CONTINUOUS HMM CONNECTED DIGIT RECOGNITION

by

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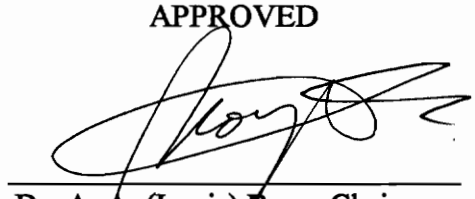

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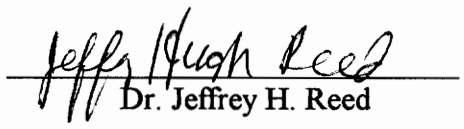

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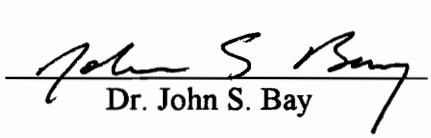

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(ABSTRACT)

In this thesis we develop a system for recognition of strings of connected digits that can be used in a hands-free telephone system. We present a detailed description of the elements of the recognition system, such as an endpoint algorithm, the extraction of feature vectors from the speech samples, and the practical issues involved in training and recognition, in a Hidden Markov Model (HMM) based speech recognition system.

We use continuous mixture densities to approximate the observation probability density functions (pdfs) in the HMM. While more complex in implementation, continuous (observation) HMMs provide superior performance to the discrete (observation) HMMs.

Due to the nature of the application, ours is a speaker dependent recognition system and we have used a single speaker's speech to train and test our system. From the experimental evaluation of the effects of various model sizes on recognition performance, we observed that the use of HMMs with 7 states and 4 mixture density components yields average recognition rates better than 99% on the isolated digits. The level-building algorithm was used with the isolated digit models, which produced a recognition rate of better than 90% for 2-digit strings. For 3 and 4-digit strings, the performance was 83 and

64% respectively. These string recognition rates are much lower than expected for concatenation of single digits. This is most likely due to uncertainties in the location of the concatenated digits, which increases disproportionately with an increase in the number of digits in the string.

Acknowledgments

If there is any part of me that I am proud of, I owe it all to my parents. This thesis is a small but important step in my efforts to live up to the standards, of being a good human being, that they have set. I cannot thank them enough for making me capable of doing whatever I am doing now.

The very thought of having Vasu, my brother, be my best friend gives me great pride. Almost every endeavor of mine has included Vasu and, strangely, I feel Vasu has not left me alone in my work towards my degree. I have spent many hours talking to my sister, Pri (even when I am alone), and every moment has been intellectually replenishing. If I have found the motivation to complete this work, it is solely due to the association with such magnetic personalities as Pri. I also feel great about having Sampath as a close friend during the course of this work.

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I thank Dr. Jeff Reed from whom I have learnt to derive intellectual satisfaction through Digital Signal Processing. I thank Dr. Reed and Dr. Bay for their time and effort in reviewing this work.

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1. Introduction

Speech is the most dominant mode of communication among human beings. It has been a part of every civilization that has been discovered. Another mode by which complex information is exchanged is by writing. However, since reading and writing require literacy, they are less universal than speaking. Lately, with the advent of resources such as the internet, communication by typing has been on the increase. Typing and writing, however, suffer from the disadvantage that they are much slower means of communication than speaking. On the other hand, man-machine communication is dominated by typing. This is because, although writing and speaking are potentially more efficient modes of man-machine communication, they require the machine to be able to recognize handwriting and speech. Research in these areas has not yet reached such a high level of maturity. Speech, however, offers the advantage of being the fastest and the most natural form of human communication. Man-machine communication through speech also promises an environment where the hands and eyes are free; the speaker and listener can enjoy unconstrained movement and remote access. These have been the main motivation for research into automatic speech recognition. The ultimate goal of automatic speech recognition is to build machines that can "hear," "understand," and "act upon" spoken input. For this, the machine must be capable of extracting information from a speech wave. The information content in a speech wave can vary between the level of being just a sound, to the level of being a tone, to the identity of the speaker and to a

meaningful communication between humans. This information is stored in the machine's internal model of the speech process. The fundamental problem of modeling speech is that it is a continuous stream of sounds with no clear boundaries between the words. Yet, humans perceive speech as a sequence of words. Hence, it is necessary to segment this continuous stream of sounds into words, subwords and gaps. The large variability in the acoustic signals corresponding to the same linguistic unit also complicates the modeling process. Speech recognizers can be classified into varying levels of complexity based on criteria such as speaker-dependency, nature and size of vocabulary, and continuity of the speech.

In a speaker-dependent system, the utterances of a single speaker are used to model the speech process. This process of characterizing speech by models is called "training." The purpose of a speaker-dependent system would be to recognize the speech uttered by the person whose speech was used in the training process. A speaker-independent system is trained by multiple speakers and is used to recognize the speech uttered by multiple speakers, some of who could be outside the training population. Due to larger variability involved, a speaker-independent system is less accurate than a speaker-dependent system. However, the speaker-dependent system needs to be retrained each time it is to be used with a new speaker.

Based on the size of the vocabulary involved, speech recognition systems can be classified as small, medium and large vocabulary systems. Typically, vocabulary sizes of 1-99, 100-999, and 1000 or more words are called small, medium, and large vocabularies

respectively. The performance and speed of recognition decrease as the vocabulary size increases. Moreover, the memory requirements and the number of confusable items in the vocabulary increase as the vocabulary size increases. The larger vocabulary recognition systems need to employ many complex constraints, such as linguistic constraints, rather than train, store and search exhaustively for each word. They also use models of subword units rather than of whole words.

Based on the continuity of the speech involved in the application, speech recognizers can be isolated-word, connected-word or continuous speech recognizers. In isolated-word recognition, the speech involved is constrained to be uttered with a sufficiently long pause (typically 200 msec) between words. With this constraint, techniques such as end-point detection can successfully locate the boundaries between the words. This simplifies the task of speech recognition. Continuous speech is much more complex to recognize due to the absence of any such simplifying constraints. The recognizer must be capable of taking care of the large variability in the articulation associated with flowing speech. Moreover, the continuous speech recognizer must deal with unknown temporal boundaries in the speech. Connected word recognition is a technique to recognize continuous speech. It is used in small vocabulary, continuous speech applications. This technique characterizes a sentence as a concatenation of models of individual words.

Let us now briefly look into the history of speech recognition systems. The earliest speech recognition systems were based upon the fact that the information required for recognizing speech existed in the acoustic signal. This was because different spoken words

were observed to give rise to different acoustic patterns. Most of the speech recognition systems employed a wide variety of filter banks to divide the speech signal into frequency bands. The outputs of the filter banks were then cross-correlated with patterns of spoken words obtained by a similar treatment of training utterances. These early systems were improved by adjusting the mean level of the speech signal so that loud speech and quiet speech have roughly the same intensity. There were also recognition systems that divided the speech signal into smaller units such as voiced and unvoiced sounds, fricatives and plosives such that a single phoneme was isolated. The recognition was then performed based on the combination of the various phonemes present. The early systems, however, did not perform very accurately. Further research suggested that the simple matching of the acoustic patterns was not enough to achieve high performance speech recognition. The same word, spoken by the same person at different times, or by different persons, varied in duration, intensity, and frequency content. So researchers began to consider the speech recognition problem as a pattern-recognition problem. Recognizers were built that attempted to normalize the intensity level, the duration, and the formant frequencies in the speech before matching with stored patterns. These methods performed better in terms of accuracy and the number of speakers whose speech was recognized. The idea of pattern-recognition also motivated researchers to employ artificial neural networks for speech recognition. However, all the early speech recognition systems were built to recognize small speech units such as vowels and not words or sentences. Later, the employment of linguistic constraints enabled development of systems that recognized words and

sentences. Recognition systems involved extraction of prosodic features (or the tonal and rhythmic aspects of speech) from the speech. This was crucial to the development of continuous speech recognition systems. The use of lexical, syntactic, semantic and pragmatic knowledge further improved the performance of continuous speech recognition systems. More recently, the uses of dynamic programming techniques, such as Dynamic Time Warping (DTW) and stochastic modeling, such as hidden Markov modeling, have led to encouraging results in the development of isolated and connected word recognition systems.

This thesis attempts to develop a connected-digit speech recognition system that can be used in applications like the hands-free telephone. The various issues investigated in this thesis include the effects of the model sizes on the recognition performance, the use of isolated word patterns for connected speech recognition, and other issues regarding the convergence of the training algorithm, such as the initial estimate and the stopping conditions. This thesis suggests a method that provides good initial models for the training procedure. We also discuss a procedure for obtaining good estimates for the stopping conditions involved in the training process. In the connected digit recognition system that we have developed in this thesis, we have incorporated the word duration statistics in a manner that is different but simpler than in many of the existing techniques. The existing techniques use an empirically scaled word duration likelihood [2]. The choice of the scale factor is crucial to successful recognition and requires a lot of trials to arrive at a "good" scale factor. The technique that we have implemented does not involve any such empirical

scale factors and consequently does not require trial and error. The development of this connected-digit recognition system involves hidden Markov models (HMM) extensively and dynamic programming strategies to a lesser extent. Before proceeding to the details of the developed system, the following chapter provides a background on the various approaches to speech recognition and the elements of a speech recognition system. Chapter 3 discusses the feature extraction block of the recognition system. In Chapter 4, we provide the necessary theoretical background about Hidden Markov Models and other issues such as training and recognition using HMMs. In Chapter 5 we demonstrate the recognition performance that we obtained when we used our system for isolated digit recognition. Chapter 6 describes the level-building algorithm that we used to perform connected digit recognition and discusses the recognition performance on strings of digits.

2. Background

In this chapter we bring out the various approaches to speech recognition and the elements of a typical speech recognition system.

2.1 Approaches to Speech Recognition

It is fundamental to speech recognition to extract the information from the speech signal. Hence, it becomes necessary to model the speech signal so that the parameters of the model characterize the information and discard any redundancy in the speech. The various approaches to speech recognition differ largely in the philosophy behind modeling of the information in speech. During speech communication, linguistic messages are converted into acoustic waveforms and transmitted. These are then received by the auditory system and converted back to a linguistic message by the speech perception system. Based on these stages of speech communication, we can identify four viewpoints of speech recognition. A combination of these four approaches can also be used in the development of a speech recognition system.

2.1.1 Acoustical Signal Approach

This approach views the speech signal as just another waveform. Therefore, we can apply various signal analysis techniques such as Fourier analysis, principal component analysis, and other mathematical methods to identify the input to the recognizer. The

principal focus of this method is the mathematical representation of the input-output characteristic of the recognizer. Each input is compared with the stored examples or templates of each class of inputs. The class that is closest to the input (in terms of an appropriate distance measure) is chosen as the identity of the input. This is the basis of various statistical modeling and pattern recognition schemes. These methods do not include any aspects of human speech production or perception. Figure 1 shows a simplified block diagram of a recognition system based on the acoustic signal approach.

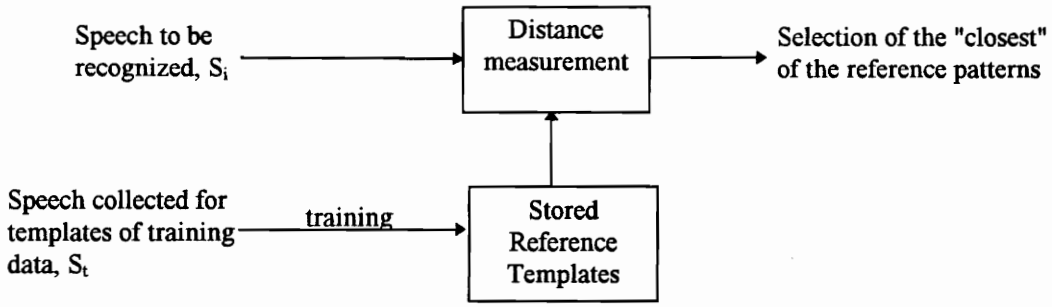

Figure 1. Block Diagram of Acoustical Signal Approach to Speech Recognition

The reference templates could include more than one example of a message in the vocabulary or one single representative obtained as an average of the various training utterances of a message. The critical part of the above system is the distance measurement, since this is fundamental to template matching. One method to determine a match between the input signal and the reference template is to compute the Euclidean distance between the two. However, it is important to choose the training data such that representatives of two different messages are quite distinct and those of the same message by all speakers, be close. Hence, it becomes necessary to collect large amounts of training data. This

increases the cost of the system in terms of storage requirements and effort needed to collect training data. This method treats every part of the speech wave as if it is equally important. However, we can provide different levels of significance to different aspects of the waveform. For example, we can provide a higher level of significance to the higher signal levels because they are less susceptible to noise. Such differential weighting can be achieved by choosing an appropriate distance measure or by extracting certain features (or parameters) from the signal and comparing only those features. These features can be extracted by many of the signal processing techniques used in waveform analysis. Due to differences in the duration of different messages or various utterances of the same message, time-warping provides a better method of comparison of the templates. Time warping can be achieved either by a simple method of time normalization of the templates before matching or by more complex methods such as Dynamic Time Warping (DTW).

2.1.2 Speech Production Approach

This approach views the information in the speech wave as a characteristic of the manner in which it was produced by the human vocal system. We now suggest a brief mechanism by which speech can be produced with the vocal apparatus. To inhale air, the rib cage expands and the diaphragm is lowered. This draws air into the lungs. Then the rib cage expands and the diaphragm is raised increasing the air pressure in the lungs. The increase in pressure forces out air through the wind pipe. In the wind pipe the air passes through the larynx, which contains the vocal cords. Due to the Bernoulli effect, the air

flow causes a drop in the pressure. This causes the laryngeal muscles to close the glottis, thereby interrupting the air flow. This interruption increases the air pressure which in turn forces the vocal cords apart. The cycle then repeats, which produces a train of glottal pulses. The vocal tract, and the oral and nasal passages pass the harmonics of the glottal waveform that are close to their natural resonances while attenuating the others. This acoustic wave radiates from the lips as speech. Some sounds such as the consonants are produced without involving the glottis. These sounds are produced by constricting the vocal tract. The constriction causes a turbulence and the air then flows out through the oral and nasal passages resulting in speech. Thus speech is produced by the excitation of the vocal tract, and/or the oral and nasal passages. Based on the nature of the excitation, speech can be broadly classified into voiced and unvoiced speech. The glottal excitation of the vocal tract produces voiced sounds. Vowels are examples of voiced sounds. Sounds produced by the constriction of the vocal tract are known as unvoiced sounds, consonants for example. When humans convey a linguistic message in the form of speech, they construct a phrase or sentence by selecting a set of sounds from a finite collection of mutually exclusive sounds. This basic set of sounds consists of the phonemes. However, due to various factors, such as accents, gender, and coarticulatory effects, a given phoneme will have a different manifestation under different conditions. These manifestations of the phonemes of a language are called phones.

Having provided the basics of speech production, we now go into the aspect of modeling speech from the production point of view. Various models have been proposed

for modeling speech [1]. Of all these models, a popular model for speech is the Discrete Time Model that is shown in Figure 2.

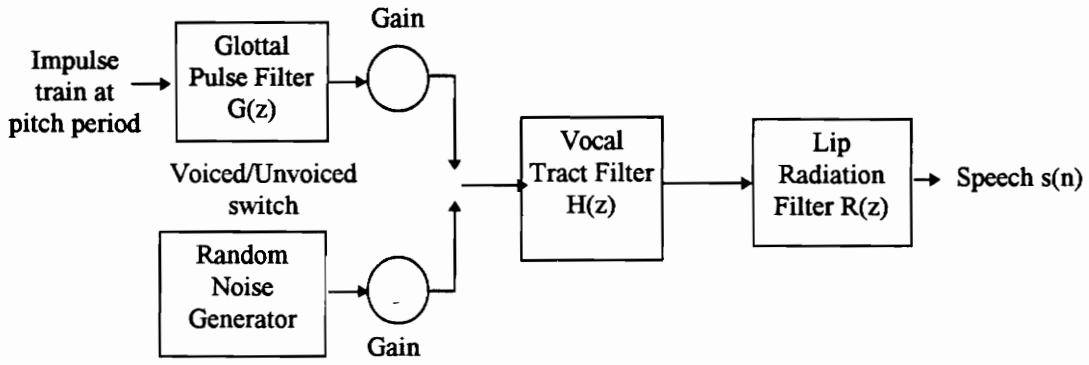

Figure 2. Discrete Time Speech Model

According to this model, the glottal pulse shaping, the human vocal tract, and the lips are modeled by the Glottal Pulse Filter G(z), the Vocal Tract Filter H(z), and the Lip Radiation Filter R(z), respectively. The voiced sounds are produced as a result of excitation by an impulse train at the pitch period and the unvoiced sounds by random noise excitation. The random noise excitation is used to model unvoiced speech because it is produced as a result of noise-like flow of air through a constriction in the vocal tract.

Thus we can model the Z-transform of speech as

$S(z) = E_1(z)G(z)H(z)R(z)$; for voiced speech with $E_1(z)$ representing an impulse train

$= E_2(z)H(z)R(z)$; for unvoiced speech with $E_2(z)$ representing noise

In general, to obtain the above system to model speech, we require an ARMA filter. However, the presence of very powerful computational techniques for deriving an all-pole

model from speech [3] provides a very good motivation for modeling speech with an all-pole transfer function. Moreover, we can obtain an all-pole filter that matches the magnitude spectrum of the speech [3]. For the purposes of coding, recognition, and synthesis, it has been found that modeling the magnitude spectrum of speech is sufficient.

2.1.3 Sensory Reception Approach

Since the sensory reception of speech is an integral part of human speech communication, its understanding could provide us with useful clues for developing a good speech recognizer. The ear is the sensory reception organ in the human body. It can be divided into three regions; the outer ear, the middle ear and the inner ear [1]. The *pinna* in the outer ear receives the speech. It then transmits the speech to the eardrum in the middle ear through the *meatus*. The eardrum is attached to a system that transduces the acoustic vibrations to mechanical vibrations. This transducer system consists of the *malleus*, the *incus* and the *stapes* that act like a hammer, an anvil, and the stirrup, respectively. The mechanical vibrations are set up in the inner ear (*cochlea*). The cochlea is a coil-like structure containing two channels called the *scalae*. The scalae are filled with a liquid called the *perilymph*. The movement of the stapes sets up a traveling wave in the perilymph. This then causes the basilar membrane to vibrate. The basilar membrane is about 35 mm long and tapers along its length. Due to this taper, the basilar membrane exhibits a resonance property that varies along its length. We can think of the basilar membrane as a broad band-pass filter with the successive points on the membrane having

an approximately constant Q characteristic. The width of the filter is inversely proportional to its resonance frequency. The basilar membrane is attached to the organ of *corti*. This contains the hair cells which transduce the mechanical vibrations to neural impulses. The ear thus performs acoustic to neural transduction and broad-band frequency analysis. Based on the functions of the ear in speech reception, we can arrive at the auditory representations of speech [1, 4]. However, the method by which the human speech reception system extracts parameters and classifies patterns is quite complex and difficult to duplicate. Hence, the sensory reception approach has not had a great impact on the development or improvement of speech recognition systems.

2.1.4 Speech Perception Approach

It has been experimentally found that certain features in speech, such as the voice onset times, formant transitions, etc., are important to the perception of speech by human beings. The speech perception approach suggests that a recognition system could duplicate the human perception system in extracting the above features. To achieve this, we have to make some sort of a perceptual categorization of the contents of speech. Experiments with perception of speech have suggested that the speech signal can be broken down into a finite number of discrete message elements [5]. It has been found that human beings are very sensitive to differences in the frequency or intensity of the different sounds provided to them. However, a listener has been found to have difficulties in identifying a single tone in isolation. Hence we can conclude that human beings exhibit

different information capacity for differential and absolute discrimination. Experiments with speech perception [5] indicate certain cues to identify certain features of speech such as consonant voicing, vowel identity, etc. Some of these cues are the voice onset time, formant transitions and single equivalent formants. The ability of a listener to identify a sound is affected by the time he or she is allowed to "learn" the sound and other linguistic constraints. The linguistic constraints help in reducing the set of sounds, from which the choice has to be made, for recognition. Although the human perception system has not been understood and modeled completely, one view of modeling considers the problem to be exactly the same as that of developing an automatic speech recognition system. A block diagram of a model based on the above view is shown in Figure 3 [Courtesy J. L. Flanagan].

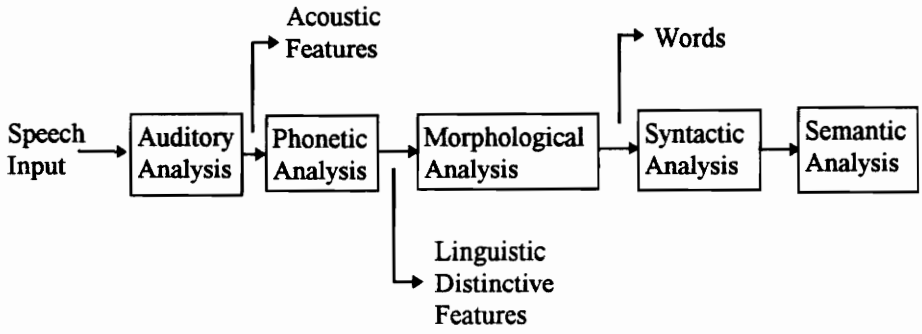

Figure 3. Model of Speech Perception

Each block of this model attempts to extract some relevant information from the speech, thereby reducing the dimensionality of the signal.

Having looked at the four philosophies of speech recognition, we now briefly describe the elements of a typical speech recognition system.

2.2 Elements of a Speech Recognition System

Speech recognition is performed as a consequence of extracting information from a speech signal. In a typical speech recognition system, there are various stages of extraction of information before a decision about the recognized speech is made. Figure 4 shows the basic block diagram of a typical speech recognition system.

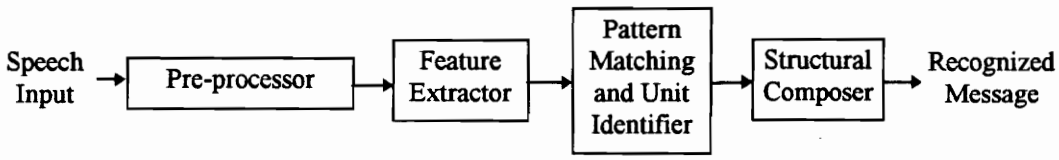

Figure 4. A typical Speech Recognition System

2.2.1. Pre-processor

The pre-processor consists of a waveform modifier that converts the analog speech input into a discrete-time or digital waveform. The pre-processor might clip the high amplitude portions of the signal, distort the duration of the signal (for example, to remove unwanted silence portions), or perform signal processing, such as filtering or de-emphasis of noise. Although the main function of the pre-processor is to make the signal suitable for the next block, we can also view it as a stage of redundancy removal.

2.2.2 Feature Extraction

This block extracts certain specific aspects of the speech wave that facilitate recognition. This extraction might result in a parametric representation of the speech wave. The parameters that are popular in speech recognition systems are the energies in frequency bands, the linear prediction coefficients (LPC), the cepstral coefficients, etc. To obtain some of these features, the feature extractor splits the speech segment into various frames by windowing. The features may also include information such as the fundamental frequency of the voice, or the number of zero crossings, or the voicing characteristic of the frame of speech. The feature extraction block also achieves data compression, resulting in reduced memory requirements. The feature extractor is sometimes called the parameter extractor.

2.2.3. Pattern Matching and Unit Identification

The pattern matching block attempts to identify certain linguistic units present in the speech to be recognized. Typically, the pattern matching block has some models of the speech units it is designed to identify. These speech units could be single words or smaller units like phonemes or syllables. These models could be simple templates of features of the speech units in the vocabulary or stochastic models such as the Hidden Markov Models (HMM). These models present in the pattern matching block are constructed as a result of a "training" procedure so that they represent the various patterns associated with the vocabulary in context. When the features are presented, the pattern matching system determines the extent to which each of the models represents the features. For example, in

the case of template matching, the test features are compared with the stored reference features using techniques like DTW. If HMM is used, the likelihood of generating the given features, by each of the models, is computed. The unit identifier chooses the model that produces the best match. Before making a decision about the recognized output, the unit identifier could use some linguistic constraints or analyze for some of the perceptual aspects of speech.

2.2.4. Structural Composer

Finally, the structural composer assembles the identified speech units into larger units that correspond to a complete message. The structural composer combines the smaller speech units with the help of a set of rules. These rules could be a set of syntactic, semantic, and/or pragmatic constraints. A structural composer could either combine words to form phrases and sentences, or work with words as the subunits. Finally, the message in the speech waveform is output.

We have provided a brief overview of the various approaches to speech recognition and the basic building blocks of a speech recognition system. Next we describe in detail the various elements of the connected-digit recognition system developed in this thesis towards the specific application of hands-free dialing of a telephone. We also present the procedures, and other design issues, involved in the training and recognition parts of the system. The following chapter provides a description of the feature extraction system. In many speech recognition systems, the pre-processor is also assumed to be a part of the

feature extractor. We make this assumption and discuss the details regarding pre-processing and feature extraction in the following chapter.

3. Feature Extraction

This chapter describes the feature extraction used in the speech recognition system developed in this thesis. As mentioned in the previous chapter, the feature extractor extracts certain specific aspects of the speech wave that represent information in the speech wave. Prior to feature extraction, the analog speech signal is converted to a digital waveform. The sampling frequency used is 11025 Hz. This sampling frequency is sufficient (greater than the Nyquist frequency) for the bandwidth of speech signals. The features extracted are the cepstral coefficients and the delta cepstral (the time derivative of the sequence of cepstral coefficients) coefficients. The reasons for choosing the cepstral and delta cepstral coefficients are discussed in Section 3.2. Figure 5 shows the block diagram of the feature extractor.

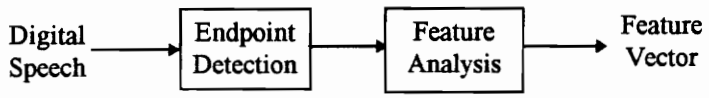

Figure 5. Feature Extractor

The basic steps involved in feature extraction are endpoint detection and feature analysis. We now go into more detail for each of these steps.

3.1 Endpoint Detection

The endpoint detection unit detects the boundaries of the spoken word. It is important to detect the boundary of the spoken word so that we can cut the unwanted portions out of the speech signal we have recorded. This removes the redundancies (such as silence portions or background noise) so that they do not affect the recognition performance of the entire system. Another advantage of endpoint detection is that it reduces the amount of subsequent processing. A simple algorithm is used to achieve end-point detection [6]. The algorithm is based on two measures: short-time energy and zero-crossing rate. The recorded string of digits is first split into m frames of 10 msec each. We then compute the energy in each frame by

$$E(m) = \sum_{k=1}^{n} \left[s_m(k) \right]^2 , \quad 1 \le m \le N \tag{3.1}$$

where n is the number of samples in a frame and N is the number of frames. We keep track of the frame where the energy exceeds a particular threshold (four times the minimum frame energy for this word [6]) and the frame where it falls below the same threshold and remains below it for more than 150 ms [6]. For the words we consider, a gap of more than 150 ms does not occur within the word and hence 150 ms is a reasonably good estimate of the duration of the silent portions at the end of the word. For each frame, we also compute the number of zero-crossings of the speech signal. As with the frame energies,

we obtain an estimate of the boundaries of the word from the number of zero-crossings. At this point we have two starting points and two final points. As an estimate of the endpoints of the words, we choose the starting point and the final point which yield the shortest duration for the word. The use of both criteria (frame energy and number of zero-crossings) improves the accuracy of detection [6]. Figure 6 shows the waveforms before and after the endpoint detection for the word "two" using both of the criteria mentioned above.

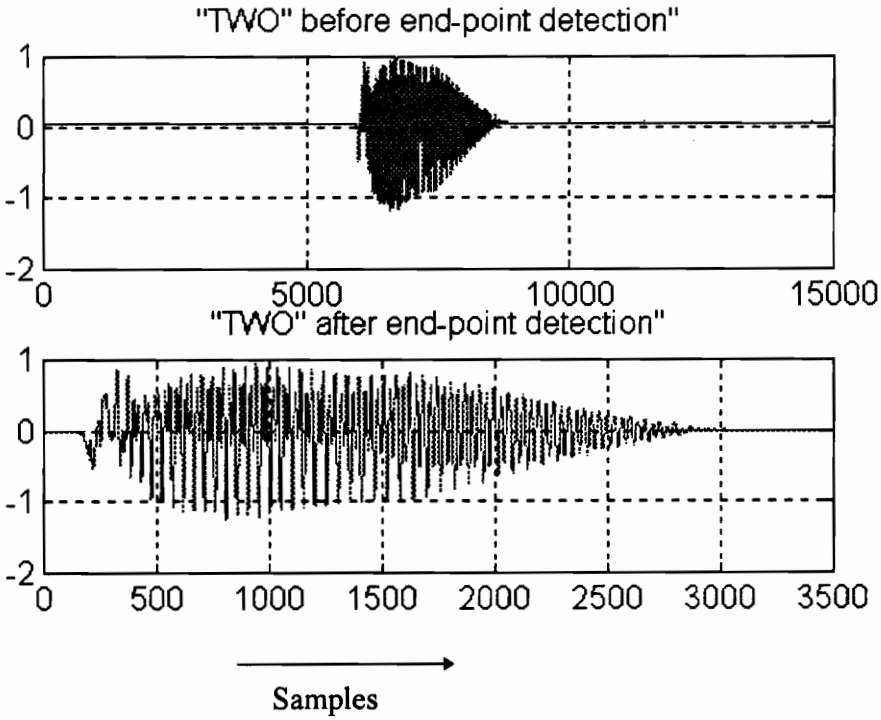

Figure 6. Plot of the word "TWO" before and after end-point detection.

3.2 Feature Analysis

This is the next step of extracting the relevant (for understanding) information in the speech waveform. As discussed in the previous chapter, one of the approaches to speech recognition is based on modeling the speech waveform from the speech production point of view. The all-pole model or the autoregressive (AR) model can be shown to be an appropriate model for speech recognition purposes [3]. The AR modeling of speech results in a set of parameters representing the magnitude spectral dynamics in the speech waveform. Figure 7 shows the block diagram of the feature analysis unit.

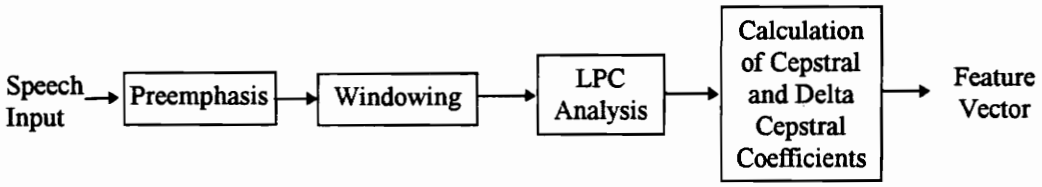

Figure 7. Feature Analysis Unit

First, we perform preemphasis of the speech by passing it through a high pass filter of the form $1-az^{-1}$, where $a = 0.95$. Most of the information in speech, for recognition purposes, lies in the vocal tract characteristics. Since the speech signal contains the glottal pulse characteristics too, we require preemphasis to cancel one of the poles of the glottal pulse so that AR modeling of the speech can model the vocal tract characteristics better. Another way to look at preemphasis is that it emphasizes the higher frequency portions of

the speech spectrum so that the higher frequency formants become more significant than before.

Parameterization of signals frequently calls for the signal to be stationary. However, speech is a non-stationary signal. Hence, before we parameterize, we split the speech signal into frames wherein the speech is considered to be quasi-stationary. We use windowing to split the speech into frames. Associated with windowing, we come across two issues: the length of the window and the type of window [3]. A longer window tends to produce a better spectral picture of the signal while inside the stationary region, whereas a shorter window provides better resolution in time. However, improvements in spectral and time resolution are in conflict with each other. Past research suggests that a window length of 45 msec is appropriate for speech signals [24]. For a given window length, we again have two competing factors in the choice of the window. We need to avoid any distortion in the selected points. Also, a window with abrupt transitions at the boundary has significantly high sidelobes in its spectrum and, when convolved with the signal in the frequency domain, brings a lot of undesirable spectral energy into the resulting spectrum. Hence, we need to choose a window that has smoother discontinuities at the boundaries. As found appropriate by past research [7], we use a Hamming window with 66% overlap.

After windowing, we obtain the tenth order AR model for the speech frames by using the Levinson-Durbin Algorithm for Linear Predictive Coding (LPC) of speech [8]. It has been found that the model order of ten is appropriate for speech [3]. The LPC analysis

obtains the AR model of speech in the process of determining a filter that predicts a future sample of speech from its past samples. In this process of prediction, it achieves data compression or removal of redundancies or extraction of the relevant information.

3.2.1 Cepstral Features

The set of cepstral features or coefficients is the complex cepstrum of the LP model of speech. It has been found that the use of a cepstral technique instead of the LPC coefficients improved speech recognition performance [9, 10]. This is because the cepstral coefficients derived from the LPC coefficients can be manipulated (for example, weighted) so that the feature vector is less affected by the glottal dynamics than the LPC coefficients [3]. Another motivation for using the cepstral parameters is that we can use the Euclidean metric between two cepstral parameter sets as a reasonably good measure of the spectral similarity of the corresponding models [3].

We use the following recursions [10] to compute the cepstral coefficients $c(i)$ from the LPC coefficients $a(i)$.

$$c(1) = -a(1), \tag{3.2}$$

$$c(i) = -a(i) - \sum_{k=1}^{i-1}\left(1 - \frac{k}{i}\right)a(k)c(i-k) \ , \ 1 < i \le p \tag{3.3}$$

The variable p is the order of the AR model. The number of cepstral coefficients has to be greater than the number of LPC coefficients. We have obtained 12 cepstral coefficients.

The cepstral coefficients are then weighted by a function given by,

$$w_c(m) = \left[1 + \frac{Q}{2}\sin\left(\frac{\pi m}{Q}\right)\right], \quad 1 \le m \le Q \tag{3.4}$$

where Q is equal to the number of cepstral coefficients used in (3.3). It has been found that weighting reduces the effect of the glottal pulse characteristics on speech modeling and thus improves the performance of speech recognition [25].

3.2.2. Delta Cepstral Features

In order to include information about the spectral changes that have occurred since the previous frame, we compute another set of features called the delta cepstral features. The delta cepstral coefficients are the derivative of the cepstral coefficients. We compute the delta cepstral coefficients by considering a window of several frames and calculating the difference in the cepstral coefficients from one frame to another within the window.

The delta cepstral coefficients are computed using

$$\Delta c_l(m) = \left[\sum_{k=-L}^{L} k c_{l-k}(m)\right] G, \quad 1 \le m < Q \tag{3.5}$$

where $(2L+1)$ is the length of the window. We have chosen $L=2$. G is a gain chosen such that the variances of the delta cepstral coefficients and the cepstral coefficients are approximately equal. We include the gain to ensure that one set of parameters does not dominate over the other when using the Euclidean metric on the final feature vector formed by appending the set of delta cepstral coefficients to the set of cepstral coefficients. This feature vector is such that it has fewer redundancies than the speech waveform we started with.

We next present the Hidden Markov Model (HMM) and other issues, such as the training and the recognition algorithm used. We have used the HMM as the model for each word for the purposes of pattern matching and unit identification.

4. The Hidden Markov Model

As mentioned earlier, one of the elements of a speech recognition system is the pattern matching block which identifies the presence of certain linguistic units present in the speech. Fundamental to speech pattern matching is representing the patterns in speech as models. Knowledge of the structure of speech together with many reference tokens (multiple utterances of the same speech) of the speech are used to obtain appropriate models. We can then compare unknown patterns of speech against these models to determine how well the available models match them. The models have to be generated from the reference data such that they accommodate the inherent variability of speech and are able to recognize speech patterns that have not been observed previously. The simplest model is a set of templates formed by storing the parameterized form (cepstral coefficients) of an utterance of each word in the vocabulary. One or more templates can be used to model each word in the vocabulary. Unknown utterances can be matched against these templates through techniques such as dynamic time warping (DTW) [11]. DTW is a good scheme to compare tokens (utterances of the same speech) of different durations. However, DTW uses the speech data in a deterministic way. Although it has been used in many practical systems, DTW requires a large number of templates to account for the large variability associated with speech. This increases the computational cost of searching to inconvenient proportions. If templates of subword units are used, the number of templates used can be decreased. However, in the presence of coarticulatory

effects, as encountered in connected-speech recognition, standardization of the subword units becomes difficult. Intuitively, a stochastic model can accommodate acoustic variability in a better way than simple template based approaches. Research in stochastic modeling of speech patterns has proceeded in two directions: the Hidden Markov Models and the Artificial Neural Networks (ANN). Hidden Markov Models were first studied by statisticians and were applied to characterize stochastic processes for which incomplete observations were available. In the mid 70s, researchers looked into applying the HMM to model speech for recognition purposes [12, 13]. Although the HMM was adapted to the speech recognition problem rather slowly (3 decades), it has had a great impact on the speech recognition schemes that have been incorporated into practical systems. In contrast, the application of ANN techniques has had lesser impact on speech recognition because research in this direction is very young. The fundamental aim of ANN techniques is to explore computing architectures that resemble the massively parallel computing found in biological neural systems. The following section introduces the HMM and provides the various notations used in this thesis.

4.1. Introduction

As mentioned above the HMM is applied to characterize random processes for which incomplete observations are available. The HMM is a means to model the problem as an observable stochastic process produced as a result of an underlying unobservable (hidden) stochastic process. In this sense, the HMM is a doubly embedded stochastic process. The

HMM is used to model speech utterances as a stochastic finite state automaton which generates the speech utterances as a result of generating a "hidden" state sequence which can be observed through an observation sequence. Typically, in small vocabulary systems the HMM models a whole word and in large vocabulary systems, the HMM models a subword unit. Since the connected digit recognition system developed in this thesis is a small vocabulary system, we have used the HMMs to model whole words.

We recall that HMM is used in the pattern matching unit and the speech utterance at this point is a string of feature vectors (in our case, the feature vector is formed by the cepstral features concatenated with the delta cepstral features). In this chapter, we will treat the string of feature vectors as an observation sequence. We denote an observation sequence as follows:

$$y(1), y(2), y(3), \ldots\ldots, y(t), \ldots\ldots, y(T).$$

where $y(.)$ is a feature vector, t is the frame index, and T is the total number of observations in the sequence.

An HMM is always associated with an observation sequence which it is more likely to generate than any other HMM. The likelihood that a given HMM generates a given observation sequence is a quantitative measure of how well the observation sequence "matches" the HMM. In the pattern matching block of the speech recognition system, we have the HMMs representing each word in the vocabulary. Given a word (or a string of feature vectors) to recognize, we compute the likelihood that each of the HMMs available would have generated the string. We find the recognized word as the one whose HMM

produces the maximum likelihood. Usually, the highest likelihood is significantly ($>10^{25}$) larger than the next highest likelihood. Since the highest likelihood "stands out" prominently, we can expect the correct model to yield the highest likelihood, even if the speech to be recognized were a little noisy. The HMM corresponding to each word in the vocabulary is formed by a process of "training," by which many utterances of a word are used to obtain the statistical makeup of the observations associated with that word.

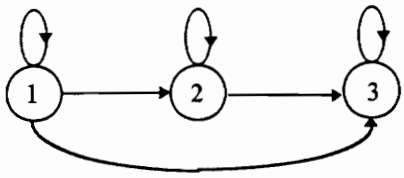

Figure 8. A typical 3-state HMM

Figure 8 shows an example of a 3-state HMM. The numbers indicate the states and the arrow heads indicate the allowable state transitions. We can imagine an HMM to be a finite state machine that generates the observation sequence by producing an observation from each state and then transiting from one state to the next until the final observation in the sequence is generated. These state transitions occur according to *state transition probabilities*. We denote the probability of making the transition from state j to state i by $a_{i,j}$. For an S-state HMM we have the *state transition matrix* as

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdot & \cdot & \cdot & a_{1,S-1} & a_{1,S} \\ \cdot & \cdot & & & & & \cdot \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & a_{i,j} & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ \cdot & & & & & \cdot & \cdot \\ a_{S,1} & a_{S,2} & \cdot & \cdot & \cdot & a_{S,S-1} & a_{S,S} \end{bmatrix}$$

We assume the state transition probabilities to be stationary in time, i.e., $a_{i,j}$ is not a function of time. We also assume that a transition must take place at any time. Therefore, any column of A should sum to unity.

As mentioned above, the state sequence is the hidden random process and we denote it by $\underline{x}$ with random variables $\underline{x}(t)$. Therefore, for an arbitrary t, we have

$$a_{i,j} \cong P\left(\underline{x}(t) = i | \underline{x}(t-1) = j\right) \tag{4.1}$$

We make the simplifying assumption that the state transition at time t is independent of the history of the state sequence prior to time t-1. Such a random sequence is called a first-order Markov process. The random variables $\underline{x}$(t) assume only integer values and hence the state sequence $\underline{x}$ is called a Markov chain.

We define the state probability vector at time t to be

$$\pi(t) \cong \begin{bmatrix} P(\underline{x}(t)=1) \\ P(\underline{x}(t)=2) \\ . \\ . \\ P(\underline{x}(t)=S) \end{bmatrix} \tag{4.2}$$

Therefore,

$$\pi(t) = A\pi(t-1) = A^{t-1}\pi(1) \tag{4.3}$$

The second stochastic process in the HMM is the observation sequence, which we denote by $\underline{y}$ with random variables $\underline{y}(t)$. An observation is generated at a particular time t, after entering a state *i*, according to the *observation probability density function* $f_{\underline{y}(t)|\underline{x}(t)}(\xi|i)$. $\underline{y}(t)$ and ξ are D-dimensional feature vectors, where D is the dimension of the feature vectors. To simplify, we assume that the random process $\underline{y}$ has independent and identically distributed random variables. This implies that $f_{\underline{y}|\underline{x}}(\xi|i)$ is independent of time. With this, we have defined the parameters that completely define a HMM. We define a HMM as

$$\mathcal{M} = \left\{S, \pi(1), A, \left\{f_{\underline{y}|\underline{x}}(\xi|i), 1 \le i \le S\right\}\right\} \tag{4.4}$$

We can realize two issues regarding the use of HMM for speech recognition. Firstly, we have to obtain the parameters of the HMM from a series of training observations. This is the problem of *training* the HMM to model a given word. Secondly, we have to compute the likelihood that a particular HMM produced the given speech observation sequence. This is the problem of *recognition*.

4.2 Discrete and Continuous Hidden Markov Models

Depending upon the form of the observation pdf's, we have two types of HMMs that can be used for speech recognition: *discrete* (*observation*) HMM and *continuous* (*observation*) HMM. The discrete observation HMM uses a discrete pdf for the observation pdf. This means that only a finite set of observations is allowed. After the feature extraction stage, the feature vectors have to be vector quantized to one of the permissible set of (say K) observation vectors. Since only K vectors are allowed, it is possible to assign to every observation vector a scalar and, consequently, the random process $\underline{y}$ becomes a scalar random variable with scalar random variables $\underline{y}(t)$ which can take only integer values in [1, K]. The observation pdf, being a discrete pdf, is completely specified by the set of weights on the impulses forming the pdf or, in other words, the probabilities of the K possible observations. The observation probability is given by

$$b(k|i) \cong P\left(\underline{y}(t) = k|\underline{x}(t) = i\right) \tag{4.5}$$

The codebook, $\{y_k, 1 \le k \le K\}$ is a characteristic of a discrete HMM and consequently we can define an observation probability matrix,

$$\boldsymbol{B} = \begin{bmatrix} b(1|1) & b(1|2) & . & . & . & b(1|S-1) & b(1|S) \\ . & & & & & & . \\ . & & & b(k|i) & & & . \\ . & & & & & & . \\ b(K|1) & b(K|2) & . & . & . & b(K|S-1) & b(K|S) \end{bmatrix} \tag{4.6}$$

The mathematical specification of a discrete HMM takes the form

$$\mathcal{M} = \{S, \pi(1), \boldsymbol{A}, \boldsymbol{B}, \{y_k, 1 \le k \le K\}\} \tag{4.7}$$

The continuous HMM represents the more general case in which the observations are continuous-valued feature vectors. The observation pdf is a multivariate continuous function. Although the observation pdf can be any continuous function (that can be a pdf), certain simplifying assumptions are usually made for the nature of these functions. For speech recognition purposes, it has been found that a finite mixture of (say M) Gaussian pdf's is a reasonably good approximation for the continuous observation pdf [14]. The continuous observation HMM takes the form,

$$\mathcal{M} = \left\{S, \pi(1), \boldsymbol{A}, \left\{f_{\underline{y}|\underline{x}}(\xi|i), 1 \le i \le S\right\}\right\} \tag{4.8}$$

The following section provides details of the continuous HMM used in this thesis with respect to issues such as training and recognition.

4.3 The Continuous Hidden Markov Model

In this thesis, the continuous HMM is used with the observation pdf approximated by a mixture (sum) of M multivariate Gaussian functions. The observation pdf is of the form

$$f_{\underline{y}|\underline{x}}(\xi|i) = \sum_{m=1}^{M} c_{im} \mathcal{N}(\xi, \mu_{im}, C_{im}) \tag{4.9}$$

where c_{im} is the coefficient for the mth component of the mixture for state i, $\mathcal{N}(.)$ is a multivariate Gaussian pdf with mean μ_{im} and covariance matrix C_{im}. The mixture coefficients are non-negative and satisfy the constraint

$$\sum_{m=1}^{M} c_{im} = 1, \qquad 1 \le i \le S \tag{4.10}$$

For simplicity of notation we define a likelihood function

$$b(\xi|i) \cong f_{\underline{y}|\underline{x}}(\xi|i) \tag{4.11}$$

Having provided the necessary introduction to HMM, we now present the details regarding the training of the HMM and recognition using the HMM. Of these two problems, the recognition problem is easier and the concepts therein serve as a convenient background to understanding the training problem. Hence, we deal with the recognition problem first.

4.3.1. Recognition

Given an observation sequence, the recognition problem deals with computing the likelihood that the models available (as a result of training) produce that observation sequence. We define some notation that will help in understanding and establishing the concepts of recognition.

We denote a partial sequence of observations in time as

$$y_{t_1}^{t_2} \cong \{y(t_1), y(t_1+1), y(t_1+2), \ldots, y(t_2)\} \tag{4.12}$$

The particular sequence of observations

$$y_1^t \cong \{y(1), y(2), \ldots, y(t)\} \tag{4.13}$$

is called the *forward partial sequence of observations at time t* and

$$y_{t+1}^{T} \cong \{y(t+1), y(t+2), \ldots, y(T)\} \tag{4.14}$$

is called the *backward partial sequence of observations at time t*. The complete sequence of observations is

$$y_1^T \cong \{y(1), y(2), \ldots, y(T)\} \tag{4.15}$$

and, for the sake of simplicity of notation, we define

$$y \cong y_1^T \tag{4.16}$$

We proceed to discuss the computation of the likelihood that a given model produces a given observation sequence.

4.3.1.1 Forward-Backward Method

The Forward-Backward (F-B) method to compute the likelihood is also called the "any path" method because it computes the likelihood that the observations could have been produced using any state sequence through the given model. Let us consider a particular state sequence of length T to be $\mathcal{I} = \{i_1, i_2, \ldots, i_T\}$. The likelihood of the observation sequence being produced from this state sequence is

$$\mathcal{L}(\mathbf{y}|\mathcal{S},\mathcal{M}) = b(\mathbf{y}(1)|i_1).b(\mathbf{y}(2)|i_2).....b(\mathbf{y}(T)|i_T) \tag{4.17}$$

Since we use the functional values of the pdf, the left hand side of (4.17) is a likelihood. The likelihood of the state sequence is

$$\mathcal{L}(\mathcal{S}|\mathcal{M}) = \pi(i_1)a(i_2,i_1)a(i_3,i_2).....a(i_T,i_{T-1}) \tag{4.18}$$

Therefore,

$$\begin{aligned}\mathcal{L}(\mathbf{y},\mathcal{S}|\mathcal{M}) = {} & b(\mathbf{y}(1)|i_1)b(\mathbf{y}(2)|i_2).....b(\mathbf{y}(T)|i_T) \\ & \times \pi(i_1)a(i_2,i_1)a(i_3,i_2).....a(i_T,i_{T-1})\end{aligned} \tag{4.19}$$

The sum of the above likelihood over all possible sequences provides the likelihood of the given model producing the observation sequence through any state sequence as

$$\mathcal{L}(\mathbf{y}|\mathcal{M}) = \sum_{all\ \mathcal{S}} \mathcal{L}(\mathbf{y},\mathcal{S}|\mathcal{M}) \tag{4.20}$$

Equation (4.20) gives a "brute force" method to compute $\mathcal{L}(\mathbf{y}|\mathcal{M})$. It has been found that this approach requires a prohibitively high amount of computation [3]. The forward-backward algorithm developed by Baum computes the above likelihood in an efficient recursive manner. The forward-backward algorithm uses a "forward-going" and a "backward-going" likelihood. We define these likelihoods as

$$\alpha\left(y_1^t, i\right) \cong \mathcal{L}\left(\underline{y}_1^t = y_1^t, \underline{x}(t) = i | \mathcal{m}\right) \tag{4.21}$$

$$\beta\left(y_{t+1}^T | i\right) \cong \mathcal{L}\left(\underline{y}_{t+1}^T = y_{t+1}^T | \underline{x}(t) = i, \mathcal{m}\right) \tag{4.22}$$

respectively. In equations (4.21) and (4.22), $\underline{y}_{t_1}^{t_2}$ denotes a partial sequence of random variables. Since the observation sequence could have reached state *i* at time *t* from any of the S possible states at time *t-1*, we have

$$\alpha\left(y_1^t, i\right) = \sum_{j=1}^{S} \alpha\left(y_1^{t-1}, j\right) a_{i,j} b\left(y(t) | i\right) \tag{4.23}$$

Clearly, (4.23) provides a recursive algorithm for computing the α's with the algorithm initiated by

$$\alpha\left(y_1^1, j\right) = \pi(j) b\left(y(1) | j\right) \qquad 1 \le j \le S \tag{4.24}$$

Similarly, the backward recursion can be defined as

$$\beta\left(y_{t+1}^T | i\right) = \sum_{j=1}^{S} \beta\left(y_{t+2}^T | j\right) a_{j,i} b\left(y(t+1) | j\right) \tag{4.25}$$

with the recursion initiated by the fictitious partial sequence y_{T+1}^{T} used in

$$\beta\left(y_{T+1}^{T}|i\right) \cong \begin{cases} 1, & \textit{if } i \textit{ is a legal final state} \\ 0, & \textit{otherwise} \end{cases} \tag{4.26}$$

where a legal final state is one at which a path through the model may end.

With the definitions of α and β we have, for a particular state i,

$$\mathcal{L}\left(y, \underline{x}(t) = i|\mathcal{M}\right) = \alpha\left(y_1^t, i\right)\beta\left(y_{t+1}^{T}|i\right) \tag{4.27}$$

Therefore,

$$\mathcal{L}\left(y|\mathcal{M}\right) = \sum_{i=1}^{S} \alpha\left(y_1^t, i\right)\beta\left(y_{t+1}^{T}|i\right) \tag{4.28}$$

The likelihood in (4.28) can be computed at any time slot, t. If we operate at the final time, $t = T$, we obtain

$$\mathcal{L}\left(y|\mathcal{M}\right) = \sum_{i=1}^{S} \alpha\left(y_1^T, i\right)\beta\left(y_{T+1}^{T}|i\right) \tag{4.29}$$

Using (4.26) in (4.29) we obtain

$$\mathcal{L}(\mathbf{y}|\mathcal{M}) = \sum_{all\ legal\ final\ i} \alpha\left(y_1^T, i\right) \tag{4.30}$$

It can be shown that the forward-backward algorithm considerably reduces the number of computations to obtain the desired likelihood as compared to the brute force method of (4.20) [3]. The following section describes the Viterbi approach to estimating the likelihood that a particular model generated a given observation sequence.

4.3.1.2 Viterbi Method

Unlike the "any path" method described above, the Viterbi approach to recognition is based on computing the likelihood that a particular HMM generated the given observation sequence through the *best* possible state sequence. The best state sequence is the one of all possible state sequences that produces the maximum likelihood. Hence the Viterbi approach seeks $\mathcal{L}(\mathbf{y},\mathcal{S}^*|\mathcal{M})$ such that,

$$\mathcal{S}^* \cong \underset{\mathcal{S}}{\text{argmax}}\, \mathcal{L}(\mathbf{y}, \mathcal{S}|\mathcal{M}) \tag{4.31}$$

where $\mathcal{I}$ is any possible state sequence. Although this method involves the added complexity of determining the best state sequence, it reduces the number of computations required to estimate the desired likelihood.

The problem of determining the best possible state sequence can be solved efficiently within the framework of dynamic programming (DP). Towards this end, consider a grid as shown in Figure 9, where the observations (from a sequence of observations) are laid out on the abscissa, and the states along the ordinate. Each point in the grid indexed by the time, state index pair (t, i). The best possible state sequence is determined as a consequence of grid searches for the path that results in an optimal objective cost function subject to some constraints.

The observations and the states are laid out along the abscissa and the ordinate respectively. The constraints are

1. Every path must advance in time by one, and only one, step for each path segment.
2. The final grid points on any path must be of the form (T,i_f) where i_f is a legal final state in the model.

We can assign two kinds of costs as we traverse along any path in the grid. They are

1. Type N cost to any node: $d_N(t,i) = b\big(y(t)|i\big)$ (4.32)
2. Type T cost to any transition: $d_T\big[(t,i)|(t-1,j)\big] \cong a_{i,j} \quad ;t>1$ (4.33)

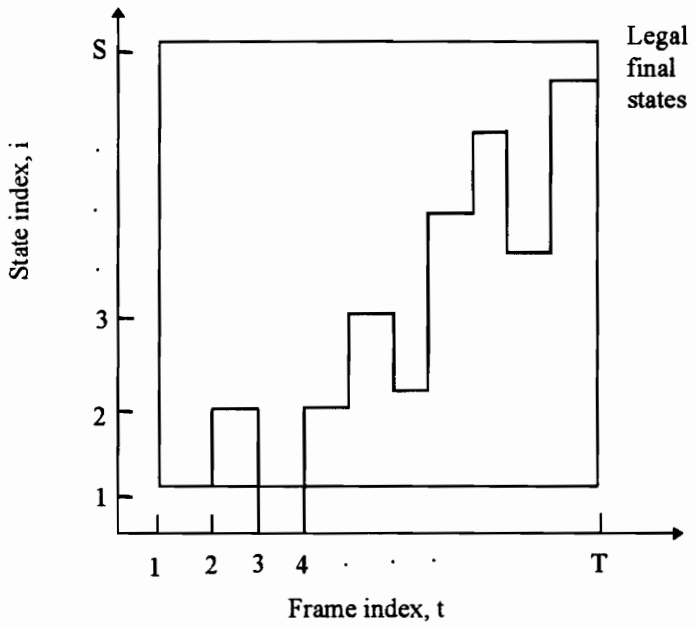

Figure 9. Grid for the HMM viewed within DP framework

The accumulated cost associated with any transition from $(t\text{-}1,j)$ to (t,i) is

$$\begin{aligned} d\left[(t,i)|(t-1,j)\right] &= d_T\left[(t,i)|(t-1,j)\right] d_N(t,i) \\ &= a_{i,j} b\left(y(t)|i\right) \end{aligned} \quad ; t > 1 \tag{4.34}$$

For simplicity of notation we assume that all the paths originate from a fictitious costless node (0,0). All paths make a transition from (0,0) with a transition cost $\pi(i)$ and arrive at $(1,i)$. Therefore for $t = 1$, the accumulated cost is

$$d\left[(1,i)|(0,0)\right] = d_T\left[(1,i)|(0,0)\right] d_N(1,i) \\ = \pi(i) b\left(y(t)|i\right) \tag{4.35}$$

Given an observation sequence between $t = 1$ and $t = T$ and a particular HMM, the joint likelihood of the observation sequence and a particular state sequence $\mathcal{I}$, of the same length, is the product of the accumulated costs of all the nodes in the path. The total cost of a path is of the form

$$D = \prod_{t=1}^{T} d\left[(t,i_t)|(t-1,i_{t-1})\right] \\ = \prod_{t=1}^{T} a_{i_t,i_{t-1}} b\left(y(t)|i_t\right) \\ = \mathcal{L}\left(y,\mathcal{I}|\mathcal{M}\right) \tag{4.36}$$

where

$$a_{i_1,i_0} = a_{i_1,0} \cong \pi(i) \tag{4.37}$$

The best state sequence $\mathcal{I}^*$ is the one that produces the maximum cost D^*.

The cost of the paths in (4.34)-(4.36) takes the form of a product of likelihoods, which in many instances becomes a very small number, which can lead to numerical problems. By using negative logarithms, we can convert these products to a process of

summation. In addition to reducing the numerical problems, a summation is computationally less expensive than a multiplication. Taking the negative logarithm converts (4.34)-(4.36) respectively to

$$\begin{aligned} d\left[(t,i)|(t-1,j)\right] &= d_T\left[(t,i)|(t-1,j)\right] + d_N(t,i) \\ &= [-\log(a_{i,j})] + [-\log(b(y(t)|i))] \end{aligned} \quad ; t > 1 \tag{4.38}$$

$$\begin{aligned} d\left[(1,i)|(0,0)\right] &= d_T\left[(1,i)|(0,0)\right] + d_N(1,i) \\ &= [-\log(\pi(i))] + [-\log(b(y(t)|i)] \end{aligned} \quad ; t = 1 \tag{4.39}$$

$$D = \sum_{t=1}^{T} d\left[(t,i_t)|(t-1,i_{t-1})\right] \tag{4.40}$$

We note that by taking the negative logarithm the cost function becomes a negative likelihood and the objective of the DP converts to finding the path that produces the lowest cost function. We now describe the use of DP to determine the path resulting in the lowest objective cost function (negative likelihood).

Let

$$D_{\min}(t,i_t) \cong \text{ distance from (0,0) to } (t,i_t) \text{ over the best path.} \tag{4.41}$$

Using Bellman's Optimality principle [3], we have

$$D_{\min}(t,i_t) = \min_{(t-1,i_{t-1})} \left\{ D_{\min}(t-1,i_{t-1}) + d\left[(t,i_t)\|(t-1,i_{t-1})\right] \right\}; \quad t > 1 \tag{4.42}$$

Equation (4.42) uses the fact that the only legal predecessor nodes to (t,i_t) are of the form $(t\text{-}1,i_{t\text{-}1})$. Moreover, all legal predecessors to (t,i_t) come from the same time slot, $(t\text{-}1)$. Therefore, (4.42) is essentially a minimization over only the previous states and hence

$$D_{\min}(t,i_t) = \min_{(i_{t-1})} \left\{ D_{\min}(t-1,i_{t-1}) + d\left[(t,i_t)\|(t-1,i_{t-1})\right] \right\}; \quad t > 1 \tag{4.43}$$

Using (4.38) in (4.43)

$$D_{\min}(t,i_t) = \min_{(i_{t-1})} \left\{ D_{\min}(t-1,i_{t-1}) + \left[-\log\left(a_{i_t,i_{t-1}}\right)\right] + \left[-\log\left(y(t)|i_t\right)\right] \right\} \tag{4.44}$$

with

$$D_{\min}(0,0) = 0 \quad ; \quad \text{for } t = 1 \tag{4.45}$$

The search ends resulting in the quantity

$$
\begin{aligned}
D^* &= \min_{i_T^*}\left\{D_{\min}\left(T, i_T^*\right)\right\} \\
&= -\log(\mathcal{L}(y, \vartheta^* | \mathcal{M}))
\end{aligned}
\tag{4.46}
$$

where

$$
i_T^* = \underset{\text{legal } i_T}{\operatorname{argmin}}\left\{D_{\min}\left(T, i_T\right)\right\} \tag{4.47}
$$

Equation (4.46) provides the negative logarithm of the joint likelihood of occurrence of the observation sequence and the best state sequence. The negative logarithm of the likelihood can be used for comparisons between various models just as well as the likelihood itself. As we will discuss later, in Section 4.3.2, describing training, it is sometimes required to obtain the state sequence that results in the least cost function. We can obtain this state sequence by backtracking from the best last state in the DP grid. Let

$$
\begin{aligned}
\psi\left(t, i_t\right) &= \text{the best last state on the optimal partial path ending at } \left(t, i_t\right) \\
&= \underset{\left(i_{t-1}\right)}{\operatorname{argmin}}\left\{D_{\min}\left(t-1, i_{t-1}\right) + \left[-\log\left(a_{i_t, i_{t-1}}\right)\right] + \left[-\log\left(y(t) | i_t\right)\right]\right\} \\
&= \underset{\left(i_{t-1}\right)}{\operatorname{argmin}}\left\{D_{\min}\left(t-1, i_{t-1}\right) + \left[-\log\left(a_{i_t, i_{t-1}}\right)\right]\right\}
\end{aligned}
\tag{4.48}
$$

The best state sequence can be obtained by backtracking from $\psi\left(T, i_T^*\right)$.

This method of determining the likelihood using DP is known as the Viterbi Algorithm, since it was first suggested by A. J. Viterbi. It can be shown that the Viterbi method is computationally less expensive than the forward-backward approach [15]. It should be noted that the best state sequence as obtained through the Viterbi method is only an estimate of the underlying state sequence because there is no way of accurately obtaining the hidden state sequence.

Having provided the theory behind the recognition process, we now present the theory behind the training process before describing the implementation of these two processes.

4.3.2. Training

While the recognition problem dealt with computing the likelihood that a particular model produces a given observation sequence, there is the need to come up with a model which will represent its designated word. In other words, we have to obtain models that will have a greater likelihood of generating the modeled word than any other model. This is the process of *training*. To train an HMM for a particular word, we need many strings of feature vectors extracted from training utterances of the word. Let these training feature strings be of the form $y = y_1^T = \{y(1), \ldots, y(T)\}$. The problem of training deals with using these training utterances of a word to obtain the state transition matrix A, and the

observation pdfs in the HMM for that word. Although there is no analytical way to obtain the HMM parameters, an iterative procedure exists to estimate them. The forward-backward algorithm (also known as Baum-Welch Reestimation) developed by Baum et al, which we introduced in Section 4.3.1.1, can be extended to provide a method that iteratively converges to a model such that the likelihood $\mathcal{L}(\boldsymbol{y}|\mathcal{M})$ is locally maximized [16]. Similarly, an extension of the Viterbi approach to recognition also provides an efficient method for estimating the parameters of an HMM. In both methods, the training procedure starts from an arbitrary initial model and recursively uses the training utterances of a word to "reestimate" or update the model until the model converges to a local optimum of the likelihood that it produces the observation sequence.

4.3.2.1. Baum-Welch Reestimation

Without going into the details of its theoretical development, we now describe the Baum-Welch Reestimation algorithm [16]. Towards this end, we introduce some notation that is needed to understand the procedure. We begin by using a single observation sequence for the reestimation. Let $\underline{u}$ be a random process, with random variables $\underline{u}(t)$, that models the state transitions at time t with

$$u_{j|i} \cong \text{label for transition from state } i \text{ to state } j \tag{4.49}$$

$$u_{j|\cdot} \cong \text{set of transitions entering state } j \tag{4.50}$$

$$u_{\cdot|i} \cong \text{set of transitions exiting state } i \tag{4.51}$$

Let $\underline{y}_j, 1 \le j \le S,$ be a random process with random variables $\underline{y}_j(t)$ modeling the observations emitted from state j at time t. We start with an arbitrary model, $\mathcal{M}$. For a given training observation sequence, $y = y_1^T$, we compute the following likelihoods:

$$\begin{aligned}
\xi(i,j;t) &\cong \mathcal{L}\left(\underline{u}(t) = u_{j|i} \,|\, y, \mathcal{M}\right) \\
&= \frac{\mathcal{L}\left(\underline{u}(t) = u_{j|i} \,|\, y, \mathcal{M}\right)}{\mathcal{L}\left(y|\mathcal{M}\right)} \\
&= \begin{cases} \dfrac{\alpha\left(y_1^t, i\right) a_{j,i} b\left(y(t+1)|j\right) \beta\left(y_{t+2}^T | j\right)}{\mathcal{L}\left(y|\mathcal{M}\right)}, & t = 1,\ldots,T-1 \\ 0, & \text{other } t \end{cases}
\end{aligned} \tag{4.52}$$

where a, α, and β are defined in (4.1), (4.21), and (4.22) respectively and b is defined in (4.11) where the continuous-valued pdf takes the form in (4.9).

$$\begin{aligned}
\gamma(i;t) &\cong \mathcal{L}\left(\underline{u}(t) \in u_{\cdot|i} \,|\, y, \mathcal{M}\right) \\
&= \sum_{j=1}^{S} \xi(i,j;t) \\
&= \begin{cases} \dfrac{\alpha\left(y_1^t, i\right) \beta\left(y_{t+1}^T | i\right)}{\mathcal{L}\left(y|\mathcal{M}\right)}, & t = 1,\ldots,T-1 \\ 0, & \text{other } t \end{cases}
\end{aligned} \tag{4.53}$$

$$\nu(i;t,l) \cong \mathcal{L}\left(\underline{x}(t) = i | \mathcal{M},\ [y(t) \text{ produced due to mixture density } l]\right)$$

$$= \frac{\alpha\left(y_1^t, i\right)\beta\left(y_{t+1}^T | i\right)}{\sum_{j=1}^{S} \alpha\left(y_1^t, j\right)\beta\left(y_{t+1}^T | j\right)} \times \frac{c_{il}\eta\left(\xi; \mu_{il}, C_{il}\right)}{\sum_{m=1}^{M} c_{im}\eta\left(\xi; \mu_{im}, C_{im}\right)} \tag{4.54}$$

We derive the following from (4.52)-(4.54):

$$\xi(i,j;\cdot) \cong \mathcal{L}\left(\underline{u}(\cdot) = u_{j|i} | y, \mathcal{M}\right) = \sum_{t=1}^{T-1} \xi(i,j;t) \tag{4.55}$$

$$\gamma(i;\cdot) \cong \mathcal{L}\left(\underline{u}(\cdot) \in u_{\cdot|i} | y, \mathcal{M}\right) = \sum_{t=1}^{T-1} \gamma(i;t) \tag{4.56}$$

$$\nu(i;\cdot,l) \cong \mathcal{L}\left(\underline{x}(\cdot) = i | \mathcal{M}, [y(\cdot) \text{ produced due to mixture density } l]\right)$$
$$= \sum_{t=1}^{T} \nu(i;t,l) \tag{4.57}$$

With these definitions, the HMM parameters for the new model, $\overline{\mathcal{M}}$ are reestimated as

$$\overline{a}_{j,i} = \frac{\text{Expected number of transitions from state } i \text{ to state } j}{\text{Expected number of transitions from state } i}$$
$$= \frac{\xi(i,j;\cdot)}{\gamma(i,\cdot)} \tag{4.58}$$

$$\bar{c}_{il} = \frac{\begin{pmatrix}\text{Expected number of times the path is in}\\ \text{state } i \text{ using } l\text{th mixture component}\end{pmatrix}}{\text{Expected number of times the path is in state } i}$$

$$= \frac{\nu(i;\cdot,l)}{\sum_{m=1}^{M} \nu(i;\cdot,m)} \tag{4.59}$$

$$\bar{\mu}_{il} = \begin{pmatrix}\text{weighted time average of observation vectors}\\ \text{weighted according to the likelihood of their}\\ \text{having been produced by mixture density } l \text{ in}\\ \text{state } i.\end{pmatrix}$$

$$= \frac{\sum_{t=1}^{T} \nu(i;\cdot,l) y(t)}{\nu(i;\cdot,l)} \tag{4.60}$$

$$\bar{C}_{il} = \frac{\sum_{t=1}^{T} \nu(i;\cdot,l)[y(t)-\mu_{il}][y(t)-\mu_{il}]^T}{\nu(i;\cdot,l)} \tag{4.61}$$

Usually, a particular state is designated as the initial state for the model and consequently there is no need to estimate the initial state probabilities. It can be shown that the new model $\bar{\mathcal{M}}$ is such that $\mathcal{L}(\boldsymbol{y}|\bar{\mathcal{M}}) \geq \mathcal{L}(\boldsymbol{y}|\mathcal{M})$ [17]. The reestimated models can be used iteratively in equations (4.58)-(4.61) until a local maximum is reached. Since $\mathcal{L}(\boldsymbol{y}|\mathcal{M})$ is a highly nonlinear function of many parameters, it may have many local maxima.

Consequently, for the training procedure to result in a good model, it is necessary to perform the iterations from different initial conditions and choose the model that corresponds to the largest of all the local maxima for $\mathcal{L}(\boldsymbol{y}|\mathcal{M})$.

We recall that, in the above reestimation procedure, we have used only one training observation sequence. However, it is insufficient to model just one observation sequence. It is essential to train a HMM with many utterances of the same word so that the statistical variations present across utterances are more completely modeled. We denote each of the multiple (say, L) observation sequences as $\boldsymbol{y}^{(l)} = \left[\boldsymbol{y}_1^{T_l}\right]^{(l)}$, where the superscript l indicates the lth observation sequence. We recall that in (4.58)-(4.61) the numerators and denominators represent an average number of certain events. Hence, it is appropriate to extend (4.58)-(4.61) to the multiple observation case by summing the events over all observations. The reestimation formulas for the multiple observation case then are

$$\bar{a}_{j,i} = \frac{\sum_{l=1}^{L} \xi^{(l)}(i,j,\cdot)}{\sum_{l=1}^{L} \gamma^{(l)}(i,\cdot)} \tag{4.62}$$

$$\bar{c}_{in} = \frac{\sum_{l=1}^{L} \nu^{(l)}(i,\cdot,n)}{\sum_{l=1}^{L}\sum_{m=1}^{M} \nu^{(l)}(i,\cdot,m)} \tag{4.63}$$

$$\overline{\mu}_{il} = \frac{\sum_{l=1}^{L}\sum_{t=1}^{T^{(l)}} \nu^{(l)}(i;\cdot,l)\, y^{(l)}(t)}{\sum_{l=1}^{L} \nu^{(l)}(i;\cdot,l)} \tag{4.64}$$

$$\overline{C}_{il} = \frac{\sum_{l=1}^{L}\sum_{t=1}^{T^{(l)}} \nu^{(l)}(i;\cdot,l)\left[y^{(l)}(t) - \mu_{il}\right]\left[y^{(l)}(t) - \mu_{il}\right]^{T^{(l)}}}{\sum_{l=1}^{L} \nu^{(l)}(i;\cdot,l)} \tag{4.65}$$

A basic problem associated with the forward-backward algorithm, as we have seen with recognition and training, is that the intermediate variables (α, β, ξ, γ and ν) involve a large number of multiplications of numbers less than unity. This could lead to underflow problems during computation. One way to circumvent this numerical problem is by scaling the forward and backward variables (α and β). The scaling is done so that the HMM parameter values are not affected throughout the reestimation procedure. We multiply $\alpha\left(y_1^t, i\right)$ and $\beta\left(y_{t+1}^T | i\right)$ by a scaling factor

$$c(t) = \frac{1}{\sum_{i=1}^{S} \alpha\left(y_1^t, i\right)} \tag{4.66}$$

so that

$$\tilde{\alpha}\left(y_1^t, i\right) = \alpha\left(y_1^t, i\right) \times c(t) \tag{4.67}$$

$$\tilde{\beta}\left(y_{t+1}^T | i\right) = \beta\left(y_{t+1}^T | i\right) \times c(t) \tag{4.68}$$

where $\tilde{\beta}$ and $\tilde{\alpha}$ are the scaled versions of β and α respectively. It can be shown that the use of these scaled values in the reestimation does not affect the HMM parameter values as the scale factors cancel out in the reestimation formulas [3]. Another drawback with the Baum-Welch reestimation method is that the maximum-likelihood estimates of the means, μ, are very sensitive to the initial estimate [14]. In the following section, we describe the Viterbi reestimation procedure which is simpler than the Baum-Welch procedure yet equally efficient.

4.3.2.2 Viterbi Reestimation

The Viterbi reestimation is based on the Viterbi decoding approach to recognition. Since the need for using multiple observations was mentioned in the previous section, the following discussion of Viterbi reestimation uses multiple observations. As with the Baum-Welch reestimation, we start with an arbitrary initial model $\mathcal{M}$ and L training sequences $y^{(l)} = \left[y_1^{T_l}\right]^{(l)}; 1 \le l \le L$ corresponding to L utterances of a particular word. We segment each of these L observation sequences into S states using the model $\mathcal{M}$, and Viterbi

decoding and backtracking as described in Section 4.3.1.2. As a result, we determine the following numbers:

$$n(u_{j|i}) \cong \text{number of transitions } u_{j|i} \tag{4.69}$$

$$n(u_{\cdot|i}) \cong \text{number of transitions exiting state } i \tag{4.70}$$

From (4.69)-(4.70) estimates of the state transition probabilities are

$$\bar{a}_{j,i} = \frac{n(u_{j|i})}{n(u_{\cdot|i})} \tag{4.71}$$

Also, for each of the S states, we have a set of observations (from the L training sequences) that occur within the state. Alternatively, at this point we have the histogram of the observations that occur within each state from which we can obtain estimates of the parameters in the observation pdf of the state. To this end we use a segmental k-means procedure to cluster the observation vectors, occurring within a single state, into M clusters. As a result, we obtain M clusters for each of the S states [18, 19]. Estimates of the observation pdf parameters from these clusters are

$$\bar{c}_{jk} = \frac{\text{number of vectors in cluster } k \text{ of the } j\text{th state}}{\text{number of vectors observed in the } j\text{th state}} \tag{4.72}$$

$\overline{\mu}_{jk}$ = sample mean of the vectors in cluster k of jth state (4.73)

$\overline{C}_{jk}$ = sample covariance matrix of the vectors in cluster k of jth state (4.74)

Instead of using the sample means and covariance matrices, it is possible to use other methods of statistical data analysis to provide estimates for the means and covariance matrices of the HMM [20]. The resulting model $\overline{m}$ is then compared with the previous model m in terms of a distance measure that reflects the statistical similarity of the HMMs. One such distance measure is [21]

$$D'(m,\overline{m}) \cong \frac{1}{\overline{T}}\left[\log \mathcal{L}(\overline{y}|m) - \log \mathcal{L}(\overline{y}|\overline{m})\right] \tag{4.75}$$

where $\overline{y}$ is a length $\overline{T}$ observation sequence generated by $\overline{m}$. It should be noted that D' doesnt conform to the requirements of a metric, since it is an asymmetric distance measure in the sense that

$$D'(m,\overline{m}) \neq D'(\overline{m},m) \tag{4.76}$$

To obtain a distance which is symmetric with respect to the models, we use

$$D(\mathcal{m},\overline{\mathcal{m}}) \cong \frac{D'(\overline{\mathcal{m}},\mathcal{m}) + D'(\mathcal{m},\overline{\mathcal{m}})}{2} \tag{4.77}$$

If the model distance at the end of an iteration is greater than a threshold, then we replace the old model $\mathcal{m}$ with the new model $\overline{\mathcal{m}}$ and repeat the process of reestimation until the distance falls below the threshold, at which point model convergence is assumed. The Viterbi reestimation using the segmental *k*-means method is faster than the Baum-Welch reestimation method. Although it is possible to use Viterbi reestimation to provide a "good" initial model for the Baum-Welch reestimation, it has been found that the HMMs resulting from both reestimation methods mentioned here yield almost the same likelihoods [2]. In this sense, we can circumvent the drawbacks of the Baum-Welch method by the exclusive use of the Viterbi reestimation method.

Having discussed the theory behind training and recognition using HMMs, in the next section we present the implementation of the training and recognition algorithms for the HMMs used in our recognition system.

4.4 Implementation

The implementation of the training and recognition procedures, described in the previous sections, and as applied to our recognition system, deals with various issues. These include assumptions regarding the structure of the HMM, the initial guesses for the models, and appropriate stopping conditions for checking convergence of the models.

4.4.1 Structure of the HMM

The structure of the HMM includes the pattern of allowable state transitions, the number of states, and the number of mixture densities used in the HMM. An ergodic model as introduced in Section 4.1 is one which allows unconstrained state transitions. Since the main motivation for using the HMM is to model the underlying acoustic phenomena, there is the need for the so called *left-to-right* model to represent the sequential ordering of events associated with speech utterances which vary in time from left to right [22]. For the case where there is a repetition of events in time, we can use more number of states and still use the left-to-right model. If the states are numbered sequentially from left to right, then the left-to-right model that we have used satisfies the constraints

$$a_{i,j} = 0, \quad \text{for all } i < j, \quad \text{and for } i > j+1 \tag{4.78}$$

In other words, we allow transitions from state j only to itself or to state j+1. We also make the following assumptions regarding the initial and final states:

$$a_{S,S} = 1 \tag{4.79}$$

$$\pi(i) = \begin{cases} 1 & \text{for } i = 1 \\ 0 & \text{for } 1 < i \le S \end{cases} \tag{4.80}$$

The assumption in (4.80) removes the need for estimating the initial state probabilities as parameters for the HMM. Apart from the propriety of the assumptions made here regarding the state transition probabilities, we have reduced the number of non-trivial entries in the state transition matrix, A. This provides a computational advantage. We have approximated the observation pdfs by Gaussian mixture densities, as in (4.9).

When continuous (observation) HMMs are used to model whole words, it is typical to use one state per analysis frame [3]. We have obtained HMMs using various numbers of states. We have also investigated the use of different numbers of states for the different digits. The number of mixture densities in (4.9) has been constrained to be a power of two to simplify the clustering algorithm used in the segmental k-means method. Various numbers of mixture densities have been used to study the effect of the number of mixture densities on recognition performance. Chapter 6 deals with the results of comparison between models with different numbers of states and different numbers of mixture density components.

4.4.2 Training

After the decision regarding the structure and the size of the HMM to be used in the implementation, we need to obtain "good" models to represent the words in our vocabulary, which are the nine digits for now. As mentioned in Section 4.3.2, we use multiple recordings of each word to provide the multiple observation sequences for the

training of the HMMs. Although it is possible to use more than one HMM, we have used only one HMM to model each word [3]. The Viterbi reestimation procedure was used extensively in the training algorithm. Figure 10 shows the block diagram of the implementation of the training algorithm. Although the Viterbi reestimation procedure is found to work well with a wide range of initial guesses, we derive the initial model from the training data with the hope of obtaining a "good" initial guess [18]. For the initial guess of the observation pdf parameters corresponding to each word, we first assume that, for all the observation sequences of the word, the different states occur an almost equal number of times. We segment all the observation sequences into state sequences based on the above assumption.

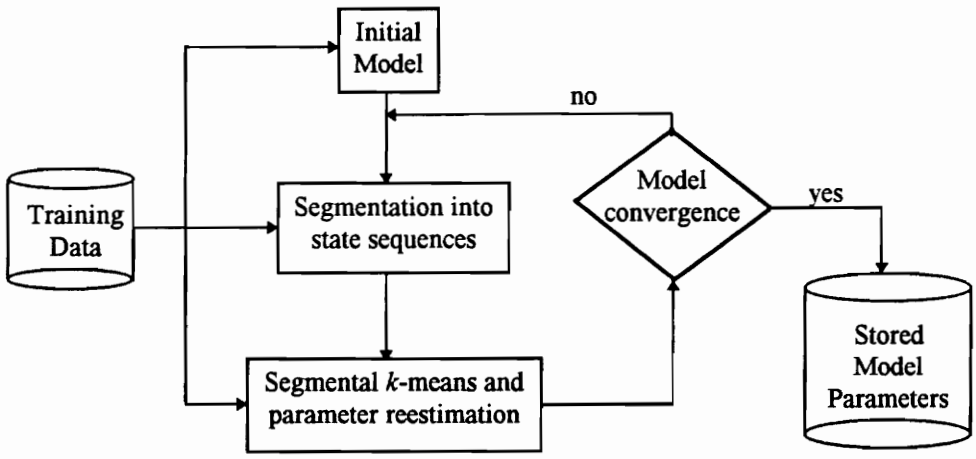

Figure. 10 Block diagram of the training algorithm

As a result, we have a set of observations that occur within each state. As mentioned in Section 4.3.2.2, we use the segmental k-means method to cluster the observations that occur within each state. From each cluster of vectors, we obtain a set of estimates $\left(\text{say, } \hat{c}, \hat{\mu} \text{ and } \hat{C}\right)$ for the observation pdf parameters of the state using (4.72)-(4.74). We use these estimates as a seed for generating random initial guesses according to

$$\overline{c} = \hat{c} \tag{4.81}$$

$$\overline{\mu} = \frac{\hat{\mu}}{2} + r\hat{\mu} \tag{4.82}$$

$$\overline{C} = \frac{\hat{C}}{2} + r\hat{C} \tag{4.83}$$

where r is random scalar variable, uniformly distributed between 0 and 1, used to generate the different initial guesses.

For the initial estimate for the state transition probabilities, we have used random values satisfying the constraints in Section 4.4.1 and the constraint that the entries of the state transition matrix must add up to unity along every column.

In order to obtain a solution as close as possible to the global optimum during the iterations, we start the reestimation from 25 different initial conditions. As a result we have, at the end of the iterations, 25 candidates ($\mathcal{M}_k$; $1 \le k \le 25$) for the final estimate of the HMM for a particular word. As the best candidate, we choose the model that provides the

"best" average (over all training sequences) likelihood of producing the training sequences according to the following equation

$$\mathcal{M}^* = \underset{\mathcal{M}_k}{\operatorname{argmin}} \left(\frac{\sum_{l=1}^{L} -\log\left(\mathcal{L}\left(\mathbf{y}^{(l)}|\mathcal{M}_k\right)\right)}{L} \right) \qquad 1 \le k \le 25 \tag{4.84}$$

where L is the total number of training observation sequences available for each word.

To check for model convergence, we have used the distance measure given in (4.77). We found that the model distance does not decrease monotonically with each iteration. However, the distance goes below a particular value and remains below it for a significant number of successive iterations. We choose this value as the threshold for stopping the iterations. Since there is scope for model distances to increase after some iterations, it is crucial to fix a suitable threshold. We have chosen a model distance of 0.1, calculated according to (4.77), between the models of two successive iterations as the stopping condition, based on experimental evaluation of the performance of the various models.

The final model $\mathcal{M}^*$ is then chosen to represent the word during recognition.

4.4.3 Recognition

At the end of the training process, we have a model for each word in the vocabulary. If we have a total of W words in the vocabulary, we end up with W HMMs at the end of

training. Figure 11 shows the block diagram of the implementation of the recognizer. To recognize an incoming word, we pass it through the endpoint detector and feature extraction blocks, as outlined in Sections 3.1 and 3.2. We determine the likelihood that each of the available models produced this incoming observation sequence. We then choose the model that yields the least negative log likelihood and the corresponding word as the recognized word.

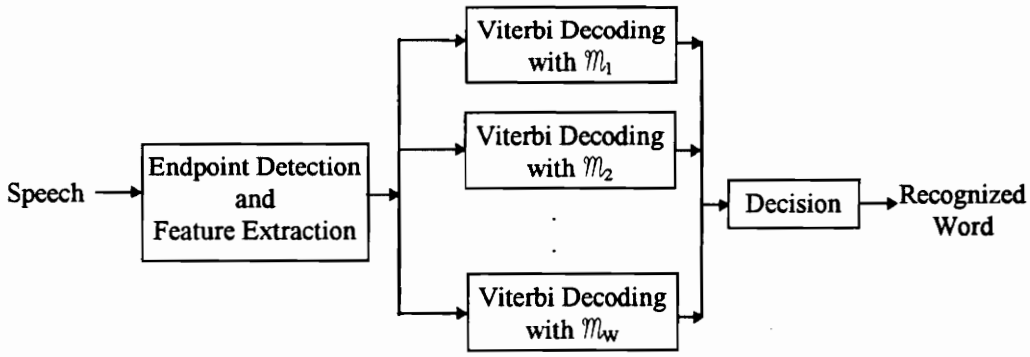

Figure 11. Block diagram of the recognition algorithm

With this, we come to the end of our discussion of the HMM and its use in recognizing the isolated words in the vocabulary. The next chapter deals with the tests conducted with the HMMs and the effects of various parameters on recognition performance.

5. Testing of the Recognition System

Having discussed the various aspects of the speech recognition system developed in this thesis, such as feature extraction and HMM training and recognition, in this chapter we present the results of the tests conducted with the training and the recognition algorithms. While the tests conducted on the training process were used to provide insights into the nature of the convergence of the training algorithm, the results of recognition can be used to discuss the effects of the number of states and the number of mixture components on the performance of the overall recognition system.

5.1 Results on Known Models

Due to the lack of knowledge of the true HMMs for the digits, we used simplified, known models to initially test the training algorithm. These known models were used to generate observation sequences based on the following procedure [14]:

i) The state index is initialized to $j = 1$ and the time index to $t = 1$.

ii) The unit interval is partitioned proportional to c_{jm}, $1 \leq m \leq M$. A random number, uniform on [0, 1], is generated and a mixture density component, l, is selected according to the subinterval in which the random number falls.

iii) A D-dimensional normal deviate, $\mathbf{y}$, of zero mean and covariance C_{jl}, is generated.

iv) The observation $O_t = y + \mu_{jl}$.

v) A unit interval is partitioned proportional to a_{jk}, $1 \le k \le S$. A random number uniform on [0, 1] is generated and the next state, j, is selected according to the subinterval in which the random number falls.

vi) The time index, t, is incremented to t+1.

vii) Steps ii through vi are repeated until a maximum allowable t is reached or the final state is generated for the maximum allowable number of times.

For a test case, the following model parameters were assumed.

$$S = 3 \qquad M = 2 \qquad D = 1$$

$$A = \begin{bmatrix} 0.8 & 0 & 0 \\ 0.2 & 0.3 & 0 \\ 0 & 0.7 & 1 \end{bmatrix}$$

$$\mu_{1\cdot} = \begin{bmatrix} 1 & 19 \end{bmatrix} \qquad \mu_{2\cdot} = \begin{bmatrix} 7 & 25 \end{bmatrix} \qquad \mu_{3\cdot} = \begin{bmatrix} 13 & 31 \end{bmatrix}$$

The covariance matrix was taken to be a diagonal matrix containing the variances of the observation vector elements, and all the variances were taken to be equal to 2.

$$c = \begin{bmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \\ 0.2 & 0.8 \end{bmatrix}$$

The above model was used to generate 100 observation sequences that were used as training data. This training data was then used as input to the training procedure. The training procedure was started from arbitrary initial models and we observed the distance between the models (according to (4.77)), out of two successive iterations for a number of

iterations. Figure 12 shows the plot of the model distances, computed according to (4.77), between successive iterations for 3 different starting models.

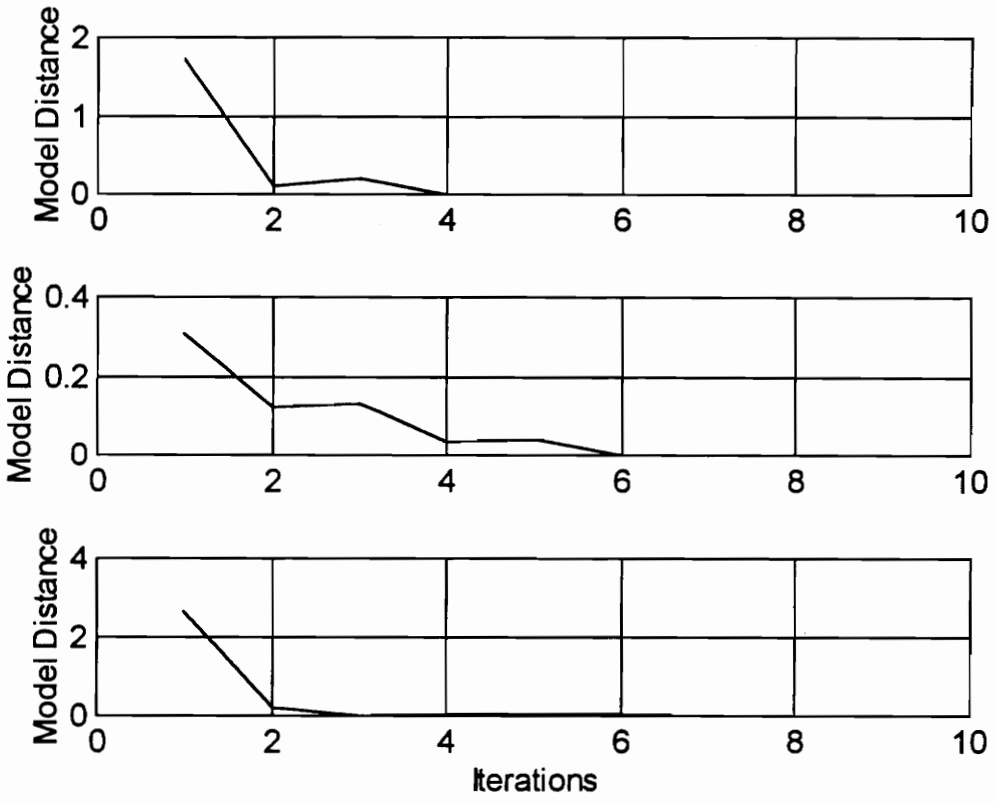

Figure 12. Model distances for successive iterations

As we see from Figure 12, there is a tendency for the model distance curve to flatten after a certain number of iterations. After this point, the model distance remains more or less constant with subsequent iterations. Thus, it can be observed that for every initial model, after a certain number of iterations, the models converge in the sense that the subsequent models are statistically similar.

For the test case used in Figure 12, the model distance converges to a value of zero. However, we found that with real speech data, due to the larger dimensions of the problem, it takes a prohibitively long time for the iterations to reach a model distance of zero. Hence, we have attempted to arrive at a convenient threshold for assuming convergence and stopping the iterations. The plots in Figure 12 show a general tendency for the model distance to decrease with successive iterations. After a number of iterations with the simple, known models it was decided that, for speech data, a threshold of 0.1 for the model distances could be used as a reasonable stopping condition for assuming convergence. As we will see from the results discussed in the next section, this threshold is quite reasonable from a recognition performance point of view.

The training with the 100 observation sequences (generated from the known HMMs), resulted in the following estimates for the various model parameters ($\hat{S}$, $\hat{M}$, and $\hat{D}$ were assumed to be the same as the true S, M, and D respectively):

$$\hat{S} = 3 \qquad \hat{M} = 2 \qquad \hat{D} = 1$$

$$\hat{A} = \begin{bmatrix} 0.77 & 0 & 0 \\ 0.23 & 0.34 & 0 \\ 0 & 0.66 & 1 \end{bmatrix}$$

$$\hat{\mu}_{1\cdot} = [1.08 \quad 18.8] \qquad \hat{\mu}_{2\cdot} = [6.9 \quad 25] \qquad \hat{\mu}_{3\cdot} = [12.6 \quad 31]$$

$$\hat{c} = \begin{bmatrix} 0.7 & 0.3 \\ 0.49 & 0.51 \\ 0.19 & 0.81 \end{bmatrix}$$

$$\hat{C}_{1\cdot} = [1.6 \quad 1.8] \qquad \hat{C}_{2\cdot} = [2.3 \quad 1.8] \qquad \hat{C}_{3\cdot} = [2.1 \quad 2.3]$$

Since the observation sequences have been generated for testing purposes, we have knowledge of the true statistics of the generated data and consequently we have the true histograms of the observations produced in each state. Figures 13-15 show the true histogram and the histogram of the observations based on Viterbi decoding for the three states.

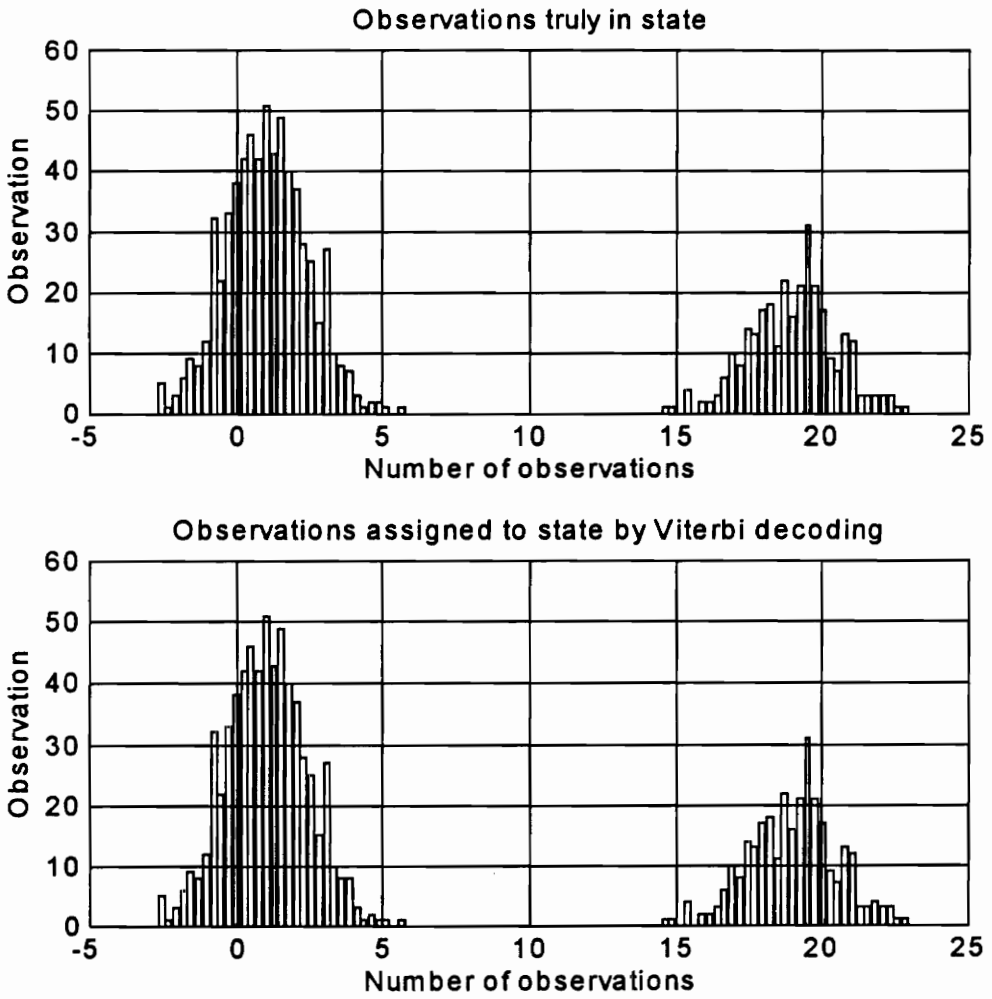

Figure 13. Histogram for State 1

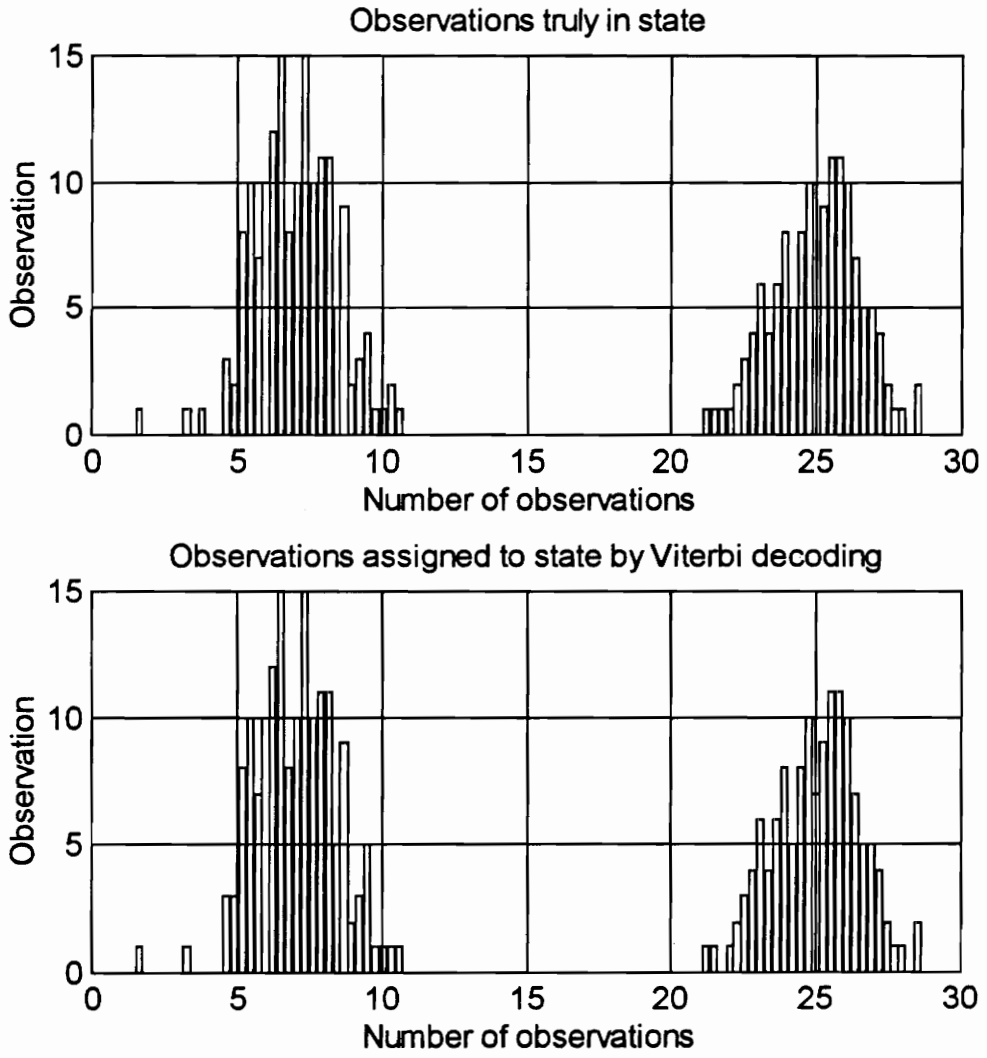

Figure 14. Histogram for State 2

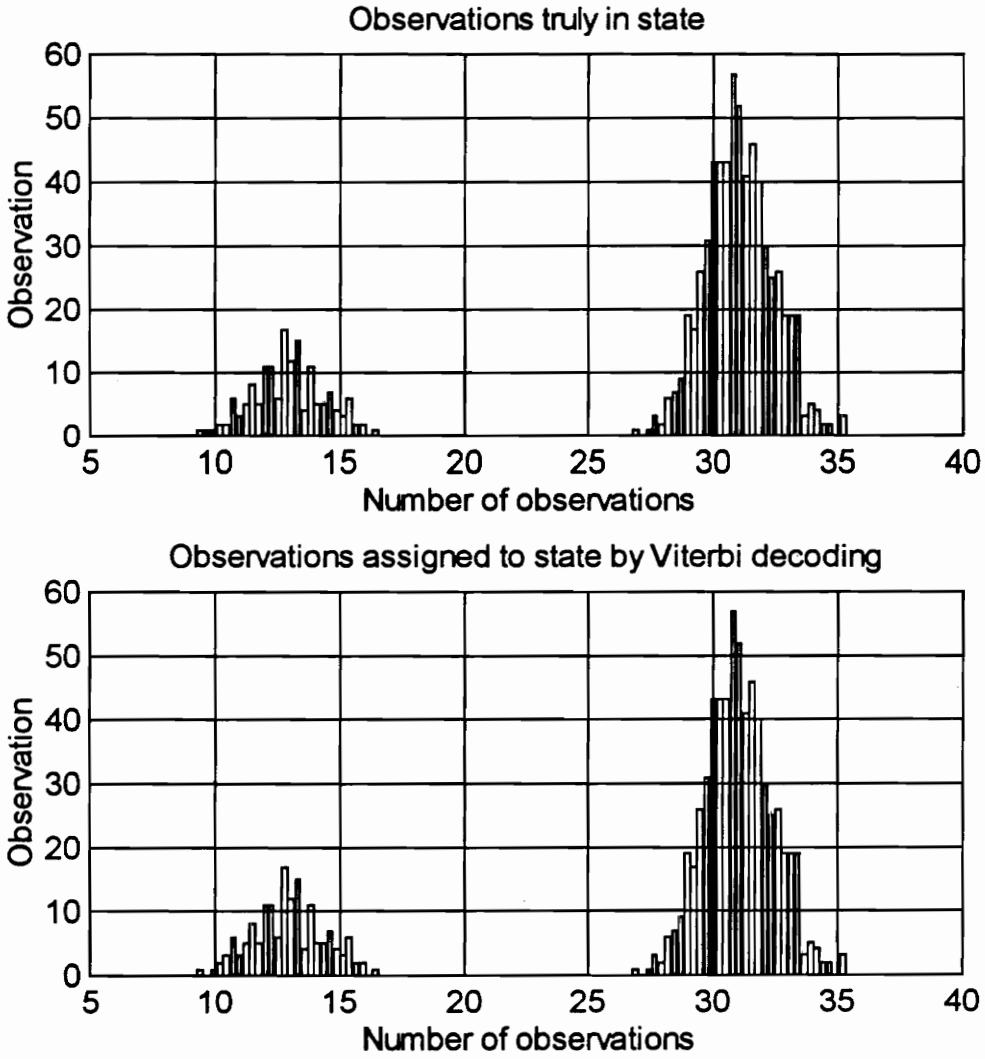

Figure 15. Histogram for State 3

From the histograms in Figures 13-15, we see that the observations assigned to the states through Viterbi decoding are almost the same as those truly generated in the states. Hence, Viterbi decoding can be used for estimating the parameters of the observation pdf. Figures 16-18 show plots of the true pdf and the reestimated observation pdf (shown by the continuous curve) for the three states.

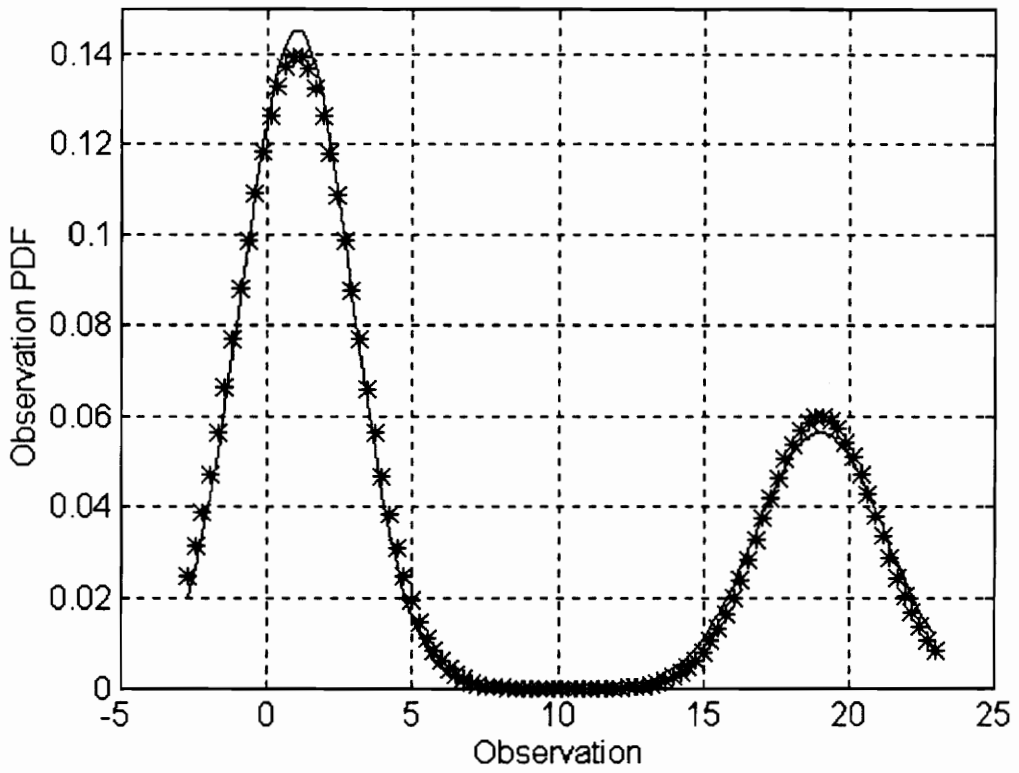

Figure 16. True (*) and Reestimated Observation PDFs for State 1

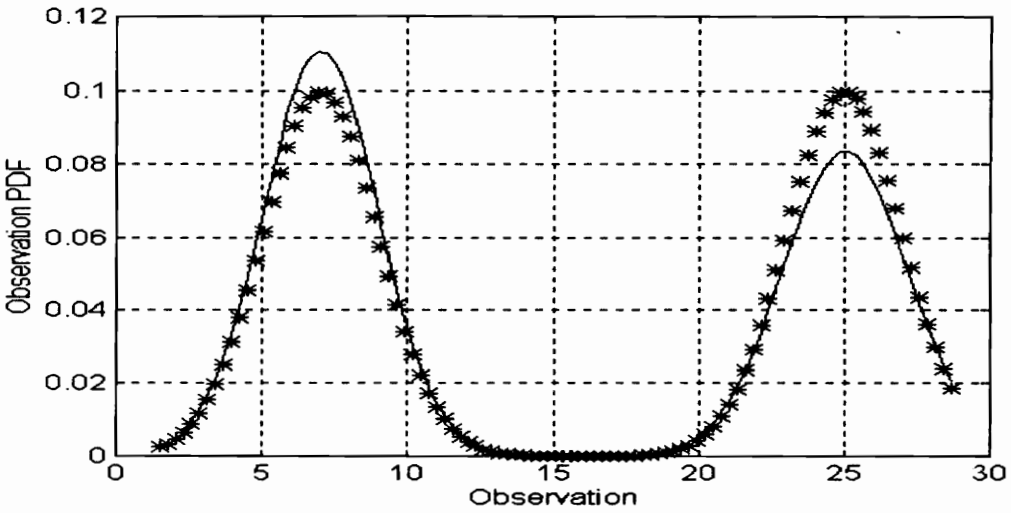

Figure 17. True (*) and Reestimated Observation PDFs for State 2

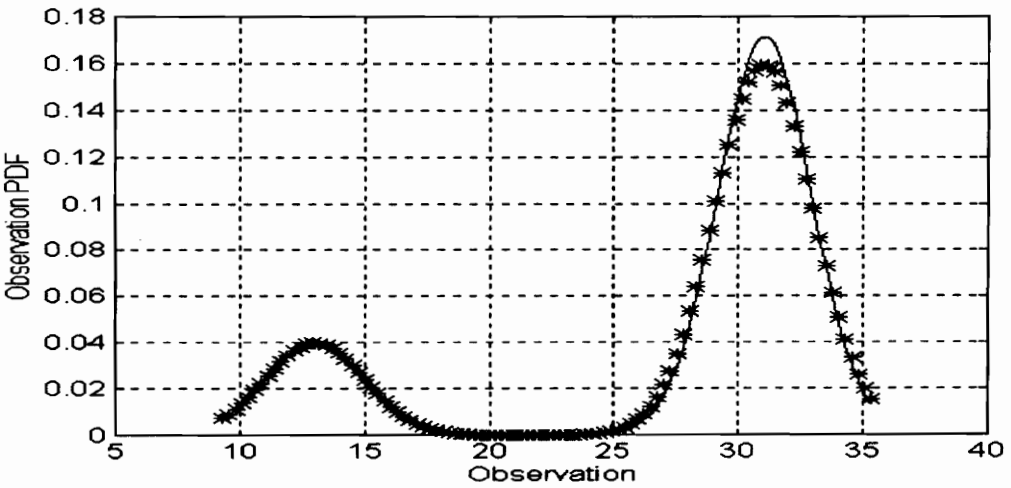

Figure 18. True (*) and Reestimated Observation PDFs for State 3

Figures 16-18 show that, for a model with "distinct" Gaussian mixture density components, the reestimation procedure results in a model that is very similar to the true model. However, similar tests conducted for a model with "overlapping" Gaussian mixture density components suggested that the reestimated model was farther away from the true model and a larger amount of training data was required to obtain a "good" reestimated model. Appendix A shows the result of the test with a model with "overlapping" Gaussian mixture density components.

To test the recognition system, we generated 5 different models. Let these models be $\mathcal{M}_i$; $1 \le i \le 5$ and let them correspond to 5 "dummy" words W_i; $1 \le i \le 5$. We let each model generate 100 observation sequences. These 500 dummy words form the input to the recognition system and the results of the recognition are shown in Table 1. The input words are along the rows and the recognized word along the columns. Each entry in Table 1 is the number of times the word, corresponding to the row containing the entry, is selected as the recognized word.

Table 1. Recognition rates for the dummy words.

	Input words for recognition				
Output	W_1	W_2	W_3	W_4	W_5
W_1	100	0	0	0	0
W_2	0	100	0	0	0
W_3	0	0	100	0	0
W_4	0	0	0	100	0
W_5	0	0	0	0	100

We observe that, for the test case, the recognition system successfully recognizes all the "dummy" observation sequences. Thus, if the true models corresponding to the observation sequences are available, the recognition system is capable of successful recognition.

Thus, the results of the tests on known models provide the proof of concept for the implementations of the training and the recognition algorithms. We now present the results of recognition with the words in the vocabulary, which are the digits 0 through 9, in our case.

5.2 Results on Words

Since the objective of this thesis is to develop a speaker dependent recognition system, we recorded two sets of 50 utterances of each digit, by a single speaker, on two different days. In order to include the variability of the recorded words, for each digit we formed the training set of 50 utterances using 25 recordings each from the two sets. Thus we have a training set of 50 utterances per digit and a testing set of 50 utterances per digit. The training sets were used to train the HMMs for the 10 digits in the vocabulary. The results of recognition on the words help us to evaluate the performance of the recognition system and to study the effects of the number of states and the number of mixture density components on recognition performance.

As with the dummy words, the results of recognition on the training and the testing sets are presented in the form of tables, where the input words are along the columns and

the recognized words along the rows. Each entry in the table is the number of times the word, corresponding to the row containing the entry, is selected as the recognized word. We varied the number of states between 5, 6, and 7, and the number of mixture density components between 1, 2, and 4. We also attempted to use 8 mixture density components. However, the segmental k-means clustering that we use for Viterbi reestimation resulted in many clusters being empty. This indicates that using 8 mixture density components leads to over-parameterization. Moreover, since the Viterbi reestimation method obtains the statistics of each mixture density component from each of the clusters, empty clusters lead to numerical problems during reestimation. We first present the results of recognition for the models with 7 states ($S = 7$) and 4 mixture density components ($M = 4$). The results with the models with other values of S and M are provided in Appendix B. We use these results to discuss the effects of S and M on performance. As an alternative to using the same number of states and mixture density components in the models for all the words in the vocabulary, we have also used different values for S and M for different words. Since it is customary to use as many states as the average number of frames in a word, for the case with a different number of states for the different digits, for each word, we have chosen the number of states to be equal to the number of frames in the shortest training utterance of that word. The results of recognition, for models with different values for S and M for different words, are also provided in Appendix B.

We have associated a figure of merit with each recognition table. The figure of merit is the sum of the elements of the diagonal in the tables. This figure of merit gives an idea

of the average percentage of correct recognition (figure of merit divided by 5) for all the words in the vocabulary.

Table 2 shows the recognition rates for the training utterances for M = 4 and S = 7. The figure of merit for Table 2 is 500 and corresponds to an average recognition rate of 100%. We can expect such a high recognition rate for the training data, as the models have been trained to "tune" to the training utterances. Table 3 shows the recognition rates for the testing utterances for M = 4 and S = 7. The figure of merit for Table 3 is 496 and corresponds to an average recognition rate of 99.2%. The recognition rate for the testing set can be expected to be less than that for the training set, as the representative models may not be tuned to some of the variability in the testing set. Hence, as expected, the trained models are a poorer representative of the testing set than of the training set. From Tables 2 and 3 and Appendix B, we see that it is possible to decrease the error rate associated with the recognition of any one word by changing the value of M and S. However, the overall recognition rate as indicated by the figure of merit, as we have chosen, depends on all the models being "good" and this figure of merit has to be observed, to arrive at a suitable value for S and M for the recognition system.

To study the effects of S and M on recognition performance, we next generalize the results obtained using 5, 6, 7, and a varying number of states. Figure 19 shows the average error percentages across the digits for the training and the testing data sets. We observe the downward trend in the error percentages with varying a number of mixture density components for a particular number of states.

Table 2. Recognition rates for M = 4, S = 7 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	50	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	0	0	0	0	0	0	50	0
9	0	0	0	0	0	0	0	0	0	50

Figure of merit = 500

Table 3. Recognition rates for M = 4, S = 7 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	49	1	0	0	0	0	0	0	0
2	0	0	48	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	1	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	1	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 496

The curves marked by 'S = X' represent the models with a different number of states for the different digits. We see that an increase in the number of mixture components generally decreases the error rates, for each of the different number of states. Figure 20 shows the trend in the average error percentages across the digits for the training and the testing data sets, with a varying number of states for a particular number of mixture density components. Although the error percentage decreases monotonically with an increase in the number of states for the testing set, there is no such trend with the training set. However, in general, an increase in the number of states seems to decrease the error rates. However, after a particular point, an increase in the number of states adversely affects the performance because, due to the larger number of parameters involved, the training algorithm may not converge to very "good" models.

We see that the decrease in error percentages with changes in the number of mixture density components is more pronounced than that with changes in the number of states. Tables 4 and 5 indicate that the trends of the recognition performance in the testing set, with changes in S and M, more or less follow the pattern in the training set. In Table 5, we see that the error rate increases when we change from using 7 states to a varying number of states. This is because, in general, in the models for the case S = X, the number of states used by each digit is more than 7. Hence, the increase in the number of states has increased the number of parameters to be reestimated. Since an increase in the number of parameters tends to increase the variance involved with each of the parameters, the

reestimation method may not result in good representative models when the number of parameters is larger than necessary.

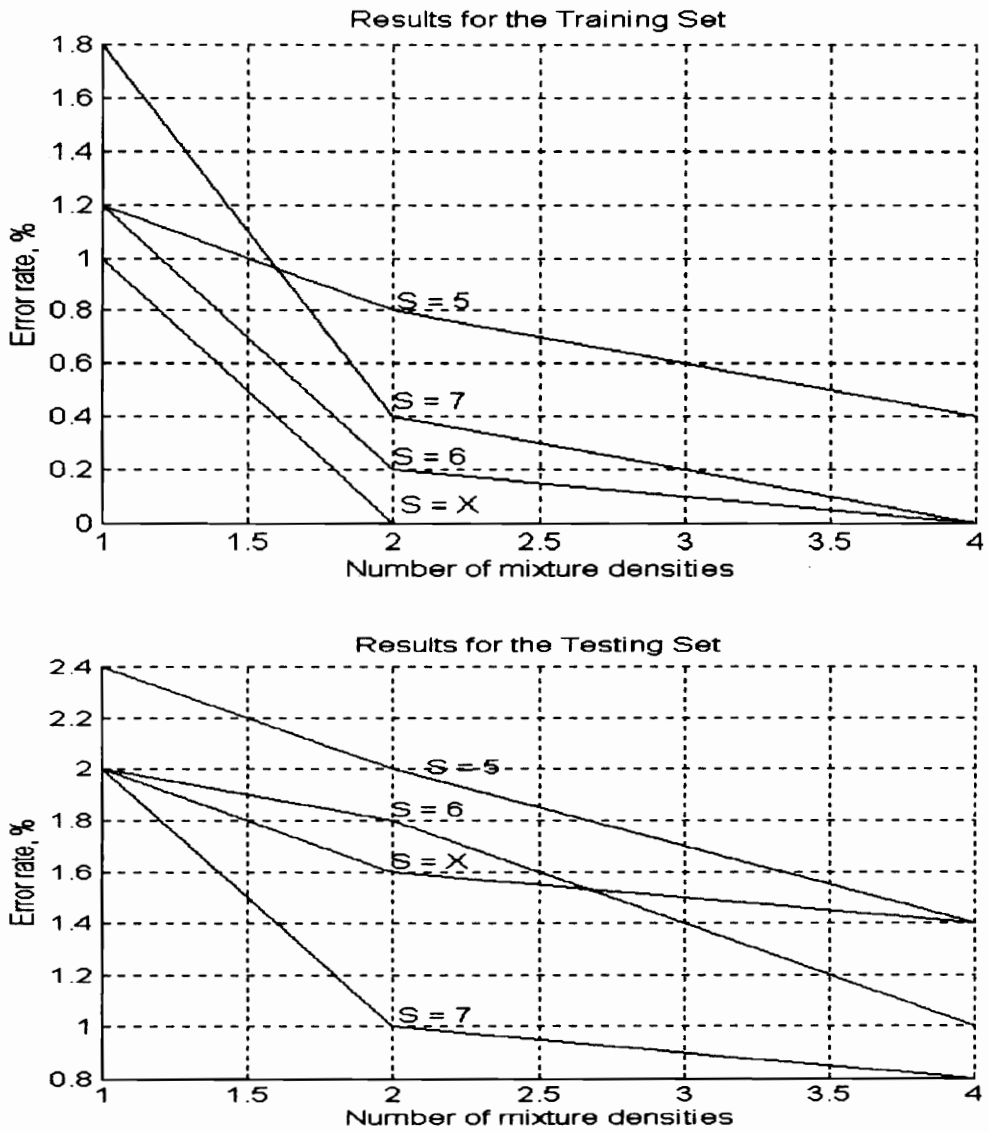

Figure 19. Plot of error rate percentages for different number of states with varying number of mixture density components.

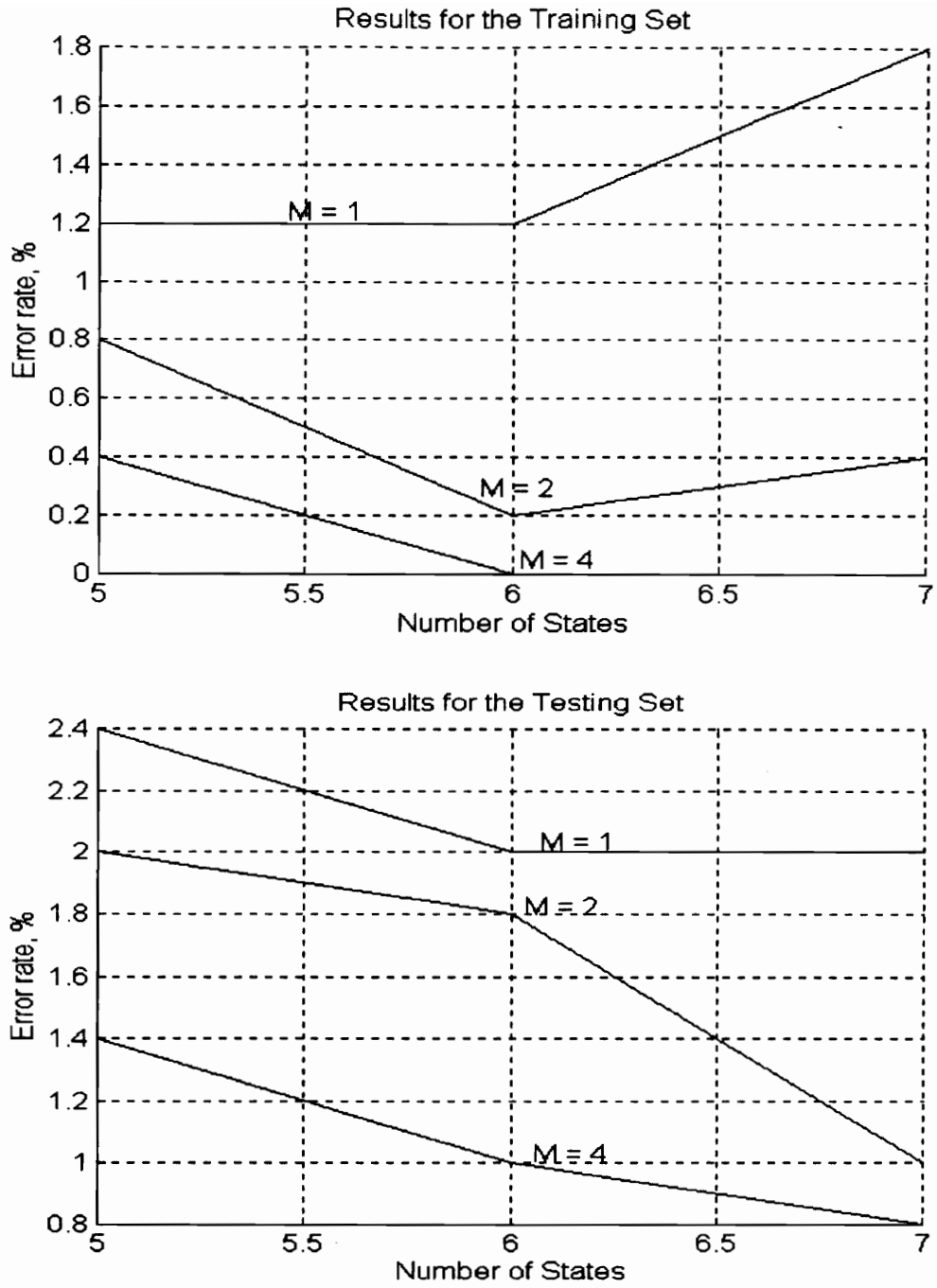

Figure 20. Plot of error rate percentages for different number of mixture density components with varying number of states.

Table 4. Error rates in recognition (training set).

Number of States	Error rates in percentage		
	1 mixture density component	2 mixture density components	4 mixture density components
5	1.2	0.8	0.4
6	1.2	0.2	0
7	1.8	0.4	0
X	1.0	0	0

Table 5. Error rates in recognition (testing set).

Number of States	Error rates in percentage		
	1 mixture density component	2 mixture density components	4 mixture density components
5	2.4	2.0	1.4
6	2.0	1.8	1.0
7	2.0	1.0	0.8
X	2.0	1.6	1.4

With the results observed in the testing set, we conclude that HMMs with 7 states and 4 mixture density components sufficiently represent the digits used in our case and result in a recognition rate of better than 99%.

6. Connected Digit Recognition

In the last two chapters, we presented the design of an isolated word recognition algorithm and demonstrated the recognition of the ten digits 0 through 9. The focus of this thesis was to develop a system that is capable of recognizing strings of digits, as in a "hands-free" telephone system. In this chapter, we present the implementation of a connected digit recognition algorithm and discuss the results of recognition on strings of digits.

6.1 Introduction

As mentioned in Chapter 1, connected digit recognition is a technique used to recognize continuous speech in a small-vocabulary application. In connected digit recognition, a string of digits is modeled as a concatenation of the models built from isolated digits. Since we use HMMs to model the isolated digits, the connected digit recognition system attempts to select the string of models that best matches the string of digits in a maximum likelihood sense. The most popular method to find the optimum set of models is a method known as the level-building (LB) method [3]. The main challenge behind finding an optimum set of digits is to determine the boundary between the words in the string. In continuous speech, this boundary is not distinct. Hence there is a need to incorporate the statistics regarding the duration of individual words, into the HMMs. It has been found in past research that a simple Gaussian duration model can be assumed for

each word in the vocabulary [23]. That is, the probability density $P_q(D)$ of duration D of the qth word is

$$P_q(D) = \frac{e^{-\left[\frac{(D-\overline{D}_q)^2}{2\sigma_q^2}\right]}}{\sqrt{2\pi}\sigma_q} \tag{6.1}$$

where the $\overline{D}_q$ and σ_q are the mean and standard deviation respectively. The mean and standard deviation can be estimated as the sample mean and standard deviation of the training data used to find the isolated word models. So, for the purpose of connected digit recognition, the word duration pdfs are included in the HMMs used to model the isolated digits. We now present the level building algorithm that we implemented to recognize a string of digits [23].

6.2 Recognition of a String using the LB Algorithm

We recall that, at this point, we have the individual word models $m_j;\ 0 \le j \le 9$ for each word in our vocabulary. Given a sequence of observations y_1^T, now corresponding to a sequence of L digits, the aim of the LB algorithm is to select the best concatenation of models $\{m_{[1]}, m_{[2]}, \ldots, m_{[l]}, \ldots, m_{[L]}\}$ that matches with the observation sequence in the

sense that the joint probability of the observation sequence and the state sequence is maximized.

Figure 21 helps us understand the LB decoding of the observation sequence into an optimal string of digits.

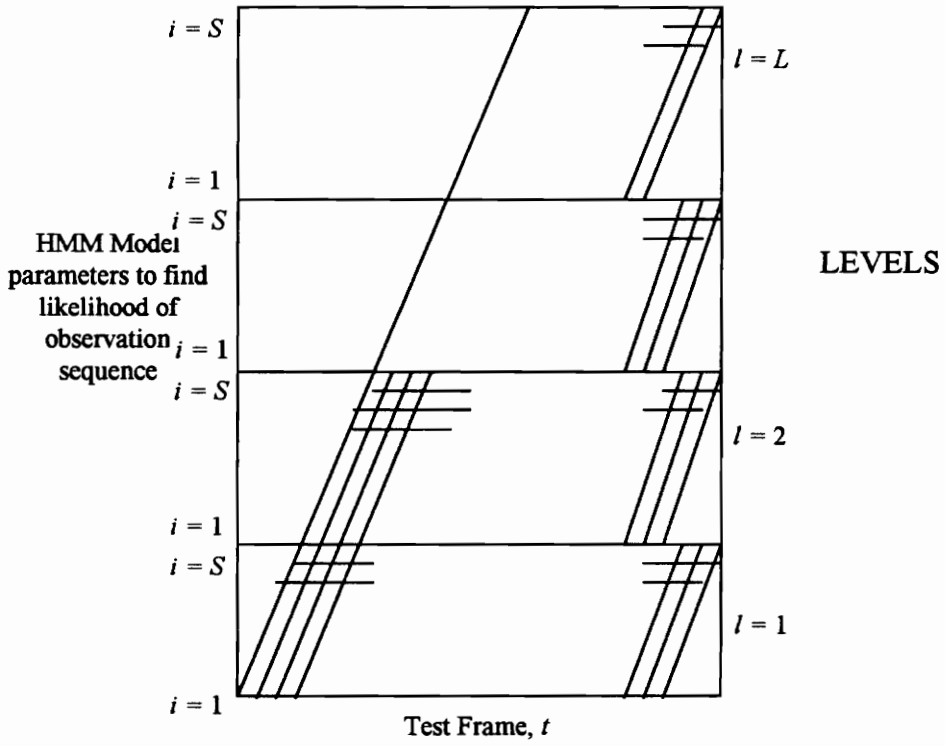

Figure 21. Implementation of LB based on HMMs for each isolated word

In Figure 21, the number of levels is equal to the number of digits in the input string. A "level" can be visualized to represent the boundary of each digit in the string. In other words, the *l*th level corresponds to the *l*th digit in the input string. Each node in the grid

represents a state index, frame index pair (i,t). Each line (or a set of lines) represent a path through a set of nodes in the grid. We begin at the level $l = 1$. At each level, we match every model $\mathcal{M}_q$ corresponding to the digit q in the vocabulary, to the observation sequence y_1^T using the Viterbi decoding method similar to the one discussed in Section 4.3.1.2. To do this match at $l = 1$, we use the following steps:

i. **Initialization** - $\delta_1(1) = b^q\left(y(1)|1\right)$, where $\delta_t(j)$ is the joint probability of partial state and observation sequences, $P\left(y_1^t, x(1)\,..\,x(t)|\mathcal{M}_q\right)$ and b^q is the observation probability using $\mathcal{M}_q$. Also,

$$\delta_1(j) = 0,\ j = 2,\ 3,\ \ldots,\ \mathrm{S}. \tag{6.2}$$

ii. **Recursion** - for $2 \le t \le T,\ 1 \le j \le S$

$$\delta_t(j) = \max_{1 \le i \le S}\left[\delta_{t-1}(i)a_{j,i}^q\right] b^q\left(y(t)|j\right) \tag{6.3}$$

iii. **Termination** - $P(l,t,q) = \delta_t(S),\ 1 \le t \le T$ (6.4)

The array P is a function of the level number l, frame number t, and the vocabulary word q. At level $l = 1$, we repeat the steps i-iii for all the words in the vocabulary. After this, we form the following arrays

$$\hat{P}(l,t) = \max_{q}\left[P(l,t,q)\right] \tag{6.5}$$

$$\hat{W}(l,t) = \operatorname{argmax}_{q}\left[P(l,t,q)\right] \tag{6.6}$$

Equation (6.5) gives the best probability $\hat{P}$ at the present level and at all times $1 \le t \le T$, and equation (6.6) gives the best word (one that yields maximum likelihood) that could be the *l*th digit in the string and end at the *t*th frame index.

For the Viterbi matching at higher levels ($l > 1$), we follow the following steps:

i. **Initialization** - $\delta_1(1) = 0$ (6.7)

$$\delta_t(1) = \max_{1\le i\le S}\left[\hat{P}(l-1,t-1),\ \delta_{t-1}(i)a^{q}_{j,i}\right] \times b^{q}\left(y(t)|1\right),\quad 2 \le t \le T \tag{6.8}$$

ii. **Recursion** - for $2 \le t \le T,\ 2 \le j \le S$

$$\delta_t(j) = \max_{1\le i\le S}\left[\delta_{t-1}(i)a^{q}_{j,i}\right] b^{q}\left(y(t)|j\right) \tag{6.9}$$

iii. **Termination** - $P(l,t,q) = \delta_t(S),\ 1 \le t \le T$ (6.10)

For the higher levels, we need to begin the Viterbi match from where the previous level ended. Hence, for the levels $l > 1$, each frame could either succeed a frame in the previous level or succeed a frame in the same level. The initialization equations (6.7) and

(6.8) let the level pick up at the most appropriate (of the above two possibilities) place from the previous level. We repeat steps i-iii for all the words in the vocabulary and then form the following arrays

$$\hat{P}(l,t) = \max_{q}\left[P(l,t,q)\right] \tag{6.11}$$

$$\hat{W}(l,t) = \operatorname{argmax}_{q}\left[P(l,t,q)\right] \tag{6.12}$$

for all the levels, $1 < l < L$, where L is the total number of levels which equals the expected number of digits in the observation string.

After we reach the highest level, L, we have $\hat{W}$, which is the array containing the best estimate of the digit ending at a particular frame, for all the frames and for all the levels. We have to obtain estimates of the boundaries of the digits corresponding to the various levels. We begin with the first level $l = 1$. We assume that all the digits in the string are equally long. Based on this assumption, we can obtain an initial estimate of the boundaries of the first digit in the string. This provides an estimate of the temporal region occupied by the level. We then examine the array $\hat{W}$ and determine the digit which occurs the maximum number of times within the boundaries that we have estimated. We have the mean and the standard deviation of the statistics of the duration of this digit. We now change the estimate of the temporal region occupied by the level to vary between the frames $\overline{D}_q - 3\sigma_q$ and $\overline{D}_q + 3\sigma_q$. The array $\hat{W}$ is again examined for the digit that occurs

the maximum number of times. This process (of searching for the digit that occurs a maximum number of times within the present boundary estimates) is repeated until the digits selected for two successive boundary estimates are the same.

Once the digit for level l is chosen, we let the initial estimate of the lower boundary (in terms of the frame index) of the next higher level $(l+1)$ be the frame $\overline{D}_{q_l^*}$ where q_l^* is the digit estimated to occupy level l. We then proceed to examine $\hat{W}$ and select the digit for the next level, $l+1$ in a manner similar to that for level l.

After we go through all the levels, we have an estimate of the string of digits corresponding to the input observation sequence. In the next section, we discuss the results of recognition on test strings.

Let us look at an example of choosing the digits in the string from the array $\hat{W}$. Let us consider a string of 2 digits. Hence, we use 2 levels (L = 2). For a test case, we have chosen the digit string "one two", to be recognized. The string we have chosen is 24 frames long. At the end of the LB algorithm, we have the following array

$$\hat{W}(l,t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 3 & 3 & 3 & 2 & 1 & 1 & 8 & 8 & 7 & 7 & 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}$$

We begin with level 1. We first assume that the first digit (level 1) is 12 frames long. So our initial boundary estimates for the 1st digit of the string are the frames 1 and 12. Between frame indices 1 and 12, we look for the digit of maximum occurrence. That is the

digit "one" (in the 1st row of $\hat{W}$). We also know that the mean duration of the digit "one" is 15 frames and the standard deviation is approximately 3 frames. So we modify the boundary estimates for the 1st digit as the 6th ($\overline{D}_q - 3\sigma_q$) and the 24th frame ($\overline{D}_q + 3\sigma_q$). We again search for the digit of maximum occurrence within frames 6 and 24. We can see that it is still the digit "one". Hence we conclude that the 1st digit of the string is "one". Therefore, we now have 1 (i.e. L-1) digits remaining in the string, and yet to be recognized. For the next level, l, we start the search from frame 15 (i.e. $\overline{D}_{q_{l-1}}$). We assume that the remaining (L-1) digits of the string are equally long. Hence, we choose the initial boundaries of the second digit as 15 and 24 and we repeat the process. We look for the digit of maximum occurrence. That is the digit "two" (in the 2nd row of $\hat{W}$). We also know that the mean duration of the digit "two" is 15 frames and the standard deviation is approximately 5 frames. According to the LB algorithm, we modify the boundary estimates (to search for the digit of maximum occurrence) for the 2nd digit to $\overline{D}_{q_{l-1}} + \overline{D}_q - 3\sigma_q$ and $\overline{D}_{q_{l-1}} + \overline{D}_q + 3\sigma_q$. However, since the maximum frame index is 24, and the length of the first digit is 15 frames, we modify the boundary estimates for the 2nd digit as the 15th and the 24th frame and search for the digit of maximum occurrence. We see that it is still the digit "two". Hence we conclude that the 2nd digit of the string is "two". Thus, we see that the string “one two” is correctly recognized.

It should be noted that knowledge of the number of digits in the string is extremely crucial to the connected digit algorithm that we have implemented. However, the HMMs,

when used within the framework of DP, have a self normalization feature [3] in the sense that irrespective of the number of levels in the LB algorithm, the length of the search path is always equal to the number of frames in the input string. This allows us to execute the LB algorithm on the same string using different values of L and choosing the set of HMMs that result in the best overall likelihood of having produced the input string. However, due to the larger amount of variability involved as compared to the situation where knowledge of the number of digits in the input string is available, we can expect the speech recognition system to yield poorer performance when the number of digits in the string is not available.

6.3 Results of recognition of strings

We now present the recognition rates observed with the recognition of strings of 2, 3, and 4 digits. We form the test strings by concatenating the observation sequences corresponding to isolated digits. We recall that we have 100 recordings of each digit (including training and testing sets). For each string length (number of digits in a string), we formed a testing set of 100 strings by randomly choosing the digits of the string from the recordings of isolated digits.

For each string length, we present the results where the isolated digit HMMs containing 1, 2, and 4 mixture density components, and 5, 6, and 7 states and a different number of states for different digits. Tables 6, 7 and 8 provide the string recognition rates

for 2, 3, and 4 digit length strings respectively. In Tables 6-8, 'X' indicates the case where different digit models use different numbers of states.

Table 6. Recognition rates for 2-digit strings.

Number of States	Recognition rates in percentage		
	1 mixture density component	2 mixture density components	4 mixture density components
5	70	76	83
6	79	81	88
7	78	82	91
X	78	82	89

Table 7. Recognition rates for 3-digit strings.

Number of States	Recognition rates in percentage		
	1 mixture density component	2 mixture density components	4 mixture density components
5	61	68	77
6	66	71	80
7	69	74	83
X	71	74	81

Table 8. Recognition rates for 4-digit strings.

Number of States	Recognition rates in percentage		
	1 mixture density component	2 mixture density components	4 mixture density components
5	49	54	58
6	53	58	61
7	53	60	64
X	51	57	63

From Tables 6-8, we observe that the trends in the recognition rates are similar to the trends observed with isolated digits, as seen in Tables 4-5. As the number of mixture density components is increased, keeping the number of states constant, the recognition rates increase significantly. Although such a trend is not very clear with an increase in the number of states, in general, a higher number of states tends to yield better performance. However, the recognition rates are significantly lower than those observed with the isolated digits. For a 2-digit string, we obtain a maximum recognition rate of 91% corresponding to models with $S = 7$ and $M = 4$. This is significantly lower than the 99.2% recognition rate observed for isolated digit recognition. Moreover, the recognition rates fall drastically for 4-digit strings, with the best (for $S = 7$, $M = 4$) recognition rate of only 64%. However, we see that models with 7 states and 4 mixture density components consistently provide the best results as far as the recognition rate of strings is concerned. The lowering of the recognition rates is probably because, due to the lack of knowledge

regarding the individual digit boundaries (unlike in isolated digit recognition), some observations belonging to adjacent digits may be included in the process of determining the best word at each level. This can lead to a wrong digit being picked to be a part of the string; it certainly would make recognition worse than for the correct individual digit. Further more, uncertainty about the boundaries of digits increases faster, relative to those on either absolute end of the string, for digits in between digits with uncertain locations. The increased uncertainty in the boundaries cannot but worsen the recognition rates for the nested digits. The consequence is a string recognition rate that is much less than for a concatenation of isolated digits with known boundaries (Based on our results: $(0.99)^L$ for an L-digit string).

7. Conclusions and Further Research

The motivation for this thesis has been to develop a speaker dependent speech recognition system that can be used to recognize strings of digits; for example, to be employed in an environment with a hands-free telephone system. In Chapters 3-6 we presented the theoretical background, the implementation, and the performance results of the recognition system. In Chapter 3, we discussed feature extraction. Chapter 4 delved into the theory behind the HMM and its application to the problem of speech recognition. We also discussed the various issues involved in the implementation of the training and the recognition algorithms in Chapter 4. In Chapter 5 we presented the results of recognition with isolated digits and discussed the effects of the number of states and the number of mixture density components on the recognition performance and error rates.

Based on the results presented in Chapter 5 and Appendix B, we concluded in Chapter 5 that HMMs with 7 states and 4 mixture density components resulted in the highest recognition rates as far as isolated digit recognition is concerned. We obtained a better than 99% recognition rate on the test data which were not used in the training process. This is significantly higher than the recognition rates obtained using a discrete HMM approach [26]. K. A. Rangarajan [26] has employed the FB algorithm and Baum-Welch reestimation for recognition and training respectively. However, to extend this system to a speaker independent or a larger vocabulary system, a larger amount of training data is needed. In such a case, although the basic structure of the recognition system can be maintained as it is here, some linguistic constraints may have to be included to achieve

similar performance. Chapter 7 details the implementation of the level-building algorithm to recognize strings of more than one digit. While we obtained better than 90% recognition rates for 2-digit strings, for a 4-digit string the recognition rates were no better than 64%. Thus, the system implemented here can be used as an isolated word recognizer, providing very high recognition rates, or it can be used to recognize 2-digit strings, at moderately high recognition rates.

To use the system for situations involving longer strings we have to involve some additional aspects into the word models. For example, the training data for each digit can be obtained from strings containing the digit [2]. As an extension to the system implemented in this thesis, to improve performance, different state and word duration models can be experimented with and employed [23]. In addition to state and word duration models, the HMMs can include models for the word energies. The latter has also been found to improve recognition performance to some extent [2].

Although other techniques that use empirical constraints to determine the boundaries between the digits in a string yield higher string recognition rates, the manner in which the digits are selected at the various levels is simpler in our system [2, 23].

In view of future real time implementation of the recognition system we observed, using MATLAB for a 10-digit vocabulary using 7 state and 4 mixture density component HMMs, that the number of flops required to perform recognition of a single isolated digit is approximately 10 million. To recognize a 3-digit string approximately 13 million operations are needed. Hence, if a Digital Signal Processor such as the ADSP-2181 (a 32

MIPS processor) is used, a 3-digit string can be recognized in approximately a third of a second and an isolated digit in less time. This is reasonably fast for the recognition system to be useful in many real-time applications. The memory requirement of the recognition system using 7 states and 4 mixture densities is for approximately 14000 words (16 bit).

APPENDIX A

We now present the results with a known model with "overlapping" mixture density components. The model parameters are assumed to be

$$S = 3 \qquad M = 2 \qquad D = 1$$

$$A = \begin{bmatrix} 0.8 & 0 & 0 \\ 0.2 & 0.7 & 0 \\ 0 & 0.3 & 1 \end{bmatrix}$$

$$\mu_{1.} = [1 \quad 5] \qquad \mu_{2.} = [3 \quad 7] \qquad \mu_{3.} = [5 \quad 9]$$

The covariance matrix was taken to be a diagonal matrix containing the variances of the observation vector elements, and all the variances were taken to be equal to 2.

$$c = \begin{bmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \\ 0.2 & 0.8 \end{bmatrix}$$

This model was used to generate 100 training observation sequences which were used in the training process. The reestimated parameters are

$$\hat{A} = \begin{bmatrix} 0.802 & 0 & 0 \\ 0.198 & 0.701 & 0 \\ 0 & 0.299 & 1 \end{bmatrix}$$

$$\mu_{1.} = [0.55 \quad 4.21] \qquad \mu_{2.} = [3.25 \quad 6.55] \qquad \mu_{3.} = [6.56 \quad 9.62]$$

$$\hat{c} = \begin{bmatrix} 0.65 & 0.35 \\ 0.51 & 0.49 \\ 0.42 & 0.58 \end{bmatrix}$$

$$\hat{C}_{1.} = [2.38 \quad 1.22] \qquad \hat{C}_{2.} = [1.31 \quad 1.27] \qquad \hat{C}_{3.} = [1.07 \quad 2.24]$$

Figures 22-24 show the plots of the true pdf and the reestimated observation pdf (shown by the continuous curve) for all three states. We observe that the reestimated pdf is significantly different from the true pdf. Figures 25-27 show the true histogram and the histogram of the observations based on Viterbi decoding for the three states. We see that the histogram based on Viterbi decoding is reasonably similar to the true hisogram. Yet the estimated pdfs in Figures 22-24 are quite different from the true pdfs. This indicates that, although the observations have been assigned to the appropriate states, due to the overlapping mixture density components, the sample mean and sample variance may not be good estimates for the mean and variance of the mixture density components.

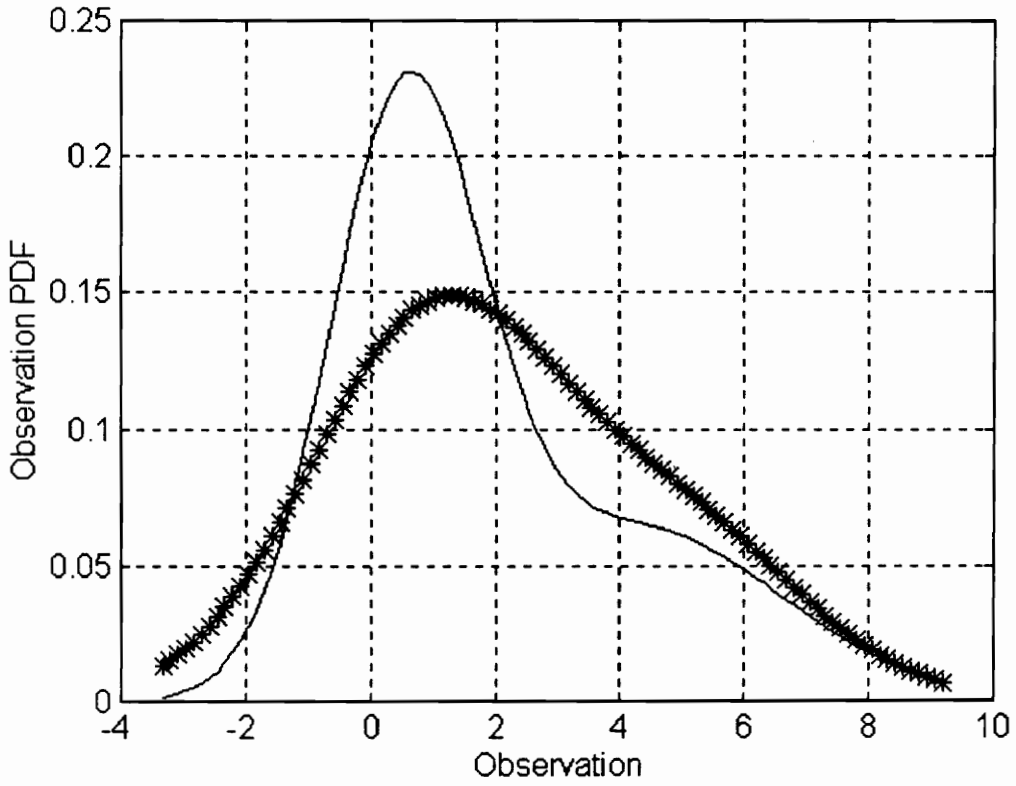

Figure 22. True (*) and Reestimated Observation PDFs for State 1

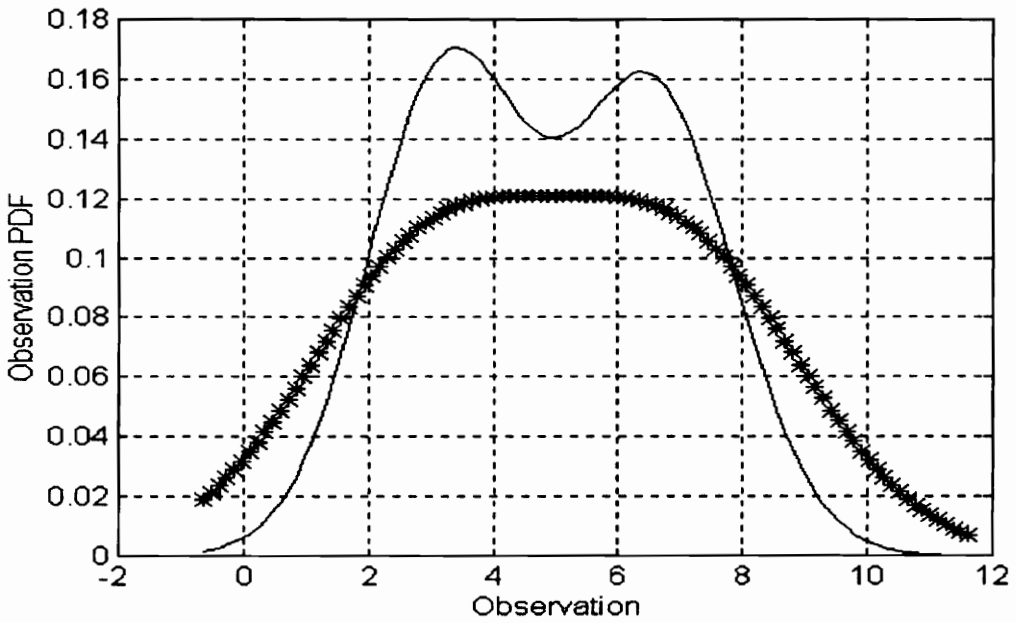

Figure 23. True (*) and Reestimated Observation PDFs for State 2

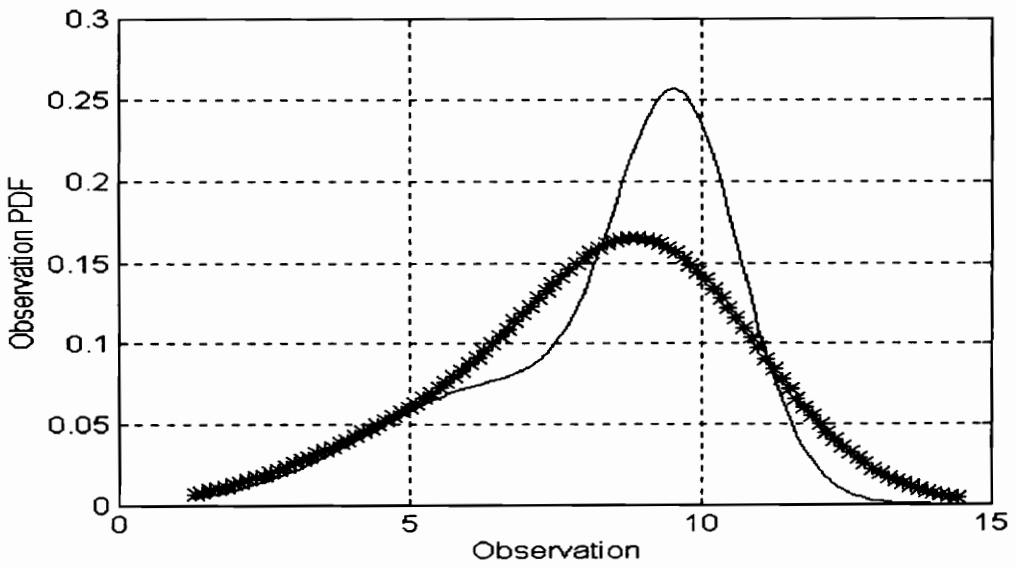

Figure 24. True (*) and Reestimated Observation PDFs for state 3

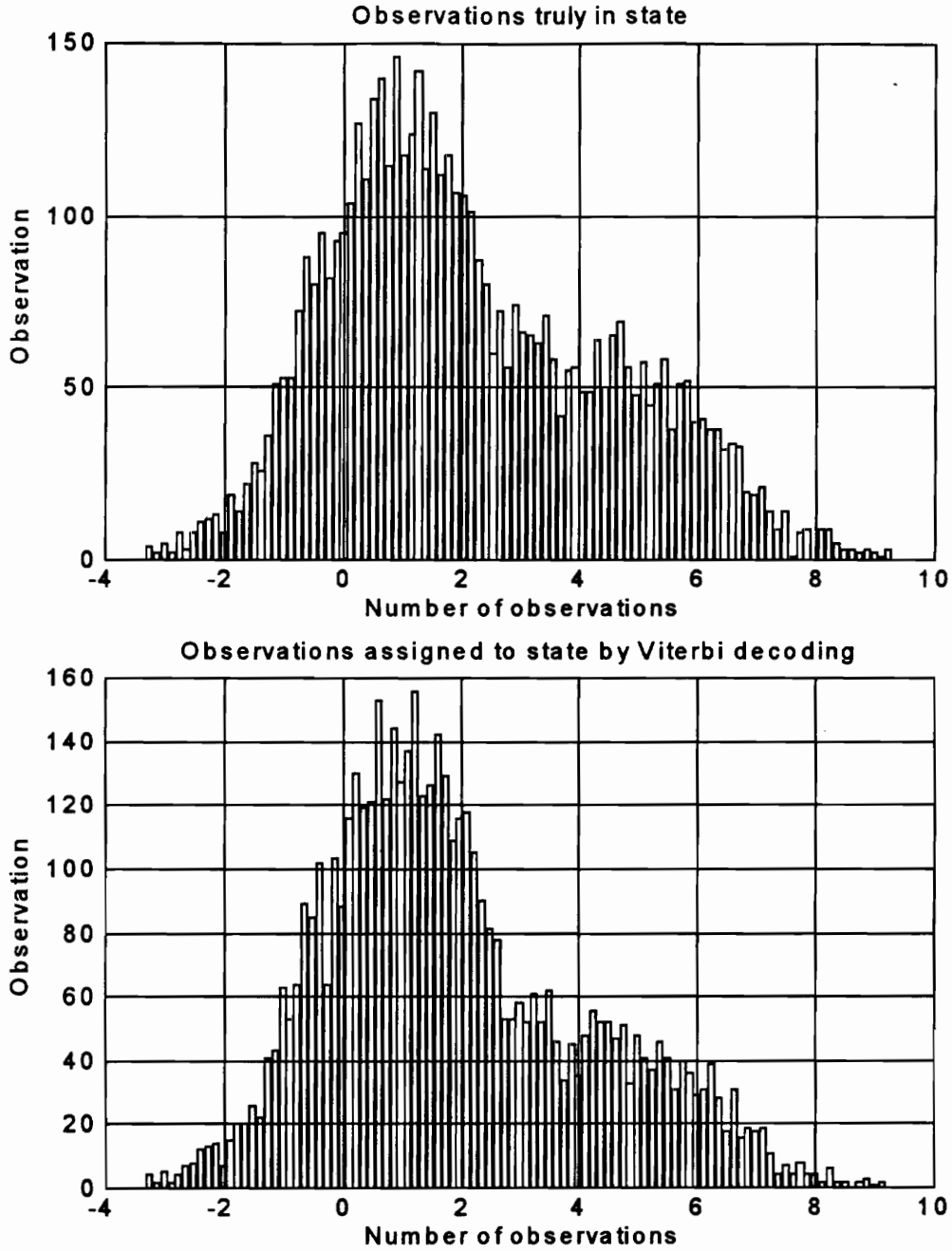

Figure 25. Histogram for State 1

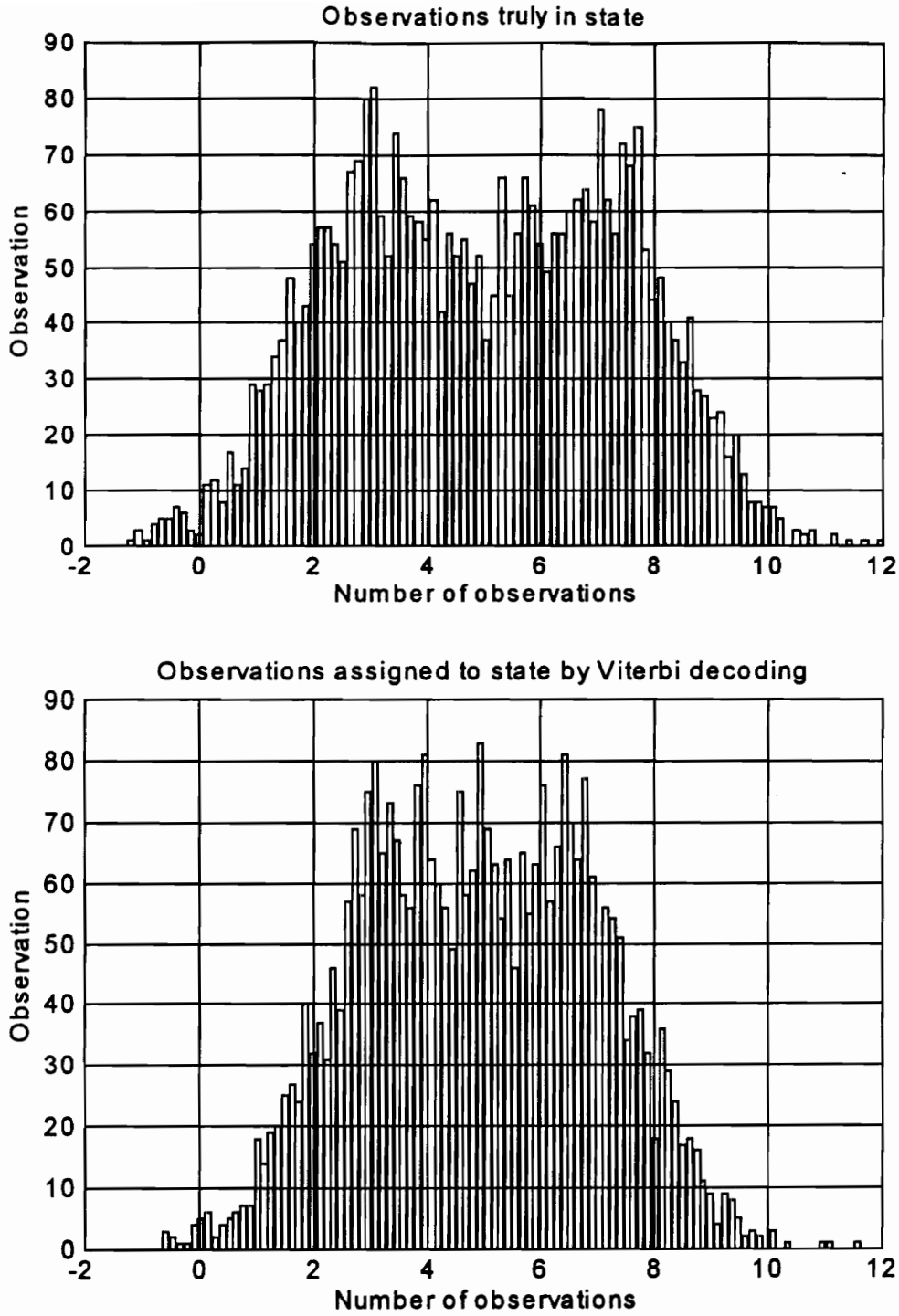

Figure 26. Histogram for State 2

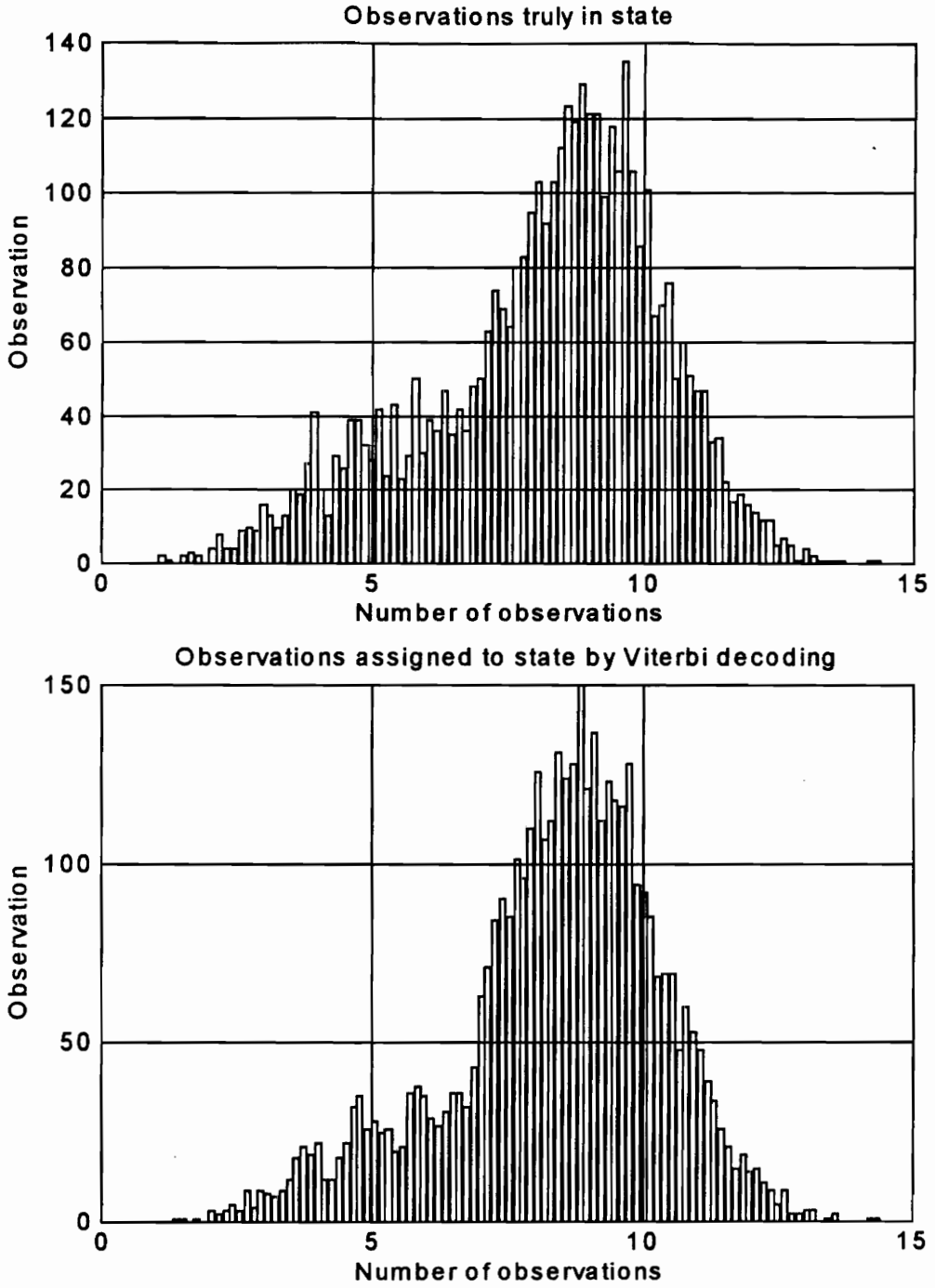

Figure 27. Histogram for State 3

APPENDIX B

We now present all the results obtained during testing of the performance of our recognition system. The results presented are for the training set and the testing set.

Table 9. Recognition rates for M = 1, S = 5 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	49	1	0	0	0	0	0	0	0
2	0	0	48	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	49	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	1	1	0	0	1	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 494

Table 10. Recognition rates for M = 1, S = 5 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	47	1	0	0	0	0	0	0	0
2	0	1	46	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	2	2	0	49	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	49	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	0	1	0	1	0	1	0	50	2
9	0	0	0	0	0	0	0	0	0	48

Figure of merit = 488

Table 11. Recognition rates for M = 2, S = 5 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	49	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	49	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	1	0	0	0	1	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 496

Table 12. Recognition rates for M = 2, S = 5 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	47	2	0	0	0	0	0	0	0
2	0	0	47	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	2	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	49	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	1	1	0	0	0	1	0	50	2
9	0	0	0	0	0	0	0	0	0	48

Figure of merit = 490

Table 13. Recognition rates for M = 4, S = 5 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	49	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	1	0	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 498

Table 14. Recognition rates for M = 4, S = 5 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	48	2	0	0	0	0	0	0	0
2	0	0	48	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	2	0	0	49	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	49	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	0	0	0	1	0	1	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 493

Table 15. Recognition rates for M = 1, S = 6 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	49	1	0	0	0	0	0	0	0
2	0	0	48	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	49	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	1	1	0	0	1	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 494

Table 16. Recognition rates for M = 1, S = 6 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	47	2	0	0	0	0	0	0	0
2	0	1	46	0	0	0	0	0	0	0
3	0	1	0	50	0	0	0	0	0	0
4	0	1	2	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	0	0	0	0	0	0	0	50	2
9	0	0	0	0	0	0	0	0	0	48

Figure of merit = 490

Table 17. Recognition rates for M = 2, S = 6 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	50	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	0	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 499

Table 18. Recognition rates for M = 2, S = 6 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	49	2	0	0	0	0	0	0	0
2	0	0	45	0	0	0	0	0	0	0
3	0	1	0	50	0	0	0	0	0	0
4	0	0	2	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	0	1	0	0	0	0	0	50	2
9	0	0	0	0	0	0	0	0	0	48

Figure of merit = 491

Table 19. Recognition rates for M = 4, S = 6 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	50	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	0	0	0	0	0	0	50	0
9	0	0	0	0	0	0	0	0	0	50

Figure of merit = 500

Table 20. Recognition rates for M = 4, S = 6 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	48	1	0	0	0	0	0	0	0
2	0	1	48	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	1	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	1	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 495

Table 21. Recognition rates for M = 1, S = 7 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	48	1	0	0	0	0	0	0	0
2	0	0	48	0	0	0	0	1	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	1	0	0	50	0	0	0	0	0
5	0	0	0	0	0	49	0	1	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	48	0	0
8	1	1	1	0	0	1	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 491

Table 22. Recognition rates for M = 1, S = 7 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	47	1	0	0	0	0	0	0	0
2	0	1	47	1	0	0	0	0	0	0
3	0	0	0	49	0	0	0	0	0	0
4	0	2	1	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	0	1	0	0	0	0	0	50	2
9	0	0	0	0	0	0	0	0	0	48

Figure of merit = 490

Table 23. Recognition rates for M = 2, S = 7 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	50	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	49	0	1	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	49	0	0
8	0	0	0	0	0	1	0	0	50	0
9	0	0	0	0	0	0	0	0	0	50

Figure of merit = 498

Table 24. Recognition rates for M = 2, S = 7 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	49	2	0	0	0	0	0	0	0
2	0	0	47	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	1	0	0	0	0	0	50	0	0
8	0	0	1	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 495

Table 25. Recognition rates for M = 4, S = 7 (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	50	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	0	0	0	0	0	0	50	0
9	0	0	0	0	0	0	0	0	0	50

Figure of merit = 500

Table 26. Recognition rates for M = 4, S = 7 (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	49	1	0	0	0	0	0	0	0
2	0	0	48	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	1	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	1	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 496

Table 27. Recognition rates for M = 1, S = X (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	49	1	0	0	0	0	0	0	0
2	0	0	49	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	49	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	1	1	0	0	0	1	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 495

Table 28. Recognition rates for M = 1, S = X (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	49	0	0	0	0	0	0	0	0	0
1	0	48	2	0	0	0	0	0	0	0
2	0	0	46	1	0	0	0	0	0	0
3	0	0	0	49	0	0	0	0	0	0
4	0	2	1	0	49	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	1	0	0	50	0	0
8	1	0	1	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 490

Table 29. Recognition rates for M = 2, S = X (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	50	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	0	0	0	0	0	0	50	0
9	0	0	0	0	0	0	0	0	0	50

Figure of merit = 500

Table 30. Recognition rates for M = 2, S = X (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	49	2	0	0	0	0	0	0	0
2	0	0	46	1	0	0	0	0	0	0
3	0	0	0	49	0	0	0	0	0	0
4	0	0	1	0	49	0	0	0	0	0
5	0	0	1	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	1	0	0	50	0	0
8	0	1	0	0	0	0	0	0	50	1
9	0	0	0	0	0	0	0	0	0	49

Figure of merit = 492

Table 31. Recognition rates for M = 4, S = X (training set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	50	0	0	0	0	0	0	0	0
2	0	0	50	0	0	0	0	0	0	0
3	0	0	0	50	0	0	0	0	0	0
4	0	0	0	0	50	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	0	0	0	0	0	0	50	0
9	0	0	0	0	0	0	0	0	0	50

Figure of merit = 500

Table 32. Recognition rates for M = 4, S = X (testing set).

	Input words for recognition									
Output	0	1	2	3	4	5	6	7	8	9
0	50	0	0	0	0	0	0	0	0	0
1	0	49	3	0	0	0	0	0	0	0
2	0	0	46	0	0	0	0	0	0	0
3	0	0	0	50	1	0	0	0	0	0
4	0	0	1	0	49	0	0	0	0	0
5	0	0	0	0	0	50	0	0	0	0
6	0	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	0	50	0	0
8	0	0	0	0	0	0	0	0	50	1
9	0	1	0	0	0	0	0	0	0	49

Figure of merit = 493

where S = X indicates that different number of states have been used for the different digits according to the following table.

Table 33. Number of States in S = X

Digit	0	1	2	3	4	5	6	7	8	9
# States	18	7	10	10	10	10	7	7	15	14

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Vita

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