Shaking and Balance of a Convertible One- and Two-Cylinder Reciprocating Compressor

by

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(ABSTRACT)

This research involves the study of a one- and two-cylinder convertible reciprocating Freon compressor for air conditioning or refrigeration purposes. The main concern is the reduction of the vibration (noise) caused during the operation of the compressor. Vibration is a main concern when the compressor is shifted from the one-cylinder operation to the two-cylinder operation mode and the reverse of this shift.

The objectives for this research are (1) to investigate the shaking force due to the reciprocating mass at high frequencies, which are up to 4600 Hz (80ω) in this research; (2) to determine the dominant force for compressor vibration among the three possible sources of shaking force due to reciprocating mass, impact forces due to clearance at the connecting rod – piston joint, and the z-axis force from the motor torque due to the rotor's conductor rods being skewed at an angle; (3) to minimize the difference in change of kinetic energies when switching between the one- and two-cylinder operating modes of the compressor.

The properties of the vibration in one- and two-cylinder operation have been studied and results have been analyzed in terms of kinetic energies generated in different setting of operation of the compressor. Dynamic simulation for the impact force is computed using SIMULINK. The Z-axis force due to the motor is computed. Results indicated that shaking force due to the reciprocating mass is the dominant force for only the first two harmonics (ω, 2ω). An optimization routine based on Hooke and Jeeves pattern search method is developed and an optimized setting of angle, force, and torque for balancing of the crankshaft to achieve objective (3) is determined.

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Chapter 1. Introduction

The compressor selected for evaluation is the Bristol H23A two-cylinder reciprocating Freon compressor for air conditioning or refrigeration purposes. Depending on the rotating direction of the crankshaft, the compressor can function in two-cylinder or one-cylinder operating modes. The purpose of the different operating modes is to minimize the energy and wear that will be incurred during the starting up – shutting down cycle of the compressor. For example, if the working environment of the compressor only calls for 40% capacity cooling, and the compressor can only run in a full (100%) capacity of two-cylinder operating mode, the duty cycle of the compressor will be at 40% with the attendant frequent on/off cycling. However, if the compressor is able to run at half (50%) capacity (i.e., the one-cylinder operating mode), the compressor will have a longer duty cycle, hence minimize the energy losses and wear during the start up of the compressor.

The driving source of the compressor is a three-phase, 5.4 kW electric motor with a rotational speed of 3450 rpm. Both compressor and electric motor are housed in a hermetic shell via two side springs, one top spring, and a shock-loop structure to isolate the compressor motions and force from the shell. Figure 1.1 shows the internal components of the compressor that are surrounded by the hermetic shell.

There are three main objectives for this research: (1) To investigate the effect of compressor shaking at high frequencies. General practice in generating the acceleration information of the slider-crank mechanism is only for the first two harmonics of the binomial series expansion, where harmonics range higher than 2ω are deemed insignificant. In this research, kinetic energies for harmonics up to the order of 80ω are generated. (2) To determine the dominant force that resulted in the shaking of the compressor. The three forces that are investigated are the shaking force due to the reciprocating mass; the impact force due to the clearance between the reciprocating (wrist pin) end of the connecting rod and piston; the possible Z-axis force in the motor due to the skew of the conductor rods of the rotor. The shaking force due to the reciprocating mass is computed by $F_S = M_{recip}A_{piston}$, where the reciprocating mass is the summation of the piston mass, the wrist pin mass, and the reciprocating point mass of the connecting rod. The impact force is computed by using SIMULINK to perform a dynamic simulation of the reciprocating end of the connecting rod with the piston when in motion. The Z-axis force of the motor is computed by assuming a certain skew angle of the conductor rods, with the Z-moment, T, of the motor computed by Zheng [7]. (3) The method that can be applied to minimize the difference in kinetic energies between the two operating modes of the compressor, so as to reduce the shaking and hence the vibration and in turn the noise of the compressor when switching from the one-cylinder to the two-cylinder operating mode and vice versa. An optimization algorithm is applied in finding the best setting in terms of angle, force and torque for the crankshaft to achieve this aim.

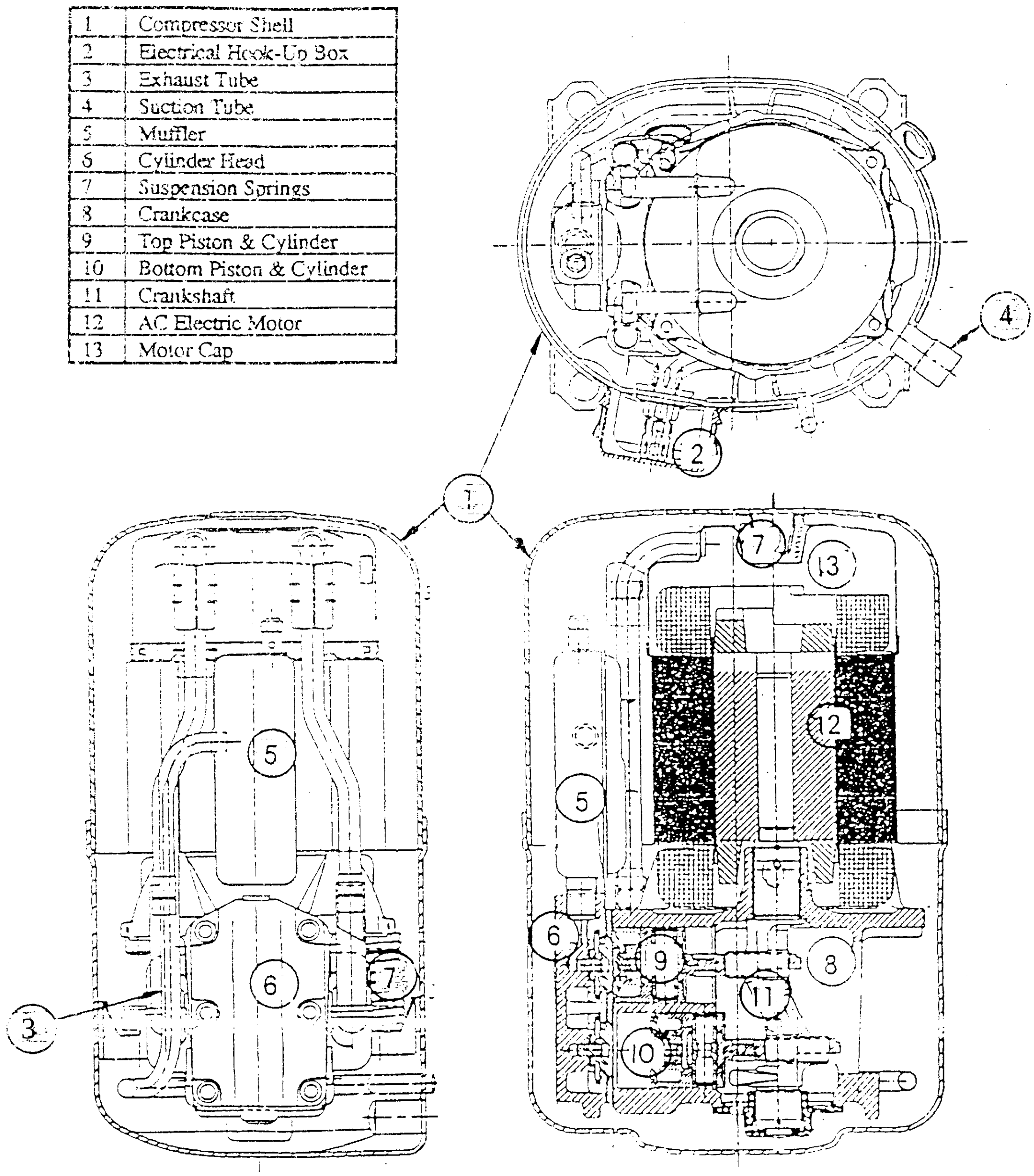


Figure 1.1 Internal components of the Bristol H25A Compressor. (From Zheng [7])

1.1 Background on Shaking Force and Shaking Couple

This section serves as a general background understanding on the generation of shaking force and shaking couple in a reciprocating machine. It is mainly extracted from Mabie and Reinholtz [1].

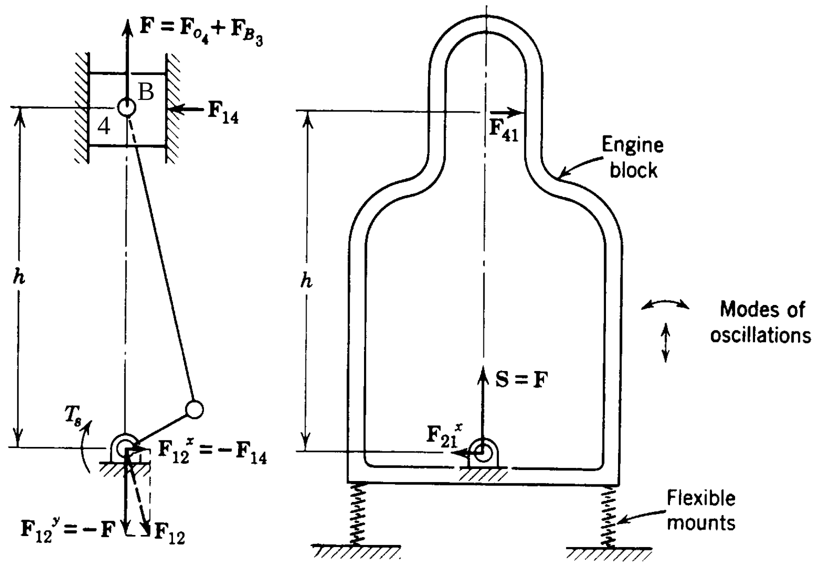


Figure 1.2 Modes of oscillations due to shaking force. (From Mabie and Reinholtz [1])

Figure 1.2 shows a free-body diagram of a slider-crank mechanism. $\mathbf{Fo}_4$ is the inertia force vector acting on the piston center (link 4), whereby the magnitude is $Fo_4 = M_4A_{g4}$. $\mathbf{F}_{B3}$ is the inertia force vector acting on the reciprocating point mass of the connecting rod, M_{B3}, which is located at the wrist pin center B and coincide with the piston center. The magnitude of $\mathbf{F}_{B3}$ is $F_{B3} = M_{B3}A_B$. It is noted that since the piston center coincides with the wrist pin center B, $A_{g4} = A_B$. $\mathbf{F}_{41}$ is the horizontal reactive force of the piston on the cylinder wall. $\mathbf{F}_{21}$ is the force of the crankshaft acting on the main bearings fixed to the engine block. The summation of the inertia forces $\mathbf{Fo}_4$ and $\mathbf{F}_{B3}$ will generate a resultant *shaking force* $\mathbf{S}$, which in turn will cause the engine block to oscillate up and down depending on the direction of the inertia forces during the phases of the mechanism cycle. Hence the effect of the reciprocating mass, which is the mass of the piston M_4 plus the reciprocating point mass M_{B3} of the connecting rod, on the engine block is the vertical shaking or vertical vibration of the engine. The shaking of the engine block is mainly due to the inertia forces of the reciprocating masses located at the wrist pin. The rotating masses, which are the masses that rotate with the crankshaft, are normally balanced and hence do not contribute to the shaking of the engine.

The shaft torque T_s, with a magnitude of $T_s = \mathbf{F}_{41}h$, generates a couple that acts on the engine block, and as the shaft torque changes in magnitude and direction during the phases of the engine cycle, it causes the block to oscillate in a rotational manner. Hence shaft torque T_s is being addressed as the Z-axis *shaking couple* for this single cylinder example. There is an additional shaking couple in the 2 cylinder machine.

1.2 Data from Previous Work

Previous analysis had been done by several students during the period from 1993 to 1995. Among them, Ramakant Arcot [3], in his work on the study of the compressor, had defined the center of mass of the system in a three-dimensional coordinate system. He had also developed the total assembly stiffness matrix for the three springs and the shock-loop used in mounting the compressor to the compressor shell. The total mass and inertia matrix defining the mass and inertia properties of the compressor assembly had also been developed. This information is the basis of Chapter 3 and is used in this research in decoupling the forced vibration equations.

Chapter 2. Development of Equations Determining the Higher Order Terms for Displacement, Velocity, and Acceleration in Slider-Crank Mechanism

2.1 The Slider-Crank Mechanism

The one- and two-cylinder convertible reciprocating compressor utilizes the slider-crank mechanism in its operation. For the application of this mechanism in a compressor, an electric motor will be used to drive the crank, in which the force will be transmitted through the connecting rod to drive the piston to compress the air. Figure 2.1(a) shows a simple layout of the four-bar slider-crank mechanism. Link 1 is the ground frame and it is fixed, link 2 is the crank, link 3 is the connecting rod, and link 4 is the slider, which in this case is the piston that is sliding along the cylinder compressing the working fluid.

2.1.1 Equations for Displacement, Velocity, and Acceleration of the Sliding Piston

As discussed in Mabie and Reinholtz [1], the displacement *X* of the sliding piston (link 4) with respect to Top Dead Center (TDC) can be determined by, from Figure 2.1b,

$$\begin{aligned} X &= (R+L)-(R\cos\theta + L\cos\phi) \\ &= R(1-\cos\theta)+L(1-\cos\phi) \end{aligned} \tag{2.1}$$

It can also be seen that

$$\begin{aligned} L\sin\phi &= R\sin\theta \\ \sin\phi &= \frac{R}{L}\sin\theta \end{aligned} \tag{2.2}$$

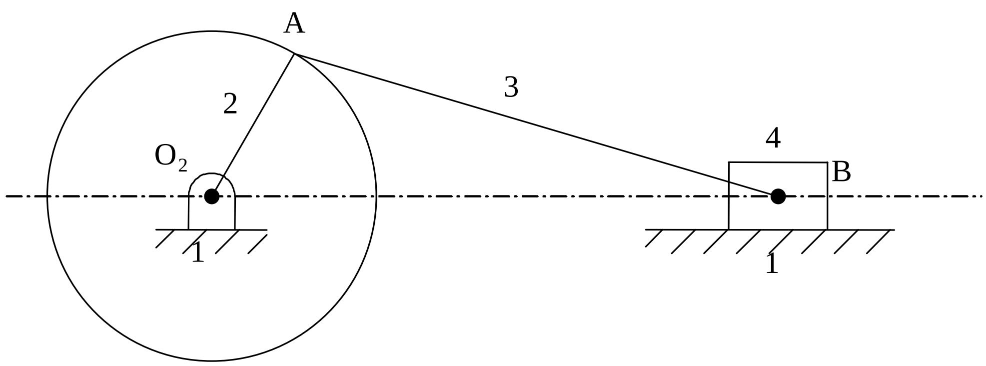


Figure 2.1a

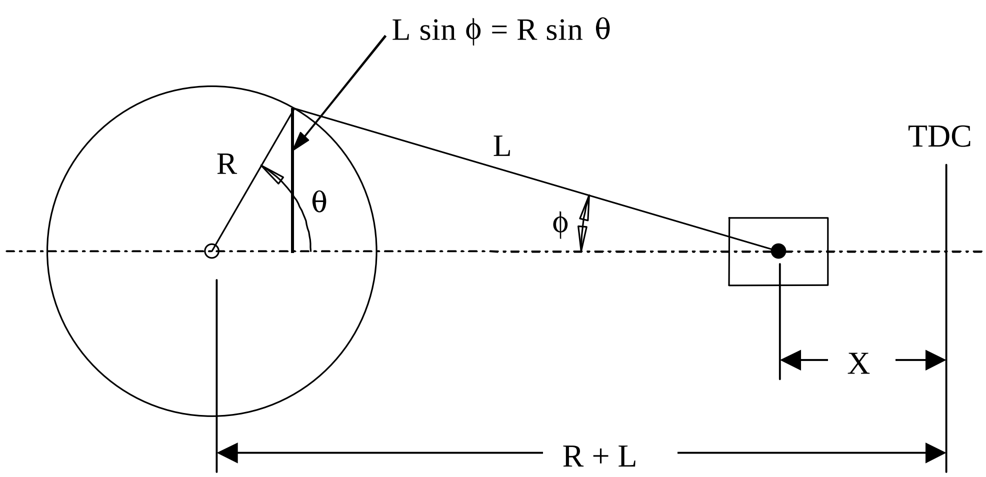


Figure 2.1b

Figure 2.1 The Slider-Crank Mechanism

By using the trigonometric identity of $\cos\phi = \sqrt{1-\sin^2\phi}$ and substituting Equation (2.2) into Equation (2.1), the displacement X can be written as

$$X = R(1-\cos\theta) + L\left[1-\sqrt{1-\left(\frac{R}{L}\right)^2 \sin^2\theta}\ \right] \qquad (2.3)$$

Equation(2.3) can be further simplified by approximating the radical $\sqrt{1-\left(\frac{R}{L}\right)^2 \sin^2\theta}$ by replacing it with the binomial series of

$$\left(1 \pm B^2\right)^{1/2} = 1 \pm \frac{1}{2}B^2 - \frac{B^4}{2\cdot 4} \pm \frac{1\cdot 3B^6}{2\cdot 4\cdot 6} - \frac{1\cdot 3\cdot 5B^8}{2\cdot 4\cdot 6\cdot 8} \pm \cdots$$

where $B = \left(\frac{R}{L}\right)\sin\theta$.

In general application, sufficient accuracy can be obtained by using only the first two terms of the series. Applying this to Equation (2.3) gives

$$\begin{aligned} X &= R\left(1-\cos\theta\right) + L\left[1-\left(1-\frac{1}{2}\left(\frac{R}{L}\right)^2 \sin^2\theta\right)\right] \\ &= R\left(1-\cos\theta\right) + \frac{R^2}{2L}\sin^2\theta \end{aligned} \qquad (2.4)$$

With θ = ω*t* where ω is constant, and taking the first and second derivatives of *X*, the velocity and acceleration of the sliding piston are, respectively

$$V = \frac{dx}{dt} = R\omega\left[\sin\theta + \frac{R}{2L}\sin 2\theta\right] \qquad (2.5)$$

$$A = \frac{d^2 x}{dt^2} = R\omega^2 \left[\cos\theta + \frac{R}{L}\cos 2\theta \right] \qquad (2.6)$$

2.1.2 Equations with the Higher Order Terms

As stated in section 2.1.1, it is of common practice to utilize only at the first two terms of the binomial series expansion in deriving the displacement, velocity, and acceleration expressions for a slider-crank mechanism. In this research, acceleration of higher order in the binomial series will be taken into consideration instead of just the first two. This allows a closer examination of the behavior of the reciprocating compressor at higher harmonic frequency. Harmonic domains of up to 80ω will be studied.

The motor that is used to drive the compressor is a three-phase, 5.4 kW electric motor rotating at 3450 rpm. This can be converted, in term of angular velocity ω, to be

$$3450 rpm \times \frac{1}{60\sec} \times 2\pi = 361.28 \, {}^{rad}\!/_{\sec}$$

As an illustration for determining the derivatives in higher order terms, an expansion of the expression up to the order of 8 will be used here. In section 3.1.1, the expression for the displacement of the slider in the slider-crank mechanism, before the simplification, has been determined to be

$$X = R(1 - \cos\theta) + L\left[1 - \sqrt{1 - \left(\frac{R}{L}\right)^2 \sin^2\theta}\,\right] \qquad (2.7)$$

By replacing the radical $\sqrt{1 - \left(\frac{R}{L}\right)^2 \sin^2\theta}$ of Equation (2.7) with the binomial series expansion of up to the order of 8, we have

$$X = R(1-\cos\theta) + L\left[1 - \left(1 - \frac{1}{2}B^2 - \frac{1}{2\cdot 4}B^4 - \frac{1\cdot 3}{2\cdot 4\cdot 6}B^6 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}B^8\right)\right]$$

$$= R(1-\cos\theta) + L\left(\frac{1}{2}B^2 + \frac{1}{2\cdot 4}B^4 + \frac{1\cdot 3}{2\cdot 4\cdot 6}B^6 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}B^8\right)$$

Substituting $B = \left(\frac{R}{L}\right)\sin\theta$ into the above expression gives

$$X = R(1-\cos\theta) + L\left[\frac{1}{2}\left(\frac{R}{L}\right)^2 \sin^2\theta + \frac{1}{2\cdot 4}\left(\frac{R}{L}\right)^4 \sin^4\theta + \frac{1\cdot 3}{2\cdot 4\cdot 6}\left(\frac{R}{L}\right)^6 \sin^6\theta + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}\left(\frac{R}{L}\right)^8 \sin^8\theta\right] \tag{2.8}$$

To find an expression for the time derivative of *X*, the powers of sine will have to be expressed in terms of multiple angles. The conversion will also allow the determination of a linear expression in which each term can be studied separately. From Jeffrey [2], this can be done by utilizing the symbolic expression of

$$\sin^{2n}\theta = \frac{1}{2^{2n}}\left\{\sum_{k=0}^{n-1}(-1)^{n-k}\,2\binom{2n}{k}\cos 2(n-k)\theta + \binom{2n}{n}\right\} \tag{2.9}$$

where n = 1, 2, …

In terms of multiple angles, Equation (2.8) can be written as

$$X = R(1-\cos\theta) + L\left[\frac{1}{2}\left(\frac{R}{L}\right)^2\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) + \frac{1}{2\cdot 4}\left(\frac{R}{L}\right)^4\left(\frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{16}\cos 4\theta\right) + \right.$$

$$\frac{1\cdot 3}{2\cdot 4\cdot 6}\left(\frac{R}{L}\right)^6\left(\frac{5}{16} - \frac{15}{32}\cos 2\theta + \frac{3}{16}\cos 4\theta - \frac{1}{32}\cos 6\theta\right) +$$

$$\left.\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}\left(\frac{R}{L}\right)^8\left(\frac{35}{128} - \frac{7}{16}\cos 2\theta + \frac{7}{32}\cos 4\theta - \frac{1}{16}\cos 6\theta + \frac{1}{128}\cos 8\theta\right)\right]$$

By simplifying and regrouping, the displacement X of the sliding piston is expressed as

$$X = \frac{5R^8}{16384L^7}\cos 8\theta - \frac{R^6\left(4L^2+5R^2\right)}{2048L^7}\cos 6\theta + \frac{R^4\left(64L^4+48L^2R^2+35R^4\right)}{4096L^7}\cos 4\theta$$
$$-\frac{R^2\left(512L^6+128L^4R^2+60L^2R^4+35R^6\right)}{2048L^7}\cos 2\theta - R\cos\theta$$
$$+\frac{R\left(16384L^7+4096L^6R+768L^4R^3+320L^2R^5+175R^7\right)}{16384L^7} \tag{2.10}$$

By taking the time derivatives of X with $\theta = \omega t$, the velocity $V = \dfrac{dX}{dt}$ is

$$V = -\frac{5R^8\omega}{2048L^7}\sin 8\theta + \frac{3R^6\omega\left(4L^2+5R^2\right)}{1024L^7}\sin 6\theta - \frac{R^4\omega\left(64L^4+48L^2R^2+35R^4\right)}{1024L^7}\sin 4\theta$$
$$+\frac{R^2\omega\left(512L^6+128L^4R^2+60L^2R^4+35R^6\right)}{1024L^7}\sin 2\theta + R\omega\sin 2\theta \tag{2.11}$$

And the acceleration $A = \dfrac{dV}{dt}$ is

$$A = -\frac{5R^8\omega^2}{256L^7}\cos 8\theta + \frac{9R^6\omega^2\left(4L^2+5R^2\right)}{512L^7}\cos 6\theta - \frac{R^4\omega^2\left(64L^4+48L^2R^2+35R^4\right)}{256L^7}\cos 4\theta$$
$$+\frac{R^2\omega^2\left(512L^6+128L^4R^2+60L^2R^4+35R^6\right)}{512L^7}\cos 2\theta + R\omega^2\cos\theta \tag{2.12}$$

It can be seen from here that the process of using Equation (2.9) in converting the powers of sine to its multiple angle equivalents by hand-calculation can be tedious and time consuming. However, with the help of a digital computer and appropriate mathematical software, this process can be made simple. The mathematical software used for converting the powers of sine to its multiple angle equivalents in this part of the thesis is *DERIVE®* for Windows, version 4.11. By *DERIVE®*, The whole expression of Equation (2.9) can be input as is, and with the appropriate value substituted for *n*, the multiple angle equivalent of the respective power of sine can be determined.

2.1.3 Equations Developed for Displacement, Velocity, and Acceleration of the Sliding Piston for Higher Order Terms

Rewriting Equation (2.8) in terms of its multiple angle equivalent, the process of re-substituting the determined multiple angle expressions into the binomial expansion proved to be a painstaking process even with the help of *DERIVE*®. In this research, the frequency domains that are under study are up to the order of 80, with the coefficients of the acceleration of concerned. Hence this re-substituting process for distance *X* and then going through the derivatives to determine the acceleration can be very tedious and painful, along with a lot of attention required.

Based on the symbolic expression of Equation (2.9), new symbolic expressions have been developed for the distance *X*, velocity *V*, and acceleration *A* of the sliding piston, or slider in the sliding-crank mechanism. These symbolic expressions allow the *X*, *V*, and *A*, to be expressed in their respective higher order terms, without going through the process of determining multiple angle equivalents of the powers of sine and cosine and the re-substitution. The three developed symbolic expressions for the distance, velocity, and acceleration, respectively, are

$$X = R(1-\cos\theta) + L\left\{\sum_{j=1}^{n}\left\{\frac{1}{2^{2j}}\left[\sum_{k=0}^{j-1}(-1)^{j-k}\,2\binom{2j}{k}\cos 2(j-k)\theta + \binom{2j}{j}\right]\cdot\left[\frac{\left(\frac{R}{L}\right)^{2j}}{2(j-0.5)}\prod_{s=1}^{j}\frac{2s-1}{2s}\right]\right\}\right\} \tag{2.13}$$

$$V = R\omega\sin\theta + L\left\{\sum_{j=1}^{n}\left\{\frac{1}{2^{2j}}\left[\sum_{k=0}^{j-1}(-1)^{j-k+1}4(j-k)\omega\binom{2j}{k}\sin 2(j-k)\theta\right]\cdot\left[\frac{\left(\frac{R}{L}\right)^{2j}}{2(j-0.5)}\prod_{s=1}^{j}\frac{2s-1}{2s}\right]\right\}\right\}$$

(2.14)

$$A = R\omega^2 \cos\theta + L\left\{\sum_{j=1}^{n}\left\{\frac{1}{2^{2j}}\left[\sum_{k=0}^{j-1}(-1)^{j-k+1}8((j-k)\omega)^2\binom{2j}{k}\cos 2(j-k)\theta\right]\cdot\left[\frac{\left(\frac{R}{L}\right)^{2j}}{2(j-0.5)}\prod_{s=1}^{j}\frac{2s-1}{2s}\right]\right\}\right\}$$

(2.15)

With the developed symbolic expressions, any higher orders of the expressions for displacement, velocity, and acceleration of the slider in a slider-crank mechanism can be determined by inputting a value for *n*. For example, to determine the expression for the acceleration of the slider up to the order of 80, expression (2.15) is input into *DERIVE*® and *n* is substituted by a value of 40. That is, value of $n = \frac{desired\_higher\_order}{2}$. The coefficient values of the acceleration at each respective harmonic can be obtained by substituting the crank radius *R* and connecting rod length *L* with the appropriate values.

2.2 Developed Symbolic Expressions Verification

The verification of the symbolic expressions developed for the distance, velocity, and acceleration for higher order terms of the slider in a slider-crank mechanism, Eqs. (2.13), (2.14), and (2.15) respectively, are done by using *DERIVE®*. Two verifications have been done, one by the truncated values, which follows the general practice of using only the first two terms comparison, and the other by the long substitution method comparison. Since acceleration is of concern in this thesis, the verifications will be carried out using the expression for acceleration. Values for the crank radius *R* will be 0.5 inch and the length of the connecting rod *L* will be 2.375 inches, with the crankshaft rotating at an angular velocity ω of 361 rad/s.

2.2.1 Verification by Truncated Values

In this verification, the truncated values are obtained by Equation (2.6)

$$A = \frac{d^2X}{dt^2} = R\omega^2\left(\cos\theta + \frac{R}{L}\cos 2\theta\right).$$

A graph for this expression is plotted in DERIVE® and is compared to the graph plotted by using the developed expression with the higher order terms up to 80. Both graphs are shown overlapping each other in Figure 2.2a, with a zoom showing the slight difference in results in Figure 2.2b.

2.2.2 Verification by Long Substitution Method

This verification is done by comparing the expression obtained by the conventional converting powers of sine to multiple angle equivalents and re-substitution into the binomial expansion, in which the procedure has been described in section 2.1.3, to the expression obtained by the symbolic expression developed. As the substitution procedure is lengthy and tedious, higher order terms to the order of 10 only will be used for the verification.

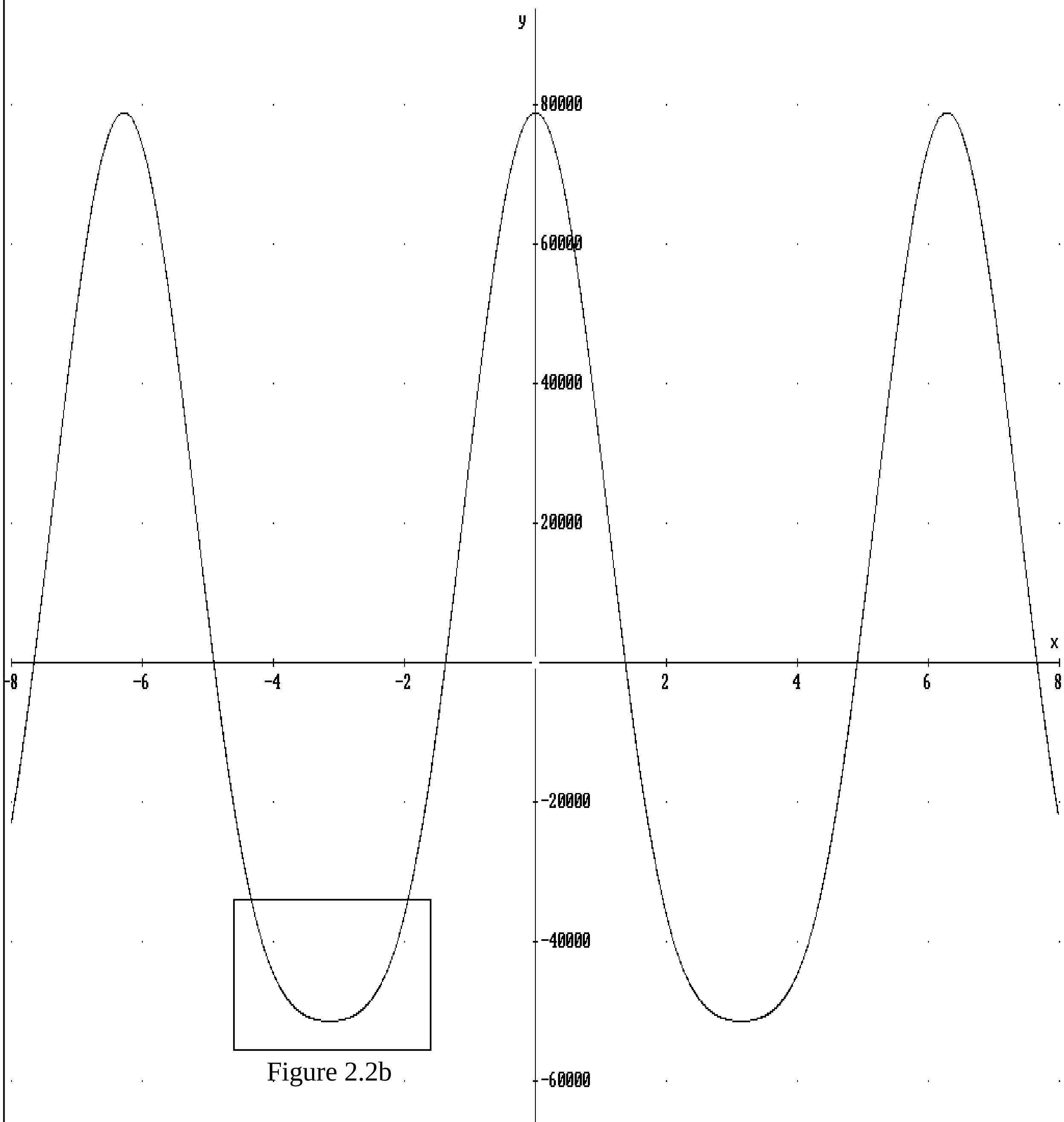


Figure 2.2a Graphs of truncated values and developed expression overlapping each other.

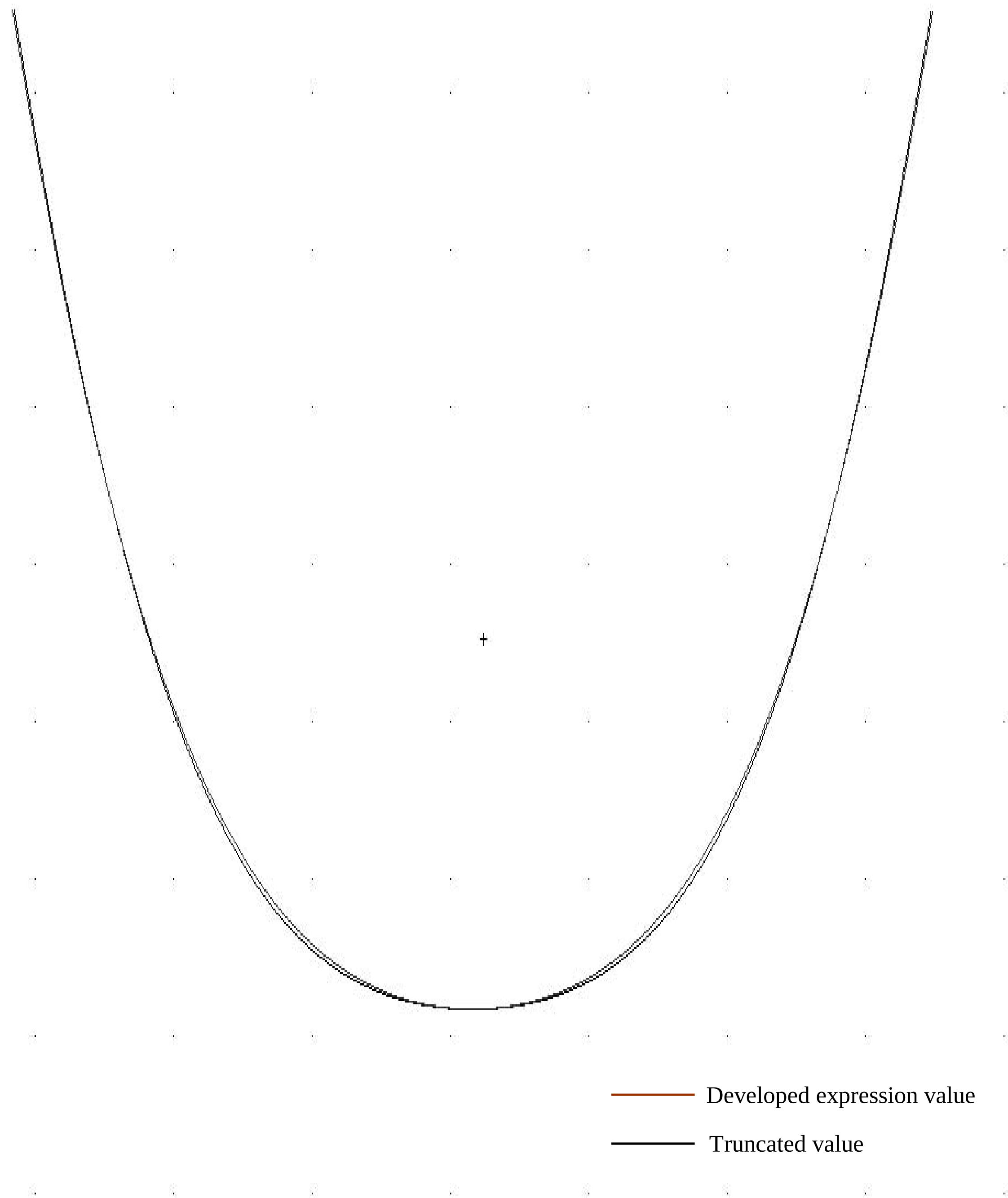


Figure 2.2b Zoom of Figure 2.2a showing a slight difference in results from the two plots.

Comparing both the expressions obtained by the long substitution and the developed expression, and the graphs plotted from both by using *DERIVE*®, they show a good match. The graphs plotted are similar to that shown in Figure 2.2a.

From both verifications, it can be concluded that the developed symbolic expressions for the higher order terms are accurate.

2.3 The Coefficients of the Harmonic Terms

With values substituted for L, R, and ω, a numerical coefficient at any desired harmonic can be obtained. However, one factor that may be overlooked with the coefficients is that, for example, the coefficient of the 10th harmonic (cos10θ) of a 10th-order expansion is different from that of a 20-order expansion. For example, with L = 2.375 inches, R = 0.5 inches, and ω = 361 rad/s, the coefficient of the 10th harmonic of a 10-order expansion is computed to be 0.000282702 whereas that of a 20-order expansion is computed to be 0.000312864.

To show that the difference in coefficients is not due to the error of the developed expressions, a demonstration of the differences is done by using hand-calculated expressions of order of 4 (n = 2) and order of 6 (n = 3) for the comparison. For the expression of 4-order, the acceleration expression is

$$A_4 = \frac{d^2X}{dt^2} = R\omega^2\cos\theta + L\left[\frac{1}{2}\left(\frac{R}{L}\right)^2\omega^2\left(2\cos 2\theta\right) + \frac{1}{8}\left(\frac{R}{L}\right)^4\omega^2\left(2\cos 2\theta - \frac{4}{2}\cos 4\theta\right)\right]$$

Simplified and regrouped,

$$A_4 = R\omega^2\cos\theta + L\left[\left\{\left(\frac{R}{L}\right)^2\omega^2 + \frac{1}{4}\left(\frac{R}{L}\right)^4\omega^2\right\}\cos 2\theta - \left\{\frac{1}{4}\left(\frac{R}{L}\right)^4\omega^2\right\}\cos 4\theta\right] \qquad (2.16)$$

The acceleration expression for the 6-order expression is

$$A_6 = \frac{d^2X}{dt^2} = R\omega^2\cos\theta + L\left[\frac{1}{2}\left(\frac{R}{L}\right)^2\omega^2\left(2\cos 2\theta\right) + \frac{1}{8}\left(\frac{R}{L}\right)^4\omega^2\left(2\cos 2\theta - \frac{4}{2}\cos 4\theta\right)\right.$$
$$\left. + \frac{1}{16}\left(\frac{R}{L}\right)^6\omega^2\left(\frac{15}{8}\cos 2\theta - 3\cos 4\theta + \frac{9}{8}\cos 6\theta\right)\right]$$

Simplified and regrouped,

$$A_6 = R\omega^2\cos\theta + L\left[\left\{\left(\frac{R}{L}\right)^2\omega^2 + \frac{1}{4}\left(\frac{R}{L}\right)^4\omega^2 + \frac{15}{128}\left(\frac{R}{L}\right)^6\omega^2\right\}\cos 2\theta\right.$$

$$\left. - \left\{\frac{1}{4}\left(\frac{R}{L}\right)^4\omega^2 + \frac{3}{16}\left(\frac{R}{L}\right)^6\omega^2\right\}\cos 4\theta + \left\{\frac{9}{128}\left(\frac{R}{L}\right)^6\omega^2\right\}\cos 6\theta\right] \qquad (2.17)$$

By looking at the coefficients of the 4th harmonic (cos4θ) in Equation (2.16) and (2.17), it is immediately noted that there is an extra term of $\left[\frac{3}{16}\left(\frac{R}{L}\right)^6\omega^2\right]$ in the coefficient of Equation (2.17) compare to Equation (2.16). This explained the difference in coefficient values of the same harmonic but different order of expansion.

Chapter 3. System Parameters and Coordinate Definition

3.1 The Center of Mass of the Assembled Compressor

A three-dimensional coordinate system for the compressor assembly had been determined by Arcot [3] (Figure 3.1). The location of the center of mass (X_g, Y_g, Z_g) of the compressor with respect to an origin coordinate axes (X_o, Y_o, Z_o) defined at the Top Dead Center of the bottom cylinder is:

$$(X_g, Y_g, Z_g) = (0.01, -3.51, 5.71)$$

In this research, the offset in X_g will be set to zero as the value is insignificant compared with that of Y_g and Z_g in order to reduce the complexity of the problem. Hence the new center of mass for the compressor will be:

$$(X_g, Y_g, Z_g) = (0, -3.51, 5.71)$$

3.2 The System Mass and Inertia Matrix and the Stiffness Matrix

The system mass and inertia matrix, M_g, and the total assembly stiffness matrix, K_g, used in this research are adapted from those that were developed by Arcot. Accordingly, the system mass and inertia matrix is:

$$[M_g] = \begin{bmatrix} M_f & 0 & 0 & 0 & 0 & 0 \\ 0 & M_f & 0 & 0 & 0 & 0 \\ 0 & 0 & M_f & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{yx} & I_{yy} & -I_{yz} \\ 0 & 0 & 0 & -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}$$

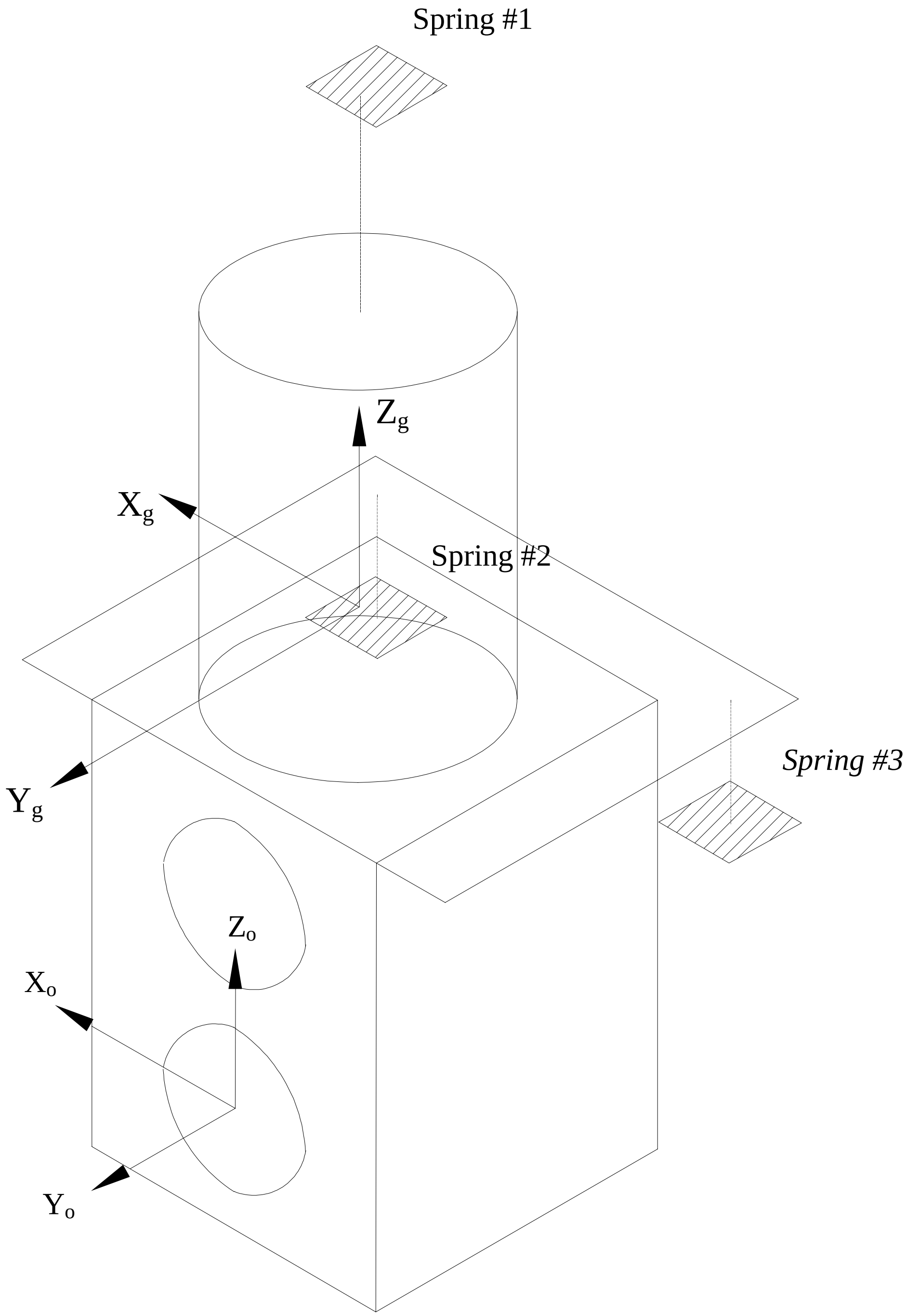


Figure 3.1 Location of the Compressor Center of Mass (X_g, Y_g, Z_g)

With the values of the mass matrix, M_f, and the inertia matrix, I, of the system obtained by Arcot,

$$[M_g] = \begin{bmatrix} .206 & 0 & 0 & 0 & 0 & 0 \\ 0 & .206 & 0 & 0 & 0 & 0 \\ 0 & 0 & .206 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.37 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.97 & .49 \\ 0 & 0 & 0 & 0 & .49 & 1.45 \end{bmatrix}$$

The units of the mass sub-matrix and the inertia sub-matrix are $\frac{lb \cdot s^2}{in}$ and $lb.in.s^2$, respectively.

The total assembly stiffness matrix, which is the sum of element stiffness matrix for Spring #1, Spring #2, Spring #3, and Spring #4 (the shockloop) is

$$[K_g] = \begin{bmatrix} 1328 & 18.9 & 14.1 & 101 & -2123 & 385 \\ 18.9 & 1355 & 36.4 & 2246 & -141 & 96.7 \\ 14.1 & 36.4 & 1079 & -115 & -159 & 32.9 \\ 101 & 2246 & -115 & 23046 & -903 & 282 \\ -2123 & -141 & -159 & -903 & 30870 & -2006 \\ 385 & 96.7 & 32.9 & 282 & -2006 & 14486 \end{bmatrix}$$

The unit for the system stiffness matrix is $\frac{lb}{in}$.

3.3 The Equivalent Masses

3.3.1 The Reciprocating Mass (M_{recip})

The weight of the individual component of the compressor had been determined and documented by Arcot [3]. The masses of these components are obtained by weight/g, whereby g is the gravitational acceleration at 386.088 in/s^2. The masses of the sliding crank assembly components contributing towards the calculation of the reciprocating mass, M_{recip}, is the summation of the pistol mass (M_p), the wrist pin mass (M_{wp}) and the equivalent translational mass M_{trans}, of the connecting rod at the wrist-pin bearing. These masses will be determined and used in the calculation of the shaking forces and moments of the compressor at different frequency domains. Two equivalent masses as discussed by Mabie and Reinholtz [1] can be used to replace the mass of the connecting rod, M_{cr}. Referring to Figure 3.2, by locating one of the equivalent masses M_{rot} at the crank bearing, and taking equilibrium of moment at point A, the equivalent mass M_{trans} can thus be determined by

$$M_{trans}\left(l_{rot} + l_{trans}\right) = M_{cr} l_{rot}$$

$$M_{trans} = M_{cr}\left(\frac{l_{rot}}{l_{rot} + l_{trans}}\right) \tag{3.1}$$

where l_{rot} is the distance of the center of mass cg of the connecting rod to the crank bearing, and l_{trans} is the distance of the connecting rod cg to the wrist-pin bearing. With W_{cr} = 0.131 lb, l_{trans} = 1.75 inches, and l_{rot} = 0.625 inches,

$$M_{trans} = \left(\frac{0.131}{386.088}\right)\left(\frac{0.625}{0.625 + 1.75}\right)$$

$$= 8.929 \times 10^{-5}\ \frac{lb.s^2}{in}$$

The reciprocating mass is then determined by summing the piston mass M_p, the wrist pin mass M_{wp}, and the translational mass M_{trans} of the connecting rod, in which

$$\text{Reciprocating mass } (M_{recip}) = M_p + M_{wp} + M_{trans}$$

$$= \frac{(0.659 + 0.075)}{386.088} + 8.9218 \times 10^{-5}$$

$$= 1.990 \times 10^{-3} \frac{lb.s^2}{in}$$

3.3.2 The Rotating Mass (*M_{rot}*)

The rotating mass M_{rot} is the other equivalent mass beside M_{trans} of the two masses mentioned in Section 3.1.3.1. This mass is used in the calculation of the forces and moments for the balancing of the crankshaft. By taking moment at point B (referring to Figure 3.2), M_{rot} can be determined by

$$M_{rot}(l_{rot} + l_{trans}) = M_{cr} l_{trans}$$

$$M_{rot} = M_{cr} \frac{l_{trans}}{(l_{rot} + l_{trans})} \tag{3.2}$$

$$= \frac{0.131}{386.088}\left(\frac{1.75}{0.625 + 1.75}\right)$$

$$= 2.500 \times 10^{-4} \frac{lb.s^2}{in}$$

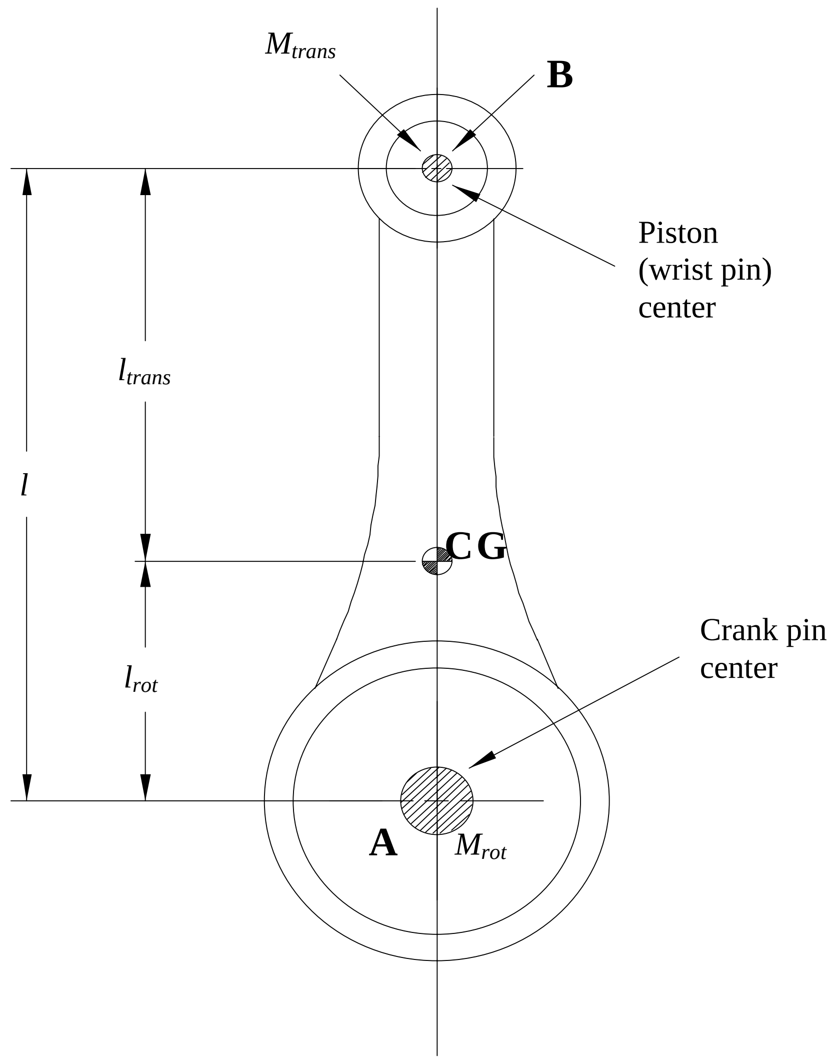


Figure 3.2 Application of equivalent masses to connecting rod.

3.4 Distance Definition of Cylinders from Center of Mass of the Compressor

Based upon the three-dimensional Cartesian coordinate system defined in section 3.1, the Z-directional distance to each of the two cylinders with respect to the center of mass of the compressor has been defined. With the top cylinder of the compressor being assigned as Cylinder A, the distance from the center of mass of the system to Cylinder A, a, is 3.835 inches. With the bottom cylinder of the compressor being assigned as Cylinder B, the distance from the center of mass of the system to Cylinder B, b, is 5.71 inches. The distance between Cylinder A and Cylinder B, c, is therefore $b - a = (5.71 - 3.835) = 1.875$ inches. (Figure 3.3)

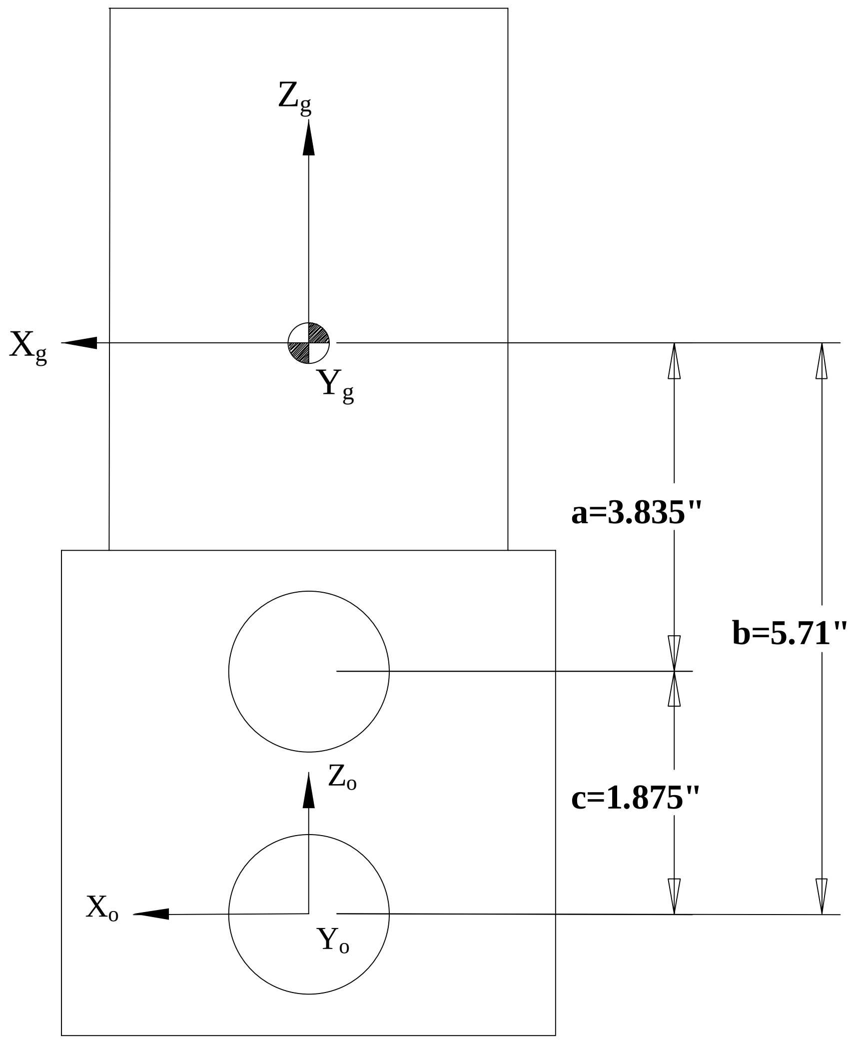


Figure 3.3 Distances of cylinders from the Center of Mass of the System.

Chapter 4. The Kinetic and Potential Energies

4.1 The Matrices Formation

4.1.1 The Modal Matrix P

As discussed by Thomson [4], the modal matrix P is an assembly of N normal modes (ϕ), or eigenvectors, in forming a square matrix where each column in the matrix represents an eigenvector. Hence for a six degree-of-freedom (6 DOF) system, in which N is 6, the modal matrix would be written as

$$P = \left[\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix}^{(1)} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix}^{(2)} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix}^{(3)} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix}^{(4)} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix}^{(5)} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix}^{(6)} \right]$$

$$= \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_4 & \phi_5 & \phi_6 \end{bmatrix}$$

In this research, the modal matrix P is obtained using MATLAB® by using the command line **[*P*,*D*] = eig(*A*)**. This will result in two matrices, P and D. Matrix P is the modal matrix as described above carrying the eigenvectors as column vectors, and D is a diagonal matrix having the eigenvalues on the diagonal. Matrix A is known as the dynamic matrix and can be obtained by multiplying the inverse of the system mass and inertia matrix, M^{-1}, with the system stiffness matrix, K. In equation form, $A = M^{-1}K$.

4.1.2 Decoupling of the Forced Vibration Equation

For a forced harmonic vibration excited by a harmonic force $F_o \sin \omega t$, the generally coupled expression can be expressed, in matrix form, as

$$M\ddot{X} + KX = F_o \sin \omega t \tag{4.1}$$

According to Thomson [4], in order to solve the 6 DOF system as a system of 1 DOF, the forced undamped 6 DOF system will have to be decoupled. By applying the coordinate transformation $X = PY$ to Equation (4.1), and premultiplying it by the transposed modal matrix P^T, the expression obtained will be

$$\left(P^T MP\right)\ddot{Y} + \left(P^T KP\right)Y = P^T F \tag{4.2}$$

Due to orthogonality, both P^TMP and P^TKP will be diagonal matrices in which the off-diagonal terms are zero. The diagonal terms in the matrix formed from P^TMP are generalized mass M_{ii} and this matrix is called generalized mass matrix. The diagonal terms in the matrix formed from P^TKP are generalized stiffness K_{ii} and this matrix is called generalized stiffness matrix. F is the force and moment vector column whereby $F = \{F_x\ F_y\ F_z\ M_x\ M_y\ M_z\}^T$. Hence for the 6 DOF system in this research, Equation (4.2) is expressed as

$$\begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix} \left[\ddot{Y}\right]_{6\times(n+1)} + \begin{bmatrix} K_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{66} \end{bmatrix} \left[Y\right]_{6\times(n+1)} = \left[P\right]^T_{6\times 6} \begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} \tag{4.3}$$

where n had been stated in section 3.1.3 as $n = \dfrac{desired\_higher\_order}{2}$.

The 6 DOF system is now uncoupled and can be solved as a 1 DOF system.

The solution for *Y* given by Thomson [4] is

$$|Y| = \frac{F_o / K}{1 - \dfrac{\omega^2}{\omega_n^2}} = \frac{F_o / K}{1 - \dfrac{M\omega^2}{K}} \tag{4.4}$$

where the natural frequency $\omega_n = \sqrt{\dfrac{K}{M}}$. Note that *Y*, F_o, *K*, and *M* are matrices. For example, Equation (4.4) can be expressed for the first ω as

$$|Y_{i1}| = \frac{F_{i1} / K_{ii}}{1 - \dfrac{M_{ii}\omega^2}{K_{ii}}} \tag{4.5}$$

For the harmonic range of (2*n)* ω, where *n* = 1,2,3… Equation (4.5) will be expressed as

$$|Y_{in}| = \frac{F_{in} / K_{ii}}{1 - \dfrac{M_{ii}(2n\omega)^2}{K_{ii}}} \tag{4.6}$$

With the solution of *Y* available, the original coordinates *X* can then be obtained by the transformation equation *X* = *PY*.

4.1.3 The Shaking Force and Moment Matrix

The notation used to identify different cylinders in operation for the shaking forces and moment will be

$$F_{cyl,S}$$

where '*cyl*' indicate the cylinder(s) that is in operation, and '*S*' indicates the shaking. For example, $F_{AB,S}$ indicates the force and moment vector for the shaking force of both Cylinders A and B in operation.

4.1.3.1 Both Cylinders A and B in Operation ($F_{AB,S}$)

As the inertial forces in both the cylinders A and B will be equal and opposite in direction, acting along the y-axis direction, the summation of forces will therefore be zero. No forces will be present along the x- and z-axes. Hence $F_x = F_y = F_z = 0$. A couple acting in the x-axis direction is present due to the coupling forces along the y-axis (Figure 4.1a). The magnitude of this couple is obtained by multiplying the reciprocating mass M_{recip} with the piston acceleration and the distance between the couple forces, which is the distance c between the two cylinders. This gives $M_x = -M_{recip}\ddot{X}c$. There is no moment present in the y- and z-axis direction, hence $M_y = M_z = 0$. The shaking force and moment vector for two cylinders in operation is therefore

$$F_{AB,S} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -M_{recip}\ddot{X}c \\ 0 \\ 0 \end{Bmatrix}$$

4.1.3.2 Only Cylinder A in Operation (Cylinder B is Idle) ($F_{A,S}$)

As only Cylinder A alone is in operation, there will be a shaking force F_s acting in the y-axis direction due to the piston in Cylinder A since the equal and opposite force by Cylinder B no longer exist (Figure 4.1b). Hence $F_y = F_s$, and $F_x = F_z = 0$. A moment in the x-axis direction

$M_x = aF_s$ will be generated, with a being the distance of Cylinder A to the center of mass of the system. M_y and M_z will be zero. The shaking force and moment vector for Cylinder A alone in operation is therefore

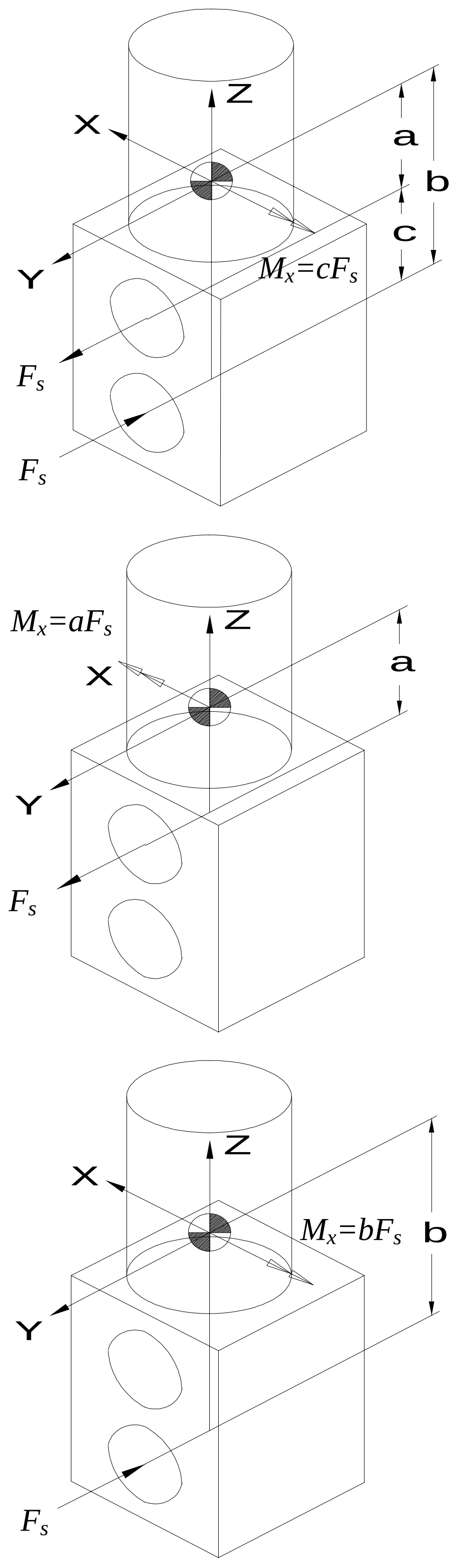
$$F_{A,S} = \begin{Bmatrix} 0 \\ F_s \\ 0 \\ aF_s \\ 0 \\ 0 \end{Bmatrix} \quad \text{or} \quad F_{A,S} = \begin{Bmatrix} 0 \\ M_{recip}\ddot{X} \\ 0 \\ aM_{recip}\ddot{X} \\ 0 \\ 0 \end{Bmatrix}$$

with $F_s = M_{recip}\ddot{X}$.

4.1.3.3 Only Cylinder B in Operation (Cylinder A is Idle) ($F_{B,S}$)

As only Cylinder B alone is in operation, there will be a shaking force F_s acting in the y-axis direction due to the piston in Cylinder B since the equal and opposite force by Cylinder A no longer exist (Figure 4.1c). Hence $F_y = F_s$, and $F_x = F_z = 0$. A moment in the x-axis direction $M_x = bF_s$ will be generated, with b being the distance of Cylinder B to the center of mass of the system. M_y and M_z will be zero. The shaking force and moment vector for Cylinder B alone in operation is therefore

$$F_{B,S} = \begin{Bmatrix} 0 \\ F_s \\ 0 \\ bF_s \\ 0 \\ 0 \end{Bmatrix} \quad \text{or} \quad F_{B,S} = \begin{Bmatrix} 0 \\ M_{recip}\ddot{X} \\ 0 \\ bM_{recip}\ddot{X} \\ 0 \\ 0 \end{Bmatrix}$$



(a) Both Cylinders A and B in

(b) Only Cylinder A in operation.

(c) Only Cylinder B in operation.

Figure 4.1 Shaking forces and moments for different operation setting.

4.1.4 Forces and Moments of Crankshaft Balanced for Different Operation Setting

The notation used to identify the cylinder(s) in operation at differently balanced crankshaft setting will be

$$F_{cyl,bal}$$

where '*cyl*' indicates the cylinder(s) that is in operation, and '*bal*' indicates the cylinder the crankshaft is balanced for. For example, $F_{AB,A}$ indicates the force and moment vector for both Cylinders A and B in operation with the crankshaft balanced for Cylinder A.

4.1.4.1 Crankshaft Balanced for 2 Cylinders Operation

4.1.4.1.1 Both Cylinders A and B in Operation ($F_{AB,2}$)

For the crankshaft being balanced for both Cylinders A and B in operation, and when both Cylinders A and B are in operation, the overall forces and moments will be zero (Figure 4.2a). Hence the force and moment vector will be

$$F_{AB,2} = \{0\}_{6x1}$$

4.1.4.1.2 Only Cylinder A in Operation (Cylinder B is Idle) ($F_{A,2}$)

When only Cylinder A is in operation (Cylinder B is idle), forces that will be generated are $F_x = -F_{rot} \cos \omega t$ and $F_y = -F_{rot} \sin \omega t$ in the x- and y-axis direction, respectively, where F_{rot} is the rotating force at crankshaft. Since there will be no force acting in the z-axis direction, hence $F_z = 0$. The moments formed are $M_x = -aF_{rot} \sin \omega t$ in the x-axis direction and $M_y = aF_{rot} \cos \omega t$ in the y-axis direction. There will be no moment in the z-axis direction, hence $M_z = 0$ (Figure 4.2b). The force and moment vector will be

$$F_{A,2} = \begin{Bmatrix} -F_{rot}\cos\omega t \\ -F_{rot}\sin\omega t \\ 0 \\ -aF_{rot}\sin\omega t \\ aF_{rot}\cos\omega t \\ 0 \end{Bmatrix}$$

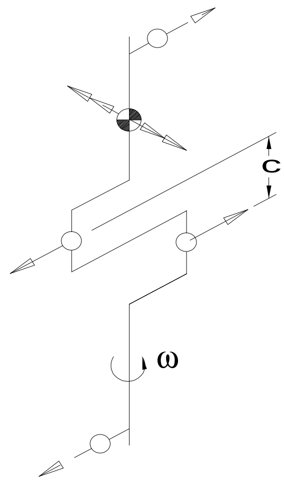
By the principle of superposition, $F_{A,2}$ can be grouped into their respective sine and cosine function for the analysis. With $F_{rot} = M_{rot}r\omega^2$, where r is the crank radius and M_{rot} is the rotating mass as defined in section 3.3.2, the force and moment vector $F_{A,2}$ will be

$$F_{A,2} = \begin{Bmatrix} 0 \\ -M_{rot}r\omega^2 \\ 0 \\ -aM_{rot}r\omega^2 \\ 0 \\ 0 \end{Bmatrix}\sin\omega t + \begin{Bmatrix} -M_{rot}r\omega^2 \\ 0 \\ 0 \\ 0 \\ aM_{rot}r\omega^2 \\ 0 \end{Bmatrix}\cos\omega t$$

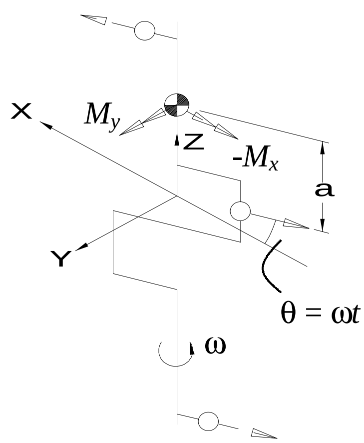
4.1.4.1.3 Only Cylinder B in Operation (Cylinder A is Idle) ($F_{B,2}$)

When only Cylinder B is in operation (Cylinder A is idle), forces that will be generated are $F_x = F_{rot}\cos\omega t$ and $F_y = F_{rot}\sin\omega t$ in the x- and y-axis direction, respectively. There will be no force acting in the z direction, therefore $F_z = 0$. The moments formed are $M_x = bF_{rot}\sin\omega t$ in the x-axis direction and $M_y = -bF_{rot}\cos\omega t$ in the y-axis direction. Since there will be no moment in the z-axis direction, hence $M_z = 0$ (Figure 4.2c). The force and moment vector will be

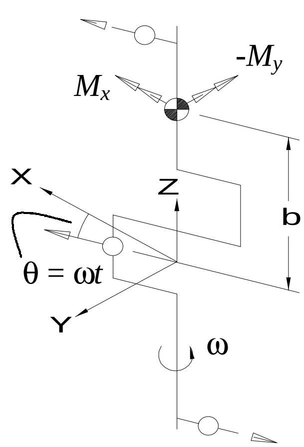
$$F_{B,2} = \begin{Bmatrix} F_{rot}\cos\omega t \\ F_{rot}\sin\omega t \\ 0 \\ bF_{rot}\sin\omega t \\ -bF_{rot}\cos\omega t \\ 0 \end{Bmatrix}$$



(a) Both Cylinders A and B in operation.



(b) Only Cylinder A in operation.



(c) Only Cylinder B in operation.

Figure 4.2 Crankshaft balanced for 2 cylinders operation.

As in the previous section, $F_{B,2}$ will be grouped into its respective sine and cosine function for the analysis. With $F = M_{rot}r\omega^2$, the force and moment vector $F_{B,2}$ will be

$$F_{B,2} = \begin{Bmatrix} 0 \\ M_{rot}r\omega^2 \\ 0 \\ bM_{rot}r\omega^2 \\ 0 \\ 0 \end{Bmatrix} \sin\omega t + \begin{Bmatrix} M_{rot}r\omega^2 \\ 0 \\ 0 \\ 0 \\ -bM_{rot}r\omega^2 \\ 0 \end{Bmatrix} \cos\omega t$$

4.1.4.2 Crankshaft Balanced for Only Cylinder A in Operation

4.1.4.2.1 Both Cylinders A and B in Operation ($F_{AB,A}$)

When both Cylinders A and B are in operation, forces that will be generated are $F_x = F_{rot} \cos \omega t$ in the x-axis direction and $F_y = F_{rot} \sin \omega t$ in the y-axis direction. As there will be no force acting in the z-axis direction, hence $F_z = 0$. The moments formed are $M_x = bF_{rot} \sin \omega t$ in the x-axis direction and $M_y = -bF_{rot} \cos \omega t$ in the y-axis direction. Moment in the z-axis direction does not exist in this case, hence $M_z = 0$ (Figure 4.3b). With $F = M_{rot}r\omega^2$, the force and moment vector will be

$$F_{AB,A} = \begin{Bmatrix} 0 \\ M_{rot}r\omega^2 \\ 0 \\ bM_{rot}r\omega^2 \\ 0 \\ 0 \end{Bmatrix} \sin\omega t + \begin{Bmatrix} M_{rot}r\omega^2 \\ 0 \\ 0 \\ 0 \\ -bM_{rot}r\omega^2 \\ 0 \end{Bmatrix} \cos\omega t$$

4.1.4.2.2 Only Cylinder A in Operation (Cylinder B is Idle) ($F_{A,A}$)

For the crankshaft being balanced for only Cylinder A in operation, and when only Cylinder A is in operation, the overall forces and moments will be zero (Figure 4.3a). Hence the force and moment vector will be

$$F_{A,A} = \{0\}_{6x1}$$

4.1.4.3 Crankshaft Balanced for Only Cylinder B in Operation

4.1.4.3.1 Both Cylinders A and B in Operation ($F_{AB,B}$)

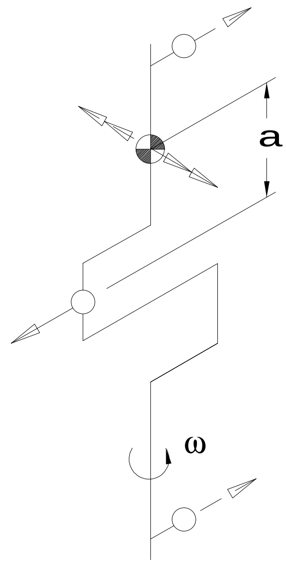
When both Cylinders A and B are in operation, the generated forces are $F_x = -F_{rot} \cos \omega t$ and $F_y = -F_{rot} \sin \omega t$ in the x- and y-axis direction, respectively. As there will be no force acting in the z-axis direction, hence $F_z = 0$. The moments formed are $M_x = -aF_{rot} \sin \omega t$ in the x-axis direct and $M_y = aF_{rot} \cos \omega t$ in the y-axis direction. There will be no moment in the z-axis direction, hence $M_z = 0$ (Figure 4.4b). The force and moment vector will be

$$F_{AB,B} = \begin{Bmatrix} 0 \\ -M_{rot} r\omega^2 \\ 0 \\ -aM_{rot} r\omega^2 \\ 0 \\ 0 \end{Bmatrix} \sin \omega t + \begin{Bmatrix} -M_{rot} r\omega^2 \\ 0 \\ 0 \\ 0 \\ aM_{rot} r\omega^2 \\ 0 \end{Bmatrix} \cos \omega t$$

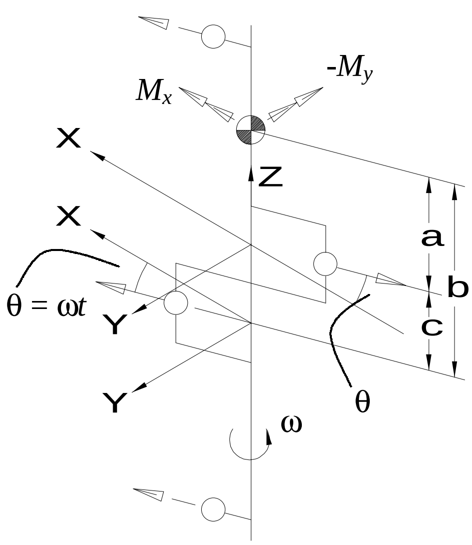
4.1.4.3.2 Only Cylinder B in Operation (Cylinder A is Idle) ($F_{B,B}$)

For the crankshaft being balanced for only Cylinder B in operation, and when only Cylinder B is in operation, the overall forces and moments will be zero (Figure 4.4a). Hence the force and moment vector will be

$$F_{B,B} = \{0\}_{6x1}$$

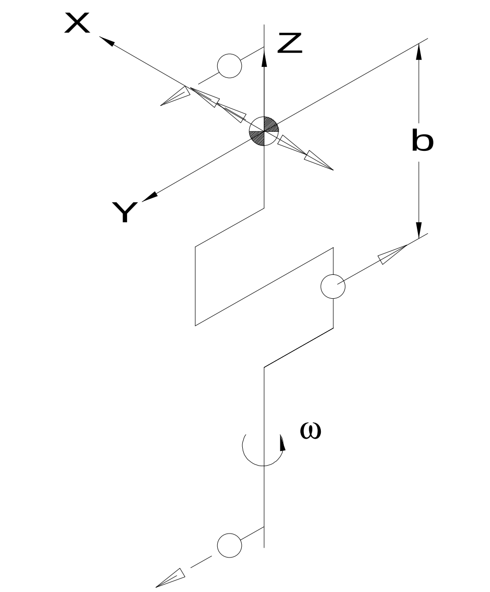


(a) Only Cylinder A in operation.

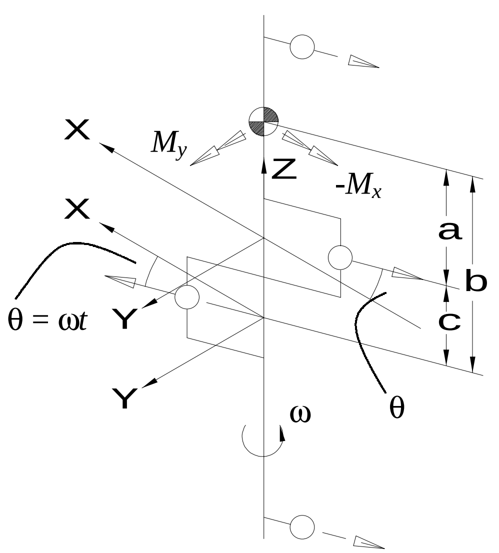


(b) Both Cylinders A and B in operation.

Figure 4.3 Crankshaft balanced for only Cylinder A in operation.



(a) Only Cylinder B in operation.



(b) Both Cylinders A and B in operation.

Figure 4.4 Crankshaft balanced for only Cylinder B in operation.

4.2 Preparation of Data for MATLAB Computation

4.2.1 The Original Coordinates *X*

In section 4.1.4, as the force and moment vector F_o is split and grouped under the same function of sine and cosine, it gives rise to two sets of vibration equations. They are

$$M\ddot{X}_s + KX_s = F\sin\omega t \qquad (4.7)$$

for the sine function, and

$$M\ddot{X}_c + KX_c = F\cos\omega t \qquad (4.8)$$

for the cosine function. The procedure of decoupling of the forced vibration system will be applied to both Equation (4.5) and (4.6) to determine the original coordinates in term of X_s and X_c, whereby $X_s = \{x_s\}\sin\omega t$ and $X_c = \{x_c\}\cos\omega t$. The original coordinates X is then determined by

$$X = \left\{\sqrt{X_s^2 + X_c^2}\right\}\sin(\omega t + \phi) \qquad (4.9)$$

where $\phi = \tan^{-1}\dfrac{X_s}{X_c}$ is the phase angle. As in the original coordinates X obtained, $\left\{\sqrt{X_s^2 + X_c^2}\right\}$ will be used towards the computation of the kinetic energies and potential energies for the harmonic domain of ω, 2ω, 4ω, …*(2n)* ω.

4.2.2 Kinetic Energy and Potential Energy

4.2.2.1 The Kinetic Energy

The kinetic energy is being expressed as

$$KE = \frac{1}{2}Mv^2$$

$$= \frac{1}{2}M\dot{X}^2$$

$$= \frac{1}{2}[\omega X]^T M[\omega X]$$

$$KE = \frac{\omega^2}{2}X^T MX \tag{4.10}$$

For the kinetic energies by shaking forces and moments, the total kinetic energy will be the summation of the individual kinetic energies obtained from the frequency domain of ω_1, ω_2, ω_4, ... to ω_{80}. Hence $KE_{shaking} = \Sigma(KE_1 + KE_2 + KE_4 + \ldots + KE_{80})$. The kinetic energies due to the shaking forces and moments for both Cylinders A and B in operation; only Cylinder A in operation; and only Cylinder B in operation are computed and listed in Table 4.1.

For the balanced crankshaft, the force and moment vector will generate a single kinetic energy value as the crankshaft is rotating at a constant angular velocity ω of 361 rad/s, with the rotating mass M_{rot} and crank radius r remain constant.

For crankshaft balanced for both Cylinders A and B in operation, no kinetic energy will be generated when both Cylinders A and B are in operation as the force and moment vector is a zero column vector (Section 4.1.4.1.1). Hence $KE_{AB,2} = 0$. However, when only Cylinder A is in operation (Section 4.1.4.1.2), the kinetic energy $KE_{A,2}$ is computed to be 2.176216×10^{-2} in-lb. When only Cylinder B is in operation (Section 4.1.4.1.3), the kinetic energy $KE_{B,2}$ computed is 3.503765×10^{-2} in-lb.

For the crankshaft balanced for only Cylinder A in operation, the force and moment vector column will be zero when only Cylinder A is in operation, hence kinetic energy $KE_{A,A}$ will be zero (Section 4.1.4.2.2). When both Cylinders A and B are in operation, the kinetic energy $KE_{AB,A}$ computed is 3.503765×10^{-2} in-lb (Section 4.1.4.2.1).

For the crankshaft balanced for only Cylinder B in operation, the force and moment vector column will be zero resulting the kinetic energy $KE_{B,B}$ to be zero when only Cylinder B is in operation (Section 4.1.4.3.2). When both Cylinders A and B are in operation, the kinetic energy

Table 4.1 Kinetic energies by shaking forces and moments at ω = 361 rad/s. (Cylinder indicated is in operation mode, and unit of *KE* is in-lb)

Omega	Cylinder A + B	Cylinder A	Cylinder B
1	7.074184E-02	6.089413E-01	9.654016E-01
2	7.392630E-04	6.158936E-03	9.777933E-03
4	2.327742E-08	1.923965E-07	3.055548E-07
6	1.675635E-12	1.382956E-11	2.196484E-11
8	1.492626E-16	1.231285E-15	1.955636E-15
10	1.466881E-20	1.209763E-19	1.921472E-19
12	1.525540E-24	1.257978E-23	1.998065E-23
14	1.646058E-28	1.357254E-27	2.155754E-27
16	1.822649E-32	1.502787E-31	2.386911E-31
18	2.057270E-36	1.696175E-35	2.694078E-35
20	2.356674E-40	1.942979E-39	3.086087E-39
22	2.731549E-44	2.252007E-43	3.576927E-43
24	3.196402E-48	2.635216E-47	4.185591E-47
26	3.770259E-52	3.108289E-51	4.936989E-51
28	4.476864E-56	3.690798E-55	5.862209E-55
30	5.346377E-60	4.407608E-59	7.000743E-59
32	6.416392E-64	5.289710E-63	8.401815E-63
34	7.733792E-68	6.375752E-67	1.012681E-66
36	9.357062E-72	7.713949E-71	1.225232E-70
38	1.135923E-75	9.364506E-75	1.487395E-74
40	1.383138E-79	1.140250E-78	1.811097E-78
42	1.688707E-83	1.392156E-82	2.211208E-82
44	2.066819E-87	1.703864E-86	2.706304E-86
46	2.535214E-91	2.090001E-90	3.319618E-90
48	3.116054E-95	2.568834E-94	4.080165E-94
50	3.837051E-99	3.163211E-98	5.024233E-98
52	4.732944E-103	3.901767E-102	6.197307E-102
54	5.847098E-107	4.820253E-106	7.656170E-106
56	7.234099E-111	5.963668E-110	9.472296E-110
58	8.962322E-115	7.388379E-114	1.173521E-113
60	1.111715E-118	9.164770E-118	1.455671E-117
62	1.380634E-122	1.138168E-121	1.807791E-121
64	1.716462E-126	1.415017E-125	2.247520E-125
66	2.136169E-130	1.761014E-129	2.797079E-129
68	2.660908E-134	2.193598E-133	3.484167E-133
70	3.316153E-138	2.733767E-137	4.342135E-137
72	4.124043E-142	3.399773E-141	5.399975E-141
74	5.046325E-146	4.160081E-145	6.607598E-145
76	5.718803E-150	4.714455E-149	7.488130E-149
78	4.895808E-154	4.035995E-153	6.410509E-153
80	1.705631E-158	1.406084E-157	2.233331E-157
Total	7.148113E-02	6.151004E-01	9.751798E-01

$KE_{AB,B}$ computed is 2.2.176216×10^{-2} in-lb (Section 1.2.4.3.1). A summary of results for the kinetic energies computed is listed in Table 4.2.

Table 4.2 Summary of the kinetic and potential energies computed for differently balanced crankshaft under different operation settings.

Crankshaft balanced for:	Cylinder(s) in operation					
	2 Cylinders (A + B)		Cylinder A only		Cylinder B only	
	KE (in-lb)	PE (in-lb)	KE (in-lb)	PE (in-lb)	KE (in-lb)	PE (in-lb)
2 Cylinders (A + B)	0	0	2.176216E-02	1.709964E-03	3.503765E-02	2.817279E-03
Cylinder A only	3.503765E-02	2.817279E-03	0	0		
Cylinder B only	2.176216E-02	1.709964E-03			0	0

4.2.2.2 The Potential Energy

The potential energy is expressed as

$$PE = \frac{1}{2}KX^2 = \frac{1}{2}X^T KX \qquad (4.11)$$

For the potential energies by shaking forces and moments, the total potential energy will be the summation of the individual potential energy obtained from the frequency domain of ω_1, ω_2, ω_4, … to ω_{80}. Hence the total potential energy is $PE_{shaking} = \Sigma(PE_1 + PE_2 + PE_4 + \ldots + PE_{80})$. The potential energies computed due to the shaking forces and moments for different operation setting are listed in Table 4.3.

For the balanced crankshaft, the force and moment vector will generate a single potential energy value as in the case for kinetic energy. For ω constant at 361 rad/s, and for the crankshaft balanced for 2 cylinders operation, no potential energy $PE_{AB,2}$ will be generated during the 2 cylinders operation as the force and moment vector will be a zero column vector (Section 4.1.4.1.1). When only Cylinder A is in operation with the crankshaft balanced for 2 cylinders operation (Section 4.1.4.1.2), the potential energy $PE_{A,2}$ computed is 1.709964×10^{-3} in-

Table 4.3 Potential energies by shaking forces and moments at ω =361 rad/s. (Cylinder indicated is in operation mode, and unit of *PE* is in-lb)

Omega	Cylinder A + B	Cylinder A	Cylinder B
1	4.663143E-03	4.690677E-02	7.336857E-02
2	1.220731E-05	1.185638E-04	1.855525E-04
4	9.614164E-11	9.258647E-10	1.449179E-09
6	3.076191E-15	2.957806E-14	4.629725E-14
8	1.541421E-19	1.481290E-18	2.318620E-18
10	9.695085E-24	9.314522E-23	1.457981E-22
12	7.001986E-28	6.726211E-27	1.052841E-26
14	5.550746E-32	5.331687E-31	8.345598E-31
16	4.705727E-36	4.519774E-35	7.074730E-35
18	4.196728E-40	4.030740E-39	6.309257E-39
20	3.894079E-44	3.739962E-43	5.854110E-43
22	3.730176E-48	3.582476E-47	5.607601E-47
24	3.667794E-52	3.522513E-51	5.513741E-51
26	3.686302E-56	3.540247E-55	5.541501E-55
28	3.774196E-60	3.624625E-59	5.673578E-59
30	3.926303E-64	3.770676E-63	5.902191E-63
32	4.141502E-68	3.977321E-67	6.225650E-67
34	4.421827E-72	4.246511E-71	6.647012E-71
36	4.772014E-76	4.582795E-75	7.173393E-75
38	5.199351E-80	4.993170E-79	7.815748E-79
40	5.713641E-84	5.487049E-83	8.588811E-83
42	6.327371E-88	6.076423E-87	9.511352E-87
44	7.056097E-92	6.776233E-91	1.060676E-90
46	7.918930E-96	7.604829E-95	1.190375E-94
48	8.939022E-100	8.584444E-99	1.343713E-98
50	1.014437E-103	9.741968E-103	1.524899E-102
52	1.156891E-107	1.110998E-106	1.739032E-106
54	1.325319E-111	1.272744E-110	1.992211E-110
56	1.524671E-115	1.464185E-114	2.291872E-114
58	1.760890E-119	1.691032E-118	2.646952E-118
60	2.041073E-123	1.960097E-122	3.068117E-122
62	2.373902E-127	2.279721E-126	3.568420E-126
64	2.769759E-131	2.659870E-130	4.163463E-130
66	3.241272E-135	3.112674E-134	4.872232E-134
68	3.803469E-139	3.652564E-138	5.717316E-138
70	4.473077E-143	4.295602E-142	6.723855E-142
72	5.258066E-147	5.049443E-146	7.903834E-146
74	6.090874E-151	5.849205E-150	9.155693E-150
76	6.544038E-155	6.284387E-154	9.836877E-154
78	5.318671E-159	5.107637E-158	7.994924E-158
80	1.761461E-163	1.691569E-162	2.647793E-162
Total	4.675351E-03	4.702533E-02	7.355412E-02

lb. When only Cylinder B is in operation (Section 4.1.4.1.3), the potential energy $PE_{B,2}$ computed is 2.817279×10^{-3} in-lb.

For the crankshaft balanced for only Cylinder A in operation, the force and moment vector column will be zero when only Cylinder A is in operation, hence the potential energy $PE_{A,A}$ will be zero (Section 4.1.4.2.2). When 2 cylinders are in operation, the potential energy $PE_{AB,A}$ computed is 2.817279×10^{-3} in-lb (Section 4.1.4.2.1).

For the crankshaft balanced for only Cylinder B in operation, the force and moment vector column will be zero resulting the potential energy $PE_{B,B}$ to be zero when only Cylinder B is in operation (Section 4.1.4.3.2). When 2 cylinders are in operation, the potential energy $PE_{AB,B}$ computed is 1.709964×10^{-3} in-lb (Section 4.1.4.3.1).

A summary of results for the potential energies computed is listed in Table 4.2.

4.3 The Total Kinetic Energy and Total Potential Energy

4.3.1 Total Kinetic Energy

The total kinetic energy at different operation setting is obtained by summing the kinetic energies from shaking forces and moments (Section 4.1.3), and the kinetic energy obtained from differently balanced crankshaft under different operation setting. This is expressed as

$$KE_{total} = KE_{cyl,S} + KE_{cyl,bal} \tag{4.12}$$

where the notation of '*cyl*', '*S*', and '*bal*' used is as explained in the earlier sections.

Table 4.4 Combination possibility between cylinder(s) in operation with crankshaft balanced for.

Shaking forces and moments (Cylinder in operation)	Crankshaft balanced for:
(1) A and B	(i) A and B
(2) A only	(ii) A only
(3) B only	(iii) B only

Table 4.4 lists the possibilities of combination for the cylinder in different operation setting with the crankshaft being balanced for.

The total kinetic energy of (1) + (i), for which the kinetic energy from shaking of both Cylinders A and B in operation, $KE_{AB,S}$, and the kinetic energy from crankshaft balanced for both cylinders in operation, $KE_{AB,2}$, will be from the kinetic energy by the shaking alone. The crankshaft has been balanced for the two cylinders operation and hence $KE_{AB,2}$ is zero. Kinetic energy of one cylinder in operation, i.e. only Cylinder A or only Cylinder B in operation will not be considered as the two cylinders operation is of interest in this case. Therefore, the total kinetic energy in this case will be

$$KE_{total} = KE_{AB,S} + KE_{AB,2}$$

$$= 7.148113\times10^{-2} + 0$$
$$= 0.07148 \text{ in-lb}$$

In the case of (1) + (ii), the total kinetic energy will be the sum of the kinetic energy due to shaking of 2 cylinders in operation ($KE_{AB,S}$) and the kinetic energy from crankshaft balanced for Cylinder A but Cylinders A and B in operation ($KE_{AB,A}$). One cylinder operation is not considered as two cylinders operation is of interest in this case. Hence

$$KE_{total} = KE_{AB,S} + KE_{AB,A}$$
$$= 7.148113\times10^{-2} + 3.503765\times10^{-2}$$
$$= 0.10652 \text{ in-lb}$$

In the case of (1) + (iii), the total kinetic energy will be the sum of the kinetic energy due to shaking of 2 cylinders in operation ($KE_{AB,S}$) and the kinetic energy from crankshaft balanced for Cylinder B but both Cylinders A and B in operation ($KE_{AB,B}$). One cylinder operation is not considered as two cylinders operation is of interest in this case. Hence

$$KE_{total} = KE_{AB,S} + KE_{AB,B}$$
$$= 7.148113\times10^{-2} + 2.176216\times10^{-2}$$
$$= 0.09324 \text{ in-lb}$$

In the case of (2) + (i), the total kinetic energy will be the sum of the kinetic energy due to shaking of only Cylinder A in operation ($KE_{A,S}$) and the kinetic energy from crankshaft balanced for 2 cylinders but only Cylinder A in operation ($KE_{A,2}$). Cases of 2 cylinders and only Cylinder B in operation are not considered as only Cylinder A in operation is of interest in this case. Hence

$$KE_{total} = KE_{A,S} + KE_{A,2}$$
$$= 6.151004\times10^{-1} + 2.176216\times10^{-2}$$
$$= 0.63686 \text{ in-lb}$$

In the case of (2) + (ii), the total kinetic energy will be the sum of the kinetic energy due to shaking of only Cylinder A in operation ($KE_{A,S}$) and the kinetic energy from crankshaft balanced for only Cylinder A and only Cylinder A in operation. Cases of 2 cylinders and only Cylinder B in operation are not considered as only Cylinder A in operation is of interest in this case. Only $KE_{A,S}$ will be present as $KE_{A,A}$ will be zero. Hence

$$\begin{aligned} KE_{total} &= KE_{A,S} + KE_{A,A} \\ &= 6.151004\times10^{-1} + 0 \\ &= 0.61510 \text{ in-lb} \end{aligned}$$

The case of (2) + (iii) will not be considered. It is of no benefit to put only Cylinder A in operation when the crankshaft is balanced for only Cylinder B in operation.

In the case of (3) + (i), the total kinetic energy will be the sum of the kinetic energy due to shaking of only Cylinder B in operation ($KE_{B,S}$) and the kinetic energy from crankshaft balanced for 2 cylinders but only Cylinder B in operation ($KE_{B,2}$) . Cases of 2 cylinders and only Cylinder A in operation are not considered as only Cylinder B in operation is of interest in this case. Hence

$$\begin{aligned} KE_{total} &= KE_{B,S} + KE_{B,2} \\ &= 9.751798\times10^{-1} + 3.503765\times10^{-2} \\ &= 1.01021 \text{ in-lb} \end{aligned}$$

The case of (3) + (ii) will not be considered. It is of no benefit to put only Cylinder B in operation when the crankshaft is balanced for only Cylinder A in operation.

In the case of (3) + (iii), the total kinetic energy will be the sum of the kinetic energy due to shaking of only Cylinder B in operation ($KE_{B,S}$) and the kinetic energy from crankshaft balanced for only Cylinder B and only Cylinder B in operation. Cases of 2 cylinders and only Cylinder A in operation are not considered as only Cylinder B in operation is of interest in this case. Only $KE_{B,S}$ will be present as $KE_{B,B}$ will be zero. Hence

$$KE_{total} = KE_{B,S} + KE_{B,B}$$
$$= 9.751798\times10^{-1} + 0$$
$$= 0.97518 \text{ in-lb}$$

A summary of the total kinetic energy obtained is tabulated in Table 4.5.

Table 4.5 Summary of the calculated total kinetic energies for different operation setting. (Unit of the total *KE* is in-lb)

	Crankshaft balanced for:		
Shaking forces and moments	(i) A and B ($KE_{cyl,2}$)	(ii) A Only ($KE_{cyl,A}$)	(iii) B Only ($KE_{cyl,B}$)
(1) A and B ($KE_{AB,S}$)	0.07148	0.10652	0.09324
(2) A Only ($KE_{A,S}$)	0.63686	0.61510	
(3) B Only ($KE_{B,S}$)	1.01021		0.97518

4.3.2 Total Potential Energy

The total potential energy for the cases of (1) + (i), (1) + (ii), (1) + (iii), (2) + (i), (2) + (ii), (3) + (i), and (3) + (iii) are calculated in the same manner as that of the total kinetic energy obtained in Section 4.3.1. A summary of the total potential energy obtained is tabulated in Table 4.6.

Table 4.6 Summary of the calculated total potential energies of different operation setting. (Unit of the total *PE* is in-lb)

	Crankshaft balanced for:		
Shaking forces and moments	(i) A and B ($KE_{cyl,2}$)	(ii) A Only ($KE_{cyl,A}$)	(iii) B Only ($KE_{cyl,B}$)
(1) A and B ($KE_{AB,S}$)	0.00468	0.00749	0.00639
(2) A Only ($KE_{A,S}$)	0.04874	0.04703	
(3) B Only ($KE_{B,S}$)	0.07526		0.07355

Chapter 5. Piston – Connecting Rod Joint Clearance Impact Forces

In order for the connecting rod (link 3) to rotate freely while joined to the sliding piston (link 4), there will have to be a certain amount of clearance between the connecting rod and the wrist pin (of piston, will be addressed as piston). This is illustrated in Figure 5.1. It is to be noted that this clearance includes the clearance between the crankshaft and the ground link; the clearance between the crankshaft and the rotating end of the connecting rod; and the clearance between the reciprocating end of the connecting rod and the piston. The clearance is lumped as the crankshaft is held fixed to the ground link, the effect of clearance will pass on to the reciprocating end. Due to this clearance, impacts will occur between the connecting rod and piston when the connecting rod is either pulling or pushing on the piston, and forces are resulted from these impacts.

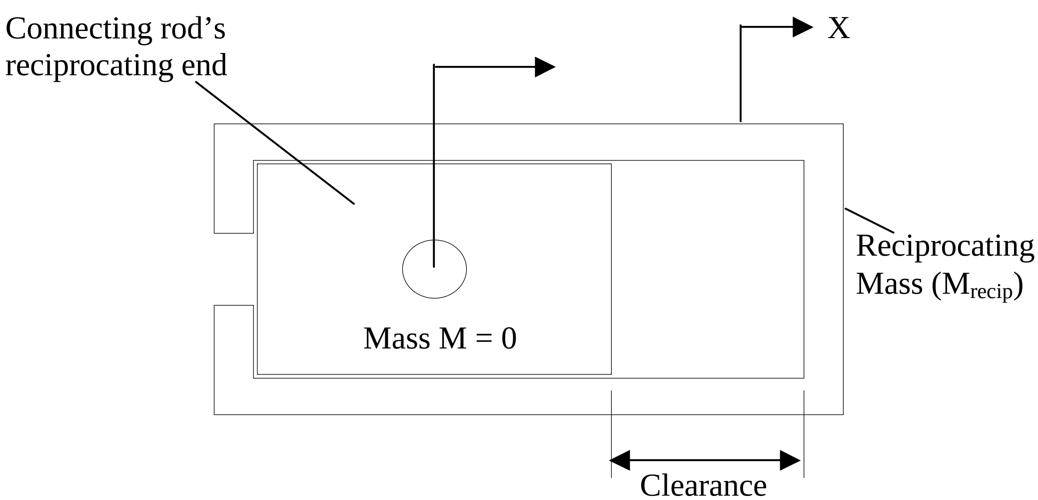


Figure 5.1 Schematic drawing of clearance between connecting rod and piston.

In Figure 5.1, the reciprocating mass (M_{recip}), which had been defined in Section 4.1.3.1, has a displacement X. The block with a mass $M = 0$, having a displacement Y, is the reciprocating end of the connecting rod that simulates the link pushing or pulling on the reciprocating mass when the slider-crank mechanism is in action.

As the shaking forces by the compressor are generated throughout a series of frequencies, determining the impact forces caused by this clearance will help eliminate the shaking forces whose values are lower than that of the impact forces in the same frequency range. In other words, higher forces will dominate. The sample plot in Figure 5.2 illustrates this idea.

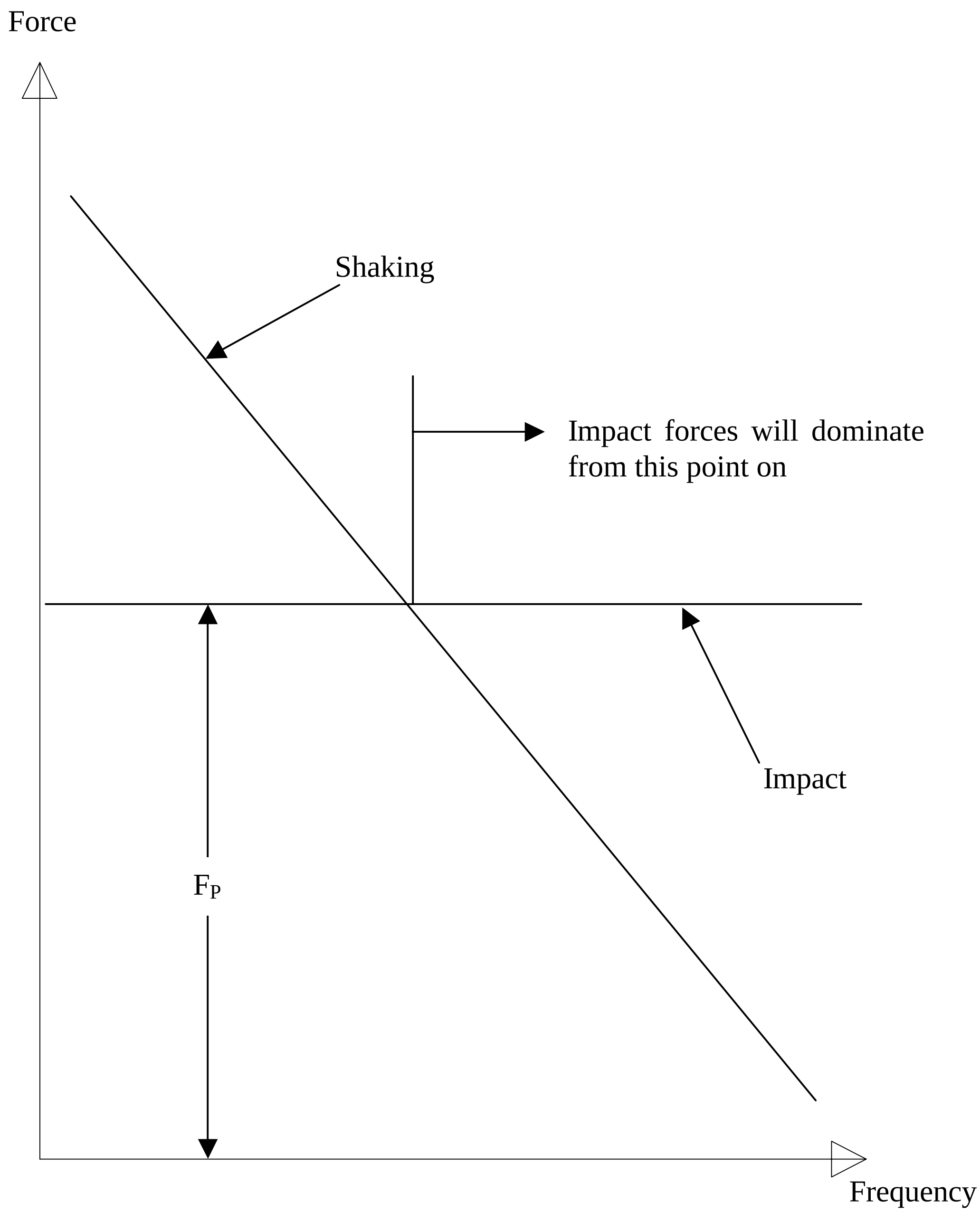


Figure 5.2 Demonstration plot of relation between shaking forces and impact forces.

5.1 The Subsystems

To determine the impact forces due to the clearance between the connecting rod and the piston when the compressor is in operation, the differential equation that governs the dynamic system has to be determined, first by generating the required free body diagram (FBD) of the system. From Figure 5.1, the reciprocating end of the connecting rod and the reciprocating mass can be rearranged and redrawn to be that as shown in Figure 5.3a. A spring *K* of high stiffness (1 10^6 lb/in) and a damper *D* of small damping value (0.1 lb-s/in) are used to link the motion of the two masses for the simulation of this dynamic system. From Figure 5.3a we can draw the FBD for the reciprocating mass, as shown in Figure 5.3b, and is ready for the derivation of the differential equation that governs the dynamic system. As the reciprocating end of the connecting rod is just prescribing a motion, hence the mass is of no importance and is therefore being assigned as $M = 0$.

With reference to the FBD in Figure 5.3, the differential equation of the dynamic system is derived as follows:

$$\sum F = M_{recip}\ddot{X} = F_K + F_D - F_P$$

$$\ddot{X} = \frac{1}{M_{recip}}\left(F_K + F_D - F_P\right)$$

$$\ddot{X} = \frac{1}{M_{recip}}\left[K\left(Y - X\right) + D(\dot{Y} - \dot{X}) - pA_P\right] \tag{5.1}$$

where K = spring stiffness (lb/in)

D = damping (lb-s/in)

p = pressure (psi)

F_K = spring force (lb)

F_D = damping force (lb)

F_p = fluid’s pressure force (lb)

A_P = piston’s face area in contact with the working fluid (in^2)

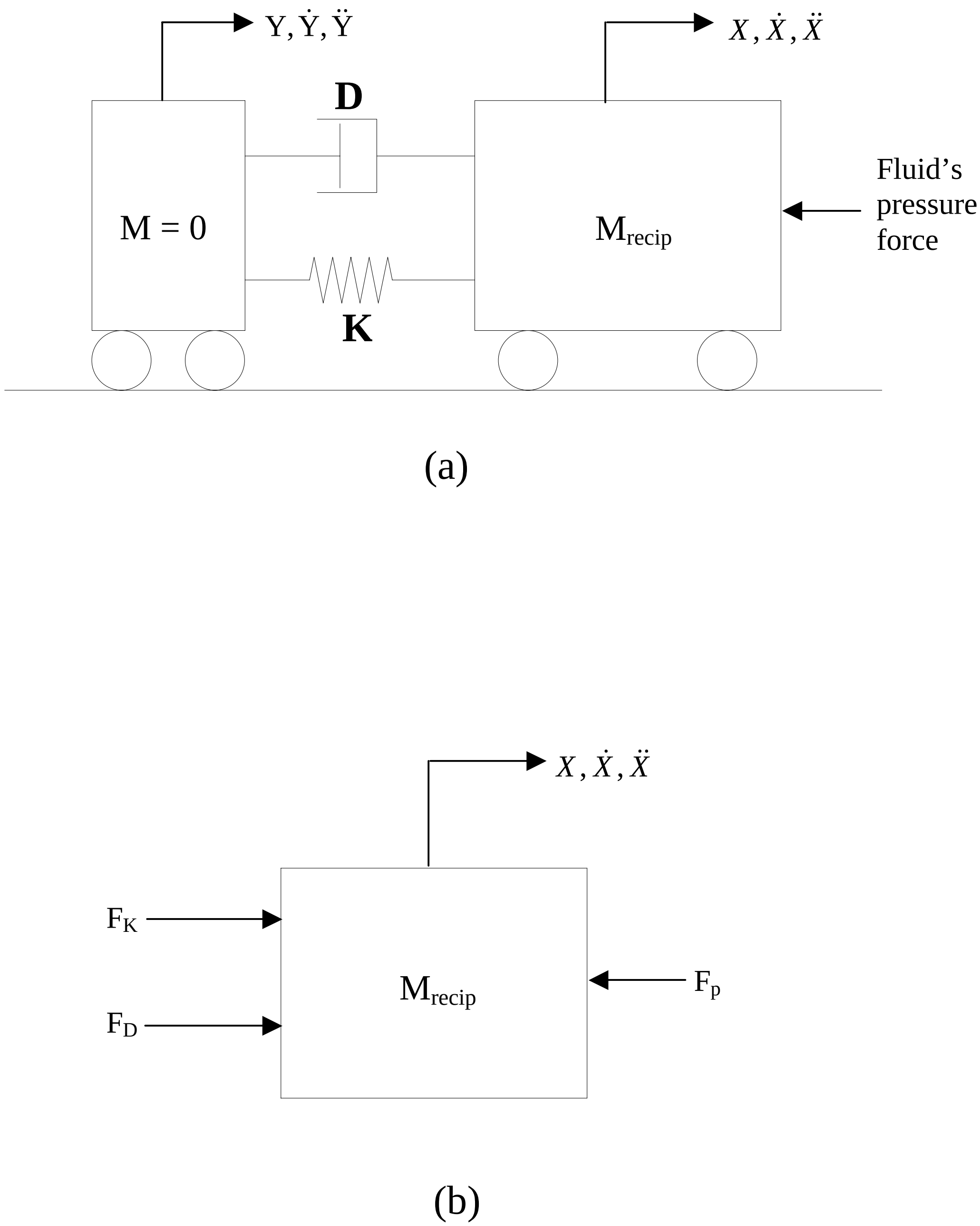


Figure 5.3 (a) The model of the dynamic system, and (b) the FBD.

With the differential equation (5.1) in place, the dynamic system can then be modeled, simulated, and analyzed by using SIMULINK® in MATLAB®, whereby SIMULINK is a simulation software package for studying the linear and non-linear dynamic systems.

Due to the complexity in setting the various working conditions in the dynamic system itself, the setting up of the SIMULINK model for the complete dynamic system will be done in parts, which consists of two subsystems, namely the modeling of the pressure force subsystem and the modeling of the spring stiffness *K* condition setting subsystem, before putting them together for the simulation and the analysis.

5.1.1 Fluid Pressure Force (F_p) Subsystem

The setup of the simulation model in SIMULINK is shown in Figure 5.4. This is one of the two subsystems in the whole impact force simulation. The input will be time *u* in seconds. This input time will go through the Gain block, whereby the Gain block will generate the output by multiplying its input by the specified angular velocity ω of the rotating crankshaft, which is 361 rad/s, to give an angular output in term radians at the given time. Dividing this output with 2π using a Product block will covert the angle in radian into number of cycles the crankshaft has rotated. By using the Rounding Function block with the "floor" option, the number of cycles input will be round off to the nearest lower integer value. For example, an input value of 72.7 will be output as 72. This new round off value of number of cycles will then be multiplied by 2π to convert it back to the radian form, and be subtracted off from the original radian value to give the crankshaft angle θ information in term of radian within the one cycle range ($0 \leq \theta \leq 2\pi$). With this information, it is ready to proceed to the next step where the pressure exerts on the piston by the working fluid when the crankshaft rotates to any given angle can then be determined. This specific task is carried out by using the Look-Up Table block, in which it will do a piecewise linear mapping of the radian value input to the pressure value output using linear interpolation. The information on the pressure output values in psi correspond to the angle input values in radian are listed in Table 5.1. The conversion of the angle from degree to radian will be performed within the Look-Up Table block. After the corresponding pressure value has been obtained, this value will be passed on to the Gain block containing the value of the piston's surface area A_p in contact with the working fluid for the multiplication task to

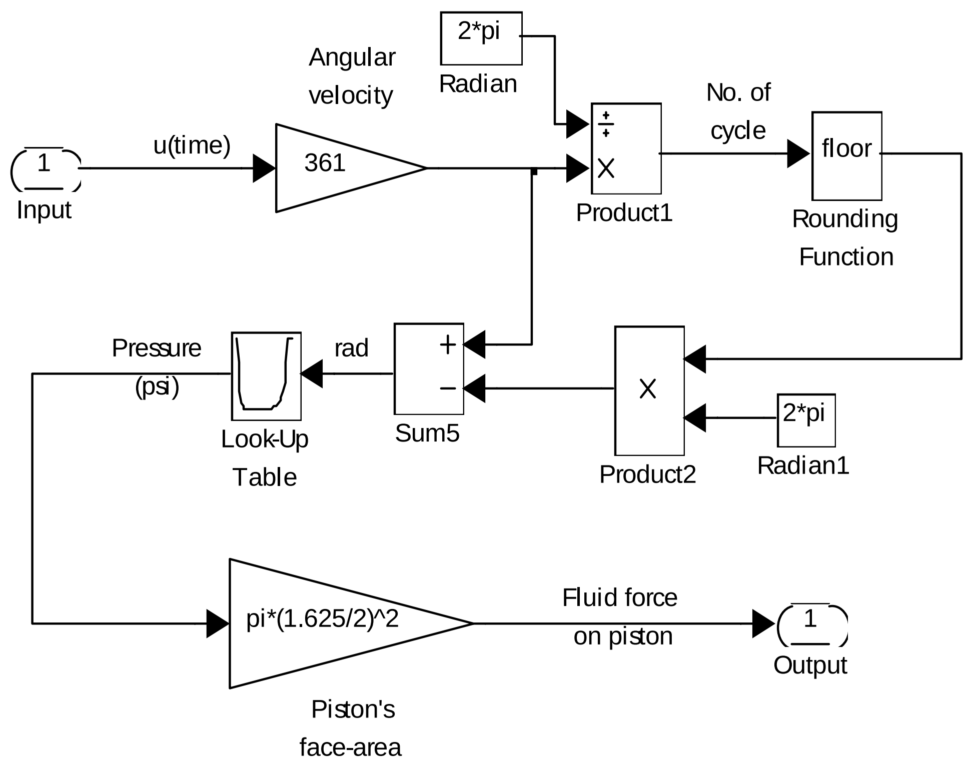


Figure 5.4 Block diagram of the fluid's pressure force subsystem model.

generate an output in force. With the diameter of the piston to be 1.625 inches, the surface area of the piston in contact with the working fluid is calculated to be $A_p = \frac{\pi d^2}{4} = 2.07394 in^2$. The fluid's pressure force F_p acting on the piston will then be determined by F_p = pressure $p \times A_p$. This will be passed on to the Output block in the subsystem.

Table 5.1 Crankshaft angle θ vs. pressure *p*. (From Zheng [7])

Angle (Degree)	Pressure (psi)
0	221
10	143.1442235
20	59.57198292
30	22.13792913
40	5.926086272
50	0
60	0
70	0
80	0
90	0
100	0
110	0
120	0
130	0
140	0
150	0
160	0
170	0
180	0
190	0.11504243
200	0.468925102
210	1.089180276
220	2.026246059
230	3.361576261
240	5.222086775
250	7.805573134
260	11.42632639
270	16.6000612
280	24.20993436
290	35.85102598
300	54.5976863
310	86.86022257
320	147.3088118
330	221
340	221
350	221
360	221

5.1.2 Spring Constant Setting Subsystem

The spring stiffness K is as shown in Figure 5.5 and is a function of ($Y – X$). The force generated by the spring goes by *force = spring constant × displacement*, or $F_K = K \times (Y - X)$. This is illustrated by the force F_K vs. displacement (Y - X) plot in Figure 5.5.

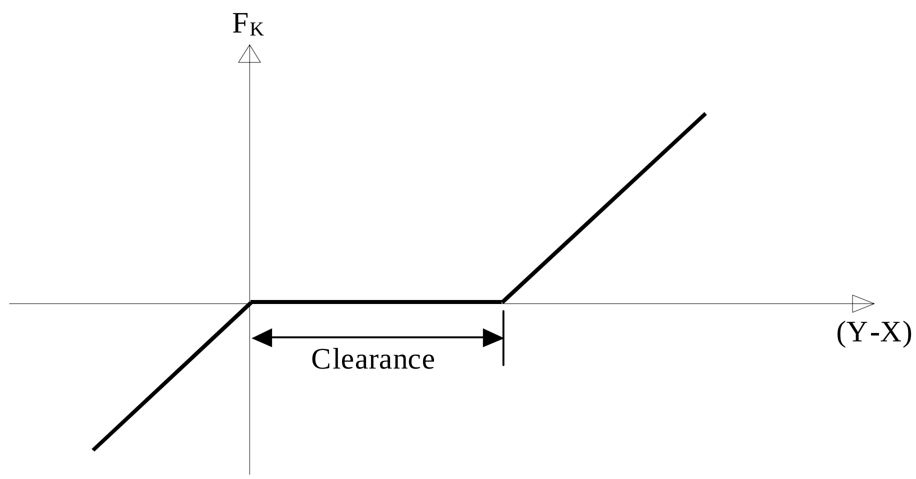


Figure 5.5 Plot of condition setting for spring stiffness *K*.

The spring constant is $K = \dfrac{\Delta F_K}{\Delta(Y - X)}$. In this subsystem, whether the impact forces exist depends on the displacement between the two masses, and the spring that link them up. From Figure 5.5, the set of conditions deciding the presence of the spring constant K is,

$$(Y - X) - Clearance > 0 \Rightarrow K \text{ is “ON”} \tag{5.2}$$

$$\left.\begin{cases}(Y - X) - Clearance \leq 0 \\ (Y - X) \geq 0\end{cases}\right\} \Rightarrow K \text{ is “OFF”} \tag{5.3}$$

$$(Y - X) < 0 \Rightarrow K \text{ is “ON”} \tag{5.4}$$

For when K is “ON”, the value of the spring constant will apply; when K is “OFF”, a value of zero will be applied. This set of conditions has been transformed into the subsystem of the simulation model by the layout of the block diagram shown in Figure 5.6.

In this subsystem, the input is the varying displacement ($Y - X$). With the clearance (CL) value set at a constant of 1.5×10^{-3} inches, both the displacement and clearance values are passed through a Sum block to generate the required $(Y - X) - CL$ value. This value will then be passed as the second input to the Switch block “Switch1”, and will be used to compare to the threshold value in deciding to either propagate the first or third input. The first input for this block is the spring constant K (1×10^{6} lb/in) and the third input is the constant of 0. The threshold value for this block is set at a very small value, i.e. 0.00000001, instead of zero to account for the “greater than” and not “greater or equal to” setting. When the second input $(Y - X) - CL$ value is greater than or equal to the threshold value, “Switch1” will propagate the first input, which is the spring constant K, to its output. Otherwise, it will propagate the third input (0). Hence “Switch1” will control the condition setting of

$$(Y - X) - CL > 0 \Rightarrow K \text{ is “ON”}$$
$$(Y - X) - CL \leq 0 \Rightarrow K \text{ is “OFF”}$$

For “Switch2”, the subsystem input of ($Y - X$) will be passed directly as the second input to this Switch block. The threshold value for Switch2 is set at zero. The first input is the constant of 0 and the third input is the spring constant K (1×10^{6} lb/in). Hence when the second input ($Y - X$) value is greater or equal to zero, Switch2 will propagate the first input of value 0 to its output. Otherwise, it will propagate the third output (spring constant K). Therefore, “Switch2” controls the condition setting of

$$(Y - X) \geq 0 \Rightarrow K \text{ is “OFF”}$$
$$(Y - X) < 0 \Rightarrow K \text{ is “ON”}$$

With “Switch1” and “Switch2” taking care of their respective condition setting, outputs from both Switch blocks will be summed using a Sum block to attain the complete set of condition setting of (5.2), (5.3), and (5.4). The output of 0 or K will then be passed on to the subsystem output.

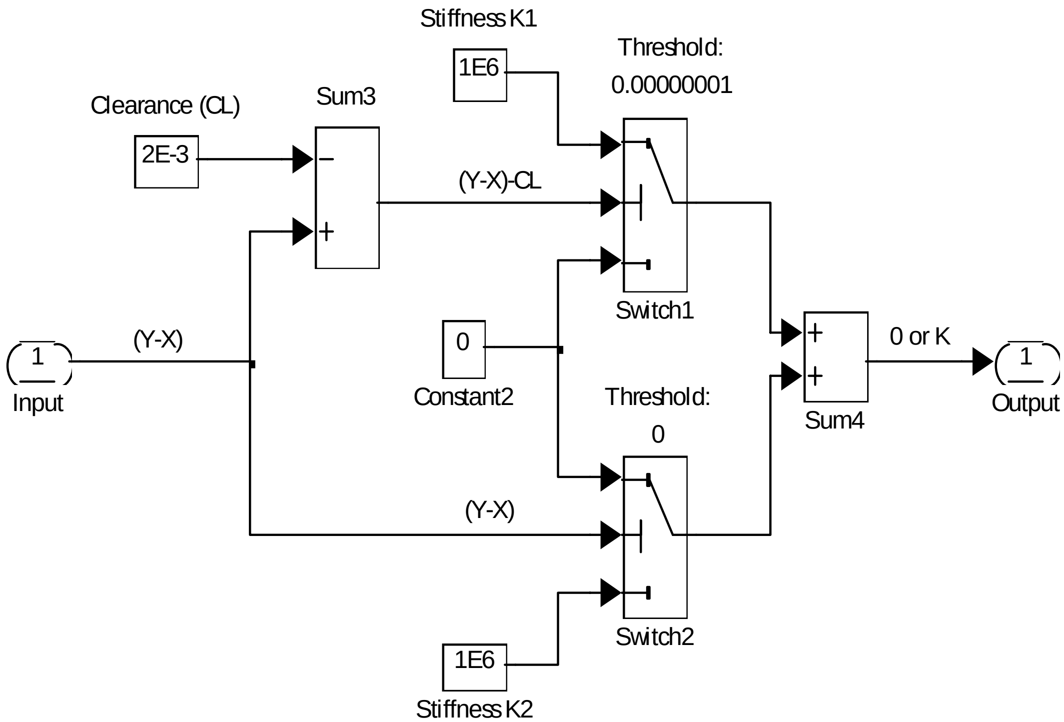


Figure 5.6 Block diagram of the spring constant *K* condition setting subsystem model.

5.2 The Impact Forces

5.2.1 The Complete SIMULINK Model

With both of the subsystems constructed, the complete model of the impact force dynamic system is constructed with reference to the model in Figure 5.3a and the SIMULINK model generated is shown in Figure 5.7.

Performing an integration on the constant of "1", whereby $\int 1\, du = u$, generates the input time u for the model. This input is used to drive both the fluid's pressure force subsystem in generating F_p and the acceleration expression generated by Equation (3.15) with frequency domain up to the order of 80, i.e. up to the expression with cos 80θ, where $\theta = \omega t$. In SIMULINK time t is expressed as u, hence $\theta = \omega u$. As the whole acceleration expression with such a high degree of order is too long to be contained in a single function Fcn block, it has been broken up into three parts to fit into the three Fcn blocks (f(u)) named "n=40_1", "n=40_2", and "n=40_3". Through the Integrator blocks and Sum blocks, $(\dot{Y} - \dot{X})$ and $(Y - X)$ are generated. The damping force F_D is obtained by passing $(\dot{Y} - \dot{X})$ through the Gain block with a damping value of 0.1 lb-s/in, in which $F_D = D(\dot{Y} - \dot{X})$. $(Y - X)$ is passed as an input into the spring constant setting subsystem to determined the presence of K as described in Section 5.1.2. The output (K or 0) from this subsystem will determine the present of spring force F_K at that given time by $F_K = K(Y - X)$. The impact forces output can be seen by using the Scope block in which plot of force versus time will be shown.

5.2.2 Conversion of Data

As the impact forces output in SIMULINK is in the time domain, it will have to be converted into the frequency domain in order to have an easy comparison with the shaking forces, which are in frequency domain. The conversion is done by using the Fast Fourier Transform (FFT) function in MATLAB. A data length equals to a power of 2 is chosen for the substantial increase in computation speed. Hence in the simulation parameters a fixed time step size (Δt) of 1.699702E-5 is used. This value is derived from

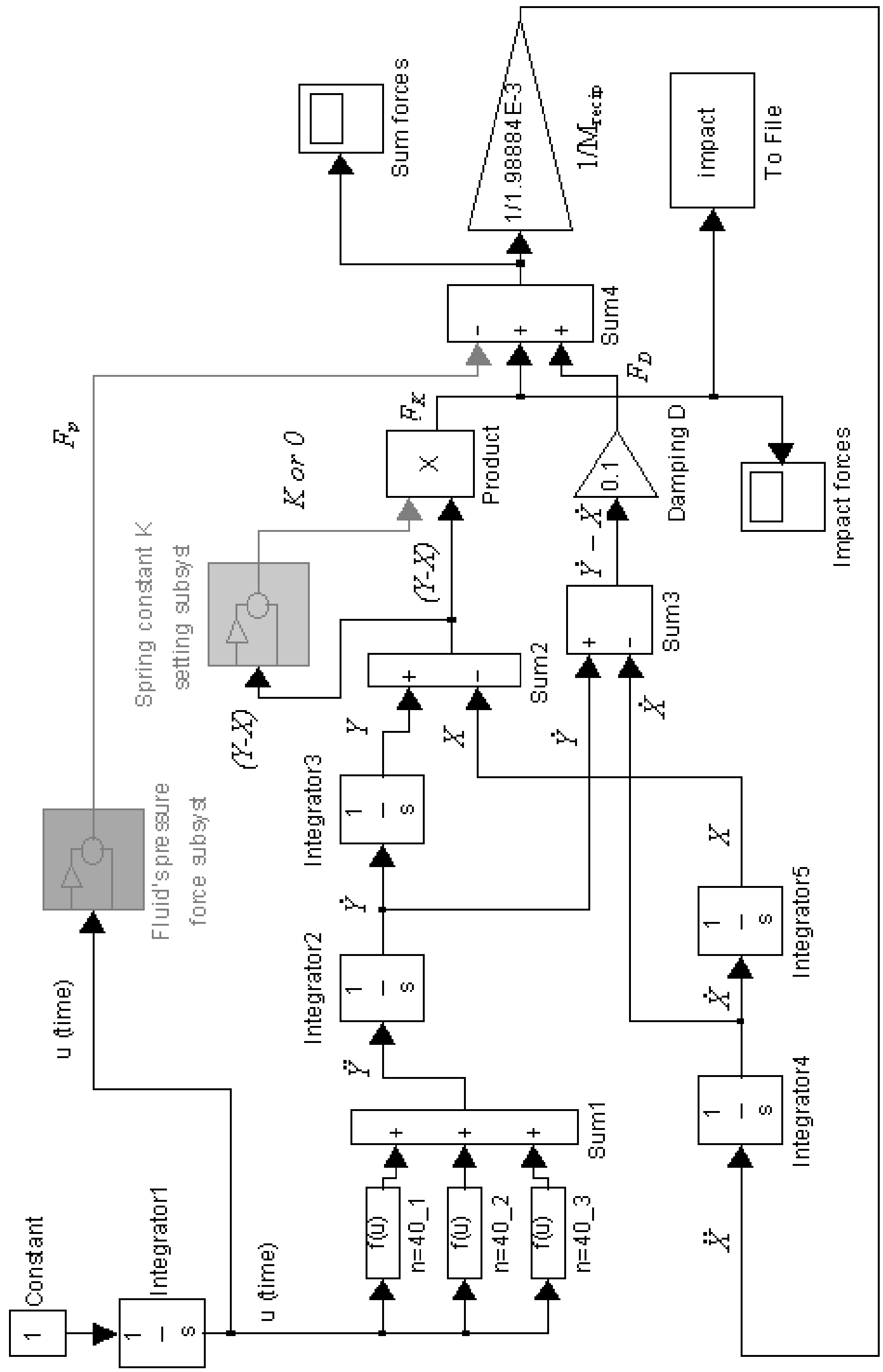


Figure 5.7 SIMULINK model for the complete modeling of impact forces.

$$\omega = 2\pi f \tag{5.5}$$

$$f = \frac{361}{2\pi}$$

$$\tau = \frac{1}{f} = 0.017405\,\text{sec} \tag{5.6}$$

$$\Delta t = \frac{\tau}{1024} = 1.699702 \times 10^{-5}\ \text{sec} \tag{5.7}$$

where ω = crankshaft angular velocity 361 rad/s

f = frequency

τ = time for 1 cycle

Δt = time step for 1024 sampling

The other setting in the simulation parameters are shown in Figure 5.8.

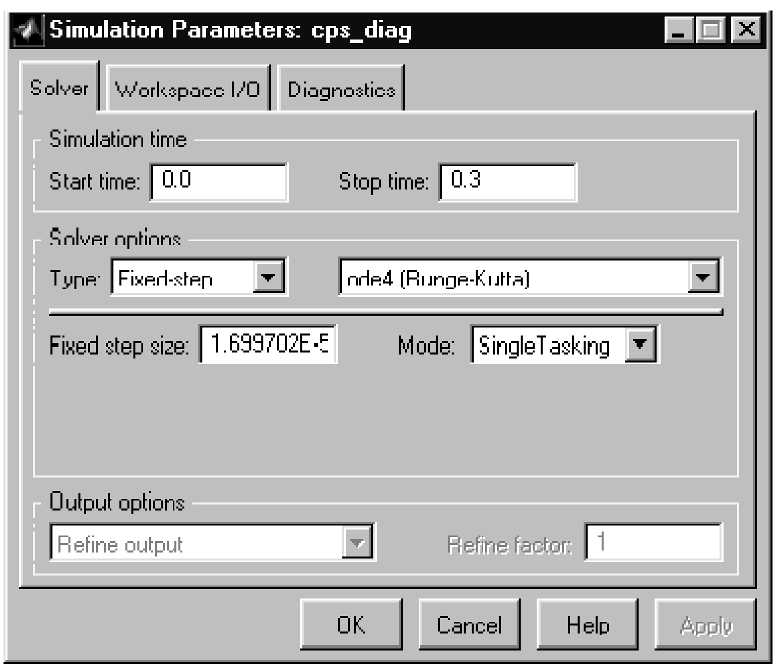


Figure 5.8 The simulation parameters.

The procedure for the conversion of data is as follows:

In SIMULINK:

1. Through the expressions of (5.5) to (5.7), the time step t of the simulation for a 1024 time step sampling is determined to be 1.699702×10^{-5} sec. The other simulation parameters are as shown in Figure 5.8.
2. Using the simulation model setup in Figure 5.7, the simulation is run and the impact force data is written as input to the To File block and saved as file impact.mat. The impact data will be written as a matrix with one column for each time step: the first row is the simulation time, and the second row is the impact force value at each corresponding time step. This matrix will be contained in the "Variable name" parameter with the default filename "ans".

In MATLAB:

1. A script file is written to load the impact.mat file and to extract the impact force values from the "ans" in the To File block in SIMULINK.
2. As the simulation time starts from 0 and ends at 0.3 sec, and from expression (5.6) the time required for one cycle is only 0.017405 sec or 1024 time steps, there will be more than one cycle of data collected for this duration of time, hence one cycle of data needs to be extracted from it. In this research, the one cycle is taken approximately from the 0.15 sec mark (data point 8825) to 0.167405 sec (data point 9848), inclusive of both points. This set of extracted data is then stored under a new filename. With that, the FFT function is invoked by

   New_name = fft(new filename),

   and the plot is made by issuing the plot command

   Plot(abs(New_name(1:512)/1024)*2).

The detail script file is in Appendix (IV). Figure 5.9 shows the plot of the average impact forces (one cycle) versus the clearance between the connecting rod end and the piston. Figure 5.10 shows the extracted one cycle of impact forces in the time domain. Figure 5.11 shows the impact forces in the frequency domain, with its average value. On the same plot are the shaking forces. Figure 5.12 is the semi-log plot of Figure 5.11, with y-axis in the log scale. Figure 5.13 is a zoom plot showing the intersection of the average impact force value and the shaking forces. The average impact force for clearances CL = 1×10^{-2} inches and CL = 1×10^{-4} inches are included to demonstrate the way the average line moves in the plot with change in average value. The clearance value used for the simulation is 1.5×10^{-3} inches.

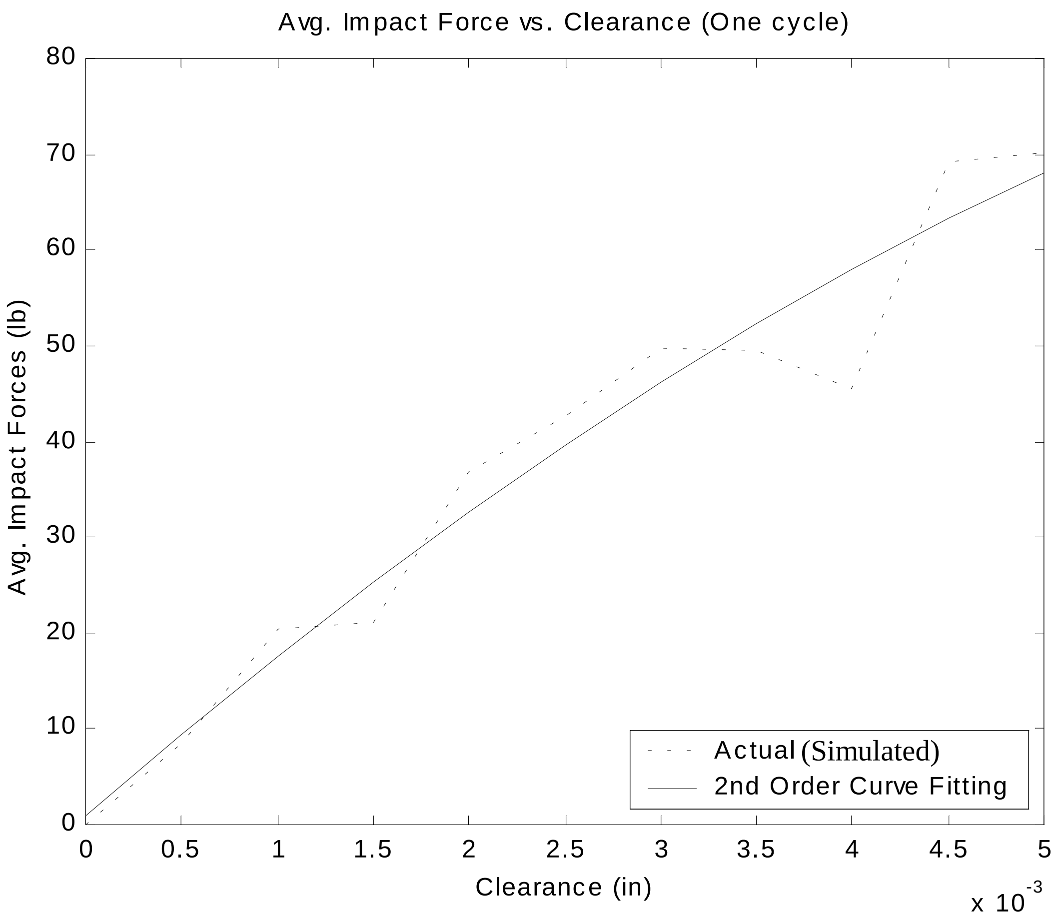


Figure 5.9 Plot of average impact force versus clearance.

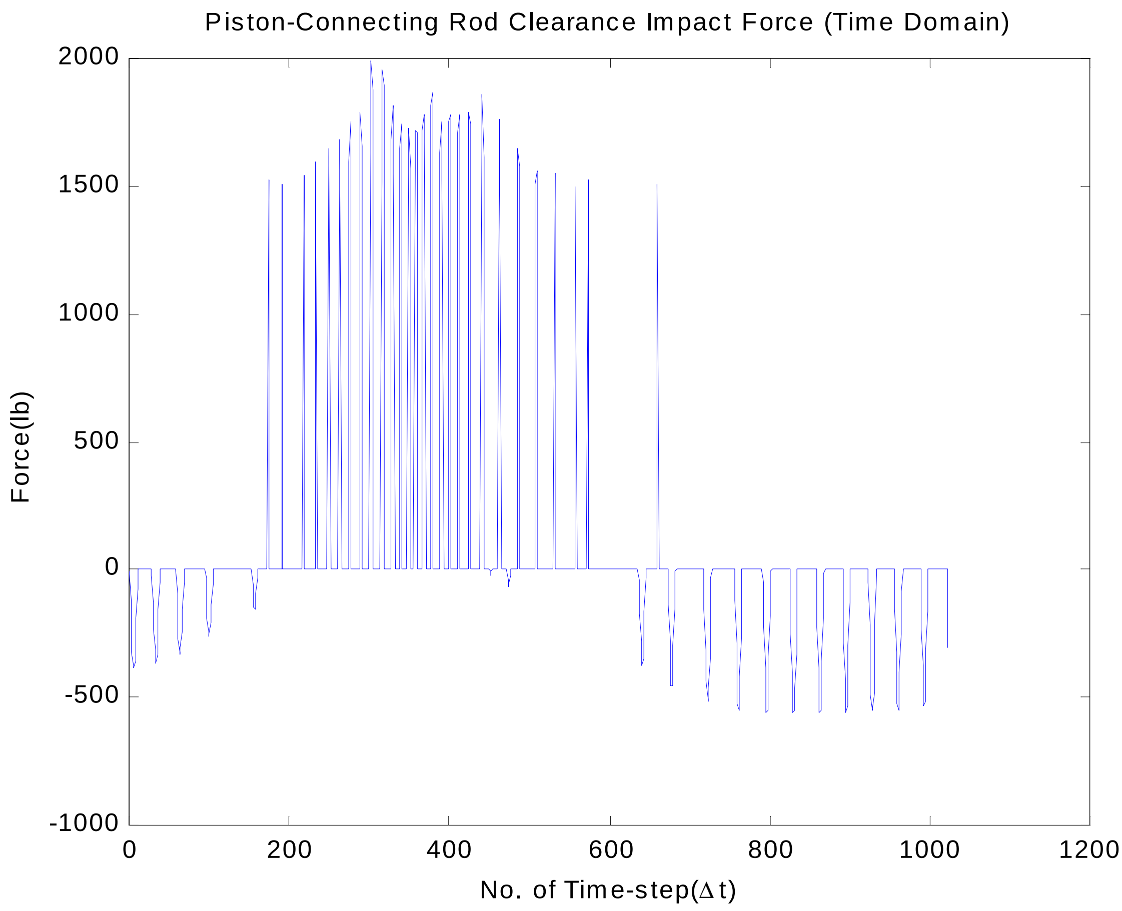


Figure 5.10 Plot of one cycle of impact forces in the time domain (as in SIMULINK).

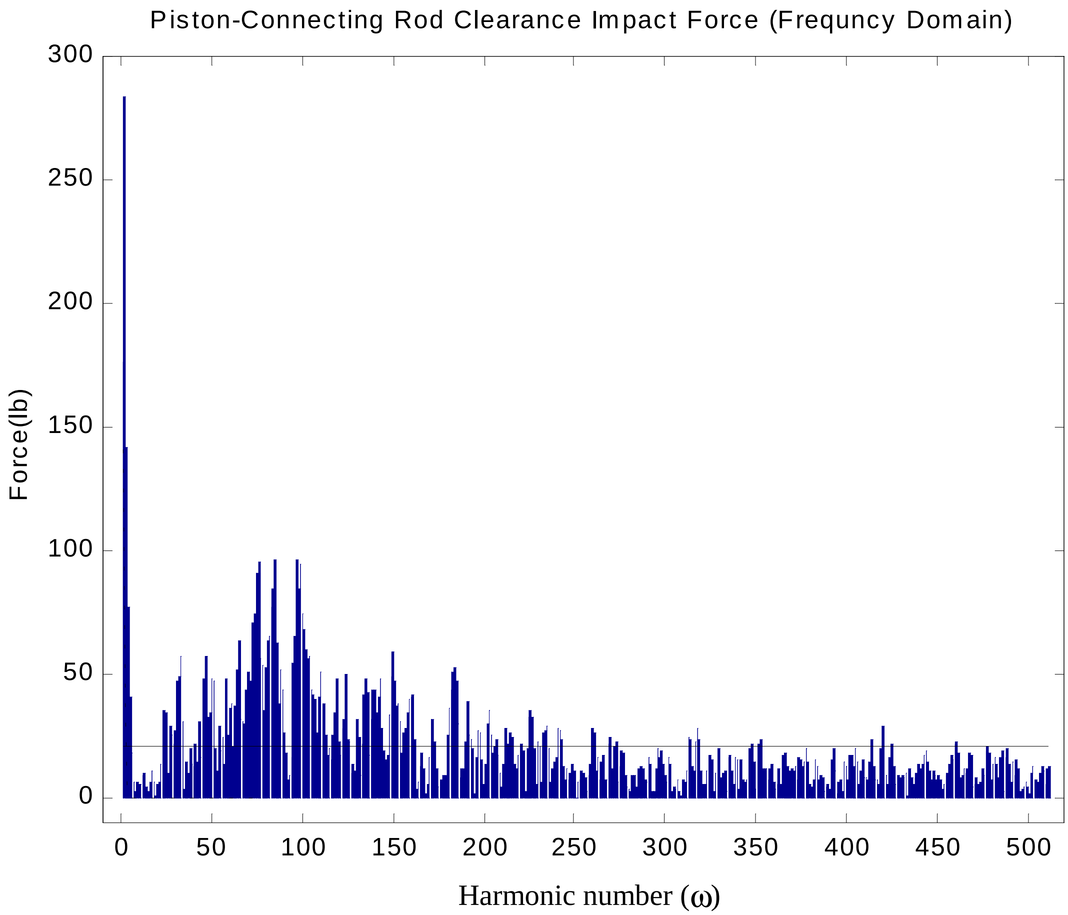


Figure 5.11 Plot of one cycle of impact forces in the frequency domain.

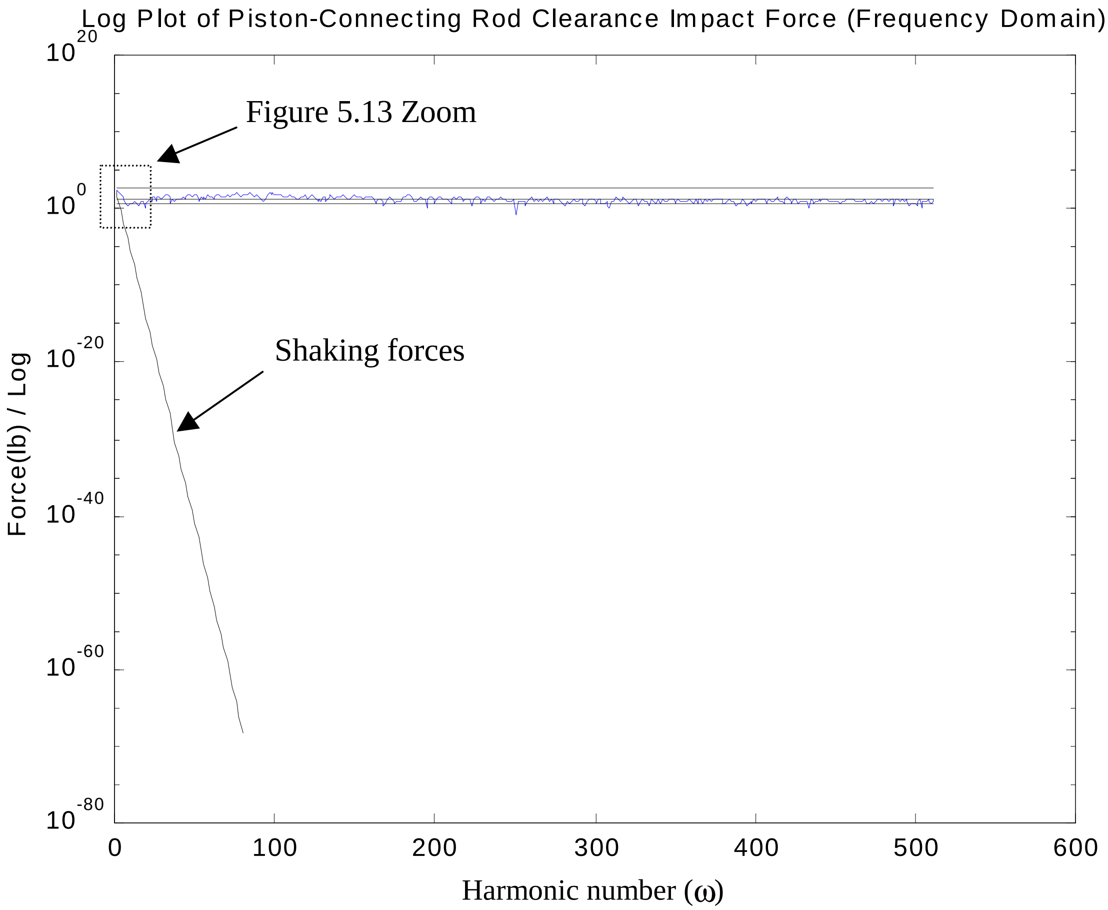


Figure 5.12 Semi-log plot of impact forces in the frequency domain.

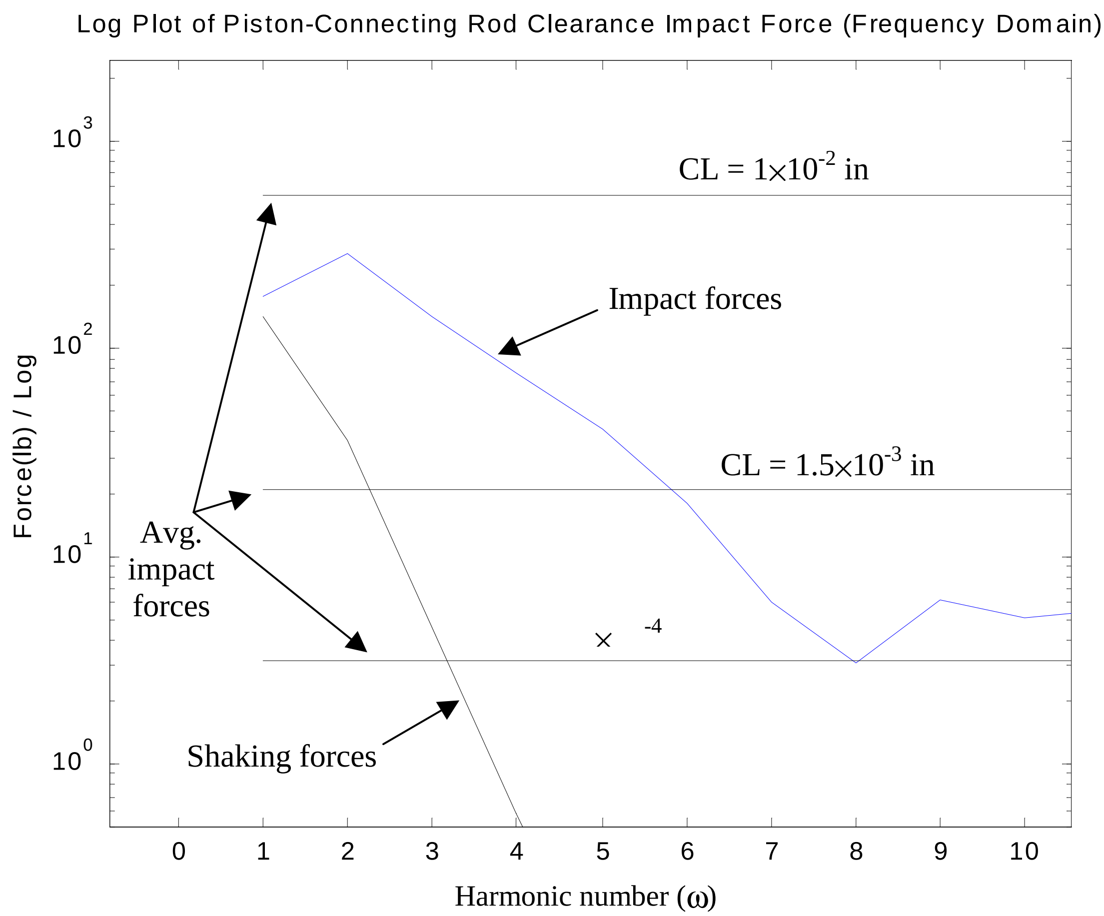


Figure 5.13 Zoom plot showing the intersection between the shaking forces and average impact force of different clearance.

Chapter 6. Z-axis Direction Force by the Motor Torque

6.1 The Motor Action

As stated in Section 3.1.2, the electric motor that is used to drive the compressor is a three-phase induction motor. As an illustration, Figure 6.1 shows a basic two-phase induction motor with two poles per phase from Gottlieb [5]. For a three-phase, two poles per phase motor, the setup will be the same as that in Figure 6.1 except that the six stator and windings will be placed at a 60° interval.

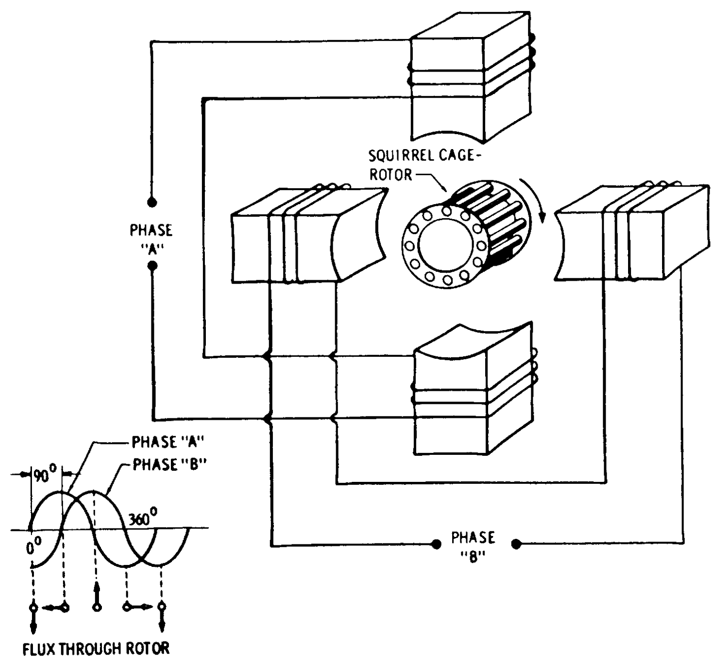


Figure 6.1 A two-phase, two-poles, squirrel-cage motor. (From Gottlieb [5])

From Cochran [6], according to Ampère's law, with the component of magnetic field and the current-carrying conductor placed mutually perpendicular to each other, a force perpendicular to both components will be generated. If this force appears as a tangential force at the circumference of the rotor, a torque or couple will be produced. Figure 6.2 shows an example of the interaction between the magnetic flux from the stator and the magnetic flux by the current-carrying conductors to generate the tangential force to produce the motor action.

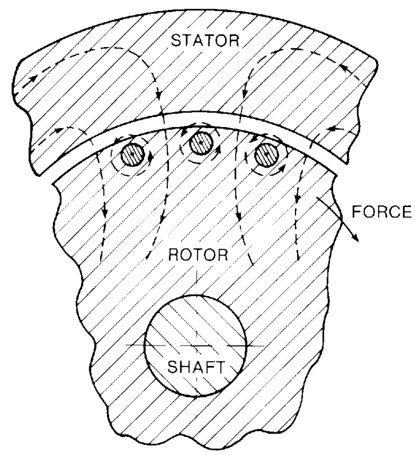


Figure 6.2 Simplified motor action. (From Cochran [6])

The torque of the motor is generated by:

$$T = F \cdot D$$
$$\Rightarrow F = \frac{T}{D} \qquad (6.1)$$

where T = motor torque (in-lb)

F = tangential force (lb)

D = diameter of the rotor (in)

Figure 6.3 illustrates the tangential force generated and acting on the conductors. In the manufacturing of the squirrel-cage rotor, it will require high precision manufacturing process and cost to fabricate the rotor that will have an exact 90° angle between the conductor rods and both the top and bottom end-rings. Hence there is a tendency that the conductor rods will skew at an angle, β, with respect to the vertical axis. Figure 6.4b demonstrates an exaggerated view of the

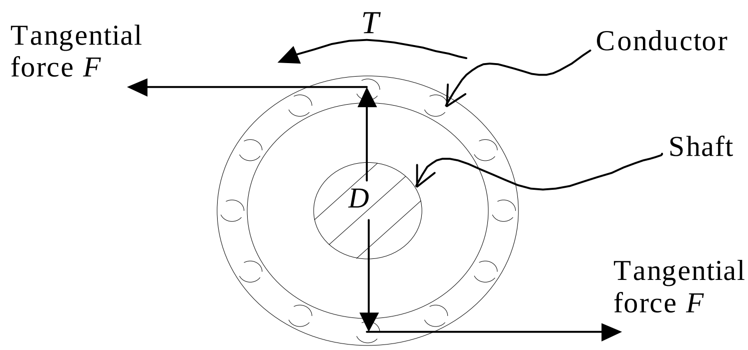


Figure 6.3 Tangential forces acting on the conductors of the rotor.

conductors skewed at an angle β. From Figure 6.4c, this skew angle will cause a force F_z acting in the z-axis direction. As a pair of forces generates the couple on the rotor, another force in the z-axis direction will be exerted on another conductor 180° apart. Hence the total forces in the z-axis direction is

$$F_z = 2F \sin \beta$$

With Equation (6.1),

$$F_z = 2\frac{T}{D}\sin \beta \tag{6.2}$$

From Cochran [6], the tangential force exerting on the conductor can be determined by

$$F = B\,I\,L \tag{6.3}$$

where B = flux density (webers/in^2)

I = current flow in the conductor (A)

L = conductor length (in)

However, as no information on B, I, and L are available for this research, the force exerted on the conductor is not determined by Equation (6.3). Instead, the forces F_z at different harmonic ω are computed based on the moment values about the z-axis direction from Zheng [7] and by using Equation (6.2). The moment values generated by Zheng are listed in Table 6.1.

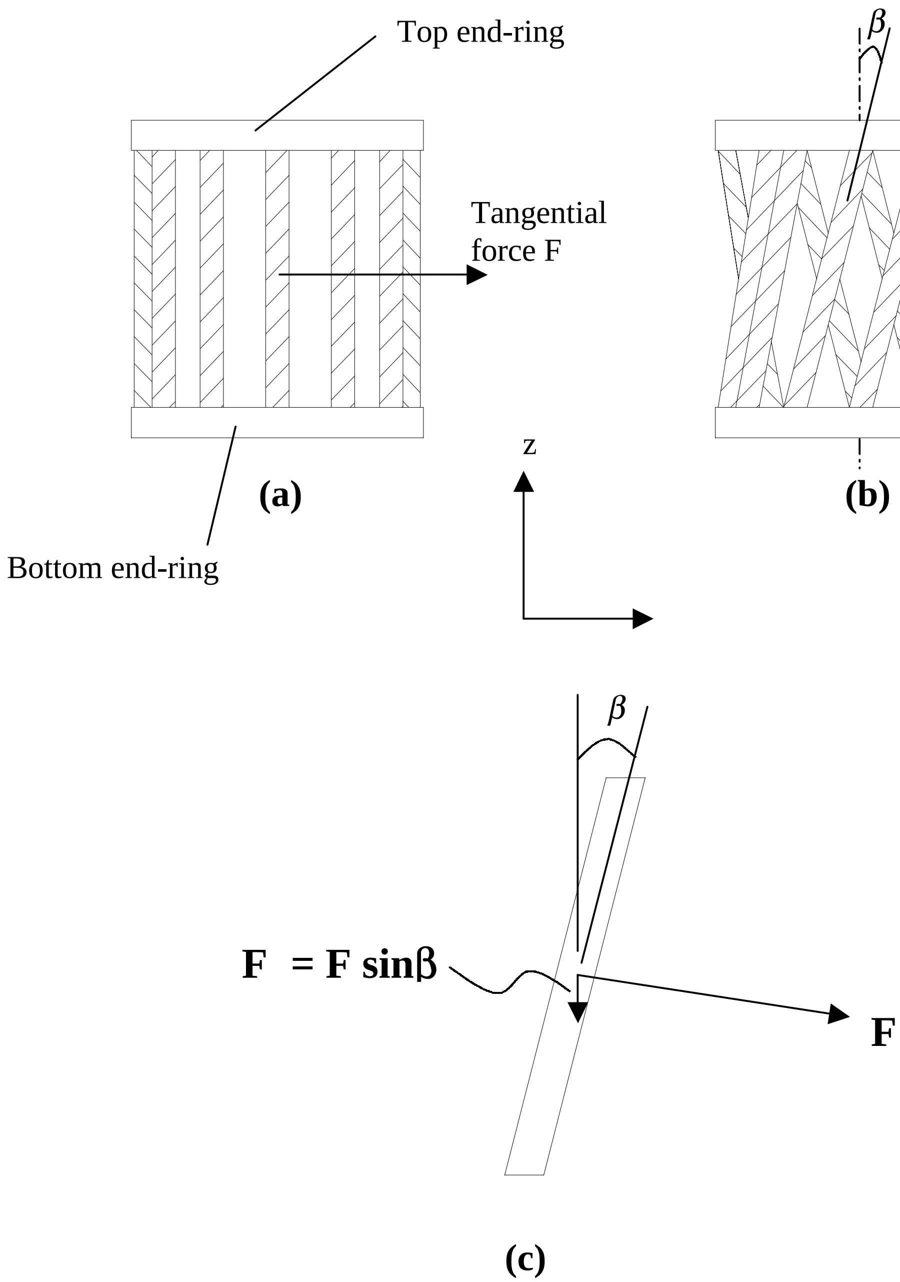


Figure 6.4 (a) Ideal squirrel-cage rotor. (b) Exaggerated view of squirrel-cage at a skew angle β. (c) The z-axis direction force due to β.

6.2 Calculation of F_z

As the moment values are listed under the sine and cosine function at different harmonic ω, the z-axis force F_z will be

$$F_z = F_o + \sum_{i=1}^{\infty} \left(F_{ic} \cos i\omega t + F_{is} \sin i\omega t \right) \tag{6.4}$$

where i = 1, 2, 3…; F_{ic} and F_{is} indicate the z-axis force generated due to the skew angle of the conductor at each harmonic under the cosine and sine function, respectively. By applying Equation (6.2), Equation (6.4) can be re-written as

$$F_z = F_o + \sum_{i=1}^{\infty} \left(\left(\frac{2T_{ic}}{D} \sin\beta \right) \cos i\omega t + \left(\frac{2T_{is}}{D} \sin\beta \right) \sin i\omega t \right) \tag{6.5}$$

In dynamic analysis, as F_o is a constant at 0 ω, it does not introduce kinetic energy and hence will be omitted. With that, the magnitude of the z-axis force can then be obtained by

$$F_z = \sum_{i=1}^{\infty} \sqrt{ \left(\frac{2T_{ic}}{D} \sin\beta \right)^2 + \left(\frac{2T_{is}}{D} \sin\beta \right)^2 } \tag{6.6}$$

6.2.1 Computation of F_z

The computation of the z-axis direction force F_z is done by using MATLAB. The diameter D for the rotor is 6 inches, and the skew angle β for the conductor is assumed to be 0.035 rad (approximately 2°). The force matrix setups for the cosine and sine function are

$$\begin{Bmatrix} 0 \\ 0 \\ F_{ic} \\ 0 \\ 0 \\ 0 \end{Bmatrix}_{6\times i}$$

for the cosine function and

$$\begin{Bmatrix} 0 \\ 0 \\ F_{is} \\ 0 \\ 0 \\ 0 \end{Bmatrix}_{6\times i}$$

for the sine function. As the harmonic range listed in Table 6.1 is from ω to 40ω,therefore i in this computation will be 40. Hence the force matrices for both the cosine and sine function will be a 6-by-40 matrix. The z-axis force F_z is computed by using Equation (6.6). The process of computing for displacement X and hence the kinetic energies is the same as that stated in Section 4.2. The computed result for F_z is 3.446 lb. A plot of force at each harmonic and the total F_z is shown in Figure 6.5.

Table 6.1 Moment about Z-axis direction in the frequency domain (from Zheng [7]).

_	Cosine Function (in-lb)	Sine Function (in-lb)
0	-1.37454E+02	0.00000E+00
1	-1.82402E+01	3.24938E+01
2	4.97732E+01	6.33141E+01
3	1.58242E+01	-4.65144E+00
4	7.86477E+01	-2.68636E+01
5	1.76394E+00	-3.19106E+00
6	-3.23154E+00	-5.62280E+00
7	-4.72647E-01	1.75284E+00
8	9.69175E+00	1.85207E+01
9	1.96588E+00	9.78834E-01
10	1.05662E+01	-1.74664E-01
11	1.72279E-01	-2.68174E-01
12	-3.46465E+00	1.87656E+00
13	-4.42965E-01	8.08455E-01
14	6.76711E-01	7.56060E+00
15	3.95510E-01	4.23005E-01
16	1.68396E+00	6.03541E-02
17	-1.37070E-01	-1.86406E-01
18	-3.30360E+00	-2.08712E-01
19	-3.52211E-01	2.48868E-01
20	-4.93303E-01	2.41382E+00
21	1.50103E-01	8.96729E-02
22	9.87604E-01	-9.68858E-01
23	-2.52519E-02	-2.14241E-01
24	-1.39574E+00	-8.77440E-01
25	-1.42187E-01	9.16269E-02
26	2.06974E-01	1.21738E+00
27	1.40152E-01	7.05616E-02
28	1.09832E+00	-3.35873E-01
29	2.42590E-02	-1.07412E-01
30	-7.50774E-01	-2.68336E-01
31	-9.51059E-02	9.05268E-02
32	-7.28636E-02	1.07194E+00
33	6.82085E-02	8.08790E-02
34	5.19242E-01	-7.19473E-02
35	-1.07511E-02	-7.60614E-02
36	-7.63753E-01	-3.09647E-01
37	-9.14587E-02	3.68388E-02
38	-2.57909E-01	5.22219E-01
39	3.51325E-02	3.72874E-02
40	4.28037E-01	-2.69016E-01

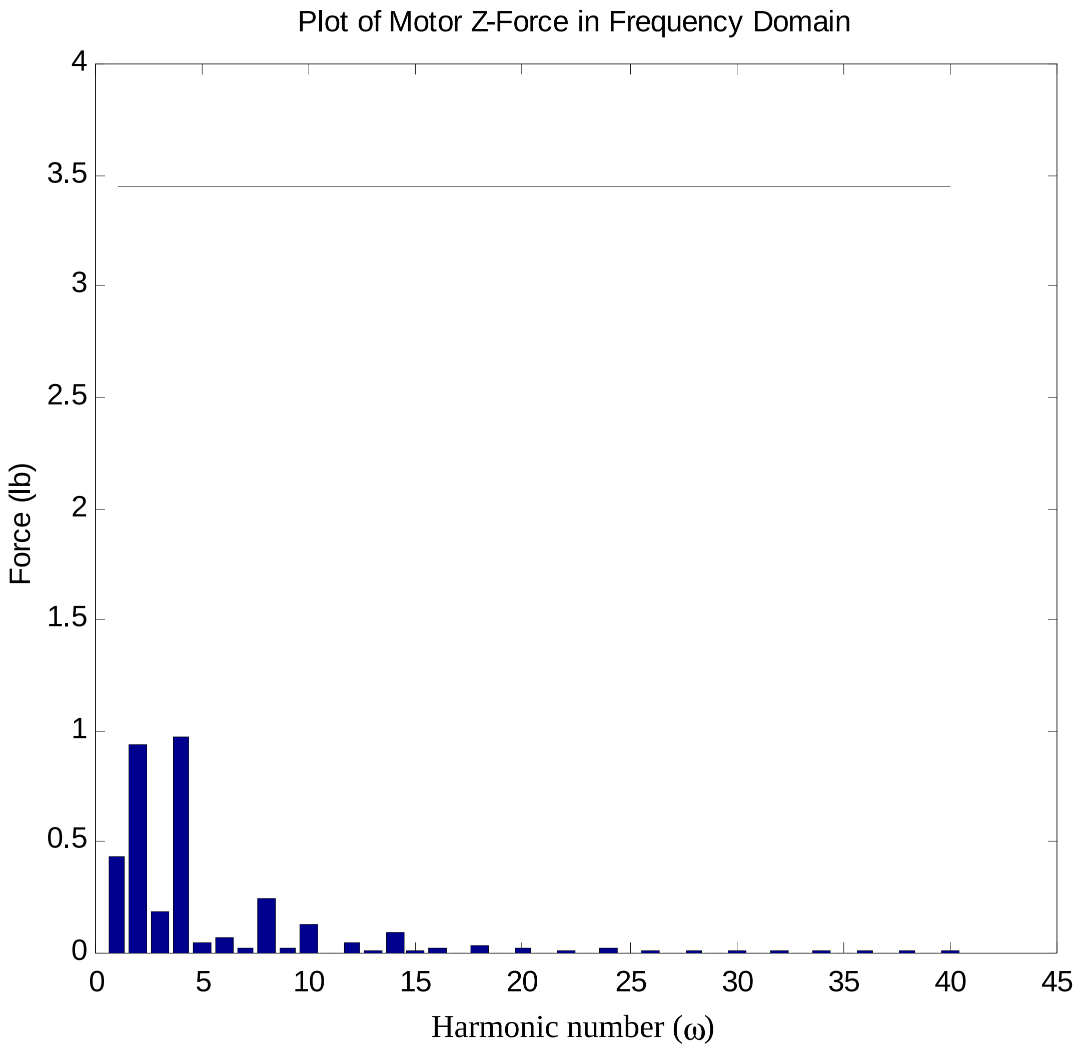


Figure 6.5 Plot of the z-axis direction force generated by the motor torque.

Chapter 7. Optimization of Crankshaft Balancing

7.1 Optimization

Optimization is the seeking of the optimum point (maximum or minimum) in a mathematical model function with the given set of variables. This function must be expressible in term of the set of variables given in order for the optimization routine to work. From Reinholtz [8], an optimization problem or a mathematical programming problem can be stated in a general form of finding a set of design variables or design vector **X**, where

$$\mathbf{X} = \{x_1, x_2, x_3 \ldots x_n\}^T$$

to maximize or minimize an objective function $f(\mathbf{X},\boldsymbol{\Theta})$. x_1, x_2, $x_3 \ldots x_n$ are the design variables and the vector $\boldsymbol{\Theta}$ consist of a set of independent parameters whereby $\boldsymbol{\Theta} = \{\Theta_1, \Theta_2, \Theta_3 \ldots \Theta_q\}^T$. The superscript T denotes a vector or matrix transpose.

There are mainly two groups of optimization methods in the nonlinear problems, whereby the objective function is nonlinear in any of the design variables. These two groups are being classified as the gradient methods and direct search methods. The gradient methods generally involve the determination of the first and sometimes the second partial derivatives of the objective function. As the first and second derivatives of a mathematical function define its gradient and turning point (of a curve), the maximum or minimum of the function is then readily obtained. The direct search methods based simply on the sequential comparison of the objective function values computed and move towards the direction where there is an improvement in the function values. Different varieties of the gradient methods and the direct search methods can be found in Siddall [9], Beveridge and Schechter [10].

In this research, the optimization method will be applied to seek the setting of angle, force, and torque on balancing the already-balanced crankshaft in order to minimize the vibration when the compressor is in operation at different operating modes. This in turn will minimize the change in the operating noise level of the compressor when switching from a two-cylinder operating mode to a one-cylinder operating mode and vice versa.

7.2 The Optimization Technique

7.2.1 Hooke and Jeeves Pattern Search Method

The objective function in this part of the research is to determine the corrective kinetic energy, KE_{corr}, to minimize the change in the difference of kinetic energies when switching between the two operating modes of one- and two-cylinder. As the function for computing the KE_{corr} involves a lengthy and complex process, where few sub-functions are put together in generating the final output value, no single explicit expression representing the objective function can be derived and hence obtaining the derivatives for the objective function is impossible. As a consequence, optimization by the gradient methods is inapplicable in this research. Alternatively, since KE_{corr} can be determined numerically, numerical evaluation of the objective function steer the optimization option towards the direct search methods, which does not call for the derivatives of the objective function and the computed function's numerical values are used in the sequential comparison to move in the direction of improves value, or optimal point.

Among the search methods in nonlinear numerical optimization, Hooke and Jeeves pattern search method stands out to be simple yet very effective optimization technique (Siddall [9]). This technique consists of two major routines: the exploratory search routine and the pattern move routine. The exploratory routine search the local proximity in the directions parallel to the coordinate axes for an improved objective function value, and the pattern routine accelerate the search by moving to an new improved position in the direction of the previous optimal point obtained by the exploratory routine. Figure 7.1 is a sample three-dimensional plot with the minimum point as indicated. In order to simplify the explanation of the algorithm, Figure 7.2 is a two-dimensional contour plot of Figure 7.1 and will be used to outline the Hooke and Jeeves optimization method. The algorithm is as follows:

1. An arbitrary or initial base point at $\mathbf{X^0} = \{x_1, x_2\}^T$ is selected, and the objective function value for this point is computed.
2. The exploratory search routine is started by giving x_1 a predetermined step Δx_1 in the positive direction, and the objective function is evaluated at this new point. If this

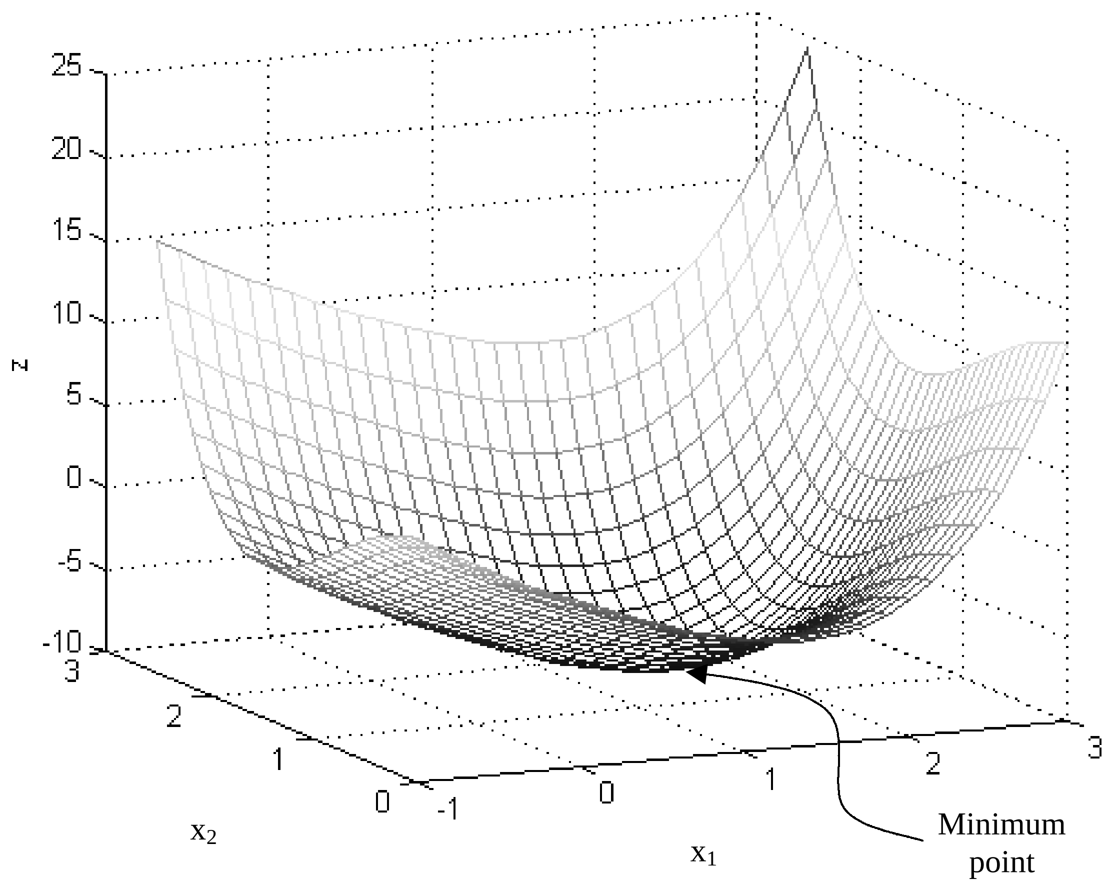


Figure 7.1 Sample 3-D plot for generating the 2-D contour plot.

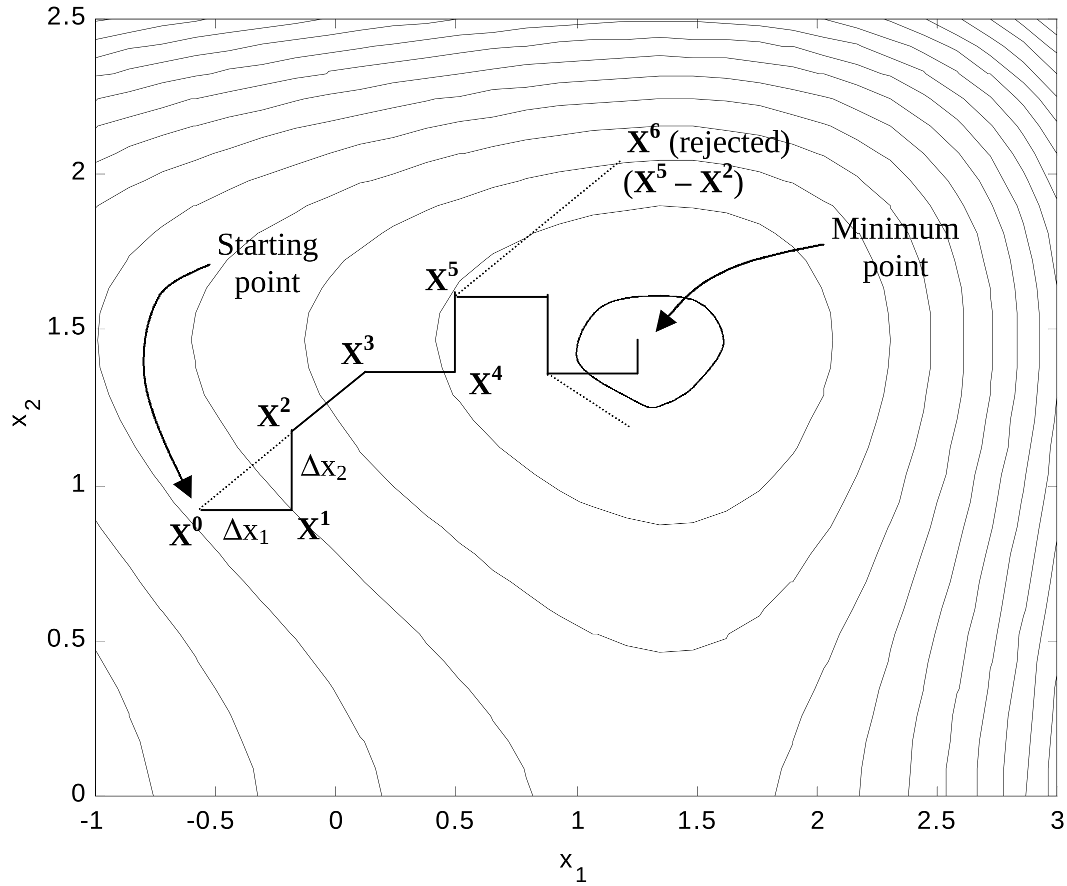


Figure 7.2 Hooke and Jeeves pattern search strategy.

improves the value of the objective function, this step will be retained and become the new base point $\mathbf{X^1}$. If not, a corresponding step in the negative direction, $-\Delta x_1$, is taken and the objective function will be re-evaluated. This point will be retained and made the new base point $\mathbf{X^1}$ if the move is successful. If both fail, no step is taken. At this stage the step size of Δx_1 will be adjusted according. For a successful step move, the value of x_1 will increase. If both steps fail, the value of Δx_1 should be decreased. From the newfound base point $\mathbf{X^1}$, a similar exploratory search is made for x_2 in the x_2 direction. The best point found in this search is labeled $\mathbf{X^2}$.

3. A pattern move is now attempted by changing each variable from the last base point, $\mathbf{X^2}$, an amount equivalent to the distance between the new base point $\mathbf{X^2}$ and the initial base point $\mathbf{X^0}$, or $(\mathbf{X^2} - \mathbf{X^0})$. This difference will commonly include the previous exploratory moves and previous pattern move.
4. If the pattern move is successful in obtaining a better objective function value, it will be retained and become the new base point $\mathbf{X^3}$. If it fails, it is canceled and replaced by a new exploratory search that will begin at point $\mathbf{X^2}$.
5. This iteration continues until the exploratory search fails to locate a better point. As stated in part (2), step size $\Delta\mathbf{X}$ will be reduced, and the search is repeated. This will keep repeating until $\Delta\mathbf{X}$ is below a certain predetermined limit (tolerance limit), in which the minimum (optimum) is assumed to have been reached.

A flow chart of this procedure developed by Reinholtz [8] is shown in Figure 7.3. One important note about nonlinear optimization programming is that a local minimum point rather than a global minimum point may be obtained, and the program cannot differentiate this. In other words, given different starting points or initial values, different local minimum will be obtained.

7.2.2 MATLAB Code for Hooke and Jeeves' Optimization

The programming code for the direct search method is developed using MATLAB, hence the flow chart in Figure 7.3 is using MATLAB programming convention. For example, the symbol "= =" means "equal to" in which two variables will be compared; and the symbol "=" is used to assign the output of an operation to a variable, where the output will be on the right hand

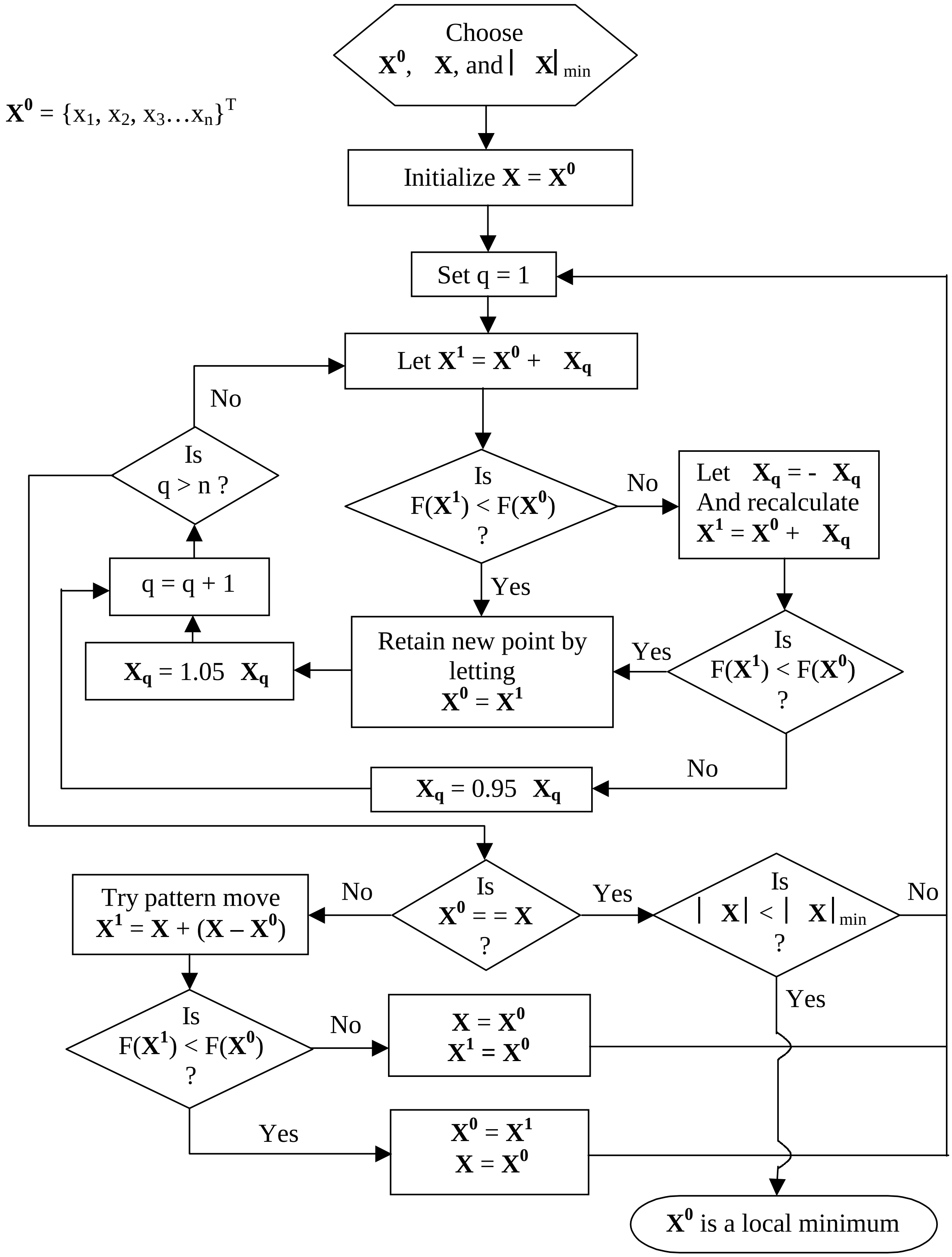


Figure 7.3 Flow chart of the Hooke and Jeeves' optimization pattern search method.
(From Reinholtz [8])

side of the "=" and the variable will be on the left hand side. The "q" in the routine is the parameter setting for the number of variables that will be optimized. There are three design variables in this optimization routine, which are the angle, force, and torque. Hence in the While loop of the routine, "n" will be set as 4 for "q<4".

The objective function is to minimize the difference in change of kinetic energy when switching the compressor from a two-cylinder operating mode to a one-cylinder operating mode, or vice-versa. The three design variables, in their optimum setting, are used to generate the corrective kinetic energy, KE_{corr}, to minimize this change. The objective function can be written as

$$f(KE) = \left\| KE_{AB,2} - KE_{A,2} \right| - KE_{corr} \right| = \text{Minimum} \qquad (7.1)$$

$KE_{AB,2}$ is the total kinetic energy generated when both Cylinders A and B are in operation, with the crankshaft balanced for two-cylinder operation. $KE_{A,2}$ is the total kinetic energy generated when only Cylinder A is in operation, with the crankshaft balanced for two-cylinder operation. With that, it is obvious that $KE_{A,2}$ will have a much higher value than $KE_{AB,2}$ since the crankshaft will be out-of-balance when only one cylinder is in operation. As the operation of the compressor can be switched to-and-fro between the two-cylinder and one-cylinder operating mode, objective function of expression (7.1) will not be feasible as its only taking care of the change when switching from two-cylinder operation to one-cylinder operation.

In order to attain the state whereby when switching from one operating mode to the other with only a minimum of kinetic energy change occurs between the switching, an alternative objective function has been formulated. Hence the new objective function is

$$f(KE) = \left\| \frac{xKE_{AB,2} + yKE_{A,2}}{2} \right| - KE_{corr} \right| = \text{Minimum} \qquad (7.2)$$

with $x + y = 2$, and both x and y are positive variables. This setting is developed with the aim of providing some flexibility for the user to set the values for x and y depending on the usage of the compressor in favor towards one of the operating mode. Graphically, this can be represented by the graph in Figure 7.4. The dashed lines in the figure are examples of the intermediate change in kinetic energy values of $\left|\frac{xKE_{AB,2} + yKE_{A,2}}{2}\right|$ with different values being assigned to *x* and *y*.

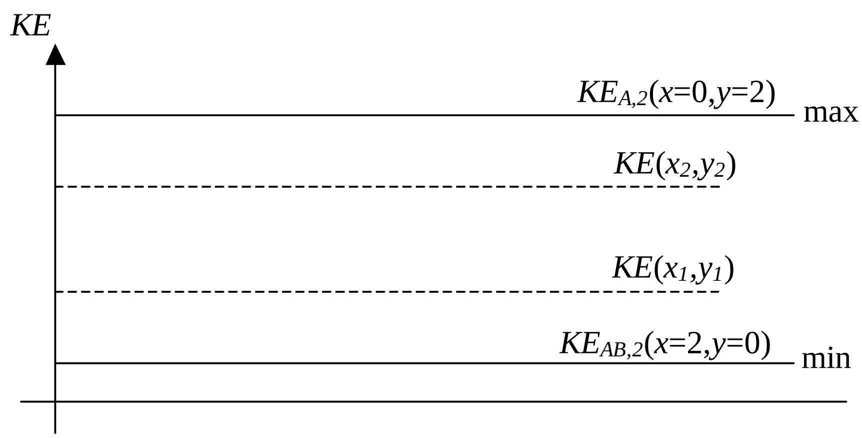


Figure 7.4 Intermediate change of kinetic energies due to the setting of *x* and *y*.

Chapter 8. Results and Discussions

8.1 The Total Kinetic Energies

A summary of the results in the total kinetic energies computed is shown in Table 8.1.

Table 8.1 Summary of the calculated total kinetic energies for different operation setting. (Unit for the total *KE* is in-lb)

	Crankshaft balanced for:		
Shaking forces and moments	(i) A and B ($KE_{cyl,2}$)	(ii) A Only ($KE_{cyl,A}$)	(iii) B Only ($KE_{cyl,B}$)
(1) A and B ($KE_{AB,S}$)	0.07148	0.10652	0.09324
(2) A Only ($KE_{A,S}$)	0.63686	0.61510	
(3) B Only ($KE_{B,S}$)	1.01021		0.97518

These are the total kinetic energies obtained from different operating mode of the compressor with the crankshaft balanced for different cylinder operation. For the case of (2) + (iii), where only Cylinder A is in operation while the crankshaft is balanced for only Cylinder B in operation, this will not be looked into as it is of no practical sense to balanced the crankshaft for the operating mode that will not be used. The same apply to the case of (3) + (ii), where only Cylinder B is in operation while the crankshaft is balanced for only Cylinder A in operation. In this research, interest is focus on switching the operating mode between the one- and two-cylinder operating modes with the crankshaft balanced for the two-cylinder in operation.

Since the crankshaft is balanced for two-cylinder operation, in the one-cylinder operating mode, the system will be off balance. With the crankshaft balanced for two-cylinder operation, when both Cylinder A and Cylinder B are in operation, the kinetic energy calculated is 0.07148 in-lb; when only Cylinder A is in operation, the kinetic energy calculated for this operation is 0.63686 in-lb. Note that if the setting is only Cylinder B in operation, the kinetic energy value is much higher compare to that of only Cylinder A in operation. This higher value of 1.01021 in-lb is expected, as Cylinder B is farther away from the system's center of mass compare to Cylinder A. Hence it will be impractical to set Cylinder B rather than Cylinder A in operation when one-cylinder operating mode is called for.

8.2 The Clearance Impact Forces

The impact forces due to the clearance between the reciprocating end of the connecting rod and the piston are generated by dynamic simulation using SIMULINK. Three different clearances are used. With the clearance of 1.0×10^{-2} inches, the average impact force computed is 3.16 lb. With the clearance of 1.5×10^{-3} inches, which is taken to be the actual clearance for use in this research, the average impact force computed is 21.11 lb. For the largest clearance of 1.0×10^{-2} inches, the average impact force computed is 546.26 lb. These average forces are plotted and shown in Figure 5.13. The purpose of using difference clearances is to demonstrate the idea of the importance of the effect of clearance can have on the overall performance of the compressor in term of shaking forces. From Figure 5.13, with comparison to the shaking forces by the reciprocating mass, it has clearly shown that if the total clearance is sufficiently large, the shaking of the compressor will be due to the impact forces instead of the reciprocating masses and the balancing of the crankshaft.

8.3 The Z-axis Force due to Torque of Motor

With a skew angle β of 2.0° of the conductor rods of the squirrel-cage rotor, the computed z-axis force is 3.45 lb. This does not appear to be of any influence in the shaking of the compressor. In fact, the assumed skew angle of 2.0° for the skewed conductor rods has exceeded the usual allowable manufacturing tolerance of ±1.0° or ±1.5°, hence the actual z-axis force can be expected to be lower than 3.45 lb. This further reduces the importance of the effect this force has on the compressor.

8.4 Optimize Balancing for the Crankshaft

The aim of the optimization routine is to reduce the change in kinetic energy when switching the operating mode of the compressor for two-cylinder operation to a one-cylinder operation and vice versa. As the current crankshaft is balanced for the two-cylinder operation, switching to a one-cylinder operating mode will cause the crankshaft to be out balance and hence the increase in the kinetic energy. Hence a new setting for balancing the crankshaft has been called for to generate a corrective kinetic energy KE_{corr} to minimize this change in energies in switching to-and-fro between the one- and two-cylinder operations. In Section 7.2.2, the objective function that has been developed for the optimization is

$$f(KE) = \left\| \left| \frac{xKE_{AB,2} + yKE_{A,2}}{2} \right| - KE_{corr} \right\| = \text{Minimum} \qquad (8.1)$$

with $x + y = 2$, and both x and y are positive variables.

To maintain the same level of change in kinetic energy when switching the operation of the compressor between the two operating modes, that is

$$KE_{A,2} - KE_{corr} = KE_{AB,2} + KE_{corr}$$
$$KE_{A,2} - KE_{AB,2} = 2\ KE_{corr} \qquad (8.2)$$

With KE_A = 0.63686 in-lb and $KE_{AB,2}$ = 0.07148 in-lb, and substituting these values into Equation (8.2), the value obtained for the corrective kinetic energy KE_{corr} is 0.28269 in-lb. With these information and the condition of $x + y = 2$, from Equation (8.1),

$$\left\| \left| \frac{x(0.07148) + y(0.63686)}{2} \right| - (0.28269) \right\| = 0$$

$$x(0.07148) + y(0.63686) = 0.56538$$

$$(2 - y)(0.07148) + y(0.63686) = 0.56538$$

$$\Rightarrow y \approx 0.747$$

and

$$x = 2 - 0.747$$
$$= 1.253$$

Hence the objective function for the optimization in this research will be

$$f(KE) = \left\| \frac{(1.253)(KE_{AB,2}) + (0.747)(KE_{A,2})}{2} \right| - KE_{corr} \right| = \text{Minimum}$$

One of the main concerns in optimization is the problem of getting optimized result of the local optimum point instead of the global optimum point. Another concern is the problem of the optimization routine "leap" over a local or global optimum point due to the step size chosen. When the optimization routine is first run in MATLAB, it was noted that given different initial values to the variables of angle, force, and torque, different minimum points were obtained. Hence it shows that local minimum points rather than global minimum point are present. Therefore, an alternative solution to this problem is to begin the search with ranges of initial values for the three variables, and the best local minimum point is selected base on the smallest objective function value obtained. This selection method coincides with that stated by Beveridge [10]. The range of initial values for the angle input is from 0.1 to 2π radians, with a 0.1 radian increment per step of computation; the range of initial values for the force input is from 68.0 to 77.0 lb, with an increment of 1.0 lb per step; and the initial values for the torque input is from 71.0 in-lb to 85.0 in-lb, with an increment of 1.0 in-lb per step. Some of the local minimum points computed are listed in Table 8.2. Values of Delta KE (ΔKE) is the objective function value of the optimization routine, and the rank number indicates the ranking of the local minimum point in term of ΔKE with 1 as the best optimized setting.

Table 8.2 Local minimum points computed by the Hooke and Jeeves optimization method.

Angle (rad)	0.2176	0.7348	2.3936	2.3340	3.4747	3.8537	5.2926
Force (lb)	72.3804	72.1619	72.7182	72.6827	72.2244	72.2211	72.2622
Torque (in-lb)	78.7485	85.5956	80.7396	79.6419	80.2494	84.2173	84.2889
Delta KE (in-lb)	2.6284E-10	5.4360E-11	2.1903E-10	1.8651E-11	1.8328E-10	1.1506E-12	2.8708E-11
Rank	7	4	6	2	5	1	3

To minimize the possible "leap" over a potential minimum point, a smaller step size has been used for the search routine and the pattern move routine in the optimization code, which are 0.95 for the search routine and 1.05 for the pattern move routine. It is to note that with a smaller step size, the computation time will be longer. With the above-mentioned setting, it can be seen that among the local minimum points in Table 8.2, the computed best optimized setting for the balancing is

$$\begin{Bmatrix} Angle \\ Force \\ Torque \end{Bmatrix} = \begin{Bmatrix} 3.8537 \\ 72.2211 \\ 84.2173 \end{Bmatrix}$$

with the objective function value being 1.1506×10^{-12} in-lb. The angle unit is in radians, the force is in lb, and the torque is in in-lb. It is to be noted that the angle is with reference to the top cylinder (Cylinder A) axis as indicated in Figure 8.1; the force and torque are with respect to the global center of mass defined in Chapter 3.

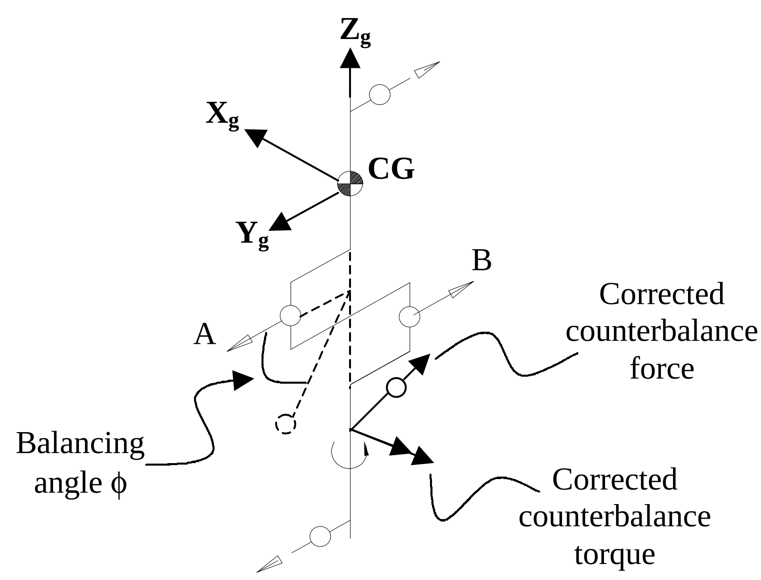


Figure 8.1 Optimum balancing angle with reference to Cylinder A axis, with a sample demonstration of optimized corrected force and torque.

Chapter 9. Conclusions and Recommendations

9.1 Conclusions

There were three objectives for this research. First, it was thought that shaking at the higher order terms, or the higher harmonics ω, would contribute a substantial amount to the kinetic energies of the compressor. As it is a normal practice that only the first two terms of the binomial series expansion in deriving the displacement, velocity, and acceleration of a slider-crank mechanism are taken into consideration for generating the shaking forces, the point or at which harmonic where the effect can be let off has not been investigated. Secondly, among the three main concerns of forces, which are the piston-crankshaft shaking force, the piston-connecting rod impact force, and the Z-axis force due to the motor torque, no investigation has been made to determine which is the dominant force among the three that will be of concern to shaking. The third objective is to find a method that can minimize the change in kinetic energies when switching between the one- and two-cylinder operating modes of the reciprocating compressor. This is so that in terms of the shaking and noise level generated by the compressor, both operating modes will appear to be the same, and will be relatively unnoticeable when switching from one operating mode to the other.

The acceleration of the piston of the compressor has been generated up to the 80th harmonic. The forces associated to this acceleration have been computed and plotted as shown in Figure 9.1. In the same figure, the impact force between the piston and the connecting rod, and the Z-axis force due to the motor torque have been plotted too. From the plot, it can be concluded that for the first two harmonic, i.e. ω and 2ω, shaking force due to the piston-crankshaft will be the dominant force. Forces at higher harmonic of more than 2ω will be eclipsed by the effect of the impact force. There may be a concern that given enough clearance between the connecting rod and the piston, the impact force will be the dominant force that causes shaking. This can be seen from the trend showed in Figure 5.13. The Z-axis force, due to the skew of the conducting rods in the motor that drive the crankshaft of the compressor, will not be of any major concern. As stated in the previous chapter, the skew angle used in this research in computing the force is larger than the allowable limit, hence the actual force generated by the motor will be even smaller than what is indicated here.

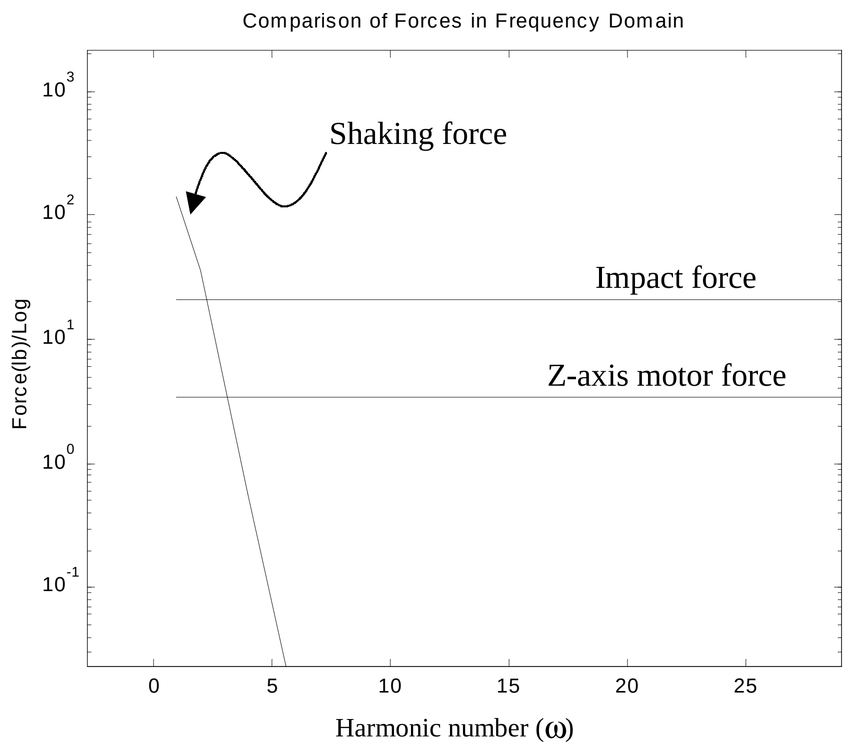


Figure 9.1 Comparison of forces in the frequency domain.

Balancing of the crankshaft will not be of importance if the dominant force that causes the shaking is due to the impact force. Instead, focus will have to be on improving the manufacturing precision of the connecting rod, the piston and the wrist pin to minimize the clearance hence reducing the impact force between them. The task of developing an optimization routine to determine the optimum setting for the angle, force, and torque in balancing the crankshaft is called for to minimize the change in kinetic energies, thus shaking, between the two operating modes of the compressor. The optimization algorithm has been developed and the computed set of optimum setting is 3.8537 radians for the angle, 72.2211 lb for the force, and 84.2173 in-lb for the torque. However, this optimum setting might not set the shaking to the minimum as there are possibilities that the cylinder(s) axis may not be perpendicular to the crankshaft axis; the crankshaft axis is not aligned with the cylinder(s) axis, etc.

9.2 Recommendations for Future Work

9.2.1 The Impact Model

A better impact model for the SIMULINK simulation could investigate the possible impact between the rotating end of the connecting rod and the rotating crankshaft. The effect of lubrication has on this part of the study can also be look into. Investigation can also be made in the area with possibility of the piston rocking within the cylinder thus causing impact between the piston and the cylinder wall.

9.2.2 Optimization

The trade off of having a more accurate optimum setting for the balancing of crankshaft is the slower, hence longer computation time required. With the advancement in the development of higher speed computer, tightening the step size in the optimization routine and using a smaller range of initial values for the angle, force, and torque can improve accuracy of the optimization results.

Optimizing the physical parameters, i.e. length and weight of connecting rod, piston weight, crankshaft radius, etc., of the compressor is not feasible in this research as these information are used in DERIVE to generate the acceleration data. The acceleration information is then manually process and put into MATLAB.

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Appendix I

Piston's acceleration data from ω, 2ω, 4ω…, 80ω in coef80.txt file

65160.5
13873.2
-157.221
2.00448
-0.0252405
0.000312864
-3.8293E-6
4.64107E-8
-5.58169E-10
6.67161E-12
-7.93424E-14
9.39642E-16
-1.10888E-17
1.30469E-19
-1.53108E-21
1.7927E-23
-2.09486E-25
2.44364E-27
-2.84601E-29
3.30997E-31
-3.84468E-33
4.46062E-35
-5.1698E-37
5.98599E-39
-6.92493E-41
8.00463E-43
-9.24575E-45
1.06718E-46
-1.23099E-48
1.4191E-50
-1.63502E-52
1.88281E-54
-2.16707E-56
2.49309E-58
-2.86682E-60
3.29452E-62
-3.77895E-64
4.29632E-66
-4.69725E-68
4.46051E-70
-2.70029E-72

Appendix II

```
%--------------------------------------------------------%
%----------MATLAB Script File for Generating the----------%
%-----KE and PE due to the Shaking Forces and Moments-----%
%--------------------------------------------------------%

clear all;
Acc=load('coef80.txt'); %Import the acceleration values for
omega(1)...omega(2n)

%System mass and inertia matrix
M = [.206 0 0 0 0 0;
   0 .206 0 0 0 0;
   0 0 .206 0 0 0;
   0 0 0 3.37 0 0;
   0 0 0 0 2.97 -.49;
   0 0 0 0 -.49 1.45];

%Total assembly stiffness matrix
K = [1328 18.9 14.1 101 -2123 385;
   18.9 1355 36.4 2246 -141 96.7;
   14.1 36.4 1079 -115 -159 32.9;
   101 2246 -115 23046 -903 382;
   -2123 -141 -159 -903 30870 -2006;
   385 96.7 32.9 382 -2006 14486];

%%System force and torque matrix
omega = 361; %Angular velocity of crank. Unit: rad/s
M_piston = 0.659/384.088; %Mass of piston. Unit: lb.s^2/in
M_wristpin = 0.075/384.088; %Mass of wrist pin
M_rcr = (0.625*0.131/2.374)/384.088; %Mass of reciprocating connecting rod
M_recip = M_piston + M_wristpin + M_rcr; %Reciprocating mass
a = 3.835; %Distance from the top cylinder to the system mass center. Unit:
inches
b = 5.71; %Distance from the bottom cylinder to the system mass center. Unit:
inches
c = 1.875; %Distance betw the two cylinders. Unit: inches
tor = max(size(Acc));

%Force in the Y-direction
for i=1:tor
   ForceY_A(i,1) = M_recip * Acc(i); %Force acting on the top cylinder
   ForceY_B(i,1) = M_recip * Acc(i); %Force acting on the bottom cylinder
end

%Generating the torque
for i=1:tor
   T_AB(i,1) = M_recip*Acc(i)*c; %Column of torque values from first omega
(omega(1))
   T_A(i,1) = ForceY_A(i) * a;   %to last omega (omega(80))
   T_B(i,1) = ForceY_B(i) * b;
end

%%Decoupling of forced vibration equation M*X_dot_dot + K*X = F
A = inv(M)*K; %The dynamics matrix
```

```
[P,D]=eig(A); %D--the diagonal matrix with the eigenvalues
              %P--the modal matrix (corresp. eigenvectors as column vectors)
de_m=P'*M*P;
de_k=P'*K*P;

%%Forming the force matrix
%Force matrix for both cylinders in operation
Force_AB = zeros(6,tor);
for i = 1:tor
   Force_AB(4,i) = -T_AB(i);
end
fo_AB=sparse(Force_AB);
F_AB = P'*fo_AB; %Force matrix

%Force matrix for cylinder A alone in operation
Force_A = zeros(6,tor);
for i = 1:tor
   Force_A(2,i) = -ForceY_A(i);
   Force_A(4,i) = -T_A(i);
end
fo_A=sparse(Force_A);
F_A = P'*fo_A; %Force matrix

%Force matrix for cylinder B alone in operation
Force_B = zeros(6,tor);
for i = 1:tor
   Force_B(2,i) = -ForceY_B(i);
   Force_B(4,i) = -T_B(i);
end
fo_B=sparse(Force_B);
F_B = P'*fo_B; %Force matrix


%%Gen. the vector column of matrix Y at omega(1-->2n), torque(1-->2n)
%Vector column of matrix Y at omega(1)
for j=1
   for i=1:6
      Y_AB(i,j)= abs((F_AB(i,j)/de_k(i,i))/(1-
(omega^2*de_m(i,i)/de_k(i,i))));
      Y_A(i,j)= abs((F_A(i,j)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
      Y_B(i,j)= abs((F_B(i,j)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   end
end

%Vector column of matrix Y at omega(2,4,6,...,2n)
for j=2:tor
   for i=1:6
      Y_AB(i,j)= abs((F_AB(i,j)/de_k(i,i))/(1-((((2*j)-
2)*omega)^2*de_m(i,i)/de_k(i,i))));
      Y_A(i,j)= abs((F_A(i,j)/de_k(i,i))/(1-((((2*j)-
2)*omega)^2*de_m(i,i)/de_k(i,i))));
      Y_B(i,j)= abs((F_B(i,j)/de_k(i,i))/(1-((((2*j)-
2)*omega)^2*de_m(i,i)/de_k(i,i))));
   end
end

%Displacement matrix
```

```
X_AB=abs(P*Y_AB); %Disp. matrix for both cylinders in operation
X_A =abs(P*Y_A); %Disp. matrix for cylinder A in operation
X_B =abs(P*Y_B); %Disp. matrix for cylinder B in operation


%%Generating the Kinetic Energy, KE, where KE = 1/2*M*(x_dot)^2
%-----------------------------------------------------------------------
%---------------------------Both cylinders------------------------------
%Generating KE of omega(1)
XAB = X_AB(:,1); %Extract the first column (omega(1)) of matrix X
KE_AB(1) = ((omega^2)/2)*(XAB'*M*XAB); %KE value for omega(1)

%Generating KE of omega(2),(4),(6),...
c_XAB = X_AB;
c_XAB(:,1)=[]; %Converting X from 6 by 41 to a 6 by 40 matrix, starting
              %with omega(2) as the first column vector
c_XAB(:,1:(tor-1));
for i=2:tor
   KE_AB(i,1) = (((((2*i)-2)*omega)^2)/2)*((c_XAB(:,i-1))'*M*c_XAB(:,i-1));
end
KE_AB %KE of both cylinders in operation.

%Total kinetic energy
Total_KE_AB = sum(KE_AB)

%%Generating the Potential Energy, U, where U = 1/2*K*X^2
%Generating PE of omega(1)
PE_AB(1) = (1/2)*(XAB'*K*XAB); %PE value for omega(1)

%Generating PE of omega(2),(4),(6),...
c_XAB(:,1:(tor-1));
for i=2:tor
   PE_AB(i,1) = (1/2)*((c_XAB(:,i-1))'*K*c_XAB(:,i-1));
end
PE_AB %PE of both cylinders in operation.

%Total potential energy
Total_PE_AB = sum(PE_AB)

%------------------------------------------------------------------
%-----------------------CylinderA----------------------------------
%Generating KE of omega(1)
XA = X_A(:,1); %Extract the first column (omega(1)) of matrix X
KE_A(1) = ((omega^2)/2)*(XA'*M*XA); %KE value for omega(1)

%Generating KE of omega(2),(4),(6)....
c_XA = X_A;
c_XA(:,1)=[]; %Converting X from 6 by 41 to a 6 by 40 matrix, starting
              %with omega(2) as the first column vector
c_XA(:,1:(tor-1));
for i=2:tor
   KE_A(i,1) = (((((2*i)-2)*omega)^2)/2)*((c_XA(:,i-1))'*M*c_XA(:,i-1));
end
KE_A %KE of only Cylinder A in operation

%Total kinetic energy
Total_KE_A = sum(KE_A)
```

```
%%Generating the Potential Energy, PE, where PE = 1/2*K*X^2
%Generating PE of omega(1)
PE_A(1) = (1/2)*(XA'*K*XA); %PE value for omega(1)

%Generating the PE of omega(2),(4),(6),...
c_XA(:,1:(tor-1));
for i=2:tor
   PE_A(i,1) = (1/2)*((c_XA(:,i-1))'*K*c_XA(:,i-1));
end
PE_A

%Total potential energy
Total_PE_A = sum(PE_A)

%-------------------------------------------------------------
%-------------------CylinderB--------------------------------
%Generating KE of omega(1)
XB = X_B(:,1); %Extract the first column (omega(1)) of matrix X
KE_B(1) = ((omega^2)/2)*(XB'*M*XB); %KE value for omega(1)

c_XB = X_B;
c_XB(:,1)=[]; %Converting X from 6 by 41 to a 6 by 40 matrix, starting
              %with omega(2) as the first column vector
c_XB(:,1:(tor-1));
for i=2:tor
   KE_B(i,1) = (((((2*i)-2)*omega)^2)/2)*((c_XB(:,i-1))'*M*c_XB(:,i-1));
end
KE_B %KE of only Cylinder B in operation

%Total kinetic energy
Total_KE_B = sum(KE_B)

%%Generating the Potential Energy, PE, where PE = 1/2*K*X^2
%Generating PE of omega(1)
PE_B(1) = (1/2)*(XB'*K*XB); %PE value for omega(1)

c_XB(:,1:(tor-1));
for i=2:tor
   PE_B(i,1) = (1/2)*((c_XB(:,i-1))'*K*c_XB(:,i-1));
end
PE_B

%Total potential energy
Total_PE_B = sum(PE_B)

%----------------------------------------------------------------------
```

Appendix III

```
%------------------------------------------------------------------------------%
%%---------Crankshaft Balanced for 2 Cylinders Operation--------------------%
%-(Force vector--forces and moments of 2 cylinders operation will be zero)--%
%------------------------------------------------------------------------------%

clear all;

%System mass and inertia matrix
M = [.206 0 0 0 0 0;
   0 .206 0 0 0 0;
   0 0 .206 0 0 0;
   0 0 0 3.37 0 0;
   0 0 0 0 2.97 -.49;
   0 0 0 0 -.49 1.45];

%Total assembly stiffness matrix
K = [1328 18.9 14.1 101 -2123 385;
   18.9 1355 36.4 2246 -141 96.7;
   14.1 36.4 1079 -115 -159 32.9;
   101 2246 -115 23046 -903 382;
   -2123 -141 -159 -903 30870 -2006;
   385 96.7 32.9 382 -2006 14486];

%%System force and torque matrix
omega = 361; %Angular velocity of crank. Unit: rad/s
l_recip = 1.75; %Length of connecting rod's cg to center of wrist pin hold
l_rot = 0.625; %Length of connecting rod's cg to center of crank hold
M_cr = 0.131/384.088; %Mass of reciprocating connecting rod
M_rot = M_cr*(l_recip/(l_recip + l_rot)); %Rotating mass
r = 0.5; %Crank radius. Unit: inches
a = 3.835; %Distance from the top cylinder to the system mass center. Unit:
inches
b = 5.71; %Distance from the bottom cylinder to the system mass center. Unit:
inches
c = 1.875; %Distance betw the two cylinders. Unit: inches
tor = 1;

%-----Grouping Force and Torque according to sin and cos function------------
%------------------------Sine-omega-t segment--------------------------------
%Generating the force values
for i=1:tor
   ForceY_As(i,1) = M_rot * r * omega^2; %Force on the top cylinder (sin)
   ForceY_Bs(i,1) = M_rot * r * omega^2; %Force on the bottom cylinder (sin)
   ForceX_Ac(i,1) = M_rot * r * omega^2; %Force on the top cylinder (cos)
   ForceX_Bc(i,1) = M_rot * r * omega^2; %Force on the bottom cylinder (cos)
end

%Generating the torque values
for i=1:tor
   T_As(i,1) = ForceY_As(i) * a; %Column of torque values from first omega
   T_Bs(i,1) = ForceY_Bs(i) * b; %(omega(1)) to last omega (omega(80))
   T_Ac(i,1) = ForceX_Ac(i) * a;
   T_Bc(i,1) = ForceX_Bc(i) * b;
end
```

```
%%Decoupling of forced vibration equation M*X_dot_dot + K*X = F
A = inv(M)*K; %The dynamics matrix
[P,D]=eig(A); %D--the diagonal matrix with the eigenvalues
              %P--the modal matrix (corresp. eigenvectors as column vectors)
de_m=P'*M*P;
de_k=P'*K*P;

%%Forming the force matrix
%Force matrix for cylinder A alone in operation
Force_As = zeros(6,tor);
Force_Ac = zeros(6,tor);
for i = 1:tor
   Force_As(2,i) = -ForceY_As(i);
   Force_As(4,i) = -T_As(i);
   Force_Ac(1,i) = -ForceX_Ac(i);
   Force_Ac(5,i) = T_Ac(i);
end
fo_As=sparse(Force_As);
fo_Ac=sparse(Force_Ac);
F_As = P'*fo_As; %Forces and moments matrix
F_Ac = P'*fo_Ac;

%Force matrix for cylinder B alone in operation
Force_Bs = zeros(6,tor);
Force_Bc = zeros(6,tor);
for i = 1:tor
   Force_Bs(2,i) = ForceY_Bs(i);
   Force_Bs(4,i) = T_Bs(i);
   Force_Bc(1,i) = ForceX_Bc(i);
   Force_Bc(5,i) = -T_Bc(i);
end
fo_Bs=sparse(Force_Bs);
fo_Bc=sparse(Force_Bc);
F_Bs = P'*fo_Bs; %Forces and moments matrix
F_Bc = P'*fo_Bc;


%%Gen. the vector column of matrix Y
for i=1:6
   Y_As(i,1)= abs((F_As(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_Bs(i,1)= abs((F_Bs(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_Ac(i,1)= abs((F_Ac(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_Bc(i,1)= abs((F_Bc(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
end

%Displacement matrix
X_As =abs(P*Y_As); %Disp. matrix for cylinder A in operation(sin)
X_Bs =abs(P*Y_Bs); %Disp. matrix for cylinder B in operation(sin)
X_Ac =abs(P*Y_Ac); %Disp. matrix for cylinder A in operation(cos)
X_Bc =abs(P*Y_Bc); %Disp. matrix for cylinder B in operation(cos)

%----------------------------------------------------------------------------

%----------Generating the total displ. X_uA and X_uB------------------------
X_A = (X_As.^2 + X_Ac.^2).^(1/2);
X_B = (X_Bs.^2 + X_Bc.^2).^(1/2);
```

```
%----------------------------------------------------------------------------

%%Generating the Kinetic Energy, KE, where KE = 1/2*M*(x_dot)^2
%----------------------Both cylinders---------------------------------------
%KE will be zero in this case as the displacement matrix [X] is zero
%----------------------------------------------------------------------------

%---------------------------CylinderA------------------------------------
%Generating KE of constant omega
%XA = X_A(:,1); %Extract the first column (omega(1)) of matrix X
KE_A(1) = ((omega^2)/2)*(X_A'*M*X_A); %KE value for omega(1)
KE_A

%%Generating the Potential Energy, PE, where PE = 1/2*K*X^2
%Generating PE of constant omega
PE_A(1) = (1/2)*(X_A'*K*X_A); %PE value for omega(1)
PE_A

%%Difference between the Kinetic and Potential Energy
%Diff. at each omega
for i = 1:tor
   diff_indv_A(i,1) = KE_A(i) - PE_A(i); %Diff. at each omega
end
%diff_indv_A
%Diff. between the total KE and total PE
%diff_total_A = Total_KE_A - Total_PE_A

%------------------------------------------------------------------------

%-----------------------------CylinderB--------------------------------
%Generating KE of omega(1)
%XB = X_B(:,1); %Extract the first column (omega(1)) of matrix X
KE_B(1) = ((omega^2)/2)*(X_B'*M*X_B); %KE value for omega(1)
KE_B

%%Generating the Potential Energy, PE, where PE = 1/2*K*X^2
%Generating PE of omega(1)
PE_B(1) = (1/2)*(X_B'*K*X_B); %PE value for omega(1)
PE_B

%%Difference between the Kinetic and Potential Energy
%Diff. at each omega
for i = 1:tor
   diff_indv_B(i,1) = KE_B(i) - PE_B(i); %Diff. at each omega
end
%diff_indv_B
%Diff. between the total KE and total PE
%diff_total_B = Total_KE_B - Total_PE_B

%---------------------------------------------------------------------------

%-------------------------Difference in Energy----------------------------
KE_A_minus_B = KE_A - KE_B %Diff. of KE betw. cyl A and cyl B in opt.
PE_A_minus_B = PE_A - PE_B %Diff. of PE betw. cyl A and cyl B in opt.

%---------------------------------------------------------------------------
```

```matlab
%-------------------------------------------------------------------%
%%-----Crankshaft balanced for Cylinder A in operation--------------%
%----(Force vector--forces and moments for A will be zero)----------%
%-------------------------------------------------------------------%

clear all;

%System mass and inertia matrix
M = [.206 0 0 0 0 0;
   0 .206 0 0 0 0;
   0 0 .206 0 0 0;
   0 0 0 3.37 0 0;
   0 0 0 0 2.97 -.49;
   0 0 0 0 -.49 1.45];

%Total assembly stiffness matrix
K = [1328 18.9 14.1 101 -2123 385;
   18.9 1355 36.4 2246 -141 96.7;
   14.1 36.4 1079 -115 -159 32.9;
   101 2246 -115 23046 -903 382;
   -2123 -141 -159 -903 30870 -2006;
   385 96.7 32.9 382 -2006 14486];

%%System force and torque matrix
omega = 361; %Angular velocity of crank. Unit: rad/s
l_recip = 1.75; %Length of connecting rod's cg to center of wrist pin hold
l_rot = 0.625; %Length of connecting rod's cg to center of crank hold
M_cr = 0.131/384.088; %Mass of reciprocating connecting rod
M_rot = M_cr*(l_recip/(l_recip + l_rot)); %Rotating mass
r = 0.5; %Crank radius. Unit: inches
a = 3.835; %Dist from top cylinder to the system mass center. Unit: inches
b = 5.71; %Dist from bottom cylinder to the system mass center. Unit: inches
c = 1.875; %Distance betw the two cylinders. Unit: inches
tor = 1;

%----Grouping Force and Torque according to sin and cos function------------
%----------------Sine-omega-t segment---------------------------------------
%Generating the force values
for i=1:tor
   ForceY_ABs(i,1) = M_rot * r * omega^2; %Force on both cylinders (sin)
   ForceY_Bs(i,1) =  M_rot * r * omega^2; %Force on the bottom cylinder (sin)
   ForceX_ABc(i,1) = M_rot * r * omega^2; %Force on both cylinders (cos)
   ForceX_Bc(i,1) =  M_rot * r * omega^2; %Force on the bottom cylinder (cos)
end

%Generating the torque values
for i=1:tor
   T_ABs(i,1) = ForceY_ABs(i) * b; %Torque values
   T_Bs(i,1) = ForceY_Bs(i) * (a+b);
   T_ABc(i,1) = ForceX_ABc(i) * b;
   T_Bc(i,1) = ForceX_Bc(i) * (a+b);
end

%%Decoupling of forced vibration equation M*X_dot_dot + K*X = F
A = inv(M)*K; %The dynamics matrix
[P,D]=eig(A); %D--the diagonal matrix with the eigenvalues
              %P--the modal matrix (corresp. eigenvectors as column vectors)
```

```
de_m=P'*M*P;
de_k=P'*K*P;

%%-------------------Forming the force matrix-----------------------------
%Force matrix for both cylinders in operation
Force_ABs = zeros(6,tor);
Force_ABc = zeros(6,tor);
for i = 1:tor
   Force_ABs(2,i) = ForceY_ABs(i);
   Force_ABs(4,i) = T_ABs(i);
   Force_ABc(1,i) = ForceX_ABc(i);
   Force_ABc(5,i) = -T_ABc(i);
end
fo_ABs=sparse(Force_ABs);
fo_ABc=sparse(Force_ABc);
F_ABs = P'*fo_ABs; %Forces and moments matrix for 2 cyl in operation
F_ABc = P'*fo_ABc;

%Force matrix for cylinder B alone in operation
Force_Bs = zeros(6,tor);
Force_Bc = zeros(6,tor);
for i = 1:tor
   Force_Bs(2,i) = 2 * ForceY_Bs(i);
   Force_Bs(4,i) = T_Bs(i);
   Force_Bc(1,i) = 2 * ForceX_Bc(i);
   Force_Bc(5,i) = -T_Bc(i);
end
fo_Bs=sparse(Force_Bs);
fo_Bc=sparse(Force_Bc);
F_Bs = P'*fo_Bs; %Forces and moments matrix for B in operation
F_Bc = P'*fo_Bc;


%%Gen. the vector column of matrix Y
for i=1:6
   Y_ABs(i,1)= abs((F_ABs(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_Bs(i,1)= abs((F_Bs(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_ABc(i,1)= abs((F_ABc(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_Bc(i,1)= abs((F_Bc(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
end

%Displacement matrix
X_ABs =abs(P*Y_ABs); %Disp. matrix for cylinder A & B in operation(sin)
X_Bs =abs(P*Y_Bs); %Disp. matrix for cylinder B in operation(sin)
X_ABc =abs(P*Y_ABc); %Disp. matrix for cylinder A & B in operation(cos)
X_Bc =abs(P*Y_Bc); %Disp. matrix for cylinder B in operation(cos)

%-------------------------------------------------------------------------

%------------Generating the total displ. X_AB and X_B---------------------
X_AB = (X_ABs.^2 + X_ABc.^2).^(1/2);
X_B = (X_Bs.^2 + X_Bc.^2).^(1/2);

%--------------------------------------------------------------------------

%%Generating the Kinetic Energy, KE, where KE = 1/2*M*(x_dot)^2
%-----------------------Both cylinders-------------------------------------
```

```
%Generating KE of constant omega
KE_AB(1) = ((omega^2)/2)*(X_AB'*M*X_AB); %KE value for constant omega

KE_AB

%%Generating the Potential Energy, U, where U = 1/2*K*X^2
%Generating PE of constant omega
PE_AB(1) = (1/2)*(X_AB'*K*X_AB); %PE value for omega(1)

PE_AB

%-----------------------------------------------------------------------

%---------------------------CylinderB-----------------------------------
%Generating KE of constant omega
KE_B(1) = ((omega^2)/2)*(X_B'*M*X_B); %KE value for constant omega

KE_B

%%Generating the Potential Energy, PE, where PE = 1/2*K*X^2
%Generating PE of constant omega
PE_B(1) = (1/2)*(X_B'*K*X_B); %PE value for constant omega

PE_B

%-----------------------------------------------------------------------
```

```
%--------------------------------------------------------------------------%
%%----------Crankshaft balanced for Cylinder B in operation-----------------%
%------(Force vector--forces and moment for Cyl B will be zero)-------------%
%--------------------------------------------------------------------------%

clear all;

%System mass and inertia matrix
M = [.206 0 0 0 0 0;
   0 .206 0 0 0 0;
   0 0 .206 0 0 0;
   0 0 0 3.37 0 0;
   0 0 0 0 2.97 -.49;
   0 0 0 0 -.49 1.45];

%Total assembly stiffness matrix
K = [1328 18.9 14.1 101 -2123 385;
   18.9 1355 36.4 2246 -141 96.7;
   14.1 36.4 1079 -115 -159 32.9;
   101 2246 -115 23046 -903 382;
   -2123 -141 -159 -903 30870 -2006;
   385 96.7 32.9 382 -2006 14486];

%%System force and torque matrix
omega = 361; %Angular velocity of crankshaft. Unit: rad/s
l_recip = 1.75; %Length of connecting rod's cg to center of wrist pin hold
l_rot = 0.625; %Length of connecting rod's cg to center of crank hold
M_cr = 0.131/384.088; %Mass of reciprocating connecting rod
M_rot = M_cr*(l_recip/(l_recip + l_rot)); %Rotating mass
r = 0.5; %Crank radius. Unit: inches
a = 3.835; %Distance from the top cylinder to the system mass center. Unit:
inches
b = 5.71; %Distance from the bottom cylinder to the system mass center. Unit:
inches
c = 1.875; %Distance betw the two cylinders. Unit: inches
tor = 1;

%------Grouping Force and Torque according to sin and cos function-----------
%--------------------Sine-omega-t segment------------------------------------
%Generating the force values
for i=1:tor
   ForceY_ABs(i,1) = M_rot * r * omega^2; %Force on both cylinders (sin)
   ForceY_As(i,1) =  M_rot * r * omega^2; %Force on the bottom cylinder (sin)
   ForceX_ABc(i,1) = M_rot * r * omega^2; %Force on both cylinders (cos)
   ForceX_Ac(i,1) =  M_rot * r * omega^2; %Force on the bottom cylinder (cos)
end

%Generating the torque values
for i=1:tor
   T_ABs(i,1) = ForceY_ABs(i) * a; %Column of torque values from first omega
   T_As(i,1) = ForceY_As(i) * (a+b); %(omega(1)) to last omega (omega(80))
   T_ABc(i,1) = ForceX_ABc(i) * a;
   T_Ac(i,1) = ForceX_Ac(i) * (a+b);
end

%%Decoupling of forced vibration equation M*X_dot_dot + K*X = F
A = inv(M)*K; %The dynamics matrix
```

```
[P,D]=eig(A); %D--the diagonal matrix with the eigenvalues
              %P--the modal matrix (corresp. eigenvectors as column vectors)
de_m=P'*M*P;
de_k=P'*K*P;

%%----------------Forming the force matrix-----------------------------
%Force matrix for both cylinders in operation
Force_ABs = zeros(6,tor);
Force_ABc = zeros(6,tor);
for i = 1:tor
   Force_ABs(2,i) = -ForceY_ABs(i);
   Force_ABs(4,i) = -T_ABs(i);
   Force_ABc(1,i) = -ForceX_ABc(i);
   Force_ABc(5,i) = T_ABc(i);
end
fo_ABs=sparse(Force_ABs);
fo_ABc=sparse(Force_ABc);
F_ABs = P'*fo_ABs; %Forces and moments matrix of AB
F_ABc = P'*fo_ABc;

%Force matrix for cylinder A alone in operation
Force_As = zeros(6,tor);
Force_Ac = zeros(6,tor);
for i = 1:tor
   Force_As(2,i) = -2 * ForceY_As(i);
   Force_As(4,i) = -T_As(i);
   Force_Ac(1,i) = -2 * ForceX_Ac(i);
   Force_Ac(5,i) = T_Ac(i);
end
fo_As=sparse(Force_As);
fo_Ac=sparse(Force_Ac);
F_As = P'*fo_As; %Forces and moments matrix of A
F_Ac = P'*fo_Ac;


%%Gen. the vector column of matrix Y at constant omega
for i=1:6
   Y_ABs(i,1)= abs((F_ABs(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_As(i,1)= abs((F_As(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_ABc(i,1)= abs((F_ABc(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
   Y_Ac(i,1)= abs((F_Ac(i,1)/de_k(i,i))/(1-(omega^2*de_m(i,i)/de_k(i,i))));
end

%Displacement matrix
X_ABs =abs(P*Y_ABs); %Disp. matrix for cylinder A & B in operation(sin)
X_As =abs(P*Y_As); %Disp. matrix for cylinder B in operation(sin)
X_ABc =abs(P*Y_ABc); %Disp. matrix for cylinder A & B in operation(cos)
X_Ac =abs(P*Y_Ac); %Disp. matrix for cylinder B in operation(cos)

%-------------------------------------------------------------------------

%---------Generating the total displ. X_AB and X_B-----------------------
X_AB = (X_ABs.^2 + X_ABc.^2).^(1/2);
X_A = (X_As.^2 + X_Ac.^2).^(1/2);

%-------------------------------------------------------------------------
```

```
%%Generating the Kinetic Energy, KE, where KE = 1/2*M*(x_dot)^2
%----------------------Both cylinders------------------------------------
%Generating KE at constant omega
KE_AB(1) = ((omega^2)/2)*(X_AB'*M*X_AB); %KE value for omega(1)

KE_AB

%%Generating the Potential Energy, U, where U = 1/2*K*X^2
%Generating PE at constant omega
PE_AB(1) = (1/2)*(X_AB'*K*X_AB); %PE value for omega(1)

PE_AB

%-----------------------------------------------------------------------

%---------------------------Cylinder A----------------------------------
%Generating KE at constant omega
KE_A(1) = ((omega^2)/2)*(X_A'*M*X_A); %KE value at constant omega

KE_A

%%Generating the Potential Energy, PE, where PE = 1/2*K*X^2
%Generating PE at constant omega
PE_A(1) = (1/2)*(X_A'*K*X_A); %PE value at constant omega

PE_A

%-----------------------------------------------------------------------
```

Appendix IV

```
%-------------------------------------------------------------------------%
%%-------Converting Imapct Force Data Generated in SIMULINK from ---------%%
%%-------Time to Frequency Domain-----------------------------------------%%
%-------------------------------------------------------------------------%

clear all;
close all;

load impact -mat; %Data file generated by SIMULINK

time=ans';
xt=time(8825:9848,2); %one cycle starting at 0.15 sec
fxt=fft(xt); %Utilizing the FFT function

%Plotting of the force values in time domain
%(Same as that in Simulink scope output)
figure(1)
plot(xt)
zoom on
title('Piston-Connecting Rod Clearance Impact Force (Time Domain)')
xlabel('No. of Time-step(\Deltat)')
ylabel('Force(lb)')

%Plotting of the force values in frequency domain
figure(2)
y=(abs(fxt(1:512)/1024)*2);
semilogy(y) %Plotting of the FFT function (Force vs Frequency)
hold on

x=(1:1:512)';
for n=1:512
   my(n)=mean(y);
end
semilogy(x,my,'k-') %Plotting of the average of the force values
hold on
zoom on
title('Log Plot of Piston-Connecting Rod Clearance Impact Force (Frequency
Domain)')
xlabel('Frequency(\omega)')
ylabel('Force(lb) / Log')

Acc=load('coef80.txt'); %Import the acceleration values for
omega(1)...omega(2n)

%%System force and torque matrix
omega = 361; %Angular velocity of crank. Unit: rad/s
M_piston = 0.659/384.088; %Mass of piston. Unit: lb.s^2/in
M_wristpin = 0.075/384.088; %Mass of wrist pin
M_rcr = (0.625*0.131/2.374)/384.088; %Mass of reciprocating connecting rod
M_recip = M_piston + M_wristpin + M_rcr; %Reciprocating mass
tor = max(size(Acc));

%Shaking Forces
for i=1:tor
```

```
   Force(i,1) = M_recip * Acc(i); %Shaking forces
end

xi=1:2;
y=abs(Force(1:2));
semilogy(xi,y,'k-')
hold on

xi=2:2:((tor-1)*2);
y=abs(Force(2:41));
semilogy(xi,y,'k-')
hold on
zoom on


figure(3)
y=(abs(fxt(1:512)/1024)*2);
bar(y) %Plotting of the FFT function (Force vs Frequency)
axis([-10 520 -10 300])
hold on

plot(x,my,'k-') %Plotting of the average of the force values
hold on
zoom on
title('Piston-Connecting Rod Clearance Impact Force (Frequncy Domain)')
xlabel('Frequency(\omega)')
ylabel('Force(lb)')

xi=1:2;
y=abs(Force(1:2));
plot(xi,y,'k-')
hold on

xi=2:2:((tor-1)*2);
y=abs(Force(2:41));
plot(xi,y,'k-')
zoom on
```

Appendix V

```
%-----------------------------------------------------------------------%
%%-------------Hooke and Jeeves Optimization Method-------------------%%
%-----------------------------------------------------------------------%


%Main routine to scan the best local minimal obtained from a range of
%input of angle, force, and torque initial values.

function main()
global x0 x x1 dx dxmin count;
clear all;
close all;
format short;
tic

%Initializing x0
%Setting the range of initial values of angle, force, and torque
aa = 0.1:0.1:6.2;
bb = 68:1:77;
cc = 71:1:85;
count=length(aa)*length(bb)*length(cc);
z = 1;
for aaa = aa
   for bbb = bb
      for ccc = cc
         x0(1,1) = aaa;
         x0(2,1) = bbb;
         x0(3,1) = ccc;
         [x0,KE0] = local_hjoptim(x0);
         for m = 1:3
            optimized(m,z) = x0(m,1);
         end
         KEnergy(z) = KE0;
         z = z+1;
      end
   end
end
%disp('Optimized setting')
%disp(optimized)
%disp('Optimized KE')
%disp(KEnergy)


%Sorting out the best optimized setting based on the least
%amount of change in KE
L = 1;
min = KEnergy(1);
for K = 1:1:(count-1)
   if KEnergy(K+1) <= min
      L = K+1;
      min = KEnergy(K+1);
   end
end
disp('Best setting')
```

```
disp(optimized(:,L))
disp(KEnergy(L))
toc

return;

%-------------------------------------------------------------------%
%%--------Subroutine for activating Hook & Jeeves routine------------%%

function [x0,KE0] = local_hjoptim(x0)
x =x0;
x1=x0;
%input
dx=[0.01;0.05;0.05]; %Step size
dxmin = 0.0001; %Tolerance or termination citeria

%Routine Start
for i = 1:1:10000
   %Search move
   q=1;
   KE0=total_KE(x0(1,1),x0(2,1),x0(3,1));
   while (q<4)
      KE0 = total_KE(x0(1,1),x0(2,1),x0(3,1));
      x1(q,1) = x0(q,1)+dx(q,1);
      KE1 = total_KE(x1(1,1),x1(2,1),x1(3,1));
      if (KE1)<(KE0)
         x0=x1;
         dx(q,1) = 1.1*dx(q,1);
      else
         dx(q,1) = -dx(q,1);
         x1(q,1) = x0(q,1)+dx(q,1);
         KE1 = total_KE(x1(1,1),x1(2,1),x1(3,1));
         if (KE1)<(KE0)
            x0 = x1;
         else
            dx(q,1)= dx(q,1)*0.95;
         end
      end
      q=q+1;
   end

   if x == x0
      if max(abs(dx))<dxmin
         %disp(x0)
         KE0 = abs(total_KE(x0(1,1),x0(2,1),x0(3,1)));
         return;
      end
   else
      %Pattern move
      x1 = x+(x-x0);
      KE0 = total_KE(x0(1,1),x0(2,1),x0(3,1));
      KE1 = total_KE(x1(1,1),x1(2,1),x1(3,1));
      if (KE1) < (KE0)
         x0 = x1;
         x = x0;
      else
         x =x0;
```

```
            x1=x0;
        end
    end

    %Check variable ranges
    if (abs(x0(1,1))>2*pi)
        disp('Exceed range of angle');
        disp(x0);
        return;
    end
    if (x0(2,1)<0)|(x0(2,1)>150)
        disp('Exceed range of Force');
        disp(x0);
        return;
    end
    if (x0(3,1)<0)|(x0(3,1)>150)
        disp('Exceed range of torque');
        disp(x0);
        return;
    end
end
disp('Exceed number of loops allowed');
disp(x0);
return;

%------------------------------------------------------------%
```

```
function [f] = total_KE(Ang,F_corr,T_corr)

%Function total_KE recalculate the corrected KE (KE_corr)
%value at the given angle in radian, force, and torque. This KE value is
%then passed back to the Hook & Jeeves optimization routine to generate
%the optimized set of data.
%
%Input: Ang -- angle in radian; F_corr -- force; T_corr -- torque
%Output: f -- Optimized KE value for a given set of input data

%Acc=load('coef80.txt'); %Import the acceleration values for
omega(1)...omega(2n)

%System mass and inertia matrix
M = [.206 0 0 0 0 0;
   0 .206 0 0 0 0;
   0 0 .206 0 0 0;
   0 0 0 3.37 0 0;
   0 0 0 0 2.97 -.49;
   0 0 0 0 -.49 1.45];

%Total assembly stiffness matrix
K = [1328 18.9 14.1 101 -2123 385;
   18.9 1355 36.4 2246 -141 96.7;
   14.1 36.4 1079 -115 -159 32.9;
   101 2246 -115 23046 -903 382;
   -2123 -141 -159 -903 30870 -2006;
   385 96.7 32.9 382 -2006 14486];

%%System force and torque matrix
omega = 361; %Angular velocity of crank. Unit: rad/s
M_piston = 0.659/384.088; %Mass of piston. Unit: lb.s^2/in
M_wristpin = 0.075/384.088; %Mass of wrist pin
M_rcr = (0.625*0.131/2.374)/384.088; %Mass of reciprocating connecting rod
M_recip = M_piston + M_wristpin + M_rcr; %Reciprocating mass
c = 1.875; %Distance betw the two cylinders. Unit: inches
%tor = max(size(Acc));
tor = 41;

%%Decoupling of forced vibration equation M*X_dot_dot + K*X = F
A = inv(M)*K; %The dynamics matrix
[P,D]=eig(A); %D--the diagonal matrix with the eigenvalues
              %P--the modal matrix (corresp. eigenvectors as column vectors)
de_m=P'*M*P;
de_k=P'*K*P;

%***************************************************************************
%%Routine calculating the correction KE (KE_corr) for minimizing the change
%%in KE from a two cylinder operating mode to a one cylinder operating mode
%%(or vice versa).

rad = Ang;
Top = length(rad); %Equivalent to max(size(degree)); # of interval
%degree = rad*(180/pi); %Converting radian to degree

%---------------------Correction KE Routine----------------------------------
%Generating the sine and cosine value of angles 0 to 2 pi radian
```

```
for i = 1:Top
   C(i,1) = cos(rad(i));
   S(i,1) = sin(rad(i));

   F_C(i,1) = F_corr * C(i,1);
   F_S(i,1) = F_corr * S(i,1);
   T_C(i,1) = T_corr * C(i,1);
   T_S(i,1) = T_corr * S(i,1);
end

%Force matrices (6 by Top) of function sin(wt) and cos(wt)
CorrF_ABs = zeros(6,Top);
CorrF_ABc = zeros(6,Top);
for i = 1:Top
   CorrF_ABs(1,i) = -F_S(i); %Function sin(wt)
   CorrF_ABs(2,i) = F_C(i);
   CorrF_ABs(4,i) = -T_C(i);
   CorrF_ABs(5,i) = -T_S(i);
   CorrF_ABc(1,i) = F_C(i); %Function cos(wt)
   CorrF_ABc(2,i) = F_S(i);
   CorrF_ABc(4,i) = -T_S(i);
   CorrF_ABc(5,i) = T_C(i);
end
Cf_ABs = P'*CorrF_ABs;
Cf_ABc = P'*CorrF_ABc;

for d=1:Top

   for j=1
      for i=1:6
         Y_CABs(i,j)=abs((Cf_ABs(i,d)/de_k(i,i))/(1-
(omega^2*de_m(i,i)/de_k(i,i))));
         Y_CABc(i,j)=abs((Cf_ABc(i,d)/de_k(i,i))/(1-
(omega^2*de_m(i,i)/de_k(i,i))));
      end
   end

   for j=2:tor
      for i=1:6
         Y_CABs(i,j)=abs((Cf_ABs(i,d)/de_k(i,i))/(1-((((2*j)-
2)*omega)^2*de_m(i,i)/de_k(i,i))));
         Y_CABc(i,j)=abs((Cf_ABc(i,d)/de_k(i,i))/(1-((((2*j)-
2)*omega)^2*de_m(i,i)/de_k(i,i))));
      end
   end

   X_CABs = abs(P*Y_CABs);
   X_CABc = abs(P*Y_CABc);

   X_CAB = (X_CABs.^2 + X_CABc.^2).^(1/2);

   KE_CAB(1) = ((omega^2)/2)*(X_CAB(:,1)'*M*X_CAB(:,1));

   for i=2:tor
      KE_CAB(i,1) = (((((2*i)-2)*omega)^2)/2)*((X_CAB(:,i))'*M*X_CAB(:,i));
   end
```

```
   Total_KE_CAB(d,1) = sum(KE_CAB); %Total corrected KE for CylAB

end
%X_CAB
%KE_CAB
%Total_KE_CAB
%----------------------------------------------------------------------------
----

KE_corr = Total_KE_CAB;
x = 1.253; %User define the value for x (or y)
y = 2 - x;

f = abs((x * 7.148112661499771e-002 + y * 6.368625806814919e-001)/2 -
KE_corr);

%For the optimized state, f will either be zero or close to zero.
%7.14811266...e-002 is the value of the total shaking KE for 2 cyl in
%operation with the crankshaft balanced for both Cyl A and B in operation
%(KE(AB,S)). 6.36825806...e-001 is the value of the total KE for only Cyl A
%in operation with the crankshaft balanced for both Cyl A and B in operation
%(KE(A,2)).
```

Vita

Chin Guan Ong was born to Mr. and Mrs. Choon Bock Ong on April 2, 1970 in Singapore. He completed his Cambridge "A" Level education at Nanyang Junior College in Singapore before he was enlisted into the Singapore Armed Forces in December of 1988 for his two-and-a-half years of national service. In August 1991, Mr. Ong enrolled in the College of Engineering at Western Michigan University in Kalamazoo, Michigan, and had successfully earned a Bachelor of Science Degree in Mechanical Engineering in May 1995. He returned to Singapore and had been working as a Mechanical Design Engineer in Western Digital (S) Pte. Ltd. before coming back to the United States in January 1998 to pursue his Masters of Science Degree in Mechanical Engineering at Virginia Polytechnic Institute and State University. He is expecting his Master's degree in March 2000.