FEASABILITY STUDY OF HYDROSTATIC

TRANSMISSIONS FOR AUTOMOBILE APPLICATIONS

by

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LIST OF SYMBOLS

Symbol	Definitions
A	Angular velocity ratio for gearbox
b,bsfc	Brake specific fuel consumption, kg/J
B	Bearings bore, m
Be	Effective bulk modulus of elasticity, N/m^2
c	Fuel economy, m/m
C_1	Fluid capacitance, m/N
CD	Drag coefficient
CDM	Motor damping coefficient
CDP	Pump damping coefficient
CFM	Motor friction coefficient
CFP	Pump friction coefficient
CSM	Motor slip coefficient
CSP	Pump slip coefficient
d	Fuel density, kg/m^3
D	Rolling radius of tires, m
DM	Ideal displacement of units, m^3/rad
E	Final drive ratio
F_{22}	Tractive effort, N
F_{23}	Force required to accelerate vehicle, N
F_{24}	Drag force on vehicle due to windage, N

Symbol	Definition
I	Mass moment of inertia, $N\text{-}m\text{-}s^2$
I_1	Inertia of vehicle, kg
J	Cost of moving to next stage of process
Jnn	Cost of arriving at stage, N
J_1	Effective rotary inertia of engine, $N\text{-}m\text{-}s^2$
J_2	Effective rotary inertia of pump, $N\text{-}m\text{-}s^2$
J_3	Effective rotary inertia of motor and drive shaft, $N\text{-}m\text{-}s^2$
J_4	Combined rotary inertia of wheels and axles, $N\text{-}m\text{-}s^2$
Kt	Tire resistance coefficient
LAMB	Swashplate angle of pump, rad
m	Mass of wheels, kg
Mc	Mass of vehicle, kg
P	Pressure difference across units, N/m
PSI	Swashplate angle of motor, rad
Q_{10}	Flow out of pump, m^3/s
Q_{11}	Leakage flow, m^3/s
Q_{12}	Flow due to change in fluid density, m^3/s
Q_{13}	Flow into motor, m^3/s
r	Radius of gyration, m
R	Rolling radius of tire, m
R_2	Viscous damping in pump, N-m-s
R_3	Leakage resistance, $N\text{-}s/m^5$
R_4	Viscous damping in motor, N-m-s
R_5	Charge pump friction, N-m-s

Symbol	Definition
R_6	Effective rolling resistance, N-m-s
R_7	Windage drag on vehicle, N-m-s
R_8	Dry friction in pump, m^3
R_9	Dry friction in motor, m^3
S	Frontal area of vehicle, m^2
T	Friction torque of wheel bearing, N-m
Tp	Tire pressure, N/m^2
T_1	Torque out of prime mover, N-m
T_2	Torque to accelerate prime mover, N-m
T_4	Torque into gearbox between prime mover and pump, N-m
T_5	Torque out of gearbox, N-m
T_6	Torque to accelerate pump, N-m
T_7	Torque lost to shear fliud in pump, N-m
T_8	Torque required to drive charge pump, N-m
T_9	Input torque to pump, N-m
T_{14}	Output torque of motor, N-m
T_{15}	Torque to accelerate motor, N-m
T_{16}	Torque to shear fluid in motor, N-m
T_{17}	Torque input to differential, N-m
T_{18}	Torque output of differential, N-m
T_{19}	Torque required to accelerate wheels, N-m
T_{20}	Torque lost to rolling resistance, N-m
T_{21}	Torque input to wheels, N-m
u	Friction coefficient of bearings

Symbol	Definition
U	Absolute viscosity, $N\text{-}s/m^2$
U_1	Pump swashplate angle control variable, m^3/rad
U_2	Motor swashplate angle control variable, m^3/rad
U_3	Engine throttle angle control variable, deg.
V	Vehicle velocity, m/s
Vdot	Acceleration of vehicle, m/s^2
We	Angular speed of engine, rad/s
Wm	Angular speed of motor, rad/s
Wp	Angular speed of pump, rad/s
Wt	Weight of vehicle, N
Ww	Angular speed of wheels, rad/s
Wedot	Angular acceleration of engine, rad/s^2
Wmdot	Angular acceleration of motor, rad/s^2
Wpdot	Angular acceleration of pump, rad/s^2
Wwdot	Angular acceleration of wheels, rad/s^2

I. INTRODUCTION

History

The petroleum shortage, which occured in the United States during 1974, significantly influenced the automotive industry. A greater emphasis was immediately centered on improved fuel economy by both the consumer and the government. During the past four years, the demand by these two institutions for more efficient automobiles has increased. The Environmental Protection Agency has set increasingly stringent standards for fuel economy while consumer sales favor the more fuel-efficient imported automobiles. In response to this impetus, the American automobile manufacturing community began investigating alternate methods of obtaining more fuel economy from their products. The academic institutions shared this interest of increasing fuel efficiency and also began research towards this end.

The methods for improving fuel efficiency can be divided into at least two broad categories; those which improve the efficiency of existing components, and those which implement components yet new to this application. The former category of methods has much to commend it in terms of lower cost and implementation time, because it only requires the expansion of research normally involved in the design and manufacture of automotive components. This made it initially appealing when the need for improved fuel economy was immediate, but the maximum improvement attainable with this type research is usually limited. The latter category of methods normally takes longer and costs more to implement, but, by taking advantage of technological advances in related fields, often yields superior long-term solutions. This thesis is the result of research of this latter category.

Purpose of Study

The objective of this thesis is twofold. One purpose is to develop a digital-computer based dynamic model of an automobile equipped with a hydrostatic transmission, and to test the feasibility of improving fuel economy by using this type of transmission. Although mechanical transmissions are inherently more efficient than their hydraulic counterparts, it is felt that a net gain in overall efficiency can be obtained by operating the engine at the point of minimum brake specific fuel consumption, its most efficient range, while maintaining the present performance. Thus, the second purpose is to utilize the model to examine methods of obtaining high overall efficiency of the drive train. One of the methods examined is dynamic programming. All simulation is performed using the IBM Continuous System Modeling Program (CSMP).

It is not the purpose of this study to develop a prototype, however, all components used in the simulation, excluding the controller, are commercially available.

This thesis is written in a format analogous to the order in which the research was performed. More specifically, after the literature is reviewed in the next chapter, the development of the model is presented, followed by the discussion of the controls and analysis of the final design. Finally, the Summary section is presented and the computer model, the regression programs, and the control program, appear in the Appendix.

II. LITERATURE REVIEW

A literature search was conducted to obtain information relevant to the use of hydrostatic transmissions for automotive uses. The objective of this review was not to determine the history and development of hydrostatic transmissions nor to denote the advantages of hydrostatic over conventional transmissions as this has been done previously (1) (1975). Rather, the objective was to obtain information beneficial in constructing models which would simulate the performance of an automobile with a conventional transmission and with a hydrostatic transmission. A review of the manufacturers literature disclosed the best apparent transmission available for use in this project.

Since the IBM Continous System Modeling Program was to be used, the describing differential equations for the test vehicle had to be written. It was decided to use the method of bond graph modeling to obtain these equations. This method of modeling is advantageous when several different types of physical systems are interfaced. Karnopp and Rosenberg (2)(1975) and Thoma (3)(1975) have written excellent books on bond graph modeling. These books included methods of handling such problems as losses due to slippage and friction in hydraulic systems and modeling of the internal combustion engine.

The performance characteristics of the internal combustion engine and the attainment of maximum fuel economy through redesign of the drive train have been the topics of many articles. Leonard (4)(1974) treated the improvement of fuel efficiency as a problem requiring a new design of the engine-transmission system. This article also provided equations for losses due to rolling resistance in tires and due to wind loading. Huebner and Gasser (5)(1973) reviewed the factors affecting vehicle fuel consumption. Caris and Richardson (6)(1953) examined the engine-transmission relationship for attaining higher efficiency. Their conclusion, which agreed with Leonard, was that an engine designed to work properly with a continuously-variable transmission can provide a substantial increase in fuel efficiency with no loss in performance. They suggested a control system which would adjust the engine setting to operate at minimum brake specific fuel consumption (bsfc). Also, they presented an account of the functions the control system would have to provide to maintain the engine at minimum bsfc. Austin et al. (7)(1974) examined fuel economy during non-urban driving and found that non-urban driving as well as urban driving can be simulated on a chassis dynamometer. Tartaglia (8)(1973) advocated the use of a relatively small prime mover coupled with a hydrostatic transmission which featured energy reclamation and storage. For a theoretical driving cycle he claimed the energy recovered through regenerative braking constitutes up to 51% of the energy used for the complete cycle.

Two articles were concerned with the redesign of the transmission only. Orshansky (9)(1974), the designer of a hydromechanical transmission, performed simulation studies to provide automobile fuel economy using this transmission. This transmission has three hydromechanical ranges and one hydrostatic, and reportedly improved overall fuel economy for a full-sized sedan by 32%. Price and Beasley (10)(1964) concluded that the slight loss in efficiency of a hydraulic compared to a mechanical (standard) transmission can be more than offset by gains in engine efficiency.

Two sources of hydraulic component analysis involved the use of a loss-coefficient model. A series of articles written by Wilson and Lemme (11)(1970) provided one method of analyzing hydrostatic transmission performance using a coefficient model. A set of describing steady-state equations for a transmission was given to be used with the loss coefficients described in an earlier paper by Wilson (12)(1964). This paper provided means of estimating the loss coefficients using graphs. Merritt (13)(1968) also presented a coefficient model for hydraulic pump and motor analysis for both the dynamic and steady-state cases. This book also contained information on hydraulic control systems.

An article written and published by the U.S. Environmental Protection Agency (14)(1974) delineated the selection and use of the EPA driving cycle for fuel economy measurements. This formed the basis for the cycle over which the model was to be simulated.

A textbook on internal combustion engines by Obert (15)(1973) provided a good source of information concerning the performance characteristics of the internal combustion engine.

Conclusion

The following conclusions were drawn from this literature search:

1. Sufficient information was not obtained from this literature search to allow the complete construction of the desired model. Although estimates of the loss coefficient for a hydrostatic transmission can be found, specific values for a commercially available unit are not in the literature. Likewise, data fully descriptive of the test prime mover were not available. The manufacturers of these respective components were contacted regarding this information.
2. A computer simulation of a purely hydrostatic transmission or of an automobile equipped with such has not been published to date.
3. Bond graph modeling is a viable method for deriving the describing differential equations of a system such as this.

III. DEVELOPMENT OF MODEL

Introduction

This chapter contains the results of the research performed in partially fulfilling the first objective of this thesis; the development of the digital computer model for simulation of the test vehicle. This chapter also includes the determination of various parameters and constants necessary in the simulation. First, several definitions are given which describe the test vehicle more closely than stated in Chapter I. Second, a brief overview of bond graph modeling is presented to introduce the reader to the concept of modeling with this technique. Finally, the model for the vehicle is developed as a composite of several subsystem models. This model is developed in terms of generalized elements which could describe any conventional vehicle. The system state equations are then derived directly from the bond graph using the general elements. These elements are then assigned specific values to simulate the desired test vehicle. The evaluation of these elements constitutes a significant amount of effort and is presented in three sections which cover the evaluation of elements associated with:

1. the external loads on the vehicle,
2. the hydrostatic transmission,
3. the characteristics of the prime mover.

Preliminary Design

Before the model could be formulated, several general decisions were made concerning the test vehicle. The type of vehicle selected for simulation is of the Ford Pinto inertia class due to the popularity of this type of automobile. Necessary engineering data was obtained for this vehicle as well as a load survey of a 2.3 litre four-cylinder engine from the Ford Motor Company.

A variable-displacement, axial-piston, hydraulic pump and motor were chosen due to their relatively high efficiency, small size, and ability to withstand large pressure drops. Complete performance data was obtained from the manufacturer for these components. It was decided that the flexibility gained from variable displacement units would more than offset the added complications of controlling the system. A simple block diagram of the preliminary design appears in Fig. 1.

A method of writing the describing state equations was needed along with means of accounting for the various losses involved with the system, which lead to the use of bond graph modeling.

Introduction to Bond Graph Modeling

Bond graph modeling is simply a means of studying dynamic systems of any type in a unified manner. The bond graph consists of subsystems connected by lines which represent power bonds between subsystems. The subsystems are physical elements which possess ports through which power may flow. These subsystems may be referred to as multiports and be

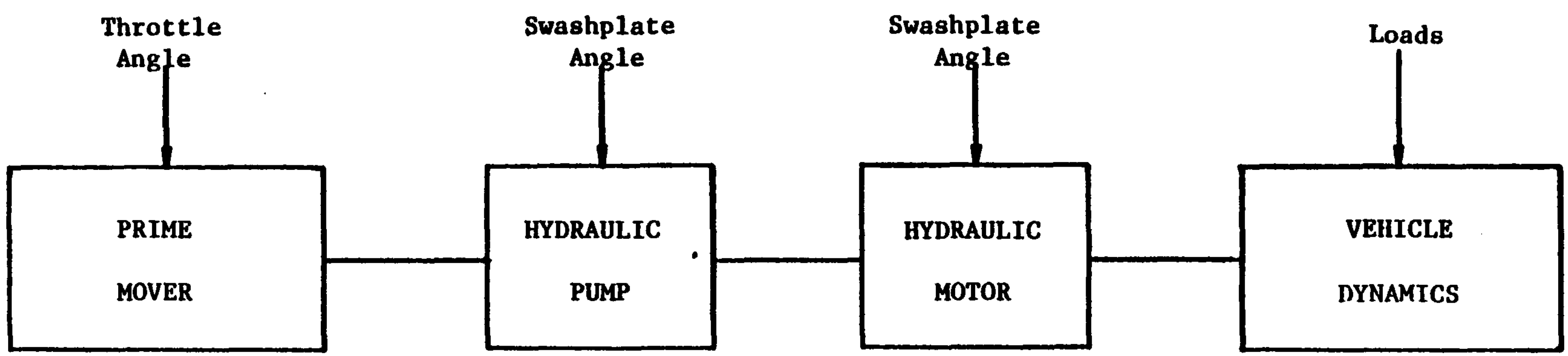


Figure 1. Block Diagram of Preliminary Design

comprised of a collection of single-port or other multiport elements. In general, when two subsystems are connected physically, two complementary variables are constrained to be equal for both subsystems. This connection is called a bond. For example, if a multiport called an electric motor is connected to a multiport called a pump by means of a rigid, massless shaft with no damping, then the torque and speed of the motor would be equal to the torque and speed of the pump. Of course, if the connecting shaft was not ideal the shaft would then be a multiport. The torque and speed of the motor would then equal the torque and speed of one end of the shaft and the torque and speed would be equal for the pump and the other end of the shaft.

The two complimentary variables which are simultaneously forced to be equal at the junction of two multiports are called power variables. This is because the product of these two variables as functions of time is the instantaneous power flowing between the two multiports. In the example above, the torque and speed are the power variables and it is clear that the product of these is power. In general, these power variables are of two types, effort and flow variables. The effort variable is denoted as $e(t)$ and the flow as $f(t)$. Examples of effort and flow variables appear in Table 1 for several types of physical systems.

By definition, the power $P(t)$ is given by:

$$P(t) = e(t) \cdot f(t).$$

Table 1. Physical Examples of Effort and Flow Variables

PHYSICAL DOMAIN	EFFORT VARIABLE	UNITS	FLOW VARIABLE	UNITS
Mechanical Translation	Force Component	N (lb)	Velocity Component	m/s (ft/s)
Mechanical Rotation	Torque Component	N-m (ft-lb)	Angular velocity Component	rad/s
Electric	Voltage Component	V	Current Component	A
Hydraulic	Pressure Component	Pa (psi)	Volume Flow Rate	m^3/s (ft^3/s)

Note that these variables are time-dependent which is necessary in describing dynamic systems.

Two other types of variables which are related to the power variables are called energy variables. In general, these are called momentum, $p(t)$, and displacement, $q(t)$. These are defined as follows:

$$p(t) = \int_0^t e(t)dt,$$

or momentum is the time integral of an effort, and

$$q(t) = \int_0^t f(t)dt,$$

or displacement is the time integral of a flow. The following relations also exist:

$$E(t) = \int_0^t P(t)dt = \int_0^t e(t) \cdot f(t)dt.$$

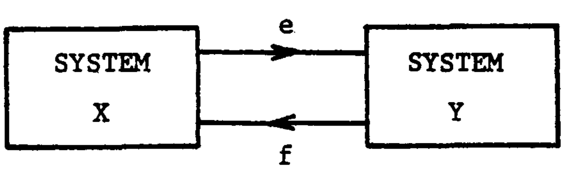
These power and energy variables are the only types of variables required to model physical systems.

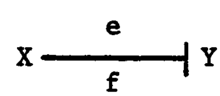
One fact that has been neglected thus far is that power can flow in either direction through a bond between multiports. Therefore a sign convention must be established. This is accomplished by a half arrow, placed on the line representing a bond, in the direction of the power flow.

Each bond between multiports has both an effort and a flow as a paired signal and it is only possible for one of these to be the input.

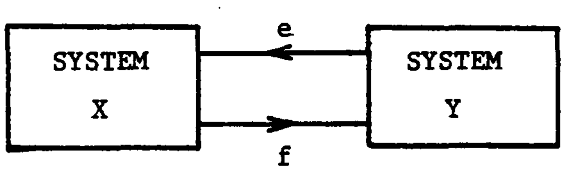
The other, therefore, must be the output. The effort and flow may be thought of as forward-effect and back-effect pairs. It must be remembered when two multiports are connected, the output of one is the input to the other, and conversely.

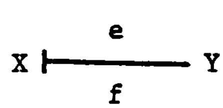
Specifying which one of the power variables is to be the output is accomplished by means of a causal stroke. This is a short line placed normal to the bond which, by definition, indicates the direction in which the effort signal is directed. By implication this also indicates the direction of the flow signal which is always opposite the effort. As an example, consider two subsystems, X and Y, joined by a bond. If effort is the output of X, then the block diagram and bond graph are:





or, if effort is the output of Y, then the block diagram and bond graph are:





Note that power flow is not assigned in this example: causality is independent of power flow. Also note the convention of writing the effort variable above the single line representing the bond and writing the flow variable below. As a further example, if power was assumed to flow from X to Y, then the examples above would be:

$$X \xrightharpoonup[f]{e}| Y, \quad \text{and} \quad X |\xrightharpoonup[f]{e} Y, \text{ respectively.}$$

In review of the material covered thus far, the concepts vital to the understanding of bond graph models are repeated below.

1. The bond graph is a method of developing a model of a system by joining subsystems called multiports.
2. These multiports are connected by bonds, represented by single lines, through which power flows.
3. This power is the product of two so-called power variables, generally referred to as effort and flow.
4. Effort and flow are familiar concepts; recognizable for the physical system being considered, eg. force and velocity for a mechanical translation system.
5. A sign convention is established using a half arrow to indicate direction of power flow in relation to a multiport.
6. The output of a multiport is established by assigning a causal stroke to the end of the bond towards which the effort is directed.

In the following section several important types of multiports are discussed along with the method of obtaining mathematical equations from the bond graph. The multiports may actually represent idealized real components such as resistors, capacitors, springs, masses, etc., in a system. More often, though, they are used in lumped-parameter models to approximate the effects of the continuum system.

The 1-port element has only one pair of effort and flow variables associated with it but may range from simple to quite complex in its behavior. Fortunately, to describe the system in this project only simple elements are required. The only types of 1-ports which are considered here are elements which dissipate or supply power, or store energy.

The resistor is an element in which the effort and flow are related by a constitutive relation which may be linear or nonlinear. Although resistive elements usually dissipate power, they may also be used to represent a supply of power. In terms of general variables, the relation for the resistor is effort equals the resistive function of the flow. In equational form this is

$$e = \Phi r(f), \quad \text{or if linear,} \quad e = Rf.$$

For example, in a linear, mechanical system

$$F = bV$$

would represent the force required to maintain a velocity through a dashpot.

Other elements which model power supplies are the so-called sources of effort and sources of flow. These names refer to the ability of these 1-ports to maintain one variable at a constant level while the other varies. Although these elements can be quite useful, they are not required in this project and are not discussed further.

There are two types of energy-storage elements dependent on the type of energy stored. One-ports which store potential energy are called capacitors, and inertance is the name given to elements which store kinetic energy. In a capacitor, the effort and displacement are related, while in an inertance the momentum and flow are related. The general relation for a capacitor is:

$$q = \Phi_c(e) \quad \text{or, if linear} \quad q = Ce.$$

For example, in a linear mechanical system $X = CF$, X is the distance a spring of compliance C is displaced due to a force F.

The general relation for an inertance is given by $p = \Phi(f)$. This reduces to the familiar momentum equation for a linear, mechanical system $p = mv$, or momentum is the product of mass and velocity.

A 2-port element is also used in modeling the subsystems and, of course, has two ports through which power may flow. The 2-ports discussed here are ideal in the sense that power is conserved; that is, output power equals input power. Furthermore, of the two possible configurations of 2-ports, the transformer and the gyrator, only knowledge of the transformer is required for this project. The bond graph symbol for the transformer is:

$$\xrightarrow[f_1]{e_1} \overset{m:1}{\mathrm{MTF}} \xrightarrow[f_2]{e_2}$$

and the constitutive laws are:

$$e_1 = me_2, \text{ and}$$

$$mf_1 = f_2.$$

The parameter, m, is called the modulus and may be a constant or a variable. The transformer is used to represent such physical devices as gear pairs, hydraulic rams, levers or electrical transformers. The transformer may also be used to interface two types of physical systems where the modulus is used as the converter. A simple example of this is the ideal hydraulic ram where force on the shaft is converted to fliud pressure and the velocity is converted to fluid flow by the modulus, A, the area of the piston.

The last types of multiports considered here, the 1-junction and the 0-junction, are components which have three or more ports and serve to interconnect other multiports into subsystems. Only two forms, each representing a series or parallel connection, are required to model any system. The connections are power-conserving and have only one output. The 1-junction, also called the effort junction or common-flow junction, has two associated constitutive relations. The first of these states that all the flow variables are equal and the second states that the algebraic sum of the effort variables is zero. The 0-junction, also known as the flow junction or common-effort junction, has similar relations but the algebraic sum of the flow is zero while all effort variables are equal. Both of these junctions are constrained to have only one output.

As mentioned previously the causal stroke plays an important role in the bond graph. Causality determines which variable of a junction is the output. It also determines if the relationship for an energy-storage element is in terms of the integral or derivative of a power variable. Capacitors or inductors with differential causality have to be algebraically related to others having integral causality before standard solution techniques can be applied. The causality assigned to transformers and junctions is determined by the causality of the connected 1-ports and the causal laws (2). This completes the information required to derive the equations for the system from the bond graph. The method of obtaining these equations is actually quite simple and is summarized below.

1. Draw the bond graph representation of the system.
2. Assign power flow arrows as logic or intuition dictates and number each bond and 1-port.
3. Assign causality trying to keep integral causality on as many energy-storage elements as possible; this usually determines the causality of the transformers, junctions, and remaining 1-ports.
4. Write the constitutive relations for each 1-port, 2-port, and junction in terms of the numbered power variables associated with each numbered bond.
5. Algebraically relate any element with differential causality to one with integral causality.

The Nth-order physical model is now described in terms of N state variables and associated algebraic relations. The state variables can easily be recognized as the outputs of the equations involving integration. The system described may be both time-varying and nonlinear.

This overview was not intended to be a complete source on the method of bond graph modeling but the reader should now have sufficient knowledge to understand the development in the remainder of this chapter. In the next section the entire model for the test vehicle is developed. An example of how the equations are obtained from the bond graph is presented along with the complete set of describing equations for the model.

Bond Graph and Coefficient Model

The bond graph method of modeling was chosen for several reasons, one of which was the ease of coupling different types of physical systems. The bond graph provides a systematic method of obtaining the describing equations in terms of general power variables and multiports including resistance, compliance, and inertance. Thus it is a simple matter to relate the mechanical rotation, translation and hydraulic subsystems which comprise the vehicle. Bond graph modeling also produces equations which are usually in the proper form for simulation by CSMP.

Using the coefficient model presented in Merritt (13) to describe the performance of the hydrostatic transmission results in the losses due to viscous and dry friction along with losses due to leakage expressed in terms of resistances. These resistances enter conveniently into the bond graph model as will later be seen.

The final model was developed by first modeling several subsystems and then combining them into an overall system. When the final bond graph is presented at the end of this section, a complete discription of each element is also given.

The transmission was the first system to be modeled. The schematic of the transmission is shown in Fig. 2. From this schematic the original bond graph was drawn, see Fig. 3. This was reduced by utilizing a number of assumptions which simplified this bond graph without sacrificing its validity. These assumptions are listed below.

1. The return line pressure is zero. This is a common assumption for return pressure of about 1.03 MPa (150 psi).
2. The pump and motor can be described by effective inertia and damping terms. Therefore the translatory and rotary mechanisms may be combined.
3. The cross-port and external leakages may be combined since the associated pressure drop across them is the same, from 1, above.
4. The resistances of the relief valves and check valves are ignored because they do not affect normal operation.
5. Seal friction in the units is negligible.

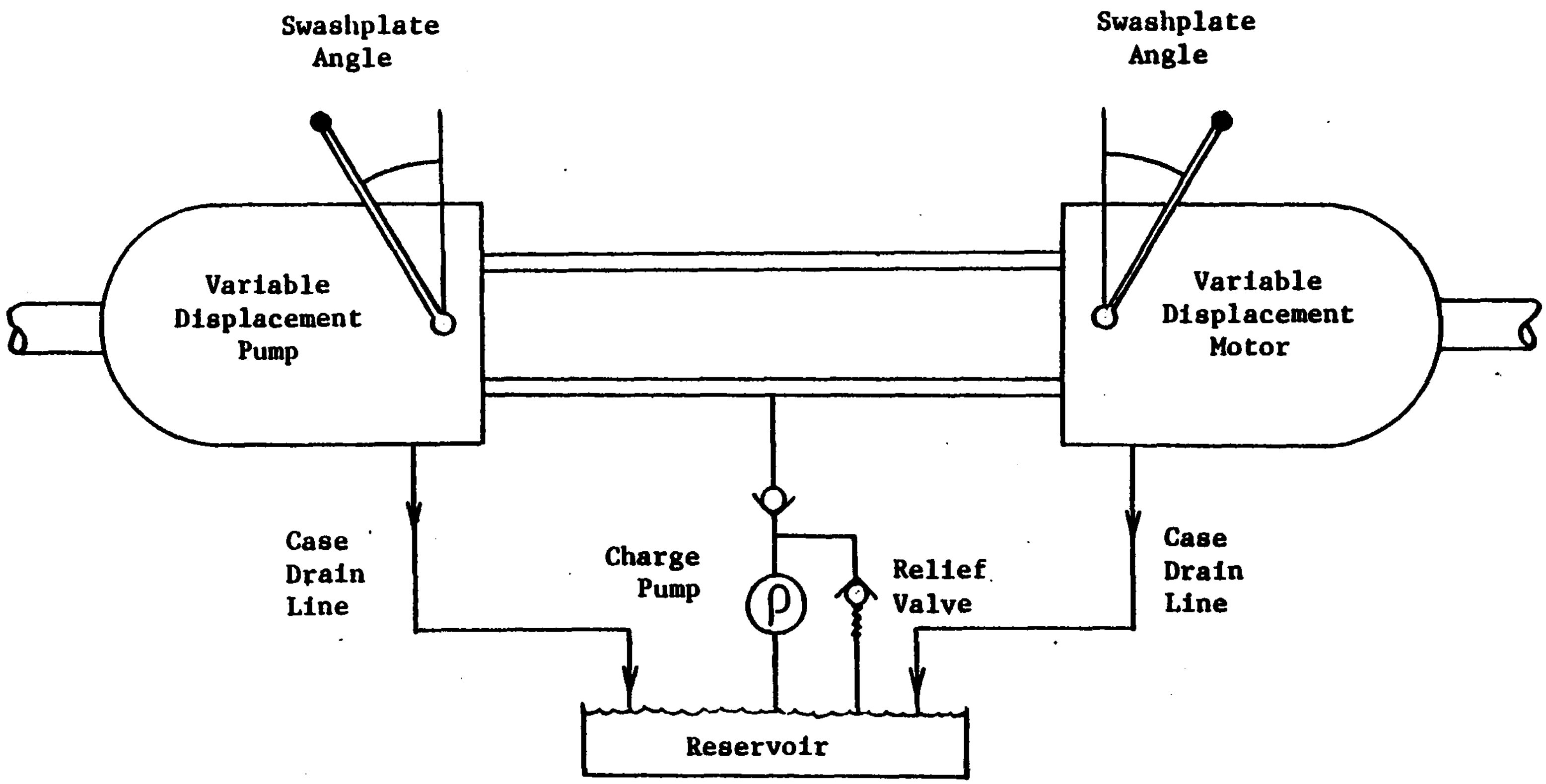


Figure 2. Schematic of Hydrostatic Transmission

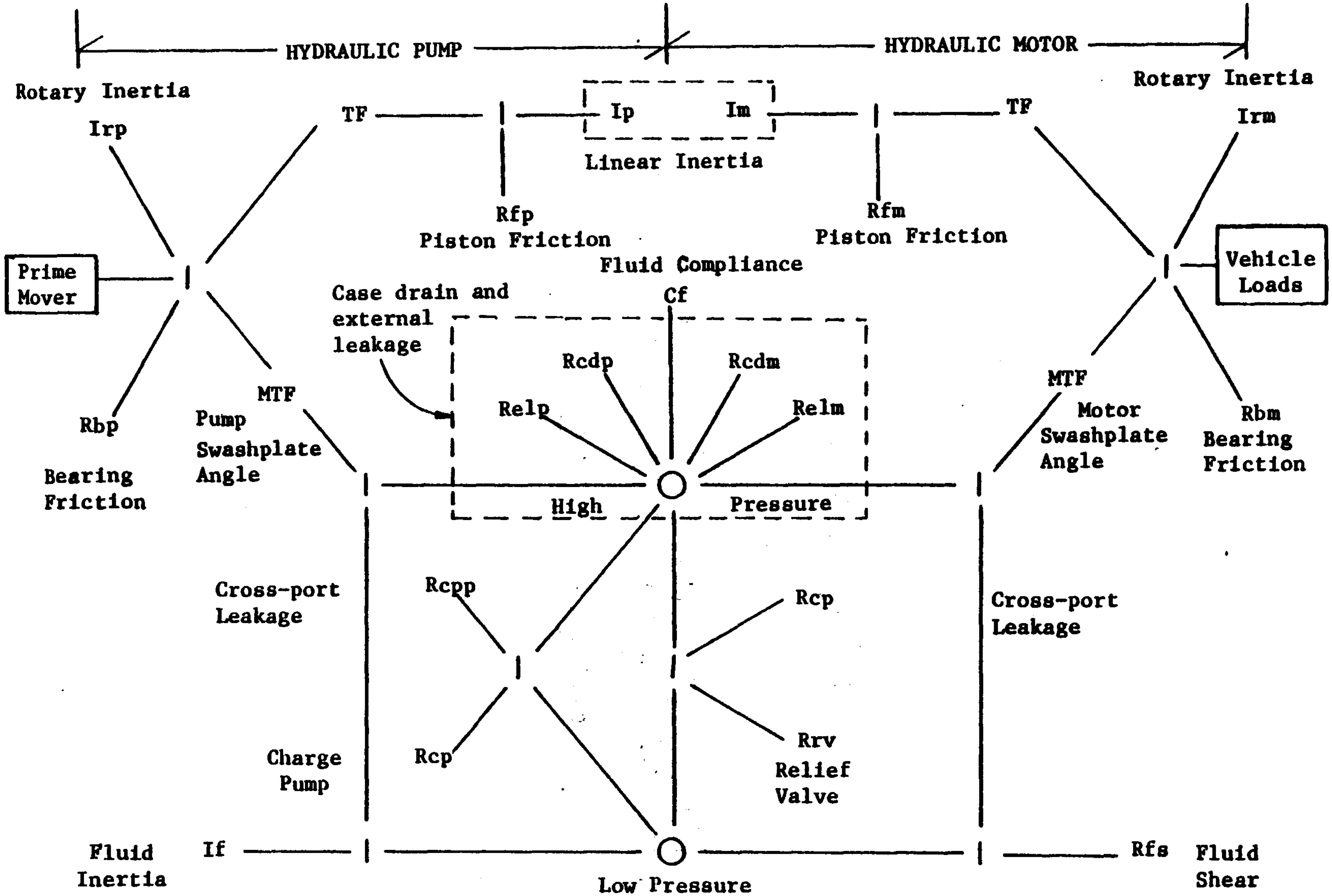
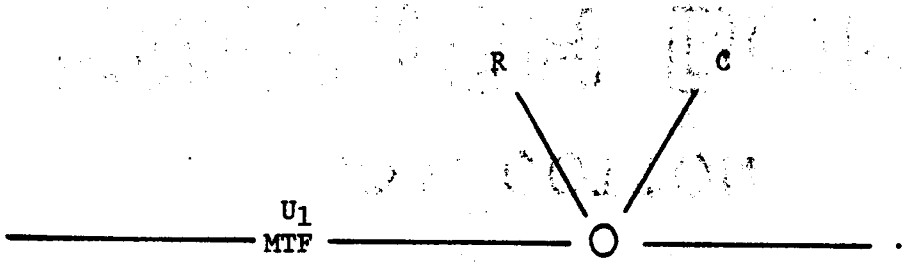


Figure 3. Original Bond Graph of Transmission

6. The compliance of the shafts and mechanisms is much lower than the compliance of the fluid and is therefore neglected.
7. Inertia of the fluid is low compared to inertia of mechanisms.

These assumptions lead to the bond graph shown in Fig. 4. This abbreviated model includes effective inertia terms for each unit, resistance terms for viscous friction, dry friction and flow leakage, and a combined compliance term for the elasticity of the fluid and hydraulic lines. The model also uses a resistance to describe the power lost to driving the charge pump. This loss is arbitrarily assigned to the main pump. This model utilizes modulated transformers to describe the variable displacement characteristic of the pump and motor. Utilizing a portion of this bond graph as an example the development of the system equations as outlined in the previous section may be demonstrated. Consider the U_1 transformer and 0-junction which represent the pump and hydraulic lines. Performing step 1. of the method of writing the equations (presented at the end of the preceding section) the bond graph for the pump and lines is redrawn below:



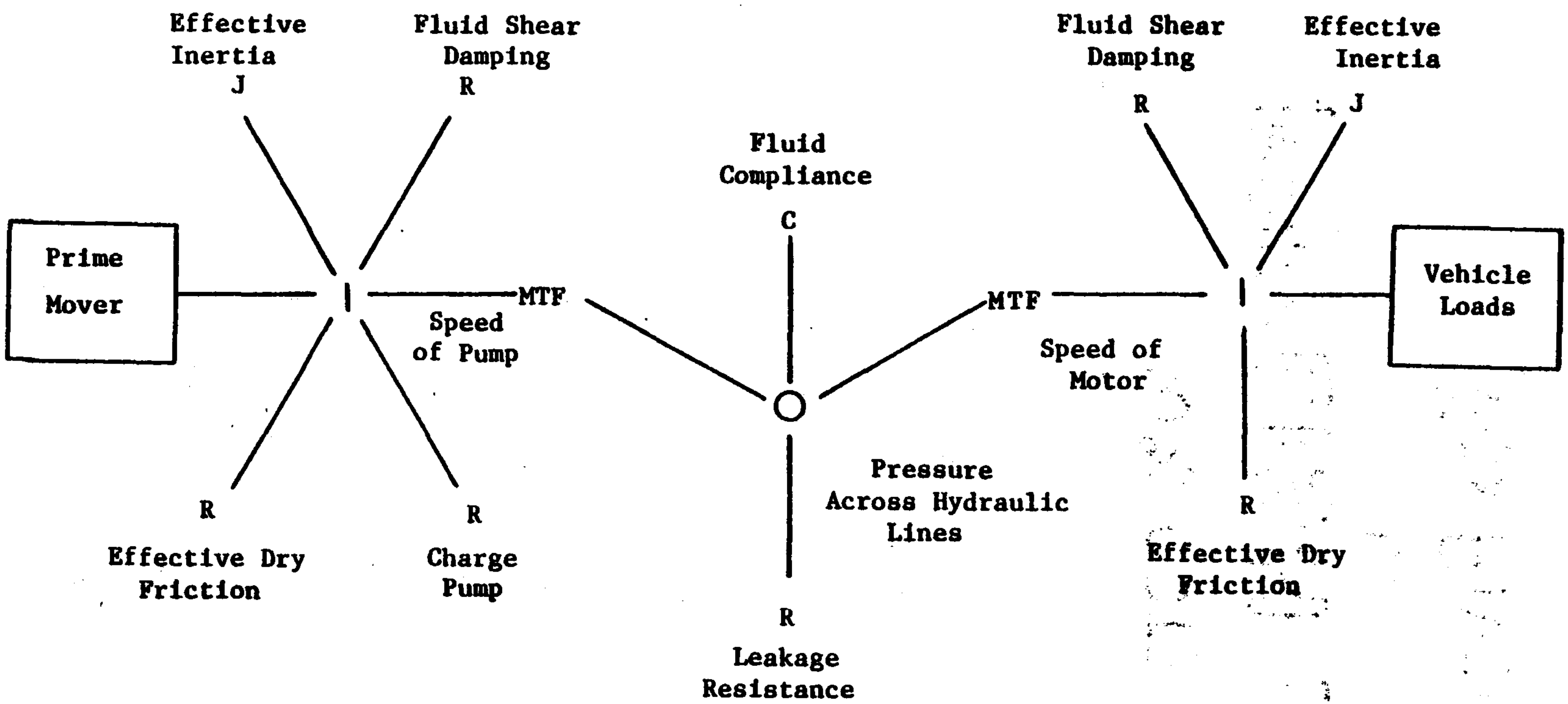
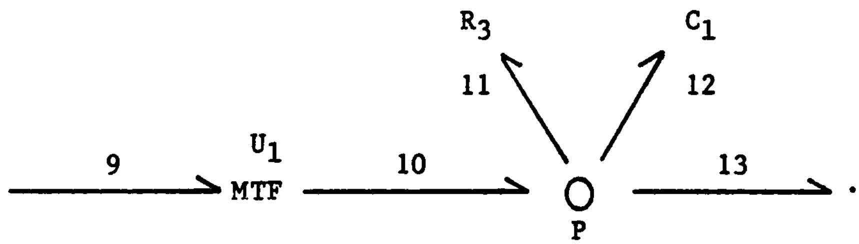
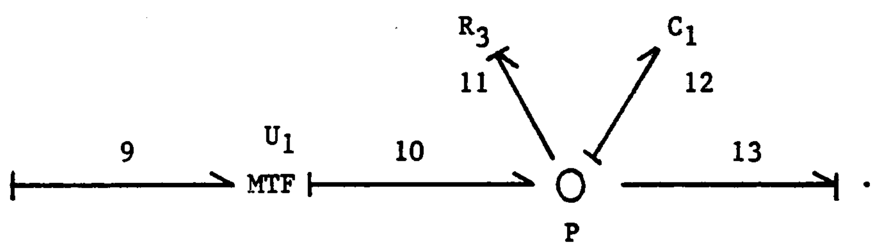


Figure 4. Final Bond Graph of Hydrostatic Transmission

The next step is assigning power flow and numbering the bonds and elements. Since the pump is connected to the prime mover, the logical power flow is as shown:



The numbers are chosen arbitrarily. Since there is only one storage element, C_1, integral causality is assigned to it, and, in accordance with step 4, the remaining causality is determined:



The causality and power flow of this bond graph imply the following:

1. the angular speed which is input to the pump is output as flow without power loss,
2. the pressure, P, across R_3 to ground causes a flow loss and a resulting power loss,
3. the amount output flow to the capacitor, C_1, determines the pressure, P,
4. pressure is the input to the multiport connected to bond 13.

The next operation is writing the constitutive relations for the 1-ports, 2-port, and junction:

for the 1-ports, $$Q_{11} = P / R_3$$

$$P = \frac{1}{C_1}\int Q_{12}dt,$$

for the 2-port $$Q_{10} = U_1 * Wp$$

$$T_9 = U_1 * P,$$

and for the junction $$Q_{12} = Q_{10} - Q_{11} - Q_{13}.$$

This is the complete set of describing equations for the hydraulic pump and hoses.

The next step in the development is to model the prime mover and the remainder of the vehicle and add this to the model of the transmission just presented. The prime mover, which is a spark-ignition, internal-combustion engine is modeled as a resistor which supplies, rather than dissipates, power. This indicates the dependence of the output torque on the speed of the engine as well as the throttle position. Effective inertial and resistive elements are included in the engine model to account for the friction and dynamic masses. Assuming the line compliance in the transmission is dominant in the system, the mechanical compliance of the engine can be neglected. It is also assumed that the torque output of the engine is uniform and not periodic.

The loads associated with the vehicle are the result of air drag, forces required for acceleration and resistance of rolling elements such as tires and bearings. For simplicity of the final model, similar elements are combined to form lumped elements. Thus, the rolling resistance due to all four tires is lumped into one resistive element. In a similar manner all wheel bearing resistance is lumped into a single resistor, the inertia of the axles and differential and all four wheels are lumped into a single inertia, and the inertia of the drive shaft is included with the inertia of the hydraulic motor. Combination of these elements is quite valid as long as they are referenced to the same velocity. Again it is assumed that the compliance of the transmission is dominant and therefore no compliance term is included in this section of the model.

Models for the prime mover and the vehicle were added to the transmission model shown in Fig. 4, resulting in the final bond graph model of the test vehicle which appears in Fig. 5. The definitions of the most significant variables are as follows:

- T_1 is the torque from the engine;
- We is the angular speed of the engine;
- T_2 is the torque required to accelerate the mass of the engine and gearbox;
- Wp is the angular speed of the pump;
- Q_{12} is the flow associated with the density change in the fluid;
- Wm is the angular speed of the motor;
- Ww is the speed of the wheels;
- F_{22} is the tractive effort;

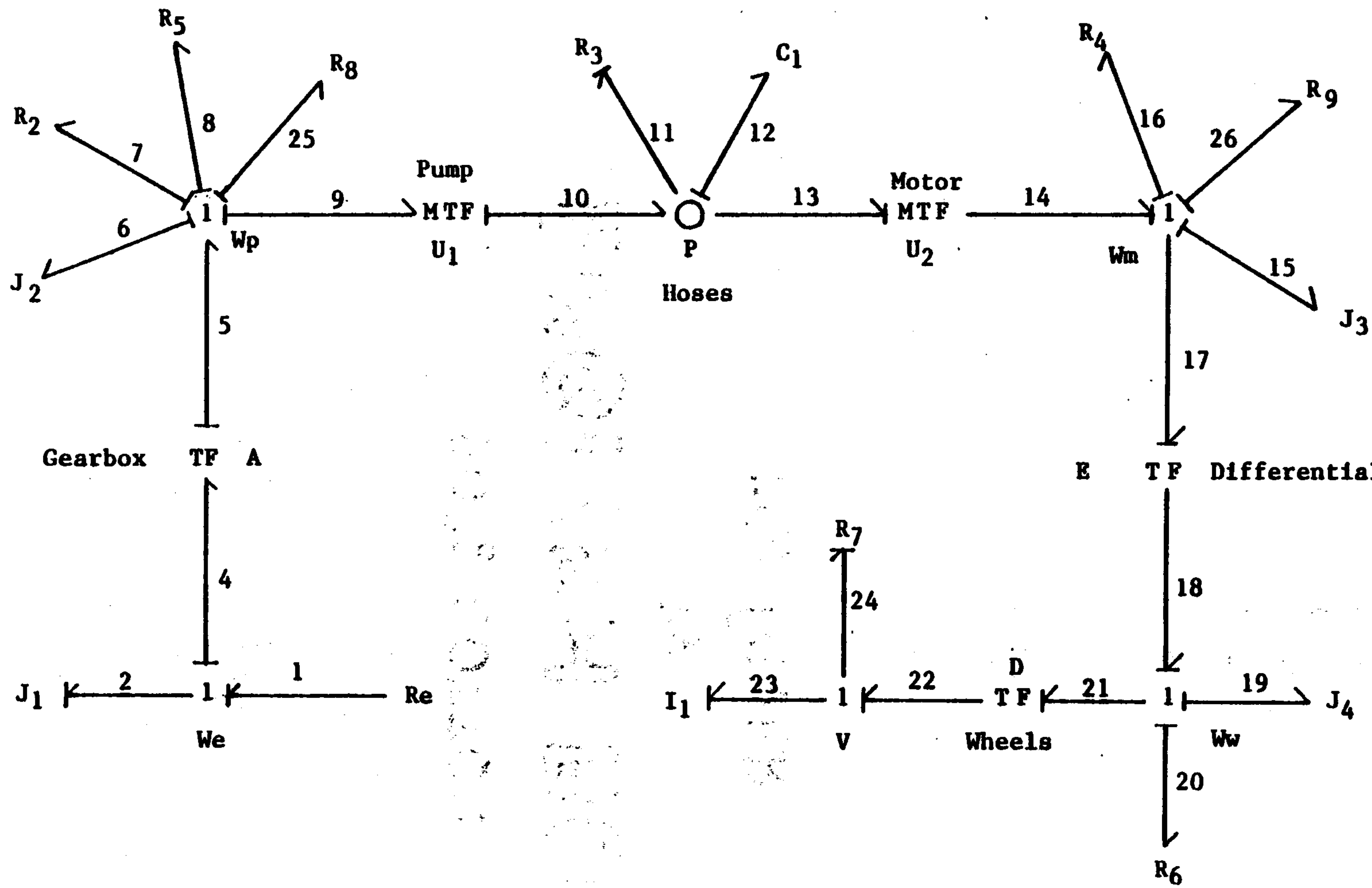


Figure 5. Final Bond Graph of Test Vehicle

F_{23} is the force required to accelerate the vehicle;

V is the velocity of the vehicle.

Using the techniques outlined earlier in this chapter, the entire set of equations for the model were obtained by writing the causal relation for each multiport in the bond graph. First, the constitutive relations for each resistance, inertance and compliance were written as follows:

J_1: $We = \frac{1}{J_1}\int T_2 dt = IT_2/J,$

J_2: $T_6 = J_2$*Wpdot,

R_2: $T_7 = R_2$*Wp,

R_5: $T_8 = R_5$*Wp,

R_8: $T_{25} = R_8$*P,

R_3: $Q_{11} = P/R_3$,

C_1: $P = \frac{1}{C_1}\int Q_{12} dt = IQ_{12}/C_1$,

R_9: $T_{26} = R_9$*P,

J_3: $T_{15} = J_3$*Wmdot,

R_4: $T_{16} = R_4$*Wm,

J_4: $T_{19} = R_4$*Wwdot,

R_6: $T_{20} = R_6$*Ww,

R_7: $F_{24} = R_7$*V ,

I_1: $V = \frac{1}{Mc}\int F_{23} dt = IF_{23}/Mc.$

Next, associated with each transformer modulus were two equations for each 2-port:

A : T_4 = A*T_5 and Wp = A*We,

U_1: $Q_{10} = U_1$*Wp and $T_9 = U_1$*P,

U_2: T_{14} = P*U_2 and $Q_{13} = U_2$*Wm,

$E: \quad T_{18} = T_{17}*E$ and $Wm = Ww*E$,

$D: \quad F_{22} = T_{21}/D$ and $Ww = V/D$.

Finally, the equations for the common-effort and common-flow junctions were obtained with the output power variable appearing on the left side of the equation. These appear below associated with the power variable which is common at each junction:

$We: \quad T_2 = T_1 - T_4$,

$Wp: \quad T_5 = T_6 + T_7 + T_8 + T_9 + T_{25}$,

$P: \quad Q_{12} = Q_{10} - Q_{11} - Q_{13}$,

$Wm: \quad T_{17} = T_{14} - T_{15} - T_{16} - T_{26}$,

$Ww: \quad T_{21} = T_{18} - T_{19} - T_{20}$,

$V: \quad F_{23} = F_{22} - F_{24}$.

As can be seen from above, these equations included three integrals; one for each energy-storage element possessing integral causality in the bond graph. The model was in proper form for simulation by CSMP except for the equations for F_{23} and T_2 which contained implicit loops. The variables F_{23} and T_2 were explicitly found algebraically; the equations are:

$$F_{23} = ((T_{14}-T_{16}-T_{26})*E/D-T_{20}/D-F_{24})/(1.+J_4/Mc/D/D+J_3*E*E/Mc/D/D),$$

$$T_2 = (T_1-A*(T_7+T_8+T_9+T_{25}))/(1.+A*A*J_2/J_1).$$

As was mentioned at the beginning of this chapter, with only a few minor changes, these equations could model a host of different types of vehicles. The major factors determining the output of these equations are the values assigned to the 1-port elements and the transformer modulii. This is the purpose of the next section.

Evaluating the Elements

As was stated previously, the 1-port elements represent the loss or supply of power and the storage of energy, and the transformers maintain a ratio between the variables associated with its two ports. This section is concerned with assigning values to these elements to properly model the test vehicle. The definitions of the constants to be evaluated appear below:

- R_e is the resistor, representing the engine, which supplies power to the system;
- R_1 is the damper representing the friction in the engine and in the gear box between the engine and pump;
- J_1 is the combined effective inertia of the engine and gearbox;
- A is the angular velocity ratio for the gearbox;
- J_2 is the effective inertia of the pump;
- R_2 is the viscous damping in the pump;
- R_5 is the resistance of the charge pump;
- R_8 is the dry friction in the pump;
- U_1 is the ideal flow per revolution of the pump;
- R_3 is the combined cross-port and external leakage resistance for both pump and motor;
- C_1 is the effective capacitance of the fluid and hydraulic lines;
- U_2 is the ideal flow per revolution of the motor;
- J_3 is the effective inertia of the motor and driveshaft;
- R_4 is the viscous damping in the motor;
- R_9 is the dry friction in the motor;

E is the final drive ratio of the differential;

J_4 is the effective inertia of the wheels, axles and differential;

R_6 is the effective rolling resistance of the tires and wheel bearings;

D is the rolling radius of the wheels;

Mc is the total mass of the vehicle;

R_7 is the air drag on the body.

For clarity in presenting this section these elements are divided into two categories dependent upon the method used for their determination. Considered, in order, are those values or relations found in the literature and those determined from data supplied by the manufacturers.

The relation for the resistance of the tires was given by Leonard (4) in terms of horsepower loss,

$$Pt = Kt*W*V/G$$

where Pt = tire drag, horse power

W = gross vehicle weight, lb

V = speed, mph

$Kt = 0.005+0.15/P + 0.000035*V/P$

P = Tire pressure, psi

G = 375 lb-mile-hp/hr.

Since power is the product of torque and angular speed, and, from the constitutive equations, torque is given as the product of a resistance and angular speed; the resistance can be found by dividing the above power expression by the angular speed squared. This is given by;

$$R_6' = Kt*Wt*R/Ww,$$

where R_6' is the rolling resistance of the tires, N-m-s

Wt is the weight of vehicle, N

R is the rolling radius of tire, m

Ww is the angular speed of wheels, rad/s

Kt = 0.005+1034.6/P+1.21*V/P

in the latter equation

V is velocity, m/s

P is pressure, Pa.

The total wheel bearing friction torque was found to be less than 0.2 N-m(2.02 in.-lb) from Palmgren (16),

$$T = u*F*B/2,$$

where T = friction torque, N-m

u = friction coefficient =.001

F = radial load, N

B = bearing bore, m.

Therefore, the rolling resistance due to the bearings is

$$R_6'' = .0075\ Wt/Ww,$$

and can be added to R_6' to give the total rolling resistance, R_6. The resistance due to air drag, R_7, was also taken from Leonard. In final form, at sea level, it is given by:

$$R_7 = 0.00028CD*S,$$

where CD is the drag coefficient

S is frontal area, m .

The fluid and line capacitance was found from Merritt (13) to be 4.75E-13 m/N (0.0002 in.^{5}/lb) by:

$$C_1 = V/Be$$

where C_1 is the capacitance, m/N

V is the internal volume = 3.277E-4 m^3(20 in.^{3})

Be is the effective bulk modulus of elasticity of the fluid = 6.895E8 N/m (100,000. psi).

This modulus is an estimate which includes the effects of entrained air bubbles and the compliance of the hoses.

The rotating inertia of the wheels was calculated using the equation of a circular cylinder;

$$I = mr^2/2.$$

where I is the mass moment of inertia about the diameter, N-m-s^2

m is the mass of the wheel, N-s^2/m

r is the radius of gyration, m.

The mass of the four wheel assemblies was found to be 15.6 N-s^2/M and the radius of gyration was estimated at .229 m, giving a total rotating inertia of 1.64 N-m-s^2 (14.7 in.-lb-s^2). The rotating inertia of the rear axles and differential was estimated to be .0056 N-m-s^2 (14.8 in.-lb-s^2).

The remainder of the values assigned to the elements was obtained from information or data supplied by the manufacturers. The following values were obtained for a Pinto from Ford Motor Company: CD = .595, Mc = 1225 kg (2700 lb), J = 0.11 N-m-s^2(0.95 in.-lb-s^2), and E = 3.45. The frontal area S, was estimated to be 1.61 m^2(2500 in.^{2}) and the rolling radius, D, was found to be 0.305 m (12 in.). Also supplied by Ford was a cross-section load survey of their 2.3 litre engine. This included, in tabular form, observed torque and observed bsfc for the entire range of engine speeds and throttle angles. Since brake, or observed data was provided, the internal damping of the engine was accounted for and the R_1 element was eliminated from the model.

It was found necessary to have functional, rather than tabular, relations for torque and bsfc as dependent variables. To this end, a multiple regression was performed on the data using the Statistical Analysis System, which resulted in second degree equations for both torque and bsfc as functions of engine speed and throttle angle. The program used for the regression appears in Appendix A. The final equations are:

$$T_1 = .593-6.3E-4J_1*X_3(1)-4.2E-3*U_3(J)+2.8E-6/J_1/J_1*X_3(I)*X_3(I)+6.1E-5$$
$$*U_3(J)8U_3(J)-1.1R-5/J_1*X_3(I)*U_3(J)$$

$$bsfc = 643.3-5.47/J_1*X_3(I)+45.6*U_3(J)-1.8E-3/J_1/J_1*X_3(I)-.465$$
$$*U_3(J)*U_3(J)+.08/J_1*X_3(I)*U_3(J)$$

where We is engine speed, rad/s;

U_3 is throttle angle, degrees.

The correlation coefficients for the equations are .91 and .97, respectively.

The remaining elements to be evaluated are associated with the hydrostatic transmission. The Sundstrand Corporation provided complete performance programs for their Series 20 pump and motor. The values of the transformer modulii, U_1 and U_2, which represent the pump and motor, relate angular speed to fluid flow and are directly related to the swashplate angles. The swashplate angle of the pump may vary from 2° to 18° while the motor may range from 7° to 18°. Maximum theoretical displacement of these units is 0.0333 l/rev (2.03 in.^{3}/rev). The effective rotating inertia of the units used by Sundstrand is,

$$J_2 = 0.00428 \text{ N-m-s}^2 \ (0.0384 \text{ in.-lb-s}^2).$$

The combined inertia of the motor and the drive shaft was found to be

$$J_3 = 0.00646 \text{ N-m-s}^2 \ (0.0580 \text{ in.-lb-s}^2).$$

The power lost in driving the charge pump is represented by the resistance, R_5. The value of this element was derived from the manufacturers data by the relation,

$$R_5 = 2.668/Wp \text{ N-m-s} \ (23.6/Wp \text{ in.-lb-s}),$$

where Wp is the pump speed in rad/s.

The data included efficiencies for the pump and motor over the entire working range of pressure, speed,and swashplate angles but did not include the loss coefficients needed for the model. Merritt (13) provides equations for torque efficiency and volumetric efficiency in terms of pressure, speed, viscosity and loss coefficients for hydraulic machines. A least-squares linear regression program was written using these equations to derive the loss coefficients from the Sundstrand data. This program appears in Appendix B.

These general loss coefficients are defined by Merritt as follows:

Cd is the viscous damping coefficient;

Cf is the dry friction coefficient;

Cs is the slip, or combined internal and external leakage coefficient.

The final values used for these coefficients result in pump and motor overall efficiencies calculated within the CSMP model which are less than 10 percent different from those provided by Sundstrand for the useful region of performance. For the pump these coefficient values are,

CDP = 1.24E05

CSP = 7.64E-9

CFP = 0.00477,

and, for the motor,

CDM = 2.11E05

CSM = 3.07E-9

CFM = 0.0298.

When these values are compared to coefficients found in the literature (11) (12), good agreement can be found for the motor coefficients, but the regressed pump coefficients indicate performance below that expected in the literature.

These loss coefficients were then used to determine the values for the resistive elements as defined by Merritt. The viscous damping values for the pump, R_2, and motor, R_4, are found as follows:

R_2 = CDP*U*DMP = 0.012486 N-m-s (0.11047 in.-lb-s)

R_4 = CDM*U*DMM = 0.021421 N-m-s (0.1895 in.-lb-s)

where U is the absolute viscosity, N-s/m ^{2}

DMP is the displacement of the pump, m^3/rad

DMM is the displacement of the motor, m^3/rad

The absolute viscosity was assumed to be 1.92E-2 N-s/m (2.78 lb-s/in.) throughout the test. The torque required to overcome the internal friction of the units is a function of pressure; the resistance elements are given by:

R_8 = CFP*DMP = 2.525E-7 m^3/rad(.01543 in.^{3}/rad)

R_9 = CFM*DMM = 1.578E-7 m^3/rad(.009628 in.^{3}/rad)

where R_8 is the pump friction

R_9 is the motor friction.

The slip coefficients are used in determining the resistances, R_3P and R_3M, which represent the leakage restrictions in the pump and motor, by the relations,

R_3P = U/DMP/CSP = 127.30 N-m-s (1126.3 lb-in.-s)

R_3M = U/DMM/CSM = 316.80 N-m-s(2802.8 lb-in.-s).

These resistances are then added in parallel to form the total leakage resistance, R_3, by

$$R_3 = R_3P*R_3M/(R_3P+R_3M) = 90.81 \text{ N-m-s} (803.4 \text{ lb-in.-s}).$$

This concludes the evaluation of the elements used in the system equations.

Using the equations and element values presented in the two proceeding sections, the CSMP model was formed. This program was named HY and the complete listing appears in Appendix C.

Summary

This chapter was concerned with the development of the test vehicle model. The method of bond graph modeling provided a clear picture of the system but is not a well-known technique. For this reason, a section was devoted to introduce the concepts of bond graph modeling and enhance the reader's comprehension of the sections that followed. The bond graph of the transmission was then developed, simplified, and added to the bond graphs of the prime mover, and the remainder of the vehicle. From this bond graph the describing equations for the entire system were then developed in terms of general elements. These elements were then evaluated using information from the literature, and information and data from the transmission and vehicle manufacturers. The system was then described entirely by equations in terms of known quantities, and could be simulated by CSMP. This completes the objective of this chapter as defined in the first section.

The model developed is third-order and, thus, requires three integrations with respect to time to simulate dynamic performance. The purpose of the next chapter is to describe this dynamic simulation using CSMP and discuss methods of control which will result in high fuel efficiency of the test vehicle.

IV. SIMULATION AND CONTROL

Introduction

The dynamic performance of the vehicle may be simulated using the model presented in Chapter III. The independent variables in these equations are the transformer modulii for the pump, U_1, and motor, U_2, and the engine throttle angle, U_3. For the steady-state velocity case, it would be a simple matter to determine the values of these variables which would yield the best efficiency. However, when the vehicle is accelerating the case becomes dynamic and the best values for the variables are no longer easily found. Thus, some method of control is desired which will operate the system, in the most efficient manner, in response to the driver's input. There may be several control designs, however, which produce high efficiencies only over a limited range of operation. The problem then becomes one of determining the control which optimizes the efficiency over the defined operating range. Optimizing a chosen performance criterion is the exact nature of optimal controls. Using this method, the performance criterion could be the overall fuel efficiency and the optimal controller could be designed to maximize it. Thus, it would appear to be the ideal approach and, for this reason, was given considerable attention. There are several serious drawbacks to these types of control, however, which are discussed in a later section.

The second objective of this thesis was to examine methods of control which result in high efficiency of the drive train. This chapter, then, presents the research performed while fulfilling this objective. More specifically, this chapter treats the simulation of the model developed in Chapter III along with the development of an optimal control program. Using a simple controller several CSMP simulations were performed using the simple control and observations made from these runs are presented. Next, the development of the optimum control program is presented. The method of obtaining the optimum controls is described and the limitations of this method are discussed. Final observations are then presented concerning the overall performance of the test vehicle as simulated by the model.

Initial CSMP Simulations

The initial simulations were made by specifying values for the control variables, U_1, U_2, and U_3. This served to debug the program and also to evaluate the values assigned to the elements in Chapter III. Equations added to the program determined the overall efficiencies of the pump and motor and provided a means of qualifying the values given to the hydraulic loss coefficients. The efficiencies obtained from the simulations were directly compared to the Sundstrand data and, if the difference was significant, new values for the coefficients were tried. This simplistic method was time-consuming but provided the loss coefficients, presented in the previous chapter, which agreed well with the Sundstrand data.

One open loop control strategy which was tried was based on the conventional method of increasing the motor speed in a hydrostatic transmission in which the pump and motor displacements are both variable. The increase in motor speed is achieved by increasing pump displacement while holding the motor angle constant and then when the pump is at maximum displacement decrease the motor angel. In the CSMP simulations this control was realized by logic which changed the pump and motor swashplate angles as the vehicle velocity changed. Since the vehicle velocity was directly related to motor speed, the vehicle velocity increased when this conventional control sequence was applied.

It should be noted that other system parameters such as engine speed or hydraulic pressure could have been used as shift points with some control strategy other than the conventional method described above. The strategy used is not claimed to be the optimum method but merely illustrates the performance obtainable from one possible control design.

After a number of combinations of shift points and swashplate angles were tried, a control sequence was found which seemed to provide high overall efficiencies of the transmission while the engine operated near minimum bsfc. In this sequence the motor swashplate angle was held at 16° while the pump swashplate angle was changed from an initial value of 6° to 8° when vehicle velocity reached 0.5 m/s (1.1 mph). The pump angle was then changed to 10° at 3.8 m/s (8.5 mph), 12° at 10.1 m/s (22.6 mph), and 14° at 16.5 m/s (37.0 mph) the vehicle velocity reached 19 m/s (42.6 mph) the pump angle was increased to 16°. The pump angle was finally changed to maximum displacement at 21.5 m/s (48.2 mph).

Using the method of control just described, a CSMP simulation was made of the test vehicle accelerating from stop to 25. m/s (56. mph) in 50 seconds. During this run the engine bsfc was maintained below 7.78E-8 kg/J (0.46 lb/bhp-hr) and the average values for the pump and motor efficiencies were .89 and .93, respectively. The pressure drop across the pump during this simulation dropped from an initial value of 38,300. MPa (5550. psi) to a final 16,900. MPa (2450. psi). The instantaneous fuel economy at any velocity may be calculated by the following formula:

$$c = v*d/(b*p)$$

where c is fuel economy, m/m^3 (mpg)

v is vehicle velocity, m/s (mph)

b is bsfc, kg/J (lb/hp - hr)

p is engine power, J/s (hp)

d is fuel density kg/m^3 (lb/gal)

Typical fuel economies thus calculated for the initial simulation were 9.1 Mm/m^3 (21.5 mpg) at 20. m/s (45.mph) and 10.4 Mm/m^3 (24.5 mpg) at 25. m/s (56. mph). It must be remembered, however, that these are instantaneous values during acceleration and may be somewhat lower than values at a constant velocity. For example, another simulation at a constant speed of 26.8 m/s (60. mph) produced fuel economy above 13.1 Mm/m^3 (31. mpg). This is higher than the standard EPA highway estimate of 28 mpg for a 1978 Pinto.

During the steady-state run at 26.8 m/s several comparisons were made between the test vehicle and the conventional vehicle equipped with a standard transmission. It was observed that the engine speed was 162. r/s (1547. rpm). This is substantially lower than the speed of the engine driving through a standard transmission in top gear, which would be 304 r/s (2900. rpm). The throttle angle was approximately 40% in each case but the torque placed on the test vehicle engine was double that placed on the engine of the conventional vehicle. The increased engine load may alleviate the problems of spark plug fouling and carbon build-up inherent with engines which operate under light loading.

The results of these initial simulations indicate that the hydrostatic transmission has the potential of improving gas economy over the standard transmission with no modifications to the engine. In addition, lower engine speed and higher loading may combine to produce a cleaner-running and longer-lasting engine. The next objective is to determine the values of the control variables which result in the optimum efficiency throughout the range of operation.

Defining the Optimal Controls

In general, the theory of optimal controls is a relatively new branch of the field and, as a result, there are not many proven methods available to the designer. Unlike classical techniques, the methods of optimal control usually require extensive use of high-speed computational devices for all but the most simple systems. The cost of obtaining the optimum values are, therefore, quite high and may not seem justified, especially when the optimum design may not be physically realizable. The main advantage of knowing the optimal control is that it demonstrates the ultimate capabilities of a specific system subject to specific constraints. Once this limit is known, the designer may elect to use a suboptimal control because of physical constraints but knows exactly the relation between the control used and the best possible control. In most optimal control schemes some form of integration over time is required for the state equations and the performance index. This is accomplished by discretizing the functions with respect to time and then approximating the integral by a summation of all the functional values at each time increment. These time increments are normally called stages. Clearly an infinite number of state and control variable values is not feasable if solution is expected by digital computer. Thus the state and control variables are expressed by a finite number of values over their ranges. These incremental values are called the quantized values.

For the purpose of this study, a method must be found which may be used with a nonlinear, dynamic system in which both the state and control variables may be constrained. Furthermore, two of the states and at least one of the controls must vary over a large range of values and the total system should be driven long enough to represent a normal acceleration to a steady-state speed. The method of direct enumeration consists of trying all possible combinations of state and control variables and storing the values of the variables which optimize the performance index. This method is simple and provides correct results, but limits the number of stages of the process due to the vast increase in the number of calculations required for each additional stage. Using the calculus of variations and Pontryagin's Minimum Principle, the optimum control values may be found for a system. This method requires the solution of differential equations which possess specific boundary conditions at both ends. Kirk (17) states that analytic solutions of these equations are generally impossible for systems higher than second order even if they are linear and time invariant. This approach leads to an open-loop type of control.

The method of dynamic programming developed by R. E. Bellman, can be used with dynamic, nonlinear systems even if the variables are constrained. Although quite similar to the direct enumeration approach, dynamic programming uses the principle of optimality to reduce the number of calculations for each stage. In fact, while the number of calculations for direct enumeration increases exponentially with the number of stages, dynamic programming calculations only increase linearly. This method requires that the system be described in state equation form and the performance measure is expressed in terms of the state and control variables. The value of this performance index is commonly called the cost. Briefly stated, the principle of optimality states that, for an optimal policy, given the initial state and the initial control decision, the remaining decisions will also form an optimal policy with respect to the state resulting from the initial decision. This principle allows the optimal path from the initial stage to the final stage of a process to be determined in a sequential manner.

The general concept underlying dynamic programming is to begin at the final stage of the process. The cost of moving from the previous stage to the final stage is then calculated for each possible state and control value. The control value which produces the optimum cost at each state is then stored. This procedure is repeated until the cost of moving from the initial stage to the final stage is known for each state. The optimum control is then given for each state at each stage by the corresponding, stored control value. In order to perform these calculations a performance measure must first be defined.

The second objective of this study was to maximize the efficiency of the drive train. Therefore, some function which reflected this efficiency was desired as the measure of performance. The general form of this performance index is

$$J = \int_0^T g(\underline{x}(t), \underline{u}(t))dt,$$

where J is the cost

T is the final time

$\underline{x}(t)$ is the state vector

$\underline{u}(t)$ is the control vector

g represents the performance function

Two candidate performance functions are listed below:

1. maximize: motor efficiency * pump efficiency / bsfc,

2. minimize: engine speed * engine torque * bsfc.

These candidate performance functions will both result in the same optimal policy when the vehicle velocity is constrained to follow a fixed path, but the second equation was chosen because it requires less computation.

The state equations have the general form

$$\frac{d\underline{x}}{dt} = \underline{A}(\underline{x}(t), \underline{u}(t)),$$

where $\underline{x}$ and $\underline{u}$ are vectors, and $\underline{A}$ is a square matrix.

These state equations and the performance index above had to be discretized for use of the digital computer. For the state equation this is done by approximating the time derivative, $\underline{x}$dot(t), of each state by

$$\underline{x}\text{dot}(t) = (\underline{x}(t+\Delta t)-\underline{x}(t))/\Delta t.$$

Assuming that t is equal to one stage, and k denotes the stage number, the state equation may be written as

$$\underline{x}(k+1) = \underline{x}(k)+\Delta t(\underline{A}(\underline{x}(k), u(k)),$$

or

$$\underline{x}(k+1) = \underline{Ad}(x(k), \underline{u}(k)),$$

where Ad is the discretized function.

In a similar manner the performance index, J, may be approximated by the summation from k = 0 to k = N-1 of $\Delta t(\underline{g}(\underline{x}(k), \underline{u}(k)))$ added to Jnn, or

$$J = Jnn + \sum_{k=0}^{N-1} \underline{Gd}(\underline{x}(k), \underline{u}(k)),$$

where N is the number of stages in the process,

Jnn is the cost of arriving at stage n,

$\underline{Gd}$ is the functional representing the cost of moving to the next stage from the present stage.

The method of dynamic programming may now be described in more detail. First, each state and control variable is given quantized values. Then, starting at the final stage, N, the cost, Jnn, of arriving at N is assigned a value for each state value. Moving back one stage, to N-1, the states at the next stage are approximated by Ad. The cost J associated with this predicted state is just Jnn. The total cost C, of moving from N-1 to N for each state value is then known by adding Jnn and J, where J is the value calculated by Gd for all possible control values. The minimum total cost, C*, is then stored for each state value along with the control which made it minimum. Moving back another stage, to N-2, the state values at N-1 are predicted by Ad and the cost associated with these values is recalled from storage. In general, the predicted state values will not fall on the quantized state values and, hence, interpolation will be required to obtain the cost. The total cost, C, is then calculated to move from N-2 to N for each state value and the minimum, C*, is stored. This procedure is followed until all stages are accounted. The end result is the set of optimal control values for each state value and stage.

There are at least two methods of following the optimal path from the first stage to the last. The first method begins by choosing an initial set of state values at stage one. If there are no other restraints, the best set to choose is I*, the one associated with the lowest cost of moving over all N stages. The control and state values associated with I* and the first stage are then used in Ad to predict the state values at the second stage. The controls associated with these new states and stage two are again used in Ad along with the new states to predict the states at stage three. This process is repeated until the final stage is reached. This provides an open-loop type of control. The disadvantage of this method is that interpolation between the state and control values may be required at each stage. This becomes difficult if the number of the state or control variables exceeds two. The second method results in a closed-loop type control. It involves regression of a functional relation for the control values as a function of the state and stage. The disadvantage of this method is the difficulty of finding a function which accurately describes the data.

The flow chart in Fig. 6, shows the computational procedures of the dynamic program. This formed the basis for the FORTRAN program named DYPR listed in Appendix D. For this program, the state and control variables were taken directly from the CSMP model of the test

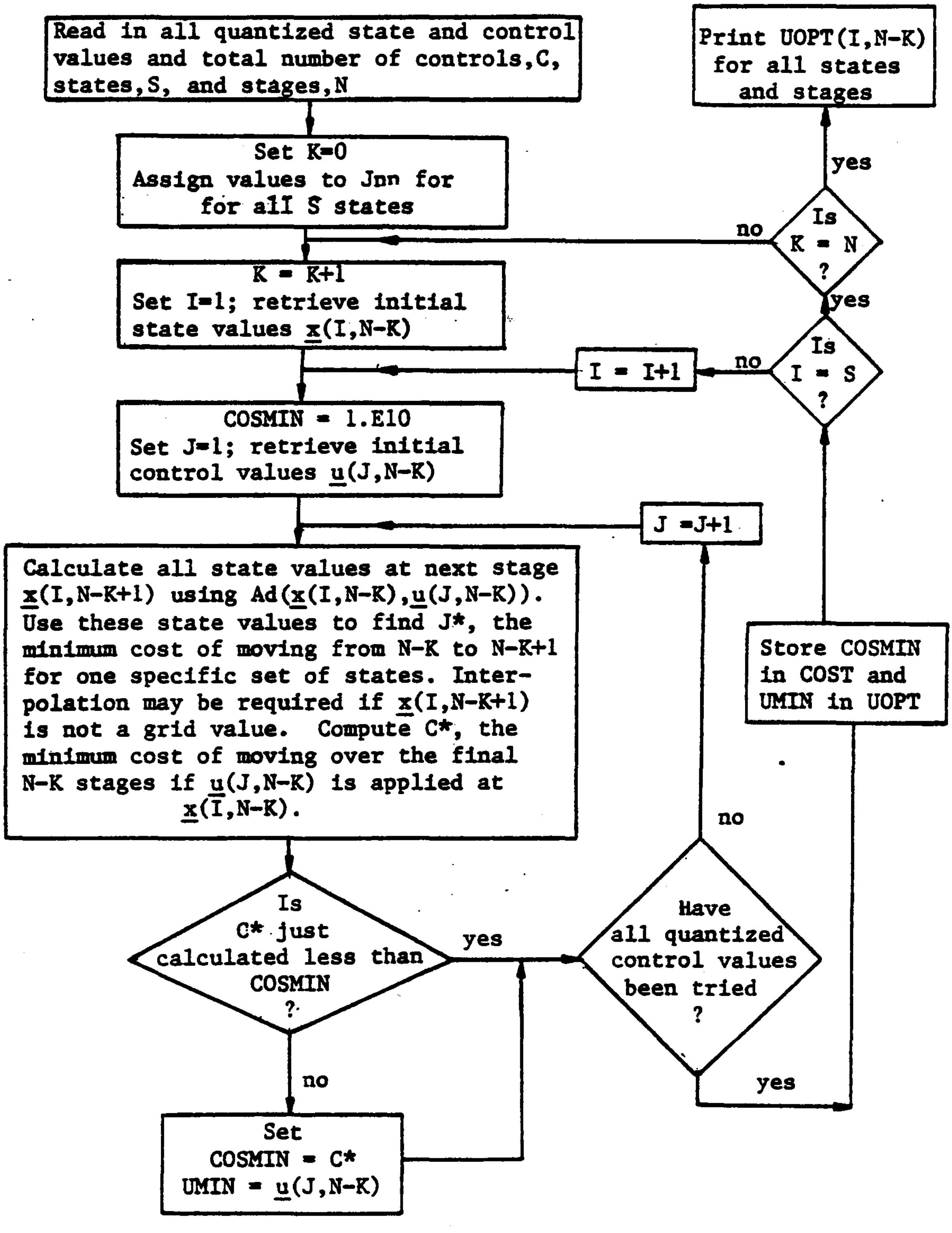


Figure 6. Flow Chart of Dynamic Program

vehicle. The functions Ad and Gd were also available directly from the CSMP model in continuous form in terms of these state and control variables but were discretized as described above. Linear-functional relationships were made for the state and control variables so they could be easily quantized within the program. Limits were then placed on the values of the state variables so that the system could not be operated outside this admissable range of states.

The system was driven by defining the value of the first state variable, x1 for each stage. The state, x1, is linearly related to the vehicle velocity and by changing it for each stage the system could be driven over any desired velocity profile.

In order to execute the program, the values for the total number of control, state, and stage values must be assigned. Originally, it was desired to use 100 quantized values for the second state, 100 for the third, 10 quantized values for the first control variable, seven for the second and 40 for the third. These values represent, respectively, increments of 34.5 kPa (50 psi) in pressure, 3.6 r/s in engine speed, 1.6° in pump and motor swashplate angle, and 2.1° in throttle angle. Even with these coarse increments, to execute the program for 50 stages using one-second time increments would require 1.4 billion loops. It may be remembered that only two of the three system states are being varied within each stage. In addition to this, approximately 80,000 storage locations are required for stage. The obvious disadvantage to dynamic programming then is the vast storage required and the number of calculations performed for a system possessing state and control values which vary over a wide range. The only means of reducing these numbers is to reduce the number of quantized state and control values. Reducing the number of control values decreases the probability of having a control which will project the system to an admissable state in the following stage. Reducing the number of state values requires that the system is capable of large, instantaneous changes in the values of its state. The only certain method of determining the minimum allowable number of quantized values is by determining the sensitivity of the system to changes in each value by trial.

For this study, a program containing a reduced number of quantized values was executed on an IBM 370 Computer. The program had 85 values for the second state, 75 for the third, 10 values for the first control, seven for the second, and 20 for the third. It was to run 30 stages for a total 267.75 million loops, about one-fifth of the program mentioned above. The program executed for nine hours of CPU time and 32.4 million kb-seconds of region time and only completed 27 stages. The first method, as described previously, of obtaining the optimal path through all the stages was employed, but since the program did not complete all 30 stages the initial state associated with the minimum cost of moving over all the stages was not known. There were other difficulties as well. When an optimal control value was applied at the associated state and stage, the predicted state at the following stage could be found, however, two problems arose. The newly predicted state may have no control associated with it which, when applied, would lead to another admissable state. It is believed that this problem can be significantly reduced if not eliminated by increasing the number of control values. The other problem occured when the predicted state values did not coincide with the available state values used in the program. Interpolation involving five variables must be performed to find the control values at the predicted state. Increasing the number of state values would reduce this problem, but in many cases, some interpolation will be required. From general observations, it is felt that the system is most sensitive to throttle angle changes. A four degree change of throttle angle produces an average of 16.27 N-m (144 in.-lb) change in torque output of the engine. It would also seem that more than 100 quantized values of state two should be used, although it is not known how many more would be adequate.

Final Observations

This chapter presented observations made from initial simulations of the model developed in Chapter III, and also the development of an optimal control program. In this section some final observations are made regarding both the simulation and control.

Simulations indicate that the test Pinto equipped with a hydrostatic transmission is capable of exceeding the EPA-estimated fuel economy of a 1978 production Pinto in Highway driving. This is significant because it was generally accepted that a mechanical transmission would be more efficient at high, steady-state speeds. This has been the only argument in favor of hydromechanical transmissions (9). The reason for this improvement lies with the variable displacement motor. For acceleration, the motor displacement should be kept maximum until the desired steady-state speed is reached. Then the motor displacement should be gradually decreased, in turn, decreasing engine speed until the point near minimum bsfc is reached. This results in a low engine speed and hence, low fuel consumption per unit distance. Unlike a mechanical overdrive, though, the hydrostatic transmission may quickly adapt itself to any change in road load or driver input.

The top speed of the vehicle is a function of the limiting speed of the hydraulic motor and the final drive ratio. The maximum acceleration is a function of the pressure drop attainable across the hydraulic units, the final drive ratio, and the maximum displacement per revolution of the motor. Thus, if a motor of larger displacement had been used in the test vehicle, with all other factors equal, top speed would have been the same but the tractive effort would have been greater. Provided there is enough torque available from the prime mover, the tractive effort may be held constant until maximum speed is attained. For the test vehicle, the maximum tractive effort is not as great as that of the production version initially and as a result requires 2.5 seconds longer to accelerate to 9 m/s (20. mph). However, by keeping the pressure high during acceleration, the simulation predicts that the test vehicle would reach 36. m/s (80. mph) in 25. seconds or 9 seconds quicker than the stock Pinto. The performance within the limits specified above is chiefly a function of the controller.

At this time it is still believed that it would be beneficial to know the optimal control for the system. If a functional relation was known for the optimal controls as a function of a state, a state variable feedback type of control might be useful. It is felt that the method of dynamic programming is the most promising of the available optimal control methods for this system. Although it is believed that the program DYPR functions correctly, the code should be reviewed and optimized to reduce the execution time required. In addition, the effects of the number of state and control quantizations should be examined. Once the results are obtained for a velocity profile, the optimal path over all the stages should be plotted. Next a functional relation for the optimal controls should be obtained. This relation could form the basis for the design of the physical controller.

Up to this point the computer programs DYPR and HY have been discussed separately. During the design process, however, it is forseen that these programs will be used in conjunction with one another. The optimum control determined by DYPR may be placed directly into HY to verify the optimum performance predicted by DYPR. In addition, HY yields the instantaneous values for hydraulic pump and motor efficiency as well as bsfc for the engine. The performance thus predicted for the vehicle employing the optimal control may provide yet another basis for designing the physical controller.

In this chapter it has been shown that the hydrostatic transmission offers the designer flexibility in obtaining performance or economy over conventional transmissions. Furthermore, the model developed in Chapter III, has been shown to be a useful tool in analyzing the dynamic performance of the test vehicle and control system.

V. CONCLUSIONS

This study has developed a model using bond graphs and the IBM CSMP language which is capable of simulating the dynamic performance of an automobile equipped with a hydrostatic transmission. The model is expressed in terms of generalized resistance, inertance, and compliance elements which represent measureable, physical effects. These elements were then assigned values to represent a Ford Pinto equipped with the Sundstrand Series 20 hydraulic pump and motor. As presented, the model requires values for the hydraulic pump and motor swashplate angles, and engine throttle angle, before a simulation can be made. Given these input values, the program is capable of outputting instantaneous values of any variable within the model along with engine bsfc and output power, overall pump and motor efficiencies, and fuel economy.

Although, in general, it is felt that the CSMP model provides a realistic simulation, the loss coefficients which describe the transmission may need improvement. The values of these coefficients used in the present model predict performance which agrees well with the performance predicted by Sundstrand and hence, confirmed their use in this study. This performance, however, may not be indicative of actual transmission performance especially in the low-speed, low-pressure, low-displacement regions of operation (18). To be sure, a more complicated model than the one used in this study is required to completely describe the hydrostatic transmission in all ranges of operation; however, such a model is not presently available in the literature.

Simulations using a simple controller indicate the test vehicle is capable of exceeding the EPA highway estimate for a 1978 Pinto with an automatic transmission. In addition, at highway speeds the test vehicle engine is turning at a lower speed than the production vehicle which should result in extended engine life. The engine in the test vehicle operates at a higher torque loading than the production version. This should not only lessen spark plug fouling but also reduce octane depreciation due to deposit build-up in the combustion chamber. During this simulation, the output torque of the engine was assumed constant and the engine speed only ranged from 82. r/s to 222. r/s. This suggests the type of operating condition to which a compression-ignition engine is suited.

Although this study was concerned with a vehicle in a low weight class, no reasons were found that would inhibit the results obtained here from being applied to a larger automobile.

Several conclusions may be drawn regarding the overall drivability of the test vehicle. The top speed of the vehicle is limited by the axle ratio and the maximum allowable speed of the hydraulic motor. The maximum acceleration is limited by the axle ratio and the pressure drop across the motor along with the maximum displacement of the motor. By keeping the pressure constant and the motor displacement fixed, a constant tractive effort may be maintained until maximum speed is reached. By altering the controller in the simulation to maintain maximum pressure, the test vehicle required 2.5 more seconds to reach 9. m/s

(20. mph) than the stock Pinto but reached 36. m/s (80. mph) 9 seconds sooner than the production car.

The results of the optimal control program were less than ideal. This is believed to be due to an insufficient number of quantized values used for the state and control variables. It may also be due to the tolerance placed on the predicted state values rendering too many predicted states inadmissable. Unfortunately, correcting these problems increases the required computation time of an already-lenghty program. It is still maintained that the program DYPR is capable of producing the optimal controls. A means of reducing execution time is required in order to incorporate a proper number of quantized state and controls values.

In final summary, the performance of a vehicle equipped with a hydrostatic transmission is a function of the hydraulic components and the design of the controller. Using a hydraulic pump and motor which both have variable displacements, more flexibility of design is acheived over a conventional transmission. The dynamic performance of vehicles incorporating hydrostatic transmission designs may now be simulated using the digital computer.

VI. RECOMMENDATIONS

There are a number of recommendations concerning observations which arose during the course of this study. These recommendations indicate possible improvements for the model and suggest further study in the directions surveyed in this thesis. Listed below are the observations and the associated recommendations.

1. A verification of the transmission model should be made. Hydraulic components similar to the Sundstrand Series 20 pump and motor should be obtained for experimental evaluation of the loss coefficients used in the model. Merritt (13) presents a procedure for determining the coefficients experimentally.

2. The execution time for the optimal control program DYPR is too high.
   A search for methods which would optimize the FORTRAN code and reduce the execution time should be made. In addition, the system sensitivity to the number of quantized values for the control and state variables should be examined to determine the least number of quantized values which still yields accurate results.

3. Some form of closed-loop control should be designed for the test vehicle.
   The optimal control values for several different velocity pro-

files should be obtained. The relation giving the optimal control as some function of the states should be found and would provide the foundation for the closed-loop control.

4. The simulations indicated that a compression-ignition engine may be a feasable prime mover.
Obtain a performance survey of a small compression-ignition engine (Volkswagen, Nissan, ...) and incorporate into the test vehicle.

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APPENDIX A

Statistical Analysis System Regression Program for T_1 and BSFC.

```
*****************************************
*****************************************
*  THIS PROGRAM USES THE SAS PACKAGE    *
*****************************************
*                                       *
*   THIS PROGRAM REGRESSESS A SECOND    *
*   DEGREE POLYNOMIAL OF TWO VARIABLES  *
*   TO THE DATA LISTED BELOW.   THE     *
*   INDEPENDENT VARIABLES ARE ENGINE    *
*   SPEED IN RAD/S AND THROTTLE ANGLE   *
*   IN DEGREES.  THE DEPENDENT VAR-     *
*   IABLES ARE OUTPUT TORQUE OF THE     *
*   ENGINE AND BSFC.                    *
*                                       *
*****************************************

DATA ONE;
INPUT WE T1 BSFC U3;
CARDS;
76.4 780. .525 14.
95.5 945.6 .457 18.
95.5 1228.8 .546 83.
114.6 788.4 .472 17.5
114.6 1052.4 .466 23.
114.6 1310.4 .474 27.
114.6 1335.6 .526 83.
143.3 630. .486 18.
143.3 838.8 .46 22.
143.3 1052.4 .43 27.
143.3 1260. .426 33.5
143.3 1386. .501 83.
191. 630. .479 23.
191. 738.4 .448 27.
191. 866.8 .435 30.
191. 1102.8 .429 35.
191. 1260. .43 41.5
191. 1468.8 .467 83.
238.8 630. .482 27.5
238.8 756. .458 30.
238.8 882. .442 32.5
238.8 1008. .433 37.
238.8 1260. .431 50.5
238.8 1411.2 .439 83.
286.5 736.8 .476 34.
286.5 838.6 .458 36.
286.5 1052.4 .444 44.
286.5 1260. .437 55.
286.5 1468.8 .465 83.
334.3 630. .51 35.
334.3 718.8 .483 37.
```

```
334.3 901.2 .465 42.5
334.3 1078.8 .435 54.
334.3 1260. .47 67.5
334.3 1468.8 .468 83.
PROC PRINT;
PROC GLM;
MODEL BSFC T1=WE U3 WE*WE U3*U3 U3*WE
/P CLM;
```

```
OUTPUT:

DEPENDENT VARIABLE: BSFC

R-SQUARE    0.833328

PARAMETER        ESTIMATE

INTERCEPT       0.59312702
WE             -0.00062546
U3             -0.00422528
WE*WE           0.00000284
U3*U3           0.00006063
WE*U3          -0.00001078



DEPENDENT VARIABLE: T1

R-SQUARE    0.952394

PARAMETER       ESTIMATE

INTERCEPT       643.30051367
WE               -5.47141507
U3               45.56418902
WE*WE            -0.00179691
U3*U3            -0.46462004
WE*U3             0.08040656
```

APPENDIX B

Regression Program for Transmission Coefficients.

```
C
C   THIS PROGRAM USES A LEAST-SQUARES LINEAR
C   REGRESSION TO DETERMINE THE LOSS COEF-
C   FICIENTS AS DEFINED BY MERRITT FOR A
C   HYDROSTATIC PUMP OR MOTOR.  THE INPUT
C   DATA FOR THIS PROGRAM IS THE SUNDSTRAND
C   SERIES 20 SIMULATION OUTPUT.
C
  100 FORMAT(3F8.0,28X,2F8.0)
      N=2000
      U=5.0E-06
      ASUM=.0E0
      BSUM=.0E0
      CSUM=.0E0
      DSUM=.0E0
      ESUM=.0E0
      FSUM=.0E0
      GSUM=.0E0
      DO 20 I=1,N
  600 READ(4,100,END=30)A,B,Q,C,D
      READ(4,100,END=30)
      READ(4,100,END=30)
      READ(4,100,END=30)
      READ(4,100,END=30)
      READ(4,100,END=30)
      READ(4,100,END=30)
      IF(Q.LT.7.8) GO TO 600
      IF(ABS(Q-8.).GT..001) GO TO 500
      IF(C.LT.1.E-7)WRITE(6,400)I,A,B,C,D
      IF(A.LT.1.E-7)WRITE(6,400)I,A,B,C,D
      B=B*3.14159/30.
      C=C/100.
      D=D/100.
      X=B/A
      Y=A/B
      Z=1/C
      BSUM=BSUM+Y
      CSUM=CSUM+Y/C
      ASUM=ASUM+X
      DSUM=DSUM+D
      ESUM=ESUM+X**2
      FSUM=FSUM+X*D
      GSUM=GSUM+Y**2
   20 CONTINUE
   30 CONTINUE
      W=I-1
      CDU=((1/W)*ASUM*DSUM-FSUM)/(ESUM-(1/W)*ASUM*ASUM)
      CF=(W-DSUM-CDU*ASUM)/W
C     CSU IS THE NAME GIVEN TO   CS/U
      CSU=(CSUM-BSUM)/GSUM
```

```
      WRITE(6,300)CDU,CF,CSU
  300 FORMAT('1','CDU=',F18.10,'     CF=',F18.10,'     CSU=',F
  400 FORMAT('0',I5,4(2X,E13.7))
  500 WRITE(6,200)Q,C,D
  200 FORMAT(3F5.1)
      STOP
      END
```

APPENDIX C

CSMP Model of Test Vehicle - HY.

```
**************************************************************
**************************************************************
*                                                            *
*                                                            *
*     THIS PROGRAM SIMULATES THE PERFORMANCE                 *
*     OF AN AUTOMOBILE WITH A HYDROSTATIC                    *
*     TRANSMISSION.  THE REQUIRED INPUT IS                   *
*     THE CONTROL FUNCTION.  THIS FUNCTION IS                *
*     A SINGLE NUMBER,J, WHICH REPRESENTS THE                *
*     THREE CONTROL VARIABLES.  THIS IS INPUT                *
*     AS THE FUNCTION OLCNTL WITH THE DESIRED                *
*     VALUE OF CONTROL FOR EACH TIME.                        *
*                                                            *
*                                                            *
**************************************************************

INITIAL
STORAGE U1(2400),U2(2400),U3(2400),UU1(10),UU2(7),UU3(33)
FUNCTION OLCNTL=(1.,25.),(2.,14.),(3.,13.)
FIXED IC1,IC2,IC3,I,J,K,L,M,N
INCON IQ120 = 1.0,IT20 = 80.00,IF230=7.0


CONSTANT                IC1=10
CONSTANT                IC2=7
CONSTANT                IC3=20
CONSTANT                C11=10.
CONSTANT                C22=7.
CONSTANT                C33=20.
CONSTANT                J1 = 0.95
CONSTANT                 A = 1.0
CONSTANT                J2 = .0384
CONSTANT                J3 = .058
CONSTANT                C1 = 0.0002
CONSTANT                 E = 3.45
CONSTANT                 D = 12.
CONSTANT                J4 = 14.8
CONSTANT                TP = 26.
CONSTANT                I1 = 8.29
CONSTANT                WT = 2700.
CONSTANT                 S = 2500.
CONSTANT                CD = .59
CONSTANT                MC = 7.00
CONSTANT                MU = 2.78E-6
CONSTANT               DMP = 2.03
CONSTANT               DMM = 2.03
CONSTANT               CDP = 1.6E05
CONSTANT               CDM = 1.7E05
CONSTANT               CSP = 2.5E-8
CONSTANT               CSM = 3.8E-9
```

```
CONSTANT                CFP = 0.08
CONSTANT                CFM = 0.07

                         R2 = CDP*MU*DMP/6.283
                        R3P = 6.283*MU/DMP/CSP
                        R3M = 6.283*MU/DMM/CSM
                         R3 = 1/(1/R3P+1/R3M)
                         R4 = CDM*MU*DMM/6.283
                         R8 = CFP*DMP/6.283
                         R9 = CFM*DMM/6.283

                         R7 = 5.4E-8*CD*S

*
* THIS NOSORT SECTION CALCULATES ALL
* POSSIBLE QUANTIZED CONTROL VALUES
*
NOSORT

      DO 40 N=1,IC1
      UU1(N)=(N*2./C11+0.03)/6.283185
   40 CONTINUE
      DO 50 N=1,IC2
      UU2(N)=(N*1.47/C22+0.56)/6.283185
   50 CONTINUE
      DO 60 N=1,IC3
      UU3(N)=N*83./C33
   60 CONTINUE
      J=1
      DO 70 I=1,IC1
      DO 70 L=1,IC2
      DO 70 M=1,IC3
      U1(J)=UU1(I)
      U2(J)=UU2(L)
      U3(J)=UU3(M)
      J=J+1
   70 CONTINUE

*
*
* THE DYNAMIC SECTION CALCULATES
* THE VARIABLE VALUES FOR EACH
* TIME ITERATION
*
*
```

```
DYNAMIC

PROCEDURE    AJ,AK,U1U,U2U,U3U=DUMMY(TIME)

        K=TIME+1
        AK=K
        J=AFGEN(OLCNTL,AK)
        AJ=J
        U1U=U1(J)
        U2U=U2(J)
        U3U=U3(J)

ENDPROCEDURE
             LAMB=U1U
              PSI=U2U
               X3=IT2
               X2=IQI2
               T1=643.3-5.47/J1*X3+45.6*U3U-1.8E-3/J1/J1*X3*X3.
               -.465*U3U*U3U+.08*X3*U3U/J1

                  R5 = 23.6/WP
                  R6 = (.00075*WT+12.0*WT*KT)/WW
                 MPH = V*0.0568
                  KT = .005 + .15/TP + .000035*MPH*MPH/TP
                  WE = IT2/J1
                  T6 = J2*WPDOT
                  T7 = R2*WP
                  T8 = R5*WP
                 T25 = R8*P
                 Q11 = P/R3
                   P = IQ12/C1
                 T26 = R9*P
                 T15 = J3*WMDOT
                 T16 = R4*WM
                 T19 = J4*WWDOT
                 T20 = R6*WW
                 F24 = R7*V*V
                   V = IF23/MC
                  WP = A*WE
                  T4 = A*T5
                 Q10 = LAMB*WP
                  T9 = LAMB*P
                 T14 = P*PSI
                 Q13 = PSI*WM
                 T18 = T17*E
                  WM = WW*E
                 F22 = T21/D
```

```
 WW = V/D
 T2 = (T1-A*(T7+T8+T9+T25))/(1.+A*A*J2/J1)

 T5 = T6+T7+T8+T9+T25

Q12 = Q10-Q13-Q11

T17 = T14-T15-T16-T26

T21 = T18-T19-T20

  F23 = ((T14-T16-T26)*E/D-T20/D-F24)...
  /(1.+J4/MC/D/D+J3*E*E/MC/D/D)

WEDOT = T2/J1
 VDOT = F23/MC
WPDOT = A*WEDOT
WWDOT = VDOT/D
WMDOT = WWDOT*E

IT2 = INTGRL( IT20,T2)
IQ12= INTGRL( IQ120, Q12)

IF23 = INTGRL( IF230, F23)


PUMPEF = (1.-1.079E-3*P/WP)/(1.+0.556*WP/P+0.03)
MOTEFF = (1.-0.556*WM/P-0.03)/(1.+1.079E-3*P/WM)
  PEFF = P*Q10/(T5*WP)
  MEFF = T17*WM/(P*Q13)

  HP = T1*WE/6600.
BSFC=.593-6.3E-4/J1*X3-4.2E-3*U3U+2.8E-6/J1/J1...
*X3*X3+6.1E-5*U3U*U3U-1.1E-5*X3*U3U/J1

MPG=MPH/HP/BSFC*6.02

  NOSORT
   CALL DEBUG(2,4.0)
TERMINAL
TIMER          FINTIM = 3.,DELT=.1,TIME=.1
METHOD     RKSFX

  PRINT P,V,HP,T1,PEFF,MEFF,BSFC,MPG
*                OUTPUT  WE
*                  LABEL    ENGINE SPEED
*                OUTPUT  WM,WP,MOTEFF,PUMPEF
*                 LABEL    PUMP AND MOTOR EFFICIENCIES VS SPE
*                OUTPUT  V,T19
*                  LABEL    VEHICLE SPEED,TORQUE TO WHEELS
```

APPENDIX D

Optimal Control Program - DYPR.

```
C     THIS PROGRAM CALCULATES THE OPTIMAL CONTROLS
C     WHICH WHEN APPLIED AT THE ASSOCIATED STATE AND
C     STAGE, MINIMIZE THE PERFORMANCE INDEX.  THIS
C     INDEX REPRESENTS THE AMOUNT OF FUEL CONSUMED
C     BY THE ENGINE IN RELATION TO THE POWER OUTPUT
C     OF THE ENGINE.
C
      DIMENSION X1(8000),X2(8000),X3(8000),U1(2800),U2(2800)
     1U3(2800),COST(8000),GJSTAR(8000),XX2(110),XX3(110),
     2UU1(11),UU2(10),UU3(45),GJSTAT(8000)
      INTEGER*2 UOPT(8000)
      NSTAGE=30
      S2=85.
      S3=75.
      C1=10.
      C2=7.
      C3=20.
      GJ1=.95
      A=1.
      GJ2=.0384
      GJ3=.058
      CC1=.0002
      CR=3.45
      D=12.
      GJ4=14.8
      TP=26.
      WT=2700.
      SS=2500.
      CD=.595
      EMC=7.00
      EMU=2.78E-6
      DMP=2.03
      DMM=2.03
      CDP=1.24E05
      CDM=2.11E05
      CSP=7.64E-9
      CSM=3.07E-9
      CFP=.0477
      CFM=.0299
      R2=CDP*EMU*DMP/6.283
      R3P=6.283*EMU/DMP/CSP
      R3M=6.283*EMU/DMM/CSM
      R3=R3P*R3M/(R3P+R3M)
      R4=CDM*EMU*DMM/6.283
      R8=CFP*DMP/6.283
      R9=CFM*DMM/6.283
      R7=1.81E-7*CD*SS
      X2MAX=1.1111
      X2MIN=0.1001
      X3MAX=368.0
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      X3MIN=91.
      DELTE=1.
      DELX2=1./S2
      DELX3=277./S3
      S=S2*S3
      IS2=INT(S2)
      IS3=INT(S3)
      CT=C1*C2*C3
      IC1=10
      IC2=7
      IC3=20
      IS=IS2*IS3
      IC=IC1*IC2*IC3
C
C     THIS PORTION CALCULATES ALL POSSIBLE STATE VALUES
C
      DO 12 M=1,IS2
      XX2(M)=1.2211-(M/S2+.1)
   12 CONTINUE
      DO 20 N=1,IS3
      XX3(N)=N*277./S3+89.
   20 CONTINUE
      DO 30 I=1,IS
      IM1=I-1
      AIM1=IM1
      KK=INT(AIM1/S3)+1
      X2(I)=XX2(KK)
      X3(I)=XX3(INT((IM1/S3-INT(IM1/S3))*S3+1.5))
   30 CONTINUE
C
C     THIS PORTION CALCULATES ALL POSSIBLE CONTROL VALUES
C
      DO 40 N=1,IC1
      UU1(N)=(N*2./C1+0.03)/6.283185
   40 CONTINUE
      DO 50 N=1,IC2
      UU2(N)=(N*1.47/C2+0.56)/6.283185
   50 CONTINUE
      DO 60 N=1,IC3
      UU3(N)=N*83./C3
   60 CONTINUE
      J=1
      DO 70 I=1,IC1
      DO 70 L=1,IC2
      DO 70 M=1,IC3
      U1(J)=UU1(I)
      U2(J)=UU2(L)
      U3(J)=UU3(M)
      J=J+1
   70 CONTINUE
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C  22 FORMAT(5X,3F10.5,I5)
C
C     THIS PORTION SETS GJSTAR TO ZERO
C
      DO 80 I=1,IS
      GJSTAR(I)=0.0
   80 CONTINUE
C
C     THIS PORTION SETS UP THE ITERATIONS FOR THE TIME,
C     STATE AND CONTROL VALUES
C
      ACOSMN=1.E5
      NM1=NSTAGE-1
      DO 1000 K=1,NSTAGE
      NMK=NSTAGE-K
      X1XN=(60.*(NMK+1)**.71+.1)*EMC
      X1MAX=1.05*X1XN
      X1MIN=.95*X1XN
      DO 100 I=1,IS
      JUMIN=0
      X1(I)=(60.*NMK**.71+.1)*EMC
      COSMIN=1.E10
      DO 10 J=1,IC
C
C     THIS PORTION CALCULATES CSTAR, THE MIN COST OVER THE
C     FINAL N-K STAGES AND STORES  MINIMUM VALUE IN COSMIN
C
      R5=23.6*GJ1/X3(I)/A
      VV=X1(I)/EMC*.0568
      AT=.005+.15/TP+.000035*VV*VV/TP
      R6=(.00075*WT+.0825*WT*AT)*EMC*D/X1(I)
      CP=1./(1.+GJ4/EMC/D/D+GJ3*CR*CR/EMC/D/D)
      X1X=X1(I)+DELTE*(((U2(J)*X2(I)/CC1-R4*CR/EMC/D*X1(I)-R
     1*CR/D-R6/EMC/D/D*X1(I)-R7/EMC/EMC*X1(I)*X1(I))*CP)
      IF(X1X.LT.X1MIN.OR.X1X.GT.X1MAX) GO TO 10
      X2X=X2(I)+DELTE*(-CR/D/EMC*U2(J)*X1(I)-X2(I)/R3/CC1+A/
     1(I))
      IF(X2X.GT.X2MAX) GO TO 10
      IF(X2X.LT.X2MIN) GO TO 10
      M=INT((X2X-.1)*S2)
      II=IS3*M-(IS3-1)
      CPP=1./(1.+A*A*GJ2/GJ1)
      T1=643.3-5.47/GJ1*X3(I)+45.6*U3(J)-1.8E-3/GJ1/GJ1*X3(I
     1.465*U3(J)*U3(J)+.08/GJ1*X3(I)*U3(J)
      X3X=X3(I)+DELTE*(-CPP*A/CC1*(U1(J)+R8)*X2(I)-CPP*A*A/G
     1X3(I)+CPP*T1)
      IF(X3X.GT.X3MAX) GO TO 10
      IF(X3X.LT.X3MIN) GO TO 10
      NINT=INT((X3X-89.)*S3/277.)
      KEYI=II+(NINT-1)
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      X3STUF=(X3X-X3(KEYI))/DELX3
      XJ1=(GJSTAR(KEYI+1)-GJSTAR(KEYI))*X3STUF+GJSTAR(KEYI)
      XJ2=(GJSTAR(KEYI+IS3+1)-GJSTAR(KEYI+IS3))*X3STUF+GJSTA
      GJINT=(XJ2-XJ1)*(X2X-X2(KEYI))/DELX2+XJ1
      WM=CR*X1(I)/EMC/D
      WE=X3(I)/GJ1
      BSFC=.593-6.3E-4/GJ1*X3(I)-4.2E-3*U3(J)+2.8E-6/GJ1/GJ1
     1+6.1E-5*U3(J)*U3(J)-1.1E-5/GJ1*X3(I)*U3(J)
      GD=DELTE*(WE*T1*BSFC)
      CSTAR=GD+GJINT
   24 FORMAT(5X,3E15.5)
      IF(CSTAR.GE.COSMIN) GO TO 10
      COSMIN=CSTAR
      JUMIN=J
   10 CONTINUE
   23 FORMAT(5X,I5,2E15.5)
      IF(K.LT.NSTAGE) GO TO 5
      ACOST=COSMIN
      IF(ACOST.GE.ACOSMN) GO TO 5
      ACOSMN=ACOST
      INCON=I
    5 UOPT(I)=JUMIN
      GJSTAT(I)=COSMIN
  100 CONTINUE
      WRITE(9) UOPT
      DO 150 I=1,IS
      GJSTAR(I)=GJSTAT(I)
  150 CONTINUE
 1000 CONTINUE
      BACKSPACE 9
      DO 9999 L=1,NSTAGE
      READ(9) UOPT
      IF(L.GE.NSTAGE) GO TO 9998
      BACKSPACE 9
      BACKSPACE 9
 9998 WRITE(6,200) L
  200 FORMAT(2X,I5)
      WRITE(6,201) UOPT
  201 FORMAT(4X,20I5)
      WRITE(6,202)
  202 FORMAT(1X/)
 9999 CONTINUE
      STOP
      END
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FEASABILITY STUDY OF HYDROSTATIC TRANSMISSION

FOR AUTOMOBILE APPLICATIONS

by

Mark Alan Nickerson

(ABSTRACT)

The feasability of obtaining improved fuel efficiency by incorporating a hydrostatic transmission was investigated. The proposed transmission was composed of positive-displacement axial-piston hydraulic devices. The displacement of the pump and motor units could be varied to allow the prime mover to operate within its most efficient range.

A dynamic model representing a Ford Pinto was developed using bond graphs and the IBM CSMP language to simulate the performance of the proposed design. The required input to this program consists of three independent, or control variables. These are the pump and motor swashplate angles and the engine throttle angle. In present form, the output of this program includes instantaneous values of vehicle velocity, pump and motor efficiencies, engine bsfc, and fuel economy.

Observations from the simulations indicate that the test vehicle is capable of exceeding the EPA highway fuel economy estimates for the 1978 production Pinto by 11%. This increase in economy is a result of operating the engine near the minimum bsfc. Although comparisons were not made in this study, it is believed that even higher increases in economy may be realized in urban-type periods of operation.