

THE EFFECT OF COMPRESSIBILITY
ON THE FRICTION HEAD LOSS DURING LAMINAR FLOW
OF ALUMINUM HYDROXIDE - FILTER AID SUSPENSIONS

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I. INTRODUCTION

Practically all of the important work on the subject of "Fluid Flow" has been for that type called true fluids. Very little information has been published pertaining to the flow of suspensions of solids in liquids.

In the chemical industries of today, a knowledge of flow characteristics of fluids that contain solids would be of great importance. A fundamental understanding of the principles involved in liquid-solid flow would aid the industries of oil well digging, water and sewage treatment, dredging, and papermaking. Today, practically all flow predictions for suspensions are based on empirical formula and arbitrary assumptions.

In liquid-solid flow, it has been found that two types of flow exist, similar to viscous and turbulent flow of true fluids. It has been fairly well established that the flow of suspensions within the turbulent zone follows rather closely the laws which have been derived for true fluids. However, the flow of suspensions in the laminar region differs widely from viscous flow of true fluids. This type of flow when applied to liquid-solid mixtures is called plastic flow.

The purpose of this thesis is to study some of the factors that influence the plastic flow of suspensions and to attempt to find a relationship between these factors.

II. LITERATURE REVIEW

Introduction

To understand the nature of the flow of suspensions, it is necessary that one fully understands fluid flow and plastic flow, and the difference between the two. Therefore, it becomes essential to define a plastic material and a fluid in the sense that the terms are used throughout this treatise.

According to Daugherty⁽²⁵⁾, when the shape of an elastic solid is altered by external forces, the body tends to return to its original configuration upon release of the external forces, because of the tangential stresses between adjacent particles. With a fluid, these tangential stresses are proportional to the velocity and vanish when the velocity approaches zero. When motion ceases the tangential stresses disappear and the fluid does not tend to regain its original shape.

The difference between a plastic and a fluid material is best illustrated by their behavior under shear stress. A true fluid will flow upon application of the slightest stress; whereas, for a plastic, the stress must exceed a certain value before flow will begin.

Fluid Flow

Nomenclature. The nomenclature used in this thesis is given in table I, page 4.

Theory. Daugherty⁽²⁶⁾ says that a perfect fluid is one in which the internal forces at any internal section are always normal to the section, even during motion. A perfect fluid is, therefore, frictionless since there can be no shearing forces. However, in an actual fluid, tangential stresses always occur whenever motion takes place. These forces oppose the sliding of one particle past another, causing fluid friction. These forces are due to a property of an actual fluid called viscosity.

Viscosity. The viscosity of a fluid is a measure of the internal friction of the fluid molecules slipping past each other, the internal resistance to fluid motion.⁽⁴¹⁾ Consider two parallel plates a distance X apart, the space between being filled with the fluid. The lower surface, of area A' , is assumed to be stationary, while the upper one, also of area A' , is moved parallel to it with a velocity V by an application of a force F . For viscous fluid motion it has been found that the force F is directly proportional to the velocity V and to the area A' and inversely proportional to the distance X . Mathematically, this statement is expressed as

$$F = \frac{\mu' V A'}{X} \quad (1)$$

where μ' is defined as the coefficient of viscosity.⁽³⁹⁾

Table I

NOMENCLATURE

Symbol Used	Meaning of Symbol	Units
A	total area of filtering surface	sq in.
A'	area of parallel plates	sq ft
C	concentration of suspended solids	lb dried solids/100 lb suspensions
D	diameter of pipe	ft
d	diameter of capillary	cm
F	force	lb
f	friction factor	dimensionless
g	acceleration due to gravity, 32.17	ft/sec-sec
ΔH	friction head loss	ft of flowing substance
m	coefficient of "plugging"	
N	length of pipe	ft
N'	length of capillary	cm
ΔP	difference in unit pressure	lb/sq ft
P	filtering pressure on cake	lb/sq in.
Δp	pressure used up during the flow overcoming the friction due to the yield value	lb/sq ft
p'	pressure	mm Hg
Q	volume rate of flow	cu ft/sec
Re	Reynolds number, $DV\rho/\mu$	dimensionless
r	distance from any point within a pipe to the center of the pipe	ft
r_0	radius within a pipe at which the shearing stress equals the yield value	ft
S	shearing stress per unit area	lb/sq ft
S_p	shearing stress in a flowing material at the boundary or pipe wall	lb/sq ft
S_r	shearing stress in a circular pipe at distance r, from the center	lb/sq ft
S_y	shearing stress at the yield point of a plastic material, called the yield value	lb/sq ft
s	coefficient of compressibility	
t	temperature	°C
V	mean velocity of flow	ft/sec
V_0	velocity of plug of radius r_0	ft/sec
V_r	velocity at any distance r, from the center of the pipe	ft/sec
v	volume of the cake	cu in/lb of filtrate
W	total weight of filtrate collected up to the variable time θ	lb
w	weight of water traversing a capillary in a unit of time	lb/sec
X	distance between two parallel plates	ft
α	coefficient of resistance of cake	
β	coefficient of resistance of filtering medium	
η	coefficient of rigidity	lb/ft-sec
η'	coefficient of rigidity	slugs/ft-sec
θ	total time of filtration	min
μ	coefficient of viscosity	lb/ft-sec
μ'	coefficient of viscosity	slugs/ft-sec
μ''	viscosity, relative to water	
ρ	density of suspension	lb/cu ft
ϕ	unknown function	

The fluid can be considered to be made up of a series of thin layers, each of which will slip slightly relative to the next layer. The force F results in a shearing stress per unit area, S , between the layers, given by

$$S = \frac{F}{A'} \quad (2)$$

Therefore, the coefficient of viscosity μ' can be defined by the equation

$$\mu' = \frac{S X}{V} \quad (3)$$

Fluidity is defined as the reciprocal of viscosity. (30)

Laminar and Turbulent Flow. For a fluid there are two different types of flow: laminar and turbulent. Binder⁽¹⁶⁾ makes the distinction that in laminar flow the fluid moves in layers or laminae, whereas, in turbulent flow there are secondary irregular motions and velocity fluctuations superimposed on the principle or average flow. In other words, during turbulent flow the individual particles of the liquid, instead of flowing in an orderly manner parallel to the axis of flow, flow in an erratic manner so that there is complete mixing.

Fluid Flow Equations. In the particular case of a fluid flowing through a circular pipe, the fluid friction will be influenced by the diameter D , length of pipe N , fluid velocity V , viscosity μ , and density ρ . Pressure and temperature of the fluid are relevant, but are not independent variables, since their

effect is likely to be covered by those of the density and viscosity of the fluid. Through a consideration of the dimensions of these quantities, it was shown that the following relationship⁽⁴¹⁾, known as the Fanning equation, exists:

$$dH = 2f \frac{v^2}{g} \frac{dN}{D} \quad (4)$$

where

$$f = \phi \left(\frac{D V \rho}{\mu} \right) \quad (5)$$

The terms in these equations are:

H = friction loss, ft-lb/lb.

f = dimensionless friction factor.

V = average linear velocity, ft/sec.

g = acceleration of gravity, 32.17 ft/sec-sec.

N = length of straight pipe, ft.

D = inside diameter of circular pipe, ft.

ρ = density of fluid, lb/cu ft.

μ = absolute viscosity of fluid, lb/ft-sec.

ϕ = unknown function.

This relationship indicates that f is a unique function of Reynold's number, $(DV\rho/\mu)$. Figure 1 is a plot of f against Reynold's number.⁽²⁷⁾ Inspection shows that the curve that represents the points is broken up into two distinct parts, a 45-degree line at the left and a very flat portion at the right. The

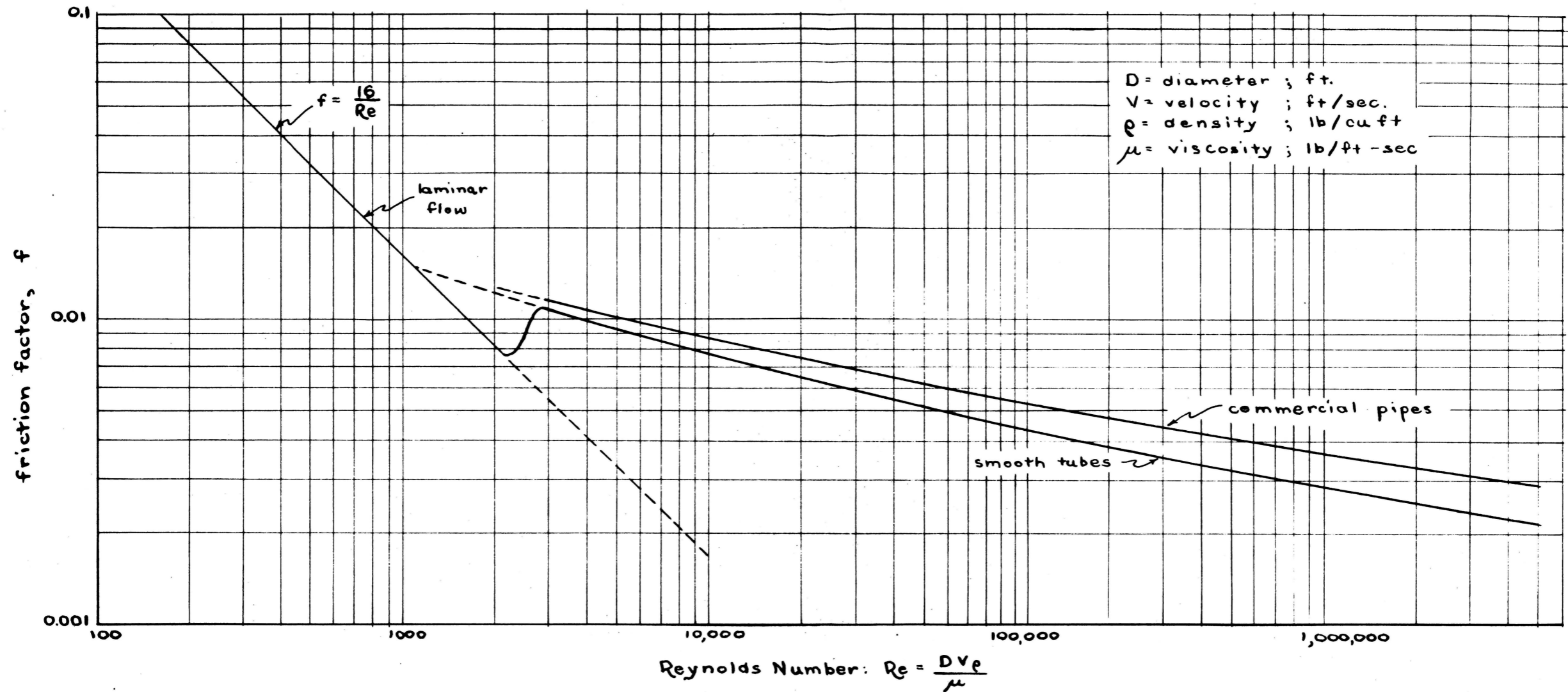


FIGURE 1
 FRICTION FACTOR DIAGRAM FOR LONG, STRAIGHT, CLEAN, ROUND PIPES

Calculated from equations of:

Drew, T.B. and R.P. Genereaux, Written Discussion, Trans. Am. Inst. Chem. Engrs., 32, 17, (1936).

45-degree line represents laminar flow which occurs at low velocities, during which the following relationship holds: (27)

$$f = \frac{16}{Re} \quad (6)$$

where Re represents Reynolds number. The flat part of the curve corresponds to the turbulent flow which exists at high velocities.

If equation (4) is rearranged and the laminar flow relationship is substituted for f in equation (6), the following equation is obtained:

$$\frac{dH}{dN} = \frac{32 \mu V}{\rho g D^2} \quad (7)$$

This equation is known as Hagen - Poiseuille's Law. Its validity has been established by careful extensive experimentation.

Critical Velocity. Critical velocity (37) is considered as that velocity below which the friction loss is directly proportional to the velocity and above which the friction loss is directly proportional to some power of the velocity between 1.7 and 2.0. Results of numerous experimental tests have shown that the relation between the variables represented by the Reynolds-Stanton diagram is widely applicable. (27)

Reynolds (27) believed that the critical velocity occurred at a definite value of the dimensionless number, $DV \rho / \mu$. This number is now called Reynolds number. Recent work (32) has shown that flow in round pipes is always laminar when the value of Reynolds number is 2100 or less.

However, the Reynolds number can exceed this value somewhat, and flow remain laminar. Usually, in industrial piping installations the flow will be turbulent above a Reynolds number of 3000⁽²⁷⁾. Between 2100 and 3000 there is uncertainty as to the type of flow which may occur. To distinguish between these two values, the velocity below which the flow is laminar is designated as the lower critical velocity; and the velocity above which the flow is turbulent is designated as the upper critical velocity.

Other Factors That Influence the Rate of Flow. The other factors that influence the rate of flow will now be discussed.

Temperature has a decided effect on the viscosity of fluids. For liquids, an increase in temperature lowers the viscosity. Poiseuille⁽¹⁷⁾ gave a formula for flow through a capillary which was independent of the capillary used:

$$w = 1,836,724,000 (1 + 0.0336793 t + 0.00022099 t^2) \frac{p' d^4}{N'} \quad (8)$$

where

w = weight of water traversing the capillary in a unit of time, lb/sec.

t = temperature in degrees C.

p' = pressure in mm of mercury

d = diameter of capillary in cm.

N' = length of capillary in cm.

The roughness of the pipe boundary surface will affect the friction drop during turbulent flow; however, it is known that the roughness of the pipe wall has little or no effect on the friction loss during the laminar flow of fluids. (6, 40)

Measurements of Fluid Flow. In order that the fluid flow equations that have been discussed to be of any practical value, the measurements of specific gravity and viscosity must be exact and accurate.

Specific gravity (or density) of liquids may be determined: (33)
(1) by the pycnometer method, weighing a known volume, or weighing equal volumes of water and the liquid and comparing; (2) by the Westphal balance method, determining the loss in weight of a plummet of known volume weighed in air and in the liquid, or by comparing the weight of a plummet of unknown volume weighed in water and in the liquid; and (3) by means of hydrometers, i.e., weighted glass floats which sink in the liquid to a depth dependent on the density, which is read at the liquid line on a calibrated stem extending above the liquid.

Measurement of viscosity (34) is done by three principal methods in the case of liquids: (1) timing the efflux of a definite volume of liquid through a short tube; (2) measuring the shear of a fluid in an annular space between concentric cylinders; and (3) timing the fall of a ball through a fluid. Viscometers of the first type are the Saybolt Universal, Engler,

Redwood, and Ostwald. The second type is represented by the Stormer and MacMichael viscometers. In the Stormer, the outer cylinder is a stationary cup, and the inner cylinder is revolved under the application of a known force, while the time is determined for a certain number of revolutions. In the MacMichael the outer cup is rotated at a known constant speed, and the force tending to turn the inner cylinder is measured in terms of the torsional stress in a supporting filament.

Plastic Flow

Plasticity is the property of a substance which enables it to be continuously and permanently altered in any direction without rupture when the stress applied exceeds a certain yield value.⁽³⁰⁾ Under the action of small shearing stresses, a plastic substance holds its shape permanently, but, under somewhat larger stresses, it may be readily deformed or molded.⁽¹⁹⁾ Since part of the applied shearing stress, S , is used up in overcoming the yield value, S_y , the equation for plastic flow is:

$$\eta' = \frac{(S - S_y) X}{V} \quad (9)$$

where

η' = coefficient of rigidity of the material, or the measure of the resistance to flow of the substance after the yield value has been overcome, slugs/ft-sec.

S_y = yield value, or force necessary to start deformation of the substance, lb/sq ft.

X = distance between two parallel plates, the space between which is filled with the plastic, ft.

V = velocity with which one plate is moved parallel to the other, ft/sec.

S = applied shearing stress per unit area, lb/sq ft.

This is the equation for plastic flow which is analogous to equation (3) for fluid flow.

The analogy between true fluids and true plastics is shown in the graphical presentation in figure 2 of the mathematical concepts of these two types of flow.⁽¹⁵⁾ In case I, the slope of the line is proportional to the coefficient of viscosity. Case II represents the true plastic, and is a graphical representation of equation (9). The slope of this line is proportional to the coefficient of rigidity whereas the intercept on the shearing stress axis is the yield value. Case III represents the flow of a pseudoplastic material. It can be seen that this curve does not obey the fundamental equation of plastic flow, because the line bends toward the origin at low rates of flow. According to Binder⁽¹⁵⁾, some suspensions fall into the category of case III.

Plastic Flow Equations. The following extension of this mathematical derivation of the flow of a plastic substance in circular pipes was presented by Rabbitt and Caldwell.⁽⁷⁾

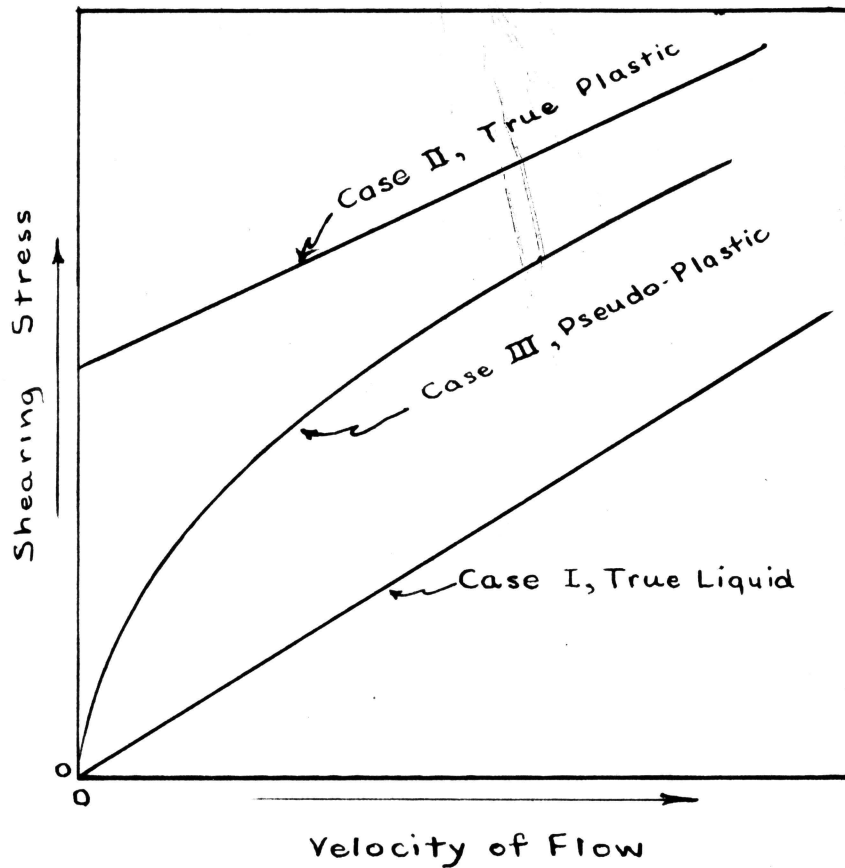


FIGURE 2
 REPRESENTATION OF LIQUID, PLASTIC
 AND PSEUDO-PLASTIC FLOW

From:
 Babbitt, H.E. and D.H. Caldwell, "Laminar Flow
 of Sludges in Pipes With Special Reference
 to Sewage Sludge" U. of Ill. Eng. Exp. Sta.
 Bull. 319, 12 (1939).

Bingham⁽²¹⁾ first formulated a mathematical analysis for plastic flow in capillary tubes. The total force producing flow between two sections of tubing of length H between the sections, and of diameter D , is $\pi D^2 \Delta P/4$ where ΔP is the difference in unit pressure between the two sections. Since there is no acceleration in steady, uniform flow, there is an equal opposing force in the opposite direction which is $\pi D H S_p$, where S_p is the shearing stress in a flowing material at the boundary or pipe wall in pounds per square foot. Therefore

$$S_p = \frac{D (\Delta P)}{4H} \quad (10)$$

Likewise, at any radius, r , from the center of a pipe

$$S_r = \frac{r (\Delta P)}{2H} \quad (11)$$

where S_r is the shearing stress in a circular pipe at a distance r , from the center, in pounds per square foot.

In plastic material confined between two parallel planes of indefinite extent which are being sheared over each other, the shearing stress will be identical at every point⁽²⁰⁾. But in flow through a tube, according to Buckingham⁽²⁴⁾, this is not the case; the shear increases continually from the center of the tube outward and only at a certain distance r_0 , does the shearing force become sufficient to overcome the yield value of the plastic. Therefore the material at the center of the tube moves as a solid plug and beyond the radius r_0 the material moves in telescoping layers.

If Δp is the pressure which is used up during the flow in overcoming the friction caused by the yield value, the yield value, S_y , is expressed by

$$S_y = \frac{D(\Delta p)}{4N} \quad (12)$$

Also

$$S_y = \frac{r_0(\Delta P)}{2N} \quad (13)$$

For a circular pipe equation (9) becomes:

$$dV = -\frac{1}{\eta'} (S_r - S_y) dr \quad (14)$$

or, substituting the value for S_r from equation (11)

$$dV = -\frac{1}{\eta'} \left[\frac{r(\Delta P)}{2N} - S_y \right] dr \quad (15)$$

The velocity, V_r , at any distance r , in the variable region, is obtained by integrating equation (15) from $r = D/2$ to $r = r$:

$$V_r = -\frac{1}{\eta'} \int_{\frac{D}{2}}^r \left[\frac{(\Delta P)r}{2N} - S_y \right] dr = \frac{1}{\eta'} \left[\frac{(\Delta P)r^2}{4N} - S_y r \right]_{\frac{D}{2}}^r$$

$$V_r = \frac{1}{\eta'} \left[\frac{\Delta P}{4N} \left(\frac{D^2}{4} - r^2 \right) - S_y \left(\frac{D}{2} - r \right) \right] \quad (16)$$

The velocity, V_0 , of the solid plug is obtained by letting $r = r_0$ in equation (16):

$$V_0 = \frac{1}{\eta'} \left[\frac{PD^2}{16N} - \frac{S_y D}{2} - \frac{r_0^2 \Delta P}{4N} + r_0 S_y \right]$$

Substituting $\frac{2N S_y}{P}$ for r_0 as obtained from equation (13)

$$V_0 = \frac{1}{\eta'} \left[\frac{\Delta PD^2}{16N} - \frac{NS_y^2}{\Delta P} - \frac{S_y D}{2} \right] \quad (17)$$

The volume of flow per unit of time is Q and

$$Q = \int_0^{\frac{D}{2}} 2\pi rV dr$$

or, if equations (16) and (17) are used

$$Q = r_0^2 V_0 + 2\pi \int_{r_0}^{\frac{D}{2}} rV_r dr \quad (18)$$

But, from equations (13) and (17)

$$\pi r_0^2 V_0 = \frac{4\pi S_y^2 N^2}{(\Delta P)^2} \left[\frac{(\Delta P)D^2}{16N} + \frac{N S_y^2}{\Delta P} - \frac{S_y D}{2} \right]$$

and from equation (16)

$$2\pi \int_{r_0}^{\frac{D}{2}} rV_r dr = \frac{2\pi}{\eta} \int_{r_0}^{\frac{D}{2}} \left[\frac{(\Delta P)}{4N} \left(\frac{D^2 r}{4} - r^3 \right) - S_y \left(\frac{Dr}{2} - r^2 \right) \right] dr$$

or,

$$2\pi \int_{r_0}^{\frac{D}{2}} rV_r dr = \frac{2\pi}{\eta} \left[\frac{D^4(\Delta P)}{256N} - \frac{D^3 S_y}{48} - \frac{P}{4N} \left(\frac{D^2 r_0^2}{8} - \frac{r_0^4}{4} \right) + S_y \left(\frac{Dr_0^2}{4} - \frac{r_0^3}{3} \right) \right]$$

and, if the value of r_0 from equation (13) is introduced

$$2\pi \int_{r_0}^{\frac{D}{2}} rV_r dr = \frac{2\pi}{\eta} \left(\frac{D^4(\Delta P)}{256N} - \frac{D^3 S_y}{48} - \frac{D^2 N S_y^2}{8 \Delta P} + \frac{DN^2 S_y^3}{(\Delta P)^2} - \frac{5N^3 S_y^4}{3(\Delta P)^3} \right)$$

If the relationships obtained for $\pi r_0^2 V_0$ and $2\pi \int_{r_0}^{\frac{D}{2}} rV_r dr$ are substituted into equation (18), it becomes

$$Q = \frac{\pi}{\eta} \left[\frac{D^4(\Delta P)}{128N} - \frac{D^3 S_y}{24} + \frac{2 N^3 S_y^4}{3(\Delta P)^3} \right]$$

or

$$Q = \frac{\pi D^4}{128N\eta} \left[(\Delta P) - \frac{4}{3} \left(\frac{4NS_y}{D} \right) + \frac{1}{3(\Delta P)^3} \left(\frac{4NS_y}{D} \right)^4 \right]$$

introduce the value of Δp given by equation (12).

$$Q = \frac{\pi D^4}{128N\eta} \left[(\Delta P) - \frac{4}{3} (\Delta P) + \frac{(\Delta P)^4}{3(\Delta P)^3} \right]$$

The mean velocity of flow, V , is:

$$V = \frac{4Q}{\pi D^2} = \frac{D^2}{32N \eta'} \left[\Delta P - \frac{4}{3} \Delta p + \frac{(\Delta p)^2}{3(\Delta P)^3} \right]$$

or, if ΔP and Δp are changed to terms of shear by equations (10) and (12),

$$V = \frac{D}{8 \eta'} \left[S_p - \frac{4}{3} S_y + \frac{S_y^4}{3 S_p^3} \right]$$

If it is desired to express the coefficient of rigidity in pounds per second foot, η , the left hand member of the equation must be divided by the gravitational constant, g . Using η instead of η' , and taking g as 32 feet per second per second, the equation reduces to

$$V = \frac{4D}{\eta} \left[S_p - \frac{4}{3} S_y + \frac{1}{3} \frac{(S_y)^4}{(S_p)^3} \right] \quad (19)$$

which may be written as

$$V = \frac{4DS_p}{\eta} \left[1 - \frac{4}{3} \left(\frac{S_y}{S_p} \right) + \frac{1}{3} \left(\frac{S_y}{S_p} \right)^4 \right]$$

From this equation, it was shown by Babbitt and Caldwell⁽⁷⁾ that the last term may be omitted with little error when the ratio of S_y/S_p is less than 0.5. The error will be 5.9 per cent when S_y/S_p is 0.5 and 1.8 per cent when S_y/S_p is 0.4. If the last term is omitted, equation (19) reduces to

$$V = \frac{4D}{\eta} \left[S_p - \frac{4}{3} S_y \right] \quad (20)$$

Theory of Plastic Flow. Bingham⁽²²⁾ set forth the theoretical explanation of the flow of plastics: "A plastic solid is made up of particles which touch each other at certain points. The space between the particles may be empty or may be filled with gas, liquid, or amorphous solid. Flow necessitates the sliding of these particles one over another according to the ordinary laws of friction, so long as the particles are large enough so that their Brownian movement is negligible." "It is by no means necessary," says Bingham, "that the particles be touching at the maximum number of points, corresponding to 'close-packing'. It is merely necessary that the particles touching each other form arches capable of carrying the load. It is evident that as aggregates of particles are formed in the process of collisions, and the size of these aggregates increases as the concentration of solid increases, there must come a time when such aggregates or clots will touch each other and form an arch or bridge across the space through which the flow is taking place. At that concentration the friction will have a finite value, and the material may be said to have a structure."

Flow of Suspensions

Theory of Sheet Flow. The following presentation of the theory of the flow of suspensions was given by Bingham⁽¹⁸⁾.

Considering the simplest conceivable case of a solid suspended in a liquid, imagine that all of the solid particles are united into sheet parallel to the direction of flow. In this case, the total flow will be the sum of the flows of the liquid between the sheets of solid.

However, if the sheets of the solid have an irregular surface, the fluids will fill in the irregularities, yet the stream lines will not pass through these pores or be appreciably distorted by their presence.

Spherical Particle Concept. The ordinary suspension consists of discrete particles⁽¹⁸⁾ and, for the sake of simplicity, a sphere that is suspended in a fluid of its own specific gravity may be considered. The shearing of the fluid which causes any cubical figure of the fluid to assume the form of a rhombohedron, will cause the sphere to rotate. The stream lines will be curved on account of the presence of the sphere, but the sphere itself moves in a linear direction with the velocity of the layer of fluid in which it is suspended. Different spheres in the same layer of fluid do not approach each other since it is assumed that they all have the same velocity.

Spheres in different strata of fluid move with unequal velocities depending on the velocity of the stratum in which each is suspended. Hence collisions must take place, depending upon the radii of the spheres, their number per unit volume, and also

upon any attraction or repulsion which may exist between them. If it is assumed that two spheres which are approaching each other in adjacent strata of fluid are rotating in the same direction, the surfaces of the two spheres must be moving in opposite directions, which are at right angles to the line joining their centers. The resistance to this shearing action which is set up as they collide will rapidly dissipate their energy of rotation as heat.

Application of Theory to Fluid Friction. It is considered that the particles are large in comparison with molecular dimensions, consequently, their contact brings the laws of ordinary friction into play.⁽¹⁹⁾ The spheres cannot rotate unless the torque exceeds a certain value. This value depends upon the pressure existing at their point of contact normal to the surfaces, and this pressure, in turn, depends not only on the rate of shear, but on the attraction or repulsion which may exist between the particles. So, when two particles come into contact, they must remain in contact for a definite period. If the spheres were without attraction or repulsion for each other, they would become separated as soon as their centers have come to be in a plane perpendicular to the plane of the strata of fluids.

During the period of contact, the spheres cannot rotate as individuals until the torque exceeds a certain value. The result is that during the time of contact the group of spheres begin to

rotate as a whole, and they pass out of the strata to which they formerly belonged and into layers of different velocities. During this period the liquid will flow around the spheres and through interstices between them. Thus other spheres tend to collide with those already in contact with each other, after which the combined mass tends to rotate as a whole. This process of accumulation increases until equilibrium is reached when these clots will have a certain average size, depending upon the number, size, and specific attraction of the particles.

Agglomeration and Plastic Flow. The important thing is that the collisions of the particles give rise to an additional loss of energy, and if these clots increase in size and number there must come a point when the clots come in contact across the entire width of the passage. At this point viscous flow of the material as a whole stops and plastic flow begins.

For a given substance and volume concentration, the number of collisions will be proportional to the number of particles, which varies inversely as the cube root of the radius. But if the angular velocity is independent of the radius, the energy of rotation will be proportional to the square of the radius, hence the loss of energy, caused by collisions, will be inversely proportional to the radius. This conclusion, if correct, indicates that very finely divided particles give comparatively viscous liquids or at higher concentrations, plastic solids.

Determination of Critical Concentration. Bingham and Durham⁽²³⁾ found that the suspension of a solid depresses the fluidity of liquid in which it is suspended by an amount proportional to the volume of the solid. However, the depression is such that a zero value of fluidity is reached at a definite but comparatively small percentage by volume of the solid. This composition was found to be the same for all temperatures but varied with the solid used. This represents composition the point of transition of viscous from plastic flow.

Gradishar, Faith and Hedrick⁽²⁸⁾ later confirmed the work of Bingham and Durham, by means of suspensions of coal in fuel oil. They concluded that a suspension may exhibit either viscous or plastic flow properties depending upon the concentration of the suspended matter. Above a critical concentration, the mechanism of flow will be transformed from viscous to plastic flow and the suspension will possess an appreciable yield value; i.e., a resistance which must be overcome before flow begins. It was shown that this critical concentration corresponded roughly to that concentration at which the particles just touched each other sufficiently to form an arch or bridge capable of resisting slight pressures, according to the previously discussed theory proposed by Bingham. The concentration at which zero fluidity occurred was shown to be practically identical to the critical concentration.

~~3~~

Development of Suspension Flow Equation. Babbitt and Caldwell(10) set out to devise a method of predicting the flow of suspensions when the flow became plastic. They found that frictional losses for turbulent flow could be evaluated by means of the Reynolds-Stanton diagram, if use of the viscosity of the dispersion medium of the suspension is made in determining Reynolds number. When the velocity was not sufficient to cause turbulence, the plastic flow equation (equation 20) had to be used. This they wrote in another form more convenient for most cases of suspension flow:

$$\frac{\Delta H}{N} = \frac{16 S_y}{3 \rho D} + \frac{\eta V}{\rho D^2} \quad (21)$$

where

H = friction head loss between two points in a pipe,
feet of flowing substance.

N = length of pipe, ft.

S_y = yield value of flowing substance, lb/sq ft.

η = coefficient of rigidity, lb/ft-sec.

V = mean velocity of flow in a pipe, ft/sec.

D = diameter of pipe, ft.

ρ = density of flowing substance, lb/cu ft.

Equation (21) was checked by experiments; the agreement between observed and computed frictional losses was sufficiently precise for practical purposes. To devise a simple means of

calculating the yield value and the coefficient of rigidity of a suspension, a modified Stormer viscometer was calibrated. A great number of Babbitt and Caldwell's tests were performed on sewage sludges, and by reason of the numerous large particles present in such sludges which tended to bind the rotating cylinder of the standard Stormer viscometer, a modified cup was devised for use with the regular Stormer viscometer.

The chief objection to Babbitt and Caldwell's work is that practically all of it was carried out on clay suspensions and sewage sludges. The applicability of their equations and methods to suspensions other than sewage sludge and clay suspensions has not been proven. J.D. Parent⁽³¹⁾ attempted to correlate the literature on the subject of flow of suspensions, and, in this correlation, he singled out Babbitt and Caldwell's work as the most conclusive in the laminar region of flow.

Another difficulty encountered when attempting to predict friction losses in the laminar flow of suspensions by Babbitt and Caldwell's method arises from the fact that the values of the yield value and the coefficient of rigidity of the suspension in question can not be readily obtained without modifying the Stormer viscometer as they described.

Compressibility and Friction Factor. Sassano⁽³⁶⁾ worked with suspensions of aluminum hydroxide and filtercel, and found that there appeared to be an apparent relationship between compressibility

coefficient and friction factor. He concluded that the aluminum hydroxide suspension which was composed of particles with a high coefficient of compressibility had a much higher friction factor than the filtercel suspensions with a low compressibility factor. Sassano measured apparent viscosity with a calibrated Stormer viscometer.

Compressibility of Suspensions

Introduction. Most suspensions are compressible to some extent. That is to say, the particles of the suspension are capable of being rearranged so that the space between the particles becomes less. This, then, is what happens, when a suspension is filtered. As the pressure on the cake becomes greater, the particles pack more closely together, decreasing the space between the particles through which the filtrate can flow, thus increasing enormously the specific resistance of the cake.

Hydrated sludges, such as the heavy metal hydrates and the calcium phosphate used for defecation of sugar sirups, are highly compressible.⁽⁴²⁾ Porous, granular suspensions, such as diatomaceous earth and calcium carbonate, are relatively non-compressible.

Development of Lewis' Filtration Equation. Whereas a great deal of experimental work has been done on the problem of the compressibility of suspensions, the theoretical solution of this problem has not been accomplished. Lewis⁽⁴²⁾ states that the

difficult nature of the problem has prevented its solution. For one thing, the suspensions that show a high degree of compressibility are hydrated precipitates; their degree of hydration and particle size are extremely sensitive to the conditions under which the precipitates are produced and change progressively with time, preventing the production of suspensions of fixed characteristics for experimental work. In addition, these suspensions are extremely sensitive to temperature changes; consequently, slight temperature changes during filtration seriously affect results.

It was originally thought that the rate of flow of a liquid through a filter cake was directly proportional to the thickness of the cake. In 1912 Almy and Lewis⁽¹⁾ proved that this was not a general rule and did not apply in many cases. In their work they conceived the idea that the specific resistance of the cake is a power function of the pressure of filtration. This was proven by their work and by subsequent work by Lewis⁽⁴²⁾. They devised a rather easy method of determining this exponent of the pressure. It was found that this exponent, designated by the letter g , is a measure of the compressibility of the suspension being filtered, hence, it is called the coefficient of compressibility.

The general equation which Lewis⁽⁴³⁾ devised for filtration is:

$$\frac{dW}{dQ} = \frac{P_0}{\mu^n (\alpha v P_0^g W + \beta A P_0^m)} \quad (22)$$

where

W = total weight of filtrate, lb, up to the variable time, θ .

θ = total time of operation, min.

P = filtering pressure on cake, lb/sq in.

μ^n = viscosity, relative to water.

v = volume of the cake, cu in/lb of filtrate.

A = total area of filtering surface, sq in.

α = coefficient of resistance of cake.

β = coefficient of resistance of filtering medium.

s = coefficient of compressibility.

m = coefficient of "plugging".

Underwood(38) showed that this equation would apply both to compressible suspensions and to incompressible suspensions, if integrated between the proper limits. The integrated equation for constant pressure filtration of compressible homogeneous sludges is:(44)

$$\frac{P\theta}{(W/A)} = \alpha v \mu^n P^s \frac{W}{A} + \beta \mu^n P^m \quad (23)$$

Theory. To visualize the compressive action, consider a single particle of suspension, just approaching the face of the cake.(42) There is exerted on this particle the pressure of the liquid around it. Its motion is free until it touches the face of the cake, when its motion is stopped. However, the liquid flows on around it, producing a frictional drag which pushes the particle

against the particle ahead of it. As the cake builds up the particle is subject to the cumulative pressure of all the particles behind it; however, the liquid pressure decreases. That is to say that as one goes deeper in the cake in the direction of the filter cloth, the liquid pressure will drop off but the compressive effect increases. This effect is noticeable when one observes the filter cake when it is removed from the press. The cake is compact and dense near the filter cloth, but more open towards the center of the cake.

Evaluation of Compressibility Coefficient. According to equation (23) the ratio $(PQ)/(W/A)$ should be proportional to the total volume of filtrate per unit area, (W/A) ; the proportionality constant varies as a power function of the pressure, the exponent being s , the coefficient of compressibility.

Thus, a relatively simple method of determining the coefficient of compressibility is furnished. The suspension is filtered at constant pressure; and the weight of filtrate, (W) , noted at various times, (Q) , during the filtration. From these data a plot of $(PQ)/(W/A)$ versus (W/A) is made. This procedure is repeated several times at different pressures. In each case, for a constant pressure, the data will conform to straight lines. The slope of each line corresponds to $\alpha \eta \mu^n P^s / 2$. If the logarithms of these slopes are plotted versus the logarithms of the pressures at which the filtration occurred, the slope of the resulting line will be equal to the compressibility

coefficient, s . Figure 3 is a plot of $(P_0)/(W/A)$ versus (W/A) for the compressible homogeneous sludge of chromium hydroxide, the work of Almy and Lewis.⁽¹⁾ Figure 4 is a plot of the logarithms of the slopes of the straight lines in figure 3 versus the logarithms of the corresponding pressures. The slope of the straight line in figure 4 is the compressibility coefficient, s , and is found to be 0.81. The values of s for various⁽⁴⁶⁾ suspensions are given in table II.

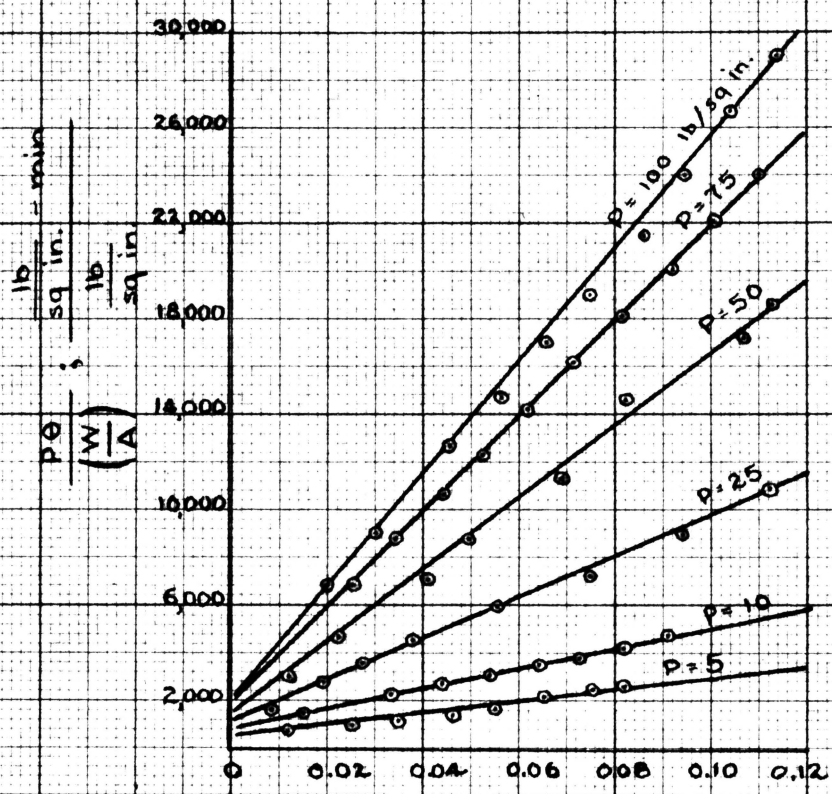
Table II

COEFFICIENTS OF COMPRESSIBILITY FOR VARIOUS SUSPENSIONS*

Type of Suspension	Coefficient of Compressibility, s
Al (OH) ₃	0.56-0.88
Fe (OH) ₃	0.69
Cr (OH) ₃	0.81
ZnCrO ₄	0.72
Fe ₂ O ₃	0.18-0.41
MgCO ₃	0.10
CaCO ₃	0.09-0.14
Kieselguhr	0.02-0.05

*Revised from:

Walker, W.H., W.K. Lewis, W.H. McAdams, and E.R. Gilliland
"Principles of Chemical Engineering." p. 355. McGraw-Hill
Book Co., Inc., New York. 1937. 3rd edition.



$$\left(\frac{W}{A}\right); \frac{\text{lb}}{\text{sq in.}}$$

FIGURE 3
 CONSTANT PRESSURE FILTRATION CURVE
 FOR $\text{Cr}(\text{OH})_3$ SUSPENSION

From:
 Walker, W.H., W.K. Lewis, W.H. McAdams, and
 E.R. Gilliland "Principles of Chemical
 Engineering". p.352. McGraw-Hill Book
 Co., Inc., New York. 1937. 3rd edition.

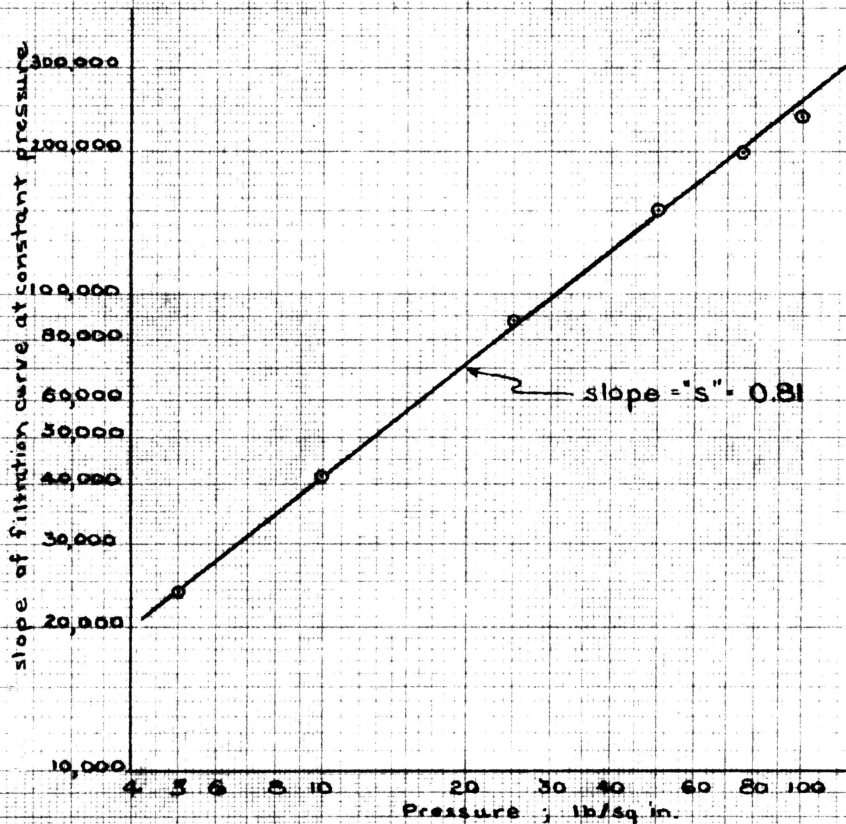


FIGURE 4
DETERMINATION OF COMPRESSIBILITY COEFFICIENT, "s",
OF $Cr(OH)_3$ SUSPENSION

Recalculated from the data of :
Almy, C., Jr. and W.K. Lewis, Factors Determining the
Capacity of a Filter Press. Ind. Eng. Chem. 4, 528-
533 (1912).

III. EXPERIMENTAL

A. Purpose of Investigation

The purpose of this investigation is to study the effect of compressibility on the friction drop in the laminar flow of aluminum hydroxide - filter aid suspensions.

B. Plan of Investigation

Preliminary Investigations. A review and study of the literature covering the subjects of fluid flow, plastic flow, flow of suspensions, measurements of viscosity, and compressibility of suspensions was made.

Study of the Use of the Stormer Viscometer. The use of the Stormer viscometer in determining the viscosity of fluids, and the yield value and the coefficient of rigidity of suspensions was studied.

Experimental Determinations Using the Stormer Viscometer. Tests on SAE 10 and SAE 30 oil were made, using the Stormer Viscometer, for the purpose of calibrating the instrument for the determination of viscosities. Absolute viscosities of these oils were determined by use of the Saybolt viscometer and specific gravity measurements.

Tests were made using the Stormer viscometer on the following suspensions of varying concentrations: (1) filter aid suspensions (non-compressible), (2) aluminum hydroxide suspensions (compressible), and (3) mixtures of filter aids and aluminum hydroxide to obtain suspensions of varying compressibility.

Construction of an Apparatus for Determining the Factors Influencing the Flow of Suspensions. An apparatus was constructed so that the flow of suspensions within pipes of circular cross-section and of varying sizes could be studied. The apparatus was so constructed that (1) the flow velocity of the suspension, (2) the temperature, and (3) the pressure loss from friction over a section of pipe could be determined.

Construction of an Apparatus for the Determination of Compressibility Coefficient. An apparatus, including a filter press, was assembled onto the suspension flow apparatus. This auxiliary apparatus was constructed so that the compressibility coefficient of the suspensions could be determined. It was so constructed that the suspensions could be filtered at a constant pressure.

Suspension Flow Determinations. The pipe apparatus was tested with water to check the mechanical functioning of the apparatus. Determinations of pressure loss, flow velocity, temperature, and age of suspension were made on the following suspensions: (1) filter aid suspensions (non-compressible), (2) aluminum hydroxide suspensions (compressible), and (3) mixtures of filter aid and

aluminum hydroxide to obtain suspensions of varying compressibility. By use of the filter press, the compressibility coefficient of each suspension was determined. Specific gravity and the concentration of the suspended solids of the above suspensions were determined by usual laboratory methods.

Interpretation of Data. An attempt to find a relationship between the factors that influence the flow of suspensions and the compressibility factor as measured by filtration was made.

A check on the methods used by Babbitt and Caldwell⁽⁵⁾ to determine the yield value and the coefficient of rigidity was made to determine if their method is applicable to aluminum hydroxide - filter aid suspensions.

C. Materials

The materials used in this investigation were as follows:

Aluminum Sulfate. Ground, standard grade, in 100-lb net weight bag. Obtained from Allied Chemical and Dye Corporation, General Chemical Division, New York, N.Y. Used for the production of the aluminum hydroxide suspensions.

Calcium Carbonate. U.S.P., code 1541, lot no. c210, in 25-lb cartons. Manufactured by General Chemical Company, New York, N.Y. Used in preliminary tests on the filter press.

Carbon Tetrachloride. Technical grade, specific gravity 1.595 at 20°C., code 1555P, in 5-gal can. Obtained from Phipps and Bird, Richmond, Va. Used as a manometer fluid.

Filter Aid. Celite 503, diatomaceous silica, in 1-lb carton. Obtained from Johns-Manville, New York, N.Y. Used for preliminary tests on settling time for various filter aids.

Filter Aid. Dicalite, diatomaceous silica, in 3-lb bags. Obtained from the Dicalite Company, New York, N.Y. Used for preliminary tests to determine settling times for various filter aids.

Filter Aid. Hyflo super-cel, diatomaceous silica, code A66050, in 50-lb bags. Obtained from Johns-Manville, New York, N.Y. Used for the production of filter aid suspensions and aluminum hydroxide - filter aid mixed suspensions.

Iodine. Resublimed, C.P., code no. S - .7300, in 2-lb bottle. Manufactured by Merck and Company, Rahway, N.J. Used to color the carbon tetrachloride for use as a manometer fluid.

Hydrochloric Acid. Reagent grade, C.P., 37 per cent HCl (minimum), specific gravity 1.178 at 60°C. (minimum). Manufactured by General Chemical Division, Allied Chemical and Dye Corporation, New York, N.Y. Used to digest the aluminum oxide in the determination of the percentage of suspended solids.

Mercury. Distilled. Obtained from F.W. Berk and Company, Wood Ridge Division, Wood Ridge, N.J. Used as a manometer fluid.

Motor Oil. SAE 10, Esso No. 1, in quart cans. Manufactured by Standard Oil Company of New Jersey, Bayway, N.J. Used for calibration of the Stormer viscometer.

Motor Oil. SAE 30, Esso No. 3, in quart cans. Manufactured by Standard Oil Company of New Jersey, Bayway, N.J. Used for calibration of the Stormer viscometer.

Sodium Hydroxide. Flake, 76 per cent sodium oxide, in 100-lb drum. Obtained from Phipps and Bird, Richmond, Va. Manufactured by Diamond Alkali Company, Pittsburgh, Pa. Used for the preparation of aluminum hydroxide suspensions.

D. Apparatus

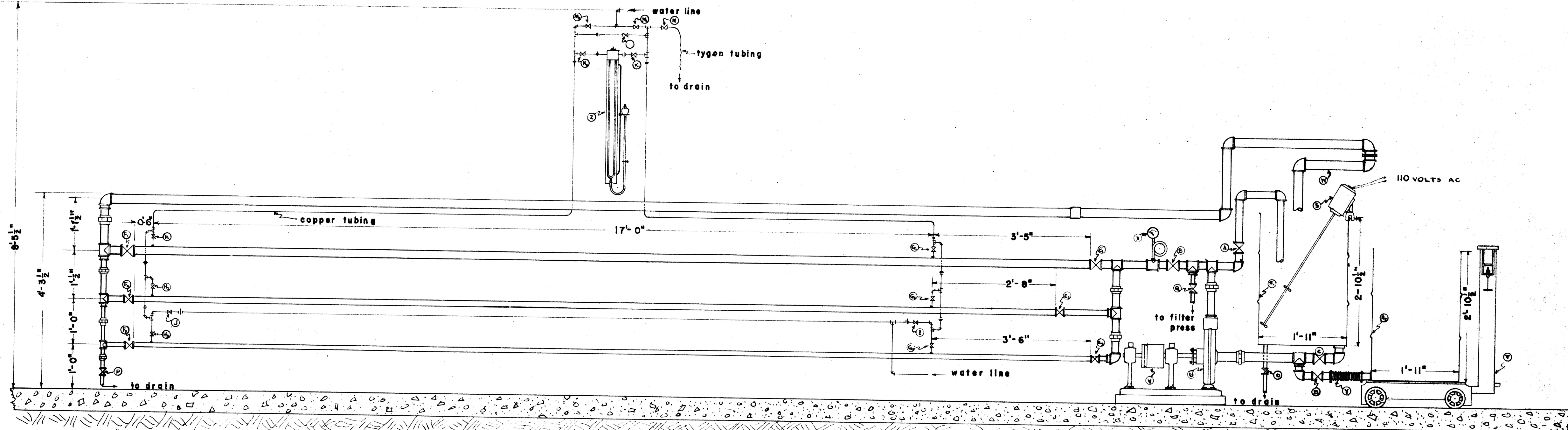
The apparatus used in this investigation consisted of a suspension flow apparatus and auxiliary equipment. The suspension flow apparatus, used to determine the friction loss of the suspension flowing through pipes and the compressibility factor of the suspension, is shown in figures 5 and 6, pages 39 and 40. It consists of the following parts:

Pump. Centrifugal, steel, 2-in. inlet and outlet, 60 gal/min capacity, size 2, figure No. 3010, style No. 621.3301. Manufactured by Gould's Pumps Inc., Seneca Falls, N.Y. Used to pump the suspension to the test lengths of pipes and the filter press.

Electric Motor. Craftsman, 2 hp, 60 cycle, 200/400 volts, 3 phase, 1740 rev/min, 5.6/2.8 amp, serial No. 844703-99. Manufactured by Sears, Roebuck and Co., Philadelphia, Pa. Used to drive the pump.

Make-Up Tank. Steel drum constructed from an empty gasoline drum, 55 gal capacity, with 2-in. and 3/4-in. outlets located in the bottom of the tank, open top. Used as a reservoir from which the suspensions were pumped.

Weighing Tank. Steel drum constructed from an empty gasoline drum, 55 gal capacity, with a 1 1/2-in. outlet located on the side near the bottom of the drum, open top. Used as a weighing container for the suspension pumped during a certain period of time.



FRONT ELEVATION

-LEGEND-

- | | |
|------------------------------------|-----------------------------------|
| Ⓐ, Ⓑ - CONTROL VALVES, GLOBE | Ⓜ - CENTRIFUGAL PUMP |
| Ⓒ THROUGH Ⓓ - CONTROL VALVES, GATE | Ⓝ - PULLEY |
| Ⓔ - MAKE-UP TANK | Ⓞ - SWING PIPE |
| Ⓕ - WEIGHING TANK | Ⓟ - PRESSURE GAGE |
| Ⓖ - AGITATOR | Ⓠ - RUBBER CONNECTING HOSE |
| Ⓢ - PLATFORM SCALES | Ⓡ - MANOMETER, WITH LEVELING BULB |

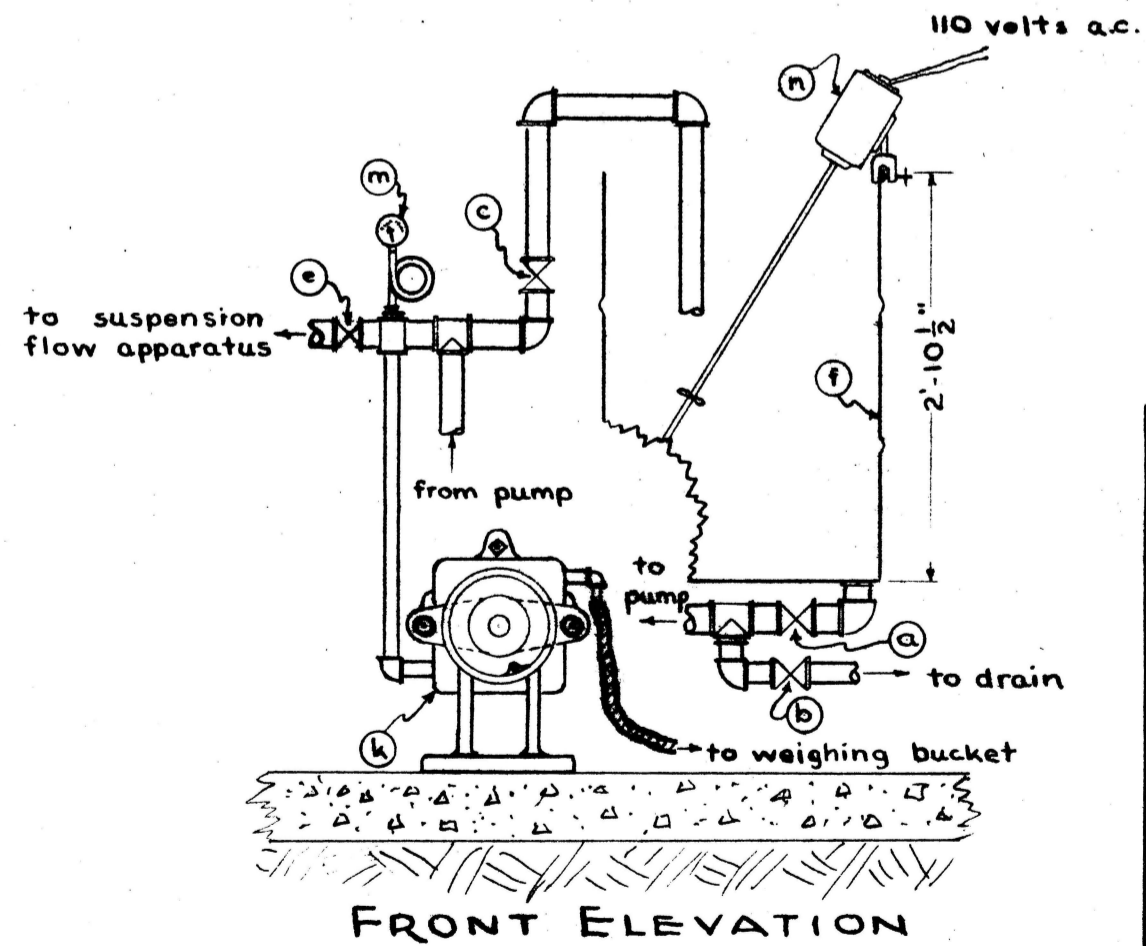
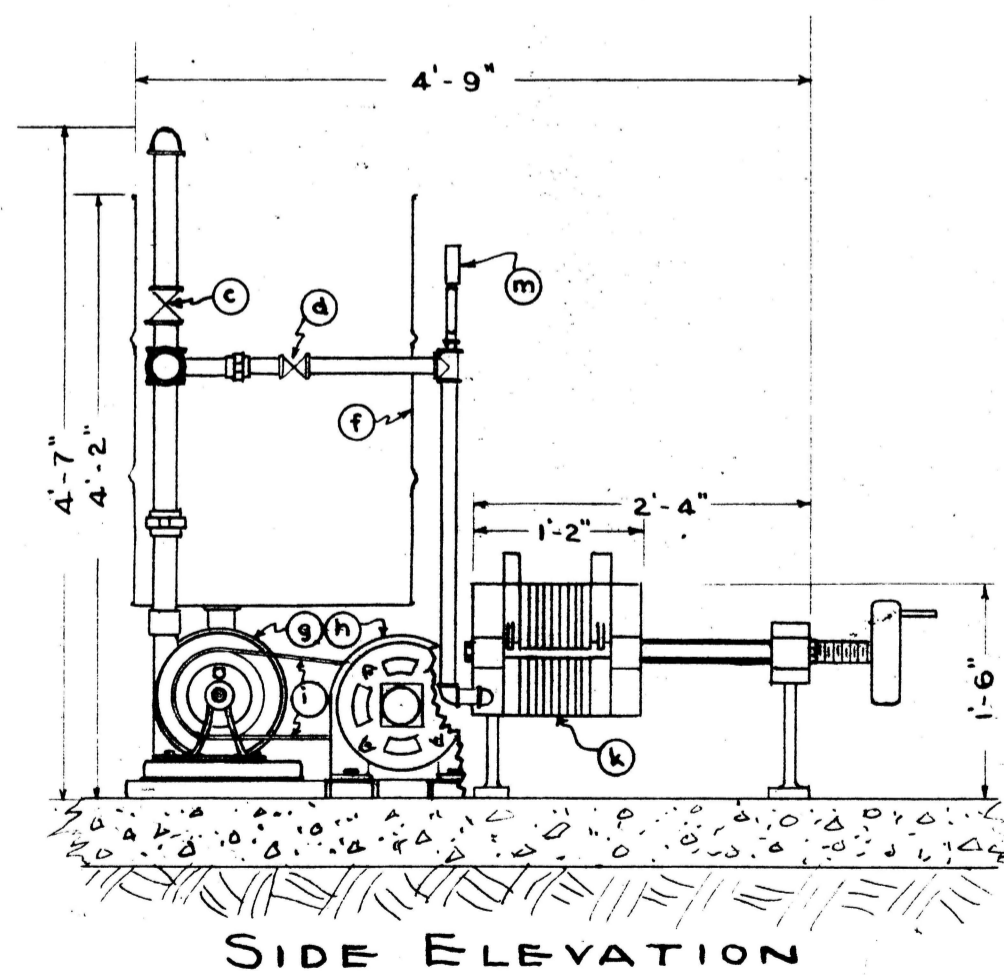
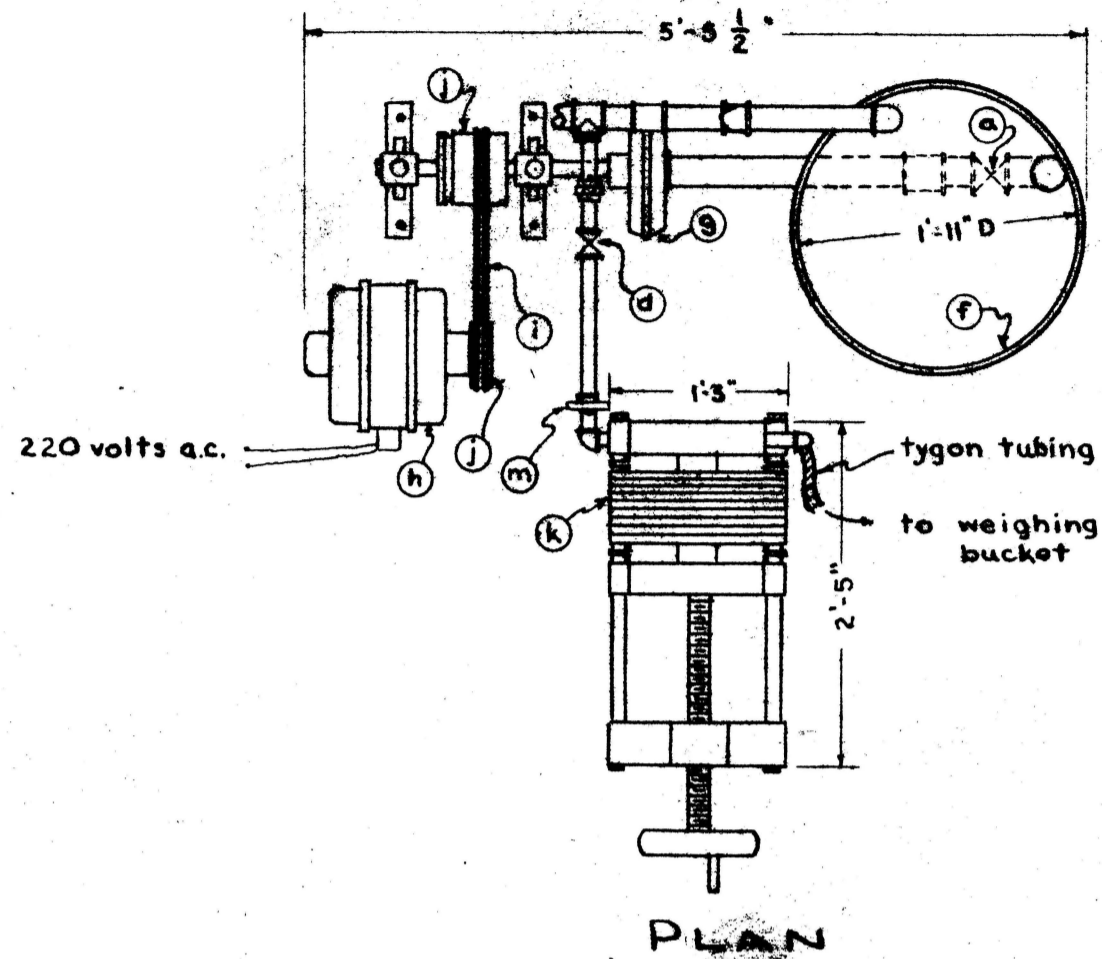
DEPARTMENT OF CHEMICAL ENGINEERING
 VIRGINIA POLYTECHNIC INSTITUTE
 BLACKSBURG, VIRGINIA

SUSPENSION FLOW APPARATUS

SCALE: $\frac{3}{4}'' = 1'$	CASE No 49
DRAWN BY: J.A.C. 1-15-49	FILE No 535
TRACED BY: J.A.C. DATE 1-21-49	FIGURE 5
CHECKED BY: DATE	
APPROVED BY: J.F.M. DATE 1-22-49	

Figure 6

- LEGEND -**
- ⓑ, Ⓒ, Ⓓ, ⓐ - CONTROL VALVES.
 - Ⓜ - MAKE-UP TANK.
 - Ⓟ - PUMP, 60 GAL / MIN CAPACITY.
 - Ⓜ - MOTOR, 220 VOLTS A.C., 3 PHASE.
 - Ⓛ - DRIVE BELT.
 - Ⓟ - PULLEYS.
 - Ⓛ - FILTER PRESS, PLATE AND FRAME TYPE.
 - Ⓛ - PRESSURE GAGE.
 - Ⓛ - AGITATOR.



DEPARTMENT OF CHEMICAL ENGINEERING VIRGINIA POLYTECHNIC INSTITUTE BLACKSBURG, VIRGINIA	
EQUIPMENT FOR THE DETERMINATION OF COMPRESSIBILITY COEFFICIENT	
SCALE: 3/4" = 1'	CASE No. 49
DRAWN BY: J.A.C. DATE 2-7-49	FILE No. 539
TRACED BY: J.A.C. DATE 2-9-49	FIGURE 6
CHECKED BY: N.F.M. DATE	
APPROVED BY: N.F.M. DATE 2-10-49	

Manometer. Filled with either mercury or carbon tetrachloride, 32 in. long, $\frac{1}{4}$ -in. pipe connections. Case was manufactured by the Meriam Co., Cleveland, Ohio. The glass tubing was constructed with a side outlet at the bottom of the manometer. This was connected, in turn, to a leveling bulb by means of rubber tubing. The manometer was used to determine the friction drop across the test lengths of pipe.

Platform Scales. Capacity 500 lb. Manufactured by the Fairbanks, Morse and Co., Chicago, Ill. Used to weigh the suspension pumped into the weighing tank.

Pump and Motor Mount. Constructed of $\frac{1}{2}$ x 1 x 2 in. channel iron. Used as a base for the pump and motor.

Tank Stand. Constructed of $\frac{1}{2}$ x 1 in. angle iron. Used to support the make-up tank.

Filter Press. Cast iron, size 10, type 41, No. 38077, five frames, closed filtrate discharge, $\frac{3}{4}$ -in. inlet located at the bottom of the press, $\frac{3}{4}$ -in. outlet located at the top of the press. Manufactured by D.R. Sperry and Co., Batavia, Ill. Used to determine the compressibility coefficient of the suspension.

Pressure Gage. Range 0 - 60 lb/sq in. gage. Manufactured by Ashcroft Gage Co., New York, N.Y. Used to measure the pressure on the test lengths of pipe.

Pressure Gage. Range 0 - 30 lb/sq in. gage, internal siphon type. Manufactured by U.S. Gage Co., New York, N.Y. Used to measure the pressure on the filter press.

Portable Mixer. Electric, number 23V8, $\frac{1}{4}$ hp, 115 volts, single phase, 60 cycle, open drip-proof type, having a universally adjustable supporting clamp for rim mounting and a stainless steel shaft with two 4 in. diameter down-thrusting stainless steel propellers operating between speed ranges of 500 and 1750 rpm. Manufactured by Alsop Engineering Corporation, Milldale, Conn. Used to keep the suspension agitated.

Fig-Tail Traps. Two required, $\frac{1}{4}$ in. nominal diameter. Used to prevent the suspension from entering the pressure gages.

Tygon Tubing. Ten ft required, clear, $\frac{3}{4}$ in. inside diameter by $\frac{1}{8}$ in. wall thickness. Obtained from Prince Rubber Co., Inc., Buffalo, N.Y. Used as a flexible connection from the filter press to the weighing bucket.

Rubber Tubing. Four ft required, $\frac{1}{4}$ in. inside diameter. Obtained from the Chemical Rubber Co., Cleveland, Ohio. Used to connect the leveling bulb to the side outlet on the bottom of the manometer.

Copper Tubing and Fittings. The following items of copper tubing and fittings were used:

Copper tubing, type M, $\frac{1}{2}$ in. outside diameter, standard hard, 40 ft required. Obtained from Noland Supply Co., Roanoke, Va. Used as manometer lines from the pipe taps on the test lengths of pipe to the manometer.

Pipe to tubing adapters, brass $\frac{1}{4}$ x $\frac{1}{4}$ in. compression type, four required, number 68F. Obtained from Noland

Supply Co., Roanoke, Va. Used to connect the copper tubing to standard $\frac{1}{2}$ in. pipe.

Standard Black Iron Pipe and Fittings. The following items of standard black iron pipe and fittings were used to construct the suspension flow apparatus as shown in figure 5, page 38. All pipe and fittings were obtained from Noland Supply Co., Roanoke, Va.

Pipe: 2 in. nominal size, 30 ft, 6 in. required; $1\frac{1}{2}$ in. nominal size, 24 ft, 2 in. required; 1 in. nominal size, 23 ft, 3 in. required; $\frac{3}{4}$ in. nominal size, 4 ft, 5 in. required; $\frac{1}{2}$ in. nominal size, 37 ft, 7 in. required; $\frac{1}{4}$ in. nominal size, 30 ft, 8 in. required; $\frac{1}{8}$ in. nominal size, 2 ft, 2 in. required.

Nipples: Four 2 x 3 in., one 2 x 2 in., one $1\frac{1}{2}$ x 6 in., three $1\frac{1}{2}$ x 4 in., one $1\frac{1}{2}$ x 3 in., seven $1\frac{1}{2}$ x 2 in., three 1 x 3 in., one 1 x 2 in., three 1 x 1 in., two $\frac{3}{4}$ x 2 in., two $\frac{3}{4}$ x 1 in., one $\frac{1}{2}$ x 6 in., two $\frac{1}{2}$ x 4 in., one $\frac{1}{2}$ x 3 in., two $\frac{1}{2}$ x 2 in., three $\frac{1}{2}$ x $1\frac{1}{2}$ in., one $\frac{1}{2}$ x 1 in., two $\frac{1}{2}$ x $\frac{1}{2}$ in., two $\frac{1}{4}$ x 6 in., one $\frac{1}{4}$ x 5 in., one $\frac{1}{4}$ x 4 in., seven $\frac{1}{4}$ x 3 in., four $\frac{1}{4}$ x 2 in., four $\frac{1}{4}$ x 1 in., three $\frac{1}{4}$ x $\frac{1}{2}$ in., four $\frac{1}{4}$ in. close, three $\frac{1}{8}$ x 2 in., fifteen $\frac{1}{8}$ x 1 in., one $\frac{1}{8}$ x $\frac{1}{2}$ in., seven $\frac{1}{8}$ in. close.

Ells, 90°: Four 2 in., five $1\frac{1}{2}$ in., two 1 in., three $\frac{3}{4}$ in., ten $\frac{1}{2}$ in., ten $\frac{1}{4}$ in., six $\frac{1}{8}$ in.

Ells, streets: One 1 in., one 3/4 in., two 1/2 in.

Ells, swing returns: one 2 in.

Tees: One 2 in., three 1 1/2 in., four 1 in., two 1/2 in., eight 1/4 in., four 1/8 in.

Unions: One 2 in., four 1 1/2 in., two 1 in., four 1/2 in., eight 1/4 in., four 1/8 in.

Reducing Tees: One 1 1/2 x 1 1/4 x 1 1/8 in., one 2 x 1 in., one 1 1/2 x 3/4 in.

Bushings: Five 2 x 1 1/2 in., one 2 x 1 in., one 2 x 1/2 in., one 1 1/2 x 1 in., two 1 x 3/4 in., one 1 x 1/2 in., three 3/4 x 1/2 in., two 1/2 x 3/8 in., two 1/2 x 1/4 in., two 3/8 x 1/4 in., eight 1/2 x 1/8 in.

Valves. The following valves were used, as shown on figure 5, page 38. All valves were obtained from Noland Supply Co., Roanoke, Va.

Gate valve, bronze, 150 lb/sq in., 2 in., Lunkenheimer, non-rising stem, one required.

Gate valves, bronze, 150 lb/sq in., 1 1/2 in., Lunkenheimer, non-rising stem, three required.

Gate valves, bronze, 125 lb/sq in., 1 in., Lunkenheimer, non-rising stem, two required.

Gate valve, bronze, 125 lb/sq in., 3/4 in., Lunkenheimer, non-rising stem, one required.

Gate valves, bronze, 125 lb/sq in., 1/2 in., Lunkenheimer, non-rising stem, four required.

Gate valves, bronze, 125 lb/sq in., $\frac{1}{4}$ in., Lunkenheimer, non-rising stem, four required.

Globe valves, bronze, 150 lb/sq in., $1\frac{1}{2}$ in., Lunkenheimer, three required.

Globe valves, bronze, 150 lb/sq in., $\frac{1}{2}$ in., Lunkenheimer, two required.

Globe valves, bronze, 150 lb/sq in., $1/8$ in., Lunkenheimer, eight required.

Needle valve, bronze, 200 lb/sq in., 1 in., Lunkenheimer, one required.

The following items of auxiliary equipment were used:

Timer. Electric, 110 volts, 60 cycle, range 0.01 to 100 min, Time-It. Manufactured by Precision Scientific Co., Chicago, Ill. Used in conjunction with the Stormer viscometer.

Viscometer. Stormer type, number 4297. Manufactured by Arthur Thomas Co., Philadelphia, Pa. Obtained from Fisher Scientific Co., Pittsburgh, Pa. Used to determine yield value and the coefficient of rigidity of the suspensions.

Viscometer. Saybolt type, Universal tip. Obtained from Fisher Scientific Co., Pittsburgh, Pa. Used in calibrating the Stormer viscometer.

Balance. Analytical type, chainomatic, capacity 200 grams, graduated to 0.1 mg. Manufactured by Seederer-Kohlbusch, Inc., Jersey City, N.J. Used for routine analytical weighings.

Specific Gravity Balance. Westphal type. Obtained from Fisher Scientific Co., Pittsburgh, Pa. Used in determining the specific gravity of the oils used in calibrating the Stormer viscometer.

pH Meter. Beckman, glass electrode, model H, serial number 2023, 115 volts, 50-60 cycles. Manufactured by National Technical Laboratories, South Pasadena, Cal. Obtained from Fisher Scientific Co., Pittsburgh, Pa. Used to determine the pH of the aluminum hydroxide suspensions.

Thermometers. Mercury-glass, engraved stem, -10° to $300^{\circ}\text{C}.$, two required. Made by Brothcom Co., New York, N.Y. Used for various temperature measurements.

Hot Plate. Electric heater, 5 amp, 110 volts, 60 watts, Chromalox. Made by Edwin L. Wiegand Co., Pittsburgh, Pa. Used for digesting aluminum hydroxide in the suspended solids analysis.

Assorted Laboratory Glassware. Various pieces of laboratory glassware, including graduated cylinders, volumetric flasks, beakers, Gooch crucibles, desiccators, Buchner funnels, and filtering flasks, were used.

E. Method of Procedure

The method of procedure employed in this investigation was as follows:

Operation of the Stormer Viscometer. On each of the suspensions tested with the suspension flow apparatus, Stormer viscosity tests were performed, where possible. The Stormer viscosity of the material was determined in the following manner:

The instrument was placed on a table in such a manner that the weight could drop without obstruction through a distance of about 40 inches. The operating weight was varied as desired by adding or removing shot. The operating weight was then weighed on a pan balance and the weight was recorded. The rotating cylinder was secured on the instrument with the set screw. The test cup was filled with the material to be tested, until the latter was 0.25 inch above the top of the vanes. The constant temperature bath was filled with water. The water bath, with test cup, was raised until the contents of the latter covered the top of the cylinder. It was secured in this position by tightening the set screw which engaged one of the vertical supports. The collar on this support was set to insure repetition of this same height on subsequent tests. The thermometer was inserted into the thermometer well. The actuating weight was raised until it nearly touched the frame above it. After stirring the material to be

tested, the brake was released and the time required for the counter to revolve through 100 revolutions was recorded. This procedure was repeated for several different operating weights.

The Stormer viscometer was calibrated against the Saybolt viscometer using SAE 10 and SAE 30 oils.

Determination of the Saybolt Viscosity. The Saybolt viscosities of the oils used in calibrating the Stormer viscometer were determined by means of the Saybolt viscometer, according to the American Society for Testing Materials, standard test number D 93-44. (4)

Determination of the Specific Gravity of the Oil. The specific gravities of the oils tested were obtained by use of the Westphal balance in the following manner:

The Westphal balance was calibrated by adding weights to the plummet until it balanced in air. The calibration was checked by determining the specific gravity of water at room temperature and comparing with the value listed in the literature. The specific gravities of the oils to be tested were determined by immersing the plummet in a cylinder of oil and adding weights until the pointer was balanced.

Calibration of the Stormer Viscometer. The absolute viscosities of the oils were calculated from the Saybolt viscosities and the specific gravities. From the Stormer viscometer data, graphs of the actuating weights versus the revolutions per second were plotted. The slopes of these lines obtained were calculated.

The Stormer viscometer was calibrated by calculating the ratio of the slopes to the absolute viscosities. This ratio was the calibration constant.

Rate of Settling Tests on Various Filter Aids. To determine the comparative rates of settling of various filter aids, the following procedure was used:

A 250-ml graduated cylinder was filled to the 40-ml mark with the filter aid to be tested. The cylinder was tapped gently on the bottom until the powder had packed well into the bottom. Then more filter aid was added until the solid was level at the 40-ml mark. The cylinder was then filled with distilled water to the 200-ml mark. The cylinder was corked and shaken until all of the powder was in suspension. The time required for the solid material to settle to a definite level was recorded for various levels. The filter aids tested were celite 503, hyflo super-cel and dicalite.

Preliminary Tests Using the Stormer Viscometer. Preliminary Stormer viscometer tests were made on a 10-per-cent suspended solids slurry of aluminum hydroxide, a 12-per-cent suspended solids slurry of aluminum hydroxide, a 5-per-cent solids suspension of aluminum hydroxide which had been washed, and a 25-per-cent suspended solids suspension of hyflo super-cel.

The 10-per-cent aluminum hydroxide suspension was prepared in the following manner: 213.5 grams of chemically pure aluminum sulfate were mixed in a 1000-ml beaker with 100 ml of

distilled water. The mixture was heated to 93°C to facilitate solution of the aluminum sulfate. In a 500-ml beaker, 77.0 grams of chemically pure sodium hydroxide were added to 85.5 ml of distilled water. The sodium hydroxide solution was added to the aluminum sulfate solution slowly, with stirring, until the pH of the suspension was 6.5.

The 12-per-cent aluminum hydroxide suspension was prepared in the same manner with the exception that the suspension was diluted with water and allowed to settle. The water was then decanted off, and fresh water was added. This procedure was repeated until most of the dissolved sodium sulfate formed in the reaction between the aluminum sulfate and the sodium hydroxide had been removed.

The five-per-cent aluminum hydroxide suspension was prepared by diluting the 12-per-cent suspension with distilled water.

The 25-per-cent filter aid suspension was prepared by adding 300 ml of distilled water to 100 grams of hyflo super-cel in a 1000-ml beaker. The slurry was stirred until a uniform consistency was obtained.

Construction of Suspension Flow Apparatus. A suspension flow apparatus was constructed as shown in figure 5, page 39. The apparatus consisted of two tanks of 55 gallons capacity each and a system of pipes through which the suspension was pumped by a

centrifugal pump. Test lengths of pipe 20 feet long and 0.5, 1.0, and 1.5 inches in diameter were provided for friction loss determinations. From the test lengths the suspension would flow through a 2-inch diameter return pipe back to the make-up tank, where a swing pipe was constructed so that the discharge could be emptied into the weighing tank. The time for flow of a given weight was used to determine the flow velocity. A system of valves was included so that any one of the test lengths of pipe could be used for the necessary determinations. Manometer leads were located at both ends of the test lengths of pipes. These leads were connected to a manometer, by which the pressure loss could be determined.

For the determination of compressibility coefficients, a filter press, pressure gage, and regulating valve, were connected to the suspension flow apparatus, as shown in figure 6, page 40.

Calibration of Suspension Flow Apparatus. The suspension flow apparatus was calibrated with water to determine if the friction drop obtained by use of this equipment would follow the accepted values; i.e., to see if there were any eccentricities in the apparatus.

Operation of the Suspension Flow Apparatus. The suspension flow apparatus was operated, using the following procedure:

1. The make-up tank was filled with the fluid or suspension to be tested, and the swing pipe was located so that the suspension would flow back into the make-up tank.

2. The manometer was filled with the fluid desired, either mercury or carbon tetrachloride tinted with iodine, by means of the leveling bulb.
3. All valves were closed.
4. Valves A, B, C, E₁, and F₁ were opened (see figure 5, page 39).
5. The motor was started and the pump was allowed to force the suspension through the test length of 1½ inch pipe.
6. Valves L and N were opened.
7. Valve J was opened, permitting water to fill up one of the manometer leads, overflowing through the tygon overflow tube.
8. Valve L was closed and valve J was closed, in that order, insuring a manometer lead filled with water.
9. Valve I was opened, permitting water to fill up the other manometer lead, overflowing through the tygon overflow tube.
10. Valve N was closed and valve I was closed, in that order, insuring that the other manometer lead was filled with water.
11. Valves G₁ and H₁ were opened.
12. Valves K₁ and K₂ were opened slowly, and the manometer reading was recorded.

13. At the same time, the swing pipe was moved so that the suspension began to flow into the weighing tank.

14. The time required for a certain weight of the suspension to flow into the weighing tank was noted and recorded.

15. The manometer reading was checked again after the rate of flow had been ascertained.

16. The temperature of the suspension was measured by means of a thermometer and recorded.

17. The procedure as outlined by steps 2 through 16 was repeated for various flow rates. The different flow rates were obtained by regulating valves A and B.

18. The entire procedure was repeated for the 1.0 and 0.5 inch test lengths of pipe by closing off the valves on the 1.5 inch test pipe and its respective manometer lead valves, and opening the corresponding valves on the other test length of pipe to be tested.

19. When the weighing tank became nearly full, the suspension was pumped back into the make-up tank by opening valve D, and closing valves B and C.

Suspension Flow Tests. Suspension flow tests were performed on the following suspensions:

1. Aluminum hydroxide suspension, 3.6 per cent suspended solids by weight. This suspension was made up in the following manner: In the make-up tank, 90.0 lb of standard aluminum sulfate were added to 25 gal of water. This was agitated by means of the

rim-mounted electric mixer until all of the aluminum sulfate had gone into solution. In a large bucket, 36.0 lb of flake sodium hydroxide, 76 per cent sodium oxide were added to 10 gal of water. This solution was allowed to dissolve and cool. The sodium hydroxide solution was poured into the make-up tank, slowly, with agitation from the electric mixer, until the suspension of aluminum hydroxide had a pH of 6.5.

2. Aluminum hydroxide suspension, 6.14-per-cent solids by weight. This suspension was prepared by filtering off about one half of the water in the 3.6-per-cent solids suspension. The filter cakes were then returned to the make-up tank and agitated until of uniform consistency.

3. This suspension was prepared by adding 3 lb of hyflo super-cel to suspension number 2. The mixture analyzed 6.20 per cent suspended solids by weight, of which 67 per cent was reported as aluminum oxide and 33 per cent was filter aid.

4. This suspension was prepared by adding 5 lb of hyflo super-cel to suspension number 3. The mixture analyzed 7.25 per cent suspended solids by weight, of which 49 per cent was reported as aluminum oxide and 51 per cent was filter aid.

5. This suspension was prepared by adding 5 lb of hyflo super-cel to suspension number 4. The mixture analysed 10.05 per cent suspended solids by weight, of which 36.2 per cent was reported as aluminum oxide and 63.8 per cent was filter aid.

6. This suspension was prepared by adding 8 lb of hyflo super-cel to suspension number 5. The mixture analyzed 13.03 per cent suspended solids by weight, of which 19.3 per cent was reported as aluminum oxide and 80.7 per cent was filter aid.

7. This suspension was prepared by making up some fresh aluminum hydroxide suspension, similar to suspension number 1, and adding this to suspension number 6. The mixture analyzed 12.02 per cent suspended solids by weight, of which 42.3 per cent was reported as aluminum oxide and 57.4 per cent was filter aid.

8. This suspension was prepared by diluting suspension number 7 with 20 gallons of water. The suspension analyzed 8.23 per cent suspended solids by weight, of which 29 per cent was reported as aluminum oxide and 71 per cent was filter aid.

Operation of the Filter Press. To determine the compressibility coefficient of each suspension tested on the flow apparatus, the filter press was operated in the following manner:

1. The pressure gage on the filter press was calibrated with a dead weight gage tester. This operation was performed before the first suspension was tested, and again after suspension number 8 was tested.

2. The suspension to be tested was thoroughly agitated by means of the mixer.

3. All valves were closed.

4. Valves (a) and (c) were opened, and the pump was started (see figure 6, page 40).

5. The end of the tygon tube from the filter press was placed in a bucket on a large balance.

6. The suspension was allowed to enter the press by slowly opening valve (d). This valve was regulated throughout the entire test so that the pressure would remain constant on the filter press.

7. The timer was started when the filtrate first started flowing into the weighing bucket.

8. At various intervals, the weight of filtrate and the time since the filtrate first started flowing into the bucket were recorded.

9. The temperature of the suspension was taken with a thermometer and recorded.

10. After 100 to 200 grams of filtrate had been collected, the pump was stopped, and valve (d) was closed.

11. The filter press was then cleaned out, and all of the filter cloths were washed.

12. After the press had been reassembled, the procedure from steps 4 through 11 was repeated, the filtering being done at a different constant pressure. This procedure was repeated for five different constant pressures.

Determination of Specific Gravity of the Suspensions. The specific gravity of each suspension tested was obtained in the following manner:

A 50-ml dry volumetric flask was weighed. The flask was filled to the 50-ml mark with distilled water and weighed again. After drying the flask, it was filled to the 50-ml mark with the suspension, the specific gravity of which was to be determined, and weighed. This test was performed in duplicate.

Analysis of the Suspensions. Each suspension tested was analyzed for suspended solids in the following manner:

A Gooch crucible was fitted onto a filtering flask, and a vacuum was applied by means of an aspirator. An asbestos suspension was poured into the Gooch crucible until a thin mat of asbestos had covered the bottom of the crucible. The crucible was then dried and weighed. A small amount of the suspension to be tested was poured into the crucible. The crucible, with contents, was weighed again. The Gooch crucible was then placed back on the filtering flask, and the vacuum was applied until all of the liquid had filtered off and only the moist solids were left in the crucible. These solids were washed with boiling water to remove the entrapped sodium sulfate, the filtering being continued during this washing. The crucible was removed, heated to redness, allowed to cool, and weighed. This procedure was repeated until no further loss of weight on drying was noted. The crucible, with contents, was then placed in a beaker of 10 per cent hydrochloric acid. The acid was heated to 90°C and the contents of the crucible were allowed to digest for 30 minutes.

The crucible was removed, placed onto the filtering flask, and the excess acid was filtered off. The crucible was removed, dried and weighed. This procedure was performed in triplicate. The first weight was total suspended solids, the last was filter aid.

F. Data and Results

The experimental data and results obtained during this investigation are presented in the form of tables and figures as indicated in the following paragraphs.

Summary of Experimental Results. Table III, page 61, presents a summary of the results obtained in this investigation. The yield values and the coefficients of rigidity listed in this table are those calculated by the graphical method as shown by figures 7, 9, 11, 13, 15, 17, 19, and 21. The coefficients of compressibility presented in table III were calculated by graphical methods shown by figures 8, 10, 12, 14, 16, 18, 20, and 22. From the results listed in table III, the empirical equations presented in tables XXI and XXII were derived.

Determinations of Yield Value and Coefficient of Rigidity. The flow data for the suspensions tested is presented in tables IV, VI, VIII, X, XII, XIV, XVI, and XVIII. From these data the coefficient of rigidity and yield value of each suspension were determined by the graphical solution shown on figures 7, 9, 11, 13, 15, 17, 19, and 21.

Determinations of Coefficient of Compressibility. The filtration data from which the coefficient of compressibility for each suspension was determined is listed in tables V, VII, IX, XI, XIII, XV, XVII, and XIX. The graphical determinations of the

coefficient of compressibility for each suspension is shown on figures 8, 10, 12, 14, 16, 18, 20, and 22.

Stormer Viscometer Tests. The experimental determinations with the Stormer viscometer produced the data presented in table XX, page 94. This data was taken on suspensions 1, 2, and 3. From this data the plot of actuating weight versus revolutions per second, as shown in figure 23, page 95, was drawn. This type of plot was not made for the other suspensions as the concentration of suspended solids became high, because reproducible data could not be obtained.

Derivation of Empirical Equations. The equations derived to show the relationship between yield value, concentration of solids and compressibility coefficient are listed in table XXI, page 96. This table shows the deviations of yield values as calculated by each equation from the experimentally determined yield values.

The equations derived to show the relationship between coefficient of rigidity, concentration of solids and the compressibility coefficient are presented in table XXII, page 97. Deviations of the coefficient of rigidity as calculated by each equation from the experimentally determined coefficient of rigidity are also listed in this table.

Table III

SUMMARY OF DETERMINATIONS OF YIELD VALUE, COEFFICIENT OF RIGIDITY,
AND COEFFICIENT OF COMPRESSIBILITY FOR ALUMINUM HYDROXIDE - FILTER AID SUSPENSIONS

Suspension Number	Temperature	Age of Aluminum Hydroxide in Suspension	Specific Gravity	Concentration of Suspended Solids, C	Percentage Aluminum Oxide in Solids, Dry Basis	Percentage Filter Aid in Solids, Dry Basis	Coefficient of Compressibility, s	Yield Value, S_y	Coefficient of Rigidity, η
	°C	days		$\frac{\text{lb dried solids}}{100 \text{ lb suspension}}$	per cent	per cent		lb/sq ft	lb/ft-sec
1	22-32	38	1.106	3.60	100.0	0.0	0.425	0.036	0.001360
2	23-31	45	1.085	6.14	100.0	0.0	0.360	0.150	0.002202
3	26-31	59	1.074	6.20	67.0	33.0	0.370	0.176	0.005169
4	23-29	75	1.130	7.23	48.5	51.5	0.200	0.212	0.005360
5	30-32	83	1.169	10.05	36.2	63.8	0.190	0.265	0.006176
6	27-28	89	1.199	13.03	19.3	80.7	0.080	0.337	0.008044
7	26-29	7	1.250	12.02	42.3	57.4	0.212	0.210	0.009780
8	24-25	16	1.180	8.23	29.0	71.0	0.089	0.081	0.002755

Table IV

FRICITION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE SUSPENSION NUMBER 1

Per cent suspended solids - 3.60
Age of suspension - 38 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity $\frac{V}{4 \times \text{Diameter}}$ $V/4D$
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	28	0.769	5.300	5.30	5.550	0.2920	25.50
0.5	28	0.392	2.705	2.33	2.440	0.1280	23.00
1.0	26	2.290	5.540	2.65	2.780	0.2460	15.85
1.0	26	1.830	4.430	1.68	1.760	0.1560	12.67
1.0	26	1.270	3.070	1.08	1.130	0.1000	8.77
				inches CCl_4			
0.5	30	0.029	0.204	19.30	0.950	0.0500	0.98
0.5	30	0.029	0.204	17.75	0.872	0.0458	0.98
0.5	30	0.086	0.596	20.10	0.987	0.0579	2.87
0.5	30	0.066	0.460	18.60	0.914	0.0480	2.22
0.5	30	0.093	0.639	20.00	0.982	0.0517	3.08
0.5	30	0.159	1.092	21.28	1.045	0.0549	5.25
0.5	30	0.143	0.985	22.48	1.110	0.0583	4.74
1.0	32	0.170	0.411	12.73	0.625	0.0553	1.18
1.0	32	0.231	0.559	12.05	0.592	0.0525	1.60
1.0	30	0.446	1.078	23.38	0.656	0.0581	3.08
1.0	30	0.246	0.595	13.43	0.660	0.0585	1.70
1.0	30	0.329	0.796	14.70	0.722	0.0640	2.28
1.5	30	0.205	0.212	2.35	0.105	0.0142	0.40
1.5	30	0.112	0.116	2.98	0.133	0.0180	0.22
1.5	30	0.282	0.291	4.03	0.180	0.0244	0.55
1.5	30	0.887	0.916	18.65	0.830	0.1125	1.71
1.5	30	0.723	0.747	13.55	0.604	0.0820	1.40
1.5	30	0.476	0.491	8.60	0.383	0.0520	0.92

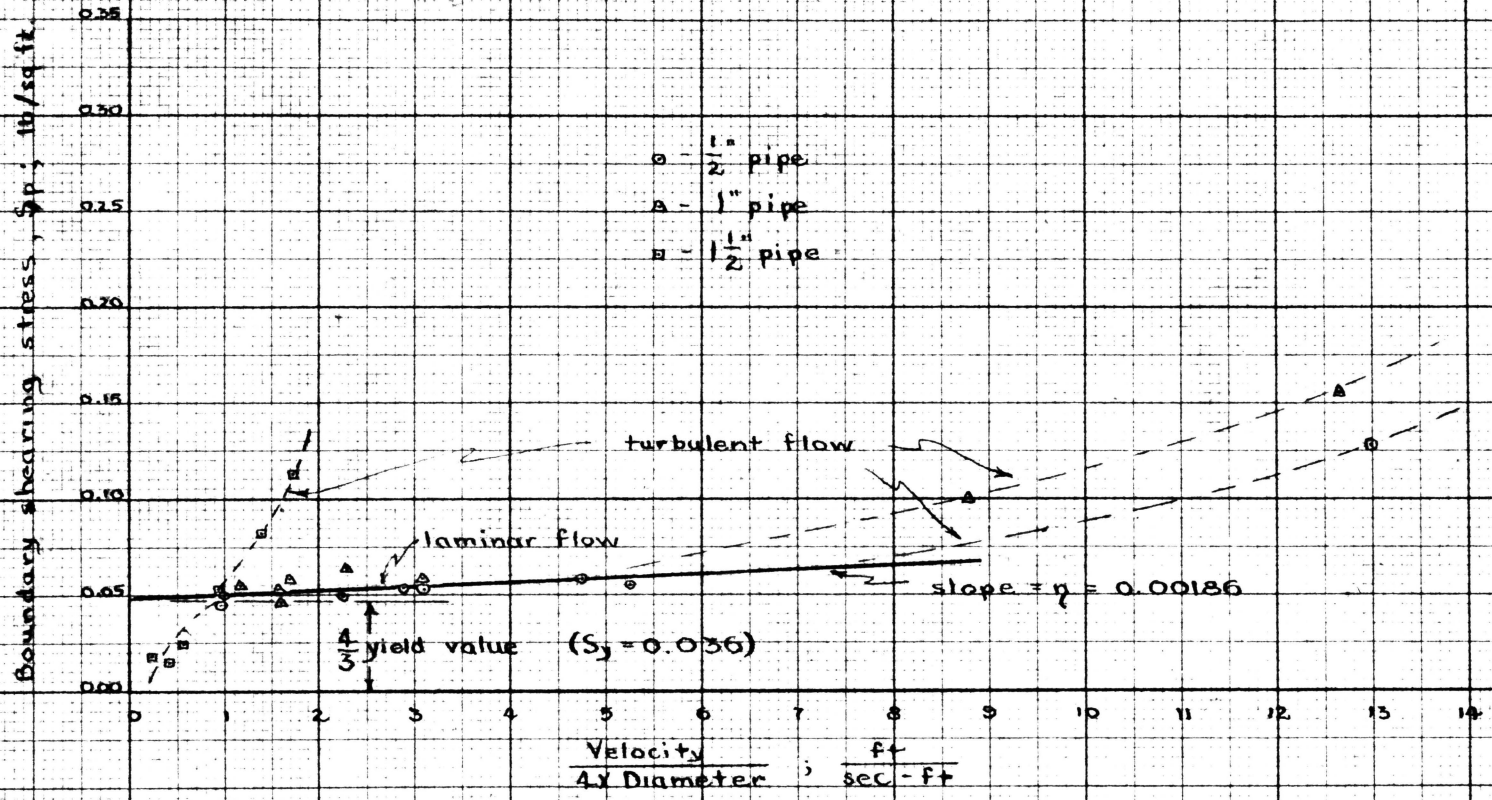


FIGURE 7

DETERMINATION OF YIELD VALUE, S_y , AND COEFFICIENT OF RIGIDITY FOR SUSPENSION NUMBER 1

Table V

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 1

Per cent suspended solids - 3.60
Age of aluminum hydroxide in suspension - 38 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ , since Filtration Began	$\frac{W}{A}$	$\frac{P \cdot \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	$\frac{\text{lb-min}}{\text{sq in.}}$	min	°C
6	200	0.38	0.000665	0.38	30
6	400	0.81	0.001332	0.81	30
6	600	1.29	0.001999	1.29	30
6	800	1.79	0.002665	1.79	30
6	1000	2.40	0.003341	2.40	30
6	1100	2.71	0.003675	2.71	30
6	1200	3.03	0.004009	3.03	30
6	1300	3.39	0.004343	3.39	30
6	1400	3.75	0.004677	3.75	30
6	1500	4.14	0.005011	4.14	30
6	1600	4.52	0.005321	4.52	30
6	1700	4.91	0.005680	4.91	30
6	1800	5.30	0.006014	5.30	30
10	200	0.21	0.000665	0.21	30
10	400	0.47	0.001332	0.47	30
10	600	0.76	0.002001	0.76	30
10	800	1.09	0.002665	1.09	30
10	1000	1.49	0.003341	1.49	30
10	1200	1.92	0.004009	1.92	30
10	1400	2.40	0.004677	2.40	30
10	1500	2.65	0.005011	2.65	30
10	1600	2.90	0.005321	2.90	30
10	1800	3.46	0.006014	3.46	30
10	2000	4.05	0.006683	4.05	30
10	2200	4.69	0.007352	4.69	30
10	2400	5.39	0.008017	5.39	30
12	200	0.25	0.000668	4,510	31
12	400	0.55	0.001336	4,950	31
12	600	0.88	0.002004	5,280	31
12	800	1.25	0.002673	5,620	31
12	1000	1.66	0.003341	5,980	31
12	1200	2.11	0.004009	6,340	31
12	1400	2.60	0.004667	6,700	31
12	1600	3.14	0.005321	7,090	31
12	1800	3.70	0.006014	7,430	31
12	2000	4.30	0.006683	7,760	31
12	2200	4.96	0.007352	8,130	31
12	2400	5.61	0.008017	8,430	31
12	3000	7.90	0.010022	9,500	31
12	3200	8.75	0.010674	9,820	31
12	3400	9.63	0.011325	10,200	31
15	200	0.22	0.000668	4,960	31
15	400	0.44	0.001336	4,950	31
15	600	0.77	0.002004	5,780	31
15	800	1.07	0.002673	6,020	31
15	1000	1.40	0.003341	6,310	31
15	1200	1.80	0.004009	6,770	31
15	1400	2.23	0.004667	7,170	31
15	1600	2.70	0.005321	7,610	31
15	1800	3.20	0.006014	8,020	31
15	2000	3.73	0.006683	8,420	31
15	2200	4.30	0.007352	8,800	31
15	2400	4.90	0.008017	9,210	31
15	3000	6.93	0.010022	10,400	31
15	3200	7.67	0.010674	10,890	31
15	3400	8.44	0.011325	11,150	31
19	200	0.17	0.000668	4,860	31
19	400	0.38	0.001336	5,420	31
19	600	0.61	0.002004	5,810	31
19	800	0.87	0.002673	6,190	31
19	1000	1.17	0.003341	6,670	31
19	1200	1.48	0.004009	7,050	31
19	1400	1.84	0.004667	7,490	31
19	1600	2.22	0.005321	7,930	31
19	1800	2.62	0.006014	8,330	31
19	2000	3.07	0.006683	8,760	31
19	2400	4.04	0.007352	9,630	31
19	3000	5.70	0.010022	10,870	31
19	3200	6.31	0.010674	11,250	31
19	3400	6.95	0.011325	11,660	31

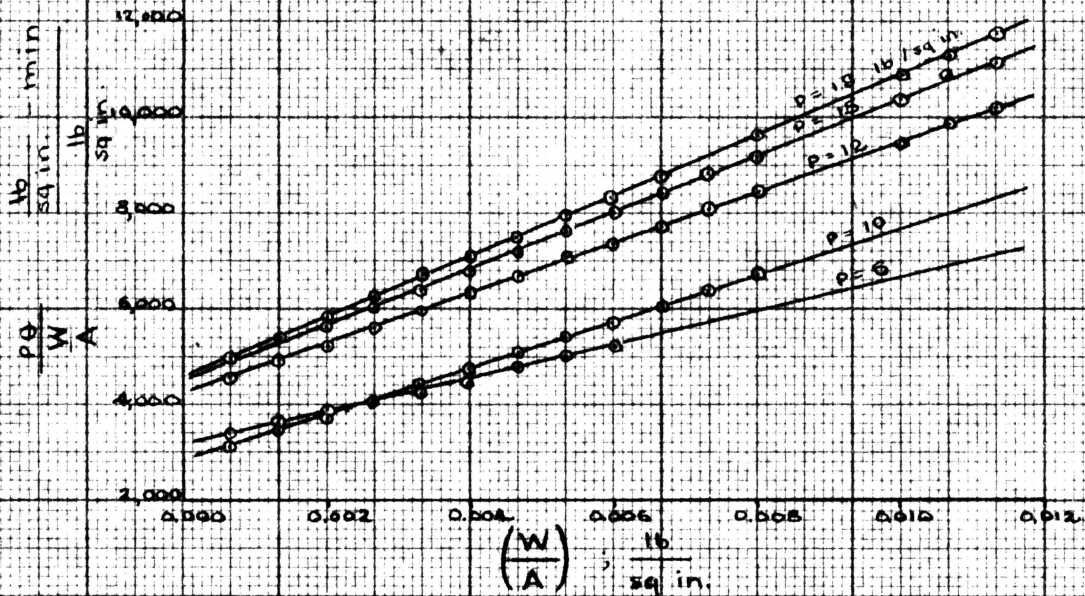


FIGURE 8 A

FILTRATION CURVE FOR SUSPENSION NUMBER 1

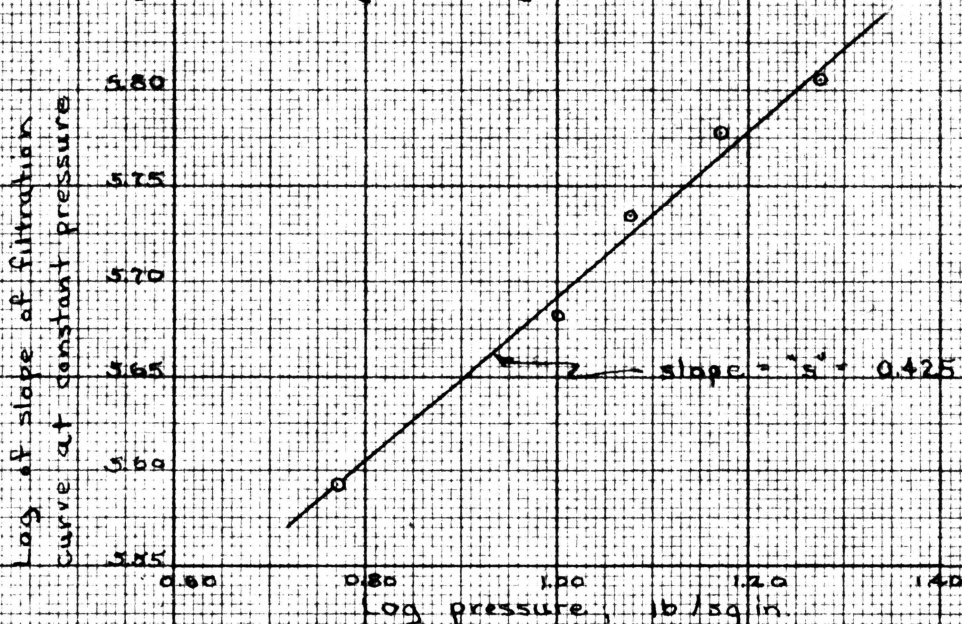


FIGURE 8 B

DETERMINATION OF COMPRESSIBILITY COEFFICIENT, s' ,
FOR SUSPENSION NUMBER 1

Table VI

FRICITION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE SUSPENSION NUMBER 2

Per Cent suspended solids - 6.14
Age of suspension - 45 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity 4 X Diameter V/4D
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	30	0.758	5.31	5.05	5.29	0.290	25.60
0.5	30	0.556	3.90	4.70	4.92	0.270	18.80
0.5	30	0.438	3.12	4.15	4.35	0.238	15.00
0.5	30	0.298	2.09	3.20	3.35	0.184	10.10
0.5	30	0.238	1.67	2.65	2.78	0.152	8.03
0.5	30	0.595	4.18	4.80	5.03	0.276	20.10
0.5	30	0.379	2.66	4.00	4.19	0.230	12.80
0.5	30	0.209	1.46	3.45	3.61	0.198	7.02
0.5	30	0.208	1.46	3.55	3.72	0.204	7.02
0.5	30	0.156	1.09	3.40	3.56	0.195	5.27
0.5	30	0.147	1.03	3.35	3.51	0.192	4.97
0.5	30	0.095	0.06	3.10	3.25	0.178	5.13
1.0	30	0.575	1.42	2.20	2.30	0.212	4.06
1.0	30	0.595	1.47	2.25	2.35	0.217	4.20
1.0	30	1.390	3.43	2.70	2.83	0.261	9.80
1.0	30	1.515	3.74	2.80	2.93	0.270	10.70
1.0	30	0.980	2.42	2.95	3.09	0.285	6.91
1.0	30	0.758	1.37	2.75	2.88	0.266	5.34
1.0	30	1.041	2.57	2.75	2.88	0.266	7.34
1.0	30	1.282	3.17	2.45	2.56	0.236	9.05
1.0	30	0.238	0.59	1.55	1.62	0.149	1.68
1.0	30	0.758	1.37	2.35	2.46	0.227	5.34
1.0	30	0.694	1.71	2.40	2.51	0.232	4.90
1.0	30	0.320	0.79	2.15	2.25	0.208	2.26
1.5	30	2.275	2.38	1.80	1.88	0.266	4.44
1.5	30	1.390	1.45	1.70	1.78	0.252	2.71
1.5	30	0.549	0.57	1.50	1.57	0.222	1.07
1.5	30	0.451	0.47	1.35	1.42	0.200	0.38

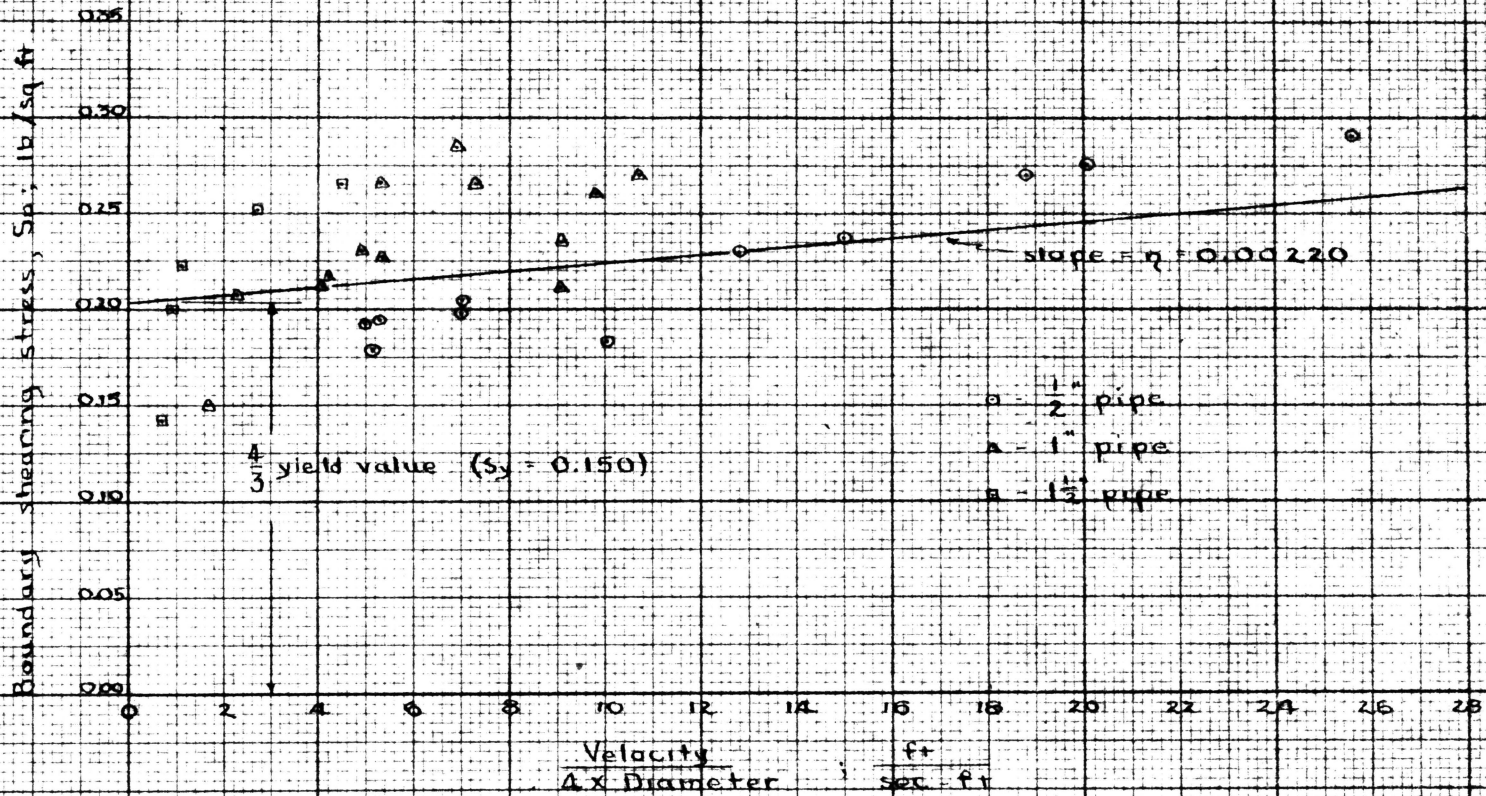


FIGURE 9

DETERMINATION OF YIELD VALUE, S_y , AND COEFFICIENT OF RIGIDITY, η , FOR SUSPENSION NUMBER 2

Table VII

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 2

Per cent suspended solids - 6.14
Age of aluminum hydroxide in suspension - 45 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ , since Filtration Began	$\frac{W}{A}$	$\frac{P \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	$\frac{\text{lb-min}}{\text{sq in.}}$	min	°C
6.0	100	0.46	0.000334	8,263	28
6.0	200	0.88	0.000668	7,904	28
6.0	300	1.39	0.001002	8,323	28
6.0	400	1.89	0.001336	8,488	28
6.0	500	2.43	0.001670	8,730	28
6.0	600	3.00	0.002004	8,982	28
6.0	700	3.61	0.002339	9,260	28
6.0	800	4.23	0.002673	9,494	28
6.0	900	4.89	0.003007	9,757	28
6.0	1000	5.58	0.003341	10,020	28
8.0	100	0.21	0.000334	5,029	29
8.0	200	0.47	0.000668	5,628	29
8.0	300	0.76	0.001002	6,067	29
8.0	400	1.06	0.001336	6,347	29
8.0	500	1.39	0.001670	6,658	29
8.0	600	1.74	0.002004	6,946	29
8.0	700	2.12	0.002339	7,250	29
8.0	800	2.53	0.002673	7,572	29
8.0	900	2.94	0.003007	7,821	29
8.0	1000	3.40	0.003341	8,141	29
8.0	1100	3.85	0.003615	8,380	29
8.0	1200	4.37	0.004009	8,720	29
8.0	1300	4.90	0.004343	9,026	29
9.0	200	0.23	0.000668	3,100	30
9.0	400	0.76	0.001336	5,120	30
9.0	500	1.08	0.001670	5,810	30
9.0	600	1.41	0.002004	6,340	30
9.0	700	1.77	0.002339	6,810	30
9.0	800	2.14	0.002673	7,210	30
9.0	900	2.50	0.003007	7,490	30
9.0	1000	2.90	0.003341	7,820	30
9.0	1100	3.33	0.003675	8,160	30
9.0	1200	3.78	0.004009	8,840	30
9.0	1300	4.24	0.004343	8,890	30
9.0	1400	4.73	0.004677	9,110	30
9.0	1500	5.22	0.005011	9,390	30
13.2	200	0.26	0.000668	5,140	31
13.2	400	0.62	0.001336	6,150	31
13.2	600	1.03	0.002004	6,790	31
13.2	800	1.54	0.002673	7,620	31
13.2	1000	2.10	0.003341	8,300	31
13.2	1100	2.41	0.003675	8,660	31
13.2	1200	2.73	0.004009	8,980	31
13.2	1300	3.08	0.004343	9,380	31
13.2	1400	3.44	0.004677	9,710	31
13.2	1500	3.81	0.005011	10,020	31
13.2	1700	4.64	0.005680	10,790	31
13.2	1800	5.06	0.006014	11,100	31
15.4	200	0.26	0.000668	6,000	31
15.4	400	0.58	0.001336	6,690	31
15.4	600	0.96	0.002004	7,360	31
15.4	800	1.41	0.002673	8,120	31
15.4	1000	1.91	0.003341	8,860	31
15.4	1100	2.18	0.003675	9,140	31
15.4	1200	2.47	0.004009	9,480	31
15.4	1300	2.79	0.004343	9,910	31
15.4	1400	3.13	0.004677	10,310	31
15.4	1500	3.48	0.005011	10,700	31
15.4	1600	4.20	0.005680	11,380	31
15.4	1800	4.60	0.006014	11,790	31
15.4	2000	5.40	0.006682	12,470	31

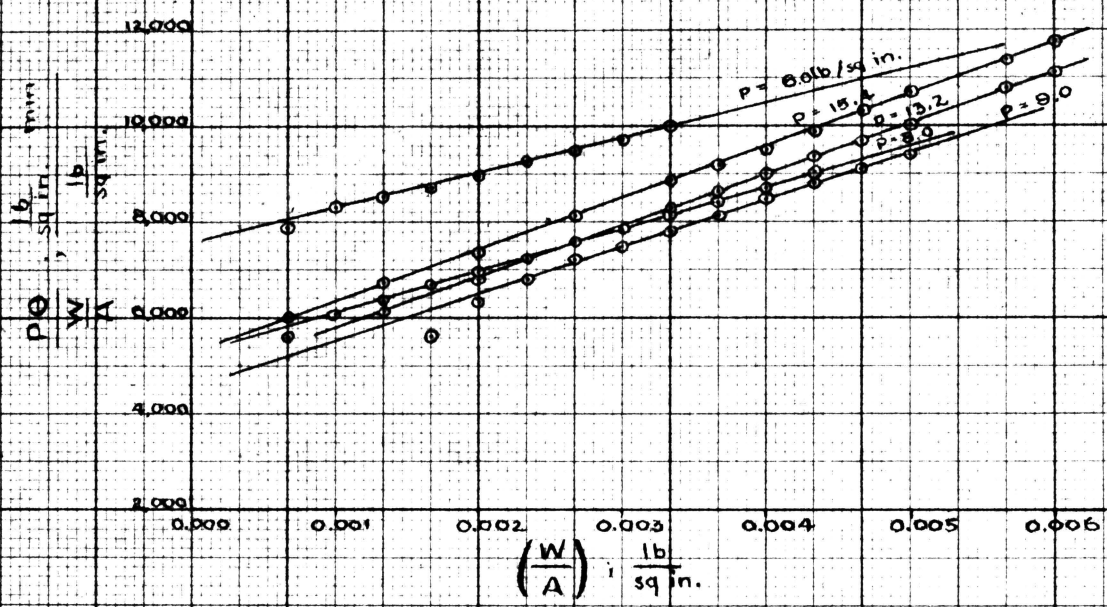


FIGURE 10A
FILTRATION CURVE OF SUSPENSION NUMBER 2

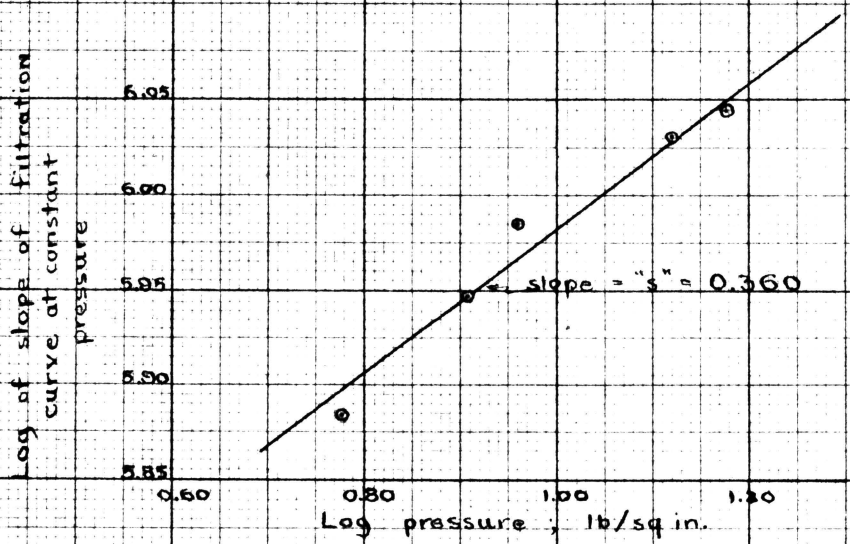


FIGURE 10B
DETERMINATION OF COMPRESSIBILITY COEFFICIENT, "s"
FOR SUSPENSION NUMBER 2

Table VIII

FRICTION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE - FILTER AID SUSPENSION NUMBER 3

Per cent suspended solids - 6.20
Per cent aluminum oxide in solids, dry basis - 67
Age of aluminum hydroxide in suspension - 59 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity $4 \times$ Diameter V/4D
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	28	0.641	4.547	5.98	6.267	0.3195	21.90
0.5	28	0.521	3.694	6.00	6.288	0.3206	17.79
0.5	28	0.471	3.344	5.85	6.131	0.3126	16.10
0.5	28	0.537	3.813	5.80	6.078	0.3098	13.36
0.5	28	0.397	2.814	5.60	5.869	0.2992	13.55
0.5	28	0.245	1.738	5.10	5.345	0.2725	8.37
0.5	28	0.252	1.791	5.20	5.450	0.2773	8.62
0.5	28	0.172	1.223	4.90	5.135	0.2618	5.89
1.0	28	2.500	6.237	3.30	3.458	0.2969	17.84
1.0	28	2.272	5.668	3.30	3.458	0.2969	16.21
1.0	28	1.389	3.465	2.83	2.965	0.2546	9.91
1.0	28	1.000	2.495	2.78	2.913	0.2501	7.14
1.0	28	0.595	1.485	2.68	2.808	0.2411	4.24
1.5	31	2.275	2.407	1.70	1.781	0.2344	4.49
1.5	31	2.380	2.518	1.65	1.729	0.2276	4.70
1.5	31	1.851	1.958	1.65	1.729	0.2276	3.65
1.5	31	1.666	1.763	1.55	1.624	0.2138	3.28
1.5	31	1.136	1.202	1.45	1.520	0.2000	2.24
1.5	31	1.111	1.175	1.55	1.624	0.2138	2.19
1.5	31	1.190	1.259	1.53	1.603	0.2110	2.34
1.5	31	0.438	4.634	1.40	1.467	0.1931	0.86
1.5	31	0.126	1.333	1.25	1.310	0.1724	0.25

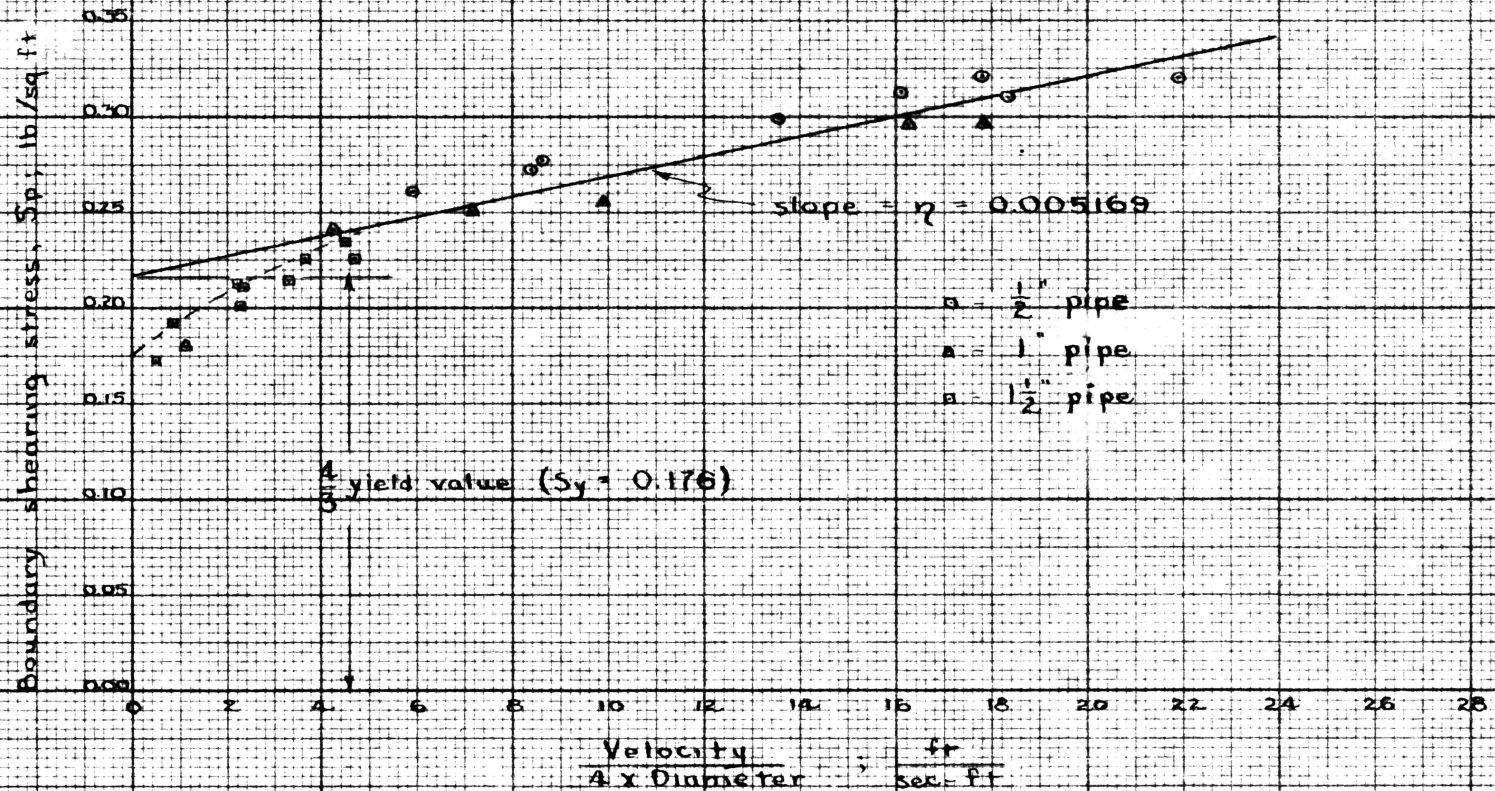


FIGURE II

DETERMINATION OF YIELD VALUE, S_y , AND COEFFICIENT OF RIGIDITY, η , FOR SUSPENSION NUMBER 3

Table IX

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 3

Per cent suspended solids - 6.20
Per cent aluminum oxide in solids, dry basis - 67.0
Age of aluminum hydroxide in suspension - 59 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ , since Filtration Began	$\frac{W}{A}$	$\frac{P \cdot \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	$\frac{\text{lb-min}}{\text{sq in.}}$	min	°C
6	100	0.55	0.000334	9,880	29
6	200	1.10	0.000668	9,880	29
6	300	1.70	0.001002	10,179	29
6	400	2.31	0.001336	10,374	29
6	500	2.94	0.001670	10,562	29
6	600	3.67	0.002004	10,988	29
6	700	4.39	0.002339	11,261	29
6	800	5.16	0.002673	11,582	29
6	900	5.99	0.003007	11,952	29
6	1000	6.77	0.003341	12,158	29
7	100	0.25	0.000334	5,240	29
7	200	0.54	0.000668	5,660	29
7	300	0.88	0.001002	6,150	29
7	400	1.20	0.001336	6,290	29
7	500	1.57	0.001670	6,590	29
7	600	1.98	0.002004	6,910	29
7	700	2.42	0.002339	7,250	29
7	800	2.90	0.002673	7,600	29
7	900	3.40	0.003007	7,910	29
7	1000	3.90	0.003341	8,170	29
7	1100	4.48	0.003675	8,530	29
7	1200	5.08	0.004009	8,860	29
7	1300	5.71	0.004343	9,210	29
7	1400	6.37	0.004677	9,540	29
7	1500	7.04	0.005011	9,820	29
10	100	0.21	0.000334	6,287	30
10	200	0.47	0.000668	7,035	30
10	300	0.72	0.001002	7,185	30
10	400	1.02	0.001336	7,634	30
10	500	1.38	0.001670	8,263	30
10	600	1.68	0.002004	8,383	30
10	700	2.05	0.002339	8,764	30
10	800	2.43	0.002673	9,090	30
10	900	2.86	0.003007	9,511	30
10	1000	3.29	0.003341	9,847	30
10	1100	3.74	0.003675	10,176	30
10	1200	4.23	0.004009	10,551	30
10	1300	4.72	0.004343	10,868	30
10	1400	5.27	0.004677	10,267	30
10	1500	5.81	0.005011	11,594	30
12	200	0.40	0.000668	7,185	30
12	400	0.83	0.001336	7,904	30
12	600	1.44	0.002004	8,622	30
12	800	2.10	0.002673	9,427	30
12	1000	2.84	0.003341	10,200	30
12	1100	3.24	0.003675	10,579	30
12	1200	3.67	0.004009	10,985	30
12	1300	4.10	0.004343	11,328	30
12	1400	4.57	0.004677	11,725	30
12	1500	5.04	0.005011	12,089	30
14	200	0.43	0.000668	9,011	30
14	400	0.92	0.001336	9,640	30
14	600	1.49	0.002004	10,409	30
14	800	2.17	0.002673	11,365	30
14	1000	2.88	0.003341	12,068	30
14	1100	3.28	0.003675	12,495	30
14	1200	3.69	0.004009	12,886	30
14	1300	4.13	0.004343	13,331	30
14	1400	4.59	0.004677	13,739	30
14	1500	5.06	0.005011	14,165	30
14	1600	5.55	0.005321	14,602	30
14	1700	6.07	0.005680	14,961	30
14	1800	6.61	0.006014	15,387	30

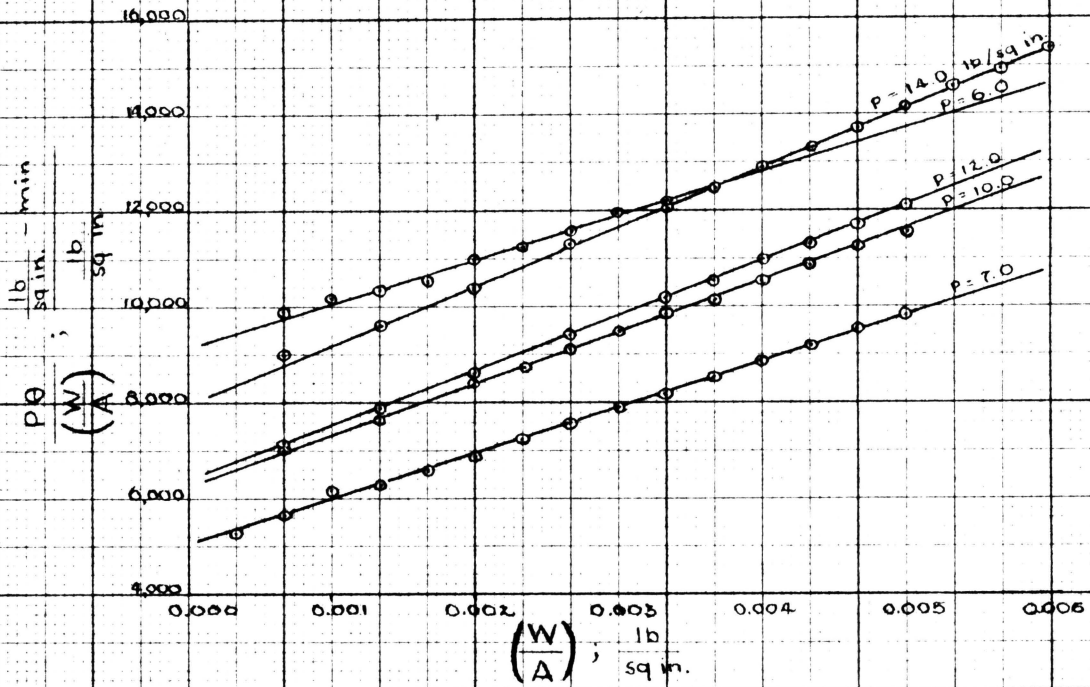


FIGURE 12 A
FILTRATION CURVE FOR SUSPENSION NUMBER 3

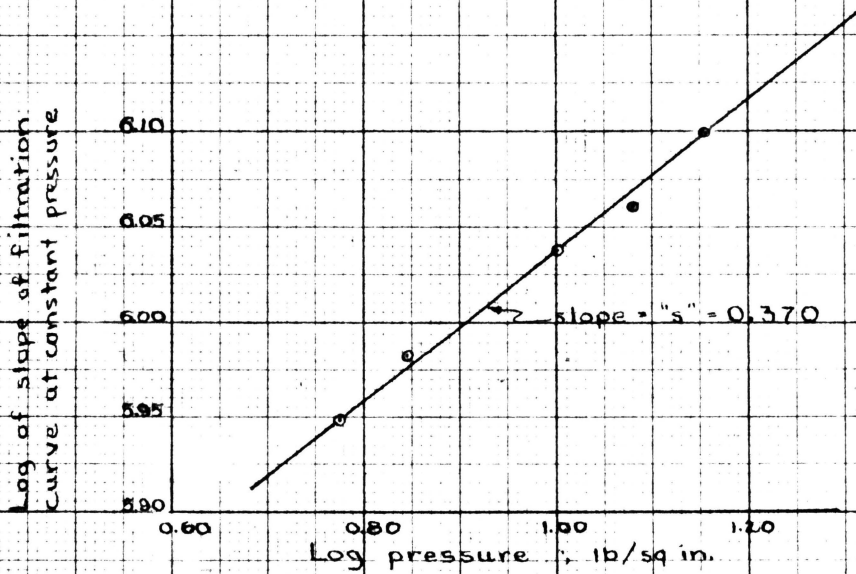


FIGURE 12 B
DETERMINATION OF COMPRESSIBILITY COEFFICIENT, "s"
FOR SUSPENSION NUMBER 3

Table X

FRICITION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE - FILTER AID SUSPENSION NUMBER 4

Per cent suspended solids - 7.25
Per cent aluminum oxide in solids, dry basis - 48.5
Age of aluminum hydroxide in suspension - 73 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity 4 X Diameter V/4D
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	29	0.773	5.21	7.33	7.67	0.411	25.10
0.5	29	0.641	4.33	6.90	7.23	0.387	20.90
0.5	29	0.617	4.17	6.85	7.17	0.384	20.10
0.5	29	0.521	3.52	6.70	7.02	0.376	16.98
0.5	29	0.347	2.34	6.43	6.73	0.361	11.28
0.5	29	0.308	2.08	6.23	6.52	0.349	10.03
0.5	29	0.227	1.53	5.88	6.16	0.330	7.37
0.5	29	0.218	1.47	5.65	5.91	0.317	7.08
0.5	29	0.091	0.61	5.45	5.71	0.306	2.96
1.0	29	2.380	5.65	4.15	4.35	0.393	16.15
1.0	29	1.470	3.49	3.75	3.93	0.355	9.97
1.0	29	2.275	5.40	3.90	4.08	0.369	15.43
1.0	29	2.500	5.94	4.03	4.22	0.382	17.00
1.0	29	1.785	4.24	3.75	3.93	0.355	12.12
1.0	29	1.315	3.12	3.55	3.72	0.336	8.93
1.0	29	0.658	1.56	3.15	3.30	0.298	4.46
1.0	29	0.278	0.66	2.88	3.01	0.272	1.89
1.0	29	0.050	0.12	2.42	2.53	0.228	0.34
1.5	29	4.170	4.20	2.10	2.20	0.305	7.84
1.5	29	2.780	2.80	1.90	1.90	0.276	5.22
1.5	29	0.438	0.44	1.75	1.83	0.254	0.82
1.5	29	0.595	0.60	1.75	1.83	0.254	1.12
1.5	29	0.320	0.32	1.65	1.73	0.240	0.60
1.5	29	0.130	0.13	1.55	1.62	0.224	0.24

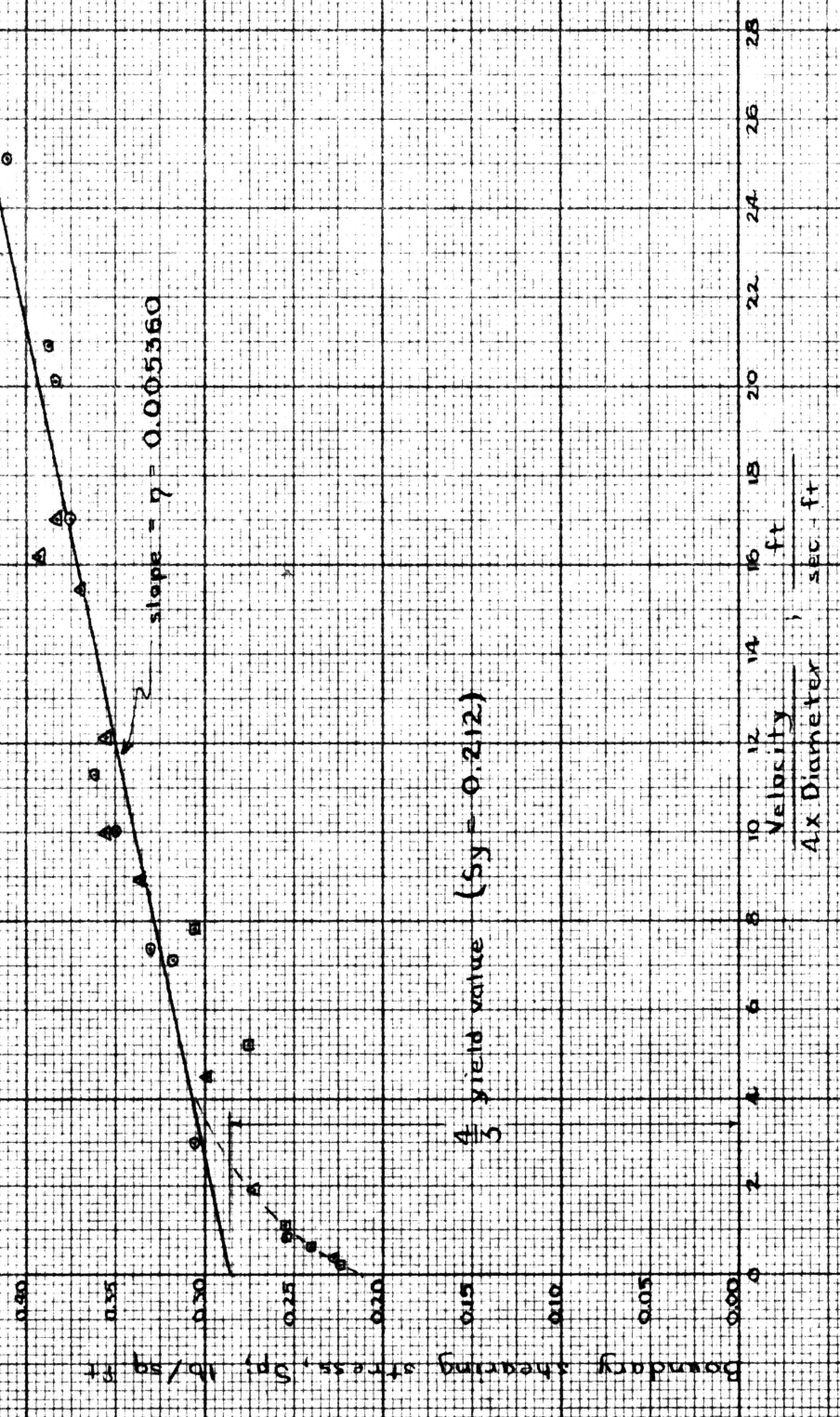


FIGURE 13
 DETERMINATION OF YIELD VALUE S_y , AND COEFFICIENT OF RIGIDITY η , FOR SUSPENSION NO. 4

Table XI

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 4

Per cent suspended solids - 7.25
Per cent aluminum oxide in solids, dry basis - 48.5
Age of aluminum hydroxide in suspension - 73 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ since Filtration Began	$\frac{W}{A}$	$\frac{P \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	$\frac{\text{lb-min}}{\text{sq in.}}$	min	°C
6.0	100	0.52	0.000334	9,341	28
6.0	200	0.97	0.000668	8,712	28
6.0	300	1.54	0.001002	9,221	28
6.0	400	2.10	0.001336	9,431	28
6.0	500	2.76	0.001670	9,916	28
6.0	600	3.41	0.002004	10,209	28
6.0	700	4.21	0.002339	10,799	28
6.0	800	4.97	0.002673	11,156	28
6.0	900	5.76	0.003007	11,493	28
6.0	1000	6.66	0.003341	11,960	28
7.5	100	0.45	0.000334	10,090	28
7.5	200	0.88	0.000668	9,900	28
7.5	300	1.40	0.001002	10,480	28
7.5	400	1.91	0.001336	10,720	28
7.5	500	2.51	0.001670	11,250	28
7.5	600	3.13	0.002004	11,720	28
7.5	700	3.79	0.002339	12,120	28
7.5	800	4.48	0.002673	12,590	28
7.5	900	5.20	0.003007	12,960	28
7.5	1000	5.97	0.003340	13,400	28
7.5	1100	7.55	0.003675	14,120	28
10.4	100	0.25	0.000334	7,550	28
10.4	200	0.48	0.000668	7,490	28
10.4	300	0.76	0.001002	7,780	28
10.4	400	1.08	0.001336	8,400	28
10.4	500	1.40	0.001670	8,720	28
10.4	600	1.75	0.002004	9,080	28
10.4	700	2.14	0.002339	9,510	28
10.4	800	2.57	0.002673	10,000	28
10.4	900	3.01	0.003007	10,400	28
10.4	1000	3.47	0.003341	10,800	28
10.4	1100	3.99	0.003675	11,300	28
10.4	1200	4.53	0.004009	11,760	28
10.4	1300	5.12	0.004343	12,280	28
10.4	1400	5.72	0.004674	12,700	28
10.4	1500	6.34	0.005011	13,130	28
12.0	100	0.18	0.000334	6,467	29
12.0	200	0.41	0.000668	7,365	29
12.0	300	0.66	0.001002	7,904	29
12.0	400	0.92	0.001336	8,263	29
12.0	500	1.22	0.001670	8,766	29
12.0	600	1.55	0.002004	9,281	29
12.0	700	1.90	0.002339	9,747	29
12.0	800	2.27	0.002673	10,190	29
12.0	900	2.67	0.003007	10,655	29
12.0	1000	3.11	0.003341	11,170	29
12.0	1100	3.55	0.003675	11,591	29
12.0	1200	4.03	0.004009	12,062	29
12.0	1300	4.54	0.004343	12,544	29
12.0	1400	5.06	0.004677	12,982	29
12.0	1500	5.61	0.005011	13,434	29
15.0	1000	2.66	0.003341	11,940	29
15.0	1100	3.03	0.003675	12,350	29
15.0	1200	3.40	0.004009	12,720	29
15.0	1300	3.78	0.004343	13,050	29
15.0	1400	4.22	0.004677	13,550	29
15.0	1500	4.69	0.005011	14,050	29
15.0	1600	5.15	0.005346	14,470	29
15.0	1700	5.65	0.005680	14,920	29
15.0	1800	6.18	0.006014	15,400	29
15.0	1900	6.73	0.006349	15,900	29
15.0	2000	7.30	0.006683	16,400	29
15.0	2100	7.89	0.007017	16,880	29
15.0	2200	8.52	0.007352	17,420	29
15.0	2300	9.19	0.007687	17,980	29

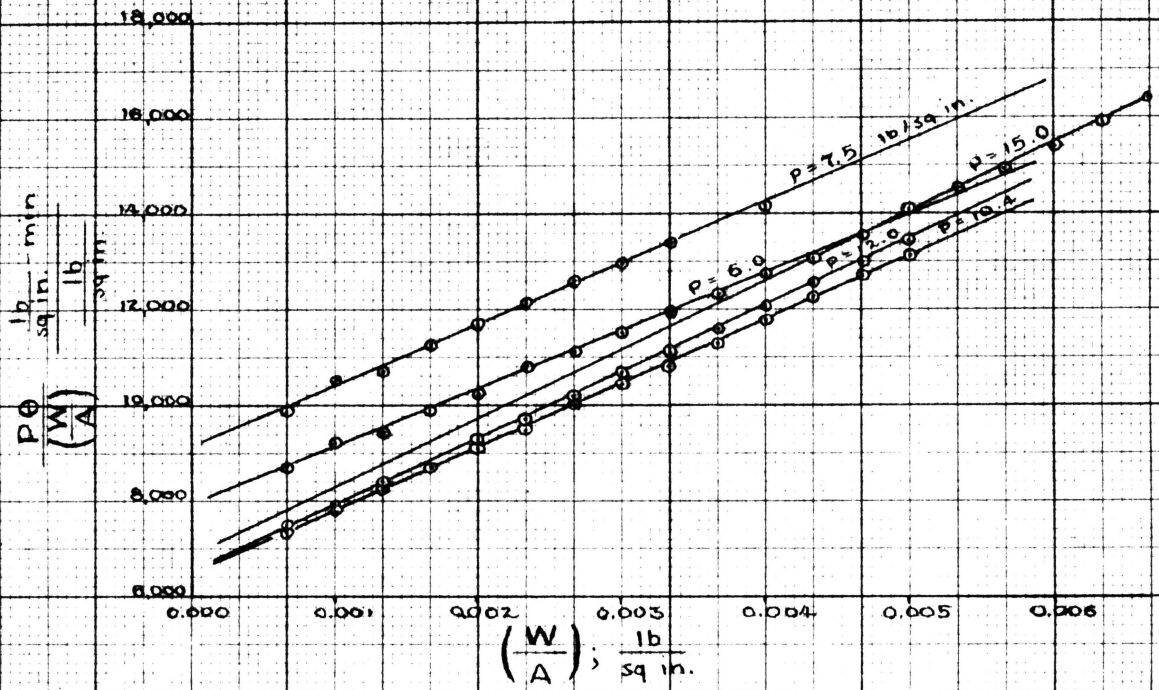


FIGURE 14 A
FILTRATION CURVE FOR SUSPENSION NUMBER 4

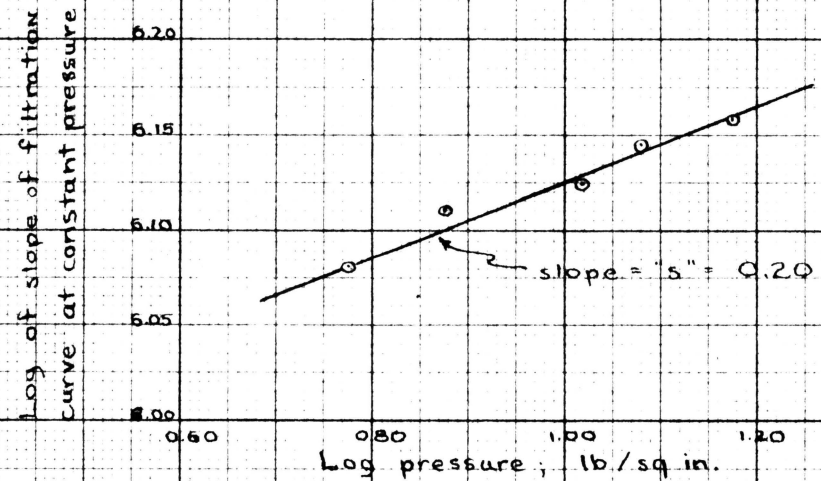


FIGURE 14 B
DETERMINATION OF COMPRESSIBILITY COEFFICIENT, "s"
FOR SUSPENSION NUMBER 4

Table XII

FRICITION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE - FILTER AID SUSPENSION NUMBER 5

Per cent suspended solids - 10.05
Per cent aluminum oxide in solids, dry basis - 36.2
Age of aluminum hydroxide in suspension - 83 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity 4 X Diameter $V/4D$
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	30	0.833	5.429	8.23	8.625	0.478	26.10
0.5	30	0.435	2.835	7.90	8.279	0.459	13.63
0.5	30	0.345	2.249	7.60	7.964	0.442	10.83
0.5	30	0.296	1.929	7.35	7.703	0.427	9.29
0.5	30	0.227	1.479	6.95	7.284	0.404	7.13
0.5	30	0.185	1.205	6.73	7.053	0.391	5.78
0.5	30	0.116	0.756	6.30	6.602	0.366	3.64
0.5	30	0.091	0.593	5.85	6.130	0.340	2.85
1.0	30	3.570	8.182	5.05	5.292	0.493	23.39
1.0	30	2.275	5.214	4.35	4.558	0.426	14.90
1.0	30	0.695	1.593	3.45	3.615	0.337	4.54
1.0	30	0.181	0.415	3.20	3.353	0.313	1.17
1.0	30	1.925	4.412	4.40	4.611	0.431	12.61
1.0	30	1.390	3.185	4.05	4.244	0.396	9.09
1.0	30	0.833	1.909	3.75	3.930	0.367	5.43
1.0	30	0.416	0.953	3.25	3.406	0.318	2.71
1.0	30	0.151	0.346	2.95	3.091	0.289	0.97
1.5	30	5.550	5.398	2.85	2.986	0.427	10.02
1.5	30	5.200	5.057	2.60	2.724	0.390	9.41
1.5	30	4.630	4.503	2.55	2.672	0.383	8.37
1.5	30	3.200	3.112	2.50	2.620	0.375	5.78
1.5	30	2.780	2.704	2.35	2.462	0.352	5.02
1.5	30	1.790	1.741	2.25	2.358	0.338	3.23
1.5	30	0.695	0.676	2.15	2.253	0.323	1.24
1.5	30	0.490	0.476	1.85	1.938	0.278	0.87

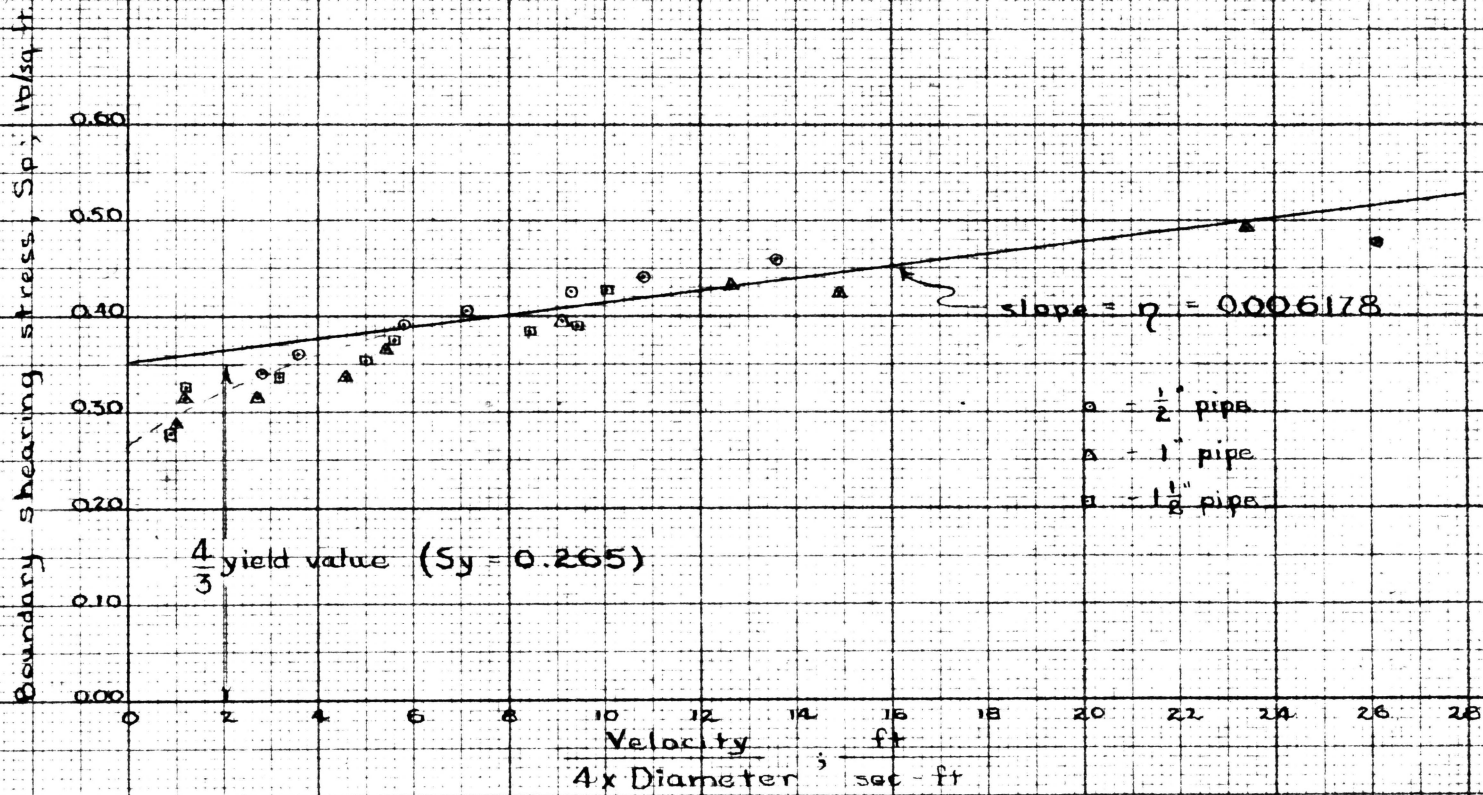


FIGURE 15

DETERMINATION OF YIELD VALUE, S_y , AND COEFFICIENT OF RIGIDITY, η , FOR SUSPENSION NUMBER 5

Table XIII

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 5

Per cent suspended solids - 10.05
Per cent aluminum oxide in solids, dry basis - 36.2
Age of aluminum hydroxide in suspension - 83 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ , since Filtration Began	$\frac{W}{A}$	$\frac{P \cdot \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	lb-min sq in.	min	°C
6.3	100	0.38	0.000334	7,160	30
6.3	200	0.82	0.000668	7,630	30
6.3	300	1.29	0.001002	8,110	30
6.3	400	1.81	0.001336	8,540	30
6.3	500	2.37	0.001670	8,950	30
6.3	600	2.98	0.002004	9,360	30
6.3	700	3.63	0.002339	9,770	30
6.3	800	4.34	0.002673	10,220	30
6.3	900	5.07	0.003007	10,600	30
6.3	1000	5.90	0.003341	11,130	30
8.5	100	0.27	0.000334	6,860	30
8.5	200	0.56	0.000668	7,140	30
8.5	300	0.90	0.001002	7,630	30
8.5	400	1.27	0.001336	8,090	30
8.5	500	1.68	0.001670	8,560	30
8.5	600	2.10	0.002004	8,900	30
8.5	700	2.60	0.002339	9,450	30
8.5	800	3.11	0.002673	9,890	30
8.5	900	3.65	0.003007	10,310	30
8.5	1000	4.25	0.003341	10,810	30
8.5	1100	4.87	0.003675	11,270	30
8.5	1200	5.51	0.004009	11,700	30
8.5	1300	6.20	0.004343	12,130	30
10.0	200	0.47	0.000668	7,040	31
10.0	300	0.77	0.001002	7,690	31
10.0	400	1.11	0.001336	8,320	31
10.0	500	1.47	0.001670	8,800	31
10.0	600	1.88	0.002004	9,380	31
10.0	700	2.31	0.002339	9,870	31
10.0	800	2.77	0.002673	10,390	31
10.0	900	3.27	0.003007	10,880	31
10.0	1000	3.79	0.003341	11,350	31
10.0	1100	4.34	0.003675	11,810	31
10.0	1200	4.91	0.004009	12,250	31
10.0	1300	5.54	0.004343	12,780	31
10.0	1400	6.19	0.004677	13,230	31
10.0	1500	6.86	0.005011	13,700	31
13.0	100	0.20	0.000334	7,790	31
13.0	200	0.41	0.000668	7,980	31
13.0	300	0.67	0.001002	8,670	31
13.0	400	0.95	0.001336	9,270	31
13.0	500	1.24	0.001670	9,660	31
13.0	600	1.57	0.002004	10,180	31
13.0	700	1.91	0.002339	10,610	31
13.0	800	2.29	0.002673	11,160	31
13.0	900	2.69	0.003007	11,600	31
13.0	1000	3.12	0.003341	12,150	31
13.0	1100	3.56	0.003675	12,590	31
13.0	1200	4.05	0.004009	13,110	31
13.0	1300	4.55	0.004343	13,610	31
13.0	1400	5.08	0.004677	14,120	31
13.0	1500	5.62	0.005011	14,580	31
13.0	1600	6.19	0.005346	15,040	31
14.0	700	1.61	0.002339	9,640	31
14.0	800	1.94	0.002673	10,150	31
14.0	900	2.31	0.003007	10,730	32
14.0	1000	2.71	0.003341	11,340	32
14.0	1100	3.12	0.003675	11,880	32
14.0	1200	3.52	0.004009	12,300	32
14.0	1300	3.99	0.004343	12,890	32
14.0	1400	4.46	0.004677	13,340	32
14.0	1500	4.96	0.005011	13,840	32
14.0	1600	5.50	0.005346	14,410	32
14.0	1700	7.05	0.005680	14,910	32
14.0	1800	6.61	0.006014	15,390	32
14.0	1900	7.21	0.006349	15,900	32

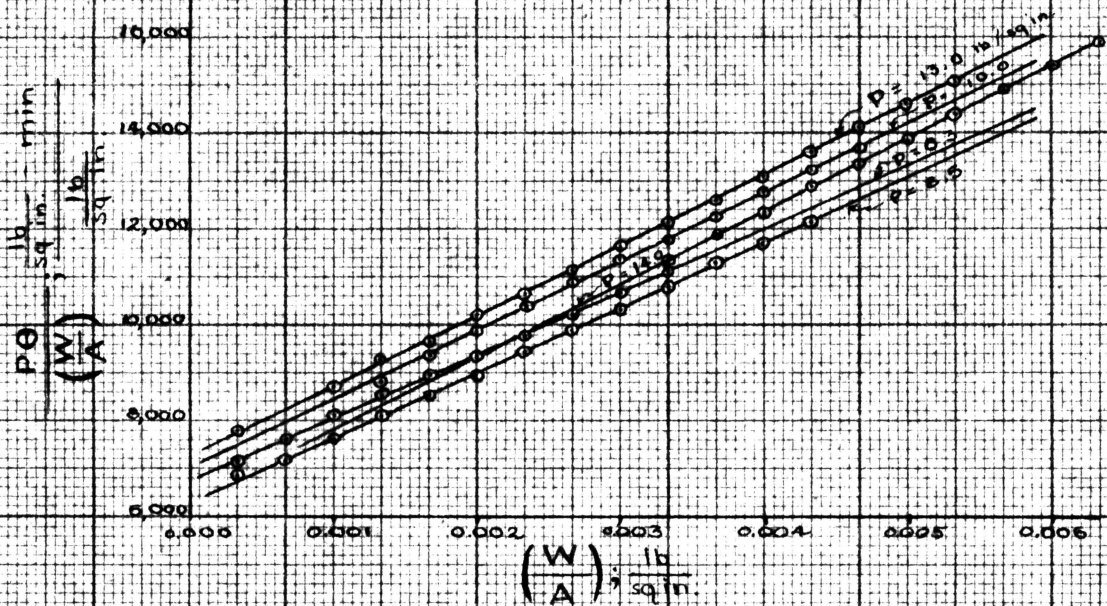


FIGURE 16 A
FILTRATION CURVE FOR SUSPENSION NUMBER 5

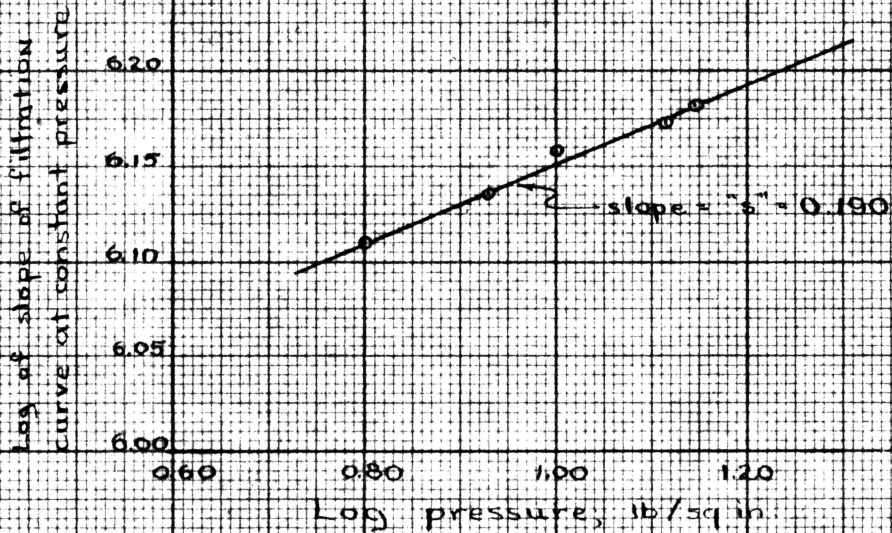


FIGURE 16 B
DETERMINATION OF COMPRESSIBILITY COEFFICIENT, "s",
FOR SUSPENSION NUMBER 5

Table XIV

FRICITION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE - FILTER AID SUSPENSION NUMBER 6

Per cent suspended solids - 13.03

Per cent aluminum oxide in solids, dry basis - 19.3

Age of aluminum hydroxide in suspension - 89 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity 4 X Diameter V/4D
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	27	0.794	5.06	11.65	12.20	0.694	24.40
0.5	27	0.746	4.76	11.35	11.90	0.677	22.90
0.5	27	0.595	3.79	11.00	11.52	0.656	18.30
0.5	27	0.725	4.62	11.40	11.94	0.680	22.25
0.5	27	0.595	3.79	11.05	11.58	0.659	18.26
0.5	27	0.556	3.54	10.75	11.27	0.641	17.05
0.5	27	0.521	3.32	10.50	11.00	0.626	16.00
0.5	27	0.397	2.53	10.20	10.69	0.608	17.20
0.5	27	0.308	1.96	9.80	10.27	0.584	9.44
0.5	27	0.250	1.59	9.10	9.54	0.543	7.68
0.5	27	0.161	1.03	8.25	8.65	0.493	4.94
0.5	27	0.077	0.49	7.80	8.16	0.465	2.37
1.0	27	3.125	7.00	7.05	7.39	0.707	20.02
1.0	27	2.940	6.59	6.85	7.18	0.686	18.85
1.0	27	2.085	4.67	6.45	6.76	0.646	13.37
1.0	27	1.667	3.73	6.00	6.28	0.600	10.68
1.0	27	0.794	1.78	6.25	5.50	0.526	5.09
1.0	27	0.595	1.33	4.90	5.14	0.494	3.81
1.0	27	0.177	0.40	4.50	4.71	0.450	1.13
1.5	27	6.950	6.59	3.90	4.08	0.599	12.29
1.5	27	5.950	5.65	3.85	4.04	0.593	10.53
1.5	27	5.210	4.94	4.00	4.19	0.615	9.22
1.5	27	5.550	5.36	3.80	3.98	0.584	10.00
1.5	27	4.630	4.39	3.70	3.88	0.569	8.20
1.5	27	3.205	3.04	3.50	3.67	0.538	5.67
1.5	27	1.924	1.83	3.25	3.40	0.489	3.40
1.5	27	0.163	0.15	2.60	2.72	0.399	0.29
1.5	27	0.834	0.79	2.80	2.93	0.430	1.47
1.5	27	0.123	0.12	2.40	2.51	0.368	0.22

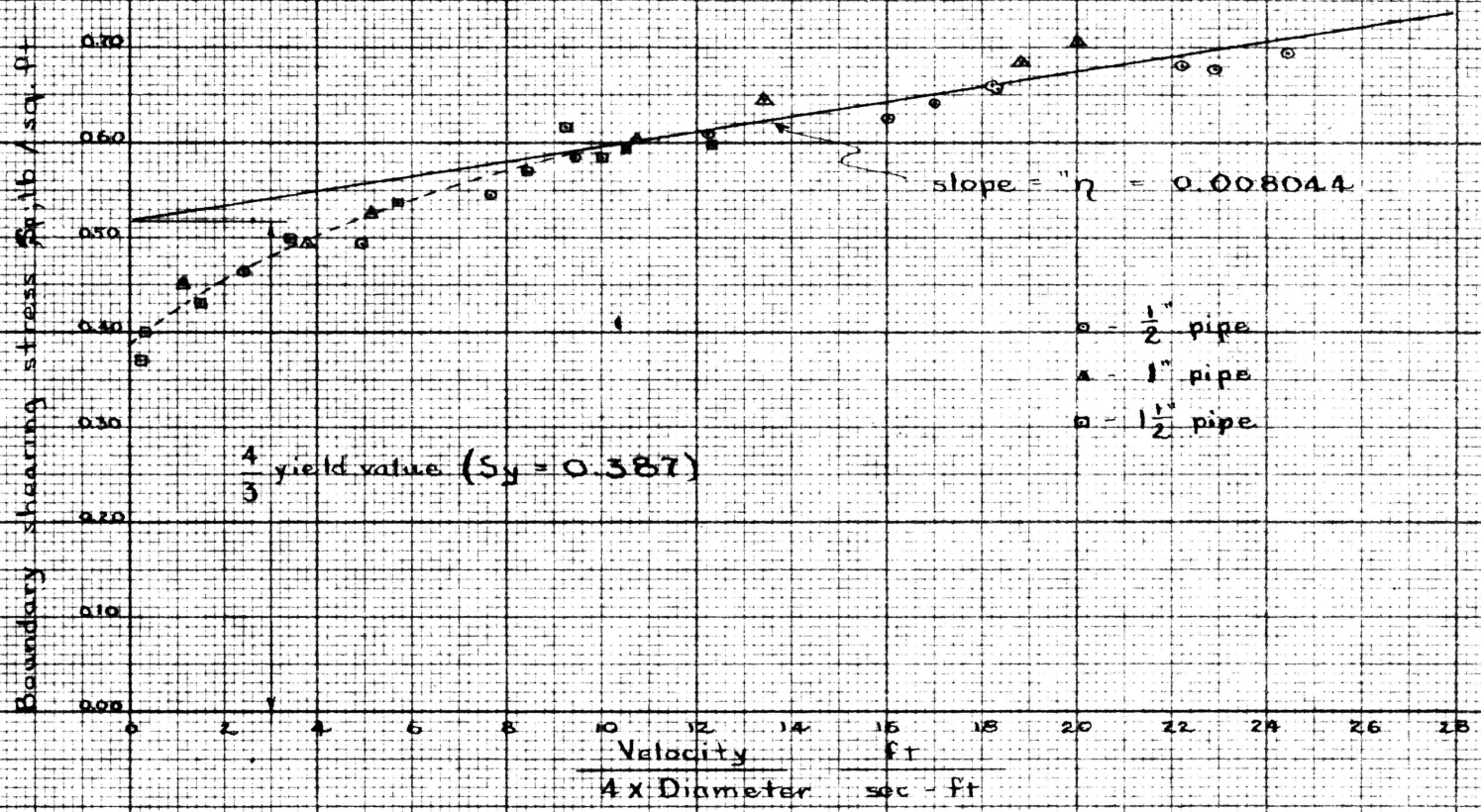


FIGURE 17

DETERMINATION OF YIELD VALUE, S_y , AND COEFFICIENT OF RIGIDITY, η , FOR SUSPENSION NUMBER 6

Table XV

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 6

Per cent suspended solids - 13.03
Per cent aluminum oxide in solids, dry basis - 19.3
Age of aluminum hydroxide in suspension - 89 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ , since Filtration Began	$\frac{W}{A}$	$\frac{P \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	$\frac{\text{lb-min}}{\text{sq in.}}$	min	°C
6.0	100	0.38	0.000334	6,826	27
6.0	200	0.80	0.000668	7,185	27
6.0	300	1.27	0.001002	7,604	27
6.0	400	1.81	0.001336	8,128	27
6.0	500	2.37	0.001670	8,514	27
6.0	600	2.98	0.002004	8,922	27
6.0	700	3.60	0.002339	9,234	27
6.0	800	4.36	0.002673	9,786	27
6.0	900	5.06	0.003007	10,096	27
6.0	1000	5.88	0.003341	10,559	27
6.0	1100	6.66	0.003675	10,873	27
8.0	100	0.20	0.000334	4,790	27
8.0	200	0.44	0.000668	5,269	27
8.0	300	0.72	0.001002	5,748	27
8.0	400	1.04	0.001336	6,227	27
8.0	500	1.38	0.001670	6,610	27
8.0	600	1.76	0.002004	7,025	27
8.0	700	2.15	0.002339	7,353	27
8.0	800	2.61	0.002673	7,811	27
8.0	900	3.10	0.003007	8,247	27
8.0	1000	3.62	0.003341	8,668	27
8.0	1100	4.19	0.004009	9,598	27
8.0	1200	4.81	0.004012	9,698	27
8.0	1300	5.42	0.004343	9,983	27
8.0	1400	6.07	0.004677	10,382	27
10.0	100	0.17	0.000334	5,089	27
10.0	200	0.35	0.000668	5,239	27
10.0	300	0.57	0.001002	5,688	27
10.0	400	0.81	0.001338	6,062	27
10.0	500	1.08	0.001670	6,460	27
10.0	600	1.39	0.002004	6,936	27
10.0	700	1.71	0.002339	7,310	27
10.0	800	2.08	0.002673	7,781	27
10.0	900	2.46	0.003007	8,180	27
10.0	1100	3.33	0.003675	9,088	28
10.0	1200	3.82	0.004009	9,528	28
10.0	1300	4.32	0.004343	9,947	28
10.0	1400	4.91	0.004677	10,498	28
10.0	1500	5.42	0.005011	10,816	28
10.0	1600	6.01	0.005346	11,242	28
12.0	100	0.11	0.000334	3,952	28
12.0	200	0.26	0.000668	4,670	28
12.0	300	0.44	0.001002	5,269	28
12.0	500	0.85	0.001670	6,107	28
12.0	600	1.10	0.002004	6,586	28
12.0	700	1.37	0.002339	7,028	28
12.0	800	1.67	0.002673	7,497	28
12.0	900	1.97	0.003007	7,861	28
12.0	1000	2.36	0.003341	8,476	28
12.0	1100	2.68	0.003675	8,751	28
12.0	1200	3.07	0.004009	9,189	28
12.0	1300	3.48	0.004343	9,615	28
12.0	1400	3.92	0.004677	10,057	28
12.0	1500	4.38	0.005011	10,488	28
12.0	1600	4.88	0.005346	10,953	28
12.0	1700	5.38	0.005680	11,366	28
12.0	1800	5.92	0.006014	11,812	28
12.0	1900	6.48	0.006349	12,247	28
14.0	200	0.25	0.000668	5,239	28
14.0	400	0.58	0.001336	6,077	28
14.0	500	0.76	0.001670	6,371	28
14.0	600	0.97	0.002004	6,776	28
14.0	700	1.21	0.002339	7,242	28
14.0	800	1.46	0.002673	7,646	28
14.0	900	1.75	0.003007	8,147	28
14.0	1000	2.05	0.003341	8,590	28
14.0	1100	2.36	0.003675	8,990	28
14.0	1200	2.71	0.004009	9,463	28
14.0	1300	3.07	0.004343	9,896	28
14.0	1400	3.45	0.004677	10,327	28
14.0	1500	3.87	0.005011	10,812	28
14.0	1600	4.28	0.005346	11,208	28
14.0	1700	4.74	0.005680	11,683	28
14.0	1800	5.20	0.006014	12,105	28

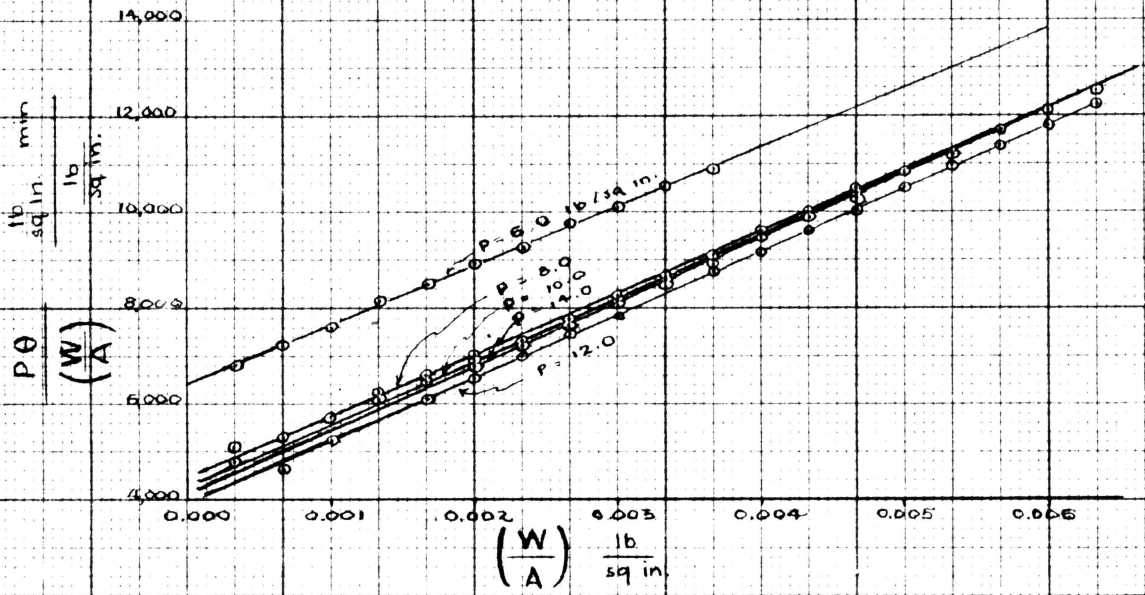


FIGURE 18 A
FILTRATION CURVE FOR SUSPENSION NUMBER 6

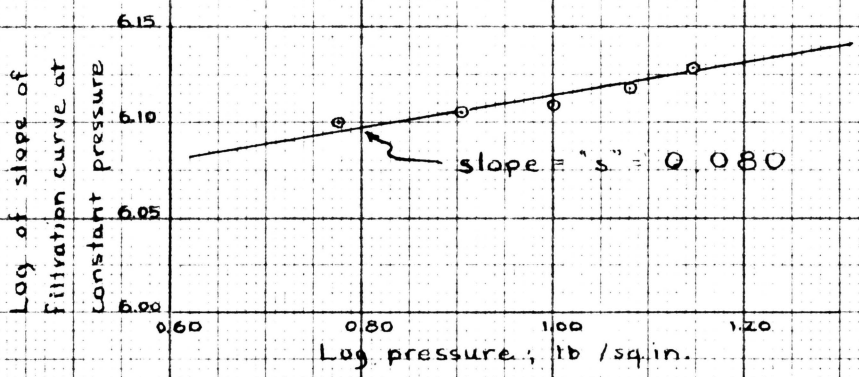


FIGURE 18 B
DETERMINATION OF COMPRESSIBILITY COEFFICIENT, "S"
FOR SUSPENSION NUMBER 6

Table XVI

FRICION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE - FILTER AID SUSPENSION NUMBER 7

Per cent suspended solids - 12.02
Per cent aluminum oxide in solids, dry basis - 42.3
Age of aluminum hydroxide in suspension - 7 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity 4 X Diameter V/4D
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	28	0.820	5.00	7.95	8.32	0.494	24.10
0.5	28	0.362	2.21	6.10	6.39	0.379	10.64
0.5	28	0.298	1.82	5.85	6.12	0.364	8.78
0.5	28	0.327	1.99	6.25	6.54	0.388	9.61
0.5	28	0.618	3.77	7.15	7.49	0.445	18.19
1.0	28	3.090	6.63	6.15	6.44	0.644	18.95
1.0	28	2.690	5.77	5.05	5.28	0.528	16.51
1.0	28	1.895	4.06	4.00	4.19	0.419	11.61
1.0	28	3.210	6.88	6.65	6.96	0.696	19.68
1.0	28	2.195	4.71	3.85	4.03	0.403	13.48
1.0	28	1.443	3.10	3.25	3.40	0.340	8.86
1.0	28	0.926	1.99	3.10	3.24	0.324	5.68
1.0	28	0.490	1.05	2.75	2.38	0.288	3.01
1.0	28	0.147	0.31	2.40	2.51	0.251	0.90
1.5	28	4.900	4.46	2.45	2.56	0.392	8.31
1.5	28	4.630	4.21	2.35	2.46	0.377	7.85
1.5	28	3.790	3.45	2.30	2.41	0.369	6.44
1.5	28	2.780	2.53	2.15	2.25	0.345	4.72
1.5	28	1.069	0.97	2.00	2.09	0.320	1.82
1.5	28	1.317	1.20	1.95	2.04	0.312	2.24
1.5	28	0.641	0.58	1.80	1.88	0.288	1.09
1.5	28	0.357	0.32	1.75	1.83	0.280	0.61

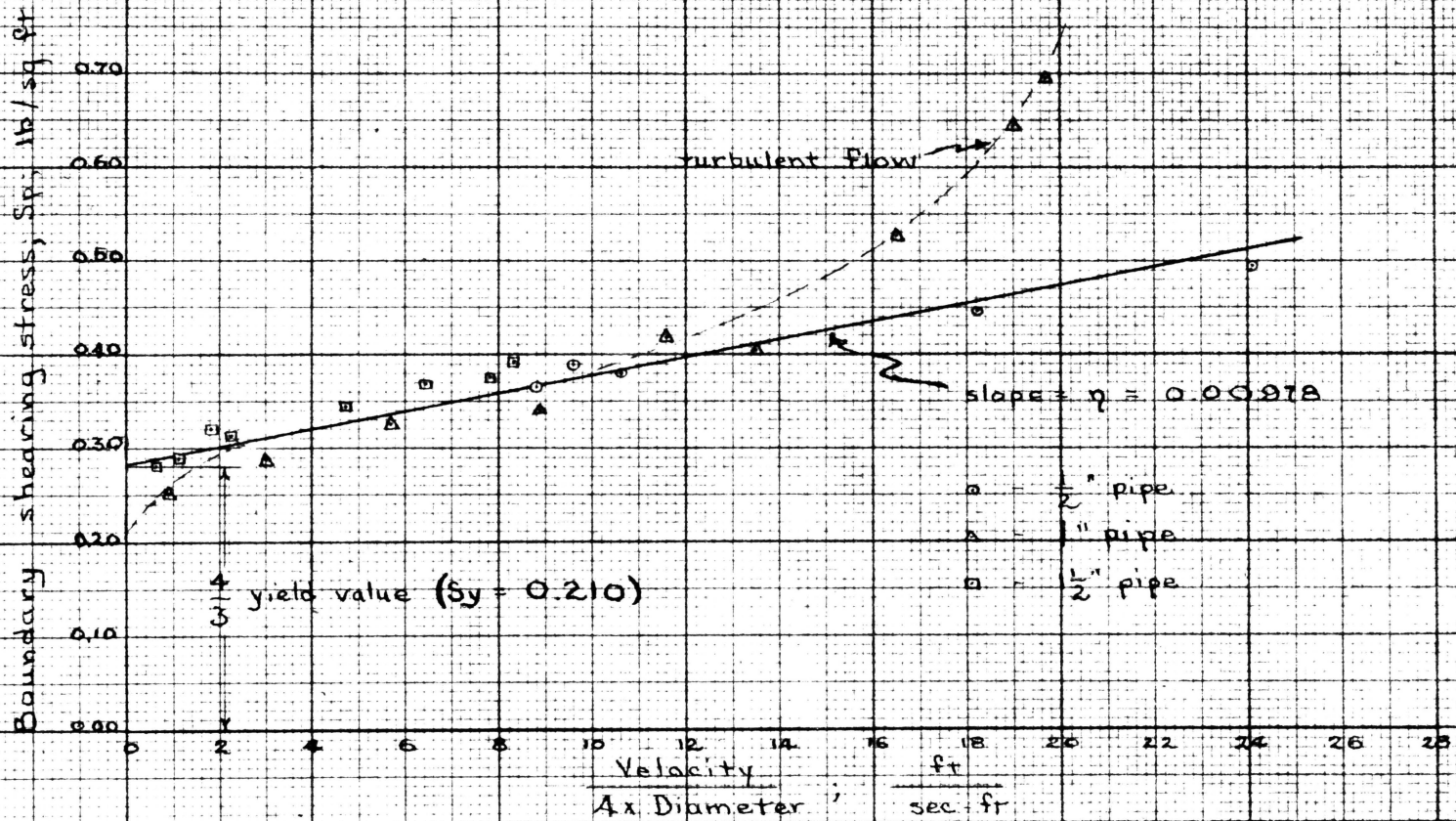


FIGURE 19

DETERMINATION OF YIELD VALUE, S_y AND COEFFICIENT OF RIGIDITY, η , FOR SUSPENSION NUMBER 7

Table XVII

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 7

Per cent suspended solids - 12.02
Per cent aluminum oxide in solids, dry basis - 42.3
Age of aluminum hydroxide in suspension - 7 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ , since Filtration Began	$\frac{W}{A}$	$\frac{P \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	$\frac{\text{lb-min}}{\text{sq in.}}$	min	°C
5.0	100	1.20	0.000334	1,798	28
5.0	200	2.69	0.000668	2,015	28
5.0	250	3.40	0.000834	2,040	28
5.0	300	4.19	0.001002	2,080	28
5.0	350	5.04	0.001169	2,158	28
5.0	400	6.04	0.000136	2,260	28
5.0	450	7.01	0.001503	2,330	28
5.0	500	8.02	0.001673	2,395	28
5.0	550	9.03	0.001837	2,460	28
5.0	600	10.01	0.002004	2,500	28
5.0	700	12.29	0.002339	2,620	28
8.0	100	0.96	0.000334	2,300	28
8.0	200	2.03	0.000668	2,435	28
8.0	300	3.27	0.001002	2,580	28
8.0	400	4.59	0.001336	2,750	28
8.0	500	5.97	0.001673	2,860	28
8.0	600	7.47	0.002004	2,985	28
8.0	700	9.14	0.002339	3,125	28
8.0	800	10.87	0.002673	3,255	28
8.0	900	12.80	0.003007	3,410	28
8.0	1000	14.80	0.003341	3,545	28
10.3	400	3.56	0.001336	2,745	28
10.3	500	4.70	0.001673	2,890	28
10.3	600	5.90	0.002004	3,030	28
10.3	700	7.20	0.002339	3,165	28
10.3	800	8.64	0.002673	3,330	28
10.3	900	10.15	0.003007	3,470	28
10.3	1000	11.49	0.003341	3,630	28
10.3	1100	13.43	0.003675	3,170	28
10.3	1200	15.14	0.004009	3,890	28
10.3	1300	17.02	0.004343	4,040	28
12.2	100	0.60	0.000334	2,190	28
12.2	200	1.27	0.000668	2,320	28
12.2	300	2.02	0.001002	2,460	28
12.2	400	2.82	0.001336	2,610	28
12.2	500	3.76	0.001673	2,740	28
12.2	600	4.76	0.002004	2,890	28
12.2	700	5.87	0.002339	3,060	28
12.2	800	7.06	0.002673	3,230	28
12.2	900	8.30	0.003007	3,370	28
12.2	1000	9.67	0.003341	3,540	28
14.0	100	0.48	0.000334	2,010	29
14.0	200	1.05	0.000668	2,200	29
14.0	300	1.68	0.001002	2,365	29
14.0	400	2.41	0.001336	2,525	29
14.0	500	3.20	0.001673	2,780	29
14.0	600	4.07	0.002004	2,845	29
14.0	700	4.96	0.002339	2,965	29
14.0	800	6.00	0.002673	3,140	29
14.0	900	7.14	0.003007	3,325	29
14.0	1000	8.32	0.003341	3,485	29

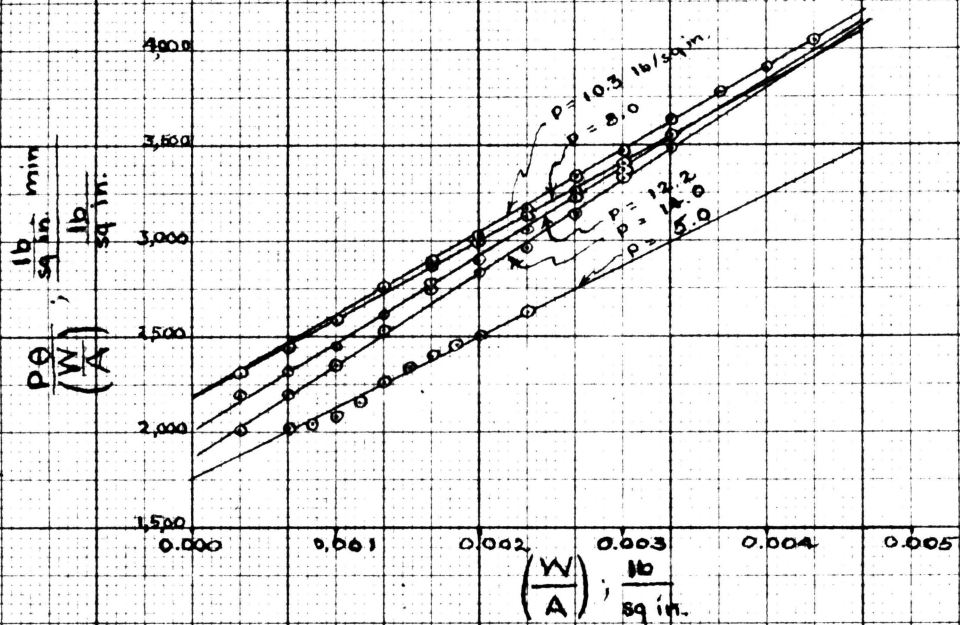


FIGURE 20 A
FILTRATION CURVE FOR SUSPENSION NUMBER 7

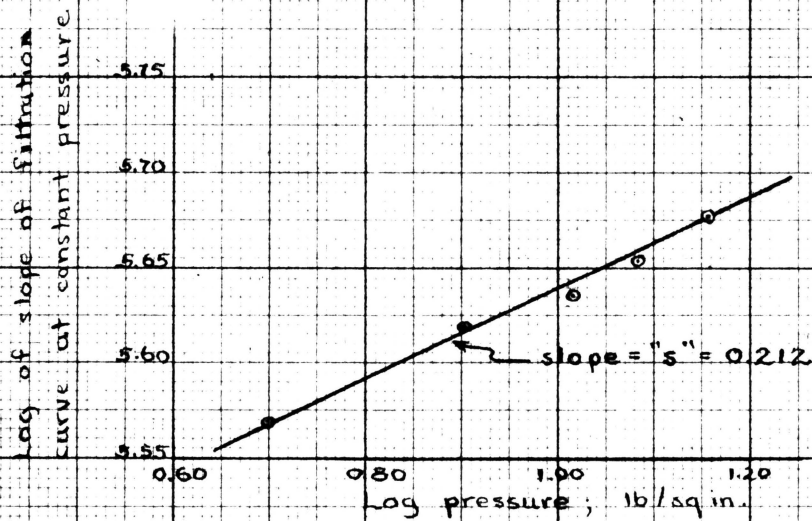


FIGURE 20 B
DETERMINATION OF COMPRESSIBILITY COEFFICIENT " s ",
FOR SUSPENSION NUMBER 7

Table XVIII

FRICITION HEAD LOSS TESTS IN PIPES OF VARIOUS SIZES
FOR ALUMINUM HYDROXIDE - FILTER AID SUSPENSION NUMBER 8

Per cent suspended solids - 8.23

Per cent aluminum oxide in solids, dry basis - 29.0

Age of aluminum hydroxide in suspension - 16 days

Nominal Diameter of Pipe	Temperature	Mass Rate of Flow	Velocity of Flow V	Manometer Reading	Friction Head Loss ΔH	Boundary Shearing Stress S_p	Velocity 4 X Diameter V/4D
inches	°C	lb/sec	ft/sec	inches Hg	feet of flowing substance	lb/sq ft	ft/sec-ft
0.5	24	0.926	5.97	6.90	7.23	0.405	28.78
0.5	24	0.641	4.14	3.50	3.66	0.205	19.93
0.5	24	0.521	3.36	2.80	2.93	0.165	16.20
0.5	24	0.287	1.85	2.20	2.30	0.129	8.91
0.5	24	0.143	0.92	2.00	2.09	0.117	4.44
1.0	24	3.470	7.87	7.60	7.96	0.752	22.54
1.0	24	3.335	7.56	6.40	6.71	0.634	21.65
1.0	24	2.605	5.91	3.80	3.98	0.376	16.90
1.0	24	0.862	1.96	1.45	1.52	0.144	5.61
1.0	24	1.785	4.05	2.30	2.41	0.228	11.59
1.0	24	0.379	0.86	1.20	1.26	0.119	2.46
1.0	24	0.298	0.68	1.15	1.20	0.114	1.94
1.5	24	7.410	7.13	3.10	3.24	0.470	13.30
1.5	24	5.210	5.01	1.80	1.88	0.273	9.35
1.5	24	2.190	2.11	0.75	0.79	0.114	3.93
1.5	24	1.390	1.34	0.75	0.79	0.114	2.49
1.5	24	0.556	0.54	0.55	0.58	0.083	1.00

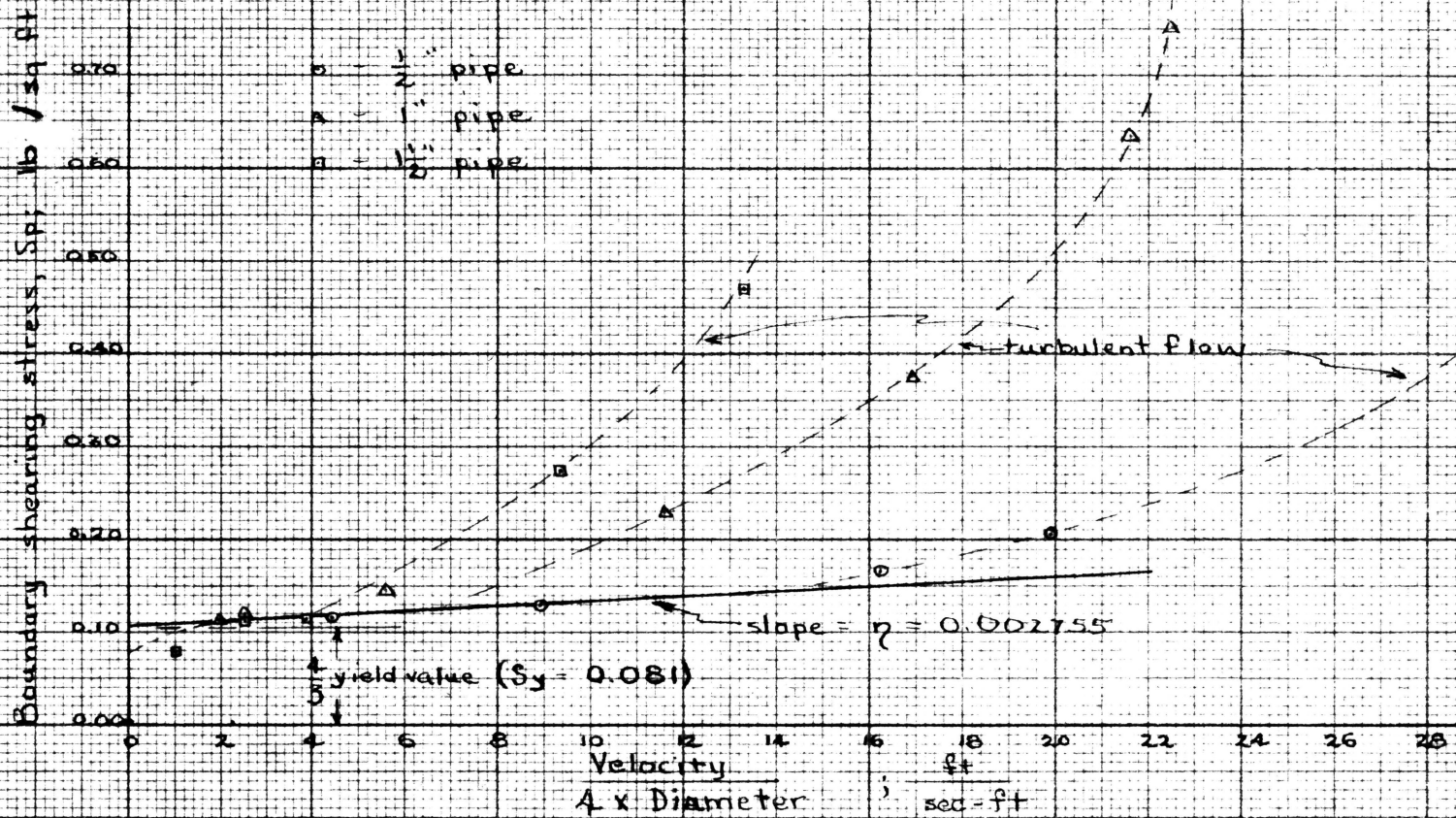


FIGURE 21

DETERMINATION OF YIELD VALUE, S_y , AND COEFFICIENT OF RIGIDITY, η , FOR SUSPENSION NUMBER 8

Table XIX

FILTRATION TESTS FOR ALUMINUM HYDROXIDE - FILTER AID
SUSPENSION NUMBER 8

Per cent suspended solids - 8.23
Per cent aluminum oxide in solids, dry basis - 29.0
Age of aluminum hydroxide in suspension - 16 days
Area of filtering surface, A, - 663 sq in.

Pressure, P, of Filtration	Wt, W, of Filtrate Collected at Time, θ	Time, θ , since Filtration Began	$\frac{W}{A}$	$\frac{P \theta}{W/A}$	Temper- ature
lb/sq in.	gm	min	$\frac{\text{lb-min}}{\text{sq in.}}$	min	°C
5.0	100	0.49	0.000334	7,230	25
5.0	200	1.07	0.000668	8,000	25
5.0	300	1.75	0.001002	8,740	25
5.0	400	2.54	0.001336	9,520	25
5.0	500	3.39	0.001670	10,120	25
5.0	600	4.26	0.002004	10,650	25
5.0	700	5.23	0.002339	11,180	25
5.0	800	6.38	0.002673	11,940	25
5.0	900	7.55	0.003007	12,550	25
5.0	1000	8.80	0.003341	13,180	25
10.2	100	0.24	0.000334	7,340	25
10.2	200	0.51	0.000668	7,790	25
10.2	300	0.81	0.001002	8,240	25
10.2	400	1.15	0.001336	8,790	25
10.2	500	1.54	0.001670	9,400	25
10.2	600	1.98	0.002004	10,090	25
10.2	700	2.45	0.002339	10,690	25
10.2	800	2.97	0.002673	11,350	25
10.2	900	3.53	0.003007	11,970	25
10.2	1000	4.12	0.003341	12,580	25
10.2	1100	4.73	0.003675	13,250	25
10.2	1200	5.46	0.004009	13,870	25
10.2	1300	6.22	0.004343	14,620	25
15.0	100	0.14	0.000334	6,290	25
15.0	200	0.30	0.000668	6,740	25
15.0	300	0.48	0.001002	7,180	25
15.0	400	0.71	0.001336	7,980	25
15.0	500	0.97	0.001670	8,740	25
15.0	600	1.25	0.002004	9,350	25
15.0	700	1.56	0.002339	10,000	25
15.0	800	1.90	0.002673	10,670	25
15.0	900	2.27	0.003007	11,320	25
15.0	1000	2.67	0.003341	12,000	25
15.0	1100	3.09	0.003675	12,620	25
15.0	1200	3.56	0.004009	13,320	25
15.0	1300	4.04	0.004343	13,980	25
15.0	1400	4.56	0.004677	14,620	25
15.0	1500	5.11	0.005011	15,300	25
15.0	1600	5.69	0.005346	15,990	25
15.0	1700	6.31	0.005680	16,650	25

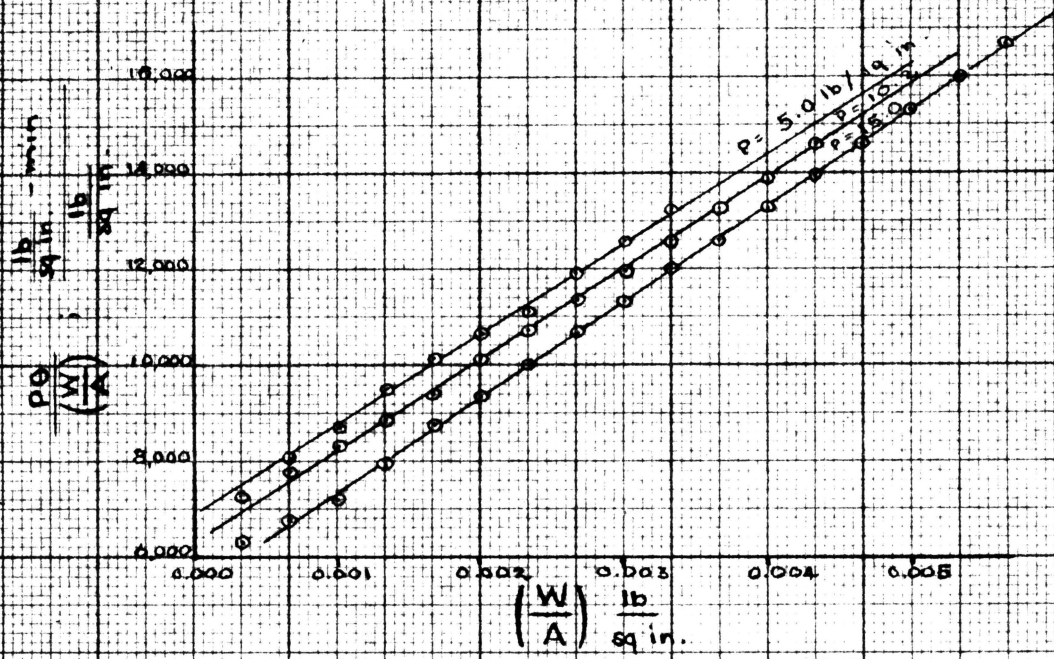


FIGURE 22 A
FILTRATION CURVE FOR SUSPENSION NUMBER 8

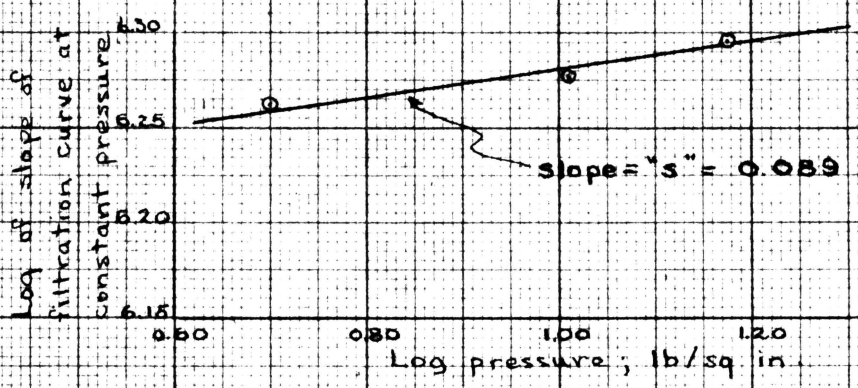


FIGURE 22 B
DETERMINATION OF COMPRESSIBILITY COEFFICIENT, 's',
FOR SUSPENSION NUMBER 8

Table XX

STORMER VISCOMETER TESTS

Suspension Number	Temperature	Actuating Weight	Time Required for 100 Revolutions	Velocity of Rotation of Cylinder
	°C	gm	sec	rev/sec
1	25	116.9	6.76	14.5
1	25	106.1	7.60	13.2
1	25	98.2	7.90	12.7
1	25	84.4	8.65	11.6
1	25	73.5	9.80	10.2
1	25	64.4	10.92	9.2
1	25	52.0	13.50	7.4
1	25	44.4	15.87	6.2
1	25	35.0	21.45	4.7
1	25	27.1	32.85	3.0
2	24	117.4	9.42	10.60
2	24	105.2	10.45	9.57
2	24	90.0	12.85	7.78
2	24	78.8	16.80	5.95
2	24	70.4	21.73	4.60
2	24	69.9	24.40	4.10
2	24	62.9	40.20	2.49
2	24	55.9	107.40	0.93
2	24	50.4	320.00	0.31
3	28	102.9	11.17	8.95
3	28	100.1	12.50	8.00
3	27	91.7	15.00	6.66
3	27	84.1	17.10	5.85
3	26	78.8	20.90	4.78
3	26	74.4	24.60	4.07
3	26	71.4	26.50	3.77
3	27	66.2	35.40	2.83
3	27	63.3	43.00	2.33
3	27	59.4	75.80	1.32

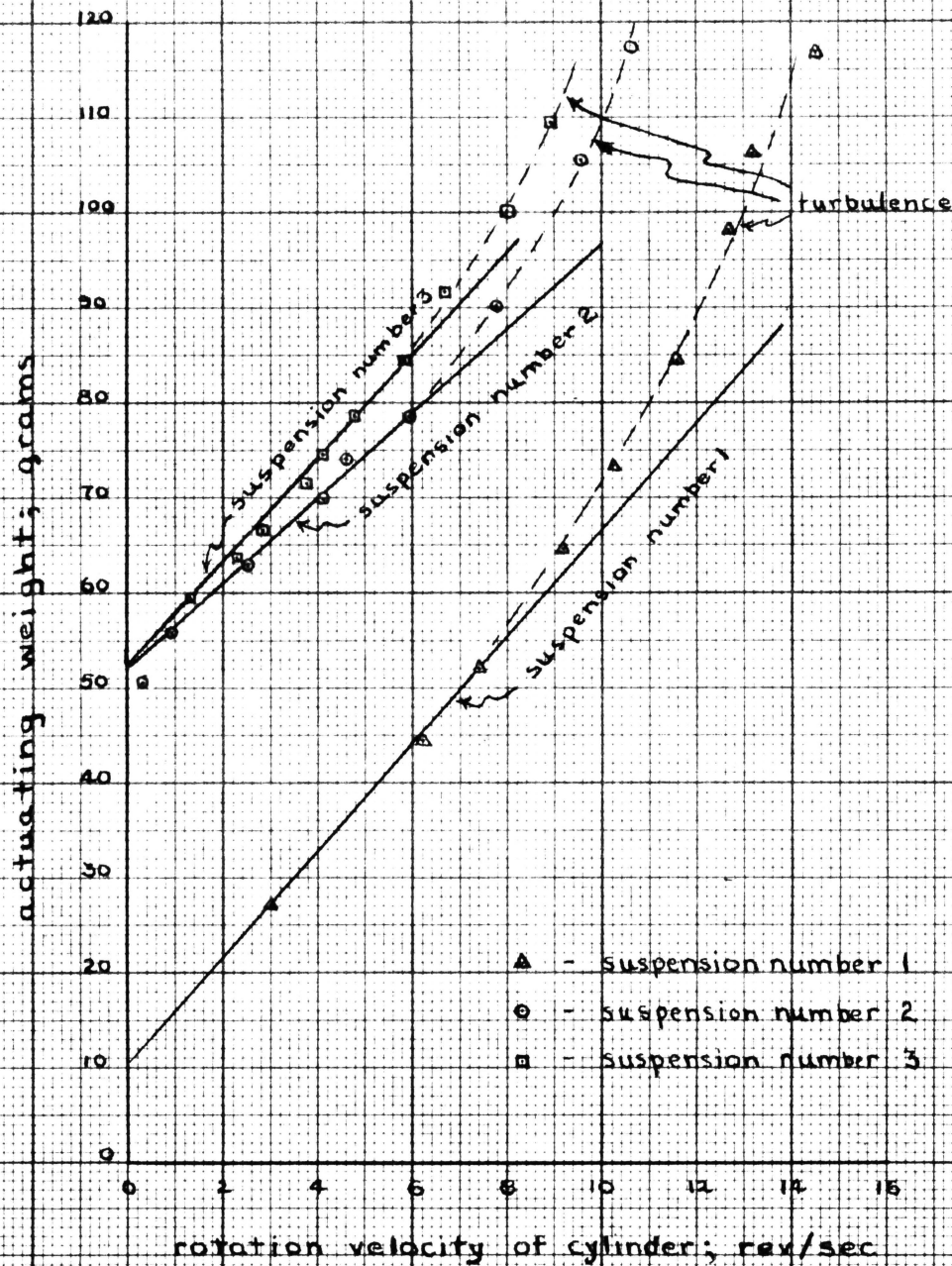


FIGURE 23
STORMER VISCOMETER TESTS
ON ALUMINUM HYDROXIDE-FILTER AID
SUSPENSIONS NUMBERS 1, 2 AND 3

Table XXI

Table XXI

EMPIRICAL EQUATION DERIVED TO SHOW A RELATIONSHIP
 BETWEEN COEFFICIENT OF COMPRESSIBILITY, s , CONCENTRATION OF SOLIDS, C , AND YIELD VALUE, S_y

Equation A: $S_y = 0.365 + 0.00795 C - 0.736 s$
 Equation B: $S_y = 0.173 + 0.0125 C - 0.876 s^2$
 Equation C: $S_y = -0.0659 + 0.00912 C - 0.157 \ln s$
 Equation D: $S_y = 0.00138 C^{1.59} s^{0.0327}$
 Equation E: $S_y = -0.1966 + 0.213 C - 0.011 C^2 - 4.520 s + 8.481 s^2$

Yield Value, Experimental Determination	Equation A			Equation B			Equation C			Equation D			Equation E		
	Calculated Yield Value	Deviation from Experimental	Percentage Deviation	Calculated Yield Value	Deviation from Experimental	Percentage Deviation	Calculated Yield Value	Deviation from Experimental	Percentage Deviation	Calculated Yield Value	Deviation from Experimental	Percentage Deviation	Calculated Yield Value	Deviation from Experimental	Percentage Deviation
lb/sq ft	lb/sq ft	lb/sq ft	per cent	lb/sq ft	lb/sq ft	per cent	lb/sq ft	lb/sq ft	per cent	lb/sq ft	lb/sq ft	per cent	lb/sq ft	lb/sq ft	per cent
0.036	0.081	-0.045	125.00	0.060	-0.024	66.66	0.101	-0.065	180.55	0.084	-0.018	50.00	0.030	0.006	16.66
0.150	0.149	0.001	0.66	0.137	0.013	8.66	0.150	0.000	0.00	0.125	0.025	16.66	0.140	-0.010	6.66
0.176	0.142	0.034	19.31	0.130	0.046	26.13	0.146	0.030	17.04	0.127	0.049	27.94	0.132	-0.006	3.40
0.212	0.276	-0.098	46.22	0.229	-0.017	6.01	0.252	-0.040	18.96	0.159	0.055	25.00	0.197	0.015	7.07
0.265	0.305	-0.040	15.09	0.267	-0.002	0.75	0.286	-0.021	7.92	0.268	-0.003	1.13	0.272	-0.007	2.64
0.337	0.410	-0.023	5.94	0.330	0.057	14.72	0.449	-0.062	16.02	0.393	-0.006	1.55	0.396	-0.009	2.32
0.210	0.305	-0.095	45.23	0.234	0.074	35.23	0.297	-0.077	36.66	0.357	-0.147	70.00	0.189	0.021	10.00
	Average 36.78			Average 22.98			Average 39.58			Average 27.45			Average 6.96		

Table XXII

Table XXII

EMPIRICAL EQUATIONS DERIVED TO SHOW A RELATIONSHIP BETWEEN COEFFICIENT OF COMPRESSIBILITY, s , CONCENTRATION OF SOLIDS, C , AND COEFFICIENT OF RIGIDITY, η .

Equation A: $\eta = -0.00829 + 0.091 C + 0.00469 s$
 Equation B: $\eta = -0.00119 + 0.09723 C + 0.00175 s^2$
 Equation C: $\eta = 0.000345 C^{1.37} s^{0.115}$
 Equation D: $\eta = 0.0639 - 0.226 C + 0.00123 C^2 + 0.394 s - 0.376 s^2$

Coefficient of Rigidity, Experimental Determination	Equation A			Equation B			Equation C			Equation D				
	Calculated Coefficient of Rigidity	Deviation from Experimental	Percentage Deviation	Calculated Coefficient of Rigidity	Deviation from Experimental	Percentage Deviation	Calculated Coefficient of Rigidity	Deviation from Experimental	Percentage Deviation	Calculated Coefficient of Rigidity	Deviation from Experimental	Percentage Deviation		
lb/ft-sec	lb/ft-sec	lb/ft-sec	Per cent	lb/ft-sec	lb/ft-sec	per cent	lb/ft-sec	lb/ft-sec	per cent	lb/ft-sec	lb/ft-sec	per cent		
0.001860	0.001975	-0.000115	6.18	0.001962	-0.000102	5.48	0.001903	-0.000032	2.79	0.007589	-0.005829	313.38		
0.002202	0.003981	-0.001779	80.79	0.003374	-0.001672	75.95	0.003686	0.001484	67.59	0.000010	0.002912	99.54		
0.005169	0.004093	0.001086	21.01	0.003354	0.001835	33.39	0.003747	-0.001422	27.51	0.002935	0.003062	155.96		
0.003350	0.004241	0.001119	30.67	0.004192	0.000763	14.32	0.004329	-0.001031	19.13	0.002841	-0.003431	64.94		
0.006178	0.006741	-0.000563	9.11	0.006790	-0.000612	9.91	0.006730	0.000552	8.93	0.004927	0.001291	20.89		
0.003044	0.003935	-0.000892	11.09	0.003035	-0.001041	12.94	0.003706	0.000662	8.23	0.003144	0.002900	36.05		
0.009780	0.009537	0.001143	11.68	0.009357	0.001323	13.52	0.009706	-0.001074	10.98	0.015008	-0.005228	53.45		
		Average	22.96			Average	22.23			Average	20.72		Average	106.31

G. Sample Calculations

The types of calculations that were made in this investigation are shown in the following examples:

Calculation of Yield Value and Coefficient of Rigidity. The data for this sample calculation is taken from table XII, page 78, for suspension number 5. The method of the determination of the yield value and the coefficient of rigidity is graphical, according to the method described by Babbit and Caldwell(5).

From the mass rate of flow, the velocity of flow, V, was calculated:

$$V = \frac{\text{mass rate of flow (lb/sec)}}{\text{density (lb/cu ft) X transverse area of pipe (sq ft)}}$$
$$V = \frac{0.833}{1.169 \times 62.2 \times 0.00211} = 5.429 \text{ ft/sec}$$

From the manometer reading in inches of mercury, the friction head loss, H, in feet of flowing substance, was calculated:

$$H = \frac{\text{manometer reading } (\rho_{Hg} - \rho_{H_2O})}{\rho_{H_2O}}$$
$$H = \frac{8.23/12 (62.2 \times 13.53 - 62.2)}{62.2}$$

$$H = 3.625 \text{ feet of flowing substance.}$$

The boundary shearing stress, S_p, was calculated according to the following equation:

$$S_p = \frac{\Delta H \rho D}{4 N}$$

where

ΔH = friction head loss, 8.625 ft of flowing substance.

ρ = density of suspension, (1.169 X 62.2) lb/cu ft.

D = diameter of pipe, 0.622/12 ft.

N = length of pipe, 17 ft.

$$S_p = \frac{8.625 \times 1.169 \times 62.2 \times 0.622/12}{4 \times 17}$$

$$S_p = 0.478 \text{ lb/sq ft.}$$

The ratio of the velocity to four times the diameter was calculated:

$$\frac{V}{4D} = \frac{5.429}{4 \times 0.622/12} = 26.10 \text{ ft/sec-ft.}$$

When these calculations were made for all of the different velocities of flow in the different sizes of pipe, a plot was made with boundary shearing stress, S_p , as the ordinate and $V/4D$ as the abscissa. This is shown in figure XV, page 79. By the method of averages, neglecting the points which slope off toward the origin at low values of the ratio $V/4D$, in this case, below 6 ft/sec-ft, a straight line was drawn through the remainder of the points. The intercept on the S_p axis is four-thirds the yield value, S_y , and the slope of the line is the coefficient of rigidity, η .

$$S_y = 3/4 \times 0.354 = 0.265 \text{ lb/sq ft.}$$

$$= \frac{0.0703}{11.378} = 0.006178 \text{ lb/ft-sec.}$$

Calculation of the Coefficient of Compressibility. The method used for the determination of the coefficient of compressibility is the graphical method presented by Walker, Lewis, McAdams, and Gilliland⁽⁴⁵⁾. The data for this example is taken from table XIII, page 80, for suspension number 5.

The filter press had a total filtering area, A , of 663 sq in. For each filtration experiment at constant pressure, P , the terms $\frac{P \theta}{(W/A)}$ and (W/A) were calculated for each weight of filtrate, W , collected at time, θ , since the filtration began.

$$\frac{W}{A} = \frac{100 \text{ gm} \times 0.002205 \text{ lb/gm}}{663 \text{ sq in.}}$$

$$\frac{W}{A} = 0.000334 \text{ lb/sq in.}$$

and

$$\frac{P\theta}{(W/A)} = \frac{6.3 \text{ lb/sq in.} \times 0.38 \text{ min}}{0.000334 \text{ lb/sq in.}}$$

$$\frac{P\theta}{(W/A)} = 7.160 \frac{\text{lb-min/sq in.}}{\text{lb/sq in.}}$$

For each constant pressure filtration a plot using $\frac{P \theta}{(W/A)}$ as the ordinate and (W/A) as the abscissa was made. These plots are shown in figure 16A, page 81. Straight lines were drawn through the points for each constant pressure filtration, using the method of selected points. The logarithms of the slopes of these filtration curves were plotted against the logarithms of the

pressures. This plot is shown in figure 16B, page 81. A straight line is drawn through these points by the method of selected points. The slope of this straight line is the compressibility coefficient, s.

$$s = 0.190 \text{ (from slope of line shown in figure 16B)}$$

Determination of a Relationship Between Compressibility Coefficient, Concentration of Solids, and Yield Value. Since it was impossible to solve for a relationship graphically, as a three dimensional plot would be necessary to plot all three variables, test forms of equations were tried to see which would give an equation which best fitted the data.

Before the test forms were set up, the following assumptions were made: (1) Yield value is a function of the coefficient of compressibility, and (2) Yield value is not a dependent function of the coefficient of rigidity.

The test forms used were:

$$S_y = b_0 + b_1 C + b_2 s. \quad (25)$$

$$S_y = b_0 + b_1 C + b_2 s^2. \quad (26)$$

$$S_y = b_0 + b_1 C + b_2 \ln s. \quad (27)$$

$$\log S_y = b_0 + b_1 \log C + b_2 \log s. \quad (28)$$

$$S_y = b_0 + b_1 C + b_2 C^2 + b_3 s + b_4 s^2. \quad (29)$$

The equations derived and their deviation from experimental results are shown in table XXI, page 96.

The following sample calculation shows how the values of the constants of equation (25) were determined. The data from which this equation was derived is given in table III, page 61. The method of solution of the equations with the experimental data is that described by Rider⁽³⁵⁾.

The summations of the following terms for suspensions numbers 1 through 7 were calculated: $\sum C$, $\sum C^2$, $\sum s$, $\sum s^2$, $\sum C s$, $\sum S_y$, $\sum C S_y$, and $\sum s S_y$. These were calculated as follows:

$$\sum C = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7.$$

$$\sum C = 0.425 + 0.360 + 0.370 + 0.200 + 0.190 + 0.080 + 0.212.$$

$$\sum C = 1.837$$

According to Rider⁽³⁵⁾, the normal equations for the solution of this general type form are:

$$\sum S_y = b_0 N + b_1 \sum C + b_2 \sum s.$$

$$\sum C S_y = b_0 \sum C + b_1 \sum C^2 + b_2 \sum C s.$$

$$\sum s S_y = b_0 \sum s + b_1 \sum C s + b_2 \sum s^2.$$

Substituting the summation values, the following equations were obtained:

$$1.436 = 7 b_0 + 58.29 b_1 + 1.837 b_2.$$

$$13.9088 = 58.29 b_0 + 556.826 b_1 + 12.9845 b_2.$$

$$0.3026 = 1.837 b_0 + 12.9845 b_1 + 0.5745 b_2.$$

Here we have three equations with three unknown constants. Solving these equations simultaneously by determinants, the values of the constants were:

$$b_0 = 0.365$$

$$b_1 = 0.00795$$

$$b_2 = 0.736.$$

Substituting these values into the type form for which they were calculated, equation (25), the following equation was obtained:

$$S_y = 0.365 + 0.00795 C - 0.736 s. \quad (30)$$

To determine the computed values of S_y , the observed values of C and s were placed in the equation and it was solved for S_y .

$$S_y \text{ (calculated)} = 0.365 + (0.00795)(3.60) - (0.736)(0.425).$$

$$S_y \text{ (calculated)} = 0.081.$$

To determine the percentage deviation, the difference between the observed value of S_y and the calculated value of S_y was divided by the observed value of S_y .

$$\text{Percentage deviation} = \frac{S_y \text{ (calculated)} - S_y \text{ (observed)}}{S_y \text{ (observed)}} \times 100$$

$$\text{Percentage deviation} = \frac{(0.081 - 0.036) 100}{0.036} = 125.00$$

Determination of a Relationship Between Compressibility Coefficient, Concentration of Solids and Coefficient of Rigidity. The determination of a relationship between compressibility, concentration of solids, and the coefficient of rigidity involved the same type of calculations as has just been described in the determinations

of a relationship between compressibility, coefficient, concentration of solids, and yield value, except that the following type forms were used:

$$\eta = b_0 + b_1 C + b_2 s. \quad (35)$$

$$\eta = b_0 + b_1 C + b_2 s^2. \quad (36)$$

$$\log \eta = b_0 + b_1 \log C + b_2 \log s. \quad (37)$$

which is equivalent to

$$\eta = 10^{b_0} C^{b_1} s^{b_2}$$

$$\eta = b_0 + b_1 C + b_2 C^2 + b_3 s + b_4 s^2. \quad (38)$$

IV. DISCUSSION

A. Construction of Equipment

Type of Apparatus. After a careful study of the literature, it was decided that the apparatus for this investigation should fulfill the following requirements:

(1) A means must be provided for determining the yield value and coefficient of rigidity for each suspension. Therefore, the apparatus must include test lengths of various sizes of pipes, manometer connections, a manometer to measure the friction head loss, a recirculating system, and a means of determining the velocity of flow.

(2) A method of determining the coefficient of compressibility must be included. Hence, the equipment should include a filter press and some means of controlling the pressure on the press.

With these aims in mind, the suspension flow apparatus, illustrated in figure 5, page 39, was constructed. It consisted of a tank reservoir for the suspension, a centrifugal pump for circulation, a motor for driving the pump, lengths of one-half-, one-, and one-and-one-half-inch pipe with connections to a manometer, a 2-inch return pipe to the weighing tank, and platform scales for weighing the slurry. In addition, a line

was connected to a plate and frame filter press so that the compressibility coefficient could be determined without removing the suspension from the make-up tank (see figure 6, page 40).

Velocity Control. In an apparatus of this type, the control of velocity may be accomplished by three methods: (1) by the use of a constant-head tank, in which the level may be changed manually as desired, (2) by means of a rotary pump with controlled speed, and (3) by means of a throttling valve and a constant speed centrifugal pump. Of these three, it was decided to use the last method, as the suspensions used in the apparatus would be of small, finely divided particles. This method of controlling velocity was found to be satisfactory with the aluminum hydroxide-filter aid suspensions used; however Babbitt and Caldwell⁽¹³⁾ do not recommend use of regulating valves for controlling velocity in the case of sewage sludges. In this work, the only difficulty encountered was at low rates of flow, near $0.2 \text{ ft}^{\text{per}}/\text{sec}$ and less, at which point it was noticed that the regulating valve, (B), became clogged.

Manometer Connections. The manometer taps on the test lengths of pipe were located 42 inches from the upstream end of the pipes and six inches from the downstream end. These distances allowed for the turbulence in the suspension to be dissipated and the stream lines to straighten during laminar flow. These distances were according to specifications set up by the American

Society of Mechanical Engineers⁽²⁾. One-quarter-inch taps were drilled into the one-and-one-half-inch pipe; one-eighth-inch taps into the one-inch-and-one-half-inch pipes, also according to American Society of Mechanical Engineers' standards⁽²⁾.

During the preliminary testing of the apparatus, it was noticed that the suspension clogged the manometer connections. To remedy this, a water line was installed, which, by opening valves (M₁) and (M₂), would flush the manometer leads.

Erratic manometer readings were also noticed, probably because of air bubbles in the manometer leads. To prevent this, a water line was installed at the bottom of the manometer leads. By opening valves (I) and (J), and allowing the water to fill the manometer leads, overflowing through valve (N), the air bubbles were eliminated.

Manometer. It was found during the preliminary work that it would be necessary to interchange manometer fluids. The two fluids tested were mercury and carbon tetrachloride tinted with iodine. For low friction head losses, the mercury did not give a large enough differential; whereas, for high friction head losses, the carbon tetrachloride differential was greater than the manometer length. For this reason, it was decided to construct a manometer, the fluids of which could be interchanged without disassembling the manometer. This was accomplished by adding a small side outlet to the bottom of the U-tube of the manometer. This side outlet was

connected to a leveling bulb by means of rubber tubing. Thus, the manometer could be filled with the fluid desired, and adjusted to the desired level. Once the desired level was obtained, a pinch-clamp was closed on the rubber tube, and the manometer functioned as a regular U-tube manometer.

Measurement of Velocity. To measure the velocity of flow, a weighing tank (R_2) was provided. It rested on platform scales (T). To determine the flow rate, the swing pipe (W) was moved so that the suspension discharged into the weighing tank. Then, the time required for a given weight of suspension to flow into the weighing tank was determined. Since the diameter of the test pipe was known, the velocity of the flow could be calculated. To empty the weighing tank, pipe connections were provided so that the suspension could be pumped back into the make-up tank (R_1). In order that the weights of the weighing tank be accurate, this connection had to be flexible, so a rubber hose (Y) was used.

Agitation. In order that the suspension be as uniform as possible, an electric mixer (S) was fitted to the make-up tank to provide agitation. In addition, such agitation was accomplished by the recirculation of the sludge through the centrifugal pump.

Clogging of Gages. Sludges have been found to give trouble by clogging gage connections. Pig-tail traps were provided to keep the suspension from entering the pressure gages.

Filter Press. A filter press was included in the flow apparatus for the purpose of determining the compressibility coefficient. This apparatus is shown in figure 6, page 40. In order that the suspension could be filtered at constant pressure, a needle valve (d) was installed in the line from the pump (g) to the filter press (k). By regulation of the needle valve (d) and the globe valve (e) on the by-pass line, the pressure on the filter could be regulated to within ± 0.2 lb/^{per}sq in. The pressure was read on the pressure gage (m). This gage was calibrated with a dead weight gage tester both before and after the determinations of compressibility were made.

The filter press was one of the plate and frame type, with five frames, and had a total filtering area of 663 sq in. The intake for delivery of slurry was located near the bottom of the press, and the discharge line for the filtrate was located at the top. This is the best type of filter press for the determinations of compressibility coefficient, according to Almy and Lewis⁽¹⁾. With this type, the press is always filled with slurry during operation. In addition, having the intake at the bottom and the outlet at the top prevents the solid particles from settling out of the suspension during filtering. A tygon tube was attached to the filtrate discharge line. The end of this tube was placed in a bucket which was placed on a pan balance. An electric timer was started as soon as filtrate began to run into the bucket; therefore,

it was possible to note the times at which various increments of filtrate had been collected.

At the end of each filtration operation at constant pressure, the press was cleaned out and all of the filter cloths were washed before beginning another filtration operation at another constant pressure. This washing was done to insure that the filter cloth would be in the same condition for each filtering operation.

Drain Lines. Drain lines were connected to the make-up tank and to the downstream end of the test lengths of pipe for the purpose of draining the apparatus when so desired. In addition, the rubber connecting hose (Y) could be detached from the weighing tank for the purpose of draining the equipment.

B. Method of Procedure

Selection of Suspensions. Since the type of apparatus that was to be used in this investigation had been decided, one of the next problems was the selection of the types of suspensions to be tested. It was known from a study of the literature that the major part of the work on laminar flow of suspensions had been done on sewage sludges, clay suspensions, and suspensions of filter aid^(5, 47). However, the use of these types of slurries would not permit enough variation in compressibility coefficient for justification of their use in this investigation. Walker, Lewis, McAdams and Gilliland⁽⁴¹⁾ report the compressibility coefficients of various types of suspensions as determined by several investigators (see table II, page 30). Of the suspensions reported, aluminum hydroxide suspensions were the most compressible, with a compressibility coefficient of 0.56 to 0.88, and kieselguhr suspensions were the least, with a compressibility coefficient of 0.02 to 0.05. On this basis, it was decided to use aluminum hydroxide as the compressible suspension, and mixtures of aluminum hydroxide and filter aid to obtain suspensions of compressibility less than that of the aluminum hydroxide suspension.

Preliminary Stormer Viscometer Tests. Rabbitt and Caldwell⁽¹²⁾ stated that the Stormer viscometer could be adapted to the measurement of yield value and coefficient of rigidity of a sludge by

slight modification. "Sewage sludges in particular have numerous large particles that render the standard Stormer cup useless by binding the rotating cylinder". Since the size of the suspended particles in an aluminum hydroxide - filter aid suspension would be very small relative to sewage sludge particles, it was thought that the standard Stormer cup could be used in the preliminary tests of this investigation.

a. Aluminum hydroxide suspension, 10 per cent suspended solids. The first suspension tested with the Stormer was a 10 per cent aluminum hydroxide sludge. It was prepared by neutralizing a concentrated aluminum sulfate solution with concentrated sodium hydroxide. On testing this suspension in the Stormer viscometer, it was found that, for a given actuating weight, the time required for the counter to revolve through 100 revolutions would vary from one run to the next, becoming less each time the operation was performed. This fact indicated that the suspension was becoming more fluid with each determination. It was believed that the failure to remove the sodium sulfate formed during the reaction of the aluminum sulfate and the sodium hydroxide might have been partly the cause of this phenomenon, since the concentration of the sodium sulfate in the suspension was near that of a saturated solution of sodium sulfate.

b. Aluminum hydroxide suspension, 12 per cent suspended solids. A 12 per cent suspension of aluminum hydroxide was prepared

in the same manner that the 10 per cent suspension had been prepared, with the exception that most of the sodium sulfate was removed by washing. The washing was done by diluting the suspension, decanting the clear liquid after the solids had settled, re-diluting the suspension, decanting the clear liquid again, and repeating the cycle of operations for a third time. The resulting suspension behaved in the same manner as did the 10 per cent suspension of aluminum hydroxide when tested in the viscometer.

Therefore, it was assumed that the aluminum hydroxide suspensions possessed thixotropic properties. Thixotropy is the phenomenon exhibited by some gels of becoming fluid when shaken⁽¹¹⁾.

Hatfield⁽²⁹⁾ states: "This thixotropic property is very important and must be considered both in the determination of the viscosity of the sludges and in the application of viscosity data to engineering problems, because (1) it is impossible to obtain a sample of quiescent sludge and get it into the viscometer beaker without some agitation or stirring, and (2) during each 100 revolutions of the viscometer cylinder the velocity of revolution has accelerated with each revolution. This acceleration is particularly noticeable at very low velocities in thick sludges."

c. Aluminum hydroxide suspension, 6 Per cent suspended solids. It was decided to test a more dilute suspension in the Stormer viscometer. A 6 per cent suspension of aluminum hydroxide was prepared by diluting the 12 per cent suspension. When tested

in the Stormer viscometer, this suspension gave constant rates of revolution for each actuating weight used. In addition, the plot of actuating weight versus number of revolutions per second showed a straight line which, on extrapolation, intercepted the actuating weight axis. This intercept indicated that this suspension possessed a yield value and was behaving as a plastic rather than a fluid.

d. Filter aid suspension, 25 per cent suspended solids.

The next suspension tested was a 25 per cent suspension of filter aid, with celite 503 as the filter aid. In the Stormer viscometer, this suspension behaved similarly to the aluminum hydroxide suspensions, in that the rate of revolution of the cylinder for any actuating weight diminished with each succeeding determination. However, in this case, it was believed that this phenomenon was due to the settling rather than thixotropy; i.e., that the rotation of the cylinder tended to throw the heavier particles of filter aid out to the walls of the cup where they settled toward the bottom. This belief is substantiated by the fact that, on agitation of the suspension in the cup, the rate of revolution of the cylinder for a given actuating weight would increase, but on subsequent tests, without agitation, would decrease again.

Consequently, the best filter aid for use in the suspension flow tests would be one with the lowest rate of settling.

Relative rates of settling were determined for three filter aids:

celite 503, dicalite, and hyflo super-cel. Of these, hyflo super-cel gave the lowest rate of settling.

Therefore, from this preliminary work, it was decided to use aluminum hydroxide in concentrations less than 10 per cent suspended solids and hyflo super-cel as the filter aid in the aluminum hydroxide - filter aid suspensions for testing in the suspension flow apparatus.

Suspension Flow Tests. In the operation of the suspension flow apparatus, it was found that accurate manometer readings could be obtained only if the manometer leads were filled with water each time before the manometer was read. It was also preferable to determine the flow rate at the same time that the manometer was read. Of course, to do this was not always possible because of the actual time it took to determine the flow rate, whereas the manometer reading was instantaneous. However, it was noticed that the manometer reading usually remained constant during the flow rate determination, except at high velocities, 10 ft^{per}/sec and greater. At these velocities, as time elapsed, the manometer reading would drop off, because of entrapped air in the suspension entering the manometer leads, for, if the manometer leads were filled with water once more, the manometer reading would be nearly the same as originally, and then would begin to drop off again. For this reason, at velocities of 10 ft^{per}/sec and above, the initial manometer reading was recorded as the friction head loss.

At velocities of 0.2 ft/^{per}sec and less, the rate of flow had to be determined as quickly as possible since the suspension began to clog the regulating valve at these low velocities.

Determinations of Compressibility Coefficients. In the determination of the coefficient of compressibility, it was found necessary to control the needle valve (valve (d), figure 6, page 40) constantly to keep the pressure on the filter press constant. Even so, continued regulation could keep the pressure constant only to within ± 0.2 lb/^{per}sq in. at pressures of 5 - 10 lb/^{per}sq in. gage. Above the pressure of 10 lb/^{per}sq in. gage, the pressure remained constant to within ± 0.1 lb/^{per}sq in. with little or no regulation of the needle valve after it once had been adjusted to the desired pressure.

The filter cloths had to be washed thoroughly of all traces of the filter cake between each determination at constant pressure, so that the succeeding determination would be carried out within the filter cloth in the same condition as it had been in the previous determination.

C. Discussion of Results

A study of the literature revealed that the best method of calculating the friction loss of a suspension during laminar flow is that of Babbitt and Caldwell^(5, 23) whose work was discussed under Flow of Suspensions. They propose the use of the equations:

$$\frac{\Delta H}{N} = \frac{16 S_y}{3 \rho D} + \frac{\eta V}{\rho D^2} \quad (21)$$

In this equation, all the terms needed to calculate the friction loss are readily determined except the yield value, S_y , and the coefficient of rigidity, η . However, if these two values are known for a suspension, the friction loss is easily calculated.

To determine the effect of compressibility on the friction loss during laminar flow of suspensions, it was decided to determine, if possible, the effect of compressibility on the yield value and the coefficient of rigidity of suspensions.

Determination of Yield Value and Coefficient of Rigidity.

The suspension flow apparatus was constructed for the purpose of determining the yield value and coefficient of rigidity of the suspensions to be tested. With this apparatus the friction loss was read on the manometer, and the velocity was calculated from the mass rate of flow, knowing the diameter of the test pipe.

The boundary shearing stress, S_p , was calculated by means of the following equation:⁽⁸⁾

$$S_p = \frac{\Delta H \rho D}{4N} \quad (24)$$

According to Babbitt and Caldwell⁽⁵⁾, if a graph is plotted with the boundary shearing stress, S_p , as the ordinate and the velocity divided by four times the diameter, $V/4D$, as the abscissa, the slope of the resulting line will represent the coefficient of rigidity, η , and the intercept on the S_p axis is $4/3$ the yield value. These plots were made for each suspension to determine its yield value and coefficient of rigidity.

a. Suspension Number 1. The plot of S_p versus $V/4D$ for suspension number 1 is shown in figure 7, page 63. This curve is quite similar to ones obtained by Babbitt and Caldwell⁽⁵⁾ in their work. It is noticed that two of the points for the one-and-one-half-inch pipe, two of the points for the one-inch pipe, and one point for the one-half-inch pipe are considerably above the remainder of the points. It was assumed that flow was turbulent when the data for these points were taken. Also, it is noticed that the three points with the least values of $V/4D$ fall well below the rest of the points. This deviation indicates that, at extremely low velocities, the suspension exhibits the properties of a pseudo-plastic material, rather than a true plastic. A pseudo-plastic material does not obey the fundamental equation of plastic flow, in that the line resulting

from a plot on a stress-velocity diagram bends toward the origin at low rates of flow (see figure 2, page 13). Therefore, these three points, and the points of turbulent flow were neglected in drawing a straight line through the remainder of the points by the method of averages. The slope of the line represented the coefficient of rigidity and was 0.00186. The yield value, as calculated from $3/4$ the intercept, was 0.036. The deviation of the furthestmost point from the straight line was 24 per cent.

b. Suspension Number 2. The graph of S_p versus $V/4D$ for suspension number 2 is shown in figure 9, page 67. As can be seen from this graph, the points are quite scattered throughout, probably because of the fact that not all of the determinations were made at the same time. During the time interval between determinations, the temperature changed. The change in temperature could cause the scattering of points. In addition, since this was one of the first determinations to be made, the technique of operating the apparatus was not as good as during the determinations on following suspensions. The straight line was determined by the method of averages. The coefficient of rigidity as calculated from the intercept was 0.150. The deviation of the point furthestmost from the straight line was 31.3 per cent.

c. Suspensions Numbers 3, 4, 5, and 6. The plots of S_p versus $V/4D$ for these suspensions showed a much smoother series of points than did the plots for previous suspensions. The straight

lines for each of these plots were determined by the method of averages, in which the points of pseudo-plastic flow were neglected. The deviations of the points furthestmost from the straight lines were 5.7 per cent, 5.6 per cent, 7.7 per cent, and 4.6 per cent for suspensions numbers 3, 4, 5, and 6 respectively. During none of these determinations did the velocity become great enough to cause turbulent flow, as figures 11, 13, 15, and 17 show.

d. Suspension Number 7. Examination of this plot of S_p versus $V/4D$ (figure 19, page 87) shows that three of the points calculated from data performed on the one-inch test pipe are considerably above the straight line that passes through the remainder of the points. This deviation was due to turbulence encountered at these velocities. The deviation of the observed points excluding those of turbulent flow and pseudo-plastic flow, furthestmost from the straight line calculated by the method of averages was 8.6 per cent.

e. Suspension Number 8. When the plot of S_p versus $V/4D$ was made for this suspension, it was found that 10 of the 17 points calculated from the data were in the turbulent region (see figure 21, page 91). There were only six points to use for the calculation of the straight line representing plastic flow. Consequently, it is believed that there is considerable room for doubt as to the accuracy of the coefficients determined by this plot. Too, the deviation of the point furthestmost from the straight line through the six "good" points was 18.9 per cent.

Determination of the Coefficient of Compressibility. Almy and Lewis⁽¹⁾ state that sludges that show the effect of compressibility to a high degree are hydrated precipitates; their degree of hydration and particle size are extremely sensitive to the conditions under which the precipitates are produced and change progressively with time. Also, this type of suspension is so exceedingly sensitive to temperature that slight temperature changes seriously affect results. For this reason, the determination of the coefficient of compressibility was carried out as soon as possible after the suspension flow tests were completed. Even so, it was noticed that there was a temperature rise of from 1° to 3°C during the filtering operation. This temperature rise was caused by the friction of the suspension in the pump. Therefore it is doubted if the coefficient of compressibility is accurate past the second decimal place.

The coefficient of compressibility was determined according to the graphical method described by Walker, Lewis, McAdams, and Gilliland⁽⁴⁵⁾. The values obtained for the coefficient of compressibility for each suspension are listed in table III, page 61.

It was mentioned that the regulating valve could be controlled only to within $\pm 0.2 \text{ lb} \frac{\text{per}}{\text{sq}} \text{ in.}$ The effect of this error on the calculation of compressibility coefficient is less than 5 per cent.

The filtration curves and the curves for the determination of compressibility coefficient were determined by the method of selected points.

If all of the errors which are introduced in this method of the calculation of the compressibility coefficient are considered, it is believed that the final values obtained were accurate to within 10 per cent in this investigation.

It is pointed out that the calculation of the compressibility coefficient for suspension number 8, figure 22, page 93, is based on data taken at only three constant pressures. Therefore, there were only three points used in the final determination of compressibility coefficient. For this reason the accuracy of this determination is considered to be less than that for the determination of compressibility coefficient for the other suspensions in which five points were used.

Necessity of Including Concentration of Suspended Solids as a Variable. From the results that were obtained by use of the suspension flow apparatus and the filter press, it was decided to determine a relationship, if possible, between compressibility coefficient, yield value and coefficient of rigidity. However, the concentration of suspended solids was varied between each suspension. It is known that the concentration of suspended matter greatly affects the yield value, and affects the coefficient of rigidity to a lesser degree.⁽⁹⁾ Babbitt and Caldwell⁽¹⁰⁾

reported the effect of percentage of solids on yield value and coefficient of rigidity for two clay suspensions and one sewage sludge. For one of the clay suspensions, as the percentage solids was increased from 15.1 to 24.8, the yield value increased from 0.059 to 0.532 and the coefficient of rigidity increased from 0.011 to 0.019. Thus, it is seen that the solids concentration does affect the yield value and coefficient of rigidity. Therefore, since the concentration of suspended matter was not held constant for all suspensions, it became necessary to include this variable in any relationship to be determined for the variables of compressibility, yield value and rigidity.

Determination of a Relationship Between Compressibility, Suspended Solids Concentration, and Yield Value. It was decided to determine separate relationships for yield value and coefficient of rigidity with compressibility and concentration of suspended solids. In the determination of an equation showing the relationship between yield value, compressibility and concentration of solids, it was impossible to use graphical methods. In order to use graphical methods, a three dimensional plot would have to be made to show all three variables. Therefore, the data was fitted into several type forms of equations, and the constants were determined by simultaneous solution.

Since the compressibility coefficient and the yield value for suspension number 8 were greatly out of line compared to the

other suspensions, the data for this suspension was not included in the determination of the equations. As has been pointed out, the accuracy of the results obtained for suspension number 8 is questionable for the reason that insufficient data were obtained. In addition, the data from which the results for suspension number 8 were taken at a temperature of 24°C. This deviation in temperature from the range of 26° to 32°C for the other suspensions would also cause a discrepancy in the results obtained for suspension number 8. These reasons justify neglecting suspension number 8 in the calculation of the equations, it is believed.

The type forms tested to determine a relationship between compressibility, yield value, and concentration of solids were:

$$S_y = b_0 + b_1 C + b_2 s \quad (25)$$

$$S_y = b_0 + b_1 C + b_2 s^2 \quad (26)$$

$$S_y = b_0 + b_1 C + b_2 \ln s \quad (27)$$

$$\log S_y = b_0 + b_1 \log C + b_2 \log s \quad (28)$$

$$S_y = b_0 + b_1 C + b_2 C^2 + b_3 s + b_4 s^2 \quad (29)$$

where

S_y = yield value of suspension, lb/sq ft.

C = concentration of suspended solids in suspension,
lb dried solids/100 lb suspension

s = coefficient of compressibility

$b_0, b_1, b_2, b_3,$ and b_4 = constants.

From these type forms the following equations were derived:

$$S_y = 0.365 + 0.00795 C - 0.736 s \quad (30)$$

$$S_y = 0.173 + 0.0125 C - 0.876 s^2 \quad (31)$$

$$S_y = 0.0659 + 0.00912 C - 0.157 \ln s \quad (32)$$

$$S_y = 0.00138 C^{1.59} s^{0.0327} \quad (33)$$

$$S_y = 0.1986 + 0.213 C - 0.011 C^2 - 4.52 s + 8.481 s^2 \quad (34)$$

Of these equations, the best relationship was obtained by the use of equation (34). For this equation, the average percentage deviation of the observed values of S_y from the computed values was 6.96 per cent. Since the determination of the compressibility coefficient was accurate only to within 10 per cent, this equation permits the prediction of yield value from compressibility coefficient and concentration of suspended matter as accurately as were the data obtained by the equipment used in this investigation. It is not believed that this equation is the only equation, or even the best one, which could be determined for the relationship between yield value, compressibility and solids concentration. It is merely the best equation which was determined to show the relationship between the variables, and is of value in predicting the yield value to within 10 per cent.

It is to be noted that this equation holds only above the critical concentration, above which plastic flow is exhibited. Below the critical concentration, the laws of hydraulics can be

used for prediction of friction loss by using the viscosity of the dispersing liquid as the viscosity of the suspension.

The equations derived and the deviation of the calculated yield values from observed yield values are shown in table XXI, page 96.

Determination of a Relationship Between Compressibility, Suspended Solids Concentration and Coefficient of Rigidity. The equations relating the suspended solids concentration, compressibility, and rigidity were developed in the same manner as were those for suspended solids concentration, compressibility and yield value. The following test forms were tried, the data from suspension number 8 being neglected for reasons previously discussed:

$$\eta = b_0 + b_1 C + b_2 s \quad (35)$$

$$\eta = b_0 + b_1 C + b_2 s^2 \quad (36)$$

$$\log \eta = b_0 + b_1 C + b_2 \log s \quad (37)$$

$$\eta = b_0 + b_1 C + b_2 C^2 + b_3 s + b_4 s^2 \quad (38)$$

where

η = coefficient of rigidity, lb/sec-ft

C = concentration of suspended solids in suspension,
lb dried solids/lb suspension

s = coefficient of compressibility

$b_0, b_1, b_2, b_3,$ and b_4 = constants.

From these, the following equations were derived:

$$\eta = -0.00329 + 0.00091 C + 0.00469 s. \quad (39)$$

$$\eta = -0.00119 + 0.00633 C + 0.00175 s^2. \quad (40)$$

$$\eta = 0.000345 C^{1.37} s^{0.113}. \quad (41)$$

$$\eta = 0.0639 - 0.0226 C + 0.00123 C^2 + 0.394 s - 0.876 s^2. \quad (42)$$

Of these, equation (41) gave the least average percentage deviation, 20.72 per cent. The deviation of each observed value from the values of the coefficient of rigidity calculated according to the preceding equations are shown in table XXII, page 88.

None of the types tested gave an average percentage deviation of less than 20 per cent. However, in each case, the major portion of the deviation was due to suspension number 2. Examination of the plot of boundary shearing stress, S_p , versus velocity divided by four times the diameter, $V/4D$, for suspension number 2 shows that the points were very scattered (see figure 7, page 63). Therefore, the validity of the value of the coefficient of rigidity as calculated from the slope approximated through these points is doubtful, whereas the yield value is probably much more accurate, as the points seem to converge toward the intercept on the S_p axis. If suspension number 2 were eliminated, the average percentage deviation for equation (41) is reduced from 20.72 per cent to 12.94 per cent.

Use of the Stormer Viscometer in Determining Coefficient of Rigidity and Yield Value. Babbitt and Caldwell⁽⁹⁾ propose the use

of a modified Stormer viscometer to determine the values of coefficient of rigidity and yield value. One objection to their method is that only a Stormer viscometer modified exactly as was theirs could be used if their calibration constants are to be of any value. In addition, their work was based solely on sewage sludges and clay suspensions. The applicability of their methods to other suspensions, such as the aluminum hydroxide - filter aid suspensions, was not known. In this investigation, each suspension was tested with the standard Stormer viscometer. This instrument was satisfactory in the case of suspensions number 1, 2, and 3. The plot of actuating weight versus revolutions per second of the rotating cylinder is shown in figure 23, page 95. However, for the remainder of the suspensions, the Stormer viscometer could not be used to determine this type of plot, because repetitive results of revolutions per second could not be obtained for any one actuating weight used. This difficulty was due to the thixotropic properties of the aluminum hydroxide, and the separation of the filter aid particles because of the centrifugal action of the viscometer.

Temperature. Babbitt and Caldwell⁽¹¹⁾ and Hatfield⁽²⁹⁾ found that a rise in temperature decreased the resistance to flow of a sewage sludge by reducing the yield value and coefficient of rigidity. As no attempt was made to study the effect of temperature on the laminar flow of suspensions, each suspension was tested at as nearly the same temperature as practical. This was

sometimes quite difficult as no means were provided in the construction of the apparatus for controlling the temperature. The variation of temperature was from 26° to 32°C for this investigation.

Age of Suspensions. It is known that the age of an aluminum hydroxide suspension definitely does have some effect on its flow properties. However, the exact manner in which aging affects the flow properties of a suspension is not known. It is believed that the effect of aging is included in the variable of compressibility. Since no attempt was made to study the effect of aging on the laminar flow of suspensions, the compressibility of each suspension was determined as soon as possible after the friction head loss tests were made. Thus, for all practical purposes, the age of the suspension at the time of the compressibility coefficient determination was the same as that at the time of the friction head loss tests. It was hoped to minimize any effect that the aging would have. The suspensions tested in this investigation varied in age of the aluminum hydroxide precipitate from 7 to 63 days.

D. Recommendations

The following recommendations are offered for future investigations:

Velocity Control. Use of a centrifugal pump and a throttling valve for controlling the velocity of flow was found to be satisfactory only between the limits of velocities of 0.2 to 9.0 ft^{per}/sec. It is suggested that a rotary pump driven by a variable speed motor be used to obtain greater control of flow rates. The use of this type of equipment would eliminate the clogging of the regulating valve which occurred at low velocities, and permit a greater range of flow rates.

Temperature Control. The accuracy of the determinations of compressibility coefficient, yield value and coefficient of rigidity would be increased by some type of temperature controlling device. A water bath surrounding the make-up tank would give somewhat better control of temperature and, consequently, correlation of results would be expected to be better.

Pressure Control on Filter Press. It is believed that a greater range of pressures for the filtration would increase the accuracy of the determination of compressibility coefficient. The use of a montajus to obtain pressures up to 50 lb^{per}/sq in. is suggested. In addition to providing a greater range of pressures,

it is believed that the pressure on the filter press would be less variable by using a monteju than with the use of a centrifugal pump, as in this investigation.

Pressure Gage. The pressure gage used in this investigation could be read to only within 0.1 lb per sq in. The use of a more sensitive pressure gage, such as a calibrated manometer, is recommended.

Further Investigations on Aluminum Hydroxide - Filter Aid Suspensions. It is suggested that additional work be done using aluminum hydroxide - filter aid suspensions for the purpose of checking the validity of the equations derived in this investigation. During future investigations, by holding the concentration of suspended solids constant, if possible, the relationship between compressibility, coefficient of rigidity, and yield value could be determined much easier and more accurately than by the method used in this investigation. A concentration of suspended solids of about 10 per cent is recommended for use, as best results were obtained near this concentration.

Effect of Particle Size. No attempt was made in this investigation to determine the effect of particle size on laminar flow. However, it is believed that the size of particles in suspension will affect the flow characteristics of a sludge. A study of this effect, together with its effect on compressibility is suggested.

E. Limitations

The following limitations were imposed on this investigation:

Type of Suspensions. The only suspensions used in this investigation were aluminum hydroxide suspensions, and mixed suspensions of aluminum hydroxide and filter aid.

Concentration of Suspended Solids. The range of the concentration of suspended solids used in this work was from 3.60 to 13.03 per cent.

Compressibility. The suspensions used varied in compressibility as shown by coefficients of 0.080 to 0.425.

Temperature. No attempt was made to control the temperature at which the determinations were made for this investigation, hence, the temperature varied from 26° to 32°C.

Velocity of Flow. By reason of the equipment used, the velocity of flow was limited to a range of from 0.1 to 9.0 ft^{per}/sec.

Pressure. In the determination of the coefficient of compressibility, the range of pressures possible was limited to 5 to 20 lb^{per}/sq in. gage. This limitation was due to the use of a centrifugal pump for the pressure producing device.

Pipe Sizes. The determinations of friction head loss in this work were limited to tests on standard black iron pipe of nominal diameter of one-half-, one-, and one-and-one-half-inches.

V. CONCLUSIONS

In this investigation of the effect of compressibility, on the friction drop during laminar flow in pipes, the suspensions used were aluminum hydroxide - filter aid suspensions, of which the concentration of suspended solids ranged from 3.6 to 13 per cent. The compressibility coefficient, as determined by filtration tests at constant pressure over the range of pressures of 5 to 20 lb/sq in., varied from 0.080 to 0.425. From the results obtained, the following conclusions are drawn:

1. The yield value of suspensions of this type may be predicted to within an accuracy of 10 per cent, by the use of the equation:

$$S_y = -0.1986 + 0.213 C - 0.011 C^2 - 4.520 s + 8.481 s^2.$$

where

S_y = yield value of suspension, lb/sq ft.

C = concentration of suspended solids in suspension,
lb dried solids/100 lb suspension

s = coefficient of compressibility.

2. The coefficient of rigidity of a suspension of this type may be determined to within an accuracy of 13 per cent, by the use of the equation:

$$\eta = 0.000345 C^{1.37} s^{0.113}$$

where

η = coefficient of rigidity of suspension, lb/ft-sec.

3. If the yield value and the coefficient of rigidity are determined by the use of the preceding equations, the friction head loss may be determined to within an accuracy of 15 per cent by the use of the following equations:

$$\frac{\Delta H}{N} = \frac{16 S_y}{3 \rho D} + \frac{\eta V}{\rho D^2}$$

where

ΔH = friction head loss, ft of flowing substance.

N = length of pipe, ft.

V = velocity of flow, ft/sec.

ρ = density of suspension, lb/cu ft.

D = diameter of pipe, ft.

4. By reason of the thixotropic properties of aluminum hydroxide and the fact that filter aid particles settle out, it is not possible to use the standard Stormer viscometer to determine the yield value and the coefficient of rigidity of aluminum hydroxide - filter aid suspensions.

VI. SUMMARY

Very little information has been published on the flow of suspensions. It has been found that the friction head loss of a suspension flowing through a pipe may be predicted fairly closely by the use of hydraulic equations in which the viscosity of the dispersing medium is used to calculate Reynolds number, if flow is turbulent. However, a simple method of determining the friction loss of a suspension if the flow is within the laminar region is not known. A better understanding of the laminar flow of suspensions would aid the industries of oil well digging, water and sewage treatment, dredging, and papermaking.

The use of the following equation to determine the friction head loss of a suspension exhibiting laminar flow is probably, the most conclusive.

$$\frac{\Delta H}{N} = \frac{16 S_y}{3 \rho D} + \frac{\eta V}{\rho D^2}$$

where

ΔH = friction head loss, ft of flowing substance.

N = length of pipe, ft.

V = velocity of flow, ft/sec.

D = diameter of pipe, ft.

ρ = density of suspension, lb/cu ft.

S_y = yield value of suspension, lb/sq ft.

η = coefficient of rigidity, lb/ft-sec.

The use of this equation necessitates the determination of the yield value and the coefficient of rigidity of a suspension to predict the friction head loss. The use of a modified Stormer viscometer in determining these coefficients has been proposed.

An obvious disadvantage to this method is that the essential coefficients needed to predict the friction head loss cannot be determined without modifying a Stormer viscometer. In addition, the application of this method of determining the yield value and the coefficient of rigidity to suspensions other than sewage sludges and clay suspensions has not been proven.

In an attempt to develop a better method of predicting the friction head loss of a suspension during laminar flow, an investigation was made on the effect of compressibility, as pertaining to filtration, on the yield value and coefficient of rigidity of a suspension. This investigation was carried out on a flow apparatus with test lengths of pipe of one-half, one, and one and one-half inches in diameter to determine the friction head loss. A filtration apparatus was used to determine the coefficient of compressibility.

From the results obtained in this investigation on aluminum hydroxide - filter aid suspensions of which the concentration of suspended solids ranged from 3.6 to 13 per cent and the coefficient of compressibility varied from 0.080 to 0.425, it was determined that:

1. The yield value, S_y , of a suspension of this type may be predicted to within an accuracy of 10 per cent by the use of the equation:

$$S_y = -0.1986 + 0.213 C - 0.011 C^2 - 4.520 s + 8.481 s^2$$

where

S_y = concentrations of suspended solids in suspension, lb dried solids/100 lb suspension.

s = coefficient of compressibility.

2. The coefficient of rigidity, η , of a suspension of this type may be determined to within an accuracy of 13 per cent, by the use of the equation:

$$\eta = 0.000345 C^{1.37} s^{0.113}$$

Experiments made during the tests with the standard Stormer viscometer gave erratic results. It is concluded that it is not possible to use this instrument to determine the yield value and coefficient of rigidity of aluminum hydroxide - filter aid suspensions.

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