A Micromechanical Model for Predicting Tensile Strength

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(ABSTRACT)

A novel micromechanical model for predicting the failure of polymeric and ceramic matrices under an applied global tensile loading is presented. The model is based on a Weibull statistical type of approach and incorporates eleven different variables to simulate different micromechanical failure phenomenon. Through the variables, the model incorporates such phenomenon as fiber-fiber interaction; matrix-fiber interaction; damage due to processing; and local stress distribution at the interphase. The effects of the load sharing constants and shape parameter on failure are studied. A software package, used in the Windows environment, was also developed to perform a numerical analysis of the model.

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1.0 Introduction

To better understand the macroscopic properties of composite materials, often it is first necessary to determine the mechanics at the microscopic level. In recent years, a considerable amount of research has been conducted in the field of micromechanical strength models. In particular, the matrix-fiber interface and its role in the failure of composite materials has received a substantial amount of attention. It is intended that the results of the research will help eliminate or, at least, reduce the uncertainty of failure in composite materials. Not only must the loading that causes failure be determined, but also the mechanism that causes the composite material to fail must be found. Some of the damage models that contribute to failure at the micromechanical level are: matrix cracking; matrix-fiber interface debonding; fiber pull-out; fiber fracture, etc. In order to reduce the uncertainty of failure associated with composite materials, a model was developed in the present effort that considers both the material's properties and some of the previously mentioned mechanisms of failure. A numerical study of the model was then performed by simulating failure paths to determine the ultimate strength of a unidirectional fiber reinforced composite under tensile loading.

The mechanics of failure in a composite material system can best be described as a series of events in which both the order of occurrence and their magnitude influence failure. Recently, a considerable amount of research has been conducted to locate the one event that controls failure of the system. Once identified and understood, the composite material could then be improved by altering the importance of the damaging event that caused failure.

Due to the presence of numerous types of mechanisms that cause failure, it is often necessary to use a statistical approach to determine and/or predict the mechanics of failure in composite material systems. Generally, the most accepted statistical representation employed is known as the Weibull approach[1]. This statistical model accurately represents the strength distribution of fiber bundles. This distribution may be caused by a combination of the following effects: manufacturing defects and/or processes; the environment; flaws in the fiber-matrix interface; flaws along the fiber; etc. A more detailed discussion of Weibull's approach can be found in Section 3.1.

When discussing composite life and failure, it is virtually impossible to do so without considering the strength of the material. For composite materials, the two phenomena are closely related. For clarity, only those models directly related to or used as a basis for the model in this study were included in the review that follows. A more detailed explanation of composite strength can be found in References [2-3].

B. W. Rosen [4] is generally acknowledged as the first researcher to consider the stress state around fiber fracture. He used a modified Weibull [1] approach to study fiber failure of composites under tensile loading. Rosen identified a region of stress near the fractured end of the fiber now known as the ineffective length. Over this length, the fiber stress increases from zero at the fiber fracture to the value in the unbroken fiber as one moves away from the fiber fracture. He also theorized that in order for a composite to fail, a statistical accumulation of fiber fractures must occur within a relatively localized area. Rosen's model was limited in that he did not let the ineffective length vary with the changing stress levels. A more detailed discussion of Rosen's model can be found in Section 3.2.

Zweben [5] also considered the stress state around a fiber fracture. He used a modified version of Rosen's approach. He did this because he felt that Rosen's approach produced too large a difference between the predicted and experimental values. Zweben included the effect of stress concentrations on composite failure in his model. He theorized that fiber discontinuities may be more important in the failure process than previously thought. Zweben also noted the possible effect of fiber coatings on fiber fracture. A more detailed discussion of Zweben's model can be found in Section 3.3.

Barker and MacLaughlin [6] examined the stress concentrations caused by discontinuous fibers for both the fiber and the matrix. They employed a model which considered such effects as fiber volume fraction, end-gap size, and modulus ratio. For the fiber, the stress concentrations reached a maximum value of 1.5 and remained relatively unchanged. The matrix stress concentration factors depended on the end-gap size and modulus ratio. The predicted and experimental values were in good agreement. This suggests that a stress concentration of approximately 1.5 is a reliable figure to consider when developing future models.

While providing a good basis for understanding the stress state around fiber fracture, both Rosen and Zweben's models are not as precise as some of the more current ones. Recently, a good accumulative damage model for both tensile and shear failure of multi-directional laminated composites was presented by Shahid and Chang [7]. Shahid and Chang assumed that under a uniform loading that the damage would accumulate uniformly. The model used the effective material properties of the damaged plies to represent the accumulating damage in the composite. Damage was determined using a modified Hashin [8] failure criteria. Like Rosen, Shahid and Chang found the tensile stiffness of the composite was not affected by fiber fracture until enough of the fractures occurred within a certain bandwidth. The numerical results from the model were in good agreement with the experimental data.

When discussing composite materials, the role of the micromechanical properties in failure can not be stressed enough. Often only a slight variation can produce a large difference in the failure of the material. The role the matrix-fiber interface is of particular interest. While much more is now known about the importance of the matrix-fiber interface, the exact mechanics that produces failure is still in question. Only through more research will this problem be better understood. Until recently, a large amount of the micromechanical models did not consider the fiber-matrix interphase, despite the experimental evidence that the mechanical properties are extremely different for the matrix, fiber, and interphase.

Gao et al. [9] performed a study in which the effect of the interphase on both the stress redistribution and damage characteristics during fracture was examined. The interphase was assumed to have a finite thickness and distinct mechanical properties. A three phase model was used to represent the material system where the three phases are: a broken core; the first intact fiber at the notch-tip; and a uniform stress field within the remaining average composite. Gao et al. found that the interfacial strength affected the stress redistribution and the ineffective length. In a more recent paper the same authors (Jayaraman et al.) [10] further investigated the effects of the interphase on the local stress field. In this study, a variation of the Equivalent Inclusion - Average Stress (EIAS) analysis [11] was used to predict the local stress field. A parametric study was then performed to show the effect of the interphase has on determining the final strength of the composite material. Jayaraman et al. found that the composite material could be optimized by optimizing the interphasial stiffness.

Begis and Blankenship [12] used a mathematical procedure known as *homogenization* to study the local stress fields in composite materials. Homogenization is a computational method in which the material constants do not vary rapidly with respect to the spatial variables. Homogenization was chosen because it was able to retain the microscopic or local effects pertaining to the failure of the composite on an average basis. Another reason for its selection was that it could be used for either a random or periodic material system. Begis and Blankenship used a periodic material system for the analysis. While the method produced reasonable results, the dependence of the solution both on an arbitrary assumption of the shapes and the layout of the fiber leaves some room for improvement. One possible solution would to be to determine a set of material constants through experimentation that could be used to make the assumptions and thus the model more realistic.

Kishore et al. [13] used a local-global approach to determine both the local mechanical and thermal stress distributions at the interface during fiber pull-out. The local-global method consisted of: a local stress analysis; global stress analysis; and a matching procedure. The local stress analysis was employed to find the analytical structure of the local stress distributions. The global stress analysis was used to determine the full-field stresses. The matching procedure was utilized to obtain a detailed understanding of the interfacial stresses. An interesting and important result of this study was that both the interfacial shear and normal local stresses vanished after a few fiber diameters. Any future model should therefore take into account the interaction of any local stresses of only a few fibers within the immediate neighborhood of the reference fiber.

Case et al. [14] developed a model using a linear superposition technique in conjunction with a fiber discount theory to predict the local stress redistribution due to single and multiple fiber fractures. The theory was then validated by using a macro-model composite conceived by Carman et al. [15]. Unlike Shear lag analysis[16] or a derivative such as the one proposed by Hedgepeth and Van Dyke [17], the Case et al. model considers the effects of constituent properties, fiber volume fraction, and the influence of the radial variation of stress on the local stress concentration. When the crack does not propagate into the matrix, the agreement between the Case et al. model and experimental data is often better than for the results obtained using either Hedgepeth and Van Dyke or Shear Lag analysis. One would expect this result since the Case et al. model utilizes material properties not included in either of the other two theories to determine fiber fracture.

Pagano and Tandon [18] used a concentric circular cylinder model with prescribed displacements as their representative volume to study the micromechanical stress in the fiber, interface, and matrix. Despite using only a three-phase model, their model was general enough that it could have included any number of cylinders to represent different phases. Some limitations of the model are: assumed perfect bonding; no fiber-fiber interaction; and damage in the composite was not considered. Despite these limitations, the Pagano and Tandon model is a good approximation and is widely used.

Aboudi [19] also developed a micromechanical model for stress distribution in unidirectional composite materials. He used fibers with square cross sections arranged in a double periodic array in a matrix. Aboudi then validated his model by calculating the global failure stress for various material systems and fiber orientations. The results demonstrated good agreement between the predicted and experimental values. While this study did not include the effect of imperfect bonding on composite failure, the model could be generalized to do so in the future. Another interesting aspect of this particular model is that the current state of the composite was used to determine future failure.

The model presented in this study was developed using a statistical approach similar to models already validated by experiment [20,21]. A computer program was developed to simulate experimental testing of a unidirectional composite under tensile loading. The results of the program were then compared to published values. Because the program was developed in Visual Basic, only a small part of the actual code could be included in Appendix A. The model considers different effects such as: fiber-fiber interaction; matrix-fiber interaction; damage due to processing; and local stress distribution at the interphase. The model was developed for use with polymeric and ceramic composite material systems in mind. Metal matrix composites were excluded because the micromechanics are considerably different from those of the other two material systems. An entirely different model would need to be developed. A metal matrix failure model is left as a possible future study.

1.1 Definition of Failure

Because the definition for failure and/or strength varies from application to application, often the first step in a study is to clearly define the problem. This occurs since unlike homogeneous materials, composites have an assortment of failure phenomena associated with their use. For one application, final failure might be defined as first ply fracture whereas for another, such as pressure vessels, it might be matrix cracking which defines final failure of the component. As one might conclude, the loading and thus the stress state in the previous example would be different which, in turn, would mean that the strength of the composite would be different. It is also possible that the strength of the composite will vary for the same application based on the failure criteria and/or theory used. As a consequence, it is often necessary to examine any results with a wary eye. As scientists it is important to remember that no number is absolute. More often the results are cautiously suggested values that should be used in design considerations. Generally, the strength of any given composite is better determined using a boundary method. The boundary method is where two known and experimentally tested theories are used to predict the range into which the strength of the composite material will fall. This is one tool used by engineers to handle the statistical scatter associated with composites.

The model in this study uses a boundary technique to determine the strength of the specimen at failure. Since a composite can fail in various ways, the model used in this study allows the user two choices to determine exactly when failure has occurred. The first method used to determine failure was local fiber strength. Once a user-defined strength parameter had been surpassed, the fiber was considered to have failed at the local level and was removed from load carrying capabilities. Final failure or fracture of the model was then reached when a user-set limit of adjacent local fiber fractures was either equaled or surpassed. A more detailed discussion of the model as well as the parameters used can be found in Section 3.4.

1.2 A Short Word on Statistics

Because statistics is such a powerful tool frequently used in analyzing composite materials, a basic understanding is necessary to fully appreciate most of the models. Although one does not need a working knowledge of statistics to understand this report, a brief overview follows so that the study may be better understood. For a more in depth account of statistics, please see any of the available books such as Reference [22].

A sample, which may be discrete or continuous, is the set of all possibilities in a probabilistic problem. Furthermore, a discrete sample may be either finite or infinite. For example, the data collected for this study was a discrete finite sample. The sample ranged in size from 100 to 400 data points. Because the failure of composite materials is often a product of random variables, the certainty associated with the results increases as the size of the sample increases. For example, one hundred tests of tensile strength will provide a more conclusive answer than just ten tests.

Each element of the intrinsic strength matrix was generated using a random number scheme available in Visual Basic and was considered to be a random variable. A random variable is one that exhibits scatter or dispersion that makes predicting a future result with any type of certainty an impossibility. Probability provides a useful tool to understanding composite materials because large scatter is typically associated with many of the mechanical phenomena of composite materials.

The complex problem of failure eliminates using many of the probability functions. One function that can be utilized, though, is the Weibull distribution. In order to gain the full benefits of the Weibull distribution, it is necessary to assume that some of the micromechanical phenomena play a larger role in failure than others. For this study, the shape parameter and the local intrinsic strength were assumed to have the largest influence on local failure. For this reason, a two parameter or random variable Weibull distribution was used as the basis to develop the model employed in this study.

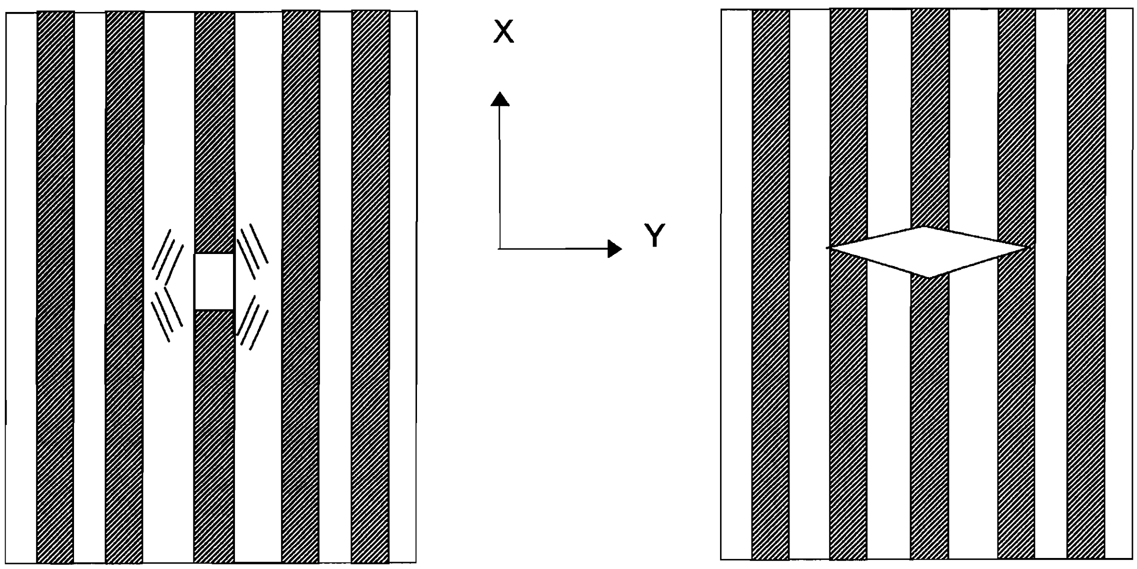
The shape parameter used in this model measures the amount of scatter associated in the intrinsic strength matrix sample. A smaller number represents little scatter of the sample while a larger number identifies more scattering. Typically, the shape parameter ranges from around three or four to 20 or so.

2.0 Matrix - Fiber Interface

Often the failure of composite materials can be linked directly to the interaction between the matrix and the fibers.[23-25] In most of the earlier studies, the matrix-fiber interface (MFI) was assumed to be perfect (i.e. the stress is equally distributed on both the matrix and fiber). While this assumption is convenient for classroom applications, a more realistic one assumes that the MFI has different material properties than either those of the matrix or fibers. These differences will produce an uneven stress distribution at the MFI when the composite is placed under a tensile global loading. An even better assumption is one in which both the strength and intrinsic flaws also produce an uneven stress distribution along each fiber. This assumption, which is used for this study, is called a random strength distribution. In most of the literature, a random distribution seemed to produce a better correlation between the actual test data and the theoretical results than other models.

2.1 Shear Yielding

Shear yielding is an important aspect to consider when modeling composite fracture. Local shear yielding is one reason a composite material system is able to survive initial fiber fracture. A matrix with little or no shear yielding when subjected to a tensile load will respond as a planar/brittle fracture as seen in part (b) of Figure 2.1. In part (a) of the same figure, one can see that a high shear yielding will often distribute the local stress concentration and localize the failure caused by matrix-fiber interface debonding and/or fiber pull-out to a relatively small part of the material system. The shear yielding allows the stress at the broken fiber to be redistributed to neighboring unbroken fibers and to other regions of the broken fiber. By redistributing the local stress to the surrounding fibers, the propagation of the crack through the composite is quickly arrested.

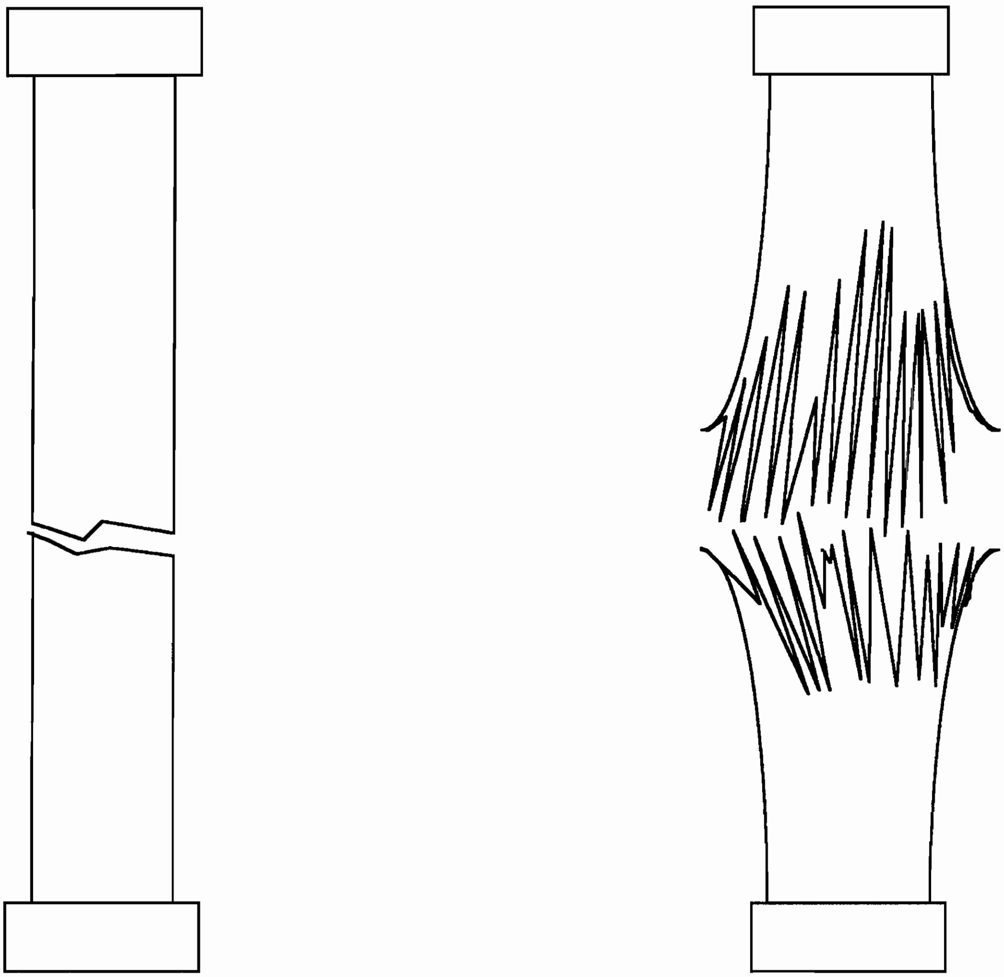


a) b)

Figure 2.1

Matrix-Fiber Interaction Failure
Part a) Shear Yielding
Part b) Planar Fracture

Without shear yielding or local debonding, the composite quickly fails in a brittle fashion soon after the first cracks appear. Brittle fracture is often identified by a failure pattern which is along pronounced lines or band widths of local fiber breaks. Band widths, which typically are not normal to the loading direction, are a result of the increased probability of more fiber fractures occurring within a region near an existent fiber fracture. If a strong yielding is present, however, bristle-like patterns are produced upon final failure. Often the fibers of this type of failure are extremely clean of matrix material. An example of each type of failure can be seen in Figure 2.2. Part a) of the figure demonstrates the type of failure associated with a matrix that has a low shear yielding strength. Part b) of the same figure illustrates the effect of a high shear strength matrix. As stated



a)
Little Shear Yielding

b)
Strong Shear Yielding

Figure 2.2

Effect of Shear Yielding
on Failure Patterns

earlier, the low shear strength in a) causes the composite to have a more pronounced failure path or band width than the one in part b). In general, the failure of low shear yielding matrix composite materials is best modeled by a fiber bundle method while failure of the intermediate and high shear strength materials are best modeled by one of the developed strength prediction theories.

Occasionally, shear yielding will cause some of the matrix to adhere to the fiber after failure. Adhesion typically represents a strong local stress redistribution effect due to an improved shear strength [26]. Most micromechanical strength models do not consider this effect. Usually the fiber-matrix interface is either considered to be perfectly debonded or perfectly bonded. One possible way to include the shear strength into a strength model is to develop load sharing constants that could be defined experimentally to simulate the debonding. One problem with this solution is that each fiber would have to be modeled independently of the others. Needless to say, the complexity of the problem would be increased. On the other hand, if a general global approximation or average of the shear strength were used then a reasonable estimate of the strength of the composite would be obtained while keeping the problem relatively simple. Such an approach was used in this study. The matrix is assumed to be an average representation of the material. If this assumption were not used then the stress states and material properties of every local element would then need to be addressed for every iteration of the program. Because a reasonable estimate of the strength is desired, this problem is left for possible future study.

2.2 Fiber Pull-Out

Fiber pull-out occurs when the local stress level in the composite reaches a point where the matrix-fiber interface begins to separate or debond. Usually, this process occurs due to high shear stresses that develop at the interface after the fiber has fractured. Another necessary component for fiber pull-out is a matrix and/or interface that is capable of shear yielding or debonding. Without this capability, the composite material, as stated previously, will often fail in a brittle fashion. Failure of the composite does not occur until enough of the fibers have debonded from the matrix. The matrix is then unable to handle the loading without the fibers and it also fails. Figure 2.3 shows an example of a fiber pull-out type failure. Typically, the fibers associated with pull-out are clean of matrix and smooth in appearance.

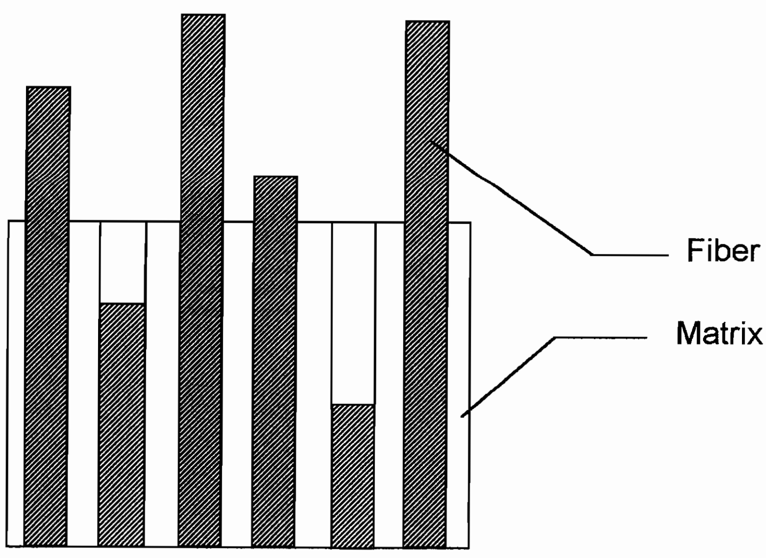


Figure 2.3
Fiber Pull-Out

Many studies have been done to alter the effect of fiber pull-out in the failure of the composites. A highly investigated approach is the modification of the fiber's coating and/or shape which changes both the material properties and response of the interphase.

Allan [27] examined the effect of fiber coatings on natural fiber composite systems. For these types of materials, fiber pull-out appears to be the dominate type of failure which is not acceptable for engineering applications. He presents some coatings that are effective at reducing fiber pull-out in experiments. A similar study of the effect of fiber coatings on failure was recently conducted by Carman and Case [28]. In their study, Carman and Case used an elasticity solution of optimal coatings to minimize the local stress state for loading transverse to the fiber direction. By doing so, the strain to failure of the optimized fibers was increased by approximately 15 times that of untreated fibers. One drawback of this method is that only one material system was studied. For a more generalized theory, a more detailed study of many material systems is necessary.

Carapella et al. [29] studied the effect of a fiber's shape on the micromechanics. The cases of a fully debonded and bonded crenulated fiber was investigated. A crenulated fiber is one that has an intentionally wavy surface. The wavy surface permits the matrix-fiber interface an additional method of bonding. (i.e. mechanical coupling in addition to the normal adhesion) Carapella et al. found for both test cases that the crenulated fibers had lower local stresses than those of the circular fibers. These results suggest that by introducing new shapes of fibers into composites one can effectively increase the stress without raising the cost of manufacturing the material which is a highly desirable result.

3.0 Strength Prediction

As with most material properties associated with composites, a complete definition of the strength of a composite material does not exist. In fact, after a careful investigation of many different papers, an individual would find many different and yet valid definitions for the strength of composite material. Often the definition is based on the application for which the composite material is used, hence, a cause for many definitions. Another cause for the various definitions of strength is that, typically, a distributed fracture and/or debonding of the matrix and/or fiber fractures often precede complete failure of the composite material. This phenomenon is unlike most engineering materials (i.e. metals) where failure and fracture are usually closely related in both time and mechanics. A different definition for failure and strength is oftentimes needed because in composites, failure and strength seem to be related by an unknown process.

For this study, the global strength of the composite shall be defined by the last allowable stress before the global failure occurs in the composite, where global failure was defined earlier in Section 1.1. As noted above, the failure of composite materials can be directly related to the intrinsic strength of the constituent materials. This is due to the random distribution of strength which is a product of flaws inherent in all composite materials which develop due to various conditions (i.e. environment, manufacturing, handling, etc.). Probability and statistics dictate that the lower strength fibers, ones with many and/or large flaws, will fail first. Many models have previously been developed using this type of statistical approach to determine damage events. These models as well as their limitations are discussed in the following sections.

3.1 Weibull Distribution Function

The Weibull distribution function [1] is the point from which almost all recent tensile composite failure models are derived. This function was originally developed to predict strength for brittle fiber bundles. It had to have the ability to accommodate the scatter inherent in brittle failure. Weibull developed the following equation, which represents the probability of the fiber fracturing at or below a given stress, with the scatter in mind:

$$G(\sigma) = 1 - \exp\left[-\left(\frac{\sigma - \sigma_1}{\sigma_o}\right)^m\right] \qquad (3.1)$$

where σ is the given stress; σ_1 is a lower strength limit shown in Figure 3.1; σ_o is the mean value of the given stress and is shown in Figure 3.1; and m is the

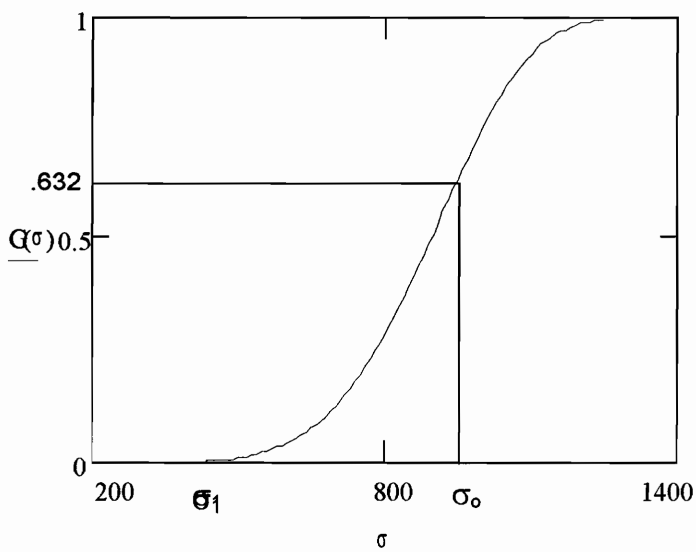


Figure 3.1

Weibull Distribution Function G(σ)

shape factor, dependent on the statistical scatter and can be calculated using the following equation:

$$\frac{s}{\sigma_m} \cong \frac{1.2}{m} \tag{3.2}$$

where s is the standard deviation for the sample space; and σ_m is the mean stress for the sample space.

In 1958, B. D. Coleman [30] derived Equation 3.3 to predict the average strength for a fiber bundle. Coleman based his equation on both the distribution of the Weibull function and a mean fiber stress to derive the following equation:

$$\frac{\sigma_b}{\sigma} = \left[\frac{1}{m\,e}\right]^{\frac{1}{m}} \frac{1}{\Gamma\left(1 + \frac{1}{m}\right)} \tag{3.3}$$

where σ_b is the fracture strength of a fiber bundle; σ is the mean strength; Γ is the gamma function; e is the naperian log base; and m is again the shape factor which is found using Equation 3.2.

One should note that Equation 3.3 can only be used to predict the fiber bundle strength since the matrix-fiber interface is not taken into consideration. Equation 3.3 is a good way to check the values obtained with the other models used for the prediction of composite failure.

Hwang and Han [31] compared the different statistical models employed in strength prediction. The goal of the study was to determine which, if any, statistical model was better for representing fatigue life and strength. The study compared the following models: normal; log-normal; and Weibull distributions. Hwang and Han found that while each model represented strength well, the log-normal and Weibull distributions were better at representing the fatigue life of composite materials. These results suggest that the log-normal and Weibull distributions are probably better suited to handle variables with large ranges.

3.2 Cumulative Weakening Model

Another strength model is the cumulative weakening model developed by B. W. Rosen [4]. In this model, the flaws are assumed to be randomly distributed in the fibers. Another assumption inherent in this model is that the statistical information for the strength of single fibers is valid. These two assumptions produce the result that each fiber fails at its weakest point first. If the matrix-fiber bond is strong and the matrix is brittle, then the first failure will quickly propagate across the composite. While this may happen, it tends to be a trivial case. Most composites do not fail in this manner. An interesting case is where the cracks do not propagate immediately. The fiber breaks, but there is a local stress relaxation (see Figure 3.2) and/or redistribution around the fiber fracture. This redistribution causes part of the fiber to be come ineffective (i.e. not carry the full load). The ineffective length is determined by the shear stress at the matrix-interface bond. As the number of fiber fractures increases, an increase in the load redistribution occurs causing the composite to become weaker. Failure is defined as the point at which a region of the composite from one side to the other can no longer sustain the applied load. Rosen developed the following equation:

$$\sigma_c^* = \frac{\sigma_o}{(lme)^{\frac{1}{\alpha}}} \tag{3.4}$$

where σ^*_C is the expected strength of the fibers; l is the ineffective length of the fiber and has been normalized by the diameter of the fiber in order to maintain the correct units; σ_O is the applied global stress; m is the shape parameter of Equation 3.2; e is the base of the natural logarithms; and alpha is an inverse measure of the dispersion of the material strength.

One drawback of the cumulative weakening model is that it is difficult to measure the ineffective length for each fiber in a composite that has failed. Also this model must be used with caution since a slight change in the ineffective length produces a large change in the predicted strength of the composite. Due to these limitations, other models such as the fiber break propagation model were developed.

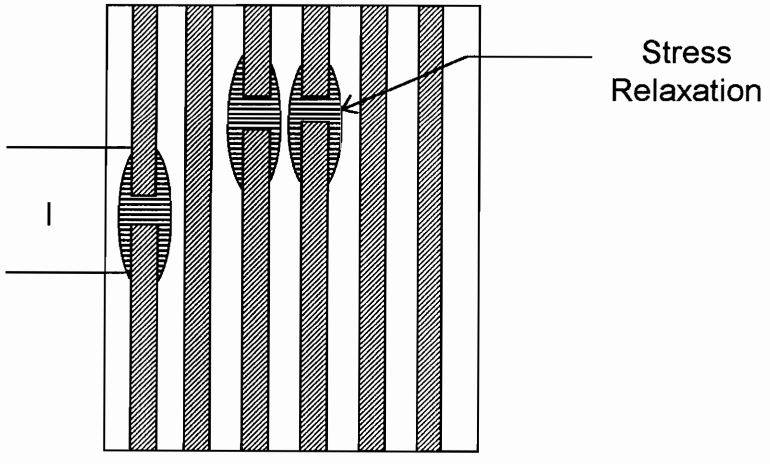


Figure 3.2

Stress Relaxation Assumed
in Deriving Equation 3.4

3.3 Fiber Break Propagation Model

The fiber break propagation model developed by Zweben [5] is much like the cumulative weakening model. Again, flaws are assumed to be randomly distributed along each fiber. When a fiber fractures, the stress of the fractured fiber is redistributed to the surrounding ones. Probability dictates that surrounding fibers will be more likely to fracture than distant fibers because the stress has increased the most on the fibers closest to the initial fiber fracture. This effect is known as a stress magnification effect. As the surrounding fibers fracture, the stress redistributes again. Under a constantly increasing load, the fibers continue to be influenced by the stress magnification effect until the composite reaches failure.

While this model is more realistic than the cumulative weakening model for predicting the actual failure of composite under tensile loading, it does have some drawbacks. One of them is that it does not take into account the stress concentration effect around the broken fibers. This produces a lower strength prediction than the one predicted by the cumulative weakening model. The fiber break propagation model is also influenced by various factors such as deformation and fracture at the fractured fiber ends.

3.4 Present Strength Prediction Model

The model employed in this study combines some aspects from both the probability of failure propagation from the cumulative weakening model and the stress magnification effect as used in the fiber break propagation model to produce a hybrid model. Like the other two models, this model is based on the fact that there are intrinsic flaws along the length of each fiber in a composite material. As a result, a probabilistic approach may be employed to determine which flaw fails first due to an applied global tensile load. Each local failure is then used to calculate the new failure criterion of each segment. This process is repeated until a failure path, which is defined in Section 2.1, is produced. However, unlike the other two models, two square matrices are used in the actual calculation for failure prediction: an intrinsic strength matrix and a failure path matrix. By using the two matrices, one is able to produce a better model than previous ones. One limitation of the model is that the smallest possible size matrix that can be used is a 3 x 3.

The stress magnification effect is simulated through the use of four load sharing constants, a - d, which determine how each segment or cell interacts with surrounding ones. These constants, which range from 0 to 1, are determined through experimentation and experience. The choice of constants can also effect the shear yielding. When higher constants are used, less shear yielding occurs causing increased fiber-matrix and fiber-fiber interaction. The actual equation is derived in the following section.

This model also considers the effect of a fiber's ineffective length on overall failure of the composite material. This is done by defining the failure path as one in which only local failures are allowed to be summed with other failures in the immediate surroundings of the local failure of interest to produce final failure of the composite material. By defining failure in this manner, the ineffective length of the fiber is correctly modeled.

4.0 Model Derivation

As a result of various studies, it is well known that the macroscopic strength of composite material systems is largely influenced by the microscopic properties of the fiber, matrix, and matrix-fiber interface. Therefore, to develop an improved macroscopic strength prediction model, it is first necessary to improve the understanding of the underlying micromechanical problem. Judging by the amount of research as well as by the number of models, this task is not at all an elementary one. Every micromechanical model is limited by the assumptions made during its development. By reducing the problem in this manner, a solution may be obtained for a set of known and thus controlled conditions. After validating the model through both numerical and experimental means, the assumptions are then relaxed to expand the usefulness of the model. As a result, all models usually are, at best, estimates of the complex micromechanical phenomena that occurs during failure.

Consider a fiber of length, L, as shown in Figure 4.1 with a stress distribution that is a function of both position along the fiber direction (i.e. x = 0 ... L) and the applied tensile global stress as given by:

$$\sigma = \sigma_g f(x) \tag{4.1}$$

where σ_g is the applied global stress on the composite material; f(x) is the local stress function associated with the fiber at a distance x from the end of the fiber; and the following boundary conditions hold: f(0) = 1; f(L) = 1. These boundary conditions are imposed so that the local fiber stress function reaches the applied average global stress at the ends of the fiber. By using this assumption, one can develop a generalized stress function for a representative section of the material of interest.

Because the local stress is found by taking the load over the local cross-section of the fiber, this function should consider the flaws along the fiber. Therefore, as demonstrated in Figure 4.1, the local stress state would be distinct for different sections along the fiber.

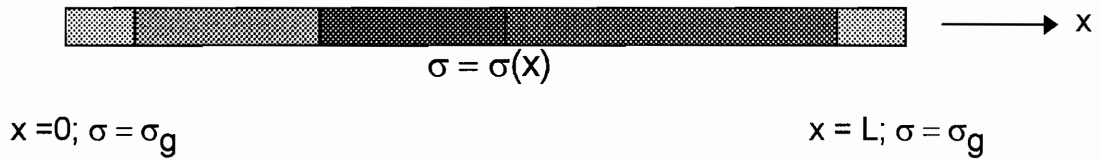


Figure 4.1
Stress Distribution as
Function of Fiber Length

It is important to note that the only assumption made in determining the local stress state is that at each end of the fiber, the stress is equal to the average global stress. Recently, a study to determine the validity of this assumption for composite material systems has been undertaken by the Materials Response Group at Virginia Polytechnic Institute & State University. Until the results of this study are obtained, the average global stress assumption will be assumed valid and used to develop the strength prediction theory.

With Equation 4.1 in mind, the fiber is now spilt into N segments or cells, each with its own intrinsic strength and flaws which, when considered with the global stress, produce a local failure criterion, F_i, as is demonstrated in Figure 4.2. Once the numeric value of F_i has surpassed a user-defined value between zero and one then the cell of interest is considered to have failed. For the case of one fiber, a single local fiber failure is sufficient for global failure. However, for most fiber-reinforced composite materials, a number of local fiber failures located in the same general area are necessary to cause global failure.

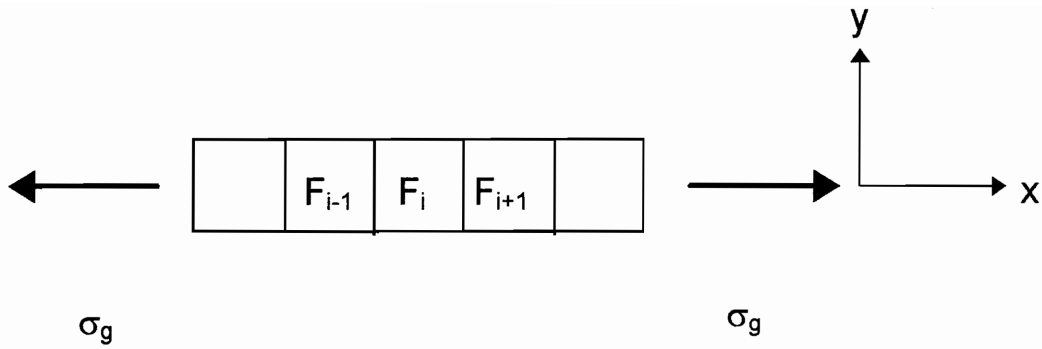


Figure 4.2

The i^{th} segment of a Fiber j

One conclusion that can be reached after examining Figure 4.2 is that the failure of the i^{th} segment can now be influenced by both the preceding and following fiber segment when the fiber is placed under a global tensile loading. In other words, the failure of the i^{th} segment, F_i, is now a function of the applied global stress; its flaws; and the intrinsic strength of that segment and its neighbors:

$$F_i = f(\sigma_g ; \Sigma X_i ; \psi_i) \tag{4.2}$$

where σ_g is the applied global stress; ψ_i is the flaw associated with the i^{th} element; and the intrinsic strength of the i^{th} segment is X_i, which is summed along the length of the fiber that influences the failure in the i^{th} segment. The failure criterion used in this study is as follows:

$$F_i = \left. \frac{\sigma_{critical}}{X} \right|_i \tag{4.3}$$

where $\sigma_{critical}$ and X are, respectively, the critical stress and intrinsic strength of the i^{th} element. Once the critical stress has either surpassed the intrinsic strength of the element or a user defined ratio of remaining strength, the local element is considered to have failed and reaches a maximum value of one. Once local failure has occurred the fiber element is removed from the load carrying capabilities of the simulation.

Because a single fiber strength theory is of little interest, it is necessary to expand Equation 4.2 to include more than one fiber. This is accomplished by introducing some assumptions to expand the single fiber theory.

The first assumption is that the composite material can be approximated by a representative square matrix or 2-D geometry as seen in Figure 4.3. This reduces the complexity and the amount of computational work while trying to maintain the fundamental aspect of the micromechanical problem. As a result, the failure function of Equation 4.3 becomes a function of i and j:

$$F_{ij} = \left. \frac{\sigma_{critical}}{X} \right|_{ij} \tag{4.4}$$

where i locates the i^{th} element on the j^{th} fiber. The area of F_{ij}'s influence on the surrounding elements must now be determined.

We will assume that the failure of the i^{th} segment is best approximated when only four segments are used, where the fibers are the two preceding (i.e. F_{ij-1}, F_{ij-2}) and two succeeding fibers (i.e. F_{ij+1}, F_{ij+2}), to determine failure of the

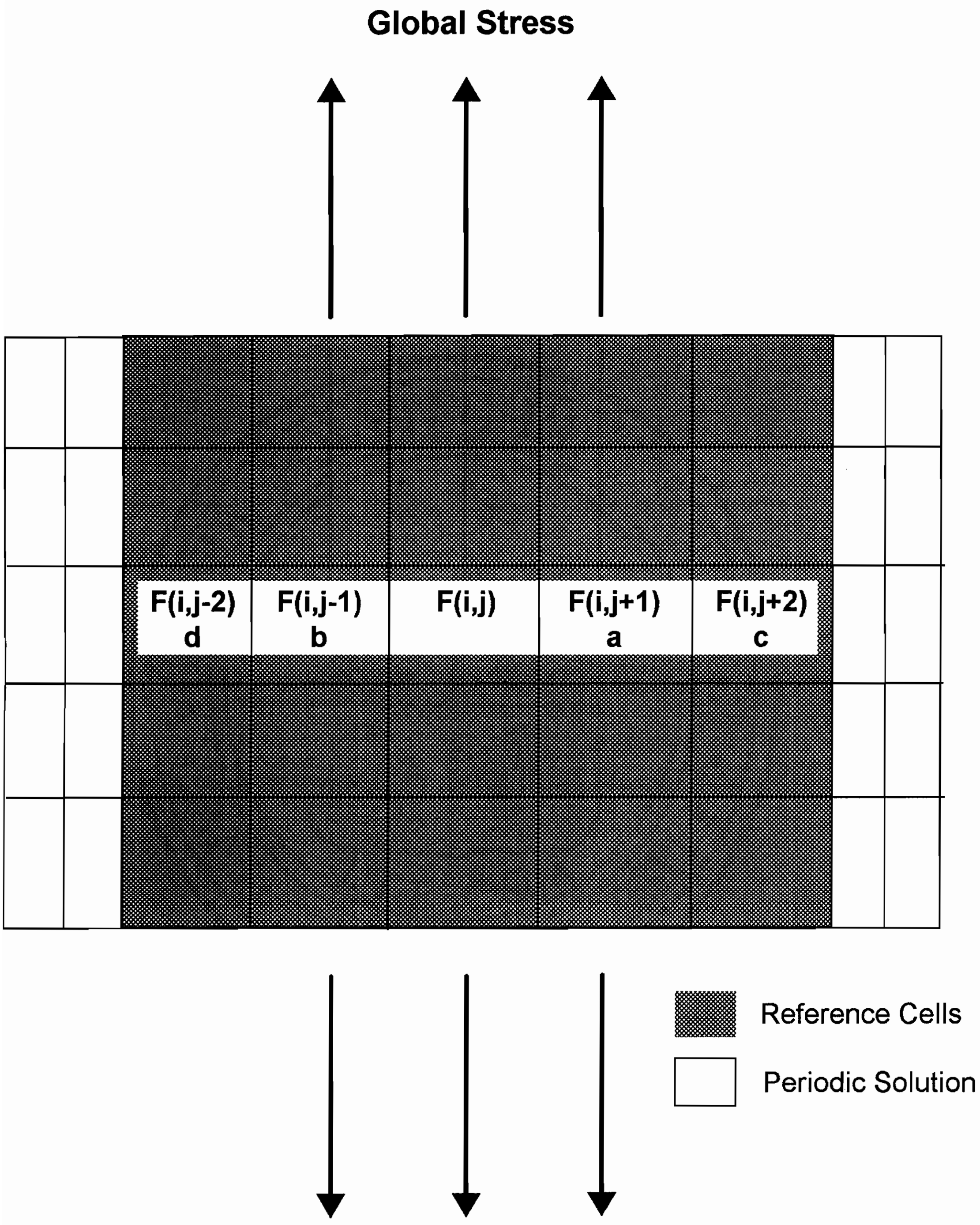


Figure 4.3

Segments Used in Failure
Path Prediction Model

reference segment. This assumption seemed to produce the best correlation between experiments performed by the Materials Response Group and theory.

The radius of the stress concentration's influence was also assumed to be limited to four fibers, two to either side of the element of interest. Because the stress concentration was modeled in this manner, it was necessary to generate the effect of the surrounding fibers on the stress concentration of the fiber of interest by using four load sharing constants, a - d. In Figure 4.3, one can see the four fiber elements employed for the model and the corresponding load sharing constant used in this study. The stress concentration for the i^{th} element is:

$$C_n = 1 + a * F_{ij+1} + b * F_{ij-1} + c * F_{ij+2} + d * F_{ij-2} \quad (4.5)$$

where C_n is a dimensionless stress concentration factor for the element and a - d are user defined load sharing constants.

The maximum value for the stress concentration, therefore, depends on the load sharing constants, a-d. For the ideal case of perfect bonding and load sharing among the fibers and matrix, the stress concentration reaches a maximum value of five. In the case of no load sharing and bonding, the minimum of one is reached. In most practical cases the stress concentration should range from around 1.2 to 1.5.

The model considers the interaction of four fibers in the direction transverse to loading. This is done to accurately predict the failure paths which are transverse to the direction of loading in unidirectional composite materials. As was discussed in the previous section, the four load sharing constants, a - d, are used to help make the model more realistic by modeling the fiber-fiber and fiber-matrix interaction. The load sharing constants also simulate the shear strength of the matrix.

Since the stress concentration depends on the ineffective length of the fiber, it is necessary to include this phenomenon into the strength prediction theory. The following equation was used to determine a nondimensional ineffective length:

$$\delta_n = g\sqrt{\frac{f}{C_n}} \tag{4.6}$$

where C_n is the stress concentration factor from Equation 4.5 and the stress constants, f and g, were proposed by Carman[28].

It is now necessary to determine the local critical stress. By using the work of Gao, et al. [9] as a basis, one can sum the stress concentrations as follows:

$$\Omega = \left(C_n{}^m + C_n{}^{m-1} + \ldots + 1\right) \tag{4.7}$$

where C_n is the stress concentration from Equation 4.5 and m is the Weibull shape factor for the material. As in Gao's model, Equation 4.7 is multiplied by the ineffective length over one plus the Weibull shape factor where the ineffective length is found using Equation 4.6 and then raised to the inverse Weibull shape factor as shown:

$$\left[\frac{\delta_n}{m+1}(\Omega)\right]^{\frac{1}{m}} \tag{4.8}$$

The last step in calculating the local critical stress is to multiply the global applied stress by Equation 4.8. Notice that the ineffective length, the stress concentration, and the intrinsic strength of each element were used in determining the local critical stress:

$$\sigma_c = \left[\frac{\delta_n}{m+1} (\Omega) \right]^{\frac{1}{m}} \sigma_g \qquad (4.9)$$

where σ_c is the critical stress for the element of interest; σ_g is the applied global stress; m is the Weibull shape factor; and δ_n is the dimensionless ineffective length for the element found using Equation 4.7.

The full size failure matrix is sandwiched by a two element wide unit value column buffer (i.e. the total matrix size is n x n+4 where n ranges from 3 on). This is done to accommodate the load sharing around the edges of the composite material. Without this buffer, the matrix would be unusable due to the fact that it will always fail in exactly the same manner regardless of what strength matrix (i.e. the matrix of X_i 's) used. In order to avoid this from happening, a periodic solution was used. In order to properly model the stress concentrations of the elements along the edges of the failure matrix, it was necessary for the failure function of the last two columns to reference the first two columns and vice versa. As stated earlier, this is known as a periodic solution and can be seen in Figure 4.3.

5.0 Results

One of the most difficult parts of any study is trying to determine which, if any, of the results are worthwhile and those destined for obscurity. Consequently, it is essential to thoroughly examine the problem before beginning to experiment. By doing so, one could determine the factors that would be most likely to produce significant results. One should not limit the study to only trying to find those results previously mentioned because, as the old adage goes, one can not see the forest through the trees. Often a notable result is produced that is completely unexpected. Therefore, it is imperative to have an open mind and eye when performing a study.

Before starting, the shape parameter, load constants and the size of the strength matrix were thought to have the biggest influence on failure. After examination, this was found to be the case.

5.1 Shape Parameter

The shape parameter plays an important role in the failure process of most composites. It represents the amount of scatter associated with the data sample being studied. It can also indicate what type of composite material is being studied. Ceramic composites are usually toward the lower end of the scale of shape parameters while metal matrix composites tend to be higher. For this experiment, the range of 1 to 15 was studied in-depth with a less exhaustive examination of theoretical numbers for validation purposes.

As one would expect from Equation 4.9, the shape parameter is an important criterion in determining the local critical stress. Multiplying each side of Equation 4.9 by the inverse of the global stress produces a normalized stress which allows one to quickly determine the effect of the shape parameter on failure.

In order to validate the program, the stress failure as a function of the shape parameter was determined for the range of 1 to 100. Although the upper bound of this range is never seen in real world applications, its use was necessary to determine whether or not the software developed for this study was working properly. The Mathcad software package was used to check the values obtained using the developed software.

In using Mathcad, it was necessary to evaluate the normalized stress as a function of both the shape parameter and the stress concentration. This was necessary to allow the Mathcad approach to simulate the numerous parameters used in the developed software. In the Mathcad approach, the shape parameter ranged from 1 to 100 while the stress concentration was held constant at a value between 1 and 2. For each value of the stress concentration used, the normalized stress approached an asymptote as the shape parameter increased. The value of the asymptote was different for each value of the stress concentration. A stress concentration of 1.3 was used to generate Figure 5.1. The value of the asymptote in Figure 5.1 is approximately 1.257. The value of 1.3 was selected because Case [21] found this to accurately model the stress concentration when three of the neighboring fibers have failed.

The next step in the process of validating the software was to run some actual test cases. Both the final stress and the normalized stress were determined as a function of the shape parameter which ranged from 1 to 15. One generally finds that the shape parameter ranges anywhere from 4 to 15 in practical applications. It was necessary to include the lower values below 4 so that the validity of the software could be determined. The rest of the parameters used in the software were chosen so that when the shape parameter had a value of 8, the resulting stress concentration was 1.301. There was virtually no difference between the

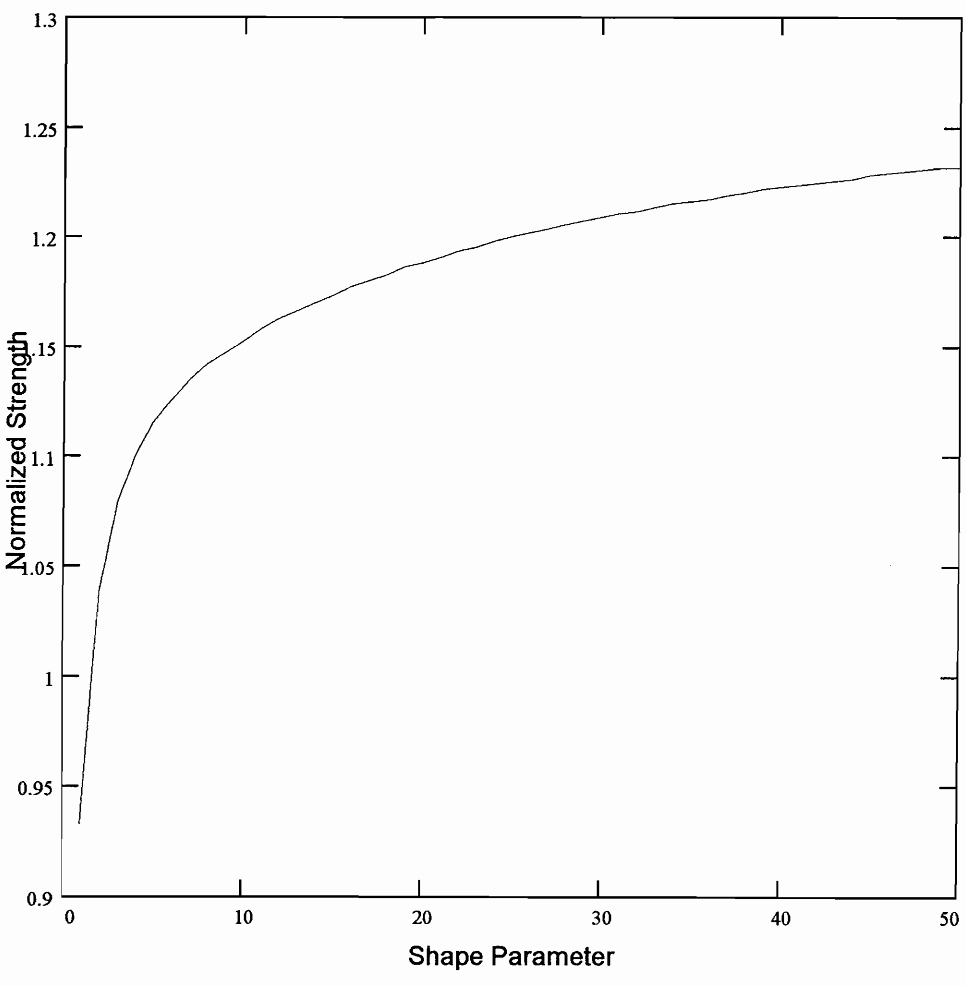


Figure 5.1

Normalized Strength as
a Function of the Shape Parameter, α

normalized stress values obtained from Mathcad and those obtained using the software. As a result, the remaining tests were completed using only the software developed for this study. The results for a sample case generated by the software can be found in Table 5.1. Similarly, the values of the parameters for the sample case are found in Table 5.2.

One conclusion reached after examining Table 5.1 is that the global stress is not very sensitive to increases in the shape parameter beyond eight. In this example, a two-fold increase in the shape parameter produces no increase in the failure load. However, one must not make the mistake of assuming that the failure processes are the same when the final stresses are identical. The failure processes are usually quite different because of the numerous parameters used in this study. For example a slight change in the load constants, a - d, might produce the same failure stress as a change in the stress constants, f and g, assuming that the rest of the parameters found in Table 5.2 are exactly identical. The failure patterns are usually dissimilar. The load constants could produce a very random failure pattern whereas the stress constants might produce a "cleaner" failure pattern. These two cases represent a strong and weak shear yielding effect, respectively and can be seen in Figure 5.2 and 5.3.

In Figures 5.4 and 5.5, one can see the result of a high and low value for alpha. In Figure 5.4, alpha was 15 while for Figure 5.5, a value of 1 was used to generate the result. The loading at failure was substantially higher for the lower value. This was expected since alpha measures the dispersion associated with the local intrinsic matrix. In other words, the matrix was closer to being a homogeneous material than a composite material. This is evident by the failure patterns in each of the figures. For a high alpha value, the failure pattern is more random than for the low alpha value. Again this is a result of the scattered associated with each matrix.

Table 5.1: Effect of Shape Parameter

Shape Parameter, α	Global Stress at Failure	Normalized Stress at Failure
1	380.3	.93
2	328.5	1.04
3	328.5	1.08
4	312.9	1.09
5	312.9	1.13
6	312.9	1.14
7	312.9	1.15
8	298.0	1.16
9	298.0	1.14
10	298.0	1.14
11	298.0	1.17
12	298.0	1.18
13	298.0	1.18
14	298.0	1.19
15	298.0	1.19

Table 5.2: Typical Parameters Used in Software

Parameters	Values
Starting Value	500
Ending Value	600
Matrix Size	10
Shape Parameter, α	8
Load Constant, a	.1000
Load Constant, b	.1000
Load Constant, c	.1000
Load Constant, d	.1000
Stress Constant, f	1.00
Stress, Constant, g	1.25
% Strength for Failure	.60
Fiber Bandwidth	4

Figure 5.2 Example of Weak Shear Yielding

Figure 5.3 Example of Strong Shear Yielding

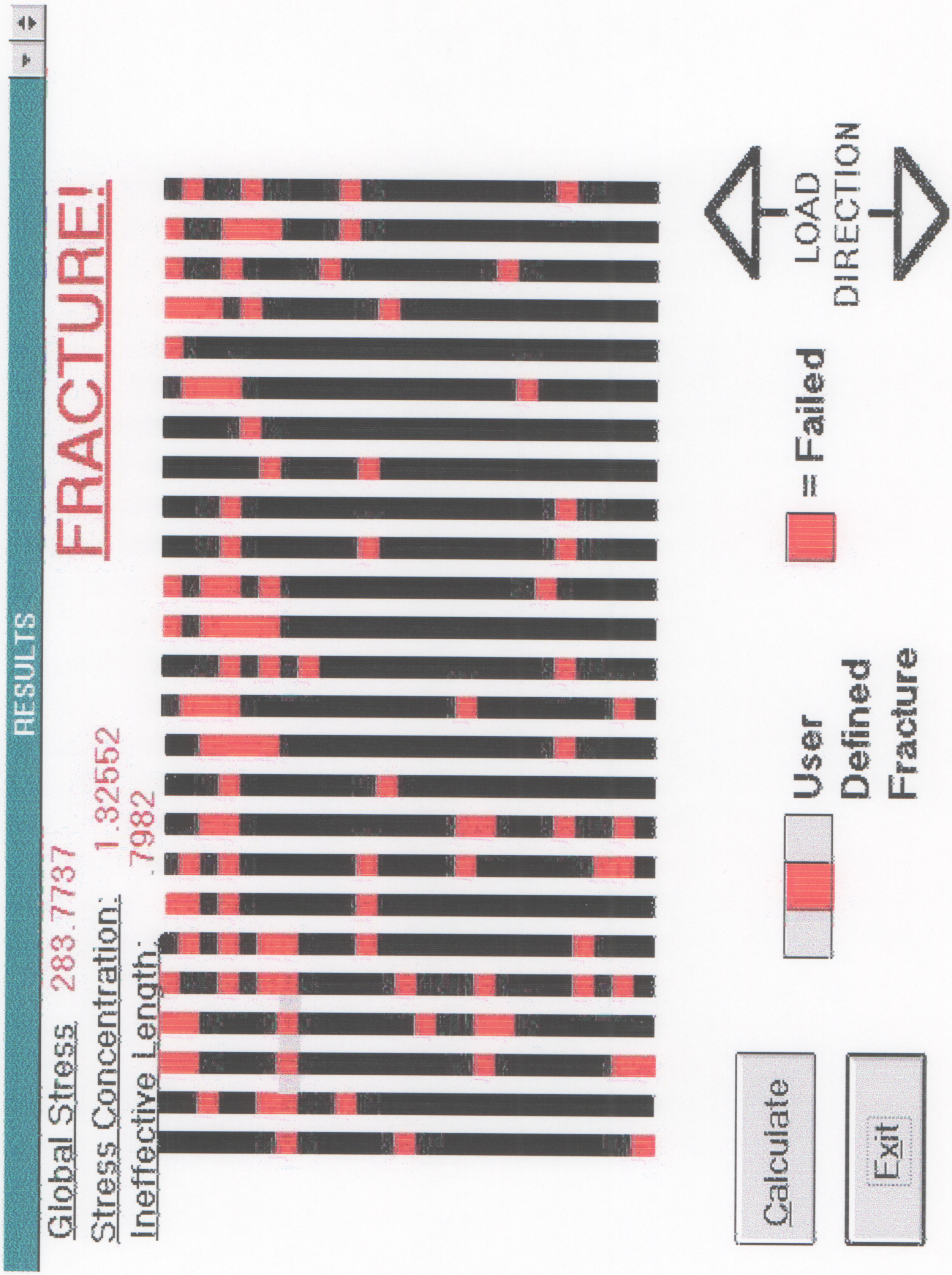


Figure 5.4 Failure Due to High α

Figure 5.5 Failure Due to Low α

5.2 Load Constants

The load constants produced a significant change in the failure process of the model. This was expected since the load constants are directly responsible for determining how the fibers share the stress concentration. In Table 5.3, one can see some general results for increasing load constants. The failure patterns associated with the lower valued load constants were typical for a strong shear yielding material systems while those of the higher valued load constants were a good representation of planar fracture.

Table 5.3: Typical Failure Loads for Different

Load Constants, a - d

Load Constants, a - d	Global Stress at Failure	Stress Concentration
0	328.5	1
.1	298.0	1.29
.2	270.0	1.64
.3	245.1	1.99
.4	222.3	2.33
.5	201.7	2.62
.6	182.9	2.93
.7	174.2	3.29
.8	158.0	3.57
.9	150.5	3.97
1	143.3	4.26

One result of particular interest is that for many of the test cases a local maximum occurs in some of the strength elements as the load constants increase. In particular, the lower valued elements of the strength matrix were reaching local minimums. One conclusion that can be made is that the program is modeling the stress redistribution after a local fiber fracture has occurred. This allows a composite material to incur more local damage before final failure. Often this extra accumulation of local damage translates into a gain in global strength. The stress redistribution effect can be seen in Figures 5.6 through 5.8. The load constants were 0, .1, and 1 in Figure 5.6, 5.7, and 5.8, respectively. When high load constants are used, the failure patterns are planar fracture. The band width is clearly present in Figure 5.8. The low load constants represent high shear yielding. The failure patterns are more random than those of the high load constants. This occurs because, while, the stress is still being redistributed in the lower valued load constants, it is at a fraction of the strength of the higher load constant values.

In Figure 5.9, one can see the total failure of the matrix. This result is reached when the failure calculation is manually performed until the entire matrix has reached failure. When a high load constant is employed then the total failure happens very quickly, often within the next calculation. When a low constant is used, the matrix will often last for 10 or more calculations.

Figure 5.6 Local Damage caused by Load Constants; a-d = 0

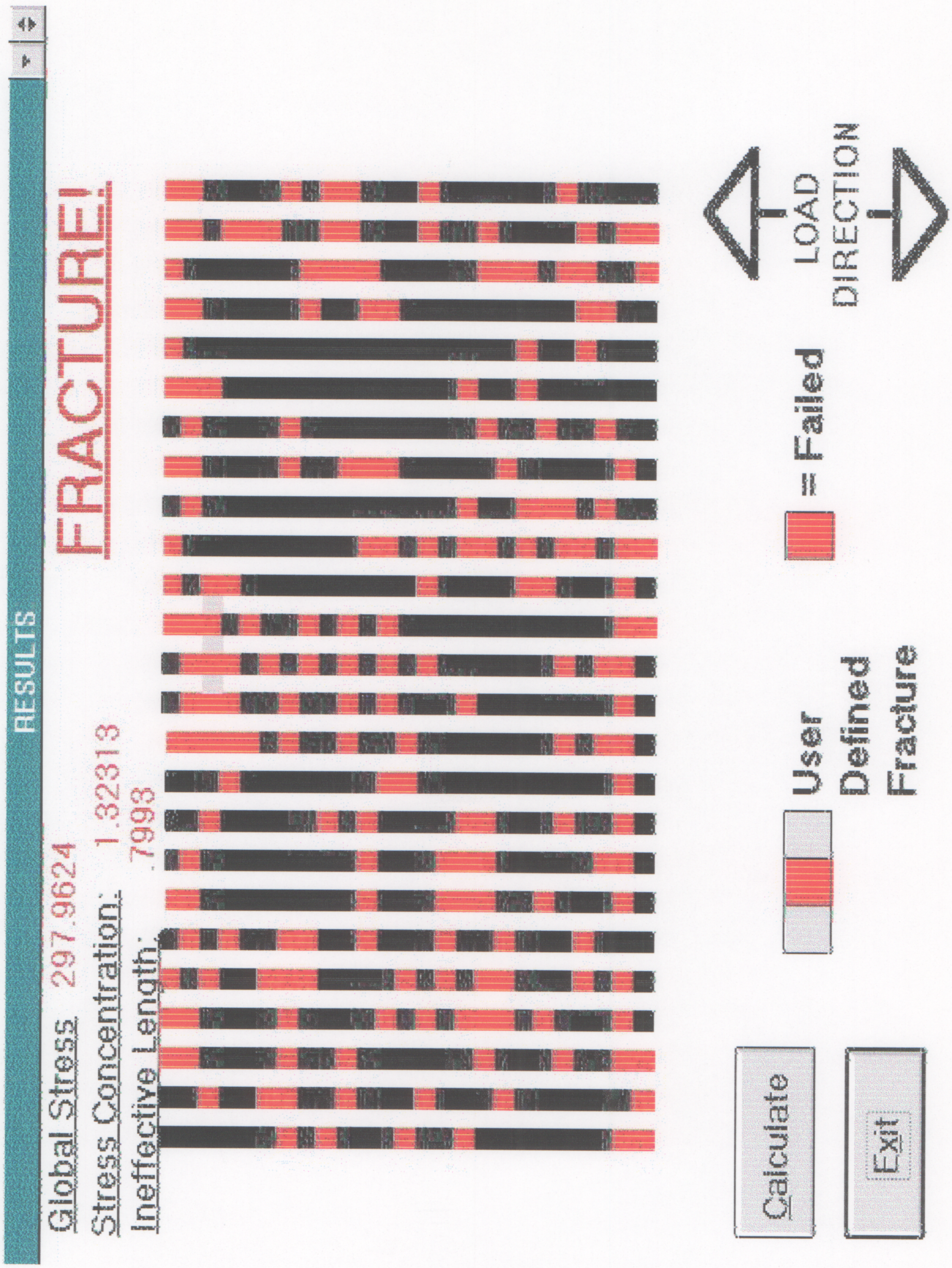


Figure 5.7 Local Damage caused by Load Constants; a-d = 0.1

Figure 5.8 Local Damage caused by Load Constants; a-d = 1

Figure 5.9 Total Failure

5.3 Matrix Size

Failure seemed to be independent of the size for matrices larger than 10 x 10. When using a probabilistic approach, a larger matrices will often fail at a lower global stress than a smaller ones. One possible explanation for this asymptote is the manner in which the program calculates failure. The calculation goes from the upper left hand corner down to the lower right hand corner and then returns to the upper left hand corner. If the program were to say, calculate in either a clockwise or counter-clockwise manner, then the results would be slightly different. In order to asses which calculation is most accurate of the real mechanics some experimental work is necessary and is left as future work.

A sample case for matrices of size 10, 15, 20, and 25 is shown in Figures 5.7 through 5.10, respectively. In order to produce similar results, one should generate the largest matrix first and then save it. This allows one to retrieve the original matrix after examining the effect of the size of the strength matrix. The most important result from the study of the matrix size is that the stress concentration remains relatively constant despite changes in the matrix size. In the example cases, the stress concentration stays within 2.5 % of the 1.3 theoretical value discussed earlier. This is another validation of the program since the stress concentration should be independent of the matrix size.

Another important aspect of the matrix size is the range between the starting and ending values used to generate the strength matrix. If this range is too large then the remaining parameters are rendered useless. For a large range between the starting and ending values, typically, a very random failure would occur because a strong element would reside next to a very weak one. Therefore, the failure would be independent of the rest of the parameters and would follow a very random pattern. This is a result of the randomizing process used in Visual Basic. In order to control this scheme, it is necessary to limit the range of the starting and ending values to a maximum of 300 at most. When the range was limited to between 100 and 150, the failure was controlled by the other parameters which was the desired result.

Figure 5.10 Failure of a 10x10 Matrix

Figure 5.11 Failure of a 15x15 Matrix

Figure 5.12 Failure of a 20x20 Matrix

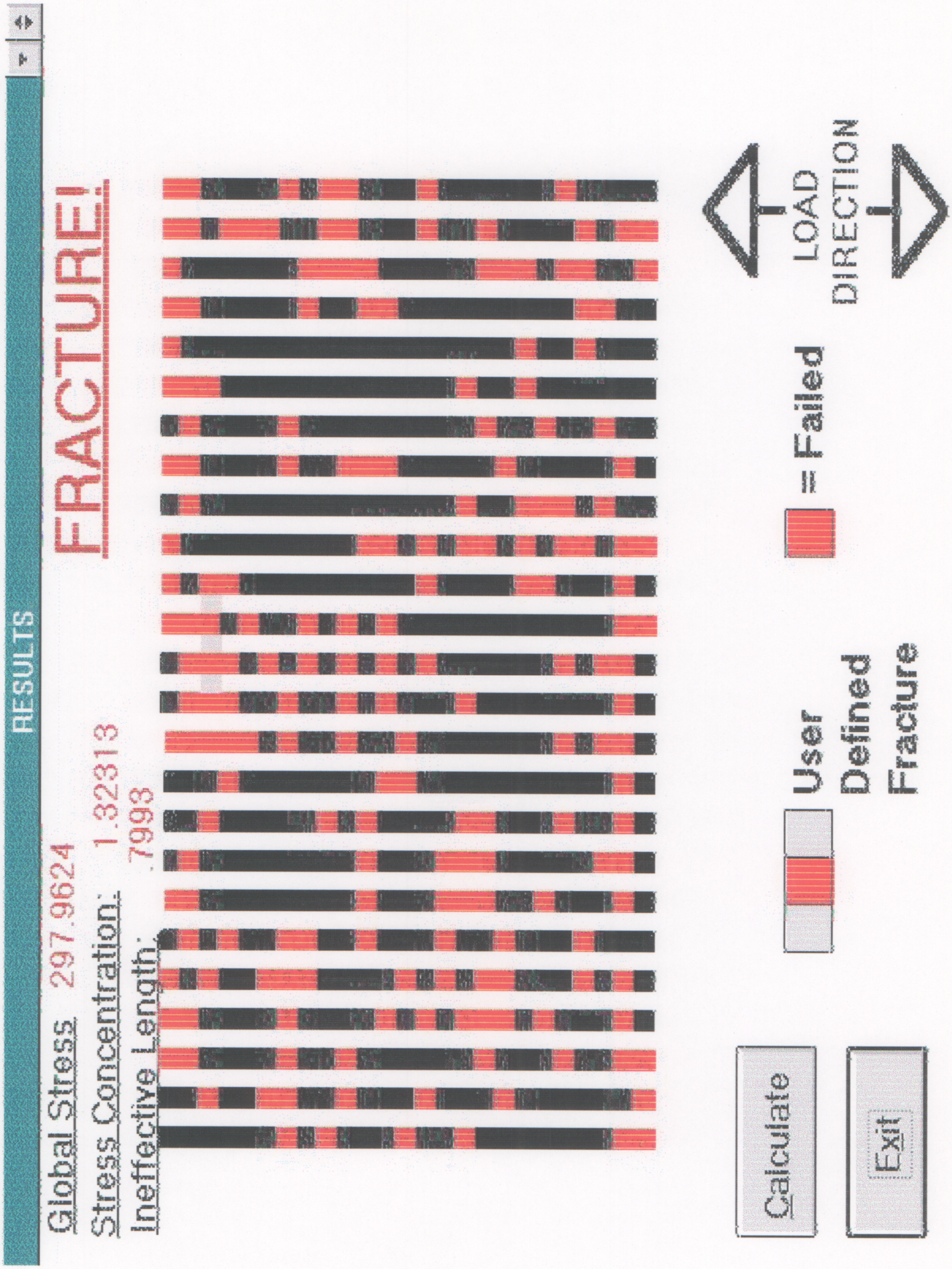


Figure 5.13 Failure of a 25x25 Matrix

6.0 Conclusions

This thesis presented a novel strength prediction approach that employed various parameters to simulate the different micromechanical phenomena associated with failure. The model utilizes eleven variables to determine the final stress at failure for a square matrix that is considered to be representative of the material. This methodology grants considerable freedom in determining which factor dominates failure. This freedom may be used to model the many different phenomena of composite failure. The author hopes that by using this approach that the uncertainty associated with the failure of composite materials can be reduced.

One such reduction of uncertainty was observed. The model can predict failure for both strong and weak shear yielding material systems. By altering the load constants, one can simulate either of the shear yielding effects. Although the results look promising, some actual experimental testing is necessary to fully validate the model. One possible approach is to use macro-model composites. These composites allow the designer to tailor a situation so that a known outcome is highly probable. This work is left for future consideration.

Another possible future study is to extend the model to three dimensions. The model could then be compared with accepted strength prediction theories such as classical laminate theory (CLT). Only then, would the model presented in this study be accepted in both academia and industry.

The range used to generate the strength matrix was found to play an important role in deterring failure. If the range was too large then the remaining parameters were ineffective. Only by limiting the range could one use the parameters to determine failure.

The software would also need to be updated to accommodate any and all changes in the model. Because it was written in Visual Basic, any changes are relatively easy to make. This was one of the best features about the programming language.

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Appendix A

```
'Strength Calculation
 While check <> True
  iter = iter + 1
 For i = 1 To vn
  For j = 3 To vn + 2
  Cn = 0!
  Cn = 1 + va * p(i, j + 1) + vb * p(i, j - 1) + vc * p(i, j + 2) + vd * p(i, j - 2)
  dN! = (vf / Cn) ^ (1 / vg)
   den! = 0
   For m = valpha To 0 Step -1
    den! = den! + Cn ^ (m)
   Next m
  inv = 1# / valpha
  sc(i, j) = vsigma / (((dN! / (valpha + 1)) * den!) ^ inv)
  p(i, j) = CCur(sc(i, j) / x(i, j))
  If p(i, j) > 1 Or p(i, j) > CCur(vfailure) Then
   p(i, j) = 1
   count = count + 1
  End If
   If vfiber = 4 Or vfiber = 5 Then
   lim = vn - 2
   ElseIf vfiber = 3 Then
   lim = vn - 1
   Else
   lim = vn
   End If

   If j > 4 And j < lim Then
    Select Case Int(vfiber)
     Case 1
      If p(i, j - 1) > CCur(vfailure) Then
       screen.MousePointer = 0
       findings.Label3.Caption = "FRACTURE!"
       findings.Show
       Exit Sub
      End If
     Case 2
      If p(i, j - 1) > CCur(vfailure) And p(i, j) > CCur(vfailure) Then
       screen.MousePointer = 0
```

```
        findings.Label3.Caption = "FRACTURE!"
        findings.Show
        holdi = i
        holdj = j
        Exit Sub
       End If
      Case 3
       If p(i, j - 1) > CCur(vfailure) And p(i, j) > CCur(vfailure) And p(i, j + 1) >
 CCur(vfailure) Then
        screen.MousePointer = 0
        findings.Label3.Caption = "FRACTURE!"
        findings.Show
        holdi = i
        holdj = j
       Exit Sub
       End If
      Case 4
       If p(i, j - 1) > CCur(vfailure) And p(i, j) > CCur(vfailure) And p(i, j + 1) >
 CCur(vfailure) And p(i, j + 2) > CCur(vfailure) Then
        screen.MousePointer = 0
        findings.Label3.Caption = "FRACTURE!"
        findings.Show
        holdi = i
        holdj = j
        Exit Sub
       End If
      Case 5
       If p(i, j - 2) And p(i, j - 1) > CCur(vfailure) And p(i, j) > CCur(vfailure) And p(i,
j + 1) > CCur(vfailure) And p(i, j + 2) > CCur(vfailure) Then
        screen.MousePointer = 0
        findings.Label3.Caption = "FRACTURE!"
        findings.Show
        holdi = i
        holdj = j
        Exit Sub
       End If
     End Select
    End If
   den! = 0
   Next j
   p(i, 1) = p(i, vn + 1)
   p(i, 2) = p(i, vn + 2)
   p(i, vn + 3) = p(i, 3)
```

```
 p(i, vn + 4) = p(i, 4)
Next i
For i = vn To 1 Step -1
 For j = vn + 2 To 3 Step -1
  Cn = 0
  Cn = 1 + va * p(i, j + 1) + vb * p(i, j - 1) + vc * p(i, j + 2) + vd * p(i, j - 2)
  dN! = (vf / Cn) ^ (1 / vg)
   den! = 0
  For m = valpha To 0 Step -1
   den! = den! + Cn ^ (m)
  Next m
  inv = 1 / valpha
  sc(i, j) = vsigma / (((dN! / (valpha + 1)) * den!) ^ inv)
  p(i, j) = CCur(sc(i, j) / x(i, j))
 Next j
Next i
 it.Caption = iter
 vsigma = vsigma * 1.035
 sigma.Caption = CCur(vsigma)
 sigma.Refresh
 norm = 2250 - Fix((vsigma / avg) * 1500)
 xn = xn + 170 / 2
 stressplot.PSet (350 + xn, norm), QBColor(0)
 stressplot.Refresh
Wend
```

Vita

Michael Patrick Mc Clain was born on November 24, 1970 in Bridgeton, New Jersey to Howard and Amanda Mc Clain. He has one younger brother whose name is Christopher. In 1989, Michael began his study of engineering and life in general at Virginia Polytechnic Institute & State University. After meandering through the first two and a half years he realized that he had no plans for the future. At this point, he rededicated himself to his work and received his Bachelor's degree in May 1994. He then decided to continue schooling to try and quench his thirst for knowledge. After two years and many sleepless nights, Michael decided that he thirst had been at least quenched for the time being. His future plans are still in the air but knows that his thirst will return which means a Ph.D. is probably in somewhere in his future.

Michael P. McClain