OPTIMAL STRESS SCREENING FOR PRODUCTS SOLD UNDER WARRANTY

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(ABSTRACT)

In the face of increasing awareness among customers and today's competitive market, the warranty of a product has become an added feature in marketing strategy. A reliable product causes less warranty support cost. However, a more reliable product costs more to manufacture. Thus, a suitable trade-off between the cost and the benefit of a development and testing program is essential to optimize the performance measure, e. g., minimize total expected cost.

Renewal theoretic models of sequences of failures over the burn-in and warranty periods and their costs are developed. Contrary to the usual asymptotic assumptions, transient behaviors of the renewal processes are considered. The expected costs associated with in-plant and field failures are balanced against the costs of implementing a burn-in program. A multi-component series system with different Weibull distributions for the components are considered. Burn-in is performed at the assembly level and the components are assumed to have different age accelerations under a common stress regimen.

Models based on analyses both at the component and the system level are constructed. Two different burn-in policies are considered. These are "fixed duration" burn-in and "failure free" burn-in. A free replacement warranty for the components with policies of both fixed warranty period and renewed warranty period after each failure is considered in the models. The profit functions under different models are optimized with respect to burn-in period, stress parameters and warranty period. The models are extended to include reliability growth over the warranty period. Finally, solution procedures for optimizing the profit functions for all cases are given.

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CHAPTER 1 INTRODUCTION

1.1 Problem Description

A warranty is a contractual obligation incurred by a manufacturer or vendor in connection with the sale of a product. The warranty specifies that the manufacturer agrees to remedy certain defects or failures in a product for a specified period after sale. Warranties are important to both customers and manufacturers. Customers need warranties to assure that the products will perform satisfactorily. Manufacturers use warranties mainly for promotional purposes. In the face of increasing awareness of reliability among customers and today's competitive market, the warranty of a product has become an added feature in marketing strategy. It is considered to imply high reliability.

Before a product is marketed it goes through a reliability improvement program. Since a warranty is typically concerned with the early life of the product, the reliability of the product is an important factor for early life performance of the product. For this reason some form of post-production reliability screening is often necessary to eliminate the early failures before they are experienced in the field causing expensive repairs, inconvenience to the customers and long term loss of goodwill. These early failures are caused by hidden flaws or defects which are not detected during the product testing stage. Although good design and manufacturing help to avoid defects considerably, it is never possible to eliminate them completely. Burn-in screening or accelerated stress screening is a systematic procedure for removing the early failures of the products while they are still in the shop. However, carrying out the burn-in process costs money in terms of set up and other fixed costs as well as the cost of carrying out the screening process and the cost of product loss during screening.. Thus, a more reliable product can be delivered only at increased cost. At the same time, a product with a longer warranty can be sold at a higher price. Therefore, from the manufacturer's point of view, there exists an optimal screening level and warranty period for the products sold.

Burn-in as a screening method is most prevalent for electronic equipment. Electronic components, most notably silicon integrated circuits (IC), tend either to last a relatively long time or to fail very early in their operation. They have a high initial failure rate that decreases rapidly with time. This period is known as the "infant-mortality period" (refer to the section on review of infant-mortality in Chapter 2). Burn-in operation approximates the early life of the component/system in a compressed time under a stress environment (refer to the section on review of accelerated testing given below). Typically (traditionally), elevated electrical and thermal conditions constitute this accelerated testing environment.

Some of the devices that have extremely high infant mortalities or for which burn-in is needed most are:

1) LSI or VLSI devices; e. g., CPU's, memories, hybrids.
2) power devices; e. g., SCR's, triacs.
3) certain devices which involve complex and expensive manufacturing process; e. g., truck/tank engines or missile systems,
4) equipment in locations where it is not possible to replace parts; e. g., space shuttles, and submarine cables.

Burn-in finds the most application when field failure must be kept to a minimum or when poor performance in the field requires extensive down time and/or repairs. Inexpensive devices where repair is simple and the consequences of failure are trivial may not need burn-in at all. Most industrial products fall somewhere between these two extremes.

For all these electronic devices, the manufacturers need to determine an optimal level of burn-in and the warranty period. Most of the available literature treats optimal burn-in and optimal warranty policies as separate problems and to our knowledge no explicit attempt has been made to combine these two problems under one modeling framework. Also, most of the work on burn-in considers the entire system as a single unit or burn-in is considered at the component level and the components are subsequently assembled in a system. Nearly all studies on stress screening focus on thermal stress only. A multiple stress environment is more appropriate as an acceleration strategy as different failure mechanisms that are responsible for the early failures of the products may be activated by different stresses. It has been observed that for some types of devices, the simultaneous use of multiple stresses provides the opportunity to obtain extreme accelerations with very modest stresses. Almost all the cost models for expected number of failures considered in the literature are developed based on either an average estimate considering cumulative probability of a failure or a hazard rate function. Neither of these two approaches are exact. A more precise model can be obtained by considering sequences of failures for the components of a system as a renewal process. In a few cases, asymptotic results based on renewal theory are used to obtain an expression for life-cycle cost for equipment. Contrary to the usual asymptotic assumptions, transient behaviors of the renewal processes are considered in this research to capture the initial transient characteristics of the failure process involved over the screening and warranty periods of the product.

1.2 Research Objectives

The aim of this research is to define a method for selecting stress screening and warranty policies for a product sold under warranty and obtain optimal burn-in duration, stress parameter levels and the warranty period for the product. A profit function reflecting the cost of screening and the associated costs of failures over the burn-in and warranty periods, and the revenue earned from such products sold, is developed and best solutions are obtained with respect to the decision variables.

A multi-component series system is considered in this study. Components are assumed to have Weibull life distributions with decreasing failure rates. Burn-in is performed at the assembly or system level and components have different age accelerations under a common multiple-stress environment. Two different burn-in policies are considered in this research effort. These are "fixed duration" burn-in and "failure free" burn-in. Under "fixed duration" burn-in, the system is subjected to a screening process for a fixed period whereas for "failure free" burn-in, it is carried out until a failure free period is obtained. A free replacement warranty for the components with policies of both fixed warranty period and renewed warranty period after each failure within the warranty period are studied.

The sequence of failures over the screening period is modeled by considering ordinary renewal process. It is assumed that the components have random ages at the end of the burn-in process and the failures over the warranty period constitute a delayed renewal process. The expressions for the total cost functions are obtained for different models. Finally, a net profit function is developed by considering the selling price of a product as a function of its warranty period.

The "non-convex" nature of the objective functions precluded the possibility of obtaining a global optimum. An optimization routine is developed by modifying the Nelder and Mead (37) algorithm to handle the upper and lower bounds on the decision variables. The optimization routine is performed with different sets of initial points identifying several local minima. The best local minima is taken to be the best solution.

It is observed that for electronic products the best solutions do not differ significantly when warranty policy changes. For systems with a relatively smaller number of components, failure free burn-in does not appear to be more cost effective or profitable than fixed period burn-in.

It is proved analytically that the stress variables in the models should always be at upper bounds when stress screening costs are assumed to be constant and independent of stress levels.

1.3 Outline of the Research

Chapter 2 provides a review of the relevant literature on burn-in optimization and warranty analysis. Several models on warranty cost and analysis are discussed. Background descriptions of early failures and burn-in are also provided.

A fixed period burn-in policy is considered in Chapter 3. For a fixed period burn-in stress screening is performed over a fixed period. Under this strategy three cost models are developed corresponding to three warranty policies. A free replacement warranty for products with policies of both fixed and renewed warranty period is considered. A warranty policy in which failed components are replaced by a more reliable component is also considered.

Chapter 4 considers a failure free burn-in policy. Under this strategy burn-in is carried out until a failure free period is obtained. Cost models are constructed for fixed as well as renewed warranty policies. Also, an expression for a lower bound for the mean time to test completion is derived.

In Chapter 5 optimization procedures and numerical examples for the models are presented. Illustrative examples of the optimization procedure for the models are given and optimal stress screening duration, stress levels and warranty period are obtained. Solutions obtained under different models are analyzed and it is concluded that for all burn-in policies optimal levels for the stress variables are at the upper bounds. The aging of a product through acceleration takes place at a minimum cost when the stresses are at the upper limits that the product can withstand without any damage. Finally, some of the models are simulated in order to validate the numerical results.

Chapter 6 contains conclusions and suggestions for future work. Theoretical and numerical results obtained in the earlier chapters are discussed. Assumptions in the present study are also discussed and scope for future research is suggested.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

This chapter provides an overview of the existing literature related to burn-in optimization and warranty analysis. This has motivated pursuit of this current research effort in combining these two closely connected problems. To our knowledge, no explicit attempt has been made to investigate these two aspects as one single problem.

As this research effort concerns the elimination of early failures or "infant-mortalities" through in-house accelerated aging, a brief review of accelerated testing is relevant to the development and understanding of the model. Also, a brief review of "infant-mortality" and the relevant work is given.

2.2 Warranty Analysis

Most of the early work in the area of warranty analysis involves estimation of warranty costs. These expenses incurred by the manufacturer during the warranty period represent reductions to profit that lag the production. Menke (32) develops one of the first models for estimation of the required warranty reserve for non-repairable products where an explicit warranty is in force. Subsequently, warranty cost modeling and analysis received considerable attention in the literature (1, 3, 21, 23, 26, 30, 38, 50).

Several models offer modifications to Menke's approach and include repairable items. The basic concern in all of these models is the estimation of the expected warranty cost for each item sold under either total rebate policy i. e., failure-free warranty, or pro-rata warranty policy. Blischke and Scheuer (6) and Nguyen and Murthy (38) derive expressions for approximate total costs over the product life cycle by applying renewal theoretic results. None of these models consider burn-in or any kind of reliability improvement process for the products sold under warranty.

Nguyen and Murthy (38) consider two general warranty policies. In policy I, the warranty is renewed only after each purchase (i. e. failure outside the free replacement period), whereas in policy II, the warranty is renewed after each failure. Two types of counter models, nonparalyzable and paralyzable, are used to represent these warranty policies respectively. Nguyen and Murthy (39) have also studied the reliability allocation to products sold under warranty. They develop a general model for obtaining the optimal reliability allocation for such products. The cost function is presented as the sum of manufacturing costs and the costs of servicing the warranty. These cost functions are assumed to be explicitly known in terms of an arbitrary set of controllable parameters and warranty duration. Certain reliability requirements define the domain of these parameters. Hence the problem is reduced to a standard non-linear optimization problem and is solved by using Lagrangian multipliers.

In a later paper, Murthy and Nguyen (33) also consider optimal development testing policies. In this study, they consider two cost models for development programs. In the first model, they assume the failure rate of the product during development testing is dependent on the modification rate. Two cases are considered under this model. For the first case it is assumed that failure rate of the product decreases continuously (Duane's model, 17) and in the second case it changes after each modification (NHPP describes the modification process). In the second model, the failure rate of the product during development testing is independent of the modification rate. In this model the failure rate of the product changes after each modification and the amount of reduction in the failure rate is independent of the modification time. Their model treats the preproduction development process which determines the failure rate of the product when it is put into production. Thus, in their model the warranty cost is dependent on the failure rate of the product after the development program is completed and the products are assumed to have an exponential life distribution.

2.3 Burn-in Cost Models and Optimization

Very rarely it is true that "money is no object" in the business world. In the few cases where this may be true and where performance is extremely important, we may be justified in using a decision model containing only physical performance criteria. Unfortunately, in most of the operations in the business world, economy is important. The ultimate objective is to carry out performance requirements in the most economic way possible. Cost reduction has been the central theme for a burn-in program since its beginning. A number of cost models have been suggested in the literature. The significant ones are reviewed here.

<u>Model 1</u>

Several authors have considered the total cost function, C_t, over the life cycle of the system (2, 31, 40, 48). The cost model is developed as:

$$\begin{aligned} C_t = C_0 + C_1 b &+ \textit{(unit repair cost during burn-in)} \\ &\times \textit{(expected number of failures during burn-in)} \\ &+ \textit{(unit repair cost during deployment)} \\ &\times \textit{(expected number of failures during deployment)}\,, \end{aligned} \tag{2.3.1}$$

where C_0 is a constant overhead cost, and C_1 is the burn-in cost per unit of burn-in time b. A nonhomogeneous Poisson Process is chosen by them to model the failure process. A declining hazard rate is considered for these models. The expected number of failures are obtained by integrating the hazard function. Ambekar (2) modified this model by incorporating the present worth analysis for the field repair costs. Marko and Schoonmaker (31) define the life-cycle cost as burn-in costs plus field repair costs. They also optimize a spare module burn-in by minimizing a part of the life-cycle cost using exhaustive search techniques. Increased costs associated with added burn-in are compared to field savings from failure reductions until an optimal solution is identified.

Plesser and Field (40) optimize the above expected cost function for optimal burn-in duration for a unit. In their approach, the assumptions made are:

- there is a clear demarcation between the infant-mortality and the steady state failure rates.
- repair actions restore the system to a bad-as-old condition.

<u>Model 2</u>

Washburn (51) presents a utility function or total cost model for a burn-in process and incorporates system effectiveness in the model. He gives the total cost function as:

$$C_t = C_1 b + C_2 . N . F(b) + C_3 k [P_E (b)]^{-1} . \tag{2.3.2}$$

The first term involves the cost of operating the burn-in facility for a time b. C_1 is the prorated cost of the facility which is amortized over its service life. It includes capital investment, interest, overhead maintenance and all related costs for physical plant and equipment. Added to these are the costs of all direct and indirect labor and a reasonable rate of return on investment.

In the second term, C_2 is the cost per unit that failed during the burn-in process. This includes the cost of producing a unit and having the burn-in. N is the number of units placed on burn-in and $F(b)$ is the cumulative probability of failure up to a burn-in time of b. Life distributions for the units follow a generalized gamma distribution. Hence, the product $C_2NF(b)$ is the total cost of the units failed in the burn-in process.

In the third term, C_3 is the cost associated with selling a unit that survives the burn-in process. It includes the cost of handling, shipping associated with delivery of the end product to the customer. k is the minimum number of units that has to satisfy a system effectiveness factor $P_E(b)$ which is a product of mission reliability and a measure of operational readiness. Operational life for the unit is assumed to begin with the burn-in process. Finally, Washburn suggests an experimental method to solve the optimization problem in terms of optimal burn-in time.

Model 3

A stochastic total cost function as a function of time for burn-in under a policy of replacement on failure is formulated by Chandrasekaran (8). For this policy the total expected cost $C_t(t)$ for a operational period of t is given as:

$$C_t(t) = C_b\,[\alpha] + C_N\,[\beta] + C_r\,[\,M(t)\,]\,, \qquad (2.3.3)$$

where

α = total hours spent by all items in the burn-in process

β = total number of items undergoing burn-in

$M(t)$ = number of replacements or renewals in a renewal process $[0, t)$ with an underlying life distribution $G(x)$ and is given as:

$$G(x) = [\,F(b + x) - F(b)\,] \,/\, \overline{F}(b)\;, \qquad (2.3.4)$$

C_b, C_r and C_N are burn-in cost, cost of replacing a failed item and cost incurred by the number of items undergoing burn-in respectively. α and β are given as:

$$\alpha = \beta \left[b \overline{F}(b) + \int_0^b t \, dF(t) \right], \tag{2.3.5}$$

b = burn-in time

$$\beta = \frac{M(t)}{\overline{F}(b)} . \tag{2.3.6}$$

The total life-cycle cost rate function is minimized under the assumption that t is a large number. In this model the underlying assumptions are:

- a single unit or a system as a whole is considered for burn-in,

- burn-in is carried out until a failure free period of b is achieved, i. e., age on the products passing the burn-in is known.

- t being a large number implied that transient characteristics of the failure process are not considered in the model. $\lim_{t \to \infty} \frac{M(t)}{t} = \frac{1}{L(b)}$ is assumed, where $L(b)$ is the mean life of an item burned-in for b units of time.

This model does not consider the failure process over the burn-in period as a renewal process. The remaining life of the component at the end of the burn-in period is given by survival distribution $\overline{F}(b)$. However, the components would have random lives at the end of the burn-in period when the screening is carried out at the system level. The model also fails to treat the transient characteristics of the failure process over the warranty period and considers its asymptotic behavior under the assumption of large t.

Model 4

Weiss and Dishon (53) express the expected lifetime of the component as a function of burn-in time b. They present the objective function as the cost per component per unit of expected lifetime, $C(b)$, as:

$$C(b) = \frac{R(b)}{\mu(b)} + \frac{(1 + \alpha)}{\mu(0)} [1 - R(b)] . \tag{2.3.7}$$

A single replacement program in which the burn-in procedure starts with the number of required components (N) and failures are replaced by brand new components at time b at the end of the burn-in process, is considered in this model. In the above function, α is the cost of a single repair in units of component cost, and $\mu(b)$ is the mean time to failure of a device subjected to a burn-in period of b. This is given as:

$$\mu(b) = \int_0^{\infty} \frac{R(t+b)}{R(b)} dt , \tag{2.3.8}$$

$R(.)$ being the survivor function.

In another case, Weiss and Dishon (53) consider a multiple replacement program in which burn-in starts with $M>N$ components. Components that fail during burn-in are replaced either immediately or at a discrete number of intermediate times. For a simple case of discrete replacement times with $T_2=b$ and $T_1=\theta b$, where $0<\theta<1$, they have the cost function expression as:

$$C(\theta, b) = 1 + R(\theta b) - R(b) + [1 - R(\theta b)]\,[2 - R(\,(1-\theta)b\,)] . \tag{2.3.9}$$

__Model 5__

Kuo (27) formulates a system life-cycle cost function $C_S(f, b_i)$ as a function of the system configuration f, different component burn-in periods b_is', costs of the components, device burn-in, shop repair, field repair, and loss of product reputation. Components are burned-in for various periods b_i's before they are placed together in a system. In this model, the system starting point for operating is assumed to be time 0. At time 0, each component is located differently at its distinct infant mortality curve. The expected fractions of failures for the components are calculated by integrating the hazard functions. The early device failures are assumed to have a Weibull distribution with a decreasing failure (hazard) rate. Kuo suggests a method for optimization of the cost function with respect to burn-in periods for the components under a set of constraints of minimal reliability requirements given as:

$$R_S(t/b_i) \geq R_{s,min}(t) , \tag{2.3.10}$$

$$R_i(t/b_i) \geq R_{i,min}(t), \qquad \forall i \tag{2.3.11}$$

where $R_S(t/b)$ and $R_i(t/b)$ are the system and device reliability respectively at operational period t. A thermal stress at an elevated temperature is considered for the burn-in process.

All these models consider the entire system as a single unit or burn-in is considered at the component level. Kuo (27) considers burn-in at the component level and the components are subsequently assembled in a system. None of the models address the problem of system level burn-in. A total life cycle cost function is considered by Chandrasekaran (8) in his model. This model is adequate when burn-in is carried out on a single unit for a fixed period and the product has a large operational period. Plesser and Field (40) assumes a known infant-mortality period for products and develop a total cost function. Weiss and Dishon (53) consider a fixed period burn-in model in which the burn-in procedure starts with a number of similar components. The failed components are replaced by new components at a discrete number of intermediate times or at the end of the burn-in process.

2.4 A Brief Review of Infant Mortality

A brief review of "infant-mortalities", that discusses the possible types of defects or flaws which may be responsible for the early failures in an electronic equipment is given.

It is widely believed that the early failures and useful lives of a population of devices come from a single distribution. The early failures or infant mortalities are considered to form the skew part of a single population. This population also consists of the distribution of a component's useful life. Factors contributing to infant mortality include:

1) surface anomalies - for example: corrosion, contamination and inversions;
2) process and quality defectives - moisture entry, corrosion, electrostatic discharges and process faults;
3) workmanship errors.

Although good design and manufacturing help considerably, the above problems cannot be eliminated entirely.
The distribution function of the infant mortality stage has been modeled as a Weibull distribution (2, 15, 25, 27, 28), a log-normal distribution (7, 52), a nonhomogeneous Poisson process (40), and an empirical distribution (12, 16, 31, 44, 45).

Contrary to the hypothesis of a single population, several researchers (22, 24, 25) have shown that the life distribution of some electrical components follow a bimodal distribution, a mixed distribution of two subpopulations. The strong portion of the bimodal distribution, centered around an expected value, is labeled the main distribution and the weak portion, started at the beginning of the life cycle known as the early failures, is the freak distribution. The freak portion has been assumed to contribute to the infant mortality. They consider the bimodality of the component lifetime to evaluate the efficiency of the burn-in process.

Two life distributions are considered for the weak population in the literature. Jensen and Petersen (24, 25) justify a Weibull life distribution while Hallberg (22) demonstrates a log-normal life distribution.

2.5 Comments

It is evident from the existing literature as discussed here that the models considered in this research proposal is unique in the aspects that:

- It unifies the problem of burn-in and warranty under one modeling framework,

- It concerns products at the post-production level where each product undergoes an aging process under accelerated stress environment. Products passing the screening test have random excess life which in turn determines the first failure time after deployment. The sequence of failures over the screening period is modeled by considering ordinary renewal process and those over the warranty period constitute a delayed renewal process.

- It captures the transient characteristics of the failure process and describes it more accurately through renewal theoretic modeling,

- Random aging of the components at the end of the burn-in process is considered rather than a fixed age attained by fixed period burn-in. Therefore, first time to failure after deployment is determined by the excess life distribution associated with the renewal process instead of the component life distribution,

- The performance measure in the form of total expected cost or net expected profit is optimized with respect to stress parameter levels in addition to burn-in period and warranty period,

- Screening is performed at the assembly or system level under a multiple stress environment.

2.6 A Brief Review of Accelerated Testing

The lives of most products, including discreet, LSI and VLSI semiconductor devices, follow the traditional bath tub curve. For semiconductor products, the time to reach the useful life period, i. e., the stable failure period, could be long. A stress environment is applied to the system to accelerate its aging process by compressing the time scale over which failures occur. The environment may be a single stress type, such as temperature, a sequence of thermal cycling, voltage and vibration or it may be a combination of such stresses.

Accelerated aging can be interpreted in terms of physical models and the acceleration factor for these models can be expressed in terms of the stress parameters. The physical models most often used are the Arrhenius relationship (19, 24, 25, 28, 35) and the Inverse Power Law (19, 35). It has been reasonably well established that acceleration due to thermal stress conforms to the Arrhenius equation, given as:

$$a = e^{\left[\frac{E_a}{K} \left(\frac{1}{T_u} - \frac{1}{T_a} \right) \right]} , \qquad (2.6.12)$$

where E_a is the component specific activation energy (in *ev*), K is the Boltzman's constant ($8.623 \times 10^{-5}\, ev/K$) and T_u and T_a are the temperatures in ^oK under use and accelerated condition, respectively.

The Inverse Power Law is used for voltage acceleration. The acceleration factor due to voltage stress is given as:

$$a = \left(\frac{V_a}{V_u} \right)^n , \qquad (2.6.13)$$

where V_a and V_u are the voltages under accelerated and use conditions, respectively.

The acceleration factor due to humidity may also be given as (19):

$$a = e^{[\ \alpha\ (RH_a - RH_u)\ ]}\ , \tag{2.6.14}$$

where α is a constant and RH_a and RH_u are the accelerated and normal relative humidity in percent.

The idea of accelerated testing is based on the assumptions that:

1) failure mechanisms involved during accelerated testing are identical to failure mechanisms present under normal operating conditions.

2) the general form or shape of the distribution is preserved under accelerated conditions.

This contraction in time due to accelerated testing is assumed to be a linear time transformation. It is a widely accepted assumption and the life distribution under accelerated aging, $F_a(t)$, under this assumption is given as:

$$F_a(t) = F(at)\ , \tag{2.6.15}$$

where the coefficient "*a*" represents aging acceleration factor, and $F(t)$ is the life distribution for the device under normal conditions. This assumption has substantial effect on the present analysis.

For a Weibull life distribution we have:

$$1 - e^{-\alpha' t^{\beta'}} = 1 - e^{-\alpha (at)^{\beta}}$$

$$\Rightarrow \qquad \alpha (at)^{\beta} = \alpha' t^{\beta'}$$

$$\Rightarrow \qquad a = (\alpha'/\alpha)^{1/\beta} t^{\beta'/\beta - 1}\ . \tag{2.6.16}$$

Since the shape parameter for the distributions under both accelerated and normal conditions are assumed to be identical, $\beta'=\beta$, and therefore acceleration is assumed to be independent of time and the acceleration factor is given as:

$$a = (\alpha'/\alpha)^{1/\beta} . \qquad (2.6.17)$$

It is noted that this expression for the acceleration factor and the expressions in terms of accelerated stress levels given in equations (2.6.12) through (2.6.14) imply a direct relationship between the stress levels and the scale parameter for the Weibull distribution α.

CHAPTER 3 DEVELOPMENT OF COST MODELS FOR FIXED PERIOD BURN-IN

3.1 Introduction

In this research, we wish to construct models for the cost structures associated with burn-in or environmental stress screening and the warranty commitment. Different burn-in strategies and warranty policies are considered. Fixed period burn-in and failure free burn-in are the two burn-in processes studied in this research.

In this chapter, cost models for the fixed period burn-in are developed. For a fixed period burn-in, stress screening is performed over a fixed period b. Under this strategy three models corresponding to three different warranty policies under which products are sold are defined. A free replacement warranty for the products with policies of both fixed warranty period and renewed warranty period is considered. A warranty policy in which failed components are replaced by a more reliable component is also considered. For all models, the same warranty periods are considered for all components in the system. These models can be extended to include different warranty periods for the components.

3.2 Assumptions

The following assumptions are made for the system under consideration and in developing the models.

1. An n component series system is considered for analysis. Failure of a component results in a failure of the entire system.
2. Stress screening is performed at the assembly or system level under a multiple stress regimen.
3. Components experience different age accelerations and the acceleration factor for the jth component, a_j, due to several stresses is given as (36):

$$a_j = \prod_{i=1}^{k} a_{ji} ,$$

where a_{ji} is the acceleration factor for component j when subjected to stress i and a_{ji} is given as:

$$a_{ji} = e^{[\, c_{ji} + d_{ji} s_{ai} \,]} .$$

where c_{ji} and d_{ji} are constants and s_{ai} denotes the accelerated level of the stress i.

4. Each component has a Weibull life distribution with scale parameter α_j and shape parameter β_j.
5. The stress screening process is monitored continuously and components that fail during burn-in are replaced immediately.
6. Repair time is measured on a different time scale, i. e., repair time is excluded from operating time, or repair time is considered to be negligible.
7. The total number of expected system failures is equal to the sum of total number of expected failures for all the components. This follows from the first assumption.

3.3 Notation

3.3 Notation

The following notation is used in the text.

$f(t)$ - failure time density function.

$F(t)$ - failure time cumulative distribution function.

$R(t)$ - Survivor function.

$h(t)$ - hazard function = $f(t)/R(t)$.

X_i - time between $(i\text{-}1)st$ and ith failure.

S_n - total time to the nth failure = $\sum_{i=1}^{n} X_i$

$f^{(n)}(t)$ - probability density function of S_n

$F^{(n)}$ - cumulative distribution function of S_n

$N(t)$ - (random) number of failures in $[0, t]$.

$M(t) \equiv E[N(t)]$ - expected number of renewals or failures in $[0, t]$.

$Y(b)$ - time from b to next renewal (excess life at b).

$G(t, b) \equiv G(t)$ - cumulative distribution function for the excess life at time b.

$g(t, b)$ - density function for the excess life at time b.

α_j - scale parameter of the Weibull life distribution for the jth component.

β_j - shape parameter of the Weibull life distribution for the jth component.

p_i - fixed cost for stress parameter i.

q_i - cost of maintaining unit difference in stress levels for unit time.

c^i_j - cost of an in-plant failure for the jth component.

C^i_j - total expected cost of in-plant failures due to jth components.

TC^i - total expected cost of in-plant failures due to all the components.

$c(\underline{S}_a,b)$ - cost of carrying out burn-in when stress levels are $\underline{S}_a$ and burn-in duration is b.

$TC^i(b)$ - total expected in-plant cost of having burn-in for a period b.

c^f_j - cost of a field failure for the jth component.

C^f_j - total expected cost of field failures due to jth components.

$TC^f(w)$ - total expected cost of field failures due to all the components.

$\overline{c}^f$ - average cost of a field failure = $\sum_{i=1}^{n} c^f_j/n$.

$TC_{fpb}(b,w)$ - total cost of a product with warranty period w burned-in for a fixed period b.

$TC_{ffb}(b,w)$ - total cost of a product with warranty period w burned-in for a failure free period b.

$NP_{fpb}(b,w)$ - net profit of a product with a warranty period w burned-in for a fixed period of b.

$NP_{ffb}(b,w)$ - net profit of a product with a warranty period w burned-in for a failure free period of b.

t_{ffb} - expected time of completion for burn-in under failure free burn-in strategy.

b - burn-in duration.

b_{ffb} - failure free burn-in duration.

w - warranty period.

$p(w)$ - selling price for the product with a warranty period of w.

subscript j refers to the jth component.

subscript s refers to system level.

subscript D refers to a delayed renewal process.

subscript r refers to a renewable warranty policy.

subscript g refers to the reliability growth over the warranty period.

3.4 Fixed Period Burn-in

Under this burn-in strategy the system is burned-in for a fixed period of time irrespective of the failures of the components during the burn-in process. As a result, components have unequal random ages at the end of the burn-in process and each component has an age less than or equal to b after burn-in. Also, the optimal screening level for the products is dependent on the warranty policies.

Three cases corresponding to three warranty policies are studied under this burn-in strategy.

<u>Model 1</u>

In this model, a free replacement policy for any failed component over a warranty period w is considered. The failed components are replaced by similar components.

That is, no reliability growth is considered for the components. The replaced components do not carry a renewed warranty.

The total expected cost $TC_{fpb}(b,w)$ of having burn-in, over a period b for a product sold with a warranty period w under the "fixed period" burn-in strategy is given as:

$$TC_{fpb}(b,\ w) \ = \ c(\underline{S}_a,b) \ + \ TC^i(b) \ + \ TC^f(w) \ . \tag{3.4.1}$$

The first term in the expression for this cost function refers to the cost of carrying out burn-in over a burn-in period of b when stress levels are at $\underline{S}_a$. In general, the stress application cost function may have any of several forms and may differ among assembly types. In this study, a function that has a fixed cost and a variable cost which increases linearly in S_{ai} and in b, is assumed:

$$c(\underline{S}_a,\ b) \ = \ \sum_i (\ p_i + q_i\ (S_{ai} - S_{oi})\ b)\ . \tag{3.4.2}$$

The second term in the total cost function (3.4.1) is the total expected cost of in-plant failures due to all the components and the last term is the total expected cost of field failures due to all the components.

In order to develop this cost model, consider the well known results for renewal process, (Ross, 43),

$$N(t) \ \geq \ n \quad \textit{if and only if} \quad S_n \leq \ t \ \ , \tag{3.4.3}$$

$$Pr\{N(t) \geq n\} = Pr\ \{S_n \leq t\ \} = F^{(n)}(t)\ , \tag{3.4.4}$$

$$Pr\{N(t) = n\} = F^{(n)}(t) - F^{(n+1)}(t) , \tag{3.4.5}$$

with $F^{(0)}(t) = 1$.

Therefore, the expected number of renewals in $[0, t]$, $M(t)$, is given as:

$$M(t) = \sum_{n=0}^{\infty} n.\ Pr\ \{\ N(t) = n\ \} . \tag{3.4.6}$$

By using Equation (3.4.5), we have $M(t)$ as:

$$M(t) = \sum_{n=1}^{\infty} F^{(n)}(t) . \tag{3.4.7}$$

Now, the expected number of renewals or failures for the *jth* component over burn-in period b is given as:

$$M_j(a_j b) = \sum_{n=1}^{\infty} F_j^{(n)}\ (a_j.b\ ) , \tag{3.4.8}$$

where $F_j(t)$, the failure time distribution for the component j is given as:

$$F_j(t) = 1 - e^{-\alpha_j . t^{\beta_j}} . \tag{3.4.9}$$

This Weibull renewal process can be evaluated numerically by expanding it into infinite series of approximate Poissonian functions of αt^{β} (Lomnicki, 29). $M(t)$ is expressed as:

$$M(t) = \sum_{n=1}^{\infty} F^{(n)}(t) = \sum_{s=1}^{\infty} D_s\,(\alpha t^{\beta}) \sum_{k=1}^{s} c_k(s)\,, \tag{3.4.10}$$

where $D_s(t)$ is given as:

$$D_s(t) = e^{-t} \sum_{r=s}^{\infty} \frac{t^r}{r!}\,, \tag{3.4.11}$$

and the $c_k(s)$ are functions of β and can be evaluated by considering certain Gamma Functions (29).

Now, as the total expected number of system failures over the burn-in period b is the sum of the expected number of failures for all the components, it is given as:

$$M_s(b) = \sum_{j=1}^{n} M_j(a_j.b)\,. \tag{3..4.12}$$

The total expected cost of in-plant failures for component j, $C_j^i(b)$ is given as:

$$C_j^i(b) = c_j^i\, M_j(a_j b)\,. \tag{3.4.13}$$

Therefore, the total expected cost of in-plant failures for all the components, $TC^i(b)$, is given as:

$$TC^i(b) = \sum_{j=1}^{n} C_j^i\,(b)\,. \tag{3.4.14}$$

Next, we wish to determine the warranty support cost for the products. In order to do this, we need to define the expected number of renewals for each component over the warranty period. The renewal processes for the components are delayed or general in type. This is because if a renewal does not occur at time b, at the end of the burn-in process, then the distribution of the time that we must wait until the first observed renewal is not the same as the distributions for subsequent times to failure.

Consider the survival or excess life distribution for a component at the end of the burn-in period b. The waiting period t for a component from the end of burn-in period b until the next failure is given as:

$$G(t, b') = Pr\{Y(b') \leq t\} = \int_0^t m(b' + t - y)[1 - F(y)]dy \; . \qquad (3.4.15)$$

where $m(\,)$ is the associated renewal density function and b' is the equivalent age for the component following age acceleration.

For this delayed renewal process,

$$Pr\{N_D(t) = n\} = G{*}F^{(n-1)} - G{*}F^{(n)} \; , \qquad (3.4.16)$$

and the expected number of renewals in $[0, w]$, $M_D(w)$, is:

$$M_D(w) = \sum_{n=1}^{\infty} G{*}F^{(n-1)}(w) \; , \qquad (3.4.17)$$

where $G(w,b')$ is given by equation (3.4.15).

Let us now consider the renewal function for the delayed renewal process:

$$G*F^{(n-1)}(w) = \int_0^w \int_0^w m(b' + w - z - y)[1-F(y)]dy\, f^{(n-1)}(z)dz$$

$$= \int_0^w \int_0^w H_1(w - z)\, f^{(n-1)}(z)dz\, \bar{F}(y)\, dy \qquad (3.4.18)$$

where $H_1(w) = m(b'+ w - y)$. If the inner integral in the expression is considered as a convolution, the following is obtained:

$$\int_0^w \int_0^w H_1(w - z)\, f^{(n-1)}(z)dz\, \bar{F}(y)\, dy$$

$$= \int_0^w H_1*F^{(n-1)}(w)\bar{F}(y)dy$$

$$= \int_0^w H_2(w- y)\bar{F}(y)dy \qquad (3.4.19)$$

where $H_2(w- y) = H_1*F^{(n-1)}(w)$, since $H_1(\,)$ and therefore, $H_1*F^{(n-1)}(\,)$ are functions of $(w\text{-}y)$. This is true as the integral is evaluated with respect to y. The following expression is obtained:

$$\int_0^w H_2(w- y)\bar{F}(y)dy = H_2*E(w)\ . \qquad (3.4.20)$$

where $E(w)$ is given as:

$$E(w) = \int_0^w \bar{F}(y)dy\ . \qquad (3.4.21)$$

Therefore,

$$M_D(w) = \sum_{n=1}^{\infty} G*F^{(n-1)}(w)$$

$$= \sum_{n=1}^{\infty} H_2*E(w)\ . \qquad (3.4.22)$$

This is an extremely complicated function to evaluate. It may appear through a cursory look at this expression that we may possibly be able to use the method suggested by Lomnicki after modifying certain functions. However, it would be extremely complicated to expand $H_2(\ )$ and $E(\ )$ into infinite series as required by the Lemma suggested by White (52) and later considered by Lomnicki (29). Therefore, the delayed renewal function is evaluated using the following approach.

The expected number of renewals or failures for any component over the warranty period w, can also be given as

$$M_D(w) = G(w,\ b') + \int_0^w M\ (w - x)dG(x,\ b')\ , \tag{3.4.23}$$

where $M(t)$ is the renewal function for the ordinary renewal process with distribution function F.

Equation (3.4.23), on integrating by parts, yields

$$M_D(w) = G(w,\ b') + \int_0^w m\ (w - x)G(x,\ b')dx\ . \tag{3.4.24}$$

An approximate evaluation of this function $M_D(w)$ can be obtained as follows.

First consider the excess or survivor distribution $G(w,\ b')$ given by equation (3.4.15). Integrating by parts,

$$G(w,\ b') = \int_0^w m(b'+w-y)[1-F(y)]dy = \int_0^w m(b'+w-\ y)dy - \int_0^w m(b'+w-\ y)F(y)]dy. \tag{3.4.25}$$

Then, $G(w,\ b')$ can be expressed as,

$$G(w,\ b') = M(b' + w) - M(b') - \int_0^w m(b' + w - y)F(y)dy\ . \qquad (3.4.26)$$

A Taylor series expansion for $m(b' + w - y)$ around b' is given as;

$$m(b' + w - y) = \sum_{k=0}^{\infty} \frac{(b')^k}{k!}\, m^{(k)}(w - y)\ . \qquad (3..4.27)$$

Ignoring the terms higher than the first order, the expression reduces to:

$$m(b' + w - y) = m(w - y) + b'm'(w - y)\ . \qquad (3.4.28)$$

This is justified because $m''(t)$ is very small for all values of $0 \leq t \leq w$. Next evaluate the integral in the expression for the excess life distribution given by equation (3.4.26).

$$\int_0^w [m(w\text{-}y) + b'm'(w\text{-}y)]F(y)dy = \int_0^w m(w\text{-}y)F(y)dy + b'\int_0^w m'(w\text{-}y)F(y)dy\ . \qquad (3.4.29)$$

Integrating by parts, the following is obtained.

$$= \int_0^w m(w - y)F(y)dy \quad + \quad b'\int_0^w m'(w - y)\ ]\ F(y)dy\ .$$

$$= - M(w - y).F(y)\Big|_0^w + \int_0^w M(w - y)f(y)dy + b'm(w - y)F(y)\Big|_0^w + b'\int_0^w m(w - y)\ f(y)dy$$

$$= 0 + M(w) - F(w) + 0 + b'\int_0^w m(w - y)f(y)dy$$

Since It is assumed that $M(t) = 0$ and $F(t) = 0$ at $t{=}0$, and

$$\int_0^w M(w - y)f(y)dy = \int_0^w \sum_{n=1}^{\infty} F^{(n)}(w - y)f(y)dy$$

$$= \sum_{n=1}^{\infty} \int_0^w F^{(n)}(w - y)f(y)dy = \sum_{n=1}^{\infty} F^{(n+1)}(w)$$

$$= \sum_{p=2}^{\infty} F^{(p)}(w) = \sum_{p=1}^{\infty} F^{(p)}(w) - F(w)$$

$$= M(w) - F(w) .$$

Again, expanding $m(w - y)$ by Taylor series around w, and neglecting the terms higher than the first order the following is obtained.

$$M(w) - F(w) + b' \int_0^w m(w - y)f(y)dy = M(w) - F(w) + b' \int_0^w [\, m(w) - m'(w)y \,]f(y)dy$$

$$= M(w) - F(w) + b' \int_0^w m(w)f(y)dy - b' \int_0^w m'(w)yf(y)dy$$

$$= M(w) - F(w) + b'm(w) \int_0^w f(y)dy - b'm'(w) \int_0^w yf(y)dy$$

$$= M(w) - F(w) + b'm(w)F(w) - b'm'(w)R(w) , \qquad (3.4.30)$$

where

$$R(w) = \int_0^w yf(y)dy .$$

For Weibull life distribution, this is evaluated as,

$$R(w) = \int_0^w yf(y)dy = \int_0^w y\alpha\beta y^{\beta - 1} e^{-\alpha y^{\beta}} dy$$

$$= \int_0^{\alpha w^{\beta}} \left(\frac{x}{\alpha}\right)^{1/\beta} e^{-x} dx$$

$$= \frac{1}{\alpha^{1/\beta}} \int_0^{\alpha w^{\beta}} x^{1/\beta} e^{-x} dx$$

$$= \frac{1}{\alpha^{1/\beta}} \; P\,(\,1/\beta + 1,\;\; \alpha w^{\beta}\,)\,. \tag{3.4.31}$$

where $P(\alpha,\ x)$ is an incomplete gamma function, given as

$$P(\alpha,\ x) \;=\; \int_{0}^{x} e^{-z}\, z^{\alpha-1} dz\,, \tag{3.4.32}$$

and can be evaluated through asymptotic expansion as follows.

Consider the (upper) incomplete gamma function which is given as

$$Q(\alpha,\ x) \;=\; \int_{x}^{\infty} e^{-z}\, z^{\alpha-1} dz\;\;, \tag{3.4.33}$$

Repeated integration by parts will lead to integrals of the form,

$$Q(\alpha,\ x) = x^{\alpha-1}e^{-x}[1 + \sum_{i=1}^{n-1} \prod_{j=1}^{i} (\frac{\alpha-j}{x})\,] + \prod_{i=1}^{n} (\alpha - j)\int_{x}^{\infty} e^{-z}\, z^{\alpha-(1+n)} dz\;\;, \tag{3..4.34}$$

The asymptotic expansion of the (upper) incomplete gamma function given by equation (3.4.34) attains maximum accuracy by summation of only the terms up to but excluding the smallest term in absolute value. The number of this term is determined by the expression $n = \alpha$ when α is an integer. Otherwise the truncated integer value of $(\alpha + x)$ is used.

Now, the (lower) incomplete gamma function $P(\alpha,\ x)$ is given as

$$P(\alpha, x) = \int_0^\infty e^{-z} z^{\alpha-1} dz - Q(\alpha, x)$$

$$= \Gamma(\alpha) - x^{\alpha-1} e^{-x} [1 + \sum_{i=1}^{n-1} \prod_{j=1}^{i} (\frac{\alpha-j}{x})] , \qquad (3.4.35)$$

Finally, the excess life distribution $G(w)$, from time b', can be evaluated as

$$G(w, b') = M(b' + w) - M(b') - M(w) + F(w) - b'm(w)F(w) + b'm'(w)R(w), \qquad (3.4.36)$$

Substituting equation (3.4.36) in equation (3.4.24) we have (putting x for w)

$$M_D(w) = G(w,b') + \int_0^w m(w\text{-}x)[M(b'+x) - M(b') - M(x) + F(x)$$
$$- b'm(x)F(x) + b'm'(x)R(x)]dx \ . \qquad (3.4.37)$$

Again, after Taylor series Expansion of $m(w\text{-}x)$ around w, and ignoring terms higher than first order we have

$$M_D(w) = G(w, b') + \int_0^w [m(w) + m'(w)(-x)][M(b' + x) - M(b')$$
$$- M(x) + F(x) - b'm(x)F(x) + b'm'(x)R(x)]dx \ . \qquad (3.4.38)$$

For decreasing failure rate of the components we can ignore terms like $m(x)m(w)$ and higher derivatives such as $m(x)m'(w)$ as they would have very small values. After performing the integration an approximation of $M_D(w)$ is obtained as:

$$M_D(w) = G(w,b') + m(w)\int_0^w M(b'+x)dx - M(b')m(w)w - m(w)\int_0^w M(x)dx + m(w)\int_0^w F(x) \quad (3.4.39)$$

$$- m'(w)\int_0^w xM(b' + x)dx + \frac{M(b')\, m'(w)\, w^2}{2} + m'(w)\int_0^w xM(x)dx - m'(w)\int_0^w xF(x)dx ,$$

where $m(t)$ is the associated renewal density function and which can be obtained as

$$m(t) = M'(t) = \frac{d}{dt} M(t) = \frac{d}{dt}[\sum_{s=1}^{\infty} c(s)\, D_s\, (\alpha t^{\beta})$$

where $c(s) = \sum_{k=1}^{s} c_k(s)$

$$= \sum_{s=1}^{\infty} c(s)\, \frac{d}{dt}[D_s\, (\alpha t^{\beta})]$$

$$= \sum_{s=1}^{\infty} c(s) \frac{d}{d(\alpha t^{\beta})} D_s\, (\alpha t^{\beta})]\, \frac{d}{dt}(\alpha t^{\beta})$$

Therefore,

$$D_s'(t) = - e^{-t} \sum_{r=s}^{\infty} \frac{t^r}{r!} + e^{-t} \sum_{r=s}^{\infty} \frac{rt^{r-1}}{r!}$$

$$D_s'(t) = - e^{-t} \sum_{r=s}^{\infty} \frac{t^r}{r!} + e^{-t} \sum_{r=s}^{\infty} \frac{t^{r-1}}{(r-1)!}$$

$$D_s'(t) = - e^{-t} \sum_{r=s}^{\infty} \frac{t^r}{r!} + e^{-t} \sum_{p=(s-1)}^{\infty} \frac{t^p}{p!}$$

$$D'_s(t) = - e^{-t} \sum_{r=s}^{\infty} \frac{t^r}{r!} + e^{-t} \frac{t^{s-1}}{(s-1)!} + e^{-t} \sum_{p=s}^{\infty} \frac{t^p}{p!}$$

$$D'_s(t) = e^{-t} \frac{t^{s-1}}{(s-1)!} \quad . \tag{3.4.40}$$

So,

$$M'(t) = m(t) = [\sum_{s=1}^{\infty} c(s) \frac{d}{d(\alpha t^{\beta})} D_s(\alpha t^{\beta})]\, \alpha\beta t^{\beta-1} \quad . \tag{3.4.41}$$

Using equation (3.4.41) and on simplification we have

$$m(t) = f(t).[\sum_{s=1}^{\infty} \frac{c(s)\ (\alpha t^{\beta})^{(s-1)}}{(s-1)!}] \quad . \tag{3..4.42}$$

And $m'(t)$ is obtained as

$$m'(t) = f'(t).[\sum_{s=1}^{\infty} \frac{c(s)\ (\alpha t^{\beta})^{(s-1)}}{(s-1)!}] + f(t)\alpha\beta t^{\beta-1}[\sum_{s=2}^{\infty} \frac{c(s)\ (\alpha t^{\beta})^{(s-2)}}{(s-2)!}]$$

$$m'(t) = \sum_{s=0}^{\infty} \frac{(\alpha t^{\beta})^s}{s!} [\ c(s+1)f'(t) + c(s+2)f(t)\alpha\beta t^{\beta-1}\ ] \quad . \tag{3.4.43}$$

$\int_0^w M(x)dx$ can be obtained as

$$\int_0^w M(x)dx = \int_0^w [\sum_{s=1}^{\infty} c(s)\ D_s\ (\alpha x^{\beta})dx$$

$$= \sum_{s=1}^{\infty} c(s) \sum_{r=s}^{\infty} \frac{1}{r!} \int_0^w e^{-\alpha x^{\beta}} (\alpha x^{\beta})^r dx.$$

Using this integral,

$$\int_0^w e^{-\alpha x^\beta} (\alpha x^\beta)^r dx$$

and substituting $(\alpha x^\beta) = z$ and changing the limits of the integral

$$= \frac{1}{\alpha^{1/\beta}\beta} \int_0^{\alpha w^\beta} e^{-z} z^{(r + 1/\beta - 1)} dz$$

$$= \frac{1}{\alpha^{1/\beta}\beta} \mathrm{P}[(\mathrm{r} + 1/\beta), \alpha \mathrm{w}^\beta]$$

where $P[\ ]$ is an incomplete Gamma function as defined earlier. Now, we have

$$\int_0^w M(x)dx = \frac{1}{\alpha^{1/\beta}\beta} \sum_{s=1}^{\infty} c(s) \sum_{r=s}^{\infty} \frac{1}{r!} P[(r + 1/\beta), \alpha w^\beta]. \tag{3.4.44}$$

Similarly we obtain

$$\int_0^w M(x)xdx = \frac{1}{\alpha^{2/\beta}\beta} \sum_{s=1}^{\infty} c(s) \sum_{r=s}^{\infty} \frac{1}{r!} P[(r + 2/\beta), \alpha w^\beta]. \tag{3.4.45}$$

$$\int_0^w M(b'+x)dx = \frac{1}{\alpha^{1/\beta}\beta} \sum_{s=1}^{\infty} c(s) \sum_{r=s}^{\infty} \frac{1}{r!} \{P[(r+1/\beta), \alpha(b'+w)^\beta] - P[(r+1/\beta), \alpha b'^\beta]\}. \tag{3.4.46}$$

$$\int_0^w M(b'+x)xdx = \frac{1}{\alpha^{2/\beta}\beta} \sum_{s=1}^{\infty} c(s) \sum_{r=s}^{\infty} \frac{1}{r!} \{P[(r+ 2/\beta), \alpha(b'+w)^\beta] - P[(r+2/\beta), \alpha b'^\beta]\}$$

$$- \frac{b'}{\alpha^{1/\beta}\beta} \sum_{s=1}^{\infty} c(s) \sum_{r=s}^{\infty} \frac{1}{r!} \{P[(r+1/\beta), \alpha(b'+w)^{\beta}] - P[(r+1/\beta), \alpha b'^{\beta}]\}. \tag{3.4.47}$$

Therefore, the expected number of field failures for the *jth* component is given by equation (3.4.39) and the expected field failure cost due to the *jth* component is given as:

$$C_j^f(w) = c_j^f M_{j,D}(w), \tag{3.4.48}$$

where $M_{j,D}(w)$ is the number of renewals or failures for the *jth* component over the warranty period w.

Finally, the total expected cost of field failures due to all the components is given as:

$$TC^f(w) = \sum_{j=1}^{n} C_j^f(w). \tag{3.4.49}$$

Now, adding equation (3.4.49) to equation (3.4.14) and $c(\underline{S}_a, b)$ we have the total cost of carrying out the burn-in process for a period of b under the strategy of a fixed burn-in period. From equations (3.4.13), (3.4.14), (3.4.48) and (3.4.49), this is given as:

$$TC_{fpb}(b, w) = c(\underline{S}_a, b) + \sum_{j=1}^{n} c_j^i \cdot M_j(a_j \cdot b) + \sum_{j=1}^{n} c_j^f \cdot M_{j,D}(w). \tag{3.4.50}$$

Therefore, the total expected cost of having burn-in is a function of b, the burn-in duration, and w, the warranty period for the product. This total cost function is to be minimized. Any of the following cases may occur.

- The warranty period, w, is known and we have b, the burn-in duration, as a decision variable. In this case we minimize the total cost function with respect to b to obtain the optimal burn-in period b^*_w.

- The burn-in period necessary to get rid of the early failures, b, is known and the warranty period w, is to be determined. This is a minimization problem with respect to w to get the optimal warranty period w^*_b.

- Both b and w are decision variables. In this case we minimize the total expected cost to determine the optimal burn-in and warranty periods (b^*, w^*).

In all the above cases it is assumed that stress levels for all the stress parameters are known when the burn-in is performed. A more general problem to consider would be when the stress levels are also included in the optimization problem as decision variables. In this case, the total expected cost function is minimized with respect to the burn-in duration and warranty period as well as the stress levels for the various stress parameters used in the burn-in process. A total cost function for this case, also expressed in terms of stress levels under stress application, is:

$$TC_{fpb}(b,w,\underline{S}_a) = c(\underline{S}_a,b) + \sum_{j=1}^{n} [c^i_j.M_j(b\prod_{p=1}^{k} e^{[c_{jp}+d_{jp}.s_{ap}]})] + \sum_{j=1}^{n} c^f_j.M_{jD}(w.) \qquad (3.4.51)$$

On substitution of equation (3.4.36) and (3.4.39) in equation (3.4.51) and simplification we have:

$$TC_{fpb}(b,w,s_{ai}) = c(\underline{S}_a,b) + \sum_{j=1}^{n} [c^i_j.M_j(b_j') + \sum_{j=1}^{n} c^f_j.[M_j(b_j'+w) - M_j(b_j')-M_j(w)$$

$$+ F_j(w) - b_j' m_j(w) F_j(w) + b_j' m_j'(w) R_j(w) + m_j(w) \int_0^w M_j(b_j' + x) dx$$

$$- M_j(b_j') m_j(w) w - m_j(w) \int_0^w M_j(x) dx + m_j(w) \int_0^w F_j(x) dx - m_j'(w) \int_0^w x M_j(b_j' + x) dx \tag{3.4.52}$$

$$+ \frac{M_j(b_j') \, m_j'(w) \, w^2}{2} + m_j'(w) \int_0^w x M_j(x) dx - m_j'(w) \int_0^w x F_j(x) dx \,] \, ,$$

where b_j' is the equivalent age for the jth component for a burn-in period of b and is given as:

$$b_j' = b \, e^{\sum_{p=1}^{k} [c_{jp} + d_{jp} . s_{ap})]} \, . \tag{3.4.53}$$

Assume that the selling price, $p(w)$, for the product is a function of its warranty period w. The net profit function $NP_{fpb}(b, w, s_{ai})$ is given as:

$$NP_{fpb}(b, w, s_{ai}) = p(w) - TC_{fpb}(b, w, s_{ai}) \, , \tag{3.4.54}$$

where $TC_{fpb}(\ )$ is given by equation (3.4.52). This net profit function needs to be maximized with respect to the decision variables b, w and s_{ai}s.

In the present study the decision variables are the burn-in duration, the warranty period and the stress levels for the stress parameters. The net profit function as given in

(3.4.54) is considered for optimization. Numerical examples are developed in chapter 5 considering a suitable revenue function for the product. It is assumed that the product is sold at a higher price when a longer warranty period is in force.

Model 2

In this model we modify Model 1 to examine a warranty policy in which the warranty is renewed for a failed component. This policy is modeled under the strategy of "fixed period" burn-in.

A total cost function for having the burn-in for a period b, for products sold with a renewable warranty of period w is

$$TC_{fpb,r}(b,\ w) \ = \ c_0 . b + \ TC^i(b) \ + \ TC^f_r(w) \ . \tag{3.4.55}$$

As the same burn-in policy is considered the first two terms in the total cost function for this model are the same as those in the previous model. The last term in the cost function, which refers to the total expected field failure costs due to all components, can be obtained using the following approach.

Consider the total expected cost due to the failures of the jth component. The probability that the jth component fails during the warranty period w is given by $G_j(w,b'_j)$. Let $\tilde{n}_j$ be the number of times the jth component fails until the failure time exceeds the warranty period w. It is a geometrically distributed random variable with

$$Pr\{\tilde{n}_j = k\} = [1 - G_j(w,\ b'_j)]\ G_j^{\,k}(w,\ b'_j) \qquad\qquad k = 0,\ 1,\ 2,\ ..... \tag{3.4.56}$$

where $G_j(w,b'_j)$ is given by equation (3.4.36). Then $E[\tilde{n}_j]$, the expected number failures for the *jth* component is

$$E[\tilde{n}_j] = \frac{G_j(w, b'_j)}{1 - G_j(w, b'_j)} . \tag{3.4.57}$$

Therefore, the total expected field failure cost for the *jth* component is

$$C^f_{j,r}(w) = c^f_j(w)\, E[\tilde{n}_j]. \tag{3.4.58}$$

and the total expected cost of field failures due to all the components is

$$TC^f_r(w) = \sum_{j=1}^{n} C^f_{j,r}(w) . \tag{3.4.59}$$

Now, for this model also, the total cost function of equation (3.4.55) can be minimized with respect to b, the burn-in duration, w, the warranty period, and also the stress levels for the stress parameters.

Model 3

The reliability growth for the components over the warranty period is considered in this model. That is, the failed components are replaced by components with a lower initial failure rate. For this model the total cost function for having the burn-in is

$$TC_{fpb,g}(b,w) = c(\underline{S}_a \cdot b) + TC^i(b) + TC^f_g(w) . \tag{3.4.60}$$

The first two terms are the same as those in the previous two models. The warranty support cost term may be obtained considering the following approach.

Let a Weibull distribution with a scale parameter α_g and a shape parameter β_g describe the failure rate reduction process during reliability growth of the components. We can also consider different distributions for reliability growth of different components. Assume that the new component which replaces the failed component has a Weibull life distribution with parameters α_{new} and β_{new} after reliability growth.

Consider the convolution for the distributions for the old and new component. For the *jth* component, the probability of two or more renewals over the warranty period, conditioned on the first failure time *t,* is given as:

$$F_j^{(2)}(w) = \int_0^w F_{j,new}(w - t)\; g_j(t)dt\,, \tag{3.4.61}$$

where $g_j(t)$ is the p.d.f. of the excess life distribution.

$$F_j^{(2)}(w) = \int_0^w [\,1 - e^{-\alpha_{j,new}(w - t)^{\beta_{j,new}}}\,]\; g_j(t)\; dt. \tag{3.4.62}$$

Now, an expression relating the distributions for the old and the new component can be obtained considering the fact that a smaller number of expected failures over the period *(w-t)* takes place because of the reliability growth over the period *t*. This reduced number of failures is a result of the fact that some failures are eliminated by the failure rate reduction process associated with reliability growth.

Therefore, the following relation can be established,

$$\alpha_j\,(w - t)^{\beta_j} - \alpha_g t^{\beta_g} = \alpha_{j,new}(w - t)^{\beta_{j,new}}\,. \tag{3.4.63}$$

The first term in the left hand side of the expression is the total expected number of failures over the time *(w-t)* in case of no reliability growth for the components. The second term is the total expected number of failures that have been avoided through the failure rate reduction process over the time t.

Substituting equation (3.4.63) in equation (3.4.62) we have:

$$F_j^{(2)}(w) = \int_0^w [\,1 - e^{-\alpha_j (w - t)^{\beta_j}} \cdot e^{\alpha_l t^{\beta_l}}\,]\, g_j(t)\, dt. \qquad (3.4.64)$$

It is extremely difficult to evaluate this integral and the associated renewal function. We could at the most obtain an approximation for this integral but currently even with the resolution of the integral, the associated renewal function is unmanageable.

In order to determine the expected number of failures for the components the following approach is considered.

Consider the pooled output from a process by superimposing the delayed renewal processes formed by the individual components. Although, the pooled output does not form a renewal process when the components have Weibull life distributions, a different approach is considered to model this process approximately. It is assumed that system failure occurs when the total expected number of failures for all the components becomes an integer value. Therefore, the first system failure would take place at $t=t_1$, if:

$$\sum_{j=1}^{n} M_{j,D}(t_1) = 1\,. \qquad (3.4.65)$$

This time t_1 is unique as $M_{j,D}(\,)$ is a strictly increasing function in time for strictly increasing $F(\,)$.

Now, at time t_1, a failed component is replaced by a component that has lower failure rate. Again, let t_2 be the time when the cumulative number of expected failures becomes *2* and this can be written as:

$$\sum_{j=1}^{n} M_{j,D}(t_2) - \alpha_g t_1^{\beta_g} = 2 . \qquad (3.4.66)$$

The total number of expected failures over the period t_2 is reduced due to the replacement of the new component at time t_1. The second term on the left hand side of the equation is the reduced number of expected failures because of the reliability growth over the time t_1. This time t_2 can also be uniquely determined once t_1 is known.

Similarly:

$$\sum_{j=1}^{n} M_{j,D}(t_2) - \alpha_g t_1^{\beta_g} - \alpha_g t_2^{\beta_g} = 3. \qquad (3.4.67)$$

For the *pth* failure or renewal at $t=t_p$, the total expected number of failures becomes p for the pooled process and this can be expressed as:

$$\sum_{j=1}^{n} M_{j,D}(t_p) - \sum_{i=1}^{p-1} \alpha_l t_i^{\beta_l} = p . \qquad (3.4.68)$$

The values of the t_is can be obtained numerically. For $t_p \geq w$, the total expected number of failures for all the components, $M_{s,D}(w)$ can be given as:

$$M_{s,D}(w) = p - 1 + \sum_{j=1}^{n} M_{j,D}(w - t_{p-1}) - \sum_{i=1}^{p-1} \alpha_l t_i^{\beta_l} \quad . \tag{3.4.69}$$

Now, the total field failure cost over the warranty period is given as:

$$TC_g^f(w) \;=\; \bar{c}^f M_{s,D}(w) \; . \tag{3.4.70}$$

Here, the average cost for the failed components is considered since we do not have any knowledge of which components fail over the warranty period. This average cost, $\bar{c}^f$, can be determined as:

$$\bar{c}^f = \sum_{j=1}^{n} c_j^f \, . \tag{3.4.71}$$

For this model also, the total cost function as given by equation (3.4.60) can be minimized with respect to the burn-in duration, *b*, the warranty period, *w*, and also the stress levels, s_{ai}'s, for the stress parameters.

Model 1 considers a fixed period burn-in and a free replacement warranty with fixed warranty period. A total cost and a net profit functions are developed. In Model 2, a renewed warranty policy is considered for a product which is subjected to a fixed period burn-in. The net profit functions for Model 1 and 2 are considered for optimization in chapter 5. A net profit function for Model 2 is developed considering the same revenue function as in Model 1. Model 3 is appropriate for new products when the components undergo a reliability growth even after the product is introduced in the market. As a result, the failed components are replaced by components with a lower inherent failure rate. A total cost function for the model is developed. This model is not considered for analysis any further as it resembles the failure free burn-in model to a large extent. The sequence of failures over the burn-in period in the failure free burn-in model is considered in a similar fashion as that over the warranty period in Model 3.

CHAPTER 4 FAILURE FREE BURN-IN

4.1 Introduction

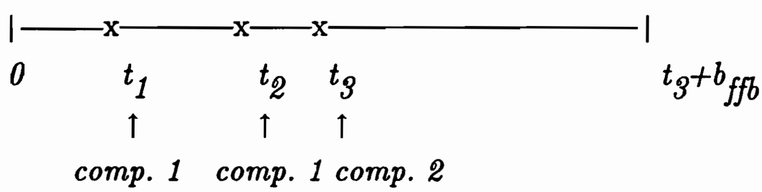
A "failure free" burn-in for the system, which continues until a failure free period is obtained, is considered in this chapter. The response to each component failure under this strategy of burn-in is to replace the failed component and restart the clock or burn-in time. Models are constructed for fixed as well as renewed warranty policies.

4.2 Development of Cost Models

<u>Model 1</u>

A "failure free" burn-in strategy for products sold with a free replacement warranty is considered in this model. It is assumed that a fixed warranty period is in force which is not renewed after a failure. It is easy to obtain an expression for the expected time of completion for the burn-in process under this strategy when burn-in is carried out at the component level rather than at the assembly or system level. It is difficult to obtain an expression for the expected time of completion for the burn-in process when it is carried out at the system level. Here, each component undergoes burn-in over a time period which is dependent on failure processes of other components. This can be explained through the following example:

Consider a system having three components and a "failure free" burn-in is being performed at the system level. Assume that the system fails three times before it attains a failure free period of b_{ffb}. This phenomenon is depicted in the following diagram:



Also assume that the first two system failures occur due to component *1*, and at $t=t_3$, component *2* fails. For this example, at the end of the failure free period, b_{ffb}, i.e., at $t=t_3+b_{ffb}$, the system would consist of components which have different ages. Component *1* has age $t_3+b_{ffb}-t_2$, component *2* has age b and component *3* has an age t_3+b_{ffb}.

It is evident that when the system consists of a large number of components, it is extremely difficult to estimate the burn-in completion time. An approximate estimate for the expected time of completion for the burn-in process for the system can be obtained considering the following assumptions:

1. The system is composed of n components where n is a large number.

2. Component failure processes are independent.

3. The interval between successive renewals in the pooled output from the component

renewal processes is very small.

4. As a result of the third assumption, it is assumed that all components behave "good-as-new" after each repair.

5. The failure rates for the components after a period of b_{ffb} without a failure are very small and therefore, chances of a system failure before it passes a failure free period of b_{ffb}, due to such components is very small.

An approximate expression for the expected time of completion for the system burn-in, t_{ffb}, can be given on the basis of the above assumptions as

$$t_{ffb} = b_{ffb} + E[\tilde{n}_1]E[x_1^{b_{ffb}}] + E[\tilde{n}_2]\,E[x_2^{b_{ffb}}] + \ldots\ldots\ldots\ldots + E[\tilde{n}_n]E[x_n^{b_{ffb}}], \tag{4.2.1}$$

where $\tilde{n}_j$ is the number of times jth component fails until failure time exceeds the burn-in period b_{ffb}. Here, it is assumed that a delay in the completion of the system burn-in is caused by failures of the components before they reach age b_{ffb}. As for the last assumption, the failure rates for the components after they attain an age b_{ffb} are very small. Obviously, the estimate given by equation (4.2.1) is a lower bound for the expected system burn-in completion time and it is not very tight. Each term in the expression represents the expected delay in the time of system burn-in completion due to each component and follows from Wald's Equation (Cox, 13) which states that

$$E[\sum_{i=1}^{\tilde{n}} z_i] = E[\tilde{n}]E[z], \tag{4.2.2}$$

$\tilde{n}_j$ is a geometrically distributed random variable with

$$Pr\{\tilde{n}_j = k\} = R_j(a_j b_{ffb}).F_j(a_j b_{ffb})^k \qquad k = 0,\ 1,\ 2,\ \ldots.., \tag{4.2.3}$$

where $F_j(t) = 1 - e^{[-\alpha_j . t^{\beta_j}]}$ and $E[\tilde{n}_j]$ is given as

$$E[\tilde{n}_j] = \frac{F_j(a_j b_{ffb})}{R_j(a_j b_{ffb})}. \tag{4.2.4}$$

And, $E[x_j^{b_{ffb}}]$ is given as

$$E[x_j^{b_{ffb}}] = \frac{1}{a_j} \int_0^{a_j b_{ffb}} t f_j(t) dt. \tag{4.2.5}$$

Equation (4.2.1) approximates the expected period of system burn-in. A more precise estimate for the time of completion for the system burn-in can be obtained following the approach we considered for the reliability growth model.

Consider the pooled output from a process obtained by superimposing the renewal processes formed by the individual components. The pooled output does not form a renewal process when the components have Weibull life distributions. As a result, the pooled output is not completely characterized by any distribution function for the interval between successive failures in the pooled output. The system failure times can be obtained using the following approach.

The total expected number of failures or renewals for all the components becomes *1* when,

$$\sum_{j=1}^{n} M_j(a_j t_1) = 1 , \qquad (4.2.6)$$

t_1 is uniquely determined as $M_j(\ )$ is a strictly increasing function in time for strictly increasing $F(\ )$. $F(\ )$ is the Weibull life distribution for the components.

Again, let t_2 be the time when the cumulative number of expected failures becomes *2,*

$$\sum_{j=1}^{n} M_j(a_j t_2) = 2, \qquad (4.2.7)$$

Following the same argument, t_2 is also unique. Now, in general the time for *pth* failure, t_p, at which the total expected number of failures becomes p for the pooled process, can be obtained from:

$$\sum_{j=1}^{n} M_j(a_j t_p) = p . \qquad (4.2.8)$$

First the t_is are to be determined numerically. A failure free period burn-in period, b_{ffb}, is said to be achieved when $t_i - t_{i-1}$ exceeds this value. An approximate estimate for the time of completion for the system burn-in is then given as:

$$t_{ffb} = t_{i-1} + b_{ffb} . \qquad (4.2.9)$$

Therefore, renewals for all of the components takes place over the period t_{ffb}. The expected number of renewals or failures for the *jth* component can be obtained from equation (3.4.8) after substituting t_{ffb} for b.

Following the same procedure as in the case of fixed period burn-in, the total expected cost of carrying out the burn-in and the total expected warranty support cost can be obtained. The total cost is then given as:

$$TC_{ffb}(b_{ffb},w) = c(\underline{S}_a,b_{ffb}) + \sum_{j=1}^{n} c_j^i.\ M_j(a_j.t_{ffb}) + \sum_{j=1}^{n} c_j^f.M_{j,d}\ (w). \tag{4.2.10}$$

$M_{j,d}(w)$, the number of renewals or failures over the warranty w is obtained using equation (3.4.39) and in the case of failure free burn-in the survival or excess life distribution for the *jth* component as represented by equation (3.4.15) can be restated as:

$$G_{j,ffb}(t,a_jt_{ffb}) = F_j(a_jt_{ffb}+ t) + \int_0^{a_jt_{ffb}} [1 - F_j(a_jt_{ffb} + t - y)]dM_j(y)\ . \tag{4.2.11}$$

An approximation for this excess life distribution can be defined using equation (3.4.36) as:

$$\begin{aligned} G_{j,ffb}(t,a_jt_{ffb}) &= M_j(a_jt_{ffb}+w) - M_j(a_jt_{ffb}) - M_j(w)+F_j(w) \\ &\quad - a_jt_{ffb}m_j(w)F_j(w) - a_jt_{ffb}m_j'(w)R_j(w)\ . \end{aligned} \tag{4.2.12}$$

The corresponding total cost function can also be expressed in terms of stress levels under stress application and this is given as:

$$TC_{ffb}(b_{ffb},w,s_{ai})=c(\underline{S}_a,b_{ffb})+\sum_{j=1}^{n}[c_j^i.M_j(t_{ffb}\prod_{p=1}^{k} e^{[c_{jp}+d_{jp}.s_{ap}]})]+\sum_{j=1}^{n}c_j^f.M_{dj}(w)\ . \tag{4.2.13}$$

Thus, the total expected cost function as given by equation (4.2.10) is expressed in terms of the expected time of completion for the system burn-in, t_{ffb}, the warranty period for the product, w, and the stress levels for the stress parameters. As t_{ffb} is expressed as a function of b_{ffb}, this total cost function is also minimized with respect to the failure free burn-in duration, b_{ffb}, warranty period, w, and the stress levels for the stress parameters used in the burn-in process. The various cases considered for fixed period burn-in may arise. Also, a similar net profit function can be constructed considering the revenue function.

Model 2

In this model, a warranty policy in which failed components are replaced by new components with renewed warranty is studied. This policy is modeled under the strategy of "failure free" burn-in.

For this model the total cost function for having a "failure free" burn-in for a period b, for products sold with a renewable warranty of period w, is given as:

$$TC_{ffb,r}(b,\ w) = c(\underline{S}_a, b_{ffb}) + TC^{i}_{ffb}(t_{ffb}) + TC^{f}_{r,ffb}(w). \tag{4.2.14}$$

The first two terms in the total cost function for this model are the same as those in the previous model as the same burn-in policy is being considered. The last term in the cost function which refers to the total expected field failure costs due to all of the components can be obtained following the same approach as that considered in the case of fixed period burn-in. Consider the total expected cost due to the failures of the *jth* component. The probability that the *jth* component fails during the warranty period w is given by $G_{j,ffb}(w,a_j,t_{ffb})$. Let $\tilde{n}_j$ be the number of times the *jth* component fails until failure time exceeds the warranty period w. It is a geometrically distributed random variable with:

$$Pr\{\tilde{n}_j = k\} = [1 - G_j(w, a_j t_{ffb})]\, G_j^k(w, a_j t_{ffb}) \qquad k = 0, 1, 2, \ldots\ldots, \tag{4.2.15}$$

where $G_{j,ffb}(w, a_j t_{ffb})$ is given by equation (4.2.12) and $E[\tilde{n}_j]$, the expected number of failures for the *jth* component is given as:

$$E[\tilde{n}_j] = \frac{G_j\,(w,\ a_j t_{ffb})}{1 - G_j\,(w,\ a_j t_{ffb})}\ . \tag{4.2.16}$$

Therefore, the total expected field failure cost for the *jth* component is given as:

$$C^f_{j,ffb,r}(w) = c^f_{j,}(w)\ E[\tilde{n}_j]\ . \tag{4.2.17}$$

The total expected cost of field failures due to all the components is given as:

$$TC^f_{ffb,r}\ (w) = \sum_{j=1}^{n} C^f_{j,ffb,r}(w)\ . \tag{4.2.18}$$

For this model, the total cost function given in equation (4.2.14) can be minimized with respect to the decision variables such as expected burn-in completion time, t_{ffb}, warranty period, w, and also the stress levels for the stress parameters under which burn-in is performed.

CHAPTER 5 OPTIMIZATION OF THE MODELS AND NUMERICAL RESULTS

5.1 Introduction

In this chapter, optimization procedures and the numerical examples for the models are discussed. The examples illustrate the usefulness of the models. Simulation models are used to establish the validity of the models. Fixed period burn-in and failure free burn-in examples are considered separately.

5.2 Optimization of the Models

The cost or the corresponding profit functions for both fixed and failure free burn-in have essentially the same structure. For failure free burn-in, the screening period is replaced by the expected time of completion for the system burn-in. As a result, the optimization strategies adopted for both the cases are the same. Modifications necessary in the optimization procedure for failure free burn-in are discussed later.

It has been observed that the objective functions for the models are "non-convex" in nature. This precludes the possibility of obtaining a global optimum using some of the well known optimization techniques. It is evident that a search routine is required to arrive at a optimal solution. Evaluations of the objective functions at different points in the design variable space indicate the objective function surfaces to be extremely flat. Search routines based on Hessian evaluations such as Newton's Method, are found to be ineffective for such flat functions. Hessian evaluations of the objective functions at different points suggested the local convexity of the functions. Gradient free line search methods such as uniform search and the golden section method assume a strictly quasiconvex function for minimization. This condition is not satisfied by the objective functions in the models. Therefore, a search method that employs functional evaluations at design points is best suited for optimizing the objective functions for the models.

A sequential simplex search method is adopted for solving the problem. This method is originally proposed by Spendley, Hext and Himsworth (49) and modified by Nelder and Mead (37). The method essentially looks at the functional values at the extreme points of a simplex. The worst extreme point is rejected and replaced by a new point along the line joining the point and the centroid of the remaining points. The process is repeated until a suitable termination criterion is satisfied. The optimization routine in this study compares the "standard deviation" for the functional values at the simplex points with a pre-set value and the process terminates when it falls below 0.0001. This method is specially suitable for problems with a small number of variables. The convergence property deteriorates as the number of variables increases. The method is well suited for optimization of the models in this study as they have at most five decision variables, depending on the problem. Readers are referred to Nelder and Mead (37) for a detailed description of the algorithm. The algorithm is modified to handle the lower and the upper bounds on the variables. and a flow-chart for the algorithm is given in Figure 1. It is assumed that the simplex structure in the algorithm is not lost when any bound is encountered. The optimization routine is performed with different sets of initial points identifying several local minima. The best of the local minima is taken to be the best solution despite the fact that it may not be the global minima. An optimization routine is written in C (Appendix A) implementing the algorithm.

5.3 Illustrative Examples

The models and the optimization procedures are illustrated by examples. An electronic telecommunication product, a circuit pack, is considered for both fixed period and failure free burn-in models. It consists of ten different types of ICs. Each IC type is present in different numbers. The number of ICs of different types, their life characteristics (Weibull scale and shape parameters) and also the cost of an in-plant failure for the components are enumerated in Table 5.1. In addition, it is assumed that the non IC components have negligible failure rates.

5.3.1 Fixed Period Burn-in

Model 1

A fixed burn-in period and a free replacement warranty with fixed warranty period are considered for this model. The set up costs and the costs of carrying out the stress screening process for different parameters are listed in Table 5.2. The stress screening costs are given as the costs of maintaining one unit differences in stress levels between accelerated and normal use condition per unit time. These cost parameters are assumed to be constant and independent of stress levels. This is a fairly reasonable assumption for the range of stress levels under which the burn-in process is carried out. The net profit function is considered as the objective function and is maximized with respect to the burn-in duration, the warranty period and the stress levels. The objective function is:

$$max\ r(w) - \sum_{i=1}^{3} p_i + q_i(S_{ai} - S_{oi})b - \sum_{j=1}^{10} c_j^i . n_j M_j(b) - \sum_{j=1}^{10} c_j^f . n_j M_{j,D}(w) \ . \qquad (5.3.1)$$

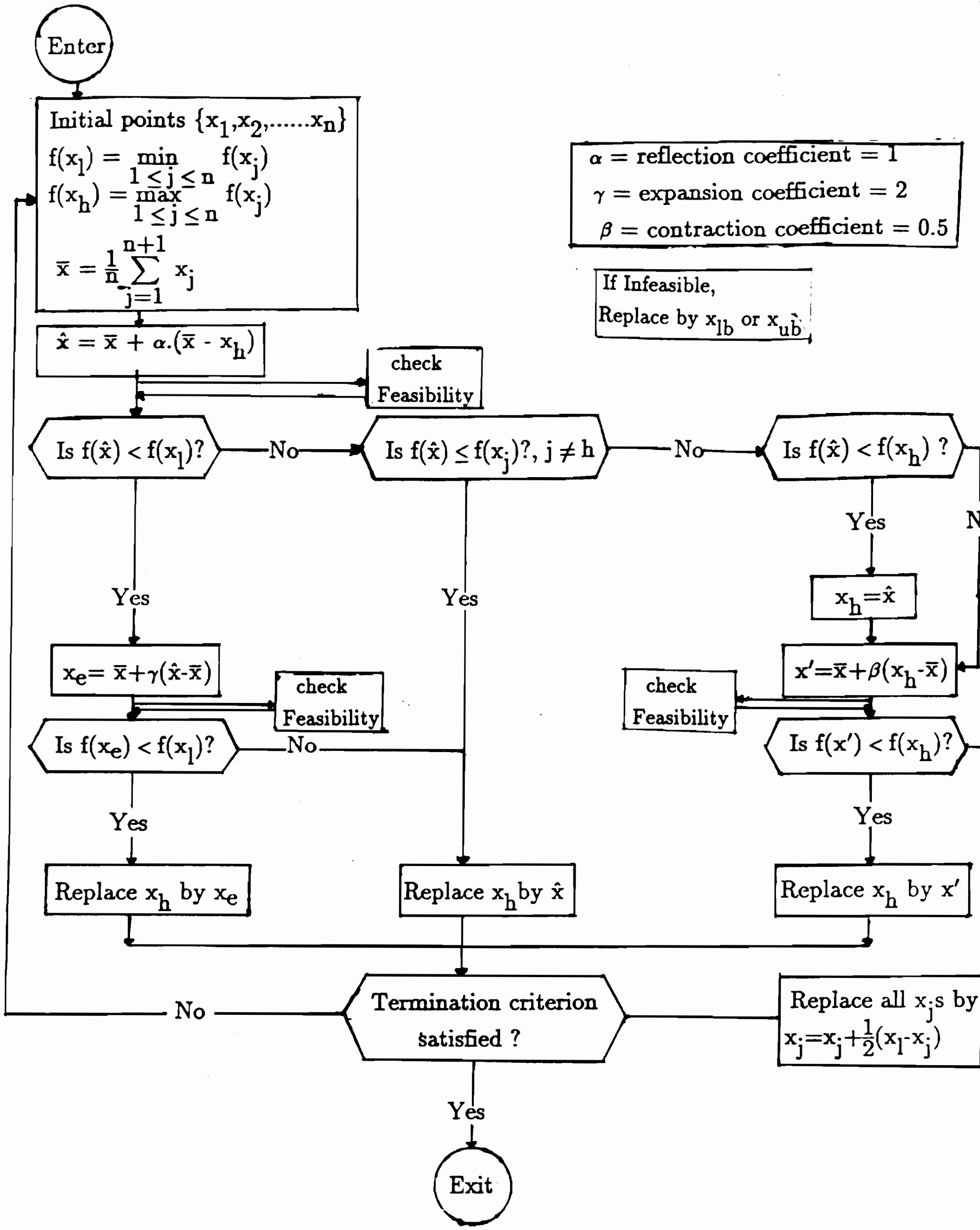


FIGURE 1 FLOWCHART FOR FUNCTION MINIMIZATION

TABLE 5.1 SYSTEM COMPONENTS AND THEIR ASSOCIATED WEIBULL PARAMETERS

Component Type	No. of Components	In-plant Failure Cost/Unit (c^i_j)	α_j	β_j
1	10	6.00	2×10^{-3}	0.25
2	5	2.50	1.2×10^{-2}	0.20
3	7	10.00	8×10^{-4}	0.25
4	1	9.00	1.2×10^{-3}	0.25
5	2	4.00	4×10^{-3}	0.10
6	15	8.00	1.06×10^{-3}	0.30
7	8	3.50	8×10^{-3}	0.20
8	6	3.00	1.2×10^{-2}	0.25
9	10	7.00	2.1×10^{-3}	0.30
10	1	5.00	2×10^{-3}	0.25

In evaluating $M_{j,D}$, all terms consisting of $m(w)$ or $m'(w)$ are neglected as they are very small for components with decreasing failure rate and small Weibull scale parameters. For example, for a component with β=0.25 and α=0.002, $m(w)$ and $m'(w)$ are 5x10^{-7} and 2.5x10^{-13}, respectively at w=10,000 hrs. $r(w)$ is the revenue function which depends on the warranty period, w. A revenue function which increases with warranty period initially at a faster rate and gradually flattens out after a certain period, is assumed. This is given as:

$$r(w) = 200 + 400[1 - e^{-0.0003w}] . \tag{5.3.2}$$

All other notations in (5.3.2) are as considered before and n_j is the number of components of the jth type. The objective function is maximized under the following constraints on the decision variables.

$$328 \leq T \leq 380 , \tag{5.3.3}$$

$$6 \leq V \leq 20 , \tag{5.3.4}$$

$$60 \leq RH \leq 100 , \tag{5.3.5}$$

$$0 \leq b \leq 3 , \tag{5.3.6}$$

$$3000 \leq w \leq 30000 , \tag{5.3.7}$$

The lower limits on the stress parameters are the normal stress conditions. The upper bounds indicate the stress limits that the components can withstand without any damage. An upper bound on the burn-in duration is considered to make the search process more efficient. Also, a lower and an upper bound on the warranty period are assumed. Five examples are considered for this model. Examples 1, 2 and 3 differ in stress screening costs. The stress screening cost is the highest in example 2 and the lowest in example 3. In all three cases the cost of a field failure $c_j^f = 100c_j^i$, c_j^i being the cost of an in-plant failure. Examples 1, 3 and 4 have the same stress screening costs but have different field failure costs. In examples 4 and 5, field failure costs of $c_j^f = 50c_j^i$, and $c_j^f = 25c_j^i$, respectively, are assumed. In all cases the setup cost is assumed to be the same.

The optimization routine is run with a set of six initial points which form a simplex in R_5. The best solutions for all five cases are given in Table 5.2. The best of the local minima is taken to be the best solution despite the fact that it may not be the global optimum. Several local minima are obtained for each case by performing the optimization routine with different sets of initial points. The components of the total cost, i. e., the stress screening cost, in-plant failure cost, and field failure cost are enumerated for all cases.

Model 2

A warranty policy that renews the warranty period for a failed component is considered. A failed component is replaced and the warranty period is renewed until the failure time exceeds the warranty period. The model has the same objective as in (5.3.1) except for the last term for field failure cost. This is given as:

$$max\ r(w) - \sum_{i=1}^{3} p_i + q_i(S_{ai} - S_{oi})b - \sum_{j=1}^{10} c_j^i . n_j M_j(b) - \sum_{j=1}^{10} c_j^f . n_j E(x_j) \ . \qquad (5.3.8)$$

where $E(x_j)$ is the expected number of times the *jth* component fails until failure time exceeds the warranty period w. The constraints are identical to the constraints in Model 1. Two examples are considered for this model. The set up costs and the costs of screening are listed in Table 5.3. In example 1, a higher field failure cost ($c_j^f = 100c_j^i$) is assumed than in example 2 ($c_j^f = 50c_j^i$). The optimal solutions for the examples are enumerated in Table 5.3.

TABLE 5.2 EXAMPLES FOR MODEL 1 (FIXED PERIOD BURN-IN)

Examples	Best Solution	
$\underline{p}$ = { 50.0, 70.0, 10.0 } $\underline{q}$ = { 3.0, 0.009, 0.0015 } $c_j^f = 100c_j^i$	T = 380 ^{o} K V = 20.0 Volt RH = 100.0 % b = 0.6061 hr. w = 5281.71 hr.	Burn-in Cost = $ 224.720 In-plant Failure Cost = $9.478 Field Failure Cost = $ 149.370 Total Cost = $ 383.580 Net Profit = $134.353
$\underline{p}$ = { 50.0, 70.0, 10.0 } $\underline{q}$ = { 5.70, 0.018, 0.006 } $c_j^f = 100c_j^i$	T = 380 ^{o} K V = 20.0 Volt RH = 100.0 % b = 0.3328 hr. w = 4282.13 hr.	Burn-in Cost = $ 228.790 In-plant Failure Cost = $8.105 Field Failure Cost = $ 180.370 Total Cost = $ 417.260 Net Profit = $ 72.32
$\underline{p}$ = { 50.0, 70.0, 10.0 } $\underline{q}$ = { 2.0, 0.0045, 0.0007 } $c_j^f = 100c_j^i$	T = 380 ^{o} K V = 20.0 Volt RH = 100.0 % b = 0.8265 hr. w = 6478.16 hr.	Burn-in Cost = $ 216.031 In-plant Failure Cost = $10.786 Field Failure Cost = $ 161.565 Total Cost = $ 388.382 Net Profit = $ 154.334
$\underline{p}$ = { 50.0, 70.0, 10.0 } $\underline{q}$ = { 3.0, 0.009, 0.0015 } $c_j^f = 50c_j^i$	T = 380 ^{o} K V = 20.0 Volt RH = 100.0 % b = 0.3328 hr. w = 7615.50 hr.	Burn-in Cost = $ 190.589 In-plant Failure Cost = $8.435 Field Failure Cost = $ 128.673 Total Cost = $ 327.698 Net Profit = $ 231.565
$\underline{p}$ = { 50.0, 70.0, 10.0 } $\underline{q}$ = { 3.0, 0.009, 0.0015 } $c_j^f = 25c_j^i$	T = 380 ^{o} K V = 20.0 Volt RH = 100.0 % b = 0.2250 hr. w = 11678.58 hr.	Burn-in Cost = $ 165.141 In-plant Failure Cost = $7.687 Field Failure Cost = $ 141.570 Total Cost = $ 314.398 Net Profit = $ 273.566

TABLE 5.3 EXAMPLES FOR MODEL 2 (FIXED PERIOD BURN-IN)

Examples	Optimal Solution	
$\underline{p} = \{ 50.0, 70.0, 10.0 \}$ $\underline{q} = \{ 3.0, 0.009, 0.0015 \}$ $c_j^f = 100c_j^i$	$T = 380\ ^oK$ $V = 20.0$ Volt $RH = 100.0$ % $b = 0.6084$ hr. $w = 5230.78$ hr.	Burn-in Cost = \$ 224.598 In-plant Failure Cost = \$9.486 Field Failure Cost = \$ 148.611 Total Cost = \$ 383.134 Net Profit = \$ 133.585
$\underline{p} = \{ 50.0, 70.0, 10.0 \}$ $\underline{q} = \{ 3.0, 0.009, 0.0015 \}$ $c_j^f = 50c_j^i$	$T = 380\ ^oK$ $V = 20.0$ Volt $RH = 100.0$ % $b = 0.3943$ hr. $w = 7550.130$ hr.	Burn-in Cost = \$ 191.593 In-plant Failure Cost = \$8.472 Field Failure Cost = \$ 128.066 Total Cost = \$ 328.131 Net Profit = \$ 230.338

5.3.2 Analysis of Fixed Period Burn-in Models

Analysis of Model 1

In examples 1 through 3, the costs of a field failure are the same. It is observed that as the stress screening cost increases or decreases the optimal burn-in duration decreases or increases, respectively. As the stress screening cost increases, it costs more to eliminate early failures in shop. As a result, a smaller number of early failures would be justified to be removed in shop. This in turn would lead to an increase in field failure cost if the warranty period remains the same. Therefore, a decrease in the optimal warranty period is observed. In this case, the revenue lost as a result of a decrease in the warranty period is also accompanied by the savings in the field failure cost. Similarly, in example 3, that has a lower stress screening cost compared to example 1, a larger number of failures take place in shop due to an increased burn-in period. This would lead to an decrease in the total expected field failure cost. As a result, optimal warranty period increases resulting more revenue. In this example, an increase in the total expected field failure cost due to a longer warranty period is offset by the increase in the revenue. The numerical results display intuitively sensible behavior in capturing the trade-off between the burn-in cost and the field failure cost, and the trade-off between the field failure cost and the revenue associated with a warranty period.

Although examples 1, 4 and 5 have the same stress screening costs, the optimal burn-in duration decreases in examples 4 and 5 as smaller cost of a field failure cost is assumed. The optimal warranty period increases as the increased revenue offsets the increase in the expected field failure cost over a longer warranty period.

It is observed that examples 2 and 4 have the same level of optimal screening even though their stress screening costs and the warranty costs are different. The trade-off level between higher stress screening cost and field failure cost in example 2 happens to be more or less the same as between lower stress screening cost and lower field failure cost in example 4. A lower field failure cost in example 4 resulted in a longer optimal warranty period increasing the value of the net profit function.

The above results conform to what is expected when the different cost parameters are varied. In example 5, when the cost of a field failure is assumed to be the least, the optimal burn-in period is the minimum. This supports the intuition that a small number of early failures needs to be removed in the shop when the field failure cost is low. This implies that the models have been successful in capturing the trade-off issues involved among different cost components in the problem.

It is evident that the solutions are independent of the setup costs. These costs behave as constants once the decision to perform screening is made. Also, it is observed in all cases that the stress levels for the stress parameter are at the upper bounds. This phenomenon is analyzed in detail below.

The sensitivity of the obtained solutions to the burn-in duration and the warranty period is examined in Table 5.4. Example 1 in Model 1 is analyzed. Four different combinations of the burn-in duration and the warranty period are considered in the neighborhood of the solution. The net profit function values are determined at all four points. Although they have different values for the total cost functions and its constituents, the net profit function values for all four cases are found to be within 1.5% of the solution obtained. This also conforms to the earlier conjecture of the overall flatness of the objective function surface.

Analysis of Optimal Stress Levels

An optimal solution establishes an optimal trade-off between the stress screening cost and the field failure cost. The stress screening cost is incurred in aging the product to an optimum level. By intuition, it appears that the optimal age could be attained by some other combinations of burn-in duration and acceleration factor where the acceleration factor is determined by the optimal stress levels. In other words the possibility of alternative optimal solutions seems reasonable. However, the following analysis reveals that the optimal aging for a product is attained at a minimum cost when the stress levels are at their upper bounds.

It is assumed that the cost parameters for different stresses are constant. This implies that the cost of maintaining an accelerated stress level is proportional to the difference

TABLE 5.4 SENSITIVITY OF THE BEST SOLUTION TO THE BURN-IN AND THE WARRANTY PERIODS

b and w	Solution
b^*, w^* = { 0.6061, 5281.71}	Burn-in Cost = \$ 224.720 In-plant Failure Cost = \$ 9.478 Field Failure Cost = \$ 149.370 Total Cost = \$ 383.580 Net Profit = \$134.353
b, w = { 0.5453, 4753.54 }	Burn-in Cost = \$ 215.173 In-plant Failure Cost = \$ 9.218 Field Failure Cost = \$ 146.081 Total Cost = \$ 370.473 Net Profit = \$133.420
b, w = { 0.5453, 5809.88 }	Burn-in Cost = \$ 215.173 In-plant Failure Cost = \$ 9.218 Field Failure Cost = \$ 173.222 Total Cost = \$ 397.614 Net Profit = \$132.385
b, w = { 0.6667, 5809.88 }	Burn-in Cost = \$ 234.124 In-plant Failure Cost = \$ 9.715 Field Failure Cost = \$ 152.653 Total Cost = \$ 396.492 Net Profit = \$133.506
b, w = { 0.6667, 4753.54 }	Burn-in Cost = \$ 234.124 In-plant Failure Cost = \$ 9.715 Field Failure Cost = \$ 127.445 Total Cost = \$ 371.284 Net Profit = \$132.325

between accelerated and normal use condition and to the time period over which the difference is to be maintained. This is a valid assumption for the range of stress levels used in a burn-in environment. An example, is the case of temperature acceleration. After the oven reaches the steady state, rate of heat loss due to convection is proportional to the difference between the ambient temperature and the temperature of the oven wall. The conduction and radiation losses are very small and are also assumed to be linearly dependent on the temperature difference. Therefore, the total heat loss is proportional to the temperature difference and the burn-in duration. As a result, the cost of maintaining a unit difference in temperature for unit time is independent of the stress level.

Consider a specific example. Suppose, there exists an alternative optimal solution, i. e., another combination of accelerated temperature level and burn-in duration (T, x), which produces the same aging as that attained using the optimal temperature at the upper bound and the associated burn-in duration (T^*, x^*). This implies:

$$e^{[\frac{E_a}{K}(\frac{1}{T_u} - \frac{1}{T})]} . x = e^{[\frac{E_a}{K}(\frac{1}{T_u} - \frac{1}{T^*})]} . x^* . \qquad (5.3.9)$$

Assuming $E_A = 0.8$, $K = 8.6\text{x}10^{-5}$ ev/K, T_u = normal use temperature = 328° K,

$$x = e^{[28.360 - \frac{9302.3}{T^*}]} . x^* / e^{[28.360 - \frac{9302.3}{T}]} . x^* . \qquad (5.3.10)$$

Again, for the combination (T, x) to be cost effective,

$$c_t.(T - 328).x \leq c_t.(T^* - 328).x^* . \qquad (5.3.11)$$

c_t is the cost of maintaining unit difference in temperature for unit time and is independent of temperature level following the assumption discussed earlier.

Substituting (5.3.10) in (5.3.11) and simplifying,

$$(T-328).\, e^{\frac{9302.3}{T}} \leq (T^{*}-328).e^{\frac{9302.3}{T^{*}}} \;.$$

Taking log of both sides and expanding,

$$ln(T-328) + \frac{9302.3}{T} \leq ln\,(T^{*}-328) + \frac{9302.3}{T^{*}} \;. \tag{5.3.12}$$

To examine the condition, consider the function

$$f(T) = ln(T-328) + \frac{9302.3}{T} \;. \tag{5.3.13}$$

Differentiating $f(T)$ and setting it equal to zero,

$$\frac{1}{T-328} - \frac{9302.3}{T^{2}} = 0 \;. \tag{5.3.14}$$

Solving the equation (5.3.14),

$$(T - 340.46).(T - 8961.83) = 0 \;. \tag{5.3.15}$$

The function $f(T)$ is strictly decreasing in the interval (340.46, 8961.83) and increasing elsewhere. Also, the maximum occurs at T = 340.46^{o} K as $f'(T) < 0$ at $T = 340.46$.

For age acceleration, the stress temperature is chosen above 340.46^{o} K when the use condition is 328^{o} K. In the interval (340.46, 8961.83), the function $f(T)$ would have a decreasing value as T increases. This contradicts the condition (5.3.12). It also suggests that the aging of a product through temperature acceleration takes place at a minimum cost when the temperature is at the upper bound.

In a similar fashion, acceleration due to voltage can be analyzed. If the same age is attained through the optimal voltage V^* and some other voltage V, the following equation is true:

$$\left(\frac{V^*}{V_u}\right)^{2.3} . x^* = \left(\frac{V}{V_u}\right)^{2.3} . x \tag{5.3.16}$$

where V_u is the voltage under use condition. $x^*$and x are the stress durations under stress voltage V^* and V, respectively. For age acceleration at minimum cost through optimal voltage V^*, the following condition must hold:

$$c_v(V^* - V_u). x^* \leq c_v.(V - V_u). x . \tag{5.3.17}$$

From (5.3.16) and (5.3.17), and on simplification:

$$\frac{V^* - 6}{(V^*/6)^{2.3}} \leq \frac{V - 6}{(V/6)^{2.3}} . \tag{5.3.18}$$

Taking the log of both sides and canceling the constant terms,

$$ln(V^* - 6) - 2.3 ln V^* \leq ln(V - 6) - 2.3 ln V . \tag{5.3.19}$$

Consider the function $g(v)$ as:

$$g(V) = ln(v\text{-}6) - 2.3 ln V \,. \tag{5.3.20}$$

On differentiation and setting the derivative equal to zero,

$$\frac{1}{(V - 6)} - \frac{2.3}{V} = 0 \,. \tag{5.3.21}$$

Solving (5.3.21), V is obtained as 10.615. Since $g''(V)$ is negative at $V = 10.615$, $g(V)$ has a maximum at that point and it strictly decreases for values $V > 10.615$. For an age acceleration, the applied voltage needs to be 10.615 when the use condition is assumed to be 6v. Therefore, the condition in (5.3.19) is true where V^* is the maximum allowable voltage and the cost of accelerating the age of a product is minimum when the acceleration is carried out at that voltage.

Following the same approach, it can be shown that the cost of attaining the same age by applying humidity decreases when the relative humidity is more than 76.39%. The cost function has a minimum when the operating level is at maximum.

Analysis of Model 2

Although a failed unit is replaced with a renewed warranty period, the optimal solution for the model does not differ very much from its counterpart in the fixed warranty period model. For products with decreasing failure rate, specially the electronic products that have very small failure rates at the end of burn-in, the total number of expected field failures does not vary significantly in the two warranty policies. This is observed in example 1 (Model 2), which is the counterpart to example 1 in the case of a fixed warranty period (Model 1). For the renewed warranty period model, the optimal burn-in duration increases by a small value. This is also accompanied by a small decrease in the optimal warranty period as the total number of expected failures would be more if the warranty period remains the same. The net expected profit for the model decreases only by a small magnitude. In example 2 for the model in which a smaller field failure cost ($c_j^f = 50c_j^i$) is assumed, the optimal screening and the warranty period decreases and increases, respectively. The same fact is noted in Model 1. Again, the optimal solutions for the same problem in both renewed and fixed warranty period models (example 2 in Model 2 and example 4 in Model 1) do not differ substantially.

5.3.3 Failure Free Burn-in

As mentioned earlier, this is a common reliability test which continues until a given contiguous failure-free time has passed. These tests give assurance to customers that the system will survive failure-free for a certain period (e.g. a mission length). This is particularly important for reliability demonstration when the system consists of a large number of components increasing the chances of an early failure. From a manufacturer's point of view, an optimal level of assurance is desired. In the examples, this failure free burn-in period is considered as a decision variable.

Optimization Procedure

The optimization for failure free burn-in is performed in a recursive manner. First, the expected time of completion for the system burn-in is calculated corresponding to a failure free burn-in period. The procedure is discussed earlier in chapter 4 and considers the pooled output obtained by superimposing the component renewal processes. The relevant equations are:

$$\sum_{j=1}^{n} M_j(a_j t_1) = 1 , \quad (5.3.22)$$

$$\sum_{j=1}^{n} M_j(a_j t_2) = 2 , \quad (5.3.23)$$

....

$$\sum_{j=1}^{n} M_j(a_j t_p) = p . \quad (5.3.24)$$

t_1, t_2,...., t_i, are the times when the cumulative number of expected failures becomes *1, 2,.....,i,* respectively. $M_j(\ )$ is the expected number of renewals for the *jth* component. The expected system burn-in completion, t_{ffb}, is then given by,

$$t_{ffb} = t_{i-1} + b_{ffb} . \quad (5.3.25)$$

t_{ffb} is substituted for b, fixed period burn-in duration, in the Fixed Period Burn-in Model for evaluation of functional values at the simplex points.

Examples

Two examples are considered for illustration. For both the examples, a free replacement warranty with fixed warranty period is considered. As it has been observed that the optimal solution does not differ significantly when a warranty policy with renewed warranty period is in force, these examples are considered representative of both policies. The setup costs and the costs of stress screening with different stresses are assumed to be the same as in the fixed period burn-in model. The objective function for this failure free burn-in model is:

$$max\ r(w) - \sum_{i=1}^{3} p_i + q_i(S_{ai} - S_{oi})t_{fib} - \sum_{j=1}^{10} c_j^i . n_j M_j(t_{fib}) - \sum_{j=1}^{10} c_j^f . n_j M_{j,D}(w) \ . \quad (5.3.26)$$

The objective function is subjected to the same set of constraints as in the earlier examples. In example 1, the system described in Table 5.1 is considered. Also, the same revenue function, $r(w)$, as in (5.3.2) is assumed. The optimal solution for this example is listed in Table 5.5. Due to the facts already explored, the optimal stress levels are at the upper bounds. The optimal net profit is obtained as \$132.62. A different system consisting of larger number of components is analyzed in example 2. The number of components of each type along with the optimal solution for the system is also listed in Table 5.5. The revenue function corresponding to this system is considered as:

$$r(w) = 200 + 860[1 - e^{-0.0003w}] \ . \quad (5.3.27)$$

A field failure cost of $c_j^f = 100c_j^i$, is assumed for both the examples. $c_j^i s$, the in-plant failure costs, for the components are the same as considered before.

5.3.4 Analysis of Failure Free Burn-in Models

Example 1 considers the same problem as in example 1 in Model 1 for the fixed period burn-in model. The optimal solutions in both cases are identical in the respect that the optimal expected system burn-in completion time in the failure free burn-in model is the same as the optimal burn-in period in the fixed period burn-in model. In the failure free model, the total number of expected system failures due to all components is 1 at 500 hours and is 2 at 4500 hours. No more system failure occurs before 30,000 hours. As a result, the failure free burn-in period either would start from 500 hours or 4500

TABLE 5.5 EXAMPLES FOR FAILURE FREE BURN-IN

Examples	Optimal Solution	
no. of components= {10, 5, 7, 1, 2, 15, 8, 6, 10, 1}		
$\underline{p}$ = { 50.0, 70.0, 10.0 }	T = 380 ^{o} K	Burn-in Cost = $ 218.070
$\underline{q}$ = { 3.0, 0.009, 0.0015 }	V = 20.0 Volt	In-plant Failure Cost = $8.794
$c_j^f = 100 c_j^i$	RH = 100.0 %	Field Failure Cost = $ 155.350
	b_{ffb} = 0.4512 hr.	Total Cost = $ 382.214
	w = 5156.25 hr.	Net Profit = $ 132.621
no. of components = {30, 25, 17, 11, 10, 15, 8, 18, 8, 10}		
$\underline{p}$ = { 50.0, 70.0, 10.0 }	T = 380 ^{o} K	Burn-in Cost = $ 344.918
$\underline{q}$ = { 3.0, 0.009, 0.0015 }	V = 20.0 Volt	In-plant Failure Cost =$25.938
$c_j^f = 100 c_j^i$	RH = 100.0 %	Field Failure Cost = $ 203.930
	b_{ffb} = 0.4794 hr.	Total Cost = $ 574.787
	w = 7133.678 hr.	Net Profit = $ 384.039

hours. For the same system as considered in fixed period burn-in, it is found that the best trade-off in the profit function takes place when the system age is 5377.27 hours due to a burn-in period of 0.6061 hours. Therefore, the solution for the expected time of completion for system burn-in in the failure free model would be the same range as no more failures take place. Also, this total expected burn-in period is attained when the failure free period starts from 500 hours after the first system failure. The best solution for the failure free burn-in period is obtained as 0.4512 hours that corresponds to a system age of 4000 hours. The expected time of completion for system burn-in is obtained as 4500 hours. The best solution for the net profit function decreases by a small value compared to that in the fixed burn-in period. This is due to the fact that failure free burn-in takes place over a total period of 4500 hours, where the best trade-off in the profit function is found to occur when the system age is 5377.27 hours.

A system consisting of a large number of components is considered in example 2. System failures occur more frequently during the burn-in period due to the presence of a larger number of components. A failure free period is attained after a longer period compared to systems with lesser number of components. This prolongs the optimal expected system burn-in completion time but the optimal failure free period decreases. For the system considered in example 2, the optimal failure free burn-in period is 0.479361 hours and it starts after the 6th system failure at an equivalent age of 7500 hours. It is noted that for the same burn-in duration, the cost of screening is the same for both the systems. It does not depend on the number of components as the burn-in is performed at the system level. However, the total expected field failure cost for a system with a larger number of components is greater. Therefore, a higher selling price for the same warranty period is assumed.

The revenue function has the same form as in example 1 except that the multiplication factor associated with the second term, which is the sum of the costs of in-plant failures due to all components, is different. Because of the fact that the stress screening cost is the same over the same duration, a larger net profit results in the optimal solution than would have been possible due to higher revenue function alone. A net profit of $384.04 is obtained as an optimal solution.

5.4 SIMULATION OF THE MODELS

A simulation study for the models is done to validate the proposed analytical models. It also gives an idea of the accuracy of the models as the expressions for the cost functions for different models use a number of approximations. The simulation experiments for the fixed period burn-in and failure free burn-in models are performed. The net profit values for the models when the simulation is performed with the best values for the stress levels, the screening period and the warranty period, are compared with the results obtained from the study. For the fixed period burn-in model, the net profit function values are also determined at different points in the neighborhood of the best solution and compared with those obtained from the analytical model. Only the models for free replacement warranty policy with a fixed warranty period are simulated. The simulation codes are written in C and are listed in Appendices C and D. Other models could also be studied with little modification to the simulation codes.

To simulate the models, random variates which correspond to component life times are generated from their Weibull distributions. In the simulation model for fixed period burn-in, the number of failures during the burn-in period and those over the warranty period are determined for each component in the system. The first failure for a component over the warranty period is determined by the remaining life at the end of the burn-in period. The cost of in-plant and field failures due to all components is added to the cost of carrying out burn-in to compute the total cost. A net profit is then evaluated considering the selling price corresponding to the optimal warranty period. The average net profit is obtained by dividing the total net profit for all simulation runs by the number of simulation runs.

For the fixed period burn-in model, three simulation experiments with run lengths of 100, 300 and 1000 were performed for example 1 in fixed warranty period model (Model 1). The average net profit function values in all three cases are listed in Table 5.6. The ratios of the average net profits for simulated models to the best solutions obtained from the model are also presented. The ratios of the average net profit for the simulated system to the best solution for run lengths of 100, 300 and 1000 are 0.724, 0.863 and 0.937 respectively. It is evident that as the number of simulation runs increases the simulated system gives more accurate results. The sensitivity of the simulation model is compared with that of the model. The average net profit function values at different points around the best solution are obtained from simulation model performed with 1000 runs. For different cases, the ratios of the average net profit to the result from the analytical model are listed in Table 5.7. The standard deviations of the net profit values for 1000 runs at all points are also enumerated. It is observed that the objective function surface in the simulation and the optimization models conform to a large extent when the number of simulation runs is large. A large value for standard deviations at all points in the simulation model can be attributed to large standard deviations of the associated Weibull distributions for the components considered in the model.

For the failure free burn-in model, first component life times are generated from their respective Weibull distributions. For each component, ten consecutive renewal points on the time axis are then calculated. Finally, all renewal points for all components are arranged into increasing order to get the system failure times. A failure free period is obtained when the difference in the failure times for two consecutive system failures exceeds the optimal value for the failure free burn-in period. Subsequently, the total system burn-in completion time is determined and the number of renewals or failures for all components during burn-in and those over the warranty period are calculated. Again, the first failure times for the components after deployment are determined by the remaining lives for the components at the end of burn-in.

The simulation model is run for example 1 for the failure free burn-in model. The expected system burn-in completion time in the simulation model is obtained as 5230.35 hours when the required failure free period is assumed to be 4000 hours following the best solution. The expected time of system burn-in completion in the optimization model is found be 4500 hours. The net profit values in all three simulation experiments with 100, 300 and 1000 simulation runs are listed in Table 5.7. Also, the ratios of the average net profit for the simulated system to optimal solution are enumerated. The ratios of the average net profit values in simulation results to the optimal solution are 0.678, 0.844 and 0.928 for number of simulation runs of 100, 300 and 1000, respectively. Again, as observed in fixed period burn-in model, the accuracy of the simulation result increases, i. e., the simulation result is closer to the predicted value, when more simulation runs are performed. It is observed that for a large number of simulation runs, the accuracy of the fixed period burn-in model and failure free burn-in model are very close. This illustrates that the approximations involved in calculating the expected system burn-in completion time in the optimization model for failure free burn-in are tight.

TABLE 5.6 SIMULATION RESULTS FOR FIXED PERIOD BURN-IN

<u>100 Simulation Runs</u> Average Net Profit = $ 97.271 Ratio of Simulation Result to Optimal Solution = 0.724
<u>300 Simulation Runs</u> Average Net Profit = $ 115.946 Ratio of Simulation Result to Optimal Solution = 0.863
<u>1000 Simulation Runs</u> Average Net Profit = $ 125.888 Ratio of Simulation Result to Optimal Solution = 0.937

TABLE 5.7 SENSITIVITY OF THE SIMULATION MODEL TO THE BURN-IN AND THE WARRANTY PERIODS

1000 Simulation Runs

b and w	Simulation Results
$b^*, w^* = \{ 0.6061, 5281.71 \}$	Average Net Profit = \$ 125.888 Ratio of Simulation Result to Best Solution = 0.937 Standard Deviation = 48.026
$b, w = \{ 0.6667, 4753.54 \}$	Average Net Profit = \$ 118.430 Ratio of Simulation Result to Best Solution = 0.885 Standard Deviation = 56.760
$b, w = \{ 0.5453, 5809.88 \}$	Average Net Profit = \$ 121.926 Ratio of Simulation Result to Best Solution = 0.908 Standard Deviation = 46.270
$b, w = \{ 0.5453, 4753.54 \}$	Average Net Profit = \$ 116.341 Ratio of Simulation Result to Best Solution = 0.866 Standard Deviation = 54.75

TABLE 5.8 SIMULATION RESULTS FOR FAILURE FREE BURN-IN

100 Simulation Runs Average Net Profit = \$ 91.091 Ratio of Simulation Result to Optimal Solution = 0.678
300 Simulation Runs Average Net Profit = \$ 113.393 Ratio of Simulation Result to Optimal Solution = 0.844
1000 Simulation Runs Average Net Profit = \$ 124.679 Ratio of Simulation Result to Optimal Solution = 0.928

CHAPTER 6 CONCLUSIONS AND FUTURE WORK

6.1 Introduction

The development and analysis of the models carried out in the previous chapters gives several insights into the behavior of the models. These findings point to the specific strengths and weaknesses in the models and suggest future research areas.

6.2 Conclusions

6.2.1 Description of the Model Development Process

An integrated approach to the warranty and stress screening problems is suggested. Burn-in screening or accelerated screening is most prevalent for electronic equipments that have decreasing failure rates. An important assumption made in the development of the models is that life distribution under acceleration conditions is considered to be a linear transformation in time scale. This implies that the general form or shape of the distribution is preserved under age acceleration.

A profit function reflecting the cost of screening and the associated costs of failures over the burn-in and warranty period and the revenue earned from such products sold is developed. The cost of failures in the model is derived considering failure sequences as a renewal process. The sequence of failures over the screening period is modeled following an ordinary renewal process that starts at time zero. It is assumed that the components have random ages at the end of the burn-in process. As a result, the failures over the warranty period constitute a delayed renewal process.

The renewal function for the components that have Weibull life distributions is considered by expanding it into an infinite series of Poissonian functions. In the derivation for the excess life distribution for the components (3.4.36), terms consisting of $m''(t)$ are ignored as they have very small values for all values of $0 \leq t \leq w$. In the expression for the delayed renewal function (3.4.39), higher derivative terms are neglected as they have very small values for components with decreasing failure rates ($\beta \leq 1$) and small scale parameters (α).

Models are developed for both fixed period and failure free burn-in policies. For fixed period burn-in, three warranty policies are considered. Optimization procedures are suggested for the models and the best solutions in the case of fixed as well as renewed warranty policies are obtained for example problems.

For the failure free burn-in model an approximate estimate for the system burn-in completion time is obtained by considering the superposition of the renewal processes formed by the failure processes of the individual components. The approximate value for the expected system burn-in completion time given by (4.2.9) is substituted for b, the burn-in duration in the fixed period burn-in model to obtain the total cost function for the model. The optimization procedure for the failure free burn-in model is the same as in fixed period model with the exception that each iteration involves determination of expected system burn-in completion time and then the corresponding total cost or net profit function value at the design point from the fixed period model.

6.2.2 Behavior of the Models

From the numerical analysis in the previous chapter, it is evident that the stress screening and the warranty period for a product depend on the costs of screening as well as on the costs of field failures. Once the decision to have burn-in is made, the setup costs are not relevant in the trade-off analysis.

For the fixed period model it is observed that as the stress screening cost increases the burn-in period decreases and vice versa. This is also associated with an increase or decrease in warranty period which would depend on the revenue function and the field failure costs. It is also noted that the burn-in duration decreases when a smaller field failure cost is assumed.

In the case of a renewed warranty, the solution for the model does not differ very much from its fixed warranty period counterpart. This is true especially for the electronic products that have very small failure rates at the end of burn-in. The total number of field failures would not vary significantly under the two warranty policies. For the renewed warranty model, the burn-in period and the warranty period increases and decreases, respectively by a small value compared to its fixed warranty period counterpart. The net profit function also decreases by a small value.

Failure free burn-in is important for reliability demonstration for systems with a large number of components. For a system that has a relatively smaller number of components, system failures take place less frequently during testing. As a result, a failure free period is achieved soon and the expected system burn-in completion time would be in the same range as the burn-in period in the fixed period burn-in model. It also appears that failure free burn-in is not more cost effective or profitable than fixed period burn-in.

It is proved analytically that the stress variables in the models should always be at upper bounds when the stress screening costs are assumed to be constant and independent of stress levels. This is a fairly reasonable assumption for the range of stress levels under which the burn-in process is carried out.

6.3 Extensions and Future Work

The models in the earlier chapters are developed based on certain assumptions. In reality, some of these assumptions may not be valid. For example, the models developed may not be valid when the stress screening process is monitored periodically. In that case, the failure detection process is not instantaneous. However, it may be possible to analyze the system by using a modified form of the models discussed in the earlier chapters after obtaining an expression for the elapsed time to test completion.

In the present study it is assumed that the component failure processes are independent. However, for some electronic systems this assumption is not valid. For example, failure of a semiconductor component in an electronic circuit could lead to failures of other components due to a sudden increase in voltage or current. It would be extremely difficult if not impossible to model this phenomenon analytically. A simulation approach could be adopted to analyze such systems.

Finally, in the entire study it is assumed that failure mechanisms involved during accelerated testing are identical to failure mechanisms under normal operating conditions. For certain devices this assumption may not be valid. Accelerated testing may activate otherwise dormant failure mechanisms in the components. In that case aggregate life distributions for the components under the accelerated condition are to be derived by combining mechanism specific failure distributions. This could be done by using competing risk theory or extreme value theory. The present modeling approach can then be adopted to analyze the system.

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Appendix A C PROGRAM FOR OPTIMIZATION OF FIXED PERIOD BURN-IN MODEL

```
# include <stdio.h>
# include <math.h>

int s = 10 ;
float b[12][12] ;
float p[3] = { 50.0, 70.0, 10.0 } ;
float q[3] = { 0.05, 0.00015, 0.000025 } ;
float xu[5] = { 328.0, 6.0, 60.0, 0.0, 0.0 } ;
float alpha[10] = { 0.002, 0.012, .0008, 0.0012, 0.004,
                    0.00106, 0.008, 0.012, 0.0021, 0.002 } ;
float beta[10] = { 0.25, 0.2, 0.25, 0.25, 0.10, 0.30, 0.20, 0.25, 0.30, 0.25 };
float ci[10] = { 6.0, 2.5, 10.0, 9.0, 4.0, 8.0, 3.5, 3.0, 7.0, 5.0 } ;
int comp[10] = { 10, 5, 7, 1, 2, 15, 8, 6, 10, 1 } ;

main ( )
{
  int i, j, k, gap ;
  float x[6][5] ;
  float x0[5], x1[5], x2[5], x3[5], x4[5], x5[5] ;
  float xlb[5] = { 328.0, 6.0, 60.0, 0.0, 0.0 } ;
```

```
   float xub[5] = { 380.0, 20.0, 100.0, 180.0, 30000.0 } ;
   float xhat[5], xext[5], xbar[5], xprime[5], xdprime[5] ;
   float tcost[6], temp, xtemp[5], ybar, sterror ;
   float tcosthat, tcostext, tcostdpr, tcostpr ;
   extern float p[3], q[3], xu[5] ;
   extern float alpha[10] ;
   extern float beta[10] ;
   extern float ci[10] ;
   extern int comp[10] ;
   float cost(float x0[]), cost(float x1[]), cost(float x2[]) ;
   float cost(float x3[]), cost(float x4[]), cost(float x5[]);
   float cost(float xhat[]), cost(float xbar[]);
   float cost(float xprime[]), cost(float xdprime[]) ;

   i = 0 ;
printf("pl. en. stss. para. lvs. at pt. 0: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[0][i],&x[0][i+1],&x[0][i+2],&x[0][i+3],&x[0][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 1: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[1][i],&x[1][i+1],&x[1][i+2],&x[1][i+3],&x[1][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 2: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[2][i],&x[2][i+1],&x[2][i+2],&x[2][i+3],&x[2][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 3: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[3][i],&x[3][i+1],&x[3][i+2],&x[3][i+3],&x[3][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 4: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[4][i],&x[4][i+1],&x[4][i+2],&x[4][i+3],&x[4][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 5: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[5][i],&x[5][i+1],&x[5][i+2],&x[5][i+3],&x[5][i+4]) ;
      for ( k=0 ; k<=4 ; ++k ) {
         x0[k] = x[0][k] ;
         x1[k] = x[1][k] ;
         x2[k] = x[2][k] ;
         x3[k] = x[3][k] ;
         x4[k] = x[4][k] ;
         x5[k] = x[5][k] ;
```

```
    }
    tcost[0] = cost(x0) ;
    tcost[1] = cost(x1) ;
    tcost[2] = cost(x2) ;
    tcost[3] = cost(x3) ;
    tcost[4] = cost(x4) ;
    tcost[5] = cost(x5) ;
newset: {
  for ( gap=2 ; gap > 0 ; gap /= 2 )
     for ( i=gap ; i < 6 ; ++i )
        for ( j=i-gap ; j>=0 && tcost[j] > tcost[j+gap] ; j -=gap ) {
                  temp = tcost[j] ;
                  for ( k=0 ; k <=4 ; ++k )
                    xtemp[k] = x[j][k] ;
                  tcost[j] = tcost[j+gap] ;
                  for ( k=0 ; k <=4 ; ++k )
                    x[j][k] = x[j+gap][k] ;
                 tcost[j+gap] = temp ;
                 for ( k=0 ; k <=4 ; ++k )
                  x[j+gap][k] = xtemp[k] ;
        }
    for ( k=0 ; k<=4 ; ++k ) {
       x0[k] = x[0][k] ;
       x1[k] = x[1][k] ;
       x2[k] = x[2][k] ;
       x3[k] = x[3][k] ;
       x4[k] = x[4][k] ;
       x5[k] = x[5][k] ;
       xbar[k] = ( x[0][k] + x[1][k] + x[2][k] + x[3][k] + x[4][k] + x[5][k] ) / 6 ;
       xhat[k] = 2 * xbar[k] - x5[k] ;

      if ( xhat[k] <= xlb[k] )
         xhat[k] = xlb[k] ;
      else if ( xhat[k] >= xub[k] )
```

```
            xhat[k] = xub[k] ;
        }
          if ( tcost[0] > (tcosthat = cost(xhat) ) )  {
              for ( k=0 ; k<=4 ; ++k )  {
                 xext[k] = 3*xhat[k] - 2*xbar[k] ;
                  if ( xext[k] <= xlb[k] )
                    xext[k] = xlb[k] ;
                 else if ( xext[k] >= xub[k] )
                    xext[k] = xub[k] ;
}
            if ( tcosthat > ( tcostext = cost(xext) ) ) {
                for ( k=0 ; k<=4 ; ++k )
                    x5[k] = xext[k] ;
                  tcost[5] = tcostext ;
            goto check ;
   }
           else if ( tcosthat <= tcostext ) {
               for ( k=0 ; k<= 4 ; ++k )
                   x5[k] = xhat[k] ;
                tcost[5] = tcosthat ;
            goto check ;
   }
}
     else if ( tcost[4] >= tcosthat ) {
         for ( k=0 ; k<= 4 ; ++k )
           x5[k] = xhat[k] ;
          tcost[5] = tcosthat ;
        goto check ;
}
else if ( tcost[4]<tcosthat && tcosthat <= tcost[5] ) {
        for ( k=0 ; k<= 4 ; ++k )
          xprime[k] = xhat[k] ;
}
else if ( tcost[4]<tcosthat && tcost[5]<tcosthat )  {
```

```
        for ( k=0 ; k<= 4 ; ++k )
            xprime[k] = x5[k] ;
}
      for ( k=0 ; k<= 4 ; ++k ) {
         xdprime[k] = 0.5 * ( xbar[k] + xprime[k] ) ;
            if ( xdprime[k] <= xlb[k] )
                xdprime[k] = xlb[k] ;
          else if ( xdprime[k] >= xub[k] )
               xdprime[k] = xub[k] ;
}
      if ( ( tcostdpr = cost(xdprime) ) > ( tcostpr = cost(xprime) ) ) {
              for ( j=0 ; j<=5 ; ++j )
                for ( k=0 ; k<= 4 ; ++k ) {
                   x[j][k] = 0.5 * ( x[j][k] + x0[k] ) ;
                    if ( x[j][k] <= xlb[k] )
                      x[j][k] = xlb[k] ;
                   else if ( x[j][k] >= xub[k] )
                     x[j][k] = xub[k] ;
     }

  for ( k=0 ; k<=4 ; ++k ) {
    x0[k] = x[0][k] ;
    x1[k] = x[1][k] ;
    x2[k] = x[2][k] ;
    x3[k] = x[3][k] ;
    x4[k] = x[4][k] ;
    x5[k] = x[5][k] ;
  }
tcost[1] = cost(x1) ;
tcost[2] = cost(x2) ;
tcost[3] = cost(x3) ;
tcost[4] = cost(x4) ;
tcost[5] = cost(x5) ;
```

```
goto check ;

    }
      else if ( tcostdpr <= tcostpr ){
        for ( k=0 ; k<= 4 ; ++k )
            x5[k] = xdprime[k] ;
         tcost[5] = tcostdpr ;
         goto check ;
      }
    }
  check: {
printf("x0= %f,%f,%f,%f,%f & c= %f\n",x0[0],x0[1],x0[2],x0[3],x0[4],tcost[0] );
printf("x1= %f,%f,%f,%f,%f & c= %f\n",x1[0],x1[1],x1[2],x1[3],x1[4],tcost[1] );
printf("x2= %f,%f,%f,%f,%f & c= %f\n",x2[0],x2[1],x2[2],x2[3],x2[4],tcost[2] );
printf("x3= %f,%f,%f,%f,%f & c= %f\n",x3[0],x3[1],x3[2],x3[3],x3[4],tcost[3] );
printf("x4= %f,%f,%f,%f,%f & c= %f\n",x4[0],x4[1],x4[2],x4[3],x4[4],tcost[4] );
printf("x5= %f,%f,%f,%f,%f & c= %f\n",x5[0],x5[1],x5[2],x5[3],x5[4],tcost[5] );
ybar = (tcost[0] + tcost[1] + tcost[2] + tcost[3] + tcost[4] + tcost[5] ) / 6 ;
      sterror = sqrt ((pow((tcost[0] - ybar), 2) + pow((tcost[1] - ybar), 2)
                + pow((tcost[2] - ybar), 2) + pow((tcost[3] - ybar), 2)
                + pow((tcost[4] - ybar), 2) + pow((tcost[5] -ybar), 2)) / 6 ) ;
    if ( sterror > 0.00001 )  {
       for ( k=0 ; k<=4 ; ++k ) {
          x[0][k] = x0[k] ;
          x[1][k] = x1[k] ;
          x[2][k] = x2[k] ;
          x[3][k] = x3[k] ;
          x[4][k] = x4[k] ;
          x[5][k] = x5[k] ;
      }
goto newset ;
    }
    else  {
  printf ("optimal to tolerences:\n" ) ;
```

```
printf("x0= %f,%f,%f,%f,%f & c= %f\n",x0[0],x0[1],x0[2],x0[3],x0[4],tcost[0] );
printf("x1= %f,%f,%f,%f,%f & c= %f\n",x1[0],x1[1],x1[2],x1[3],x1[4],tcost[1] );
printf("x2= %f,%f,%f,%f,%f & c= %f\n",x2[0],x2[1],x2[2],x2[3],x2[4],tcost[2] );
printf("x3= %f,%f,%f,%f,%f & c= %f\n",x3[0],x3[1],x3[2],x3[3],x3[4],tcost[3] );
printf("x4= %f,%f,%f,%f,%f & c= %f\n",x4[0],x4[1],x4[2],x4[3],x4[4],tcost[4] );
printf("x5= %f,%f,%f,%f,%f & c= %f\n",x5[0],x5[1],x5[2],x5[3],x5[4],tcost[5] );
   }
   }
}

float cost(xa)
float xa[5];

{
  int i, k, n ;
  float fcost, incost, wcost, tlcost, bp, neprofit ;
  float renew(float, float, float) ;

  fcost = 0.0 ;
  incost = 0.0 ;
  wcost = 0.0 ;
   bp = xa[3] / 60.0 * exp(28.3607 - 9302.3256/xa[0] - 4.1211 +
         2.3*log(xa[1]) -3.66 + 0.061*xa[2] ) ;
  printf ("%f, %f, %f, %f, %f, %f \n", xa[0], xa[1], xa[2], xa[3], xa[4], bp);
        for ( k=0 ; k <= 2 ; ++k )
          fcost = fcost + p[k] + q[k] * ( xa[k] - xu[k] ) * xa[3] ;
             for ( n=0 ; n<= 9 ; ++n ) {
               incost = incost + comp[n] * ci[n] * renew(bp, alpha[n], beta[n] ) ;
              wcost = wcost + 100*comp[n]*ci[n]*(renew((bp+xa[4]),alpha[n],beta[n] )
                     - renew(bp,alpha[n],beta[n] ) - renew(xa[4], alpha[n], beta[n])
                       + 1 - exp(-alpha[n] * pow(xa[4], beta[n]) ) ) ;
}
   tlcost = fcost + incost + wcost ;
   neprofit = tlcost - 200 - 400.0 * ( 1 - exp(-0.0003*xa[4]) ) ;
```

```
 printf ( "%f, %f, %f, %f, %f \n", fcost, incost, wcost, tlcost, neprofit ) ;
return(neprofit) ;
}

float renew (t, ralpha, rbeta)
float t, ralpha, rbeta ;

{
   extern int s ;
   int m, n, p, k, i ;
   extern float b[12][12] ;
   float weib, convo, renewal ;
   float delta(int, float) ;
   float D(int, float) ;
   float c(int, float) ;

   renewal = 0.0 ;
   weib = 0.0 ;
    for ( m=0 ; m <= s ; ++m )
          for ( n=0 ; n <= s ; ++n )
               b[m][n] = 0.0 ;
      for ( p=0 ; p <= s ; ++p )
           b[0][p] = delta(p, rbeta) ;
       for ( k=1 ; k <= (s-1); ++k )
            for ( p=k ; p <= s ; ++p )
             for ( i=(k-1) ; i <= (p-1) ; ++i ) {
                b[k][p] = b[k][p] + b[k-1][i] * delta((p-i), rbeta) ;
    }
     weib = ralpha * pow(t, rbeta) ;
       for ( i=1 ; i <= s ; ++i )
            renewal = renewal + c(i, rbeta) * D(i, weib) ;
  return(renewal) ;
}
```

```
float c(arg, cbeta)
int arg ;
float cbeta ;

{
   int i ;
   float sub1 ;
   float phi(int, int, float) ;

   sub1 = 0.0 ;
   for ( i=1 ; i <=arg ; ++i )
       sub1 = sub1 + phi(i, arg, cbeta) ;
 return (sub1) ;
}

float phi(index, arg, pbeta)
int index, arg ;
float pbeta ;

{
   int i, j ;
   float first, second, d ;
   float a(int, int, float) ;

   d = 0.0 ;
   first = 0.0 ;
   second = 0.0 ;
   if ( index == arg )
      d = a(index, arg, pbeta) ;
   else if ( index <= arg ) {
     for ( i=index ; i <= arg ; ++i )
          first = first + a(i,arg, pbeta) ;
    for ( j=index ; j <= (arg - 1) ; ++j )
        second = second + a(j, (arg - 1), pbeta) ;
```

```
        d = first - second ;
    }
    return (d) ;
}

float a(index, arg, abeta)
int index, arg ;
float abeta ;
{
    int i ;
    float sub2 ;
    float power(float, int) ;
    float factor(int) ;
    float delta(int, float) ;

    sub2 = 0.0 ;
    for ( i=index ; i <= arg ; ++i ) {
        sub2 = sub2 + power(-1, index + i) * factor(arg) / factor(i)
                /factor(arg - i) * b[index][i] /delta(i, abeta) ;
    }
   return (sub2) ;

}
float D(index, arg)
int index ;
float arg ;
{
    int i ;
    float sub3 ;
    float power(float, int) ;
    float factor(int) ;

    sub3 = 0.0 ;
    for ( i=index ; i <= s ; ++i )
```

```
        sub3 = sub3 + power(arg, i) / factor(i) * exp(-arg) ;
  return (sub3) ;
}

float delta(x, dbeta)
int x ;
float dbeta ;
{
   float d ;
   float gamma(float) ;
   float factor(int) ;

   d = gamma(dbeta * x + 1) / factor(x) ;
 return (d) ;
}

float gamma(x)
float x ;
{
   float g ;
   float power(float, int) ;

   if ( x < 2 )
      g = 1 - 0.5748646 * (x - 1) + 0.9512363 * power((x - 1), 2)
          - 0.6998588 * power((x - 1), 3) + .4245549 * power((x - 1), 4)
          - 0.1010678 * power((x - 1), 5) ;
   else if ( x >= 2 )
      g = (x - 1) * gamma(x - 1) ;
 return (g) ;
}

float power(x, n)
int n ;
float x ;
```

```
{
   int i ;
   float p ;

   p = 1 ;
   if ( n > 0 )
     for ( i=1 ; i <=n ; ++i )
         p = p * x ;
  return (p) ;
}

float factor(n)
int n ;

{
   int i, f ;

   f = 1 ;
   if ( n > 0 )
     for ( i=1 ; i<=n ; ++i )
        f = f * i ;
 return (f) ;
}
```

Appendix B C PROGRAM FOR OPTIMIZATION OF FAILURE FREE BURN-IN MODEL

```
# include <stdio.h>
# include <math.h>
#define del 0.000001

int s = 10 ;
float b[12][12], bp ;
float nsysf[40] ;
float ti[40] ;
float p[3] = { 50.0, 70.0, 10.0 } ;
float q[3] = { 3.0, 0.009, 0.0015 } ;
float xu[5] = { 328.0, 6.0, 60.0, 0.0, 0.0 } ;
float alpha[10] = { 0.002, 0.012, .0008, 0.0012, 0.004,
                    0.00106, 0.008, 0.012, 0.0021, 0.002 } ;
float beta[10] = { 0.25, 0.2, 0.25, 0.25, 0.10, 0.30, 0.20, 0.25, 0.30, 0.25 };
float ci[10] = { 6.0, 2.5, 10.0, 9.0, 4.0, 8.0, 3.5, 3.0, 7.0, 5.0 } ;
int comp[10] = { 10, 5, 7, 1, 2, 15, 8, 6, 10, 1 } ;

main ( )
{
  int i, j, k, t, n, r, gap ;
  float x[6][5] ;
```

```
float xlb[5] = { 328.0, 6.0, 60.0, 0.0, 0.0 } ;
float xub[5] = { 380.0, 20.0, 100.0, 3.0, 30000.0 } ;
float xhat[5], xext[5], xbar[5], xprime[5], xdprime[5] ;
float fbp[6] ;
float tcost[6], temp, xtemp[5], ybar, sterror ;
float tcosthat, tcostext, tcostdpr, tcostpr ;
extern float p[3], q[3], xu[5] ;
extern float alpha[10] ;
extern float beta[10] ;
extern float ci[10] ;
extern float nsysf[40] ;
extern float ti[40] ;
extern int comp[10] ;
float renew(float, float, float) ;
float y[5] ;
float eqbr(float y[]) ;
float cost(float y[]) ;

i = 0 ;
printf("pl. en. stss. para. lvs. at pt. 0: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[0][i],&x[0][i+1],&x[0][i+2],&x[0][i+3],&x[0][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 1: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[1][i],&x[1][i+1],&x[1][i+2],&x[1][i+3],&x[1][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 2: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[2][i],&x[2][i+1],&x[2][i+2],&x[2][i+3],&x[2][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 3: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[3][i],&x[3][i+1],&x[3][i+2],&x[3][i+3],&x[3][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 4: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[4][i],&x[4][i+1],&x[4][i+2],&x[4][i+3],&x[4][i+4]) ;
printf("pl. en. stss. para. lvs. at pt. 5: temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[5][i],&x[5][i+1],&x[5][i+2],&x[5][i+3],&x[5][i+4]) ;
    for ( t=0 ; t<=39 ; ++t ) {
        nsysf[t] = 0.0 ;
        ti[t] = (t+1) * 500.0;
```

```
    for ( n=0 ; n<=9 ; ++n )
      nsysf[t] = nsysf[t] + comp[n] * renew(ti[t], alpha[n], beta[n]) ;
      printf ( "%f, %f\n", ti[t], nsysf[t] ) ;
  }
    for ( i=0 ; i <=5 ; ++i ) {
      for ( k=0 ; k<=4 ; ++k )
        y[k] = x[i][k] ;
        y[3] = eqbr(y) ;
       tcost[i] = cost(y) ;
   }
newset: {
     for ( gap=2 ; gap > 0 ; gap /= 2 )
         for ( i=gap ; i < 6 ; ++i )
            for ( j=i-gap ; j>=0 && tcost[j] > tcost[j+gap] ; j -=gap ) {
                    temp = tcost[j] ;
                    for ( k=0 ; k <=4 ; ++k )
                      xtemp[k] = x[j][k] ;
                    tcost[j] = tcost[j+gap] ;
                    for ( k=0 ; k <=4 ; ++k )
                     x[j][k] = x[j+gap][k] ;
                   tcost[j+gap] = temp ;
                   for ( k=0 ; k <=4 ; ++k )
                      x[j+gap][k] = xtemp[k] ;
  }
      for ( k=0 ; k<=4 ; ++k ) {
        xbar[k] = ( x[0][k] + x[1][k] + x[2][k] + x[3][k] + x[4][k] + x[5][k] ) / 6 ;
        xhat[k] = 2 * xbar[k] - x[5][k] ;
           if ( xhat[k] <= xlb[k] )
              xhat[k] = xlb[k] ;
          else if ( xhat[k] >= xub[k] ) {
             xhat[k] = xub[k] ;
  }
          y[k] = xhat[k] ;
  }
```

```
y[3] = eqbr(y) ;
  if ( tcost[0] > (tcosthat = cost(y)) )   {
    for ( k=0 ; k<=4 ; ++k )  {
        xext[k] = 2*xhat[k] - xbar[k] ;
         if ( xext[k] <= xlb[k] )
           xext[k] = xlb[k] ;
        else if ( xext[k] >= xub[k] ) {
           xext[k] = xub[k] ;
}
  y[k] = xext[k] ;
}
y[3] = eqbr(y) ;
if ( tcosthat > ( tcostext = cost(y) ) ) {
     for ( k=0 ; k<=4 ; ++k )
       x[5][k] = xext[k] ;
     tcost[5] = tcostext ;
  goto check ;
}
 else if ( tcosthat <= tcostext ) {
    for ( k=0 ; k<= 4 ; ++k )
      x[5][k] = xhat[k] ;
     tcost[5] = tcosthat ;
     goto check ;
 }
}
 else if ( tcost[4] >= tcosthat ) {
    for ( k=0 ; k<= 4 ; ++k )
       x[5][k] = xhat[k] ;
       tcost[5] = tcosthat ;
    goto check ;
}
else if ( tcost[4]<tcosthat && tcosthat <= tcost[5] ) {
    for ( k=0 ; k<= 4 ; ++k ) {
       xprime[k] = xhat[k] ;
```

```
            y[k] = xprime[k] ;
        }
    }
    else if ( tcost[4]<tcosthat && tcost[5]<tcosthat ) {
        for ( k=0 ; k<= 4 ; ++k ) {
            xprime[k] = x[5][k] ;
            y[k] = xprime[k] ;
        }
    }
    y[3] = eqbr(y) ;
    tcostpr = cost(y) ;
    for ( k=0 ; k<= 4 ; ++k ) {
        xdprime[k] = 0.5 * ( xbar[k] + xprime[k] ) ;
        if ( xdprime[k] <= xlb[k] )
            xdprime[k] = xlb[k] ;
        else if ( xdprime[k] >= xub[k] )
            xdprime[k] = xub[k] ;
        y[k] = xdprime[k] ;
    }
    y[3] = eqbr(y) ;
    if ( ( tcostdpr = cost(y) ) > tcostpr ) {
        for ( j=0 ; j<=5 ; ++j )
            for ( k=0 ; k<= 4 ; ++k ) {
                x[j][k] = 0.5 * ( x[j][k] + x[0][k] ) ;
                if ( x[j][k] <= xlb[k] )
                    x[j][k] = xlb[k] ;
                else if ( x[j][k] >= xub[k] )
                    x[j][k] = xub[k] ;
            }
        for ( i=1 ; i <=5 ; ++i ) {
            for ( k=0 ; k<=4 ; ++k )
                y[k] = x[i][k] ;
            y[3] = eqbr(y) ;
            tcost[i] = cost(y) ;
```

```
            }
          goto check ;
 }
   else if ( tcostdpr <= tcostpr ){
        for ( k=0 ; k<= 4 ; ++k )
             x[5][k] = xdprime[k] ;
          tcost[5] = tcostdpr ;
     goto check ;
   }
  }
  check: {
  for ( i=0 ; i<=5 ; ++i )
  printf("x %d = %f,%f,%f,%f,%f & cost = %f\n",
                  i,x[i][0],x[i][1],x[i][2],x[i][3],x[i][4],tcost[i] );
 ybar = (tcost[0] + tcost[1] + tcost[2] + tcost[3] + tcost[4] + tcost[5] ) / 6 ;
          sterror = sqrt ((pow((tcost[0] - ybar), 2) + pow((tcost[1] - ybar), 2)
          + pow((tcost[2] - ybar), 2) + pow((tcost[3] - ybar), 2)
          + pow((tcost[4] - ybar), 2) + pow((tcost[5] -ybar), 2)) / 6 ) ;

    if ( sterror > 0.00001 )
       goto newset ;
    else  {
   printf ("optimal to tolerences:\n" ) ;
      for ( i=0 ; i<=5 ; ++i )
        printf("x %d = %f,%f,%f,%f,%f & cost = %f\n",
                        i,x[i][0],x[i][1],x[i][2],x[i][3],x[i][4],tcost[i] );
  }
}
}

float eqbr(z)
float z[5] ;

{
```

```
    int t, r ;
    float bp, ft[21], ffb ;

    r = 0 ;
     for ( t=0 ; t<=20 ; ++t )
          ft[t] = 0.0 ;
   bp = z[3] * exp(28.3607 - 9302.3256/z[0] - 4.1211 +
          2.3*log(z[1]) -3.66 + 0.061*z[2] ) ;
    for ( t=0 ; (ti[t]-ft[r]) <= bp; ++t ) {
         if ( nsysf[t] >= (r+1-del) && nsysf[t] <= (r+2) ) {
               ++r ;
              ft[r] = ti[t] ;
         }
      }
      printf ( " %f, %d\n", ft[r], r ) ;
      ffb = ( ft[r] + bp ) / exp(28.3607 - 9302.3256/z[0] - 4.1211 +
            2.3*log(z[1]) -3.66 + 0.061*z[2] ) ;
  printf ("%f, %f, %f, %f, %f, %f\n", z[0], z[1], z[2], z[3], z[4], ffb);
return(ffb) ;
}

float cost(xa)
float xa[5] ;

{
  int i, k, n ;
  float fcost, incost, wcost, tlcost, bp, neprofit ;
  float renew(float, float, float) ;

  fcost = 0.0 ;
  incost = 0.0 ;
  wcost = 0.0 ;
     bp = xa[3] * exp(28.3607 - 9302.3256/xa[0] - 4.1211 +
              2.3*log(xa[1]) -3.66 + 0.061*xa[2] ) ;
```

```
  printf ("%f, %f, %f, %f, %f, %f \n", xa[0], xa[1], xa[2], xa[3], xa[4], bp);
        for ( k=0 ; k <= 2 ; ++k )
            fcost =  fcost + p[k] + q[k] * ( xa[k] - xu[k] ) * xa[3] ;
        for ( n=0 ; n<= 9 ; ++n ) {
           incost = incost + comp[n] * ci[n] * renew(bp, alpha[n], beta[n] ) ;
          wcost = wcost + 100*comp[n]*ci[n]*(renew((bp+xa[4]),alpha[n],beta[n] )
                 - renew(bp,alpha[n],beta[n] ) - renew(xa[4], alpha[n], beta[n])
                 + 1 - exp(-alpha[n] * pow(xa[4], beta[n]) ) ) ;
         }
   tlcost = fcost + incost + wcost ;
  neprofit = tlcost - 200 - 400*(1 - exp(-0.0003*xa[4])) ;
  printf ( "%f, %f, %f, %f, %f \n", fcost, incost, wcost, tlcost, neprofit ) ;
 return(neprofit) ;
}

float renew (t, ralpha, rbeta)
float t, ralpha, rbeta ;

{
  extern int s ;
  int m, n, p, k, i ;
  extern float b[12][12] ;
  float weib, convo, renewal ;
  float delta(int, float) ;
  float D(int, float) ;
  float c(int, float) ;

  renewal = 0.0 ;
  weib = 0.0 ;
     for ( m=0 ; m <= s ; ++m )
       for ( n=0 ; n <= s ; ++n )
          b[m][n] = 0.0 ;
    for ( p=0 ; p <= s ; ++p )
       b[0][p] = delta(p, rbeta) ;
```

```
      for ( k=1 ; k <= (s-1); ++k )
          for ( p=k ; p <= s ; ++p )
             for ( i=(k-1) ; i <= (p-1) ; ++i ) {
                b[k][p] = b[k][p] + b[k-1][i] * delta((p-i), rbeta) ;
             }
    weib = ralpha * pow(t, rbeta) ;
       for ( i=1 ; i <= s ; ++i )
           renewal = renewal + c(i, rbeta) * D(i, weib) ;
  return(renewal) ;
}

float c(arg, cbeta)
int arg ;
float cbeta ;

{
   int i ;
   float sub1 ;
   float phi(int, int, float) ;

   sub1 = 0.0 ;
   for ( i=1 ; i <=arg ; ++i )
     sub1 = sub1 + phi(i, arg, cbeta) ;
   return (sub1) ;
}

float phi(index, arg, pbeta)
int index, arg ;
float pbeta ;
{
   int i, j ;
   float first, second, d ;
   float a(int, int, float) ;
```

```
    d = 0.0 ;
    first = 0.0 ;
    second = 0.0 ;
    if ( index == arg )
        d = a(index, arg, pbeta) ;
   else if ( index <= arg )  {
       for ( i=index ; i <= arg ; ++i )
           first = first + a(i,arg, pbeta) ;
         for ( j=index ; j <= (arg - 1) ; ++j )
             second = second + a(j, (arg - 1), pbeta) ;
      d = first - second ;
  }
 return (d) ;
}

float a(index, arg, abeta)
int index, arg ;
float abeta ;
{
  int i ;
  float sub2 ;
  float power(float, int) ;
  float factor(int) ;
  float delta(int, float) ;

  sub2 = 0.0 ;
  for ( i=index ; i <= arg ; ++i )  {
     sub2 = sub2 + power(-1, index + i) * factor(arg) / factor(i)
            /factor(arg - i) * b[index][i] /delta(i, abeta) ;
   }
 return (sub2) ;
}

float D(index, arg)
```

```
int index ;
float arg ;
{
   int i ;
   float sub3 ;
   float power(float, int) ;
   float factor(int) ;

   sub3 = 0.0 ;
   for ( i=index ; i <= s ; ++i )
       sub3 = sub3 + power(arg, i) / factor(i) * exp(-arg) ;
   return (sub3) ;
}

float delta(x, dbeta)
int x ;
float dbeta ;
{
   float d ;
   float gamma(float) ;
   float factor(int) ;

   d = gamma(dbeta * x + 1) / factor(x) ;
  return (d) ;
}

float gamma(x)
float x ;

{
   float g ;
   float power(float, int) ;
   if ( x < 2 )
       g = 1 - 0.5748646 * (x - 1) + 0.9512363 * power((x - 1), 2)
```

```
            - 0.6998588 * power((x - 1), 3) + .4245549 * power((x - 1), 4)
            - 0.1010678 * power((x - 1), 5) ;
    else if ( x >= 2 )
        g = (x - 1) * gamma(x - 1) ;
  return (g) ;
}

float power(x, n)
int n ;
float x ;

{
   int i ;
   float p ;

   p = 1 ;
   if ( n > 0 )
      for ( i=1 ; i <=n ; ++i )
           p = p * x ;
  return (p) ;
}

float factor(n)
int n ;

{
   int i, f ;

   f = 1 ;
   if ( n > 0 )
     for ( i=1 ; i<=n ; ++i )
          f = f * i ;
  return (f) ;
}
```

Appendix C "C" PROGRAM FOR SIMULATION OF FIXED PERIOD BURN-IN MODEL

```
# include <stdio.h>
# include <math.h>
# include <stdlib.h>

int s = 10 ;
float p[3] = { 50.0, 70.0, 10.0 } ;
float q[3] = { 0.05, 0.00015, 0.000025 } ;
float xu[5] = { 328.0, 6.0, 60.0, 0.0, 0.0 } ;
float alpha[10] = { 0.002, 0.012, .0008, 0.0012, 0.004,
                    0.00106, 0.008, 0.012, 0.0021, 0.002 } ;
float beta[10] = { 0.25, 0.2, 0.25, 0.25, 0.10, 0.30, 0.20, 0.25, 0.30, 0.25 };
float ci[10] = { 6.0, 2.5, 10.0, 9.0, 4.0, 8.0, 3.5, 3.0, 7.0, 5.0 } ;
int comp[10] = { 10, 5, 7, 1, 2, 15, 8, 6, 10, 1 } ;

main ( )
{
  int i, j, k, ii, jj, in, fn ;
  float x[5], ts[1000], tc[10] ;
  float bp, fcost, tts, vari, tlcost, fl, avtsc, sddv, neprofit ;
  extern float p[3], q[3], xu[5] ;
  extern float alpha[10] ;
```

```
  extern float beta[10] ;
  extern float ci[10] ;
  extern int comp[10] ;
  float weib(float, float) ;

printf("pl. en. stss. para. lvs. : temp, volt, mois, br, w:\n" ) ;
scanf("%f %f %f %f %f",&x[0],&x[1],&x[2],&x[3],&x[4]) ;

 fcost = 0.0 ;
 tts = 0.0 ;
 vari = 0.0 ;
 bp= x[3] / 60.0 * exp(28.3607 - 9302.3256/x[0] - 4.1211 +
      2.3*log(x[1]) -3.66 + 0.061*x[2] ) ;
   for ( k=0 ; k <= 2 ; ++k )
       fcost = fcost + p[k] + q[k] * ( x[k] - xu[k] ) * x[3] ;
          for ( i=0 ; i<=999 ; ++i ) {
               ts[i] = 0.0 ;
            for ( j=0 ; j<=9 ; ++j ) {
               tc[j] = 0.0 ;
             for ( jj=0 ; jj<=comp[j] ; ++jj ) {
             fl = 0.0 ;
             in = 0 ;
             fn = 0 ;
             while ( fl <= (bp + x[4]) ) {
                fl = fl + weib(alpha[j], beta[j]) ;
                 if ( fl <= bp )
                   ++in ;
                else if ( fl <= (bp + x[4]) )
                   ++fn ;
             }
            tc[j] = tc[j] + in * ci[j] + 100 * ci[j] * fn ;
    }
   ts[i] = ts[i] + tc[j] ;
 }
```

```
    tts = tts + ts[i] ;
  }
    avtsc = tts / 1000 ;
    for ( ii=0 ; ii<=999 ; ++ii )
      vari = vari + pow((ts[ii]  - avtsc), 2) ;
     sddv = sqrt(vari/1000) ;
    printf("average total cost due to system failures is = %f\n", avtsc ) ;
    printf("sddv for all simulation runs is = %f\n", sddv ) ;
    tlcost = fcost + avtsc ;
    neprofit = tlcost - 200 - 400.0 * ( 1 - exp(-0.0003*x[4]) ) ;
    printf("neprofit is = %f\n", neprofit ) ;
}

float weib(ralpha, rbeta)
float ralpha, rbeta ;

{
  float u, p, t ;

  u = rand( ) / pow(2, 15) ;
  p = ( - log(u) / ralpha ) ;
  t = pow(p, (1/rbeta)) ;
 return(t) ;
}
```

Appendix D C PROGRAM FOR SIMULATION OF FAILURE FREE BURN-IN MODEL

```
# include <stdio.h>
# include <math.h>
# include <stdlib.h>

int s = 10 ;
float p[3] = { 50.0, 70.0, 10.0 } ;
float q[3] = { 0.05, 0.00015, 0.000025 } ;
float xu[5] = { 328.0, 6.0, 60.0, 0.0, 0.0 } ;
float alpha[10] = { 0.002, 0.012, .0008, 0.0012, 0.004,
                     0.00106, 0.008, 0.012, 0.0021, 0.002 } ;
float beta[10] = { 0.25, 0.2, 0.25, 0.25, 0.10, 0.30, 0.20, 0.25, 0.30, 0.25 };
float ci[10] = { 6.0, 2.5, 10.0, 9.0, 4.0, 8.0, 3.5, 3.0, 7.0, 5.0 } ;
int comp[10] = { 10, 5, 7, 1, 2, 15, 8, 6, 10, 1 } ;

main ( )
{
  int i, j, k, r, ii, jj, js, gap, in, fn ;
  float x[5], ts[1000], tbp[1000], tc[10], ft[700], tyft[10][20][10] ;
  float ffbp, tbr, fcost, tts, temp, tlcost, fl, avtsc, ttbp, atbp, neprofit ;
  extern float p[3], q[3], xu[5] ;
  extern float alpha[10] ;
```

```
   extern float beta[10] ;
   extern float ci[10] ;
   extern int comp[10] ;
   float weib(float, float) ;

printf("pl. en. stss. para. lvs. : temp, volt, mois, ffbr, w:\n" ) ;
scanf("%f %f %f %f %f",&x[0],&x[1],&x[2],&x[3],&x[4]) ;
fcost = 0.0 ;
tts = 0.0 ;
ttbp = 0.0 ;
ffbp= x[3] / 60.0 * exp(28.3607 - 9302.3256/x[0] - 4.1211 +
      2.3*log(x[1]) -3.66 + 0.061*x[2] ) ;
r = 1 ;
    for ( i=0 ; i<=999 ; ++i ){
       ts[i] = 0.0 ;
      for ( j=0 ; j<=9 ; ++j ) {
           for ( jj=0 ; jj<=comp[j] ; ++jj ) {
            fl = 0.0 ;
              for ( js=0 ; js<=9 ; ++js ) {
                 fl = fl + weib(alpha[j], beta[j]) ;
                 tyft[j][jj][js] = fl ;
                 ft[r] = fl ;
                 ++r ;
            }
       }
    }

    for ( gap=2 ; gap > 0 ; gap /= 2 )
        for ( ii=gap ; ii<=650 ; ++ii )
            for ( r=ii-gap ; r>=0 && ft[r] > ft[r+gap] ; r -=gap ) {
                   temp = ft[r] ;
                   ft[r] = ft[r+gap] ;
                   ft[r+gap] = temp ;
            }
```

```
    r = 0 ;
    while ( (ft[r+1] - ft[r]) <= ffbp  )
        ++r ;
    tbp[i] = ft[r] + ffbp ;
    for ( j=0 ; j<=9 ; ++j ) {
        tc[j] = 0.0 ;
      for ( jj=0 ; jj<=comp[j] ; ++jj ) {
        js = 0 ;
        in = 0 ;
        fn = 0 ;
          while ( tyft[j][jj][js] <= (tbp[i] + x[4]) ) {
               if ( tyft[j][jj][js] <= tbp[i] )
                   ++in ;
              else if ( tyft[j][jj][js] <= (tbp[i] + x[4]) )
                   ++fn ;
            ++js ;
        }
   tc[j] =  tc[j] + in * ci[j] + 100 * ci[j] * fn ;
  }
   ts[i] = ts[i] + tc[j] ;
 }
   tts = tts + ts[i] ;
   ttbp = ttbp + tbp[i] ;
}

   avtsc = tts / 1000 ;
   atbp = ttbp/ 1000 ;
   printf("average total cost due to system failures is = %f\n", avtsc ) ;
   printf("av. length of system burn-in completion time is = %f\n", atbp ) ;
   tbr= tbp * 60.0 / exp(28.3607 - 9302.3256/x[0] - 4.1211 +
          2.3*log(x[1]) -3.66 + 0.061*x[2] ) ;
    for ( k=0 ; k <= 2 ; ++k )
        fcost =  fcost + p[k] + q[k] * ( x[k] - xu[k] ) * tbr ;
```

```
    tlcost = fcost + avtsc ;
    neprofit = 200 + 400.0 * ( 1 - exp(-0.0003*x[4]) ) - tlcost ;
    printf("neprofit is = %f\n", neprofit ) ;
}

float weib(ralpha, rbeta)
float ralpha, rbeta ;
{
  float u, p, t ;

  u = rand( ) / pow(2, 15) ;
  p = ( - log(u) / ralpha ) ;
  t = pow(p, (1/rbeta)) ;
  return(t) ;
}
```

VITA

Tapas Ranjan Kar was born in Calcutta, India, on 1st January, 1965, to Manoranjan Kar and Indu Kar. After completing higher Secondary he was accepted into I. I. T. Kharagpur in August, 1983. He graduated with a B. Tech. (Hons.) in Mining Engineering in May, 1987 and enrolled at Virginia Tech in Fall, 1987. He completed his M. S. degree in Mining and Minerals Engineering in December, 1989. Currently he is pursuing his career with a consulting company.

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