AN INTEGRATED SYSTEM OF STAND MODELS FOR LOBLOLLY PINE

by

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INTRODUCTION

Loblolly pine (Pinus taeda L.) is the single most important commercial timber species in the South and among the most intensively managed tree species in the world. The forest manager may now select from a number of site preparation and planting alternatives, plant genetically improved stock, and apply intermediate cultural treatments including thinning, fertilization, and various forms of insect and disease control to meet specific management objectives. Harvesting options and alternative product mixes must also be considered. Thus, increases in management intensity have increased the complexity of resource management decisions. New technology, changing utilization standards, energy considerations, environmental concerns, changing land use patterns, and changing economic conditions are but a few of the unknowns which dictate that resource management remain a flexible and dynamic process.

Stand models, which provide accurate estimates of stand growth and yield, have become essential for evaluating the numerous management and utilization alternatives for loblolly pine. Decisions must be made for individual stands, for entire forests, and for broad regional planning. The projection period and level of stand detail required may vary in each case. No single stand model may be expected to provide information efficiently for all levels of decision making. Thus it is not surprising that a wide variety of models, of varying degrees of complexity and detail, has been developed for loblolly pine.

Existing stand models form a continuum of complexity and detail, or "resolution," ranging from relatively simple whole stand yield prediction equations to very complex simulation models of the development and interactions of individual trees in a stand. In choosing appropriate stand models for growth and yield estimates, the user should be concerned with the reliability of estimates, the flexibility to reproduce desired management alternatives, the ability to provide sufficient detail for decision making, and the efficiency in providing this information. In practice a number of different stand models may be required, each tailored to a specific set of decision criteria. Unfortunately, however, there has been little effort to date to ensure consistent growth and yield estimates from models of different levels of resolution. Users must deal with these inconsistencies or be forced to sacrifice efficiency or flexibility by selecting a single model for all projections.

An integrated system of stand models is needed which provides consistent growth and yield estimates at various levels of resolution. Although existing models have been developed to address specific information needs, they each attempt, in one form or another, to describe the development of stands of trees. This common goal in modeling suggests that much can be done toward developing a unified mathematical structure for modeling the biological development of stands of trees which can be applied at any level of resolution.

The benefits of an integrated model system are many. First, and most practical, is that estimates of stand growth and yield would be structurally related and response surfaces would "behave" similarly at all levels of resolution. By further constraining the models to provide numerical consistency, the problems related to inconsistent estimates would be eliminated. Second, an integrated system would be data efficient. Existing information from a number of tree and stand level studies could be incorporated into such a system. For example, effects of certain cultural treatments on individual trees could be incorporated at the tree level which may be used to provide stand level yield estimates. Third, and related to this, an integrated system of models would help in focusing attention to real data needs and ordering research priorities. Finally, an integrated system could be used to examine hypotheses of tree and stand growth under intensive management.

OBJECTIVES

The goals of this study were to investigate relationships among stand models and develop methods for integrating stand models of different levels of resolution. More specific objectives were to:

1. investigate and examine mathematical relationships among the existing approaches to stand modeling for loblolly pine;
2. develop an integrated system of stand models for loblolly pine in which models of different levels of resolution are related by a unified mathematical structure; and
3. investigate requirements for numerically consistent growth and yield estimates among models in an integrated system.

PREVIOUS WORK

Stand Modeling

Modern quantitative analysis of forest stand growth and yield dates to MacKinney and Chaiken's (1939) application of multiple regression methods to the problem of variable density yield estimation. Since that time a variety of approaches has been taken to growth and yield estimation, each arising from the need for specific types of information. Current modeling approaches may be thought to lie on a continuum with respect to structural complexity and output detail. This continuum may be broken into three broad categories: 1) whole stand models, 2) size class distribution models, and 3) individual tree models.

Whole Stand Models

Since MacKinney and Chaiken (1939) a number of investigators have used multiple regression to construct stand aggregate growth and/or yield expressions (e.g., Bennett 1970, Bennet *et al*. 1959, Brender and Clutter, 1970, Burkhart *et al*. 1972a, 1972b, Clutter 1963, Coile and Schumacher 1964, Goebel and Shipman 1964, Schumacher and

Coile 1960, Sullivan and Clutter 1972). These models provide growth and/or yield estimates for the whole stand (or some merchantable portion) as a function of stand level attributes such as age, density, and site index. Clutter (1963) introduced the notion of compatibility in growth and yield equations by recognizing the derivative-integral relationship between stand growth and yield, thus providing structurally compatible growth and yield models. Sullivan and Clutter (1972) later refined the compatible models to provide numerically consistent growth and yield estimates.

The multiple regression models above were fitted empirically and, in general, predict well within the ranges of observed data. Another whole stand approach has been to specify a theoretical growth rate model and solve the resulting differential equations for stand yield estimates (Turnbull 1963, Pienaar 1965, Moser 1967, Moser and Hall 1969, Pienaar and Turnbull 1973). This approach has been considered to lead to more rational growth and yield relationships. Leary (1970) and Moser (1972) extended this methodology to include sets of simultaneous differential equations representing different stand components.

Size Class Models

A number of modeling approaches have been developed which consider the stand in terms of the distribution of the number of trees per unit area by size class. In most cases diameter class has been used. Classical stand table projection methods, in which the number of trees per diameter class are projected over time, may be considered to fall this category. Bruner and Moser (1973) and Peden *et al*. (1973) independently developed Markov models for stand projection in which the transition probabilities describing growth into the next diameter class, mortality, and other factors form a first-order Markov process.

The most common stand models in this general category are those in which the distribution of diameters in a stand has been described by a probability density function (pdf). In this approach the number of trees in each diameter class is estimated using the fitted pdf, mean tree height is estimated for each diameter class, and volume in each class is calculated by substituting into tree volume equations. Unit area yield estimates are obtained by summing over diameter classes of interest. Only stand level attributes, such as age, site index, and stand density are used as input to these models, but the technique provides detail on stand structure.

Studies involving the use of pdf's to represent diameter distributions have differed mainly in the pdf used. Early studies (e.g., Bennett and Clutter 1968, Burkhart and Strub 1974, Lenhart 1972, Lenhart and Clutter 1971) used the beta density function to describe diameter distributions. More recently the Weibull density function has been adopted (Clutter and Belcher 1978, Dell *et al*. 1979, Feduccia *et al*. 1979, Lohrey and Bailey 1977, Smalley and Bailey 1974a, 1974b, Smith 1978). Hafley and Schreuder (1977) proposed Johnson's S_B distribution for describing diameter and height. Johnson's bivariate S_{BB} distribution has been used to describe joint diameter-height distributions (Schreuder and Hafley 1977).

The above studies apply to pure even-aged stands with no intermediate cuttings. Recently Hyink (1979) and Stiff (1980) have shown the utility of the Weibull density function for describing diameter distributions in mixed uneven-aged stands. In addition, they provide a means to project the pdf parameters over time to provide a pdf-based growth and yield model.

Individual Tree Models

Stand models which use the individual tree as the basic growth unit may be classified as individual tree models.

Munro (1974) further segregated this class of models into distance dependent and distance independent categories depending on whether or not individual tree locations are required tree attributes. Distance independent models may simulate tree growth either individually or by size classes, usually as a function of present size and stand level attributes. Detailed tree records are maintained providing a great deal of information on stand structure. No general form has been followed in the construction of individual tree distance independent models so it is difficult to make general statements about their structure. Examples of distance independent models are found in the work of Goulding (1972), Stage (1973), Dale (1975), Botkin *et al.* (1970), and Alder (1979).

Distance dependent models that have been developed, although varying in detail, have, in general, shared a common structure. Initial tree and stand attributes are input or generated and each tree is assigned a coordinate location. The growth of each tree is simulated as a function of its size, the site quality, and a measure of competition from neighbors. The competition index varies from model to model but in general is a function of the tree's size in relation to the size of and distance to competitors (hence, the need for individual tree locations).

Mortality may be controlled either probabilistically or deterministically as a function of competition and/or other individual tree attributes. Models of this type have been developed by Newnham and Smith (1964), Arney (1974), Ek and Monserud (1974), Hegyi (1974), Daniels and Burkhart (1975), Daniels et al. (1979b), and others.

Individual tree distance dependent models provide very detailed records of stand structure and development and are well suited for inclusion of routines to simulate cultural treatments. Silvicultural treatments involving spatial and competitive influences, for example, thinning, have been included in a number of individual tree distance dependent models since it is generally felt that response follows directly from the competition relationships included. Fertilization and site preparation options have also been included in some models (e.g., Daniels and Burkhart 1975).

Because distance dependent models are generally closely tied to biological growth processes, a number of studies have used these models to examine hypotheses on tree and stand growth under conditions for which data do not presently exist. Mitchell (1978), for example, used his Douglas-fir model (Mitchell 1975) to examine density and spacing effects on yield. Nance and Bey (1979) investigated hypothetical rotation-age yield gains in loblolly pine due to genetically improved stock by incorporating a number of assumptions on the growth and interaction of improved trees into the model of Daniels and Burkhart (1975). Recently, Thurmes (1980) used this model to simulate effects of intensive cultural practices in investigating optimal management strategies for loblolly pine.

Competition and Point Density Indices

Distance dependent individual tree models consider the effects of size and spatial patterns of neighbors on tree growth through the use of some measure of competition. Most competition indices are considered to be a measure of the degree to which growth resources may be limited by the number, size, and proximity of neighbors. The resources involved and their absolute availability may not be well defined, but may include, at least conceptually, light, water, nutrients, and physical growing space. Other competition measures have been developed which attempt to quantify the relative availability of resources. Point density mesures are an attempt to quantify the density affecting a given point (or tree) in a stand. These measures may be thought of as competition indices which express a tree's competitive status in terms usually associated with stand level density measures, such as trees per acre or basal area per acre.

Staebler presented perhaps the first individual tree competition index around 1950 (Gerrard 1969). He assumed that each tree had a circular area of influence, which was a function of its size, and that the degree to which this influence area was overlapped by those of its neighbors was a measure of competitive stress. Staebler used a linear function of dbh to define the radius of the influence area and measured competition by the linear overlap of influence areas. Most published competition indices since that time have been based on a modification of Staebler's work, with changes in the definition of the influence radius and measure of overlap.

Newnham and Smith (1964), for example, based the influence radius on crown radius and measured competition in terms of the angles subtended by overlapping crowns. Subsequent investigators have used the area of influence overlap (e.g., Gerrard 1969, Keister 1971, Arney 1974). Bella (1971) and Ek and Monserud (1974) used weights in summing area overlap so that given the same overlap area, larger trees would be considered to have a larger competitive effect than small trees.

Hegyi (1974) introduced a simple, but effective index, based on the sum, over all competitors within a fixed radius, of the ratios of competitor to subject tree dbh, divided by their distance apart. Daniels (1976) later modified this index to consider as competitors trees included by a given angle guage sweep around the subject tree.

Point density measures have been developed which quantify individual tree competition in terms adapted from stand density measures. Spurr's (1962) point density is an extension of point sampling methods to define a measure of basal area per acre affecting each point (or tree). Brown (1965) introduced a measure, termed Area Potentially Available (APA), which partitions the total stand area (and thus associated resources) into that potentially available for each tree. Thus APA is essentially a measure of the inverse of density, in trees per unit area, affecting each tree. Brown calculated APA by bisecting intertree distances to construct polygons of available area. Jack (1962) and Mead (1965) independently developed similar methods. Moore et al. (1973) modified the APA index by weighting the division of tree distances by relative tree sizes. Recently, Pelz (1978) introduced further modifications including a three dimensional competition model calculated by multiplying APA by total tree height.

Glover and Hool (1979) introduced a distance independent index based on the ratio of a tree's individual basal area to the average basal area in the stand. While not an index of competition *per se*, this index does quantify the relative size position of an individual tree in a stand which may be highly related to its competitive position. It was found to be useful in mortality prediction for loblolly pine.

Other approaches to competition indices have been taken, but will not be discussed due to their limited applicability for the present problem. (See Daniels and Burkhart 1975 for additional information about competition indices.) A number of studies have involved comparisons of various competition indices (see, e.g., Gerrard 1969, Johnson 1973, Opie 1968, Daniels 1976, Alamdag 1978, Noone and Bell 1980). In general, the results of these comparisons have been extremely variable. Performance of different competition indices appears to be species, and perhaps, data-set specific. Calculation procedures for many of the indices above will be discussed in a subsequent section.

Unifying Modeling Approaches

To date, much energy has been used to debate modeling alternatives while little effort has gone into comparing them and integrating them into a unified theory. The efficient use of stand modeling alternatives was stressed by Daniels *et al*. (1979a). They compared three models for loblolly pine, a whole stand model, a beta pdf-based diameter distribution model, and an individual tree distance dependent model and noted that all performed similarly for simple yield estimates. However the individual tree model, while providing greater detail and flexibility in simulating management alternatives, took nearly 1,400 times as long to execute as the whole stand model. Although guidelines are given for the efficient use of these models, they still lack an overall unifying structure and may provide inconsistent growth and yield estimates when interchanged.

A number of investigators have recognized the mathematical relationships between the probability density functions often used to describe diameter distributions and different modeling approaches. Strub and Burkhart (1975) developed a class-interval-free method for obtaining expected stand yield estimates from pdf's used to describe diameter distributions. Thus, they provided a link between pdf-based diameter distribution models and whole stand yield models. Work is currently underway in developing methods for "recovering" pdf parameters from whole stand attributes (Hyink 1980, Frazier[1] 1980). Recently, Bailey (1980)

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[1] Frazier, J. R. Personal Communications.

presented the diameter growth relationships implied by a number of assumed pdf's, providing a partial link between diameter distribution methods and individual tree growth models.

An attempt at an integrated system of stand models is included in the CARP (Computer Assisted Resource Planning) System developed in Canada (Glew *et al.* 1976). A compatible system of growth models, ranging from simple yield equations to individual tree models, was developed specifically to take advantage of the merits of each modeling approach.

The most comprehensive example of a truly integrated system of stand models is the growth model included in FREP (Forest Resources Evaluation Program) (Leary 1979). FREP was designed as a general system of models to project forest growth from inventory records, at varying levels of resolution, for nearly every conceivable stand type or condition in the Lake States. The system is driven by a system of simultaneous difference equations. In the simplest case one difference equation may be used for whole stand growth estimates for stands of single species composition. Mixed stands are modeled using two or more equations of the same form. The system is designed such that, theoretically, it may be expanded to include one equation for each tree to yield an individual tree level model. The difference equation used describes a growth potential multiplied by a modifier to estimate realized tree (individual or aggregate) growth. When the size of and distances to neighbors are included in the growth modifier function the model becomes distance dependent. Thus, the FREP system provides a unified theoretical structure for models ranging from whole stand models to individual tree distance dependent models.

METHODS

Model Structure

An integrated system of stand models must involve an overall mathematical structure common to all levels of resolution desired. A "telescoping" system is proposed in which a highly detailed overall model is developed and its components are "collapsed" around this common structure to provide structurally compatible models at each successively lower stage of resolution.

The model of highest resolution in this system should be closely tied to biological growth processes and must be of suffient detail and flexibility to ensure that it will provide growth and yield estimates for the most intensive management regimes under consideration, for example, those involving spatial and competitive influences. Thus, we are led to initially consider a distance dependent individual tree model, such as that developed by Daniels and Burkhart (1975). In such models tree growth and survival probability are simulated as functions of size, site factors, and an index representing the tree's competitive position. This competition index, generally a function of the size of and distance to neighboring trees, quantifies the degree to which growth resources are available, or limited, for individual tree growth.

A point density measure which possesses a readily interpretable stand-level density analog (e.g., trees or basal area per unit area) is highly desirable as an initial choice for the competition index. "Collapsing" point density to stand density eliminates the dependence on intertree distances, resulting in a *distance independent individual tree model*. Previous studies (e.g., Johnson 1973, Moore *et al*. 1973) have indicated that point density measures based on stand density units compare favorably with other competition indices in terms of their relationship with tree growth. A distance independent index of a tree's relative size position (Glover and Hool 1979) may also be important in maintaining individual tree growth characteristics when collapsing from a distance dependent to a distance independent model.

The adoption of point density and distance independent measures as competition models provide a major link between distance dependent and distance independent stand models. Because of the importance of these relationships, an intensive evaluation and comparison of "collapsable" point density measures, other competition indices, and distance independent size position indices was conducted and will be presented in subsequent sections.

The individual tree distance independent model can be collapsed dimensionally to consider trees in size classes, thus providing a size class projection model. In this next lower stage of resolution individual tree growth equations are applied to mean dimensions in each size class. Survival probability simplifies to the proportion of trees in the size class surviving to the next year. A relative size position index would be applicable to size class dimensions. Depending on the mathematical form of the growth models chosen pdf-based size class models may also be obtained using relationships described by Bailey (1980).

Finally, a stand level model, at the lowest level of resolution considered, is obtained by collapsing the dimensions of the model to one "average" tree. The size position index collapses to a constant. Tree growth equations are applied to the stand's average tree dimensions. The survival equation indicates the proportion of all trees surviving to the next year.

In each case, unit area yields are obtained by applying known tree volume equations to tree or average dimensions, summing over the portion of the stand of interest (for all but whole stand models) and applying appropriate expansion factors.

The central unifying mathematical structure in all model forms discussed above lies in the use of a common set of tree growth and survival functions. As resolution is reduced arguments in these functions are collapsed from tree to stand interpretations and the number of times the functions are evaluated is reduced with the dimensionality of the model. Because these functions provide the common mathematical structure for models of all resolution they must be carefully chosen. In a subsequent section variables affecting growth will be examined in light of a number of different growth concepts and models.

Whereas structural compatibility between models of different resolution levels may be attained in the above manner, the problem of numerical consistency remains. In collapsing from tree to stand level attributes inconsistencies may be introduced which are related to assuming central values of the distributions of tree attributes. In general, given a set of items with many attributes, for example a tree, the item associated with the mean of one attribute is not necessarily the same as that associated with the mean of another attribute. If the distribution of each attribute is symetrical then this property is assured. In the present context, consider that the tree of mean diameter is not necessarily the tree of mean point density, basal area, height, or volume. Inconsistencies introduced in this manner are serious if, by adopting central values, systematic biases are introduced. This may be the case when assuming the tree of average growth dimensions (e.g., diameter and height) represents the tree of average volume, for example. Numerical consistency will be addressed in terms of parameter estimation and model application in later discussions.

Problems arise because the dimensions of measurement and growth (diameter or basal area and height) are not the dimensions of ultimate interest (volume). The choice of basal area as the growth unit, rather than diameter, may be argued on the basis that it is generally considered to be more closely related to volume than is diameter. Surely, when considering increasing xylem cell area as a growth process basal area is just as valid a growth unit as is diameter. Strub (1976) presented additional justification for considering basal area as a modeling unit and noted that distributions of basal area was often symetrical, compared to skewed diameter distributions.

Data

The data used in this study are from a mutual competition study at the North Louisiana Hill Farm

Experiment Station near Homer, Louisiana. These data were obtained through a cooperative agreement between Virginia Tech, L.S.U., and Weyerhaeuser Company for the purposes of testing competition models and preliminary development of integrated stand model structures.

The mutual competition study at the Hill Farm Experiment Station was established in the winter of 1957-1958. Existing pine and hardwood stems on an abandoned crop field were cut and stumps were burned. Seedlings raised at the Hill Farm from seed collected in natural stands in Northern Louisiana were planted at a 6 x 6 foot spacing. In the Fall of 1962 twenty, 0.25 acre plots with buffer strips were established. Four of these plots were thinned to 1000 trees per acre and the other sixteen were thinned to 600 trees per acre. Twelve of these sixteen plots were thinned to 300 trees per acre when a .1 inch diameter growth difference was detected between the two treatments. Of these twelve plots, eight were thinned to 200 trees per acre using the same diameter growth criterion. Similarly, four of these eight plots were thinned to 100 trees per acre. Thus, by 1965, each of the five treatments, 1000, 600, 300, 200, and 100 trees per acre were represented by four replications (Table 1) (Sprinz et al. 1979).

TABLE 1. Summary of stand development for the Hill Farm mutual competition study[1]

Treatment (trees/acre)	Age (years)							
	5	6	7	8	9	10	13	20
	-------------Trees Surviving per Acre---------------							
1000	991	990	990	990	988	981	927	662
600	600	600	600	600	599	599	593	523
300	594	594	300	299	299	299	296	287
200	596	596	299	201	201	201	201	198
100	596	596	299	99	99	99	99	98
	-----------Basal Area per Acre (ft^2/acre)-------------							
1000	25.4	40.0	59.1	78.2	93.5	109.4	127.8	161.3
600	17.2	29.8	45.9	63.5	78.0	92.7	113.8	159.4
300	16.5	28.6	25.4	37.7	49.1	60.6	85.6	137.2
200	15.8	26.4	23.8	25.7	34.5	45.0	58.3	120.4
100	17.4	29.8	26.0	15.3	21.7	30.3	51.5	96.4

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[1] Data summaries from Sprinz *et al.* 1979.

Diameter at breast height (dbh) was measured on all live trees annually from 1962 through 1967 and again in 1970 and 1977. Total height was initially recorded for each tree, but subsequent measurements included only a sample of total tree heights. Mortality was tabulated by measurement year for each tree. In 1977 each plot was stem-mapped and tree crown class and stem quality codes were assigned.

The development of the mutual competition study plots, by density treatment, has been well-documented by Sprinz *et al*. (1979). They noted that the density treatments significantly affected dbh growth and height growth (Table 2). Mortality was negligible for the three lowest densities, but reached 33 percent by age 20 for the 1000 trees per acre treatment and 13 percent for the 600 trees per acre treatment. Basal area per acre was also affected by treatment (Table 1). The lower density plots had a higher rate of basal area increase. By age 20 the 1000 trees per acre and 600 trees per acre treatments had converged in terms of basal area, primarily due to mortality (Sprinz *et al*. 1979).

In the present study, these growth data were prepared for calculation of a number of competition indices. Missing heights were filled in using the relationship

$$\ln H = b_0 + b_1/D$$

TABLE 2. Average tree size development for the Hill Farm mutual competition study[1]

Treatment (trees/acre)	Age (years) 5	6	7	8	9	10	13	20
	---------------Average Dbh (inches)---------------							
1000	2.2	2.7	3.3	3.8	4.2	4.5	5.0	6.7
600	2.3	3.0	3.7	4.4	4.9	5.3	5.9	7.5
300	2.2	3.0	3.9	4.8	5.5	6.1	7.3	9.4
200	2.2	2.8	3.8	4.8	5.6	6.4	7.9	10.6
100	2.3	3.0	4.0	5.3	6.3	7.4	9.8	13.4
	---------------Average Height (feet)---------------							
1000	11.1	14.6	18.6	22.6	26.2	29.8	38.3	53.7
600	11.8	15.8	20.1	24.4	28.4	32.1	40.6	54.2
300	11.9	15.6	19.8	24.0	27.8	31.5	40.8	59.2
200	11.4	15.0	19.3	23.6	27.2	30.9	40.7	59.4
100	11.9	15.8	19.7	24.3	27.3	31.5	41.0	62.0

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[1] Data summaries from Sprinz _et al_. (1979).

where

H = total height

D = diameter breast height.

Comparisons of these estimated regression lines for each plot, at each age, indicated no problems, such as crossing within the range of prediction for adjacent ages. Therefore separate regressions were used for each plot at each age to fill in missing height information.

Competition and Point Density Calculations

The proposed integrated system of models requires a distance dependent index of competition at the highest level of resolution. To collapse the highest level model to lower levels of resolution, the competition measure must take on a distance independent or stand level interpretation. In this section various competition indices, point density measures, and distance independent indices were evaluated and compared on the basis of tree growth and survival relationships using the Hill Farm data.

A computer program, COMP5, was written in standard FORTRAN to calculate several variations of five different general families of competition and point density indices including

1. distance-independent size position indices (Glover and Hool 1979);
2. distance dependent area overlap competition indices (e.g., Arney 1973, Ek and Monserud 1974);
3. distance dependent size ratio/distance competition indices (Hegyi 1974, Daniels 1976);
4. Spurr's point density (Spurr 1962); and
5. Area Potentially Available (APA) or polygon indices (Brown 1965, Moore _et_ _al_. 1973, Pelz 1978).

COMP5 was used to calculate each index for each tree on each plot at each measurement period after treatments were established resulting in over 50 measures of competition on each of the over 14,000 growth records. Edge effects on distance dependent index values for border trees were controlled by a translation of plot borders (Monserud and Ek 1979). Predicted heights were used only where actual heights were missing. The indices examined are described below.

Distance Independent Indices

Glover and Hool (1979) defined their distance independent Basal Area Index as

$$G_i = \frac{\pi (D_i/2)^2}{\pi\{[(\sum_{j=1}^{n} D_j)/n]/2\}^2}$$

or equivalently,

$$= \frac{D_i^2}{\bar{D}^2}$$

where

G_i = the Basal Area Index for tree i

D_i = the dbh of tree i

$\bar{D}$ = the arithmetic mean dbh on the plot.

They interpreted this index as the ratio of a tree's basal area to the basal area of a tree with mean dbh, on a given plot.

A variation to this index was calculated which considers the ratio of a tree's basal area to the mean basal area on the plot, or

$$G_{Bi} = \frac{D_i^2}{(\sum_{j=1}^{n} D_j^2)/n}$$

Similar indices were also calculated based on ratios of tree height to mean height, tree height to mean height of dominant and codominant trees, and squared dbh times height to its mean.

Crown ratio, defined as the ratio of live crown length to total height, is a natural distance independent index of a tree's photosynthetic potential. It is not a competition index, but it does integrate the effects of competition in a measure which directly indicates a tree's vigor and it is highly related to growth (Daniels and Burkhart 1975, Leary 1979).

Crown lengths were not measured annually in the Hill Farm data so that crown length and thus crown ratio were not observed variables. Feduccia <u>et al</u>. (1979) presented equations for estimating crown ratio for loblolly pine in the Hill Farm region. These equations were used to estimate crown ratio for each tree at each age:

$$CR_i = 1. - \exp.(D_i/\bar{D})^{-\lambda} \ln(1-\overline{CR})$$

where

CR_i = crown ratio

D_i = dbh

$\bar{D}$ = mean dbh

$\overline{CR} = b_0 + b_1 \log_{10} H_D + b_2 \log_{10} A + b_3 \log_{10} TS$

$\lambda = c_0 + c_1 \log_{10} H_D + c_2 \log_{10} A + c_3 \log_{10} TS$

H_D = average height of dominant and codiminant trees,

A = plantation age

TS = surviving number of trees per acre

b_k, c_k = regression coefficients presented by Feduccia et al. 1979.

Area Overlap Indices

The area overlap indices of Arney (1973) and Ek and Monserud (1974) were calculated. Both are based on the overlap of open grown crown areas (Figure 1). The open grown crown radius was predicted for each tree as a function of dbh (Strub *et al.* 1975). Arney's (1973) index was calculated as

$$A_{1i} = 100 \cdot \frac{\sum_{j=1}^{n} a_{ij} + CA_i}{CA_i}$$

where

A_{1i} = Arney's area overlap index for subject tree i

a_{ij} = area of overlap between subject tree i and competitor j

CA_i = open grown crown area of subject tree i

n = the number of competitors (defined by overlap).

Note that Arney's (1973) index has a minimum value of 100 when there is no overlap of projected open grown crowns.

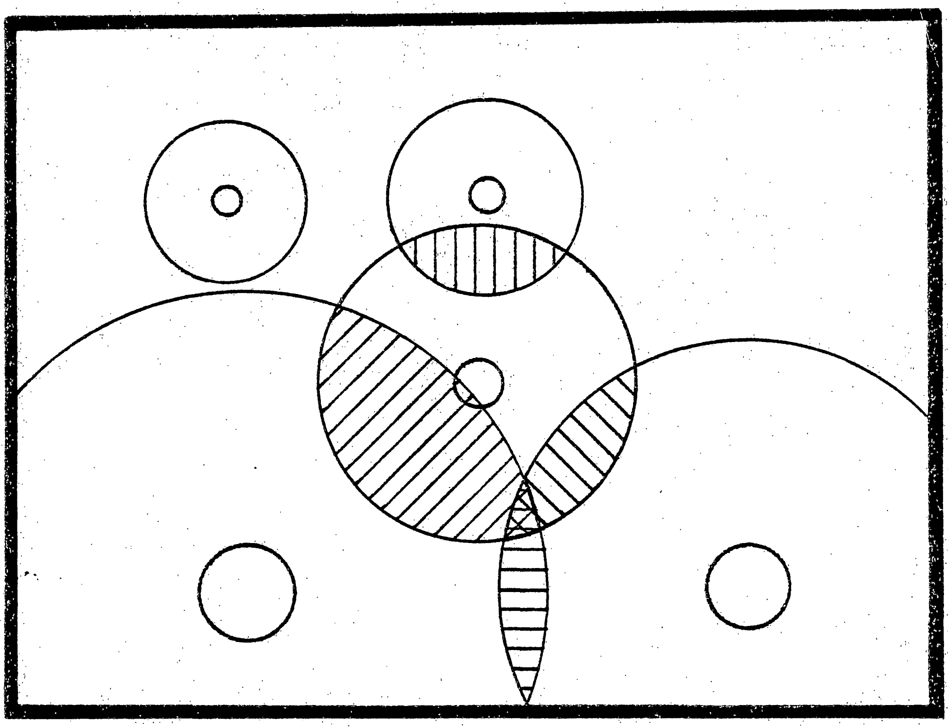

FIGURE 1. Selection of competitors for influence area overlap competition indices.

In the index of Ek and Monserud (1974), the overlap terms are weighted by the relative size of competitors to the subject tree as

$$A_{2i} = \sum_{j=1}^{n} (a_{ij} \frac{R_j H_j}{R_i H_i})/CA_i$$

where

A_{2i} = Ek and Monserud's weighted index for tree i
H = total height
R = open grown crown radius.

Size Ratio/Distance Indices

Hegyi (1974) introduced a simple competition index calculated as

$$C_{1i} = \sum_{j=1}^{n} (D_j/D_i)/L_{ij}$$

where

C_{1i} = Hegyi's index of competition for subject tree i
D = dbh
L_{ij} = distance between subject tree i and competitor j
n = number of competitors.

Hegyi originally defined n as the number of trees within a fixed radius of the subject tree. Daniels (1976) modified the index to include as competitors all trees in a fixed angle gauge sweep around the subject tree (Figure 2). The above index was calculated, using a basal area factor (BAF) 10 (ft^2/acre) angle gauge to define competitors, as were modifications based on squared dbh and/or squared distance.

Spurr's Point Density

Spurr (1962) introduced a measure of point density which considered the basal area per unit area affecting one point or tree. This index was calculated as

$$S_{1i} = 75.625 \sum_{j=1}^{n} (j - 1/2) \frac{D_j^2}{L_{ij}^2}$$

where n includes all competitors in an angle guage sweep around the subject tree (Figure 2). Spurr originally considered 4, 5, and 6 diopter sweeps. In the present study, sweeps of 3.03 (corresponding to a 10 BAF angle guage), 4 and 5 diopters were compared.

Point density expression S_{ij} does not include the subject tree. A modified point density estimate that includes the subject tree was also calculated and is given by

$$S_{2i} = 75.625 \sum_{j=1}^{n} (j + 1/2) \frac{D_j^2}{L_{ij}^2}$$

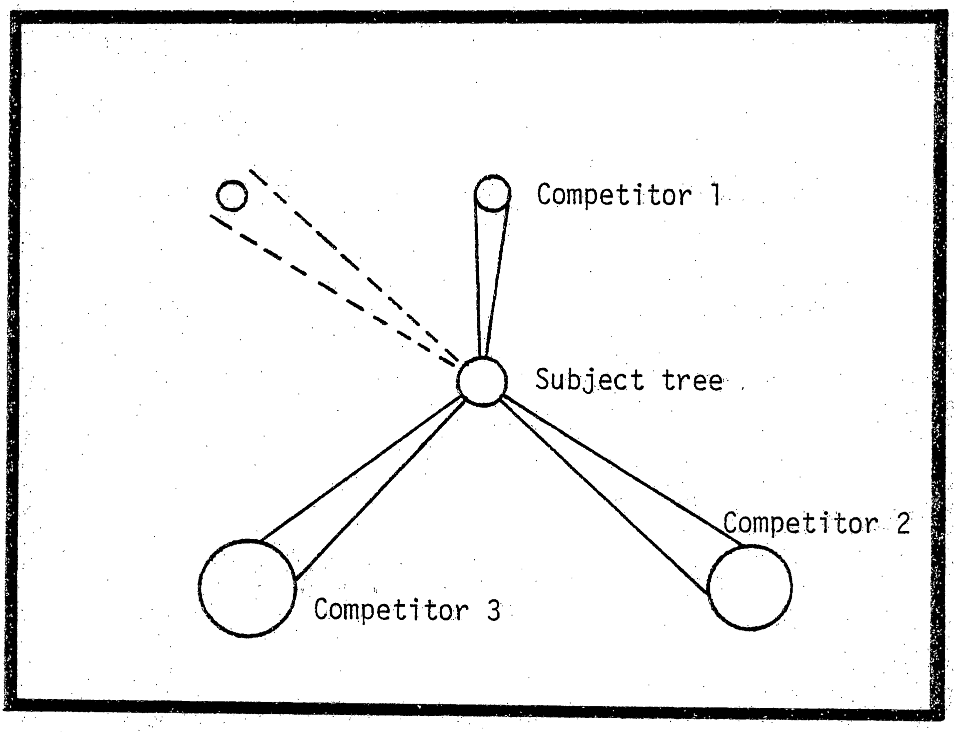

FIGURE 2. Selection of competitors for distance ratio/size competition indices and Spurr's point density.

Area Potentially Available or Polygon Indices

Brown (1965) introduced the polygon method as a point density measure. Polygons are constructed around the subject tree by the intersection of the perpendicular bisectors of the distance between the subject tree and competitors (Figure 3) and polygon area is calculated from the cordinates of the polygon vertices. Polygons are mutually exclusive and collectively exhaustive of the total area. Thus, the individual polygon areas sum to the total area.

Moore *et al.* (1973) modified the index so that the division of distances was weighted by tree size, as

$$l_{ij} = \frac{D_i^2}{D_i^2 + D_j^2} \cdot L_{ij}$$

where

l_{ij} = the distance from subject tree i to the weighted mid point between tree i and competitor j

L_{ij} = the distance between subject tree i and competitor j

D = dbh.

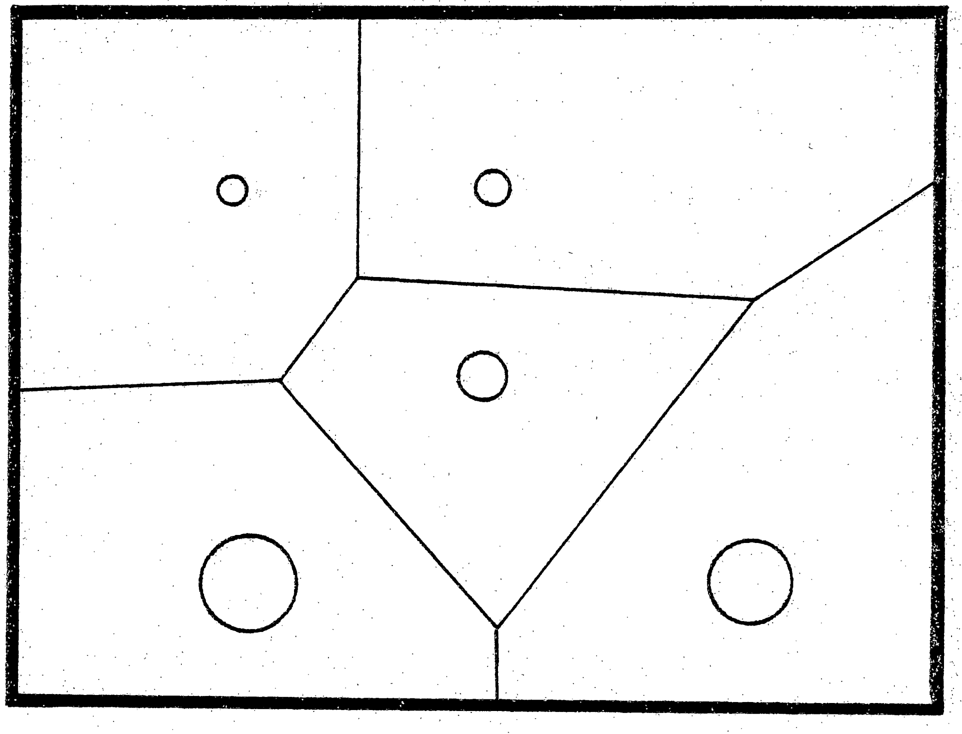

FIGURE 3. Polygons constructed by bisecting intertree distances.

Pelz (1978) introduced other weighting factors, such as dbh, total height, height times dbh, and height times squared dbh, as well as a three dimensional index calculated as the polygon area multiplied by total height.

The assymetric division of intertree distances resulting from weighting by tree sizes may lead to areas not assigned to any tree or open areas (Figure 4). Thus, weighted polygon construction leads to polygons which are mutually exclusive, but may not be collectively exhaustive of the stand area.

Indices were calculated based on Brown's (1965) original method of constructing polygons and based on polygons with the division of intertree distances weighted by dbh, dbh squared (or basal area), dbh squared times height, open grown crown radius, and open grown crown area. Polygon area was calculated for each of these variations. In addition, the three-dimensional growing space indices of Pelz (1978) were calculated by multiplying each of the above area indices by total height.

Mead (1965) introduced the notion that not only the area of the polygon, but the shape of the polygon and position of the plant within the polygon may be important in describing growth. He presented two additional measures associated with the polygon area which describe

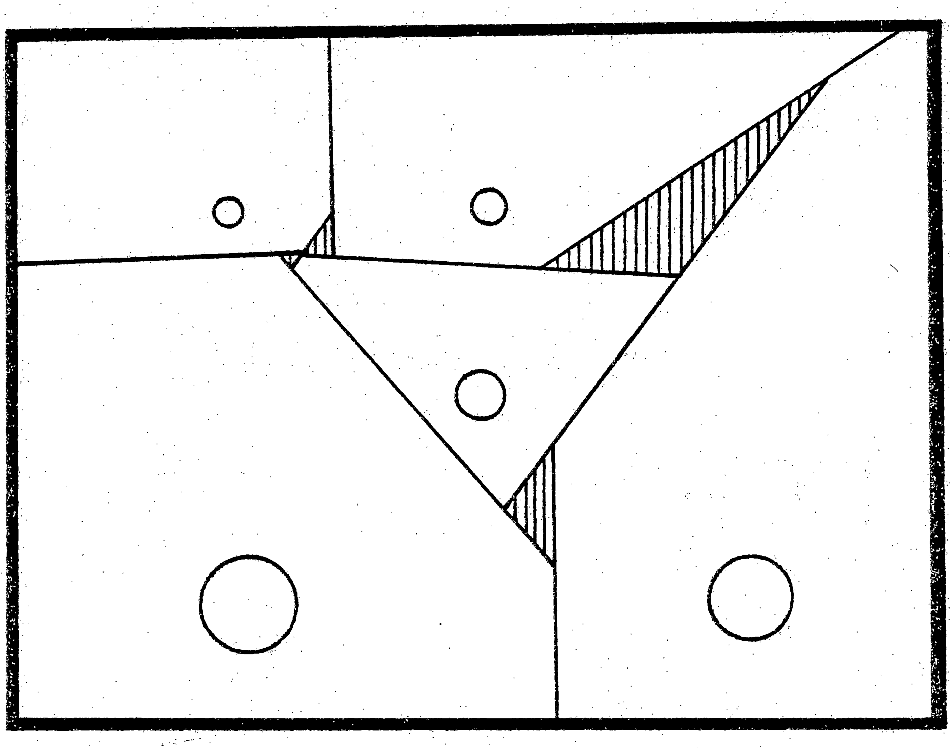

FIGURE 4. Polygons constructed by weighting the division of intertree distances by tree size.

"eccircularity" or the extent to which the polygon is eliptical rather than circular, and "abcentricity" or the degree to which the plant tends toward the vertices, rather than the center, of the polygon. Both of these auxiliary indices were calculated for each polygon.

Competition Analyses

The relationship between competition and tree growth is of major importance in selecting a competition index for stand modeling. All of the indices calculated were evaluated and compared on the basis of simple correlation and contribution to multiple correlation (in the presence of tree size and stand density measures) with annual dbh and basal area growth. Only observations which included no border effects for all indices were included in these analyses. Average growth values were used for observations in which the measurement period was greater than one year.

For all valid trees and growth measurements the product-moment or Pearson correlation coefficient (r) was calculated between each index and observed annual increment in dbh and basal area to provide estimates of overall correlation. In addition, changes in correlation between the indices and growth were examined by density treatment and over age.

Whereas simple correlations provide useful comparisons, in practice, competition measures are generally used to predict growth when other growth predictors, such as present tree size and stand density, are already known. Contributions to multiple correlation, in the presence of tree size and stand density measures, were examined in an iterative manner with multiple linear regression procedures, using a model of the form

$$\Delta X = b_0 + b_1\,CI + b_2\,S + b_3\,SD$$

where

ΔX = observed increment (in dbh or basal area)
CI = competition or point density measure
S = tree size (dbh or basal area)
SD = stand density measure (basal area per acre and trees per acre)

This model was estimated with consecutively increasing numbers of terms to evaluate the contribution of the competition measure in the presence of, first, tree size, and then tree size and stand density. The model was also estimated in each case without the competition term to determine the contribution of tree size and stand density.

Indices were evaluated and compared on the basis of their contribution to the coefficient of determination (R^2), interpreted here as the squared multiple correlation coefficient. The relative significance level (p) of the b coefficients were also examined.

Distance dependent indices were further compared on the basis of their contribution to correlation in the presence of a distance independent index and crown ratio. Two additional terms, b_4G, and b_5CR were added to the above model, where G represents a distance independent index similar to that of Glover and Hool (1979) and CR is crown ratio.

Execution time is a major concern in deciding among models of different levels of resolution, particularly distance dependent models. The time required by program COMP5 to calculate the indices above at each measurement period, was determined for each plot. These time estimates reflect not only the efficiencies of the indices themselves, but of the algorithms used to calculate them. More efficient algorithms may exist for any of the indices. However, these comparisons offer some insight into the relative costs of competition calculations.

Finally, the competition indices were evaluated on the basis of point density considerations. The relationship between point density measures and stand density measures was examined. Various modifications, combinations, and auxiliary measures associated with these relationships were also investigated.

Growth Relationships

The linear models presented earlier are useful in determining the relative contribution of different variables to multiple correlation with growth and thereby screening alternative variables which may be useful in growth modeling. However, relationships between measures of tree size, competition, and growth are almost certainly nonlinear in nature. Further, each variable used in growth modeling must be examined in light of the components of growth which it attempts to describe. By including relevant growth descriptors in a logical manner, nonlinear growth models may be constructed empirically which have some biological basis and behave well with respect to observed growth phenomena.

One concept which has been useful in modeling individual tree growth is that growth can be represented by the product of potential growth multiplied by a modifier function (Daniels and Burkhart 1975, Leary 1979). The potential for growth may be considered to be based on the quality and quantity of resources available for growth and/or growth in the absence of competition. The modifier function then serves to modify the potential to correspond to the individual's ability to use these resources, current size, and competitive position.

In the present context, a tree's potential may be a function of site quality, age, open grown size relationships, and its share of the total resource base. The modifier may be a function of a tree's individual size or relative size, photosynthetic potential, and a measure of competition from neighbors.

Another useful concept in growth modeling relates growth rate to the difference between constructive and destructive metabolism. The Chapman-Richards generalization of Von Bertalanffy's classical growth equation (Pienaar and Turnbull 1973) quantifies this relationship as

$$\frac{dX}{dt} = b_1X^m - b_2X$$

where

X = a measure of size
t = time
b_1 = constructive metabolism rate
b_2 = destructive metabolism rate
m = allometric constant.

The equation states that the rate of growth in x is the difference between constructive metabolism, as represented by a rate constant times an allometric relationship with size, and destructive metabolism, represented by a rate constant times size.

By combining the two concepts above, an empirical growth model can be derived. If one considers b_1 and b_2 in the Chapman-Richards model functions of the resources available for metabolism, then a potential function may be factored out to yield the empirical growth function

$$\Delta X = \Delta Xp(b_1X^m - b_2X)$$

where

ΔX = growth

ΔX_p = potential growth

Thus the form of the modifier in the original relationship is made to be a function of the difference between constructive and destructive terms.

If potential is based on resources then an area based index such as APA may be incorporated to determine the "piece of the pie" allocated to the tree and may be associated with b_1 and/or b_2 above. The constructive coefficient, b_1, may be related to the tree's relative crown size or crown ratio. Finally the destructive or negative coefficient b_2 may be related to competition or resource demand.

By considering components of growth and measures associated with growth in a logical manner the concepts of growth potential, realized growth, resource quality, resource quantity, competition, and constructive and destructive components of metabolism, have been incorporated in an empirical model of tree growth. In addition, the role of observed and calculated variables in describing growth was defined by their logical relationship with model coefficients.

RESULTS AND DISCUSSION

Competition Analyses

Each of the competition and point density indices calculated and previously discussed proved to be significantly correlated with annual dbh and basal area growth ($p < .0001$). In many cases a single variation within a family of indices proved uniformly superior to all others in the family in terms of simple and multiple correlations with growth (Appendix I, II). This was true for Spurr's (1962) point density and the size ratio/distance indices of Daniels (1976). In both cases correlations were greatest when a 10 (ft^2/acre) BAF angle guage was used to choose competitors. Spurr's point density calculated by excluding the subject tree had higher correlation than when calculated by including the subject tree. As was found by Daniels (1976), the ratio of diameters divided by distance was more highly correlated with growth than were similar measures involving diameter and/or distance squared. Weighted polygon or APA indices (e.g., Moore et al. 1973) had uniformly higher correlations than unweighted APA (Brown 1965) and three-dimensional indices (Pelz, 1979). In other cases, however, several indices within a family had very similar results. For instance, it would be difficult to choose between relative size indices based on basal area, height, or squared diameter times height or between APA indices weighted by basal area or squared diameter times height (Appendix I, II).

Simple Correlations with Growth

In general, the distance dependent indices were more highly correlated with growth than the distance independent measures (Table 3). Crown ratio, however, had the highest overall correlation with dbh growth of any measure examined (.7703) and was as highly correlated with basal area growth as any except the APA measures. Of the distance independent relative size indices, the ratio of basal area to mean basal area (G_B) was more highly correlated with growth than the original squared diameter to squared mean diameter ratio of Glover and Hool (1979). Ratios based on height to mean height, and squared dbh times height to its mean had correlations similar to those of the basal area ratio index (G_B). The ratio of height to mean dominant height (average height of dominants and codominants) (G_{HD}) had the highest correlations of this family of indices (Table 3).

TABLE 3. Correlation of various competition and point density indices with annual growth of loblolly pine.

Type of Index	Symbol	Authors	Remarks[1]	Correlation Coefficient (r) with growth in Dbh	B
				n = 6490	
Distance Independent					
Relative Size	G_B	Variation of Glover and Hool (1979)	$B_i/\bar{B}$	.2355	.4332
	G_{HD}		H_i/HD	.3057	.4937
Crown Ratio	CR		CL_i/H_i	.7703	.4772
Distance Dependent					
Area Overlap	A_1	Arney (1973)	Unweighted overlap	-.6604	-.4347
	A_2	Ek and Monserud (1974)	Weighted overlap	-.6316	-.4920
Dbh Ratio/ Distance	C_1	Daniels (1976)	$\Sigma(\frac{D_j}{D_i})/L_{ij}$	-.6872	-.5574
Spurr's Point Density	S_1	Spurr (1962)	Excluding subject tree	-.7143	-.4175
	S_2	Spurr (1962)	Including subject tree	-.6545	-.3807
Area Potentially Available	P_1	Brown (1965)	Unweighted polygon	.3329	.6367
	P_2	Moore *et al.* (1973)	Weighted by basal area	.3888	.7463

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[1] Where B = tree basal area; H = tree height; HD = average height of dominant and codominant trees; CR = crown ratio; CL = crown length; D = dbh; L = distance.

The indices representing competitive stress, those with negative correlations with growth (area overlap, size ratio/distance, Spurr's point density), were much more highly correlated with diameter growth than APA, which represents competitive advantage. But APA variations had considerably higher correlations with basal area growth, than did any of the other measures.

Interestingly, among distance dependent indices, the highest overall correlations with dbh growth (-.7143) and basal area growth (.7438) were with Spurr's (1962) point density and weighted APA (Moore et al. 1973), respectively; both are measures of point density. The other distance dependent indices (area overlap and size ratio/distance indices) had correlations with growth which were higher in magnitude, but similar in rank to those reported by Daniels (1976) for loblolly pine.

The presence of five different density treatments (1000, 600, 300, 200, and 100 trees per acre) offered the opportunity to examine competition-growth correlations by density class. Simple correlations were calculated by density treatment. For each index, correlations with both diameter and basal area were nearly constant over density, with a slight peak at the 300 trees per acre treatment. The indices maintained the same relative ranking, over all densities, as initially shown in Table 3.

Correlations between growth and competition measures were also examined over age. It was hypothesized that the correlation between growth and distance dependent competition measures should be small at young ages and increase as intraspecific competition begins. This is shown rather dramatically in Figure 5.

The competitive stress indices (A_1, C_1, and S_1) and the unweighted APA index (P_1) each start out with very low correlations with basal area growth at age five and increase until they peak at about age nine or ten. Growth during the early years is apparently more highly related to a tree's relative size as shown by the distance independent size position index (G_B). This index is more highly correlated with basal area growth than the others until about age eight and reaches a low point at age ten. The weighted APA index (P_2), both a measure of a tree's competitive position (growing area) and size dominance (due to weighting), apparently incorporates both relationships and maintains the highest correlation at all ages.

It should be noted that below age seven only the highest densities (1000 and 600 trees per acre) are represented. Data from the 300 trees per acre treatment enter the calculations at age seven and those from the remaining treatments (200 and 100 trees per acre) enter at

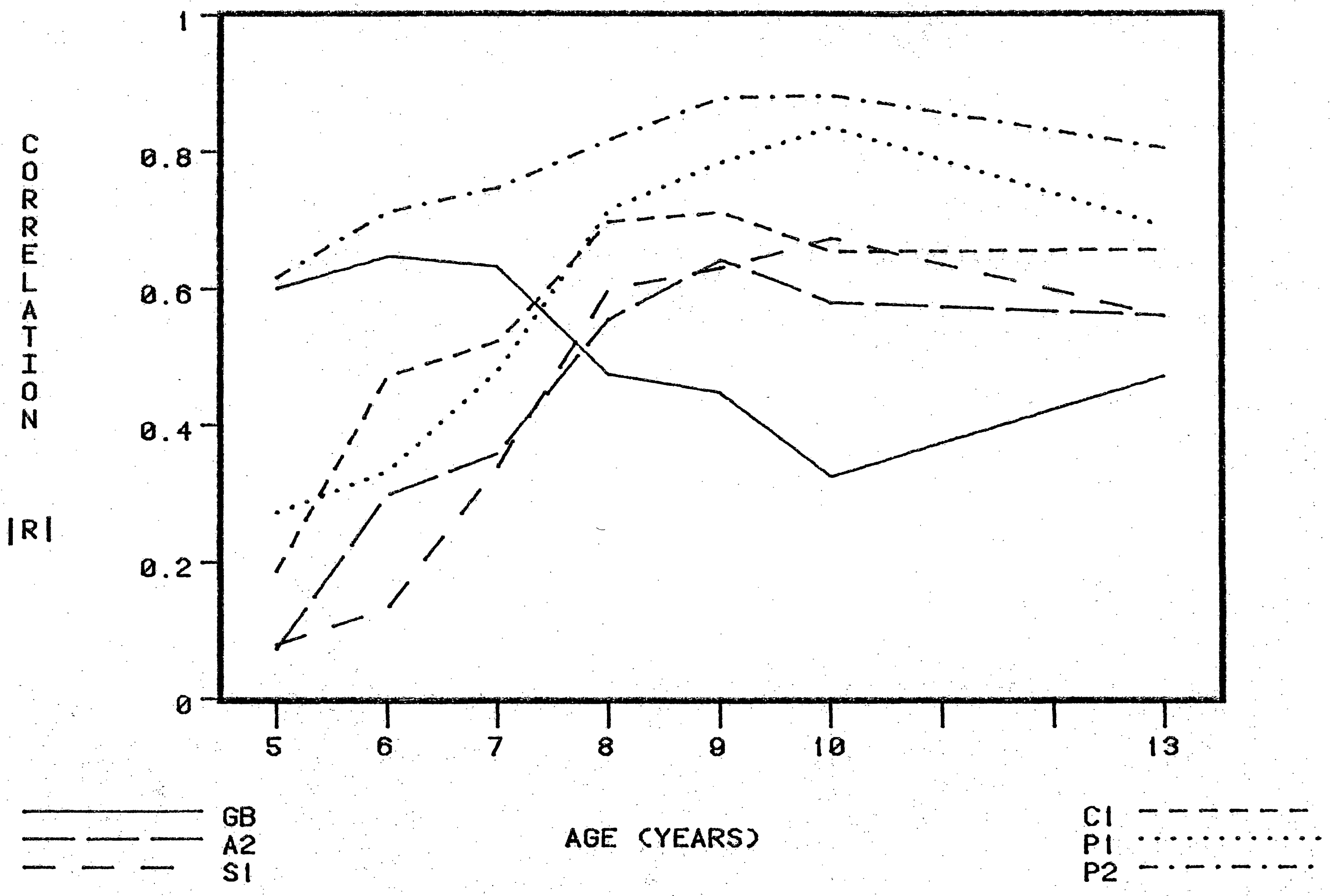

FIGURE 5. Absolute value of correlation coefficients of competition indices with basal area growth by age.

age eight. However, it is likely that data from these low density plots would tend to increase the growth correlations for the distance independent index, while decreasing the correlations for the distance dependent indices. The opposite was observed. Also, the values of basal area growth at age ten represent the average growth from age ten to thirteen and those at age thirteen represent the average growth from age thirteen to twenty. This might explain the slight decreases in correlation values after age nine.

Strub _et_ _al_. (1975) demonstrated a relationship between crown closure, as indicated by Crown Competition Factor (CCF) (Krajicek _et_ _al_. 1961), and the age when average diameter growth first deviates from that of open grown trees. This typically occurred at about age eight or nine. In their loblolly pine growth simulation, Daniels and Burkhart (1975) used this result to determine the age when intra-specific competition begins and when to start annual growth calculations incorporating a competition index. The results of competition and growth correlations here tend to substantiate earlier findings of Strub _et_ _al_. (1975) and application by Daniels and Burkhart (1975).

Multiple Correlations

Competition measures are seldom used as solitary independent variables and, therefore, should be compared in the presence of other, more common tree size and stand density measures. These comparisons were made using linear regression by including first tree size, then tree size and stand density along with the competition measure of interest. Distance dependent measures were further evaluated by including more easily obtainable distance independent measures in this linear model.

All of the indices compared made a highly significant contribution to the multiple coefficient of determination, R^2, (squared multiple correlation coefficient) in the presence of dbh, for diameter growth, and basal area, for basal area growth (Tables 4 and 5). At this stage, crown ratio provided the most additional explanation of variability in diameter growth ($R^2 = .6121$) (Table 4). The highest R^2 among distance dependent indices was that for Spurr's point density (S_2). The APA indices, particularly the unweighted index (P_1), and the distance independent relative size indices (G_B, G_{HD}) provide the least information on diameter growth at this stage. For basal area growth (Table 5) weighted APA (P_2) indices had the highest contributions to R^2, followed by crown ratio. Distance independent indices again had the lowest R^2 values.

TABLE 4. Squared multiple correlation coefficients between loblolly pine diameter growth and various competition and point density indices in the presence of tree size and stand density.

Index	Squared Multiple Correlation Coefficients (R^2) with Dbh Growth for Model Including[1]				
	D	D, TS	D, BA	D, TS, CR, G_B	D, BA, CR, G_B
Distance Independent					
G_B	.1737	.5465	.6129	--	--
G_{HD}	.2529	.5012	.6296	--	--
CR	.6121	.6121	.6157	--	--
Distance Dependent					
A_1	.4373	.4413	.6027	.6410	.6309
A_2	.4231	.4777	.6079	.6427	.6298
C_1	.5093	.5390	.6176	.6473	.6350
S_1	.5362	.5450	.6082	.6476	.6352
S_2	.4382	.4686	.6052	.6425	.6305
P_1	.2491	.2962	.6029	.6434	.6348
P_2	.4085	.4281	.6121	.6520	.6419
Check[2]	.0352	.2647	.6025	.6381	.6270

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[1] The model $\Delta D = b_0 + b_1 D + b_2(TS \text{ or } BA) + b_3 G_B + b_4 CR + b_5 CI$ was fitted including increasing numbers of terms, where D = dbh; TS = trees surviving per acre; BA = stand basal area per acre; G_B = relative basal area index; CR = crown ratio; and CI = competition index.

[2] Fit without the index as a check on the contribution of the index.

TABLE 5. Squared multiple correlation coefficients between loblolly pine basal area growth and various competition and point density indices in the presence of tree size and stand density.

Index	Squared Multiple Correlation Coefficients (R^2) with Growth in B for Model Including[1]				
	B	B, TS	B, BA	B, TS, G_B, CR	B, BA, G_B, CR
Distance Independent					
G_B	.2155	.5316	.5252	--	--
G_{HD}	.2608	.4776	.5489	--	--
CR	.4907	.5137	.4972	--	--
Distance Dependent					
A_1	.3877	.4225	.4938	.5948	.5358
A_2	.3498	.4394	.4954	.5915	.5282
C_1	.3988	.4648	.4985	.5913	.5285
S_1	.4569	.4983	.4991	.6057	.5369
S_2	.3848	.4527	.4954	.5974	.5312
P_1	.4063	.4226	.5550	.6576	.6475
P_2	.5764	.5862	.6325	.6856	.6688
Check[2]	.0995	.3292	.4937	.5913	.5280

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[1] The model $\Delta B = b_0 + b_1B + b_2(TS \text{ or } BA) + b_3G_B + b_4CR + b_5CI$ was fitted including increasing numbers of terms, where B = tree basal area; TS = trees surviving per acre; BA = stand basal area per acre; G_B = relative basal area index; CR = crown ratio; and CI = competition index.

[2] Fit without index as a check on the contribution of the index.

The addition of stand density measures significantly increased the explanation of variability in diameter and basal area growth. Whereas all but Arney's (A_1) index contributed significantly for diameter growth, with dbh and basal area per acre included, these contributions were small compared with that of basal area per acre (BA) (Table 4). Trees per acre (TS) contributed relatively little in the presence of dbh and each of the indices except the relative size ratio indices (G_B, G_{HD}). Spurr's point density (S_1) had the highest contribution of the distance dependent indices in the presence of dbh and either measure of stand density. In both cases, however, a distance independent measure was superior (G_{HD} with TS and CR with BA). For basal area growth (Table 5) the weighted APA index was clearly superior in the presence of tree basal area (B) and either TS or BA. In the presence of size and BA most of the other distance dependent indices had significant but relatively small contributions. The relative size index based on basal area had the highest R^2 of the distance independent indices. All indices had a relatively high R^2 compared with B and TS alone.

Up to this point in the comparisons, distance dependent indices have shown little advantage over distance independent measures in describing variability in growth, either singly or in the presence of tree size and stand density. However, when the distance independent measures G_B and CR were included in the model, a number of indices, particularly Spurr's point density (S_1) and the APA indices (P_1, P_2), made a sizeable improvement in R^2. Further, the APA indices had the highest partial contribution of all other variables for basal area growth (Table 5).

Thus, while some distance dependent competition indices contribute little to describing variability in growth when considered in the presence of tree size, stand density, and distance independent measures, others, particularly the APA indices and perhaps Spurr's point density, are well suited for growth prediction in this typical stand modeling situation. Whether the contribution added is worth the cost of additional calculations depends on modeling objectives.

Finally, it should be noted that measures related to those explicitly discussed and tabulated can and do provide as much information in growth description. Of the size ratio indices, almost identical results were obtained with ratios based on basal area, diameter squared times height, average height of dominants and codominants, and average height. Similarly, the weighted APA index based on diameter squared times height, performed similarly to that based on basal area (see Appendices).

Point Density Considerations

The integrated system of stand models, as discussed earlier, requires a measure of competition which may be reduced to a stand density measure. Of the indices examined only the polygon or APA indices and Spurr's point density fall into this category. Arney stated that the mean of his area overlap index reduced to crown competition factor (CCF) (Krajicek *et al.* 1961). However, his index has a lower bound of 100 and, therefore, cannot mathematically reduce to CCF, at least not before CCF reaches 100.

Relationships with growth indicate that a weighted APA index is a logical choice for inclusion in the integrated model framework. Because it does not sum to the total stand area, however, its inverse does not directly reduce to the stand density measure of stems per unit area as is the case for unweighted APA. The sum of weighted APA values and stand area were found to be proportional. Weighted APA values were then "corrected" to sum to the total area by the relationship

$$P'_i = P_i\left(\frac{\text{Area}}{\sum_{i=1}^{n} P_i}\right)$$

where

P'_i = the corrected polygon area

P_i = the original polygon area

Area = total stand area

N = the number of trees

The interpretation of P' is not that of the unweighted APA which sums to the total area by being collectively exhaustive of the area. Rather, it implies an overlap of potential growth areas while maintaining the desirable property of defining open areas not available to any tree.

The simple and multiple correlation analyses discussed above were repeated using the adjusted APA estimates, P', for all weighted indices. Simple correlations and contributions to multiple correlation were consistently higher for these adjusted values than those presented in the original analyses. Further, the use of P' poses no difficulty in modeling. In constructing individual tree growth models, values of P' are used directly as an independent variable. In implimenting the model, P' may be calculated from P, the known stand area, and the sum of P.

The auxiliary indices of Mead (1965) associated with each polygon were also examined further. Although both the index of eccircularity and the index of abcentricity, given polygon area, were significantly related to diameter and basal area growth ($p < .05$), even in the presence of tree size and stand density measures, their contributions to multiple correlation with growth was so small compared with the other variables that further analyses were abandoned. These measures may be useful in modeling seeded stands where irregular spatial patterns would produce more variability in polygon shape and tree position within the polygon.

Spurr's point density provides an estimate of basal area per unit area by variably weighting the sum of angles subtended by neighboring tree boles and may be considered an extension of Bitterlich methods (Spurr 1962). This estimate does not directly reduce mathematically to the stand measure. But the relationships between point density and basal area per acre are linear and very strong. Regressing basal area per acre on the mean of Spurr's point density produced the equations

$$BA = 13.806 + 1.0650\ \overline{S}_1$$
$$r^2 = .9957 \quad S_{y.x} = 2.893$$

and

$$BA = -.47383 + .93523\ \overline{S}_2$$
$$r^2 = .9902 \quad S_{y.x} = 4.363$$

where

BA = basal area per acre (ft.^{2}/acre)

$\overline{S}_1$ = mean point density estimate excluding the subject tree (ft.^{2}/acre)

$\overline{S}_2$ = mean point density estimate including the subject tree (ft.^{2}/acre).

Residuals from this analysis had no observable trends with age, stand density or mean tree sizes. The strength of these relationships indicates that perhaps mean point density could be corrected to correspond to basal area per acre when considered in the integrated model framework.

In a similar manner, relationships between Arney's (1973) competition index and CCF were examined. Conceptually, if not mathematically, both may be considered estimates of open grown crown overlap; Arney's overlap is calculated tree by tree, whereas in CCF the overlap is implicit. Plots of CCF versus the mean of Arney's index indicated a close linear relationship for CCF values greater than 100. For these cases the regression of CCF on the mean of Arney's index was

$$\mathrm{CCF} = -.32985 + .98767\,\bar{A}$$

$$r^2 = .9890 \quad S_{y.x} = 5.031$$

where

CCF = crown competition factor

$\bar{A}$ = mean of Arney's (1973) competition index.

The slope coefficient was not significantly different from unity.

Execution Time

One of the primary factors affecting the choice of different stand models is cost. Distance dependent individual tree models are both more costly to construct and to use than simpler models. Computer costs in executing such models can be significant and are directly related to the iterative calculation of competition values. The time required by the central processing unit (CPU) at the Virginia Tech VM/370 installation to calculate each of the distance dependent indices was measured by plot and age as an index of relative cost.

Execution time was nearly identical for area overlap, size ratio/distance, and Spurr's point density indices (Table 6). This is due in large part to the similarity of the algorithms used to calculate each of these indices. The APA indices required considerably more time to compute, due in part to an admittedly inefficient algorithm and in part to the large number of calculations required to construct polygons.

Execution time per plot was a function of density (Table 2); more trees require more calculations. Whereas this relationship was nearly linear for APA indices indicating that execution time per tree was nearly constant, time increased geometrically with density for the overlap,

TABLE 6. Mean computer time required to calculate distance dependent competition and point density indices.

Index	Execution Time per .25 Acre Plot (seconds x 100) Density Treatment (Trees/Acre) 1000	600	300	200	100
Area Overlap (A_1, A_2)	237	95	27	13	4
Size Ratio/Distance (C_1)	245	100	29	15	4
Spurr's Point Density (S_1, S_2)	231	97	29	15	4
Area Potentially Available (P_1, P_2)	574	343	157	108	48

size ratio/distance, and Spurr indices. This, again, is a function of the algorithms employed to locate competitors.

In general, area overlap indices considered fewer competitors but required more computations per competitor than either the size ratio/distance or Spurr indices resulting in similar time requirements for these three indices. The number of competitors increased with increasing age as trees' influence areas expanded, resulting in increasing execution time with increasing age. Execution time for APA indices decreased over time due to mortality.

Growth and Survival Models

The Hill Farm growth data used in the correlation analyses provided an excellent opportunity to examine variables affecting growth. However, for a number of reasons, the data are not especially well suited for fitting operational growth and survival equations. First, the data represent only one site and thus site quality effects and interactions cannot be incorporated into the growth and survival functions. Second, the range in ages (five to twenty years), while very good for modeling early growth, is too young for modeling later stand development. One could not expect equations derived from these data to behave well for older ages. Finally, although trees were measured annually up to age ten, gaps of three and seven years exist in the measurement periods from ages ten to twenty. The use of average growth values for these later years make the data appear very linear and tend to mask relationships in these very important years of stand growth. In addition, the year of mortality was not known for trees which died during these measurement gaps.

Although limitations exist, the Hill Farm data were used for demonstration purposes. Equations were fitted to observed height and basal area growth using the general growth model

$$\Delta X = \Delta X_p(b_1 X^m - b_2 X)$$

which was discussed earlier. The coefficients b_1 and b_2 above were replaced with functions of variables chosen from the analyses of growth correlation and included crown ratio (CR), relative basal area ratio (G_B), height to dominant height ratio (G_{HD}), and adjusted APA weighted by basal area (P_2'). Other variables, including different relative size ratios and APA variations, may be equally suitable.

Height Growth

Height growth was quite variable in the Hill Farm data. Relationships between observed height increments and competition, stand density, and tree size measures were weak.

Potential height growth was considered to be the change in observed average dominant height. An individual tree may grow more or less than this potential depending on its attributes. The final form of the height increment equation was

$$\Delta H_i = \Delta H_p e^{-b_1/APA_i}(b_0 G_{Hi}{}^{b_2} - b_3 H_i)$$

or

$$\Delta H_i = \Delta HD e^{-b_1/APA_i}[(b_0/\overline{H}^{b_2}) H_i{}^{b_2} - b_3 H_i]$$

where

ΔH = annual height increment

ΔH_p = potential height increment = ΔHD

HD = average height of dominant and codominant trees

H = tree height

APA = adjusted, weighted polygon index

$G_H = H_i/\overline{H}$.

The use of G_H rather than H was chosen to facilitate later analyses. As seen in the second formulation, the desirable effects of incorporating H are maintained. Values of the coefficients b_0 - b_3 and associated fit statistics are presented in Table 7.

Basal Area Growth

Basal area growth was highly related to a number of tree and stand measures as was shown earlier. The maximum basal area for a tree of given height and age was considered to be that if open grown. A relationship between open grown basal area and height and age was developed using data from 81 open grown loblolly pine trees (see Daniels and Burkhart 1975) which took the form

$$B_{OG} = b_0 A + b_1 A H^2$$

where

B_{OG} = open grown basal area
A = age
H = total height.

Potential basal area growth was then calculated by the difference, with respect to age of the B_{OG} function as

$$\Delta B_p = \Delta B_{OG} = b_0 + b_1 \{(A+1)(H+\Delta H)^2 - AH^2\}$$

TABLE 7. Individual tree growth and survival functions for integrated loblolly pine stand models.

Equation[1]	R^2	Sy·x
$\Delta H = 1.047\,\Delta Hp\, e^{-\frac{0.4464}{10000\cdot APA}} (G_H^{0.2819} - .003361H)$	0.2507	1.094
$Bp = 0.01415\,A + 1.644\,A\,H^2/100000$	0.8928	0.4177
ΔB[2] $= 1.722\,\Delta Bp\, e^{-\frac{2.463}{10000\cdot APA}} (CR^{3.009} G_B^{0.2155} - 0.01371B)$	0.7417	0.008622
ΔB[3] $= 1.740\,\Delta Bp\, e^{-\frac{2.774}{10000\cdot APA}} (CR^{2.964} G_B^{0.2603} - 0.04036B)$	0.7455	0.008554
$S = (1 + CR^{-8.255}(1 - \frac{A_1}{A_2})^{1.276} e^{-(6.816 + 7.771 APA\cdot 10000)})^{-1}$	--	--

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[1] Where H = height, Hp = potential height, APA = area potentially available, $G_H = H/\bar{H}$, $\bar{H}$ = mean height, B = basal area, $B_{OG} = Bp$ = open grown or potential basal area, CR = crown ratio, $G_B = B/\bar{B}$, $\bar{B}$ = mean basal area, A = age, HD = height of dominant and codominant trees, S = tree survival probability.

[2] ΔBp calculated with observed height and height increment.

[3] ΔBp calculated with observed height and potential height increment.

The above equation was evaluated for potential basal area growth by substituting age and observed height. In order to make ultimate basal area growth projections less dependent on observed height, the above equation was also evaluated using average height of the dominant and codominant trees.

The final growth model for tree basal area took the form

$$\Delta B_i = \Delta B_p e^{-b_1/APA_i} (b_0 CR_i^{b_2} G_{Bi}^{b_3} - b_4 B_i)$$

where

ΔB_i = basal area growth
ΔB_p = potential basal area growth
APA = adjusted, weighted polygon index
CR = crown ratio
G_B = the ratio of basal area to mean basal area
B = basal area.

Again, the use of the relative basal area ratio G_B facilitates later analyses while still maintaining the effects of B. Values of the coefficients b_0 - b_4 as well as appropriate fit statistics are included in Table 7.

Survival Probability

Survival was very high in the early years of the Hill Farm mutual competition study; mortality was significant only after age ten for the most dense plots. Unfortunately, the year of mortality was not recorded for ages between ten and thirteen or between thirteen and twenty. This precluded the development of operational individual tree survival probability functions and hindered comparisons of survival predictors. Using variables chosen from the analysis of growth correlations a survival function was fitted to demonstrate components of the integrated system of stand models.

Individual tree survival probability was defined using the logistic function as

$$S_i = [1 + CR^{b_1} (1 - A_1/A_2)^{b_2} e^{-(b_3 + b_4 APA)}]^{-1}$$

where

A_1 = age at beginning of projection period

A_2 = age at end of projection period

Coefficients (Table 7) were estimated from binary survival data using maximum likelihood procedures.

AN INTEGRATED SYSTEM OF STAND MODELS

The increment and survival probability equations developed earlier provide the central core for an integrated system of stand projection models ranging from individual tree distance dependent models to whole stand growth projection models. Although the limitations in available data precluded the development of operational models, the equations which were developed do provide a means of demonstrating an integrated system of stand models for loblolly pine.

Individual Tree Models

Distance Dependent

The individual tree increment and survival functions developed earlier (Table 7) may be incorporated directly into a distance dependent individual tree model such as that of Daniels and Burkhart (1975). Initial tree sizes and coordinates may be read as input to the model or generated. Methods for initially generating loblolly pine stands for individual tree simulation are described in detail by Daniels and Burkhart (1975) and Daniels *et al*. (1979b).

Once planting locations are assigned, mortality may be predicted using a survival function and assigned at random. The survival function of Feduccia *et al.* (1979) may be applicable for this purpose because it was developed for the same geographical region as the Hill Farm.

A two parameter Weibull distribution was used to generate dbh in the studies of Daniels and Burkhart (1977) and Daniels *et al.* (1979b). This same distribution may be modified to consider basal area as the variate. Strub (1976) demonstrated the utility of the normal distribution for describing basal area distributions, and related its parameters, μ and σ^2, to stand attributes as

$$\tilde{\mu} = \bar{B} = b_0 + b_1 HD + b_2 / TS$$

$$\tilde{\sigma} = b_0 + b_1 HD / TS$$

where

HD = average height of dominant and codominant trees

TS = surviving number of trees per acre.

Height may be predicted using the equation of Daniels and Burkhart (1975).

Once an initial stand has been determined, annual growth calculations may begin. Competition or growing space, in terms of weighted APA, is calculated at the beginning of each year. Crown ratio may be estimated using the equations of Feduccia, et al. (1979) and the relative size ratio indices G_B and G_H may be calculated. Survival probability, S_i, (Table 7) is evaluated for each tree and mortality is assigned using Bernouli trials. Growth of survivors is then calculated.

Potential height growth is evaluated as the change in average height of dominants and codominants obtained as the first difference of an appropriate site index equation. The site index equation of Popham et al. (1979) may be appropriate for this purpose since it is based on data from the same geographic region. The height growth equation in Table 7 may then be used to assign individual tree height growth. Basal area potential growth may be considered as the change in open grown basal area for either current height, age, and realized height growth, or for potential height growth. Individual basal area growth is then assigned using the appropriate equation in Table 7.

Competition, survival, and growth are computed annually. Summaries of stand development may be obtained at any desired age. Yield values may be obtained by applying any desired individual tree volume or weight functions to the array of simulated heights and basal areas, summing over trees of interest, and applying appropriate area expansion factors.

Distance Independent

The distance independent model is obtained from the dependent model by simply collapsing the point density measure, APA, to the inverse of stand density, 1/TS. Of course there is no need for spatial pattern generation or associated arrays for coordinate locations for this model. Otherwise, the model structure is similar to that described for the distance dependent model.

By collapsing APA to 1/TS the height and basal area growth functions reduce to

$$\Delta H_i = b_0(HD_{A+1} - HD_A)e^{-b_1 TS}(G_H{}^{b_2} - b_3 H_i)$$

and

$$\Delta B_i = b_0(B_{OG,A+1} - B_{OG,A})e^{-b_1 TS}(CR_i{}^{b_2} G_{Bi}{}^{b_3} - b_4 B_i)$$

The survival function takes the form

$$S_i = [1 + CR_i{}^{b_1}(1 - A_1/A_2)^{b_2} e^{-(b_3 + b_4/TS)}]^{-1}$$

Size Class Models

Direct Projection by Size Class

The classical approach to size class projection models is to consider the number of trees in each of several size classes of fixed width and project the number of trees per size class. A typical case is the classical stand table projection method in which the number of trees in fixed (e.g., one or two inch) diameter class are projected over time.

An alternative approach is to consider variable width size classes, each with roughly equal numbers of trees initially, and to project the mean attributes and survival in each class. The size ratio indices may be considered to represent the mean size in the class, j, over the stand mean or

$$G_j = \frac{X_j}{\bar{X}}.$$

Thus given an initial distribution of numbers of trees, total height, and basal area in each of j size classes, along with stand variables age, density, and site index, the equations in Table 7 may be used to project attributes in each class. These equations would take the form shown above for distance independent individual tree models except that individual tree variables ΔH, H, G_{HD}, ΔB, B, G_B, CR, and S would represent mean attributes within a size class, rather than individual tree attributes.

Using the above notation a projection model for basal area per acre by size class takes the form

$$BA_{t+1} = \sum_{j=1}^{n} \{[\bar{B}_j + b_0(\Delta B_p)e^{-b_1 TS}(CR_j^{b_2} G_{Bj}^{b_3} - b_4\bar{B}_j)]N_j S_j\}$$

where

n = the number of size classes

N_j = the number of trees in class j

S_j = survival probability for class j

$\bar{B}_j$ = mean basal area in class j

Note that all variables on the right hand side of the equation are at time t. The above model assumes that mortality is uniform with a size class.

Pdf Relationships

Bailey (1980) noted that growth functions are implied by assuming a common family of probability density functions to model tree size over the life of the stand. He derived these implied growth relationships from transformations which, when applied to a random variable, kept the new variable in the same family of pdf's. Growth functions thus derived were presented for a number of pdf's commonly used to model tree size distributions including the exponential, normal, beta, Johnson's S_B, log normal, Weibull, and generalized gamma distributions. The growth functions derived earlier will be examined with respect to pdf relationships.

By collapsing the individual tree increment equations to stand level attributes, the models obtained are

$$\Delta\bar{H} = b_0(\Delta H_p)e^{-b_1 TS}(1-b_3\bar{H})$$

and

$$\Delta\bar{B} = b_0(\Delta B_p)e^{-b_1 TS}(\overline{CR}^{b_2} - b_4\bar{B}).$$

These equations take the general form

$$\Delta X = c_0 + c_1 X$$

to yield the linear size projection model

$$X_{t+1} = c_0 + (c_1+1)X_t$$

where c_0 and c_1 are functions of stand level variables. For basal area

$$c_0 = b_0(\Delta B_p)e^{-b_1 TS}\overline{CR}^{b_2}$$

and

$$c_1 = -b_0 b_4(\Delta B_p)e^{-b_1 TS}.$$

The results of Bailey (1980) indicate that such a transformation is appropriate to maintain a number of distributions including the exponential, beta, Johnson's S_B, and normal. Strub (1976) has already demonstrated the use of the normal pdf for modeling basal area distributions. Thus the normal pdf will be considered for further development of this example.

The relationships between the growth function and pdf not only indicate that the pdf family is maintained over time, but provide a direct means of projecting pdf parameters from the growth function. Considering a random variable

$$X \sim N(\mu, \sigma^2)$$

it is known that

$$a_0 + a_1 X \sim N(a_0 + a_1\mu, a_1^2\sigma^2).$$

Thus, given

$$B_t \sim N(\mu_B, \sigma_B^2)$$

the basal area growth function developed here implies that

$$B_{t+1} \sim N(c_0 + (c_1+1)\mu_B, (c_1+1)^2\sigma_B^2).$$

In other words, the growth function provides the ability to project the parameters of the normal distribution (and others) over time, given initial parameter values. Initial parameter estimates have already been considered for the normal (Strub 1976).

The above formulation assumes that mortality is equally distributed over the basal area distribution or that no mortality occurs. But mortality rates are generally higher for smaller trees. Thus the mean basal area for living and growing trees is higher than the observed mean basal area. An adjustment to the mean then must be made for mortality and may take the form

$$B_{t+1} = c_0 + (c_1+1)KB_t$$

where K is an adjustment factor for B_t to account for differential mortality with respect to size. Note that the linear form of the growth relationship is maintained. A more complete explanation of the adjustment factor K and its derivation are presented in the section on whole stand models.

The pdf-based basal area projection system described here may be used as are typical diameter distribution models for stand yield estimates. That is, the number of trees in each basal area class (perhaps classes of fixed width of .05 to .1 ft.^{2} are appropriate) is estimated using the integrated pdf and known stand density. Volume (or weight) per class is estimated by substituting class midpoint basal area and mean height into tree volume expressions and multiplying by the number of trees per class. Stand yield estimates are then made by summing over size classes of interest. Mean height per class may be projected, as discussed earlier for size class models, or estimated from tree and stand variables. Alternatively local volume tables based solely on basal area may be used to estimate volume per class. The class-interval-free methods of Strub and Burkhart (1975) would also be applicable for direct whole stand yield estimates from the pdf.

Whole Stand Models

When collapsed to stand level measures the basal area projection model takes the form

$$B_{t+1} = c_0 + (c_1+1)B_t$$

where c_0 and c_1 are defined earlier. The height projection model becomes

$$H_{t+1} = a_0 + (a_1+1)H_t$$

where

$$a_0 = b_0(\Delta H_p)e^{-b_1 TS}$$
$$a_1 = -a_0 b_3.$$

The survival function now represents the proportion of TS surviving until the next year. This relationship is written as

$$TS_{t+1} = S\ TS_t$$

where

$$S = [\,1 + \overline{CR}^{b_1}(1-A_1/A_2)^{b_2}\ e^{-(b_3\ -\ b_4/TS)}]^{-1}.$$

An initial stand level model for projecting basal area per unit area may be derived as

$$\begin{aligned} BA_{t+1} &= [BA_t + TS_t\ (c_0 + c_1\overline{B}_t)]S \\ &= [c_0 TS_t + (c_1+1)BA_t]S \\ &= [c_0 + (c_1+1)\overline{B}_t]TS_t S \end{aligned}$$

where

BA = basal area per unit area

$\overline{B}_t$ = mean basal area = BA_t/TS_t

Thus BA_{t+1} is a linear function of BA_t. This model implicitly assumes that the proportional reduction in basal area per unit area due to mortality is the same as that for trees per unit area, or in other words, that the mean basal area, after mortalilty, is unchanged. This assumption must surely be questioned; mortality is generally higher for small trees, resulting in an increase in mean basal area after mortality.

Instead of reducing BA_t by S, a factor R, interpreted as the proportion of basal area surviving, was applied to BA_t to account for mortality. The model now becomes

$$\begin{aligned} BA_{t+1} &= c_0 TS_t\ S + (c_1+1) BA_t\ R \\ &= c_0 TS_t\ S + (c_1+1)\ TS_t\ S\ \bar{B}'_t \\ &= [c_0 + (c_1+1)\bar{B}'_t]\ TS_t\ S \end{aligned}$$

where

$$\begin{aligned} \bar{B}'_t &= \text{mean basal area after mortality} \\ &= BA_t R/(TS_t\ S) \\ &= \bar{B}_t(R/S)\ . \end{aligned}$$

Thus $\bar{B}'_t = \bar{B}_t(R/S)$

The relationships between R and other variables were examined. The ratio $K = \bar{B}'/\bar{B} = R/S$ was found to be almost constant in the Hill Farm data ($K = 1.0008$). Since R must be related to survival, S, it was hypothesized that R was a function of some power of S, $R = b_0 S^{b_1}$. Thus, the relationship developed was

$$R = S^{.5341} \quad (r^2 = .9028)$$

or

$$K = S^{-.4659}$$

The model finally becomes

$$BA_{t+1} = [c_0 + (c_1+1)K\bar{B}_t]\ TS_t\ S$$

The projection equation for average basal area becomes

$$\bar{B}_{t+1} = c_0 + (c_1+1)K\bar{B}_t$$

which was shown earlier for pdf relationships. Note that the model retains the linear relationship with B_t.

A similar model for height is

$$\bar{H}_{t+1} = a_0 + (a_1+1)K_H\bar{H}_t$$

where

$$R_H = S^{.8218} \quad (r^2 = .9903)$$

$$K_H = S^{-.1782}$$

Thus equations to project basal area per unit area, trees per unit area, mean basal area and mean height have been derived from individual tree growth and survival models. Stand yield estimates may be made by substituting mean sizes into known individual tree volume expressions and multiplying by the number of trees per unit area as

$$Y = TS\ V(\overline{B}, \overline{H}).$$

Alternatively relationships between projected basal area per acre and mean height may be used to estimate volume yields.

Considerations for Numerical Consistency

The integrated system of stand models developed here relies on a common mathematical structure for models of different levels of resolution. Thus models at each level should behave similarly with respect to growth projections. However, these models may still provide numerically inconsistent estimates when interchanged. Numerical consistency may be obtained for some resolution levels and not others. For instance, if growth and survival are allocated stochastically in individual tree models, then individual estimates would be inconsistent although the mean of several evaluations of the model could be consistent with other models. Three approaches to numerical consistency may be considered: 1) identifying relationships and chosing among model variables; 2) applying constraints in estimating model parameters; and 3) applying constraints in the application of models. All three approaches may be required.

It was noted that the variables of growth and measurement are not necessarily those of ultimate interest. Basal area was chosen over dbh for growth modeling because it is more closely related to volume. Other choices may also be made. The relative size indices G_H, and G_B were chosen so that the growth models, when collapsed, represented average height and average basal area. It may be desirable to use a size index based on squared dbh times height (or B·H). Collapsing this index to one at the stand level implies the tree of average B·H, where B·H is generally considered to be linear with total volume.

Other variables must also be considered. It may or may not be reasonable to assume that the tree of mean APA or crown ratio represents the tree of mean basal area, basal area growth, or volume. In the example presented mean crown ratio was associated with mean dbh using the equations of Feduccia *et al.* (1979). Thus the relationships among different variables must be examined. Modifiers may be incorporated to adjust variable means to apply to mean basal area or volume. A variable of this type was incorporated for stand level projections to adjust mean sizes in accounting for differential mortality with respect to size. A detailed examination of these relationships would require complete specification of all size and attribute distributions, which may not be practical.

A second approach may be considered in which conditions and/or constraints are applied in estimating model coefficients. For example growth models may be conditioned in such a way as to pass through given points. Constraints may further be imposed so that means or sums of individual tree values follow some predicted or observed stand level attribute values.

Constraints may also be applied during the application of models. If coefficients are estimated at the stand level, as well as at the individual tree level, then predicted stand level response could be used to scale predicted tree level values and "force" numerical consistency with stand level attributes. This procedure has the advantage that stand level data, from a much broader base than would be available at the tree level, could be incorporated to broaden the applicability of the integrated system. Of course, in this context, the individual tree models serve only to allocate variability of predicted aggregate response and tree-level models lose some of their appeal in terms of flexibility.

CONCLUSIONS AND RECOMMENDATIONS

An integrated system of stand models for loblolly pine was developed and demonstrated which incorporates virtually all of the stand projections methods currently in use today. Data were not available to fully calibrate this system of models for operational use, but model components were identified and preliminary parameter estimates were presented. It is hoped that these studies will help to promote the development of more general operational stand models, encourage others to investigate further the relationships between tree growth and stand dynamics, and provide guidelines for future data collection and analysis.

In many ways the present study may be considered a "pilot" or feasibility study for future development of integrated systems of stand models. Indications are that such systems are well within reach and that future modeling efforts should address more general objectives than in the past. Further development of integrated model system may take place in a number of areas in parallel.

Data from a broader inference base must be available before an integrated system, as described here, can be fully calibrated and tested. A number of data sets currently exist which may be used to calibrate and test the distance independent components. Distance dependent data covering a large portion of the range of loblolly pine is currently being collected through the VPI and SU Loblolly Pine Growth and Yield Cooperative.

In the meantime work may continue on the development of model components. Further development of biologically rational growth relationships must continue. The example presented here was just one of many possible formulations possible. The rational behavior of growth functions is crucial to the link between tree level and stand level models. Development of differential growth equations may allow variable projection periods and provide additional flexibility over the annual growth projections considered here. Survival relationships may also be improved as well as the representation of mortalilty related changes in size distributions. The pdf relationships may also be developed further. Growth relationships for more distribution families may be examined. Joint height-basal area distributions may be possible as well.

Questions related to numerical consistency are wide open for study. The relationships between attributes at both the tree and stand levels must be examined closely.

Closer examination of the distributions of attributes will help in model construction. Parameter estimation and model application procedures, must also be examined with respect to numerical consistency.

Future work may also concentrate on development of flexible stand management routines such as those used by Daniels and Burkhart (1975) and Thurmes (1980). Identification of appropriate tree and stand components for inclusion of these routines is also important.

As management intensity increases, and information needs become more varied, stand models must become more flexible. Integrated systems of stand models should provide much greater flexibility in meeting these needs than have single purpose approaches used in the past.

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APPENDICES

APPENDIX I

Correlation of various competition and point density indices with annual growth of loblolly pine.

TYPE OF INDEX	SYMBOL	AUTHORS	REMARKS	CORRELATION COEFFICIENT (1) WITH GROWTH IN D	B
				n = 6490	
Distance Independent					
Relative Size	G	Glover and Hool (1979)	$(D_i/\overline{D})^2$	.2329	.4027
	G_B	Variation	$B_i/\overline{B}$	.2355	.4332
	G_H	"	$H_i/\overline{H}$	.2537	.3950
	G_{BH}	"	$B_iH_i/\overline{BH}$	.2333	.4329
	G_{HD}	"	H_i/HD	.3057	.4937
Crown Ratio	CR		Cl_i/H_i	.7703	.4772
Distance Dependent					
Area Overlap	A_1	Arney (1973)	Unweighted	-.6604	-.4347
	A_2	Ek and Monserud (1974)	Weighted	-.6316	-.4920
Dbh Ratio/Distance	C_I	Daniels (1976)	$\Sigma(D_i/D_j)/l_{ij}$	-.68722	-.5574
Spurr's Point	S_1	Spurr (1962)	Excluding sub. tree	-.7143	-.4175
Density	S_2	Spurr (1962)	Including sub. tree	-.6545	-.3807
Area Potentially	P_1	Brown (1965)	Unweighted	.3329	.6367
Available	P_2	Moore *et al.* (1973)	Weighted by B	.3888	.7463
	P_3	Variation	Weighted by BH	.3872	.7487
	P_4	"	Weighted by H	.3638	.6855
	P'	"	P_2 modified to sum to total area	.3934	.7480
Three Dimensional	PH_1	Pelz (1978)	P_1 x H_i	.0852	.5122
Indices	PH_2	"	P_2 x H_i	.1359	.5800
	PH_3	"	P_3 x H_i	.1420	.5803
	PH_4	"	P_4 x H_i	.1023	.5350
Tree Size					
	D			-.1876	.3571
	B			-.1860	.3154
	H			-.4705	NS
Stand Level Measures					
	TS			-.3332	-.5707
	BA			-.7388	-.4032
	CCF	Krajicek, *et al.* (1961)		-.7536	-.5184

[1] Where B = tree basal area; H = tree height; HD = average height of dominant and codominant trees; CR = crown ratio; CL = crown length; D = dbh; L = distance; TS = surviving trees per acre; BA = basal area per acre; CCF = crown competition factor; NS = nonsignificant (.05).

APPENDIX II

Squared multiple correlation coefficients between loblolly pine growth and various competition and point density indices in the presence of tree size and stand density.

INDEX	Squared Multiple Correlation Coefficient (R^2) with Dbh Growth for Model Including[1]						
	D	D,TS	D,BA	D,TS,G_B,CR	D,BA,G_B,CR	D,TS,G_{HD},CR	D,BA,G_{HD},CR
DISTANCE INDEPENDENT							
G	.1577	.5442	.6121				
G_B	.1737	.5465	.6129				
G_H	.1724	.5071	.6222				
G_{BH}	.1693	.5306	.6130				
G_{HD}	.2529	.5012	.6296				
CR	.6121	.6121	.6157				
DISTANCE DEPENDENT							
A_1	.4373	.4413	.6027	.6410	.6309	.6511	.6478
A_2	.4231	.4777	.6029	.6427	.6296	.6487	.6432
C_1	.5093	.5390	.6176	.6473	.6350	.6452	.6481
S_1	.5362	.5450	.6082	.6476	.6352	.6589	.6524
S_2	.4382	.4686	.6052	.6425	.6305	.6522	.6463
P_1	.2491	.2962	.6029	.6434	.6348	.6518	.6509
P_2	.4018	.4281	.6121	.6520	.6419	.6631	.6597
P_3	.4085	.4415	.6143	.6528	.6422	.6627	.6587
P_4	.3089	.3378	.6053	.6483	.6399	.6545	.6536
P'	.4069	.4336	.6131	.6524	.6423	.6636	.6602
PH_1	.1240	.2651	.6044				
PH_2	.2141	.3166	.6025				
PH_3	.2160	.3259	.6025				
PH_4	.1496	.2721	.6034				
CHECK[2]	.0352	.2647	.6025	.6381	.6270	.6464	.6418

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[1] The model $D = b_0 + b_1D + b_2(TS \text{ or } BA) + b_3(G_B \text{ or } G_{HD}) + b_4CR + b_5CI$ was fitted including increasing numbers of terms, where D = dbh; TS = trees surviving per acre; BA = stand basal area per acre; G_B = relative basal area index; G_{HD} = relative height to dominant height index; CR = crown ratio; and CI = competition index.

[2] Fit without index as a check on the contribution of the index.

AN INTEGRATED SYSTEM OF STAND MODELS FOR LOBLOLLY PINE

by

Richard F. Daniels

(ABSTRACT)

An integrated system of stand models was developed for loblolly pine in which models of different levels of resolution are related by a unified mathematical structure. A "telescoping" system is presented in which a highly detailed overall model is specified and its components "collapsed" around a common set of growth and survival functions to provide structurally compatible models at each successively lower stage of resolution.

The most detailed model is a distance dependent individual tree model which simulates the growth and competitive interaction of trees in a stand. Tree basal area and height growth were modeled using a modified Chapman-Richards function in terms of potential growth, current size, relative size, crown ratio, and an index of competition. Potential growth was considered a function of site quality, age, and open-grown size relationships. Tree survival probability was described using a logistic function in terms of age, crown ratio, and competition.

The competition or point density index is a function of the size and location of neighbors. Published indices were

evaluated and compared on their simple correlation with growth, multiple correlation with growth in the presence of other tree and stand measures, computer execution time, and relationships to stand level density measures. The area potentially available (APA) for each tree was chosen as the most suitable. The APA index is calculated as the area of the polygon constructed from lines which divide the distance between a tree and its neighbors. Mean APA, or average area per tree, is estimated by the inverse of trees per unit area, permitting point density to collapse to stand density, resulting in a distance independent individual tree model.

This model was collapsed dimensionally to consider trees grouped in size classes. Tree growth and survival equations are applied to the mean attributes of each size class, resulting in a size class projection model. At the lowest level of resolution, the dimensions of the model are collapsed to one "average" tree. A stand level projection model results from applying the tree growth and survival equations to the stand's average tree attributes.

At the stand level, the basal area growth function provides a transformation which, for a number of probability density functions (pdf's), will regenerate the initial pdf family. Considering a normal pdf to describe basal area distributions, a pdf-based size distribution model is

presented, in which the projected parameters are expressed in terms of the growth function coefficients. Applications to other pdf families are discussed.

Preliminary tree growth and survival equation coefficients were estimated using data from a loblolly pine stand density study in North Louisiana. Structurally compatible models at each level of resolution are detailed. Considerations for numerically consistant estimates from models of different levels of resolution are discussed in terms of model specification, estimation, and implementation. Recommendations for model application and future model development are presented.

APPENDIX II (continued)

Squared multiple correlation coefficients between loblolly pine growth and various competition and point density indices in the presence of tree size and stand density.

INDEX	Squared Multiple Correlation Coefficients (R^2) with Basal Area Growth for Model Including[1]						
	B	B,TS	B,BA	B,TS,G_B,CR	B,BA,G_B,CR	B,TS,G_{HD},CR	B,BA,G_{HD},CR
DISTANCE INDEPENDENT							
G	.1955	.5281	.5194				
G_B	.2155	.5316	.5252				
G_H	.1949	.4786	.5280				
G_{BH}	.2118	.5264	.5263				
G_{HD}	.2608	.4776	.5489				
CR	.4907	.5137	.4972				
DISTANCE DEPENDENT							
A_1	.3877	.4225	.4938	.5948	.5358	.5824	.5579
A_2	.3498	.4394	.4954	.5915	.5282	.5785	.5500
C_1	.3988	.4646	.4985	.5913	.5285	.5785	.5500
S_1	.4569	.4983	.4991	.6057	.5369	.5963	.5609
S_2	.3848	.4527	.4954	.5974	.5312	.5858	.5540
P_1	.4063	.4226	.5550	.6576	.6475	.6398	.6460
P_2	.5764	.5862	.6325	.6856	.6688	.6810	.6791
P_3	.5833	.5988	.6387	.6861	.6660	.6801	.6748
P_4	.4708	.4788	.5810	.6703	.6603	.6487	.6534
P'	.5786	.5893	.6336	.6849	.6676	.6809	.6785
PH_1	.2704	.3535	.5187				
PH_2	.4077	.4663	.5463				
PH_3	.4052	.4760	.5450				
PH_4	.3063	.3772	.5255				
CHECK[2]	.0995	.3292	.4937	.5913	.5280	.5785	.5498

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[1] The model $B = b_0 + b_1B + b_2(TS \text{ or } BA) + b_3(G_B \text{ or } G_{HD}) + b_4CR + b_5CI$ was fitted including increasing numbers of terms, where B = tree basal area; TS = trees surviving per acre; BA = stand basal area per acre; G_B = relative basal area index; G_{HD} = relative height to dominant height index: CR = crown ratio; and CI = competition index.

[2] Fit without index as a check on the contribution of the index.