

Development of Reduced-Order Flame Models for Prediction of Combustion Instability

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ABSTRACT

Lean-premixed combustion has the advantage of low emissions for modern gas turbines, but it is susceptible to thermoacoustic instabilities, which can result in large amplitude pressure oscillations in the combustion chamber. The thermoacoustic limit cycle is generated by the unsteady heat release dynamics coupled to the combustor acoustics. In this dissertation, we focused on reduced-order modeling of the dynamics of a laminar premixed flame. From first principles of combustion dynamics, a physically-based, reduced-order, nonlinear model was developed based on the proper orthogonal decomposition technique and generalized Galerkin method. In addition, the describing function for the flame was measured experimentally and used to identify an empirical nonlinear flame model. Furthermore, a linear acoustic model was developed and identified for the Rijke tube experiment. Closed-loop thermoacoustic modeling using the first principles flame model coupled to the linear acoustics successfully reproduced the linear instability and predicted the thermoacoustic limit cycle amplitude. With the measured experimental flame data and the modeled linear acoustics, the describing function technique was applied for limit cycle analysis. The thermoacoustic limit cycle amplitude was predicted with reasonable accuracy, and the closed-loop model also predicted the performance for a phase shift controller. Some problems found in the predictions for high heat release cases were documented.

To my wife Jing

My parents

And my grandfather

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Nomenclature

| | |
|----------|---|
| A | Pre-exponent factor or the input velocity amplitude |
| C | Characteristic acoustic speed |
| C_p | Constant pressure specific heat |
| D | Combustor diameter in geometry |
| D_x | Matrix of partial derivatives with respect to states in x |
| E_a | Activation energy |
| f | Frequency in Hz |
| G_x | Transfer function |
| j | Imaginary number indices |
| J | Jacobian matrices |
| k | Wave number or control gain |
| L | Length of combustor or chimney |
| N_x | Describing function |
| P | Acoustic pressure |
| q | Total heat release rate (Watts) |
| R | Reaction rate term |
| t | Time variable |
| T | Temperature or period of oscillation |
| u | Total velocity |
| x | Spatial Variable |
| y | System output in time domain |
| Y | Mass fraction of species |
| δ | Dirac delta function |
| ρ | Density |
| ϕ | Equivalence Ratio |
| γ | Specific heat |
| ω | Frequency in radians per second |
| ϕ | Equivalence ratio |
| ψ | Streamline coordinates |

Subscripts & Superscripts

| | |
|---------------|-------------------------------------|
| $(x)'$ | Perturbation of variable x |
| (\bar{x}) | Mean variable x |
| (\tilde{x}) | Fundamental component of signal x |
| $(x)^{(n)}$ | n th derivative of variable x |
| $(x)_f$ | Referring to at the flame location |
| $(x)^T$ | Transpose of matrix or vector x |