Appendix A

Original fan model

The original fan noise model implemented into the TBIEM3D code considered a collection of spinning point dipoles to model the loading component of the fan noise. One point dipole was used to model the loading force on each blade of the fan (Morfrey 1964). The point dipoles were located close to the duct wall (i.e. toward the tip of the blade) where the loading force (and hence the loading noise) is maximum (Dring et al. 1982). The derivations of this fan model based on spinning point dipoles were first done by Lan (1993). Lan also modeled the thickness component of the fan noise using spinning point monopoles. The volume displacement of each fan blade was modeled by a point monopole. Hence the loading and thickness components of the noise generated by each fan blades were modeled by a spinning point monopole and a spinning point dipole, as described in Figure A.1. The analytical expression derived by Lan for the incident pressure field generated by this fan model is presented next .

$$p_i(r,\psi,z,t) = \sum_{n_n=-\infty}^{\infty} Q_i^{n_h}(r,Z)\, e^{-i\kappa M Z}\, e^{i n_h N (t-\psi)} \tag{A.1}$$

where

$$Q_i^{n_h}(r,Z) = A^{load} + A^{thick} . \tag{A.2}$$

A^{load} and A^{thick} are given by

$$A^{load} = \frac{N}{2\pi\beta}\nabla.\left\{-F_r[\frac{\partial}{\partial r'}G_{n_h}(r,r',Z)]_{r'=0} - \frac{1}{\beta}F_z[\frac{\partial}{\partial Z'}G_{n_h}(r,r_0,Z-Z')]_{Z'=0}\right\}$$

$$-\frac{r_0 N}{2\pi\beta}\left(\frac{i\kappa\ M F_z}{\beta r_0} + \frac{i n_h N\ F_\psi}{r_0^2} + \frac{F_r}{r_0^2}\right)G_{n_h}(r,r_0,Z) \qquad \text{(A.3)}$$

and

$$A^{thick} = -\frac{\gamma_0 N}{2\pi\beta}\ G_{n_h}(r,r_0,Z-Z')_{Z'=0}$$

$$\left\{\left(\frac{V}{\beta}\right)^2\left[\frac{\partial^2}{\partial Z'^2} + 2i\kappa M\ \frac{\partial}{\partial Z'} + (\kappa M)^2\right] + i\,n_h N\frac{2V}{\beta}\frac{\partial}{\partial Z'} - (n_h N)^2\right\} \qquad \text{(A.4)}$$

where r_0 is the radial location of the point sources, N is the number of blades, V= M c is the velocity of translation of the engine fan, γ_0 represents the volume occupied by one fan blade, and $\vec{F} = F_r\ \hat{i}_r + F_\psi\ \hat{i}_\psi + F_z\ \hat{i}_z$ is the force applied by one blade to the fluid. The Green's function G and the parameters κ and β are defined by Eq. (3.39), Eq. (3.27) and Eq. (3.18).