.122	0.202	0.423*	0.389*	26
9 X 11	0.080	0.034	0.559**	0.394*	25
7 X 9 (=5X7)	0.007	-0.129	0.629**	0.387*	26
9 X 11 (=5X7)	0.129	-0.017	0.604**	0.367	25
Huddleston			· · · ·		÷
5 X 7	0.232	0.212	0.308	0.184	21
7 X 9	0.224	0.076	0.655**	0.295	18
9 X 11	0.296	0.237	0.557**	0.262	26
7 X 9 (=5X7)	0.273	0.156	0.671**	0.431	18
9 X 11 (=5X7)	0.219	0.302	0.606**	0.456	26
			•		

Table 6. Correlations Between Variables for Phi \geq .60

* p < .05 ** p < .005 *** p < .0005

. .

DISCUSSION

Phase I

Number of errors. It would certainly be gratifying to be able to point to one particular font as the "best" font regardless of character/matrix size. It is evident from Figure 30 that the Lincoln/Mitre font is slightly superior to the Huddleston font for the 7 X 9 and 9 X 11 character sizes. The two fonts are roughly equal for the reduced 7 X 9 and 9 X 11 sizes. However, for the 5 X 7 dot matrix, the Huddleston is markedly superior to all other fonts.

An earlier study by Maddox, *et al.* (1977) showed the Maximum Dot font to be superior to the Lincoln/Mitre font at the 5 X 7 matrix size. This early study, however, did not control for the time required to write each character vis-a-vis the number of dots in the character. That is, characters written in the Maximum Dot font have, on the average, more dots than the same characters written in the Lincoln/Mitre font. In the earlier study, the time taken to write each character was proportional to the number of dots in the character. Thus, more time was required to write characters from the Maximum Dot font than those from the Lincoln/Mitre. This extra time meant more viewing time for the subjects and a possible confound in the data.

In the present study, delays were built in so that the average time required to write *any* character from *any* font was constant. With this refinement, the Maximum Dot and Lincoln/Mitre fonts are found to

be essentially the same as far as performance at the 5 X 7 matrix size is concerned. While the superiority of the Maximum Dot font is no longer found, neither is the claimed superiority of the Lincoln/Mitre font manifested. The loci of the confusions in the Lincoln/Mitre font are nearly identical in both studies.

What of the character/matrix size? It is evident from Figure 29 that as the number of dots available for character construction increases, the number of identification errors decreases when overall character size is held to the size occupied by the 5 X 7 matrix. This is a wholly expected result. Previous experimentation (Snyder and Maddox, 1978) has suggested that performance improvements can be expected as dot matrix characters more and more closely approximate stroke characters. This concept has great face validity in that the characters can be made more and more distinct as more dot locations are made available.

A result which was not entirely anticipated is the performance improvement which resulted from the addition of points within a compressed character size. Upon reflection, however, this effect is precisely the same as that described above. That is, the compressed character size contributes even more to the similarity between dot matrix and stroke characters than does the addition of dots at the same density. A look at the series of figures showing the different character/matrix sizes for any given font provides a strong subjective impression of the stroke effect with the high density character/matrix size. Confusion matrices. While the average numbers of errors can give an overall view of the confusions inherent in certain fonts, the matrices generated in this study can pinpoint the loci of these confusions. Although these matrices lack the conciseness of a statistical table and are, in fact, rather tedious to study, they provide a great deal of information. Generally, confusion matrices are used to learn which characters in a given font should be modified to reduce the number of confusions in that font. Another use of the matrices might be to design a composite font for certain applications. A composite font is one made up of characters from several different fonts. If the least confused characters from each font are used in the composite, then, presumably, the composite font will have fewer overall confusions than the fonts from which the characters are drawn.

It must be noted, however, that these confusion matrices apply only to the specific task of tachistoscopic identification of single alphanumerics. The relation between performance on this task and performance on other common tasks (e.g., reading) is not known. This task was used in the present research for simplicity and ease of comparison with results from past studies. Such a task does not provide the subject with the benefit of context which might be available in some other tasks. Despite these shortcomings, tachistoscopic identification is a traditional method of evaluating the goodness of font designs. In any case, composite font designs generated from such research should, in turn, be evaluated to determine what, if any, new confusions are introduced by the combination of several font styles.

Phase II

In the second phase of this research an attempt was made to relate observer performance on a tachistoscopic recognition task with (1) the 2-D Fourier spatial frequency spectra of the characters, and (2) a nonparametric similarity measure, the Phi coefficient. On a relative basis, the 2-D Fourier correlation between characters was not, in general, a very good predictor of subject performance. By the same token, the digital Phi coefficient provided moderate correlations with confusions. It seems intuitive that the 2-D Fourier spectrum of a character would be a fairly good measure of subjective similarity between it and other characters. Why, then, did this method not predict any better than it did?

First, it should be remembered that the stimuli which were used in this study were dot matrix characters. The dots that make up such characters are arranged in preset locations within the matrix lattice. Thus, the 2-D Fourier spectrum of each character contains substantial power in components which correspond to the basic matrix size. For example, in a 5 X 7 dot matrix character, the frequency spectrum shows considerable power corresponding to 5 cycles horizontally and 7 cycles vertically. This happens *regardless of* the details of the character within these limits. The similarity this produces among characters can be seen indirectly in the average 2-D correlations listed in Table 2. These correlations are much higher than one would expect to see on the basis of character similarity alone. They reflect the similarity of the basic matrix size and undoubtedly overshadow any more subtle character differences.

In attempting to correlate performance with either of the two similarity measures, we are basically looking for some monotonic relationship which would allow the number of confusions to be related to the magnitude of the similarity. This assumes that confusion is a more or less continuous phenomenon. However, previous studies have led researchers to postulate a threshold type phenomenon to be operating for this type of task. Models proposed by Townsend (1971) to account for performance in tachistoscopic recognition tasks assume a threshold for confusion. The only real differences among these models relate to what happens when the subject is unsure as to which character was seen. Common to the models is a mechanism whereby the subject is *positive* which character was seen or is thrown into some state of uncertainty as the the character shown. The point at which the subject is no longer positive can be considered to be a threshold of confusion.

In terms of the present research, a threshold phenomenon would manifest itself by subjects making very few confusions until the similarity between characters reached some threshold value. After this value was attained, the character would be confused very often with similar characters. This effect has been noted in similar studies. Dahljelm (1976) attempted to design a low confusion font for stroke characters on the basis of a truncated 2-D spatial frequency spectra comparison. In taking data with which to construct confusion matrices, he found that only 2-5 character pairs per font had 10 or more errors,

even though the total errors per font ranged from approximately 250 to over 400. This is consistent with the present study in which a total of 630 character pairs were used in the analysis of each font-size combination. A total of 80 confusions could have occured for any pair. Thus, 8 or more confusions represent 10 percent of the possible number of errors. The Maximum Dot font exhibited a low of 7 pairs (9 X 11 reduced size) and a high of 18 pairs (5 X 7 size) of characters which were confused more than 10 percent of the time. The Huddleston font had a low of 3 pairs (9 X 11 reduced size) and a high of 10 pairs (5 X 7 size).

If a threshold type phenomenon is actually occurring for the tachistoscopic recognition task, then linear correlation could be misleading. In effect, the similarity between characters could increase with *no* effect on confusion. Once the similarity reached a certain (threshold) value, confusions would proliferate but further increases would again have no effect on additional confusion. The increases in correlation coefficients with selective data sorting are evidence of this type of phenomena. Generally, it can be said that if the similarity measure between two characters reached a certain value, then those characters are likely to be confused. If the value is below a certain value, then those characters are not likely to be confused.

Both the 2-D correlations and the Phi coefficients show evidence of the thresholding phenomenon. However, the Phi coefficient is unaffected by the basic matrix size. Unlike the 2-D spatial frequency spectrum, the Phi coefficient does not attribute similarity to two characters simply because they both happen to be 7 X 9 dot matrix characters. This allows more subtle character differences to be manifested in the Phi coefficient while they remain hidden in the 2-D spectrum correlations.

Something must be said for cases of character pairs which are confused but are not objectively similar as well as pairs which exhibit high objective similarity but are rarely confused. Both cases are highly visible in the data and tend to depress correlations (not to mention researchers). In attempting to account for such apparently paradoxical results, the task must be kept in mind. Subjects are presented with single characters for *very* short periods of time and forced to name the character which was shown. This is an alien task to most subjects in that they have probably not performed it before. However, the kernel of the task is *very* familiar to subjects and has been practiced for many years.

For example, very few subjects, when presented with the letter "O" would say that they saw the letter "Q". Even though the 2-D spectra of the two characters are highly correlated and the Phi coefficient is quite high, the two characters may be rarely confused. Although this appears to be inconsistent, it should be noted that subjects have been practicing the discrimination between "O" and "Q" since they learned the English alphabet. On the other hand, subjects will sometimes confuse characters which have very low values of objective similarity. Particularly in dot matrix presentations, which are generally unfamiliar to subjects, certain pair discriminations might be required much more frequently than in normal activities. In an experimental setting, all discriminations are required to be made the same number of times. Thus, subjects can be expected to make apparently anomalous confusions as a result of the task and experimental design, not only as a result of physical similarities.

Any method of measuring intercharacter similarity must eventually be evaluated in relation to the other methods available. Methods for measuring stroke character similarity have been reported for many years. These methods range from feature lists or critical features (Geyer and DeWald, 1973; Holbrook, 1975; Townsend, 1971) to characteristics of the spatial frequency spectra of alphanumerics (Dahljelm, 1976; Kabler, 1975).

Examination of these techniques reveals that nearly all of them are tedious, time consuming, and calculationally complex. In addition, the feature list methods require the characters to be systematically analyzed for the presence or absence of certain features. This is done on the assumption that subjective confusion is the result of overloaded or aliased pattern recognizers in the visual system. Fourier techniques are more complex mathematically, but are predicated on the existence of some spatial frequency analysis mechanism within the visual system.

The Phi coefficient assumes nothing about the visual system directly. It is simply a measure of the proportion of dots that two dot matrix characters have in common. It should be obvious that the more dots two characters have in common, the more highly correlated their 2-D frequency spectra will be. This is indeed the case. The Phi coefficient and the 2-D coefficient correlations are very highly correlated with each other. The Phi coefficient has the advantage of not being artificially elevated by the basic dot lattice of each character. This advantage is manifested in the rather strong relationship between the Phi coefficient and subject performance.

SUMMARY AND CONCLUSIONS

Font Selection

Of the four dot matrix fonts used in this research, two, Lincoln/ Mitre and Huddleston, produced consistently better recognition and identification performance than either the Maximum Dot or Maximum Angle fonts. Except at the 5 X 7 matrix size, the choice of either Lincoln/Mitre or Huddleston is more or less a matter of preference. At the 5 X 7 matrix size, the Huddleston font is recommended over all others tested.

The possibility of selecting characters from each font to be used in a composite font is not to be overlooked. However, caution must be exercised so that new confusions are not introduced in the process. Any composite font should be tested before widespread use is initiated. A test similar to the experimental task used in this research is recommended, although a more practical task (e.g., reading) might certainly be desirable.

Intercharacter Similarity

Two intercharacter similarity measures were related to performance in the present research. The spatial frequency spectra correlations required the characters to be digitized, Fourier analyzed, and correlated. The time consuming nature of the digitization and the complex and computer-intensive analyses would seem not to be justified by the strength of the relationship to performance. What began as a

simple test of a perceptual model turned into a test of mathematical techniques. Whether or not the human visual system functions as a spatial frequency analyzer remains a disputed question. At least for this particular task and stimulus type, the methods of 2-D Fourier analysis introduce more questions than they resolve.

One quite useful tool to emerge from this research is the Phi coefficient. This nonparametric correlation coefficient has been shown to be a very good rough index of confusion among dot matrix characters. It has much to recommend it: the Phi coefficient is mathematically simple, requires minimal character analysis, and has high face validity. Such a measure makes an excellent "rule-of-thumb" design tool for intercharacter confusion prediction.

General

This research has answered some of the more pressing questions pertaining to dot matrix symbology. In the performance measurement phase of the study, four fonts and five matrix/character sizes were combined factorially. This is the most comprehensive study of its type ever performed for dot matrix characters. The results relate observer performance to font and character/matrix size in a logical and empirical manner.

The methodology developed for the two-dimensional photometric scanning and subsequent analysis extends far beyond the immediate research problem. These techniques will find use in image processing applications where the intensity distributions are much more complex

than the simple dot matrix patterns encountered here.

There are certainly many questions yet to be resolved in dot matrix display design. Among these research areas are the effects of upper and lower case dot matrix characters on observer performance and the relationship of tachistoscopic character recognition to other perceptual tasks.

APPLICATIONS

Although this research was not intended to be primarily applications oriented, several of the results and much of the methodology can be put to practical use. As design tools, the results of the tachistoscopic font studies are of immediate use. Considering the paucity of hard data with which to design dot-matrix character sets, the fontsize interaction results as well as the confusion matrices generated for each font and size should be very useful.

As a more abstract design aid, the digital Phi coefficient will provide at least a coarse selector of candidate alphanumerics for minimum confusion. This coefficient is easy to compute, has high face validity, is fairly simple to understand, and correlates fairly well with performance.

In addition to the results of this research, the methodology developed during the course of the research should also find widespread applicability. Specifically, the hardware and software needed for repeatable and accurate two-dimensional scans of intensity distributions have the potential for more general use. Such two-dimensional digitizations are required in fields as disparate as X-ray crystallography and aerial vegetation surveys. The use of such techniques in display evaluation should become relatively commonplace.

Finally, the two-dimensional transform algorithm used in this research is a versatile tool which should find expanded use. This

routine, originally from the University of Arizona, allows small computer users to decompose very large arrays (up to 512 X 512) into the two-dimensional frequency domain. Until recently this was possible only with large scale computer systems or array processors.

REFERENCES

- Albert, D.E. Prediction of intelligibility of contextual and noncontextual dot matrix characters. Unpublished Master of Science thesis, Virginia Polytechnic Institute and State University, 1975.
- Almagor, M. Temporal encoding in the visual system. Unpublished Doctor of Philosophy dissertation, Virginia Polytechnic Institute and State University, 1977.
- Barr, A.J., Goodnight, J.H., Sall, J.P., and Helwig, J.T. A user's guide to SAS 76. Raleigh: Sparks Press, 1976.
- Bracewell, R. The Fourier transform and its applications. New York: McGraw-Hill, 1965.
- Brigham, E.O. The fast Fourier transform. Englewood Cliffs: Prentice-Hall, 1974.
- Burton, G.J. Visual detection of patterns periodic in two dimensions. Vision Research, 1976, 16, 991-998.
- Campbell, F.W., Kulikowski, J.J., and Levinson, J. The effect of grating orientation on the visual resolution of gratings. *Journal* of Physiology, 1966, 187, 427-436.
- Campbell, F.W. and Robson, J.G. Application of Fourier analysis to the visibility of gratings. Journal of Physiology, 1968, 197, 551-566.
- Charman, R.P. and Olin, A. Tutorial image quality criteria for aerial camera systems. *Photographic Science and Engineering*, 1965, 9, 385-397.
- Cooley, J.W. and Tukey, J.W. An algorithm for the machine calculation of complex Fourier series. *Mathematical Computation*, 1965, 19, 297-301.

Cornsweet, T.N. Visual perception. New York: Academic Press, 1970.

Dahljelm, H.D. Investigation of alphanumeric symbol legibility determination by use of Fourier spatial frequency components. Master of Science thesis, Air Force Institute of Technology, GE/EE/76D-20, 1976.

- DePalma, J.J. and Lowry, E.M. Sine-wave response of the visual system. II. Sine-wave and square wave contrast sensitivity. Journal of the Optical Society of America, 1962, 52, 328-335.
- Geyer, L.H. and DeWald, C.G. Feature lists and confusion matrices. Perception and Psychophysics, 1973, 14, 471-482.
- Ghiselli, E.E. Theory of psychological measurement. New York: McGraw-Hill, 1964.
- Graham, C.H. Vision and visual perception. New York: John Wiley and Sons, 1965.
- Haeusing, M. Color coding of information on electronic displays. Proceedings of the 6th Congress of the International Ergonomics Association, 1976, 210-217.
- Holbrook, M.B. A comparison of methods for measuring the interletter similarity between capital letters. *Perception and Psychophysics*, 1975, 17, 532-536.
- Huddleston, H.F. cited in Ellis, B., Burrell, G.J., Wharf, J.H., and Hawkins, T.D.F. The format and colour of small matrix displays for use in high ambient illumination. *Proceedings of* the S.I.D., 1975, 16, 250-258.
- Jahn, T.L. Visual critical flicker frequencies as functions of intensity. Journal of the Optical Society of America, 1946, 36, 83.
- Kabler, P.W. Fourier spatial frequency components as descriptors for recognition of variable font typescript. Master of Science thesis, Air Force Institute of Technology, GE/EE/75D-34, 1975.
- Keesee, R.L. Prediction of modulation detectability thresholds for line scan displays. WPAFB, Ohio: Aeromedical Research Laboratory, AMRL-TR-76-38, December, 1976.
- Kelly, D.H. Pattern detection and the two-dimensional Fourier transform: flickering checkerboards and chromatic mechanisms. *Vision Research*, 1976, 16, 277-287.
- Kelly, D.H. and Magnuski, H.S. Pattern detection and the two-dimensional Fourier transform: circular targets. Vision Research, 1975, 15, 911-915.

- Lowry, E.M. and DePalma, J.J. Sine-wave response of the visual system. I. The Mach phenomenon. *Journal of the Optical Society of America*, 1961, *51*, 740-746.
- Maddox, M.E. Prediction of information transfer from computergenerated dot matrix displays. Unpublished Master of Science thesis, Virginia Polytechnic Institute and State University, 1977.
- Maddox, M.E., Burnette, J.T., and Gutmann, J.C. Font comparisons for 5 X 7 dot-matrix characters. *Human Factors*, 1977, 19, 89-93.
- Maffei, L. and Fiorentini, A. The visual cortex as a spatial frequency analyzer. Vision Research, 1973, 13, 1255-1267.
- McCormick, E.J. Human factors engineering. New York: McGraw-Hill, 1970.
- Pantle, A. Visual information processing of complex imagery. WPAFB, Ohio: Aeromedical Research Laboratory, AMRL-TR-74-43, July, 1974.
- Schnitzler, A.D. Theory of spatial-frequency filtering by the human visual system. I. Performance limited by quantum noise. Journal of the Optical Society of America, 1976, 66, 608-617.(a)
- Schnitzler, A.D. Theory of spatial-frequency filtering by the human visual system. II. Performance limited by video noise. Journal of the Optical Society of America, 1976, 66, 617-625.(b)
- Snyder, H.L. Image quality and observer performance. In L.M. Biberman (Ed.) Perception of displayed information. New York: Plenum Press, 1973, 87-118.
- Snyder, H.L. and Maddox, M.E. Information transfer from computergenerated dot-matrix displays. Blacksburg, Virginia: Virginia Polytechnic Institute and State University, HFL-78-3/ARO-78-1, October, 1978.

Townsend, J.T. Theoretical analysis of an alphabetic confusion matrix. *Perception and Psychophysics*, 1971, 9, 40-50.

VanCott, H.P. and Kinkade, R.G. Human engineering guide to equipment design. Washington, D.C.: U.S. Government Printing Office, 1972.

The two page vita has been removed from the scanned document. Page 1 of 2 The two page vita has been removed from the scanned document. Page 2 of 2

TWO-DIMENSIONAL SPATIAL FREQUENCY CONTENT AND CONFUSIONS AMONG DOT MATRIX CHARACTERS

Ъÿ

Michael Edward Maddox

(ABSTRACT)

A two-phase study was conducted which related the confusions among dot matrix characters to the two-dimensional spatial frequency similarity of these characters.

During the first phase of the study, subjects were shown single alphanumeric characters from four different dot matrix fonts. In addition to the font variable, the size of the character was varied. All common matrix sizes, 5 X 7, 7 X 9, and 9 X 11, were used. The design of the study allowed the effects of matrix size (number of dots) and character size (angular subtense) to be separated in the analysis. Data from this phase of the research were analyzed in terms of both correctness and character confusion frequencies. The ANOVA of the number of correct character recognitions provided interesting interaction effects among font and matrix/character size. These results are discussed in terms of display design considerations.

The second phase of the study consisted of digitizing and analyzing all characters from two of the fonts used in the first phase. The fonts chosen represent the most and least confusable of the four, based on the performance data obtained. These characters were scanned photometrically using a computer-controlled X-Y stage. The resultant digitized arrays were subjected to a 512 X 512 point fast Fourier transform (FFT). The Fourier coefficients were correlated for all possible character pairs within each font-matrix/character size cell. These correlations provided an objective similarity measure among characters based upon their 2-D spatial frequency spectra.

In addition to the spatial frequency similarity measure, a simple digital Phi coefficient was calculated for each character pair. This coefficient is simply a nonparametric correlation coefficient between two digital arrays.

The final analysis performed in this study was the correlation of observed performance (confusions) with objective similarity measures (2-D spectra and Phi coefficients). A strong relationship between objective and subjective confusability would be a very useful design aid for display manufacturers. The obtained correlational relationships are discussed in terms of their utility for design and their implications for visual system models based on spatial frequency analysis.