Willans Line Based Equivalent Consumption Minimization Strategy for Charge Sustaining Hybrid Electric Vehicle

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(ABSTRACT)

Energy management strategies for charge sustaining hybrid electric vehicles reduce fuel power consumption from the engine and electric power consumption from the motor while meeting output power demand. The equivalent consumption minimization strategy is a real time control strategy which uses backward facing models and an equivalence ratio to calculate the lowest total fuel power consumption. The equivalence ratio quantifies the fuel power to battery power conversion process of the hybrid electric vehicle components and therefore quantifies electric power consumption in terms of fuel power consumption. The magnitude of the equivalence ratio determines when the hybrid electric vehicle commands a conventional, electric, or hybrid mode of operation. The equivalence ratio therefore influences the capability of the control strategy to meet charge sustaining performance. Willans line models quantify the input power to output power relationship for powertrain and drivetrain components with a linear relationship and a constant offset. The hybrid electric vehicle model performance is characterized using three Willans line models in the equivalent consumption minimization strategy. The slope of the Willans line models, or marginal efficiency, is used to generate a single equivalence ratio which quantifies the fuel to battery energy conversion process for the hybrid electric vehicle. The implementation of a Willans line based equivalent consumption minimization strategy reduces total fuel power consumption while achieving charge sustaining performance over mild and aggressive drive cycles.

Willans Line Based Equivalent Consumption Minimization Strategy for Charge Sustaining Hybrid Electric Vehicle

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(GENERAL AUDIENCE ABSTRACT)

The charge sustaining hybrid electric vehicle in this paper generates output power with an internal combustion engine powered by a fuel tank and an electric traction motor powered by a battery pack. Hybrid electric vehicle energy management strategies generate torque commands to meet output power demand based on the minimum total input power from both the fuel tank and battery pack. Willans line models simplify the energy management strategy by quantifying the output power to input power relationship, or efficiency, of each component with a linear slope and constant offset. The use of Willans line models quantifies the efficiency of the hybrid electric vehicle with three linear relationships. Energy management strategies also ensure the battery pack starts and ends at the same operating condition to maintain charge sustaining performance. Charge sustaining hybrid electric vehicles therefore use the battery pack as an energy buffer and do not need to be charged by an external power supply since all energy comes from fuel. The output to input power relationship of Willans line models quantifies the power conversion of the hybrid electric vehicle and coupled to a term which accounts for changes in the battery pack. The use of Willans line models in hybrid electric vehicles effectively generates torque commands to the engine and motor while improving fuel economy and maintaining charge sustaining performance.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Research has been conducted to develop energy management strategies which reduce the energy consumption of hybrid electric vehicles. The equivalent consumption minimization strategy is a real time implementable hybrid electric vehicle energy management strategy. The equivalent consumption minimization strategy uses a backward facing powertrain model to calculate the equivalent input fuel power required to meet output power demand at each instantaneous time step. The equivalent consumption minimization strategy discretizes the backward facing powertrain model into an array of operating points which are used to calculate the input power cost at each discrete operating point [1]. The limits of the discretized operating points align with the limits of the powertrain components to ensure physical component limitations are not exceeded. The discretized array of input power operating points is used to select the input power operating point with the lowest magnitude or cost to reduce fuel consumption. The equivalent consumption minimization strategy selects the minimum input power for the internal combustion engine and electric traction motor at each time step by relating the consumption of chemical power from fuel to the consumption of electric power from the battery pack with an equivalence ratio. The equivalence ratio therefore calculates the equivalent fuel power consumption required from the battery pack. Once the input power with the minimum total input fuel power is identified, the hybrid supervisory controller commands the internal combustion engine torque and electric traction motor torque associated with the minimum input fuel power operating point to the respective vehicle controllers.

The magnitude of the equivalence ratio guides the equivalent consumption minimization strategy to command a conventional mode of operation which only uses the internal combustion engine, an electric mode of operation which only uses the electric traction motor, or a combination of the two to generate a hybrid mode of operation based upon the instantaneous driver demand. When the equivalence ratio has a lower magnitude value, the value of equivalent fuel input power from the battery pack is reduced, and therefore drives an electric mode of operation. During electric modes of operation the energy from the battery pack is quickly depleted and can impact the battery pack state of charge and charge sustaining performance if that energy is not captured at a later point in time. When the equivalence ratio has a higher magnitude value, the value of equivalent fuel input power from the battery pack is increased, and therefore drives a conventional mode of operation. Therefore the fuel economy and the charge sustaining capability of a hybrid electric vehicle are strongly correlated to the value of the equivalence ratio. Charge sustaining performance will be discussed in a later section.

The first contribution of this paper defines a single equivalence ratio based upon powertrain and drivetrain component models to improve fuel economy while also maintaining charge sustaining performance, independent of the drive cycle. The second contribution of this paper reduces the number of discrete operating points the equivalent consumption minimization strategy cost function evaluates at each instantaneous time step while also directly calculating the associated torque command. The emphasis of this paper is therefore placed on the design and development of the hybrid electric vehicle energy management strategy and is presented as follows. The first chapter presents a high level view of energy management strategies for hybrid electric vehicles. The second chapter evaluates research conducted on hybrid electric vehicle energy management strategies specifically as it pertains to the equivalent consumption minimization strategy. The third chapter addresses the topology of the hybrid electric vehicle. The fourth chapter discusses the implemented driver model and methods used to generate plant models for the hybrid electric vehicle powertrain and drivetrain components. The fifth chapter addresses the background and a detailed description of the equivalent consumption minimization strategy as well as the role charge sustaining performance has in the energy management strategy strategy. The sixth chapter discusses the concept and development of Willans line models for the vehicle powertrain and drivetrain components. The seventh chapter details how the Willans line models are implemented in the equivalent consumption minimization strategy for the hybrid electric vehicle energy management strategy. The eighth chapter presents and analyzes the results of the Willans line based equivalent consumption minimization strategy. The ninth chapter details the conclusions from the Willans line based equivalent consumption minimization strategy.

Chapter 2

Review of Literature

The definition of the equivalent consumption minimization strategy requires the use of backward facing models and forward facing models to generate torque commands to the appropriate components on the hybrid electric vehicle [1]. The torque commanded to each component is selected to achieve the output power demand. The equivalent consumption minimization strategy uses a single implementation method to generate torque commands. The backward facing models in the equivalent consumption minimization strategy calculate an array of input power values which range the from maximum to minimum limitations of each component. The array of input powers of each component are added together to meet the output power demand. The combination of input powers which are then fed through a forward facing model to generate a torque command may not respect constraints imposed on the hybrid electric vehicle control strategy. Constraints on the equivalent consumption minimization strategy include but not limited to current output limits, torque output limits, and battery pack state of charge limits. The equivalent consumption minimization strategy can therefore generate operating points which are invalid and can not be used to generate torque commands to the appropriate components. The number of discrete points evaluated between the maximum and minimum operating limits is a configurable parameter in the equivalent consumption minimization strategy. The number of invalid points is therefore expected to increase as the number of discrete points between the maximum and minimum operating limits increases as well.

The equivalence ratio is a parameter associated with the equivalent consumption minimization strategy and is an area of interest because it influences the fuel economy and charge sustaining performance of a hybrid electric vehicle. Prior research associated with the equivalent consumption minimization strategy focuses on adaptive or variable equivalence ratio determination. Adaptive equivalence ratio determination is defined as a controller which requires prior real time drive cycle knowledge to predict the instantaneous equivalence ratio. A variable equivalence ratio is defined as a controller which does require prior real time drive cycle knowledge to predict the instantaneous equivalence ratio.

The adaptive equivalence ratio method in [2] defines a battery pack state of charge penalty factor which is added to the instantaneous equivalence ratio. The variation in battery pack state of charge is evaluated over a specific prior time interval and updates the equivalence ratio based on the variation at each time interval. The equivalence ratio therefore changes in the equivalent consumption minimization strategy calculations for the next time interval in the drive cycle. The method detailed in [2] therefore changes the definition of the equivalence ratio over the course of a drive cycle. While this method requires additional computational power to manage prior equivalence ratio state of charge performance, the control strategy presented in [2] yields the desired reduction in energy consumption and maintains charge sustaining performance in parallel.

The adaptive equivalence method in [3] uses a neural network trained by dynamic programming simulation results to predict future value of the equivalence ratio in the equivalent consumption minimization strategy. The neural net in [**?**] uses real time information to generate equivalence ratio values and is therefore defined as an adaptive equivalence ratio. The equivalence ratio quantified in [3] evaluates the powerflow of the electric components to determine the value of the equivalence ratio over a finite amount of time. The neural network based equivalence ratio yields favorable fuel economy results for different driving scenarios.

The ability of the neural network based equivalence ratio to maintain charge sustaining performance however is not specifically confirmed or quantified with this methodology.

The variable equivalence ratio method in [4] defines a lower bound equivalence ratio based upon the lower heating value of the fuel and an upper bound equivalence ratio based upon the lower heating value of the fuel and the efficiencies of the powertrain components. The variable equivalence ratio varies between the upper and lower bounds by using a proportional-integral-derivative controller to regulate the error between the actual and reference battery pack state of charge. The equivalent consumption minimization strategy presented in [4] uses an input power cost function which requires an iterative approach to tune the initial value of the equivalence ratio to achieve desirable results. The performance of the equivalence ratio in [4] solely focuses on improving the fuel economy and does not focus on maintaining charge sustaining performance because analysis is not performed on a charge sustaining hybrid electric vehicle.

The equivalent consumption minimization strategy is coupled to a rule-based strategy in [5] which uses an equivalence ratio lookup table to generate torque command to vehicle components. A particle swarm optimization is used in [5] to quantify the optimal equivalence ratio for the instantaneous power demand and battery pack state of charge at each point over a given drive cycle. The particle swarm optimization is therefore independent of drive cycle since it evaluates instantaneous output power demand and battery pack state of charge. The performance of the equivalent consumption minimization based strategy in [5] focuses on using the variable equivalence ratio to improve fuel economy while also achieving charge sustaining performance.

Chapter 3

Hybrid Electric Vehicle Topology

This paper evaluates a hybrid electric Chevrolet Blazer which is converted from a conventional Chevrolet Blazer. A naturally aspirated internal combustion engine coupled to a nine speed automatic transmission is powered by gasoline fuel to generate torque on the front axle of the hybrid electric vehicle. An electric traction motor coupled to a single speed gearbox is powered by a battery pack to generate positive and negative torque on the rear axle of the hybrid electric vehicle. Other than through the road, the engine on the front axle and the motor on the rear axle are not coupled to one another. The topology of the hybrid electric vehicle is depicted in Figure 3.1. The engine has a power rating of approximately 150 kW and a maximum torque of 225 Nm. The motor has a peak power rating of approximately 80 kW and a maximum torque of 250 Nm. The battery pack has a peak power rating of 90 kW and a total energy capacity of 5.0 kWh with a usable energy capacity of approximately 2.0 kWh. Table 3.1 details the specifications of the hybrid electric Chevrolet Blazer components. This paper does not address the selection process for the components integrated into the hybrid electric vehicle.

Table 3.1: Hybrid electric Chevrolet Blazer component specifications.

Component	Power (kW)	Torque (Nm)	Energy Capacity (kWh)	Usable Energy (kWh)
Engine	150	225	NA	NA
Motor	80	250	NA	NA
Battery Pack	90	NA	5.0	2.0

The topology of the hybrid electric vehicle has five distinct modes of operation. The hybrid

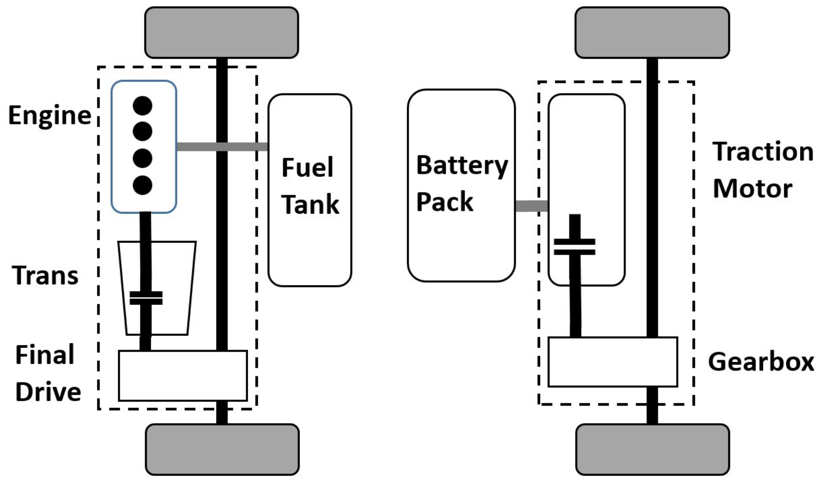

Figure 3.1: Powertrain and drivetrain component orientation as well as associated energy sources for the hybrid electric vehicle.

electric vehicle can operate with an engine only mode of operation, electric traction motor propulsion mode of operation, electric traction motor generator mode of operation, engine assisting mode of operation, and engine loading mode of operation. Each hybrid electric vehicle mode of operation has a unique powerflow. Powerflow is defined as the path in which the power conversion process takes place. Engine only mode uses chemical power from the fuel to produce mechanical output power on the front axle. Electric traction propulsion mode uses electric power from the battery pack to produce mechanical output power on the rear axle. Electric traction motor generator mode (regenerative braking) uses the mechanical power from the hybrid electric vehicle to generate electric output power on the rear axle. Electric traction motor generator mode charges the battery pack and is the only method the vehicle has to capture electric power. Engine assisting mode uses the chemical power from fuel to produce mechanical output power on the front axle and the electric power is used to produce mechanical output power on the rear axle. Engine loading mode uses the chemical power from fuel to produce mechanical output power on the rear axle and the electric traction motor acts as a generator by commanding negative mechanical power to produce electric power on the rear axle. When the electric traction motor is acting as a generator in engine loading mode of operation, the internal combustion engine must generate additional torque to meet output power. The selected mode of operation is based upon the output power demand and fuel consumption minimization. The mode of operation selection process will be discussed in detail in the following chapters.

Chapter 4

Hybrid Electric Vehicle Model

4.1 Driver Model

The performance of the hybrid electric vehicle is evaluated with the use of a known velocity profile known as a drive cycle. The drive cycle velocity profile demands a specific output power which is based on the characteristics of the hybrid electric vehicle. It is the responsibility of the driver model to generate a command to the hybrid electric vehicle controller to produce an actual output power which tracks the desired output power. The actual output power produced by the hybrid electric vehicle propels the vehicle at a specific velocity. It is therefore the responsibility of the driver model to command the hybrid electric vehicle to generate a specific output power which regulates the output velocity of the vehicle to the reference or drive cycle velocity.

A model predictive controller is implemented to generate commands to the hybrid vehicle controller and track the reference velocity profile. Model predictive controllers use the definition of a dynamic system to predict its desired behavior over a finite time horizon. The dynamic system for this model predictive controller is the hybrid electric vehicle and attributes associated with ability of a human driver to respond to changes in the drive cycle velocity profile. The implemented model predictive controller utilizes the hybrid electric vehicle powertrain, driver preview distance, and driver response time to characterize the dynamic system. The desired behavior of the dynamic system is to maintain an output velocity consistent with the drive cycle velocity profile. The longitudinal driver model for the hybrid electric vehicle model generates an accelerator pedal position which the hybrid electric vehicle controller interprets and produces a wheel torque or output power demand to meet the output power required by the drive cycle velocity profile. The resulting output velocity generated by hybrid electric vehicle is fed back into the model predictive controller longitudinal driver model to predict the control decision or output power demand over the next finite time horizon. Reference Appendix A for more detail on the model predictive controller driver model. [6]

The emphasis of this paper is not placed on the model predictive controller for the longitudinal driver model. The model predictive controller facilitates the development of the simulation environment needed to evaluate the performance of the hybrid electric vehicle and associated energy management strategy. The emphasis of this paper is placed on the hybrid electric vehicle energy management strategy which will be discussed in the following chapters.

4.2 Vehicle Plant Model

A tractive force model is used to quantify the forces acting against the hybrid electric vehicle. The tractive force model determines the forces the hybrid electric vehicle powertrain and drivetrain components must generate to overcome the forces acting against the vehicle and meet output power demand. Equation 4.1 quantifies the tractive force acting against the hybrid electric vehicle. Grade forces can be included in the tractive force equation but were not considered in this paper.

$$F_{tr} = A + BV + CV^2 + ma \tag{4.1}$$

In Equation 4.1, F_{tr} is the total tractive force acting against the vehicle, V is the velocity of the vehicle, A, B, and C are the road load coefficients, m is the mass of the vehicle, and a is the acceleration of the vehicle. The A term in the tractive force equation is equivalent to a mass dependent rolling resistance force acting against the vehicle, the B term is a velocity dependent rolling resistance force acting against the vehicle, and the C term is the aerodynamic drag dependent force acting against the vehicle. Environmental Protection Agency road load coefficients [7] for an AWD V6 conventional Chevrolet Blazer are selected for simulation purposes. The AWD V6 conventional Chevrolet Blazer mass dependent rolling resistance term is scaled by the mass of the hybrid electric Chevrolet Blazer to account for the mass of the electric traction motor and battery pack. The remaining AWD V6 conventional Chevrolet Blazer road load coefficients are not scaled because they are not associated with the change in mass of the vehicle. Reference Appendix A for more detail on the vehicle plant model.

4.3 Internal Combustion Engine and Transmission Plant Model

The internal combustion engine plant model uses steady state dynamometer test data to quantify the performance of the 2.5L internal combustion engine. Test data obtained by the Environmental Protection Agency [8] on the same 2.5L engine integrated in the hybrid electric Chevrolet Blazer is used to generate a brake specific fuel consumption map depicted in Figure 4.1. The brake specific fuel consumption map depicts the maximum engine torque produced at each tested engine speed, contour lines of constant brake specific fuel consumption, and the optimal brake specific fuel consumption point or the engine operating point with highest efficiency. The instantaneous speed and torque of the engine determine the brake specific fuel consumption.

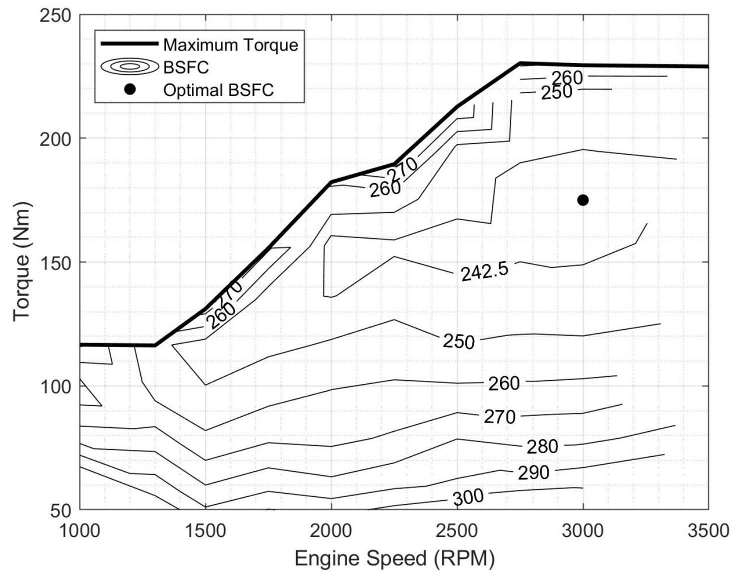

Figure 4.1: Internal combustion engine brake specific fuel consumption map.

An accessory load is added to the engine load to account for the additional load from the alternator imposed on the engine. The added electrical load from the alternator is translated into a mechanical load on the engine using a power balance model. The additional mechanical load on the engine ensures the accessories associated with the conventional vehicle and the low voltage electrical load associated with hybrid vehicle components remain powered during operation of the hybrid electric vehicle. The additional mechanical load on the engine is approximately 1 kW when idle engine speeds exceed 700 RPM. Reference Appendix A for more detail on the engine plant model.

The nine speed automatic transmission uses steady state dynamometer data to quantify the efficiency of the transmission. Test data obtained by General Motor is used to generate a four-dimensional lookup table which quantifies the efficiency of the transmission. The four-dimensional lookup table uses the engine torque, engine speed, the transmission gear, and the temperature to quantify the efficiency of the transmission.

4.4 Electric Traction Motor and Single Speed Gearbox Plant Model

The electric traction motor system is comprised of the electric traction motor, inverter, and single speed gearbox. A power balance model partially characterizes the efficiency of the electric traction motor system. Equation 4.2 details the power balance model which characterizes the performance of the electric traction motor and inverter.

$$P_{output} = P_{input} - P_{loss} \tag{4.2}$$

In Equation 4.2, P_{output} is the mechanical power produced by the electric traction motor and inverter, P_{input} is the electrical power produced by the battery pack, and P_{loss} are the losses inherent to the electric traction motor and inverter. The same power balance model is used to characterize the performance of the electric traction motor and inverter when they behave as a motor and a generator. The losses associated with the electric traction motor and inverter are quantified in Equation 4.3.

$$P_{loss} = k_c T^2 + k_i \omega + k_w \omega^3 + C_{loss} \tag{4.3}$$

In Equation 4.3, k_c is the copper loss coefficient, T is the torque output of the electric traction motor, k_i is the iron loss coefficient, ω is the rotational speed output of the electric traction motor, k_w is the electric traction motor windage loss coefficient, and C_{loss} is the constant loss term associated with the electric traction motor. The same loss model is used to characterize the losses when the electric traction motor and inverter behave as a motor and a generator.

Data provided by the manufacturer required interpolation did not properly characterize the performance of the electric traction motor and inverter. An electric traction motor and inverter with known characteristics is scaled to develop a refined efficiency map which properly characterizes the performance of the electric traction motor and inverter. The data provided by the manufacturer is used to ensure the refined motor efficiency map is similar to the manufacturer data. Figure 4.2 depicts the refined motor efficiency map. Reference Appendix A for more detail on the motor and inverter plant model.

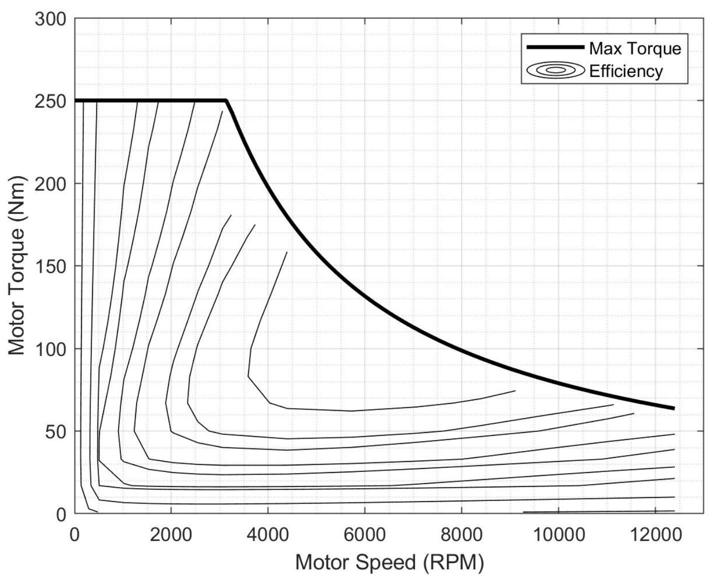

Figure 4.2: Electric traction motor efficiency map.

Performance data for the single speed gearbox is not provided by the supplier. However, the performance of the single speed gearbox is assumed to have slightly better performance than the nine speed automatic transmission since it is a drivetrain component without multiple gears. The single speed gearbox is assumed to have an average operating efficiency of approximately 98%. The performance of the single speed gearbox is modeled using a linear trend with a constant offset.

4.5 Battery Pack Plant Model

The battery pack plant model quantifies the power and the state of charge of the battery pack. The battery pack current output, voltage output, and internal resistance of the battery pack calculates the battery pack power. Equation 4.4 details the instantaneous current output of the battery pack.

$$I(t) = \frac{V(t)}{2R_0} - \sqrt{\left(\frac{V(t)}{2R_0}\right)^2 - \frac{P_{output}(t)}{R_0}} \tag{4.4}$$

In Equation 4.4, $V(t)$ is the instantaneous voltage output of the battery pack, R_0 is the internal resistance of the battery pack, and P_{output} is the instantaneous power demand. The battery pack is assumed to have a constant internal resistance over the narrow state of charge range and normal expected operating temperatures. The power output of the battery pack is dependent upon the instantaneous voltage of the battery pack. The instantaneous voltage of the battery pack is a function of the battery pack state of charge. The state of charge of the battery pack is measured by relating the instantaneous stored charge to the total charge capacity of the battery pack. The battery pack is assumed to have a constant total charge capacity. Figure 4.3 depicts the relationship between the state of charge and instantaneous voltage of the battery pack.

The battery pack state of charge is a function of the amount of current discharged from the battery pack or current charged to the battery pack. The rate of change of battery pack state of charge is detailed in Equation 4.5.

$$S\dot{O}C = -\frac{I(t)}{\eta_{coul}\, Q_{nom}} \tag{4.5}$$

In Equation 4.5, $S\dot{O}C$ represents the rate of change of the battery pack state of charge, $I(t)$ is the instantaneous current produced by the battery pack, η_{coul} is the Coulombic efficiency

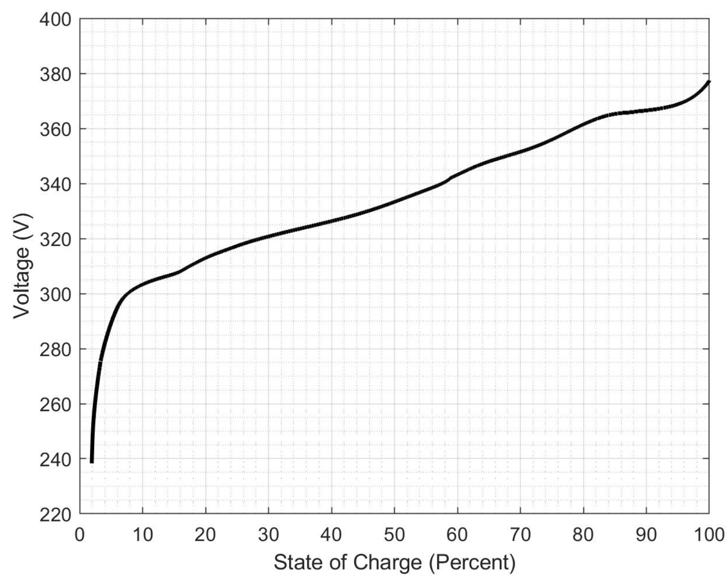

Figure 4.3: Battery pack voltage as a function of battery pack state of charge.

of the battery pack, and Q_{nom} is the total charge capacity of the battery pack. Reference Appendix A for more detail on the battery pack plant model.

Chapter 5

Energy Management Strategy

Energy management control strategies for hybrid electric vehicles seek to achieve optimal or minimal energy consumption while meeting output power demand. Optimal energy consumption is calculated at each time step in the form of a cost function. The minimum value of the cost function dictates the commands generated for the appropriate vehicle controllers and does so at each time step.

Dynamic programming is a control algorithm which calculates the optimal energy consumption with a backward facing model. Since dynamic programming uses a backward facing model it requires complete knowledge of a simulation or drive cycle before running simulations. Dynamic programming therefore can not be implemented on a real time controller. Local and global constraints on battery pack state of charge, component limits, and other factors can be imposed on the dynamic programming algorithm. Dynamic programming guarantees optimal results and therefore respects local and global constraints when calculating the minimal energy consumption in a simulation or over a drive cycle. Reference Appendix B for more detail on dynamic programming.

Pontryagin's minimum principle is an optimal control algorithm based upon the fundamentals of dynamic programming but is calculated in a forward facing model. Pontryagin's minimum principle therefore can be implemented on a real time controller. However, an iterative tuning method is used in conjunction with Pontryagin's minimum principle to obtain optimal results which cannot be instantaneously performed on a real time controller. Similar to dynamic programming, Pontryagin's minimum principle is capable of respecting local and global constraints on the aspects such as battery pack state of charge and component limits, but still requires tuning to guarantee optimal results. Reference Appendix B for more detail on Pontryagin's minimum principle.

The equivalent consumption minimization strategy attempts to solve the optimal control problem by computing a forward facing instantaneous minimization calculation. Local constraints on battery pack state of charge can be imposed on the equivalent consumption minimization strategy. However, global constraints can not be placed on the equivalent consumption minimization strategy because the instantaneous nature of the algorithm can not guarantee optimal results. The equivalent consumption minimization strategy can therefore only produce optimal results relative to the capabilities of the algorithm. For instance, if the control algorithm exceeds the limitation of the local constraint on battery pack state of charge, the equivalent consumption minimization strategy does not yield optimal results. Although the equivalent consumption minimization strategy yields sub-optimal results, the results can be approximately the similar to the results obtained by dynamic programming and Pontryagin's minimum principle based energy management strategies.

The equivalent consumption minimization strategy calculates the minimum input power at each instantaneous time step. The calculation of the minimal input power to meet output power demand requires knowledge of the powertrain component capabilities with a backward facing model. The backward facing model calculates the input power required to meet the output power demand from each powertrain component and generates the cost function at each instantaneous time step. The equivalent consumption minimization strategy applies an equivalence ratio to the electric power input to the electric traction motor and converts the consumption of electric power into terms of equivalent fuel power supplied to the internal combustion engine. The equivalence ratio represents the fuel to battery power conversion process for the given powertrain components. The equivalence ratio therefore facilitates a calculation of the total equivalent fuel power at each instantaneous time step. The equivalent input power consumption equation for the equivalent consumption minimization strategy is detailed in Equation 5.1.

$$P_{eqv} = P_{fuel} + (s)P_{elec} \tag{5.1}$$

In Equation 5.1, P_{eqv} is the total equivalent fuel input power, P_{fuel} is the fuel input power, s is the equivalence ratio, and P_{elec} is the electric input power. The equivalent fuel input power, whether to the engine or motor, experiences inherent losses while attempting to meet the output power demand. The equivalence ratio therefore also be evaluates the efficiency of the hybrid electric vehicle as it converts energy back and forth between the fuel tank and battery pack.

The equivalent consumption minimization strategy imposes a local constraint on the battery pack state of charge over the course of a simulation or drive cycle. The local constraint on battery pack state of charge maintains charge sustaining performance. Charge sustaining hybrid electric vehicles maintain a small difference in battery pack charge between the initial and final point of a simulation or drive cycle so the hybrid electric vehicle does not need to be charged with an external energy source. The capability of a hybrid electric vehicle control strategy to achieve charge sustaining performance is quantified by the net amount of electric energy consumed to the amount of fuel energy consumed over a drive cycle as detailed in Equation 5.2.

$$\frac{\Delta E_{batt}}{\Delta E_{fuel}} \leq 0.01 \tag{5.2}$$

In Equation 5.2, ΔE_{batt} is the net change in electric energy over a drive cycle, and ΔE_{fuel} is the change in fuel energy over a drive cycle. For a hybrid electric vehicle to remain charge sustaining the net change in electric energy to fuel energy ratio should remain less than one percent over a finite time horizon, typically a drive cycle. When charge sustaining hybrid electric vehicles achieve charge sustaining performance the power used to meet drive cycle output power demands comes from the chemical power from the fuel tank. The battery pack therefore acts as an energy buffer since the net change of electric energy over a drive cycle is approximately zero. There are cases not detailed in this paper in which the charge sustaining metric will not be achieved, such as extended driving scenarios at a grade. Figure 5.1 depicts three different battery pack state of charge trajectories over a drive cycle and Figure 5.2 depicts the associated charge sustaining performance metric associated with each battery pack state of charge variation. The red battery pack state of charge trajectory in Figure

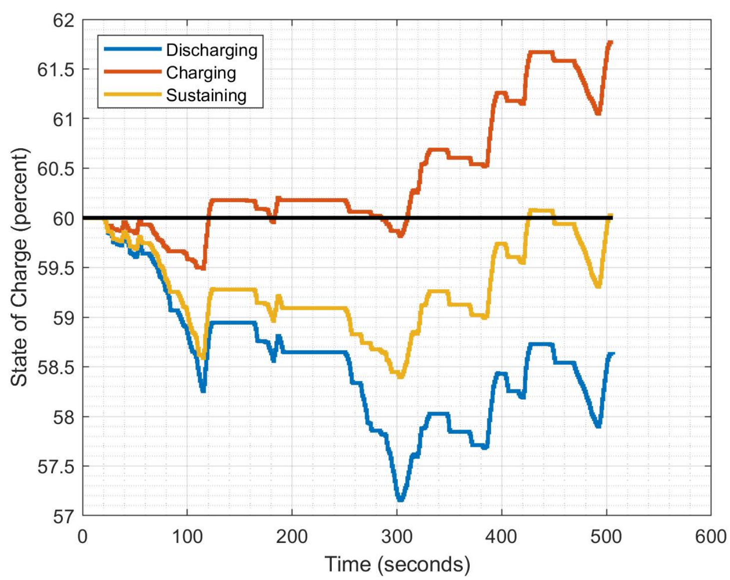

Figure 5.1: Three different battery pack state of charge trajectories over a single drive cycle.

5.1 depicts an energy management strategy which places a higher value on the equivalent input fuel power from the battery pack. The energy management strategy therefore burns too much fuel power from the fuel tank and charges the battery pack. The red dot in Figure 5.2 associated with the red battery pack state of charge trajectory in Figure 5.1 therefore does not achieve charge sustaining performance. The blue battery pack state of charge

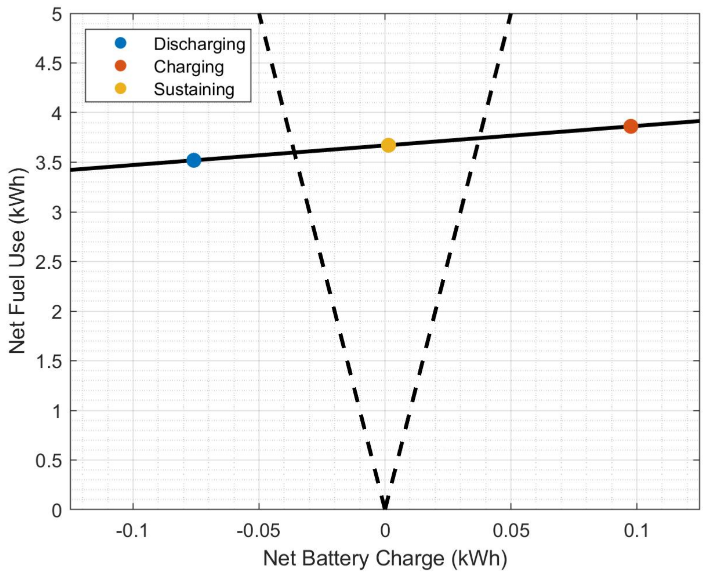

Figure 5.2: Charge sustaining metric for the three different battery pack state of charge trajectories over a single drive cycle.

trajectory in Figure 5.1 and associated blue dot in Figure 5.2 depict an energy management strategy which does not burn enough fuel from the fuel tank and therefore does not achieve charge sustaining performance. The yellow battery pack state of charge in Figure 5.1 and associated yellow dot in Figure 5.2 are indicative of an energy management strategy which appropriately calculates the fuel to battery conversion process over a drive cycle to achieve charge sustaining performance.

The battery pack state of charge constraint constraint is implemented in the energy management strategy to maintain charge sustaining performance. The equivalence ratio quantifies the fuel to battery conversion process on the hybrid electric vehicle, it does not quantify or account for changes in battery pack state of charge. An additional yet separate penalty term is added onto the equivalence ratio to account for variations in the battery pack state of charge. It is important to note, the definition and value of the equivalence ratio does not change, the addition of the penalty factor is a different parameter added to the equivalence ratio. The equivalence ratio with battery pack state of charge penalty factor accounts for both the efficiency of the hybrid electric vehicle and the changes in battery pack state of charge. The penalty factor added to the equivalence ratio in the calculation of the equivalent consumption minimization strategy is detailed in Equation 5.3.

$$P_{eqv} = P_{fuel} + (s + p(SOC))\, P_{elec} \tag{5.3}$$

In Equation 5.3, P_{eqv}, is the equivalent fuel input power, P_{fuel} is the fuel input power, s is the equivalence ratio, $p(SOC)$ is the battery pack state of charge penalty factor, and P_{elec} is the electric input power. The addition of the battery pack state of charge penalty factor behaves as a local constraint to help enforce the charge sustaining performance constraint.

Chapter 6

Willans Line Model

The equivalent consumption minimization strategy uses backward facing models in the vehicle controller to calculate the total required input power to meet the output power demand. Backward facing powertrain and drivetrain models require the use of lookup tables or higher order polynomials to model component performance. Modeling a component with lookup tables or higher order polynomials however is not always feasible on real time vehicle controllers due to storage and processing implications, respectively. Willans line models [9] [10] [11] [12] [13] quantify the performance of powertrain and drivetrain components with a linear relationship. Willans line models achieve a linear relationship by quantifying the amount of input power required to generate a desired output power. Willans lines models quantify the incremental amount of input power required to generate an additional increment amount of output power with a linear relationship which represents the marginal efficiency of the powertrain or drivetrain component. The losses inherent to powertrain or drivetrain components are represented by a constant offset of the linear relationship. The inherent losses must be overcome before output power is produced. The Willans line model is detailed in Equation 6.1.

$$P_{output} = \frac{1}{\eta_m} P_{input} + P_{loss} \tag{6.1}$$

In Equation 6.1, P_{output} is the output power a powertrain or drivetrain component produces, η_m is the marginal efficiency of the powertrain component, P_{input} is the amount of power that must be supplied to generate the output power, and P_{loss} are the losses inherent to the powertrain or drivetrain component. Figure 6.1 depicts a Willans line marginal efficiency and constant offset. The Willans line marginal efficiency is a constant and therefore quantifies the amount of additional input power required to generate an additional amount of output power at any operating point. The efficiency of a powertrain component quantifies the relationship of input power and output power at a single operating point. The Willans line model also captures the entire performance envelope, or efficiency, of a powertrain or drivetrain component as a function of output power. A visual representation of the efficiency and marginal efficiency of a given component are depicted in Figure 6.1. For

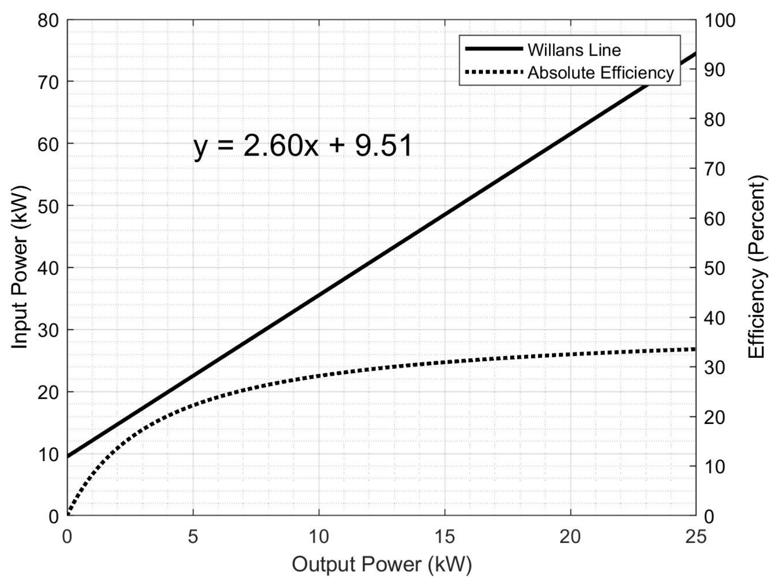

Figure 6.1: Willans line model with associated efficiency.

visual representation, Figure 6.1 characterizes output power up to 25 kW but the linear approximation of the Willans line extends up to the capability of a given powertrain or drivetrain component. As depicted, marginal efficiency and efficiency are not synonymous with one another. Marginal efficiency quantifies the incremental amount of input power required to generate an additional increment of output power. The efficiency quantifies the magnitude of losses that must be overcome at the given output power and how much of the input power is converted into output power. The following sections detail the process used to generate a Willans line model for the previously mentioned powertrain and drivetrain components integrated in the hybrid electric vehicle.

6.1 Internal Combustion Engine

The dynamometer test data from the Environmental Protection Agency [8] used to generate the brake specific fuel consumption map is also used to generate Willans line models for the 2.5L naturally aspirated internal combustion engine. The Willans line model for the engine is developed by calculating the input power to output power relationship. The input power of the engine is quantified by the mass flow rate of fuel into the engine cylinders and the lower heating value of the Tier 3 E10 fuel (42.94 MJ/kg). The output power of an engine is quantified by the torque produced at the given engine speed. The output power of an engine is a function of engine speed, therefore engines have unique Willans line models at discrete engine speeds. The Willans lines model for the discrete engine speeds tested on the 2.5L naturally aspirated internal combustion engine are depicted in Figure 6.2.

The internal combustion engine Willans line models exhibit a linear relationship at lower magnitudes of output power when operating under stoichiometric combustion. Stoichiometric combustion occurs when the amount of mass air flow into each cylinder is chemically balanced with the amount of mass fuel flow into the cylinders of the engine. Willlans line model at higher output power is characterized by an engine operating under enrichment. Enrichment combustion occurs when the amount of mass fuel flow into the engine cylinders exceeds the stoichiometric or chemically balanced amount of mass air flow into the internal combustion engine. Therefore, under enrichment combustion an additional increment of fuel or input power does not generate an additional equal incremental amount of output power.

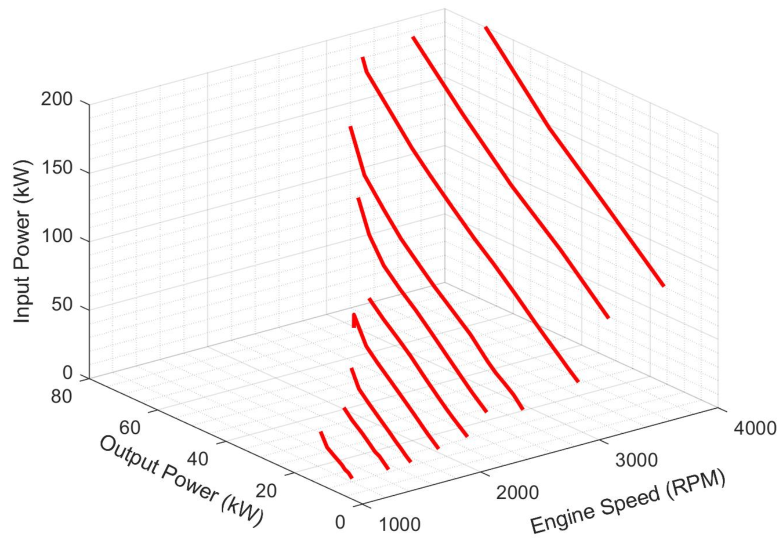

Figure 6.2: Raw internal combustion engine dynamometer test data converted into Willans line model.

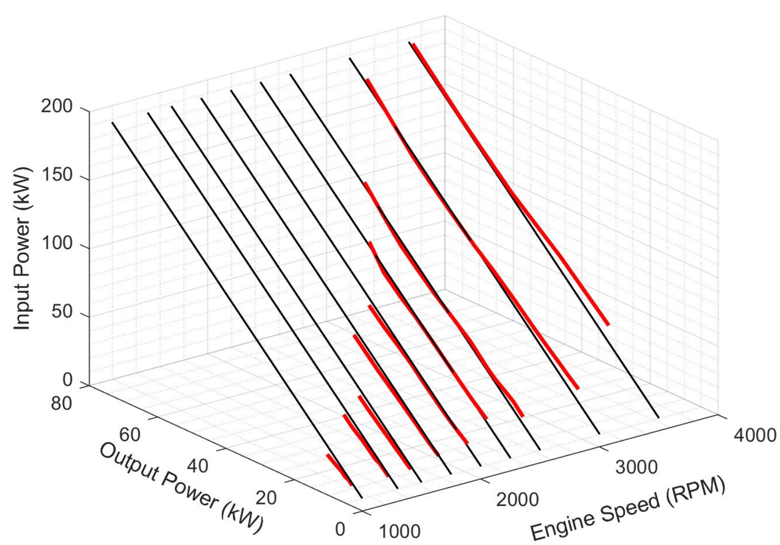

Figure 6.3: Internal combustion engine Willans line models with single Willans line fit model.

Since the enrichment operating region of the engine does not respect the linear Willans line model, it is not accounted for when generating a Willans line model for the discrete engine speeds. The linear Willans line model is a good approximation since the engine does not operate at wide open throttle for extended amounts of time during drive cycle tests.

The transmission controller uses a lookup table based on accelerator pedal position and vehicle speed to command scheduled gear shifts. The transmission controller map is tuned to ensure the internal combustion engine will not operate at high engine speeds for extended amounts of time. The behavior of the engine speed was evaluated over the course of several drive cycles and was found to have a desired engine speed of approximately 1500 RPM with vehicle accelerations experiencing engine speeds of approximately 3000 RPM. The Willans line data is therefore evaluated for engine speeds between 1000 RPM and 3600 RPM.

Phlips [14] demonstrates that Willans lines at different discrete engine speeds can be simplified into a single Willans line model. A single Willans line model is developed to characterize the entire performance of the 2.5L naturally aspirated internal combustion engine. The single Willans line model is applied to each discerete engine speed Willans line and is depicted in Figure 6.3. The single Willans line model is based on the combustion and engine speed assumptions for the 2.5L naturally aspirated internal combustion engine which is depicted in Figure 6.4.

The incremental amount of chemical power required to produced an incremental amount of output power is quantified through the marginal efficiency or the internal combustion engine Willans line model slope. The required amount of chemical input power to overcome engine losses is quantified by the offset of the Willans line model. The Willans line model representing the engine operating region has an approximate marginal efficiency of 38% and a constant offset of approximately 10 kW.

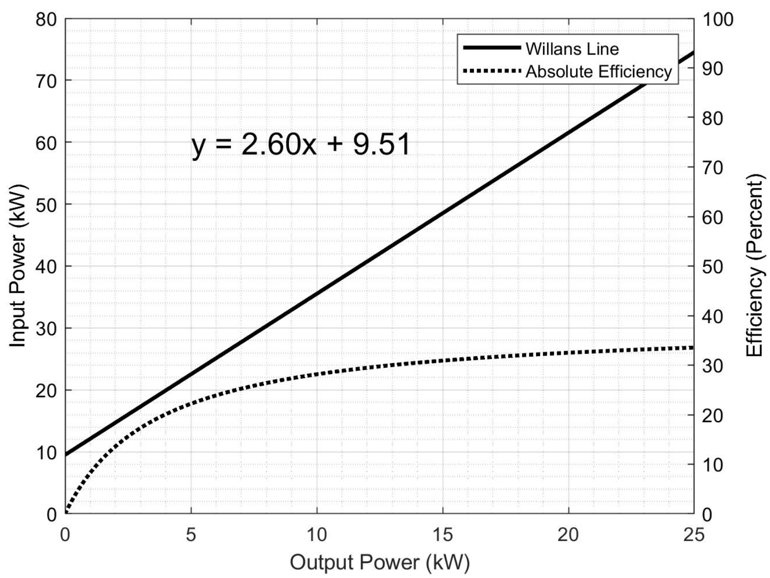

Figure 6.4: Single Willans line model and associated efficiency for internal combustion engine.

6.2 Nine Speed Automatic Transmission

The nine speed automatic transmission transmits the mechanical input power produced by the internal combustion engine and generates mechanical output power as a function of the transmission gear ratios. Phlips [15] demonstrates that, similar to an engine, a transmission operating at varying speeds can be simplified into a single Willans line model. Transmissions transmit power and are a therefore a drivetrain component. Drivetrain components have a relatively high marginal efficiency and a low loss term in comparison to the Willans line model terms for an engine. The performance of the transmission is assumed to have characteristics similar to the information gathered by Phlips [15] for modeling purposes. The marginal efficiency of the transmission is 97% and the loss coefficient is approximately 0.2 kW.

6.3 Electric Traction Motor

The refined manufacturer test data is used to generate Willans line models for the electric traction motor system. The motor system converts electric power produced by the battery pack into mechanical energy and is therefore categorized as a powertrain component. A similar process of quantifying the input power required to generate a desired output power for the internal combustion engine is used to develop a Willans line model for the motor system. The input power to the motor system is quantified by the current and voltage output of the battery pack. The output power from the motor system is quantified by the torque produced at the given motor speed. The output power of the motor system is a function of motor speed, therefore motors have different Willans line models at discrete speeds.

Figure 6.5 depicts the Willans line model developed for the motor system. The Willans line for the motor system exhibits a linear relationship at lower magnitudes of output power since the motor system experiences low magnitudes of loss relative to the magnitude of the produced torque. At higher magnitudes of output power at discrete motor system speeds, the Willans lines does not exhibit a strictly linear relationship. The portion of the motor system Willans line model at higher output power which is not strictly linear is characterized by the torque squared related loss term. The linear Willans line model is however a decent approximation of the high output power region which is not strictly linear. On the other hand, the motor system does not expend a lot of energy at high torque points. The behavior of the electric traction motor system at high torque output is therefore not included in the development of discrete motor system Willans line models.

The speed of the electric traction motor system is related to vehicle speed since the single speed gearbox is directly correlated to the rotation of the axle. Drive cycle tests do not demand vehicle speeds exceeding 80 mph and with a gear ratio of 11.6, the motor system does not exceed speeds of approximately 10,000 RPM. The Willans line model data is therefore evaluated between motor system speeds of 0 RPM and 10,000 RPM. The single Willans line model data based on the torque loss dependency and electric traction motor speed applied to each discrete motor system speed is depicted in Figure 6.5. Similar to the internal

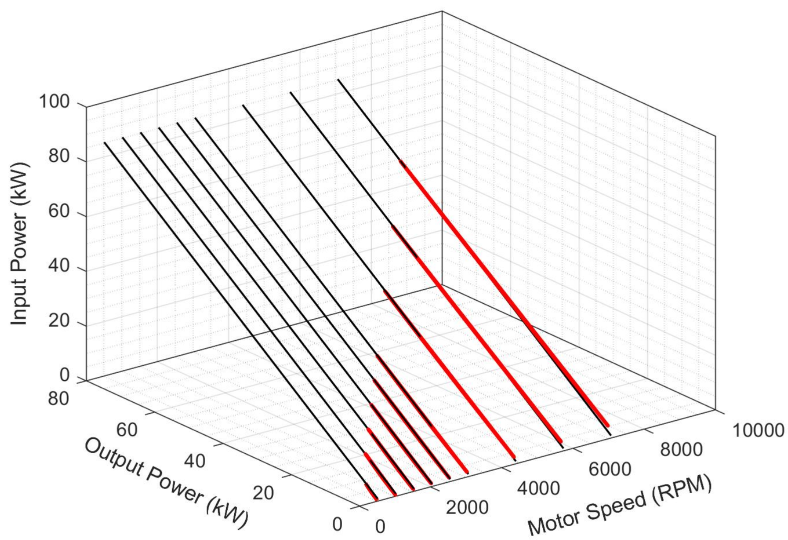

Figure 6.5: Electric traction motor Willans line models with single Willans line fit model.

combustion engine, the electric traction motor system is simplified into a single Willans line model. The incremental amount of electrical power required to produced an incremental amount of output power is quantified through the marginal efficiency of the motor system Willans line model slope. The required amount of electrical input power to overcome the motor system losses is quantified by the constant offset of the Willans line model. The single Willans line model for the motor system behaving as a motor is depicted in Figure 6.6. The Willans line model representing the electric traction motor system while motoring has an approximate marginal efficiency of 95% and a constant offset of approximately 1 kW.

The electric traction motor system is a bidirectional power source. When the motor system

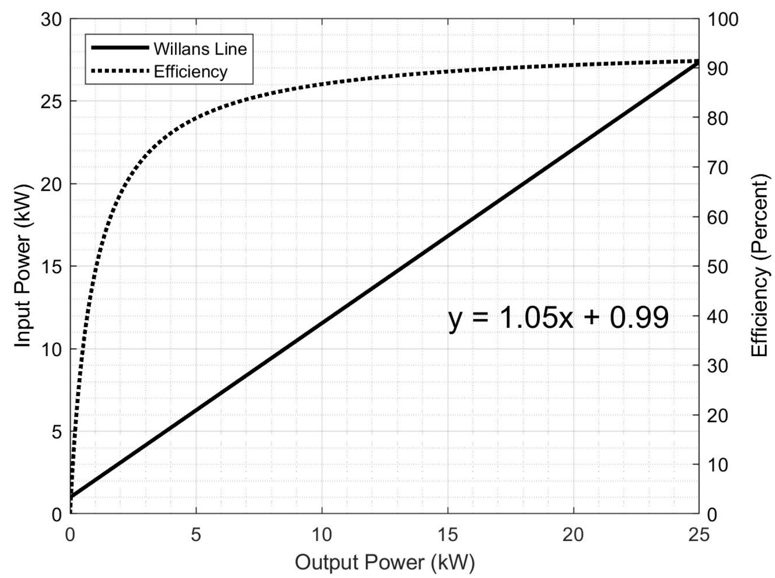

Figure 6.6: Single Willans line model and efficiency for electric traction motor while behaving as a motor.

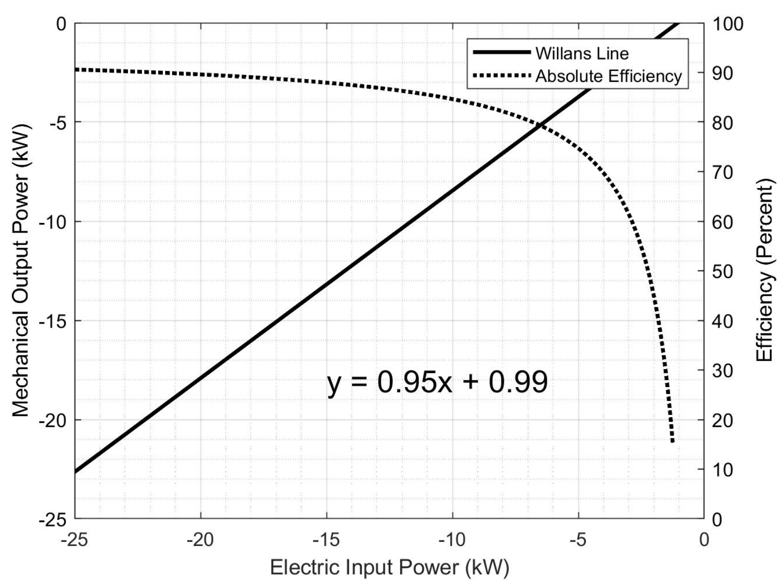

Figure 6.7: Single Willans line model and efficiency for electric traction motor while behaving as a generator.

is a generator the output power is electric power. When the motor system produced electric output power, the powerflow pathway is deemed to follow a negative direction. The motor system when it behaves as a generator is quantified with a separate Willans line. A similar process is used to produce a single Willans line model for the motor system as a motor is followed to produce a Willans line model for the motor system as a generator. The single Willans line model for the motor system behaving as a generator is depicted in Figure 6.7. Since the slope of the generator Willans line model is different than the motor Willans line model, the definition of efficiency is therefore different for the negative powerflow pathway. Since the powerflow is in the negative pathway, the reciprocal of marginal efficiency is taken to be the marginal efficiency of the motor system as a generator.

6.4 Single Speed Gearbox

The single speed gearbox transmits the mechanical output power produced by the electric traction motor system and generates a mechanical output power as a function of the gearbox ratio. Similar to the nine speed automatic transmission, a gearbox operating at varying speeds can be simplified into a single Willans line model. Gearboxes are drivetrain components and therefore have a relatively high marginal efficiency and relatively low loss related term with respect to Willans line models when compared to a motor system. Data specific to the gearbox coupled to the electric traction motor is not provided by the supplier. The performance of the gearbox is assumed to have characteristics similar yet slightly higher than the performance of the transmission since it only has a single static gear. The marginal efficiency of the single speed gearbox is 98% and the loss coefficient is approximately 0.15 kW.

Similar to the motor, the gearbox can also transmit power in the negative powerflow direction. A similar process used to generate a positive powerflow gearbox Willans line model is used to generate a negative powerflow gearbox Willans line model. Since the motor is converting power in the negative direction, the equivalence ratio or slope of the Willans line is the marginal efficiency. The marginal efficiency of the single speed gearbox in the negative powerflow direction is 98% and the loss coefficient is approximately 0.15 kW. Similar to the motor system, the efficiency of the gearbox in the negative powerflow pathway is defined differently. Since the powerflow is in the negative pathway, the reciprocal of marginal efficiency is taken to be the marginal efficiency of the gearbox in the negative powerflow pathway.

Chapter 7

Willans Line Based Energy Management Strategy

The Willans line model marginal efficiency quantifies the capability of a given component to convert or transmit an input power once it overcomes inherent losses. As demonstrated earlier, the Willans line model also quantifies the efficiency of a given component. The marginal efficiency is therefore similar to the equivalence ratio since both the marginal efficiency and equivalence ratio quantify the efficiency of the hybrid electric vehicle components. The equivalence ratio of a component is quantified by taking the reciprocal of the marginal efficiency or the slope of the Willans line model. Equation 7.1 details the relationship between the equivalent consumption minimization strategy equivalence ratio and the Willans line model marginal efficiency.

$$P_{eqv} = P_{fuel} + s(t)P_{elec} = P_{fuel} + \frac{1}{\eta_m}P_{elec} \tag{7.1}$$

In Equation 7.1, P_{eqv} represents the total equivalent fuel input power, P_{fuel} represents the input chemical power, P_{elec} represents the input electrical power, $s(t)$ is the equivalence ratio, and η_m is the Willans line model marginal efficiency.

The hybrid electric vehicle components on the front and rear axles have unique Willans line models. Phlips [15] has demonstrated that component marginal efficiencies can be combined to generate a single Willans line model which quantifies the input to output power efficiency or performance of an axle. The following sections detail the process which is used to generate Willans line models for the front and rear axles of the hybrid electric vehicle. The combination of Willans line models for each axle quantifies the input to output power performance of the conventional components on the front axle and the electric components on the rear axle.

7.1 Engine Only Operating Mode

Propulsion for engine only operation is achieved by producing the required torque with the internal combustion engine, which is transmitted to the nine speed automatic transmission, and finally transmitted to the front axle. The engine only operating mode input power is the chemical power from E10 fuel and the output power is the mechanical power generated by the front axle. The front axle must therefore overcome the losses inherent to the engine and transmission before the front axle generates a force greater than or equal to the inertial and road load forces acting against the hybrid electric vehicle to accelerate or maintain speed, respectively. The Willans line models for the engine and transmission are linearly combined to generate a single Willans line model to quantify the front axle input power to output power relationship. The linear combination of the engine and transmission Willans line models require consideration of both the marginal efficiency and constant offset. Equation 7.2 details the process for linearly combining the engine and transmission Willans line models.

$$P_{out} = \left(\eta_{m_E}\eta_{m_T}\right) P_{in} + \left(\frac{P_{loss_T}}{\eta_{m_E}} + P_{loss_E}\right) \tag{7.2}$$

In Equation 7.2, P_{out} represents the mechanical power produced by the front axle, η_{m_E} represents the marginal efficiency of the engine, η_{m_T} represents the marginal efficiency of the transmission, P_{in} represents the chemical input power to the engine, P_{loss_T} represents the losses inherent to the transmission, and P_{loss_E} represents the losses inherent to the engine. Figure 7.1 depicts the single Willans line model for the engine only operation on the front axle. The engine only operating mode has a marginal efficiency and constant offset of approximately 37% and 10 kW, respectively.

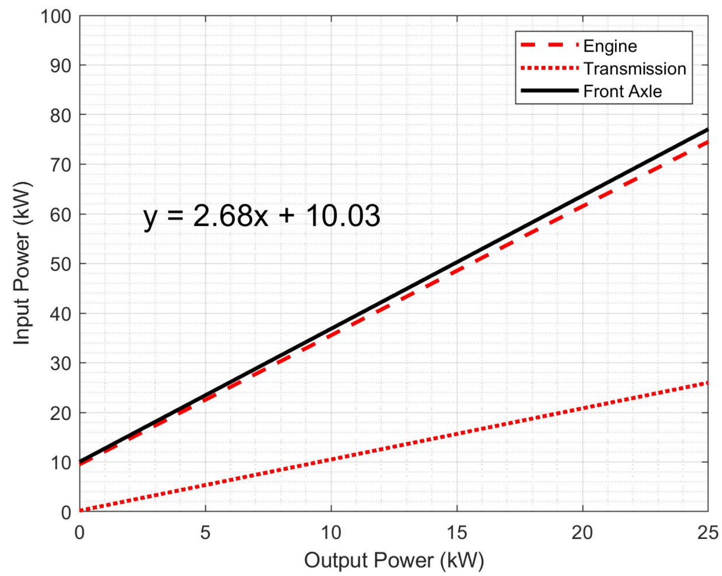

Figure 7.1: Linear combination of internal combustion engine and transmission Willans line models to quantify front axle Willans line model.

7.2 Electric Traction Motor Propulsion Operating Mode

Electric traction motor propulsion is achieved by producing the required torque with the electric traction motor, which is transmitted to the single speed gearbox, and then transmitted to the rear axle. The rear axle motor propulsion operating mode input power is the electrical power produced by the battery back and the output power is the mechanical power produced by the rear axle. The rear axle must therefore generate enough force to meet or overcome the losses inherent to the motor and gearbox before the vehicle maintains its current speed or accelerates, respectively. The Willans line models for the motor and gearbox are linearly combined to generate a single Willans line model to quantify the rear axle input power to output power relationship. A process similar to the one used to generate a single Willans line model for the engine only operating mode is followed to generate a single Willans line model for the motor propulsion operating mode. Figure 7.2 depicts the single Willans line model for the rear axle motor propulsion operating mode. The motor propulsion operating mode has a single Willans line model with a marginal efficiency and linear offset of approximately 93% and 1 kW, respectively.

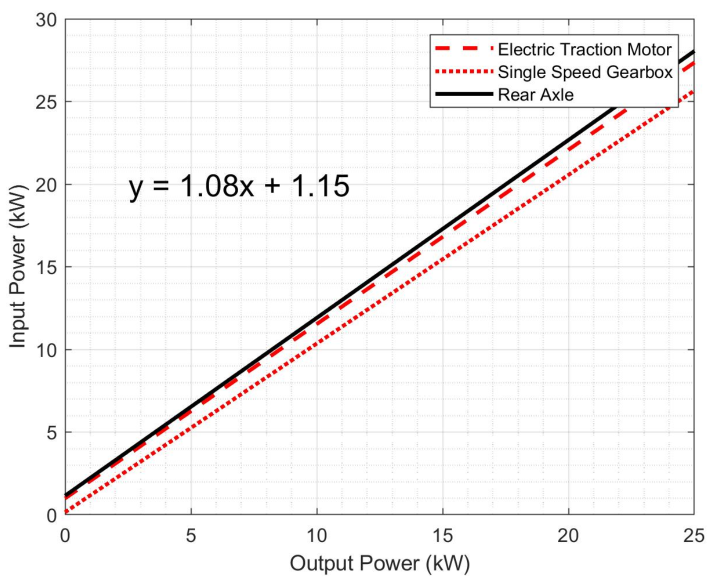

Figure 7.2: Linear combination of electric traction motor and single speed gearbox Willans line models to quantify rear axle propulsion Willans line model.

7.3 Electric Traction Motor Generator Operating Mode

Electric traction motor generator mode of operation is achieved by harnessing the mechanical power of the hybrid electric vehicle to charge the battery pack using the single speed gearbox and electric traction motor on the rear axle. The rear axle motor generator operating mode input power is the mechanical power of the hybrid electric vehicle and the output power is the electric power produced by the motor. Even though the powerflow for motor generator operating mode is in the negative powerflow direction, the rear axle must overcome the losses inherent to the motor and gearbox before the battery pack can be charged. The Willans line model for the motor in the negative powerflow direction is combined with the gearbox in the negative powerflow direction to generate a single Willans line model for the input power to output power relationship on the rear axle. A process similar to the one used to generate a single Willans line model for the engine only operating mode is followed to generate a single Willans line model for the motor generator operating mode. Figure 7.3 depicts the single Willans line model for the rear axle motor generator operating mode. Since the powerflow is in the negative direction, the marginal efficiency of this operating mode is the equivalence ratio or slope of the Willans line. The electric traction motor generator operating mode has a marginal efficiency and linear offset of approximately 93% and 1 kW, respectively.

7.4 Engine Assisting Operating Mode

Engine assisting propulsion is achieved by producing the required amount of torque with the internal combustion engine and nine speed transmission on the front axle along with torque from the electric traction motor and the single speed gearbox on the rear axle. The engine assist operating mode input power is the chemical power from the Tier 3 E10 fuel and the

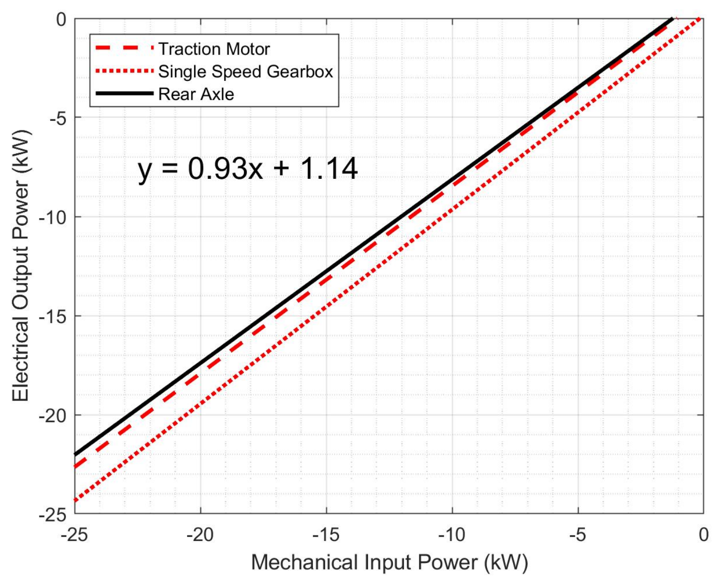

Figure 7.3: Linear combination of electric traction motor and single speed gearbox Willans line models to quantify rear axle generator Willans line model.

electric power from the battery pack. The engine assist operating mode output power is the mechanical power produced by the front axle and the mechanical power produced by the rear axle. The previously developed Willans lines for the front axle engine only mode of operation and rear axle motor propulsion mode of operation are used to quantify the input to output power relationship for each axle of the engine assist operating mode. The Willans lines for the engine only mode of operation and motor propulsion mode of operation are not linearly combined into a single Willans line since the gearing on the front and rear axle are different from one another. For implementation purposes the power produced by the electric traction motor during engine assist operating mode is assumed to be a constant magnitude of approximately 15 kW. Figure 7.4 depicts the Willans line models which calculate the input to output power relationship for the engine assisting operating mode.

The power demand is handled differently when transitioning from another mode of operation to the engine assist mode of operation. For instance if 15 kW of output power is commanded

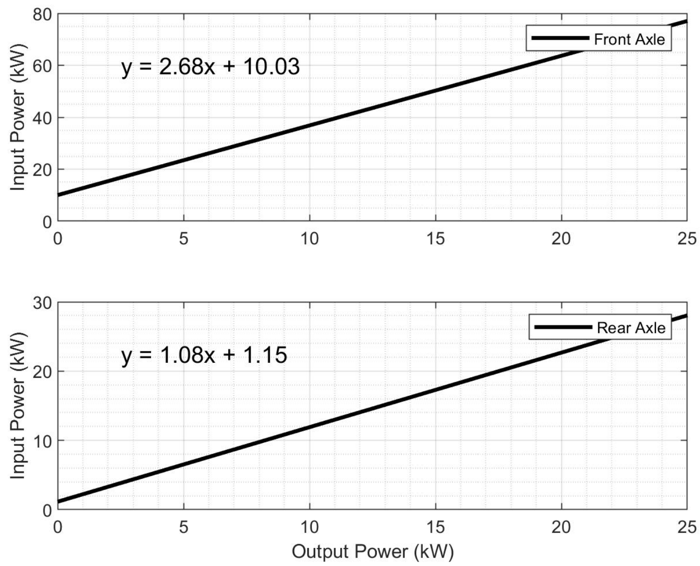

Figure 7.4: Front axle and rear axle propulsion Willans lines.

during engine only operation, the engine assist mode of operation may request 10 kW of output power on the front axle from the engine and 5 kW of output power on the rear axle from the motor to meet the total 15 kW demand. The commanded torque and associated output power produced by each component during the engine assist mode of operation is dictated by the equivalent consumption minimization strategy input power cost function. There are therefore a number of ways to generate 15 kW of output power with the front and rear axle during engine assist mode of operation.

7.5 Engine Loading Operating Mode

Propulsion for engine loading operating mode is achieved by producing the required amount of torque with the internal combustion engine and nine speed transmission on the front axle plus an additional amount of power to compensate for the negative torque from the electric traction motor and the single speed gearbox on the rear axle. The input power for engine loading operating mode is the chemical power from the Tier 3 E10 fuel on the front axle and vehicle mechanical power on the rear axle. The output power for engine loading operating mode is mechanical power on the front axle and electric power to the battery pack on the rear axle. The previously developed Willans lines for the front axle engine only mode of operation and rear axle motor generator mode of operation are used to quantify the input to output power relationship for the engine loading operating mode. The Willans lines for the two modes of operation are not combined into a single Willans line since the gearing on the front and rear axle are different from one another. For implementation purposes the power generated by the electric traction motor during engine load operating mode is assumed to be a constant magnitude of approximately -20 kW. Figure 7.5 depicts the Willans line models which calculate the input to output power relationship for the engine loading operating mode.

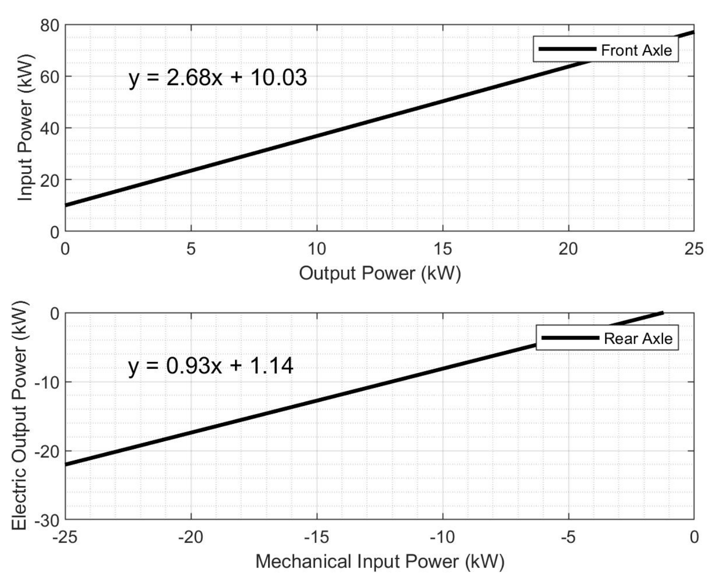

Figure 7.5: Front axle and rear axle generator Willans line.

The power demand for engine loading experiences a similar power transition as a shift to engine assist mode of operation. For instance if 15 kW of output power is commanded, the engine loading mode of operation may request 20 kW of output power on the front axle from the engine and -5 kW of input power on the rear axle from the motor to meet the total 15 kW demand. The engine load mode of operation produces mechanical output power on the front axle, the front axle therefore uses a positive powerflow pathway. Since the engine load mode of operation produces electric output power on the rear axle, the rear axle uses a negative powerflow pathway. The commanded torque and associated output power produced by each component during the engine loading mode of operation is dictated by the equivalent consumption minimization strategy input power cost function. There are therefore a number of ways to generate 15 kW of output power with the front and rear axle during engine loading mode of operation.

7.6 Willans Line Based Equivalent Consumption Minimization Strategy

The Willans line models for the rear axle when operating as a motor and a generator have two different slopes. The axes and therefore the slopes for the positive Willans line model powerflow pathway and the negative Willans line model powerflow pathway are different from one another. When the axes for the negative powerflow pathway are changed to match the positive powerflow pathway Willans line model axes, the two models are identical to one another. The marginal efficiency or the Willans line model slope on the rear axle can therefore be quantified with a single value. The single marginal efficiency or Willans line model slope therefore quantifies the battery power conversion process on the rear axle when the electric traction motor operates as a motor and a generator.

The equivalence ratio for the equivalent consumption minimization strategy quantifies the fuel to battery power conversion process or efficiency of the hybrid electric vehicle. The reciprocal of the Willans line model marginal efficiency or modeled slope of the front axle and the rear axle quantify the fuel power conversion and battery power conversion of the hybrid electric vehicle, respectively. The Willans line model slope is therefore analogous to the equivalence ratio for the equivalent consumption minimization strategy. The fuel to battery conversion process for the hybrid electric vehicle is quantified with a single value by taking the product of the front axle Willans line model slope and rear axle Willans line model slope. The Willans line model marginal efficiency of the front axle and rear axle are approximately 37% and 93%, respectively. The Willans line model slope for the front axle is therefore 2.68. The Willans line model slope for the rear axle is therefore 1.08. The product of the front and rear axle Willans line model slope generates an equivalence ratio value of approximately 2.9. The single equivalence ratio quantifies the fuel to battery power conversion process of the hybrid electric vehicle and is implemented as a static value in the equivalent consumption minimization strategy.

As previously stated, the equivalent consumption minimization strategy equivalence ratio only quantifies the fuel to battery conversion process for the hybrid electric vehicle. The equivalence ratio does not account for variations in the battery pack state of charge. The equivalence ratio is therefore paired with a penalty factor associated with battery pack state. The penalty factor is comprised of a polynomial scaled penalty and a proportional-integral controller penalty. The battery pack state of charge penalty factor is implemented to maintain the local constraint on battery pack state of charge. Maintaining the local constraint on battery pack state of charge facilitates the ability to achieve the global constraint of achieving charge sustaining performance. The fifth order polynomial scaled portion of the state of charge penalty factor is enforced by setting allowable limits on battery pack state of charge. The fifth order polynomial is offset by the initial battery pack state of charge of 60%

with a minimum threshold of 50% battery pack state of charge and a maximum threshold of 70% state of charge. Figure 7.6 depicts the fifth order polynomial scaled penalty term on the battery pack state of charge.

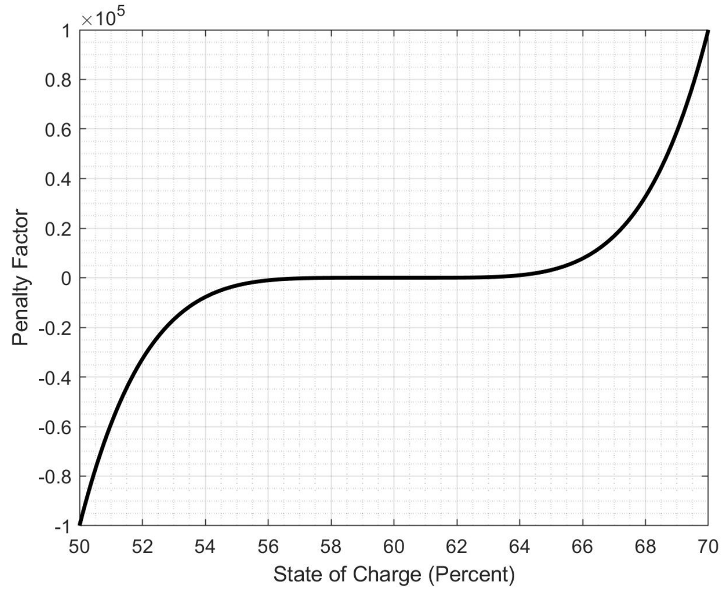

Figure 7.6: Battery pack state of charge polynomial penalty factor.

The polynomial scaled portion of the state of charge penalty factor does not change until the instantaneous battery pack state of charge approaches the minimum or maximum limits. If the instantaneous state of charge approaches the maximum threshold, the equivalence ratio will begin to decrease to promote electric only or engine assist modes of operation to discharge the battery pack. Similarly, if the instantaneous state of charge approaches the minimum threshold, the equivalence ratio will begin to increase to promote engine only or engine loading modes of operation to charge the battery pack. The proportional-integral controller is implemented on the battery pack state of charge variation as well to track the targeted or initial battery pack state of charge value to achieve charge sustaining performance. Equation 7.3 details the proportional-integral controller penalty term on the battery pack state of charge.

$$u(t) = K_p e + K_i \int_0^t e(t)dt \tag{7.3}$$

In Equation 7.3, K_p represents the proportional term of the controller, e is the error between the target and actual battery pack state of charge, K_i is the integral term of the controller, and $u(t)$ is the total control authority commanded by the proportional-integral controller. The proportional integral term is multiplied by the polynomial scaled penalty term to generate a single penalty term on battery pack state of charge. Equation 7.4 details the equivalence ratio term with the battery pack state of charge penalty terms in the input power cost function for the equivalent consumption minimization strategy.

$$P_{eqv} = P_{fuel} + [s + (p(SOC))(u(t))]P_{elec} \tag{7.4}$$

In Equation 7.4, P_{eqv} is the total equivalent fuel power, P_{fuel} is the input fuel power, s is the static Willans line based equivalence ratio, $p(SOC)$ is the polynomial scaled battery pack state of charge penalty term, $u(t)$ is proportional-integral controller battery pack state of charge penalty term, and P_{elec} is the input electric power. The summation of the equivalence ratio and instantaneous battery pack state of charge penalty term is called the modified equivalence ratio in this paper. The design of the modified equivalence ratio acts to ensure the local constraint on battery pack state of charge is maintained while the control strategy tracks the target battery pack state of charge as it commands different modes of operation over a drive cycle.

The Willans line models characterize the input power to output power relationship for the front and rear axles of the hybrid electric vehicle. The linear input to output power relationships are used to simplify the input power cost function associated with the equivalent consumption minimization strategy. The equivalent consumption minimization strategy is simplified by using the input power to output power relationship of the front axle, rear axle propulsion, rear axle generator, and a combination of the three relationships to directly evaluate the input power cost for the hybrid electric vehicle five modes of operation. Figure 7.7 depicts a diagram of the Willans line based equivalent consumption minimization strategy. In Figure 7.7, the wheel torque or output power demand is populated into the Willans line

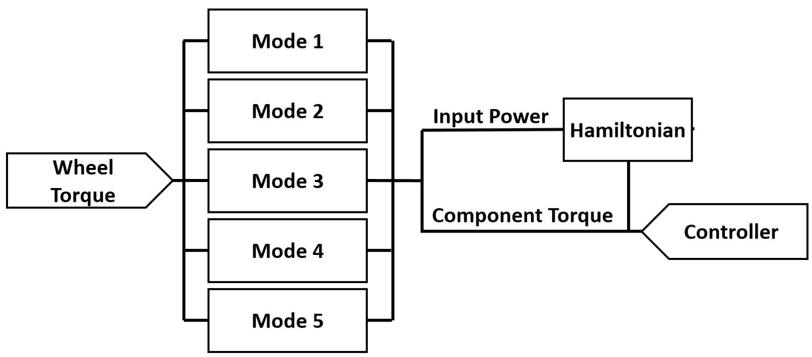

Figure 7.7: Willans line based equivalent consumption minimization strategy logic flow diagram.

model for each mode of operation to determine the input power for each. The output power demand is also used simultaneously in each mode of operation to calculate the torque command for each associated hybrid electric vehicle controller. The input power for each mode of operation is concatenated with one another to generate a cost function which is evaluated by the Hamiltonian function or input power cost function. The input power cost function then selects the input power with the lowest cost, while accounting for the instantaneous value of the modified equivalence ratio, and selects the index associated with that input power to command the component torques to the associated component controllers. The This process is completed at each instantaneous time step over a drive cycle.

The performance of the hybrid electric vehicle and the control decisions commanded to the hybrid electric vehicle components are therefore simplified into three linear relationships. The Willans line based equivalent consumption minimization strategy facilitates the use of the output power demand to generate torque commands to each vehicle controller. Each powertrain component is capable of meeting drive cycle demands on its own and therefore any torque command to the powertrain components can not be categorized as infeasible operating points. Simplifying the equivalent consumption minimization strategy into a cost function based on the hybrid electric vehicle modes of operation removes the need to evaluate the feasibility of a given operating point. The output power demand and instantaneous speed of each powertrain component is used to calculate the torque command for each component based on the selected minimum of the input power cost function. The Willans line based equivalent consumption minimization strategy therefore does not require the use of an additional forward facing model to generate a torque command from the backward facing model input power.

The hybrid vehicle controller uses the Willans line based consumption minimization strategy to command the vehicle controllers in the right direction to minimize fuel consumption. The Willans line based equivalent consumption minimization strategy therefore reduces fuel consumption. Since the vehicle is a charge sustaining hybrid electric vehicle, the Willans line based equivalent consumption minimization strategy reduces fuel consumption since all energy over a drive cycle comes from the fuel tank.

Chapter 8

Results and Discussion

The Willans line based equivalent consumption minimization strategy is evaluated over four different drive cycles which consist of two city and two highway drive cycles. Figure 8.1 depicts the two city drive cycles and Figure 8.2 depicts the two highway drive cycles.

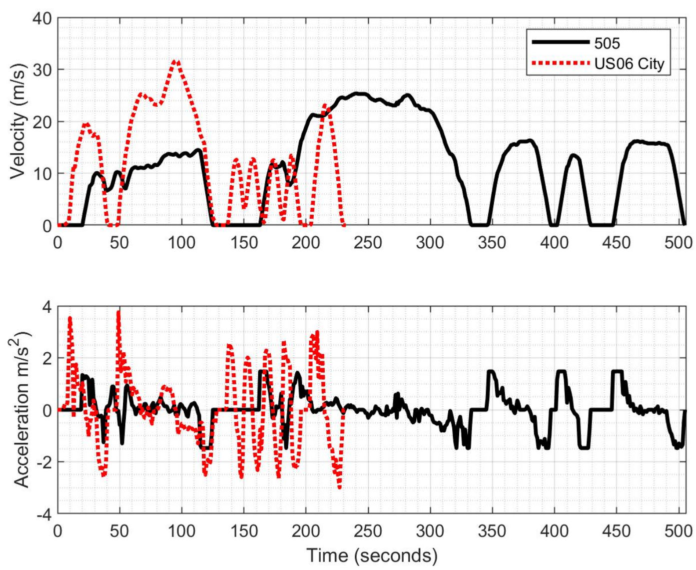

Figure 8.1: Velocity and acceleration profile for mild 505 drive cycle and aggressive US06 City drive cycle.

The selected drive cycles are evaluated because each has varying characteristics which stress the Willans line based equvialent consumption minimization strategy in different ways to generate a preliminary evaluation of the strategy. The four drive cycles are considered mild and aggressive due to the differences in each drive cycle velocity and acceleration magnitude.

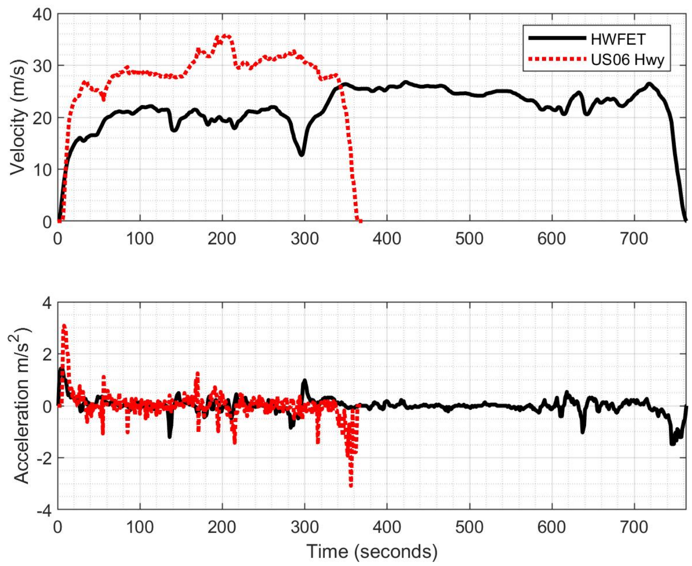

Figure 8.2: Velocity and acceleration profile for mild HWFET drive cycle and aggressive US06 Highway drive cycle.

Table 8.1 details the maximum velocity and maximum acceleration of the four drive cycles. As detailed in 8.1, the 505 drive cycle is a mild drive cycle since it has a velocity profile

Table 8.1: Maximum velocity, maximum acceleration, and distance of 505, US06 City, HWFET, and US06 Highway drive cycles.

Drive Cycle	505	US06 City	HWFET	US06 Highway
Velocity (m/s)	25.3	31.6	26.8	35.9
Acceleration (m/s^2)	1.5	3.8	1.4	3.1
Distance (mi)	17.96	8.85	51.28	31.15

with a moderate maximum velocity and moderate maximum acceleration. The US06 City drive cycle is considered an aggressive drive cycle because it has a high maximum velocity and acceleration over a short duration and distance. The HWFET drive cycle which is a mild drive cycle since it has a velocity profile with a moderate maximum velocity and acceleration over a longer duration of time and distance. The US06 Highway drive cycle which is considered an aggressive drive cycle since it has a high maximum velocity and high maximum acceleration.

8.1 Willans Line Based Equivalent Consumption Minimization Strategy Performance

The Willans line based equivalent consumption minimization strategy is implemented in loop with the desired drive cycle profile, the driver model, and the hybrid electric vehicle plant models to simulate the performance of the strategy. Figure 8.3 depicts the performance of the Willans line based equivalent consumption over several 505 drive cycles. The top plot

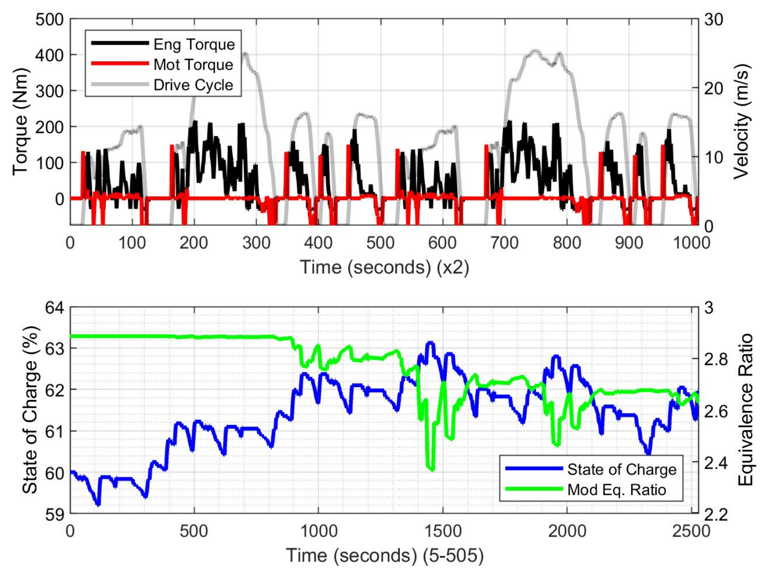

Figure 8.3: Top plot depicts torque split and drive cycle profile over two 505 drive cycles. Bottom plot depicts state of charge and modified equivalence ratio over five 505 drive cycles.

in Figure 8.3 depicts two 505 drive cycles back to back with the torque split between the internal combustion engine and electric traction motor as well as the 505 drive cycle profile. One of the more valuable aspects of hybridizing a conventional vehicle is eliminating the engine operating points at high brake specific fuel consumption or low efficiency at instances when the hybrid electric vehicle transitions from a zero to non-zero velocity. The Willans line based equivalent consumption minimization strategy eliminates low engine efficiency operation by commanding motor torque at the zero to non-zero velocity transitions which is attributed to the cost of the equivalent fuel power from the battery pack being less than the cost of the fuel power from the fuel tank. Although not depicted, the driver model maintains trace misses between the drive cycle profile and actual velocity of the hybrid electric vehicle to remain within +/- 2 mph of variation over the course of the 505 drive cycles. The Willans line based equivalent consumption minimization strategy is capable of commanding a hybrid electric vehicle without an array of operating points evaluated in the equivalent consumption minimization strategy. The top plot therefore indicates that the Willans line based equivalent consumption minimization strategy effectively commands torque to the engine and motor.

The bottom plot of Figure 8.3 depicts the battery pack state of charge and the variation of the modified equivalence ratio over the course of five 505 drive cycles. The battery pack state of charge begins to increase over the first few 505 drive cycles and then decrease over the last 505 drive cycles. The change in battery pack state of charge is attributed to the polynomial penalty factor which reduces the instantaneous value of the modified equivalence ratio and the proportional-integral controller which reduces the moving average of the modified equivalence ratio to discharge the battery pack and track the initial battery pack state of charge. The bottom plot therefore indicates that the Willans line based equivalence ratio and in turn the modified equivalence ratio facilitate a hybrid electric vehicle which achieves charge sustaining performance.

The Willans line based equivalent consumption minimization strategy succesfully generated a torque split command and maintained charge sustaining performance for the aggressive US06 City, mild HWFET, and aggressive US06 Highway drive cycles. The results of the Willans line based equivalent consumption minimization strategy will be discussed in the following sections.

The Willans line based equivalent consumption minimization strategy needs to be evaluated and compared to a equivalent consumption minimization strategy which uses an array of operating points. The comparison of equivalent consumption minimization strategies will quantify the the benefits of the Willans line based equivalent consumption minimization strategy in torque split command and charge sustaining performance.

8.2 Willans Line Validation

The powertrain and drivetrain component Willans lines are a model of the associated plant models. The component based Willans line models are the Willans line models detailed in the prior chapters. The input power to output power relationship which characterizes the components is used to quantify the average input power and output power over to generate a data point for each drive cycle. The four previously mentioned drive cycles are used to generate a Willans line model of the plant model performance. Plant model Willans line models are therefore generated by evaluating the results of drive cycle simulations. The hybrid electric vehicle is simulated with an engine only mode of operation to quantify the fuel input power and mechanical output power produced on the front axle to evaluate the accuracy of the component Willans line model. Figure 8.4 depicts the correlation between the component and plant model based Willans line for the front axle. The component based Willans line model has a lower offset and a higher slope than the plant model Willans line which has a higher offset and lower slope. The component and plant based Willans line model efficiencies are also similar to one another with an efficiency of approximately 30% and 34%, respectively. The component based Willans line for the engine is therefore a good approximation of the engine plant model performance on the front axle. Additional work needs to be completed to verify the component and plant Willans line models and associated efficiencies when the electric traction motor behaves as a motor and as a generator.

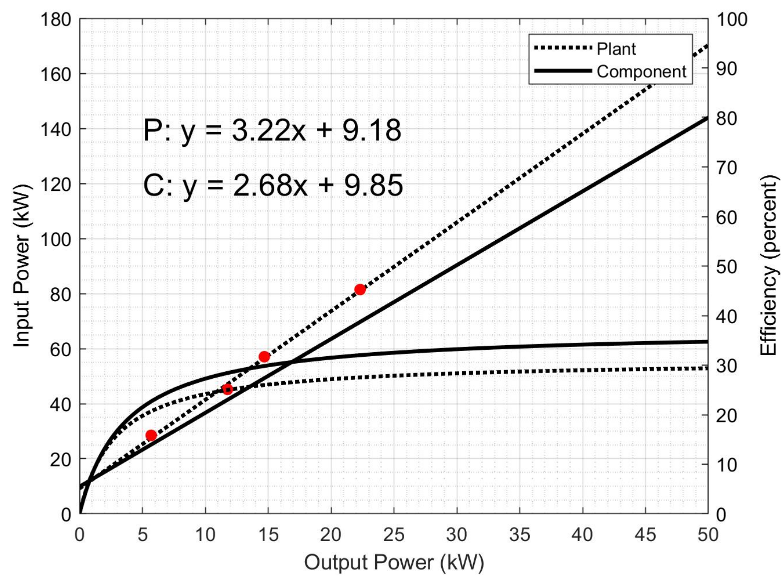

Figure 8.4: Front axle component based Willans line model compared to respective plant model based Willans line model.

8.3 Charge Sustaining Performance Results

The modified equivalence ratio is designed to drive the battery pack state of charge to a value which seeks to achieve charge sustaining performance. The Willans line based equivalence ratio captures the performance of the hybrid electric vehicle components and the penalty factor addresses the changes in battery pack state of charge drives. The performance of the hybrid electric vehicle and the change in battery pack state of charge are two different aspects of the control strategy and are therefore treated as such. The combination of the equivalence ratio and battery pack state of charge penalty factor dictates the Willans line equivalent consumption minimization strategy will command specific modes of operation to seek charge sustaining performance. The charge sustaining performance of the hybrid electric vehicle is achieved with the implementation of the modified equivalence ratio in the equivalent consumption minimization strategy. A separate controller does not exist to regulate the charge sustaining performance metric.

The trajectory of the battery pack state of charge and the variation of the modified equivalence ratio is evaluated over each of the four previously defined drive cycles. Figure 8.5 depicts the changes in battery pack state of charge and the variation in the modified equivalence ratio over the city drive cycles. In Figure 8.5, the top plot depicts the battery pack

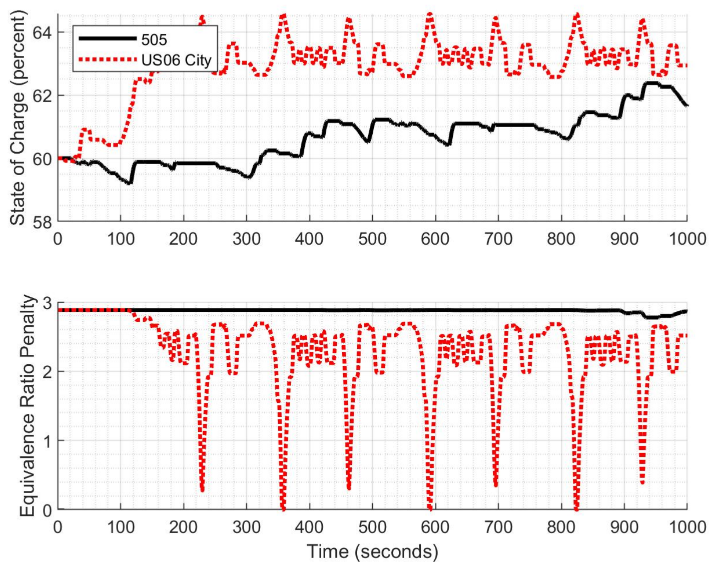

Figure 8.5: Battery pack state of charge trajectory and modified equivalence ratio variation over 505 and US06 City drive cycles.

state of charge trajectories, and the bottom plot depicts the variation in modified equivalence ratio. The mild 505 drive cycle has a steady increase in battery pack state of charge however the modified equivalence ratio does not vary because the limits of the penalty factor have not been infringed upon and the integrator term in the proportional-integral controller has not built up enough error to impact the value of the modified equivalence ratio. The modified equivalence ratio for the 505 drive cycle therefore remains constant while maintaining charge sustaining performance. The aggressive US06 City drive cycle is indicated by the large swing in battery pack state of charge at the beginning of the drive cycle. The polynomial penalty factor prevents the battery pack state of charge from continuing to increase by decreasing the value of the modified equivalence ratio to promote an electric motor propulsion or engine assist mode of operation. Although the battery pack state of charge remains high for the US06 City drive cycle the integral term of the proportional-integral controller does not build enough error to drive the moving average of the modified equivalence ratio down while seeking to maintain charge sustaining performance. Similar results are obtained from the mild and aggressive highway drive cycles. Table 8.1 details the fuel economy and associated charge sustaining metric of the four drive cycles. All drive cycle simulations were ensured to

Table 8.2: Fuel economy results of hybrid electric Chevrolet Blazer and associated charge sustaining metric.

Drive Cycle	Fuel Economy	Charge Sustaining Metric
505	30.7	0.89%
US06 City	18.0	-1.41%
HWFET	34.5	-0.10%
US06 Highway	24.3	0.49%

maintain or nearly meet charge sustaining performance results. The US06 City drive cycle is the only drive cycle that nearly meets charge sustaining performance because the drive cycle is completed over a short duration and distance and is therefore difficult but capable of being achieved. The results obtained in Table 8.1 are all obtained with a single tuning of the polynomial and proportional-integral controller aspects of the battery pack state of charge penalty factor. The Willans line based equivalent consumption minimization strategy with the modified equivalence ratio is therefore robust to drive cycles and does not require individual tuning to ensure charge sustaining performance over various drive cycles.

8.4 Fuel Economy Performance Results

The fuel economy results of the city and highway drive cycles are calculated for each simulation. The fuel economy of the hybrid electric Chevrolet Blazer is compared to the fuel economy of a conventional Chevrolet Blazer [7] because it has similar vehicle characteristics.

The hybrid Chevrolet Blazer also maintains All-Wheel Drive functionality similar to the conventional electric Chevrolet Blazer. The tabulated results for the conventional Chevrolet Blazer and hybrid electric Chevrolet Blazer are specified in Table 8.3.

Table 8.3: Fuel economy results of conventional Blazer and hybrid electric Chevrolet Blazer.

Drive Cycle	Conventional Blazer	Hybrid Blazer
505	23.8	30.7
US06 City	13.6	18.0
HWFET	33.4	34.5
US06 Highway	25.2	24.3

The hybrid electric Chevrolet Blazer improved fuel economy over the mild and aggressive city drive cycles. The increase in city drive cycle capability is attributed to the Willans line based equivalent consumption minimization strategy commanding electric traction motor propulsion and engine assist modes of operation at zero to non-zero velocity profile transitions and as well as the ability of the motor to regain battery power during regenerative braking events. The hybrid electric Chevrolet Blazer did not significantly improve the conventional Blazer highway drive cycle fuel economy because there are not as many launch events to take advantage of and there are not may events to significantly regain battery power during regenerative braking events.

Phlips [15] has previously generated a Willans line model to quantify the fuel economy performance of a conventional vehicle. Phlips uses this fuel economy Willans line model to evaluate conventional vehicles which have different engine technologies. A similar process is followed to model the fuel economy based Willans line model of the hybrid electric Chevrolet Blazer. Since the hybrid electric vehicle is charge sustaining, all power or energy comes from the fuel tank. The fuel economy input power to output power relationship is therefore evaluated by calculating the charge sustaining fuel economy input power and the associated average output power of the vehicle. Figure 8.6 depicts the fuel economy Willans line model for the hybrid electric Chevrolet Blazer and the fuel economy Willans line model generated by EPA fuel economy results [7] for the conventional Chevrolet Blazer.

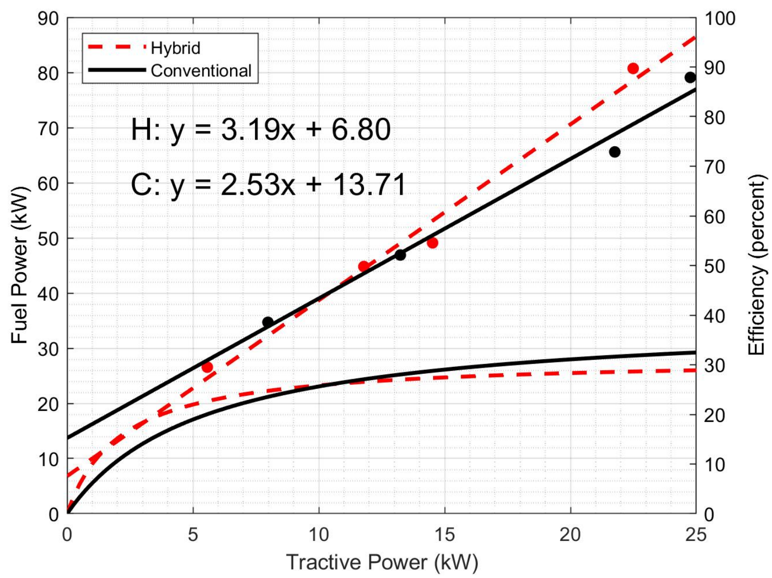

Figure 8.6: Hybrid electric vehicle fuel economy Willans line model.

The hybrid Chevrolet Blazer and conventional Chevrolet Blazer have similar fuel economy Willans line models. The fuel economy Willans line model for the hybrid electric Chevrolet Blazer has a lower constant offset with a higher slope relative to the conventional Chevrolet Blazer. The differences in hybrid and conventional fuel economy Willans line models is reflected in the output power based efficiency. The hybrid electric Chevrolet Blazer however, has a higher efficiency at low output power which is attributed to the addition of hybrid components with higher average efficiencies. The hybrid electric Chevrolet Blazer has a lower efficiency at high output power which is attributed to the additional inertial mass of hybrid components at higher vehicle velocities. It is important to note that the charge sustaining hybrid electric fuel economy Willans line is not confirmed to be an appropriate method to compare conventional and hybrid vehicle technologies. Additional work needs to be completed to verify if the fuel economy Willans line model for a charge sustaining hybrid electric vehicle is a sufficient method to compare to a conventional vehicle fuel economy Willans line model.

Chapter 9

Conculusion

This paper introduces the concept of a Willans line model and how it applies to both powertrain and drivetrain components. The Willans line models was simplifies calculating the input power to output power conversion process of the hybrid electric vehicle into the combination of three Willans line models. The Willans line model facilitates the input power cost function calculation and removes the need for additional models to generate a torque command to the appropriate controllers and therefore simplifies the hybrid electric vehicle energy management strategy. The definition of the Willans line model slope and its correlation to the equivalence ratio for the equivalent consumption minimization strategy quantifies the fuel conversion process and the battery conversion process for the hybrid electric vehicle. An analogous equivalence ratio was quantified by taking the product of the Willans line model slopes to generate a single static equivalence ratio which characterizes the fuel to battery conversion process of the hybrid electric vehicle. The hybrid electric vehicle energy management strategy fuel economy results and charge sustaining performance are separate but dependent aspects which generate a modified equivalence ratio. The Willans line based equivalent consumption minimization strategy with modified equivalence ratio successfully generates torque commands to the internal combustion engine and electric traction motor. The Willans line based equivalent consumption minimization strategy improves city drive cycle and marginally improves highway drive cycle fuel economy while maintaining charge sustaining performance.

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Appendices

Appendix A

First Appendix

A.1 Simulation Model

In a classic control feedback loop, an input is provided to a controller which generates an input to a plant model. Sensors on board the system measure the output of the plant model which is used by the controller to modify its next commanded input to the plant model. A drive cycle test requires the drive cycle velocity be provided as a input. The driver model therefore determines what command is sent to the vehicle controller and what input the controller provides to the vehicle plant model. The emphasis of this paper is placed on the design, implementation, testing, and validation of the energy management strategy on the vehicle controller. The next section details the driver model and plant model required to simulate a drive cycle simulation. The following sections detail the subsystem plant models needed to quantify the performance of the vehicle.

A.2 Driver Model

The driver model for the hybrid electric vehicle uses a model predictive controller to command an accelerator pedal position to the vehicle controller equivalent to the driver demand to regulate the desired longitudinal hybrid electric vehicle velocity. The model predictive controller uses a linear time invariant model to develop an observer for the longitudinal velocity of the hybrid electric vehicle. Equation A.1 details the linear time invariant system state and Equation A.2 details the observer of the linear time invariant system.

$$\dot{x} = Ax + Bu \tag{A.1}$$

$$y = mx \tag{A.2}$$

Where A in Equation A.1 is the state matrix of the system, x is the state vector, B is the input control vector, and u is the control input to the system. In Equation A.2, y is the scalar output observer and m is the observer vector. The model predictive controller uses the observer to minimize the control authority. Equation A.3 details the control input to the system which is minimized over a given interval.

$$J = \frac{1}{T}\int_{t}^{t+T} \{[f(\eta) - y(\eta)]\, W(\eta - t)\}^2 d\eta \tag{A.3}$$

In Equation A.3, W is a weighting function and $f(\eta)$ is the previewed input, and t to $(t+T)$ is the preview interval. When the preview interval is known the optimal control input is quantified in Equation A.4.

$$u^0(t) = u(t) + \frac{\epsilon(t+T^*)}{T^*K} \tag{A.4}$$

In Equation A.4, $u^0(t)$ is the optimal control input, $u(t)$ is the control input, $\epsilon(t+T^*)$ is the observer based aspect of the control input, T^* is the preview time interval, and K is a scaling term.

MacAdam [6] uses a model predictive controller to regulate the longitudinal velocity of a vehicle and is found to display very strong correlation to that of a human driver. Therefore the implementation of a model predictive controller with a preview distance of approximately two meters is used for Environmental Protection Agency drive cycle simulations for a hybrid electric vehicle.

A.3 Plant Model

A.3.1 Vehicle Plant Model

The vehicle plant model is a physics based model that quantifies the amount of force required to accelerate or decelerate the vehicle. The physics model is a linear steady state model that calculates the forces acting against the vehicle so the controller can command the appropriate amount of force, in the form of torque, from the internal combustion engine and electric traction motor to overcome the forces acting against the vehicle. Non-linearities from a vehicle level standpoint are assumed to be negligible. Four forces act against the vehicle, the inertial force, the rolling resistance force, and the aerodynamic drag force, and the grade force. The tractive force acts against the inertial, rolling resistance, aerodynamic drag, and grade forces to accelerate or maintain the desired speed of the vehicle. Figure A.1 details the coordinate system and the orientation of the forces acting on the vehicle. A summation

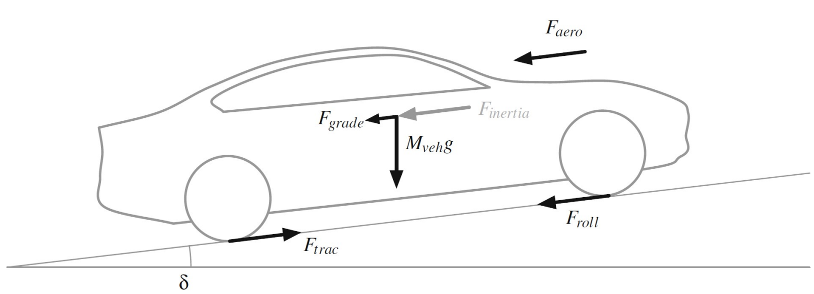

Figure A.1: Free body diagram of physics based vehicle plant model.

of forces in the positive longitudinal direction quantifies the amount of tractive force the vehicle powertrain components must produce to meet or exceed the forces acting against it.

The equation for the free body diagram is detailed in Equation A.5. The free body diagram is then expanded upon into quantifiable metrics in Equation A.6.

$$F_{TR} = F_{INT} + F_{RR} + F_{DRAG} + F_{GR} \tag{A.5}$$

$$F_{TR} = ma + (mgC_{rr0} + mgC_{rr1}) + \frac{1}{2}\rho C_D A V^2 + mgsin(\theta) \tag{A.6}$$

Where m is the mass of the vehicle, a is the acceleration of the vehicle, g is the gravitational constant, C_{rr0} is a coefficient of rolling resistance, C_{rr1} is also a coefficient of rolling resistance, ρ is the density of air, C_D is the coefficient of drag of the vehicle frontal area, A is the frontal area of the vehicle, V is the velocity of the vehicle, and $sin(\theta)$ is the angle of the vehicle orientation.

The tractive force equation derived from the free body diagram quantifies the demands of the vehicle, however, several of the constants such as the coefficient of drag and the frontal area of the vehicle are not easily attainable. The Environmental Protection Agency does however run fuel economy tests on vehicles for emissions purposes and as a result of doing so collect data to calculate the equivalent inertial, rolling resistance, and drag related terms. Environmental Protection Agency fuel economy tests are run on a dynamometer, therefore the grade related force is assumed to be negligible. The fuel economy tests conduct coast down tests to calculate the equivalent road load terms that can also be used to calculate the tractive force required by the vehicle The inertial force is equivalent to the first road load coefficient, A. The two rolling resistance forces are equivalent to the second road load coefficient, B. The aerodynamic drag force is equivalent to the third road load coefficient, C. The road load equation derived from Environmental Protection Agency testing is detailed in Equation A.7.

$$F_{TR} = A + BV + CV^2 + ma \tag{A.7}$$

Road load coefficients for vehicles tested by the Environmental Protection Agency are available to the public. The hybrid vehicle designed by Virginia Tech is not going to have a coast down test conducted by the Environmental Protection Agency, and therefore will not have road load coefficients that quantify the inertial, rolling resistance, and aerodynamic force terms of the hybrid Chevrolet Blazer. The Environmental Protection Agency has conducted coast down tests on the 2.5L Front-Wheel-Drive and 3.6L All-Wheel-Drive options. Virginia Tech received a stock 3.6L All-Wheel-Drive conventional vehicle and converted it into a 2.5L Front-Wheel-Drive hybrid vehicle. The coefficients for the 3.6L All-Wheel-Drive conventional Chevrolet Blazer road load coefficients were chosen to given they are approximately closer to the mass of the hybrid Chevrolet Blazer. The 3.6L All-Wheel-Drive specific road load coefficients were modified based on approximations to generate road load coefficients closer to the hybrid Chevrolet Blazer.

Granted the inertial and rolling resistance road load coefficient terms are mass dependent, the respective terms are scaled by the mass of the hybrid Chevrolet Blazer. Virginia Tech has weighed the hybrid Chevrolet Blazer and scaled the inertial and rolling resistance road load terms based on the Environmental Protection Agency test mass and the measured mass of the hybrid Chevrolet Blazer. The road load coefficient associated with aerodynamic drag does not have a mass dependent term, but it does have a coefficient of drag and a frontal area term. The exterior of the hybrid Chevrolet Blazer will not be modified and the stock orientation of the hybrid Chevrolet Blazer will not be significantly impacted due to the integration of hybrid components. The impact to the coefficient of drag and frontal area of the hybrid Chevrolet Blazer is therefore deemed to be negligible.

Several assumptions were made about the road load model of the vehicle plant model. The road load model does not account for tire slip because it is assumed that the drive cycle demand will not require accelerations in excess of the capability of the vehicle to generate tractive forces that overcome the rolling resistance of the hybrid Chevrolet Blazer. This assumption was based upon the added mass of the hybrid Chevrolet Blazer in addition to the average low accelerations and short high accelerations inherent to Environmental Protection Agency drive cycles. The tires on the hybrid Chevrolet Blazer are also assumed to be directly coupled to the axle without any additional degrees of freedom that would require additional modeling.

A.3.2 Internal Combustion Engine Plant Model

The internal combustion engine plant model utilizes steady state dynamometer test data to quantify the performance of the 2.5L internal combustion engine. The dynamometer testing measures the brake specific fuel consumption, or mass flow rate of fuel, at a given engine speed and engine torque. The brake specific fuel consumption map generated by the Environmental Protection Agency for the General Motors 2.5L internal combustion is depicted in Figure A.2. The discontinuities in the brake specific fuel consumption map in Figure **??** are a result of the data obtained and not indicative of the actual performance at low torque and high engine speed.

A.3.3 Electric Traction Motor Plant Model

A power balance was utilized to quantify the performance of the electric traction motor plant model. The power balance dictates the electric traction motor input power from the battery pack equal the mechanical output power of the electric traction motor and the inherent electric traction motor losses. The electric traction motor power balance equation is detailed

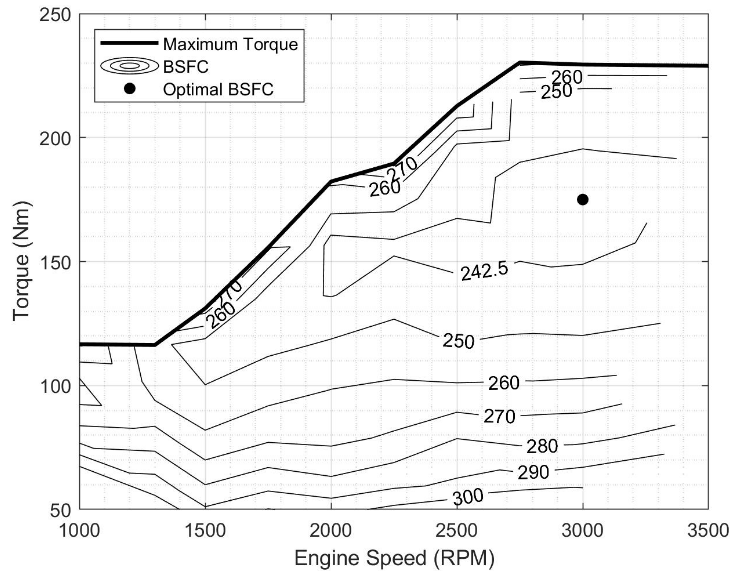

Figure A.2: Environmental Protection Agency brake specific fuel consumption map for General Motors 2.5L LCV internal combustion engine.

in Equation A.8. The electric traction motor losses are quantified in Equation A.9.

$$P_{in} = P_{loss} + P_{out} \tag{A.8}$$

$$P_{loss} = k_c T^2 + k_i \omega + k_w \omega^3 + C \tag{A.9}$$

Where k_c is the copper related loss constant, T is the torque produced by the electric traction motor, k_i is the inductance related loss constant, ω is the speed of the electric traction motor, k_w is the windings related loss constant, and C is the loss constant inherent to the electric traction motor.

The granularity of the electric traction motor efficiency data is limited and can not be implemented into a lookup table without significant interpolation to characterize the performance of the electric traction motor. An electric traction motor with known characteristics is chosen as a reference point to tune and enhance the limited amount of electric traction motor efficiency data provided. The known electric traction motor efficiency data is tuned to match the provided supplier electric traction motor efficiency data by scaling the known electric traction motor maximum torque and base speed by the supplier electric traction motor maximum torque and base speed. Equation A.10 is utilized to scale the known electric traction motor power losses to match the supplier electric traction motor power losses.

$$P_{loss,scale} = \left[\frac{(T)(\omega)}{T_{ref}\omega_{ref}}\right]\left[k_c T_{ref}^2 \left(\frac{T}{T_{base}}\right)^2 + k_i \omega_{ref}\left(\frac{\omega}{\omega_{base}}\right) + k_i \omega_{ref}^3 \left(\frac{\omega}{\omega_{base}}\right)^3 + C\right] \quad \text{(A.10)}$$

The efficiency map generated for the electric traction motor is depicted in Figure A.3.

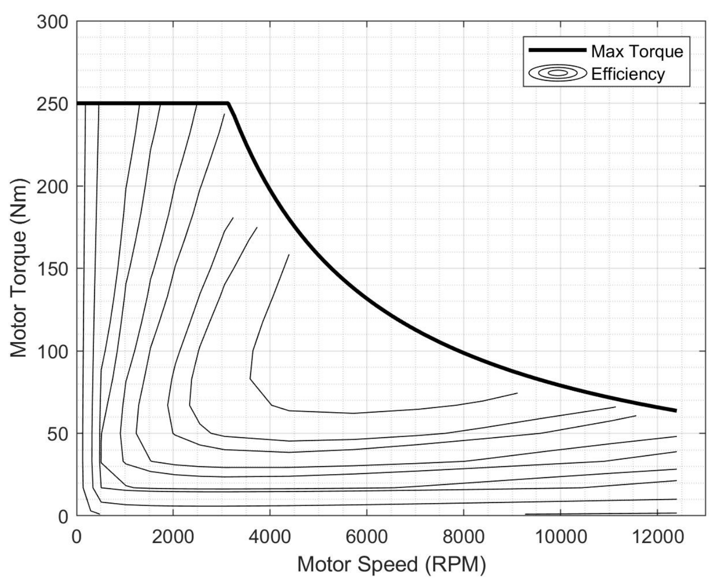

Figure A.3: Enhanced efficiency map for the supplier electric traction motor based upon known electric traction motor characteristics.

A.3.4 Battery Pack Plant Model

The battery pack is a secondary energy source on the hybrid Chevrolet Blazer and is managed to ensure the battery pack operates properly. Proper operation of the battery pack is maintained by operating the battery pack within a specific region of the battery pack energy capacity. The energy capacity of the battery pack is quantified by the state of charge (SOC) of the battery pack. The battery pack state of charge relates the instantaneous energy capacity of the battery pack to the usable energy capacity of the battery pack. Equation A.11 quantifies the battery pack state of charge.

$$SOC(t) = \frac{Q(t)}{Q_{nom}} \tag{A.11}$$

Where SOC in Equation A.11 is the battery pack state of charge, $Q(t)$ is the instantaneous energy capacity of the battery pack, and Q_{nom} is the usable energy capacity of the battery pack. The usable energy capacity of the battery pack is defined by the state of charge operating region. Automotive battery packs typically have a state of charge region of operation between twenty percent and eighty percent state of charge, therefore limiting the total energy capacity of the battery pack to sixty percent of the battery pack energy capacity. The total energy capacity of the battery pack is assumed to be a constant value and therefore the usable energy capacity of the battery pack is also assumed to be a constant value.

A zero order model of Ohms Law is used to quantify the power of the battery pack. The power of the battery pack is a function of the battery pack state of charge. Figure A.4 depicts the change in nominal voltage of the battery pack as a function of battery pack state of charge. The nominal voltage of the battery pack as a function of battery pack state of charge is obtained by multiplying the number of battery cells in series by the nominal voltage of a single battery cell.

A power balance model is used to quantify the output power of the battery pack. Equation A.12 details the potential power, output power, and losses of the battery pack.

$$P_{output} = P_{potential} - P_{losses} = V_{nom}I(t) - R_0 I(t)^2 \tag{A.12}$$

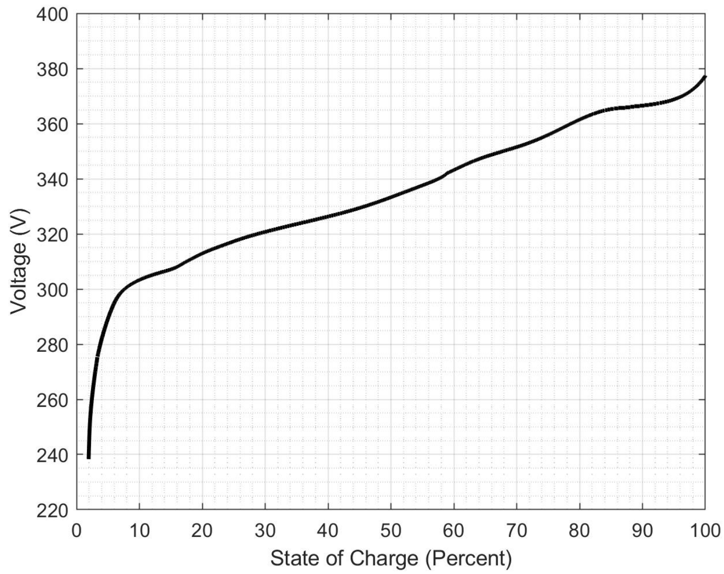

Figure A.4: Battery pack nominal voltage as a function of battery pack state of charge.

The measured output power of the battery pack, P_{output} in Equation A.12, must overcome the internal losses of the battery pack before power is charged or discharged from the battery pack. The internal losses of the battery pack are quantified by the internal resistance R_0 and current output of the battery pack. The battery pack is assumed to have a constant internal resistance.

Equation A.12 is used to solve for the current of the battery pack at a battery pack power demand. Equation A.13 quantifies the current of the battery pack at a given battery pack power demand.

$$I(t) = \frac{V_{nom}}{2R_0} - \sqrt{\left(\frac{V_{nom}}{2R_0}\right)^2 - \frac{P_{output}}{R_0}} \tag{A.13}$$

The current of the battery pack is dictates the battery pack state of charge. Equation A.3.4 quantifies the rate of change of battery pack state of charge.

$$S\dot{O}C = -\frac{I(t)}{\eta\, Q_{nom}}$$

Where I in Equation A.3.4 is the current of the battery pack, η is the Coulombic efficiency of the battery pack, and Q_{nom} is the nominal energy capacity of the battery pack. Given the battery pack is assumed to have a constant usable energy capacity and the internal resistance of the battery is constant, integration of battery pack state of charge is deemed an acceptable method to quantify the battery pack state of charge over time.

Appendix B

Second Appendix

B.1 Optimal Control Strategy

B.1.1 Dynamic Programming

Hybrid electric vehicle powertrain and drivetrain components have limitations though which impose state and control trajectory constraints on the cost function at each time step. Dynamic programming is one method which calculates the optimal control trajectory while respecting the local and global constraints. Dynamic programming uses a backward solving approach which discretizes all operating points and accounts for the impact of all constraints associated with each operating points in the form of a cost term. Dynamic programming evaluates the cost function for the last time step in a drive cycle and each cost function prior to that time step until the minimal cost for an entire drive cycle is calculated. Dynamic programming therefore requires complete knowledge of a drive cycle before simulating the performance of a hybrid electric vehicle. Equation B.1 details the backwards calculation of the minimum control based on the prior control authority to generate the global minimum control input vector.

$$\mu(x_k, k) = min\left(L_k(x_k, u) + Y_{k+1}(f_k(x_k, u_k), u_k)\right) \tag{B.1}$$

Dynamic programming is used to calculate the global minimum energy consumption by using a backward facing simulation. The optimal control problem however requires complete knowledge of the vehicle demand or drive cycle to calculate and command a the minimum energy consumption at each discrete time step.

B.1.2 Pontryagin's Minimum Principle

Dynamic programming is used to develop a forward facing model which can generate optimal solutions. Pontryagin's minimum principle presents a forward solving approach which calculates a control trajectory based on an instantaneous cost function. Pontryagin's minimum principle guarantees optimal energy consumption when the control trajectory generates the global, not local, minimal cost. Pontryagin's minimum principle however does not guarantee optimal energy consumption because the minimal results may exist outside the local constraints. The co-state also requires a shooting method strategy to determine the ideal co-state value which yields the optimal energy consumption control trajectory, if it exists. Equation B.2 details the Hamiltonian cost function that Pontryagin's minimum principle uses to generate optimal results.

$$H = P_{fuel}(t) + \lambda(t) P_{elec}(t) \tag{B.2}$$

In equation B.2, H is the summation of input powers, P_{fuel} is the input power produced by E10 fuel, λ is the co-state variable, and P_{elec} is the input power produced by electric power.