3.0 Literature Review

Error measurement tools are an integral component of any work done in the areas of geometric trimming, filleting, approximation of curves and surfaces, and merging of adjacent surface patches [Glou89] [Hosc90] [Hosc89] [Hosc88] [Hosc87] [Jone91] [Mine98] [Patr88] [Rock89] [Roja94].

A lot of the literature in geometric modeling has been devoted to curve and surface design. Farin [Fari97] and Yamaguchi [Yama88] have comprehensively discussed the subject of parametric polynomial curve and surface design. The suitability of B-spline geometry for use in geometric modeling, especially in light of the localized shape control and degree control properties has also been discussed. A rigorous derivation of the B-spline basis is attributed to deBoor [Fari97].

Visual trimming can be characterized as hiding the trimmed part of the surface while displaying the surface. Cassale and Bobrow [Casa89a] [Casa89b] as well as Farouki [Faro87] have discussed visual trimming of surface patches.

Geometric trimming results in a new mathematical description of the surface patch. Hoscheck et al. divide the parametric domain of the given NURBS patch into rectangular

components and fit Bézier patches through each subdivision [Hosc90] [Hosc89] [Hosc88] [Hosc87]. The conversion is achieved through knot insertion and a change of basis. Hoscheck et al. [Hosc90] and Rockwood et al. [Rock89] have developed norms for calculating the approximation error. The former calculates the maximum distances between the original and the approximated surface patch, and the later calculates the maximum distances between the approximated surface and the given curves on the original surface.

Rojas [Roja94] uses Fleming's constraint based inversion algorithm to generate B-spline surfaces passing through a given set of points before geometrically trimming them [Flem92a] [Flem92b]. Rojas uses the volume differential between the trimmed surface and an evaluation of the original surface approximated up to the trimming curve. The volume calculation involves arithmetic calculations that introduce numerical precision error. He has also developed an alternate error measurement scheme that uses the length of perpendicular edge instead of the volume computation. The edge length method results in similar results but is free of numerical precision errors. A visualization toolkit that manipulates the display of the generated surfaces has also been implemented.

Approximation of rational curves and surfaces by non-rational ones has also been discussed extensively in the literature. Almost all attempts have been accompanied by a description of an error bound to limit or at least measure the approximation error. Bardis and Patrikalakis [Bard89] establish a global error bound for approximation of a NURB surface by a lower order non-uniform integral B-spline patch. Schaback [Scha93] derives

uniform and numerically accessible error estimates for functions defined by a control net and a partition of unity. The basic idea behind the method is to use refined control net (achieved by subdivision or knot insertion) for a function as an approximation to the function. Floater [Floa95] introduces an explicit error bound for approximating rational tensor product bi-quadratic Bézier surfaces with splines.

Once the necessary manipulations have been applied to a patch, visual inspection is the easiest way to visualize the error as compared to the original patch. Graphical tools incorporating PHIGS have been successfully used for this purpose in the past [Flem92a] [Flem92b] [Jone91] [Roja94].