CHAPTER VIII. Results

The simulation of microwave heating of a ceramic rod that travels vertically through a microwave applicator is presented in this section. The electric field strength profile is assumed to be constant along the rod length but varies with time such that the power the rod absorbs is held constant. The program accomplishes this by calculating the absorbed power at every time step. If the difference between the calculated power and the required power is greater than a small specified value (0.05 is used for the plots in this thesis), then a new value of the electric field is calculated by

$$E_{new} = E_{old} \sqrt{\frac{\text{Power}_{\text{required}}}{\text{Power}_{\text{calculated}}}} . \tag{4.1}$$

The newly calculated electric field strength is used to recalculate the volumetric heat source at the same time step. The resulting power is the required power within the specified value of 0.05. As discussed in the previous section, a steady field prohibits temperatures higher than 800^{o}C without encountering thermal runaway. By forcing the absorbed power in the model to be constant, higher steady temperatures are obtained. The absorbed power is determined from

$$P_{abs} = \int \dot{q}\left[T(x,r,t)\right] dV \cong \sum_{i=1}^{M}\sum_{j=1}^{N} \dot{q}\left[T(x,r,t)\right] \Delta V \tag{4.2}$$

where $\dot{q}$ is the volumetric heating source from microwave energy, V is the volume of the section of the rod in the microwave cavity, M is the number of finite increments along the length, and N is the number of increments along the radius. Simulations were performed with thirty four millimeters as the cavity length for the purpose of comparison to experimental results from a thirty four millimeter long cavity. The results are shown for both 2.0 mm and 4.67 mm diameter mullite rods.

1. Temperature Distribution in Stationary Rods

1.1 Two Millimeter Diameter Mullite Rod

Figure 4.1 shows the temperature distribution in a 2.0 mm stationary mullite rod subjected to a uniform field at steady state for various absorbed power levels. The figure shows only the section of the rod that is heated. It is noted that portions of the rod protrude out of both ends of the cavity and are included in the domain of the solution. These portions do not appear in the plot, however. The rod remains motionless throughout the heating and absorbs power at a constant rate until steady state is reached. The temperatures shown represent steady state surface temperatures along the length of the rod.

As expected, the maximum temperature of the rod occurs halfway through the cavity. Axial conduction along the length lowers the temperature near the cavity entrance and exit, and the symmetry is due to a uniform field and a zero velocity. Higher temperatures are achieved by increasing the power. As temperatures increase the profile becomes more uniform due to a dominance in radiation compared to axial conduction.

1.2 Mullite Rods of 4.67 mm Diameter

The same conditions of the two millimeter diameter rod are applied to the 4.67 mm diameter mullite rods. Figure 4.2 shows the surface temperature distribution for a 4.67mm stationary mullite rod at steady state for various absorbed powers.

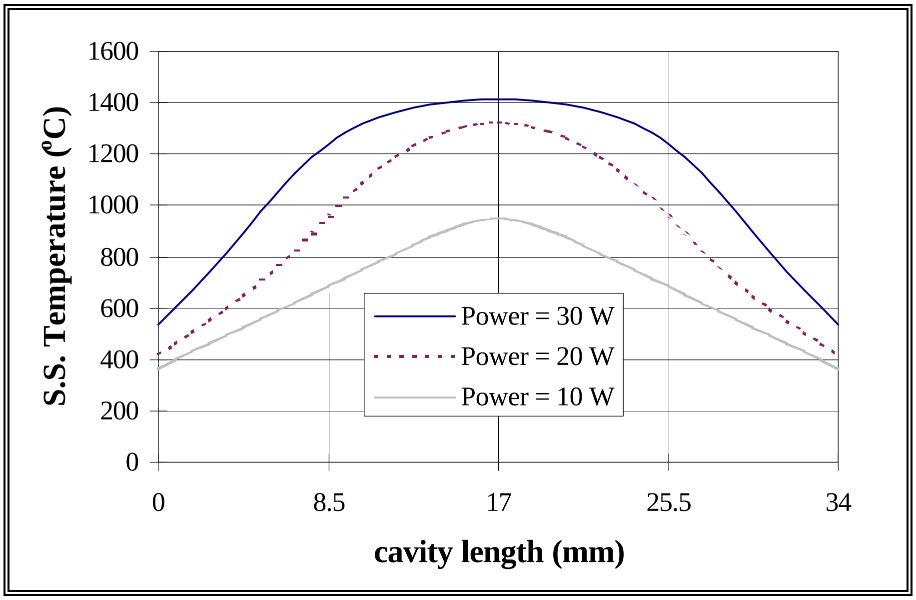

Figure 4.1 Steady State Surface Temperature Distribution along a 2 mm Diameter Mullite Rod at Different Absorbed Powers with Respect to the Length of the Rod in the Cavity

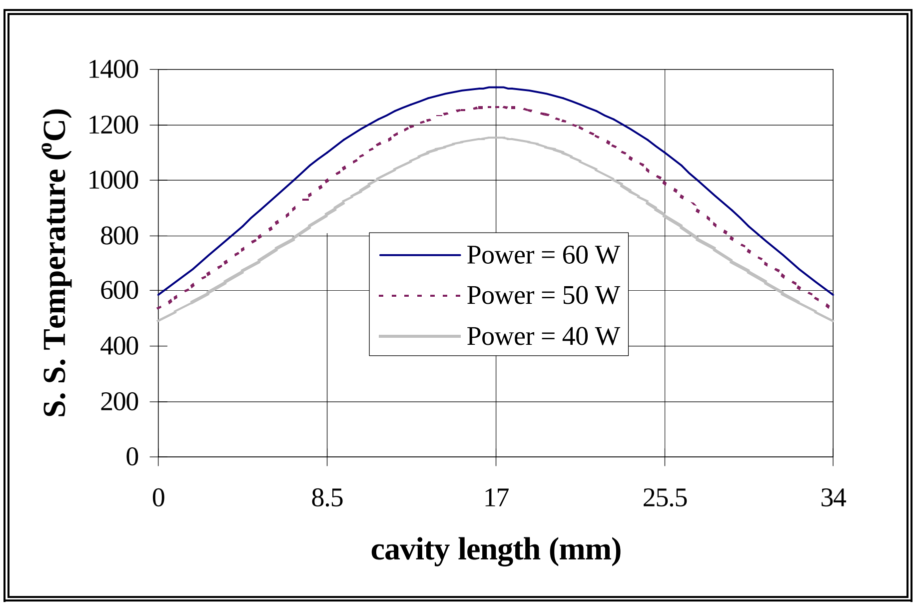

Figure 4.2 Steady State Surface Temperature Distribution along a 4.67 mm Diameter Mullite Rod at Different Absorbed Powers with Respect to the Length of the Rod in the Cavity

The temperature distribution is qualitatively the same as for the 2 mm case. However, more power is required to raise the temperature to match temperatures for the 2mm case. An absorbed power of 50 W is required to raise the temperature of a 4.67mm rod to about 1300^{o}C compared to 20 W for 2mm rods.

For both cases (2mm and 4.67mm), a localized maximum temperature occurs at the center of the cavity. Noting that the electromagnetic fields are modeled as uniform fields, the hotter regions are present as a result of axial conduction along the rod and heat loss by convection and radiation. This suggests that localized hot spots seen in experiments may not be entirely due to nonuniformities in the field as originally thought.

1.3 Radial Temperature Distribution

A nonuniform temperature distribution also exists radially. The radial temperature distribution is shown in Fig. 4.3 for a 2mm rod with a constant 20 W absorbed power and a 4.67mm mullite rod with a 50 W absorbed power after steady state is reached. The steady state maximum surface temperature was similar for both cases, hence, the temperature difference ($T(r)-T_{surf}$) at the maximum is plotted in Fig. 4.3 for comparison. The thicker of the two rods experiences a temperature difference twice as large. The calculated steady maximum temperature of a 2mm mullite rod heated constantly with an absorbed power of 20 W is 1323^{o}C at the surface and the center of the rod is 24^{o}C hotter. A 4.67mm rod reaches a surface temperature of 1269^{o}C at an absorbed power of 50 W and the center of the rod is 50^{o}C hotter. The model assumes uniform volumetric heating; however, the temperature gradient is caused by heat loss at the surface by radiation and convection. The thicker rod has a greater temperature drop which can be explained by greater thermal resistance from the rod center to the surface due to a greater radial area the heat has to travel through.

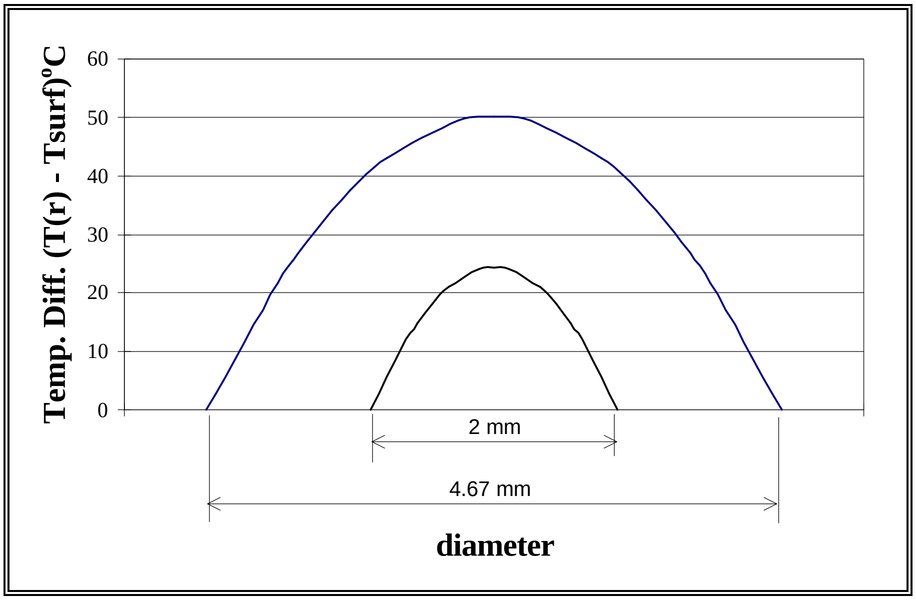

Figure 4.3 Radial Temperature Distribution along a 2mm and 4.67mm Rod at Center

1.4 1D Model vs 2D Model

A two dimensional analysis was developed to consider both axial and radial conduction. For thin rods, a one dimensional model may be sufficient, but large temperature gradients are present for thicker rods. A "quick" method for determining whether a rod is thin or thick is by examining the Biot number:

$$Bi = \frac{h_{r+c}\ r}{2\ k}, \tag{4.3}$$

where h_{r+c} is the sum of the radiation and convection heat transfer coefficient, r is the radius, and k is the thermal conductivity. Since the heat transfer coefficient and the thermal conductivity depend on temperature, the Biot number also is temperature dependent. It is general practice to assume that the one dimensional assumption is valid when the Biot number is less than 0.1. Table 4.1 lists some Biot numbers at selected temperatures for a 2mm and 4.67mm mullite rod.

Table 4.1 Biot Numbers for 2mm and 4.67mm Mullite Rod

Temp. (^{o}C)	Bi (2mm)	Bi (4.67mm)
1000	0.026	0.061
1300	0.036	0.083
1400	0.042	0.099
1600	0.058	0.136

As the mullite rod becomes hotter, the Biot number increases signifying that the one dimensional model becomes less valid. The 4.67mm rod has a higher Biot number than the thinner rod as expected, and it is prudent to use a two dimensional model for this case since the Biot number becomes larger than 0.1 for temperatures above 1400^{o}C.

A second method in determining whether the one dimensional analysis is valid is to solve the governing equations using both models and compare results. Figure 4.4 shows the steady state temperature profile for a microwave heated mullite rod for: the center temperature using the 2-D model, the surface temperature using the 2-D model, and the temperature using the 1-D model. The rod has a diameter of 4.67mm and the absorbed power is assumed to be constant at 55 W. Interestingly, the surface temperature of the 2-D model corresponds to the temperature of the 1-D model. The center temperature however exceeds the surface temperature by 54^{o}C. In cases where the surface temperature matches the 1-D model temperature, plots in this thesis were generated using the 1-D model analysis in order to save computer time.

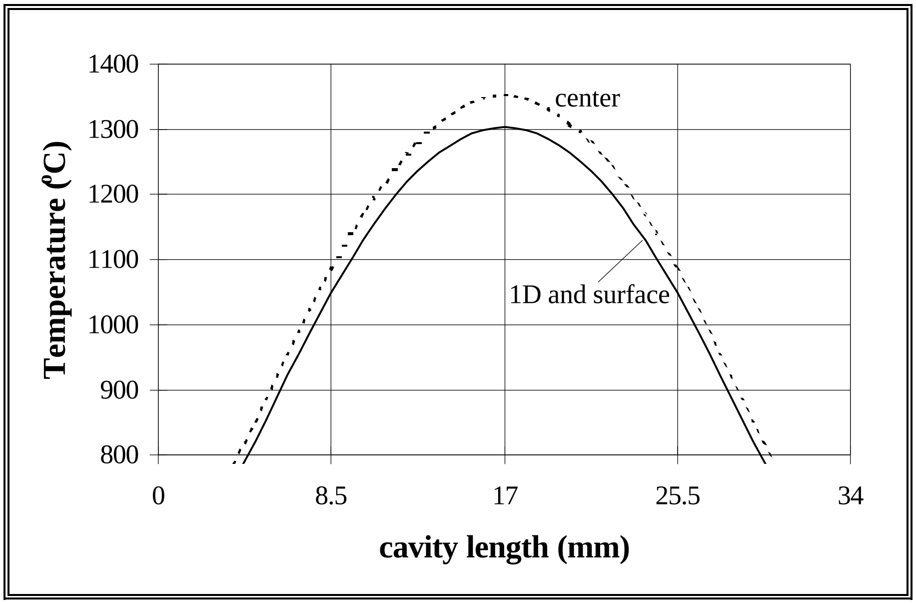

Figure 4.4 Temperature Distribution Comparison of 1-D and 2-D models using a 4.67mm Diameter Mullite Rod

2. Temperature Distribution in Moving Rods

2.1 Two Millimeter Diameter Mullite Rod

Only the midsection of a rod reaches elevated sintering temperatures for the stationary rod case. Continuous processing is desired so that the entire rod at some time reaches high temperatures. Figure 4.5 shows the temperature distribution along a 2mm diameter mullite rod as it passes through the applicator at a constant velocity. The temperatures represent the surface temperatures along the length of the portion of the rod that is being heated. Distributions for several different velocities are shown to compare the effect velocity has on heating. The velocity is slow enough that forced convection is negligible, thus convection heat loss is only due to natural convection from a vertical cylinder.

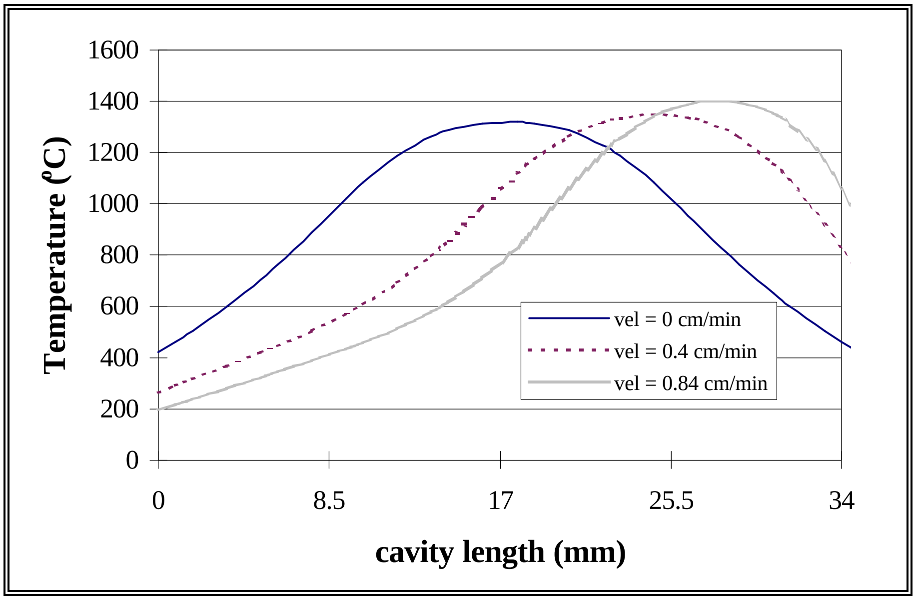

Figure 4.5 Steady State Temperature Distribution along a 2mm Diameter Rod for Various Velocities and an Absorbed Power of 20W

Compared to the stationary rod, the maximum temperature for a moving rod is located farther down the rod in the direction of motion because the movement carries with it the energy that was absorbed and moves the maximum temperature away from the center. Furthermore, for an increase in velocity, the maximum temperature increases. It is probable that this effect is caused by forcing the absorbed power to be the same for each case.

2.2 Mullite Rods of 4.67 mm Diameter

Figure 4.6 shows the effect that velocity has on steady state temperature for a 4.67mm mullite rod.

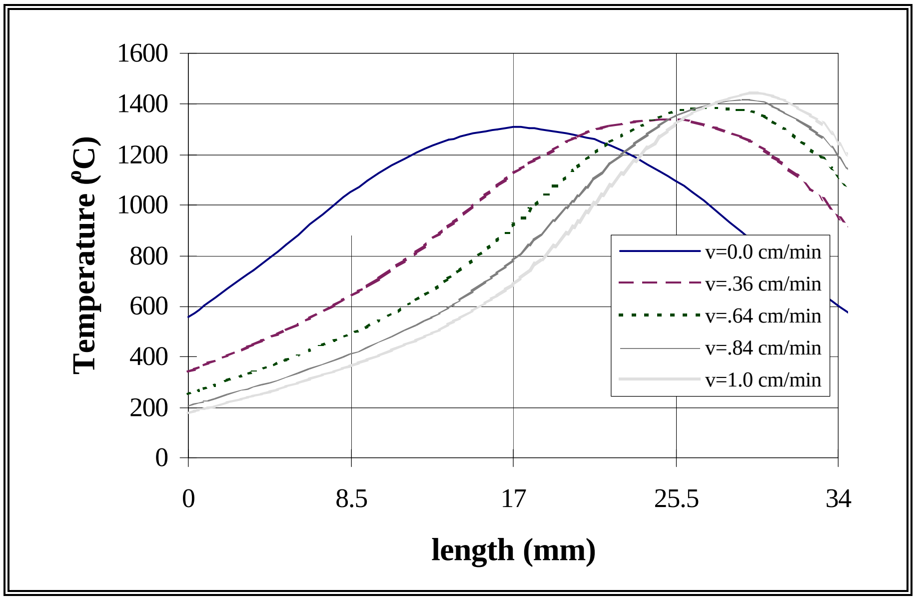

Figure 4.6 Steady State Temperature Distribution along a 4.67 mm Diameter Rod for Various Velocities and an Absorbed Power of 55 W

For an increase in velocity, the exit temperature increases and the entrance temperature decreases. The temperature that corresponds to the section of the rod in the center of the cavity also decreases with an increase in velocity. This concept is important to understand when comparing the simulation to data from the LANL experiments (which will be discussed later in this thesis.) For experimental data, the temperature often appears to drop with an increase in velocity. Since the temperature sensor measures the temperature of the section of the rod that corresponds to the center of the cavity, an increase in velocity can move a hot section away from the temperature sensor resulting in an apparent decrease in temperature. In reality the maximum temperature may be the same or even higher but is out of view of the sensor.

The effect that velocity has on a 2mm and 4.67mm rod are similar. For example, a velocity of 0.84 cm/min moves the maximum temperature to about 3/4 of the way through the cavity for both cases.

3. Transient Temperature Distribution

3.1 Two Millimeter Diameter Mullite Rod

The temperature distribution along a 2 mm mullite rod is plotted in Fig. 4.7 for a rod starting at ambient temperature for time intervals of ten seconds from 0 to 40 seconds. The absorbed power is held constant at 20 W.

The two millimeter mullite rod reaches steady state in 153 seconds within a margin, Δ, of 0.01 %. Δ is defined as

$$\Delta = \left| \frac{T - T_{NEW}}{T_{NEW} - T_i} \right|, \tag{4.4}$$

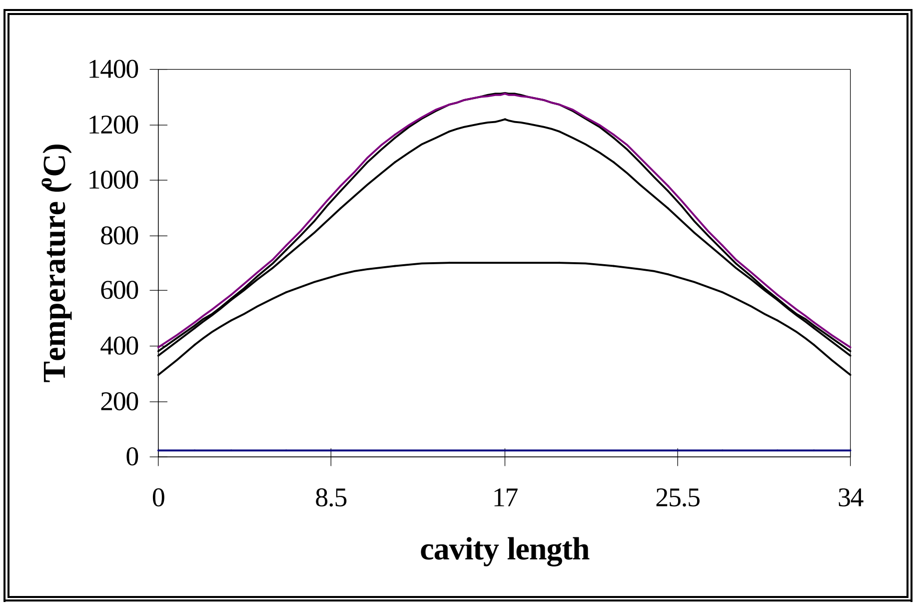

Figure 4.7 Transient Temperature Distribution over a 2mm Mullite Rod with an Absorbed Power of 20 W at 10 sec Intervals

where T is the temperature of the rod, T_{NEW} is the temperature of the rod at a time step of 0.01 sec later, and T_i is the initial temperature of the rod. As a rod is heated and approaches steady state, the absolute difference between the temperature at time t, and the temperature at time t + Δt, approaches zero. The time necessary to reach steady state depends on the convergence requirement. For example, if the criterion is increased from 0.01% to 0.1%, steady state is reached in only 81 seconds.

The temperature profile initially is uniform at the ambient temperature. As time passes, the rod is heated uniformly thus revealing a uniform temperature region near the center of the cavity. The decrease in temperature near the cavity ends results from axial conduction. Finally, a steady state profile is reached with a local maximum occurring in the center. From Fig 4.6, it appears that steady state is reached within about 30 seconds; however, the convergence requirement is not fully met until about a minute later.

3.2 Mullite Rod of Diameter 4.67 Millimeters

The transient temperature distribution for the thicker rod is plotted in Fig 4.8 at 10 sec time intervals from time = 0 to 70 seconds. Comparing Fig 4.8 with Fig 4.6 shows that it takes longer to heat a 4.67 mm mullite rod to a steady state temperature of about 1200^{o}C than a 2 mm rod. Steady state for the 4.67 mm rod is reached in 135 sec within 0.1% or 254 sec within 0.01%. The temperature shape is similar to that of the 2mm case. Initially, the temperature increases rapidly, but as steady state is approached the temperature increases slowly until there is no temperature change at all.

Figure 4.9 shows the distribution in 10 sec intervals of the 4.67 mm moving rod. The same rod moving at 0.84 cm/sec under the same conditions arrives at steady state in 263 sec within 0.1% error. The location of the maximum temperature changes with time until steady state is obtained.

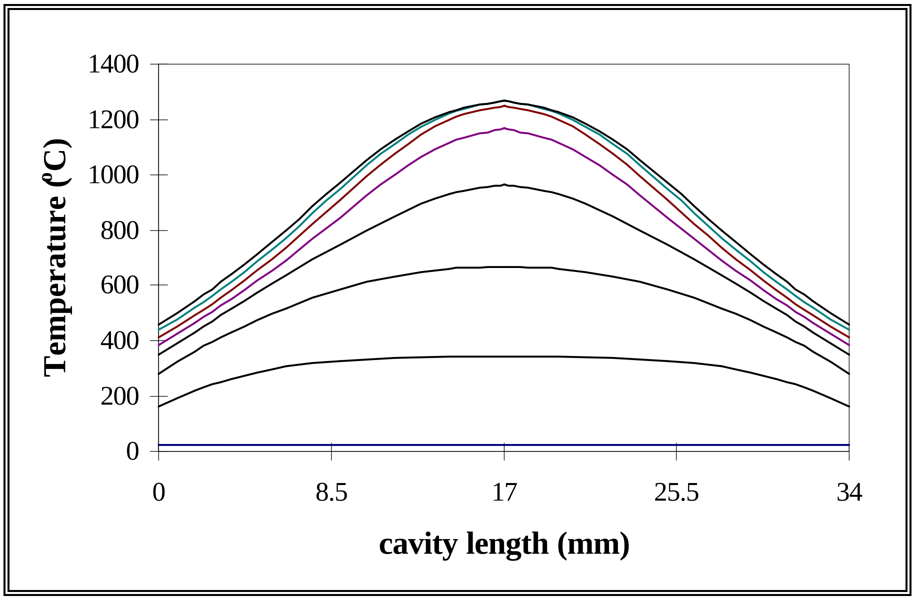

Figure 4.8 Transient Temperature Distribution along a 4.67 mm Mullite Rod with an Absorbed Power of 50 W at 10 sec Intervals

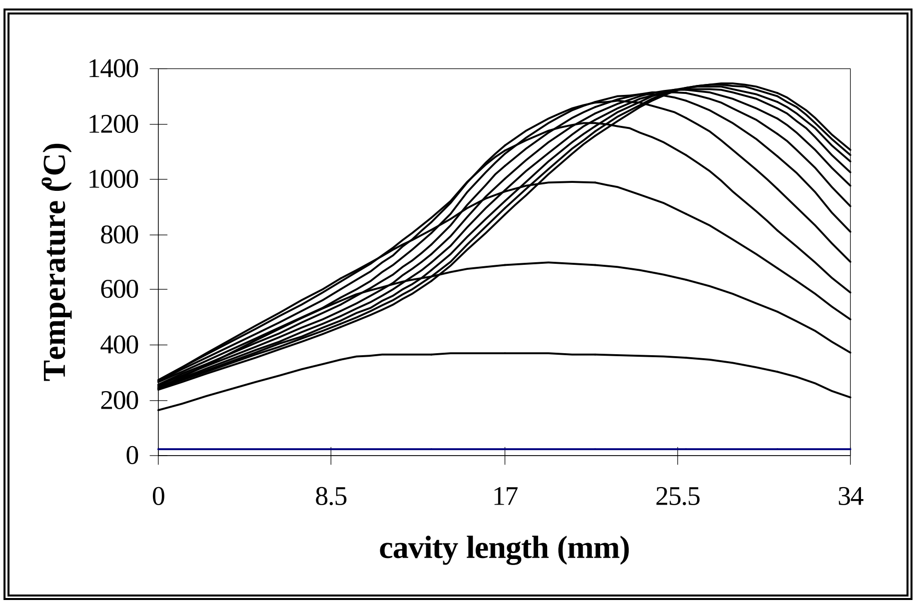

Figure 4.9 Temperature Distribution along a 4.67 mm Mullite Rod at a Power of 50 W and velocity of 0.84 cm/sec at 10 sec Intervals

4. Electromagnetic Field Requirement for Constant Power

When the electromagnetic field is held constant with respect to time at values greater than the critical value, thermal runaway occurs before sintering temperatures are reached. A solution to this problem is to ensure that an appropriate absorbed power level is maintained during processing. This requires that the electric field vary with time as shown in Fig. 4.10 for both the 2mm and 4.67mm diameter mullite rods at absorbed powers of 20 and 50 W respectively. The electric field is plotted in V/m and is assumed to be constant along the length of the rod, but is time dependent.

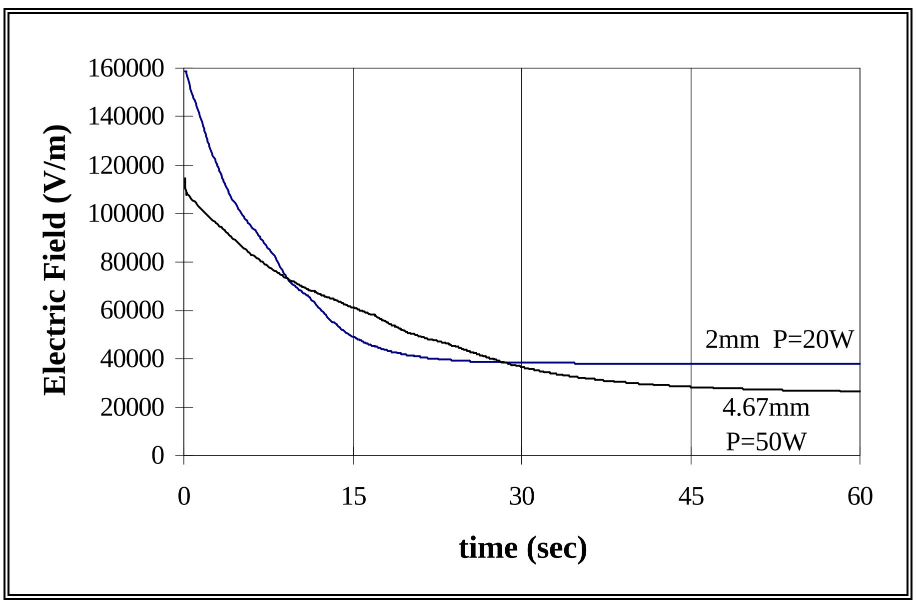

Figure 4.10 Electromagnetic Fields Required to Hold the Absorbed Power Constant at 20W and 50W for a 2mm and 4.67mm Diameter Mullite Rod Respectively

A large electric field is required for initial heating since mullite is nearly transparent to microwaves at low temperatures. The dielectric loss is related to microwave energy absorption; therefore, a low dielectric loss results in small microwave energy absorption. Since the dielectric loss increases with temperature, less electric field strength is necessary to hold the absorbed power constant. The definition of the absorbed power demonstrates this principle:

$$P_{abs} = C_1 = \int \dot{q} \, dV = \int (2\pi f \varepsilon_o \varepsilon''(T) \, |E|^2) \, dV \quad . \qquad (4.2)$$

By letting $C_2 = 2\pi f\varepsilon_o$,

$$C_1 = C_2 \int (\varepsilon''(T)|E|^2) \, dV \, . \qquad (4.5)$$

This equation shows that for an increase in the dielectric loss, $\varepsilon''(T)$, the electric field, E, must decrease for a constant power.

The same equation (Eq. 4.5) also demonstrates the reason why a larger electric field is required for a 2mm rod at steady state than a 4.67mm rod (Figure 4.10). For a larger volume, V, and the same dielectric loss, $\varepsilon''(T)$, at steady state, the electric field must be smaller in magnitude in order to maintain a constant power.

When designing a microwave applicator, an ideal distribution for the field may resemble the same shape shown in Fig 4.10. Using such a field distribution, steady state temperatures higher than the critical temperature are obtained without varying the electric field with time. The distribution can be determined by x = vt where x is the distance along the cavity, v is a constant velocity, and t is time. The electric field at calculated x will be the corresponding electric field at t from Fig 4.10.

5. Effect of Thermal Emissivity

The developed model depends on the accuracy of the modeled mullite material properties. If the properties are incorrect, the calculated temperatures will also be incorrect. Most of the collected data for the properties were either located from reliable data or measured, except for emissivity. A range of emissivities were plotted in Fig. 2.4 for alumina. Assuming that the emissivity of mullite falls in that range, an average emissivity was developed for the model. Figure 4.11 shows the effects that changing the emissivity has on the final steady state temperature for a 4.67mm mullite rod when heated with an absorbed power of 55 W. The highest temperature line represents the temperature profile assuming that the lowest emissivity of Touloukian's plot (Fig. 2.4) is the emissivity of mullite. Likewise, the lowest curve is developed when the highest emissivity is assumed. Depending on the emissivity used in the model, the maximum temperature may be 100^{o}C higher or lower, a possible error of 7.3%.

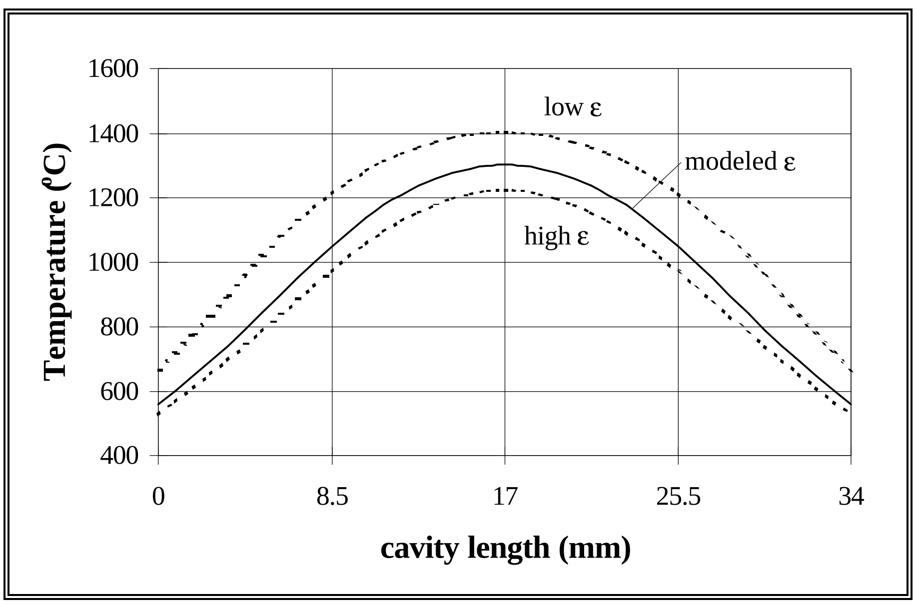

Figure 4.11 Temperature Distribution of 4.67mm Mullite Rod at an Absorbed Power of 55 W with Different Emissivities

6. Comparison of Model Results to Experimental Data

6.1 Two Millimeter Diameter Mullite Rod

Vogt et al. [12] performed laboratory experiments on microwave heating of 2mm and 4.67mm diameter mullite rods as described earlier. Figure 4.12 shows measured values of absorbed power, rod temperature, and rod velocity for the 2mm diameter rod and a comparison to the simulated data. The simulation #1 represents the temperature at the midpoint of the cavity. Simulation #2 represents the same simulation except that the temperature measurements occur at a location 66.7% from the cavity entrance. The difference between the location of the two cases is about 5mm in a 34mm long applicator. The change produces a significant difference in the resulting temperatures. In the actual experiments, the temperature sensor was located about the midpoint of the cavity and measured an integrated temperature over a small area near the midpoint. Not knowing the exact location causes difficulty in comparing to simulated data.

The power representation of the actual and simulated data differ in the figure. The measured power represents the power input minus the reflected power where:

$$P_{input} = P_{reflect} + P_{abs} + P_{loss} \, . \tag{4.6}$$

The power measured in the experiments thus can also be stated as the sum of the absorbed power and the power loss to the cavity walls. The simulated power is input data and was chosen to match the measured power, however, it only represents the absorbed power and does not include losses. The difference in powers is the most likely explanation for the temperature differences between actual and simulated data. If the power loss is significant, the simulated temperature would match more closely to the actual temperature since less absorbed power is required in the model.

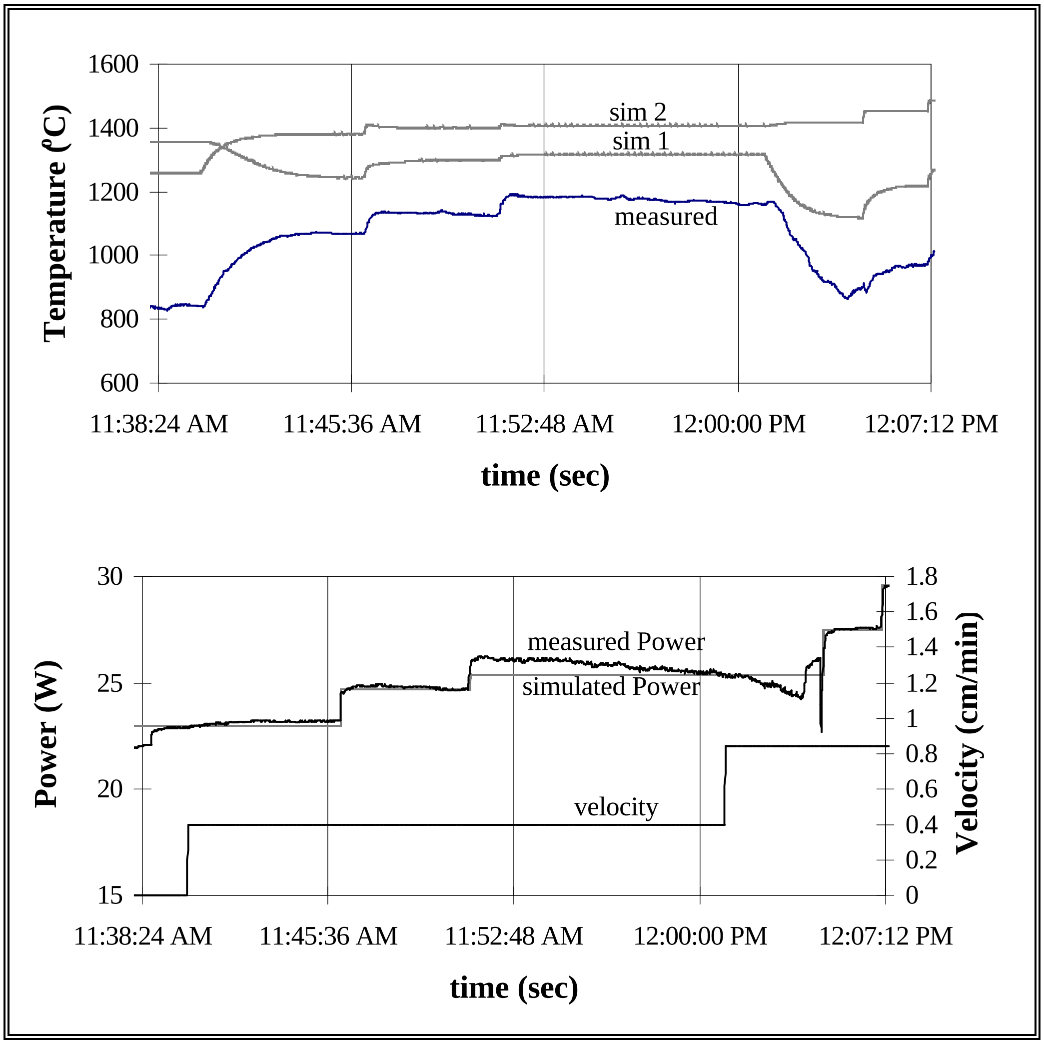

Figure 4.12 Comparison of Actual Temperature Measurements with Simulated Data for the 2mm Diameter Case

The absorbed power initially begins at about 23 W and increases in increments until the process is completed at a power of 27 W. At each increase of power, the temperature also increased. The single plotted velocity represents both actual and simulated velocities. The rod is initially stationary until 11:40 a.m. when the rod begins to move steadily at 0.4 cm/min. At this time, a hot section moved into view of the sensor which produced an apparent increase in temperature. Depending on the location of the sensor in the simulated data, a hot section either moved into or out of view. At 12:01, the velocity again increased to 0.84 cm/min. The velocity increase carried away the hot section from view of the temperature sensor resulting in a temperature drop. It is interesting to note that the temperature decrease was not apparent in simulation #2 where the temperature sensor is located past the midpoint of the cavity.

6.2 Mullite Rod of 4.67 Millimeter Diameter

Figure 4.13 shows experimental data and simulation results for a 4.67mm diameter mullite rod. Simulation #1 and simulation #2 represent different sensor locations as in the 2mm case. Simulation #1 corresponds to calculations as if the temperature sensor were at the midpoint of the cavity, and simulation #2 represents the calculations for a sensor 66.7% of the distance from the cavity entrance. The heating was done on a stationary rod, therefore temperature changes are a result of changes in power only. At each power increase, both simulated and measured temperatures also increased.

The simulated data match qualitatively with the actual data; however the simulated temperatures are higher than the actual. This probably is the result of the modeled power which does not represent actual absorbed power since it assumes no losses. Another possible explanation is that the emissivity of mullite is not accurately known. Figure 4.11 demonstrates the great effect that a small change in emissivity has on temperature. If the actual emissivity is as high as the upper branch of Touloukian's plot (Fig. 2.4), the measured and simulated temperatures would match more closely, but would not account for the total difference. This suggests that there were significant power losses in the cavity during the experiments.

Figure 4.14 shows the effect that a notable power loss has on the output. In this case the power loss is assumed to be half of the measured power equal to the absorbed

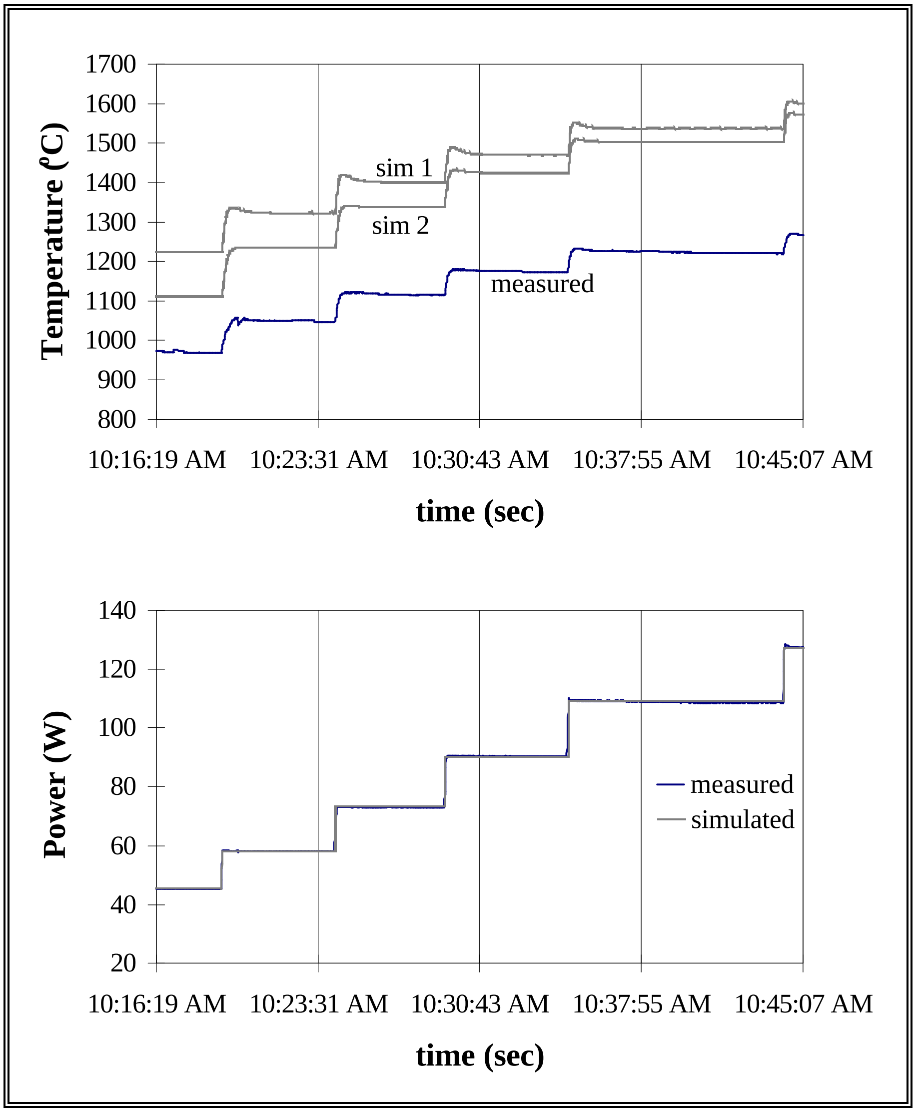

Figure 4.13 Comparison of Actual Temperature Measurements with Simulated Data for the 4.67mm Diameter Case

power. This assumption allows the temperatures to more closely match. Power loss increases with temperature; however, Fig 4.14 suggests that the power loss needs to be lower at low temperatures and higher at high temperatures in order for the temperatures to match even more closely.

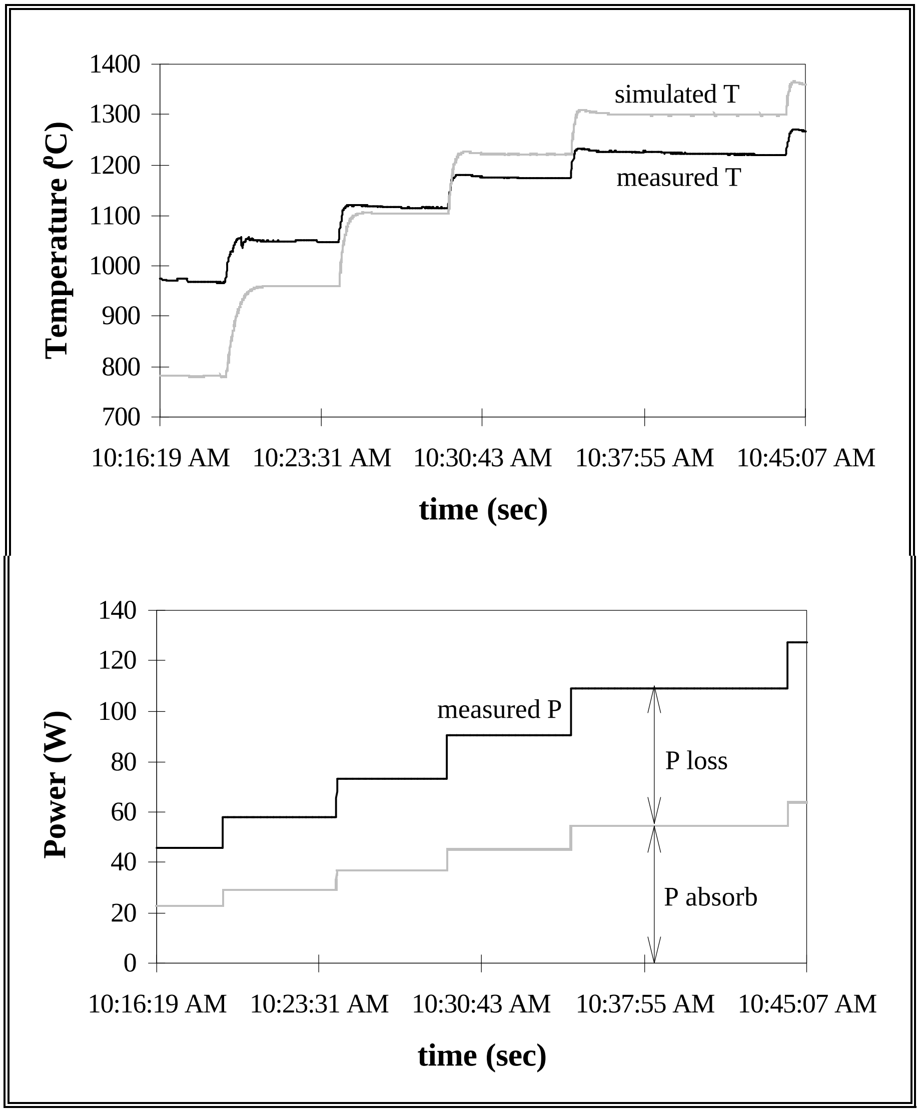

Figure 4.14 Effect of Significant Power Loss on Temperature