THE EFFECTS OF THE PRESSURE ARCH UPON MULTIPLE SEAM MINING

by

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I. - INTRODUCTION

The coalfields of the United States, and particularly those of the Appalachian region of the eastern United States, consist of many contiguously placed, mineable coal seams. Both historically and presently, the determination of the mining sequence of these contiguously placed coal seams has been based largely on the ownership of the coal seam, the availability of the coal seam, and the general economic situation at the time in question. If, however, the mining of any particular coal seam occurs without regard to any subsequent ground control problems which may arise, serious and hitherto unaccounted problems may affect any future mining operations in seams above or below that which was previously mined. This lack of concern for future operations may often be detrimental to the recovery, cost and safety of the mining operation of adjacent seams.

The ground control mechanisms which should be considered in an area of multiple seam or multiple horizon mining can be divided into four distinct areas. These areas are the arching of stresses around mine openings, the effects of subsidence from a previously mined lower seam upon an overlying seam, the transfer of pillar loads from one seam to another, and massive interseam failure (Haycocks and Karmis, 1981).

The concept of arching attempts to explain the interaction of stresses that results from mine openings rather than those which develop due to the pillars. The most common arching theory is the pressure arch theory which states that a high pressure zone arches over the opening in an elliptical shape, while the area inside this high pressure arch is de-stressed with respect to the normal cover load.

In multiple seam mining, the high arching stresses as well as the de-stressed area within the arch may affect the mining conditions in an adjacent overlying seam. The effects of arching upon multiple seam mining under sub-critical subsidence conditions are reported in this study.

The stresses around underground mine openings of various widths and depths of cover are modelled by use of the finite element computer program. The finite element program yields the horizontal and vertical stresses as well as the principal stresses and their inclinations across the model. The finite element method was verified by using photoelastic modelling which exhibited similar configurations of the pressure arch. The finite element method was then used to model an actual case study and produced results very close to the observed data. A series of design tables is included to give an approximation of the stresses that may be encountered in a variety of mining conditions.

II. - LITERATURE REVIEW

2.1 - Arching Theory

The concept that the rock above an underground mine opening would form an arch in order to support itself is one that is common to both the hard rock and coal mining industries. This arching must be considered in the design of any multiple horizon mining operation.

The concept of arching was first discussed by Fayol (1885) in his study of the effects of underground coal mining upon the surface. Fayol, who was one of the early investigators of mining subsidence, concluded that "the movement of strata is limited by a kind of dome which has for its base the area worked out, and that the amount of movement is less as it is further from the center of the excavation." Fayol postulated that when the areal extent of excavation was vast the height of the dome was not greater than 200 times the extracted seam height. When the area of excavation is of limited extent Fayol concluded that the height of the dome is between two and four times the width of the excavated area. More specifically, the height of the dome is twice the width of the excavation when the seam height is less than two meters (6.5 feet) and increases as the height of the extracted seam becomes greater. Fayol's

conclusions were based on model studies that dealt with a stratified medium. He felt that the roof consisted of a series of independent beams with fixed ends which fail due to the inward shearing of the beam ends. This action gives rise to the dome shape as shown in Figure 1. This dome theory that Fayol presented has become the basis of two-dimensional pressure arch theory.

One of the earliest proponents of arching, or pressure arch theory, was B. S. Randolph (1915). Randolph stated that when the coal is first mined, the first stress on the overlying strata is very similar to that on a uniformly loaded horizontal beam. Since the overlying strata tend to have a low tensile strength, an arch-shaped line of stress carries the weight of the overlying strata to the solid material on either side of the excavation. This line of stress would be in equilibrium and support all the material above it. Along and on both sides of this line of stress there is a zone of material upon which more or less pressure is exerted, its width depending on the total strength and elasticity of the material. This pressure tends to hold the material in place. Randolph felt that the position and type of forces acting on the arch would affect the shape of the line of stress or arch. If a hydrostatic pressure situation existed, the arch would be in the shape of a semi-circle. If there was a larger load toward the center of the

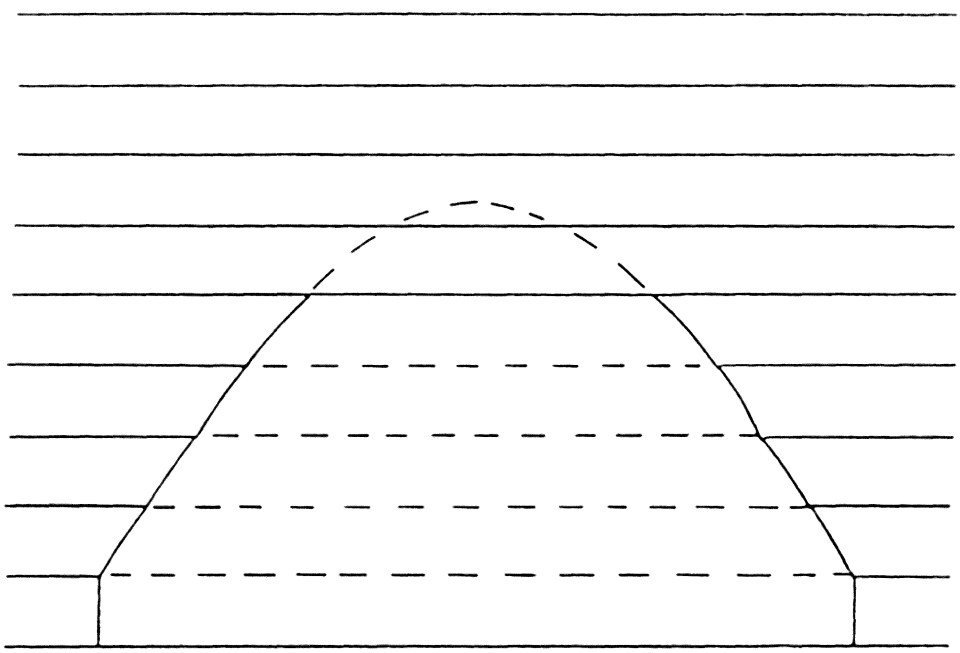


Figure 1. Arch Formed in Stratified Rock (after Fayol, 1885).

excavation, the arch would be in the shape of a parabola. Also, if the overlying strata is comprised of different materials, the position of the line of stress will be altered and the shape of the arch changed in order to maintain a state of equilibrium.

In 1937 J. R. Dinsdale presented a major work on the pressure arch theory. First, several terms must be defined before they are used to explain Dinsdale's paper. Before any excavation is made the ground at any depth is subjected to a pressure equal to the weight of the column of the ground above. This pressure is called the "superincumbent" or "undisturbed" pressure. When an excavation is opened, a pressure ring forms around the opening and the weight of the ground above, part of the superincumbent pressure, is deflected to the solid rock at the sides of the excavation. Since the sides of the excavation form the abutments of the pressure arch, the pressure on them is called the "abutment pressure". Inside this pressure ring there is a core of decompressed and fractured ground which is called the "intradosal" ground. Around the outside of this core of decompressed or de-stressed ground there is a zone of firm, compressed ground called the "extradosal" ground. These terms are illustrated in Figure 2.

According to Dinsdale, the height of the intradosal ground above the excavation is very dependent upon the

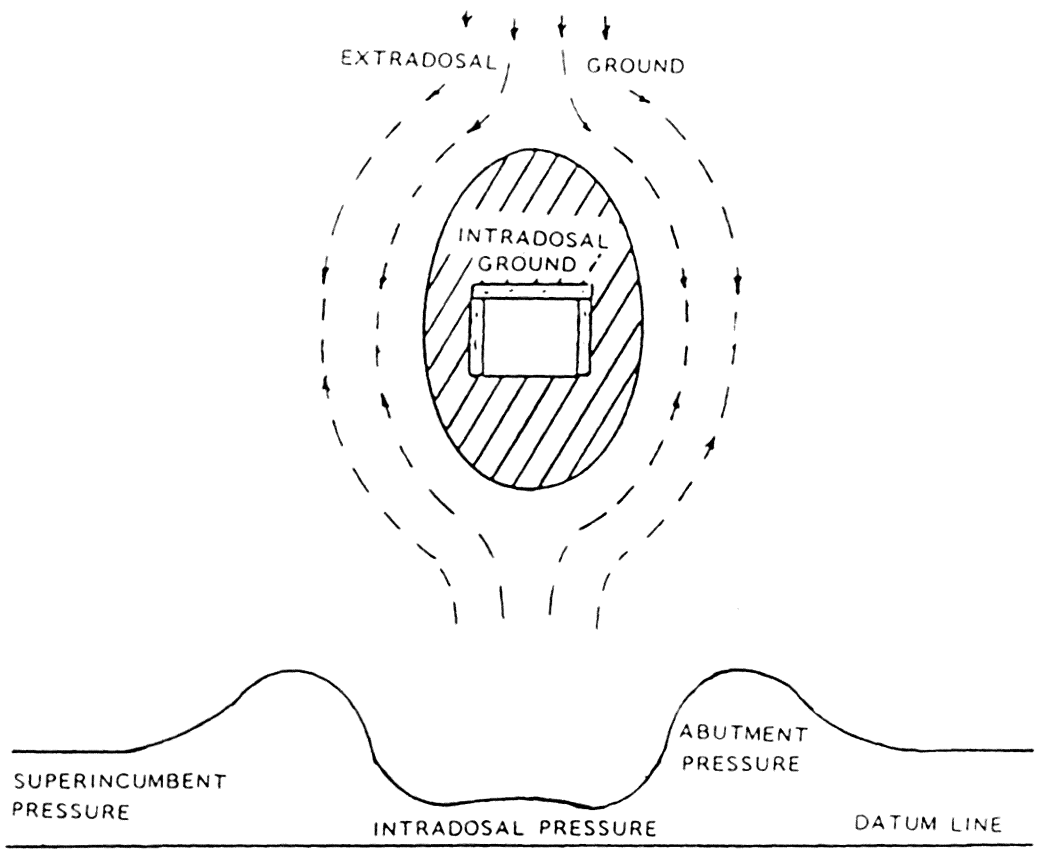


Figure 2. Pressure Arch Theory Terminology (after Dinsdale, 1937).

resistance of the material forming the arch to crushing and the intensity of pressure to which the arch is subjected. The stability of the pressure arch is maintained until failure takes place at either of the abutments or along the arch itself. Failure continues until a new pressure arch forms and equilibrium is restored.

Where pillars are left in place in the excavation, each pillar forms an abutment for the minor pressure arches which span the openings between the pillars as shown in Figure 3. If one of these pillars fails a sudden merging of adjacent minor pressure arches takes place. This merging results in the formation of a major pressure arch with a coinciding increase in the pressure on the abutments of the major arch. This occurrence is illustrated in Figure 4.

Dinsdale also noted that when the excavations are continually widening, as in the case of longwall mining, the pressure arch renews itself as the span of the arch increases. Eventually, the pressure arch is incapable of spanning the opening and obtaining support along the solid sides of the excavation. When this occurs, the arch and the extradosal ground it supports will collapse and result in the subsidence of the surface.

Denkhaus (1964) surveyed strata movement theories, including arching theory. Among these theories are the rigid dome theory, the presence of a fracture dome in elastic rock and its modifications and beam theory.

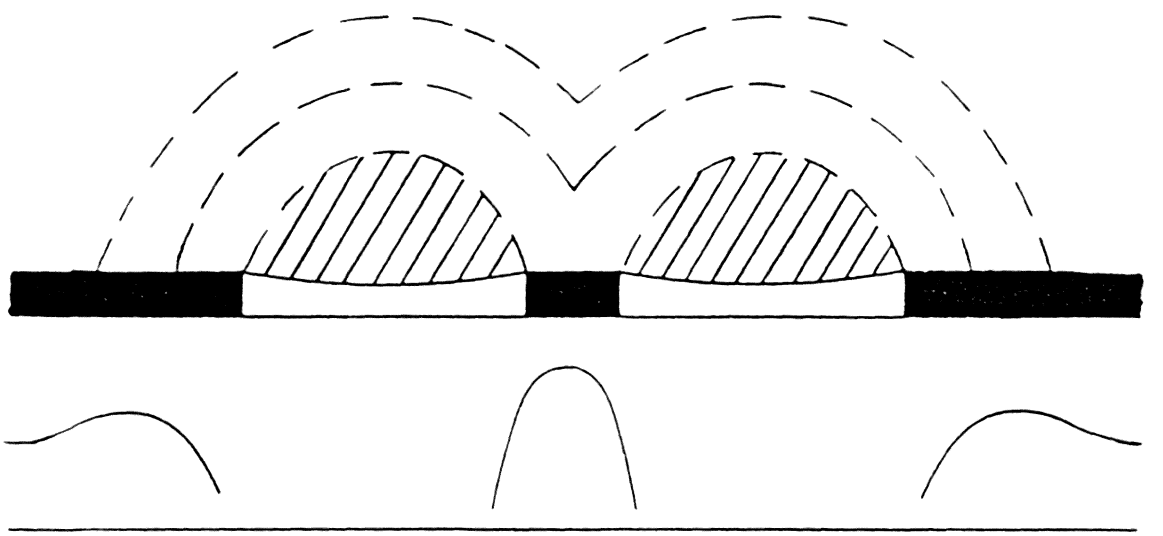


Figure 3. Minor Pressure Arches Across Excavation (after Dinsdale, 1937).

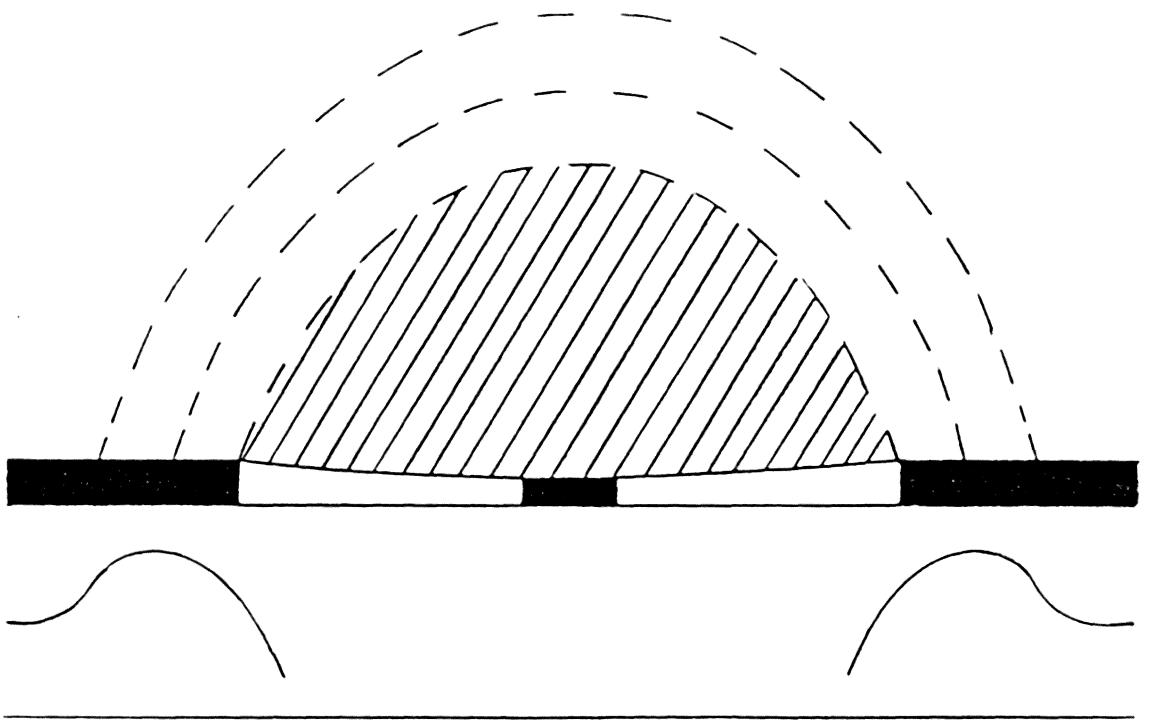


Figure 4. Major Pressure Arch Formed From Merger of Minor Arches (after Dinsdale, 1937).

The rigid dome theory is the simplest of these theories since it is based on the equilibrium of forces at the boundary of the dome. This theory does not take deformation into account, however, it does consider failure which occurs when the load reaches a critical value called the failure load. In order to simplify the theory, a dome which is normally three dimensional is reduced to a two dimensional problem, as in the case of a longwall panel where the length of the panel is of sufficient length to eliminate it from the calculations. Also, a distinction must be made between sufficiently cohesive rock and insufficiently cohesive rock.

In the case of sufficiently cohesive rock, the core of the dome or the intradosal ground will not separate from the dome boundary. For this reason the rock along the dome boundary must carry the weight of the core of the dome. If the span of the dome becomes too large, the weight of the core will exceed the cohesive resistance of the rock and a sudden collapse of the core or a major portion of it may occur. Any support which is under the dome would have to take a large amount of weight suddenly.

If the rock is insufficiently cohesive, portions of the core will separate from the dome boundary either gradually or at short intervals as the span of the dome increases. Of the two conditions, this is the more favorable with respect to ground control considerations. As the span of the dome

increases, the core may form smaller domes within the main dome in order to support itself.

Denkhaus then presented the following formulas to determine the span at any depth, the maximum height for a stable dome and the corresponding maximum span. In the case of insufficiently cohesive rock, the formula for span is:

$$s = \sqrt{\frac{8\sigma_c d}{w}\ (1-h/d)\log(1-h/d)} \qquad \text{(Eq. 2.1)}$$

where s is the span, h is the height of the dome, d is the depth of the opening below the surface, w is the specific weight of the material, and σ_c is the uniaxial compressive strength of the rock. The formulas for the maximum height for a stable dome and its corresponding span are:

$$H_{max} = 0.63d \qquad \text{(Eq. 2.2)}$$

$$S_{max} = \sqrt{\frac{2.96\sigma_c d}{w}} \qquad \text{(Eq. 2.3)}$$

For sufficiently cohesive rock, the formulas change to the following:

$$s = \sqrt{\frac{8\sigma_c h}{w}\ (1-h/d)} \qquad \text{(Eq. 2.4)}$$

$$H_{max} = 0.5d \qquad \text{(Eq. 2.5)}$$

$$S_{max} = \sqrt{\frac{2\sigma_c d}{w}} \qquad \text{(Eq. 2.6)}$$

Note: when the rock is sufficiently cohesive the core of the dome remains in situ and the shape of the dome is parabolic. The above formulas are based on the assumption that the rock is both homogeneous and isotropic as well as other assumptions which are presented in the original and not discussed in this report.

As the height and the span of the dome increase, the probability that non-homogeneities and anisotropics exist in the rock increases. Their presence would weaken the actual strength of the rock medium.

The rigid dome theory does not give any information about the stress or movement ahead of the face nor around or within the dome. It does, however, set limits to a zone of movement; displacement does not occur outside the dome but it may occur within the dome. This, of course, is because the rigid dome theory does not account for deformation of the rock, as it is considered a rigid medium.

The second theory that Denkhaus discusses is that of the fracture dome in elastic rock. In this theory the rock is considered to be a homogeneous isotropic elastic body. According to Denkhaus, "If a failure criterion for rock, that is the condition under which rock fails for a given state of stress, is applied to the stress distribution around an excavation, it is possible to predict the shape and size of the fracture zone that may surround the

excavation." In this theory, it is assumed that a fracture zone will assume a shape and size so that at the dome boundary neither the tensile stress exceeds the uniaxial tensile strength nor the compressive stress exceeds the uniaxial compressive strength.

For this theory an axis ratio, 2h/s, can be obtained from the following formulas:

$$2h/s = (1+\sigma t/wd)/2k - 0.5 \qquad \text{(Eq. 2.7)}$$

$$2h/s = 2/(k - 1 + \sigma c/wd) \qquad \text{(Eq. 2.8)}$$

where k is the ratio of lateral to vertical initial rock pressure, σt is the uniaxial tensile strength and the other symbols are the same as those used previously. The fracture dome will take the shape of an ellipse that will use the largest of the two ratios above. Note: when k equals zero, ratio 2h/s equals infinity.

The advantage to the fracture dome in elastic rock theory is that the ground within the ellipse or the intradosal ground is fractured and therefore cannot be taken as a homogeneous elastic medium, while the extradosal ground is still a homogeneous elastic medium whose stress and displacements may still be predicted by using the theory of elasticity. The weakness of the elastic dome theory is that it is based on an oversimplification of the way in which rock actually fails.

There are two major factors that are usually ignored in elastic dome theory that should be considered. First, whenever failure is initiated, the stress distribution around the opening changes. The new stress distribution of the changed geometry should be determined and the failure criterion reapplied and repeated until no further propagation of fracture occurs. Second, the forces exerted from the fractured rock in the intradosal zone onto the dome boundary must be taken into account. Due to its own weight, the fractured rock in the intradosal zone tends to sag, until blocks of the fractured material wedge between each other and the side of the dome boundary. Therefore, the vertical downward forces of the individual blocks may have components causing thrust between the blocks and against the dome boundary and other components tending to shear blocks out of the arch system formed by the thrust. This shear is resisted by the friction between the blocks which in turn is increased by the thrust so that the arch system remains stable. These arch systems are known as Voussoir arches. This dome shape with Voussoir arches is illustrated in Figure 5.

Another theory discussed by Denkhaus is beam and plate theory. In both the rigid dome and elastic dome theory, the rock was considered to be a homogeneous and isotropic medium. One type of non-homogeneity and anisotropy that can

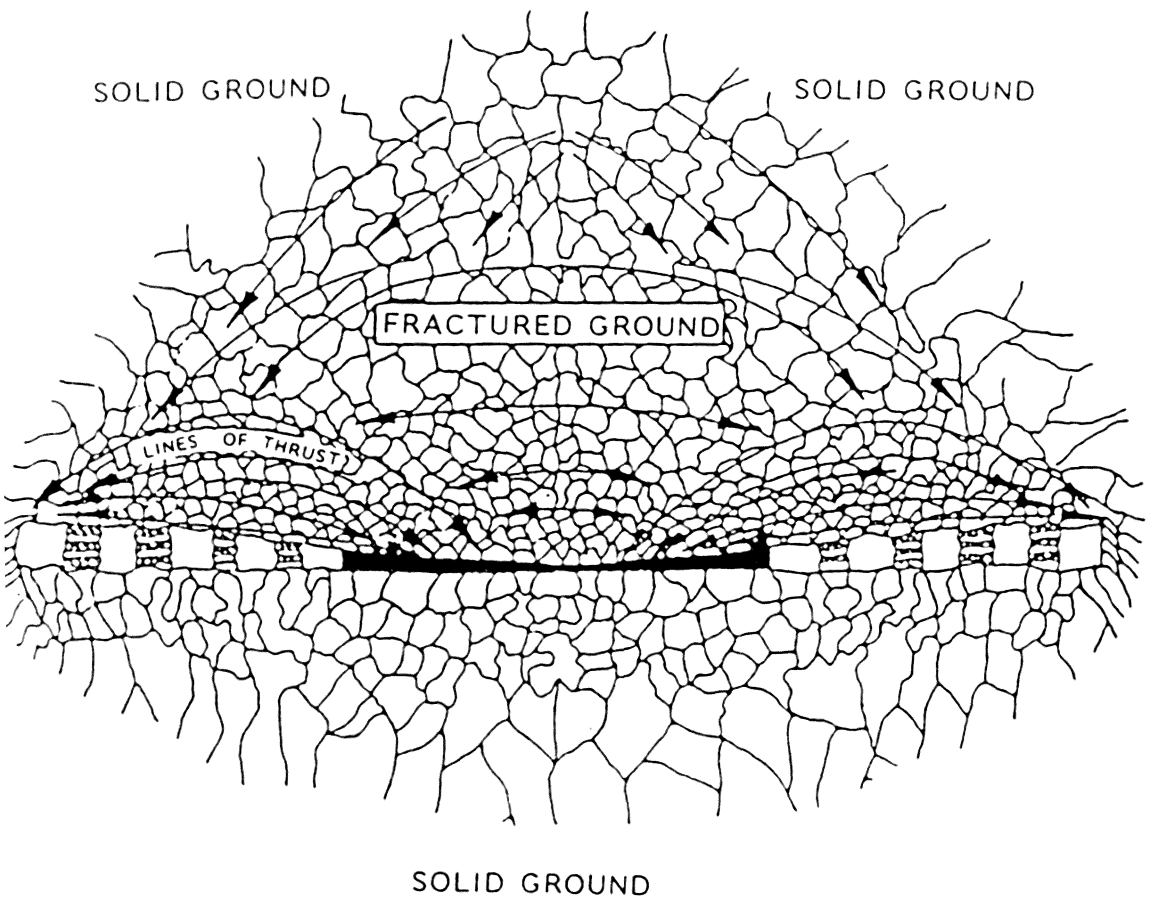


Figure 5. Hypothetical Shape of Dome with Intradosal Voussoir Systems (after Denkhaus, 1964).

more easily be dealt with theoretically is the stratification of a rock medium. This is a case of a series of plates (three dimensional) or beams (two dimensonal) which overlie each other.

In beam theory, the maximum stress occurs at the face and, if the stress is greater than the strength, failure will occur there. If the beams are assumed to be clamped at both ends, failure will occur at the two ends. However, if the beams are somewhat elastic, the point of maximum stress and, therefore, failure moves toward the midpoint along the face. This process continues upwards through the strata until it stops.

The amount of deflection in each beam depends upon the thickness of the strata and its elastic deformation modulus. Therefore, it is possible for a lower strata may deflect more than the overlying beam so that a gap (called the Weber cavity) occurs between the two beams.

The dome itself is defined as the area within which the layers deflect and possibly fail and where Weber cavities may occur, while no deflection or failure occurs in the the extradosal ground outside the dome. Note that the beam theory is based on the assumption that no bond exists between the successive layers so that they may deflect freely.

2.2 - Arch Shape and Size

As already shown in this report, the shape of the pressure arch has been found to be one of three shapes: (1) elliptical, (2) parabolic or (3) a modified shape. Mohr (1956) believed in an elliptically shaped dome where the ratio of the height of the dome to the semi-width of the dome would vary with the ratio of the vertical stress to the horizontal stress. Mohr's relationship is as follows:

$$\frac{B}{L} = \frac{ZZ}{XX} = \frac{1-PR}{PR} \qquad \text{(Eq. 2.9)}$$

where B= the height of the dome from the central axis
L= the semi-width of the dome
ZZ= the vertical stress
XX= the horizontal stress
PR= Poisson's Ratio

This finding appears to be accurate for arches in massive and intact rock and can be determined by measuring the principal stresses present underground. According to this equation, as a hydrostatic stress field condition is reached, the ratio of the axes equals one and the shape of the dome becomes a semi-circular arc as proposed by Irving (1946).

Another important factor in the determination of the shape of the pressure arch is the nature of the strata in which the arch is formed. According to Randolph there will be a high arch in strong strata with few joints. Therefore,

under a strong sandstone bed, the fall of the arch will be so slow that it allows a large span to be excavated from the underlying seam. The arch span enlarges until the stress is so great that failure occurs either at the pillar face or back in the excavation. This continues until the resulting roof falls allow the side of the pressure arch to rest on the gob, thereby reducing the span of the pressure arch.

Another important work on the pressure arch theory was performed by the National Coal Board (1954) which found that with an increase in the width of the underground opening, the size of the pressure arch will increase until a certain width is reached at which the superincumbent pressure can no longer be supported by the abutments along the solid sides of the excavation. At this point, the excavation is spanned by a pressure arch with one abutment on the solid coal and the other resting on the gob. The National Coal Board theorized that the pressure arch extends as far below the seam as it does above the seam and when the abutment of an arch in one seam is superimposed upon an arch abutment from an adjacent seam, the stresses will be additive. The superposition of pressure arch abutments is illustrated in Figure 6. The National Coal Board also indicated that the pressure in a certain area may be below that of the superincumbent pressure. This would occur where an upper seam lies within the de-stressed zone of a pressure arch

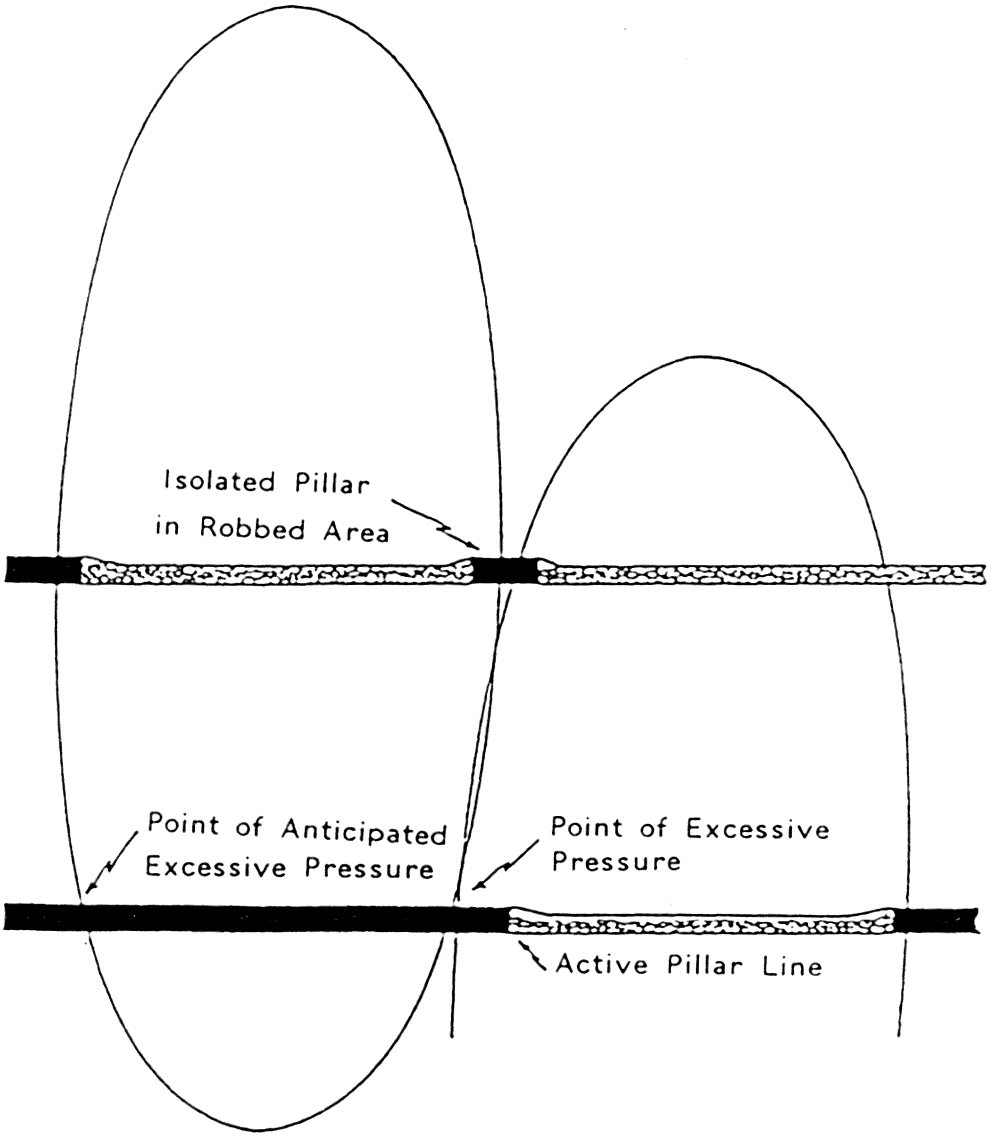


Figure 6. Superposition of Arch Stresses (after Stemple, 1956)

created by the mining of a superjacent or subjacent coal seam. The National Coal Board felt that the strata is under great pressure for about 20 to 30 yards beyond the outer edge of the de-stressed intradosal zone. However, beyond this distance, the pressure rapidly reduces to the normal superincumbent load. The height of the pressure arch was held to be about twice the width of the pressure arch. The National Coal Board created a formula to obtain the width of the maximum pressure arch for depths between 400 and 2,000 feet:

$$W = D/20 + 20 \qquad \text{(Eq. 2.10)}$$

where W = width of the maximum pressure arch in yards
and D = depth of seam below surface in feet

For example, if Seam "C" is 600 feet underground and being mined, it is possible for a pressure arch to form to a height of 300 feet above Seam "C". If Seam "B" is only 120 feet above "C" it may be affected by this maximum pressure arch of Seam "C". If Seam "A" is 350 feet above Seam "C" it will not be affected by the pressure arch, as shown in Figure 7. A comparison of the size of pressure arches according to both theory and experience is illustrated in Figure 8.

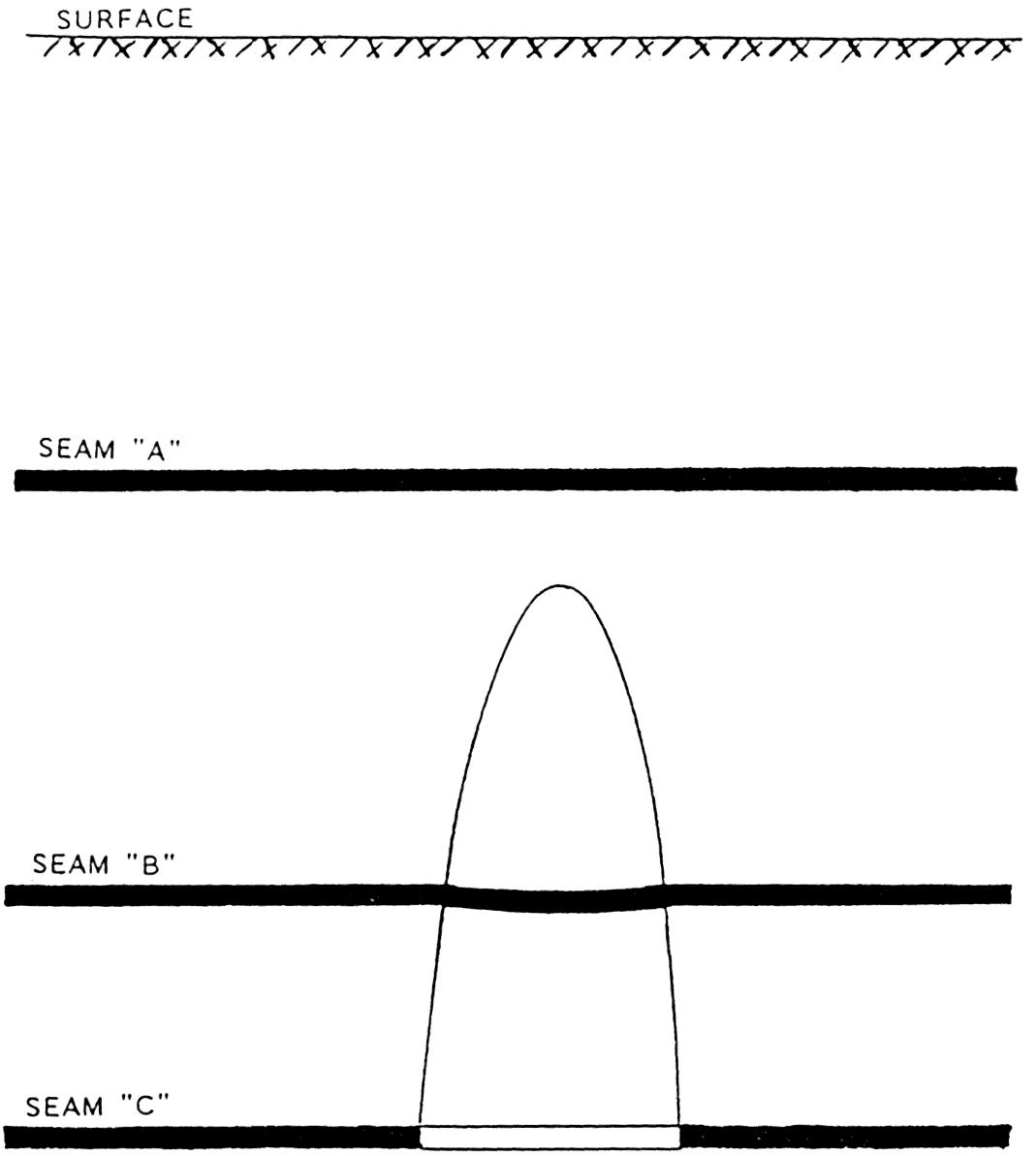


Figure 7. Extent of Major Pressure Arch

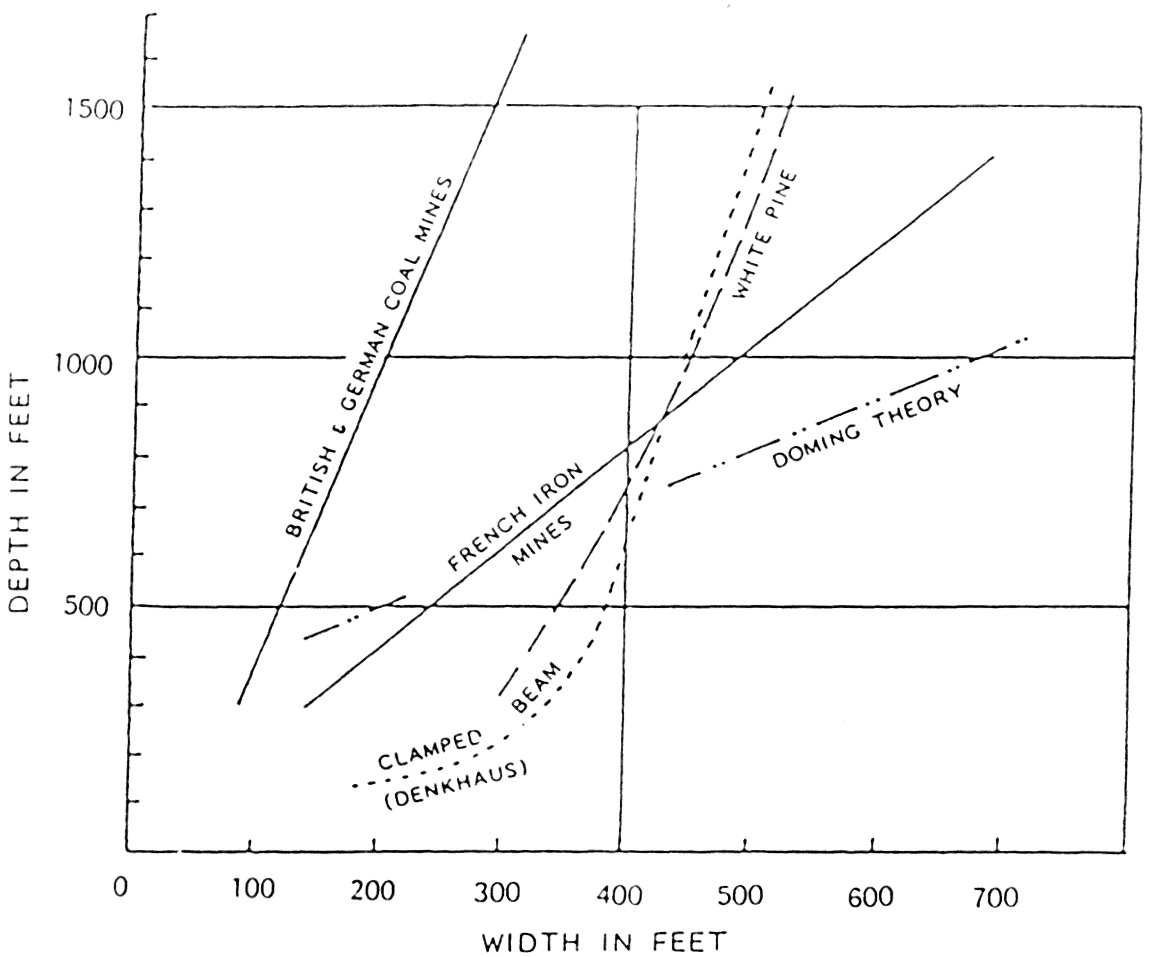


Figure 8. Comparison of Pressure Arch Size, Theory and Experience (after Barrientos and Parker, 1969).

2.3 - Finite Element Method

The finite element method is a mathematical technique in which a continuous solid is modeled by the use of a mesh containing a finite number of elements interconnected at the corner or nodal points. The nodal points and elements are then assigned numbers, and the coordinates of the nodal points are determined. Throughout experimental displacement of the model, the straight lines which form the elements are forced to remain straight. The stresses and strains that arise from forcing the elements to remain straight are evaluated according to equilibrium equations. The basic equilibrium equation used in finite element analysis is (Desai and Abel, 1972):

$$[k]\{q\} = \{Q\} \qquad \text{(Eq. 2.11)}$$

where $[k]$ is the element property matrix,
$\{q\}$ is the vector of nodal displacements,
$\{Q\}$ is the vector of element model forcing parameters.

. These equilibrium equations are then assembled in order to obtain a relationship of the following form (Desai and Abel, 1972),

$$[K]\{r\} = \{R\} \qquad \text{(Eq. 2.12)}$$

where $[K]$ is the assemblage property matrix,
$\{r\}$ is the assemblage vector of nodal displacements, and
$\{R\}$ is the assemblage vector of nodal forcing parameters.

The boundary conditions and the specific loading conditions for any particular point can be incorporated into the finite element analysis to solve for the nodal displacements. The stresses and strains can then be determined by using a transformation matrix such as the one below (Desai and Abel, 1972):

$$\{S\} = [c][B]\{q\} \qquad \text{(Eq. 2.13)}$$

where $\{S\}$ is the stress in any element,
$[c]$ is the stress-strain matrix,
$[B]$ is the transformation matrix, and
$\{q\}$ is as above in Equation 2.11

Further discussion of the finite element method is beyond the scope of this thesis. Additional information on the finite element method may be obtained from Desai and Abel, 1972, and Zienkiewicz, 1977 as well as many other works.

The output from most finite element programs is in the form of displacement at the nodes, the vertical and horizontal stresses, the principal stresses and the angles at which they occur, and the shear stress for any given element.

2.3.1 - Advantages of the Finite Element Method

It is important to understand why the finite element method is preferred over other methods of stress analysis. Several of these advantages are listed by Wilson,1965.

1) The method is completely general with respect to the geometry and material properties.

2) Complex bodies composed of many different materials can easily be modeled.
3) Anisotropic materials are automatically included in the formulation and thereby allows filament structures to be handled easily.
4) Displacements or stress boundary conditions may be specified for any nodal point of the finite element mesh.
5) Arbitrary thermal, mechanical and accelerated loads can also be included in the analysis.
6) In regards to the accuracy, it has been shown mathematically that the solution of a finite element analysis converges to the exact solution as the number of elements for a particular size mesh is increased. In this way the desired degree of accuracy can be obtained.
7) In the mechanics of the finite element method, a symmetric, positive, definite matrix which most often occurs in a banded form can be solved by using a minimum of computer storage.

2.3.2 - Modeling Criteria for the Finite Element Method

Perhaps the most important aspect of the finite element method is the discretization of the model body itself. The choice of the number, size, shape and configuration of the

elements should be made in order to simulate the original body as closely as possible and to obtain a higher degree of accuracy in the results of the finite element method.

Although a certain degree of engineering judgment is essential in the discretization of the model, there are several guidelines suggested by Desai and Abel (1972) that make this process easier.

1) In the location of the nodes, subdivision lines and planes are usually placed where abrupt changes in geometry, loading and material properties occur.
2) A finer subdivision is necessary in regions where stress concentrations may be expected.
3) Curved boundaries of the model can be approximated as a step-like edge formed from the sides of the elements adjacent to the boundary, if straight-sided elements are to be employed. Isoparametric elements with curved sides can be used in lieu of straight-sided elements.
4) To minimize the computer time and storage for the overall stiffness matrix, the band width should be as small as possible. The band width is given as,

$$B = (D + 1)f \qquad \text{(Eq. 2.14)}$$

where B is the semi band width

D is the maximum difference occuring for all elements of the assemblage, and
f is the number of the degrees of freedom at each node.

For this reason the numbering system of the nodes is very important in terms of computer time and storage. Figure 9 (b) shows the better way to number the nodes and elements for the model shown.

5) The aspect ratio of the elements affects the accuracy of the finite element solution. The aspect ratio is the largest dimension of the element to the smallest dimension. Long, narrow elements are usually avoided in a finite element mesh, whereas equilateral quadrangles are most often used. Figure 10 shows the inaccuracy of solutions as a function of the aspect ratio.

6) For a body of infinite lateral dimensions, side boundaries can be assumed and the side nodes are allowed to move only vertically, not horizontally.

Kulhawy (1974) showed that for geotechnical problems, such as those under consideration in this study, arbitrary, linear strain quadrilateral elements are the most appropriate. Kulhawy recommends a minimum of 125-150 elements for the analysis of simple structures in

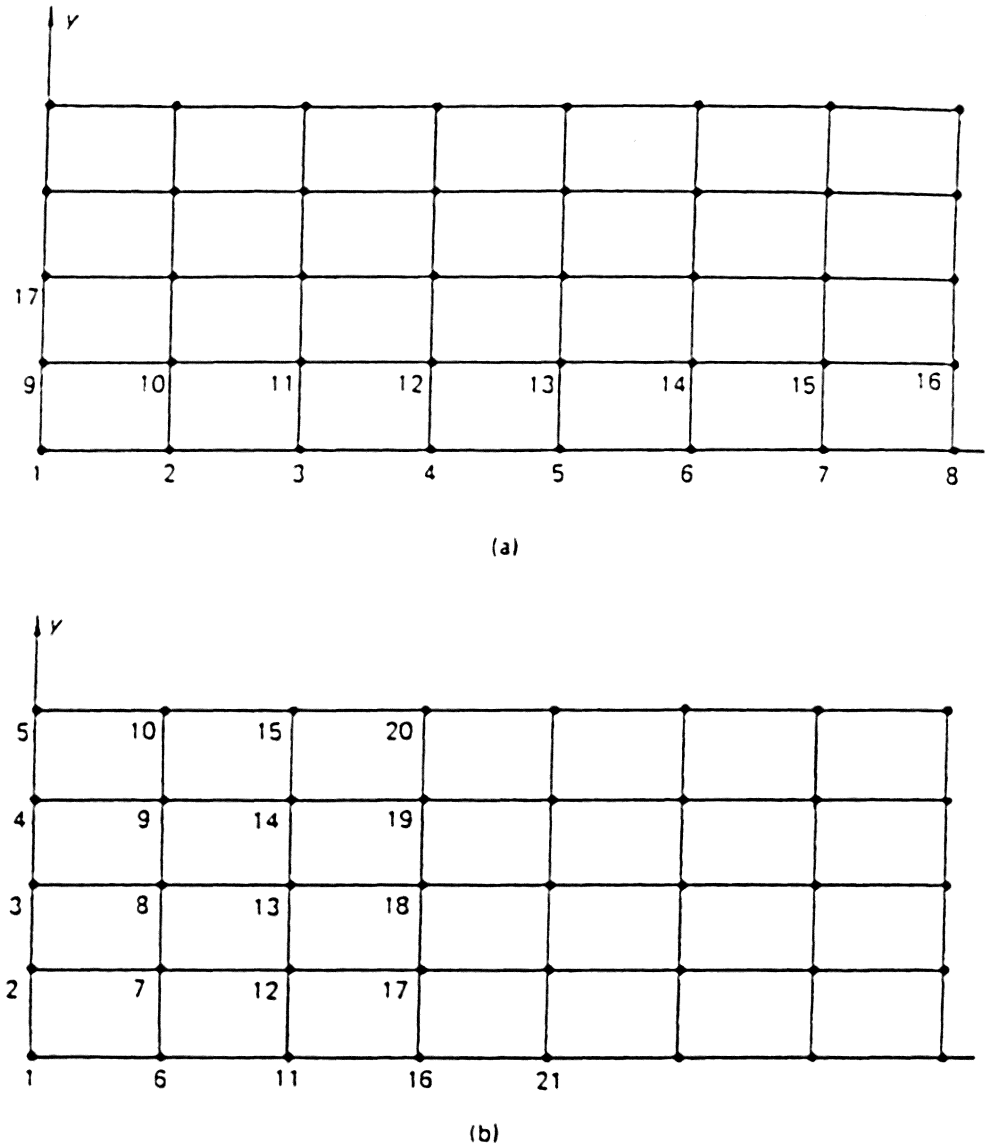


Figure 9. Numbering of Nodes to Reduce Band Width (after Desai and Abel, 1972).

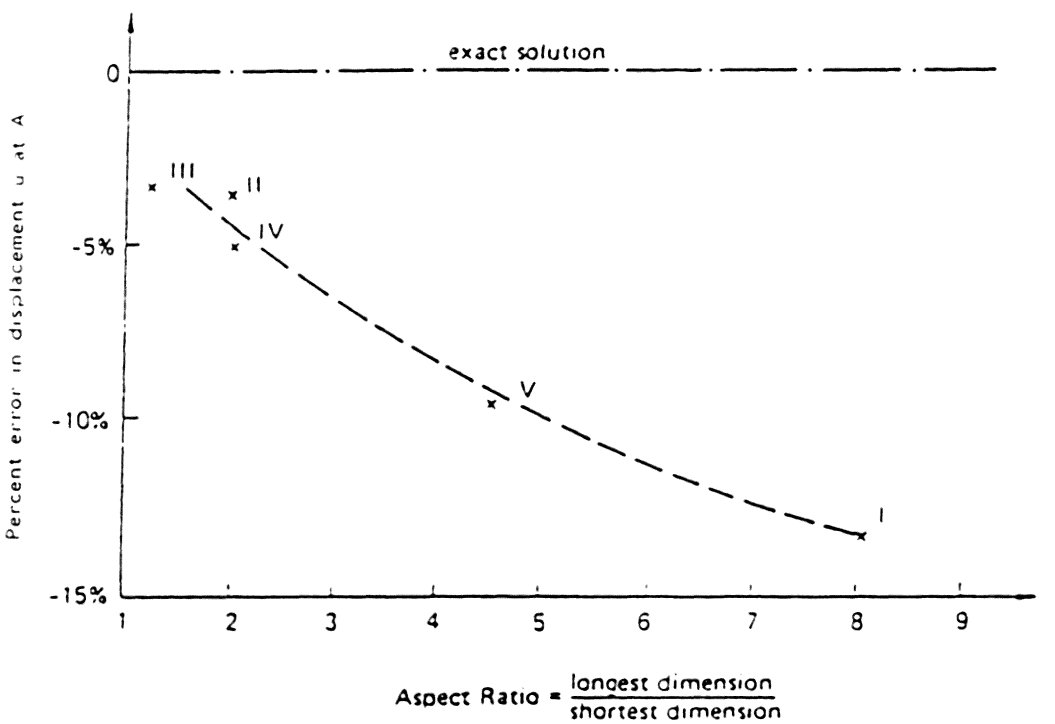


Figure 10. Inaccuracy of Solution as a Function of Aspect Ratio (after Desai and Abel, 1972).

homogeneous rock and that the boundaries of the finite element mesh should be at least six radii away from the center of the opening in order to minimize the boundary effects upon the solution as shown in Figure 11.

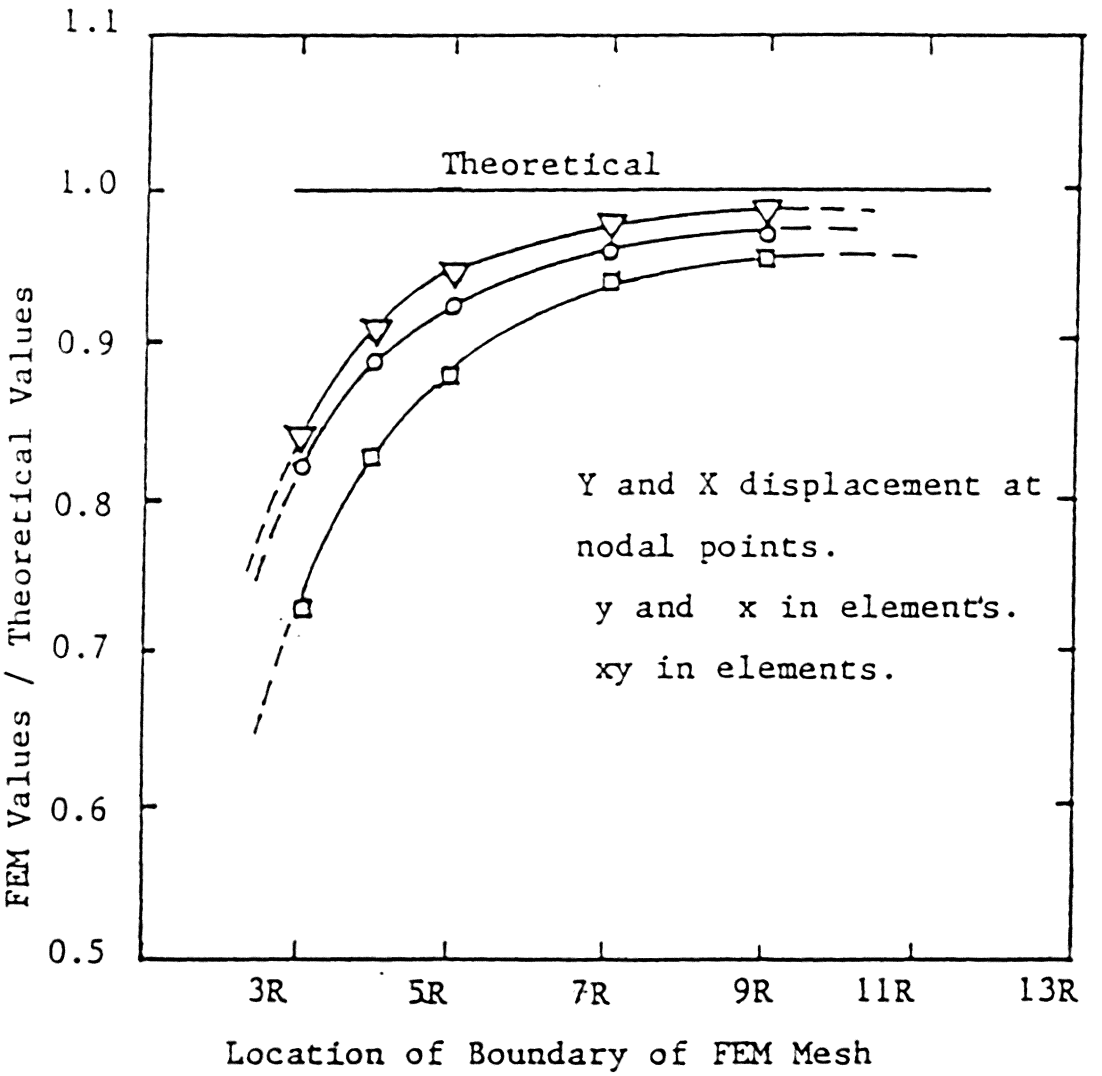


Figure 11. Effect of External Boundary Location on Accuracy of Finite Element Method Solution (after Kulhawy, 1974).

III. - CASE STUDIES

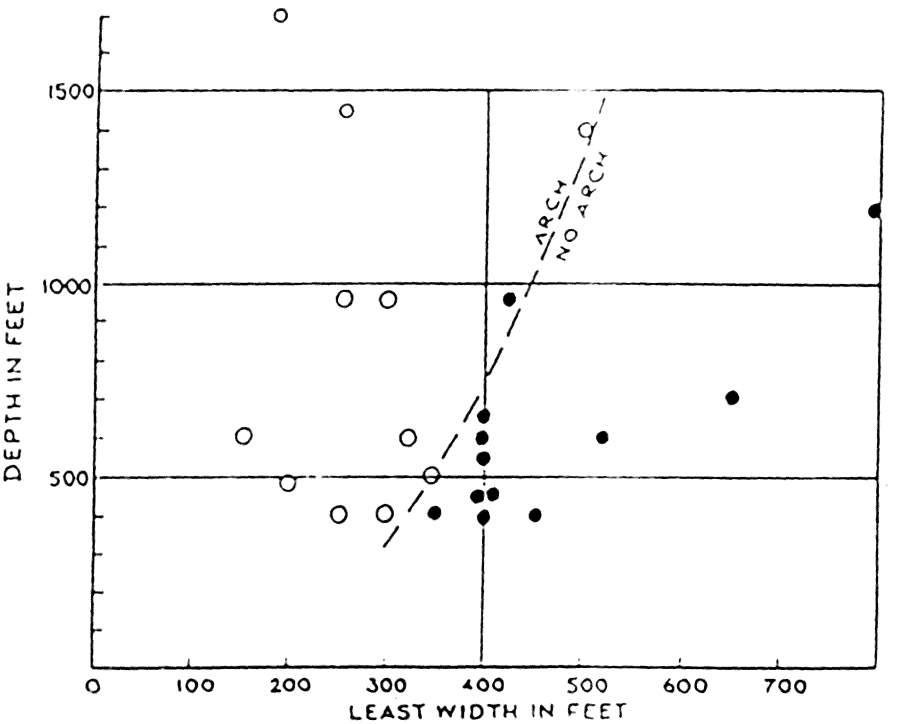
In order to analyze the effects of the pressure arch upon multiple seam mining, several case studies were collected from published literature.

3.1 - White Pine Copper Mine

The study by Barrientos and Parker (1974) was conducted at the White Pine copper mine in Michigan and relates how simple observations and measurements were used to define the limits of the pressure arch.

Mining conditions at the White Pine mine were stable until the excavation reached a critical width of approximately 300 feet. Observations were made at many underground failures, some of which were not planned but the majority of which were created by pillar robbing operations. At this point it became clear to Barrientos and Parker that, within some critical width of excavation, pillars could fail without a major collapse of the roof occurring and that this critical width of excavation increased slightly with an increase in depth. The results of these observations are shown in Figure 12.

In July 1966, full scale workings of various dimensions were developed at White Pine in order to test the pressure arch theory and to define the maximum width of the arch.



Underground collapse appeared at surface ●
Underground collapse did not appear at surface O

Figure 12. Critical Width versus Depth
(after Barrientos and Parker, 1974).

Several conclusions were drawn from the information supplied by the test panels:

1) A pressure arch can be formed,
2) The maximum width of the arch at a depth of 550 feet was at least 308 feet,
3) Pillars within the panel must be small enough to yield or no arch would form,
4) Load was transferred to abutment pillars,
5) Distribution of load could be arranged through design of pillar stiffness,
6) High extraction (in this case 92%) can be attained within the narrow panel,
7) Effective modulus of deformation of small pillars was less than half of the laboratory value (Barrientos and Parker, 1974).

The extent of the pressure arch which formed as a result of the mining conducted at White Pine is shown in Figure 13. Barrientos and Parker believed that the rock mass could not exert any significant resistance to tensile stress. However, due to many measurements and observations they felt that it was the lateral compressive forces which held the jointed rock mass in place in the roof. They used these findings to further explain the pressure arch theory. According to Barrientos and Parker the weight of the overburden above a cave or yielding pillars can be supported

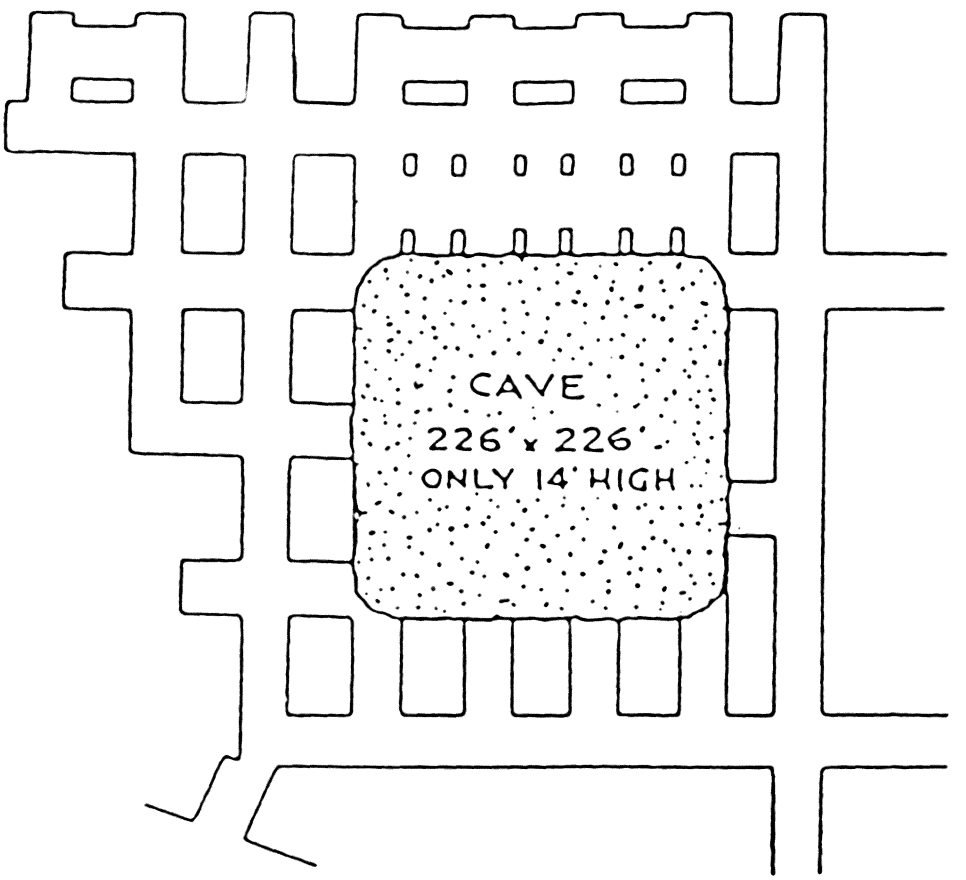


Figure 13. Extent of Pressure Arch Formed as Result of Mining at White Pine (after Barrientos and Parker, 1974).

from the sides if the lateral stress is great enough and the weight of the rock is not too great. Since the weight of the rock mass within the pressure arch, per square inch of side support area, depends upon the unsupported span, for each different lateral stress condition there will be a different critical width of excavation. Since the lateral stress is comprised of two parts, a primitive stress that exists near the surface and changes little with depth and the Poisson component of vertical stress that does change with depth, the width of the pressure arch can be expected to increase as depth increases. The two components of lateral stress are illustrated in Figure 14.

3.2 - Cadley Hill Colliery

This case study was conducted at the Cadley Hill Colliery of the Main Coal District. The Main Coal seam lies at a depth of approximately 1600 feet and averages 14 feet 6 inches thick. Approximately 190 feet below the Main Coal lies the four foot thick Woodfield seam, 60 feet below that is the 5 foot 8 inch Stockings seam and about 50 feet below that is the 4 foot Eureka seam.

The roadways of the Main Coal workings crossed over areas that were worked in the lower Stockings seam and the following observations were made:

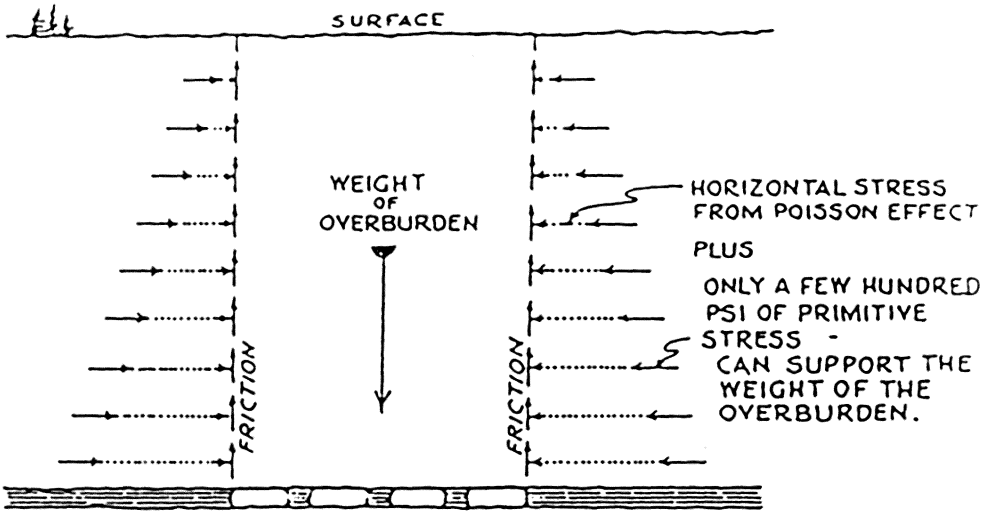


Figure 14. Two Components of Lateral Stress
(after Barrientos and Parker, 1974).

1) The ground immediately above the worked panel of the lower Stockings seam was de-stressed and the roadways in the Main Coal seam were in excellent shape.

2) The ground immediately above or to one side of the edge of the gob in the Stockings seam was highly stressed and the roadways driven in the Main Coal seam deteriorated badly.

Specific examples of the effect of areas of high stress and low stress upon an upper seam include a return gate which was driven in the Main Coal seam directly above a pillar edge in the Eureka seam experienced over five feet of floor heave before the face retreated. A Main Coal main gate that was above the Stockings gob stood in good shape until the entries passed the edge of the gob and began to deteriorate. A cross-section of the Main Coal seam and the Woodfield workings is shown in Figure 15. These examples of the effect of the stresses around a pre-existing lower mining opening upon an upper seam should be noted in the future in the mine design of an upper seam working over a previously mined lower seam.

3.3 - Monongalia County, West Virginia

This case study is one of many documented by Stemple (1956). In this instance the lower seam is the Pittsburgh

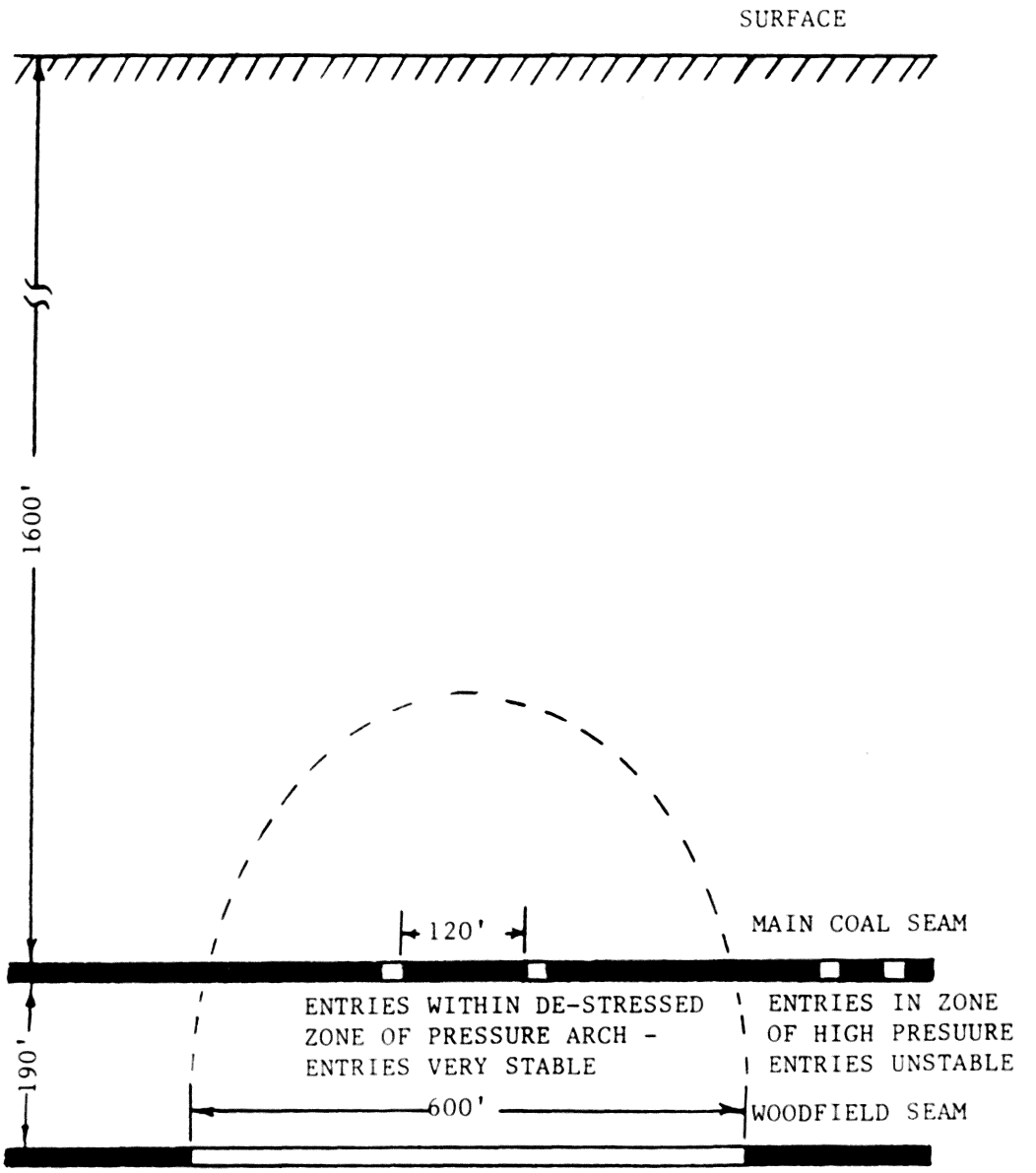


Figure 15. Cross-section of the Main Coal and Woodfield Workings.

seam which averages 96 inches in thickness. The upper seam is the Sewickley seam which is about 66 inches thick. The Pittsburgh seam is mined by driving twenty foot rooms on ninety foot centers with an overall extraction ratio of 60% and up to 85% where pillaring has occurred. The Sewickley seam is mined with twenty foot rooms on seventy foot centers with a resulting extraction ratio of approximately 75 percent. The innerburden between the two seams is about ninety feet thick and is comprised of mostly shales and limestone. The overburden is approximately 400 feet thick and is comprised of shales, fireclays, limestones and sandstones.

This case study is an example of the disturbance that may occur in an upper seam over a line between the solid coal and the gob in the lower seam. However, the greatest damage is not directly over the edge of the remaining coal in the lower seam. Instead, the most damage occurred in an area out over the gob away from the solid coal, usually at a distance of 100 to 300 feet from the edge of the solid coal. A cross-section view of the case study is shown in Figure 16.

The location of the damage is due to the fact that the overlying strata above the gob fail successively further out over the gob. This results in an arch shape which carries the damage out to a point away from the edge of the solid coal in the lower seam.

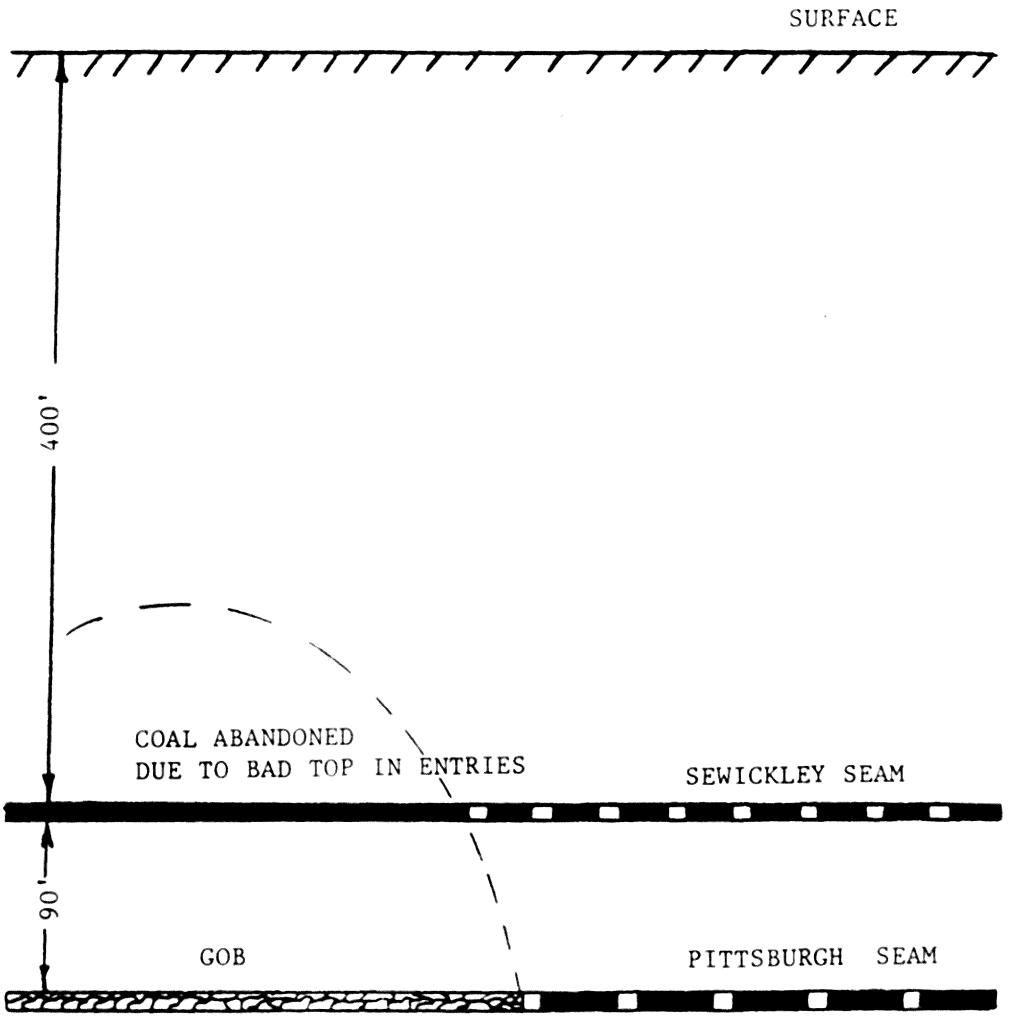


Figure 16. Cross-section of Sewickley and Pittsburgh Seams Showing Line of Damage Above Gob of the Lower Seam.

IV. - FINITE ELEMENT MODEL DEVELOPMENT

4.1 - The Finite Element Computer Program

In an effort to evaluate the effect of the pressure arch upon multiple seam mining, finite element computer modelling was used to simulate certain mining conditions. In this way, with proper input, the program accurately yields the principal stresses and their directions across the model.

The computer program used in this study was developed and published by Desai and Abel (1972). This program was selected for both its simplicity and generality. The program can be used for the linear, elastic, plane strain, or plane stress analysis of isotropic bodies. The finite element program was written in FORTRAN IV and can be used with both quadrilateral and triangular element shapes.

Most of the calculations are performed in the various subroutines within the computer program. The main program itself reads the title of the problem, computes the semi-band width (as explained in Section 2.3.2 of this text), solves the overall equilibrium equations, and prints the displacements. Following is a list of the subroutines within this finite element program with an explanation of their functions.

1) DATAIN: Reads and prints the input data.
Performs error checks on the input.

2) ASSEMBL: Initializes and assembles the overall stiffness matrix, and the load vector. Temporarily stores information needed for later calculations. Introduces geometric boundary conditions on the model.

3) QUAD: Computes the stress-strain matrix, stiffness matrix, strain-displacement matrix and body force vector of either a quadrilateral or triangular matrix.

4) CST: Compute the strain-displacement matrix, body force vector, and stiffness matrix for a constant strain triangle element.

5) GEOMBC: Applies the prescribed boundary conditions for each individual node.

6) BANSOL: Triangularizes the overall stiffness matrix by symmetric Gauss-Doolittle decomposition or solves for the displacement vector corresponding to a particular load vector.

7) STRESS: Computes the strains, stresses and the principal stresses. Prints the stresses at the centroid of the elements.

More information concerning this program is available in Desai and Abel (1972) and in Appendix C-1.

4.1.1 - Preparation of Finite Element Model

The first step in preparing a finite element model is to sketch what the model is supposed to represent. In this case, eighteen different models were prepared. In each case the height of the mining opening was set at five feet. The width of the opening was set at a minimum of 100 feet and was increased by multiples of 100 feet to a maximum of 600 feet. The depth of the mine opening below the surface was set at 500 feet, 750 feet and 1000 feet. The lateral extent of the model was limited to a maximum of 2000 feet, that is, 1000 feet from the centerline of the mine opening.

In order to make the results of the finite element program as accurate as possible, the values used to define the material properties of the model itself must also be as accurate as possible. In this study, three different materials were used, a sandstone, a shale and a coal.The material properties for these different strata are shown in Table 1.

The finite element program is then run using the necessary input as described in Appendix C-1. The program will produce values for the horizontal stress, vertical stress, shear stress, principal stresses and their directions for each element of the model.

Table 1

Material Properties Used in the Analysis

Rock Type	Modulus of Elasticity	Poisson Ratio	Density
Shale	1.61×10^8 psi	0.10	158.0 pcf
Coal	0.72×10^8 psi	0.25	125.0 pcf
Sandstone	3.00×10^8 psi	0.15	144.0 pcf

4.2 - The Mesh-Plot Program

The finite element program requires large amounts of input in order to make the required calculations. If, however, an error occurs in the input, the finite element program will run until the error is detected thereby incurring a loss of by time and computer funds. For this reason, an auxiliary program was created (Bhattacharya, 1980) to detect any errors in the input of the nodal points and elements themselves. This mesh-plot program generates a plot of the finite element mesh.

The mesh-plot program was written in FORTRAN and is tailored for use at the facilities of the Virginia Tech Computing Center. The mesh-plot program is capable of plotting the finite element mesh on either a Versatec 1200 electrostatic plotter or a CALCOMP 1051 drum plotter.

The formatting of the input is identical for both the fininte element program and the mesh-plot program. In this way the element and nodal point data for the finite element program can be checked in the mesh-plot program before the finite element program is run.

The mesh-plot program checks for negative element areas and missing element or nodal numbers. Further information on the mesh-plot program is furnished in Appendix C-2. An example of the output from the mesh-plot program is shown in Figure 17.

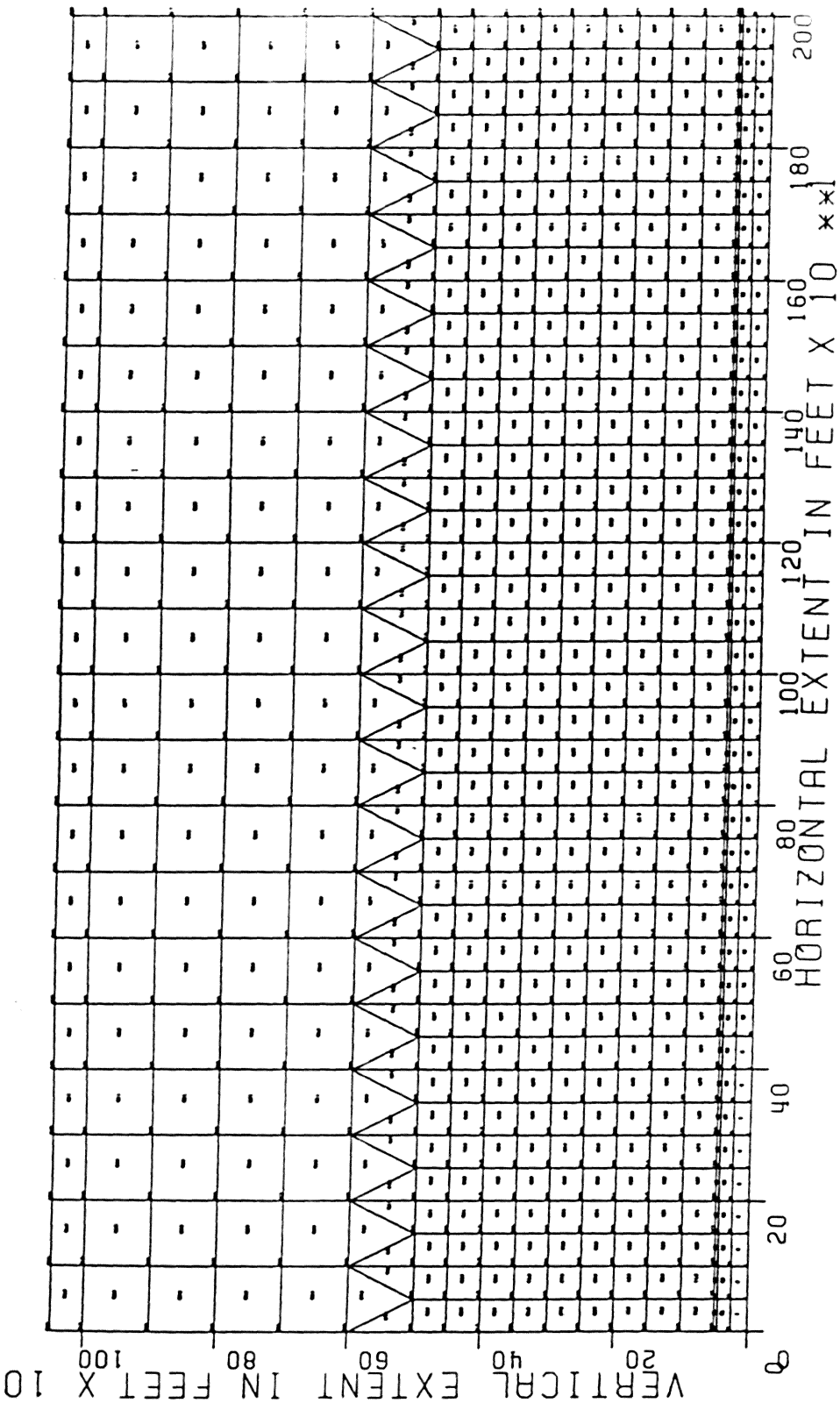


Figure 17. Example of Mesh-Plot Output

4.3 - The Stress-Plot Program

As part of the output of the finite element program, principal stresses at each element and their directions are produced. The stress-plot program plots the scaled magnitudes of the principal stresses and directions of these stresses at their proper relative positions (Bhattacharya, 1980). This program has been modified to read as input the output from the finite element program. This eliminates any errors which may occur in the transfer of data from one format to another caused by operator error.

The stress-plot program plots the principal stresses by calculating the coordinates of the ends of the stress vector as shown in Figure 18. The sign convention for the stresses is identical to that used in the finite element program, that is, tensile stress is positive and compressive stress is negative. An example of a stress vector diagram is shown in Figure 19. Additional information on the stress-plot program is given in Appendix C-3.

4.4 - The General Purpose Contouring Program

Another method of showing the stresses caused by an underground opening is through the use of contours. This is accomplished with the help of the General Purpose Contouring

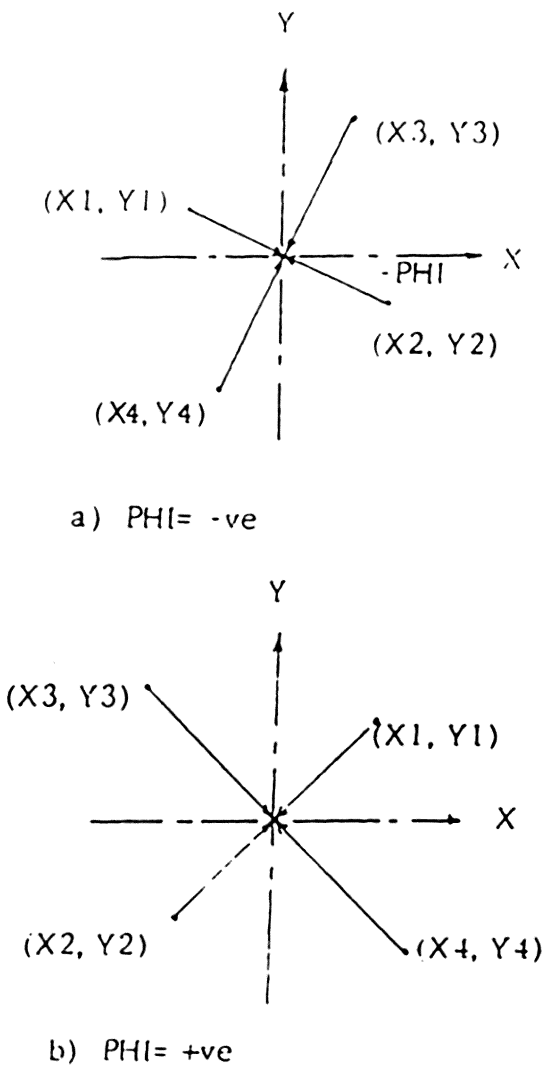


Figure 18. Sign Convention for Plotting Stress Vectors (after Coates, 1969).

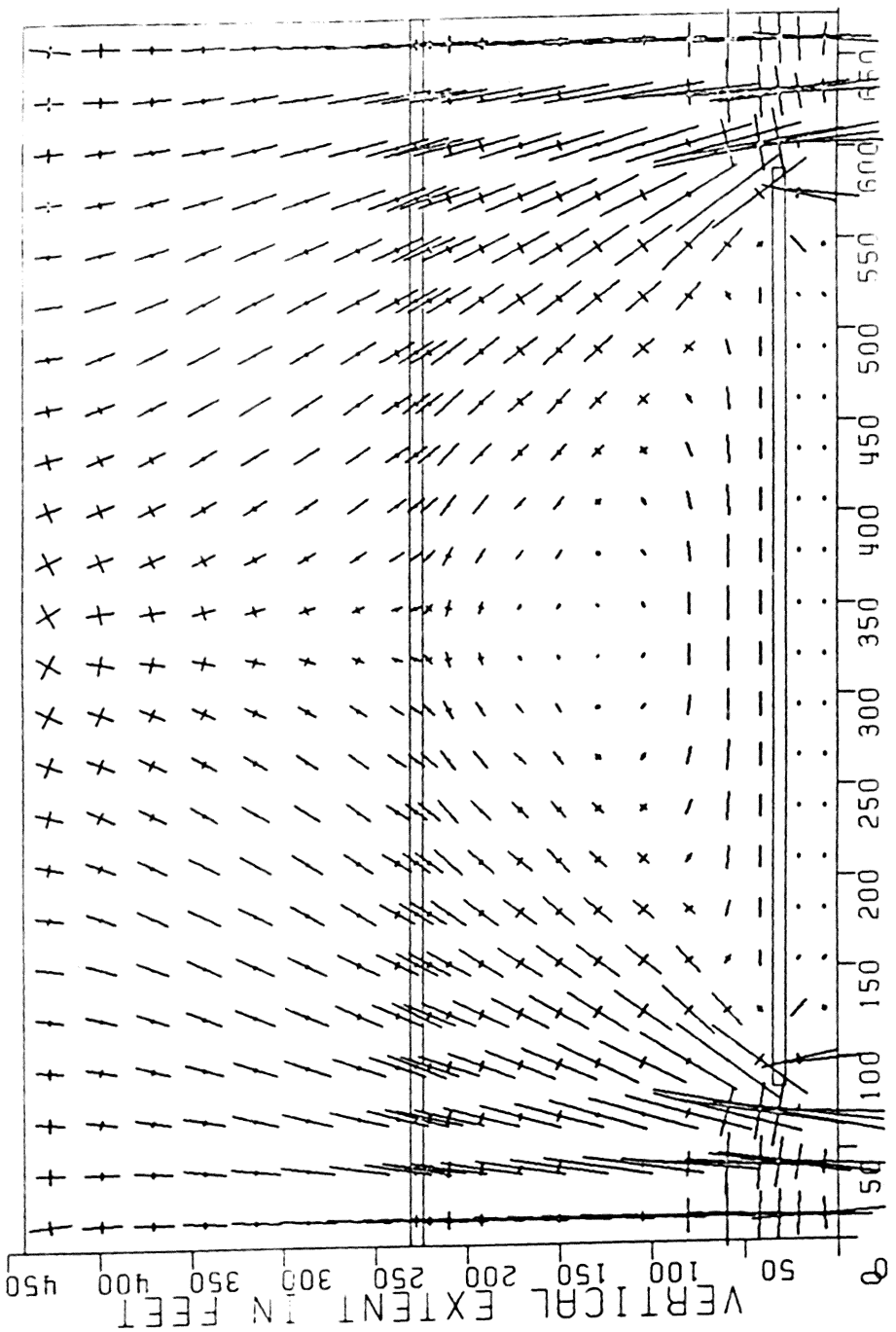


Figure 19. Example of Stess Vector Diagram

Program (GPCP) developed by Batten and available through the California Computer Products, Inc. (1971).

The program graphically displays the functions of two variables as contour maps. GPCP is a very flexible program that allows the user to: specify a function at random points or at an array; delete the contour plotting in specific areas; plot bold index contour lines with or without labels; suppress contour lines in areas of high contour density; and make notations on the plot.

The General Purpose Contouring Program is written in FORTRAN and uses either a Versatec or CALCOMP plotter. Since the GPCP package is made available from California Computer Products, Inc. directly to the Virginia Tech Computing Center, a listing of the program is not available to the public. A sample of GPCP input is included in Appendix C-4. An example of the output from GPCP is shown in Figure 20.

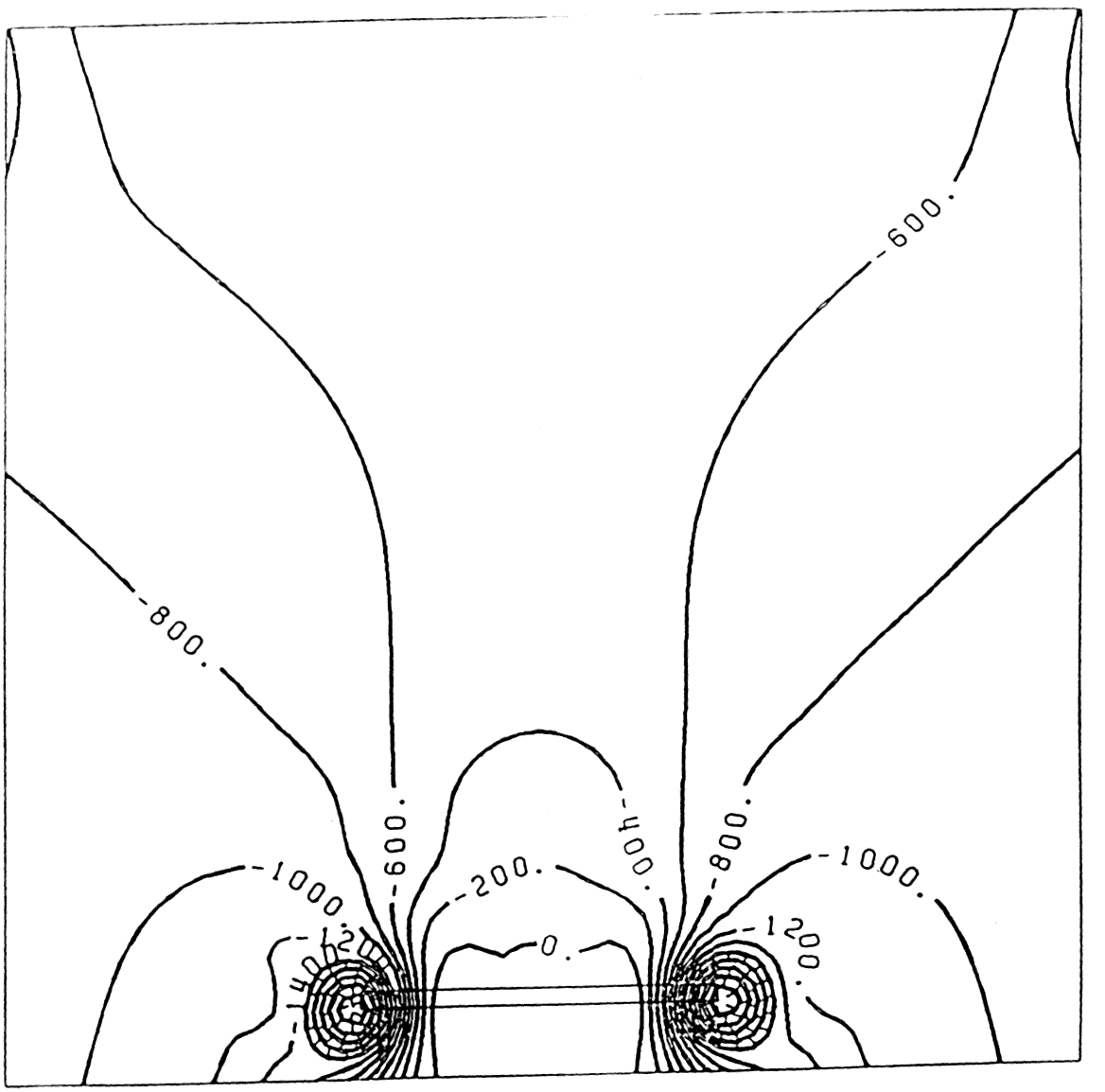


Figure 20. Example of GPCP Output

V. - PHOTOELASTIC MODEL DEVELOPMENT

The basis of photoelasticity is that when a birefringent material is loaded and viewed through the plates of a polariscope, fringes appear which are directly proportional to the resulting shear stresses in the material. To elaborate, after a photoelastic model is loaded, the material is viewed with the light from a dark field circular polariscope. Alternating light and dark patterns can be seen across the stressed areas of the model. The dark bands on the model represent patterns of light interference and are known as fringes. These fringes are assigned an order, or number, depending upon the boundary conditions of the model. The value of the fringe order is directly proportional to the maximum shear stress at any point or the difference between principal stresses at that point, as shown in the following equation.

$$\tau_{max} = \frac{S1 - S2}{2} = \frac{n\,f}{t} \qquad \text{(Eq. 5.1)}$$

where τ_{max} = maximum shear stress

S1 = major principal stress

S2 = minor principal stress

n = fringe order

f = material fringe factor

t = material thickness

By using photoelastic methods it is also possible to determine the principal stresses across the model. By adjusting the polariscope a plane polarized light field gives the inclination of the principal stresses to the vertical for any point on the model. The shear stress along the x-y plane is defined in terms of the following variables:

$$\tau_{xy} = \frac{nf}{t} \sin 2\theta \qquad \text{(Eq. 5.2)}$$

where τ_{xy} = shear stress in x-y plane

θ = principal stress inclination

and other variables as in Eq. 5.1.

In this study, the photoelastic modelling was limited to two dimensional analysis. According to Coates (1965), two dimensional approximations are adequate for any engineering purpose when the length of the opening is greater than twice the opening width. This finding was substantiated by Heuze (1970) who found only minor changes in the stresses along a line of pillars in a room and pillar mines. An additional constraint of plane stress analysis was also placed on the model. Plane stress theory will give accurate results when, as noted by Frocht (1948), the lateral dimensions of the photoelastic model are at least four times the thickness.

In an effort to obtain a more accurate stress pattern in the photoelastic model, two horizontal slots were machined into each edge of the photoelastic material pillar in order

to model the yield zone which lies along the perimeter of coal pillars. This had the effect of reducing the normally high stresses that are found at the edge of the opening. The photoelastic material, polyurethane rubber (PSM-4) was machined according to the techniques outlined by Neall and Haycocks (1974) and Neall (1975).

In order to model the stresses surrounding an underground opening, several photoelastic models were used. A crossed circular polariscope was used to eliminate the isoclinic lines from the fringe pattern since the directions of the principal stresses were not desired from this model. The principal stress difference, however was desired as this would show the areas of high and low stress across the model. The fringe pattern is known as an isochromatic fringe pattern. When the model is viewed with white light, where all the wavelengths of the visible spectrum are present, the isochromatic fringe pattern will appear as a series of colored bands. A black fringe will appear only when the principal stress difference is zero and a zero order of extinction occurs for all wavelengths of light. This black area correlates with the de-stressed region within a pressure arch. Examples of isochromatic fringe patterns which show the de-stressed zone above the model openings are shown in Figures 21 and 22.

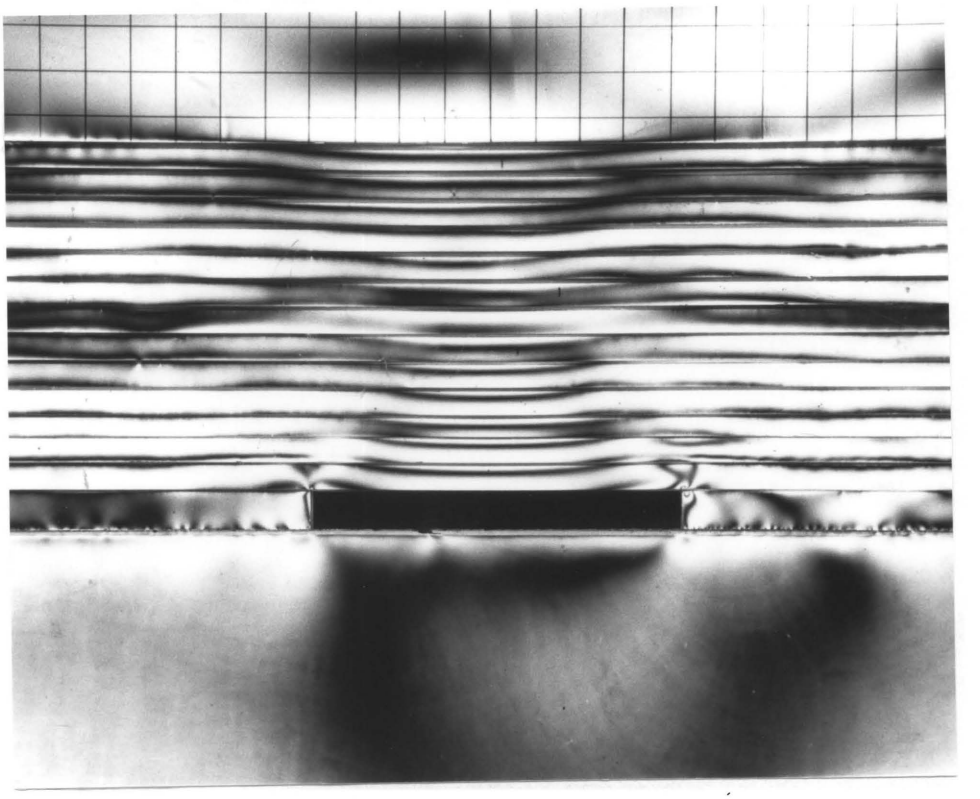


Figure 21. Example No. 1 of Isochromatic Fringe Pattern

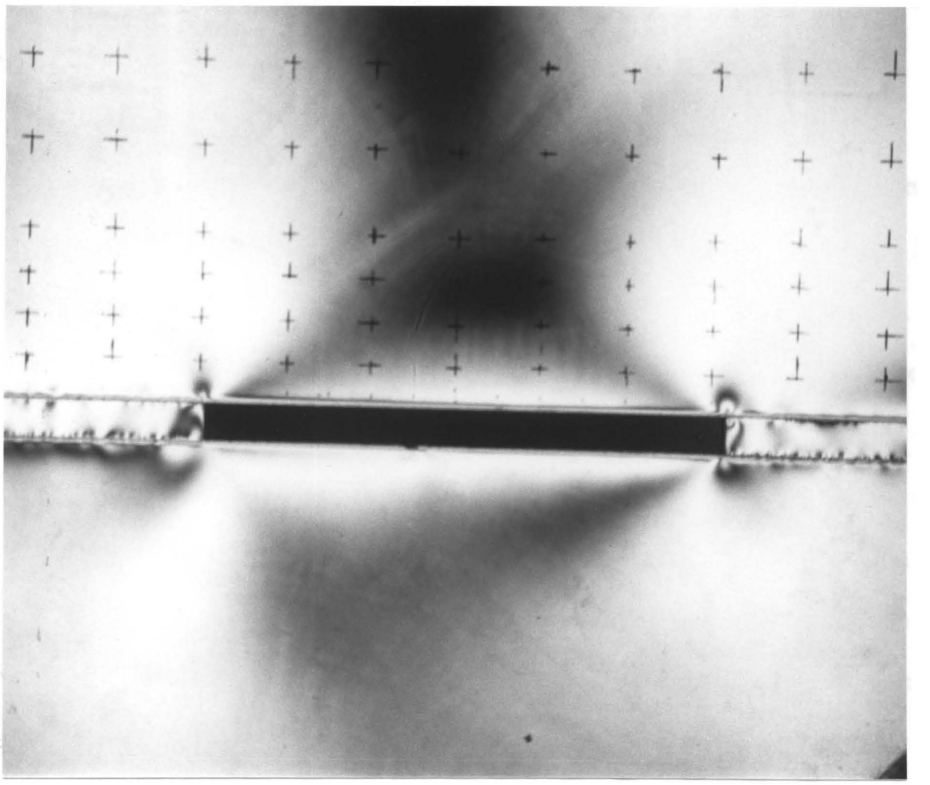


Figure 22. Example No. 2 of Isochromatic Fringe Pattern

VI. - CASE STUDY CORRELATION

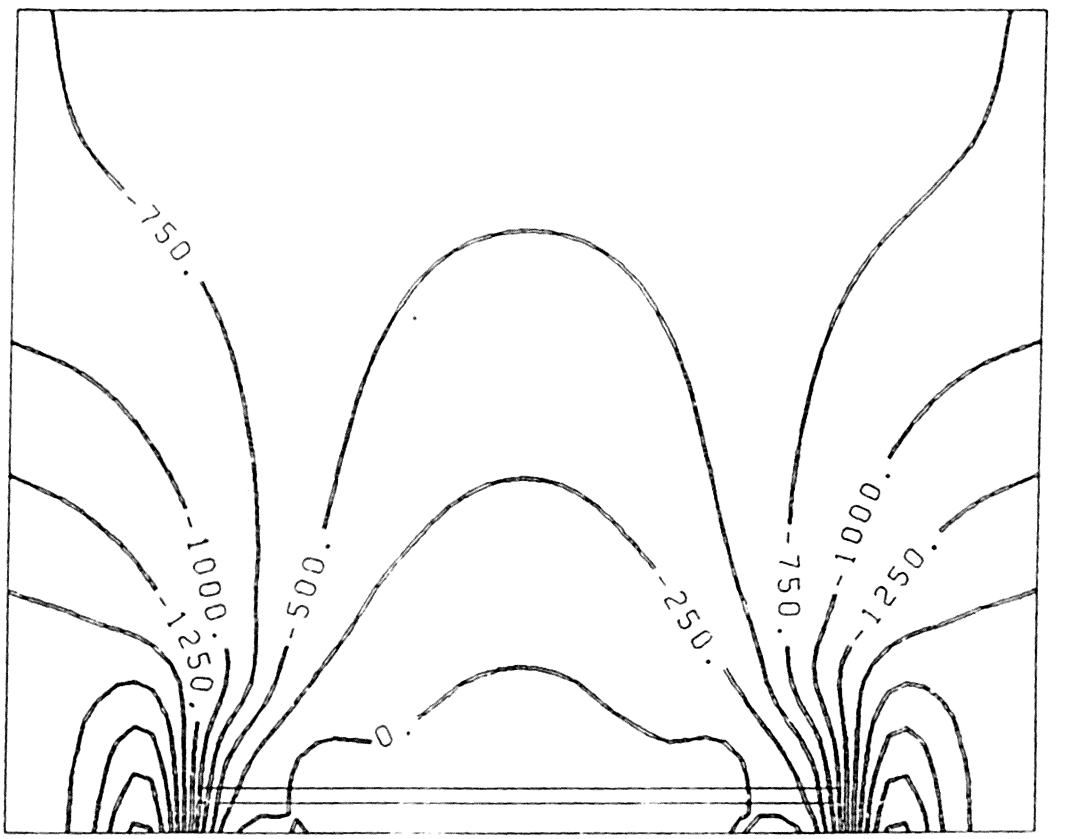
In order to check the accuracy of the finite element computer method, it was necessary to run the program with input derived from a documented case study and to check the results obtained from the finite element analysis against those noted in the case study.

For this purpose a case study was selected that measured the stresses around an underground opening. Howell et al (1976) documented the stresses around a shortwall mining area comprised of six adjacent panels at the Hendrix No. 22 Mine in Kentucky. Each of the faces were 170 feet wide on 270 foot centers, with an extracted seam height of four feet. The depth of cover over the shortwall area ranged from 770 feet above panel 1 to a depth of 170 feet over panel 5. The overburden consisted of 115 feet of shale and coal strata directly above the mined seam, a hundred foot thick sandstone layer above that and layers of shale and sandstone reaching to the surface. The results were obtained from a borehole, drilled from the surface, above the centerline of panel 1, which was the first panel to be mined.

According to Howell, the first 33 feet above the shortwall face consisted of a large tensile zone, which was due to caving of material into the excavation. Between 33

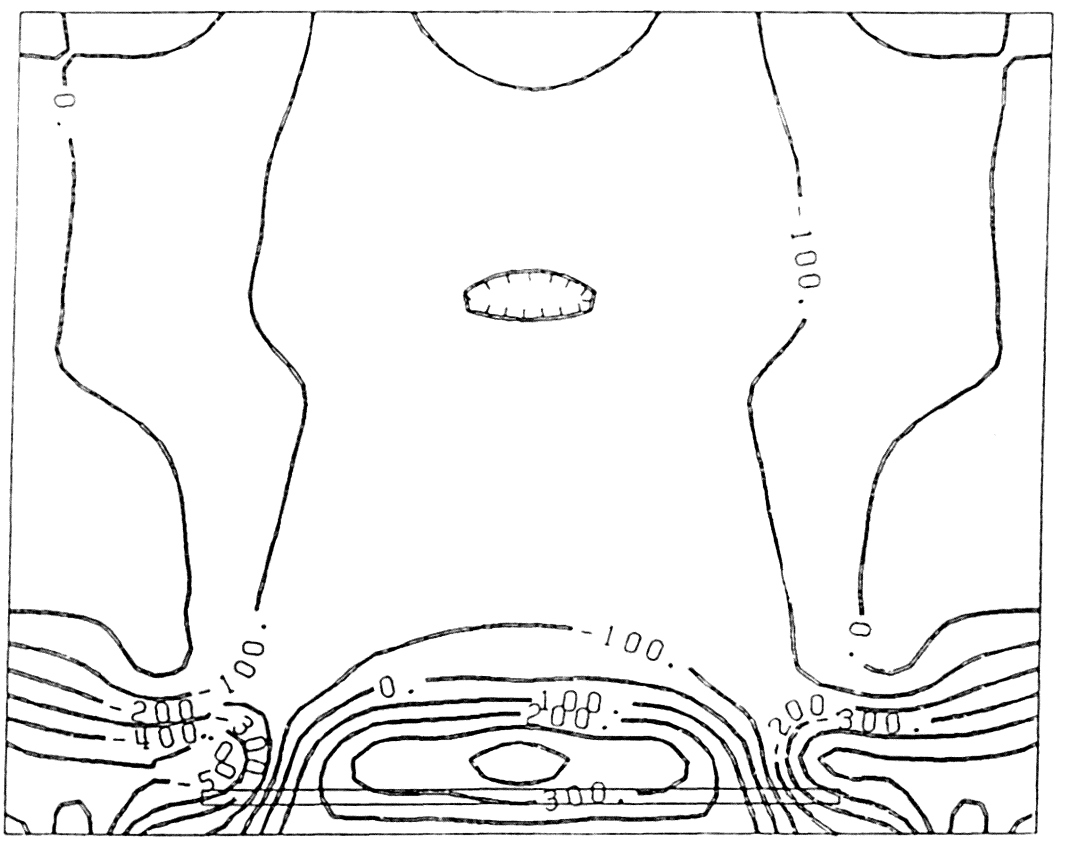
and 49 feet above the opening the strains were compressive, perhaps due to the rotation of blocks as they caved. The strata fell in large blocks due to the sandy nature of the shale. Above this compression zone the strains again became tensile, perhaps due to bed separation at the base of the thick sandstone layer. This indicates that the sandstone bed may have bridged over the excavation, acting as a beam.

The conditions described in the case study were duplicated as closely as possible in the input of the finite element program. A mine opening of 170 feet wide by 4 feet in height was modeled and the model duplicated the geologic column of the Hendrix mine as closely as possible. The results from the finite element program and subsequent contouring of the output show that a tension zone extends approximately 30 feet above the mine opening. This could be seen in Figures 23 and 24 which are contours of the vertical and horizontal stresses respectively. However, the finite element program used in this research could not duplicate the bed separation which may have occurred between the different materials in the overburden. From this correlation it was decided that the finite element program was sufficiently accurate.



SCALE: 1 INCH = 50 FEET

Figure 23. Contour of Vertical Stresses, Hendrix Mine Case Study.



SCALE: 1 INCH = 50 FEET

Figure 24. Contour of Horizontal Stresses, Hendrix Mine Case Study.

VII. - FINITE ELEMENT RESULTS

The finite element program was run for varying widths of mine opening, depths of cover and overburden material. The output from the finite element program for each of these different mining conditions is summarized in Appendices A and B. The width of opening ranged from 100 to 600 feet in increments of 100 feet. The depth of overburden was set at 500, 750 and 1000 feet. It is believed that the range of these values would cover the majority of mining conditions in the Appalachian region. As an additional variable to be considered, the overburden was selected as either completely sandstone or completely shale as these would be the two extremes to be encountered in the mining of coal seams.

From the various output it was determined that a de-stressed zone lies above each opening. In this region the reported stresses in the overburden are actually less than the stresses that would be present if no mine opening was present. The vertical stresses and horizontal stresses both exhibit this trend. The extent and magnitude of this de-stressed zone is dependent upon both the width of opening and depth below the surface. For example, for an opening 200 feet wide under 500 feet of overburden, a tension zone exists to a height of approximately 50 feet above the opening along the centerline of the mine opening. At a

distance of 70 feet above the opening the vertical stresses are just ten percent of the superincumbent pressure. At 175 feet above the opening the vertical stresses are only 55 percent of the superincumbent pressure. This trend holds for all the models, where a region of reduced stress overlies the mine opening.

For a given depth below the surface, the tension zone increases in both width and height as the width of the mine opening increases. For an opening 300 feet wide under 500 feet of overburden, a tension zone extends 90 feet above the opening; for a 400 foot opening, 125 feet above the opening; for a 500 foot opening, 175 feet; and, for a 600 foot wide opening, the tension zone extends 225 feet above the opening.

For a given width of opening, the height of tensile zone decreases as the depth of overburden increases. For a 300 foot opening under 500 feet of overburden, a tension zone reaches approximately 90 feet above the mine opening. For an opening of the same width under 750 feet of overburden, the tensile zone is about 30 feet high. Under 100 feet of cover the tensile zone is approximately 20 feet in height.

For points within the intradosal ground or pressure arch, research shows that the stresses decrease as the the width of opening increases for any given depth. In other words, as the width of opening increases the zone above the opening is subject to decreasing stresses creating an envelope above

the opening where the vertical stresses within the overlying rock are minimal.

However, just as the intradosal stresses decrease with an increase in opening size, so do the abutment pressures increase around the intradosal zone. For a 100 foot opening under 500 feet of overburden, the abutment pressures around the opening are just 10 percent more than the superincumbent pressure. The abutment pressures around a 600 foot opening under the same depth of cover can approach and surpass twice the superincumbent pressure. As the depth of cover for a given opening width increases, the abutment pressures get closer to the superincumbent pressures.

The different materials of the overburden did not have a major effect upon the finite element output for the different opening widths and depth of overburden. The results obtained using a completely shale overburden are within ten percent of those obtained using a sandstone overburden. Therefore, the values cited above are average values for a given mining condition.

The de-stressed zone above the mine openings as shown in the finite element analysis correlates to that exhibited in the photoelstic models. This correlation can also be seen in the stress trajectories drawn for both the finite element model and for the photoelastic model as shown in Figures 25 and 26. This zone of decreased stress should be important in the mine design of future operations in an upper seam.

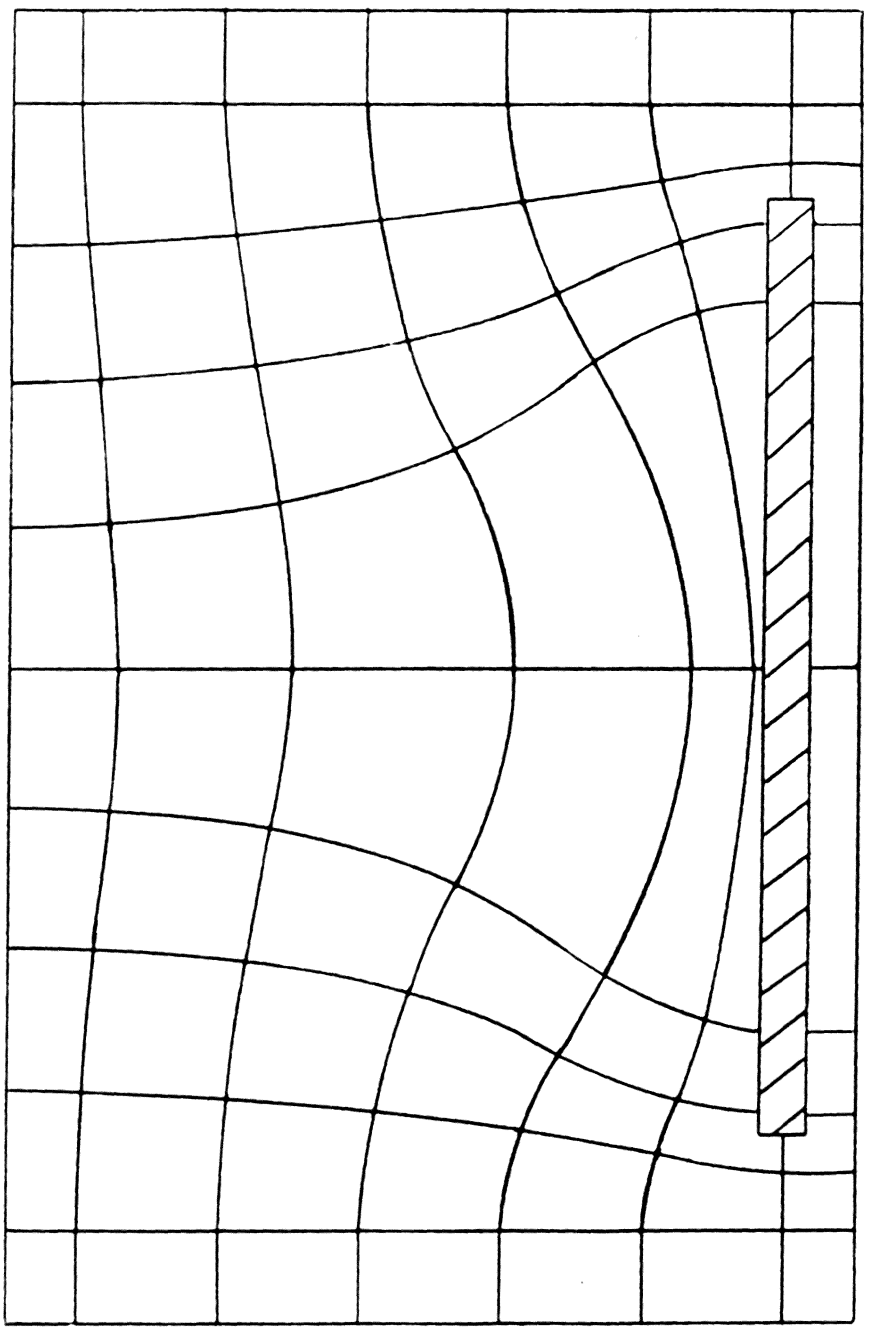


Figure 25. Stress Trajectories from a Finite Element Model.

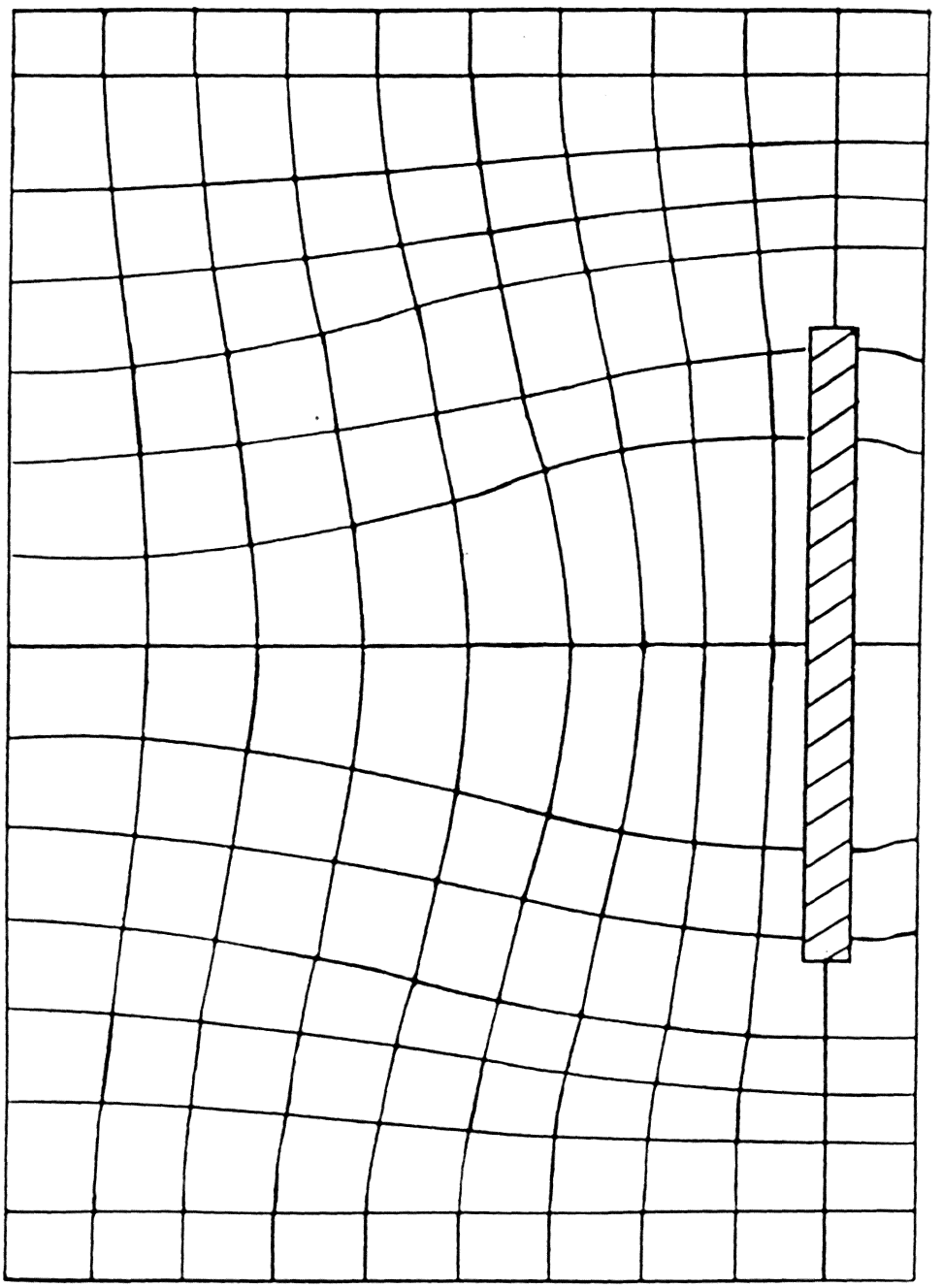


Figure 26. Stress Trajectories from a Photoelastic Model.

By using the tables in Appendices A and B, it is also possible to correlate information obtained from the case studies with experimental values. The case study shown in section 3.4 of this report had a lower seam opening width of 500 feet, a depth of overburden of approximately 1000 feet and an interseam thickness of about 200 feet. The case study reported that the upper seam entries experienced no roof control problems as they were in a de-stressed zone above the lower panel. The tables of Appendix A show that the vertical stress at that point is only about forty percent of the superincumbent pressure that may be expected if there were no opening below.

VIII. - SUMMARY

In order to ascertain the effects that a pressure arch has upon the mining of an upper seam, it is necessary, first, to evaluate the stresses created by an underlying mine opening.

The excavation of an opening underground disturbs the pre-existing stress field within the rock. These changes in the stress field result in both the deformation and displacement of the strata above the mine opening. The extent and degree of these changes in the stress field affects the possible future mining conditions in an overlying seam. If an excavation is formed in an upper seam at a later date, the location of the opening with respect to the pressure arch formed by a previous lower opening is very important.

For this reason research was conducted in an attempt to evaluate the extent and degree of the change of the stress field due to an underground opening. Case studies were collected and analyzed to determine those changes in the stress field that have been documented by previous authors. After this, the finite element program was utilized to find trends in the changes of the stress field due to various widths of mine opening, depths of cover and overburden materials. These findings are summarized in Appendices A

and B. These tables are to be used as a design tool for future mining operations in a previously mined area. These tables must be used as approximations as each different mine will have different geologic conditions betwen the two seams.

Although the tables obtained in this reseach are approximations, it is felt that the finite element method which is presented can be applied with confidence anywhere if the input to the program is as accurate as possible in regards to the material properties, geologic column and size of the mine openings.

The results of the finite element analysis was checked qualitatively by using photoelstic modelling. Photoelastic modelling produced a similar de-stressed zone above the mine opening that was found in the finite element analysis. It is also possible to use the photoelastic modelling method to obtain the principal stresses and their inclinations for points across the model. This would give further information with which to check the finite element analysis method. However, this option was beyond the scope of this project and its timetable.

IX. - CONCLUSIONS AND RECOMMENDATIONS

9.1 - Conclusions

Based on the findings of this reseach the following conclusions can be made:

1) The underground mining of a seam alters the native stress field within the rock.

2) This altered stress field is in the form of a pressure arch which forms above the opening and is divided into two distinct zones.

   a) The first of these zones is the de-stressed intradosal zone above the opening where the stresses are now below the superincumbent pressures present before mining.

   b) The second zone is the pressure ring which surrounds the de-stressed zone. In this pressure ring the pressures are higher than the native stresses.

3) If in the future an upper seam is to be extracted, the position of the new mine opening in relation to the pressure arch around the lower opening is of importance.

   a) If the upper opening is driven in the intradosal zone, lower pressures can be expected than in the surrounding rock and, possibly, fewer roof control

problems will be encountered.

b) If the upper opening is driven through the pressure ring of an existing arch, higher pressures will be encountered along with a probable increase in roof control problems.

9.2 - Recommendations

In areas where multiple seam mining is to take place, the determination of the stress field that may be encountered should be an important factor in the design of the mine layout. An accurate account of these stresses would enable the mine engineers to avoid areas of high stress or at least to be prepared for them when they are encountered. In order to obtain results that are as accurate as possible, the input into the program should also be accurate. This would be the result of actual core information as well as accurate values for the modulus of elasticity, Poisson's ratio and the weight density of each material in the overburden.

Any theoretical approach to a rock mechanics problem is an accurate approximation only if the theoretical approach can be applied to a known situation and give an accurate representation of the actual values. In order to ensure that this finite element approach is accurate, case studies which evaluate the present stress fields should be collected from literature or, more importantly, from actual field data.

The photoelastic modelling method also requires more work. In order to more fully understand how the stresses in

a photoelastic model correlate to those obtained in the finite element analysis, information about the values of the principal stresses as obtained from a photoelastic model must be compared to those of an equivalent finite element model. This technique exists and should be further investigated in future research.

Another aspect that must be considered in this area is the variable of time. It must be accurately determined if the period of time between mining the lower seam and mining the upper seam will alter the stress field in magnitude and extent.

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APPENDIX A

VALUES OF SIGMA (Y) FOR VARIOUS DEPTHS AND WIDTHS

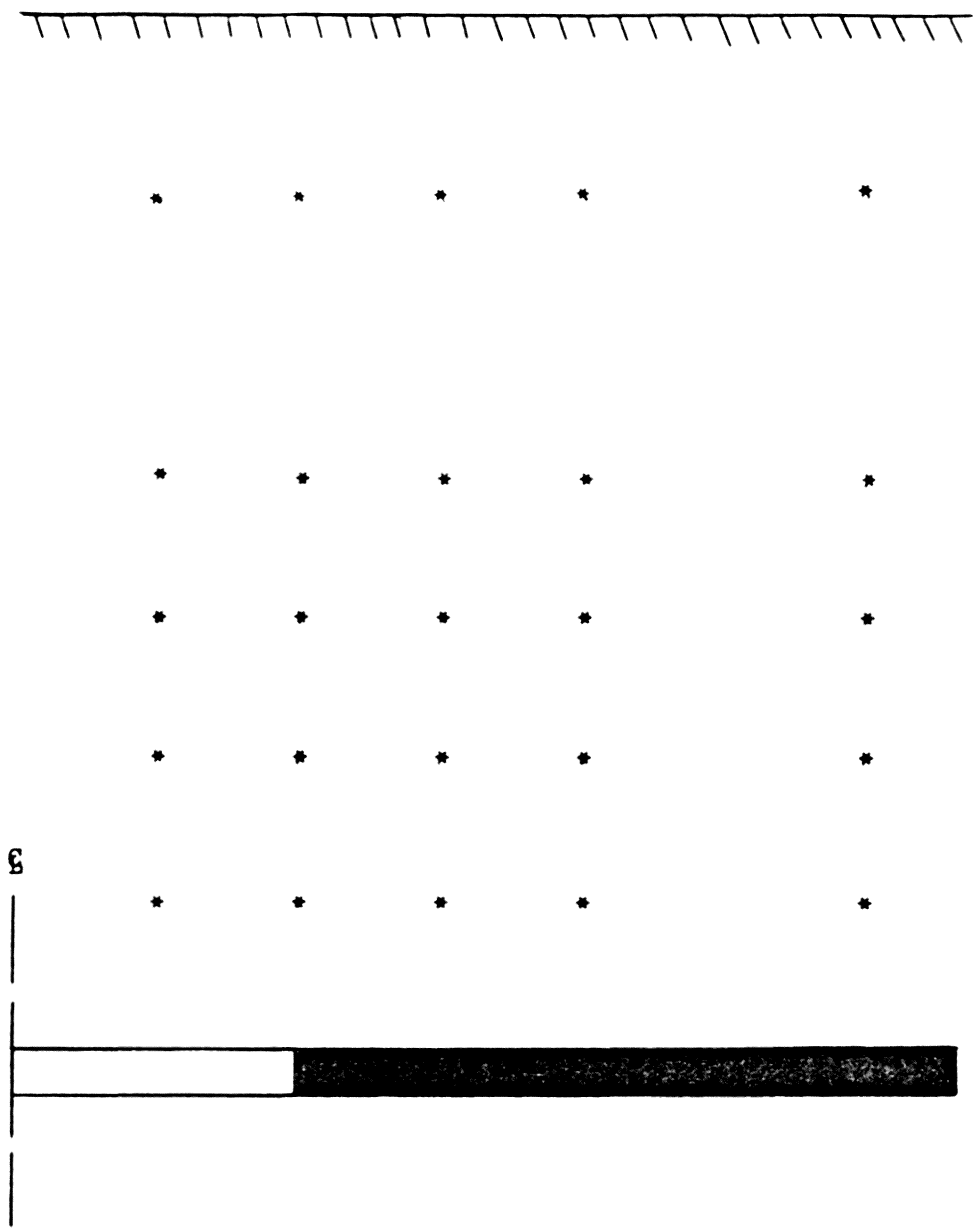

Figure 27. Example Grid of Location of Values from Tables of Appendices A and B.

Table A-1

Sigma (Y) for Opening 100' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
4 1/2	450'	97.94	98.17	98.40	98.73	99.53
4	400'	96.74	97.07	97.40	98.03	99.33
3 1/2	350'	95.43	95.92	96.40	97.37	99.17
3	300'	94.10	94.94	95.78	97.02	99.24
2 1/2	250'	92.30	93.68	95.06	96.72	99.66
2	200'	89.79	91.92	94.04	96.50	99.84
1 1/2	150'	85.69	89.56	93.44	97.10	101.64
1	100'	78.12	86.57	95.02	99.98	105.44
3/4	75'	73.18	84.94	96.71	102.24	107.29
1/2	50'	64.48	84.31	104.14	107.02	108.33
1/4	25'	55.79	83.68	111.58	111.79	109.37

H/W = Height above seam divided by width of opening

Table A-2

Sigma (Y) for Opening 200' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
2	400'	87.46	91.43	96.23	100.70	105.80
1 1/2	300'	78.43	86.28	95.42	103.00	109.62
1	200'	66.72	83.06	99.48	109.62	113.12
3/4	150'	56.20	84.24	107.70	117.11	114.80
1/2	100'	45.39	88.82	117.72	121.67	113.36
1/4	50'	23.61	103.94	138.66	127.16	112.33

Table A-3

Sigma (Y) for Opening 300' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
1 1/2	450'	85.54	94.16	102.60	107.35	108.94
1	300'	63.94	87.54	109.16	118.76	119.73
3/4	225'	53.09	88.09	116.55	124.90	121.27
1/2	150'	36.16	95.94	133.41	131.78	120.66
1/4	75'	16.61	114.90	149.72	131.71	116.83

Table A-4

Sigma (Y) for Opening 400' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1	400'	75.28	97.93	114.57	117.14	105.48
3/4	300'	59.31	98.53	124.89	126.98	108.34
1/2	200'	43.95	105.42	138.41	133.33	108.86
1/4	100'	22.81	124.71	155.08	133.38	107.40

Table A-5

Sigma (Y) for Opening 500' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
3/4	375'	65.52	106.80	128.80	122.80	100.40
1/2	250'	43.06	115.64	148.40	132.60	100.76
1/4	125'	16.32	137.06	165.87	132.53	100.00

Table A-6

Sigma (Y) for Opening 600' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
3/4	450'	78.73	109.80	121.43	111.25	96.76
1/2	300'	47.28	124.89	151.40	125.95	92.86
1/4	150'	16.17	157.86	173.07	126.00	92.00

Table A-7

Sigma (Y) for Opening 100' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
7	700'	**.**	99.00	99.08	99.17	99.34
6 1/2	650'	**.**	98.66	98.83	99.00	99.34
6	600'	**.**	98.33	98.58	98.83	99.33
5 1/2	550'	**.**	98.50	98.78	99.06	99.64
5	500'	**.**	97.72	97.95	98.12	98.73
4 1/2	450'	96.65	96.89	97.12	97.32	98.01
4	400'	96.31	96.66	97.02	97.44	98.36
3 1/2	350'	95.53	95.96	96.40	97.08	98.38
3	300'	94.48	95.08	95.68	96.62	98.44
2 1/2	250'	92.94	93.94	94.92	96.26	98.78
2	200'	90.52	92.27	94.02	96.14	99.65
1 1/2	150'	86.52	89.97	93.42	96.73	101.45
1	100'	79.04	86.88	94.72	99.34	104.47
3/4	75'	74.07	85.11	96.15	101.41	106.30
1/2	50'	65.04	84.18	103.32	106.12	107.46
1/4	25'	56.00	83.24	110.48	110.82	108.62

. = Percentage undeterminable from model used

Table A-8

Sigma (Y) for Opening 200' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
3 1/2	700'	96.00	96.75	97.50	98.55	100.30
3	600'	94.00	95.00	96.00	97.66	100.66
2 1/2	500'	91.38	92.92	95.12	97.54	101.68
2	400'	87.36	90.06	93.59	97.28	102.75
1 1/2	300'	82.09	87.21	93.49	99.13	105.34
1	200'	72.64	84.38	96.52	104.54	108.83
3/4	150'	65.10	84.36	101.32	109.58	110.27
1/2	100'	54.84	87.55	110.28	115.76	111.00
1/4	50'	42.52	96.08	123.45	120.52	110.86

Table A-9

Sigma (Y) for Opening 300' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
2	600'	88.50	92.00	96.50	100.66	105.33
1 1/2	450'	79.16	86.06	93.88	100.89	105.73
1	300'	67.91	84.66	100.80	109.69	112.04
3/4	225'	59.43	85.34	107.24	115.24	113.05
1/2	150'	46.88	90.92	119.54	121.54	112.88
1/4	75'	27.11	104.60	136.74	125.26	111.78

Table A-10

Sigma (Y) for Opening 400' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1 1/2	600'	82.66	91.34	100.67	106.67	107.67
1	400'	66.44	86.94	105.70	114.98	112.76
3/4	300'	56.92	88.24	112.76	120.70	113.56
1/2	200'	44.40	94.84	125.30	126.40	112.92
1/4	100'	24.24	112.93	142.65	128.28	111.10

Table A-11

Sigma (Y) for Opening 500' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1	500'	66.63	91.20	111.41	118.00	110.44
3/4	375'	54.93	92.14	119.47	124.67	111.87
1/2	250'	40.86	99.58	130.19	128.96	111.37
1/4	125'	18.16	119.44	150.56	130.48	109.20

Table A-12

Sigma (Y) for Opening 600' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1	600'	72.67	96.67	114.67	116.33	106.66
3/4	450'	56.71	94.81	119.29	121.12	105.68
1/2	300'	40.00	104.94	138.73	131.81	110.02
1/4	150'	16.98	127.25	155.68	130.97	107.93

Table A-13

Sigma (Y) for Opening 100' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
9 1/2	950'	**.**	99.53	99.54	99.55	99.57
9	900'	**.**	99.40	99.42	99.45	99.50
8 1/2	850'	**.**	99.20	99.27	99.35	99.50
8	800'	**.**	99.00	99.12	99.25	99.50
7 1/2	750'	**.**	98.84	98.94	99.04	99.25
7	700'	**.**	98.67	98.76	98.84	99.00
6 1/2	650'	**.**	98.46	98.57	98.67	98.88
6	600'	**.**	98.25	98.38	98.50	98.75
5 1/2	550'	**.**	98.16	98.36	98.58	98.97
5	500'	**.**	97.70	97.90	98.06	98.56
4 1/2	450'	96.99	97.16	97.32	97.54	98.19
4	400'	96.60	96.80	97.01	97.42	98.29
3 1/2	350'	95.86	96.21	96.55	97.12	98.27
3	300'	94.88	95.41	95.94	96.76	98.36
2 1/2	250'	93.43	94.29	95.15	96.38	98.63
2	200'	91.05	92.63	94.20	96.16	99.92
1 1/2	150'	86.84	90.16	93.48	96.60	101.11
1	100'	79.30	86.96	94.63	99.11	104.06
3/4	75'	74.49	85.24	96.00	101.14	105.89
1/2	50'	65.35	84.11	102.87	105.70	107.04
1/4	25'	56.21	82.98	109.74	110.26	108.20

Table A-14

Sigma (Y) for Opening 200' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
4 1/2	900'	97.60	97.85	98.10	98.55	99.45
4	800'	96.00	96.50	97.00	97.75	99.25
3 1/2	700'	94.67	95.34	96.00	97.00	99.00
3	600'	93.25	94.12	95.00	97.50	99.25
2 1/2	500'	91.36	92.53	94.23	96.24	99.89
2	400'	88.00	90.19	93.22	96.37	101.19
1 1/2	300'	83.02	87.50	93.00	97.98	103.72
1	200'	73.75	84.52	95.65	103.13	107.34
3/4	150'	66.18	84.26	100.18	108.04	108.99
1/2	100'	55.78	87.04	108.93	114.38	109.98
1/4	50'	43.01	95.30	122.17	119.43	110.26

Table A-15

Sigma (Y) for Opening 300' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
3	900'	95.19	96.05	97.40	98.78	101.00
2 1/2	750'	91.08	92.84	95.28	97.84	102.34
2	600'	86.69	89.50	93.81	97.88	104.00
1 1/2	450'	80.27	90.21	92.50	98.94	105.65
1	300'	69.44	84.00	98.16	106.47	110.09
3/4	225'	61.03	84.46	104.26	111.94	111.42
1/2	150'	48.38	89.40	116.14	118.50	111.74
1/4	75'	28.32	102.60	132.97	123.24	111.35

Table A-16

Sigma (Y) for Opening 400' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
2	800'	88.25	91.75	96.50	101.00	105.50
1 1/2	600'	79.62	87.00	95.88	103.12	109.25
1	400'	67.24	84.34	100.83	110.28	112.74
3/4	300'	58.41	85.62	107.60	116.05	113.57
1/2	200'	46.02	91.64	119.82	122.30	113.22
1/4	100'	25.59	108.84	137.65	125.56	111.79

Table A-17

Sigma (Y) for Opening 500' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1 1/2	750'	80.81	89.66	99.60	106.58	110.50
1	500'	65.28	85.75	104.92	114.21	115.27
3/4	375'	55.68	87.20	112.16	120.28	115.88
1/2	250'	42.44	94.28	125.68	126.17	114.88
1/4	125'	20.00	113.14	144.00	128.00	112.69

Table A-18

Sigma (Y) for Opening 600' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1 1/2	900'	86.70	94.70	103.00	107.00	108.00
1	600'	65.25	88.38	109.25	118.25	118.38
3/4	450'	55.99	89.81	115.62	123.74	118.98
1/2	300'	40.92	97.39	130.36	130.12	118.23
1/4	150'	18.78	118.56	148.46	130.31	114.88

Table A-19

Sigma (Y) for Opening 100' by 5', 500' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
4 1/2	450'	98.18	98.40	98.64	98.92	99.61
4	400'	97.38	97.69	98.02	98.58	99.54
3 1/2	350'	96.38	96.83	97.28	98.03	99.48
3	300'	95.10	95.77	96.44	97.43	99.46
2 1/2	250'	93.59	94.66	95.72	97.16	99.77
2	200'	91.42	93.20	94.98	97.14	100.67
1 1/2	150'	87.66	91.09	94.52	97.79	102.32
1	100'	80.72	88.50	96.27	100.52	104.99
3/4	75'	76.13	87.07	98.01	102.62	106.58
1/2	50'	67.90	86.66	105.42	107.05	107.30
1/4	25'	59.67	86.24	112.82	111.48	108.02

Table A-20

Sigma (Y) for Opening 200' by 5', 500' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
2	400'	88.36	92.04	96.68	101.00	105.50
1 1/2	300'	79.80	87.22	95.96	103.10	109.14
1	200'	68.50	84.43	100.11	109.52	112.32
3/4	150'	58.39	85.88	108.20	116.64	113.86
1/2	100'	47.79	91.13	119.89	120.91	112.33
1/4	50'	26.20	108.16	138.23	125.78	111.39

Table A-21

Sigma (Y) for Opening 300' by 5', 500' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
1 1/2	450'	85.96	94.48	102.74	107.34	108.60
1	300'	65.46	88.55	109.14	118.12	118.60
3/4	225'	54.86	89.32	116.67	124.13	120.15
1/2	150'	38.26	97.78	133.17	130.56	119.34
1/4	75'	18.53	118.82	148.72	130.03	115.34

Table A-22

Sigma (Y) for Opening 400' by 5', 500' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1	400'	76.24	98.58	114.44	116.72	105.38
3/4	300'	60.84	99.34	124.63	126.15	107.85
1/2	200'	45.86	106.90	137.99	132.09	108.17
1/4	100'	24.83	127.50	153.54	131.92	106.76

Table A-23

Sigma (Y) for Opening 500' by 5', 500' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
3/4	375'	66.62	107.87	129.01	122.08	100.22
1/2	250'	57.88	116.84	147.83	131.42	100.44
1/4	125'	18.20	139.86	164.03	131.10	99.64

Table A-24

Sigma (Y) for Opening 600' by 5', 500' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
3/4	450'	79.12	110.48	121.66	111.12	96.66
1/2	300'	48.82	126.06	150.67	124.86	93.96
1/4	150'	17.97	154.02	169.37	125.57	91.99

Table A-25

Sigma (Y) for Opening 100' by 5', 750' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
7	700'	**.**	99.07	99.14	99.20	99.34
6 1/2	650'	**.**	98.74	98.86	98.96	99.18
6	600'	**.**	98.42	98.57	98.72	99.03
5 1/2	550'	**.**	98.88	99.05	99.35	99.70
5	500'	**.**	98.09	98.32	98.46	98.92
4 1/2	450'	97.07	97.26	97.45	97.73	98.44
4	400'	97.02	97.20	97.40	97.86	98.76
3 1/2	350'	96.34	96.65	96.96	97.53	98.71
3	300'	95.47	95.94	96.42	97.22	98.83
2 1/2	250'	94.18	94.99	95.80	96.98	99.14
2	200'	92.03	93.54	95.06	96.88	99.92
1 1/2	150'	88.34	91.45	94.56	97.44	101.48
1	100'	81.50	88.74	95.99	99.92	104.10
3/4	75'	76.96	87.22	97.49	101.84	105.69
1/2	50'	68.46	86.60	104.75	106.30	106.56
1/4	25'	59.96	85.98	112.01	110.75	107.42

Table A-26

Sigma (Y) for Opening 200' by 5', 750' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
3 1/2	700'	96.17	96.90	97.63	98.54	100.18
3	600'	94.17	95.38	96.60	98.12	100.55
2 1/2	500'	92.06	93.52	95.48	97.78	101.62
2	400'	88.34	90.85	94.20	97.64	102.76
1 1/2	300'	83.42	88.24	94.17	99.46	105.15
1	200'	74.50	85.77	97.20	104.66	108.38
3/4	150'	67.23	85.93	101.92	109.40	109.70
1/2	100'	57.33	89.58	110.75	115.10	110.26
1/4	50'	45.35	99.08	123.72	119.26	110.12

Table A-27

Sigma (Y) for Opening 300' by 5', 750' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
2	600'	89.92	92.35	96.90	100.46	105.10
1 1/2	450'	80.14	86.76	94.22	100.98	105.55
1	300'	69.54	85.87	101.34	109.70	111.58
3/4	225'	61.28	86.68	107.64	114.93	112.50
1/2	150'	49.02	92.66	119.69	120.79	112.24
1/4	75'	29.42	107.42	136.30	14.09	111.07

Table A-28

Sigma (Y) for Opening 400' by 5', 750' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1 1/2	600'	83.24	91.74	100.55	106.62	107.84
1	400'	67.65	87.78	105.95	114.80	112.34
3/4	300'	58.50	89.42	113.02	120.33	113.05
1/2	200'	46.22	98.90	125.26	125.67	111.96
1/4	100'	26.16	115.87	141.53	127.20	110.53

Table A-29

Sigma (Y) for Opening 500' by 5', 750' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1	500'	67.58	91.88	111.58	117.58	107.52
3/4	375'	56.38	93.20	119.56	124.18	111.30
1/2	250'	42.55	100.95	132.90	130.40	113.80
1/4	125'	19.83	122.19	150.19	129.48	108.70

Table A-30

Sigma (Y) for Opening 600' by 5', 750' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1	600'	72.90	97.20	114.82	116.34	106.32
3/4	450'	57.79	95.51	119.34	120.49	105.05
1/2	300'	41.63	106.26	138.53	130.99	109.38
1/4	150'	18.53	130.07	154.74	129.87	107.37

Table A-31

Sigma (Y) for Opening 100' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
9 1/2	950'	**.**	99.79	99.79	99.79	99.79
9	900'	**.**	99.36	99.36	99.36	99.36
8 1/2	850'	**.**	99.36	99.36	99.36	99.36
8	800'	**.**	99.36	99.36	99.36	99.36
7 1/2	750'	**.**	99.20	99.23	99.27	99.34
7	700'	**.**	99.03	99.10	99.18	99.33
6 1/2	650'	**.**	98.84	98.94	99.03	99.22
6	600'	**.**	98.66	98.77	98.88	99.11
5 1/2	550'	**.**	98.64	98.79	98.94	99.25
5	500'	**.**	98.16	98.32	98.44	98.84
4 1/2	450'	97.52	97.67	97.82	98.02	98.57
4	400'	97.22	97.41	97.60	97.98	98.73
3 1/2	350'	96.62	96.90	97.18	97.70	98.71
3	300'	95.78	96.24	96.68	97.40	98.75
2 1/2	250'	94.51	95.26	96.02	97.07	99.00
2	200'	92.44	93.82	95.20	96.89	99.70
1 1/2	150'	88.84	91.76	94.68	97.37	101.19
1	100'	82.00	89.00	96.01	99.65	103.56
3/4	75'	77.45	87.45	97.45	101.44	104.94
1/2	50'	68.82	86.82	104.81	106.10	106.22
1/4	25'	60.20	86.18	112.17	110.77	107.50

Table A-32

Sigma (Y) for Opening 200' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
4 1/2	900'	97.27	97.72	98.18	98.64	99.54
4	800'	96.80	97.26	97.72	98.40	99.78
3 1/2	700'	95.44	96.05	96.66	97.57	99.39
3	600'	93.62	94.53	95.44	96.81	99.32
2 1/2	500'	92.10	93.22	94.84	96.69	100.04
2	400'	89.03	91.06	93.88	96.82	101.28
1 1/2	300'	84.37	88.57	93.74	98.42	103.65
1	200'	75.62	85.97	96.48	103.38	107.10
3/4	150'	68.33	85.91	100.82	107.86	108.67
1/2	100'	58.24	89.24	109.51	113.58	109.37
1/4	50'	45.78	98.32	122.58	118.18	109.12

Table A-33

Sigma (Y) for Opening 300' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
3	900'	95.68	96.36	97.72	99.09	100.91
2 1/2	750'	91.75	93.46	95.86	98.32	102.51
2	600'	87.47	90.21	94.30	98.18	103.87
1 1/2	450'	81.16	86.72	93.07	99.17	104.84
1	300'	71.02	85.10	97.74	106.48	109.71
3/4	225'	62.94	85.82	104.82	111.88	111.03
1/2	150'	50.54	91.07	116.74	118.17	111.50
1/4	75'	30.54	105.32	133.00	122.16	109.85

Table A-34

Sigma (Y) for Opening 400' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
2	800'	89.04	92.70	97.26	101.37	105.71
1 1/2	600'	80.41	87.70	96.24	103.19	108.88
1	400'	68.47	85.27	101.17	110.24	112.27
3/4	300'	59.96	86.70	107.96	115.79	113.03
1/2	200'	47.90	93.26	120.16	121.90	112.85
1/4	100'	27.48	111.74	137.10	124.31	111.15

Table A-35

Sigma (Y) for Opening 500' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1 1/2	750'	81.57	90.30	99.88	106.53	110.20
1	500'	66.41	86.54	105.13	114.05	114.80
3/4	375'	57.01	88.22	112.42	120.00	115.38
1/2	250'	44.10	95.71	125.90	125.56	114.30
1/4	125'	21.66	115.72	142.69	127.59	111.97

Table A-36

Sigma (Y) for Opening 600' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (Y) as a Percentage of the Overburden Pressure at set Distances from Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1 1/2	900'	87.06	94.80	103.01	106.88	108.02
1	600'	66.08	89.08	109.37	118.02	117.91
3/4	450'	57.08	90.70	115.75	123.37	118.26
1/2	300'	42.40	98.71	130.42	129.46	117.52
1/4	150'	20.22	121.32	147.62	129.50	114.47

APPENDIX B

VALUES FOR SIGMA (X) FOR VARIOUS DEPTHS AND WIDTHS

Table B-1

Sigma (X) for Opening 100' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
4 1/2	450'	-21.35	-20.65	-19.95	-18.70	-15.95
4	400'	-25.50	-24.90	-24.30	-23.25	-21.00
3 1/2	350'	-31.40	-30.68	-29.95	-28.78	-26.40
3	300'	-38.60	-37.50	-36.40	-34.80	-31.80
2 1/2	250'	-47.00	-45.08	-43.15	-40.72	-36.65
2	200'	-56.70	-53.00	-49.30	-45.58	-40.30
1 1/2	150'	-67.85	-60.25	-52.65	-47.45	-42.42
1	100'	-79.25	-61.90	-44.55	-43.75	-46.95
3/4	75'	-84.70	-60.30	-35.90	-40.20	-50.80
1/2	50'	-44.46	-48.50	-52.55	-57.52	-64.42
1/4	25'	-4.21	-36.70	-69.20	-74.85	-78.05

H/W = Height above seam divided by width of opening

Negative values are compressive

Positive values are tensile

Table B-2

Sigma (X) for Opening 200' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
2	400'	-46.20	-39.38	-29.92	-19.78	-2.76
1 1/2	300'	-55.80	-45.75	-34.28	-25.35	-23.45
1	200'	-74.48	-49.95	-30.22	-24.52	-35.92
3/4	150'	-74.22	-35.70	-17.88	-28.20	-53.56
1/2	100'	-73.95	-21.44	-5.54	-31.88	-71.21
1/4	50'	-27.05	+0.75	-28.95	-78.35	-97.90

Table B-3

Sigma (X) for Opening 300' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
1 1/2	450'	-86.95	-53.60	-12.90	-24.18	+66.02
1	300'	-71.95	-43.02	-17.90	-8.02	-8.44
3/4	225'	-79.60	-38.35	-12.50	-17.85	-39.65
1/2	150'	-78.86	-27.37	-10.14	-38.72	-75.10
1/4	75'	-26.88	+3.04	-41.81	-99.29	-125.25

Table B-4

Sigma (X) for Opening 400' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1	400'	-132.25	-78.85	-24.38	+11.74	+32.42
3/4	300'	-95.92	-46.32	-12.08	-5.26	-13.40
1/2	200'	-70.9	-10.42	+3.97	-24.75	-55.50
1/4	100'	-7.85	+17.30	-32.00	-80.10	-97.45

Table B-5

Sigma (X) for Opening 500' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
3/4	375'	-157.00	-69.80	+4.25	+36.20	+39.55
1/2	250'	-103.50	-16.83	+7.62	-16.80	-36.82
1/4	125'	-15.80	+28.30	-32.60	-92.35	-104.00

Table B-6

Sigma (X) for Opening 600' by 5', 500' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
3/4	450'	-295.00	-81.00	+89.35	+148.50	+123.50
1/2	300'	-131.75	-28.10	+22.28	+14.29	+2.65
1/4	150'	-20.19	+26.82	-43.67	-104.05	-107.92

Table B-7

Sigma (X) for Opening 100' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
7	700'	**.**	-17.80	-17.30	-16.80	-15.80
6 1/2	650'	**.**	-23.65	-23.20	-22.75	-21.85
6	600'	**.**	-29.50	-29.10	-28.70	-27.90
5 1/2	550'	**.**	-38.78	-38.30	-37.83	-36.88
5	500'	**.**	-48.05	-47.50	-46.96	-45.86
4 1/2	450'	-50.26	-51.20	-50.56	-49.92	-48.65
4	400'	-59.10	-58.48	-57.85	-56.90	-54.82
3 1/2	350'	-67.80	-66.85	-65.90	-64.38	-61.40
3	300'	-77.15	-75.52	-73.90	-71.60	-67.35
2 1/2	250'	-87.45	-84.58	-81.70	-78.08	-72.02
2	200'	-99.45	-93.82	-88.20	-82.58	-74.58
1 1/2	150'	-113.00	-101.58	-90.15	-82.30	-74.60
1	100'	-126.50	-100.65	-74.80	-73.52	-78.08
3/4	75'	-133.00	-96.60	-60.20	-66.55	-82.15
1/2	50'	-71.38	-77.24	-83.10	-90.52	-100.82
1/4	25'	-9.75	-57.88	-106.00	-114.50	-119.50

. = Percentage undeterminable from model used

Table B-8

Sigma (X) for Opening 200' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
3 1/2	700'	-46.40	-42.75	-39.10	-33.10	-20.65
3	600'	-48.90	-45.90	-42.90	-38.45	-29.80
2 1/2	500'	-60.41	-57.35	-53.96	-49.67	-42.14
2	400'	-74.32	-68.90	-61.68	-54.70	-45.92
1 1/2	300'	-95.32	-82.75	-68.30	-57.22	-50.58
1	200'	-117.75	-86.30	-59.35	-48.62	-59.08
3/4	150'	-122.88	-74.65	-44.10	-43.78	-71.15
1/2	100'	-104.35	-46.48	-31.20	-56.08	-92.40
1/4	50'	+1.00	-58.48	-89.70	-106.32	-119.75

Table B-9

Sigma (X) for Opening 300' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
2	600'	-75.02	-66.10	-48.26	-38.80	-15.60
1 1/2	450'	-84.63	-71.15	-55.91	-43.12	-33.22
1	300'	-110.50	-73.52	-43.10	-34.40	-48.98
3/4	225'	-121.00	-63.00	-26.10	-30.20	-65.35
1/2	150'	-110.00	-31.28	-7.68	-45.55	-98.52
1/4	75'	-56.80	+2.70	-50.40	-105.05	-130.00

Table B-10

Sigma (X) for Opening 400' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1 1/2	600'	-106.50	-82.20	-52.20	-24.55	+11.34
1	400'	-105.92	-68.25	-36.98	-25.62	-35.82
3/4	300'	-113.95	-53.98	-19.10	-24.62	-63.10
1/2	200'	-105.08	-20.98	-0.96	-43.40	-100.00
1/4	100'	-28.78	+10.70	-51.35	-114.92	-145.25

Table B-11

Sigma (X) for Opening 500' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1	500'	-126.98	-83.47	-28.71	-27.26	+7.21
3/4	375'	-117.00	-53.25	-14.50	-18.40	-50.05
1/2	250'	-106.50	-15.60	+4.27	-46.20	-106.70
1/4	125'	-38.70	+26.55	-47.40	-130.00	-166.50

Table B-12

Sigma (X) for Opening 600' by 5', 750' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1	600'	-179.00	-95.10	-8.80	+48.10	+93.30
3/4	450'	-120.09	-62.62	-11.08	+0.66	-6.58
1/2	300'	-108.40	-15.02	+3.13	-48.92	-104.22
1/4	150'	-25.80	+28.48	-60.65	153.00	-192.75

Table B-13

Sigma (X) for Opening 100' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
9 1/2	950'	**.**	-14.93	-14.69	-14.45	-13.97
9	900'	**.**	-21.20	-20.98	-20.75	-20.30
8 1/2	850'	**.**	-27.95	-27.74	-27.52	-27.10
8	800'	**.**	-34.70	-34.50	-34.30	-33.90
7 1/2	750'	**.**	-42.00	-41.78	-41.55	-41.10
7	700'	**.**	-49.30	-49.05	-48.80	-48.30
6 1/2	650'	**.**	-57.00	-56.68	-56.35	-55.70
6	600'	**.**	-64.70	-64.30	-63.90	-63.10
5 1/2	550'	**.**	-73.18	-72.75	-72.30	-71.34
5	500'	**.**	-81.67	-81.20	-80.70	-79.59
4 1/2	450'	-88.16	-89.09	-88.47	-87.73	-85.86
4	400'	-98.25	-97.40	-96.55	-95.28	-92.48
3 1/2	350'	-108.00	-106.50	-105.00	-103.15	-99.22
3	300'	-118.00	-115.75	-113.50	-110.50	-104.75
2 1/2	250'	-129.50	125.75	-122.00	-117.00	-108.75
2	200'	-143.00	-135.75	-128.50	-120.75	-110.00
1 1/2	150'	-158.50	-143.50	-128.50	-117.75	-107.50
1	100'	-174.50	-140.00	-105.50	-103.75	-109.75
3/4	75'	-182.00	-133.50	-85.00	-93.50	-114.00
1/2	50'	-98.75	-106.62	-114.50	-124.25	-137.50
1/4	25'	-15.50	-79.75	-144.00	-155.00	-161.00

Table B-14

Sigma (X) for Opening 200' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
4 1/2	900'	-38.20	-36.60	-35.00	-32.25	-26.05
4	800'	-47.50	-46.15	-44.80	-42.45	-37.40
3 1/2	700'	-60.90	-59.15	-57.40	-54.75	-49.45
3	600'	-77.30	-74.50	-71.70	-67.90	-61.10
2 1/2	500'	-94.85	-91.37	-87.39	-82.58	-74.84
2	400'	-116.00	-109.00	-99.60	-90.40	-79.10
1 1/2	300'	-142.00	-125.25	-105.95	-91.25	-82.18
1	200'	-168.50	-126.65	-90.92	-76.58	-89.72
3/4	150'	-173.25	-109.28	-68.95	-68.25	-103.45
1/2	100'	-147.15	-70.25	-49.88	-82.52	-129.25
1/4	50'	-5.65	-84.12	-125.22	-147.00	-163.50

Table B-15

Sigma (X) for Opening 300' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
3	900'	-62.35	-57.40	-48.70	-38.60	-16.60
2 1/2	750'	-70.92	-66.20	-58.48	-50.25	-34.85
2	600'	-92.78	-84.60	-73.20	-63.15	-49.80
1 1/2	450'	-123.64	-105.36	-87.05	-71.92	-64.28
1	300'	-161.50	-113.08	-72.60	-60.85	-79.45
3/4	225'	-174.00	-98.30	-49.30	-53.75	-97.90
1/2	150'	-159.00	-55.16	-23.30	-71.62	-132.00
1/4	75'	-87.00	-8.35	-76.60	-146.50	-176.00

Table B-16

Sigma (X) for Opening 400' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
2	800'	-87.45	-74.80	-57.30	-38.60	-7.72
1 1/2	600'	-109.00	-89.30	-67.15	-50.30	-38.80
1	400'	-149.25	-100.90	-62.12	-49.92	-70.90
3/4	300'	-165.50	-87.02	-41.10	-47.60	-98.15
1/2	200'	-155.00	-45.44	-17.28	-69.72	-140.00
1/4	100'	-56.08	-3.10	-80.22	-159.00	-192.50

Table B-17

Sigma (X) for Opening 500' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1 1/2	750'	-112.88	-85.90	-52.29	-23.25	-0.56
1	500'	-138.59	-94.72	-57.99	-44.97	-56.01
3/4	375'	-161.00	-80.55	-33.70	-43.00	-95.45
1/2	250'	-157.5	-39.69	-11.32	-73.38	-150.50
1/4	125'	-74.60	-3.82	-75.40	-174.50	-215.00

Table B-18

Sigma (X) for Opening 600' by 5', 1000' Sandstone Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1 1/2	900'	-164.00	-101.65	-25.30	+44.00	+122.00
1	600'	-142.00	-84.10	-34.90	-17.25	-20.15
3/4	450'	-154.91	-76.60	-31.30	-38.68	-79.77
1/2	300'	-155.50	-37.14	-13.20	-79.95	-156.75
1/4	150'	-63.10	+11.88	-90.15	-199.00	-243.25

Table B-19

Sigma (X) for Opening 100' by 5', 500' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
4 1/2	450'	-18.35	-17.68	-17.00	-15.85	-13.30
4	400'	-20.75	-20.18	-19.60	-18.62	-16.52
3 1/2	350'	-24.75	-24.08	-23.40	-22.30	-20.08
3	300'	-28.70	-28.95	-27.90	-26.40	-23.58
2 1/2	250'	-36.40	-34.55	-32.70	-30.40	-26.55
2	200'	-44.15	-40.55	-36.95	-33.38	-28.38
1 1/2	150'	-53.55	-45.98	-38.40	-33.42	-28.72
1	100'	-64.05	-46.00	-27.95	-27.88	-31.52
3/4	75'	-69.40	-43.65	-17.90	-23.35	-34.50
1/2	50'	-29.77	-32.46	-35.15	-39.60	-45.90
1/4	25'	+9.86	-21.27	-52.40	-55.85	-57.30

Table B-20

Sigma (X) for Opening 200' by 5', 500' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
2	400'	-42.92	-35.88	-26.10	-15.67	+1.62
1 1/2	300'	-48.65	-38.08	-26.08	-16.88	-10.42
1	200'	-63.98	-37.62	-16.95	-11.30	-23.68
3/4	150'	-62.28	-19.30	-1.39	-13.19	-39.84
1/2	100'	-60.57	-0.97	+14.17	-15.08	-56.01
1/4	50'	-12.68	+26.90	-9.00	-60.75	-80.90

Table B-21

Sigma (X) for Opening 300' by 5', 500' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
1 1/2	450'	-88.75	-53.58	-10.86	+27.88	+71.22
1	300'	-66.35	-35.18	-8.55	+1.35	+0.36
3/4	225'	-71.20	-26.10	+1.15	-5.34	-28.85
1/2	150'	-68.64	-7.00	+8.70	-23.28	-62.01
1/4	75'	-12.42	+29.70	-23.32	-84.60	-110.88

Table B-22

Sigma (X) for Opening 400' by 5', 500' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1	400'	-133.50	-76.75	-19.53	+17.95	+38.50
3/4	300'	-91.55	-37.90	-1.96	+4.45	-4.76
1/2	200'	-61.58	+5.74	+19.68	-11.67	-44.08
1/4	100'	+6.46	+41.50	-14.44	-65.02	-83.18

Table B-23

Sigma (X) for Opening 500' by 5', 500' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
3/4	375'	-160.00	-65.95	+12.20	+44.85	+46.95
1/2	250'	-85.20	-3.10	+21.35	-5.61	-26.98
1/4	125'	-2.74	+53.00	-16.70	-79.60	-91.25

Table B-24

Sigma (X) for Opening 600' by 5', 500' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
3/4	450'	-310.25	-80.15	+100.08	+160.00	+130.50
1/2	300'	-130.50	-17.22	+34.72	+24.86	+11.89
1/4	150'	+1.48	+50.72	-29.35	-93.22	-96.38

Table B-25

Sigma (X) for Opening 100' by 5', 750' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
7	700'	**.**	-15.00	-14.55	-14.10	-13.20
6 1/2	650'	**.**	-19.00	-18.58	-18.15	-17.30
6	600'	**.**	-23.00	-22.60	-22.20	-21.40
5 1/2	550'	**.**	-28.52	-28.11	-27.70	-26.90
5	500'	**.**	-34.03	-33.62	-33.21	-32.39
4 1/2	450'	-39.19	-38.61	-38.02	-37.44	-36.26
4	400'	-44.60	-44.00	-43.40	-42.52	-40.58
3 1/2	350'	-51.25	-50.32	-49.40	-47.98	-45.18
3	300'	-58.55	-56.98	-55.40	-53.22	-49.20
2 1/2	250'	-66.80	-64.00	-61.20	-57.75	-52.00
2	200'	-76.55	-71.15	-65.75	-60.35	-52.75
1 1/2	150'	-88.50	-77.12	-65.75	-58.25	-51.08
1	100'	-102.05	-74.95	-47.85	-47.65	-52.92
3/4	75'	-109.00	-70.35	-31.70	-39.75	-56.20
1/2	50'	-48.45	-52.30	-56.15	-62.70	-71.92
1/4	25'	+12.1	-34.25	-80.65	-85.65	-87.65

Table B-26

Sigma (X) for Opening 200' by 5', 750' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
3 1/2	700'	-44.60	-40.90	-37.20	-31.15	-18.60
3	600'	-43.00	-39.95	-36.90	-32.45	-23.75
2 1/2	500'	-50.51	-47.13	-43.75	-39.26	-31.49
2	400'	-62.02	-55.12	-47.68	-40.50	-31.52
1 1/2	300'	-78.28	-65.00	-50.00	-38.60	-32.00
1	200'	-97.98	-64.15	-35.90	-25.18	-36.45
3/4	150'	-101.98	-49.15	-17.28	-17.96	-46.62
1/2	100'	-82.20	-16.27	-16.73	-28.68	-65.88
1/4	50'	+26.00	-32.05	-58.40	-75.90	-91.70

Table B-27

Sigma (X) for Opening 300' by 5', 750' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
2	600'	-70.70	-61.40	-47.15	-33.10	-9.22
1 1/2	450'	-74.48	-60.01	-43.73	-30.34	-20.20
1	300'	-95.05	-55.32	-23.05	-14.36	-30.08
3/4	225'	-103.00	-40.15	-1.20	-6.45	-43.80
1/2	150'	-90.05	-1.42	+21.75	-19.37	-69.95
1/4	75'	-35.80	+40.35	-22.20	-77.50	-104.00

Table B-28

Sigma (X) for Opening 400' by 5', 750' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
1 1/2	600'	-104.20	-78.80	-47.20	-18.22	+19.00
1	400'	-95.28	-54.45	-21.18	-9.48	-20.92
3/4	300'	-99.80	-33.98	+2.91	-3.73	-44.90
1/2	200'	-86.35	+7.60	+27.18	-19.16	-79.08
1/4	100'	-4.60	+48.80	-22.55	-89.90	-121.50

Table B-29

Sigma (X) for Opening 500' by 5', 750' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1	500'	-123.48	-75.36	-28.82	-0.34	+20.63
3/4	375'	-107.0	-36.75	+4.54	-0.55	-34.85
1/2	250'	-89.85	+11.21	+30.65	-24.37	88.70
1/4	125'	-15.80	+65.70	-20.00	-107.30	-145.00

Table B-30

Sigma (X) for Opening 600' by 5', 750' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1	600'	-183.00	-93.00	-1.11	+58.90	+106.00
3/4	450'	-130.82	-50.28	+4.909	+16.45	+7.63
1/2	300'	-94.35	+9.54	+27.25	-29.70	-88.68
1/4	150'	-2.02	+67.05	-34.80	-133.32	-174.25

Table B-31

Sigma (X) for Opening 100' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	25'	50'	75'	100'	150'
9 1/2	950'	**.**	-12.43	-12.19	-11.95	-11.47
9	900'	**.**	-16.70	-16.50	-16.30	-15.90
8 1/2	850'	**.**	-21.50	-21.31	-21.12	-20.75
8	800'	**.**	-26.30	-26.12	-25.95	-25.60
7 1/2	750'	**.**	-31.60	-31.40	-31.20	-30.80
7	700'	**.**	-36.90	-36.68	-36.45	-36.00
6 1/2	650'	**.**	-42.60	-42.30	-42.00	-41.40
6	600'	**.**	-48.30	-47.92	-47.55	-46.80
5 1/2	550'	**.**	-54.62	-54.19	-53.75	-52.82
5	500'	**.**	-60.95	-60.46	-59.95	-58.83
4 1/2	450'	-67.46	-66.52	-65.89	-65.18	-63.41
4	400'	-73.55	-72.72	-71.90	-70.75	-68.18
3 1/2	350'	-80.95	-79.72	-78.50	-76.58	-72.82
3	300'	-89.15	-87.08	-85.00	-82.02	-76.60
2 1/2	250'	-98.75	-94.92	-91.10	-86.45	-78.72
2	200'	-110.00	-102.72	-95.45	-88.25	-78.08
1 1/2	150'	-124.00	-108.90	-93.80	-83.78	-74.15
1	100'	-140.00	-104.12	-68.25	-67.92	-74.88
3/4	75'	-148.00	-96.90	-45.80	-56.50	-78.35
1/2	50'	-66.90	-72.15	-77.40	-86.25	-98.42
1/4	25'	+14.20	-47.40	-109.00	-116.00	-118.50

Table B-32

Sigma (X) for Opening 200' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	50'	100'	150'	200'	300'
4 1/2	900'	-34.30	-32.70	-31.10	-28.30	-22.05
4	800'	-39.60	-38.20	-36.80	-34.45	-29.35
3 1/2	700'	-49.00	-47.25	-45.50	-42.75	-37.35
3	600'	-61.40	-58.55	-55.70	-51.85	-45.00
2 1/2	500'	-75.14	-71.37	-67.15	-62.16	-54.08
2	400'	-92.62	-85.35	-75.50	-66.10	-54.52
1 1/2	300'	-115.50	-97.82	-77.70	-62.48	-53.42
1	200'	-139.25	-94.68	-57.02	-42.52	-56.85
3/4	150'	-143.22	-73.62	-31.12	-31.70	-68.88
1/2	100'	-115.88	-28.62	-8.62	-44.48	-92.85
1/4	50'	+28.40	-48.00	-82.45	-105.28	-125.50

Table B-33

Sigma (X) for Opening 300' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	75'	150'	225'	300'	450'
3	900'	-59.90	-54.80	-45.72	-35.25	-12.50
2 1/2	750'	-62.09	-57.10	-49.04	-40.55	-24.60
2	600'	-77.95	-69.40	-57.55	-47.05	-33.20
1 1/2	450'	-101.93	-84.10	-65.30	-48.71	-37.24
1	300'	-137.00	-84.55	-41.90	-30.00	-50.20
3/4	225'	-148.00	-64.95	-13.20	-19.10	-66.25
1/2	150'	-130.00	-13.45	+17.95	-34.63	-98.48
1/4	75'	-57.80	+42.65	-37.90	-108.85	-141.00

Table B-34

Sigma (X) for Opening 400' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	100'	200'	300'	400'	600'
2	800'	-82.30	-68.95	-50.50	-30.85	+1.48
1 1/2	600'	-96.15	-74.75	-51.15	-33.45	-21.60
1	400'	-129.00	-77.00	-35.42	-22.85	-45.78
3/4	300'	-142.25	-56.40	-7.70	-15.66	-69.92
1/2	200'	-127.95	-4.93	+22.75	-34.68	-109.25
1/4	100'	-22.62	+48.52	-40.48	-124.35	-159.75

Table B-35

Sigma (X) for Opening 500' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	125'	250'	375'	500'	750'
1 1/2	750'	-107.75	-78.55	-42.85	-12.11	+23.95
1	500'	-124.27	-75.19	-34.98	-21.17	-33.84
3/4	375'	-141.00	-53.10	-3.07	-14.06	-70.65
1/2	250'	-132.00	-0.99	+27.10	-40.82	-122.75
1/4	125'	-42.60	+65.45	-37.30	-143.00	-185.00

Table B-36

Sigma (X) for Opening 600' by 5', 1000' Shale Overburden

Height Above Seam		Sigma (X) in Pounds per Square Inch at set Distances from the Centerline of Mine Opening				
H/W	feet	150'	300'	450'	600'	900'
1 1/2	900'	-170.00	-103.05	-21.70	+52.10	+134.50
1	600'	-131.00	-68.85	-16.50	+1.93	-1.78
3/4	450'	-83.40	-45.98	-3.67	-12.64	-57.60
1/2	300'	-132.50	-0.57	+22.90	-50.32	-132.25
1/4	150'	-29.98	+63.98	-53.68	-170.00	-216.00

APPENDIX C

USER'S MANUAL FOR COMPUTER PROGRAMS

Appendix C-1

The Finite Element Program

1. Definition of Variables

AK	Assemblage stiffness matrix.
AREA	Area of triangular element.
B	Matrix [B], for quadrilateral.
BODYF	Nodal load contribution from TBODY.
BT	Matrix [B], for triangle.
C	Stress-strain matrix, [C].
CB	Matrix product [C] [B].
CF	Common factor in the computation of stress-strain matrix, C.
E	Modulus of elasticity, E. In STRESS, the strains are at the centroid of the element.
EL	Length of element side where surface traction is prescribed.
FAC	Factor for averaging element strains.
IBAND	Semi-band width of assemblage equations, IBAND < MAXBW.
IE(M,I)	Element identification array, M is element number, I < M < NEL. I = 1,2,3,4 denote corner nodes of element, and I = 5 denotes MTYP for the element.
ISC,JSC	Nodal numbers of i and j for side on which surface traction is prescribed.
ISTOP	Index used to count data errors.
I,J,K,L	Indices of the four nodes of quadrilateral in QUAD.

I,J,K	Indices of the three nodes of triangular element in CST.
KODE(I)	Index of displacement and concentrated load conditions at node I.
KKK	In BANSOL, index designating function to be used. KKK = 1 for triangularization of stiffness. KKK = 2 for backward solution using triangularized stiffness.
MAXBW	Maximum semi-band width allowed.
MAXDOF	Maximum degrees of freedom, MAXDOF = 2 X MAXNP.
MAXEL	Maximum number of elements allowed.
MAXMAT	Maximum number of materials allowed.
MAXNP	Maximum number of nodes allowed.
MAXSLC	Maximum number of surface traction cards allowed.
MDIM	Maximum band width, MDIM = MAXBW.
MTYP	Material type number, 1 < MTYP < NMAT.
NBODY	Option for body force. NBODY = 0 for no weight. NBODY = 1 for weight force in the negative Y direction.
NDIM	Maximum degrees of freedom, NDIM = MAXDOF.
NEL	Number of elements, NEL < MAXEL.
NEQ	Total number of equations, NEQ = 2 X NNP < MAXDOF.
NMAT	Number of different materials, 1 < NMAT < MAXMAT.
NNP	Number of nodal points, NNP < MAXNP.
NOLINE	In STRESS, index to limit output to fifty lines per page.

NOPT	Option for plane strain/stress. NOPT = 1 for plane strain, NOPT = 2 for plane stress.
NPROB	Problem number.
NSLC	Number of surface traction cards, NSLC < MAXSLC.
PR	Poison's Ratio.
PXI,PXJ	Nodal contributions of surface tractions in the X direction.
PYI,PYJ	Nodal contributions of surface tractions in the Y direction.
Q	Load vector of quadrilateral element. In STRESS, the element displacement vector.
QK	Stiffness matrix of the quadrilateral element.
R	Assemblage load vector. Also computed displacements for the assemblage in MAIN, BANSOL and STRESS.
RO	Weight density of the material.
SIG	Array for stresses.
SURTRX SURTRY	X and Y components of prescribed distributed tractions.
TBODY	Total weight of triangular element.
TH	Thickness, h.
TITLE	Array for title of the problem (72 alphanumeric characters).
TK	Stiffness matrix of the triangular element.
TOTALA	Total area of quadrilateral element.
U	Prescribed displacement in the X or Y direction in GEOMBC.

ULX(I)	Concentrated load or displacement in the X and Y
VLY(I)	directions at node I.
X(I),Y(I)	X and Y coordinates of node I.
XQ,YQ	Coordinates of the nodes of a quadrilateral or triangular element. (XQ(5) and YQ(5) are coordinates of the centroid.)

2. Input Data

The input data are called for exclusively in the SUBROUTINE DATAIN. This information is divided into the title card, the basic parameters, material properties, nodal point data, surface traction cards and an exit card.

a) Title card:

FORMAT (I5,1X,18A4) - 1 card per problem

Col.	Symbol	Definition
1-5	NPROB	Problem number
6		Blank
7-78	TITLE	Title of Problem

b) Basic parameters:

FORMAT (6I5) - 1 card per problem

Col.	Symbol	Definition
1-5	NNP	Number of nodal points
6-10	NEL	Number of elements
11-15	NMAT	Number of different materials
16-20	NSLC	Number of surface traction cards
21-25	NOPT	Option for plane strain/stress = 1 for plane strain = 2 for plane stress
26-30	NBODY	Option for body force = 0, no weight force = 1, weight force in negative Y direction

c) Material properties:

FORMAT (4E10.3) - NMAT cards per problem

Col.	Symbol	Definition
1-10	E	Modulus of Elasticity
11-20	PR	Poisson's Ratio
21-30	RO	Weight density of the material
31-40	TH	Thickness

d) Nodal point data:

FORMAT (2I5,4F10.1)

Col.	Symbol	Definition
1-5	M	Nodal point number
6-10	KODE	Option for free or fixed nodes: = 0, for fully free nodes = 1, for nodes fixed in the X-direction = 2, for nodes fixed in the Y-direction = 3, for fully fixed nodes
11-20	X	X-coordinate of node
21-30	Y	Y-coordinate of node
31-40	ULX	Prescribed load/displacement in the X direction
41-50	VLY	Prescribed load/displacement in the Y direction

e) Element data:

FORMAT (6I5)

Col.	Symbol	Definition
1-5	M	Element number

Col.	Symbol	Definition
6-10	IE(M,1)	
11-15	IE(M,2)	Corner node numbers for the
16-20	IE(M,3)	number M element
21-25	IE(M,4)	
26-30	IE(M,5)	Material type

f) Surface traction:

FORMAT (2I5,4E10.3)

Col.	Symbol	Definition
1-5	ISC(I)	Number i nodal point
6-10	JSC(J)	Number j nodal point
11-20	SURTRX(I)	Surface traction in the X-direction of the i node
21-30	SURTRX(J)	Surface traction in the X-direction of the j node
31-40	SURTRY(I)	Surface traction in the Y-direction of the i node
41-50	SURTRY(J)	Surface traction in the Y-direction of the j node

g) Exit card:

FORMAT (I1)

Col.	Symbol	Definition
1	N	Termination option = 0, for termination of program operation = 1,2,3,..., for execution of next problem

2.1 Notes on Input Data

1. Units must be consistent.

2. Usually one card is needed for each node. However, if some nodes fall on a straight line and spaced at an equal distance, data for only the first and last nodes of this group are necessary. The intermediate nodal point data are generated by linear interpolation. This applies only to the nodal point coordinates.

3. Usually one card is needed for each element. However, if some of the elements are lined up so that their corner node indices each increase by one when compared to the previous element, only the data for the first and last elements of the set are necessary.

```
C     FINITE ELEMENT PROGRAM                                                    MAIN  10
C     INTRODUCTION TO THE FINITE ELEMENT METHOD - DESAI-ABEL                    MAIN  20
C     EXAMPLE CODE (PROGRAM) FOR PLANE STRAIN/STRESS                            MAIN  30
      LOGICAL OUT                                                               MAIN  40
      DIMENSION TITLE(18)                                                       MAIN  50
      COMMON NNP,NEL,NMAT,NSLC,NOPT,NBODY,MTYP,E(6),PR(6),RO(6),TH(6),IEMAIN  60
     1(800,5),X(800),Y(800),ULX(800),VLY(800),KODE(800),ISC(80),JSC(80),MAIN  70
     2SURTRX(80,2),SURTRY(80,2)                                                 MAIN  80
      COMMON /ONE/ QK(10,10),Q(10),B(3,10),C(3,3),BT(3,6),XQ(5),YQ(5)           MAIN  90
      COMMON /TWO/ IBAND,NEQ,R(1600),AK(1600,144)                               MAIN 100
      DATA MAXEL,MAXNP,MAXMAT,MAXBW,MAXSLC/800,800,6,144,80/                    MAIN 110
                                                                                MAIN 120
C                                                                               MAIN 130
C     PROBLEM  IDENTIFICATION  AND  DESCRIPTION                                 MAIN 140
C                                                                               MAIN 150
1     READ 7, NPROB,(TITLE(I),I=1,18)                                           MAIN 160
      IF (NPROB.LE.0) GO TO 5                                                   MAIN 170
      PRINT 8, NPROB,(TITLE(I),I=1,18)                                          MAIN 180
      CALL DATAIN (MAXEL,MAXNP,MAXMAT,MAXSLC,ISTOP)                             MAIN 190
      MAXDOF=2*MAXNP                                                            MAIN 200
C                                                                               MAIN 210
C     COMPUTE  MAX. NODAL DIFFERENCE AND SEMI-BANDWIDTH, EQ. (6-1)              MAIN 220
      MAXDIF=0                                                                  MAIN 230
      DO 2 I=1,NEL                                                              MAIN 240
      DO 2 J=1,4                                                                MAIN 250
      DO 2 K=1,4                                                                MAIN 260
      LL=IABS(IE(I,J)-IE(I,K))                                                  MAIN 270
      IF (LL.GT.MAXDIF) MAXDIF=LL                                               MAIN 280
2     CONTINUE                                                                  MAIN 290
      IBAND=2*(MAXDIF+1)                                                        MAIN 300
      NEQ=2*NNP                                                                 MAIN 310
      IF (IBAND.GT.MAXBW) GO TO 4                                               MAIN 320
      IF (ISTOP.GT.0) GO TO 5                                                   MAIN 330
      CALL ASEMBL (ISTOP)                                                       MAIN 340
      IF (ISTOP.GT.0) GO TO 5                                                   MAIN 350
C                                                                               MAIN 360
C     TRIANGULARIZE STIFFNESS MATRIX, EQ. (2-2), KKK=1                          MAIN 370
      CALL BANSOL (1,AK,R,NEQ,IBAND,MAXDOF,MAXBW)                               MAIN 380
C                                                                               MAIN 390
C     SOLVE FOR DISPLACEMENTS CORRESP. TO LOAD VECTOR R, EQ.(2-3), KKK=2MAIN 400
      CALL BANSOL (2,AK,R,NEQ,IBAND,MAXDOF,MAXBW)                               MAIN 410
      DXPL=0.0                                                                  MAIN 420
      IF (DXPL.EQ.0.0) GO TO 3                                                  MAIN 430
      PRINT 9, (I,R(2*I-1),R(2*I),I=1,NNP)                                      MAIN 440
      WRITE (6,6) (X(I),Y(I),R(2*I-1),R(2*I),I,I=1,NNP)                         MAIN 450
C                                                                               MAIN 460
3     CALL STRESS                                                               MAIN 470
      GO TO 1                                                                   MAIN 480
4     PRINT 10, IBAND,MAXBW                                                     MAIN 490
      GO TO 1                                                                   MAIN 500
5     STOP                                                                      
                                                                                MAIN 510
C                                                                               MAIN 520
6     FORMAT (2F10.3,2E15.7,I10)                                                MAIN 530
7     FORMAT (I5,1X,18A4)
```

```
8      FORMAT (/8H1PROBLEM,I5,3H.. ,18A4/)                                    MAIN 540
9      FORMAT (37H1OUTPUT TABLE 1.. NODAL DISPLACEMENTS//13X,4HNODE,9X,11HMAIN 550
     1U = X-DISP.,9X,11HV = Y-DISP./(5X,I12,2E20.8))                          MAIN 560
10     FORMAT (///12H BANDWIDTH =,I4,25H EXCEEDS MAX. ALLOWABLE =,I4//30HMAIN 570
     1 GO ON TO NEXT PROBLEM          )                                       MAIN 580
       END                                                                    MAIN 590
```

```
C     FINITE ELEMENT PROGRAM - SUBROUTINE DATAIN                                DATA  10
      SUBROUTINE DATAIN (MAXEL,MAXNP,MAXMAT,MAXSLC,ISTOP)                       DATA  20
      COMMON NNP,NEL,NMAT,NSLC,NOPT,NBODY,MTYP,E(6),PR(6),RO(6),TH(6),IEDATA  30
     1(800,5),X(800),Y(800),ULX(800),VLY(800),KODE(800),ISC(80),JSC(80),DATA  40
                                                                               DATA  50
     2SURTRX(80,2),SURTRY(80,2)                                                DATA  60
C                                                                              DATA  70
      ISTOP=0                                                                  DATA  80
      READ 22, NNP,NEL,NMAT,NSLC,NOPT,NBODY                                    DATA  90
C                                                                              DATA 100
      PRINT 23, NNP,NEL,NMAT,NSLC,NOPT,NBODY                                   DATA 110
C                                                                              DATA 120
C     CHECKS TO BE SURE INPUT DATA DOES NOT EXCEED STORAGE CAPACITY            DATA 130
      IF (NNP.LE.MAXNP) GO TO 1                                                DATA 140
      ISTOP=ISTOP+1                                                            DATA 150
      PRINT 24, MAXNP                                                          DATA 160
1     IF (NEL.LE.MAXEL) GO TO 2                                                DATA 170
      ISTOP=ISTOP+1                                                            DATA 180
      PRINT 25, MAXEL                                                          DATA 190
2     IF (NMAT.LE.MAXMAT) GO TO 3                                              DATA 200
      ISTOP=ISTOP+1                                                            DATA 210
      PRINT 26, MAXMAT                                                         DATA 220
3     IF (NSLC.LE.MAXSLC) GO TO 4                                              DATA 230
      ISTOP=ISTOP+1                                                            DATA 240
      PRINT 27, MAXSLC                                                         DATA 250
4     IF (ISTOP.EQ.0) GO TO 5                                                  DATA 260
      PRINT 28, ISTOP                                                          DATA 270
      STOP                                                                     DATA 280
C                                                                              DATA 290
5     READ 29, (E(I),PR(I),RO(I),TH(I),I=1,NMAT)                               DATA 300
      PRINT 30                                                                 DATA 310
      PRINT 31, (I,E(I),PR(I),RO(I),TH(I),I=1,NMAT)                            DATA 320
C                                                                              DATA 330
C     READ AND PRINT NODAL DATA (REF. 1)                                       DATA 340
      PRINT 32                                                                 DATA 350
      N=1                                                                      DATA 360
6     READ 33, M,KODE(M),X(M),Y(M),ULX(M),VLY(M)                               DATA 370
      IF (M-N) 7,10,8                                                          DATA 380
7     PRINT 34, M                                                              DATA 390
      PRINT 35, M,KODE(M),X(M),Y(M),ULX(M),VLY(M)                              DATA 400
      ISTOP=ISTOP+1                                                            DATA 410
      GO TO 6                                                                  DATA 420
8     DF=M+1-N                                                                 DATA 430
      RX=(X(M)-X(N-1))/DF                                                      DATA 440
      RY=(Y(M)-Y(N-1))/DF                                                      DATA 450
9     KODE(N)=KODE(N-1)                                                        DATA 460
      X(N)=X(N-1)+RX                                                           DATA 470
      Y(N)=Y(N-1)+RY                                                           DATA 480
      ULX(N)=0.0                                                               DATA 490
      VLY(N)=0.0                                                               DATA 500
10    PRINT 35, N,KODE(N),X(N),Y(N),ULX(N),VLY(N)                              
                                                                               DATA 510
      N=N+1                                                                    DATA 520
      IF (M-N) 11,10,9                                                         DATA 530
11    IF (N.LE.NNP) GO TO 6
```

```
C     READ AND PRINT ELEMENT PROPERTIES, TABLE 6-4                              DATA 540
      PRINT 36                                                                  DATA 550
      L=0                                                                       DATA 560
12    READ 38, M,(IE(M,I),I=1,5)                                                DATA 570
13    L=L+1                                                                     DATA 580
      IF (M-L) 14,16,15                                                         DATA 590
14    PRINT 37, M                                                               DATA 600
      PRINT 39, M,(IE(M,I),I=1,5)                                               DATA 610
      ISTOP=ISTOP+1                                                             DATA 620
      GO TO 12                                                                  DATA 630
15    IE(L,1)=IE(L-1,1)+1                                                       DATA 640
      IE(L,2)=IE(L-1,2)+1                                                       DATA 650
      IE(L,3)=IE(L-1,3)+1                                                       DATA 660
      IE(L,4)=IE(L-1,4)+1                                                       DATA 670
      IE(L,5)=IE(L-1,5)                                                         DATA 680
16    PRINT 39, L,(IE(L,I),I=1,5)                                               DATA 690
      IF (M-L) 17,17,13                                                         DATA 700
17    IF (NEL-L) 18,18,12                                                       DATA 710
18    CONTINUE                                                                  DATA 720
C                                                                               DATA 730
C     READ AND PRINT SURFACE LOADING(TRACTION) CARDS                            DATA 740
      IF (NSLC.EQ.0) GO TO 20                                                   DATA 750
      PRINT 40                                                                  DATA 760
      DO 19 L=1,NSLC                                                            DATA 770
      READ 41, ISC(L),JSC(L),SURTRX(L,1),SURTRX(L,2),SURTRY(L,1),SURTRY(DATA 780
     1L,2)                                                                      DATA 790
19    PRINT 42, ISC(L),JSC(L),SURTRX(L,1),SURTRX(L,2),SURTRY(L,1),SURTRYDATA 800
     1(L,2)                                                                     DATA 810
20    IF (ISTOP.EQ.0) GO TO 21                                                  DATA 820
      PRINT 43, ISTOP                                                           DATA 830
C                                                                               DATA 840
21    RETURN                                                                    DATA 850
C                                                                               DATA 860
22    FORMAT (615)                                                              DATA 870
23    FORMAT (35HOINPUT TABLE 1.. BASIC PARAMETERS  //5X,40H NUMBER OF NDATA 880
     1ODAL POINTS. . . . . .,15/5X,40H NUMBER OF ELEMENTS. . . . .DATA 890
     2 . . . . . .,15/5X,40H NUMBER OF DIFFERENT MATERIALS . . . . .,15/DATA 900
     35X,40H NUMBER OF SURFACE LOAD CARDS. . . . . .,15/5X,40H 1 = PLANEDATA 910
     4 STRAIN, 2 = PLANE STRESS. . .,15/5X,40H BODY FORCES(1 = IN -Y DIRDATA 920
     5EC., 0 = NONE),15)                                                        DATA 930
24    FORMAT (////33H TOO MANY NODAL POINTS, MAXIMUM =,15)                      DATA 940
25    FORMAT (////30H TOO MANY ELEMENTS, MAXIMUM = ,15)                         DATA 950
26    FORMAT (////30H TOO MANY MATERIALS, MAXIMUM =,15)                         DATA 960
27    FORMAT (////40H TOO MANY SURFACE LOAD CARDS, MAXIMUM = ,15)               DATA 970
28    FORMAT (////28H EXECUTION HALTED BECAUSE OF,15,13H FATAL ERRORS/)         DATA 980
                                                                                DATA 990
29    FORMAT (4E10.3)
30    FORMAT (36HOINPUT TABLE 2.. MATERIAL PROPERTIES//10H  MATERIAL,5X,DATA1000

     110HMODULUS OF,6X,9HPOISSON'S,7X,8HMATERIAL,7X,8HMATERIAL/4X,6HNUMBDATA1010
     2ER,5X,10HELASTICITY,8X,7H  RATIO,8X,7HDENSITY,6X,9HTHICKNESS)            DATA1020
                                                                                DATA1030
31    FORMAT (I10,4E15.4)
32    FORMAT (34H1INPUT TABLE 3.. NODAL POINT DATA //5X,5HNODAL,48X,7HX-DATA1040
     1DISP.,8X,7HY-DISP./5X,5HPOINT,6X,4HTYPE,14X,1HX,14X,1HY,8X,7HOR LODATA1050
     2AD,8X,7HOR LOAD)                                                          DATA1060
                                                                                DATA1070
33    FORMAT (215,4F10.1)
```

```
34     FORMAT (5X,17HERROR IN CARD NO.,I5/)                                    DATA1080
                                                                               DATA1090
35     FORMAT (2I10,4E15.4)                                                    DATA1100
36     FORMAT (34H1INPUT TABLE 4.. ELEMENT DATA      //11X,31HGLOBAL INDIC
      1ES OF ELEMENT NODES/3X,7HELEMENT,7X,1H1,7X,1H2,7X,1H3,7X,1H4,2X,8H
      2MATERIAL)                                                               DATA1120
37     FORMAT (5X,25HERROR IN ELEMENT CARD NO.,I5/)                            DATA1130
                                                                               DATA1140
38     FORMAT (6I5)                                                            DATA1150
39     FORMAT (I10,4I8,I10)                                                    DATA1160
40     FORMAT (37H1INPUT TABLE 5.. SURFACE LOADING DATA//17X,33HSURFACE LO
      1OAD INTENSITIES AT NODES/4X,6HNODE I,4X,6HNODE J,10X,2HXI,10X,2HXJ
      2,10X,2HYI,10X,2HYJ)                                                     DATA1180
                                                                               DATA1190
41     FORMAT (2I5,4E10.3)                                                     DATA1200
42     FORMAT (2I10,4E12.4)                                                    DATA1210
43     FORMAT (///45H ASSEMBLY AND SOLUTION WILL NOT BE PERFORMED.,I5,21H
      1  FATAL CARD ERRORS  )                                                  DATA1220
                                                                               DATA1230
       END
```

```
C     FINITE ELEMENT PROGRAM - SUBROUTINE ASSEMBL(ISTOP)                    ASML  10
      SUBROUTINE ASEMBL (ISTOP)                                             ASML  20
      COMMON NNP,NEL,NMAT,NSLC,NOPT,NBODY,MTYP,E(6),PR(6),RO(6),TH(6),IEASML  30
     1(800,5),X(800),Y(800),ULX(800),VLY(800),KODE(800),ISC(80),JSC(80),ASML  40
     2SURTRX(80,2),SURTRY(80,2)                                             ASML  50
      COMMON /ONE/ QK(10,10),Q(10),B(3,10),C(3,3),BT(3,6),XQ(5),YQ(5)       ASML  60
      COMMON /TWO/ IBAND,NEQ,R(1600),AK(1600,144)                           ASML  70
      DIMENSION LP(8)                                                       ASML  80
C                                                                           ASML  90
      REWIND 1                                                              ASML 100
C     INITIALIZE                                                            ASML 110
      ISTOP=0                                                               ASML 120
C     INITIALIZE PARTS OF MATRICES C AND BT                                 ASML 130
      BT(1,4)=0.0                                                           ASML 140
      BT(1,5)=0.0                                                           ASML 150
      BT(1,6)=0.0                                                           ASML 160
      BT(2,1)=0.0                                                           ASML 170
      BT(2,2)=0.0                                                           ASML 180
      BT(2,3)=0.0                                                           ASML 190
      C(1,3)=0.0                                                            ASML 200
      C(2,3)=0.0                                                            ASML 210
      C(3,1)=0.0                                                            ASML 220
      C(3,2)=0.0                                                            ASML 230
C                                                                           ASML 240
C     INITIALIZE OVERALL STIFFNESS MATRIX AK AND OVERALL LOAD VECTOR R      ASML 250
      DO 1 I=1,NEQ                                                          ASML 260
      R(I)=0.0                                                              ASML 270
      DO 1 J=1,IBAND                                                        ASML 280
1     AK(I,J)=0.0                                                           ASML 290
C                                                                           ASML 300
C     COMPUTE  ELEMENT  STIFFNESSES AND LOADS ONE BY ONE                    ASML 310
C                                                                           ASML 320
      DO 10 M=1,NEL                                                         ASML 330
      IF (IE(M,5).GT.0) GO TO 2                                             ASML 340
      ISTOP=ISTOP+1                                                         ASML 350
      GO TO 10                                                              ASML 360
2     CALL QUAD (M,AREA)                                                    ASML 370
      IF (AREA.GT.0.0) GO TO 3                                              ASML 380
      ISTOP=ISTOP+1                                                         ASML 390
      PRINT 19, M                                                           ASML 400
C                                                                           ASML 410
C     CONDENSE ELEMENT STIFF. FROM 10X10 TO 8X8, EQ.(5-64), AND ELEMENT     ASML 420
C     LOADS FROM 10X1 TO 8X1, EQ.(5-64D).   (REF.2)                         ASML 430
3     IF (IE(M,3).EQ.IE(M,4)) GO TO 7                                       ASML 440
      DO 6 J=1,2                                                            ASML 450
      IJ=10-J                                                               ASML 460
      IK=IJ+1                                                               ASML 470
      PIVOT=QK(IK,IK)                                                       ASML 480
      DO 5 K=1,IJ                                                           ASML 490
      F=QK(IK,K)/PIVOT                                                      ASML 500
                                                                            ASML 510
      QK(IK,K)=F                                                            ASML 520
      DO 4 I=K,IJ                                                           ASML 530
      QK(I,K)=QK(I,K)-F*QK(I,IK)
```

```
4       QK(K,I)=QK(I,K)                                                 ASML 540
5       Q(K)=Q(K)-QK(IK,K)*Q(IK)                                        ASML 550
6       Q(IK)=Q(IK)/PIVOT                                               ASML 560
C                                                                       ASML 570
C                                                                       ASML 580
C       STORE MULTIPLIERS,PIVOTS,CONDENSED LOADS, STRAIN-DISP. AND STRESS-ASML 590
C       MATRICES ON SCRATCH TAPE NO. 1 (TO BE USED LATER TO COMPUTE STRAINASML 600
C       STRESSES )                                                      ASML 610
7       WRITE (1) ((QK(I,J),J=1,10),I=9,10),Q(9),Q(10),((B(I,J),J=1,10),I=ASML 620
       11,3),((C(I,J),J=1,3),I=1,3),XQ(5),YQ(5)                         ASML 630
                                                                        ASML 640
C                                                                       ASML 650
C       ASSEMBLE STIFF.  AND  LOADS , DIRECT STIFF. METHOD, SEC. 6-5.   ASML 660
C                                                                       ASML 670
        LIM=8                                                           ASML 680
        IF (IE(M,3).EQ.IE(M,4)) LIM=6                                   ASML 690
        DO 8 I=2,LIM,2                                                  ASML 700
        IJ=I/2                                                          ASML 710
        LP(I-1)=2*IE(M,IJ)-1                                            ASML 720
8       LP(I)=2*IE(M,IJ)                                                ASML 730
        DO 9 LL=1,LIM                                                   ASML 740
        I=LP(LL)                                                        ASML 750
        R(I)=R(I)+Q(LL)                                                 ASML 760
        DO 9 MM=1,LIM                                                   ASML 770
        J=LP(MM)-I+1                                                    ASML 780
        IF (J.LE.0) GO TO 9                                             ASML 790
        AK(I,J)=AK(I,J)+QK(LL,MM)                                       ASML 800
9       CONTINUE                                                        ASML 810
10      CONTINUE                                                        ASML 820
C                                                                       ASML 830
C       ADD EXTERNALLY APPL. CONC. NODAL LOADS TO R                     ASML 840
        DO 12 N=1,NNP                                                   ASML 850
        IF (KODE(N).EQ.3) GO TO 12                                      ASML 860
        K=2*N                                                           ASML 870
        IF (KODE(N).EQ.1) GO TO 11                                      ASML 880
        R(K-1)=R(K-1)+ULX(N)                                            ASML 890
        IF (KODE(N).NE.0) GO TO 12                                      ASML 900
11      R(K)=R(K)+VLY(N)                                                ASML 910
12      CONTINUE                                                        ASML 920
C                                                                       ASML 930
C       CONVERT LINEARLY VARYING SURFACE TRACTIONS TO STATIC EQUIVALENTS, ASML 940
C       AND ADD TO OVERALL LOAD VECTOR  R, EQ.(5-61A).                  ASML 950
        IF (NSLC.EQ.0) GO TO 14                                         ASML 960
        DO 13 L=1,NSLC                                                  ASML 970
        I=ISC(L)                                                        ASML 980
        J=JSC(L)                                                        ASML 990
        II=2*I                                                          ASML1000
        JJ=2*J                                                          
                                                                        ASML1010
        DX=X(J)-X(I)                                                    ASML1020
        DY=Y(J)-Y(I)                                                    ASML1030
        EL=SQRT(DX*DX+DY*DY)                                            ASML1040
        PXI=SURTRX(L,1)*EL                                              ASML1050
        PXJ=SURTRX(L,2)*EL                                              ASML1060
        PYI=SURTRY(L,1)*EL                                              ASML1070
        PYJ=SURTRY(L,2)*EL
```

```
      R(II-1)=R(II-1)+PXI/3.0+PXJ/6.0                                   ASML1080
      R(JJ-1)=R(JJ-1)+PXI/6.0+PXJ/3.0                                   ASML1090
      R(II)=R(II)+PYI/3.0+PYJ/6.0                                       ASML1100
      R(JJ)=R(JJ)+PYI/6.0+PYJ/3.0                                       ASML1110
13    CONTINUE                                                          ASML1120
C                                                                       ASML1130
C     INTRODUCE KINEMATIC CONSTRAINTS (GEOMETRIC BOUNDARY CONDITIONS),  ASML1140
C     EQ.(6-18).          REF. 1.                                       ASML1150
C                                                                       ASML1160
14    DO 17 M=1,NNP                                                     ASML1170
      IF (KODE(M).GE.0.AND.KODE(M).LE.3) GO TO 15                       ASML1180
      ISTOP=ISTOP+1                                                     ASML1190
      GO TO 17                                                          ASML1200
15    IF (KODE(M).EQ.0) GO TO 17                                        ASML1210
      IF (KODE(M).EQ.2) GO TO 16                                        ASML1220
      CALL GEOMBC (ULX(M),2*M-1)                                        ASML1230
      IF (KODE(M).EQ.1) GO TO 17                                        ASML1240
16    CALL GEOMBC (VLY(M),2*M)                                          ASML1250
17    CONTINUE                                                          ASML1260
      END FILE 1                                                        ASML1270
      IF (ISTOP.EQ.0) GO TO 18                                          ASML1280
      PRINT 20, ISTOP                                                   ASML1290
18    RETURN                                                            ASML1300
C                                                                       ASML1310
19    FORMAT (/5X,17H AREA OF ELEMENT ,I5,14H  IS NEGATIVE /)           ASML1320
20    FORMAT (////42H SOLUTION WILL NOT BE PERFORMED BECAUSE OF,I5,15H  ASML1330
     1 DATA ERRORS /)                                                   ASML1340
      END                                                               ASML1350
```

```
C     FINITE ELEMENT PROGRAM - SUBROUTINE QUAD(M,TOTALA)                    QUAD  10
      SUBROUTINE QUAD (M,TOTALA)                                            QUAD  20
      COMMON NNP,NEL,NMAT,NSLC,NOPT,NBODY,MTYP,E(6),PR(6),RO(6),TH(6),IEQUAD  30
     1(800,5),X(800),Y(800),ULX(800),VLY(800),KODE(800),ISC(80),JSC(80),QUAD  40
     2SURTRX(80,2),SURTRY(80,2)                                             QUAD  50
      COMMON /ONE/ QK(10,10),Q(10),B(3,10),C(3,3),BT(3,6),XQ(5),YQ(5)       QUAD  60
      COMMON /TWO/ IBAND,NEQ,R(1600),AK(1600,144)                           QUAD  70
C                                                                           QUAD  80
      I=IE(M,1)                                                             QUAD  90
      J=IE(M,2)                                                             QUAD 100
      K=IE(M,3)                                                             QUAD 110
      L=IE(M,4)                                                             QUAD 120
      MTYP=IE(M,5)                                                          QUAD 130
      TOTALA=0.0                                                            QUAD 140
C                                                                           QUAD 150
C     CONSTRUCT STRESS-STRAIN MATRIX C,EQ.(3-16C).  FOR PLANE STRAIN        QUAD 160
C     NOPT=1, AND FOR PLANE STRESS  NOPT=2.  PRESENT  CODE  IS FOR          QUAD 170
C     ISOTROPIC MATERIALS                                                   QUAD 180
      IF (NMAT.EQ.1.AND.M.GT.1) GO TO 2                                     QUAD 190
      IF (NOPT.EQ.2) GO TO 1                                                QUAD 200
      CF=E(MTYP)/((1.0+PR(MTYP))*(1.0-2.0*PR(MTYP)))                       QUAD 210
      C(1,1)=CF*(1.0-PR(MTYP))                                              QUAD 220
      C(1,2)=CF*PR(MTYP)                                                    QUAD 230
      C(2,1)=C(1,2)                                                         QUAD 240
      C(2,2)=C(1,1)                                                         QUAD 250
      C(3,3)=CF*(1.0-2.0*PR(MTYP))/2.0                                      QUAD 260
      GO TO 2                                                               QUAD 270
1     CF=E(MTYP)/(1.0-PR(MTYP)*PR(MTYP))                                    QUAD 280
      C(1,1)=CF                                                             QUAD 290
      C(1,2)=PR(MTYP)*CF                                                    QUAD 300
      C(2,1)=C(1,2)                                                         QUAD 310
      C(2,2)=CF                                                             QUAD 320
      C(3,3)=CF*(1.0-PR(MTYP))/2.0                                          QUAD 330
2     LIM=4                                                                 QUAD 340
      IF (K.EQ.L) LIM=3                                                     QUAD 350
      XQ(5)=0.0                                                             QUAD 360
      YQ(5)=0.0                                                             QUAD 370
      DO 3 N=1,LIM                                                          QUAD 380
      NN=IE(M,N)                                                            QUAD 390
      XQ(N)=X(NN)                                                           QUAD 400
      YQ(N)=Y(NN)                                                           QUAD 410
      XQ(5)=XQ(5)+X(NN)/FLOAT(LIM)                                          QUAD 420
3     YQ(5)=YQ(5)+Y(NN)/FLOAT(LIM)                                          QUAD 430
C                                                                           QUAD 440
C     INITIALIZE QUAD. STIFFNESS, LOAD VECTOR AND STRAIN-DISPLACEMENT VEQUAD 450
      DO 5 II=1,10                                                          QUAD 460
      Q(II)=0.0                                                             QUAD 470
      DO 4 JJ=1,10                                                          QUAD 480
4     QK(II,JJ)=0.0                                                         QUAD 490
      DO 5 JJ=1,3                                                           QUAD 500
5     B(JJ,II)=0.0                                                          QUAD 510
      IF (K.NE.L) GO TO 6                                                   QUAD 520
      CALL CST (1,2,3,TOTALA)                                               QUAD 530
      GO TO 7                                                               QUAD 540
```

```
6       CALL CST (1,2,5,AREA)                                    QUAD 550
        TOTALA=TOTALA+AREA                                       QUAD 560
        CALL CST (2,3,5,AREA)                                    QUAD 570
        TOTALA=TOTALA+AREA                                       QUAD 580
        CALL CST (3,4,5,AREA)                                    QUAD 590
        TOTALA=TOTALA+AREA                                       QUAD 600
        CALL CST (4,1,5,AREA)                                    QUAD 610
        TOTALA=TOTALA+AREA                                       QUAD 620
/       RETURN                                                   QUAD 630
        END                                                      QUAD 640
```

```
C     FINITE ELEMENT PROGRAM - SUBROUTINE CST(I,J,K,AREA)                    CST   10
      SUBROUTINE CST (I,J,K,AREA)                                            CST   20
      COMMON NNP,NEL,NMAT,NSLC,NOPT,NBODY,MTYP,E(6),PR(6),RO(6),TH(6),ICST   30
     1(800,5),X(800),Y(800),ULX(800),VLY(800),KODE(800),ISC(80),JSC(80),CST   40
     2SURTRX(80,2),SURTRY(80,2)                                              CST   50
      COMMON /ONE/ QK(10,10),Q(10),B(3,10),C(3,3),BT(3,6),XQ(5),YQ(5)        CST   60
      COMMON /TWO/ IBAND,NEQ,R(1600),AK(1600,144)                            CST   70
      DIMENSION CB(3,6), LC(6), LT(3), TK(6,6)                               CST   80
                                                                             CST   90
C                                                                            CST  100
      LT(1)=I                                                                CST  110
      LT(2)=J                                                                CST  120
      LT(3)=K                                                                CST  130
C                                                                            CST  140
C     COMPUTE  STRAIN-DISPLACEMENT MATRIX B FOR TRIANGLE, EQ. (5-35A)        CST  150
      BT(1,1)=YQ(J)-YQ(K)                                                    CST  160
      BT(1,2)=YQ(K)-YQ(I)                                                    CST  170
      BT(1,3)=YQ(I)-YQ(J)                                                    CST  180
      BT(2,4)=XQ(K)-XQ(J)                                                    CST  190
      BT(2,5)=XQ(I)-XQ(K)                                                    CST  200
      BT(2,6)=XQ(J)-XQ(I)                                                    CST  210
      BT(3,1)=BT(2,4)                                                        CST  220
      BT(3,2)=BT(2,5)                                                        CST  230
      BT(3,3)=BT(2,6)                                                        CST  240
      BT(3,4)=BT(1,1)                                                        CST  250
      BT(3,5)=BT(1,2)                                                        CST  260
      BT(3,6)=BT(1,3)                                                        CST  270
      AREA=(BT(2,4)*BT(1,3)-BT(2,6)*BT(1,1))/2.0                             CST  280
C                                                                            CST  290
C     COMPUTE   C*B                                                          CST  300
      DO 1 II=1,3                                                            CST  310
      DO 1 JJ=1,6                                                            CST  320
      CB(II,JJ)=0.0                                                          CST  330
      DO 1 KK=1,3                                                            CST  340
1     CB(II,JJ)=CB(II,JJ)+C(II,KK)*BT(KK,JJ)                                 CST  350
C                                                                            CST  360
C     COMPUTE (B**T)*C*B, EQ.(5-45A)                                         CST  370
      DO 2 II=1,6                                                            CST  380
      DO 2 JJ=1,6                                                            CST  390
      TK(II,JJ)=0.0                                                          CST  400
      DO 2 KK=1,3                                                            CST  410
2     TK(II,JJ)=TK(II,JJ)+BT(KK,II)*CB(KK,JJ)                                CST  420
C                                                                            CST  430
C     ADD TRIANGLE STIFNESS TO QUADRILATERAL STIFFNESS, EX.(6-2).            CST  440
C     ADD TRIANGLE STRAIN-DISPLACEMENT MATRIX TO QUADRILATERAL  STRAIN-      CST  450
C     DISPLACEMENT MATRIX                                                    CST  460
      DO 3 II=1,3                                                            CST  470
      LC(II)=2*LT(II)-1                                                      CST  480
3     LC(II+3)=2*LT(II)                                                      CST  490
      DO 5 II=1,6                                                            CST  500
      LL=LC(II)                                                              CST  510
      IF (AREA.EQ.0.0) GO TO 7                                               CST  520
      FK=1.0/(4.0*AREA)                                                      CST  530
      FB=2.0*FK
```

```
      DO 4 JJ=1,6                                                       CST  540
      MM=LC(JJ)                                                         CST  550
4     QK(LL,MM)=QK(LL,MM)+TK(II,JJ)*TH(MTYP)*FK                         CST  560
      DO 5 JJ=1,3                                                       CST  570
5     B(JJ,LL)=B(JJ,LL)+BT(JJ,II)*FB                                    CST  580
C                                                                       CST  590
C     DEVELOP  BODY  FORCE  VECTOR,  EQ.(5-61B)                         CST  600
      IF (NBODY.EQ.0) GO TO 7                                           CST  610
      TBODYF=AREA*RO(MTYP)*TH(MTYP)                                     CST  620
      BODYF=-TBODYF/3.0                                                 CST  630
      DO 6 II=1,3                                                       CST  640
      JJ=2*LT(II)                                                       CST  650
6     Q(JJ)=Q(JJ)+BODYF                                                 CST  660
7     RETURN                                                            CST  670
      END                                                               CST  680
```

```
C     FINITE ELEMENT PROGRAM - SUBROUTINE STRESS                                STRS  10
      SUBROUTINE STRESS                                                         STRS  20
      COMMON NNP,NEL,NMAT,NSLC,NOPT,NBODY,MTYP,E(6),PR(6),RO(6),TH(6),IESTRS  30
     1(800,5),X(800),Y(800),ULX(800),VLY(800),KODE(800),ISC(80),JSC(80),STRS  40
     2SURTRX(80,2),SURTRY(80,2)                                                 STRS  50
      COMMON /ONE/ QK(10,10),Q(10),B(3,10),C(3,3),BT(3,6),XQ(5),YQ(5)           STRS  60
      COMMON /TWO/ IBAND,NEQ,R(1600),AK(1600,144)                               STRS  70
      DIMENSION SIG(6), SIGI(3)                                                 STRS  80
C                                                                               STRS  90
      REWIND 1                                                                  STRS 100
      PRINT 10                                                                  STRS 110
      NOLINE=47                                                                 STRS 120
C                                                                               STRS 130
C     RETRIEVE MULTIPLIERS, PIVOTS, MATRICES B AND C, AND CENTROIDAL COOSTRS 140
C     FOR ELEMENT                                                               STRS 150
      DO 9 M=1,NEL                                                              STRS 160
      READ (1) ((QK(I,J),J=1,10),I=1,2),Q(9),Q(10),((B(I,J),J=1,10),I=1,STRS 170
     13),((C(I,J),J=1,3),I=1,3),XC,YC                                           STRS 180
C                                                                               STRS 190
C     SELECT NODAL DISPLACEMENTS FOR THE ELEMENT                                STRS 200
      LIM=4                                                                     STRS 210
      IF (IE(M,3).EQ.IE(M,4)) LIM=3                                             STRS 220
      DO 1 I=1,LIM                                                              STRS 230
      II=2*I                                                                    STRS 240
      JJ=2*IE(M,I)                                                              STRS 250
      Q(II-1)=R(JJ-1)                                                           STRS 260
1     Q(II)=R(JJ)                                                               STRS 270
C                                                                               STRS 280
C     RECOVER CONDENSED DISPLACEMENTS FOR THE QUADRILATERAL, EQ. (5-64G)STRS 290
      IF (LIM.EQ.3) GO TO 3                                                     STRS 300
      DO 2 K=1,2                                                                STRS 310
      JK=K+8                                                                    STRS 320
      IK=JK-1                                                                   STRS 330
      DO 2 L=1,IK                                                               STRS 340
2     Q(JK)=Q(JK)-QK(K,L)*Q(L)                                                  STRS 350
C                                                                               STRS 360
C     COMPUTE ELEMENT STRAINS, EQ.(5-35A)                                       STRS 370
      LIM=10                                                                    STRS 380
      FAC=0.25                                                                  STRS 390
      GO TO 4                                                                   STRS 400
3     LIM=6                                                                     STRS 410
      FAC=1.0                                                                   STRS 420
4     DO 5 I=1,3                                                                STRS 430
      E(I)=0.0                                                                  STRS 440
      DO 5 J=1,LIM                                                              STRS 450
5     E(I)=E(I)+B(I,J)*Q(J)*FAC                                                 STRS 460
C                                                                               STRS 470
C     COMPUTE ELEMENT STRESSES  , EQ.(5-35B)                                    STRS 480
      DO 6 I=1,3                                                                STRS 490
      SIG(I)=0.0                                                                STRS 500
      DO 6 J=1,3                                                                STRS 510
6     SIG(I)=SIG(I)+C(I,J)*E(J)                                                 STRS 520
C     COMPUTE PRINCIPAL STRESSES AND THE ANGLE WITH THE POSITIVE X AXIS STRS 530
      SP=(SIG(1)+SIG(2))/2.0                                                    STRS 540
```

```
      SM=(SIG(1)-SIG(2))/2.0                                                    STRS 550
      DS=SQRT(SM*SM+SIG(3)*SIG(3))                                              STRS 560
      SIG(4)=SP+DS                                                              STRS 570
      SIG(5)=SP-DS                                                              STRS 580
      SIG(6)=0.0                                                                STRS 590
      IF (SIG(3).NE.0.0.AND.SM.NE.0.0) SIG(6)=28.648*ATAN2(SIG(3),SM)           STRS 600
C     PRINT STRESSES, 50 LINES PER PAGE                                         STRS 610
      IF (NOLINE.GT.0) GO TO 7                                                  STRS 620
      PRINT 11                                                                  STRS 630
      NOLINE=49                                                                 STRS 640
7     NOLINE=NOLINE-1                                                           STRS 650
C                                                                               STRS 660
C     RE-USE STORAGE SPACE IN UN-NEEDED ARRAYS FOR STRESSES & CENTROIDS         STRS 670
C                                                                               STRS 680
      DO 8 JJJ=1,5                                                              STRS 690
      ISUB1=MOD(JJJ+1,2)*120+M                                                  STRS 700
      ISUB2=(JJJ+1)/2                                                           STRS 710
8     AK(ISUB1,ISUB2)=SIG(JJJ)                                                  STRS 720
      STX=SIG(1)/144.                                                           STRS 730
      STY=SIG(2)/144.                                                           STRS 740
      STXY=SIG(3)/144.                                                          STRS 750
      STMAX=SIG(4)/144.                                                         STRS 760
      STMIN=SIG(5)/144.                                                         STRS 770
9     WRITE (6,12) M,XC,YC,STX,STY,STXY,STMAX,STMIN,SIG(6)                      STRS 780
      END FILE 1                                                                STRS 790
      RETURN                                                                    STRS 800
C                                                                               STRS 810
10    FORMAT (47H1OUTPUT TABLE 2.. STRESSES AT ELEMENT CENTROIDS//1X,7HESTRS 820
     1LEMENT,2X,1HX,4X,1HY,3X,8HSIGMA(X),2X,8HSIGMA(Y),2X,8HTAU(X,Y),2X,STRS 830
     28HSIGMA(1),2X,8HSIGMA(2),2X,5HANGLE)                                      STRS 840
11    FORMAT (1H1,7HELEMENT,2X,1HX,4X,1HY,3X,8HSIGMA(X),2X,8HSIGMA(Y),2XSTRS 850
     1,8HTAU(X,Y),2X,8HSIGMA(1),2X,8HSIGMA(2),2X,5HANGLE)                       STRS 860
12    FORMAT (I4,2F7.2,1P6E10.2)                                                STRS 870
      END                                                                       STRS 880
```

```
C     FINITE ELEMENT PROGRAM - SUBROUTINE GEOMBC(U,N)                    GEOM  10
      SUBROUTINE GEOMBC (U,N)                                            GEOM  20
      COMMON /TWO/ IBAND,NEQ,R(1600),AK(1600,144)                        GEOM  30
C     THIS SUBROUTINE MODIFIES THE ASSEMBLAGE STIFFNESS AND LOADS FOR THGEOM  40
C     PRESCRIBED DISPLACEMENT U AT DEGREE OF FREEDOM N, EQ.(6-18B). (REFGEOM  50
      DO 2 M=2,IBAND                                                     GEOM  60
      K=N-M+1                                                            GEOM  70
      IF (K.LE.0) GO TO 1                                                GEOM  80
      R(K)=R(K)-AK(K,M)*U                                                GEOM  90
      AK(K,M)=0.0                                                        GEOM 100
1     K=N+M-1                                                            GEOM 110
      IF (K.GT.NEQ) GO TO 2                                              GEOM 120
      R(K)=R(K)-AK(N,M)*U                                                GEOM 130
      AK(N,M)=0.0                                                        GEOM 140
2     CONTINUE                                                           GEOM 150
      AK(N,1)=1.0                                                        GEOM 160
      R(N)=U                                                             GEOM 170
      RETURN                                                             GEOM 180
      END                                                                GEOM 190
```

```
C     FINITE ELEMENT PROGRAM - SUBROUTINE BANSOL                          BANS  10
      SUBROUTINE BANSOL (KKK,AK,R,NEQ,IBAND,NDIM,MDIM)                    BANS  20
C     SYMMETRIC BAND MATRIX EQUATION SOLVER.  (REF. 2)                    BANS  30
C                                                                         BANS  40
C     KKK = 1 TRIANGULARIZES THE BAND MATRIX AK, EQ. (2-2)                BANS  50
C     KKK = 2 SOLVES FOR RIGHT HAND SIDE R, SOLUTION RETURNS IN R, EQ.(2BANS  60
C                                                                         BANS  70
      DIMENSION AK(1600,144), R(1)                                        BANS  80
      NRS=NEQ-1                                                           BANS  90
      NR=NEQ                                                              BANS 100
      IF (KKK.EQ.2) GO TO 3                                               BANS 110
      DO 2 N=1,NRS                                                        BANS 120
      M=N-1                                                               BANS 130
      MR=MINO(IBAND,NR-M)                                                 BANS 140
      PIVOT=AK(N,1)                                                       BANS 150
      DO 2 L=2,MR                                                         BANS 160
      CP=AK(N,L)/PIVOT                                                    BANS 170
      I=M+L                                                               BANS 180
      J=0                                                                 BANS 190
      DO 1 K=L,MR                                                         BANS 200
      J=J+1                                                               BANS 210
1     AK(I,J)=AK(I,J)-CP*AK(N,K)                                          BANS 220
2     AK(N,L)=CP                                                          BANS 230
      GO TO 6                                                             BANS 240
3     DO 4 N=1,NRS                                                        BANS 250
      M=N-1                                                               BANS 260
      MR=MINO(IBAND,NR-M)                                                 BANS 270
      CP=R(N)                                                             BANS 280
      R(N)=CP/AK(N,1)                                                     BANS 290
      DO 4 L=2,MR                                                         BANS 300
      I=M+L                                                               BANS 310
4     R(I)=R(I)-AK(N,L)*CP                                                BANS 320
      R(NR)=R(NR)/AK(NR,1)                                                BANS 330
      DO 5 I=1,NRS                                                        BANS 340
      N=NR-I                                                              BANS 350
      M=N-1                                                               BANS 360
      MR=MINO(IBAND,NR-M)                                                 BANS 370
      DO 5 K=2,MR                                                         BANS 380
      L=M+K                                                               BANS 390
C     STORE COMPUTED DISPLACEMENTS IN LOAD VECTOR R                       BANS 400
5     R(N)=R(N)-AK(N,K)*R(L)                                              BANS 410
6     RETURN                                                              BANS 420
      END                                                                 BANS 430
```

Appendix C-2

The Mesh-plot Program

1. Input Data

a) Option card:

FORMAT (I1)

Col.	Symbol	Definition
1	OPTAX	Plotting option and option for end of plot = 0, if the Y-coordinate is limited by the height of the plotter = 1, if the X-coordinate is limited by the height of the plotter = 9, to indicate the end of the plot

b) Basic parameters - card 1:

FORMAT (3F7.1,3F5.3,2I4,F10.3)

Col.	Symbol	Definition
1-7	XTIC	Distance between tic marks on the plot axes in inches
8-14	XLNG	Length of the X-axis in inches
15-21	YLNG	Length of the Y-axis in inches
22-26	SIZFAC	Scaling factor of the mesh
27-31	H1	Height of the element numbers to be printed in inches
32-36	H2	Height of the nodal point numbers to be printed in inches
37-40	CTR	Counter variable = 1
41-44	NTR	Second counter variable =1

Col.	Symbol	Definition
45-54	XMAX9	Maximum extent in X-direction - used only when OPTAX = 1

c) Basic parameters - card 2:

FORMAT (2I4,I2)

Col.	Symbol	Definition
1-4	NOELE	Number of elements
5-8	NOSND	Number of nodal points
9-10	NTEST	= 0, does not test safety factor

d) Nodal point data:

FORMAT (I5,5X,2F10.1)

Col.	Symbol	Definition
1-5	N	Nodal point number
6-10		Blank
11-20	XX(N)	X-coordinate of nodes
21-30	YY(N)	Y-coordinate of nodes

e) Element data:

FORMAT (5I5)

Col.	Symbol	Definition
1-5	N	Element numbers
6-10	IE(N,1)	
11-15	IE(N,2)	Node numbers of corner nodes of
16-20	IE(N,3)	the number n element
21-25	IE(N,4)	

```
C     MESH-PLOT PROGRAM                                                      MSPL  10
      DIMENSION DX(4), DY(4), DXX(12), DYY(12), F(800), XX(800), YY(800)MSPL  20
                                                                             MSPL  30
     1, IE(800,4)
      COMMON XORD(800),YORD(800),NDI(800),NDJ(800),NDK(800),NTR,NDL(800)MSPL  40
                                                                             MSPL  50
     1,X10,Y10
      COMMON /OPTA/ OPTAX                                                    MSPL  60
      INTEGER CTR,OPTAX                                                      MSPL  70
      CALL PLOTS (0,0,50)                                                    MSPL  80
      CALL PLOT (5.,.5,-3)                                                   MSPL  90
      ERROR=0.                                                               MSPL 100
      READ (5,31) OPTAX                                                      MSPL 110
      IF (OPTAX.EQ.0) GO TO 1                                                MSPL 120
      READ (5,32) XTIC,YLNG,XLNG,SIZFAC,H1,H2,CTR,NTR,XMAX9                  MSPL 130
      GO TO 2                                                                MSPL 140
1     CONTINUE                                                               MSPL 150
      READ (5,32) XTIC,XLNG,YLNG,SIZFAC,H1,H2,CTR,NTR                        MSPL 160
                                                                             MSPL 170
2     CONTINUE                                                               MSPL 180
      IICTR=1                                                                MSPL 190
      NTR=1                                                                  MSPL 200
      IFLAG=0                                                                MSPL 210
      IF (CTR.EQ.0) GO TO 3                                                  MSPL 220
      IICTR=CTR                                                              MSPL 230
      CTR=CTR-1                                                              MSPL 240
      ICTR=CTR                                                               MSPL 250
      IFLAG=1                                                                MSPL 260
3     CONTINUE                                                               MSPL 270
      ISKP=0                                                                 MSPL 280
      IF (H1.LE.0.OR.H2.LE.0) ISKP=1                                         MSPL 290
C      SIZFAC IS THE SCALING FACTOR OF THE MESH                              MSPL 300
C      H1 IS THE HEIGTH OF THE ELEMENT NUMBER TO BE ORINTED BEFORE FACTOMSPL 310
C      H2 IS THE HEIGTH OF THE NODAL POINT NUMBER OR THE SAFETY FACTOR NMSPL 320
C      NBER TO BE PRINTED BEFORE FACTORING                                   MSPL 330
C      IF NTEST = 0, THE NODAL POINT NUMBER WILL BE PRINTED                  MSPL 340
C      IF NTEST = 1, THE SAFETY FACTOR WILL BE PRINTED                       MSPL 350
C      XTIC IS THE DISTANCE BETWEEN THE TIC MARKS IN INCHES                  MSPL 360
C       XLNG IS THE LENGTH OF THE X AXIS INCHES                              MSPL 370
C      YXLNG IS THE LENGTH OF THE Y AXIS INCHES                              MSPL 380
C     CHOOSING COORDINATES                                                   MSPL 390
C          IF OPTAX=0 THE Y COORDINATE IS LIMITED BY THE HIGHT OF THE       MSPL 400
C         PLOTTER.  IF OPTAX=1 THE X COORDINATE IS LIMIATED.                MSPL 410
C      IF H1 OR '2 ARE 0 OR NEGATIVE THE NODAL AND ELEMENT NUMBERS WILL      MSPL 420
C          NOT BE PRINTED                                                   MSPL 430
C          IF A BLOW UP OF A SECTION IS WANTED CTR IS THE FIRST NODE IN      MSPL 440
C          THAT SECTION. IF THE NODES ARE NOT SEQUENCED  NSEQ MUST BE       MSPL 450
C          SPECIFIED AS 1.                                                  MSPL 460
C     ONE (1) BLANK CARD IS REQUIRED AFTER THE NODAL POINT CARDS AND ALSMSPL 470
C     AFTER THE ELEMENT CARDS....                                           MSPL 480
      WRITE (6,33) SIZFAC                                                    MSPL 490
      XLNG=XLNG*SIZFAC                                                       MSPL 500
      XLS=XTIC*SIZFAC                                                        MSPL 510
      YLNG=YLNG*SIZFAC                                                       MSPL 520
      READ (5,34) NOELE,NOSND,NTEST                                          MSPL 530
      M=1                                                                    MSPL 540
4     READ (5,35) N,XX(N),YY(N)
```

```
      IF (N-M) 5,8,6                                                    MSPL 550
5     GO TO 4                                                           MSPL 560
6     DF=N+1-M                                                          MSPL 570
      RX=(XX(N)-XX(M-1))/DF                                             MSPL 580
      RY=(YY(N)-YY(M-1))/DF                                             MSPL 590
7     XX(M)=XX(M-1)+RX                                                  MSPL 600
      YY(M)=YY(M-1)+RY                                                  MSPL 610
8     M=M+1                                                             MSPL 620
      IF (N-M) 9,8,7                                                    MSPL 630
9     IF (M.LE.NOSND) GO TO 4                                           MSPL 640
      DO 10 N=1,NOSND                                                   MSPL 650
      XORD(N)=XX(N)                                                     MSPL 660
      YORD(N)=YY(N)                                                     MSPL 670
10    CONTINUE                                                          MSPL 680
      IF (OPTAX.EQ.0) GO TO 12                                          MSPL 690
      DO 11 IJ=IICTR,NOSND                                              MSPL 700
      AAA=XORD(IJ)                                                      MSPL 710
      XORD(IJ)=YORD(IJ)                                                 MSPL 720
      YORD(IJ)=XMAX9-AAA                                                MSPL 730
11    CONTINUE                                                          MSPL 740
12    CONTINUE                                                          MSPL 750
      CTR=0                                                             MSPL 760
      IF (IFLAG.EQ.1) CTR=ICTR                                          MSPL 770
      LL=0                                                              MSPL 780
13    READ (5,36) N,(IE(N,I),I=1,4)                                     MSPL 790
14    LL=LL+1                                                           MSPL 800
      IF (N-LL) 15,17,16                                                MSPL 810
15    GO TO 13                                                          MSPL 820
16    IE(LL,1)=IE(LL-1,1)+1                                             MSPL 830
      IE(LL,2)=IE(LL-1,2)+1                                             MSPL 840
      IE(LL,3)=IE(LL-1,3)+1                                             MSPL 850
      IE(LL,4)=IE(LL-1,4)+1                                             MSPL 860
17    NDI(LL)=IE(LL,1)                                                  MSPL 870
      NDJ(LL)=IE(LL,2)                                                  MSPL 880
      NDK(LL)=IE(LL,3)                                                  MSPL 890
      NDL(LL)=IE(LL,4)                                                  MSPL 900
      IF (N-LL) 18,18,14                                                MSPL 910
18    IF (NOELE-LL) 19,19,13                                            MSPL 920
19    CONTINUE                                                          MSPL 930
      CALL SAXIS (0.,0.,25HHORIZONTAL EXTENT IN FEET,-25,XLNG,0.,0.,XTICMSPL 940
     1,XLS,0.)                                                          MSPL 950
      CALL SAXIS (0.,0.,23HVERTICAL EXTENT IN FEET,23,YLNG,90.,0.,XTIC,XMSPL 960
     1LS,0.)                                                            MSPL 970
      CALL SYMBOL (-1.,3.,.25,12HELEMENT PLOT,90.,12)                   MSPL 980
      CALL PLOT (0.,0.,-3)                                              MSPL 990
      DO 25 M=NTR,NOELE                                                 MSPL1000

      I=NDI(M)                                                          MSPL1010
      J=NDJ(M)                                                          MSPL1020
      K=NDK(M)                                                          MSPL1030
      L=NDL(M)                                                          MSPL1040
      DX(1)=XORD(I)                                                     MSPL1050
      DY(1)=YORD(I)                                                     MSPL1060
      DX(2)=XORD(J)                                                     MSPL1070
      DY(2)=YORD(J)                                                     MSPL1080
```

```
      DX(3)=XORD(K)                                                    MSPL1090
      DY(3)=YORD(K)                                                    MSPL1100
      DX(4)=XORD(L)                                                    MSPL1110
      DY(4)=YORD(L)                                                    MSPL1120
      DO 20 IJ=1,4                                                     MSPL1130
      DX(IJ)=DX(IJ)*SIZFAC                                             MSPL1140
      DY(IJ)=DY(IJ)*SIZFAC                                             MSPL1150
20    CONTINUE                                                         MSPL1160
      CALL PLOT (DX(1),DY(1),3)                                        MSPL1170
      DO 21 L=1,4                                                      MSPL1180
21    CALL PLOT (DX(L),DY(L),2)                                        MSPL1190
      CALL PLOT (DX(1),DY(1),2)                                        MSPL1200
      IF (DX(4).EQ.DX(3).AND.DY(4).EQ.DY(3)) GO TO 22                  MSPL1210
      X10=(DX(2)+DX(1))/2                                              MSPL1220
      Y10=(DY(3)+DY(2))/2                                              MSPL1230
      GO TO 23                                                         MSPL1240
22    DX(4)=DX(1)                                                      MSPL1250
      DY(4)=DY(1)                                                      MSPL1260
      II=I                                                             MSPL1270
      JJ=J                                                             MSPL1280
      KK=K                                                             MSPL1290
      DXX(1)=XORD(II)                                                  MSPL1300
      DYY(1)=YORD(II)                                                  MSPL1310
      DXX(2)=XORD(JJ)                                                  MSPL1320
      DYY(2)=YORD(JJ)                                                  MSPL1330
      DXX(3)=XORD(KK)                                                  MSPL1340
      DYY(3)=YORD(KK)                                                  MSPL1350
      DXX(4)=.5*(DXX(1)+DXX(2))                                        MSPL1360
      DYY(4)=.5*(DYY(1)+DYY(2))                                        MSPL1370
      DXX(5)=.5*(DXX(2)+DXX(3))                                        MSPL1380
      DYY(5)=.5*(DYY(2)+DYY(3))                                        MSPL1390
      DXX(6)=.5*(DXX(3)+DXX(1))                                        MSPL1400
      DYY(6)=.5*(DYY(3)+DYY(1))                                        MSPL1410
      DXX(7)=.5*(DXX(4)+DXX(5))                                        MSPL1420
      DYY(7)=.5*(DYY(4)+DYY(5))                                        MSPL1430
      DXX(8)=.5*(DXX(5)+DXX(6))                                        MSPL1440
      DYY(8)=.5*(DYY(5)+DYY(6))                                        MSPL1450
      DXX(9)=.5*(DXX(6)+DXX(4))                                        MSPL1460
      DYY(9)=.5*(DYY(6)+DYY(4))                                        MSPL1470
      DXX(10)=.5*(DXX(7)+DXX(8))                                       MSPL1480
      DYY(10)=.5*(DYY(7)+DYY(8))                                       MSPL1490
      DXX(11)=.5*(DXX(8)+DXX(9))                                       MSPL1500

      DYY(11)=.5*(DYY(8)+DYY(9))                                       MSPL1510
      DXX(12)=.5*(DXX(9)+DXX(7))                                       MSPL1520
      DYY(12)=.5*(DYY(9)+DYY(7))                                       MSPL1530
      X10=DXX(10)*SIZFAC                                               MSPL1540
      Y10=DYY(10)*SIZFAC                                               MSPL1550
23    XM=M                                                             MSPL1560
      IF (ISKP.EQ.1) GO TO 24                                          MSPL1570
      CALL NUMBER (X10,Y10,H1,XM,0.,-1)                                MSPL1580
24    CONTINUE                                                         MSPL1590
25    CONTINUE                                                         MSPL1600
      IF (NTEST.EQ.0) GO TO 27                                         MSPL1610
      WRITE (6,37)                                                     MSPL1620
```

```
      DO 26 I=IICTR,NOSND                                              MSPL1630
      X=XORD(I)-.5                                                     MSPL1640
      Y=YORD(I)+.5                                                     MSPL1650
      READ (5,38) I,F(I)                                               MSPL1660
      XI=F(I)                                                          MSPL1670
      X=X*SIZFAC                                                       MSPL1680
      Y=Y*SIZFAC                                                       MSPL1690
      CALL NUMBER (X,Y,H2,XI,0.,1)                                     MSPL1700
      WRITE (6,39) I,F(I)                                              MSPL1710
26    CONTINUE                                                         MSPL1720
      GO TO 30                                                         MSPL1730
27    CONTINUE                                                         MSPL1740
      IF (ISKP.EQ.1) GO TO 29                                          MSPL1750
      DO 28 I=IICTR,NOSND                                              MSPL1760
      X=XORD(I)-.25                                                    MSPL1770
      Y=YORD(I)+.05                                                    MSPL1780
      ZI=I                                                             MSPL1790
      X=X*SIZFAC                                                       MSPL1800
      Y=Y*SIZFAC                                                       MSPL1810
      CALL NUMBER (X,Y,H2,ZI,0.,-1)                                    MSPL1820
28    CONTINUE                                                         MSPL1830
29    CONTINUE                                                         MSPL1840
C                                                                      MSPL1850
C                                                                      MSPL1860
30    CALL PLOT (XLNG+5.0,0.0,999)                                     MSPL1870
C                                                                      MSPL1880
C                                                                      MSPL1890
      STOP                                                             MSPL1900
C                                                                      MSPL1910
31    FORMAT (I1)                                                      MSPL1920
32    FORMAT (3F7.1,3F5.3,2I4,F10.3)                                   MSPL1930
33    FORMAT (9H0 SIZFAC=,F10.4)                                       MSPL1940
34    FORMAT (2I4,I2)                                                  MSPL1950
35    FORMAT (I5,5X,2F10.1)                                            MSPL1960
36    FORMAT (5I5)                                                     MSPL1970
37    FORMAT (1H ,29H       N.P.       SAFETY FACTOR)                  MSPL1980
38    FORMAT (I10,F6.1)                                                MSPL1990
39    FORMAT (I11,8X,F6.1)                                             MSPL2000
                                                                       MSPL2010
      END
```

Appendix C-3

The Stress-Plot Program

1. Input Data

a) Title card:

FORMAT (I5,1X,18A4)

Col.	Symbol	Definition
1-5	NPROB	Problem number
6		Blank
7-78	TITLE	Title of plot

b) Basic parameters:

FORMAT (2I5,2F10.7)

Col.	Symbol	Definition
1-5	NOSEL	Number of elements
6-10	NBNDY	Number of boundary points
11-20	SC	Scaling factor for the plot
21-30	SCALE	Scaling factor for the stress vectors

c) Stress data:

FORMAT (4X,2F7.2,29X,1P3E10.2)

Col.	Symbol	Definition
1-4		Blank
5-11	X	X-coordinates of element centroids
12-18	Y	Y-coordinates of element centroids
19-47		Blank
48-57	STMAX(M)	Maximum principal stress of element M
58-67	STMIN(M)	Minimum principal stress of element M

Col.	Symbol	Definition
68-77	PHI(M)	Angle of inclination of principal stresses of element M

d) Boundary data:

FORMAT (2F10.4)

Col.	Symbol	Definition
1-10	XXA(N)	X-coordinate of boundary point N
11-20	YYA(N)	Y-coordinate of boundary point N

e) Exit card or title card of next plot:

FORMAT (I5,1X,18A4)

Col.	Symbol	Definition
1-5	NPROB	Problem number = 0, indicates end of plot = any integer, continues next plot
6		Blank
7-78		Title of next plot

Note: This program is capable of plotting four separate quadrilateral boundaries. If more boundaries are desired, they can be plotted by the additional of more logical "IF" statements in the program. It should also be noted that the first boundary point of each separate boundary should be repeated after the following three boundary points have been specified.

```
C     STRESS VECTOR PROGRAM                                                    STVP  10
      DIMENSION XORD(1500), YORD(1500), STMAX(1500), STMIN(1500), PHI(15STVP  20
     100), ANGLE(1500), XXA(1500), YYA(1500)                                   STVP  30
      DIMENSION TITLE(18)                                                      STVP  40
1     READ (5,19) NPROB,(TITLE(I),I=1,18)                                      STVP  50
      IF (NPROB.LE.0) GO TO 18                                                 STVP  60
      CALL PLOTS (0,0,50)                                                      STVP  70
      CALL PLOT (5.,.25,-3)                                                    STVP  80
      READ (5,20) NOSEL,NBNDY,SC,SCALE                                         STVP  90
      DO 3 M=1,NOSEL                                                           STVP 100
      READ (5,21) X,Y,STMAX(M),STMIN(M),PHI(M)                                 STVP 110
      RITE=0.0                                                                 STVP 120
      IF (RITE.EQ.0.0) GO TO 2                                                 STVP 130
      WRITE (6,21) X,Y,STMAX(M),STMIN(M),PHI(M)                                STVP 140
2     XORD(M)=X*SC                                                             STVP 150
      YORD(M)=Y*SC                                                             STVP 160
3     CONTINUE                                                                 STVP 170
      CALL SAXIS (0.,0.,23HVERTICAL EXTENT IN FEET,23,4.5,90.,0.,25.,.25STVP 180
     1,0.)                                                                     STVP 190
      CALL SAXIS (0.,0.,25HHORIZONTAL EXTENT IN FEET,-25,6.75,0.,0.,25.,STVP 200
     1.250,0.)                                                                 STVP 210
      CALL SYMBOL (-.8,.5,.07,68H500 X 7 FOOT OPENING,  PENG  CASE STUDYSTVP 220
     1, STRESS SCALE 1 IN =10000PSI,90.,68)                                    STVP 230
      DO 4 N=1,NBNDY                                                           STVP 240
      READ (5,22) XXA(N),YYA(N)                                                STVP 250
      XCO=XXA(N)*SC                                                            STVP 260
      YCO=YYA(N)*SC                                                            STVP 270
      IF (N.EQ.1) II=3                                                         STVP 280
      IF (N.EQ.6) II=3                                                         STVP 290
      IF (N.EQ.11) II=3                                                        STVP 300
      IF (N.EQ.16) II=3                                                        STVP 310
      IF (N.EQ.21) II=3                                                        STVP 320
      IF (N.EQ.26) II=3                                                        STVP 330
      IF (N.EQ.31) II=3                                                        STVP 340
      IF (N.EQ.36) II=3                                                        STVP 350
      IF (N.EQ.41) II=3                                                        STVP 360
      IF (N.EQ.46) II=3                                                        STVP 370
      IF (N.EQ.51) II=3                                                        STVP 380
      IF (N.EQ.56) II=3                                                        STVP 390
      IF (N.EQ.61) II=3                                                        STVP 400
      CALL PLOT (XCO,YCO,II)                                                   STVP 410
      II=2                                                                     STVP 420
4     CONTINUE                                                                 STVP 430
      PI=3.14159/180.                                                          STVP 440
      DO 17 I=1,NOSEL                                                          STVP 450
      ANGLE(I)=ABS(PHI(I))*PI                                                  STVP 460
      IF (PHI(I)) 5,6,6                                                        STVP 470
5     X1=XORD(I)-ABS(STMAX(I))*COS(ANGLE(I))*SCALE                             STVP 480
      Y1=YORD(I)+ABS(STMAX(I))*SIN(ANGLE(I))*SCALE                             STVP 490
      X2=XORD(I)+ABS(STMAX(I))*COS(ANGLE(I))*SCALE                             STVP 500

      Y2=YORD(I)-ABS(STMAX(I))*SIN(ANGLE(I))*SCALE                             STVP 510
      X3=XORD(I)+ABS(STMIN(I))*SIN(ANGLE(I))*SCALE                             STVP 520
      Y3=YORD(I)+ABS(STMIN(I))*COS(ANGLE(I))*SCALE                             STVP 530
      X4=XORD(I)-ABS(STMIN(I))*SIN(ANGLE(I))*SCALE                             STVP 540
```

```
      Y4=YORD(I)-ABS(STMIN(I))*COS(ANGLE(I))*SCALE                    STVP 550
      GO TO 7                                                         STVP 560
6     X1=XORD(I)+ABS(STMAX(I))*COS(ANGLE(I))*SCALE                    STVP 570
      Y1=YORD(I)+ABS(STMAX(I))*SIN(ANGLE(I))*SCALE                    STVP 580
      X2=XORD(I)-ABS(STMAX(I))*COS(ANGLE(I))*SCALE                    STVP 590
      Y2=YORD(I)-ABS(STMAX(I))*SIN(ANGLE(I))*SCALE                    STVP 600
      X3=XORD(I)-ABS(STMIN(I))*SIN(ANGLE(I))*SCALE                    STVP 610
      Y3=YORD(I)+ABS(STMIN(I))*COS(ANGLE(I))*SCALE                    STVP 620
      X4=XORD(I)+ABS(STMIN(I))*SIN(ANGLE(I))*SCALE                    STVP 630
      Y4=YORD(I)-ABS(STMIN(I))*COS(ANGLE(I))*SCALE                    STVP 640
7     IF (STMAX(I)) 8,9,9                                             STVP 650
8     CALL NEWPEN (2)                                                 STVP 660
      GO TO 10                                                        STVP 670
9     CALL NEWPEN (3)                                                 STVP 680
10    CALL PLOT (X1,Y1,3)                                             STVP 690
      CALL PLOT (X2,Y2,2)                                             STVP 700
      IF (STMIN(I)) 11,12,12                                          STVP 710
11    CALL NEWPEN (2)                                                 STVP 720
      GO TO 13                                                        STVP 730
12    CALL NEWPEN (3)                                                 STVP 740
13    CALL PLOT (X3,Y3,3)                                             STVP 750
      CALL PLOT (X4,Y4,2)                                             STVP 760
      CALL NEWPEN (1)                                                 STVP 770
      IF (PHI(I)) 14,15,15                                            STVP 780
14    XP=((X3+X4)/2.)-.05                                             STVP 790
      YP=((Y1+Y2)/2.)+.05                                             STVP 800
      ANGP=180.-ABS(PHI(I))                                           STVP 810
      XQ=((X3+X4)/2.)+.05                                             STVP 820
      YQ=YP                                                           STVP 830
      ANGQ=90.-ABS(PHI(I))                                            STVP 840
      GO TO 16                                                        STVP 850
15    XP=((X3+X4)/2.)-.05                                             STVP 860
      YP=((Y1+Y2)/2.)+.05                                             STVP 870
      XQ=((X3+X4)/2.)+.05                                             STVP 880
      YQ=YP                                                           STVP 890
      ANGP=PHI(I)                                                     STVP 900
      ANGQ=270.+PHI(I)                                                STVP 910
16    CONTINUE                                                        STVP 920
17    CONTINUE                                                        STVP 930
      CALL PLOT (30.,10.,999)                                         STVP 940
      GO TO 1                                                         STVP 950
18    STOP                                                            STVP 960
C                                                                     STVP 970
19    FORMAT (I5,1X,18A4)                                             STVP 980
20    FORMAT (2I5,2F10.7)                                             STVP 990
21    FORMAT (4X,2F7.2,29X,1P3E10.2)                                  STVP1000

22    FORMAT (2F10.4)                                                 STVP1010
      END                                                             STVP1020
```

Appendix C-4

The General Purpose Contour Program

1. Input Data

a) Job card:

FORMAT (A4,1X,18A4,A3)

Col.	Symbol
1-4	JOB
5	Blank
6-80	Title of plot

b) Flexible mode card:

FORMAT (A4)

Col.	Symbol
1-4	FLEX
5-80	Blank

c) Refinement card:

FORMAT (A4,1X,F5.0)

Col.	Symbol
1-4	REF
5	Blank
6-10	Degree of contour smoothness Values: 2 through 20 Default: 5

d) New pen assignment card:

FORMAT (A4,1X,I5)

Col.	Symbol
1-4	NWPN

Col.	Symbol
5	Blank
6-10	Pen number = 1, black pen; = 2 blue pen; = 3, red pen; = 4 green pen

e) Size card:

FORMAT (A4,1X,4F5.0,F10.0,F5.0,2F10.0,F5.0,F10.0)

Col.	Symbol
1-4	SIZE.
5	Blank
6-10	Scaling factor in X-direction (unit/inch)
11-15	Scaling factor in Y-direction (unit/inch)
16-20	X-coordinate of plotter origin (inches)
21-25	Y-coordinate of plotter origin (inches)
26-35	X-coordinate of origin of plot (units)
36-40	Increment between tic marks on X-axis (units)
41-50	Maximum extent in the X-direction (units)
51-60	Y-coordinate of the origin of plot (units)
61-65	Increment between tic marks on Y-axis (units)
66-75	Maximum extent in the Y-direction (units)
76-80	Blank

f) Control point card 1:

FORMAT (A4,1X,2F5.0,2I5,45X,2I5)

Col.	Symbol
1-4	CNTL
5	Blank

Col.	Symbol
6-20	Can be left blank if the locaton of the data points are not to be plotted
21-25	= 2 or 6, if location of data points are not to be plotted
26-70	Blank
71-75	= 8, does not plot data point centers
76-80	= 8, does not plot data point centers

g) Control point card 2:

FORMAT (A4,1X,13A4,A3)

Col.	Symbol
1-4	CNTL
5	Blank
6-80	Specify the format of the contour cards Example: (3X,2F7.2,10X,1P1E10.2,30X,13A1) Note: the format must contain exactly 16 quantities, even though they are not all used.

h) Control point card 3:

FORMAT (A4,1X,3I5)

Col.	Symbol
1-4	CNTL
5	Blank
6-10	= 1
11-15	= 2
16-20	=3
21-80	Blank

i) Contour data cards:

FORMAT as specified in Control point card 2

Col.	Symbol
see	X-coordinate of data point
above	Y-coordinate of data point
	Data value

j) Block end card:

FORMAT (A4)

Col.	Symbol
1-4	BEND

k) Blanking card:

FORMAT (A4,1X,2I5)

Col.	Symbol
1-4	BLNK
5	Blank
6-10	= 1, for beginning a blanking block
11-15	Number of blanking points

Note: A sub-region within the contuoring region can be blanked, i. e. contouring can be suppressed in that area, by specifying a number of blanking points in such a manner as to traverse the region to be blanked in a clockwise direction.

l) Boundary card:

FORMAT (A4,1X,6F10.0) - number of cards as specified in columns 11-15 of Blanking card

Col.	Symbol
1-4	BND
5	Blank
6-15	X-coordinate of first point
16-25	Y-coordinate of first point
26-35	X-coordinate of second point
36-45	Y-coordinate of second point
46-55	X-coordinate of third point
56-65	Y-coordinate of third point
66-80	Blank

m) Blanking card:

FORMAT (A4,1X,2I5)

Col.	Symbol
1-4	BLNK
5	Blank
6-10	= 0, for termination of blanking
11-15	= 0
16-80	Blank

n) Phase 4 card:

FORMAT (A4)

Col.	Symbol
1-4	PHS4
5-80	Blank

o) Bold contours card:

FORMAT (A4,1X,F5.0)

Col.	Symbol
1-4	BOLD
5	Blank
6-10	Number of levels between bold contours = 5 by default
11-80	Blank

p) Skip contours card:

FORMAT (A4,1X,F5.0)

Col.	Symbol
1-4	SKIP
5	Blank
6-10	Minimum distance between adjacent light contour lines
11-80	Blank

q) Basic contour levels card:

FORMAT (A4,1X,F5.0)

Col.	Symbol
1-4	BLEV
5	Blank
6-10	Contour interval
11-80	Blank

r) Border card:

FORMAT (A4)

Col.	Symbol
1-4	BRDR
5-80	Blank

s) Pause card:

FORMAT (A4)

Col.	Symbol
1-4	PAUS
5-80	Blank

t) Plot map card:

FORMAT (A4)

Col.	Symbol
1-4	PLOT
5-80	Blank

u) Line card:

FORMAT (A4,1X,I5,7F5.0,I5)

Col.	Symbol
1-4	LINE
5	Blank
6-10	= 0, coordinates relative to plotter origin
	= 1, coordinates relative to origin of plot
11-15	X-coordinate of first point
16-20	Y-coordinate of first point
21-25	X-coordinate of second point
26-30	Y-coordinate of second point

Col.	Symbol
31-35	Distance between tic marks Blank if tic marks not required
36-40	Height of tic marks Blank if tic marks not required
41-45	Lateral displacements of line (Only required for stereoscopic maps)
46-50	Integer value that determines line type = +1, for plotting solid line
51-80	Blank

Note: Line cards are used to draw lines from one point to another. Several lines cards may be used to draw additional boundaries within the main boundary.

v) End card:

FORMAT (A4)

Col.	Symbol
1-4	END
5-80	Blank

w) Stop card:

FORMAT (A4)

Col.	Symbol
1-4	STOP
5-80	Blank

```
JOB   SIGMA (Y)
FLEX
REF       5
NWPN      1
SIZE  80.   80.   1.    1.       0.      25.     672.        0.0     25.0    441.
CNTL                          6
CNTL (3X,2F7.2,10X,1P1E10.2,30X,13A1)
CNTL       1     2     3
  1  14.00    7.00 -9.14E+02 -3.95E+03  2.33E+02 -8.96E+02 -3.97E+03  4.37E+00
509 630.00 427.00  4.14E+02 -7.95E+02 -1.60E+01  4.14E+02 -7.95E+02 -7.60E-01
510 658.00 427.00  3.26E+02 -1.10E+03 -2.26E+02  3.61E+02 -1.14E+03 -8.80E+00

BEND
BLNK
PHS4
BOLD
SKIP .03
BLEV   500
BRDR
PAUS
PLOT
LINE      1  84.   28.   84.   35.                    +1
LINE      1  84.   35. 588.   35.                    +1
LINE      1 588.   35. 588.   28.                    +1
LINE      1 588.   28.   84.   28.                    +1
LINE      1   0.    0.    0. 441.                    +1
LINE      1   0.  441. 672. 441.                    +1
LINE      1 672.  441. 672.    0.                    +1
LINE      1 672.    0.    0.    0.                    +1
LINE      1   0.  224.    0. 231.                    +1
LINE      1   0.  231. 672. 231.                    +1
LINE      1 672.  231. 672. 224.                    +1
LINE      1 672.  224.    0. 224.                    +1
END
STOP
```

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THE EFFECTS OF THE PRESSURE ARCH UPON MULTIPLE SEAM MINING

by

Stephen David Hudock

(ABSTRACT)

The coal fields of Appalachia consist of many contiguously placed, mineable coal seams. Common practice in mining multiple seams is to extract the seams in a descending order. However, the mining sequence may still be based on seam ownership, availability of the seam and the general economic situation, not on ground control considerations.

One of the major ground control mechanisms that must be considered in the design of a mine is the arching of stresses around a previously mined lower seam. This investigation deals with the extent and magnitude of the stresses above an underground opening. The finite element approach was utilized to determine the extent and magnitude of stresses for various widths of mine opening, depths of cover and overburden material. This information can then be used as an approximation of the stresses that may be encountered in an actual mining situation.