Sensitivity Analysis

of

Ship Longitudinal Strength

by

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(ABSTRACT)

The present work addresses the usefulness of a simple and efficient computer program (ULTSTR) for a sensitivity analysis of ship longitudinal strength, where this program was originally developed for calculating the collapse moment. Since the program is efficient it can be used to obtain ultimate strength variability for various values of parameters which affect the the longitudinal strength, viz., yield stress, Young's modulus, thickness, initial imperfections, breadth, depth, etc.

The results obtained with this approach are in good agreement with those obtained by use of a more complex nonlinear finite element program USAS, developed by American Bureau of Shipping.

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This thesis is dedicated to my loving parents.

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NOMENCLATURE

A_B	Total Effective Area of Bottom.
A_D	Total Effective Cross Section Area of the Deck Including Stiffeners.
A_S	Total Effective Area of One Side.
$\overline{A}_f$	Average Area of Flange.
$\overline{A}_s$	Average Area of Side.
b	Actual Width of the Plates.
b_e	Effective Width of the Plates.
B	Breadth of the Section, Ship.
d_i	Distance from the Final Neutral Axis to the Centroid of Small Portion of the Effective Area on the Cross Section.
D	Depth of the Section, Ship.
E	Young's Modulus.
E_t	Tangent Modulus.
$\mathbf{f}_M$	Probability Density Function (PDF) of Safety Margin.
$\mathbf{f}_R$	Probability Density Function (PDF) of Strength.
$\mathbf{f}_S$	Probability Density Function (PDF) of Load.
$\mathbf{f}_{R,S}$	Joint Probability Density Function (PDF) of Strength and Load.
$\mathbf{f}_\theta$	Probability Density Function (PDF) of Safety Factor.
$\mathbf{F}_R$	Cumulative Density Function (CDF) of Strength.

$\mathbf{F}_S$	Cumulative Density Function (CDF) of Load.
g	Distance from the Center of the Deck to the Plastic neutral Axis.
M	Safety Margin.
M_p	Fully Plastic Moment.
M_s	Still Water Bending Moment.
M_u	Ultimate Bending Moment.
M_y	Initial Yield Moment.
$M_{\min}$	Minimum Wave Bending Moment.
$M_{\max}$	Maximum Wave Bending Moment.
N	Correction Factor for Predictive Model.
N_g	Correction Factor for Multiple Variable Predictive Model.
p_i	Basic or Mean Values of the Parameters.
$\mathbf{P}_F$	Probability of Failure.
$\bar{r}$	Estimated Mean Value of R due to Inherent Randomness.
R	Strength, Resistance or Supply Capacity of the Structure.
$\bar{s}$	Estimated Mean Value of S due to Inherent Randomness.
S	Load Acting on the Structure.
t_f	Flange Thickness.
t_s	Side Thickness.
x_l	Lower Limit of Random Variable X.
x_u	Upper Limit of Random Variable X.
X	Any Random Variable.
$\overline{X}$	Mean Value of $\hat{X}$.
$\hat{X}$	Predicted Value of Random Variable X.
Z	Section Modulus.
Z_e	Conventional Elastic Section Modulus.

Z_p	Plastic Section Modulus.
COV	Coefficient of Variation $= \frac{\text{Standard Deviation}}{\text{Mean}}$.
α_i	Nondimensionalized Sensitivity Derivatives.
α_i^*	Direction Cosines.
β	Reliability Index.
σ	Ultimate stress.
σ_R	Standard Deviation of Strength.
σ_y	Material Tensile Yield Stress.
$\overline{\sigma}_y$	Mean Value of σ_y.
σ_{Y_c}	Compression Yield Strength of Material.
σ_{Y_i}	Standard Deviation of Random Variable Y_i .
Δ_X	COV of Total Uncertainty in the Prediction of X.
δ_B	COV of Breadth.
δ_D	COV of Depth.
δ_R	COV of Strength.
δ_S	COV of Load.
δ_t	COV of Thickness.
δ_{t_f}	COV of flange Thickness.
δ_{t_s}	COV of Side Thickness.
δ_X	COV of Random Variable X.
δ_{Y_i}	COV of Y_i.
δ_{ob}	COV of Strength due to Objective Uncertainties.
δ_{sb}	COV of Strength due to Subjective Uncertainties.
δ_{σ_y}	COV of Tensile Yield Stress.
δ_{Z_e}	COV of Section Modulus Z_e .

δ_{ϕ}	COV of Failure Stress Ratio.
δ_{ϕ_1}	COV Associated with Variability in Plate Thickness, Breadth and Corrosion.
δ_{ϕ_2}	COV Associated with Residual Stress.
δ_{ϕ_3}	COV Associated with E or E_t .
Δp_i	Increment to the Mean Values of the Parameters.
$(\Delta A_e)_i$	Small Portion of the Effective Area on the Cross Section.
Δ_R	Random Error in $\bar{r}$.
Δ_S	Random Error in $\bar{s}$.
Δ_X	Random Error in Predicted Mean Value of $\overline{X}$.
Δ_g	COV of N_g.
θ	Safety Factor.
ρ	Correlation Coefficient.
ϕ	Failure Stress Ratio.
Φ	Cumulative Distribution Function Corresponding to the Normal Distribution.
μ_R	Mean Value of R.
μ_S	Mean Value of S.
ν	Mean Value of N.
ν_g	Mean Value of N_g.
ν_R	Systematic Error in $\bar{r}$.
ν_S	Systematic Error in $\bar{s}$.

Chapter 1. Introduction

Great strides have been made in the ability of engineers to accurately compute stresses, deflection, and buckling loads. If loads and strengths are exactly known, then improved structural analysis could permit significant reduction in the factor of safety and result in economic benefits [1]. However, it is generally recognized that structural problems are often nondeterministic. Structural safety is clearly a function of maximum load (or combination of loads) that may be imposed over the useful life of the structure. Structural safety will also depend on the strength or load-carrying capacity of the structure or its components. As the lifetime maximum load and the actual capacity of the structure are difficult to predict exactly, and any prediction is subject to uncertainty, the absolute assurance of the safety of the structure is not possible. Realistically, safety may be assured only in terms of the probability that the available strength will be adequate to withstand the lifetime maximum load. Structures are designed and built despite imperfect information and knowledge. In fact, for many problems load and strength can be described as statistical variables only, and, therefore the safety of structures must also be statistical variables. In short , problems of structural design must be resolved in the face of uncertainty and as a consequence risk of failure is virtually unavoidable. Most of the work in this area, however, has been confined to estimating the reliability level of a geometrically defined system. It is only recently that some progress has been made in incorporating the probability concept in the design process itself [2]. Failure should be interpreted with respect to some predefined limit state; it may be an excessive deflection, major cracking in a beam or girder, or the total collapse of the structure. Therefore, depending on the limit state under consideration, the concept of failure probability is applicable to both the safety and the performance of a structure. It has been proposed for some time that the rational criteria for the safety of a structure is its reliability or probability of survival. Statistical distribution of loads and strengths must be considered in determining the reliability. The entire structure must be examined to predict its probability of survival.

The reliability of a structure is its ability to fulfill its design purpose for some specified period of time. In order to asses the safety of real structures, concepts and methods for evaluating the reliability of structural systems are required. In real structures the failure of structural systems is possible due to a number of failure modes, i.e. the occurrence of any one of several possible modes constitute failure of the system [3]. Consequently all possible failure modes must be included in the analysis of the system reliability; otherwise, the system reliability will be overestimated. Developments in structural reliability have largely concentrated on single mode failure events, such as for specialized systems in which the failure of systems is defined by a single limit state.

Most system reliability studies are based on "failure mode approach" (FMA); i.e., based on the ways in which a system may fail under an applied load. Recently, Bennett and Ang [3] suggested the "stable configuration approach" (SCA), which is based on ways that different configurations derivable from the original system or any of its subsystems can carry a load.

For structures as complex as a ship hull, experimental data simply can not be obtained in quantities required to establish design criteria and the desired statistics [4]. So far, very few ship hulls have been tested to collapse due to enormous cost associated with such tests. Thus, a reliable analytical method of prediction of the longitudinal ultimate strength of the hull is necessary. During the past two decades a great deal of activity has taken place in predicting, in a realistic manner, the sea load to which the ship hull is subjected. The main initiatives behind these activities are the possible deficiencies in the standard calculation of the longitudinal bending moment, particularly under certain circumstances. Due to the random nature of seas, spectral analysis of ocean waves had been performed and the ship system was assumed to be linear [5]. The principle of superposition was then used to obtain the motions and load spectra in random seas. When the extreme loads exceed a minimum resistive strength, the failure occurs. Therefore, both of these factors should be considered simultaneously in the analysis for a safe design. This leads to the concept of reliability and the consideration of probability of failure.

The problem of ensuring adequate reliability of ship structures is not a new one [6]. It existed even before the development of formal reliability theory and was simply expressed by the requirement that such a structure would fulfil its design function throughout its operating life.

Practical use of reliability theory in shipbuilding is far behind compared to other branches of technology. The major reasons for this slow development are [6]:

a) The statistical study of the seaway, which is the main source of loading on the ship's hull, began only about 30 years ago and results suitable for practical use were obtained only during the last few years.

b) the very complex character of interactions of the ship with its environment and the interaction between the different parts of the hull.

c) The lack of statistical data on the mechanical characteristics of shipbuilding materials, and their changes during the building, operations and repairing of the ships.

Another barrier to the use and implementation of reliability methods in ship structures is the lack of methodology to incorporate and reflect properly the dynamic responses in the total load on a ship hull girder [7]. In addition, the progress in determining the ultimate strength of the hull girder and the factors affecting it has been slow. Statistical data pertaining to factors such as material properties, fabrication tolerances, initial deformations, residual stresses and corrosion are in general available, but not in sufficient quantities to permit unquestionable and reliable statistical estimates of their probability distributions and other relevant parameters.

Due to the random character of sea loads, the limitation on production and fabrication techniques, and certain inadequacies in the present day methods of design analysis, additional strength must be provided for the purpose of safety of the ships [8]. In addition, uncertainties associated with the design parameters, which can be grouped in general under applied loads and resistive strength; lead to the conclusion that the design problem is probabilistic rather than deterministic, i.e. the loads and strengths are not single unique values, but are quantities which exhibit statistical variations and may have wide dispersions. The additional strength is therefore necessary to provide a certain level of safety against extreme conditions of those parameters. Safety in this context denotes the margin against failure of the total structure to carry loads.

The first attempts to apply probabilistic methods to ship structural analysis was made about 15 years ago, mainly by Mansour and Faulkner [8]. These early approaches were based on a structural reliability theory that was far from being completed and referred to both longitudinal and local strength with a failure equation in the form

$$Z\,.\sigma - M_u = 0 \tag{1.1}$$

where Z is the section modulus, σ is an appropriate ultimate stress and M_u is the extreme vertical bending moment. Such an oversimplified formulation of structural behavior has been criticized, especially by practical designers. This point can now be overcome on the basis of a general approach which enables the use of any general failure equation, where different loads and load effect processes can combine in arbitrary ways.

Three levels of analysis are very popular [9], the first being called "semiprobabilistic" the second "distribution free" and third "fully probabilistic". In the development of structural reliability theory it was shown that exact calculation of the failure probability (level 3) is almost always numerically impracticable and that,moreover, advanced level 2 methods can be developed, appropriate to include all available information on the probability distribution of design variables. Since all these methods are based on a linearization of the limit state equation they are called first order reliability methods (FORM) and they lead to an approximate estimate of the failure probability at satisfactory level.

Since the design problem is not deterministic some degree of doubt has to be recognized and explicitly expressed in the design procedure [8]. This is one of the differences in relation to the conventional "factor of safety" approach in which any risk of failure is considered inadmissible. It is important to emphasize that accepting the existence of risk of failure does not necessarily mean reduction in safety. Since in this method uncertainties are accounted for, it provides a more rational and controlled approach. Therefore reliability methods should ensure greater or more certain real safety by reducing or recognizing areas of ignorance.

Sensitivity analysis of ultimate longitudinal strength due to variation in parameters like yield stress, thickness etc. are very important for reliability analysis. It was suggested by Kaplan [10] that since the program "ULTSTR", developed by Adamchak [11], can provide values of collapse moment or ultimate strength of a ship with small computational effort, it can be used effectively and efficiently to determine the changes in strength associated with variation in parameters characterizing the ship section. The suggested procedure involves parametric changes in quantities such as the material yield strength, dimension of plate and stiffener elements, etc., with the ultimate strength determined in each case. These values of ultimate strength can be used to calculate the sensitivity.

The first such attempt to use computational means for determining such sensitivities was made by American Bureau of Shipping (ABS) using their nonlinear finite element program "USAS" (Ultimate Strength Analysis of Structure) [4]. However, this program was very expensive in terms of computer time. Although we get very good estimates of collapse moment, such accuracy may not be required for preliminary design.

In this study, sensitivity of longitudinal strength was calculated for two tankers due to variation in yield stress, Young's modulus, thickness, initial imperfections, breadth and depth.

The present report describes the application of the simpler method using the ULTSTR program to obtain these sensitivities and compares results with the ABS study.

Chapter 2 gives an overview of the fundamentals of structural reliability. Chapter 3 describes the failure modes in ship structure. Chapter 4 includes the uncertainties in ship longitudinal strength and gives a brief description of the program ULTSTR. Procedure for sensitivity calculation is also included in this chapter. Chapter 5 gives the results and discussions and chapter 6 contains the conclusion.

Chapter 2. Fundamentals of Structural Reliability

2.1. Overview

Since uncertainty is unavoidable in designing a structure, the assurance of its safety or performance can not be absolute; realistically, structural safety or performance may be assured in terms of probability.

The problem of reliability of engineering systems may be cast essentially as a problem of "supply" versus "demand" [12]. In other words, problems of engineering reliability may be formulated as the determination of capability (supply) of an engineering system to meet certain requirements (demand). In the consideration of safety of the structure, we are concerned with ensuring that the strength of the structure (supply) is sufficient to withstand the lifetime maximum load (demand).

The reliability of a structural system is achieved through the use of factors or margin of safety and adopting conservative assumptions in the process of design; that is, by ascertaining that a "worst" or minimum supply condition will remain adequate (by some margin) under a "worst", or maximum demand requirements.

In the determination of the strength and/or the load effect, estimation or prediction of the actual state of nature is necessary; invariably, such predictions are subject to error and uncertainty. In the case of fatigue resistance, it is a matter of the number of load cycles or load repetitions that a structural element can sustain without incurring appreciable damage. Again, prediction of actual fatigue life may be subject to significant error and uncertainty. In light of such uncertainties, the available supply and actual demand can not be determined precisely; these may be described as belonging to the respective ranges of possible supply and demands. In order to explicitly represent or reflect the significance of uncertainty, the available supply and required demand may be modeled as random variables. In these terms, therefore, the reliability of a system may be more realistically measured in terms of probability. For this analysis we define following random variables:

R = strength, resistance or supply capacity

S = load or demand requirement

The objective of reliability analysis is to ensure the event $(R > S)$ through out the life or certain specified time of the structural system. This assurance is possible only in terms of probability $P(R > S)$. This probability, therefore, represents a realistic measure of the reliability of the system; conversely, the probability of the complimentary event $(R < S)$ is the corresponding measure of unreliability, that is, $P(R < S)$ is the probability of failure.

If the probability distribution of R and S are known then the required probability may be formulated as follows:

$$\mathbf{P}_F = P\,(R < S) = \sum_{all s} P(R < S/S = s)\; P(S = s) \qquad (2.1.1)$$

Where $\mathbf{P}_F$ is the probability of failure.

If the supply and demand, R and S, are statistically independent then

$$P\,(R < S \,/\, S = s) = P\,(R < S) \qquad (2.1.2)$$

and

$$\mathbf{P}_F = \sum_{all s} P\,(R < S)\; P(S = s) \qquad (2.1.3)$$

For continuous R and S, we can write

$$\mathbf{P}_F = \int_0^{\infty} \mathbf{F}_R(s)\mathbf{f}_S(s)ds \qquad (2.1.4)$$

Where $\mathbf{f}_S(s)$ is the probability density function of S and $\mathbf{F}_R(s)$ is the cumulative distribution function of R, both evaluated at $S = s$.

Eqn. (2.1.4) is the convolution with respect to S and may be explained also with reference to Fig. (2.1) as follows:

If S = s, the conditional probability of failure would be $\mathbf{F}_R$ (s), but since S = s (or more precisely s < S < s+ds) is associated with probability $\mathbf{f}_S$ (s) ds, the integration over all values of S yield Eqn. (2.1.4). Reliability may also be formulated by convolution with respect to R yielding

$$\mathbf{P}_F = \int_0^{\infty} [1 - \mathbf{F}_S(r)]\mathbf{f}_R(r)dr \qquad (2.1.5)$$

Where $\mathbf{f}_R(r)$ is the probability density function of R and $\mathbf{F}_S(r)$ is the cumulative distribution function of S, both evaluated at $R = \mathrm{r}$.

Eqns. (2.1.4) and (2.1.5) represent the overlapping region (convolution) between $\mathbf{f}_R$ (r) and $\mathbf{f}_S$ (s), as shown in Fig. (2.1), and thus $\mathbf{P}_F$,depends on the relative position of $\mathbf{f}_R$(r) and $\mathbf{f}_S$ (s); i.e., as the relative positions become further apart, the $\mathbf{P}_F$ decreases; and as $\mathbf{f}_R$ (r) and $\mathbf{f}_S$ (s) become closer to each other, $\mathbf{P}_F$ increases. This is shown in Fig. (2.2). The relative position between $\mathbf{F}_R$ (r) and $\mathbf{f}_S$ (s) may be measured by the ratio μ_R/μ_S , which may be called the "central safety factor" or the difference $(\mu_R - \mu_s)$ which is the mean "safety margin". μ_R and μ_S are the mean values of R and S respectively.

$\mathbf{P}_F$ also depends on the degree of dispersion of $\mathbf{f}_R$ (r) and $\mathbf{f}_S$ (s) as may be seen in Fig. (2.3). These dispersions may be expressed in terms of coefficients of variation (COV) δ_R and δ_S . ($\delta_R = \frac{\mu_R}{\sigma_R}$ and $\delta_S = \frac{\mu_S}{\sigma_S}$). σ_R and σ_S are the standard deviations of R and S respectively. Therefore measure of safety or reliability ought to be a function of the relative position of $\mathbf{f}_R$ (r) and $\mathbf{f}_S$ (s) as well as of the degree of dispersions.

The failure probability $\mathbf{P}_F$ also depends on the form of $\mathbf{f}_R$ (r) and $\mathbf{f}_S$ (s), however, available information may be just enough to evaluate mean and standard deviations. The quantitative evaluation of the true $\mathbf{P}_F$ often poses a major problem because determination of the correct form of $\mathbf{f}_R$ (r) and $\mathbf{f}_S$ (s) may be very difficult.

If R and S are not statistically independent then probability of failure can be expressed in terms of the joint probability density function as follows:

$$\mathbf{P}_F = \int_0^{\infty}\left[\int_0^{s} \mathbf{f}_{R,S}(r,s)dr\right]ds \tag{2.1.6}$$

$$\mathbf{P}_F = \int_0^{\infty}\left[\int_0^{r} \mathbf{f}_{R,S}(r,s)ds\right]dr \tag{2.1.7}$$

Where $\mathbf{f}_{R,S}(r,s)$ is the joint probability density function of R and S.

2.2. Margin of Safety

For a given limit state, failure may be defined as the event (R - S < 0). As R and S are random variables, the "safety margin" M = R - S is also a random variable with PDF $\mathbf{f}_M$ (m). The probability of failure, therefore, may be expressed as

$$\mathbf{P}_F = \int_{-\infty}^{0} \mathbf{f}_M(m)dm = \mathbf{F}_m(0) \tag{2.2.1}$$

Where $\mathbf{f}_M(m)$ is the probability density function of safety margin M. Graphically this represents the area of $\mathbf{f}_M$ (m) below 0, as shown in Fig. (2.4).

2.3. The Factor of Safety

Alternatively, failure may also be defined as the event (R/S < 1.0). The ratio R/S is the "Factor of Safety" against the particular limit state. Denoting

$$\theta = \frac{R}{S} \tag{2.3.1}$$

which is also a random variable with PDF $\mathbf{f}_\theta(\theta)$ as shown in Fig. (2.5), the failure probability is

$$\mathbf{P}_F = \int_{0}^{1.0} \mathbf{f}_\theta(\theta)d\theta \tag{2.3.2}$$

2.4. Generalization

The reliability of a structural system involves multiple variables. In particular, supply and demand may, respectively, be function of several variables. For such cases the supply and demand problem must be generalized. This generalization is often necessary when the problem must be formulated in terms of basic design variables. For this purpose a performance function, or state function can be defined as

$$g(\underset{\sim}{X}) = g(X_1, X_2, ..., X_n) \qquad (2.4.1)$$

where $\underset{\sim}{X} = (X_1, X_2, ..., X_n)$ is a vector of basic state or design variables of the system, and the function $g(\underset{\sim}{X})$ determines the performance or state of the system. Accordingly, the limiting performance requirement may be defined as $g(\underset{\sim}{X}) = 0$ which is the limit state of the system. Safety is defined as $g(\underset{\sim}{X}) > 0$ and failure as $g(\underset{\sim}{X}) < 0$. Geometrically the limit state equation, $g(\underset{\sim}{X}) = 0$, is an n-dimensional surface that may be called the "failure surface". One side of the failure surface is the safe state, $g(\underset{\sim}{X}) > 0$ whereas the other side of the failure surface is the failure state, $g(\underset{\sim}{X}) < 0$.

Assuming that the variables are uncorrelated [3], we can define a set of uncorrelated reduced variates

$$X_i' = \frac{(X_i - \mu_{X_i})}{\sigma_{X_i}}, \; i = 1,2,........,n \qquad (2.4.2)$$

Where μ_{X_i} and σ_{X_i} are the means and standard deviations of X_i.

The safe state and failure state may also be portrayed in the space of the above reduced variates, separated by the appropriate limit state equation. In terms of reduced variates X_i', the limit state equation would be

$$g(\sigma_{X_1} X_1' + \mu_{X_1}, \ldots\ldots\ldots, \sigma_{X_n} X_n' + \mu_{X_n}) = 0 \qquad (2.4.3)$$

From the Fig. (2.6) we see that as the limit state surface (or failure surface), $g(\underline{X}) = 0$, moves further away from or closer to the origin, the safer region, $g(\underline{X}) > 0$, increases or decreases accordingly. Therefore, the position of the failure surface relative to the origin of the reduced variates should determine the safety or reliability of the system. The position of the failure surface may be represented by the minimum distance from the surface $g(\underline{X}) = 0$ to the origin of the reduced variates. This minimum distance is the approximate measure of safety, this corresponds to the distance at the "most probable failure point", $\underline{x}'^*$, on the limit state surface $g(\underline{X}) = 0$. The distance from a point $X' = (X'_1, X'_2, \ldots\ldots\ldots\ldots, X'_n)$ on the failure surface $g(\underline{X}) = 0$ to the origin of X' is

$$D = \sqrt{(X'^2_1 + X'^2_2 + \ldots\ldots X'^2_n)} \qquad (2.4.4)$$

The point on the failure surface, $(x'^*_1, x'^*_2, \ldots\ldots\ldots, x'^*_n)$, having the minimum distance to the origin may be determined by minimizing the function D, subject to the constraint $g(\underline{X}) = 0$. The minimum distance is obtained as [Fig. (2.7)]

$$\beta = \sum_i \alpha_i^* x'^*_i \qquad (2.4.5)$$

where

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial X'_i}\right)^*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)^{*2}}} \qquad (2.4.6)$$

are the direction cosines along X'_i evaluated at $\underline{x}'^*$

x'^*_i are the ith component of $\underline{x}'^*$

β is the reliability index.

If the random variables are all individually Gaussian, the probability of failure is

$$P_F = \Phi(-\beta) \qquad (2.4.7)$$

Where $\Phi(x)$ is the cumulative distribution function corresponding to the normal distribution [12].

In Eqns. (2.1.4, 2.1.5, 2.2.1, 2.3.1) it is assumed that all the required information is available which includes parameters such as means or medians and the variances or

COV's, as well as the form of the distribution of $\mathbf{f}_R(r)$ and $\mathbf{f}_S(s)$. In practice, none of this required information is available [13]. The parameters are estimated from required data and forms of PDF are inferred from similar data. However, the available data is not sufficient to correctly estimate the parameters. Therefore, error of estimation is unavoidable. Resistance R (or supply), and load S (or demand) are often functions of the respective resistance and load variables. For example

$$R = g(R_1, R_2, \ldots\ldots\ldots\ldots, R_n) \tag{2.4.8}$$

$$S = h(S_1, S_2, \ldots\ldots\ldots\ldots, S_n) \tag{2.4.9}$$

Available data is limited to the individual variable R_j and S_j . Theoretically all information about R and S can be obtained from these data including PDF by solving multiple integral. However, evaluation of such multiple integral is very difficult. Therefore, for many engineering purposes, approximation that is consistent with the state and quality of available information is used. For this purpose, the mean and variance (or COV's) of R and S are described approximately as a function of its constituent variables. The form of R_j and S_j may be prescribed judgementally taking into account any relevant physical or mathematical factors that are pertinent to a given problem.

For a set of prescribed form of R_j and S_j , the failure probability becomes largely a function of the central values and of dispersions of R and S as indicated before. The main problem in determination of P_F then is the determination of these central values and dispersions.

These central values and dispersions may be determined as functions of these constituent variables, through the function $R = g(R_1, R_2, \ldots\ldots\ldots\ldots, R_n)$ and $S = g(S_1, S_2, \ldots\ldots\ldots\ldots, S_n)$, respectively. The dispersions thus obtained represent the degree of uncertainty associated with inherent randomness.

In this process, there are bound to be errors associated with the imperfections of the estimation procedures, and thus additional uncertainty is introduced. Errors in the estimated dispersions is generally neglected as these are of secondary importance (relative to the importance in the errors in central values). The practical situation therefore, may be portrayed graphically as in Fig. (2.8). Fig. 2.8 (a) represents the inherent randomness of R and S as measured by the COV's δ_R and δ_S, with estimated mean values $\bar{r}$ and $\bar{s}$. Because $\bar{r}$ and $\bar{s}$ may contain random errors, they may be represented also as random variables $\overline{R}$ and $\overline{S}$, with the PDF's as in Fig. 2.8 (b). The COV's Δ_R and Δ_S represent the random errors in the estimated $\bar{r}$ and $\bar{s}$, whereas the systematic errors are $v_R = \mu_R/\bar{r}$ and $v_S = \mu_S/\bar{s}$, in which μ_R and μ_S are the correct mean values.

2.5. The Failure Mode Approach

The reliability of a multicomponent system is essentially a problem involving multiple modes of failure, i.e., the failure of different components, or different sets of components constitute distinct and different failure modes of the system. The consideration of multiple modes of failure, therefore, is fundamental to the problem of system reliability. The identification of the individual failure modes and the evaluation of the respective failure probabilities may be a problem in themselves. The occurrence of any one of the failure modes constitutes failure of the system. Failure is defined as exceeding a prescribed limit state of the system. If a performance function with k failure modes is defined as

$$g_j(\underline{X}) = g_j(X_1, X_2, \ldots\ldots\ldots\ldots\ldots, X_n); j = 1, 2, \ldots\ldots\ldots, k \qquad (2.5.1)$$

such that the individual failure events are

$$E_j = [g_j(\underline{X}) < 0] \qquad (2.5.2)$$

Then the compliments of E_j are the safe events; i.e.,

$$\overline{E}_j = [g_j(\underline{X}) > 0] \qquad (2.5.3.)$$

In case of two variables, the above events may be portrayed as in Fig. (2.9), in which three failure modes represented by the limit state equation as $g_j(\underline{X}) = 0$, j = 1, 2, 3.

The safety of the system is the event in which none of the potential failure modes occurs, i.e.

$$\overline{E} = \overline{E}_1 \cap \overline{E}_2 \cap \ldots\ldots\ldots\ldots \cap \overline{E}_k \qquad (2.5.4)$$

Conversely, the failure event would be

$$E = E_1 \cup E_2 \cup \ldots\ldots \cup E_k \tag{2.5.5}$$

Therefore, the failure of a system is the union of all the possible failure modes, and the system failure probability is

$$P_F = P(E_1 \cup E_2 \cup \ldots\ldots \cup E_k) \tag{2.5.6}$$

The evaluation of P_F in Eqn. (2.5.6) requires the identification of all the potential failure modes. For a system built of a number of discrete element components, the individual modes of failures will be functions of the failures of the components; whereas, in the case of a "continuous system" the failure modes will depend on the system properties.

The individual failure modes; $E_1, E_2, \ldots\ldots\ldots, E_k$ are generally correlated or statistically depended. For this reason, exact point estimate of P_F from Eqn. (2.5.6.) is seldom possible. In such cases lower and upper bounds of P_F are estimated [3]. The simplest bounds are those that are a function of single mode probabilities, namely

$$\max P(E_i) \leq P_F \leq 1 - \prod_i P(\overline{E}_i) \tag{2.5.7.}$$

The lower bound represents the system failure probabilities if all the failure modes are perfectly correlated, where as the upperbound would be the failure probability if all the failure modes are statistically independent.

2.6. Analysis of Uncertainty

The evaluation of safety and reliability requires information on uncertainty, as represented by the standard deviation or COV's as discussed before. The determination of these uncertainties constitute an essential task in the evaluation of structural reliability. The question of safety or reliability arise principally because of the presence of uncertainty. A basic model for representing the different types of uncertainty is discussed below [12].

2.6.1. Basic Model.

Suppose X is a single variable, which is the true state of nature and whose actual realization is unknown. Prediction or estimation of X is invariably necessary; for this purpose we introduce a predictive model $\hat{X}$. Since $\hat{X}$ will have imperfection, the resulting prediction, therefore, will contain errors and a correction N is introduced such that

$$X = N\hat{X} \tag{2.6.1.1}$$

If the state of nature is random, the model $\hat{X}$ should naturally be a random variable with mean and COV, $\overline{X}$ and δ_X respectively which represent the inherent variability.

The necessary correction N may also be considered a random variable, whose mean value v represents the mean correction for the systematic error in the predicted mean value $\overline{X}$, whereas the COV of N, Δ_X , represents the random error in the predicted mean value $\overline{X}$. Δ_X should include all the errors in X , including the error in the estimated δ_X , in addition to the error in the estimated mean value $\overline{X}$. However, within the 1st order approximation, Δ_X may be limited to the error in $\overline{X}$.

It is reasonable to assume that N and $\hat{X}$ are statistically independent. Therefore, we obtain

$$\mu_X = v\overline{X} \tag{2.6.1.2}$$

The total uncertainty in the prediction of X then becomes

$$\Omega_X = \sqrt{\delta_X^2 + \Delta_X^2} \tag{2.6.1.3}$$

In case of multivariable system

$$Y = g(X_1, X_2, ...,X_n) \tag{2.6.1.4}$$

the correct functional relation $g(X_1, X_2, ...,X_n)$ may not be known precisely. A model function $\hat{g}(X_1, X_2, ...,X_n)$ is introduced and a correction N_g may be necessary such that,

$$Y = N_g \hat{g}(X_1, X_2, \dots\dots\dots, X_n) \tag{2.6.1.5}$$

in which N_g has mean value v_g and COV Δ_g . On the basis of 1st order approximation, the mean value of Y is

$$\mu_Y \simeq v_g \hat{g}(\mu_{X1}, \mu_{X2}, \dots\dots\dots, \mu_{X3}) \tag{2.6.1.6}$$

where v_g is the bias in $\hat{g}(\mu_{X1}, \mu_{X2}, \dots\dots\dots, \mu_{X3})$ and

$$\mu_{X_i} = v_i \overline{X}_i \tag{2.6.1.7}$$

Also, the total COV of Y is

$$\Omega_y^2 \simeq \Delta_g^2 + \frac{1}{\mu_g^2} \sum_i \sum_j \rho_{ij} c_i c_j \sigma_{x_i} \sigma_{x_j} \tag{2.6.1.8}$$

in which

$c_i = \dfrac{\partial g}{\partial x_i}$, evaluated at $(\mu_{X1}, \mu_{X2}, \dots\dots\dots, \mu_{X3})$

ρ_{ij} = correlation coefficient between x_i and x_j

2.6.2. Ranges of Known Statistical Parameters for Prescribed Distributions

Depending on the range of possible values of random variable, the mean value of the variable and the underlying uncertainty may be evaluated by prescribing a suitable distribution within the range. If x_l and x_u are the lower and upper limit of a random variable X, then the mean and COV of X may be determined as follows (Fig. 2.10).

a) for uniform PDF :

$$\overline{X} = \frac{1}{2}(x_l + x_u) \tag{2.6.2.1}$$

$$\delta_X = \frac{1}{\sqrt{3}}\left(\frac{x_u - x_l}{x_u + x_l}\right) \tag{2.6.2.2}$$

b) For symmetrical triangular distribution :

$$\overline{X} = \frac{1}{2}(x_l + x_u) \tag{2.6.2.3}$$

$$\delta_X = \frac{1}{\sqrt{6}}\left(\frac{x_u - x_l}{x_u + x_l}\right) \tag{2.6.2.4}$$

c) For biased triangular distribution :

i.For bias towards the higher values :

$$\overline{X} = \frac{1}{3}(x_l + 2x_u) \tag{2.6.2.5}$$

$$\delta_X = \frac{1}{\sqrt{2}}\left(\frac{x_u - x_l}{2x_u + x_l}\right) \tag{2.6.2.6}$$

ii.For bias towards the lower values :

$$\overline{X} = \frac{1}{3}(2x_l + x_u) \tag{2.6.2.7}$$

$$\delta_X = \frac{1}{\sqrt{2}}\left(\frac{x_u - x_l}{x_u + 2x_l}\right) \tag{2.6.2.8}$$

d) Limits covering $\mp$ 2 standard deviations :

$$\overline{X} = \frac{1}{2}(x_l + x_u) \tag{2.6.2.9}$$

$$\delta_X = \frac{1}{2}\left(\frac{x_u - x_l}{x_u + x_l}\right) \tag{2.6.2.10}$$

Chapter 3. Failure Modes in Ship Structures

3.1. Overview

The design of any structure, as stated in chapter 2, involves consideration of the uncertainties that arise in regard to the external actions imposed on the structure as well as the strength and response properties of the structural elements. These different uncertainties can be taken into account by introducing probability concepts into the structural design procedure.

The basic theory tells us that if we can clearly and completely define a probability distribution for loads (demand) and strength (capability), it is possible to calculate the probability of failure or collapse. A design strength standard can then be established on the basis of an acceptable failure probability, which can be interpreted in terms of factor of safety or an allowable load factor.

When considering structural failure, all possible modes of failure which are mutually exclusive have to be considered [8]. The risk of failure is then the sum of the separate risks. Criteria have been established to determine whether the failure modes may be treated as statistically dependent or independent for bell-type distributions. Total ship loss may arise from two broad damage mechanisms affecting a large and critical region of the cross section.

A. Fracture

  i. Brittle fracture

  ii. Fatigue

B. Ductile

  i. Tension yield

  ii. Compression plastic instability

  iii. Incremental collapse

A brief description of above mentioned failure modes are given below.

3.2. Fracture Modes

Brittle fracture is very difficult to deal with because of large uncertainties associated with the material quality (notch - toughness in relation to temperature). When brittle fracture occurs, there is no appreciable elongation, and experience showed that if a crack was initiated in a structure such as, say, the deck of a ship, this crack could be propagated through the structure and may cause ultimate failure. In welded ships, because of the continuity of the structure, there is no barrier to stop a crack which is initiated travelling right round the ship. Brittle fracture of a material such as mild steel is dependent to a great extent on the temperature, there being a certain temperature above which it will not occur. One of the ways of achieving this temperature is to keep a certain minimum manganese-carbon ratio, and such steel are often referred to as notch-tough steel. Brittle mode of failure has been brought under control on the basis of improved design details and workmanship and use of notch-tough strakes as crack stoppers to provide "fail safe" margin.

Transverse fatigue cracks do occur in ships [9]. By themselves, they do not constitute a serious risk in ship loss terms, unless the condition for the spontaneous fracture are also present. Otherwise, the propagation of fatigue cracks will be slow and should be detectable for repair long before propagation by weakened cross section can develop. Fatigue failure is associated with the total history of the cyclic wave loads (the low and high frequency component) rather than just one or a few extreme values. Such repeated loads are random in nature and some consideration must be given to this as it will affect the fatigue life.

3.3. Ductile Modes

Incremental collapse can be dismissed from formal considerations of risk of failure. This phenomenon has been recognized long time back and methods for calculating the shakedown limit (M_s) have been established for simple beam structures [9]. If the bending moment exceeds the critical limit M_s then the structure never shakes down. A shakedown state is achieved in a structure subjected to repeated loadings within prescribed limits if it always responds elastically after some initial plastic flow [15]. If a shakedown state is not achieved, then failure may occur due to alternating yield or incremental collapse. The shakedown limit is always lower or at most equal to the static collapse or ultimate load when the influence of buckling is disregarded. Thus it would appear desirable that the shakedown limit should be used as a basis of failure evaluation rather than the ultimate strength.

From the view point of longitudinal strength calculations, a ship or marine vehicle can be idealized as a beam [15]. In this case, it is more convenient to formulate the shakedown theorem in terms of bending moment. Moment curvature relationship of a beam is shown in fig(3.1) [14]. In this relation it is assumed that the yield and fully plastic moment M_y and M_p are of the same magnitude for flexure in either sense. Thus an initially stress free beam behaves elastically along 'oa' or 'oe' and with further loading the slope of the bending moment - curvature relation is progressively reduced along 'ab' or 'ef' as the bending moment tends to M_p in magnitude. The elastic range of bending moment for initially stress free beam is 2 M_y and it is assumed that the elastic range remains constant regardless of the history of loading. For example, if bending moment is reduced after loading along 'oab', the relation 'bcd' is followed, where the elastic range of moment along 'bc' is still 2 M_y.

For this type of bending moment - curvature relation the condition for shakedown to occur are as follows.

To prevent alternating plasticity

$$M_{\max} - M_{\min} \leq 2M_y \tag{3.3.1}$$

and to prevent incremental collapse

$$M_s + M_{\max} \leq M_p \tag{3.3.2}$$

$$M_s + M_{\min} \geq - M_p \tag{3.3.3}$$

where M_s is the still water bending moment, $M_{\max}$ is the maximum hogging (sagging) wave bending moment, $M_{\min}$ is the minimum sagging (hogging) wave bending moment.

It is evident from Eqn. (3.3.1) that if the moments vary between equal positive and negative values, then maximum bending moment must be less then or equal to the initial bending moment M_y .

Deflection of the structure increases in each cycle due to progressive yielding. If a sufficiently large number of cycles of this load occurs, unacceptably large deflection would build up rendering the structure useless. The structure would then be said to have failed by incremental collapse. It would strictly be considered in any plastic design procedure and in risk of failure calculations. In ship bending there is no prior knowledge of the sequence of loading, which is therefore termed variable repeated loading. Fortunately, however, it can be shown that failure by incremental collapse are usually far less likely to occur in practical structures than failure by plastic collapse (tension yield and compression instability). This is especially so in longitudinally framed ships, where the shakedown load M_s is only slightly less than M_u (ultimate moment), and where the loading is approximately exponential so that the number of load excursions higher than M_s may be expected to be very small anyway.

It is therefore only necessary to provide adequate safeguard against the occurrence of plastic collapse in order that the safety of the structure in respect of incremental collapse should also be ensured.

Plastic collapse will also involve both tension and compression actions, and to this extent they are not strictly independent modes of collapse. Indeed, allowing for the ability for the box beam to redistribute stresses in the cross-section due to changes in stiffness, resulting from plate deformation or from plasticity or both, it can be shown that the ultimate moment of the cross section is always dominated by the problem of compression instability. Due to this, intensive efforts are often directed towards a better understanding of the compression behavior of plates of welded stiffened panels and of plated grillages and efforts have recently been directed towards the statistics of compression strength.

3.4. Failure Due to Yielding and Plastic Flow of the Hull

If the critical buckling stresses of deck grillage and columns consisting of stiffeners and effective plating are greater than the yield stress of the material, the following equations should be utilized for estimating the ship hull girder strength [7].

3.4.1. Initial Yield Moment

It is assumed that the ultimate condition is reached when the entire deck or bottom has reached the yield state, i.e.,

$$M_y = Z_e \sigma_y \qquad (3.4.1.1)$$

where

M_y = initial yield moment

Z_e = conventional elastic section modulus at deck or bottom

σ_y = tensile yield strength of the material

3.4.2. Fully Plastic Collapse Moment

Here it is assumed that, when the entire cross section (including hull sides) has reached the yield state (assuming plastic material), then the ultimate condition is reached [19], .i.e.,

$$M_p = Z_p \sigma_y \tag{3.4.2.1}$$

where

M_p = fully plastic moment

Z_p = plastic section modulus

$$= A_D g + 2A_S\left(\frac{D}{2} - g + \frac{g^2}{D}\right) + A_B(D - g)$$

where

A_D = total effective cross section area of the deck including stiffeners

A_B, A_S = total effective area of bottom and one side respectively

D = depth of the section

g = distance from the center of the deck area to the plastic neutral axis

$$\frac{g}{D} = \frac{A_B + 2A_S - A_D}{4A_S}$$

3.4.3. Effective Plastic Collapse Moment

Here only the effective area of the compressed parts of the hull is used in calculating the plastic moment, i.e., an allowance is made for the possibility of buckling of plating between stiffeners under the compressive loads. If buckling of plates between stiffeners occur in the compressed parts of the hull, the effective plastic section modulus is less than that given in (3.4.2.). In this case the requirement of zero axial force provide the condition for location of neutral axis , i.e.,

$$F_X = 0 \qquad \text{or} \qquad \int_A \sigma_y \mathrm{dA} = 0 \tag{3.4.3.1}$$

where dA represents a small portion of effective area. This condition implies that the total effective area under compression and that under tension are equal. All longitudinal stiffeners and plating under tension are considered to be fully effective. The effective plating moment is then obtained from

$$M_{Pe} = \sigma_y \sum_i (\Delta A_e)_i d_i \tag{3.4.3.2}$$

where

$(\Delta A_e)_i$ = small portion of the effective area on the cross section

d_i = distance from the final NA to the centroid of $(\Delta A_e)_i$

Chapter 4. Uncertainties in Ship Longitudinal Strength

When considering ship's longitudinal strength and requirements for adequate structural design, the load acting on the ship represent the "demand" and the ship structural strength represents "capability" of the structure. Uncertainties associated with demand and capability which affect the ship structural design are discussed below.

4.1. Uncertainties in demand

With reference to longitudinal hull bending the principal loads acting on a ship's hull may be summarized as follows [10];

- static bending moment resulting from uneven distribution of weights and buoyancy in still water.
- static bending moment caused by the waves generated by the ship's forward motion in calm water.
- quasi-static or low frequency bending moment caused by relatively

long encountered waves

- dynamic (vibratory) bending moment caused by wave - hull impacts or high frequency wave forces.
- thermal loads resulting from uneven temperature gradients.

Of all of the above loads, the one receiving the most attention has been the quasi-static wave bending moment. Waves causing such bending moments could only be understood and described by statistics and probability theory. A specific sea condition can be described by its directional spectrum, defining the component wave frequencies and directions present. Uncertainties arise from:

- variability in directional properties of the wave spectra, with only limited data available.
- combined effect of two storms, or sea and swell.
- variability of spectral shapes for a given significant height (considerable data is available for limited ocean areas, but more data are needed).
- possibility of "freak" waves, usually as a result of effects of shoaling water nearby coasts, currents, etc.

4.2. Uncertainties in capability

Because of the limitations on the control of the properties of steel and other materials used in shipbuilding and because of limitation on production and fabrication procedures of ship components, the strength of apparently identical ships will not be, in general, identical. In addition uncertainties associated with residual stresses arising from welding, the presence of small holes, etc., may also,affect the strength of the ship. These limitations and uncertainties indicate that a certain variability in strength about some mean value will result. This will in turn introduce an element of uncertainty as to what is the actual strength of the ship that should be compared with the estimated applied load.

Additional uncertainties in the strength will arise due to uncertainties associated with the assumptions and methods of analysis used to calculate that strength. Further uncertainties are associated with numerical errors in the analysis. These errors may accumulate in one direction or possibly tend to cancel each other. Time dependents effects, such as deterioration due to corrosion, cracking, wear and tear, and variability in thermal stresses may also affect the strength.

The variation or "imperfections" will occur throughout a given structure as well as from one structure to another, and may be regarded as error in the mathematical sense [8]. These errors may be divided into two categories, systematic and random. Systematic errors affect the expectation whereas random errors influence the dispersion.

Another convenience is Ang's division of random uncertainties into "objective" and "subjective" [7].

4.3. Objective Uncertainties

These are uncertainties associated with random variables for which statistical data can be collected and examined. They can be quantified by coefficients of variation(COV), derived from available statistical information. The main objective uncertainties are:

i. uncertainties associated with the main dimensions of the hull such as beam and depth of the cross sections and height of different decks

ii. uncertainties associated with the material yield strength and Young's modulus of elasticity of different component of the section such as plates, girders, and stiffeners.

iii. uncertainties associated with the distribution of residual stresses due to welding.

iv. uncertainties associated with manufacturing imperfections, flaws, plate fairness, etc.

v. uncertainties associated with scantlings of components such as plate thickness, stiffeners, girders, and face plate dimensions.

4.4. Subjective uncertainties

These are uncertainties associated with the lack of information and knowledge. They can be determined only on the basis of previous experience. The following subjective uncertainties cause a variability in the moment that may cause failure by yielding or buckling:

i. uncertainties associated with the degree of effectiveness of the plating due to shear lag effects.

ii. uncertainties associated with the usual Navier hypothesis that plane sections remain plane and perpendicular to the neutral axis.

iii. uncertainties related to the presence of major discontinuities; openings, superstructures, etc.

iv. uncertainties associated with the residual strength after buckling and the effect of the initial deformations on the buckling loads.

vi. uncertainties associated with compression nonlinearities; effective width, inelasticity, residual stresses and shakedown effect.

4.5. Evaluation of the Factors Affecting Hull Strength

Neglect of local peak stresses arising from shear lag is of no consequence in tension due to plastic redistribution [8]. Even in compression some stress redistribution will occur before failure due to changes in stiffness. There is also a tendency for deep longitudinal structure where peak stresses occur to be stronger than average. Hence, although elastic measurements in ships do exhibit appreciable shear lag, there seem sound reasons, despite the apparent optimism, for neglecting shear effects when evaluating ductile strength.

Major discontinuities, on the other hand, can raise serious uncertainties. Little data exist, and no general rule apply other than to recommend ignoring the area of the openings and improving inelastic buckling strength locally by using higher yield strength plating and closer stiffening.

Residual strength has been studied for ships. Assumptions suppose that once a critical gross panel has failed the remaining cross section will be unable to sustain any load shedding. Disregarding residual strength may be pessimistic when tension dominates.

Neglect of inelasticity and residual stress effects, including shakeout at sea, introduces a minor pessimism. Both effects are reasonably well allowed for the gross panel strength modelling as effective width effects.

Total uncertainties is added statistically assuming the two groups to be independent random variables so that

$$\delta^2 = \delta_{ob}^2 + \delta_{sb}^2 \tag{4.5.1}$$

where δ_{ob} and δ_{sb} are the objective and subjective COV's respectively. This division into objective and subjective was also found convenient when considering systematic uncertainties. For example, the systematic errors introduced by nominal yield stress and plate thickness can be assessed objectively from measured histogram and these estimates should be used in design.

Errors arising from time dependent effects may be very important, especially where fracture modes of failure are likely. Corrosion certainly warrants further attention, and midlife estimate must be made for cross-section scantlings at the very least, when evaluating full risk failure.

The uncertainties associated with variation in material properties, dimensions of structural elements, corrosion, etc. are among the objective uncertainties that can be determined from an established representation of the ship strength. Let the strength R (or the ultimate bending moment) be a function of several parameters Y_i and denoted by

$$R = f(Y_1, Y_2, \ldots\ldots\ldots\ldots, Y_n) \tag{4.5.2}$$

It will be assumed that the function and all their derivatives are continuous. It is required to find the probability density function of R, which is the frequency distribution for any given strength level. This could be directly evaluated by generating functions or by convolution methods, but this requires solving multiple integrals. An approximate method which is commonly adopted makes use of the Taylor's series, expanded preferably about the mean values of Y_n . Thus,

$$R = R(\overline{Y}_1, \overline{Y}_2, ..., \overline{Y}_n) + \sum_{i=1}^{n} \left(\frac{\partial R}{\partial Y_i} \right)_{Y_i = \overline{Y}_i} (Y_i - \overline{Y}_i) \tag{4.5.3}$$

where second and higher order terms are dropped since it is assumed that all values of Y_i are near $\overline{Y}_i$. The moments of S give the mean (μ_R) and standard deviation (σ_R) [7]:

$$\mu_R \simeq f(\overline{Y}_1, \overline{Y}_2, ...\overline{Y}_n) \tag{4.5.4}$$

$$\sigma_R^2 = \sum_{i=1}^{n} \left(\frac{\partial R}{\partial Y_i} \right)^2 {\sigma_{Y_i}}^2 \tag{4.5.5}$$

where $\overline{Y}_i$ and $\sigma^2_{Y_i}$ are the mean and variance of the random variable effecting the ship strength. The partial derivatives in Eqn. (4.5.5) should be evaluated at the mean value $\overline{Y}_i$. In terms of objective COV, Eqn. (4.5.5) may be written as

$$\delta_R^2 = \left(\frac{\sigma_R}{\mu_R} \right)^2 = \sum_{i=1}^{n} \left(\frac{\partial R}{\partial Y_i} \frac{\overline{Y_i}}{\mu_R} \right) \delta_{Y_i}^2 \tag{4.5.6}$$

where δ_R is the strength objective COV and δ_{Y_i} are the COV of Y_i.

Therefore, as an approximation the mean and variance of a function can easily be found from the mean and variance of the variable Y_i, provided these variables are independent and their second order partial derivatives are small [8].

It should be noted that the derivatives which lead to Eqns. (4.5.3 - 4.5.5) do not require the knowledge of the probability density function of Y_i, nor is the PDF of R known even those for Y_i are known. This additional information require less elementary probability technique. If the variables Y_i are not statistically independent, then the theory of correlated errors should be applied. Weak dependencies are ignored in the interest of simplicity.

The procedure described above is the basis for determining the COV values for the different possible modes of ship structural failure described in chapter 3. This analysis was used to determine the objective uncertainty considering initial yield moment as the failure mode [7]. A brief description of the procedure is given below.

If the strength is defined as

$$R = Z_e \sigma_y \tag{4.5.7}$$

where

Z_e = conventional elastic section modulus

σ_y = material tensile yield stress

Making use of Eqn. (4.5.6), we get

$$\delta_R^2 = \left[\frac{\overline{Z}_e \overline{S}_y}{\mu_s} \right]^2 \delta_{Z_e}^2 + \left[\frac{\overline{Z}_e \overline{S}_y}{\mu_s} \right]^2 \delta_{\sigma_y}^2 \tag{4.5.8}$$

Since $\mu_R = \overline{Z}_e \overline{\sigma}_y$ = mean of the strength; we get

$$\delta_R{}^2 = \delta_{Z_e}{}^2 + \delta_{\sigma_y}{}^2 \tag{4.5.9}$$

Depending on the expression for Z_e , δ_{Z_e} can be calculated. For a symmetric section, Z_e can be written as [7]

$$Z_e = \frac{D}{2} \left(t_f B + \frac{1}{3} t_s D \right) \tag{4.5.10}$$

where

D = depth of the section

B = breadth of the section

t_f = flange thickness

t_s = side thickness

assuming that COV's of flange thickness and side thickness are same, i.e., $\delta_{t_f} = \delta_{t_s} = \delta_t$, we get

$$\delta_{Z_e}^2 = \left[\frac{\gamma + \frac{2}{3}}{\gamma + \frac{1}{3}} \right]^2 \delta_D^2 + \left[\frac{\gamma}{\gamma + \frac{1}{3}} \right]^2 \delta_B^2 + \left[\frac{\gamma^2 + \frac{1}{9}}{(\gamma + \frac{1}{3})^2} \right]^2 \delta_t^2 \tag{4.5.11}$$

where δ_B , δ_D and δ_t are COV's of depth, breadth and plate thickness respectively.

$$\gamma = \frac{\overline{A}_f}{\overline{A}_s}$$

where

$\overline{A}_f$ = is average area of flange

$\overline{A}_s$ = is average area of side

When considering the cases corresponding to failure arising as a result of buckling, the strength mode is given by

$$R = Z_e \sigma_{Y_c} \phi \tag{4.5.12}$$

where

σ_{Y_c} = compression yield strength of the material

ϕ = failure stress ratio

$= \dfrac{\text{(average failure stress)}}{\text{(yield strength} \sigma_{Y_c})}$

ϕ depends on the effectiveness of the plating after buckling and can also be written as

$$\phi = \frac{b_e}{b} \tag{4.5.13}$$

where

b_e = effective width

b = actual width

then strength COV is given as

$$\delta_R^2 = \delta_{Z_e}^2 + \delta_{\sigma_{Y_c}}^2 + \delta_\phi^2 \tag{4.5.14}$$

where δ_ϕ is the COV of ϕ.

The COV of ϕ is dependent on the variability of several parameters such as the plate thickness t, the breadth b, the modulus E or E_t, residual stresses and corrosion. We can write

$$\delta_\phi^2 = \delta_{\phi_1}^2 + \delta_{\phi_2}^2 + \delta_{\phi_3}^2 \tag{4.5.15}$$

in which

δ_{ϕ_1} - is the COV associated with variability in plate thickness, breadth and corrosion

δ_{ϕ_2} - is the COV associated with residual stress

δ_{ϕ_3} - is the COV associated E or E_t

4.6. Sensitivity Analysis

An advanced computational procedure for determining the ultimate strength of a ship hull from the point view of compressive failure is given by the work of Smith [14]. The particular analysis is based upon a finite element approach which also allows the consideration of residual stresses and initial deformations. However, this method requires extensive computational effort, so that applying such a procedure to obtain results for a large set of parametric variations in ship section properties would not be practical.

A similar but simpler procedure was developed by Adamchak [11], known as the computer program ULTSTR. This program can be used to estimate the collapse moment of a ship hull under longitudinal bending. Since the ULTSTR program can provide the values of the collapse moment or ultimate strength of a ship with small computational effort, it can be used effectively and efficiently to determine the changes in strength associated with variation in parameters characterizing the ship section. This was suggested by Kaplan [10]. The parametric variation results allow simpler linear determination of changes in ship strength associated with such changes in basic parameters. Parametric changes considered for this study are yield stress, Young's modulus, thickness, imperfections, breadth, and depth.

4.6.1. Program ULTSTR

This program was developed to predict the approximate collapse strength of the hull girder in ductile failure under longitudinal bending. Section yielding, interframe Euler beam-column buckling, and interframe stiffener tripping are included as the probable ductile modes of failure. The program also accounts for the effect of material having different yield strength in plating and stiffeners, for lateral pressure loading, and for initial out of plane distortion due to fabrication.

In this program, it is assumed that collapse results from a sequence of failure of local components rather than from an overall simultaneous instability of the complete cross section. This allows one to address the collapse behavior of the hull by concentrating on the collapse behavior of the local components that make up the cross section, where such components are represented as a single plate-beam combination, an individual gross panel or a complete cross stiffened grillage. This is very convenient, since the collapse behavior of the local components is technically tractable, although to varying degrees. This does not imply that the collection of solutions for the collapse of local components is absolutely comprehensive and totally consistent, but rather than their behavior is understood well enough to allow development of a collapse model which provides the speed and accuracy needed in preliminary design and which can be utilized in a practical and meaningful fashion to address the major structural considerations. On the other hand, treating this problem as an overall instability of the complete cross section presents a major practical obstacle. Numerically this problem can be treated using finite element methods but due to size and complexity of the mathematical model, it is beyond the practical limits of time, cost and capacity of today's computing system. In the majority of cases ductile hull collapse is due to a sequence of local component failures rather than a simultaneous occurrence. The most probable ductile failure modes are primarily local phenomena in which there is relatively little direct influence from other major component of the cross-section.

The actual solution approach involves dividing the hull cross section into a set of "gross panel" element and "hard corner" elements and then imposing a curvature on the hull in small finite increments Fig. (4.1). Each increment of curvature is assumed to produce a linear strain distribution through the depth of the cross section. The location of zero strain corresponds to what is referred to as the "instantaneous" or "incremental" neutral axis. The assumptions of linear strain through the cross section is common practice in naval architecture and is certainly "sufficiently" valid for a stress levels at or below the so called design values. This is also consistent with the degree of engineering accuracy expected and with the approximations that of necessity have been made concerning other aspects of the program.

At each value of curvature, the program evaluates the equilibrium state of each gross panel and hard corner element relative to its state of stress and stability corresponding to its particular value of strain. It then computes the total moments on the cross section by summing the moment contributions (stress x effective area x lever arm) of all the elements that make up the section. In this manner a moment curvature relationship is defined. Since the stress distribution, unlike that of strain, is not necessarily linear across the depth of the section, the location of the instantaneous neutral axis must be determined in an iterative fashion from the condition that the net force on the cross section must be zero. This force is computed in the same fashion as bending moment,i.e., by summing the contribution of the instantaneous neutral axis from increment to increment. The cumulative strain distribution that results is still linear through the depth of the cross section, since it represents the superposition of number of linear increments.

Gross panel elements in the cross section can fail either through material yielding (in either tension or compression) or through some form of structural instability (in compression only). The instability failure modes presently incorporated include :

i. Euler beam-column buckling
ii. Stiffener lateral-torsional buckling

Although it is impossible to determine what failure mode may be most critical for a particular gross panel element, it is assumed that once instability is detected in a given mode, the behavior follows through to failure in that same mode. Interaction among failure mode has been ignored in this program.

Regardless of the specific type of failure involved, the general nature of an element's behavior can be described in terms of a "load shortening" curve. This curve has three distinct zones of behavior. The first represents a zone of stable behavior in which the load applied to the element is less than the critical value corresponding to its preferred mode of failure. Since load dependent effectiveness relationship is utilized in this program for gross panel elements, the curve in this range will not generally be absolutely linear for such elements, although the deviation from linearity in this region will need careful scrutiny for detection. The second zone, or plateau, occurs after an element has reached its critical load. On this plateau, the element will continue to deform without any increase in loading. This critical load may correspond to one of the possible forms of buckling or to the condition of material yielding. As the Fig. (4.2) indicates, some elements have no third zone of behavior but remain indefinitely in the second zone after reaching their critical load. The third and final zone is characterized by a drop off in the element's load carrying capability as deformations increases. This necessity for reducing load to maintain equilibrium is called "unloading". This can significantly affect the behavior of the overall hull cross section.

The effect of including unloading in element behavior can be seen in Fig. (4.3) which shows a typical moment-curvature diagram for a complete hull cross-section. Fig. (4.3) shows that including unloading in element behavior will lead to a similar type of behavior in the composite cross section. In fact, to get unloading in the composite cross section requires unloading in the element; including only the first two types of behavior in the element will lead to a moment-curvature diagram for the cross section that asymptotically approaches limits for the moments. The critical issue, however, is whether the peaks differ significantly from the moment asymptotes. At present there is not enough experience with this subject to resolve this issue.

4.6.2. Sensitivity Calculations

Two tankers were chosen for which a sensitivity study was performed. Six parameters were considered for the sensitivity study i.e., yield stress, Young's modulus, plating thickness, initial imperfections, breadth, and depth.

The way to obtain sensitivity parameters was rather straight forward [4]. The sensitivity can be measured by means of a sensitivity parameter which corresponds to a nondimensionalized partial derivative:

$$\alpha_i = \left[\frac{M_u(p_1, p_2, \ldots\ldots\ldots, p_i + \Delta p_i, \ldots\ldots\ldots, p_n) - M_u(p_1, p_2, \ldots\ldots\ldots, p_i, \ldots\ldots\ldots, p_n)}{\Delta p_i} \right]$$

$$* \frac{p_i}{M_u(p_1, p_2, \ldots\ldots\ldots, p_n)} \qquad (4.6.2.1)$$

evaluated at the point $p = (p_1, p_2, \ldots\ldots\ldots, p_n)$. M_u is the ultimate longitudinal moment. This definition of sensitivity is used in reliability analysis. In practice we calculate the ultimate strength of the ship using ULTSTR program, then change the value of one selected parameter and calculate the ultimate strength with ULTSTR again. Even though many calculations are required in the process, this program takes very little computer time and cost. For the purpose of this study, the following values of the four parameters were selected to correspond to the basic point at which the sensitivity is evaluated:

yield stress σ_y = 242.5 MPa (35 200 psi)

Young's modulus E = 2.07 X 10^5 MPa (30 X 10^6 psi)

plating thickness = actual plating thickness

imperfection amplitude = 10 mm (0.4 inch)

These values correspond to the statistical means of the variables, except thickness. The same basic parameters were chosen for the ABS study also.

The following values of Δp were chosen for our study:

yield stress σ_y	=	3.0 MPa
Young's modulus E	=	0.02 X 10^5 MPa
plating thickness	=	1% of t
imperfection amplitude	=	2 mm
breadth	=	1% of B
depth	=	1% of D

Main parameters of the tankers are given in Table 1. Midship section drawings are shown in Fig. (4.4) and Fig. (4.5). The number of gross panel elements and hard corners are given in Table 2.

The material in case of both the ships is steel with a yield stress of 221 MPa (32 ksi), Young's modulus of 2.07 X 10^5 MPa (30 X 10^6 psi), Poisson's ratio of 0.3, and a specific gravity of 7.85 .

The range of parametric variation is given in Table 3.

Chapter 5.Result and Discussions

5.1. Overview

The sensitivity study was carried out for two tankers. Tanker1 is the same for which a sensitivity analysis was carried out by American Bureau of Shipping using their nonlinear finite element program USAS. The second tanker (referred as Tanker2) was taken from Ref. [8]. Calculations are carried out for different values of yield stress, Young's modulus, thickness, initial imperfection, breadth and depth.

5.2. Numerical Results

Each run with ULTSTR took about 30 to 45 seconds of execution time on IBM 3090 compared to 6 hour CPU time on IBM 4341 taken by USAS for a typical run [4].

The values of ultimate moment obtained by ULTSTR were about 15 - 18 % less than that obtained by USAS. In ULTSTR the limiting moment is reached when the stress at either top deck or bottom of the hull reaches yield stress. In such case the plates in the intermediate decks, the inner bottom, the longitudinal bulkhead, and the side shell most likely do not reach the yield stress and as a result it can be expected that the hull can be loaded beyond this point until it finally collapses. Hence the limiting moment obtained by ULTSTR are always below the USAS value. A simplified program similar to ULTSTR was developed by Billingsley [18], called FLEXM. Ultimate moment obtained by this program was about 12 - 14 % less than that obtained by USAS as indicated in ref. [4].

Sensitivity values obtained for both the tankers are given in Table 4 (for sagging) and Table 5 (for hogging).

The high sensitivity of tankers due to variation in yield stress is most probably due to the fact that the failure mode of the longitudinally framed tanker is dominated by plasticity. The values obtained by ULTSTR are quite comparable to that obtained by USAS.

The influence of variation in Young's modulus on the ultimate strength should not be very high. However, with ULTSTR we get the derivative as 0.16 compared to 0.04 obtained by USAS. This difference is mainly due to consideration of the tangent modulus concept in the computation of effective width in the program ULTSTR. Effective width relationship plays an important role throughout the collapse mode theories.

High sensitivity of longitudinal strength due to variation in thickness is due to direct effect of it on the section modulus. It should be noted that for these calculations the thickness was varied uniformly for all the elements of the ship hulls, including both the plates and beams.

The derivative of ultimate moment with respect to the amplitude of initial imperfection is very small. The reduction in ultimate moment is very small, but we must realize that this variation represent only a single ray in the multidimensional space of all the imperfections from the origin (which corresponds to the perfect structure).

The derivatives of ultimate moment due to variation in breadth and depth were also studied. It can be seen that the collapse moment is much more sensitive to depth than breadth. This is expected since the dependence of section modulus on depth is of second order whereas that on breadth is linear. This difference in derivatives in the two tankers is mainly due to the difference in scantlings in the midship sections. The same analysis was done for the hogging condition and similar results were obtained (see Table 5).

Coefficient of variations (COV) of ultimate longitudinal strength due to subjective uncertainties is obtained using Eqn. (4.5.6). This study was not carried out by ABS. It is assumed that all the variables are independent. The effect of initial imperfection was neglected due to very low sensitivity of the ultimate strength. The following COV values are used for the variables listed below, as given in [8] and [10]:

yield stress .... 7 %

Young's modulus .... 2 %

thickness .... 4 %

B, D .... 0.2 %

The affect of parametric variations from the basic values (yield's stress of 242.5 MPa, Young's modulus of 2.07 X 10^5 MPa, actual plating thickness and imperfection amplitude of 10mm)on the ultimate longitudinal strength is shown in Fig. 5.1 to Fig.

5.24. The ratio of ultimate moments to basic ultimate moment (which corresponds to basic values of parameters) have been plotted against yield's stress/ basic yield stress, Young's modulus/basic Young's modulus, thickness multiplier (thickness/basic thickness), imperfection amplitude/basic imperfection amplitude, breadth multiplier and depth multiplier.

The slopes of these plots represent the nondimensionalized sensitivity given in Table 4 and Table 5. It can be observed that the nature of all the plots are linear within the range of parametric variation considered for this study. Range of these variables listed in Table 3 cover the COV values obtained for these variables from statistical analysis.

The COV's of ultimate longitudinal strength due to the above mentioned five variables are given in Table 6. We can observe that the COV values are quite consistent for both the tankers for both sagging and hogging condition.

The COV of strength due to objective uncertainties for Tanker 2 was also calculated by Mansour and Faulkner [8]. They obtained COV of 8.6% for both sagging and hogging conditions. However, they considered COV of σ_y as 8 % compared to 7 % considered by us. In our calculations, if we use COV of 8 % for σ_y, the COV of strength for Tanker 2 due to objective uncertainties is obtained as 8.7% for both sagging and hogging conditions.

Chapter 6. Conclusion

At the preliminary design stage, since we cannot afford to expend great effort, we can use program ULTSTR to calculate the ultimate moment. This program evaluates the ultimate moment with very low computational effort. Results obtained with this program are in generally good agreement with those obtained by the much more advanced and expensive program effort like USAS.

We can also see that the yield stress variability dominates all the objective uncertainties considered for tankers. Variability of plating thickness is of intermediate importance where as the effect of Young's modulus, breadth and depth can be ignored.

The COV values of ultimate longitudinal strength due to five objective uncertainties seem to be quite reasonable for tankers. This COV values can be used for reliability analysis.

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TABLE 1. Main Particulars of Tankers

	Tanker1	Tanker2
Length	230.00 m	236.00 m (775.00 ft)
Breadth	38.10 m	32.16 m (105.15 ft)
Depth	16.60 m	19.05 m (62.50 ft)
Draft	12.60 m	14.33 m (47.00 ft)
Displacement	91,000.00 tons	91,200.00 tons
Web frame spacing	4.115 m	3.12 m (10.25 ft)
Framing	Longitudinal	Longitudinal

TABLE 2. Gross and Hard Corner Panels

	Tanker1	Tanker2
Gross Panel Elements	67	58
Hard Corner Elements	10	10
No. of Stiffeners	22	10

TABLE 3. Range of Parametric Variation

	Tanker1	Tanker2
Yield stress σ_y	221.00 - 260.00 MPa	30.0 X 10^3 - 40.0 X 10^3 psi
Young's Modulus E	1.93 X 10^5 - 2.21 X 10^5 MPa	28.0 X 10^6 - 32.0 X 10^6 psi
Thickness t	0.85t - 1.15t	0.85t - 1.15t
Initial imperfection	0.00 - 20.00 mm	0.0 - 2.0 in
Breadth	36.20 - 40.00 m	100.23 - 110.78 ft.
Depth	15.77 - 17.43 m	59.38 - 65.53 ft.

TABLE 4. Sensitivity Values (Sagging)

(α_i in Eqn. 4.6.2.1)

	Tanker1		Tanker2
	ULTSTR	USAS(ABS)	
σ_y	0.85	0.98	0.88
E	0.16	0.04	0.13
t	1.22	1.01	1.16
Imperfection	-0.007	-0.003	-0.008
B	0.483	-	0.645
D	0.99	-	1.14

TABLE 5. Sensitivity Values (Hogging)

(α_i in Eqn. 4.6.2.1)

	Tanker1	Tanker2
σ_y	0.84	0.95
E	0.23	0.0541
t	1.21	1.01
Imperfection	-0.002	-0.002
B	0.360	0.567
D	1.05	1.19

TABLE 6. COV values of Ultimate Longitudinal Strength

	Tanker1	Tanker2
Sagging	7.87 %	7.89 %
Hogging	7.84 %	7.94 %

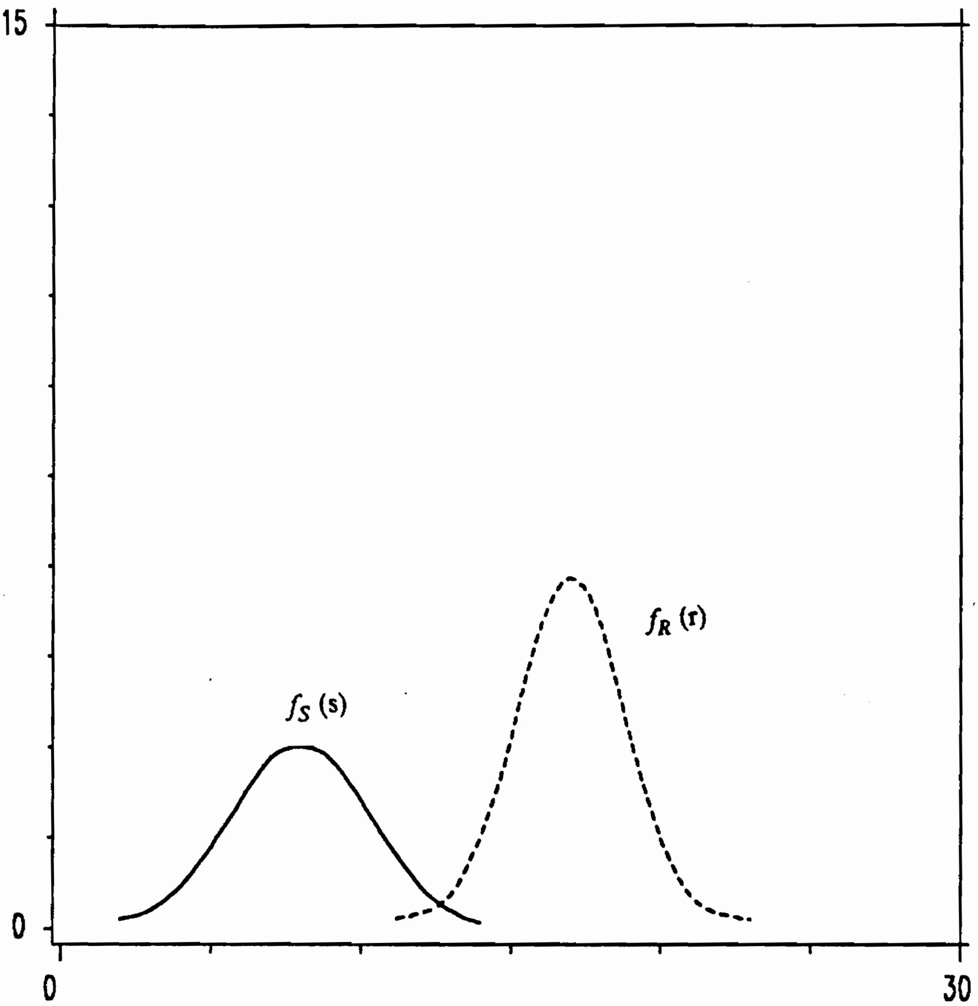


Figure 2.1. PDF for R and S

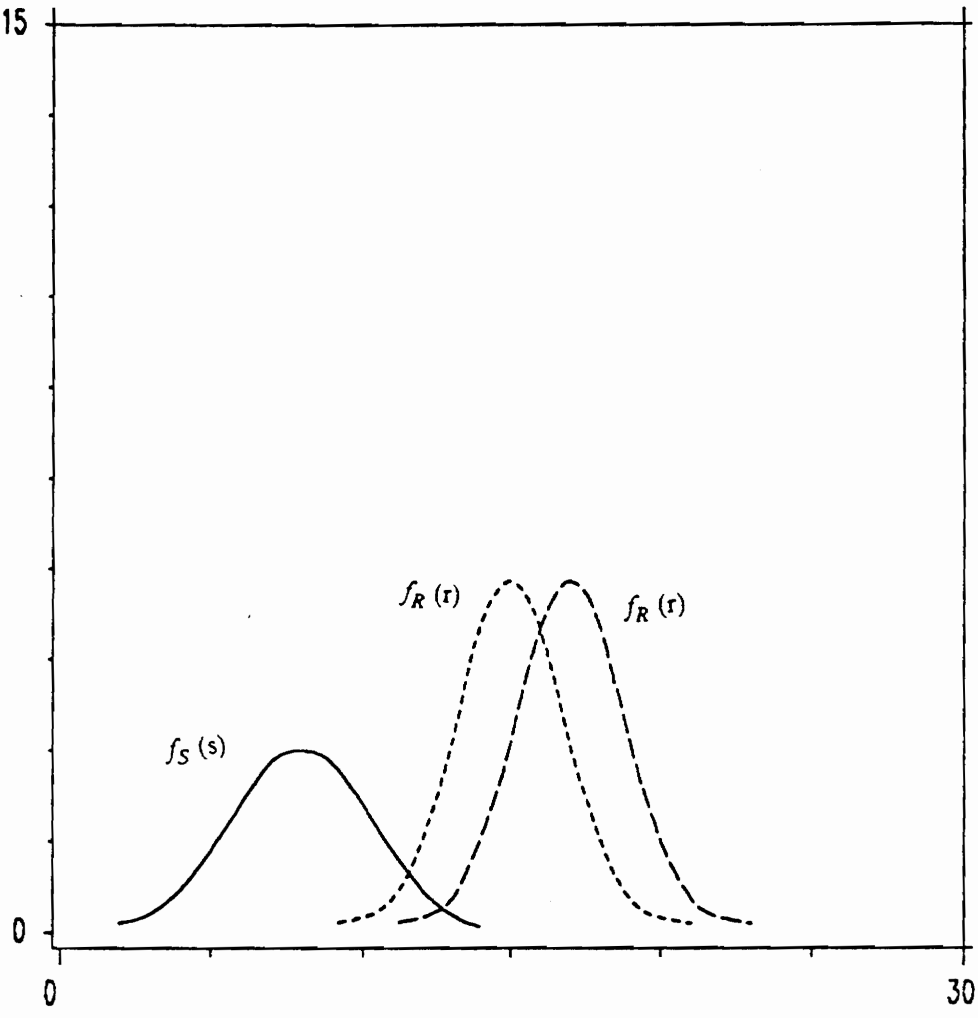


Figure 2.2. Effect of relative Position Between f_R (r) and f_S(s) on P_F

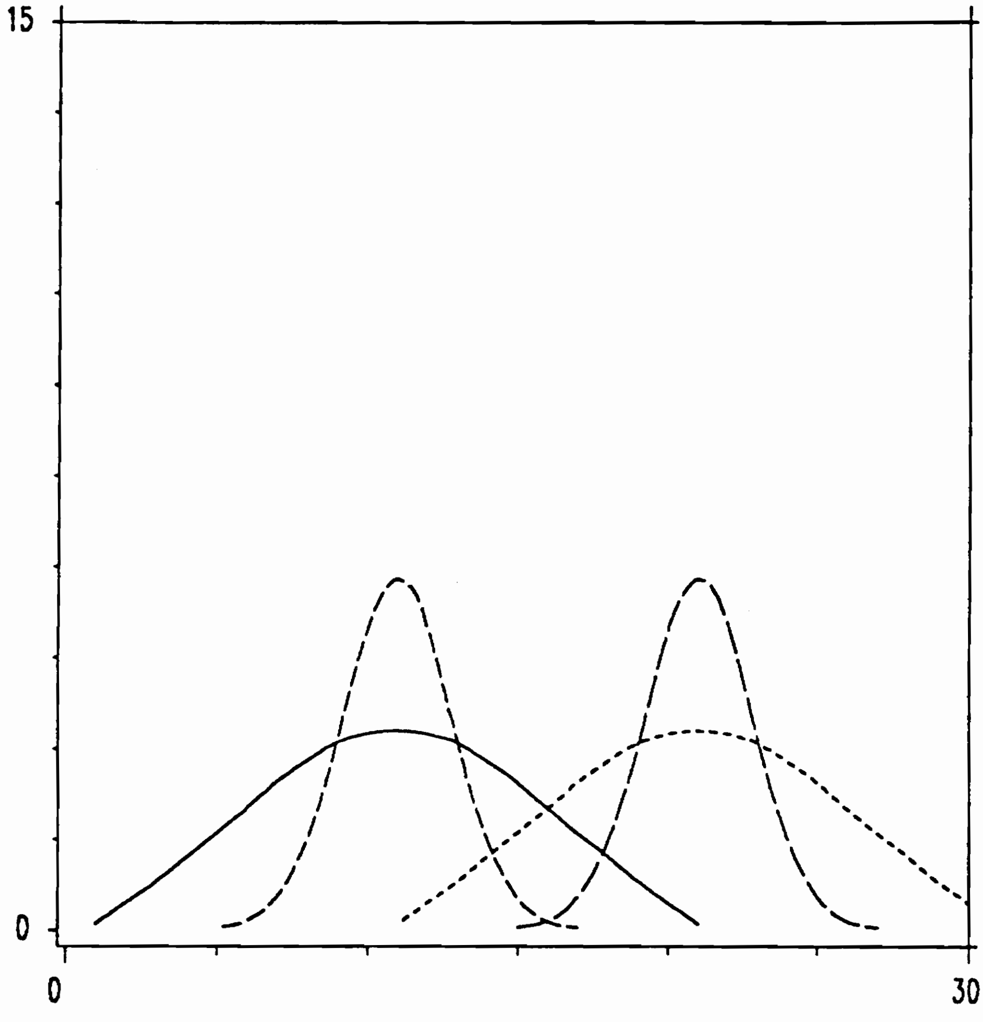


Figure 2.3. Effect of Dispersions in f_R (r) and f_S(s) on P_F

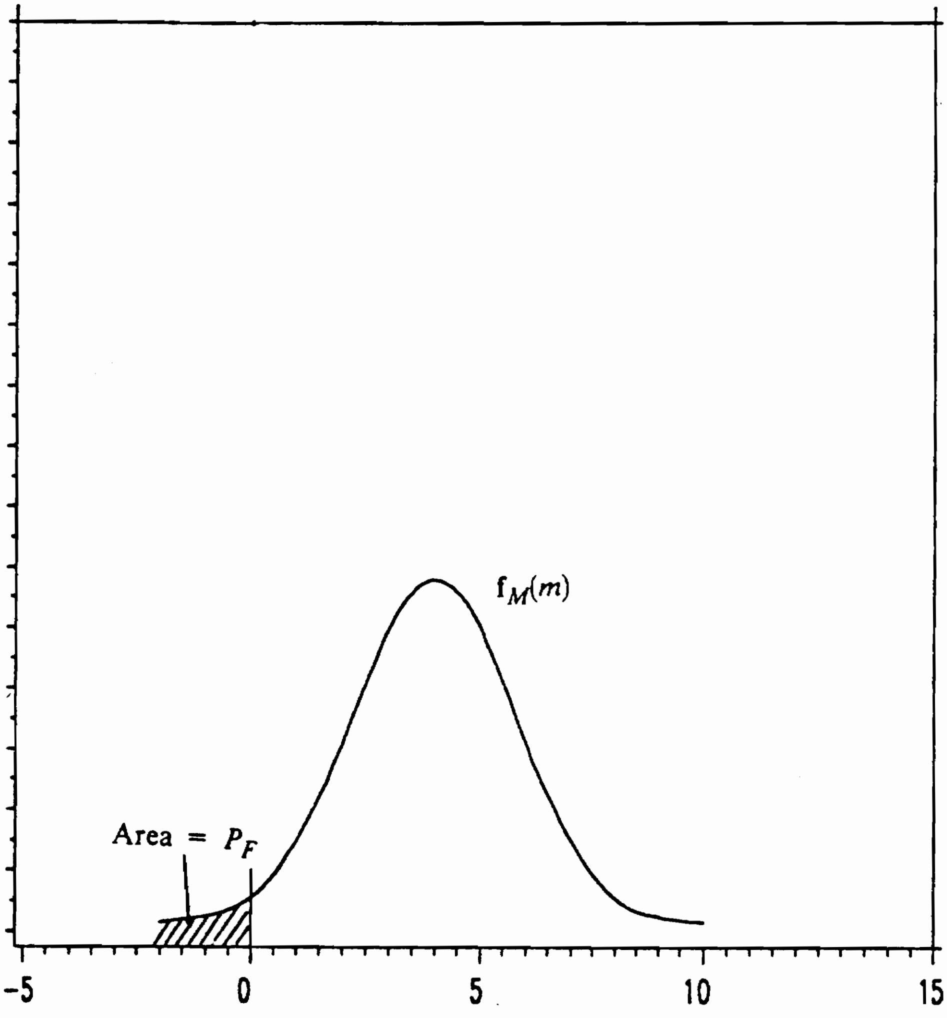


Figure 2.4. Failure Probability and PDF of M

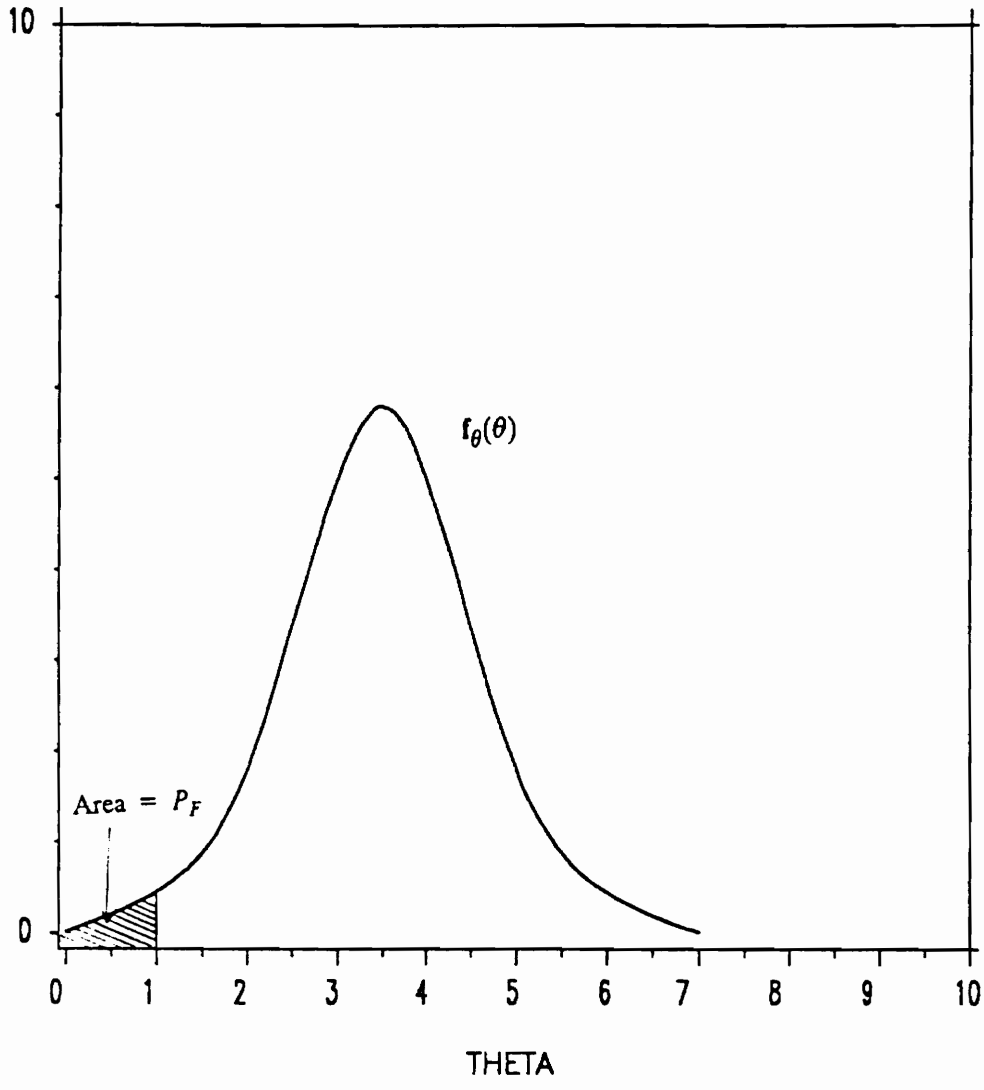


Figure 2.5. Failure Probability and PDF of θ

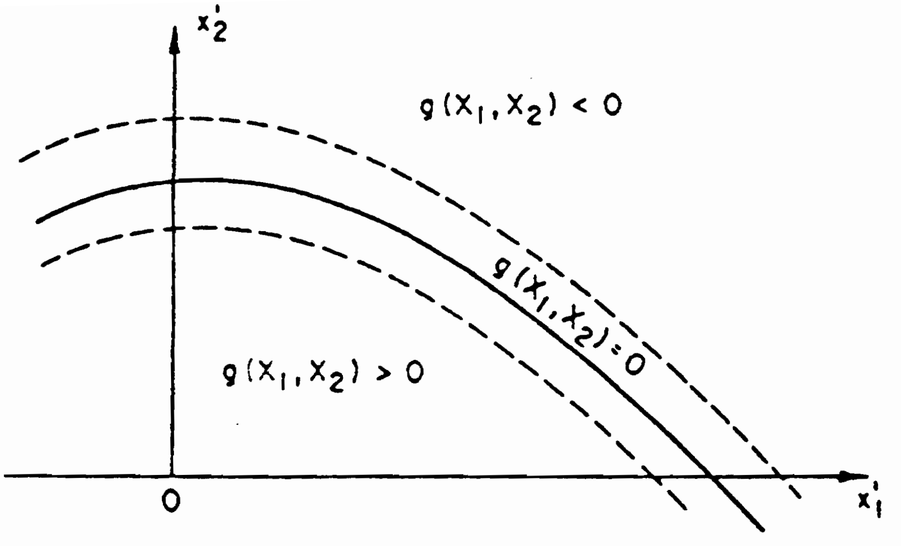


Figure 2.6. Safe and Failure States in Space of Reduced Variates

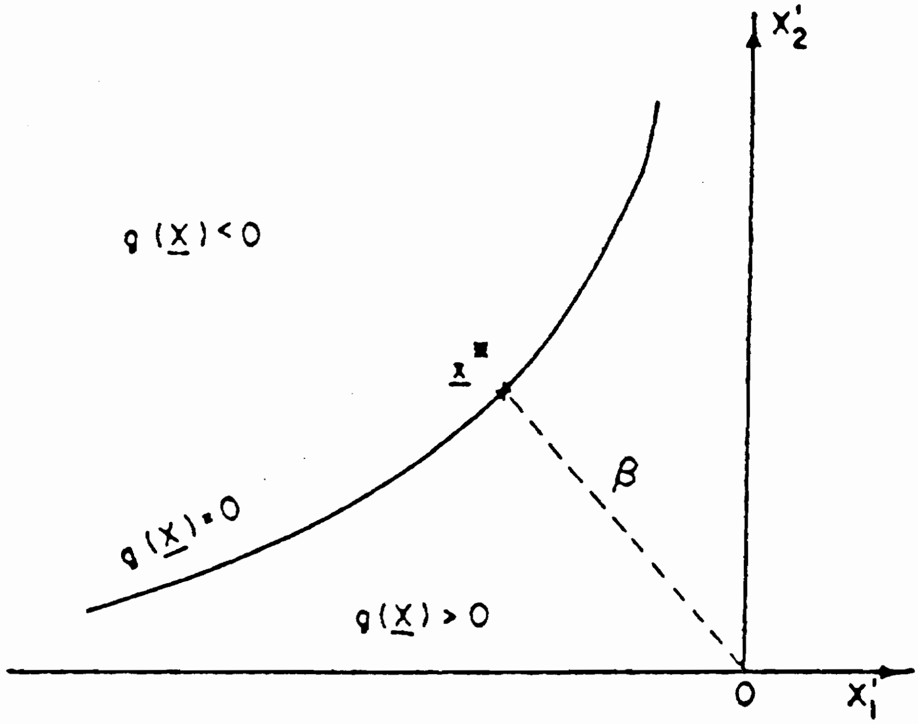


Figure 2.7. Limit-state Surface in Reduced Variable Space

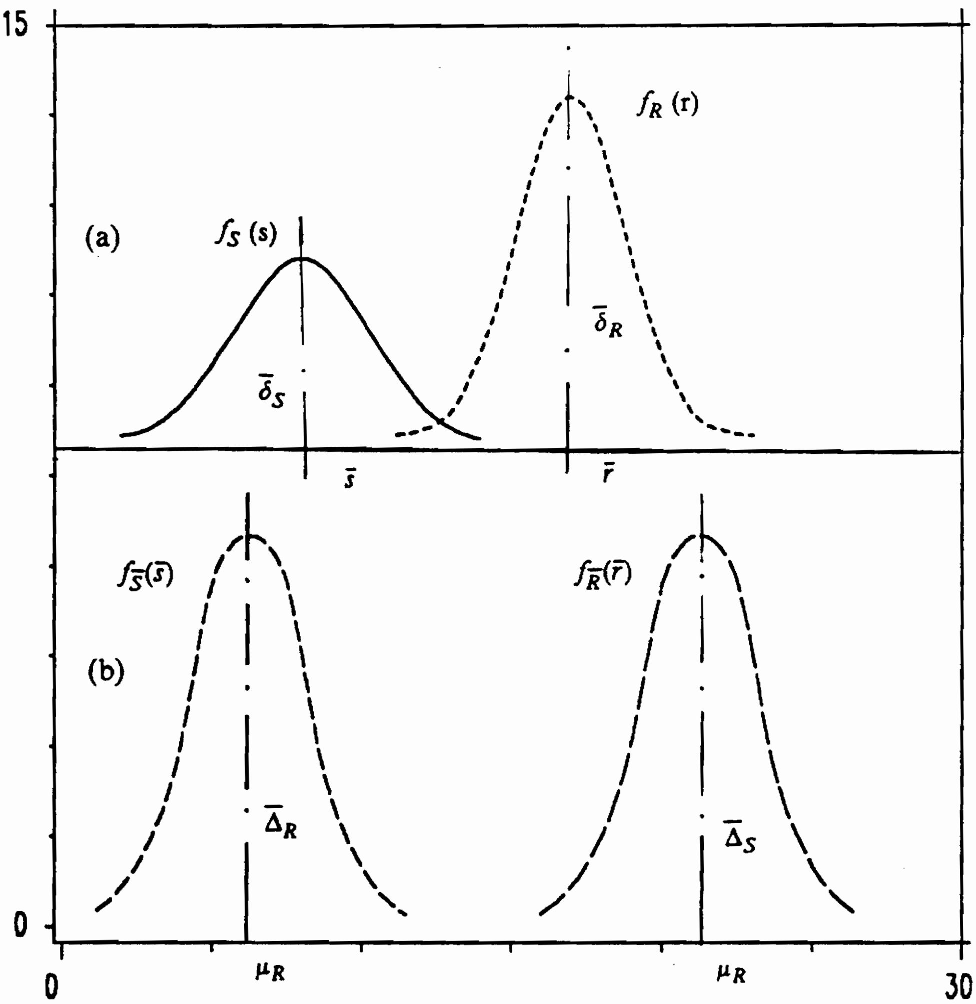


Figure 2.8. (a) Uncertainty due to Randomness; (b) Uncertainty due to Prediction Error

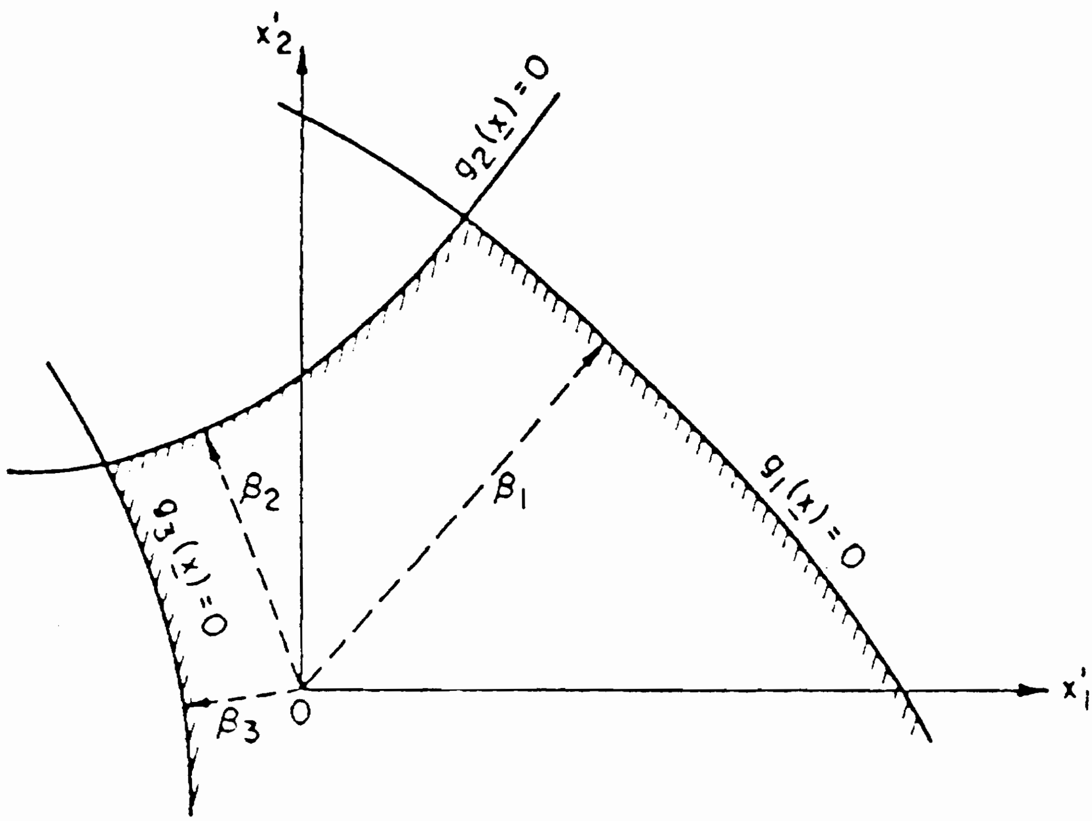


Figure 2.9. Multiple Modes of Failure

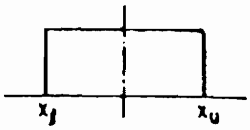
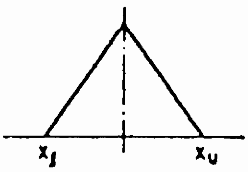
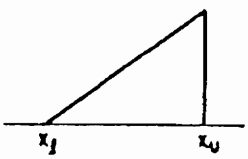
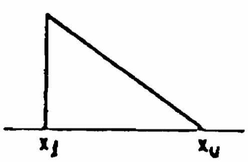
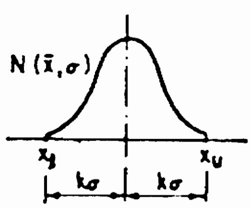
PDF	Mean Value, $\bar{x}$	c.o.v., δ_x or Δ_x
	$\frac{1}{2}(x_l + x_u)$	$\frac{1}{\sqrt{3}}\left(\frac{x_u - x_l}{x_u + x_l}\right)$
	$\frac{1}{2}(x_l + x_u)$	$\frac{1}{\sqrt{6}}\left(\frac{x_u - x_l}{x_u + x_l}\right)$
	$\frac{1}{3}(x_l + 2x_u)$	$\frac{1}{\sqrt{2}}\left(\frac{x_u - x_l}{2x_u + x_l}\right)$
	$\frac{1}{3}(2x_l + x_u)$	$\frac{1}{\sqrt{2}}\left(\frac{x_u - x_l}{x_u + 2x_l}\right)$
	$\frac{1}{2}(x_l + x_u)$	$\frac{1}{k}\left(\frac{x_u - x_l}{x_u + x_l}\right)$

Figure 2.10. Statistics of Prescribed Distributions

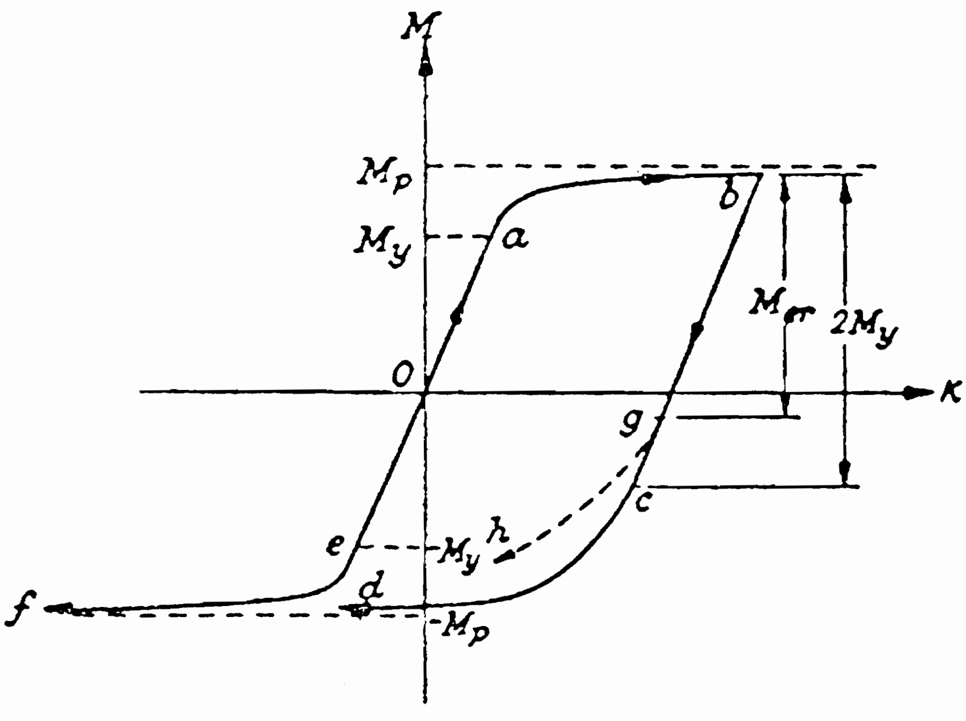


Figure 3.1. General type of Bending Moment Curvature Relation[14]

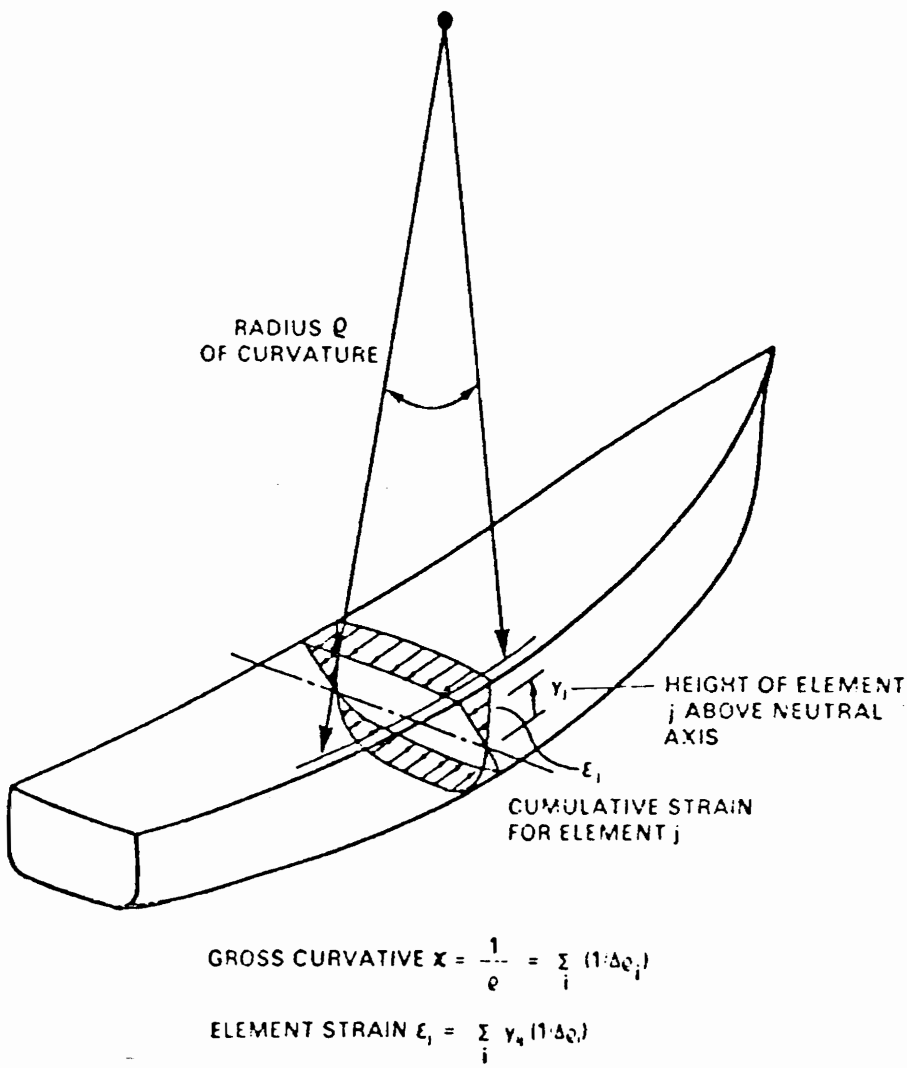


Figure 4.1. Incremental Concept for Hull Bending

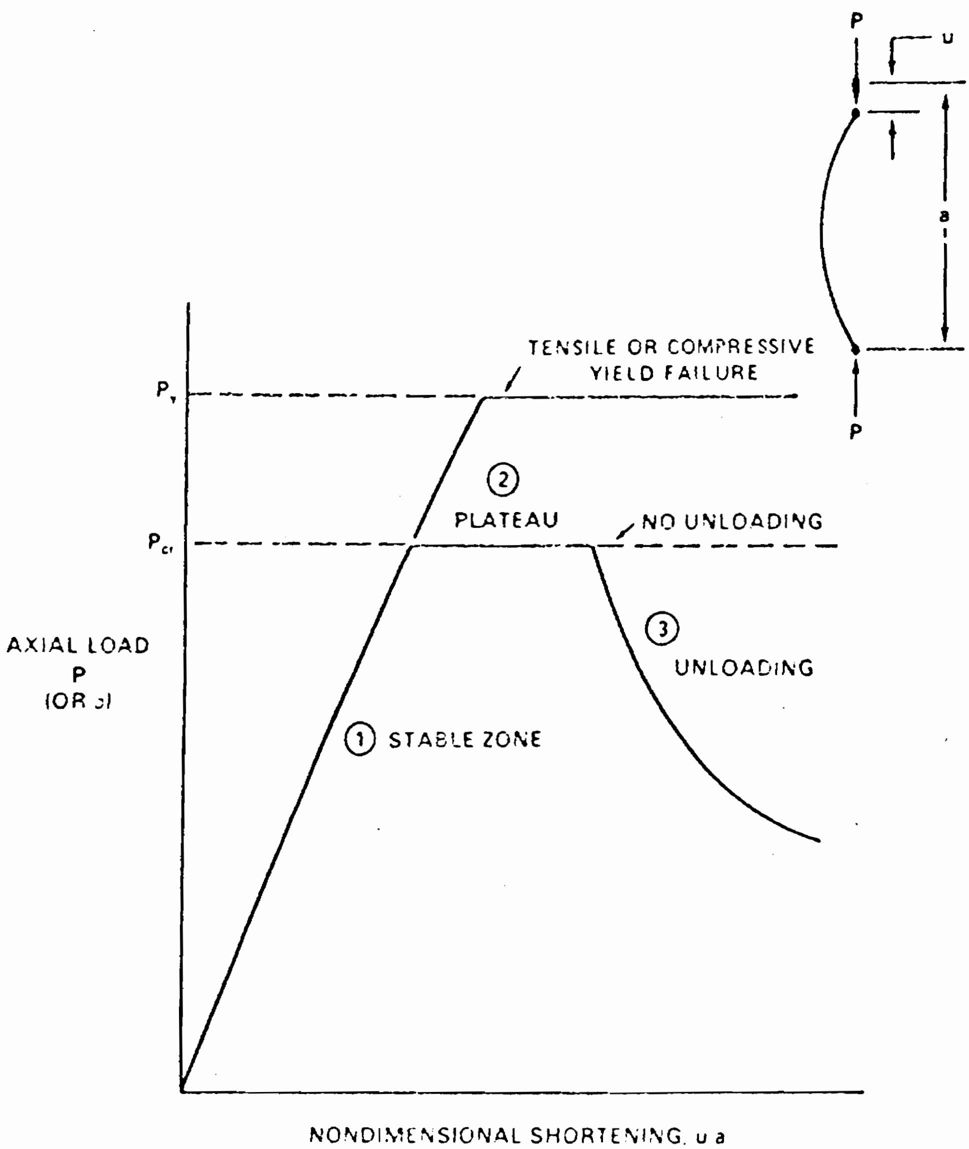


Figure 4.2. Typical Gross Panel Load Shortening Curves [11]

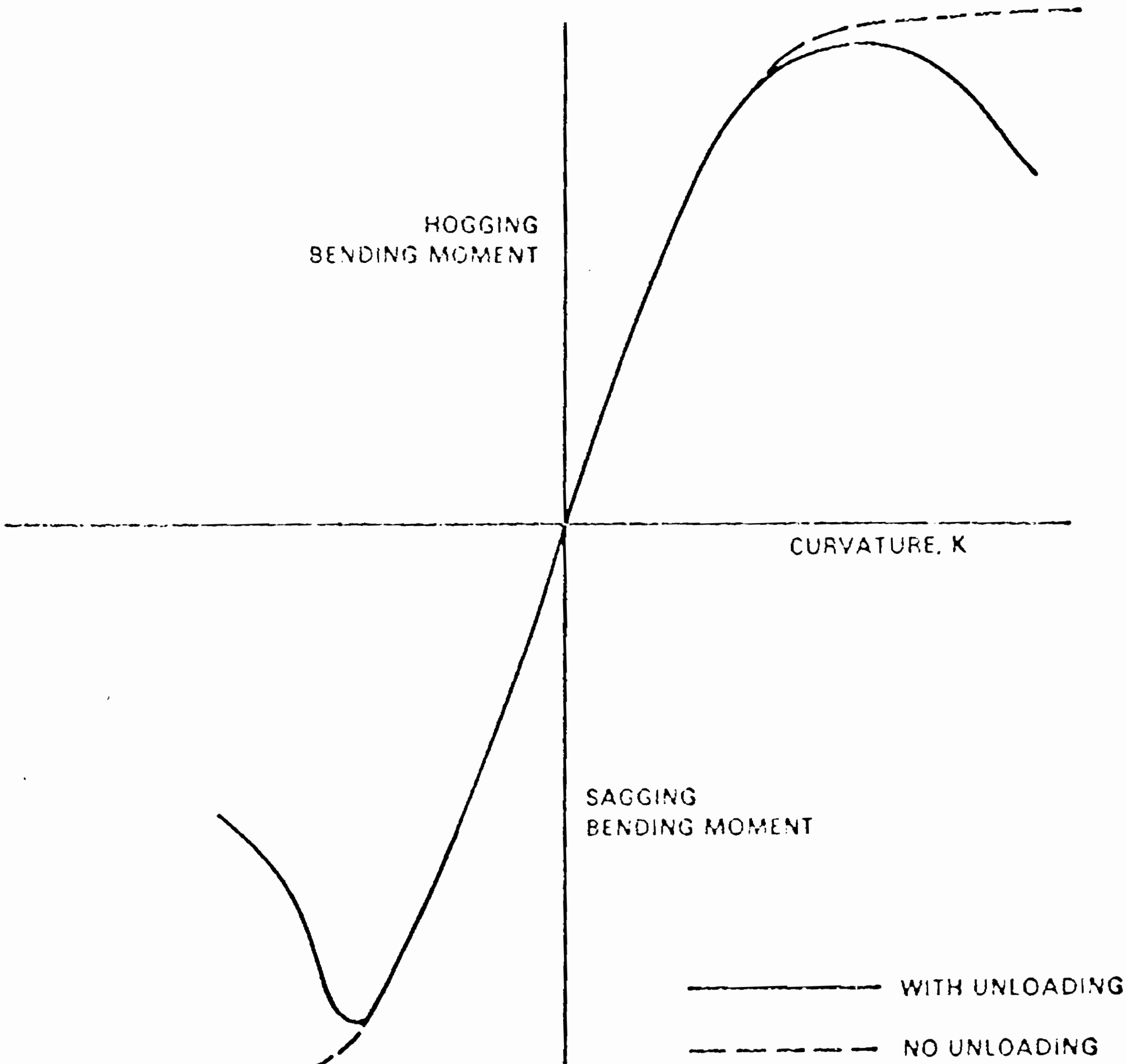


Figure 4.3. Typical Moment-Curvature Diagram for a Hull Cross Section [11]

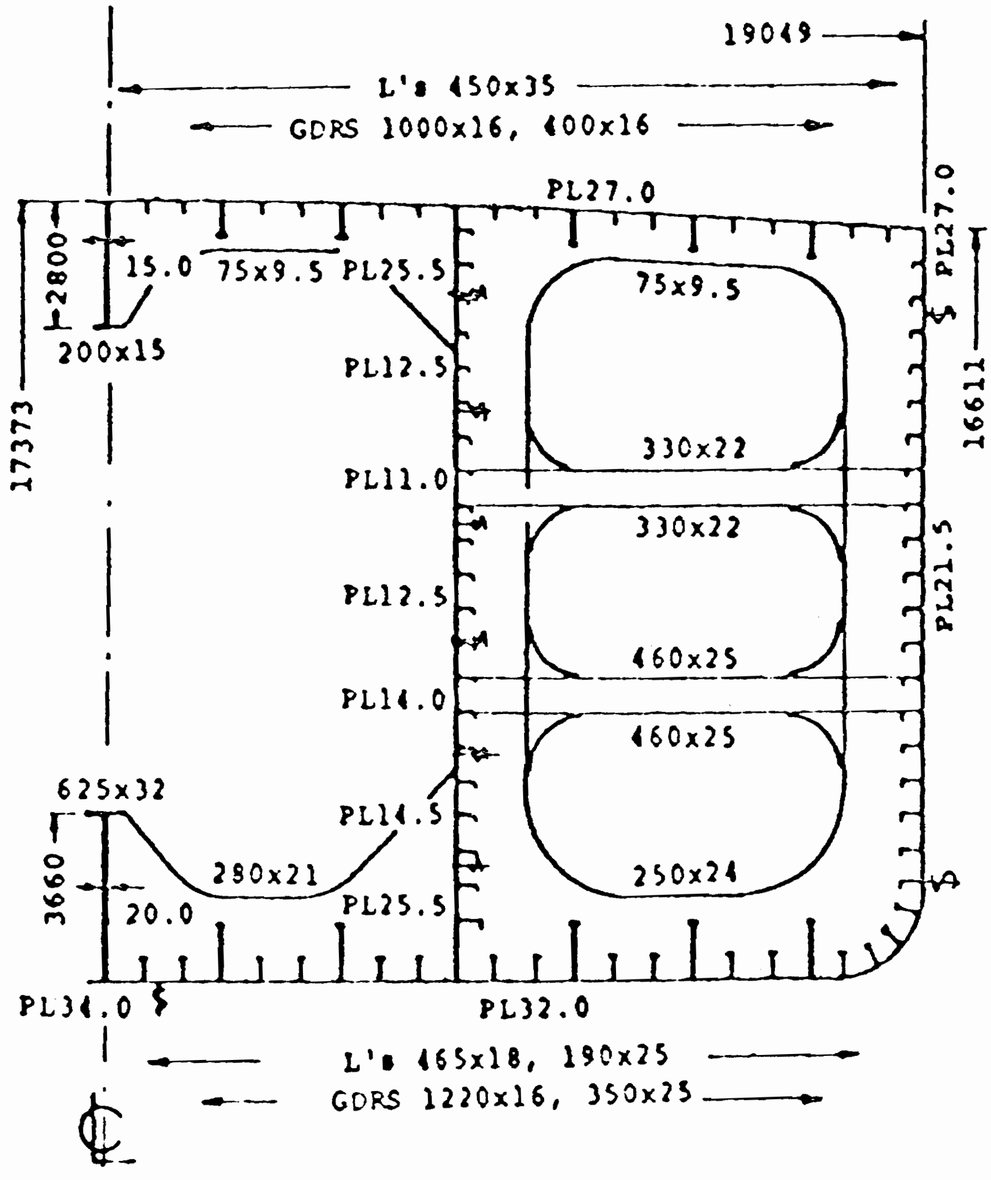


Figure 4.4. Midship Section - Tanker 1

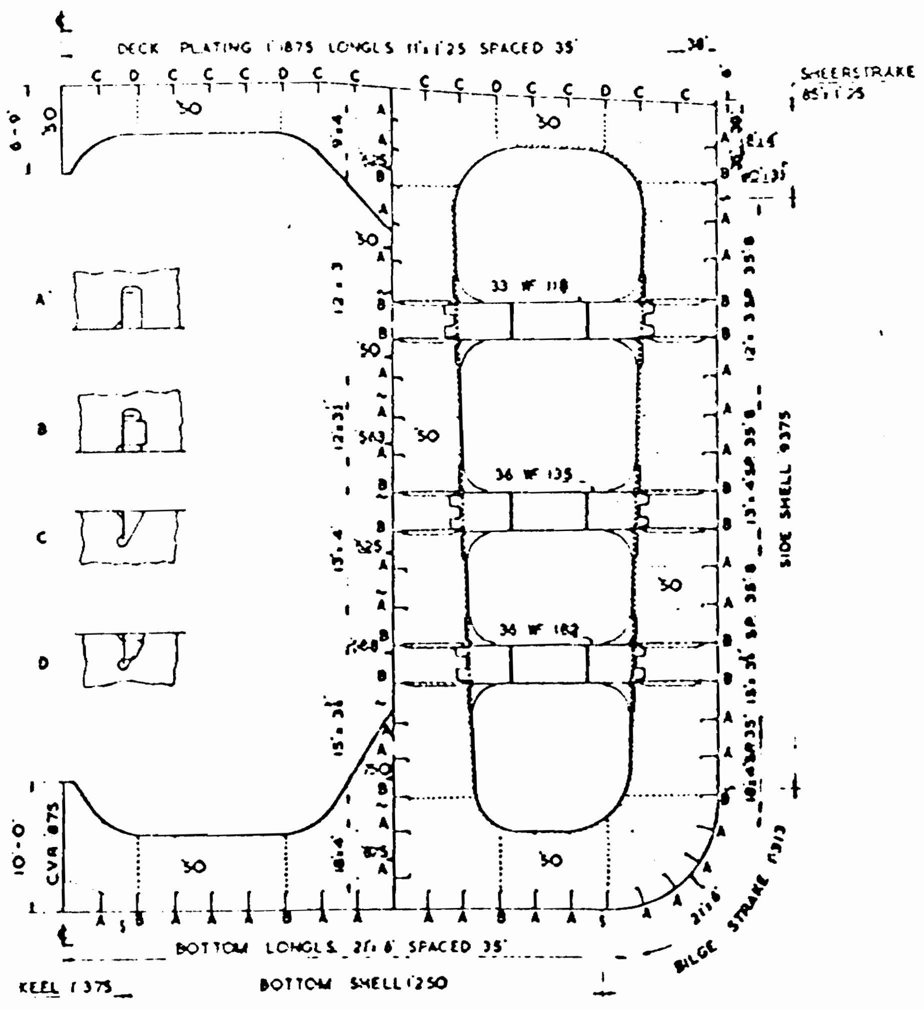


Figure 4.5. Midship Section - Tanker 2

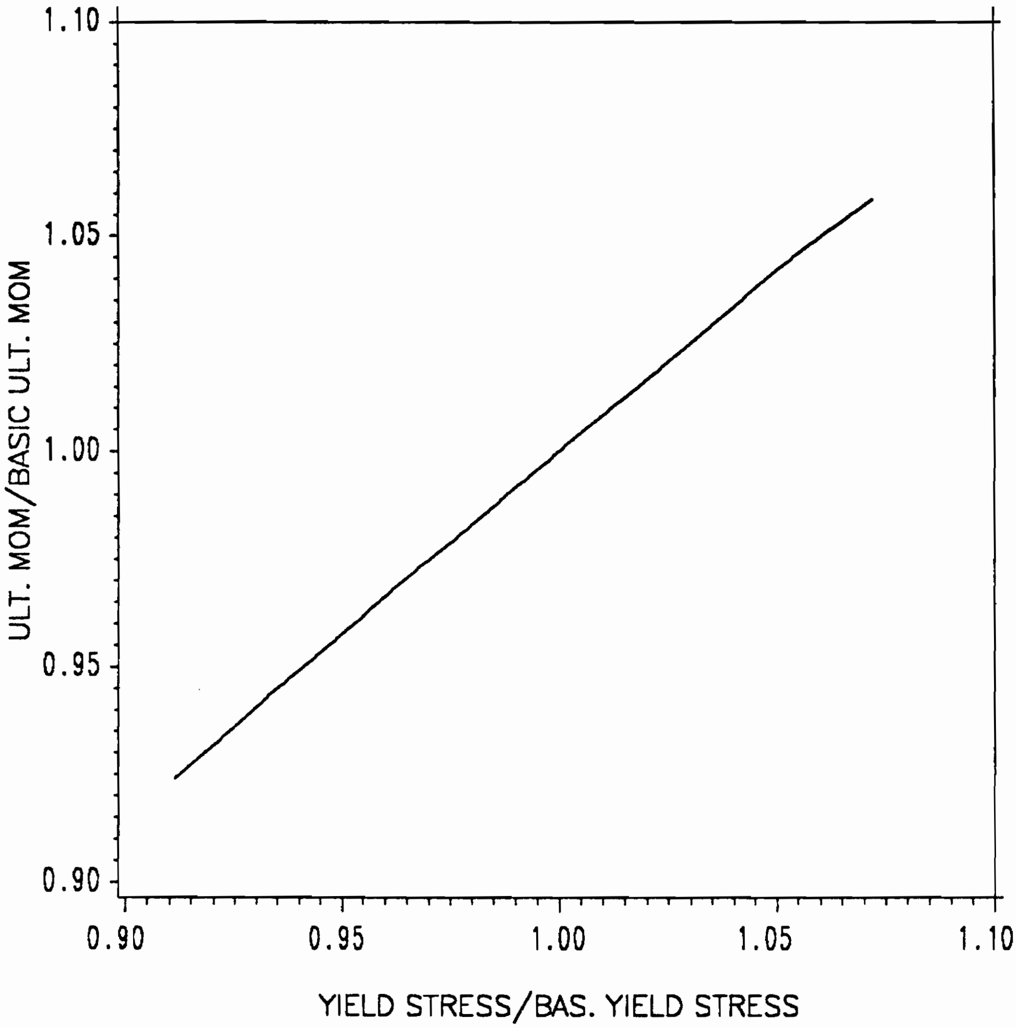


Figure 5.1 Yield Stress Ratio VS. Ultimate Moment Ratio (Tanker1 - Sagging)

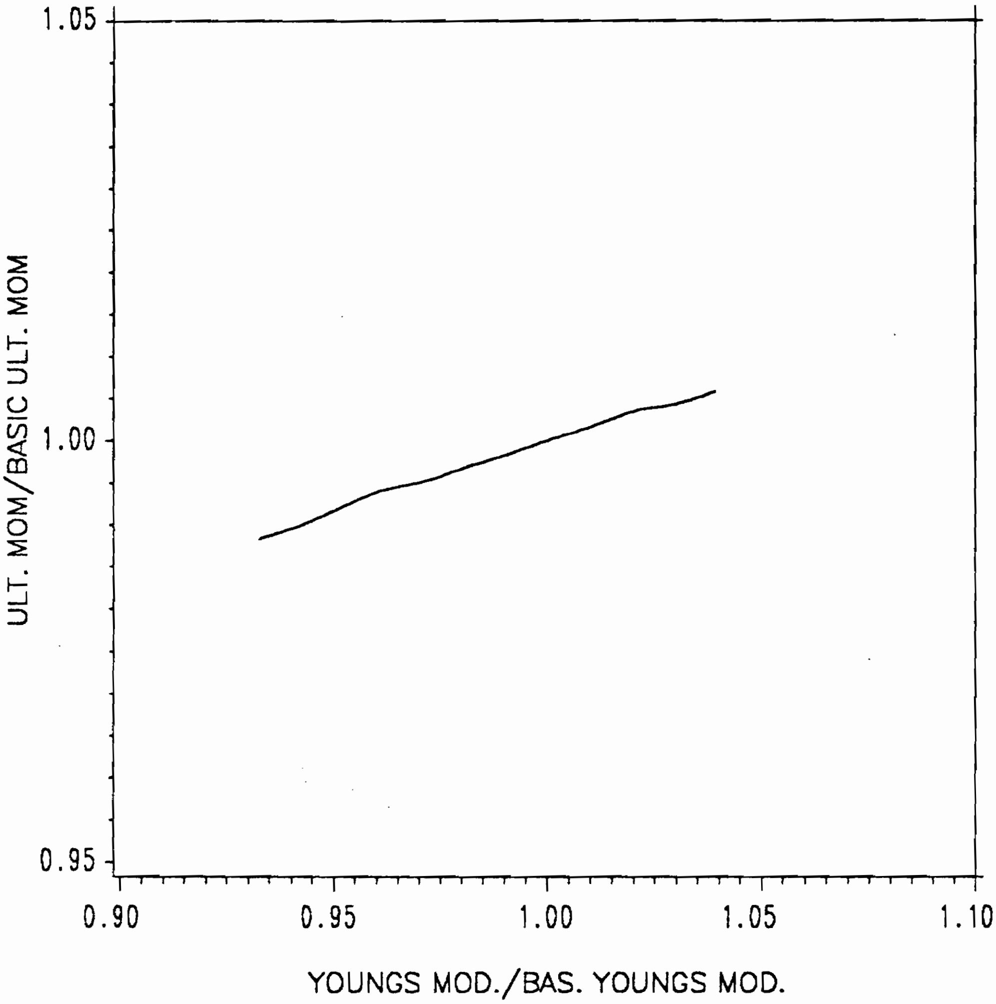


Figure 5.2 Young's Mod. Ratio VS. Ultimate Moment Ratio (Tanker1 - Sagging)

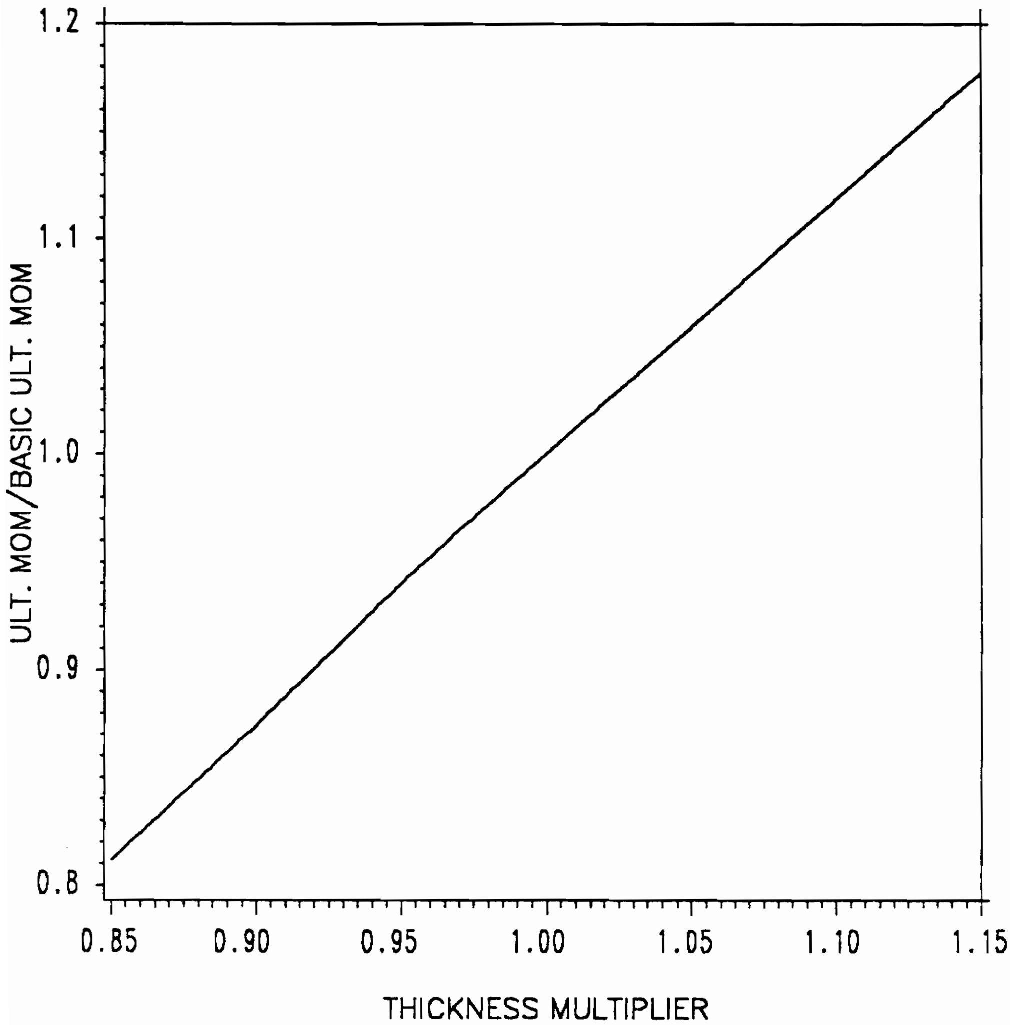


Figure 5.3 Thickness Ratio VS. Ultimate Moment Ratio (Tanker1 - Sagging)

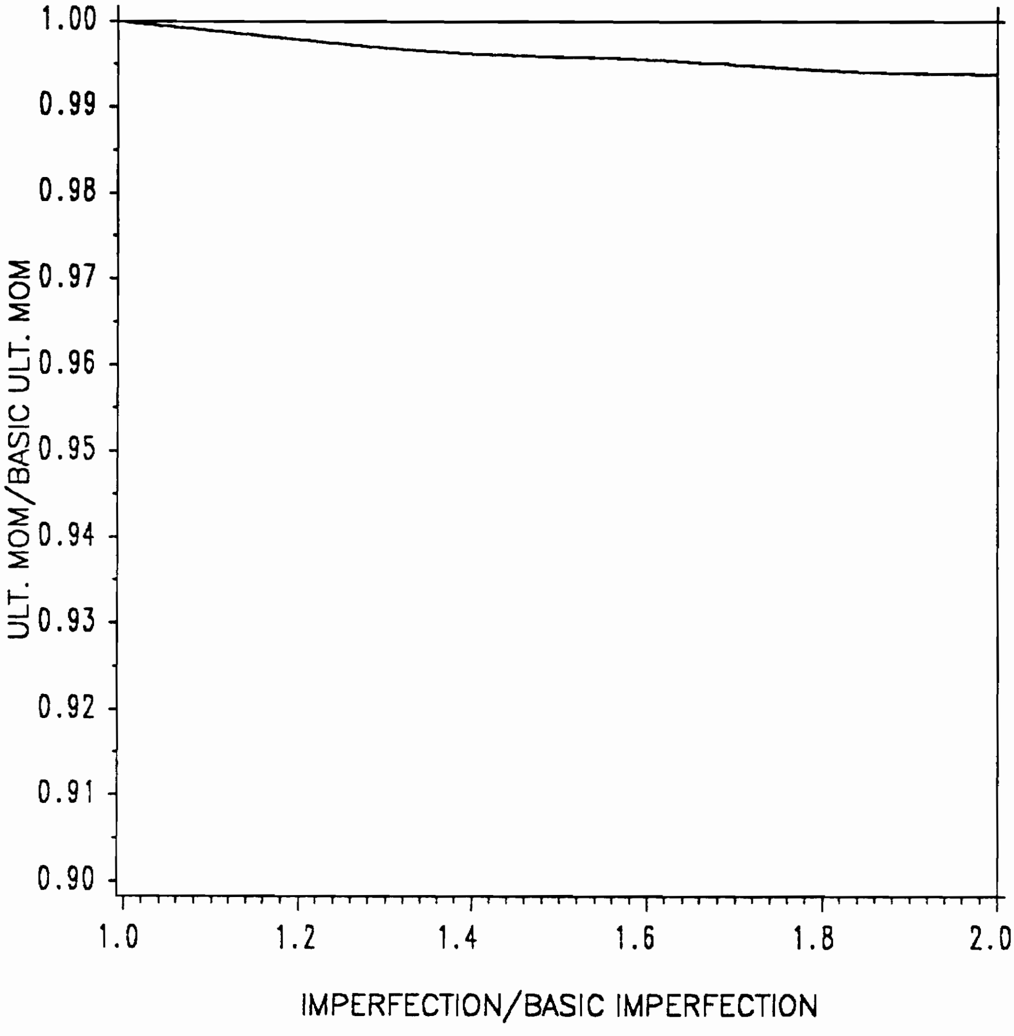


Figure 5.4 Imperfection Ratio VS. Ultimate Moment Ratio (Tanker1 - Sagging)

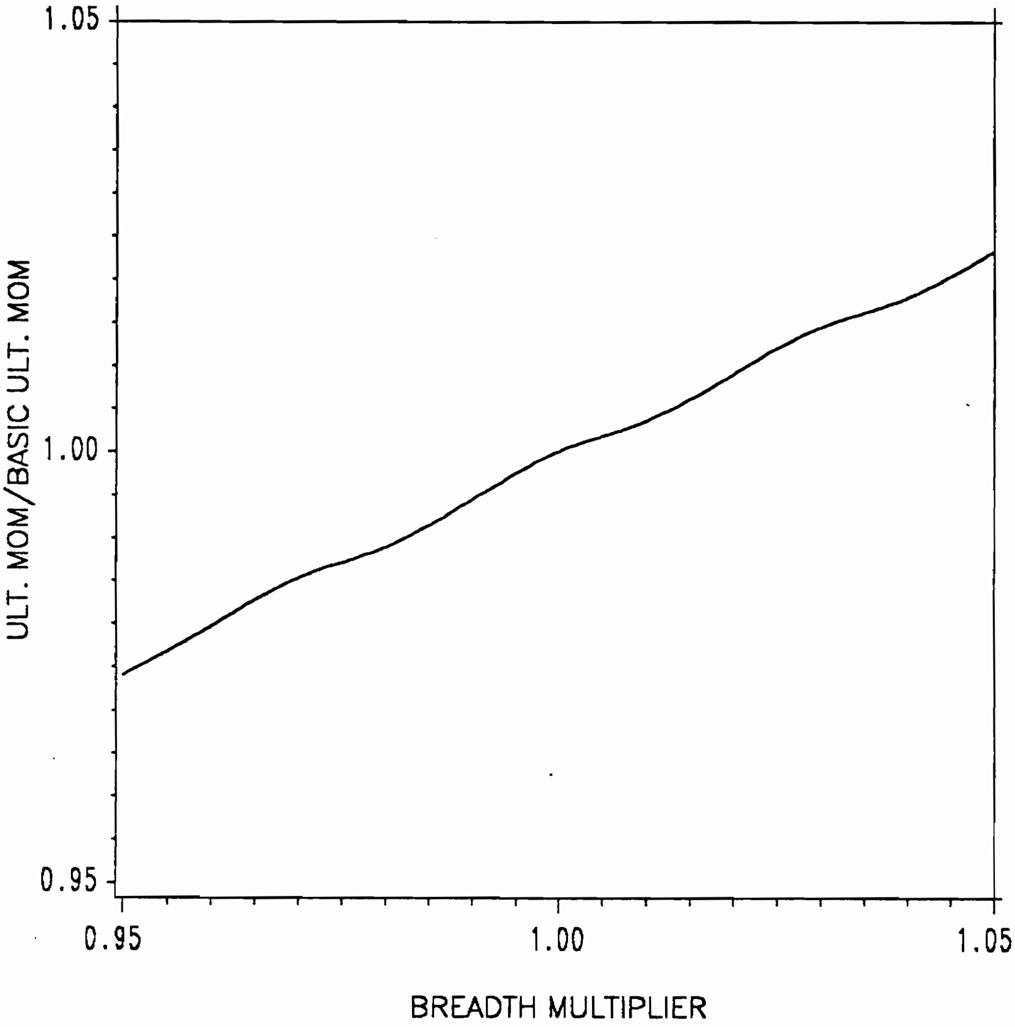


Figure 5.5 Breadth Multiplier VS. Ultimate Moment Ratio (Tanker1 - Sagging)

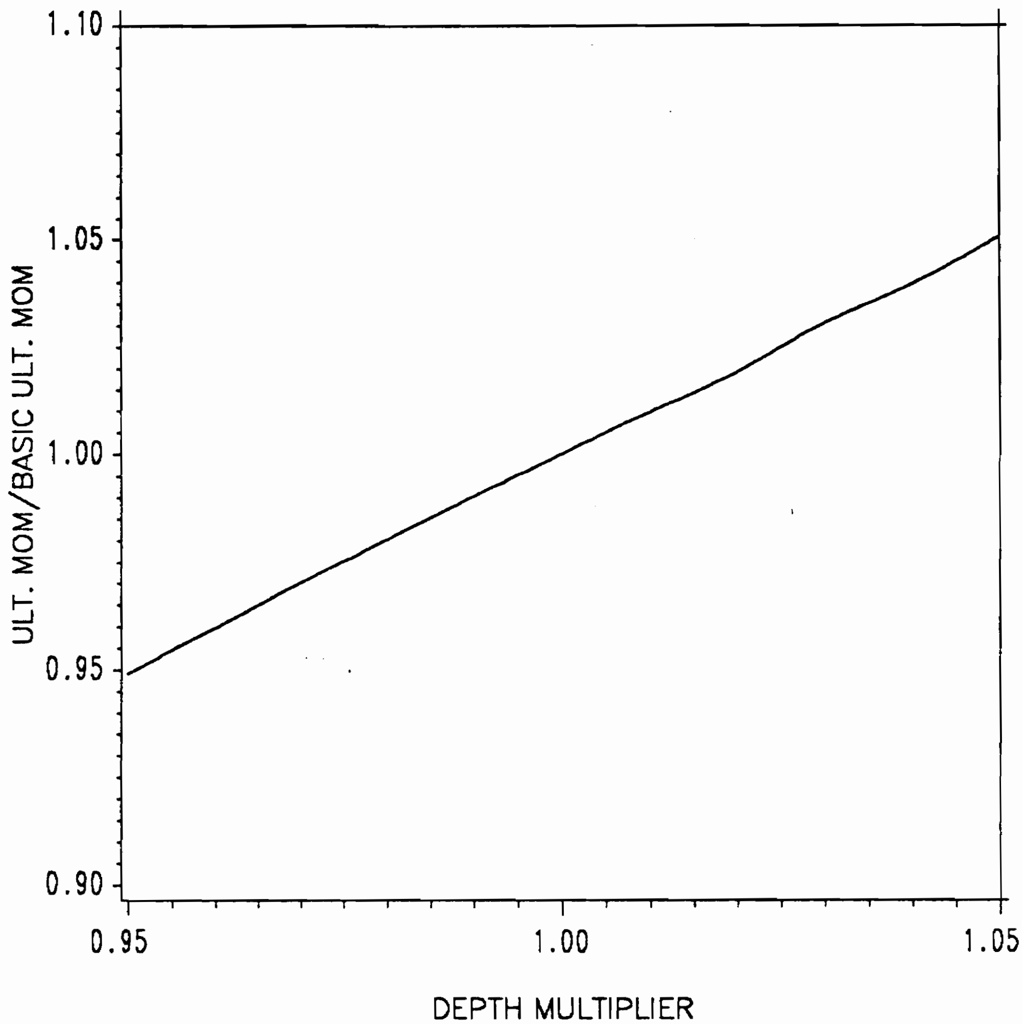


Figure 5.6 Depth Multiplier VS. Ultimate Moment Ratio (Tanker1 - Sagging)

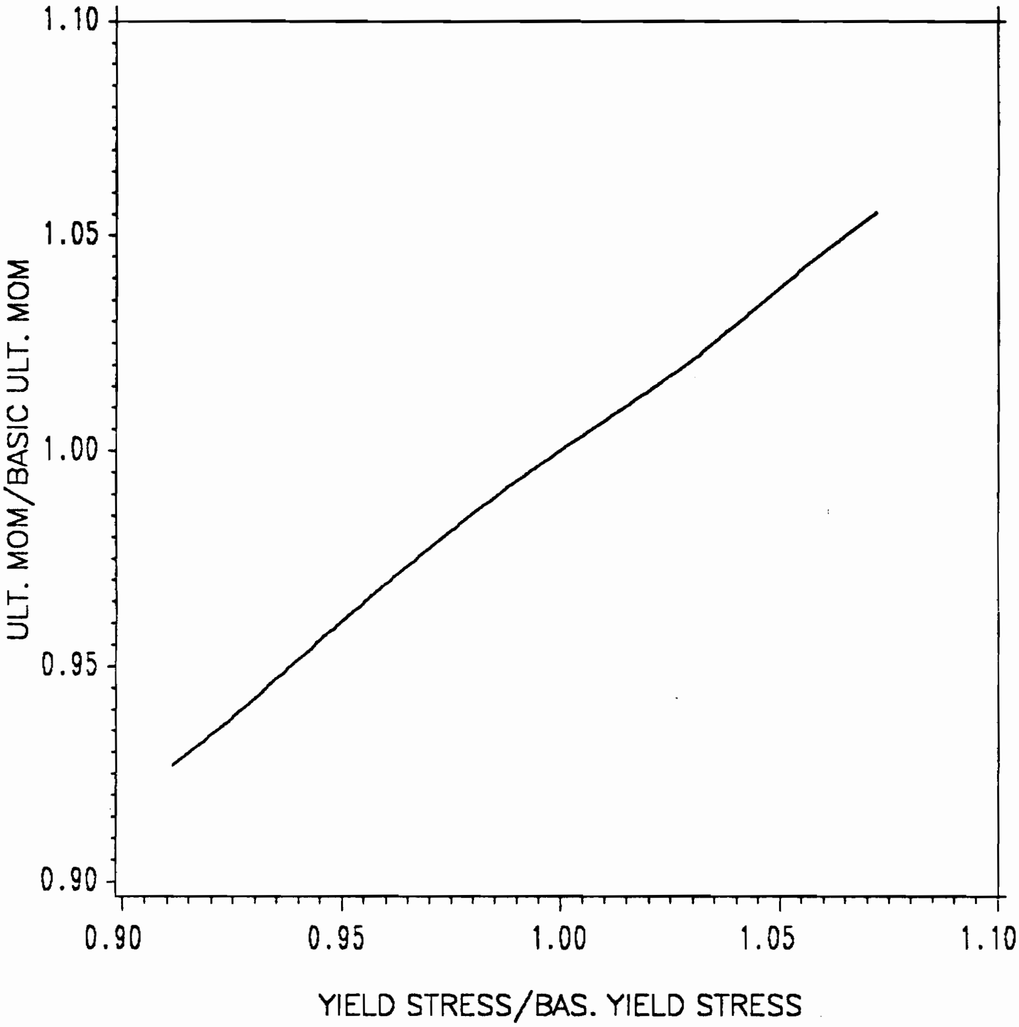


Figure 5.7 Yield Stress Ratio VS. Ultimate Moment Ratio (Tanker1 - Hogging)

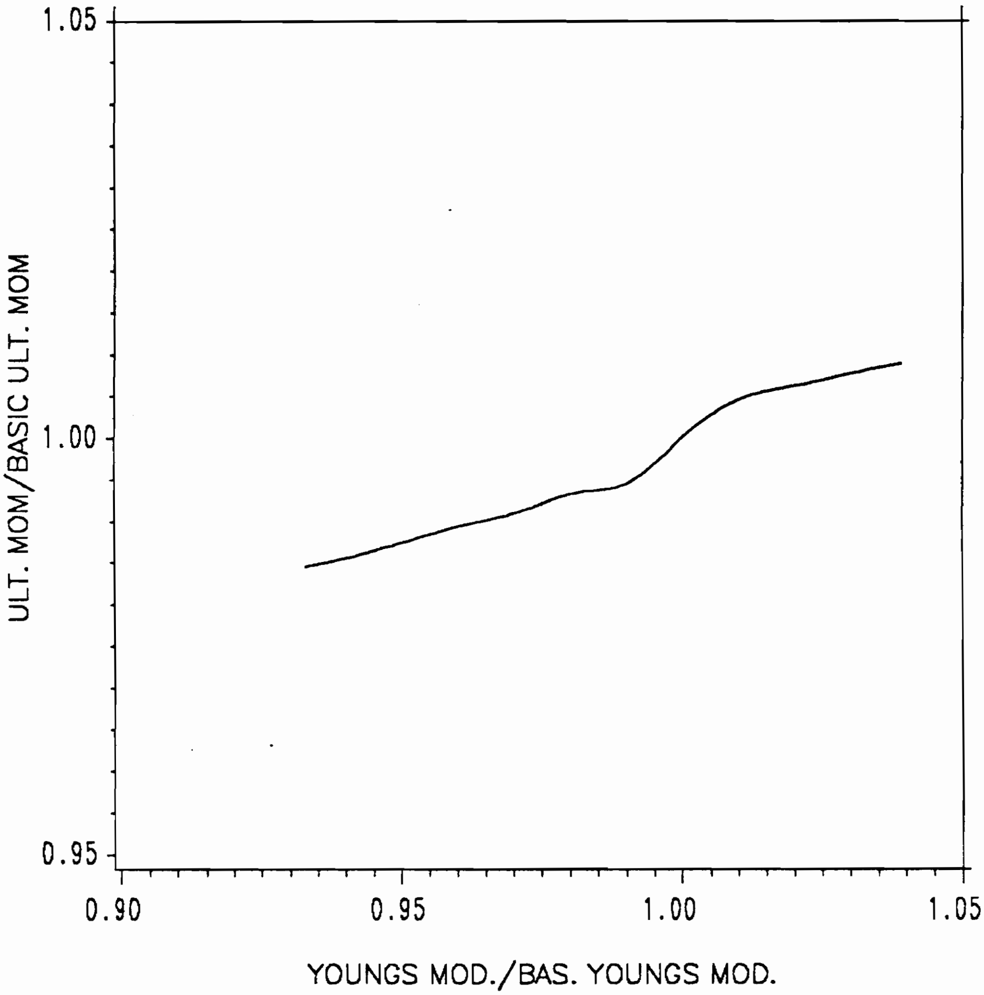


Figure 5.8 Young's Mod. Ratio VS. Ultimate Moment Ratio (Tanker1 - Hogging)

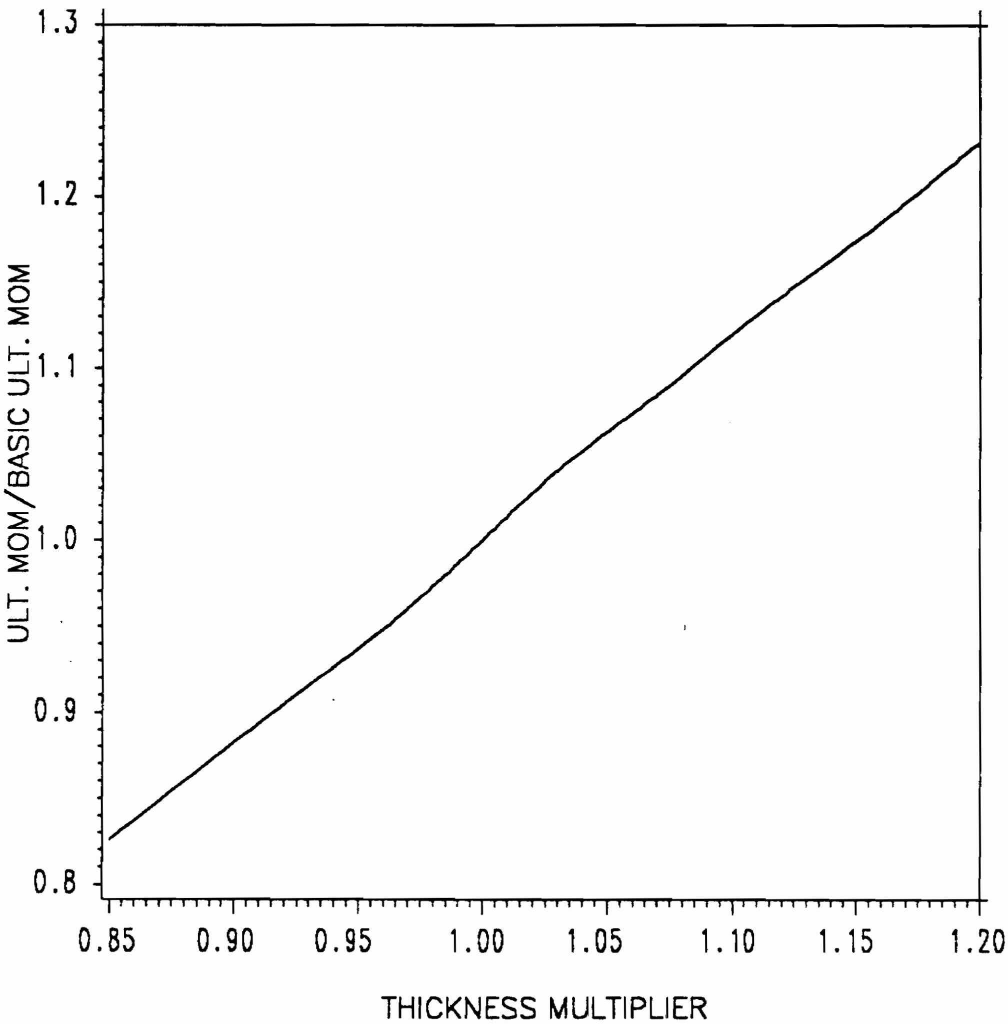


Figure 5.9 Thickness Ratio VS. Ultimate Moment Ratio (Tanker1 - Hogging)

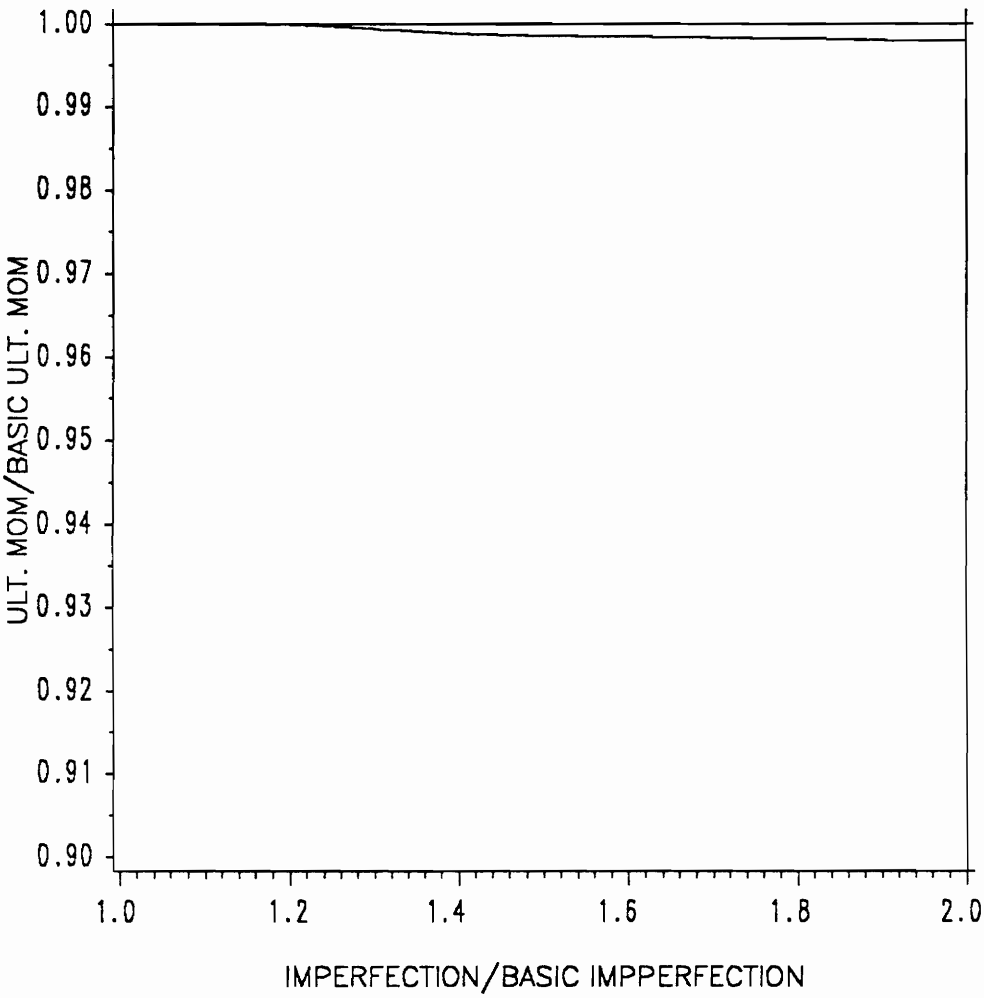


Figure 5.10 Imperfection Ratio VS. Ultimate Moment Ratio (Tanker1 - Hogging)

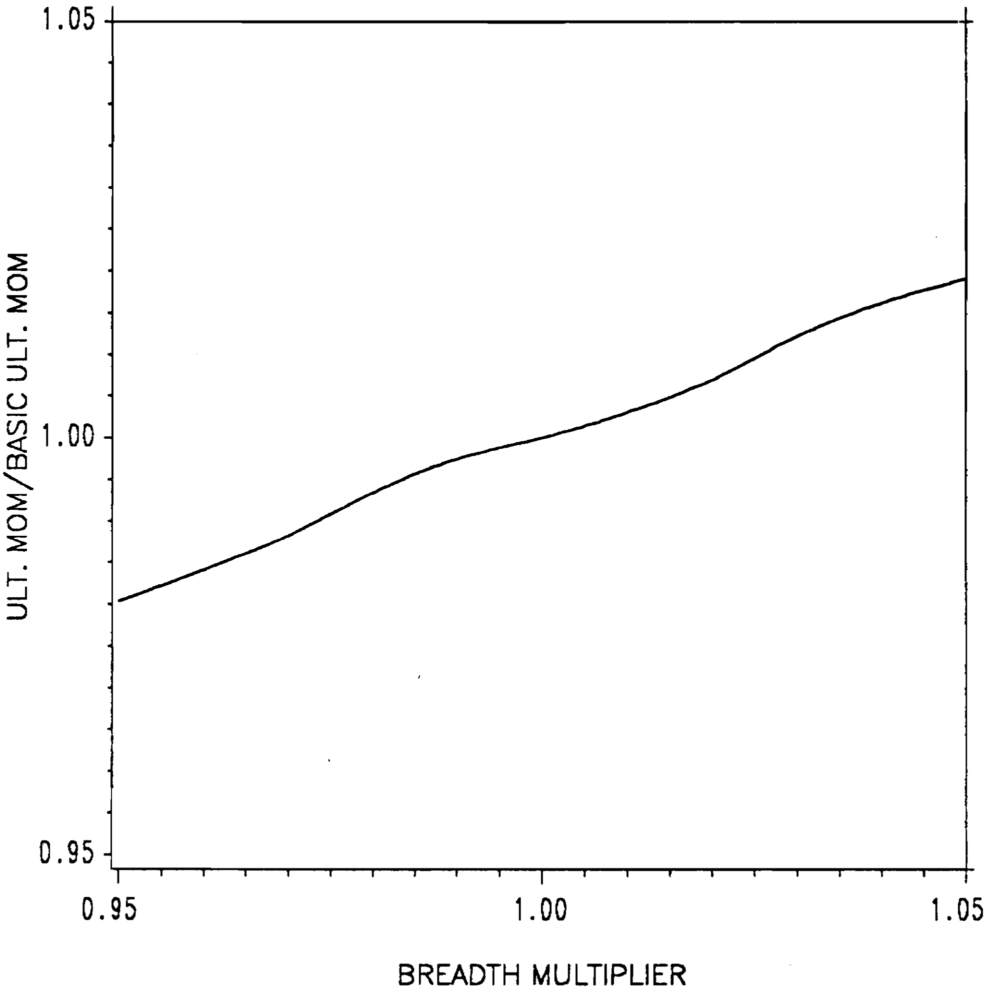


Figure 5.11 Breadth Multiplier VS. Ultimate Moment Ratio (Tanker1 - Hogging)

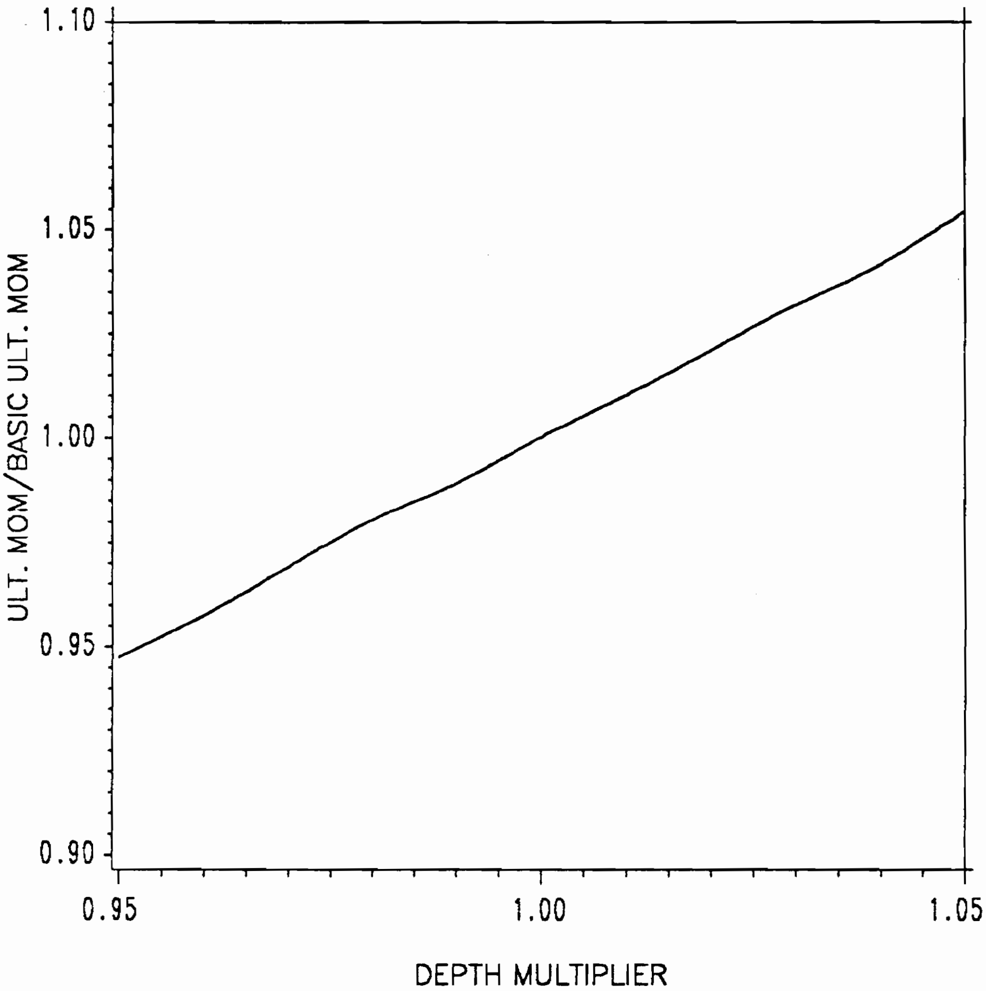


Figure 5.12 Depth Multiplier VS. Ultimate Moment Ratio (Tanker1 - Hogging)

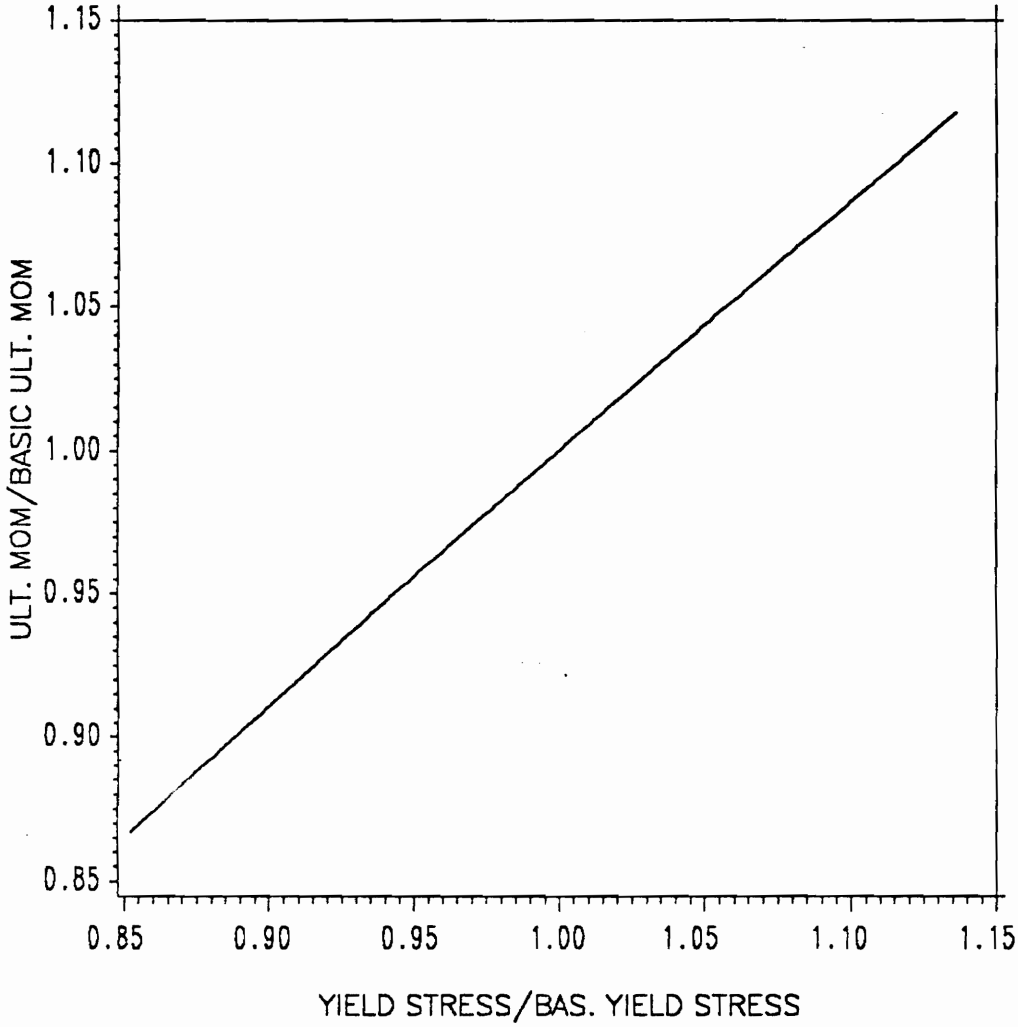


Figure 5.13 Yield Stress Ratio VS. Ultimate Moment Ratio (Tanker2 - Sagging)

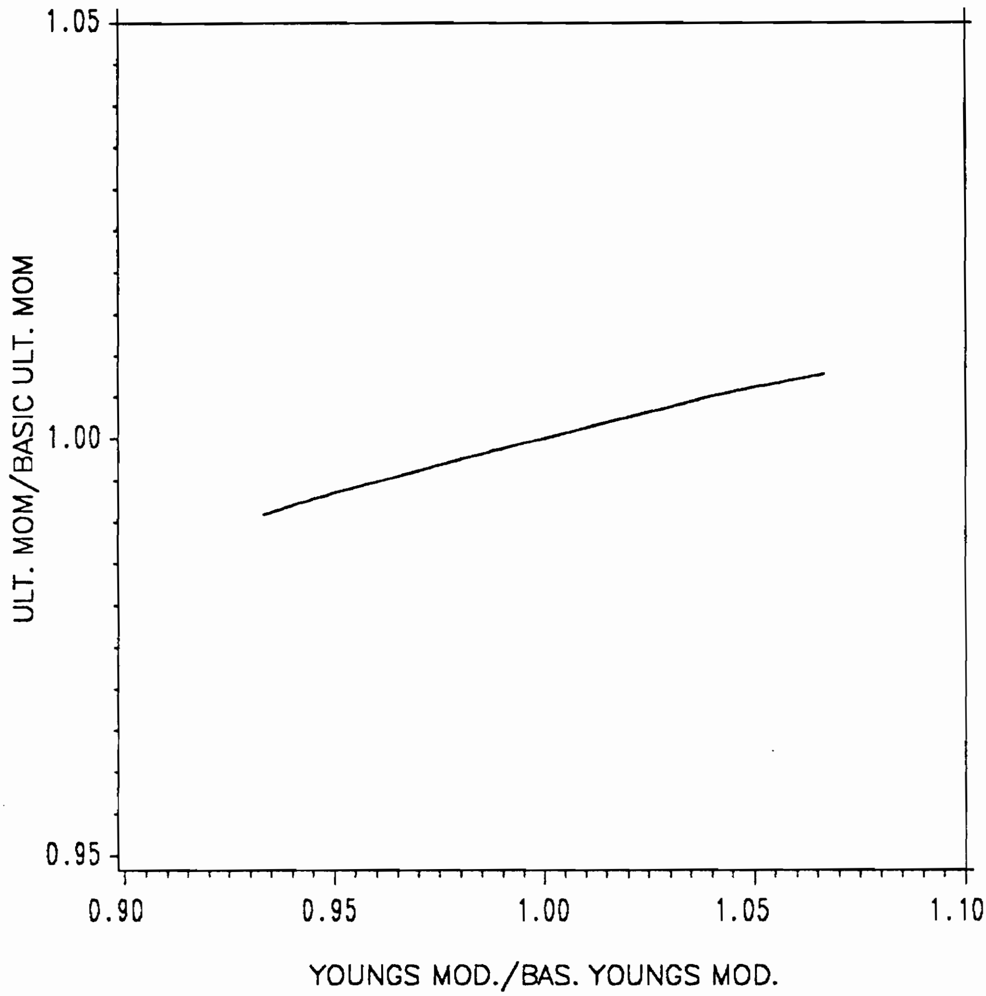


Figure 5.14 Young's Mod. Ratio VS. Ultimate Moment Ratio (Tanker2 - Sagging)

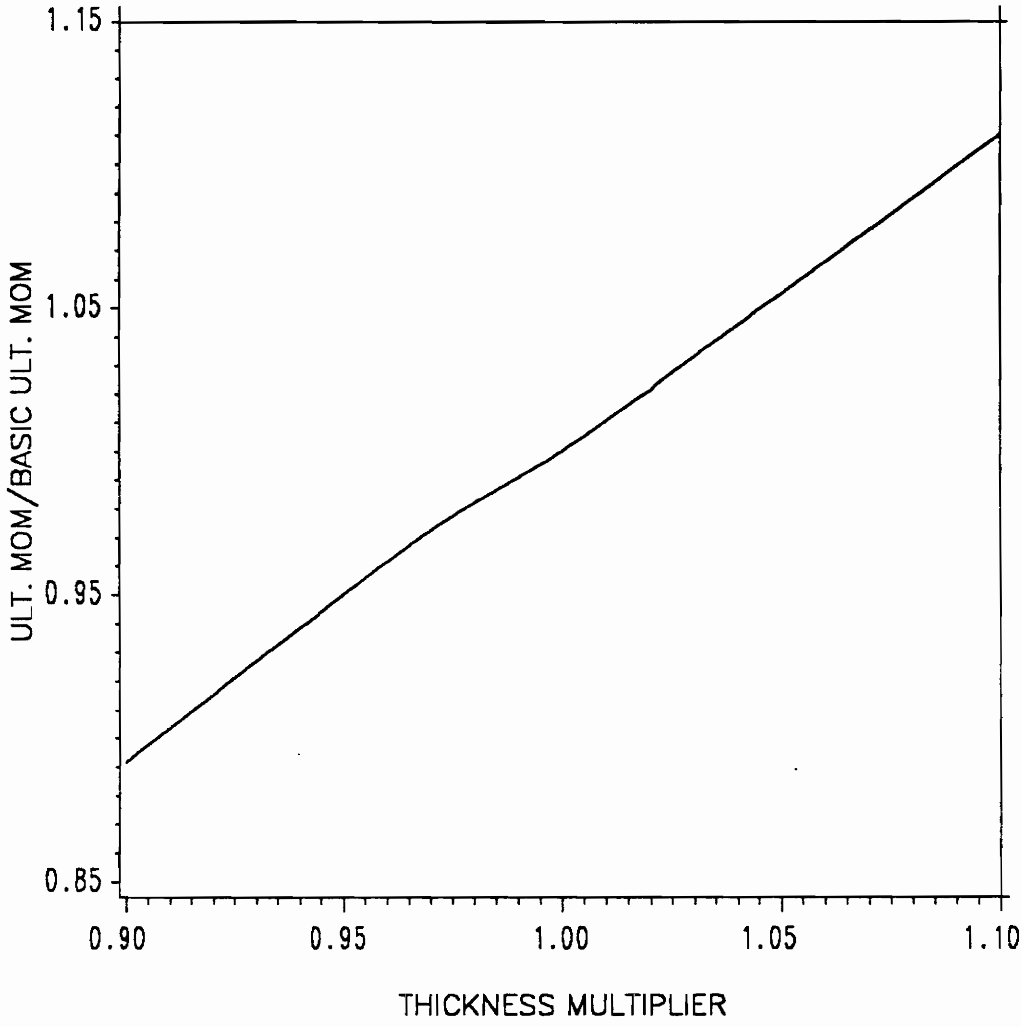


Figure 5.15 Thickness Ratio VS. Ultimate Moment Ratio (Tanker2 - Sagging)

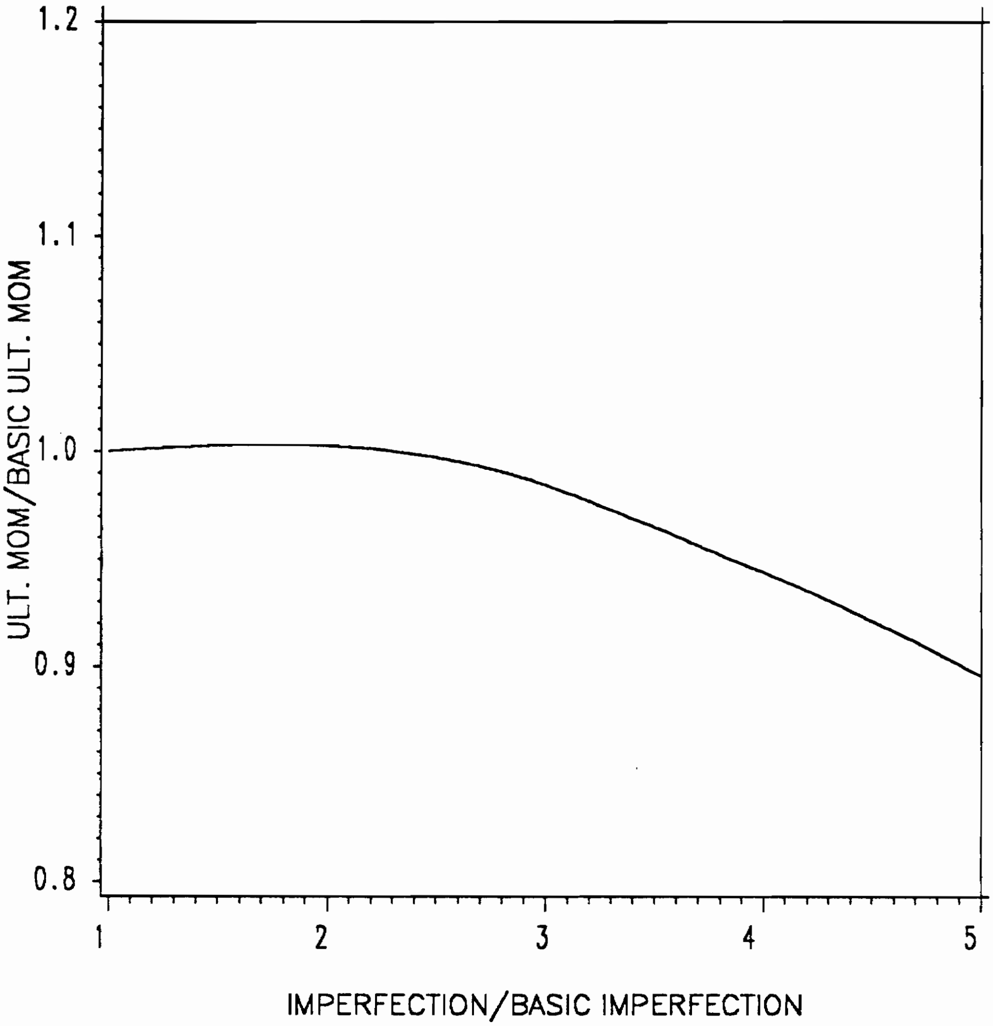


Figure 5.16 Imperfection Ratio VS. Ultimate Moment Ratio (Tanker2 - Sagging)

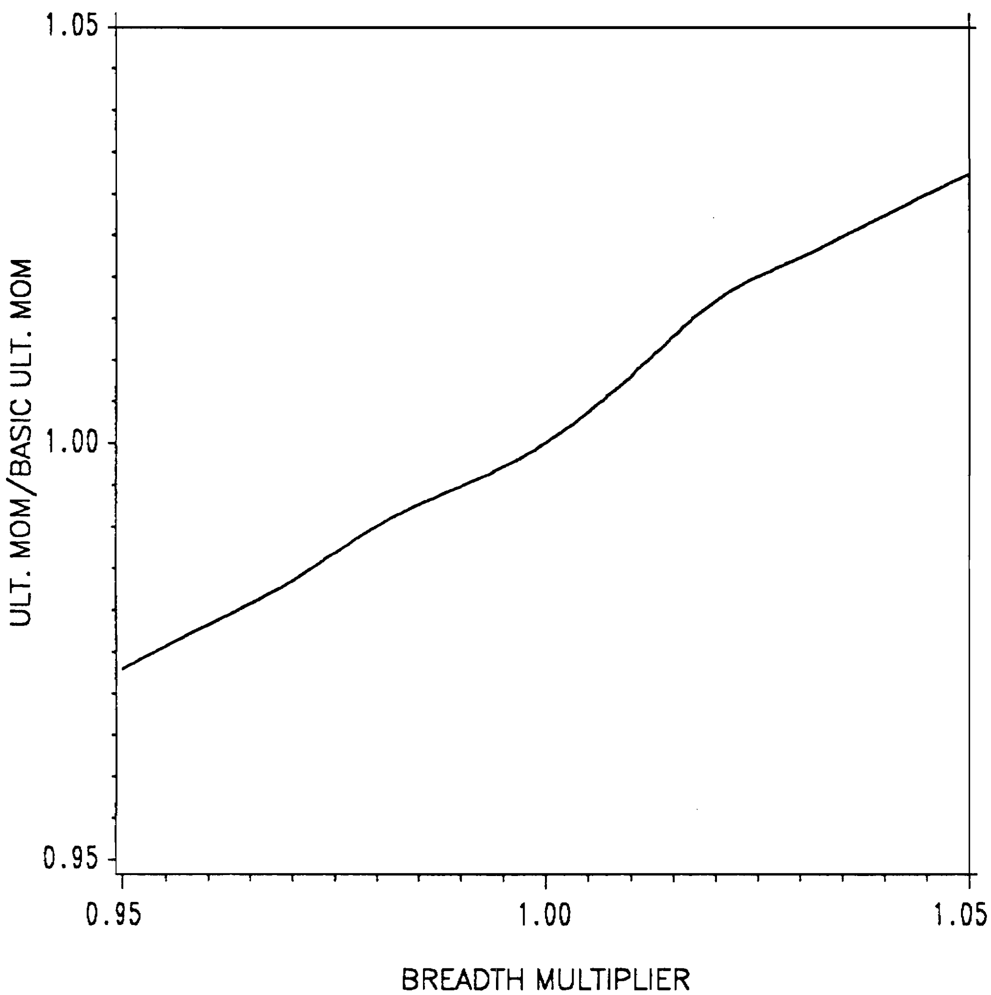


Figure 5.17 Breadth Multiplier VS. Ultimate Moment Ratio (Tanker2 - Sagging)

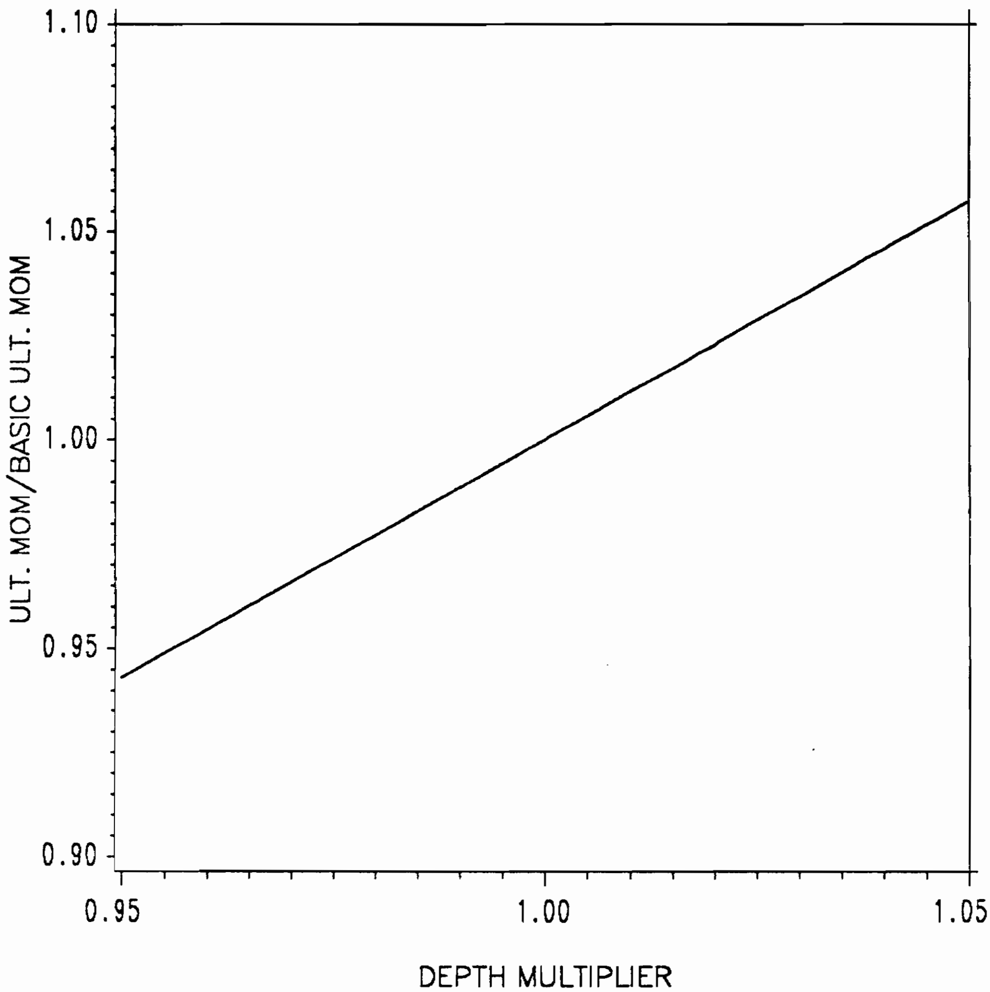


Figure 5.18 Depth Multiplier VS. Ultimate Moment Ratio (Tanker2 - Sagging)

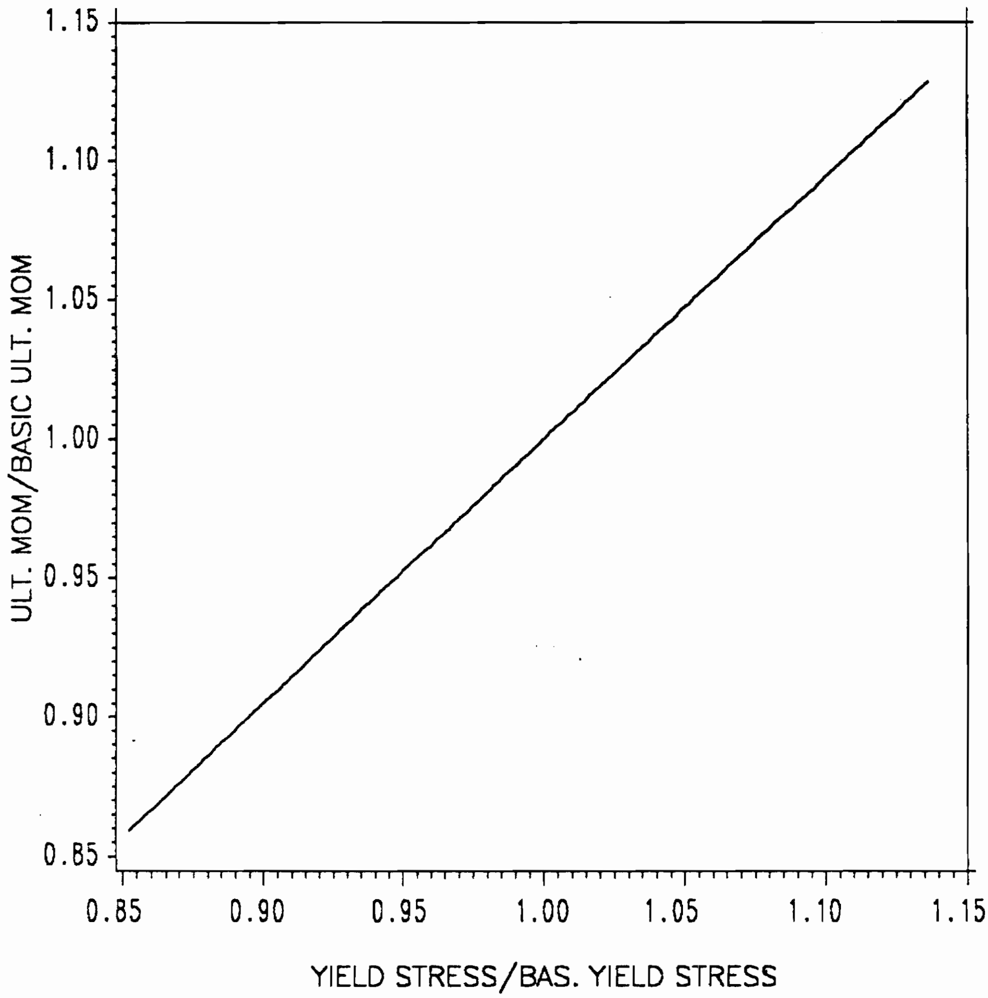


Figure 5.19 Yield Stress Ratio VS. Ultimate Moment Ratio (Tanker2 - Hogging)

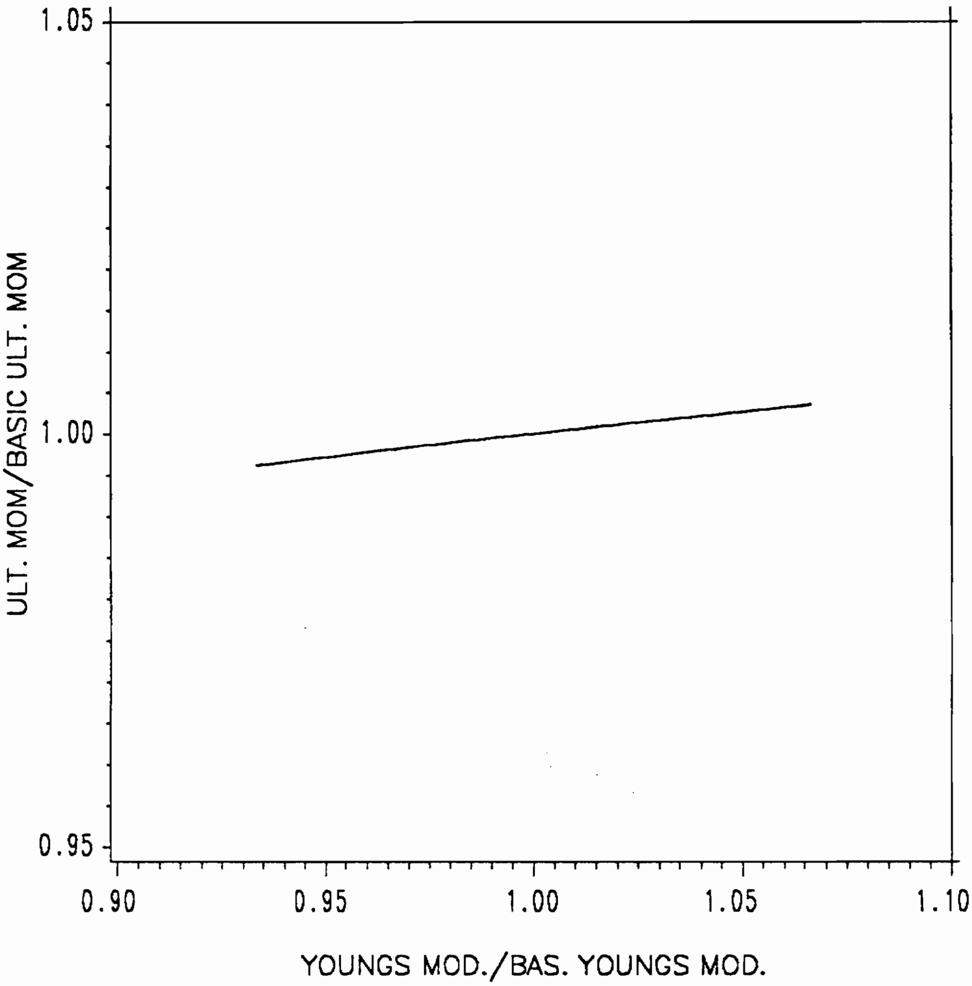


Figure 5.20 Young's Mod. Ratio VS. Ultimate Moment Ratio (Tanker2 - Hogging)

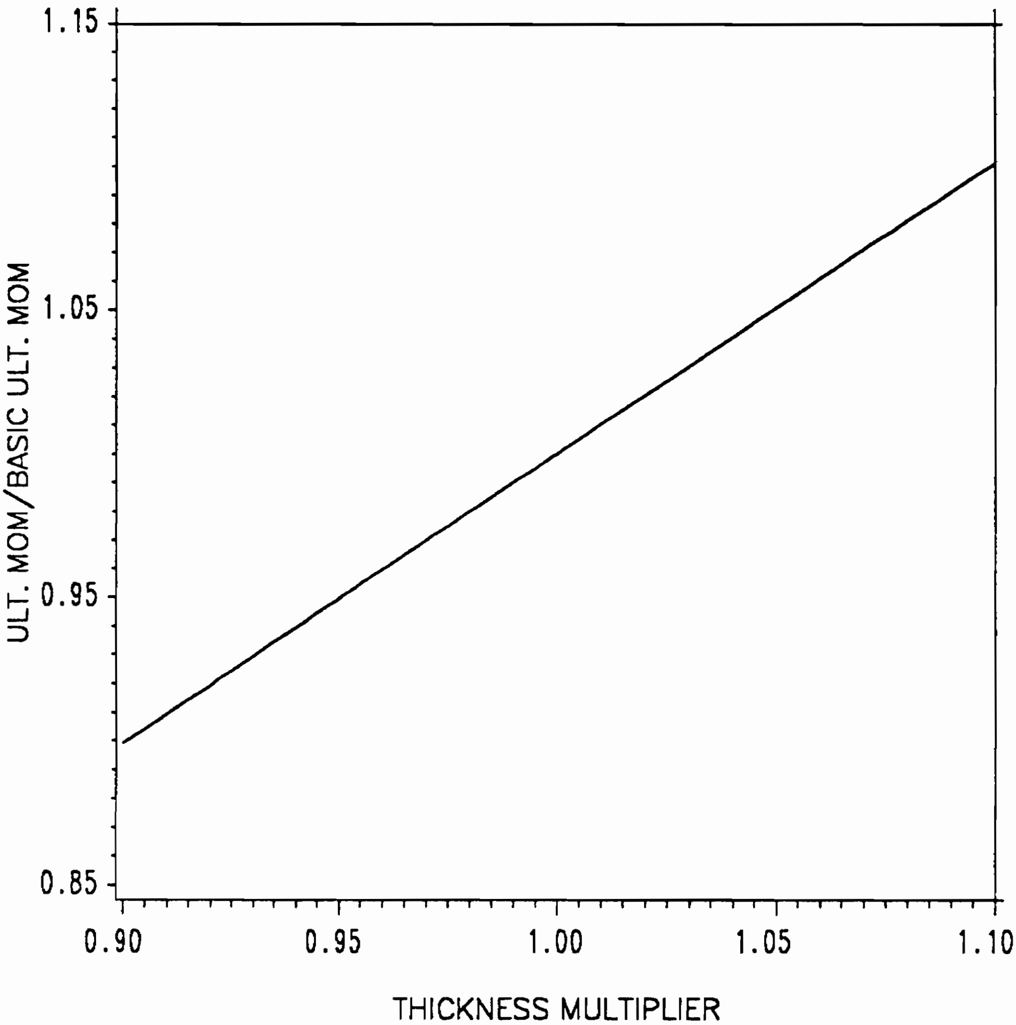


Figure 5.21 Thickness Ratio VS. Ultimate Moment Ratio (Tanker2 - Hogging)

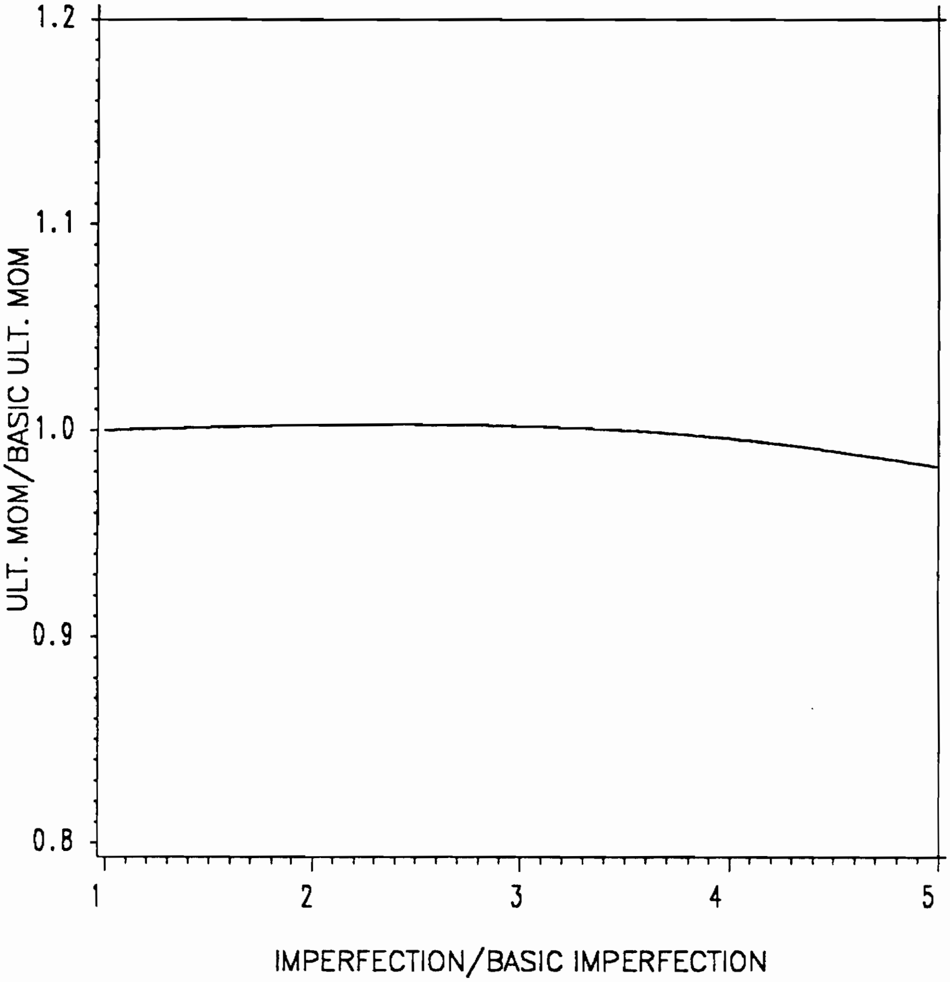


Figure 5.22 Imperfection Ratio VS. Ultimate Moment Ratio (Tanker2 - Hogging)

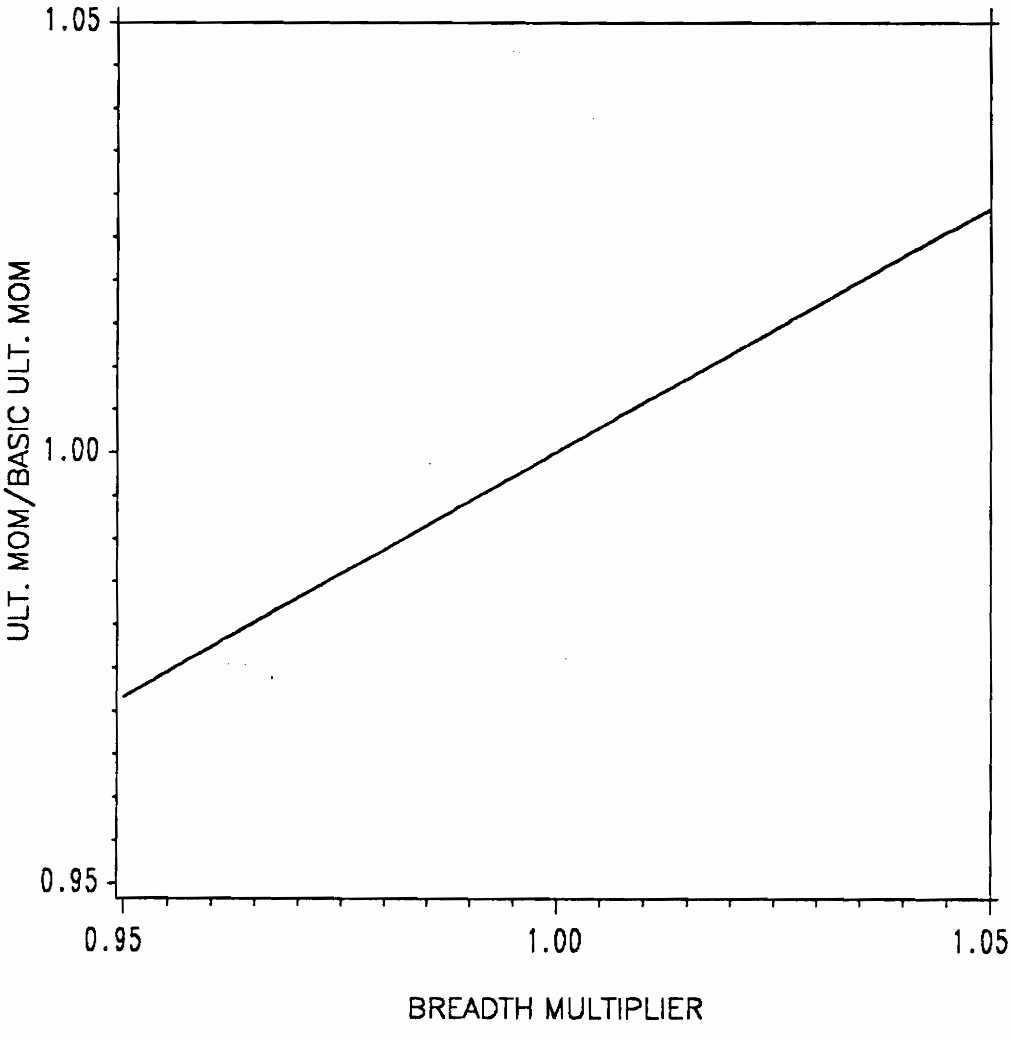


Figure 5.23 Breadth Multiplier VS. Ultimate Moment Ratio (Tanker2 - Hogging)

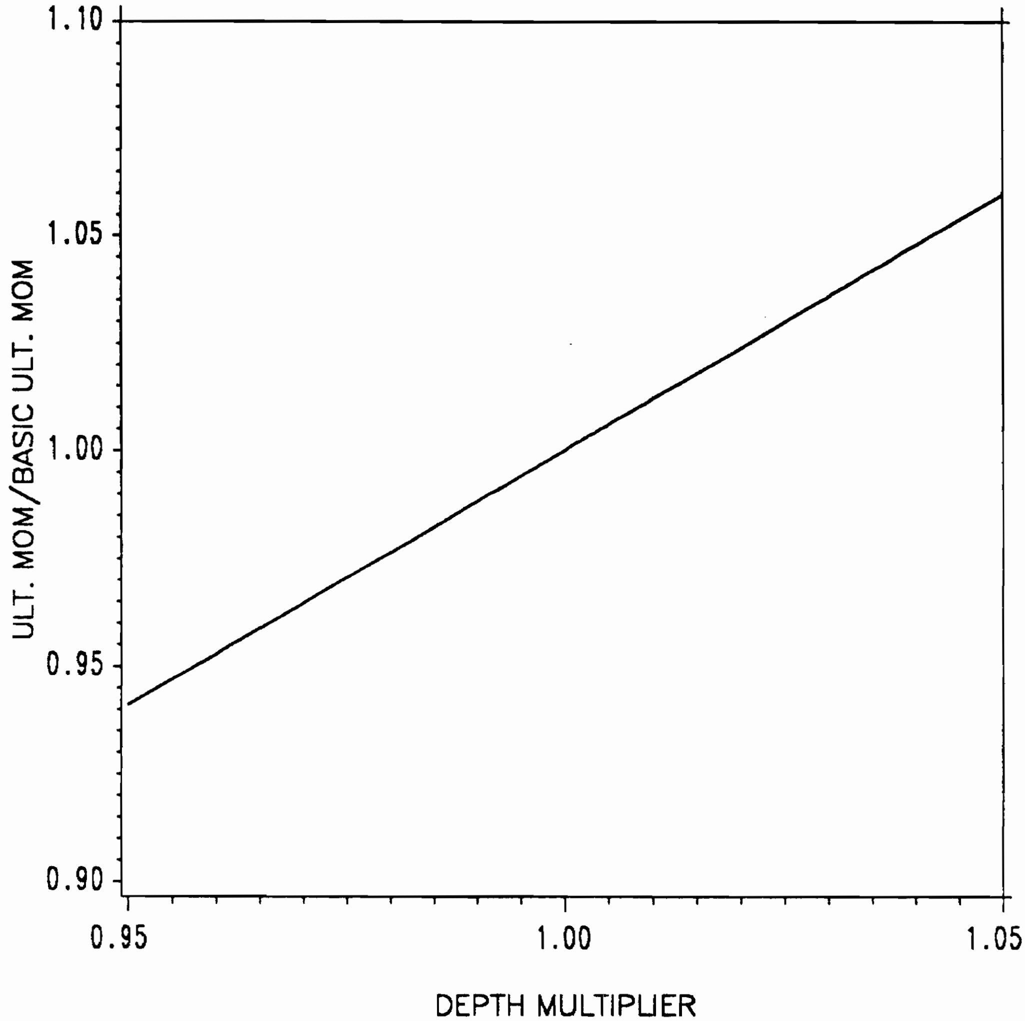


Figure 5.24 Depth Multiplier VS. Ultimate Moment Ratio (Tanker2 - Hogging)

Vita

The author was born in Nagpur, India on the 18th day of March, 1960. He received his Bachelor of Technology degree in Naval Architecture from the Indian Institute of Technology, Kharagpur, India in May, 1981. Following that, he worked in the design department of Mazagon Dock Ltd., Bombay, India for five years. In September 1986, he enrolled in the Master of Science program in the Aerospace and Ocean Engineering department at Virginia Polytechnic Institute and State University.