Trade-offs Between Equity and Efficiency in Prioritizing Critical Infrastructure Investments: A Case of Stormwater Management Systems

Abstract

Critical infrastructures in many countries face the problem of aging and, thus, require significant upgrades to continue serving their purpose for the next few decades, especially in the face of extreme weather events caused by global climate change. Given the urgent need for such improvements and the substantial funding gaps being experienced, prioritizing investments in critical infrastructures is a challenging task. Furthermore, the need to assure equitable solutions, as well as to consider deep uncertainty due to climate change, adds to the complexity of the problem. We seek to address this complexity by developing a set of models that explicitly consider both horizontal and vertical equity, along with efficiency, in prioritizing stormwater infrastructure improvement projects. While horizontal equity seeks to provide equal resources to everyone, vertical equity aims to allocate relatively more resources to vulnerable groups who are disproportionately susceptible to shocks and are more likely to fall into chronic poverty. By differentiating between losses in horizontal equity and vertical equity due to efficiency considerations, the models provide a practical approach to find the right balance among efficiency, horizontal equity, and vertical equity. The initial models are then extended into regret-based optimization models to help address the issue of deep uncertainty. A case study of stormwater infrastructure improvement in the City of Miami is presented, through which the performance of the models is explored both with and without the projected sea-level rise scenarios. The findings highlight the value of the proposed approach in promoting equity while maintaining efficiency.

Key words: critical infrastructures; efficiency; vertical equity; horizontal equity; deep uncertainty

1 Introduction

Critical infrastructures include systems and assets that are essential to the functioning of a society, such that their incapacitation or destruction would result in severe consequences for national security, economic security, or public health and safety (Cybersecurity and Infrastructure Security Agency 2023). In many countries, critical infrastructures face the problem of aging and require significant improvements to continue providing their services (Drzik 2019). For instance, stormwater systems, which, as one of the main categories of critical infrastructures, are designed to manage and control stormwater runoff during rainfall events, have a rating of D in the U.S.: "*poor condition and mostly below standard, with many elements approaching the end of their service life*" (ASCE 2021). Because of such poor stormwater infrastructure, and extreme rainfall and storms, flooding is the most common and costly natural disaster in the U.S., and it caused on average \$17 billion in direct damage annually from 2010 to 2018 (FEMA 2020). Climate change, increasing rainfall trends, and urbanization are only expected to exacerbate the issue. Many cities, like Chicago and Philadelphia, require a substantial budget to upgrade the aging stormwater systems underneath densely populated urban areas (ASCE 2021). Other critical infrastructures in the U.S., such as roads, bridges, levees, dams, and energy systems, also have similar issues and abysmal ratings.

Despite the immediate need to upgrade critical infrastructures, funding gaps are often substantial. For example, the American Society of Civil Engineers' (ASCE) economic analysis shows a gap of $434 billion over ten years to upgrade stormwater, wastewater, and drinking water, in particular, and a gap of $2.59 trillion over ten years to upgrade the U.S. critical infrastructures overall (ASCE 2021). This indicates that the historical $1.2 trillion infrastructure bill passed in 2021 will not be sufficient to fully upgrade the current systems (Boak 2021). Inevitably, federal and local governments have to decide how to prioritize improvement projects to upgrade the critical infrastructures. Although allocating limited resources among development projects is not a new problem, the importance of ensuring equitable investments and addressing deep uncertainty stemming from climate change make the problem of prioritizing critical infrastructure investments a challenging one.

Ensuring equitable critical infrastructure investments and climate change adaptation is a critical element in improving social cohesion, health, and peace, as well as economic productivity (Pelling and Garschagen 2019; van den Berg 2021). Traditionally, disaster loss is often measured by direct and indirect damages that are calculated in monetary terms, and projects are evaluated based on cost-benefit analysis. Relying only on the economic loss and cost-benefit analysis for planning purposes, however, often favors wealthy people, neighborhoods, or districts (Hallegatte et al. 2016; Hallegatte and Walsh 2021; Pelling and Garschagen 2019; van den Berg 2021). For example, using this approach, a project in a low-income neighborhood receives funding only when the probability of a disaster is drastically high (van den Berg 2021). Low-income neighborhoods, however, are more vulnerable to both small and large shocks and are more likely to fall into chronic poverty after disasters (Fothergill and Peek 2004; Winsemius et al. 2018). Therefore, socially-vulnerable communities should be prioritized in climate change adaptation interventions (Pelling and Garschagen 2019).

Equity, in general, refers to a "fair distribution" of benefits or costs (Camporeale et al. 2019), and can be defined in terms of either horizontal or vertical equity (Arnette and Zobel 2019; Karakoc et al. 2020). Horizontal equity is "the equal treatment of equals" (Joseph et al. 2016). However, it falls short in acknowledging the differentiating nature of the current social disparities and the elevated requirements of disadvantaged individuals. Vertical equity, on the other hand, is "the unequal, but equitable, treatment of unequals" (Joseph et al. 2016). Thus, an investment portfolio is considered fair if it provides relatively more resources to more vulnerable groups in order to offset existing inequalities. Both horizontal and vertical equity, as well as efficiency, should be taken into account when allocating limited resources to improve critical infrastructures.

In addition to ensuring equity, addressing deep uncertainty is also a challenge in prioritizing critical infrastructure investments, especially when preparing for future extreme weather events. Critical infrastructures are inherently vulnerable to extreme weather and climate change. Extreme climate events, such as excessive precipitation, heat events, and sea-level rise, severely impact normal operations of critical infrastructures. Further, critical infrastructures often have a long life span, and therefore, future possibilities should be factored into their design and improvement (Wang et al. 2019). However, incorporating climate change projections into decision-making models is not a trivial task. Coastal communities in many areas of the world, for instance, are threatened by sea-level rise, which has immediate implications for critical infrastructures. Sea-level rise depends on various factors and its projections are so uncertain that it is often impossible to assign probability distributions to them. Projections for the sea-level rise in Southeast Florida, for example, differ by more than 1 meter by the year 2120 based on different standard models (Southeast Florida Regional Sea Level Rise Work Group 2019). This condition, where experts cannot agree on associated probability distributions, is known as deep uncertainty (Hallegatte et al. 2012; Oddo et al. 2020).

Underestimating climate change uncertainties creates suboptimal decisions for the present and an extensive need for improvements in the future. At the same time, relying solely on the worst-case scenarios makes preparation more challenging and, on a larger scale, less efficient, especially under budgetary constraints (Hausfather and Peters 2020). Furthermore, decisions based on a single set of best estimate probabilities (or the most likely scenario) cause higher future costs or regrets if the reality turns out to be different (Lawrence et al. 2020). Therefore, a better approach in response to the deep uncertainty is to make decisions that perform well across possible future scenarios.

In this study, we address the equity and deep uncertainty issues in prioritizing critical infrastructure improvement projects by developing equity-based deterministic and regret-based decision-making models. These models are initially designed in the context of improving stormwater systems as an important type of critical infrastructures. However, our treatment of equity and deep uncertainty can also be extended to other contexts with little to no modification (see Section 6.3). In summary, the main contributions of our work are as follows.

- We develop a novel model that simultaneously considers efficiency, horizontal equity, and vertical equity in prioritizing critical infrastructure investments, where decision-makers can adjust the weights of these factors based on their preferences.

- We differentiate between loss in horizontal equity due to efficiency and vertical equity considerations (i.e, price of efficiency and vertical equity) and loss in vertical equity due to efficiency and horizontal equity considerations (i.e, price of efficiency and horizontal equity). We further provide a practical approach to find the right balance among the above two measures and price of fairness (i.e., loss in efficiency due to fairness considerations). Ensuring the appropriate balance among these measures can assist in avoiding perceived injustices and potential public relations difficulties that may arise when prioritizing critical infrastructure improvement projects in practice.[1] By incorporating both dimensions of equity alongside efficiency, decision-makers can ensure that their choices align with broader social values and priorities, reducing the likelihood of public backlash and negative public relations outcomes.

- We provide an equity-based regret function to address deep uncertainty and calculate the relative cost of deterministic behavior. We further discuss the implications of such an approach for equity considerations.

- Finally, we present a real case study of the City of Miami and share the underlying data to allow other researchers to contribute to the broader conversation about equity in allocating limited resources for climate adaptation investment.

The remainder of the paper is organized as follows. We begin our discussion with a review of the literature related to resource allocation problems with equity considerations and addressing deep uncertainty in Section 2. Section 3 then provides an overview of a city-level comprehensive stormwater management plan and introduces the selection process of stormwater infrastructure

[1] A recent report by Flavelle (2020) highlights the political and public relations obstacles that can arise when selecting and implementing critical infrastructure improvement projects.

improvement projects as a critical step in the master plan. This is followed in Section 4 by a formal problem definition and the presentation of different alternative models for prioritizing stormwater infrastructure improvement projects. Section 5 presents a case study of the City of Miami and compares the performance of the different models. Section 6 discusses the main implications of the research, and Section 7 concludes the paper with a discussion of future research directions.

2 Background

In this section, we provide an overview of the existing literature on critical infrastructure investments, equity considerations in resource allocation problems, and addressing deep uncertainty.

2.1 Critical infrastructure investments

Critical infrastructure investments are essential for ensuring the resilience of communities against future extreme events. Cost-benefit analysis (CBA) is widely used to evaluate critical infrastructure improvement projects in practice (Wang et al. 2019; Hallegatte et al. 2016). Eijgenraam et al. (2014) use CBA and mixed-integer nonlinear programming to find the optimal strategy for upgrading the dikes protecting different flood-prone areas. Their model minimizes total costs, including investment costs and the expected loss from flooding, while deciding the timing, height, and location of the dikes. Asplund and Eliasson (2016) also employ a CBA approach to evaluate road and railway investment projects in Sweden and Norway. Their analysis accounted for various sources of uncertainty in calculating costs and benefits, providing a more comprehensive evaluation of the projects.

A number of studies have highlighted the need for incorporating equity into CBA and have proposed models that account for vulnerability in critical infrastructure investments. Kind et al. (2017), for example, propose a revised CBA that accounts for social welfare and observe that incorporating equity and vulnerability significantly changes decisions about who to target and what actions to take. Ciullo et al. (2020) also propose an efficient monetary approach that uses the equal distribution of risk reduction in service areas as the main instrument for ensuring equity, where risk reduction is defined in monetary terms. In this formulation, areas with higher initial risks would benefit from larger risk reductions. While these approaches provide a fairer allocation or distribution of risks compared to traditional CBA, the monetization of equity remains a challenging issue. Furthermore, these methods do not provide a clear approach to understanding the trade-offs between efficiency and different types of equity considerations.

2.2 Equity in resource allocation problems

The issue of incorporating equity into resource allocation problems has been extensively studied in the literature. Nevertheless, there remains a lack of agreement on the most appropriate methods for measuring and implementing equity in different contexts mainly because fairness is subjective in nature. We review related prior studies in two parts: horizontal equity and vertical equity.

2.2.1 Horizontal equity

Horizontal equity, also known as equitability, refers to equal treatment of similar individuals or groups. Most of the previous equity modeling efforts have focused on addressing horizontal equity in resource allocation problems. The Rawlsian approach of maximizing the minimum outcome level in allocation (Rawls 1971) is one of the most established and commonly used approaches to incorporating horizontal equity in optimization problems (Karsu and Morton 2015; Alem et al. 2022). Other common methods in the literature are divided into the inequality index-based and aggregation function-based approaches, such as using the well-known Gini coefficient and its variants (Alem et al. 2022; Eisenhandler and Tzur 2019; Rodríguez-Pereira et al. 2021; Park and Berenguer 2020), Atkinson's index (Bertsimas et al. 2012; McCoy and Lee 2014), and proportional fairness function (Kelly et al. 1998). (Karsu and Morton 2015) and Chen and Hooker (2021) provide comprehensive reviews of these methods and their main properties.

In practice, there is often a trade-off between efficiency and fairness, where improving fairness may come at the cost of efficiency. Therefore, a relevant question in resource allocation problems with equity considerations is how to balance the trade-off between being fair and being efficient. Models based on Atkinson's index (known as α-fairness), in particular, are a general form of models which aim to provide a fair distribution of resources by balancing efficiency and horizontal equity. Bertsimas et al. (2011, 2012) are among the first to analytically consider the loss of efficiency due to fairness considerations (i.e., the price of fairness) and the loss of fairness due to efficiency considerations (i.e, the price of efficiency) and they offer a practical approach to quantifying the trade-offs between efficiency and (horizontal) equity, which enables decision-makers to strike a balance that aligns with their preferences.

2.2.2 Vertical equity

Vertical equity refers to unequal but equitable treatment of unequal groups where more vulnerable and disadvantaged groups receive a better treatment. There is a growing call for incorporating vertical equity in resource allocation problems, such as climate change adaptation initiatives and humanitarian investments, where individuals in disadvantaged communities rely more heavily on the resources and are disproportionately affected by allocation decisions (Klinsky et al. 2014; Pelling and Garschagen 2019). Many federal and local governments now aim to prioritize vulnerable communities in long-term investments (see, e.g., Executive Order No. 14008 (2021); California Governor's Office of Planning and Research (2018), and Seattle Office of Sustainability and Environment (2017)). These calls emphasize the importance of taking into account vertical equity, in addition to horizontal equity, when allocating resources to improve critical infrastructures. Nevertheless, vertical equity has received relatively little attention in the academic literature and detailed reviews of available adaptation plans by van den Berg (2021) and Berke et al. (2023) show a wide discrepancy between highlighting vertical equity as an important goal and operationalizing vertical equity.

In a recent study, Breugem and Van Wassenhove (2022) consider vertical equity in resource allocation problems by incorporating asymmetric fairness constraints, where each priority group is ensured a minimum percentage of the total utility. They provide an upper bound on the price of fairness and show that the vector of priorities has a crucial impact on the price of fairness. They further calculate the upper bound on the price of fairness in two real-world applications (i.e., COVID-19 vaccine allocation and health-delivery optimization) where prioritization (vertical equity) is critical. However, they do not explore the loss of fairness imposed by efficiency considerations (i.e., the price of efficiency) and the relationship between horizontal and vertical equity.

In the humanitarian operations literature, a number of studies have incorporated social vulnerability indicators in the decision-making models to ensure vertical equity in allocating limited resources (Alem et al. 2021; Arnette and Zobel 2019; Karakoc et al. 2020). Social vulnerability indicators were first introduced by Cutter et al. (2003) and calculated based on socioeconomic and demographic data. These indicators were then adopted by the Centers for Disease Control and Prevention (CDC) and the Agency for Toxic Substances and Disease Registry (ATSDR) to create the CDC/ATSDR Social Vulnerability Index for Disaster Management (SVI) (Flanagan et al. 2011). SVI is defined based on 15 U.S. census variables and is designed to help decision-makers identify areas that may need more support before, during, or after disasters. As recommended by the CDC, SVI is an appropriate variable to capture vulnerability and to be used for planning for disasters and climate change adaptation. Similar variables have also been defined for other countries, such as Norway (Holand et al. 2011), Portugal (Guillard-Gonçalves et al. 2015), and Brazil (Alem et al. 2021). Vulnerability scores have recently attracted attention from academic researchers too. Alem et al. (2021) apply vulnerability scores in pre-positioning of limited resources to supply victims' needs after disasters, Arnette and Zobel (2019) use vulnerability scores to generate equitable solutions in pre-positioning required assets in advance of natural disasters, and Karakoc et al. (2020) use vulnerability scores to prioritize vulnerable areas in the restoration process of disrupted interdependent infrastructure networks.

While previous studies in the literature offer certain approaches, such as using fairness constraints or incorporating SVI, to attain vertical equity, the issues of achieving both horizontal and vertical equity while ensuring efficiency, and characterizing the trade-offs among these measures, have not been sufficiently addressed in the literature. Addressing both horizontal and vertical equity while ensuring efficiency is important, particularly in the context of critical infrastructure investment, as it helps to avoid potential public relations issues associated with under-serving areas that happen to have low vulnerability and relatively low service needs (Flavelle 2020). To the best of our knowledge, our study is the first to differentiate between the loss in horizontal equity and the loss in vertical equity, and to provide a practical approach to enable decision-makers to balance efficiency against both types of equity in infrastructure investments.

2.3 Deep uncertainty

Deep uncertainty, in part, refers to the condition where experts cannot agree on (1) the probability distributions utilized to represent uncertainty in the key factors that drive change over time, or (2) the interrelations among those factors (Lempert et al. 2003, 2006). The infrastructure investment problem is a well-known example of a resource allocation problem under conditions of deep uncertainty as infrastructure investments require long-term commitments and the future conditions/environment under which they will serve is hard to predict. Lempert et al. (2003, 2006) and Walker et al. (2001) are among the first to posit that traditional stochastic methods are not well-suited for making investment decisions in the presence of deep uncertainty resulting from climate change. Instead, they propose decision-making methods that can accommodate deep uncertainty by accounting for the set of all possible future scenarios.

While traditional models aim to find the optimal decision under specific or expected conditions, decision-making methods under deep uncertainty look for strategies or policies that perform relatively well under all or most plausible future conditions (Cox Jr 2012; Reis and Shortridge 2020). A "good" decision in this context may be defined, for example, as one in which its cost does not exceed a certain threshold in all cases (namely, absolute performance criterion) or one that performs close to all or some of the optimal deterministic decisions (namely, relative performance criterion)

(Lempert and Groves 2010). In the latter case, the closeness to the optimal deterministic decision is typically evaluated in terms of regret. Regret metrics assess a decision by comparing its performance in a specific state of the world with the performance of the best possible option in that state. Further, a robust decision may be obtained by minimizing the maximum regret across all the possible states of the world (Kwakkel and Haasnoot 2019). Therefore, finding the optimal decision often involves creating a set of scenarios using a combination of plausible values of uncertain parameters and solving the optimization model for each scenario (Cox Jr 2012; Reis and Shortridge 2020).

Researchers have proposed various methods for making decisions under deep uncertainty to address the challenge of allocating limited resources for climate adaptation investments (Dittrich et al. 2016; Reis and Shortridge 2020; Kalra et al. 2014). Further, recent studies show that underserved communities are less resilient to severe events and will suffer the most from climate change (EPA 2021). However, the issues of deep uncertainty and equity have not been adequately addressed in a simultaneous and integrated manner. Considering deep uncertainty while ensuring equity is crucial for critical infrastructure investments because relying on specific estimates of future infrastructure performance may lead to adverse impacts on certain groups or communities if the actual environmental conditions evolve differently from what was anticipated.

In a recent study, Lempert et al. (2020) address both climate change uncertainties and equity issues in transportation infrastructure planning. They define equity as person-trips for low and middle-income cohorts and consider a plan to be successful if it meets or exceeds a preset number of person-trips in various plausible scenarios. This problem, however, is essentially different from prioritizing critical infrastructure investment projects under budget constraints considered in our study, where the decision-makers need to choose the right projects among a number of viable projects with different costs. Furthermore, the equity definition by Lempert et al. (2020) is specific to the transportation sector. In our study, we provide an equity-based regret function and a model which aims to identify the optimal decision that minimizes the maximum equity-based regret across all potential future scenarios. We also calculate the relative costs of being deterministic and discuss its implications in terms of equity considerations.

3 Problem Context: Stormwater Master Plan

To better explain the problem at hand, we first provide an overview of a city-level comprehensive stormwater management (master) plan. For details, see the City of Miami Stormwater Master Plan (City of Miami 2021) and the City of Riviera Beach Stormwater Master Plan (City of Riviera Beach 2010). The primary goal of a stormwater master plan is to help a city understand the current state of its stormwater system and provide well-designed alternatives to address present and future potential problems. These plans are updated every 5 to 10 years due to different needs such as land-use changes, project completions, and climate change impacts. Stormwater master plans usually include four phases, as shown in Figure 1. Phase 1 entails reviewing available data about the existing stormwater management system, topography, and critical infrastructures, identifying gaps in data, and collecting required data. The final output of the data collection phase is a stormwater atlas that includes a (digital) map of the city's stormwater management system, an elevation map, and critical infrastructures' information and floor elevations.

The flood modeling phase includes developing mathematical equations and models to simu-

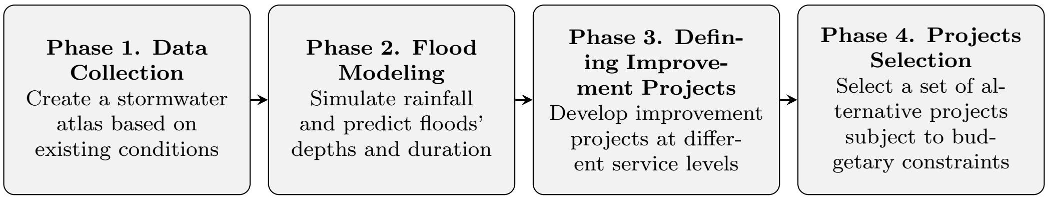


Figure 1: Four phases in a typical stormwater master plan

late rainfall and predict runoff quantity and quality[2] based on the city's data, including details of the stormwater system, the topography, soils capacities, and groundwater levels. Stormwater runoff simulation is widely performed using the United States Environmental Protection Agency's Stormwater Management Model (EPA-SWMM) (Rossman 2010). The developed model is typically fine-tuned and validated against recent historical storm events.

After validation, the model is used to simulate storm events with different degrees of intensity, duration, and system conditions. The model's outputs are then used to calculate the stormwater system's service level in different service areas under specific storm events and climate change scenarios. Service level indicates a system's ability to maintain its functionality under an event (Chasey et al. 1997; Ruparathna et al. 2017). Service levels are defined and measured within the context of the specific infrastructure, and different entities may have varying definitions and measurements of this concept. In a stormwater management system, service level is used to evaluate the effectiveness of drainage infrastructure in handling runoff from rainfall or sea-water flooding events and is measured based on metrics such as the number of buildings and roads affected. Cities aim to maintain a certain service level for their critical infrastructures in response to such events.

The third phase of a stormwater master plan includes developing citywide projects to address flooding at different service levels specified by decision-makers. The alternative projects should be defined considering different service level goals because of possible budget constraints and should be assessed under different climate change scenarios. In phase four, decision-makers typically select only a subset of alternative projects because, due to budgetary constraints, implementing all projects at the highest service level is not always feasible. Selecting the best alternatives for enhancing stormwater systems is a challenging problem for municipalities due to the complexity and uncertainty involved in the decision-making process (Jha et al. 2006; Marcelo et al. 2018) and the fact that these projects are long-term investments designed to last for decades.

4 Equity-Based Decision-Making Models

Consider a representative city that is prone to flooding in a set of service areas (i.e., neighborhoods), denoted by $N = \{1, 2, \ldots, n\}$, and aims to improve its stormwater management system. After conducting initial analysis following steps 1 to 3 in the stormwater master plan described in Section 3 (see Figure 1), the local government has defined a set of improvement projects $L = \{0, 1, 2, \ldots, l\}$, where project 0 means no improvement at no cost. Further, project $j \in L$ in service area $i \in N$ improves the performance of the corresponding service area to service level SL_{ij} at cost c_{ij}. A higher level of performance improvement within an area increases the service level, but it also costs

[2]The quality of runoff water refers to the characteristics and condition of the runoff water that flows over surfaces such as roads, rooftops, and landscapes. Polluted runoff may include dirt, oils, pesticides, fertilizers, and other substances and is a significant threat to clean water (EPA 2023a,b).

more, i.e., $c_{ij} < c_{ij+1}$ and $SL_{ij} < SL_{ij+1}$ for all $i \in N$ and $j \in \{1, \dots, l-1\}$. Further, the total available budget, denoted by B, is limited. We assume that in each service area i, project l, which leads to the highest level of service SL_{il}, is able to achieve the performance goal for its associated service area, and thus, there is no performance advantage in any additional investment.

The government's goal is to select a set of improvement projects in different service areas in an equitable and efficient manner such that the total cost does not exceed the available budget B. In the next section, we develop mathematical models that incorporate equity considerations in this problem. Appendix A.1 in the Supplementary Materials provides a summary of the notation.

4.1 Deterministic models

We first consider, as a benchmark, the case where the government's goal is to maximize total benefits while accounting for costs. In this approach, known as the utilitarian allocation, the government maximizes the total service level across all service areas. Let $x_{ij} \in \{0, 1\}$ denote the decision to choose project j in service area i. The problem can be formulated as follows:

$$\begin{aligned} \textbf{UT:} \max \sum_{i \in N} \sum_{j \in L} & SL_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{i \in N} \sum_{j \in L} c_{ij} x_{ij} \leq B, \\ & \sum_{j \in L} x_{ij} = 1 \qquad \forall i \in N, \\ & x_{ij} \in \{0, 1\} \qquad \forall i \in N, \forall j \in L. \end{aligned} \tag{1}$$

The constraints in (1) guarantee that the total cost does not exceed the budget, and exactly one level of improvement is selected in each service area (recall that level zero corresponds to no improvement). For convenience, in the remainder of the paper, we denote the set of all feasible solutions to problem (1) by X_0, i.e.,

$$X_0 = \left\{ x_{ij} \in \{0, 1\} \;\middle|\; i \in N, j \in L, \sum_{i \in N} \sum_{j \in L} c_{ij} x_{ij} \leq B, \text{and} \sum_{j \in L} x_{ij} = 1 \quad \forall i \in N \right\}. \tag{2}$$

Model **UT** provides a simple approach for choosing the "best" set of projects to invest. However, it does not take equity into consideration in its allocation of resources to the different service areas. It therefore may generate solutions in which a number of service areas receive no improvements in their stormwater management capabilities, despite the relative needs of the local populations, if those improvements happen to be more costly than in other areas. To resolve this issue of *horizontal inequity*, we consider a family of models based on α-fair welfare functions (Bertsimas et al. 2012), which maximizes the following parametric class of utility functions:

$$U_\alpha(x) = \begin{cases} \displaystyle\sum_{i \in N} \frac{(\sum_{j \in L} SL_{ij} x_{ij})^{1-\alpha}}{1-\alpha} & \text{for } \alpha \geq 0, \alpha \neq 1, \\ \displaystyle\sum_{i \in N} \log(\sum_{j \in L} SL_{ij} x_{ij}) & \text{for } \alpha = 1. \end{cases}$$

The α-**FAIR** model is then defined as follows:

$$\alpha\textbf{-FAIR:} \max_{x \in X_0} U_\alpha(x). \tag{3}$$

The proposed model provides a trade-off between efficiency and horizontal equity. The parameter α is referred to as the inequality aversion parameter, and increasing its value leads to a more equitable distribution. If $\alpha = 0$, the model is equivalent to the utilitarian model (**UT**) and as $\alpha \to \infty$, the model converges to the Rawlsian max-min model which is assumed to provide the "fairest" solution in terms of horizontal equity (Bertsimas et al. 2012). Because of the importance of the special case where $\alpha \to \infty$, we define it separately. The new model guarantees that the budget is spent first on service areas with the lowest service levels.

$$\textbf{H-FAIR:} \max_{x \in X_0} \min_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}. \tag{4}$$

H-FAIR achieves horizontal equity by allocating the same baseline service level to each service area. However, it fails to take into account the differential needs of each service area. Because socio-economically vulnerable communities are more likely to be significantly impacted by disasters, such as significant flood events, acknowledging the need for *vertical equity* suggests that they should receive priority over less vulnerable populations when all else is equal.

One approach to address both horizontal equity and vertical equity in this problem is to incorporate social vulnerability levels of the service areas (through SVI scores) directly into problem (4) and then minimize the maximum equity-weighted deviation of service levels from their highest level over all service areas. Weighting a service area by its social vulnerability score prioritizes the more vulnerable areas, and leads the model to improve the level of service in those areas. However, although such approach provides a level of vertical equity, it is important to recognize that it can potentially leave less vulnerable areas with little or no service at all. One approach used in the humanitarian logistics literature to address this concern is to add a constraint that requires a minimum level of service to be offered in each service area (Hu et al. 2017; Rodríguez-Espíndola et al. 2018). Such a constraint provides horizontal equity by helping to ensure that each area receives the same baseline level of service, although it can also be problematic if it subsequently makes the problem infeasible due to resource limitations (Holguín-Veras et al. 2013). Further, it is crucial to allow decision-makers to adjust the weight attributed to horizontal equity. With this in mind, the model presented below includes support for both horizontal and vertical equity by adding the minimum service levels from the optimal solution of problem (3) as a set of new constraints. Let SL_i^α denote the optimal service level in area i obtained by α-**FAIR**. The following model provides a desirable minimum service level in all areas while still prioritizing the more vulnerable ones:

$$\begin{aligned} \textbf{HV-FAIR}(\alpha, \beta)\textbf{:} \min_{x \in X_0} \max_{i \in N} & SVI_i \sum_{j \in L} (1 - SL_{ij}) x_{ij} \\ \text{s.t.} \quad & \sum_{j \in L} SL_{ij} x_{ij} \geq \beta SL_i^\alpha, \quad \forall i \in N, \end{aligned} \tag{5}$$

where $\beta \in [0, 1]$, referred to as the vertical inequity parameter, controls the balance between vertical equity and horizontal equity. The constraints in (5) guarantee that each area' service level is at least a β fraction of the optimal service level obtained by **α-FAIR**. While providing a minimum service level to all service areas, **HV-FAIR** ensures that a percentage of the budget is assigned to the most vulnerable areas. Therefore, the weight attributed to vertical equity can be adjusted using β. If $\beta = 1$, then all service areas will have a service level at least equal to their optimal service level determined by **α-FAIR**. As β reduces, more budget is allocated to vulnerable areas, and for $\beta = 0$, the entire budget is distributed solely based on vertical equity considerations.

4.1.1 Solution Algorithms

In this section, we briefly discuss the solutions to the models presented. The **UT** model represents a multiple-choice knapsack problem (MCKP), which has been extensively studied in the literature. MCKP is $\mathcal{NP}$-hard because it contains the classical knapsack problem as a special case. Nevertheless, the problem can be solved in pseudo-polynomial time using dynamic programming (Dudziński and Walukiewicz 1987). For a comprehensive review of solution algorithms for MCKP, see Kellerer et al. (2004). For the **α-FAIR** model, note that the objective function is a submodular function. Therefore, the existing results in the literature on maximizing a submodular set function with knapsack constraints are applicable (Sviridenko 2004; Fluschnik et al. 2019). We now present a polynomial-time algorithm for the **H-FAIR** problem. As long as the budget allows, the algorithm improves the service level of the area with the lowest service level by the minimum possible amount.

Algorithm 1

Step 1. Let $\mathbf{x}^* = (x_{ij})_{i \in N, j \in L}$. Set $x_{i0} = 1$ for all $i \in N$ and $x_{ij} = 0$ for all $j \geq 1$ and for all $i \in N$.

Step 2. Let $\eta_i = \{j \in L \mid x_{ij} = 1\}$ for all i, and $\xi = \min\{\text{argmin}_i \, SL_{i\eta_i}\}$. If $SL_{\xi,\eta_\xi} = 1$, then *Stop*; otherwise, go to *Step 3.*

Step 3. If $\sum_{i \neq \xi} c_{i\eta_i} x_{i\eta_i} + c_{\xi\eta_\xi+1} \leq B$, then set $x_{\xi\eta_\xi} = 0$, $x_{\xi\eta_\xi+1} = 1$, and go to *Step 2.* Otherwise, *Stop.*

Theorem 1. *The solution* $\mathbf{x}^*$ *given by Algorithm 1 is an optimal solution to* **H-FAIR**.

The proof is provided in Appendix A.2. Similar to Algorithm 1, we can also develop an algorithm that, given the optimal service levels SL_i^α obtained by **α-FAIR**, solves the **HV-FAIR** problem in polynomial time. The solution algorithm is essentially the same as Algorithm 1 except that the initial project selected for each area i is the one that ensures that the service level of that area is at least βSL_i^α. For brevity, the algorithm is provided in Appendix A.2.

HV-FAIR provides the decision-maker with the flexibility to choose suitable weights for efficiency, horizontal equity, and vertical equity by controlling the inequality aversion parameter α and the vertical inequity parameter β. However, there are trade-offs involved in finding the right balance among these measures. To help decision-makers in selecting the optimal weights, we define the associated measures below.

4.1.2 Price of fairness

Achieving fairness often comes at the expense of efficiency. The price of fairness is defined as the loss in efficiency due to fairness considerations (Bertsimas et al. 2012). To quantify this loss, let $x^{\mathbf{UT}}$ denote the optimal solution to **UT**. Then, for any given feasible solution $x \in X_0$, the price of fairness is defined as the relative decrease in total service level between $x^{\mathbf{UT}}$ and x.

$$PF(x) := \frac{\sum_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}^{\mathbf{UT}} - \sum_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}}{\sum_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}^{\mathbf{UT}}}. \tag{6}$$

4.1.3 Price of efficiency and vertical equity

We define the price of efficiency and vertical equity as the loss in horizontal equity due to the efficiency or vertical equity considerations. To quantify this measure for a given solution, we use the relative difference between the highest possible minimum service level, which is achieved by **H-FAIR**, and the minimum service level achieved by that solution. Let $x^{\mathbf{H-FAIR}}$ denote the optimal solution to **H-FAIR**. Then, the price of efficiency and vertical equity of any feasible solution $x \in X_0$ is

$$PEV(x) := \frac{\min_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}^{\mathbf{H-FAIR}} - \min_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}}{\min_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}^{\mathbf{H-FAIR}}}. \tag{7}$$

4.1.4 Price of efficiency and horizontal equity

The price of efficiency and horizontal equity is defined as the loss in vertical equity due to the efficiency or horizontal equity considerations. Recall from Section 4.1 that our measure of vertical inequity used in the **HV-FAIR** model is the maximum weighted deviation of service level from its ideal level. Consequently, the vertical equity measure is defined as the vertical inequity measure subtracted from 1. To quantify the price of efficiency and horizontal equity of a given solution, we use the relative difference between the highest possible vertical equity measure, which is achieved by **HV-FAIR**$(\alpha, 0)$ for any $\alpha \geq 0$, and the value of vertical equity measure achieved by that solution. Let $x^{\textbf{HV-FAIR}}$ denote the optimal solution to **HV-FAIR**$(\alpha, 0)$ for any $\alpha \geq 0$. Note that when $\beta = 0$, the solution to the **HV-FAIR** model does not depend on the value of α. After simplifying the numerator, the price of efficiency and horizontal equity can be presented as follows.

$$PEH(x) := \frac{\max_{i \in N} SVI_i \sum_{j \in L} (1 - SL_{ij}) x_{ij} - \max_{i \in N} SVI_i \sum_{j \in L} (1 - SL_{ij}) x_{ij}^{\mathbf{HV-FAIR}}}{1 - \max_{i \in N} SVI_i \sum_{j \in L} (1 - SL_{ij}) x_{ij}^{\mathbf{HV-FAIR}}}. \tag{8}$$

So far, we have assumed that there is no uncertainty in future conditions and, therefore, the service level that can be achieved by implementing each project is deterministic. In the next section, we introduce an equity-based regret function to develop models that incorporate uncertainty in selecting the best improvement projects.

4.2 Min-max regret model

It is important to recognize that the amount of flooding protection provided by a particular stormwater management project over the lifetime of the investment depends on uncertain factors such as the intensity of future storms and the amount of sea-level rise (Little et al. 2015). From a practical perspective, calculating the service level of a project in an area (i.e., $SL_{ij}, i \in N, j \in L$ in our model) requires estimating the impact of the project on future flooding events by analyzing the output from multiple replications of a simulated climate scenario. Let $\mathcal{S}$ denote the set of all scenarios. Further, let $SL_{ij}^s \in [0,1]$ denote the level of service for service area i when project $j \in L$ is implemented under the conditions represented by scenario $s \in \mathcal{S}$. To define the regret-based version of our model, let $\hat{X}_0(\alpha,\beta,s)$ denote the set of all feasible solutions to **HV-FAIR**(α,β) under scenario $s \in \mathcal{S}$, i.e.,

$$\hat{X}_0(\alpha,\beta,s) =$$
$$\left\{ x_{ij} \in \{0,1\} \;\middle|\; i \in N, j \in L, \sum_{i\in N}\sum_{j\in L} c_{ij}x_{ij} \le B, \sum_{j\in L} x_{ij} = 1 \text{ and } \sum_{j\in L} SL_{ij}x_{ij} \ge \beta SL_i^\alpha \quad \forall i \in N \right\},$$

where SL_i^α denotes the optimal service level in area i obtained by α-**FAIR**. The regret-based model aims to minimize the maximum equity-based regret across all scenarios. We define an equity-based regret function as follows. Let $\rho^*(\alpha,\beta,s)$ denote the optimal objective function value of **HV-FAIR**(α,β) under scenario s. Therefore,

$$\rho^*(\alpha,\beta,s) \le \max_{i\in N} SVI_i \sum_{j\in L}(1 - SL_{ij}^s)x_{ij},$$

for all $x \in \hat{X}_0(\alpha,\beta,s)$. Then, the regret of a given solution $x \in \hat{X}_0(\alpha,\beta,s)$ under scenario s is

$$REGR(x,\alpha,\beta,s) = \max_{i\in N} SVI_i \sum_{j\in L}(1 - SL_{ij}^s)x_{ij} - \rho^*(\alpha,\beta,s),$$

and the min-max equity-based regret version of **HV-FAIR**(α,β) is defined as

$$\textbf{RBM}(\alpha,\beta)\textbf{:} \quad \min_{x\in\cap_{s\in S}\hat{X}_0(\alpha,\beta,s)} \; \max_{s\in S} REGR(x,\alpha,\beta,s). \tag{9}$$

4.2.1 Price of being deterministic and incorporating uncertainty

The deterministic models, introduced in Section 4.1, rely solely on a single scenario (such as the current condition or the best estimate of future conditions) and, thus, if the actual situation turns out to be significantly different, these models are likely to stray from the ideal decision. On the other hand, incorporating uncertainty through the regret-based model, introduced in Section 4.2, also comes at a cost. By relying on uncertainty around the sea-level scenarios, the solution obtained by **RBM** may also be sub-optimal for minimizing the maximum weighted deviation of service level from its ideal level under the conditions that actually occur in reality. Therefore, it is important to evaluate the expected loss in the outcome due to incorporating uncertainty in the models. To that end, let F denote a distribution over the set of all scenarios $\mathcal{S}$. For any given feasible solution $x \in X_0$ and any given scenario $s \in \mathcal{S}$, define $g(x,s)$ as the maximum weighted deviation of service levels in solution x from the ideal levels under scenario s, i.e.,

$$g(x,s) = \max_{i\in N} SVI_i \sum_{j\in L}(1 - SL_{ij}^s)x_{ij}.$$

Further, let $x_s^{\mathbf{HV-FAIR}}$ and $x^{\mathbf{RBM}}$ denote, respectively, the optimal solutions to **HV-FAIR**, given scenario s, and **RBM**, where α and β are fixed. Without loss of generality, assume that $s_1 \in \mathcal{S}$ is the scenario that represent the current conditions, i.e., the scenario used to obtain solutions to the deterministic models. Following El-Amine et al. (2018) and Sadeghzadeh et al. (2020), we define the price of investing in $x_{s_1}^{\mathbf{HV-FAIR}}$ as the difference between the expected outcome achieved by $x_{s_1}^{\mathbf{HV-FAIR}}$ across all scenarios and the expected outcome achieved by a clairvoyant decision-maker who has perfect information on the realization of future scenarios:

$$\pi^{\mathbf{HV-FAIR}} = \mathbb{E}_{s\sim F}\left[g(x_{s_1}^{\mathbf{HV-FAIR}}, s)\right] - \mathbb{E}_{s\sim F}\left[g(x_{s}^{\mathbf{HV-FAIR}}, s)\right]. \tag{10}$$

Similarly, we define the price of investing in $x^{\mathbf{RBM}}$ as the difference between the expected outcome achieved by $x^{\mathbf{RBM}}$ across all scenarios and the expected outcome achieved by a clairvoyant decision-maker:

$$\pi^{\mathbf{RBM}} = \mathbb{E}_{s\sim F}\left[g(x^{\mathbf{RBM}}, s)\right] - \mathbb{E}_{s\sim F}\left[g(x_{s}^{\mathbf{HV-FAIR}}, s)\right]. \tag{11}$$

Higher values of $\pi^{\mathbf{HV-FAIR}}$ and $\pi^{\mathbf{RBM}}$ indicate that **HV-FAIR** and **RBM** solutions differ further from the expected optimal outcome (i.e., the expected optimal maximum weighted deviation of service level from its ideal value).

5 Case Study

In this section, we show how the proposed models can be used to prioritize critical infrastructure investments in the city of Miami, Florida, in a fairness-aware efficient manner (later, we show the robustness of our results using a comprehensive numerical experiment). The City of Miami has been dealing with serious flooding issues and is projected to face significant sea-level rise in the next few decades. The case study is divided into four parts. Section 5.1 provides an overview of the study area and its current stormwater master plan. Section 5.2 summarizes the results of applying the deterministic optimization models. We compare the performance of equity-based models against the current practice in Section 5.2.1. We also discuss the balance among efficiency, horizontal equity, and vertical equity and examine two possible approaches to determine the appropriate weightings between efficiency and fairness in practice in Section 5.2.2. Section 5.3 provides the results of applying the regret-based model under three sea-level rise scenarios and discusses when to use regret-based models. Finally, in Section 5.4, we conduct a comprehensive numerical experiment to confirm our findings in the case study.

5.1 Study area

In 2019, Miami consolidated its sea-level rise committee into a climate resilience committee, which now is responsible for recommending actions to enable the city to thrive in the face of climate change threats. To address its flooding issues and prepare for climate change, the city recently performed a comprehensive and well-documented study of these issues and updated its stormwater master plan. The results of this study show that the city's stormwater systems are in need of significant improvements. In summary, the flood modeling phase reveals that more than 20% of the city's land area, including 250 miles of streets, would be flooded in a representative 10-year storm. It is also estimated that around 5,390 buildings, including 28 critical structures (emergency operations, police, fire, hospital, evacuation shelter, government, etc.), would be inundated in a 100-year storm.

In order to define improvement projects that can address these potential issues, the city has been divided into specific service areas, each consisting of a neighborhood or group of neighborhoods with a common shared drainage area. Figure A.2 in Appendix A.4 displays the 78 actual service areas identified by the city. Appendix A.4 also provides the list of service areas along with their primary neighborhoods. The city has defined two sets of alternative improvement projects for each service area, which correspond to two different service level goals: a higher-cost alternative and a lower-cost alternative (with a correspondingly lower service level). These projects each consist of smaller improvement projects that together are intended to achieve the desired service level goals. These improvement projects include installing gravity drainage wells, new gravity storm sewers, and new exfiltration systems, and adding pumped injection wells. Appendix A.5 contains a summary of the alternative projects within a particular service area.

The projects in the higher-cost alternative service level category encompass combinations of smaller subprojects at a level that avoids flooding over road crowns in a representative 10-year storm. They also maintain flooding below the estimated first-floor elevation of the buildings in a representative 100-year storm. The projects under the higher-cost alternative service level category, across all service areas, cost approximately \$5.05 billion. The lower-cost alternative projects include improvements at a level that avoids flooding over road crowns in a representative 5-year storm, and that maintains flooding below buildings' estimated first-floor elevation in a representative 100-year storm. These projects cost approximately \$3.62 billion. Table 1 provides descriptive statistics of the projects' costs across service areas for both alternatives. The cost of each project and more details on estimated service level scores under each alternative project in the 78 service areas are provided in Appendix A.4.

Table 1: Descriptive statistics of the projects' costs

	lower-cost alternative projects (\$M)	higher-cost alternative projects (\$M)
Average	46.393	64.702
Stdandard deviation	38.225	48.741
Minimum	2.794	9.372
Median	41.144	53.747
Maximum	227.675	289.805
Total	3618.691	5046.776

5.2 Project selection under the deterministic scenario

Given the complete set of higher-cost and lower-cost alternative infrastructure improvement projects, we now compare the performance of the deterministic optimization models introduced in Section 4.1. The following analysis solves a sequence of problems based on changing the available budget from \$0 to \$5.1 billion in \$100 million increments. We compare the performance of the models based on three main criteria discussed in Sections 4.1.2 to 4.1.4. Note that although α and β can assume different values, the results of **HV-FAIR** are reported assuming $\alpha \to \infty$ and $\beta \in \{0, 0.6\}$. These particular combinations were chosen as they highlight the differences between the models more effectively. Also, we explored a range of α values in α-**FAIR** model and opted for 2.5 due to its ability to effectively portray the contrast between **UT** (where α in the α-**FAIR** model is set to 0) and **H-FAIR** (where α in the α-**FAIR** model approaches infinity).

Figure 2 illustrates the relative performance of the models in terms of price of fairness at different budget levels. By definition, the price of fairness of **UT** is equal to zero, and as expected, α-**FAIR**($\alpha = 2.5$) has a lower price of fairness than the other three models across all budget levels.

When the budget is extremely low (i.e, close to zero), price of fairness of all models is zero because the budget is not sufficient to improve the service level in any of the areas. All fairness-aware

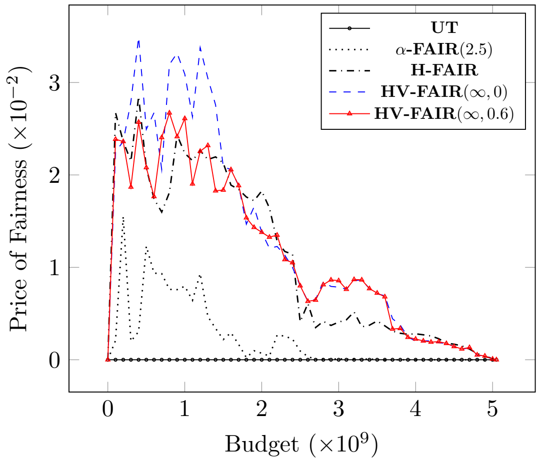
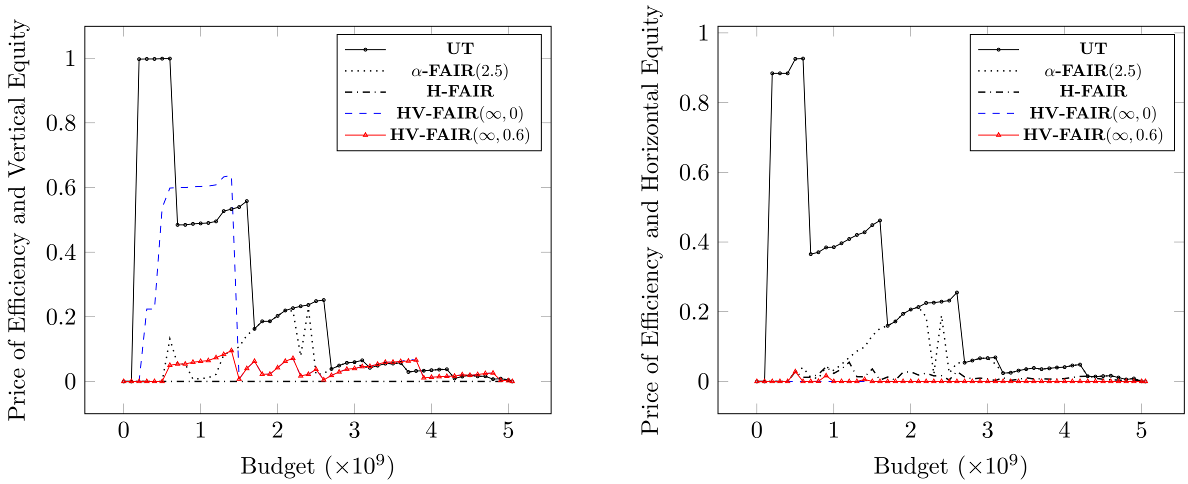


Figure 2: Performance of different models in terms of price of fairness

models have a relatively high price of fairness when the budget is limited (i.e, below a certain threshold but not extremely low) because these models target areas with lower service level or higher social vulnerability index from the outset, and budget constraints make it challenging to enhance performance in other areas. As budget surpasses that threshold, the price of fairness of all fairness-aware models reduces significantly because now the higher budget level allows these models to also invest in projects in areas that are less vulnerable or have higher service levels. Interestingly, **HV-FAIR**$(\infty, 0.6)$, which considers both horizontal and vertical fairness, exhibits a price of fairness that is similar to the other two extreme models (**H-FAIR** and **HV-FAIR**$(\infty, 0)$). This means that the balance between the horizontal and vertical fairness can be achieved without imposing additional compromise in efficiency.

HV-FAIR$(\infty, 0.6)$ performs exceptionally well in the other two criteria (i.e., the price of efficiency and vertical equity, and price of efficiency and horizontal equity), which is not the case with the other models. Figure 3a shows the price of efficiency and vertical equity at each budgetary level. Under severe budget limitations (less than 25% of the full budget required), **UT** and **HV-FAIR**$(\infty, 0)$ exhibit a very high price of efficiency and vertical equity. In contrast, **HV-FAIR**$(\infty, 0.6)$ performs well across all budget levels. Further, α**-FAIR**(2.5), does not exhibit consistently good performance across different ranges of budget levels. This shows that the performance of α**-FAIR**(2.5), in terms of price of efficiency and vertical equity, depends heavily on the budget level.

Figure 3b compares the price of efficiency and horizontal equity of different models at each budgetary level. Similar to the case with the price of efficiency and vertical equity, **UT** also shows poor performance in terms of price of efficiency and horizontal equity when the budget is limited. **H-FAIR** and **HV-FAIR**$(\infty, 0.6)$ exhibit a relatively low price of efficiency and horizontal equity, while α**-FAIR**(2.5) shows inconsistent performance across various budget levels. While **H-FAIR** performs comparably to the two **HV-FAIR** models in terms of the price of efficiency and horizontal equity, **H-FAIR** may not prioritize the most vulnerable areas. We demonstrate this by calculating the minimum service level obtained by these three models in the 10 most vulnerable service areas (sorted by social vulnerability index SVI). As Figure 4 shows, the two **HV-FAIR** models meet the



(a) (b)

Figure 3: Performance of different models in terms of (a) price of efficiency and vertical equity and (b) price of efficiency and horizontal equity

expectation of more effectively addressing vertical inequity by performing better than **H-FAIR** in the most vulnerable areas, even at the highest levels of available budget.

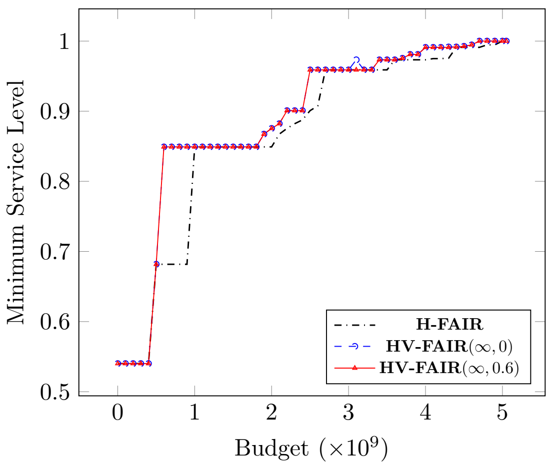


Figure 4: Minimum service level in the 10 most vulnerable areas

We also report the number of projects selected at different improvement levels by the different models in Appendix A.6. In summary, the results show that **H-FAIR** and the two **HV-FAIR** models select fewer projects (total number of lower-cost and higher-cost projects) compared to **UT** and **α-FAIR** (see Figure A.3c). This is because **H-FAIR** and **HV-FAIR** focus on improving service areas with a lower service level or a higher vulnerability index, which might involve investing in more expensive projects and, consequently, they allocate the limited budget to a smaller set of projects. Interestingly, when the budget is greater than approximately 50% of the full required budget, **H-FAIR** and the two **HV-FAIR** models select more lower-cost projects than do **UT** and **α-FAIR**, while **UT** and **α-FAIR** select more higher-cost projects. When more budget is available, investing in a higher number of lower-cost projects ultimately leads to more equitable solutions overall, both in terms of horizontal and vertical equity. This is because such an approach allows for a wider range of communities to receive resources and benefits (because each project corresponds to exactly one service area), thereby promoting a more even distribution of resources.

5.2.1 *Equity-based models vs. current practices*

In the first phase of its current improvement process, the City of Miami has prioritized improving 11 particular service areas using the lower-cost projects. These projects are selected based on factors such as visible and repeated flooding issues, addressing areas of known capacity shortfalls, and aligning with the guiding themes of the Miami Forever Bond. The Miami Forever Bond - a source of funding for stormwater system improvement projects - considers equity as a factor in its guiding themes alongside economic return, modernization, safety, and wellness in order to "distribute investment benefits fairly across the city and income levels while maintaining cohesiveness of social fabric and diversity" (Miami Forever Bond 2018). However, despite this emphasis on equity and listing different factors, equity has not yet been directly addressed or formally discussed and quantified during the planning process. According to the master plan, the required budget for the selected projects is approximately \$667 million, which is around 20% of the total budget required to fulfill the service level goal under the lower-cost alternative project. Table 2 summarizes the optimal solutions obtained by our models with a budget of \$667 million and compares them with the projects currently selected by the city. The results show that all of the models generate solutions

Table 2: Comparison between the proposed models and the city at \$667 million budget level

Model	Price of fairness (%)	Price of efficiency and vertical equity (%)	Price of efficiency and horizontal equity (%)
UT	0.00	48.38	36.52
α-**FAIR**(2.5)	0.86	13.40	3.96
H-FAIR	1.52	0.00	1.19
HV-FAIR$(\infty, 0)$	2.69	59.93	0.00
HV-FAIR$(\infty, 0.6)$	2.22	5.27	0.00
The city	4.93	100.00	92.64

that outperform the city's current plan in terms of the three performance criteria. In particular, the current plan falls short in improving the more vulnerable areas or areas with very low service levels. This is reflected in the high loss of both horizontal and vertical equity associated with the plan. There are two distinct underlying factors that might have negatively impacted the performance of the city's approach. First, with regard to equity, the city's plan does not explicitly outline and quantify the concept of equity. This is not surprising since decision-makers have recently placed higher importance on achieving equity in investments related to critical infrastructures and climate change, such as the Justice40 Initiative by the United States government in 2021. Second, with regard to efficiency, the city's project prioritization process involved a number of criteria that impact efficiency. These additional criteria include the number of complaints originating from service areas, visible and repetitive problems, and the presence of "shovel-ready" projects (i.e, projects that are prepared for immediate implementation) (City of Miami 2021).

The results also show that **HV-FAIR**$(\infty, 0.6)$ is similar to **H-FAIR** in providing a low price of fairness, while also performing well in terms of price of efficiency and vertical equity, and price of efficiency and horizontal equity. Further, **HV-FAIR**$(\infty, 0.6)$ matches the best minimum service level across the ten most vulnerable areas (see Figure 4), while **H-FAIR** only does better than the city's current plan. At this particular budget level, none of the other proposed models (except for **H-FAIR**) perform well in all the three criteria. Although **H-FAIR** provides a better solution in terms of horizontal equity, the solution of **HV-FAIR**$(\infty, 0.6)$ is superior in terms of vertical equity. This suggests that while both solutions have their strengths, **HV-FAIR**$(\infty, 0.6)$ may be a better option for ensuring fairness across different levels of society. This observation further highlights the fact that a significant trade-off exists among efficiency, horizontal equity, and vertical equity.

Table 3: Characteristics of projects selected by the different models and the city at $667 million budget level

Model	Total No. of projects selected	Avg. Cost ($M)	Avg. current service level of selected areas	Avg. SVI of selected areas
UT	15	44.4	0.78	0.79
α-**FAIR**(2.5)	11	60.6	0.64	0.73
H-FAIR	7	94.4	0.55	0.78
HV-FAIR$(\infty, 0)$	5	121.4	0.70	0.92
HV-FAIR$(\infty, 0.6)$	6	94.7	0.40	0.81
The city	11	60.6	0.85	0.79

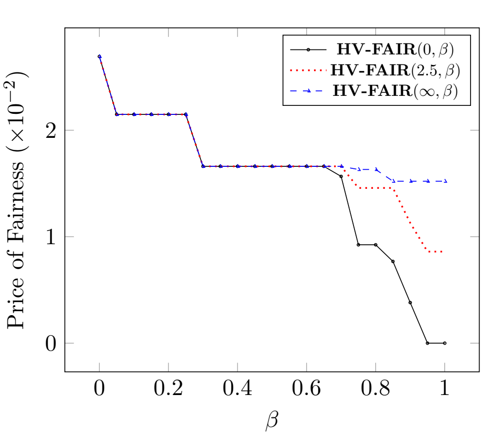
To better highlight the difference between projects selected by the different models and the projects selected by the city, we report the total number of projects selected, the average cost of projects, the average current service level of the selected areas, and the average of the SVI of the selected areas for each case in Table 3. The total numbers of improvement projects funded by **H-FAIR** and both **HV-FAIR** models at both the lower-cost and higher-cost alternative levels are smaller than those of the other models. As Table 3 indicates, this is because the average cost of projects selected by these models is more significant and the budget is spent on improving areas with the combination of the lowest service levels and the highest vulnerability. Projects selected by **HV-FAIR**$(\infty, 0.6)$, in particular, are in service areas with a relatively very low average existing service level and high vulnerability. Projects selected by **UT** and the city are less costly and not particularly in lower service areas or with higher SVI.

5.2.2 Balancing efficiency, horizontal equity, and vertical equity

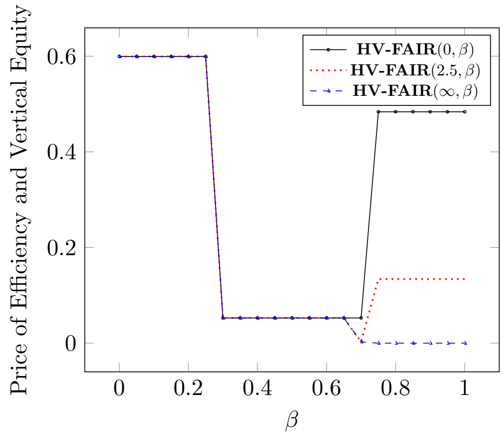
As the results in previous sections show, there are substantial trade-offs among efficiency, horizontal equity, and vertical equity in a real-world resource allocation problem. Finding the right balance among these measures is essential to an efficient and equitable allocation of resources. **HV-FAIR** provides the decision-maker with a tool to achieve this balance by adjusting the inequality aversion parameter α and the vertical inequity parameter β. **HV-FAIR** is a two-stage model that encompasses all the other models presented in Section 4. For instance, it includes the **UT** model as a special case when $\alpha = 0$ and $\beta = 1$, and includes the α-**FAIR** model as a special case when $\beta = 1$. Recall that α controls the trade-off between efficiency and horizontal equity, and β controls the trade-off between efficiency and vertical equity. Therefore, the right balance among these measures can be obtained by setting the right values of α and β in **HV-FAIR**.

Controlling the trade-off between efficiency and horizontal equity through the inequality aversion parameter α has been studied in the literature. Bertsimas et al. (2012) show that as α increases, an upper bound on the price of fairness increases, while the price of efficiency and vertical equity decreases. Our results, aligned with previous research, reveal that an increase in the value of α corresponds to a rise in the price of fairness, while the price of efficiency and vertical equity decreases. One novel contribution of our model, however, is that in addition to controlling the balance between the price of fairness, and the price of efficiency and vertical equity, it also controls the level of vertical equity through the vertical inequity parameter β. To explore this, we change the value of β while holding the budget fixed at $667 million when $\alpha = 0$, $\alpha = 2.5$, and as $\alpha \to \infty$.

Figure 5a plots the price of fairness against β for the three different cases of α. Not surprisingly, the results show that as the weight of vertical equity decreases (i.e., as β increases), the price of fairness also decreases for all the three cases of α. However, the price of fairness becomes more heavily influenced by changes in α as β exceeds a certain threshold. This can be seen, in Figure 5a, by noticing that as β exceeds 0.65, models with lower values of α result in lower prices of fairness (the price of fairness is zero when $\alpha = 0$ and $\beta = 1$). When β is small, the model heavily focuses on vertical equity, and therefore, changes in α may not impact the price of efficiency. However, when β is sufficiently high, the model's emphasis on vertical equity is lower, and consequently, the impact of changes in α on the price of fairness is more evident. Likewise, the price of efficiency and vertical equity declines from its peak value as β increases from 0 to 0.65 for all the three cases of α (Figure 5b). However, increasing β beyond 0.65 affects the price of efficiency and vertical equity differently depending on α (the price of efficiency and vertical equity is zero when $\alpha \to \infty$ and $\beta = 1$). Furthermore, as expected, Figure 5c shows that as the weight of vertical equity decreases (i.e., as β increases), the price of efficiency and horizontal equity also increases for all three cases of α.



(a)



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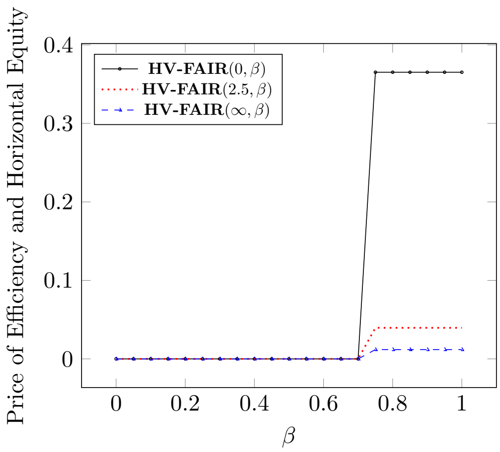

(c)

Figure 5: Impact of vertical inequity parameter β on efficiency, horizontal equity, and vertical equity

We now explain two practical approaches for balancing the trade-off among efficiency, horizontal equity, and vertical equity by choosing the proper values of the inequality aversion parameter α and the vertical inequity parameter β. As discussed in Bertsimas et al. (2012), one possible approach to determine the appropriate weightings between efficiency and fairness in practice is to incorporate a tolerance level for inefficiency and/or unfairness. To demonstrate this approach, suppose that the decision-maker aims to maximize efficiency while ensuring that the losses in horizontal and vertical equity due to efficiency do not exceed 10%. Figures 5b and 5c suggest that, in this case, β should be between 0.3 and 0.7 for $\alpha \in \{0, 2.5\}$, and between 0.3 and 1 as $\alpha \to \infty$. Figure 5a then indicates that the proper choices of α and β are 0 and 0.7, respectively (because these values provide the lowest possible price of fairness while still meeting the criteria for addressing unfairness). This results in a price of fairness of 1.5%, a 5% loss of horizontal equity, and no compromise on vertical equity.

Another possible approach is to allow for earmarking a portion of the available funds for improving socially-vulnerable areas. For example, Executive Order No. 13985 (2021) declared a Federal Government policy of pursuing a whole-of-government approach to advancing equity, and Executive Order No. 14008 (2021) on Tackling the Climate Crisis at Home and Abroad began the Justice40 Initiative (section 223) whose goal is to deliver 40% of overall benefits from relevant federal investments addressing climate change to disadvantaged communities. To demonstrate how our approach can be used to achieve such goal, suppose the decision-maker aims to maximize efficiency while assigning θ% of the budget to disadvantaged communities. One can calculate the minimum budget required to satisfy the constraints in (5) based on different values of β when $\alpha = 0$, and then determine the appropriate value of β. Figure 6 shows the percentage of the budget assigned to vertical equity (θ) for different values of β when the available budget is \$667 million. The results show that if the decision-maker's goal is to assign, for example, 40% of the budget to improve disadvantaged communities while maximizing efficiency, then the proper value of β is around 0.7 (i.e., in Figure 6, 40% of the budget corresponds to $\beta = 0.7$), which results in a price of fairness of around 1% (note that, in Figure 5a, the price of fairness is around 1% for $\beta = 0.7$ and $\alpha = 0$).

There are two additional alternative models that the decision-maker may use to select projects while addressing equity: (1) maximizing efficiency while allocating a predetermined fraction of the budget to support disadvantaged communities, and (2) maximizing efficiency while providing a minimum service level to each service area. In Appendix A.7, we elaborate on these two straightforward models and show the superiority of our proposed approach over them.

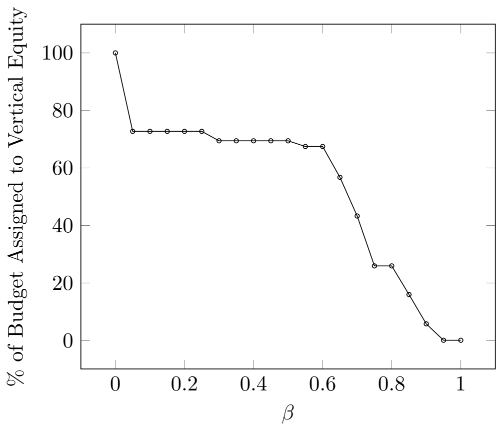

Figure 6: Percentage of the budget assigned to vertical equity based on values of β

5.3 Project selection under deep uncertainty

The results in the previous sections show the effectiveness of our models in improving stormwater infrastructure investments in terms of efficiency and equity under the assumption that investments are not heavily influenced by the uncertainties in future climate projections. However, it is known that growing sea levels are gradually diminishing the performance of the stormwater management systems in many coastal cities (City of Miami 2021). Therefore, it is insightful to study the effectiveness of such investments taking into account the projections of sea-level rise (SLR). To support local governments, regional entities, and other partners in developing appropriate adaptation strategies and infrastructure design, the Southeast Florida Regional Sea Level Rise Work Group (2019) have developed a unified SLR projection for Southeast Florida. The stormwater master plan for the City of Miami evaluates two specific SLR scenarios: 18 inches and 30 inches. They approximately correspond to the years 2035 to 2060 for the 18 inches scenario and the years 2050 to 2090 for the 30 inches scenario. More details about SLR projections, flood modeling simulation, and estimated service levels under the SLR scenarios, and for each alternative project in the 78 service areas, are provided in Appendix A.8.

Given the estimated service levels, we apply the regret-based model, introduced in Section 4.2, to the stormwater infrastructure investment problem considering three scenarios (Scenario 1: current sea level, Scenario 2: 18 inches sea-level rise, and Scenario 3: 30 inches sea-level rise), and three values of the vertical inequity parameter β (0, 0.5, and 0.6). Note that an optimal solution to the regret-based model provides the smallest maximum deviation from each of the optimal deterministic solutions under the different scenarios, i.e., the smallest maximum regret across all scenarios. Figure 7 shows how maximum regret changes with the budget for the three values of β. When the budget is extremely limited, regret is relatively low because the set of feasible solutions under any scenario is very small and, therefore, the optimal decisions under different scenarios are very similar. In the extreme case, where the budget is zero, there is only one feasible solution under any scenario (i.e., $x_{i0} = 1$ for all $i \in N$ and $x_{ij} = 0$ for all $i \in N$ and $j \geq 1$) and, consequently, regret is zero under any scenario. As the budget increases, maximum regret also increases. This is because the increase in the budget provides more flexibility, and the optimal decisions may differ under different scenarios. When the budget exceeds a specific limit (around \$1.5 billion in our case study), the optimal solutions for the different values of β result in positive regrets, which implies that no one solution is optimal under all potential scenarios. As the budget increases further, the maximum regret starts decreasing. This is because this additional flexibility now allows the decision-maker to select solutions that perform relatively well under all potential scenarios. In the extreme case, where the budget is sufficient to support all the projects that provide the highest service level, regret is zero under any scenario.

The non-monotonicity of regret highlights the importance of using regret-based models (as opposed to deterministic ones) to balance the performance across multiple scenarios in most real-world applications where the budget level is moderate. The following result, whose proof is provided in Appendix A.2, establishes the non-monotonicity of regret.

Theorem 2. *For any given α and β, the optimal maximum regret obtained by* $\mathbf{RBM}(\alpha, \beta)$ *in* (9) *is non-monotone in the budget B.*

The above result implies that the optimal maximum equity-based regret may increase with more budget. We also note that when regret is used as a metric, inequity may also increase under some scenarios as the budget increases. This is because the regret-based model aims to minimize the maximum regret across all scenarios and, therefore it is possible that, as more budget becomes available, a solution that minimizes maximum regret results in higher inequity under a certain scenario. Appendix A.3 provides more details on the non-monotonicity of inequity in the budget when maximum regret is used as the metric.

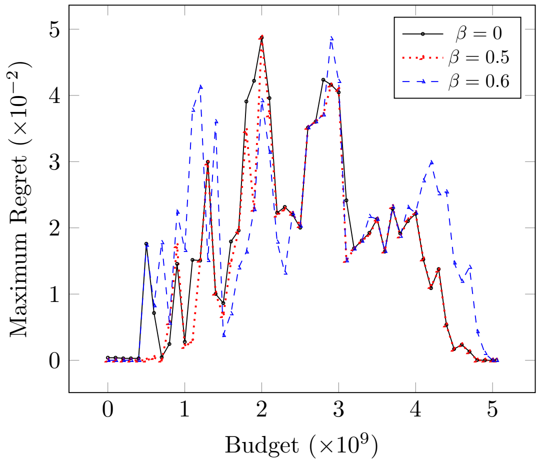

Figure 7: Maximum regret per budget

To more precisely illustrate the advantages of using the regret-based models, Table 4-Panel A shows the maximum weighted deviation of service levels from its highest possible level that results from solving the deterministic model (**HV-FAIR**) and **RBM** under the three different scenarios, given a fixed budget of \$4 billion, as $\alpha \to \infty$, and $\beta = 0$ (In Table 4, $x_{s_i}^{\mathbf{HV-FAIR}}$ denotes the optimal solution obtained by **HV-FAIR** under scenario s_i, $i = 1, 2, 3$, and $x^{\mathbf{RBM}}$ denotes the optimal solution obtained by **RBM**). Thus, for example, if scenario s_1 actually occurred and the optimal solution $x_{s_1}^{\mathbf{HV-FAIR}}$ obtained by **HV-FAIR** under scenario s_1 was implemented, then the regret level across all service areas would obviously be 0. If scenario s_2 (scenario s_3) were to actually occur, however, then this same policy would instead result in a regret of 0.108 (0.208 in case of scenario s_3). The highlighted cells and the cells with underlined values, respectively, show each row's best and second best values of regret levels.

Table 4: Regret level of the optimal solutions of **HV-FAIR** and **RBM** when the budget is \$4 billion

Panel A. $\beta = 0$

	Regret			
Scenario	$x_{s_1}^{\mathbf{HV-FAIR}}$	$x_{s_2}^{\mathbf{HV-FAIR}}$	$x_{s_3}^{\mathbf{HV-FAIR}}$	$x^{\mathbf{RBM}}$
1	0.000	0.022	0.183	0.022
2	0.108	0.000	0.034	0.000
3	0.208	0.117	0.000	0.007
Max. regret	0.208	0.117	0.183	0.022

Panel B. $\beta = 0.5$

	Regret			
Scenario	$x_{s_1}^{\mathbf{HV-FAIR}}$	$x_{s_2}^{\mathbf{HV-FAIR}}$	$x_{s_3}^{\mathbf{HV-FAIR}}$	$x^{\mathbf{RBM}}$
1	0.000	0.022	0.022	0.022
2	0.108	0.000	0.000	0.000
3	0.208	0.117	0.000	0.007
Max. regret	0.208	0.117	0.022	0.022

Panel C. $\beta = 0.6$

	Regret			
Scenario	$x_{s_1}^{\mathbf{HV-FAIR}}$	$x_{s_2}^{\mathbf{HV-FAIR}}$	$x_{s_3}^{\mathbf{HV-FAIR}}$	$x^{\mathbf{RBM}}$
1	0.000	0.022	0.083	0.022
2	0.108	0.000	0.042	0.021
3	0.155	0.065	0.000	0.018
Max. regret	0.155	0.065	0.083	0.022

As can be seen in Table Table 4-Panel A, although the solution obtained by **RBM**, $x^{\mathbf{RBM}}$, may not be the best one in any of the scenarios (i.e., it may result in positive regret under all scenarios), it is at least the second best in each case. On the other hand, the solutions obtained by the deterministic model (**HV-FAIR**) are often the best only under their specific scenario and may not be even in the top two solutions for the other scenarios. We observed similar behavior considering other values of β (See Table 4–Panel B and Panel C).

5.3.1 When to use regret-based models?

As discussed earlier in Section 4.2.1, it is important to recognize that a regret-based solution comes at a cost. We now compare the price of being deterministic and incorporating uncertainty based on the formulas provided in Section 4.2.1. To illustrate this, we let p denote the probability that the current scenario s_1 does not occur in the future, and we assume that scenarios s_2 and s_3 are equally likely, i.e., $p_{s_1} = 1 - p$ and $p_{s_2} = p_{s_3} = p/2$. To highlight the difference between the prices, we further assume that the budget is fixed at \$4 billion. Figure 8 shows the price of being deterministic model ($\pi^{\mathbf{HV-FAIR}}$) vs. the price of incorporating uncertainty ($\pi^{\mathbf{RBM}}$) when β is 0.

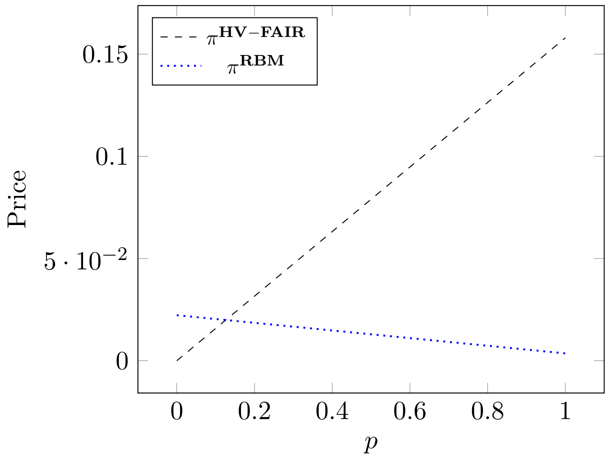

Figure 8: Price of being deterministic vs. incorporating uncertainty ($\beta = 0$)

As expected, the price of the regret-based model only changes slightly when changing the probability of the sea-level rise scenarios (p) because the regret-based model is designed to minimize the maximum regret over all scenarios. However, as p increases, the price of the deterministic model increases significantly compared to the regret-based model. The results suggest that if the probability of sea-level rise is significant (more than 13%, in this case), then the decision-makers should use the regret-based model. This is especially important in the case of the City of Miami since all current SLR models predict a significant rise in the next decades. We used similar methods to compare the price of the deterministic model ($\pi^{\mathbf{HV-FAIR}}$) with that of the regret-based model ($\pi^{\mathbf{RBM}}$) when β is 0.5 and 0.6. While the resulting graphs show similar behaviors, the intercept point increases from 13% to around 16% (see Figures A.5a and A.5b in Appendix A.9).

5.4 Robustness of the Results

In this section, we conduct a comprehensive numerical experiment to better illustrate the trade-offs among efficiency, horizontal equity, and vertical equity, and to confirm our findings in the case study described in Section 5.2. In these experiments, we consider four different problem sizes based on the number of service areas $n \in \{25, 50, 75, 100\}$. For each problem size, we randomly generate 200 instances as follows. First, for each service area i, we consider five service levels $SL_{ij}, j = 0, \ldots, 4$, where $SL_{i0} = 0$ and $SL_{i4} = 1$. The remaining three service levels ($SL_{ij}, j = 1, 2, 3$) are then drawn from a uniform distribution on $[0, 1]$ and sorted in ascending order. The cost of improving the highest service level (c_{i4}) in each service area is generated from a uniform distribution $[0, 1]$, and the cost of improving other service levels is estimated using linear interpolation (note that c_{i0} is set to 0). Finally, each service area i is assigned a social vulnerability index SVI_i in a similar manner.

The average price of fairness of the optimal solutions obtained by **UT**, **H-FAIR**, and **HV-FAIR**($\infty, \beta \in \{0, 0.5, 1\}$) across 200 instances with a problem size of 25 is reported in Appendix A.10 (see Figure A.6a). The graph illustrates how the average price of fairness varies with the increase in the allocated budget, which ranges from 0% to 100% of the total required budget. As we observed in Section 5.2, all fairness-aware models exhibit a considerably high price of fairness when the allocated budget is severely limited, and as the budget surpasses a certain threshold, the price of fairness of all fairness-aware models decreases significantly. Figures A.6b and A.6c provide the average loss of horizontal equity (i.e., the average price of efficiency and vertical equity) and the average loss of vertical equity (i.e., the average price of efficiency and horizontal equity) at each budgetary level. Consistent with our results in Section 5.2, under severe budget limitations, both **UT** and **HV-FAIR**($\beta = 0$) models demonstrate a high price of efficiency and vertical equity. On the other hand, **HV-FAIR** leads to relatively desirable outcomes across all three measures at all budget levels when using a moderate value of β. One interesting observation is that **HV-FAIR**($\beta = 1$) weakly outperforms **H-FAIR** in all three measures. This can be attributed to the fact that, in **HV-FAIR**($\beta = 1$) (when $\alpha \to \infty$), the service levels of all service areas are at least as low as those in **H-FAIR**. We observe the same behavior, consistent with our findings in the case study described in Section 5.2, using larger problem sizes (see Appendix A.10 for details).

Next, we examine the trade-off among the three measures by changing the value of β, while holding the budget fixed. Figures A.10a-A.10c in Appendix A.11 plot the average of each of these measures against β for two values of α when the allocated budget is 40% of the total required budget. For each measure, the average is calculated over 200 instances, with a problem size of 25. Overall, the findings align with those from our case study in Section 5.2.2. As β increases, the price of fairness reduces, and the rate of decrease is higher when α is smaller. Moreover, the price of efficiency and vertical equity declines from its peak value as β rises from 0 to a certain threshold. However, for $\alpha = 0$, the price of efficiency and vertical equity increases as β exceeds the threshold, similar to our case study results. Finally, the price of efficiency and horizontal equity increases in β. The results obtained from larger problem sizes exhibit similar trends (see Appendix A.11).

6 Discussion and Managerial Insights

This study provides several important insights into addressing equity and uncertainty in prioritizing critical infrastructure investments. In the following, we summarize the main insights.

6.1 Trade-offs among efficiency, horizontal equity, and vertical equality

First of all, with respect to the influence of the vertical inequity parameter β on the three efficiency and equity metrics, our observations in both the case study and the numerical experiments show that when the inequality aversion parameter α is large in the **HV-FAIR** model, the price of fairness does not change substantially as the weight of vertical equity increases (i.e., as β decreases). This observation implies that if the decision-maker is already placing a significant emphasis on horizontal equity, then incorporating vertical equity does not necessarily result in substantial further loss in efficiency. Note that as β increases, the feasible set of **HV-FAIR** becomes smaller, and the feasible solutions have service levels closer to the optimal service levels obtained by α**-FAIR**. When α is sufficiently high, the optimal solutions over the restricted feasible set generated by higher values of β have a similar price of fairness to those over the larger set generated by smaller values of β.

Additionally, as expected, when the weight of vertical equity is extremely high (i.e., when β is close to zero), loss of horizontal equity (i.e., price of efficiency and vertical equity) and loss of efficiency (i.e., price of fairness) are both very high. However, our observations show that, in such situations, a small increase in β may significantly reduce both the price of efficiency and vertical equity and the price of fairness. This implies that, under certain circumstances, a little compromise in vertical equity can go a long way in terms of improving efficiency and horizontal equity.

We also discuss two practical approaches for balancing the trade-off among efficiency, horizontal equity, and vertical equity based on the decision makers' preferences: (1) incorporating a tolerance level for inefficiency and/or unfairness, and (2) allowing for earmarking a portion of the available funds for improving socially-vulnerable areas similar to the Justice40 Initiative introduced by Executive Order No. 14008 (2021). Our study demonstrates how the **HV-FAIR** model allows decision-makers to incorporate the above practical approaches into their decision-making process by choosing the proper values of the inequality aversion parameter α and the vertical inequity parameter β. Therefore, by adjusting the values of α and β in the **HV-FAIR** model, decision-makers can ensure that their decisions are consistent with their predetermined efficiency and equity goals.

6.2 Budgetary implications

In both the case study and the numerical experiments, we observe that all the fairness-aware models have a relatively high price of fairness when the budget is limited (i.e., below a specific threshold but not exceptionally low). This is because these models aim to prioritize areas with lower service levels or higher social vulnerability index values, which makes it difficult to enhance performance in other regions due to budget constraints. However, if the budget exceeds the threshold, the price of fairness for all fairness-aware models reduces considerably because the higher budget level enables these models to invest in projects in areas that have higher service levels or are less vulnerable.

Additionally, we observe that the **HV-FAIR** model with a carefully-selected vertical inequity parameter β, which accounts for both horizontal and vertical equity, exhibits a price of fairness similar to the two extreme models of **H-FAIR** (which only considers horizontal equity) and **HV-FAIR**$(\alpha, 0)$ for any α (which only considers vertical equity). This implies that within a predetermined loss in efficiency, certain decisions may exhibit relatively acceptable performance in both dimensions- horizontal and vertical equity- without significantly under-performing in either aspect. While somewhat intuitive, this nuanced aspect carries an important implication for practical implementation, especially when decision-makers may have a predefined tolerance for loss in efficiency.

We also provide insights on how the number of selected projects is impacted by the choice of the equity model and the available budget. In summary, the results show that models with a high emphasis on horizontal or vertical equity (i.e., **H-FAIR** and **HV-FAIR** as $\alpha \to \infty$ or $\beta \to 1$) choose fewer total numbers of projects compared to the utilitarian model (**UT**) and the α**-FAIR** model with a small inequality aversion parameter α. This is because the fairness-aware models with a high emphasis on horizontal or vertical equity focus on improving service areas with lower service levels or higher vulnerability index values, which might involve investing in more expensive projects. As a result, the limited budget is spread across a lower number of projects. Interestingly, if a greater budget is available, allocating it to a higher number of lower-cost projects results in more equitable outcomes overall. This is because such an approach enables more communities to receive resources and benefits, resulting in a more equitable distribution of resources.

Furthermore, with respect to using regret-based models for prioritizing infrastructure investments, we establish that regret is not monotone in the available budget. In other words, additional budget does not necessarily reduce regret. The non-monotonicity of regret underscores the significance of employing regret-based models, rather than deterministic ones, to maintain optimal performance across various scenarios in real-world applications where the budget level is moderate. Our findings also indicate that when regret is used as a metric, inequity may also increase under some scenarios as the budget increases. This observation can be attributed to the regret-based model's objective of minimizing maximum regret across all scenarios. As more budget becomes available, a solution that minimizes maximum regret may lead to higher levels of inequity under certain scenarios.

6.3 Generalizability of the approach

Finally, it is important to note that although we presented our models in the context of stormwater systems, our proposed approach to ensure equity and address deep uncertainty is generalizable and can be applied to prioritize investment problems in other types of critical infrastructures, and in the context of other types of potential disasters. The proposed approach can be summarized in four steps, as illustrated in Figure 9.

Step 1 involves defining the set of improvement projects, their costs, and the service levels they offer. The service level should be defined and measured within the context of the specific critical infrastructure being considered, and different entities may have varying definitions and measurements of this concept. For example, in the transportation systems sector, service level refers to "*a collection of measures of automobile congestion and travel time delay, and it is among the longest-standing and most widely adopted metrics for reporting transportation system performance in the country*" (U.S. Department of Transportation 2017, p. 2). Adey et al. (2019) provide a comprehensive guideline to measure such service levels in the transportation sector. In contrast, in the case of an electric power system, the service level of a service area in a given scenario might be instead defined based on the percentage of customers and businesses that maintain power despite the disruptive impacts of a possible disaster such as hurricane. An even more general example of the use of service levels to manage public services and infrastructures is provided by the City of San Francisco, which gives target service levels for several general infrastructure categories in support of city planning efforts (AECOM 2014). In this case, the two most frequent metrics for measuring the service level associated with recreation and open space infrastructure are the number of parks in a given area and the distances of households from those parks. Just as with stormwater infrastructure, the combination of such metrics provides a foundation for comparing different budget-constrained infrastructure investments not only on the basis of efficiency but also in terms of equity.

In Step 2 of the proposed approach, **UT**, **H-FAIR**, α**-FAIR**, and **HV-FAIR** are solved with

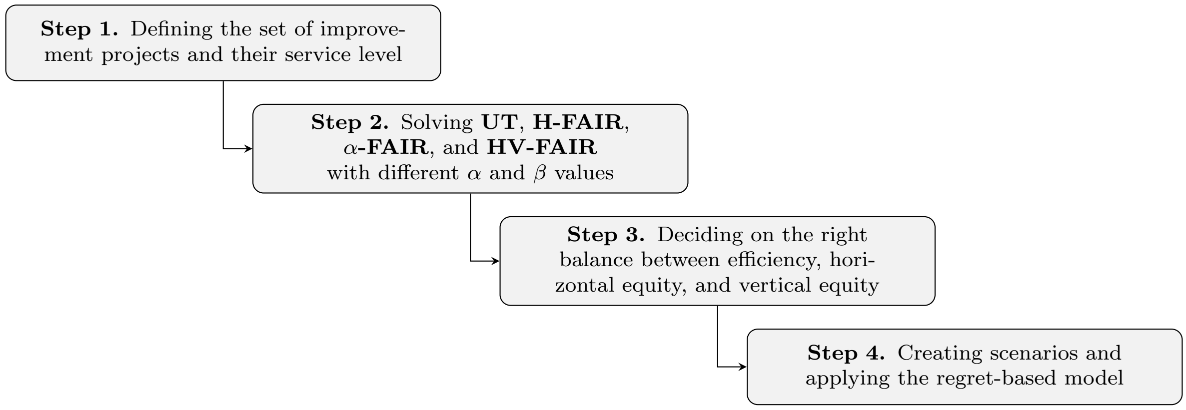

Figure 9: Four steps of the proposed approach

varying values for the inequality aversion parameter α and the vertical inequity parameter β. Subsequently, price of fairness, price of efficiency and vertical equity, and price of efficiency and horizontal equity for different solutions are calculated as explained in Section 4.1. In Step 3, the decision-maker can assess the trade-offs and determine the optimal balance among efficiency, horizontal equity, and vertical equity, by choosing proper values of α and β. The discussion on finding the right balance among efficiency, horizontal equity, and vertical equity is provided in Section 5.2.2. If extreme weather and climate change are anticipated to affect the communities and the performance of projects, it is important to generate future scenarios that measure the degree to which various improvements would actually safeguard against the impacts of future catastrophic events, as given in Step 4. Then, the regret-based model, as explained in Section 4.2, is utilized to identify a decision that minimizes the maximum equity-based regret across all possible future scenarios.

7 Concluding Remarks

As equity is subjective in nature, it is crucial to equip decision-makers with a tool that permits finding the right balance among efficiency, horizontal equity, and vertical equity. In this study, we provide equity-based models that enable decision-makers to achieve this balance in prioritizing critical infrastructure investments. Specifically, our **HV-FAIR** model encompasses two parameters (namely, the inequality aversion parameter and the vertical inequity parameter) that can be adjusted to attain the desired balance. Additionally, recent studies suggest that underserved communities, which are less resilient to individual disasters, will also suffer the most from climate change (EPA 2021). A solution that is deemed efficient and equitable based on current conditions may not remain optimal or equitable in the future if the anticipated climate change scenarios, such as projections of sea-level rise, are actually realized. Therefore, decision-makers should consider the significant impact of deep uncertainty on critical infrastructure investments, particularly given those structures' long life spans, and not rely only on assessing their performance under current conditions.

Our case study in this paper is the City of Miami, which is a coastal city. While our approach to prioritizing infrastructure investment projects can be applied to other cities, in doing so it is important to recognize the geographical features of the city. Coastal and inland cities particularly differ in how they are impacted by floods. For example, coastal cities are more vulnerable to flooding due to storm surges and sea-level rise, while inland cities may be more vulnerable to flash flooding and flooding from nearby rivers. As a result, the specific improvement projects required to address flooding may differ significantly between these two types of cities. Furthermore, coastal cities are likely to face greater uncertainty from climate change and, therefore, there might be a greater need for decision-making under uncertainty in such cities.

In this study, we use the CDC/ATSDR Social Vulnerability Index for Disaster Management (SVI) (Flanagan et al. 2011) to capture the vulnerability of service areas. While SVI has been widely used in the literature, it is not the only measure of vulnerability. Alternative metrics, such as the Gini Index for measuring income inequality or the Social Deprivation Index (Butler et al. 2013), may also be used to measure vulnerability. Employing different measures of vulnerability may slightly change the outcomes in terms of vertical equity. Nevertheless, our proposed approach can also be implemented using other measures of vulnerability with little to no modification.

Finally, our proposed model to address deep uncertainty aims to minimize the maximum equity-based regret across all potential future scenarios. While our use of the min-max regret method provides a rigorous approach to reduce the potential cost of being deterministic, future studies could explore alternative techniques, such as the robust optimization approach, to address deep uncertainty and provide further insights. These alternative approaches, however, may require additional data about the service levels of the improvement projects under different conditions.

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E-Companion for "Trade-offs Between Equity and Efficiency in Prioritizing Critical Infrastructure Investments: A Case of Stormwater Management Systems"

A.1 Notation

Sets:

N	Set of service areas ($N = \{1, 2, \ldots, n\}$)
L	Set of projects ($L = \{0, 1, 2, \ldots, l\}$)

Parameters:

c_{ij}	Cost of improvement project $j \in L$ in service area $i \in N$
SL_{ij}	service level in service area $i \in N$ after selecting project $j \in L$
B	Total budget
SVI_i	Social vulnerability index of service area $i \in N$
α	Inequality aversion parameter
β	Vertical inequity parameter

Decision variable:

x_{ij}	Binary variable that equals 1 if improvement project $j \in L$ is selected in area $i \in N$ and 0 otherwise

A.2 Proofs of Technical Results

Proof of Theorem 1. Recall from Algorithm 1 that $\mathbf{x}^* = (x_{ij})_{i \in N, j \in L}$. Suppose to the contrary that $\mathbf{x}^*$ is not an optimal solution. Then, there exists a solution $\hat{\mathbf{x}} \in X_0$ such that $\hat{\mathbf{x}} \neq \mathbf{x}^*$ and

$$\min_{i \in N} \sum_{j \in L} SL_{ij} \hat{x}_{ij} > \min_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}. \tag{A.1}$$

Let η_i denote the project selected in area i under solution $\mathbf{x}^*$, i.e., $\eta_i = \{j \in L \mid x_{ij} = 1\}$ for all i, and $\hat{\eta}_i$ denote the project selected in area i under solution $\hat{\mathbf{x}}$, i.e., $\hat{\eta}_i = \{j \in L \mid \hat{x}_{ij} = 1\}$ for all i. Further, let ζ denote the area with the lowest service level under solution $\mathbf{x}^*$, i.e., $\zeta = \min\{\mathrm{argmin}_i \, SL_{i\eta_i}\}$ (note that in case of multiple areas with the lowest service level, ζ is the one with the smallest index). By definition, we have $x_{\zeta\eta_\zeta} = 1$ and $SL_{\zeta\eta_\zeta} = \min_{i \in N} \sum_{j \in L} SL_{ij} x_{ij}$. Further, from (A.1), we have $\min_{i \in N} \sum_{j \in L} SL_{ij} \hat{x}_{ij} > SL_{\zeta\eta_\zeta}$. Therefore, since $\sum_{j \in L} \hat{x}_{ij} = 1$ and $SL_{ij} > SL_{ij-1}$ for $j \geq 1$, there exists $l \geq \eta_\zeta$ such that $\hat{x}_{\zeta l} = 1$. In words, the area that has the lowest service level under solution $\mathbf{x}^*$ must have a higher service level under solution $\hat{\mathbf{x}}$. From Algorithm 1, it is straightforward to see that

$$\sum_{i \neq \zeta} c_{i\eta_i} x_{i\eta_i} + c_{\zeta l} > B. \tag{A.2}$$

Further, because $\hat{\mathbf{x}}$ is a feasible solution, we have

$$\sum_{i \neq \zeta} c_{i\hat{\eta}_i} \hat{x}_{i\hat{\eta}_i} + c_{\zeta l} \leq B. \tag{A.3}$$

From (A.2) and (A.3), there exists $k \in N$ such that $k \neq \zeta$ and $c_{k\hat{\eta}_k} \hat{x}_{k\hat{\eta}_k} < c_{k\eta_k} x_{k\eta_k}$. Therefore, because $c_{ij} > c_{ij-1}$ for all $i \in N$ and $j \geq 1$, we have $\hat{\eta}_k \leq \eta_k - 1$ and consequently

$$SL_{k\hat{\eta}_k} \leq SL_{k\eta_k - 1}. \tag{A.4}$$

Further, from Algorithm 1, the service level of area k is increased from $SL_{k\eta_k-1}$ to $SL_{k\eta_k}$ in an iteration of the algorithm where $SL_{k\eta_k-1}$ is the area with the lowest service level. Thus, we have

$$SL_{k\hat{\eta}_k-1} \leq SL_{\zeta\eta_\zeta}. \tag{A.5}$$

Now, from (A.4) and (A.5), we have $SL_{k\hat{\eta}_k} \leq SL_{\zeta\eta_\zeta}$. Consequently,

$$\min_{i\in N} \sum_{j\in L} SL_{ij}\hat{x}_{ij} \leq SL_{k\hat{\eta}_k-1} \leq SL_{\zeta\eta_\zeta} = \min_{i\in N} \sum_{j\in L} SL_{ij}x_{ij},$$

which contradicts (A.1). This contradiction completes the proof. ■

Algorithm 2 (An algorithm for the **HV-FAIR** problem)

Step 1. Let $\mathbf{x^{HV}} = (x_{ij})_{i\in N, j\in L}$ and $\nu_i = \{j \in L \mid \text{argmin}_j\, SL_{ij} \geq \beta SL_i^\alpha\}$. Set $x_{i\nu_i} = 1$ for all $i \in N$ and $x_{ij} = 0$ for all $j \neq \nu_i$ and for all $i \in N$.

Step 2. Let $\eta_i = \{j \in L \mid x_{ij} = 1\}$ for all i, and $\xi = \min\{\text{argmax}_i\, SV_i(1 - SL_{i\eta_i})\}$.

Step 3. If $\sum_{i\neq\xi} c_{i\eta_i}x_{i\eta_i} + c_{\xi\eta_\xi+1} \leq B$, then set $x_{\xi\eta_\xi} = 0$, $x_{\xi\eta_\xi+1} = 1$, and go to *Step 2.* Otherwise, *Stop.*

Theorem A.1. *The solution* $\mathbf{x^{HV}}$ *given by Algorithm 2 is an optimal solution to* **HV-FAIR**.

Proof of Theorem A.1. From Algorithm 2, it is straightforward to see that $\mathbf{x^{HV}}$ is a feasible solution to **HV-FAIR**, i.e., $x_{ij} \in X_0$ for all $i \in N$ and for all $j \in L$, and

$$\sum_{j\in L} SL_{ij}x_{ij} \geq \beta SL_i^\alpha.$$

The proof of the optimality of $\mathbf{x^{HV}}$ is virtually identical to that of $\mathbf{x}^*$ in Theorem 1. ■

Proof of Theorem 2. We first show that for any given α and β, there exists a threshold $\underline{B} > 0$ such that for any budget level $B < \underline{B}$, the optimal maximum regret is equal to zero. To see this, let $\underline{B}$ denote the minimum cost among all projects that have a positive cost, i.e, $\underline{B} = \min_{i\in N, j\geq 1} c_{ij}$ (Recall that $c_{i0} = 0$ for all $i \in N$ and $cij > 0$ for all $i \in N$ and $j \geq 1$). Consider any $\alpha \geq 0$ and $\beta \in [0, 1]$. Then, for any $B < \underline{B}$, the set of feasible solutions to the α-FAIR model, defined in (3), is

$$\begin{aligned} X_0 &= \left\{ x_{ij} \in \{0,1\} \;\middle|\; i \in N, j \in L, \sum_{i\in N}\sum_{j\in L} c_{ij}x_{ij} \leq B, \text{and} \sum_{j\in L} x_{ij} = 1 \quad \forall i \in N \right\} \\ &= \{x_{i0} = 1 \quad \forall i \in N, x_{ij} = 0 \quad \forall i \in N, \forall j \geq 1\}. \end{aligned}$$

Consequently, we have $SL_i^\alpha = SL_{i0}^s$ for all $i \in N$, where SL_i^α is defined as the optimal service level in area i obtained by α-FAIR. Theretofore, for any scenario $s \in \mathcal{S}$, the set of feasible solutions to

HV-FAIR(α, β) is

$$\hat{X}_0(\alpha,\beta,s) =$$
$$\Big\{x_{ij} \in \{0,1\} \mid i \in N, j \in L, \sum_{i\in N}\sum_{j\in L} c_{ij}x_{ij} \le B, \sum_{j\in L} x_{ij} = 1 \text{ and } \sum_{j\in L} SL_{ij}^s x_{ij} \ge \beta SL_{i0}^s \quad \forall i \in N\Big\}$$
$$= \{x_{i0} = 1 \quad \forall i \in N, x_{ij} = 0 \quad \forall i \in N, \forall j \ge 1\}. \tag{A.6}$$

Recall that $\rho^*(\alpha,\beta,s)$ denotes the optimal objective function value of **HV-FAIR**(α,β) under scenario s. Therefore,

$$\rho^*(\alpha,\beta,s) = \max_{i\in N} SVI_i(1 - SL_{i0}^s).$$

Let $REGR(x,\alpha,\beta,s)$ denote the regret of a given solution $x \in \hat{X}_0(\alpha,\beta,s)$ under scenario s, i.e.,

$$REGR(x,\alpha,\beta,s) = \max_{i\in N} SVI_i \sum_{j\in L}(1 - SL_{ij}^s)x_{ij} - \rho^*(\alpha,\beta,s). \tag{A.7}$$

From (A.6) and (A.7), for any $x \in \hat{X}_0(\alpha,\beta,s)$ and for any $s \in \mathcal{S}$, we have $REGR(x,\alpha,\beta,s) = 0$. Consequently, for any $B < \underline{B}$,

$$\min_{x\in\cap_{s\in S}\hat{X}_0(\alpha,\beta,s)} \max_{s\in S} REGR(x,\alpha,\beta,s) = 0. \tag{A.8}$$

Now, it is sufficient to show that there exists a threshold $\overline{B}$ such that $\underline{B} \le \overline{B} < \infty$ and for any $B \ge \overline{B}$, the optimal maximum regret is equal to zero. To see this, let $\overline{B}$ denote the total cost of all projects with the highest service levels in each area, i.e, $\overline{B} = \sum_{i\in N} c_{il}$. Obviously, we have $\underline{B} \le \overline{B} < \infty$. Let $\bar{\mathbf{x}}$ denote the solution where all the projects with the highest service level in each are are selected, i.e., $\bar{x}_{il} = 1$ for all $i \in N$ and $\bar{x}_{ij} = 0$ for all $i \in N$ and $j \le l - 1$. We have $\sum_{j\in L} \bar{x}_{ij} = 1$ for all $i \in N$ and

$$\sum_{i\in N}\sum_{j\in L} c_{ij}\bar{x}_{ij} = \sum_{i\in N} c_{il} = \overline{B} \le B.$$

Consequently, $\bar{\mathbf{x}}$ is a feasible solution to α-FAIR, i.e., $\bar{\mathbf{x}} \in X_0$. From (3), it is straightforward to see that $U_\alpha(\bar{\mathbf{x}}) \ge U_\alpha(x)$ for all $x \in X_0$. Therefore, $\bar{\mathbf{x}}$ is the optimal solution to the α-FAIR model and the optimal service levels under any scenario $s \in \mathcal{S}$ are $SL_i^\alpha = SL_{il}^s$ for all $i \in N$. Therefore, for any $\beta \in [0,1]$ and for all $i \in N$, we have

$$\sum_{j\in L} SL_{ij}^s \bar{x}_{ij} = SL_{il}^s = SL_i^\alpha \ge \beta SL_i^\alpha.$$

Consequently, for any $\alpha \ge 0$, $\beta \in [0,1]$, and $s \in \mathcal{S}$, if $B \ge \overline{B}$, then $\bar{\mathbf{x}}$ is a feasible solution to **HV-FAIR**(α,β), i.e,

$$\bar{\mathbf{x}} \in \hat{X}_0(\alpha,\beta,s). \tag{A.9}$$

Further, we have

$$\max_{i\in N} SVI_i \sum_{j\in L}(1 - SL_{ij}^s)\bar{x}_{ij} = \max_{i\in N} SVI_i \sum_{j\in L}(1 - SL_{il}^s) \le \max_{i\in N} SVI_i \sum_{j\in L}(1 - SL_{ij}^s)\bar{x}_{ij},$$

for all $x \in \hat{X}_0(\alpha, \beta, s)$. Therefore, $\bar{\mathbf{x}}$ is the optimal solution to **HV-FAIR**(α, β) under any scenario $s \in \mathcal{S}$, i.e,

$$\rho^*(\alpha, \beta, s) = \max_{i \in N} SVI_i \sum_{j \in L} (1 - SL^s_{il}).$$

Thus, for any scenario $s \in \mathcal{S}$,

$$REGR(\bar{\mathbf{x}}, \alpha, \beta, s) = \max_{i \in N} SVI_i \sum_{j \in L} (1 - SL^s_{ij})\bar{x}_{ij} - \rho^*(\alpha, \beta, s) = 0. \quad \text{(A.10)}$$

Therefore, for any $B \geq \overline{B}$, from (A.9) and (A.10), we have

$$\min_{x \in \cap_{s \in S} \hat{X}_0(\alpha, \beta, s)} \max_{s \in S} REGR(x, \alpha, \beta, s) = \max_{s \in S} REGR(\bar{\mathbf{x}}, \alpha, \beta, s) = 0. \quad \text{(A.11)}$$

Note that in the trivial case where regret is zero for all the values of B, the optimal maximum regret is weakly non-monotone in the budget B. Suppose that regret is positive for some values of B. Then, from (A.8), we observe that for $B < \underline{B}$, the optimal maximum regret is zero and from (A.11), we observe that for $B \geq \overline{B}$ too, the optimal maximum regret is zero. Therefore, the optimal maximum regret is non-monotone in the budget B. ■

A.3 Non-monotonicity of Inequity in the Budget

In Section 5.3, we noted that when regret is used as a metric, inequity may also increase under some scenarios as the budget increases. Here, we show this by providing an example. Consider the following example with three areas, two projects in each area, and three possible future scenarios. Service levels of different areas when each project is implemented under each scenario is as follows:

Scenario 1: $SL^{s_1}_{10} = 0.5, SL^{s_1}_{11} = 0.6, SL^{s_1}_{20} = 0.2, SL^{s_1}_{21} = 1.$

Scenario 2: $SL^{s_2}_{1,0} = 0.2, SL^{s_2}_{11} = 0.6, SL^{s_2}_{20} = 0.3, SL^{s_2}_{21} = 0.9.$

Scenario 3: $SL^{s_3}_{10} = 0.3, SL^{s_3}_{11} = 0.4, SL^{s_3}_{20} = 0.2, SL^{s_3}_{21} = 0.9.$

Also, the cost to implement each project is as follows: $c_{10} = 0, c_{11} = 10, c_{20} = 0, c_{21} = 20$. Further, $SV_1 = SV_2 = 1$ and $\beta = 0$. We want to see the impact of increased budget on inequity. Consider two levels of budget $B = 10$ and $B = 20$. If budget is 10, then the optimal solution that minimizes the maximum regret is $x_{10} = 0, x_{11} = 1, x_{20} = 1, x_{21} = 0$ (i.e., to invest in area 1) simply because investment in area 2 is not feasible. Consequently, inequity, defined as the maximum weighted deviation of service level from its ideal level, under scenario 1 is

$$\mathcal{Q}^{s_1} = \max_{i \in N} SVI_i \sum_{j \in L} (1 - SL^{s_1}_{ij})x_{ij} = 0.8.$$

Similarly, we have $\mathcal{Q}^{s_2} = 0.7$ and $\mathcal{Q}^{s_3} = 0.8$. Now, if budget is 20, then the decision-maker can choose to invest either in area 1 or area 2. Investing in area 1 leads to a maximum regret of 0.3 across all scenarios and investing in area 2 results in a maximum regret of 0.1. Therefore, the optimal solution is to invest in area 2, i.e., $x_{10} = 1, x_{11} = 0, x_{20} = 0, x_{21} = 1$. Consequently, we have $\mathcal{Q}^{s_1} = 0.5, \mathcal{Q}^{s_2} = 0.8, \mathcal{Q}^{s_3} = 0.7$. Therefore, as the budget increases from 10 to 20, inequity increases under scenario 2. The results obtained in our case study also show the non-monotonicity of inequity in the budget. Figure A.1 illustrates how inequity, defined as the maximum weighted deviation of the service level from its ideal level and calculated based on the optimal solution of **RBM**, changes with the budget when $\alpha \to \infty$ and $\beta = 0$.

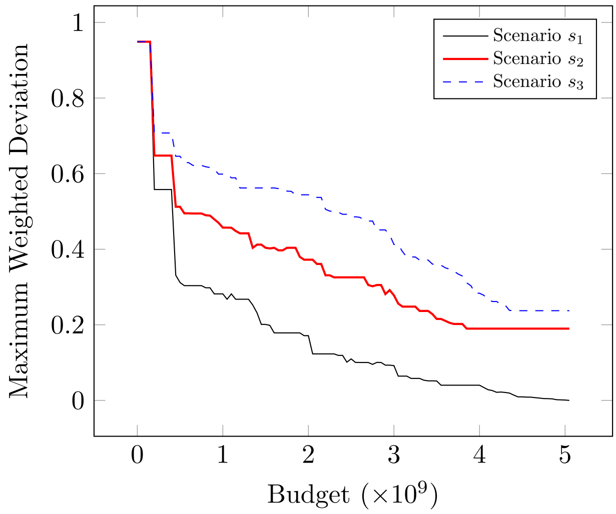


Figure A.1: Maximum weighted deviation of service level from its ideal level under different scenarios for the optimal solution of **RBM**($\alpha \to \infty$ and $\beta = 0$)

A.4 List of service areas and their characteristics

In order to define improvement projects, the city has been divided into specific service areas, each consisting of a neighborhood or group of neighborhoods with a common shared drainage area. Figure A.2 displays the 78 actual service areas identified by the city. The City of Miami uses a measure of service level as the primary mechanism to evaluate the performance of stormwater management systems under different conditions and improvement projects. The service level for each service area is defined based on the following flooding number:

$$F = C_1 Len_{10} + C_2 Bldg_{100} + C_3 Str_{crit}, \tag{A.12}$$

where Len_{10} is the length of road in linear feet flooded above crown in that service area for a 10-year storm, normalized by the total length of road flooded in the city, $Bldg_{100}$ is the total number of buildings in that service area flooded above the estimated first floor elevation for a 100-year storm, normalized by the total number of buildings flooded in the city, and Str_{crit} is the number of critical structures (emergency operations, police, fire, hospital, evacuation shelter, government, etc.) in that same service area that would be flooded in the 100-year storm, normalized by the total number of critical structures with such a flooding issue in the city. C_1, C_2, and C_3 are weights used to reflect the importance of each measure (each is equal to 1 in the current master plan). The current average value for F across all service areas is 3.85% with a standard deviation of 4.22% and a median of 2.42%. The maximum and minimum values are 24.38% and 0.16%, respectively.

The defined service levels capture both the size and the severity of the damage in service areas. Pertaining to size, the flooding number represents the combined total of road length flooded, the overall count of flooded buildings, and the number of critical structures inundated. These numbers are normalized against the corresponding number for the entire city, rather than just the service

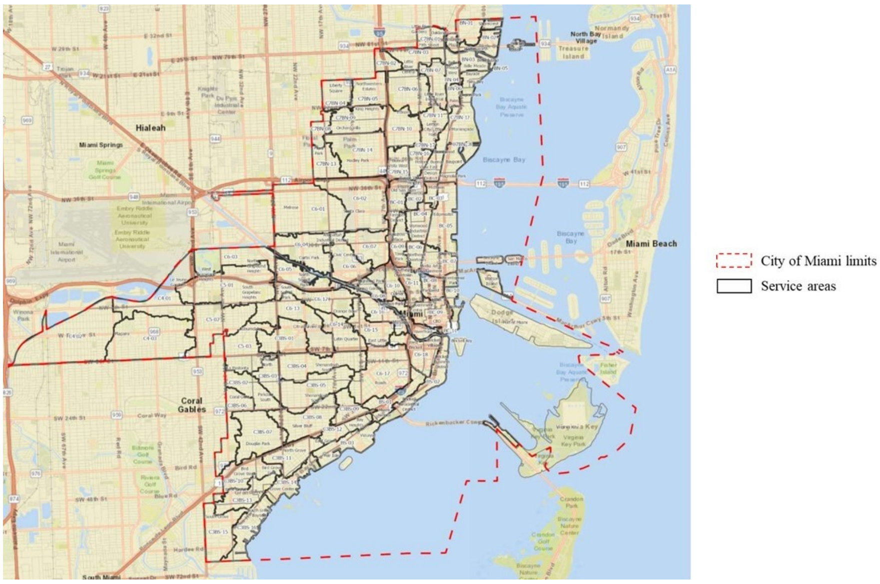


Figure A.2: The 78 service areas in the City of Miami (City of Miami 2021)

area. As an example, consider a service area that only has one structure, and suppose that this structure is extremely prone to flooding. Such a service area would not necessarily be assigned a high priority, because its flooding number depends also on the number of structures that are prone to be flooded in the entire city. In terms of severity, floods are taken into account only once they surpass particular thresholds. For example, the number of structures flooded is determined based on floods above the estimated first-floor elevation.

The master plan uses a simple method to prioritize the service areas into six categories, based on the existing value of this measure: highest priority (5 areas), high priority (11 areas), moderately high priority (7 areas), moderate priority (15 areas), moderately low priority (11 areas), and lower priority (29 areas). However, as highlighted in the master plan, simply prioritizing projects based on the flooding numbers does not guarantee efficiency and/or equity. We therefore apply our models to this situation, to generate a more efficient and equitable portfolio of options to use as the basis for investing in improvement projects.

In Miami's master plan, a higher flooding number F corresponds to a lower service level. Therefore, to apply our models to this data, we need to normalize and transform the flooding number associated with each service area.[3] Let F_i denote the flooding number of service area i, and F_{max} denote the largest flooding number over all service areas. We define the service level of service area i as follows:

$$SL_i = 1 - \frac{F_i}{F_{max}} \tag{A.13}$$

Based on the current data provided by the city, the minimum and the maximum values of F_i are 0 and 0.244, respectively. In the following, we use this data to first investigate the problem under the existing conditions scenario (i.e., a deterministic model with no sea-level rise), and then consider the result of incorporating sea-level rise scenarios.

[3] To ensure that the results are not affected by the normalization and transformation, we additionally assessed the performance of different methods based on the flooding number and compared them with the total flooding number of the service areas selected by the city. Our results show that the performances of different approaches remain unaffected by the normalization and transformation. For brevity, we do not provide the results here.

The results of running the flood modeling simulation show that the higher-cost alternative projects nearly achieve the service level goal of no flooding over the road crowns in the 10-year recurrence storm. The length of roads flooded citywide is reduced by 99% (from 250 miles to 2.1 miles) under these projects. The higher-cost alternative projects also reduce the number of flooded structures by approximately 96% under the 100-year storm, but not necessarily stop it entirely. Additional improvements are often impractical, mainly due to very high marginal costs in a number of areas that have very low elevations. In this study, since the service level goals are nearly achieved under these projects, therefore, we assume that the higher-cost alternative project in each service area provides a service level of 1. Note that in the areas that improvements are not practical, a possible outside alternative would be to offer incentives and encourage residents and businesses in low elevation areas to relocate to safer areas. This approach is commonly referred to as property buyouts and has been implemented by the federal government and by many states (Congressional Research Service 2022; Atoba et al. 2021).

Although the lower-cost alternative projects are mainly designed based on the 5-year storm, to make a valid comparison with the higher-cost alternative projects, we instead consider the service level that they would provide under the 10-year recurrence storm for roads and the 100-year recurrence storm for structures. The results of running the flood modeling simulation for the lower-cost alternative projects shows that 54.4 miles of roads will be flooded in the 10-year storm (a 78% reduction from the existing conditions) and 799 structures, including eight critical infrastructures, will be flooded in the 100-year storm (an 85% reduction from the current conditions).

The following file contains a list of service areas along with their respective characteristics and details of alternative projects: `https://www.dropbox.com/s/wt8k982v1hgjy6c/List%20of%20service%20areas.xlsx?dl=0`

A.5 Summary of the alternative projects within Biscayne North Basin (BN)

In the City of Miami, a higher-cost alternative in each service area is a collection of different improvement measures that lead to the desired 10-year storm standard, and a lower-cost alternative in each service area is an adjusted or reduced set of measures that lead to the 5-year storm standard.

In the following, we provide an example of the alternative projects within Biscayne North Basin (BN) (City of Miami 2021):

In addition to the exfiltration and gravity wells, the major improvements to meet the higher-cost alternative primary service level goal include:

- New stormwater collection piping both west and east of Dixie Hwy, a new 166 cfs pump station (PS) with a new force main[4] (FM) to the finger canal to the Bay, three new interconnected 150, 166, and 285 cfs stormwater pump stations (SWPSs) both north and south of 79th Street and on both sides of NE 10 Ave and NE 78th St to pump out the area between ridges of Dixie Hwy and NE 82 St, and new injection wells.
- New storm sewers along NE 82nd St to NE 86th St for BN-01.

[4]Force mains are pipelines that convey pressurized wastewater from the discharge side of a pump or pneumatic ejector to a discharge point.

- New Belle Meade Island collection system with a new 100 cfs PS and discharging to 6 new injection wells.
- New storm sewers in collection areas from the railroad tracks to Biscayne Blvd directing flow northward.
- Upgrade of the Belle Meade PS from the existing capacity of 100 cfs to 250 cfs and new gravity mains to collect stormwater on SE 7th Ave, with 12 injection wells for BN-03.
- New South Belle Meade 100 cfs PS with 10 injection wells for BN-05.
- Coordination with the parallel and in progress Shorecrest stormwater infrastructure improvements (by others).

Some of the primary differences from the higher-cost alternative to meet the lower-cost alternative service goal include:

- In the BN-01 area, collection is now routed to the new Shorecrest PS 4 sized at 140 cfs, eliminating PS 3, PS 1 is reduced by 200 cfs with less gravity pipe required, and PS 2 is reduced to 270 cfs.
- Belle Meade Island PS is reduced to 90 cfs.
- Belle Meade PS increases by 75 cfs with 9 injection wells as other areas where the improvement project was reduced can now be consolidated into it.
- South Belle Mead PS is reduced to 80 cfs.
- The gravity systems along NE 4th Ct are deleted.

The master plan provides a summary of flood areas that the flooding issue is difficult to resolve:

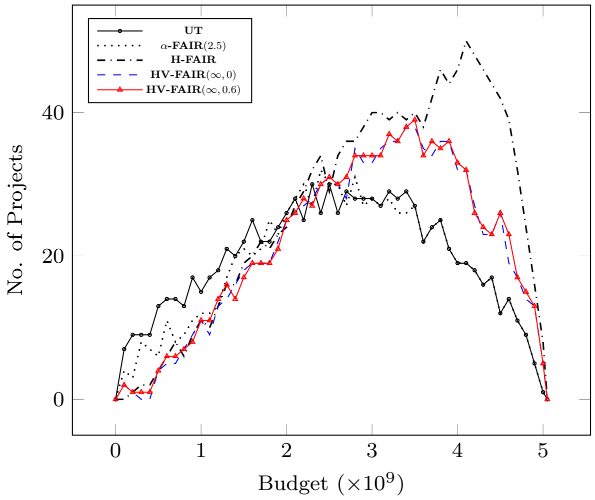
- The areas behind the Marine Max and the Boatworks Miami marinas at NE 79th St and NE 7th Ave were built for access down to the waterway and are at an elevation too low for additional improvement projects to resolve these flooding issues.

A list of off-site issues:

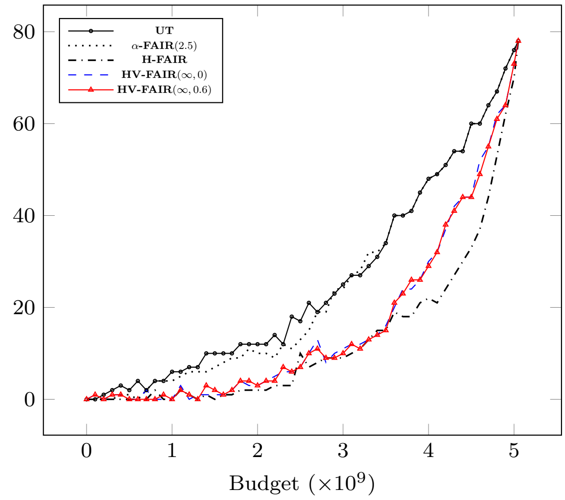
- The seawalls in the adjacent areas of Miami Shores must also be raised for approximately two blocks past the City limits to NE 89th Street or this off-site area will continue to flood into the Shorecrest area of Miami.
- In the Biscayne Plaza Area, the seawall along the C-7 canal needs to be raised or bermed in Miami Shores, El Portal, and Unincorporated Miami-Dade County to prevent off-site flow from traveling down NE 4th place into the City of Miami.

A.6 Number of projects selected at different improvement levels

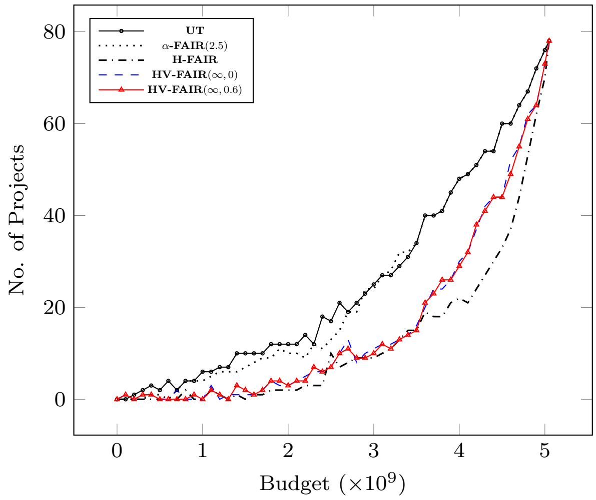
Figures A.3a to A.3c show, respectively, the number of projects selected at the lower-cost, the higher-cost, and both alternative levels by each model.



(a) Lower-cost level



(b) Higher-cost level



(c) Lower-cost and higher-cost levels

Figure A.3: Number of projects selected at different improvement levels

A.7 Additional alternative models

In Section 5.2.2, we mentioned that there are two additional alternative models that the decision-maker may use to select projects while addressing equity: (1) maximizing efficiency while allocating a predetermined fraction of the budget to support disadvantaged communities, and (2) maximizing efficiency while providing a minimum service level to each service area. In this section, we elaborate on these two straightforward models and discuss the results.

The first model can be formulated as follows:

$$
\begin{aligned}
\textbf{E1:} \max \sum_{i \in N} \sum_{j \in L} & SL_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N_D} \sum_{j \in L} c_{ij} x_{ij} \geq \theta B, \\
& \sum_{i \in N} \sum_{j \in L} c_{ij} x_{ij} \leq B, \\
& \sum_{j \in L} x_{ij} = 1 \qquad \forall i \in N, \\
& x_{ij} \in \{0, 1\} \qquad \forall i \in N, \forall j \in L,
\end{aligned}
\tag{A.14}
$$

where N_D is the set of disadvantaged service areas ($N_D \in N$), and θ is the fraction of the budget allocated to disadvantaged communities. The first constraint in (A.14) guarantees that the budget assigned to disadvantaged service areas is greater than or equal to θ fraction of the total budget.

The above model, although straightforward, requires the decision-maker to categorize communities into a binary classification of disadvantaged and non-disadvantaged ones. While this simplification allows for a straightforward implementation, it might overlook the nuanced realities and variations that exist within communities. Furthermore, determining a specific threshold on the vulnerability score to categorize communities is often a challenging task, especially when the vulnerability scores of the communities are densely distributed. This is because, in such a case, any classification of the communities will result in a situation where two communities with very similar vulnerability scores are not classified in the same category. In contrast, our proposed model presents distinct advantages by sidestepping these issues and allowing the decision-makers to balance efficiency, horizontal equity, and vertical equity. Another drawback of the **E1** model is that the first constraint in (A.14) can make the problem infeasible when the total required investment in disadvantaged service areas is less than θ fraction of the available budget. For instance, when there is only one disadvantaged service area, and the improvement cost within that area accounts for a fraction of the available budget that is smaller than θ, allocating θ fraction of the budget to the disadvantaged service area becomes unattainable.

The second model aims to maximize efficiency while providing a minimum service level in each service area. This model can be formulated as follows:

$$\begin{aligned}
\textbf{E2:} \max & \sum_{i\in N}\sum_{j\in L} SL_{ij}x_{ij} & & \\
\text{s.t.} \quad & \sum_{j\in L} SL_{ij}x_{ij} \geq \lambda & \forall i \in N, & \\
& \sum_{i\in N}\sum_{j\in L} c_{ij}x_{ij} \leq B, & & \qquad (A.15) \\
& \sum_{j\in L} x_{ij} = 1 & \forall i \in N, & \\
& x_{ij} \in \{0,1\} & \forall i \in N, \forall j \in L, &
\end{aligned}$$

where λ is the minimum service level in each service area. The first constraint in (A.15) ensures the minimum service level requirement. However, this particular constraint can be problematic as it may make the problem infeasible due to resource limitations, and therefore, necessitates careful parameter selection. Pre-selection of the minimum service level may also result in a very high price of fairness. Furthermore, similar to the **E1** model, this approach does not provide the decision-makers with a tool to balance efficiency, horizontal equity, and vertical equity.

To better highlight some of the aforementioned points, we present the optimal solutions obtained by **E1** and **E2** at the $667 million budget level, and compare them with the projects currently selected by the city and those from the models presented in Section 4. Results are shown in Table A.1. In **E1**, we assume that 40% of the available budget is allocated to service areas classified as disadvantaged, determined by their social vulnerability index value. As mentioned before, the **E1** model requires a predetermined threshold, denoted by T_{SVI}, on the vulnerability score to classify the service areas into disadvantaged and non-disadvantaged ones. The first vulnerability score threshold we consider is based on the value suggested by the Department of Homeland Security (DHS 2022), where a service area with a social vulnerability index value greater than 0.6 is designated as disadvantaged. We also report the outcome corresponding to higher thresholds in Table A.1. For thresholds below 0.6, results are similar to those at 0.6. Further, for the **E2** model, we examine a wide range of minimum service levels.

Table A.1: Performance of **E1** and **E2** compared with the other models at the $667 million budget level

Model	Price of fairness (%)	Price of efficiency and vertical equity (%)	Price of efficiency and horizontal equity (%)
UT	0.00	48.38	36.52
α-**FAIR**(2.5)	0.86	13.40	3.96
H-FAIR	1.52	0.00	1.19
HV-FAIR$(\infty, 0)$	2.69	59.93	0.00
HV-FAIR$(\infty, 0.6)$	2.22	5.27	0.00
The city	4.93	100.00	92.64
E1$(T_{SVI} = 0.60)$	0.00	48.38	36.52
E1$(T_{SVI} = 0.70)$	0.00	48.38	36.52
E1$(T_{SVI} = 0.80)$	0.00	48.38	36.52
E1$(T_{SVI} = 0.90)$	0.00	48.38	36.52
E1$(T_{SVI} = 0.95)$	0.14	99.88	92.64
E2$(\lambda = 0.1)$	0.00	48.38	36.52
E2$(\lambda = 0.2)$	0.00	48.38	36.52
E2$(\lambda = 0.3)$	0.00	48.38	36.52
E2$(\lambda = 0.4)$	0.00	48.38	36.52
E2$(\lambda = 0.5)$	0.82	13.40	3.96
E2$(\lambda = 0.6)$	0.88	5.27	1.19
E2$(\lambda \in \{0.7, 0.8, 0.9, 1\})$	infeasible	infeasible	infeasible

Our results show that, for most values of the vulnerability score threshold (i.e., 0.6, 0.7, 0.8., and 0.9), the optimal solution obtained by **E1** is the same as that obtained by **UT**. This is due to the fact that, in our case study, the set of disadvantaged service areas (N_D) encompasses a majority of the service areas, leading to the convergence of **E1** and **UT**. This observation underscores the crucial nature of the definition of disadvantaged communities when applying **E1**. However, when the vulnerability score threshold, T_{SVI}, is set at 0.95, the model has a significantly higher price of efficiency and horizontal equity as well as price of efficiency and vertical equity relative to **UT**. This is because in **E1**, vulnerability is treated as a binary variable, potentially leading to the neglect of those disadvantaged areas with extremely low service levels. Conversely, the **HV-FAIR** model prioritizes service areas based on both service levels and the social vulnerability index. Finally, as anticipated, **E2** with a relatively high minimum service level ($\lambda = 0.6$) results in a relatively low price of fairness, and price of efficiency and vertical equity, but it demonstrates less favorable outcomes in terms of the price of efficiency and horizontal equity compared to **HV-FAIR**. Higher values of λ render the problem infeasible, and values of λ lower than or equal to 0.4 lead to results similar to those of **UT**.

A.8 Details on deep uncertainty and projections of sea-level rise

In 2011, the Southeast Florida Regional Climate Change Compact developed a unified Sea-Level Rise (SLR) projection for Southeast Florida to support local governments, regional entities, and other partners in developing appropriate adaptation strategies and infrastructure design (Southeast Florida Regional Sea Level Rise Work Group 2019). This projection was updated in 2019 by an ad hoc workgroup consisting of scientific experts from academia and the government. The prediction of sea-level rise is dependent on many factors, such as the inherent response of the earth systems, global reactions to future changes, and technological advances. Because these factors all involve uncertainties, scenarios are used to estimate future sea levels based on different options and factors. The proposed unified SLR projection incorporates three curves to be used for applications (the National Oceanic and Atmospheric Administration (NOAA) High Curve, the NOAA Intermediate High Curve, and the median of the Intergovernmental Panel on Climate Change (IPCC)). Figure A.4 shows the projections based on these three different curves.

The stormwater master plan for the City of Miami evaluates two specific SLR scenarios: 18 inches and 30 inches. They approximately correspond to the years 2035 to 2060 for the 18 inches scenario and the years 2050 to 2090 for the 30 inches scenario. Given these scenarios, the flood modeling simulation was rerun for both the higher-cost and lower-cost alternative improvement projects. In summary, the results indicate that the higher-cost projects still perform relatively well in case of sea-level rise. The higher-cost projects reduce the length of roads flooded citywide from 250 miles to 3.6 miles (99% reduction) under the 18 inches scenario and to 6.9 miles (97% reduction) under the 30 inches scenario. These projects also reduce the number of flooded structures under the 100-year storm from 5,390 to 519 (90% reduction) under the 18 inches scenario and to 1,034 (81% reduction) under the 30 inches scenario. The lower-cost alternative projects, on the other hand, result in significantly more flooding, especially in coastal areas and around rivers. Under the 18 inches SLR scenario, 95.4 miles of roads (62% reduction) and 1,490 structures (72% reduction) are flooded when the lower-cost alternative projects are selected. These numbers are much higher under the 30 inches sea level scenario: 163.7 miles of roads (35% reduction) and 2,743 structures

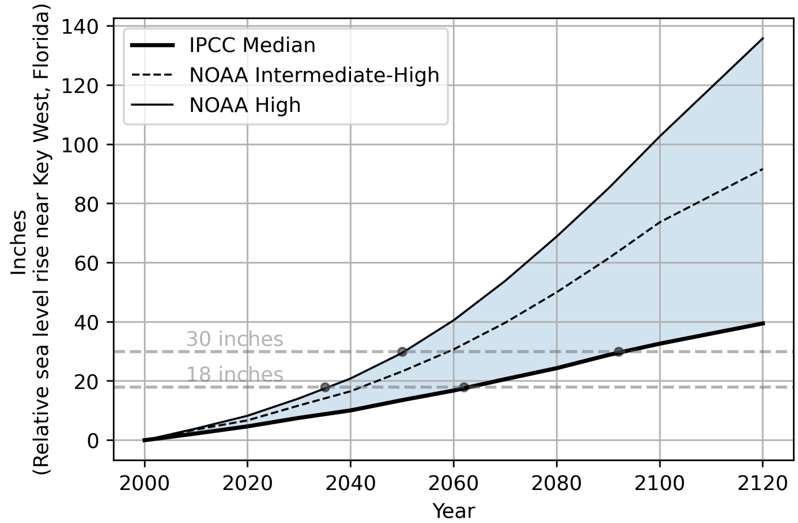
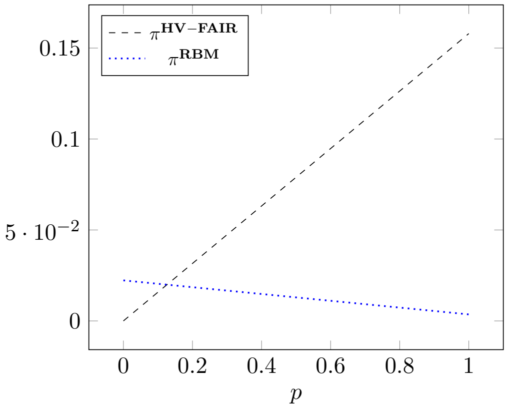


Figure A.4: SLR projections in Southeast Florida (Data from Southeast Florida Regional Sea Level Rise Work Group (2019))

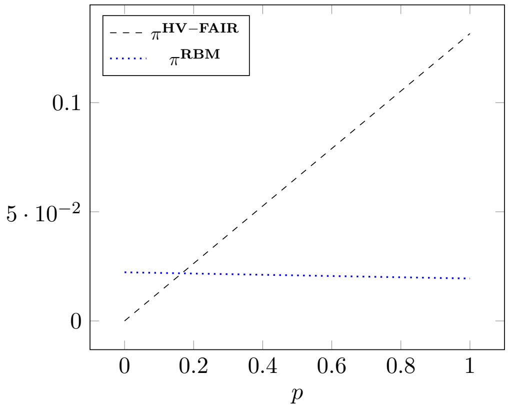
(49% reduction).

The City of Miami's stormwater master plan provides the inundation maps of alternative projects under the 10-year and 100-year design storms, the total miles of roads, and the total number of structures flooded for each sea-level rise scenario and project improvement alternative. However, the master plan does not offer details of the miles of roads and the number of structures flooded under the sea-level rise scenarios for each of the 78 service areas separately. Furthermore, the service level values for service areas without any improvement under the sea-level rise scenarios are not provided in the master plan. That being the case, we used the inundation maps and the total numbers to estimate the missing data for each service area under the sea-level rise scenarios. The inundation maps clearly show more flooding around the coastal areas and rivers, and therefore, those areas are assigned lower service level values. Based on the information provided in the inundation maps with project improvements, we estimate that the sea-level rise would reduce the service level in service areas by 30% and 50% in the case of the 18 and 30 inches scenarios, respectively, compared to the service level of service areas with no sea-level rise under the current conditions. Estimated service levels under each alternative project and the sea-level rise scenarios in the 78 service areas are provided in Appendix A.4.

A.9 Comparing the regret-based model and the deterministic model with different weights of vertical equity



(a) $\beta = 0.5$



(b) $\beta = 0.6$

Figure A.5: Price of the regret-based model vs. the corresponding deterministic model

A.10 Average prices against budget with different problem sizes

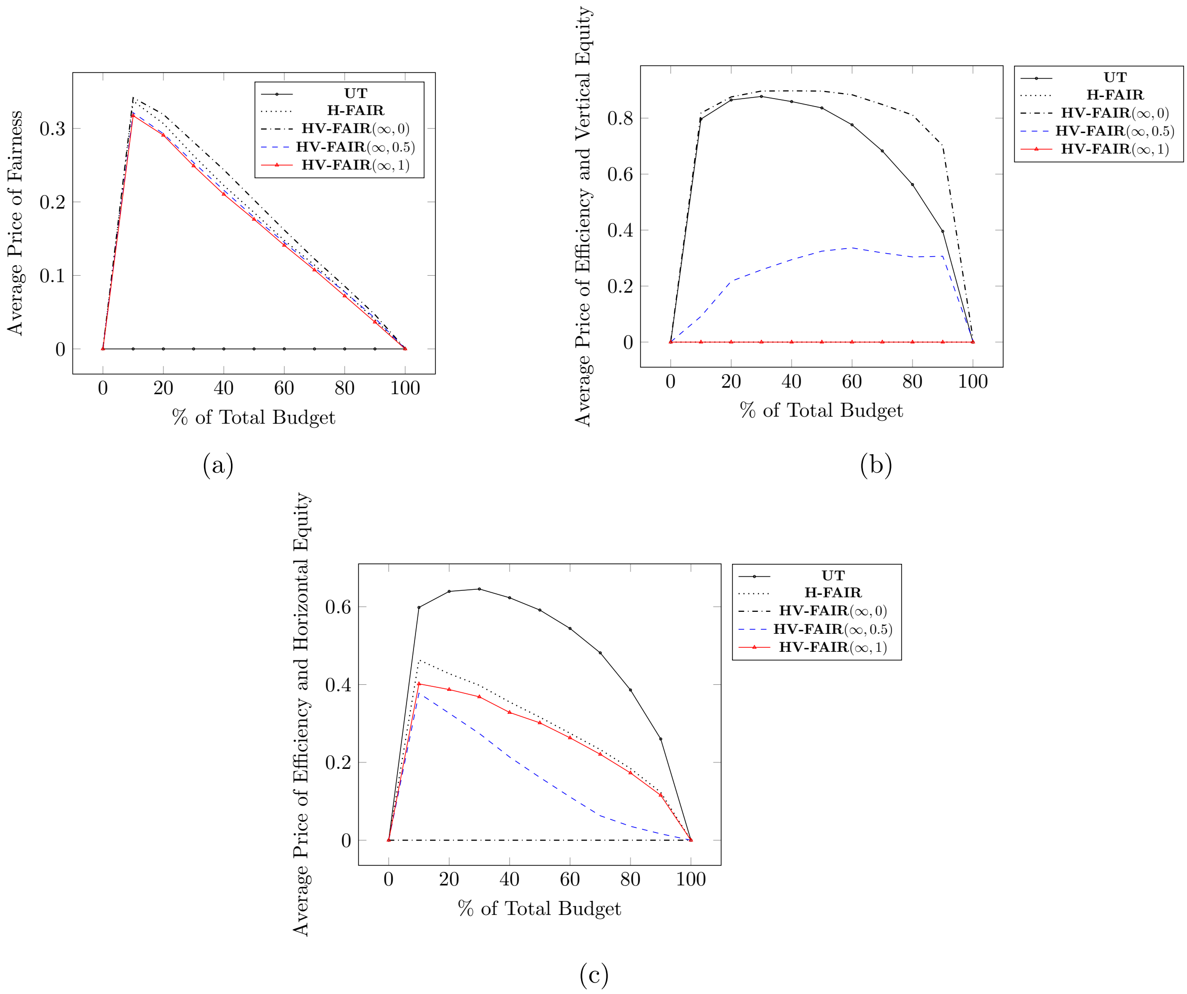
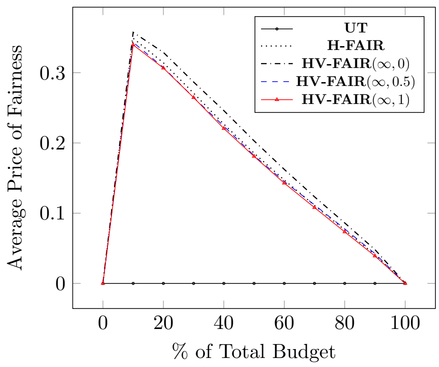
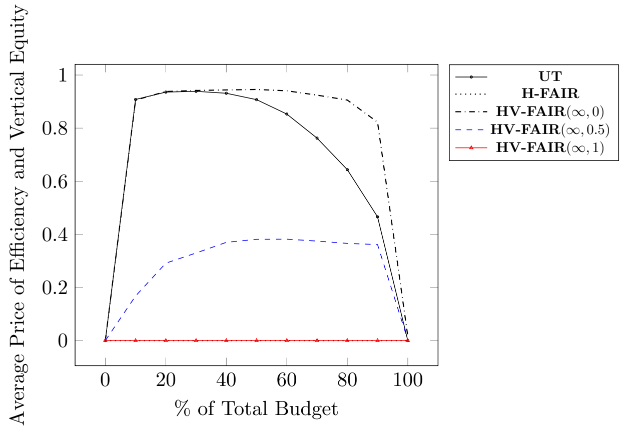


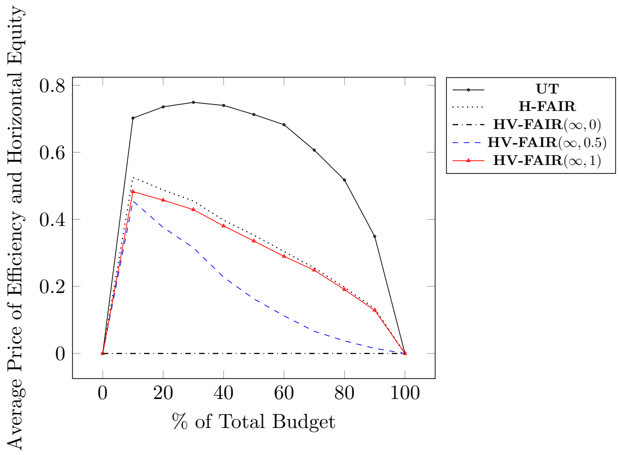
Figure A.6: (a) average price of fairness, (b) average price of efficiency and vertical equity, and (c) average price of efficiency and horizontal equity ($n = 25$)



(a)



(b)



(c)

Figure A.7: (a) average price of fairness, (b) average price of efficiency and vertical equity, and (c) average price of efficiency and horizontal equity ($n = 50$)

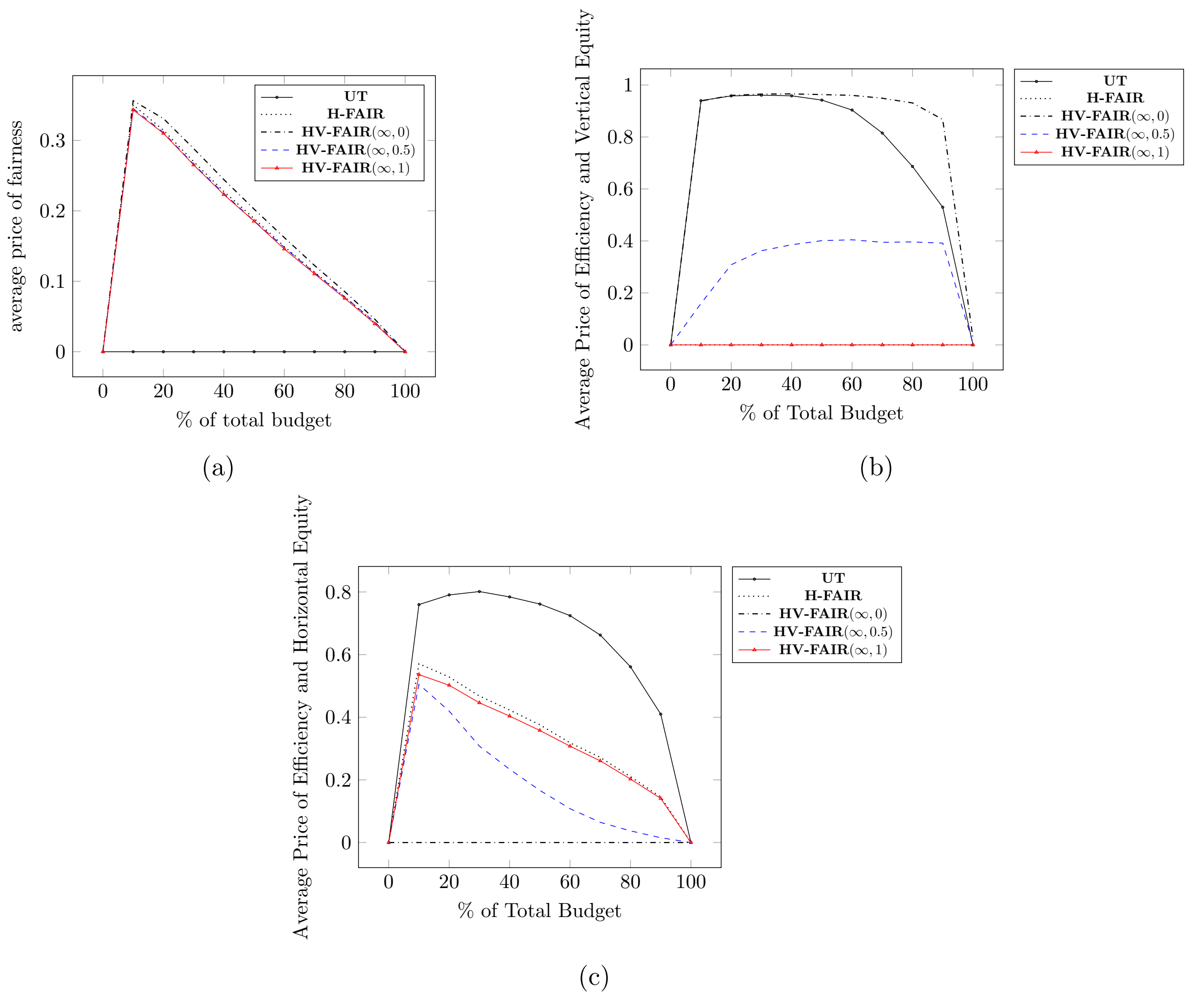
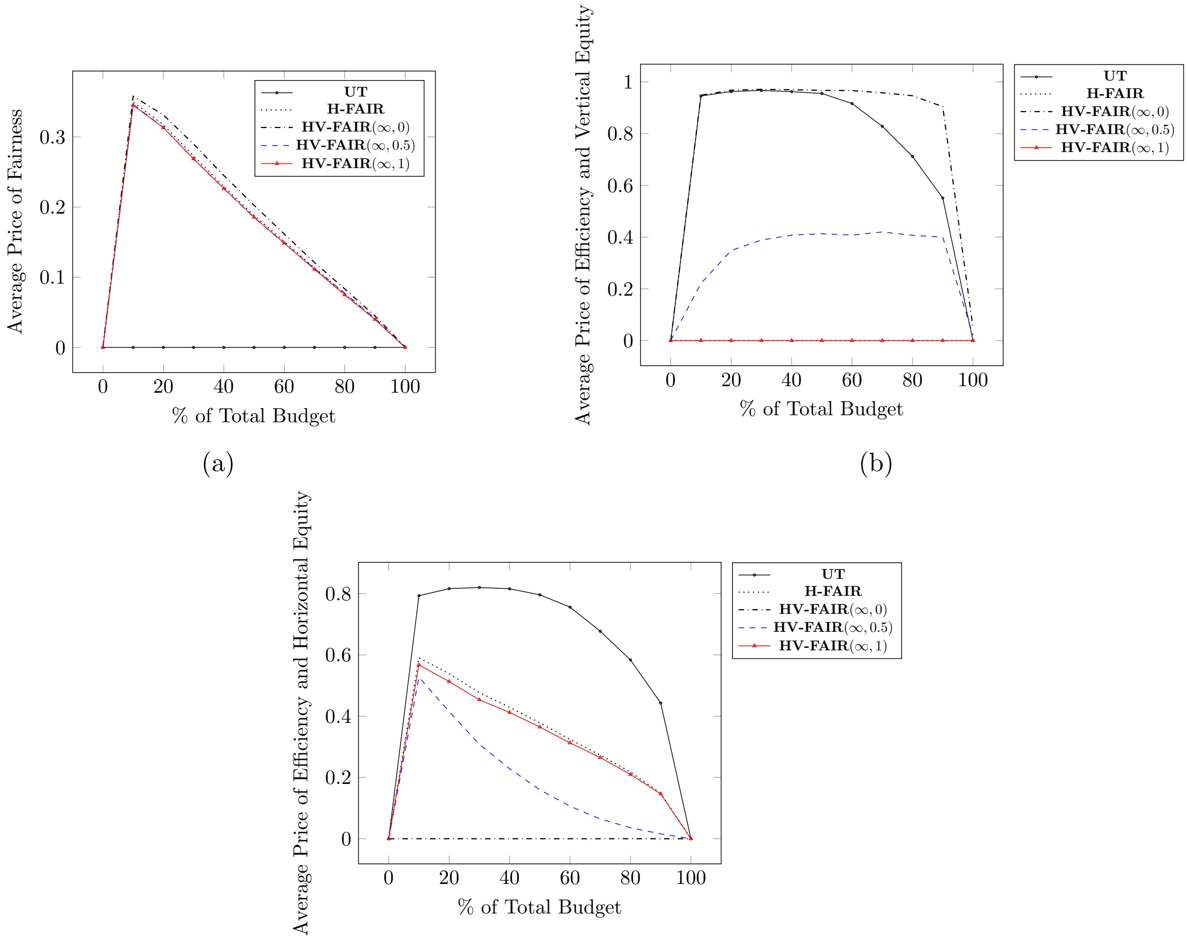


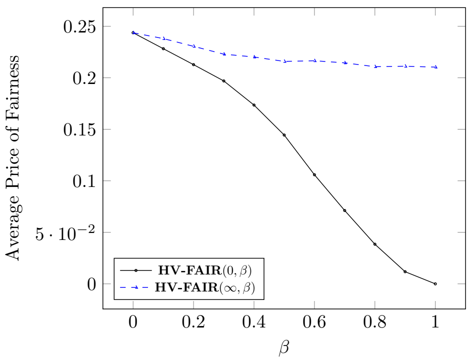
Figure A.8: (a) average price of fairness, (b) average price of efficiency and vertical equity, and (c) average price of efficiency and horizontal equity ($n = 75$)



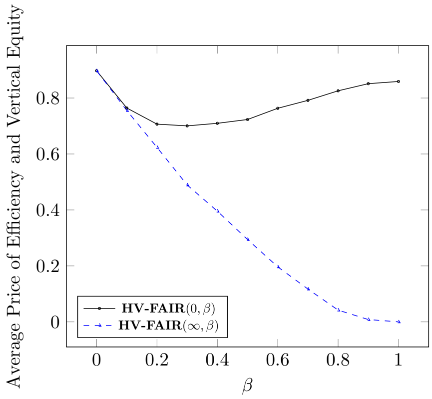
(a) (b) (c)

Figure A.9: (a) average price of fairness, (b) average price of efficiency and vertical equity, and (c) average price of efficiency and horizontal equity ($n = 100$)

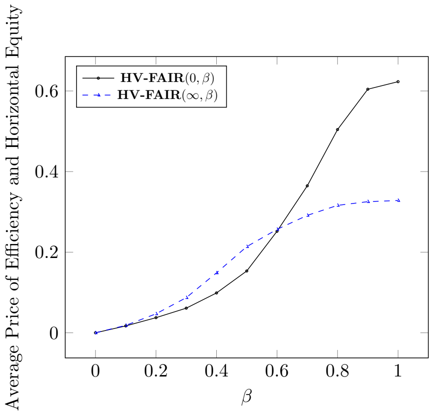
A.11 Impact of the weight of vertical equity on average prices with different problem sizes



(a)

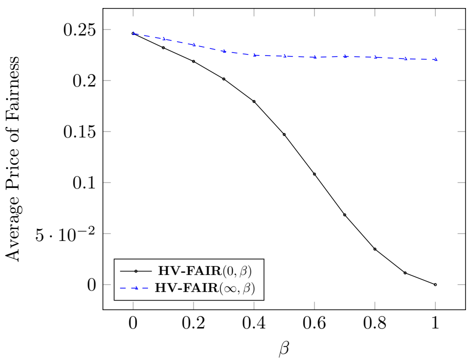


(b)

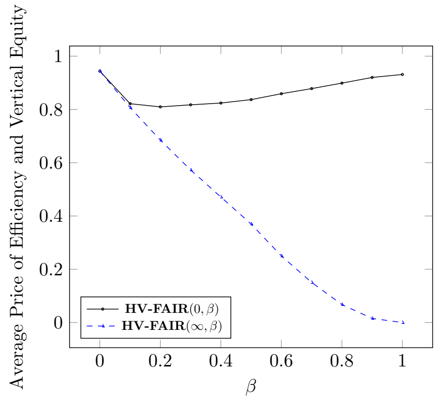


(c)

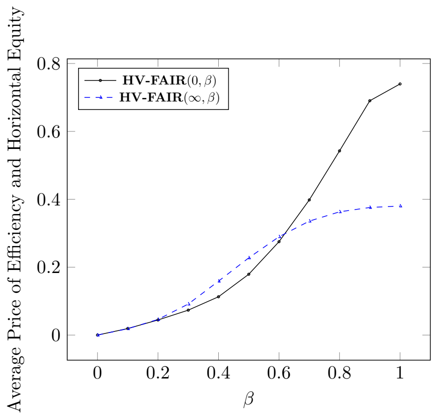
Figure A.10: Impact of β on three average prices ($n = 25$)



(a)

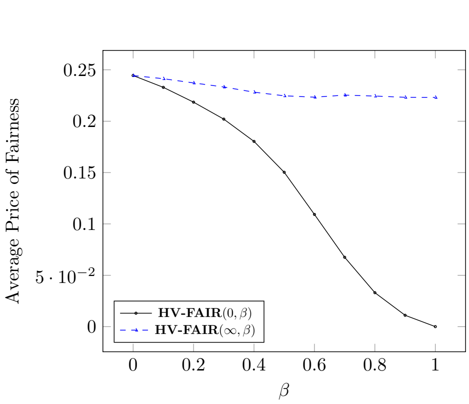


(b)

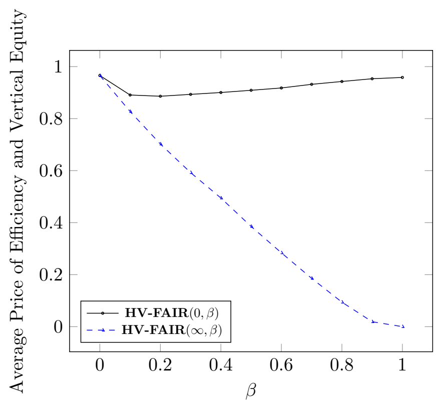


(c)

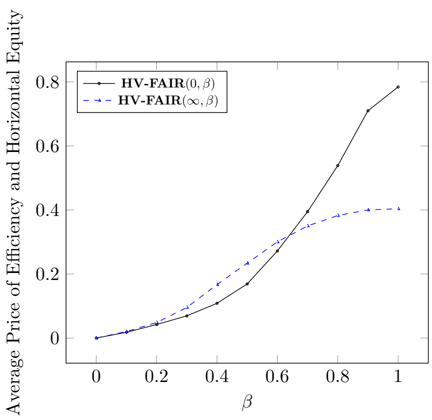
Figure A.11: Impact of β on three average prices ($n = 50$)



(a)



(b)



(c)

Figure A.12: Impact of β on three average prices ($n = 75$)

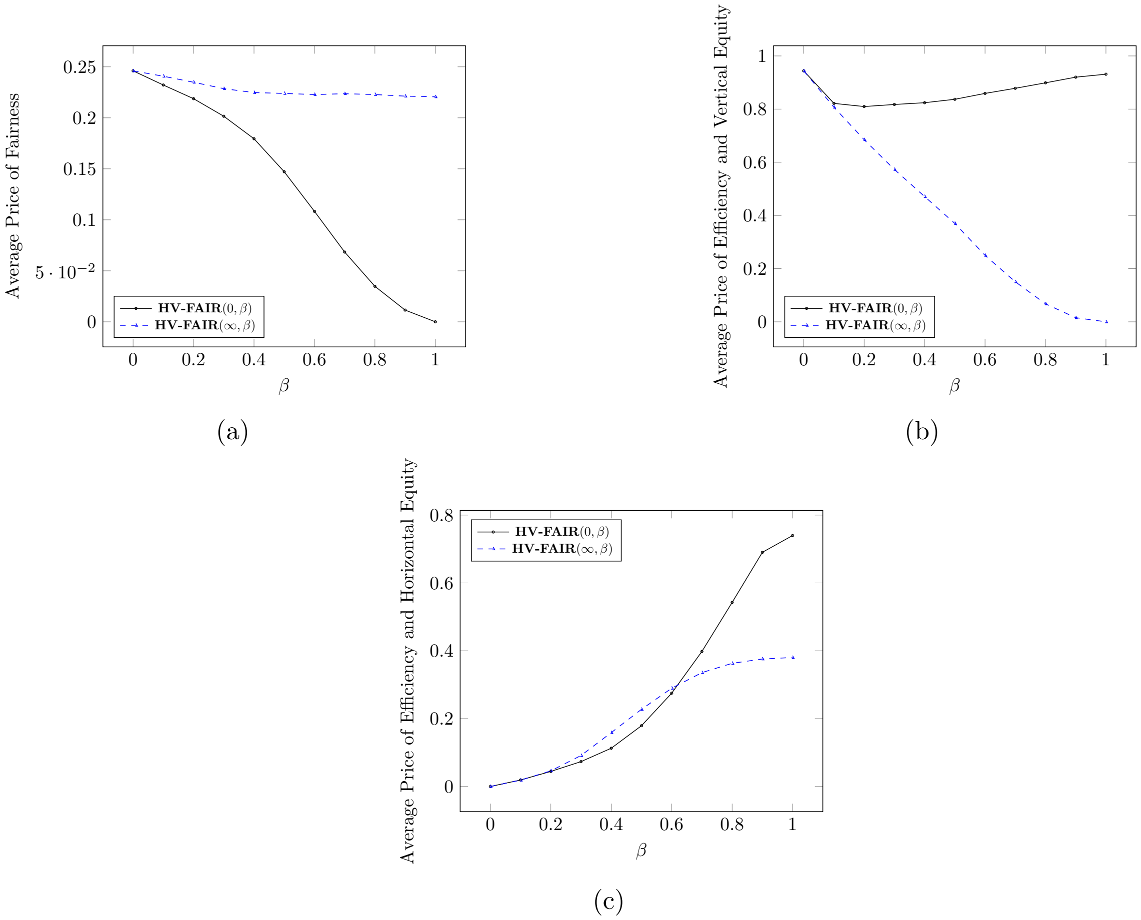


Figure A.13: Impact of β on three average prices ($n = 100$)