Ratchet Model of Baryogenesis

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We propose a toy model of baryogenesis which applies the 'ratchet mechanism,' used frequently in the theory of biological molecular motors, to a model proposed by Dimopoulos and Susskind.

Keywords: baryogenesis, ratchet mechanism, Dimopoulos-Susskind model

1. Introduction

The ratio of baryon-number to photon-number densities in our universe has been established via Big-Bang Nucleosynthesis (BBN) [1–3] and WMAP [4, 5] to be

$$\eta \;=\; \frac{n_B}{n_\gamma} \;\approx\; 6\times 10^{-10}\;. \tag{1}$$

The more precise numbers are

$$\begin{aligned} \eta_{10}(\mathrm{BBN:D/H}) \;&=\; 5.8\ \ \pm 0.3\ \ ,\\ \eta_{10}(\mathrm{WMAP\!:\,7yr}) \;&=\; 6.18\pm 0.15\;, \end{aligned} \tag{2}$$

where $\eta_{10} = 10^{10}\,\eta$, and the BBN value is determined from the deutron abundance reported in Ref. [6, 7]. As we can see, the agreement is very good.

The objective of baryogenesis is to explain how the above number can come about from a universe initially with zero net baryon number. Since the pioneering work of Sakharov [8], very many proposals have been made as to what this baryogenesis mechanism could be.[a] Among the early ones was a model by Dimopoulos and Susskind [15] in which baryon number is generated via the coherent semi-classical time-evolution of a complex scalar field. Similar mechanisms have been employed by Affleck and Dine [16], Cohen and Kaplan [17], and Dolgov and Freese [18], of which the Affleck-Dine mechanism has been popular and intensely studied due to its natural implementability in SUSY models. Two of us have also considered the

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[a]For recent reviews, see Refs. [9–14].

application of the Dimopoulos-Susskind model to the cosmological constant problem [19].

In this talk, I will discuss the Dimopoulos-Susskind model, how it satisfies Sakharov's three conditions for baryogenesis, in particular, how it uses the expansion of the universe to satisfy the third, and then propose the 'ratchet mechanism' [20–22] as an alternative for driving the model away from thermal equilibrium.

2. The Dimopoulos-Susskind Model

Consider the action of a complex scalar field given by

$$S \;=\; \int d^4x\,\sqrt{-g}\,\Big[\,g^{\mu\nu}\partial_\mu\phi^\dagger\partial_\nu\phi - V(\phi,\phi^\dagger)\,\Big]\,. \tag{3}$$

If the potential $V(\phi,\phi^\dagger)$ is invariant under the global change of phase

$$\phi \;\to\; e^{i\xi}\phi\,,\qquad \phi^\dagger \;\to\; e^{-i\xi}\phi^\dagger\,, \tag{4}$$

then the corresponding conserved current is

$$B_\mu \;=\; \sqrt{-g}\Big(i\,\phi\,\overleftrightarrow{\partial_\mu}\,\phi^\dagger\Big)\,. \tag{5}$$

If we identify B_0 with the baryon number density, then adding to the action a potential which is not invariant under the above phase change, such as

$$V_0(\phi,\phi^\dagger) \;=\; \lambda\,\big(\,\phi+\phi^\dagger\big)\big(\,\alpha\,\phi^3+\alpha^*\phi^{\dagger 3}\big)\,,\qquad |\alpha| \;=\; 1\,, \tag{6}$$

would lead to baryon number violation. Furthermore, unless $\alpha=\pm 1$, this potential also violates C and CP since ϕ transforms as

$$\begin{aligned}\phi(t,\vec{x}) \;&\xrightarrow{C}\; \phi^\dagger(t,\vec{x})\,,\\ \phi(t,\vec{x}) \;&\xrightarrow{CP}\; \pm\phi^\dagger(t,-\vec{x})\,,\end{aligned} \tag{7}$$

where the sign under CP depends on the parity of ϕ. (P is not violated.)

In Ref. [15], Dimopoulos and Susskind subject ϕ to the potential

$$V_n(\phi,\phi^\dagger) \;=\; \lambda\,\big(\phi\,\phi^\dagger\big)^n\big(\,\phi+\phi^\dagger\big)\big(\,\alpha\,\phi^3+\alpha^*\phi^{\dagger 3}\big)\,. \tag{8}$$

The purpose of the factor $(\phi\,\phi^\dagger)^n$ is simply to give the coupling constant λ a negative mass dimension. Setting $\phi=\phi_r\,e^{i\theta}/\sqrt{2}$, the baryon number density becomes

$$n_B \;=\; B_0 \;=\; \sqrt{-g}\;\phi_r^2\,\dot{\theta}\,, \tag{9}$$

which shows that to generate a non-zero baryon number n_B, one must generate a non-zero $\dot{\theta}$. The potential in the polar representation of ϕ is

$$V_n(\phi_r,\theta) \;=\; \lambda\left(\frac{\phi_r^2}{2}\right)^n\phi_r^4\,\cos\theta\,\cos(3\theta+\beta)\,, \tag{10}$$

where we have set $\alpha=e^{i\beta}$. The θ-dependence of this potential for fixed ϕ_r is shown in Fig. 1 for the case $\beta=\pi/2$. Note that under B, C, and CP, the phase θ

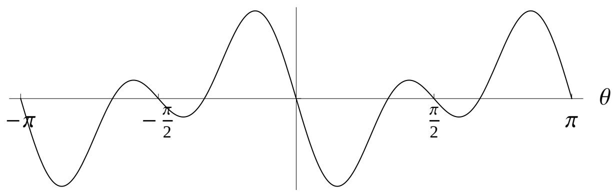

Fig. 1. θ-dependence of the B, C, and CP violating potential, Eq. (10), for the case $\beta = \pi/2$.

transforms as

$$\begin{aligned} \theta(t,\vec{x}) &\xrightarrow{B} \theta(t,\vec{x}) + \xi\ , \\ \theta(t,\vec{x}) &\xrightarrow{C} -\theta(t,\vec{x})\ , \\ \theta(t,\vec{x}) &\xrightarrow{CP} -\theta(t,-\vec{x})\ . \end{aligned} \tag{11}$$

If the parity of ϕ is negative, then θ will also be shifted by π under CP. So in terms of θ, the violation of B is due to the loss of translational invariance, and the violation of C and CP are due to the loss of left-right reflection invariance which happens when $\beta \neq 0, \pi$. The question is, can the asymmetric force provided by this potential make θ flow in one preferred direction thereby generate a non-zero $\dot{\theta}$? For that, one must move away from thermal equilibrium.

In the original Dimopoulos-Susskind paper [15], this shift away from thermal equilibrium is accomplished by the expansion of the universe. Consider a flat expanding universe with the Friedman-Robertson-Walker metric:

$$ds^2 \;=\; dt^2 - \left(\frac{a(t)}{a_0}\right)^2 d\vec{x}^2\ . \tag{12}$$

During a radiation dominated epoch, the scale factor evolves as

$$\frac{a(t)}{a_0} \;\sim\; \sqrt{2t}\ . \tag{13}$$

Introducing the conformal variable $\tau = \sqrt{2t}$, the line-element becomes

$$ds^2 \;=\; \tau^2\left(d\tau^2 - d\vec{x}^2\right)\ , \tag{14}$$

while the action simplifies to

$$S \;=\; \int d^3\vec{x}\,d\tau\left[\partial_\mu\hat{\phi}^\dagger\partial^\mu\hat{\phi} - \frac{1}{\tau^{2n}}V_n(\hat{\phi},\hat{\phi}^\dagger) + \cdots\right]\ . \tag{15}$$

Here, the scalar field has been rescaled to $\hat{\phi} \equiv \tau\phi$, and the ellipses represent total divergences and terms that depend only on $|\hat{\phi}|$.

At this point, a simplifying assumption is made that the dynamics of $|\hat{\phi}|$ is such that it is essentially constant and does not evolve with τ, leaving only the phase of $\hat{\phi}$ as the dynamic variable.[b] Setting $\hat{\phi} = e^{i\theta}/\sqrt{2}$, the action within a domain of spatially constant θ becomes

$$S = \int d^3\vec{x}\, d\tau \left[\frac{1}{2} \left(\frac{d\theta}{d\tau} \right)^2 - \frac{1}{\tau^{2n}} V_n(1, \theta) \right] . \tag{16}$$

The equation of motion for θ within that domain is then

$$\frac{d^2\theta}{d\tau^2} + \frac{1}{\tau^{2n}} \frac{\partial V_n}{\partial \theta} = 0 . \tag{17}$$

To this, a friction term, which is assumed to come from the self-interaction of $\hat{\phi}$, is added by hand as

$$\frac{d^2\theta}{d\tau^2} + \frac{1}{\tau^{2n}} \frac{\partial V_n}{\partial \theta} + \frac{\lambda^2}{\tau^{4n}} \frac{d\theta}{d\tau} = 0 , \tag{18}$$

where the coefficient of $d\theta/d\tau$ has been fixed simply by dimensional analysis. If $n > 0$, both force and friction terms vanish in the limit $\tau \to \infty$, and it is possible to show that a non-zero $n_B \sim d\theta/d\tau$ survives asymptotically, its final value depending on the initial value of θ. This initial value is expected to vary randomly from domain to domain, resulting in different asymptotic baryon numbers in each, and when summed results in an overall net baryon number. On the other hand, if $n = 0$, which would make the self-interactions of ϕ renormalizable, the friction term will eventually bring all motion to a full stop.

3. The Ratchet Mechanism

A striking feature of the Dimopoulos-Susskind model is its similarity with the problem of biased random walk one encounters in the modeling of biological motors [20–22]. An example of a biological motor is the myosin molecule which walks along actin filaments. This molecule is modeled as moving along a periodic sawtooth-shaped potential, similar to that shown in Fig. 1. Thermal equilibrium inside a living organism is broken by the presence of ATP (adenosine triphosphate) whose hydrolysis into ADP (adenosine diphosphate) and P (phosphate) provides the energy required to fuel the motion:

$$\mathrm{ATP} \;\to\; \mathrm{ADP} + \mathrm{P} + \mathrm{energy} . \tag{19}$$

This is often modeled as a randomly fluctuating temperature of the thermal bath: the molecule is excited out of a potential well during periods of high-temperature, allowing it to diffuse into the neighboring ones, and then drops back into a well during periods of low-temperature. Due to the asymmetry of the potential, this sequence can lead to biased motion depending on the depth and width of the repeating potential wells, and the height and frequency of the temperature fluctuations.

[b] This assumption that $|\hat{\phi}|$ is constant would require the magnitude of the unscaled field $|\phi|$ to evolve as $1/\tau = 1/\sqrt{2t}$.

Analogy with such 'temperature ratchet' models suggests a possible way to drive the evolution of θ in the Dimopoulos-Susskind model without relying on the non-renormalizability of the self-interaction of ϕ, or the expansion of the universe directly. Let us assume the existence of ATP- and ADP-like particles A and B which interact with ϕ via the reaction

$$A + \phi \;\leftrightarrow\; B + \phi + Q\;, \tag{20}$$

where Q is the energy released in the reaction. A and B are assumed to be stable (or highly meta-stable) states that have fallen out of thermal equilibrium at an earlier time in the evolution of the universe. Though they interact with ϕ, giving or taking energy away from it, their masses are such that the decay

$$A \to B + \phi + \bar{\phi} \tag{21}$$

is kinematically forbidden.

In order to isolate the effect of the presence of a bath of these particles, we neglect the expansion of the universe and subject $\phi = \phi_r\, e^{i\theta}/\sqrt{2}$ to the $n = 0$ renormalizable Dimopoulos-Susskind potential $V_0(\phi_r, \theta)$. We again adopt the simplifying assumption that the evolution of ϕ_r is suppressed. Though the interactions between ϕ and the A and B particles occur randomly, we model their effect by a periodically fluctuating kinetic energy of θ [20] :

$$K(t) \;=\; K_0\Bigl[1 + A\sin(\omega t)\Bigr]^2\;. \tag{22}$$

This function oscillates between $K_{\min} = K_0(1-A)^2$ and $K_{\max} = K_0(1+A)^2 = K_{\min} + Q$. Therefore,

$$Q \;=\; 4K_0 A\;. \tag{23}$$

Then, the equation of motion of θ in our model will be given by the Langevin equation

$$\phi_r^2\,\ddot{\theta} \;=\; -\frac{\partial V_0}{\partial\theta} - \eta\,\dot{\theta} + \sqrt{4\eta K(t)}\,\xi(t)\;, \tag{24}$$

where η is the coefficient of friction, and $\xi(t)$ is Gaussian white noise:

$$\langle \xi(t)\rangle \;=\; 0\;, \qquad \langle \xi(t)\xi(s)\rangle \;=\; \delta(t-s)\;. \tag{25}$$

The above Langevin equation is equivalent to the following Fokker-Planck equation governing the evolution of the probability density $p(\theta, t)$ and the probability current $j(\theta, t)$:

$$\begin{aligned} 0 &= \frac{\partial p(\theta,t)}{\partial t} + \frac{\partial j(\theta,t)}{\partial\theta}\;, \\ j(\theta,t) &= -\frac{1}{\eta}\left[\frac{\partial V_0}{\partial\theta}\,p(\theta,t) + 2K(t)\frac{\partial p(\theta,t)}{\partial\theta}\right]\;. \end{aligned} \tag{26}$$

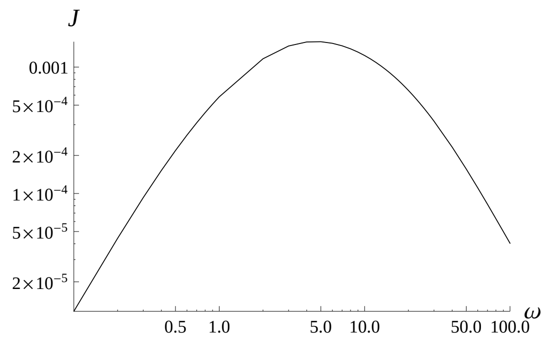

Fig. 2. ω-dependence of J for the case $\beta = \pi/2$, $K_{\min} = 0.5$, $Q = \eta = \phi_r = \lambda = 1$.

The quantity of interest for baryon number generation is the period-averaged probability current

$$J \;=\; \frac{1}{\mathcal{T}}\int_0^{\mathcal{T}} j(\theta,t)\,dt\;, \tag{27}$$

which is asymptotically independent of θ and approaches a constant, a non-zero value signifying a non-zero baryon number. For the sake of simplicity, we set $\beta = \pi/2$, and ϕ_r, λ, and η all equal to one. We then solved these equations numerically for various values of $K_{\min}$, ω, and Q, and have found that non-zero J can be generated for a very wide range of parameter choices. As an example, we show the ω-dependence of J for the case $K_{\min} = 0.5$ and $Q = 1$ in Fig. 2. Further details of our analysis can be found in Ref. [23].

4. What is the ATP-like particle?

Whether the ratchet mechanism we are proposing here can be embedded into a realistic scenario remains to be seen. Of particular difficulty may be maintaining a sufficiently large population of the ATP-like particles to drive the ratchet. But what can these ATP-like particles be? Several possibilities come to mind: First, it could be the inflaton at reheating, transferring energy to the ϕ field via parametric resonance. Second, they could be heavy Kaluza-Klein (KK) modes in some extra-dimension model. And third, perhaps they could be technibaryons transferring energy to technimeson ϕ's. Finally, regardless of what their actual identities are, if the ATP-like particles are highly stable and still around, they may constitute dark matter, thereby connecting baryogenesis with the dark matter problem. These, and other possibilities will be discussed elsewhere [24].

Acknowledgments

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