CHAPTER VII. Electromagnetic Field Strength

1. Role of the Electromagnetic Field

One of the goals of this research is to develop strategies for heating a ceramic rod to a high steady state temperature while avoiding thermal runaway. Some questions that arise are, "Is there a required electric field that will accomplish this goal?" and "If so, can the required electric field be determined?" The electric field obviously plays a major role in microwave heating since the heat source depends on it through

$$\dot{q} = 2\pi f \varepsilon_o \varepsilon''(T) |E|^2 . \tag{3.1}$$

By controlling the electric field it is hoped that we can also develop an appropriate steady state temperature. If high temperatures can be achieved using microwave energy, it must be accomplished by controlling the electric field strength and shape, since the material properties cannot be controlled.

2. Calculation of the Electric Field

The required electric field strength can indeed be calculated given any desired steady state temperature and assuming a uniform field. First we assume that a steady state temperature can be reached. This assumption allows us to simplify the governing equation by removing the time dependence. Thus the conduction equation

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) - \rho v\frac{\partial}{\partial x}(C_p T) + \frac{1}{r}\frac{\partial}{\partial r}(kr\frac{\partial T}{\partial r}) + \dot{q} = \rho C_p \frac{\partial T}{\partial t} \qquad (3.2)$$

simplifies to

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) - \rho v\frac{\partial}{\partial x}(C_p T) + \frac{1}{r}\frac{\partial}{\partial r}(kr\frac{\partial T}{\partial r}) + \dot{q} = 0. \qquad (3.3)$$

Next, we assume that the rod is sufficiently long that the temperature is uniform in some finite region in the axial direction. This allows us to eliminate x-derivatives from Eq. 3.3 to obtain

$$\frac{1}{r}\frac{\partial}{\partial r}(kr\frac{\partial T}{\partial r}) + \dot{q} = 0. \qquad (3.4)$$

If the rod is too short, this assumption is not valid. Due to axial conduction, a steeper temperature change occurs at the entrance and exit of the microwave cavity than near the center. For a long cavity, the axial temperature profile shows a flat region near the center and a temperature decrease near the ends. Since the entrance and exit are close together in a short cavity, this flat region cannot occur since most of the heat is conducted to the ends. An illustration of this principle is depicted in Fig. 3.1.

Equation 3.4 can now be solved given the following two boundary conditions:

$$@\ r=0, \qquad \frac{\partial T}{\partial r} = 0 \qquad (3.5)$$

and

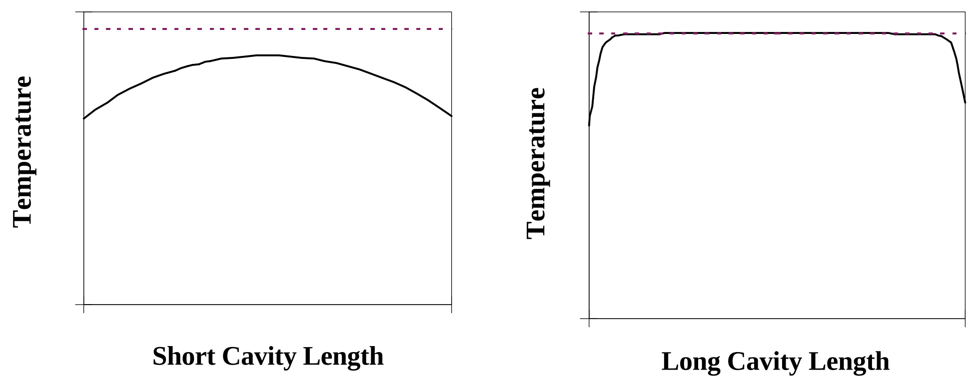

Figure 3.1 Comparison of Temperature Profile of Stationary Rods in Long and Short Microwave Cavities

$$@\ r=R, \qquad -k\frac{\partial T}{\partial r} = h(T_s - T_\infty) + h_r(T_s - T_{wall})\,. \qquad (3.6)$$

Integration of Eq. 3.4, assuming $\dot{q}$ = constant, yields

$$T(r) = -\frac{1}{4k}\dot{q}\,r^2 + \frac{h}{h+h_r}T_\infty + \frac{h_r}{h+h_r}T_{wall} + \frac{1}{4k}\dot{q}\,R^2 + \frac{1}{2(h+h_r)}\dot{q}\,R\,. \qquad (3.7)$$

For any given heat source $\dot{q}$ a steady state temperature profile can be calculated using Eq. 3.7. However, we want to find the electric field that will produce a given steady temperature. Substituting Eq. (3.1) into Eq. (3.7), we solve the resulting equation for E to find

$$E = \left\{ \frac{T(r) - \dfrac{h}{h + h_r} T_\infty - \dfrac{h_r}{h + h_r} T_{wall}}{\left[\dfrac{R^2}{4k}\left(1 - \dfrac{r^2}{R^2}\right) + \dfrac{R}{2(h + h_r)} \right] 2\pi f \varepsilon_o \varepsilon''} \right\}^{1/2} . \qquad (3.8)$$

Equation 3.8 can be reduced further if it assumed that the cavity wall temperature is equal to the ambient temperature ($T_{wall} = T_{amb}$) and that the rod temperature is evaluated at the surface so that r = R. These two assumptions are incorporated into Eq. 3.8 to give

$$E = \left[\frac{\left(T(R) - T_\infty\right)\left(h + h_r\right)}{R\pi f \varepsilon_o \varepsilon''} \right]^{1/2} . \qquad (3.9)$$

For any steady temperature, the corresponding electric field can be determined. For example, if a 2mm diameter rod needs to be heated to 1000^{o}C, we can find the electric field that will generate that temperature by simply substituting the correct information into Eq. 3.8 or Eq. 3.9. A computer program to calculate the "steady state" electric fields is included in the appendix, and Fig 3.2 shows the electric field as a function of temperature for 2 mm and 4.67 mm diameter mullite rods of length 34 mm.

Figure 3.2 shows an increase in field strength at low temperatures, a decrease at mid-ranged temperatures and again an increase at high temperatures. At temperatures less than about 750^{o}C, higher temperatures require higher electric field strengths. A larger field strength is required because the dielectric loss varies little within this temperature range. Equation 3.9 shows that if the dielectric loss remains constant, the electric field must increase for an increase in the steady state temperature. It is noted that in Eq. 3.9, the radiation heat transfer coefficient, h_r, increases with temperature and the convection heat transfer coefficient, h, remains approximately constant.

The required field strength decreases with steady state temperature in the temperature range of 750°C to 1200°C. In this temperature range, the dielectric loss appears to increase exponentially with temperature (Fig. 2.3), thus decreasing the required field according to Eq. 3.9.

Finally, for temperatures greater than 1200°C, the required electric field strength again increases with temperature. The model used for the dielectric loss of mullite assumes a linear increase with temperature at temperatures greater than 1000°C. However, the radiation heat transfer coefficient, hr, is proportional to the temperature raised to the fourth power. Equation 3.9 demonstrates that more rapid increases in hr compared to the loss, ε", will produce an increase in the required electric field.

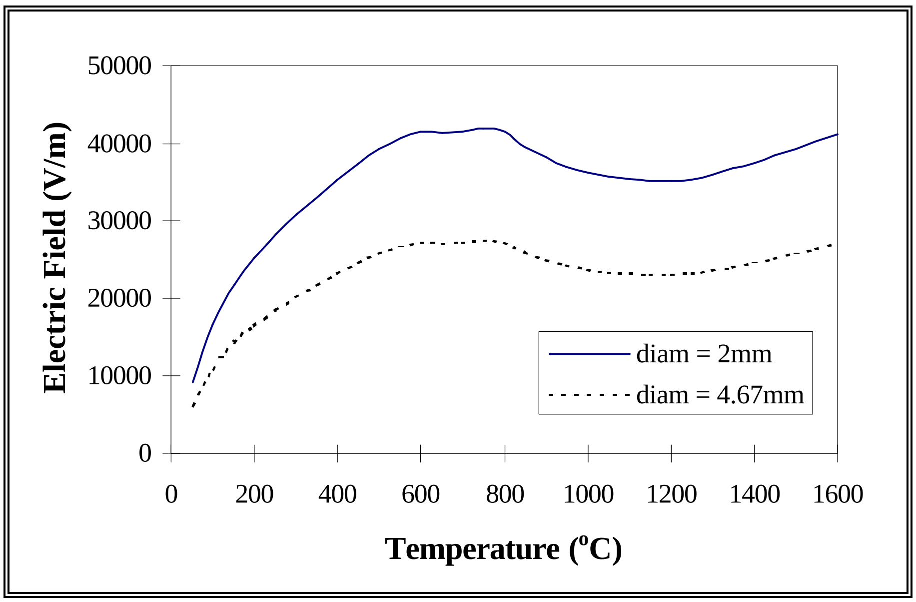

Figure 3.2 Electric Fields that Produce Steady State Temperatures for a 2mm and 4.67mm Diameter, 34mm Long Mullite Rod

Figure 3.2 shows a relative maximum at a temperature of about 750^{o}C. This temperature is defined in various technical papers including Ref. [4] as the critical temperature. Temperatures lower than the critical temperature are attainable using the appropriate constant electric field. However, higher temperatures cannot be realized using a constant electric field strength. For a given electric field strength in this range, there exist at least two possible steady state temperatures. In a heating process, temperatures rise until the lower of the two steady state temperatures is reached. Increasing field strength beyond this point leads to thermal runaway (off the figure to the right). This is a possible and reasonable explanation for the difficulties in heating ceramics to high temperatures. Different approaches have been devised in attempting to overcome this dilemma and can be found in modern microwave processing literature. Vogt et. al. [12] achieved higher temperatures by controlling the absorbed power. The approach that is used in this research is to produce the same result by controlling the absorbed power in the computational model.