Chapter 3 Results and Discussion

Two bluff body wake generators were installed in the wind tunnel. The cylindrical rod was used as a control wake generator, where results would be compared to the wake generated by the ring model. There were three basic sets of measurements done for each model. The first measurement set included single point one dimensional velocity profile measurements at 20 streamwise stations for $50 \leq x/d < 500$. For each of these profiles, spectra measurements were taken at the center, half width, and at edge of the wake. The second set of experiments included a 2D-plane cross-sectional profile measurement at $x/d = 126$ and $x/d = 450$. The third set of the experiments was a two-point measurement at $x/d = 126$ and $x/d = 450$.

Measurements were made for an approach free-stream velocity U_∞ of 27.7 m/s, corresponding to Reynolds number, based on the 3/16" rod diameter d used in both models, of 8040. The momentum thickness θ of the plane and ring wakes averaged over all the measured profiles were 0.106" and 0.109" respectively, implying momentum thickness Reynolds numbers of 4545 and 4674.

The results for the profile measurements in the range of $50 \leq x/d < 500$ will be presented using the wake-centered coordinate systems shown in figures 3.1 (a) and (b) for the plane and ring wake respectively. For both wakes the coordinate x is measured downstream from the center of the wake generator, y is measured in the traverse direction perpendicular to the wake centerline, and z is measured parallel (or tangent to) that centerline. Mean and fluctuating velocities are expressed in terms of the components (U,V,W) and (u,v,w) measured in these directions. Except where otherwise stated, the angle α between the y-axis of the two flows was 60 degrees.

The contour plots for the grid measurements are plotted such that the flow is coming towards you. The positive y component points upward, with positive x axis pointing out of the paper, and z completes the right hand rule with positive z pointing towards the left.

3.1 Two dimensionality and axisymmetry of the flows

Profiles of mean velocity deficit $(U_\infty - U)/U_\infty$, turbulence normal stress $\overline{u^2}$ and Reynolds shear stress $\overline{uv}$ measured in the plane and ring wakes at $x/d = 126$ are shown in figures 3.2 - 3.4. Profiles for the plane wake measured at the tunnel centerline and spanwise positions ± 20 diameters from that location are shown. The near Gaussian mean velocity profiles, double-peaked $\overline{u^2}$ and antisymmetric $\overline{uv}$ profiles are closely reminiscent of the results of numerous previous studies. Differences between the profiles measured at different spanwise locations are slight and well within the measurement uncertainty indicating a closely two-dimensional flow. This two-dimensionality is also apparent in similar profiles measured at $x/d = 450$. Figures 3.2 (b) to 3.4 (b) include profiles of the ring wake measured at $\alpha = 30^o, 70^o, 140^o, 220^o, 240^o$, and 320°. The profiles show a near axisymmetric flow is an indication that the influence of the wires on the ring wake was small. The largest deviations from axisymmetry are found in the $\overline{u^2}$ profile, where there is scatter for $-1 \le \eta < 1$.

Cross-sectional measurements of the mean flow at $x/d = 126$ and $x/d = 450$ are shown in figures 3.5 - 3.6 for the plane and ring wakes. Contour plots of the mean velocity for the plane wake measured at $x/d = 126$ and $x/d = 450$, are shown in figures 3.5 (a) and (b). The contour plots for axial velocity component, U normalized on the free stream approach velocity U_∞, for both stations show that the wake is essentially two-dimensional. Figures 3.6 (a) and (b) shows the contour plots of the mean velocity in rectangular coordinates for the ring wake at both stations. The axial velocity component shows that the ring wake is essentially axisymmetric.

3.2 Effect of wire interference on the ring wake

The effect of the wires used to mount the ring was investigated by looking at profiles at $\alpha = 0^\circ, 90^\circ, 180^\circ$, and 270° at $x/d = 126$. Profiles of mean velocity deficit $(U_\infty - U)/U_\infty$, turbulence normal stress $\overline{u^2}/U_\infty^2$, and Reynolds shear stress in polar form $\overline{uv_r}/U_\infty^2$ for ring wake at $x/d = 126$ are shown in figures 3.7 (a) - (c). The mean velocity

deficit profiles in figure 3.7 (a) show that the profile is similar to that on figure 3.2 (b). The velocity deficit at the center of the wake is in the range of 7.5% - 9% compared to 9%-10% for the other stations in figure 3.3 (b). Clearly, there is an additional deficit induced due to the presence of the wires, however the difference is negligible falling between 1 - 1.5 %. The profiles for the Reynolds normal stress in figure 3.7 (b) is similar to the other stations shown in figure 3.3 (b). The difference is mainly in the lower peak values towards the negative η, however the difference is small. The Reynolds shear stress profiles are shown in figure 3.7 (c) show the effect of the wire is seen clearly on the profiles for $\alpha = 0°$ and $180°$, with lower peak negative and positive respectively. The profiles at $\alpha = 90°$ and $270°$ are nearly symetrical with their peak to peak value to the levels of profiles. Overall the profiles show a near axisymmetric flow is an indication that the influence of the wires on the ring wake was small.

3.3 Self-preservation in the Mean Flow

Following Wygnanski *et al*. (1986) we plot the profiles measured between $50 \leq x/d < 500$ in figures 3.8 (a) and (b) in terms of the normalized parameter $f(\eta) = (U_\infty - U)/U_o$ against the normalized distance coordinate $\eta = y/L_o$. Here U_o is the maximum velocity deficit at the wake centerline and L_o is the distance from the centerline to the point where the deficit is half U_o. Also plotted as a solid line is the function given by Wygnanski *et al.* (1986) that describes the self-similar velocity profile of their plane-wake flows:

$$f(\eta) = exp[-0.637\eta^2 - 0.056\eta^4]$$

Even though the axial mean velocity profiles for a two dimensional wake of a circular cylinder are not expected to become self-preserving until about 400 diameters downstream (see Wygnanski et al. (1986)) the plane wake profiles measured at all stations collapse neatly on to the theoretical profile. The ring wake profiles (which, strictly speaking, should not reach such a state) also collapse but show more scatter, particularly towards the negative η edge of the wake. This perhaps could be as the result of the lateral curvature but it is not easy to clearly distinguish such an effect from the

uncertainty in the data. Note that at the furthest downstream location represented here (x/d = 455) the half width L_o of the ring wake was 1.0", one third of the radius of the ring. For this profile the point η = 3 thus lies at the central axis of the ring wake. The evolution of the mean velocity profile parameters U_o and L_o is shown in figure 3.11, where $(L_o/\theta)^2$ and $(U_\infty/U_o)^2$ are plotted vs. x/d. The downstream variation momentum thickness θ (see figure 3.12 (a)) was nearly constant for both wakes. The averaged θ over all the measured profiles was 0.106" and 0.109" respectively for the plane and ring wakes, implying momentum thickness Reynolds numbers of 4545 and 4674 respectively. Figure 3.12 shows linear variations, indicating, as one would expect, that the half wake width and reciprocal of the axial velocity deficit varying in a square root manner for the plane wake. This figure shows the growth rate of both wakes to be different. The average slopes $d\,(L_o/\theta)^2/d(x/d)$ and $d(U_\infty/U_o)^2/d(x/d)$ are 0.1566 and 0.6164 for the ring wake, about 20% greater than the values of 0.1309 and 0.6233, respectively, for the plane wake. The preservation of the momentum deficit of the mean-velocity profiles in a plane two-dimensional wake (Wygnanski *et al*. (1986)) can be shown by plotting θ/L_o versus U_o/U_∞. Self-similar velocity profiles should fit to the function

$$\frac{\theta}{L_o} = \frac{U_o}{U_\infty}\left(\phi_1 - \frac{U_o}{U_\infty}\phi_2\right)$$

where

$$\phi_n = \int_{-\infty}^{\infty} f^n(\eta)\,d\eta \quad (n = 1,2)$$

The values obtained by Wygnanski et al. (1986) for ϕ_1 and ϕ_2 where 2.06 and 1.505 respectively. For the present plane wake ϕ_1 and ϕ_2 were 2.045 and 1.46 and the measured profile parameters (figure 3.12 (b)) fit closely with some scatter to this function for the plane wake. Although this analysis, strictly speaking, should not apply for the ring wake the values of ϕ_1 and ϕ_2 for this flow (2.06 and 1.45 respectively) are quite close to the plane-wake values for increasing x/d values (x/d increases from right to left in figure 3.12 (b)).

3.4 Self-Preservation in the Turbulent Flow

3.4.1 Reynolds Stresses

All six terms of the turbulence stress profiles, expressed in similarity functions are plotted in figures 3.13 - 3.18, for both the plane and ring wakes. The normal turbulent stress similarity functions: $g_{11}(\eta) = \overline{u^2}/U_o^2$, $g_{22}(\eta) = \overline{v^2}/U_o^2$, and $g_{33}(\eta) = \overline{w^2}/U_o^2$ are shown in figures 3.13 - 3.15. The turbulent shear stresses: $g_{12}(\eta) = \overline{uv}/U_o^2$, $g_{23}(\eta) = \overline{vw}/U_o^2$, and $g_{13}(\eta) = \overline{uw}/U_o^2$ are shown in figures 3.16 - 3.18. Also included in the figures is the streamwise variation of peak values of the similarity functions.

The normal turbulent stresses g_{11}, g_{22}, and g_{33} for the plane wake (figures 3.13 (a) - 3.15 (a)), are shown to achieve approximate self-similarity for $x/d > 230$. The degree to which the wakes achieve the self - preserving state can be shown by plotting the downstream variation $\sqrt{\overline{u^2}}|_{\max}/u_o$, $\sqrt{\overline{v^2}}|_{\max}/u_o$, and $\sqrt{\overline{w^2}}|_{\max}/u_o$. This is shown in figures 3.13 (c) - 3.15 (c), where the peak value reaches a constant value as self-similarity is achieved. This result is within the range of those published by Wygnanski *et al.* (1996) for a variety of wake generators. Their result show that the g_{11} functions approaches self-preservation at $x/d > 200$. The turbulent shear stress g_{12}, for the plane wake in figure 3.16 (a) reaches a self-preserving state earlier compared to g_{11} at for $x/d > 180$. Similar conclusion was reached by Wygnanski *et al.* (1986), the fact that the g_{12} function approaches self-similarity earlier compared to the g_{11} function. The other profiles g_{23}, and g_{13} for the plane wake (figures 3.17 (a) and 3.18 (a)), appear to show that self-similarity is achieved much earlier compared to g_{11} function.

For the ring wake, the turbulent normal stress profiles (figures 3.13 (b) - 3.15 (b)), the asymmetry in the profiles is quite prominent initially and decaying for increasing diameters downstream. The profiles do not achieve self-similarity (as one might expect). This is shown in figures 3.13 (c) - 3.15 (c), where the peak values of g_{11}, g_{22}, and g_{33} never quite asymptotes to a constant value in the streamwise direction. Similarly, the turbulent shear stress profiles g_{12}, g_{23}, and g_{13} in figures 3.16 (b) - 3.18 (b) show self-

similarity is not achieved. Interestingly, the g_{11} function approaches self-preservation more slowly, and it is not clear that it has been reached, even at the most downstream station. For the ring wake, neither of the similarity functions appear to become completely self-preserving (as one might expect) but, downstream of $x/d = 365$ the profiles are very similar to those of the plane wake.

In an ideal two-dimensional plane wake, the profiles of $g_{23}(\eta) = \overline{vw}/U_o^2$ and $g_{13}(\eta) = \overline{uw}/U_o^2$ should be zero due to the symmetry constraint on w. These profiles for both wakes show very small values, however not exactly zero values illustrating the departure from exact two-dimensionality. One possible reason could be due to uncertainty in measurement. The profiles however suggest a connection with the W mean velocity component (figure 3.10). Although the W profiles are small percent of the axial velocity deficit, they suggest the presence of streamwise vortices in both wakes. This is illustrated in the W velocity jump across the wake between 6 and 2% U_o for the plane wake and between 8 and 3% U_o for the ring wake. The jump in W is higher in the near wake and decreasing further down stream. For the plane wake, at $x/d = 50$, the velocity jump is about 6% U_o, implying a vortex sheet strength of 6% Uo = 0.9% U_∞. Such a vortex sheet could be generated by a variation of about ± 0.08 degrees in yaw angle across the span of the circular cylinder model. Such variation seems more likely, as most wind tunnels have flow angularities at least of this magnitude. Similarly for the ring wake at the same station, we find the variation of about 0.1% degrees.

Choosing appropriate scaling factors can minimize the variations in the profiles seen in the normal and shear Reynolds stresses. The peak value of the Reynolds stress, $\overline{u^2}|_{max}$ was chosen as a factor by which the stresses where to be normalized. The associated length-scaling factor, L_1 was chose to be as half width of the distance where the value of the Reynolds normal stress is 25% $\overline{u^2}|_{max}$. This value was chosen so as to avoid the uncertainty in the measurement near the peak values and at the edge of the wake.

The plots using this normalization technique for the Reynolds normal stress vs. $\eta_t = y/L_1$ are shown in figures 3.19 (a) - (f), and for the Reynolds shear stress in figure

3.20 (a) - (f) for both the plane and ring wakes. As shown in these figures the scaling chosen collapses the normal and shear stresses better for both the plane and ring wakes. For the Reynolds normal stress case, $\overline{u^2}$ collapses better compared to $\overline{v^2}$, and $\overline{w^2}$ for both wakes in figures 3.19 (a) - (f). The profiles for the plane wake show no asymmetry behavior, while for the ring wake, the asymmetry in the profiles is quite prominent initially and decaying for increasing diameters downstream.

A way to examine how well the profiles collapse is by looking at the downstream variation of $\sqrt{\overline{u^2}_{CL}/\overline{u^2}_{max}}$, $\sqrt{\overline{v^2}_{CL}/\overline{u^2}_{max}}$, and $\sqrt{\overline{w^2}_{CL}/\overline{u^2}_{max}}$, where the subscript CL represents the centerline of the wake, on figures 3.21 (a) - (c). These stastics should be constant where the profiles collapse into a single curve. It is observed that $\overline{u^2}$ and $\overline{v^2}$ collapse earlier for $x/d \geq 200$ compared to $\overline{w^2}$. For the plane wake, $\overline{w^2}$ the ratio $\sqrt{\overline{w^2}_{CL}/\overline{u^2}_{max}}$ is constant for $x/d > 300$.

For the cross terms of the Reynolds stress profiles in figures 3.20 (a) - (f), we see most notably an improvment in the collapse of the shear stress. This point can be illustrated further by looking at the variation of $\sqrt{+\overline{uv}_{max}/\overline{u^2}_{max}}$, $\sqrt{\overline{vw}_{CL}/\overline{u^2}_{max}}$, and $\sqrt{\overline{uw}_{CL}/\overline{u^2}_{max}}$ as shown in figure 3.21 (d) - (f). The ratio of the maximum poisitive peak in figure 3.21 (d), illustrates the shear profiles for both wakes collapsing for $x/d > 150$. The collapse of vw and uw profiles is very good for the plane wake. For the ring wake, the we see an abrubt drop of $\overline{vw}$ and $\overline{uw}$ profiles at $x/d = 455$ and 475.5 respectively. This however could be explained in the fact that the levels of $\overline{w^2}$ (figure 3.19 (f)) and subsequently $\sqrt{\overline{w^2}}$ are lower at those streamwise positions, therefore any shear terms with w term in it is bound to have a lower value also and this is what apparently is carried over to the $\overline{vw}$ and $\overline{uw}$ profiles. This could possibly be an inherent consequence of the ring wake development itself, which we are not aware of.

Overall a better collapse of the profiles was achieved using such normalization method than the normalization scales from the mean flow. For the plane wake, a marked improvement in collapsing the data is seen in $\overline{u^2}$ and $\overline{uv}$ profiles. The improvement for the ring wake is significant in all the Reynolds stresses profiles. The success in using

such a scaling parameter illustrates that the turbulence field has it's own length and velocity scales associated with it different from the mean flow. Such improvement can be noted by comparing the scaling parameters as they vary in the downstream direction. The variation of these scales with downstream distance is shown on figure 3.22 (a) and (b) for both L_1 and $\overline{u^2}_{peak}$ respectively. Also included in the figures is the downstream variation of L_o and U_o. The variation of L_1 with x/d is better compared to L_o as there is less uncertainty involved for the ring wake. As is the case for L_o, L_1 grows at the same rate for both wakes. The variation of the peak turbulent intensity varies as the inverse square root of x/d much as the maximum velocity deficit U_o does. And as can be seen in figure 3.22, there is hardly any difference in the variation for both the wakes, the variation almost identical. And the uncertainty in the variation with downstream distance is almost negligible compared to the wide scatter seen in U_o.

Normal and shearing stresses $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$, $\overline{uv}$, $\overline{vw}$, and $\overline{uw}$ grid contours for the plane wake at $x/d = 126$ are shown in figure 3.23 (a) - (f) respectively. Similarly figure 3.24 shows profiles for the plane wake at $x/d = 450$. All quantities are normalized on U_∞^2. Normal stress $\overline{u^2}$ shows the common double peak on either side of the center of the wake (figure 3.23 (a)), which is mainly do to the presence of vortices on either side of the wake. The normal stress $\overline{u^2}$ dominates and the $\overline{v^2}$ and $\overline{w^2}$ more or less of equal magnitude. This is shown for both profiles. The shear stress $\overline{uv}$ is by far dominant compared to $\overline{vw}$ and $\overline{uw}$. The contour plots show that the profile is symmetric. Comparing the Reynolds shearing stresses for both profiles, the peak magnitude drops by as much as 40%, by the time the flow reaches $x/d = 450$.

Grid plots of the Reynolds stresses in polar form $\overline{u^2}$, $\overline{uv}_r$ for the ring wake are shown in figures 3.25 for x/d = 126 and 450. Compared to other normal Reynolds stresses $\overline{u^2}$ dominates and decreases by an order of magnitude at $x/d = 450$. For the shear stress, $\overline{uv}_r$ plays a dominant role as expected and peak value falls off by as much as 64% at $x/d = 450$, which is greater than that of the plane wake.

3.4.2 Third Order Turbulent Products

The third order turbulent stress in similarity functions: $\overline{u^3}/U_o^3$, $\overline{v^2u}/U_o^3$,

$\overline{w^2u}/U_o^3$, $\overline{v^3}/U_o^3$, $\overline{u^2v}/U_o^3$, $\overline{w^2v}/U_o^3$, $\overline{w^3}/U_o^3$, $\overline{u^2w}/U_o^3$, and $\overline{v^2w}/U_o^3$ versus η are

plotted in figures 3.26 - 3.34 respectively for both the plane and ring wakes. Overall the
collapse is much better for the plane wake compared to the ring wake as was the case for
the second order turbulent quantities. As was the case for the Reynolds stress profiles,
self preservation is achieved for the plane wake but not for the ring wake. Looking at the
profiles for the plane wake, the general view that higher order take much longer to
achieve self preservation is not necessarily true. Indeed compared to the Reynolds stress
terms in similarity functions i.e. g_{11}, g_{22}, g_{33}, g_{12}, g_{23}, and g_{13}, that the third order
turbulent quantities seem to reach self preservation at about the same streamwise position
for $x/d > 200$. Self-preservation for the ring wake is never quite achieved, with the
profiles showing that the wake has not reached an equilibrium state. The self-similar
nature of the profiles can be improved by using the scaling parameters $\left(\overline{u^2}_{max}\right)^{3/2}$ for the
turbulent quantity and L_1 as the length scale, where $\eta_t = y/L_1$. The turbulent quantities
normalized using the above mentioned scaling parameters, are shown in figures 3.35 -
3.37. Overall the collapse of the triple order quantities is improved for both wakes.

Triple turbulent quantities $\overline{u^3}$, $\overline{v^2u}$, and $\overline{w^2u}$ for the plane wake at both stations
$x/d = 126$ and $x/d = 450$, are shown in figure 3.38. (a) - (f). Similarly Figure 3.39 (a) - (f)
show contour plots of $\overline{v^3}$, $\overline{u^2v}$, and $\overline{w^2v}$ and figure 3.40 (a) - (f) show contour plots of
$\overline{w^3}$, $\overline{u^2w}$, and $\overline{v^2w}$. All those terms were normalized on U_∞^3. Overall the dominant triple
product term is the $\overline{u^3}$ followed by $\overline{v^3}$. The terms $\overline{u^3}$, $\overline{v^2u}$, and $\overline{w^2u}$ are symmetric
about the center of the wake, while the terms $\overline{v^3}$, $\overline{u^2v}$, and $\overline{w^2v}$ are not. The terms $\overline{w^3}$,
$\overline{u^2w}$, and $\overline{v^2w}$ are negligible compared to the other third order terms. The drop in
magnitude of all corresponding plots from $x/d = 126$ to $x/d = 450$ is an order of
magnitude. The plots also show the uniformity of the flow in the spanwise direction.

The grouping of the above terms is not done without reason. As Fabris (1981) described in measuring the triple order products, the terms $\overline{u^3}$, $\overline{v^2u}$, and $\overline{w^2u}$ could be thought of as the longitudinal transport of $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$ i.e. by u-fluctuations. Similarly $\overline{v^3}$, $\overline{u^2v}$, and $\overline{w^2v}$ as lateral transport of $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$, i.e., by v-fluctuations and the terms $\overline{w^3}$, $\overline{u^2w}$, and $\overline{v^2w}$ as streamwise transport of $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$ by w-fluctuations. The terms $\overline{v^3}$, $\overline{u^2v}$, and $\overline{w^2v}$ play the dominant role in the transport of the turbulent kinetic energy as will be discussed in the next section. The lateral transport of $\overline{u^2}$ and $\overline{v^2}$ is about three times as intense as that of $\overline{w^2}$, which is similar to what Fabris (1981) found out.

Figures 3.41 - 3.43 show the triple products of the ring wake in cylindrical coordinates. Figure 3.41 (a) - (f) show the components $\overline{u^3}$, $\overline{v_r^2u}$, and $\overline{v_\theta^2u}$ for both stations at $x/d = 126$ and $x/d = 450$. Figures 3.42 (a) - (f) show the components $\overline{v_r^3}$, $\overline{u^2v_r}$, and $\overline{v_\theta^2v_r}$ for both stations at $x/d = 126$ and $x/d = 450$. Figures 3.43 (a) - (f) show the components $\overline{v_\theta^2}$, $\overline{u^2v_\theta}$, and $\overline{v_r^2v_\theta}$ for both stations at $x/d = 126$ and $x/d = 450$. The terms $\overline{u^3}$, $\overline{v_r^2u}$, and $\overline{v_\theta^2u}$ in figures 3.42 show that they are negatively valued on both sides of the center of the wake. Similarly as in the plane wake, the terms could be thought of as the longitudinal transport of $\overline{u^2}$, $\overline{v_r^2}$, and $\overline{v_\theta^2}$ by the u-fluctuations. Comparing all other terms $\overline{u^3}$ plays a dominant role. The $\overline{v_r^2}$, $\overline{u^2v_r}$, and $\overline{v_\theta^2v_r}$ terms in figure 3.42 show distinct rings as in the figure 3.41, with the inner ring having negative values and the outer ring with positive values. These terms are similar to $\overline{v^3}$, $\overline{u^2v}$, and $\overline{w^2v}$ terms in the plane wake and play a similar role in the transport of *tke*. Whereas the lateral transports of $\overline{u^2}$ and $\overline{v^2}$ is about three times as intense as that of $\overline{w^2}$ in the plane wake case, the radial transports of $\overline{u^2}$, $\overline{v_r^2}$ are an order of magnitude as intense as that of $\overline{v_\theta^2}$. The terms $\overline{v_\theta^2}$, $\overline{u^2v_\theta}$, and $\overline{v_r^2v_\theta}$ in figures 3.43 don't show any pattern in behavior.

3.5 Turbulent Kinetic energy

The turbulent kinetic energy (*tke*), $K = (\overline{u^2} + \overline{v^2} + \overline{w^2})/2$ normalized on the maximum deficit velocity U_o^2, was computed for each profile for both wakes. Figures 3.44 (a) and (b) show K vs. η for the plane and ring wake respectively. The profiles for the plane wake show that a state of equilibrium in the *tke* is reached for $x/d \geq 230$ where the profiles collapse into a single curve. This is not surprising in that it was shown earlier self-preservation for the mean velocity and turbulent flow was achieved for $x/d > 200$ for the plane wake. Thus we can state that the plane wake is in a dynamic equilibrium where loss and gain in energy is in equilibrium. The peak value of *tke* is found to be at $\eta = \pm 0.6$ in the region where the *tke* is in equilibrium. The ring wake however does not achieve any sense of equilibrium as figure 3.44 (b) shows, and this is not surprising as the Reynolds stress components did not achieve self-preservation as was shown previously. The clearly demonstrates the differences in both these wakes.

Figures 3.45 (a) - (d) show contour plots of the turbulent kinetic energy, K/U_∞^2, for both wakes at $x/d = 126$ and $x/d = 450$ is greatest approaching the center of the wake and tails of to zero towards the edge of the wake as the fluctuations die out. The *tke* decreases by approximately 27% at $x/d = 450$. The greatest *tke* production is seen near the region, where $\overline{uv}$ has the highest peak. The plots also show the uniformity of *tke* in the spanwise direction.

The *tke* production is calculated using the following equation

$$-\overline{u^2}\frac{\partial U}{\partial x_1} - \overline{v^2}\frac{\partial V}{\partial y_1} - \overline{w^2}\frac{\partial W}{\partial z_1} - \overline{uv}\left(\frac{\partial U}{\partial y_1} + \frac{\partial V}{\partial x_1}\right) - \overline{vw}\left(\frac{\partial V}{\partial z_1} + \frac{\partial W}{\partial y_1}\right) - \overline{uw}\left(\frac{\partial U}{\partial z_1} + \frac{\partial W}{\partial x_1}\right)$$

where

$$\frac{\partial U}{\partial x} = -\frac{\partial V}{\partial y} - \frac{\partial W}{\partial z} \quad \text{from continuity, and assuming} \quad \frac{\partial W}{\partial x} = \frac{\partial V}{\partial x} = 0$$

Figure 3.46 (a) and (b) show the contour plot the *tke* production for the plane wake at $x/d = 126$ and $x/d = 450$ respectively. Major contributor to the in the production of tke is the Reynolds shear stress. Positive peak production for $x/d = 126$ is observed to occur at $y/d \approx 3.2$, which corresponds to $\eta = 1$. Negative *tke* production is seen close to

the center of the wake. It is also shown that the *tke* production is uniform in the spanwise direction. At *x/d* = 450, *tke* production has fallen off by an order of magnitude.

Figure 3.46 (c) and (d) show the *tke* production for the ring wake at *x/d* = 126 and *x/d* = 450 respectively. The tke production was calculated using $\overline{uv_r}\partial U/\partial r$, since it is the major contributor. The levels of production at *x/d* = 126 for the ring wake is much less compared to the plane wake. The level of production falls of by an order of magnitude for the ring wake from *x/d* = 126 to *x/d* = 450.

3.5.1 Turbulent kinetic en ergy balance

The gradients of the third order turbulent quantities enter into the balance of the turbulent energy equation:

$$V\frac{\partial K}{\partial y} - (-\overline{uv})\frac{\partial U}{\partial y} + \frac{\partial(\overline{v}K)}{\partial y} + \varepsilon = 0$$

where $K = (\overline{u^2} + \overline{v^2} + \overline{w^2})/2$. The first term is change of *tke* by the mean velocity components and the second term production of *tke* by shear stresses. The third term is the change in tke due to turbulent diffusion and the fourth term change in *tke* due to dissipation.

All the terms were computed in the range $50 \le x/d < 500$ for both wakes. These terms were computed by completing the difference to make the balance equation equal to zero. The various terms in the balance equation was normalized by multiplying L_o/U_o^3, are plotted in figures 3.47 - 3.51 for the plane wake and figures 3.52 - 3.56 for the ring wake. The terms in the figures were fitted with a polynomial of an 8th order. All four terms in the balance equation are plotted for all *x/d* station in figures 3.57 and 3.58 for the plane and ring wakes respectively.

The general picture in the region *x/d* > 200 for both wakes is that for $\eta > 1.2$, turbulent transport brings *tke* to the outer edge of the wake where the maximum is at $\eta \approx 1.7$, where it is balanced by dissipation. Just inboard of the half wake width, where tke production is maximum at $\eta \approx 0.85$, additional *tke* is contributed due to the advection of *tke* from the mean flow. This *tke* is in turn taken away by turbulent diffusion away

towards the outer edges of the wake. In the central part of the wake, *tke* is mainly due to the deposition by advection from the mean flow, in which case some of it is dissipated and some of it transported outwards to the edge of the wake due to turbulent diffusion. This overall picture of the *tke* balance agrees with that of Antonia *et al.* (1987) at $x/d = 400$.

The equilibrium of the plane wake is clearly shown in figures 3.57 (a) & (b) and 3.58 (a) - (b), where all the four terms of the *tke* budget are plotted in the range $50 \leq x/d < 500$. The terms collapse for $x/d > 200$ indicating that the plane wake is in equilibrium. This is not surprising due to the fact that the terms in the *tke* balance equation do reach a self-preserving state for the plane wake for $x/d > 200$. Figure 3.57 (c) & (d) and 3.58 (c) - (d) show the terms for the ring wake and the profiles show that they don't collapse as the wake is not yet in equilibrium. The only profile that seems to reach an approximate equilibrium is the *tke* production term.

3.6 Spectra Results

Spectra measurements were made at the center, half wake width, and edge of both wakes. This measurements where done in the range $50 \leq x/d < 500$ for both the plane and ring wake. The autospectra and cross-spectra results for both wakes, G_{uu}, G_{vv}, G_{ww}, G_{uv}, G_{vw}, and G_{uw} are normalized by multiplying $U_{\infty} /(2\pi U_{o}^{2} L_{o})$ and plotted against non-dimensional wave number obtained by multiplying the frequency by $2\pi L_{o} /U_{\infty}$. Similarly the cross-spectra was normalized in the same way. Included in the legend is the position of where the measurement point was taken in terms of η, at each successive streamwise location.

Figures 3.59 show the autospectra of the *u*, *v*, and *w* fluctuations in the plane wake. Figures 3.59 (a) - (c) show autospectra of the *u*, *v*, and *w* fluctuations at $\eta \approx 0$; (d) - (f) at $\eta \approx 1$; and (g) - (i) at $\eta \approx 2$ respectively. The autospectra shows the common broad band distribution across the frequency spectrum with the -5/3 slope for the inertial subrange. The presence of periodic passage of an eddy structure is shown by the distinct peak in the G_{vv} autospectra at non-dimensional wave number of 0.5. Also it is shown

from the figures that the normalized autospectra does indeed collapse to a single curve for increasing streamwise distance. Not surprisingly, G_{vv} and G_{ww} take longer to reach self-preservation compared to G_{uu}. It is interesting to note that variation in the self-similar profiles is greater around the peak in the G_{vv} spectra. The inertial subrange with the -5/3 slope covers about a decade of wave numbers. The autospectra for the ring wake in figures 3.63 (a) - (c) show similar result as that for the plane wake. However the collapse of the data is poor compared to the plane wake case. This shows that self-preservation doesn't hold for all scales of motion for the ring wake.

The autospectra for the plane wake at $\eta = 1$ show similar result as has been discussed above at the center wake. The only difference is a weakly defined peak in G_{ww} that is not present at the center of the wake. At the edge of the wake (figures 3.59 (g) - (i)) the energy level decreases for all velocity fluctuations, with the distinct peak in the G_{vv} still present. It is also noted that the range of wavenumbers where the -5/3 slope holds has narrowed. Similar results are shown for the ring wake in figure 3.63 (d) - (i) with the exception of poor collapse and progressively worse at the edge of the wake.

Figures 3.60 (a) - (c) show cross-spectra of u & v, v & w, and u & w at $\eta \approx 0$; (d) - (f) at $\eta \approx 1$; and (g) - (i) at $\eta \approx 2$ respectively for the plane wake. The profiles of the cross-spectrum G_{uv} at $\eta \approx 1$ achieve self-similarity and at $\eta \approx 1$ to a lesser extent. The collapse is poor at the center of the wake and this is not surprising since the Reynolds stress is zero. The cross-spectrum of the u and w show self-similarity at $\eta \approx 0$ and to a lesser extent at $\eta \approx 1$. For the ring wake, similar result is shown in figures 3.64 (a) - (i) and again here also the collapse is poor compared to the plane wake.

The coherency functions γ_{uv}^2, γ_{vw}^2, and γ_{uw}^2 for the plane wake are shown in figures 3.61 (a) - (c) for $\eta \approx 0$, (d) - (f) for $\eta \approx 1$, and (g) - (i) for $\eta \approx 2$ respectively. The coherency is zero essentially for all except for γ_{uv}^2 at $\eta \approx 1$ and $\eta \approx 2$. The γ_{uv}^2 at $\eta \approx 1$ shows that the coherence has three distinct parts. The highest coherence with an average value of 0.4 falls in the wave numbers greater than the passage wave number frequency of 0.5. This illustrates that the scales most responsible for the Reynolds shear stress come from scales with wavenumbers lesser than the ones associated with the

passage wavenumber frequency. The second distinct part is the transition where the coherency drops to zero. This region of wave numbers covers between 0.5 and 20, which covers the inertial subrange as noted before. For wave number greater than 20, coherency is zero. Similar results are shown in figures 3.65 (a) - (i) for the ring wake.

The phase of the u and v fluctuations in degrees are shown in figures 3.62 (a) - (c) for the plane wake at $\eta \approx 0$, $\eta \approx 1$, and $\eta \approx 2$ respectively. Figures 3.66 (a) - (c) show similar plots for the ring wake. All other phases are not shown for brevity, as they are essentially random when the coherence is zero, as illustrated in figure 3.62 (a). The phase between u and v fluctuating velocities at $\eta \approx 1$ is zero for the most part i.e. in phase for wavenumbers ranging from 0.02 to 3. For wavenumbers greater than 3, the signals are out of phase by $\pm$ 180°.

3.7 Two-point Turbulence measurements

3.7.1 Measurement and data reduction

Four sets of two-point measurements were done for both wakes. The measurements where done in two sets for each wake at $x/d = 126$ and $x/d = 450$. For each set of the measurement, one of the hot-wire probes was held as a fixed y position, in which case the position chosen was at the center wake. Another hot-wire probe was then traversed around the fixed probe at some 460 points (see figure 2.10 (a) and (b)). The moveable probe was yawed at 11° to the fixed probe axis, where nominal tip probe separation was positioned 0.1 in.. The spacing of the grid points in Δy and Δz decreases closer to the fixed probe, where smallest spacing is 0.05 in.. The reason for this spacing was because the correlation function goes to zero as probe separation increases. The overall width of the grid was $\eta = \pm 3.33$ at $x/d = 126$, $\eta = \pm 2.06$ at $x/d = 450$ adequately covering the width of the wake at both measurement stations. At each point for the plane wake, 150 records, each 3x1024 points in length at 50kHz sampling rate was taken. The only difference for the ring wake was we took 50 records at each point due to time constraint. Both probes where sampled by HSDAS-12 A/D converter, where there was a

delay in sampling of 10μsec between each probe. This was taken into account in the data reduction process.

The data reduction process was performed to obtain an accurate description of the two-point correlation tensor from the time series recorded in binary form from the two hot-wire probes. This was accomplished in a series of steps described in detail by Devenport *et al.*'s report on their plane wake measurement of a NACA 0012 airfoil which can be obtained at http://www.aoe.vt.edu/flowdata.html (choose "Plane wake" under the Data list). Following is in summary form of the data reduction process.

The first order of reduction was to obtain the instantaneous velocity components of the two probes. Once the components are obtained, they were reprocessed by breaking them into 1024 point records (with 50% overlap) and computing the cross-spectrum tensor between two velocity signals over 250 realizations. Also, the average autospectra of the fixed probe was computed to provide data for $\Delta y = \Delta z = 0$. The average cross-spectrum was then phase corrected for the difference in sampling delay between the two probes, and also rotated to account for the different alignment of the probes. Then it was Fourier transformed to obtain 1024-point estimate of the time-delay correlation tensor represented as $R_{ij} = R_{ij}(y, \Delta y, \Delta z, \tau) = \overline{u_i(y,z,t)u_j(y+\Delta y, z+\Delta z, t+\tau)}$, where the overbar represents the average expected value. The time delay τ is defined as positive for an event that occurs in the moveable probe at $(y + \Delta y, z + \Delta z)$ after an event at the fixed probe at (y,z).

The final reduced data (some 500Mbytes) was not amenable for an easy manipulation of the correlation tensor due to the file size and also the irregular spacing of the moveable probe in y, Δy, and Δz, which makes computational time prohibitive. Therefore, an easily referenced data of the original correlation tensor was obtained by linear interpolation on a smaller 3-dimensional rectangular grid. Verification of such interpolation, was made by comparing the Reynolds stresses and spectra of the component velocity fluctuations with grid measurements made at the same streamwise station.

3.8 Two-point results and discussion

Discussion of the results of the two-point measurement is presented in this section. First we will take a look at the auto and cross correlation of the velocity components at zero time delay, zero Δy and zero Δz separation. Such simple representations provide only limited information of the four-dimensional correlation tensor. In order to get a more physical insight contained in the tensor, linear stochastic estimation is applied.

3.8.1 Form of correlation tensor

Figure 3.67 - 3.69 show the normal (R_{ii}) components of the correlation function at zero time delay, zero Δy, and zero spanwise separation for fixed probe position at $\eta = 0$ at x/d = 126 and x/d = 450 for the plane wake. Similarly figures 3.70 - 3.72 show the same for the ring wake. The normal correlations for zero time delay show that they are fairly symmetrical about Δy and Δz as is expected. The normal components R_{vv} and R_{ww} in figures 3.67 (b) and (c) have a preferred orientation in the vertical and spanwise directions respectively. Similar result is shown for correlations at x/d = 450. The cross-correlation R_{uv} shows as expected a positive and negative correlation on each side of the wake. Figures 3.68 show the correlation function $R_{ii}(0,0,\Delta z,\tau)$ as a function of time, at fixed probe position of $\eta = 0$, zero-Δy separation. The normal R_{uu} correlation shows a strong preference on the streamwise direction, while R_{vv} and R_{ww} are symmetrical. The correlations R_{uu} and R_{vv} show negative values at large Δz separations and time delay τ respectively. The correlations of R_{uu} and R_{vv} in figure 3.69 for zero-spanwise separation show a preferred orientation in Δy and in the mean flow direction.

The correlations also satisfy the backflow required for continuity as shown in Townsend (1976), which states that given any two points on a plane separated by separation vector $\vec{r}$, the correlation of the velocities normal to the plane must be negative for some separation values of $\vec{r}$. Thus R_{uu} for zero time delay, R_{vv} for separation $\Delta y = 0$,

and R_{ww} for zero spanwise separation have negative values at some separation satisfying continuity.

Figure 3.70 - 3.72 shows contours normal correlations for the ring wake at $x/d = 126$ and $x/d = 450$. The R_{uu} functions for zero time delay (figure 3.70 (a)), zero Δy (figure 3.71 (a)), and zero spanwise separation (figure 3.72 (a)), show that they have similar forms as for the plane wake at $x/d = 126$. While the functions R_{vv} and R_{ww} are different in form compared to the plane wake, suggesting a different turbulent structure over all.

The eduction from such correlation function about eddies is quite tenuous at best and one can only infer about some organized motion of some type. Grant showed that two counter rotating vortex eddies show correlations functions give similar qualitative behavior. Since the correlation function is an average of realizations over different sizes of eddies, it does not give a full picture of the instantaneous eddy structure. In order to obtain a more realistic physical information about the eddy structure, an objective method of eduction using linear stochastic estimation was performed.

3.8.2 Linear Stochastic estimation

Linear stochastic estimation (LSE) uses the correlation tensor to create the best linear estimate of the velocity field. LSE according to Adrian (1996), in general deals with an estimation of unknown quantity y from a set of variables $\mathbf{E}(E1, E2,...E_N)$, in which case some information is known that may or may not affect the unknown variable. In our case we want to estimate the instantaneous velocity field from the two-point correlation function itself,

$$u_j(\Delta y, \Delta z, \tau)\big|_{LSE} = R_{ij}(y, \Delta y, \Delta z, \tau)\frac{u_i(y)}{u_i(y)^2}$$

Thus by choosing a reference velocity $u_1(y)$, a three-dimensional velocity vector field is obtained. Figures 3.73 - 3.83 show results using such calculation. The plots show sectional planar cuts of the velocity field for each choice of velocity reference.

For the plane wake at $x/d = 126$, figure 3.73 shows an eddy structures with a streamwise vorticity counter rotating on either side of the wake. These structures are also

observed at $x/d = 450$ (figure 3.78). This are similar eddy structures observed by Grant (1958), Townsend (1979), and Mamford (1983) among others of the double-roller eddy type counter-rotating on either side of the wake centerline. Eddies with spanwise vorticity are also observed near the center of the wake some of them spanning the whole width of the wake as shown in figure 3.76 at $x/d = 126$ and figure 3.81 at $x/d = 450$ for the plane wake. The LSE for the ring wake in figures 3.82 and 3.83 show predominantly the presence of eddies with spanwise vorticity. Other plots for the ring wake are not shown, as there was no observed recognizable eddy structure.

3.9 Uncertainty Analysis

The uncertainty analysis presented here was referenced from Devenport *et al*. (1998) report on a plane wake from a NACA 0012 airfoil. The same apparatus and technique was used for our experiment. The uncertainty estimated shown in table 1 were computed using the method of Kline and McClintock (1953). The sources of uncertainty include

- A/D conversion error 0.366mV (from manufacturers specs)
- inaccuracy in velocity calibration curve fit slope and intercept (inferred from successive calibrations using the same probe)
- uncertainty in pitch and yaw angle of angle calibrator (inferred from repeated angle calibrations, 0.11 degrees)
- interpolation over angle step size (5 degrees)
- drift during calibration
- limited number of samples taken
- total sampling time compared to integral time scale of turbulence
- uncertainty in Pitot-static head used to determine reference velocity (0.01" water)

Chapter 4 Conclusions

Single and Two-point hot-wire measurements were in a plane and ring wakes to understand the effect of lateral curvature on a turbulent wake in the self-preserving region of the flow. Measurements were made in the Virginia Tech 3' x 2' subsonic wind tunnel. The cylinder rod was used as a control wake generator, where results would be compared to the wake generated by the ring model introducing curvature effects. Single point, spectra, plane grid and two-point measurements were taken using four sensor hot-wire in the range of $50 \leq x/d < 500$. All mean velocities, turbulent quantities, spectra and correlation tensor were obtained to study the development of the ring wake and effect of lateral curvature on the far wake region.

4.1 Single point conclusions

The single-point measurements in the range of $50 \leq x/d < 500$ reveal that both wakes were closely two-dimensional. The Reynolds number based for the flow was 8040 and the momentum thickness Reynolds number was 4545 and 4674 for the plane and ring wakes respectively. The axial velocity when normalized by the half width wake and maximum deficit for both wakes was universal. The scaling parameters based on the mean flow, show the dependence $U_o \sim x^{-1/2}$, $L_o \sim x^{1/2}$ for both wakes as noted in theory by Townsend (1956), Tennekes and Lumley (1972), and George (1989).

In the turbulent field, the plane wake reaches self-preservation for $x/d > 200$, while the ring wake never achieves such a state as was expected due to the extra length scale involved with the curvature of the ring. The Reynolds shear stress profile achieves self-preservation faster compared to the normal axial Reynolds stress profile, which agrees with Wygnanski *et al*. (1986). The profiles for $\overline{vw}$, and $\overline{uw}$ show a non-zero value for both wakes. This is thought to arise more likely from the variation of yaw angle of the models in the spanwise direction.

The collapse in the self-similar profiles in the Reynolds stress and triple turbulent quantities is improved by choosing $\overline{u^2}|_{max}$ for the velocity scale and for the length scale half width of the Reynolds axial normall stress where $\overline{u^2} = 0.25\overline{u^2}|_{max}$. The improvement

is apparent in both wakes, especially for the ring wake illustrating the turbulent flow has it own velocity and length scales different from that of the mean flow. The triple products essentially show the same result as the Reynolds stress profiles.

The turbulent kinetic energy (*tke*) profiles were calculated to get an insight into the dynamic equilibrium of the wakes as they evolve downstream. The profiles show that such equilibrium is reached for the plane wake for $x/d \geq 230$. The ring wake however does not reach such a state, which is not surprising, since the normal Reynolds stresses do not achieve self-preservation. The peak value of turbulent kinetic energy occurs at $\eta = \pm 0.6$ for both wakes, while peak *tke* production occurs at $\eta \approx 1$ where the shear stress has its maximum value.

Further investigation of the *tke*, was performed by looking at t*ke* budget equation consisting of the production, dissipation, convection, and diffusion terms. The dissipation was highest at the center of the wake and decreasing towards the edge of the wake. The turbulent diffusion term diffuses *tke* from region of high production towards the out edge of the wake. The convection term brings in *tke* from the mean flow, with highest values at the center of the wake. All the terms in the *tke* budget for the plane wake reach an approximate equilibrium state. This result is not reflected for the ring wake.

Spectra measurements taken at the center wake, half wake width, and the edge of the wake reveal the presence of periodic passage of an eddy structure is shown by the distinct peak in the G_{vv} autospectra at non-dimensional wave number of 0.5. The autospectra for the plane wake collapses to a single curve when normalized illustrating that the wake is self-preserving for all scales of motion. The coherency is essentially zero for all except for *uv* where it shows two distinct average coherence of 0.4 is seen at the lower wavenumbers, thus most contributing to the Reynolds shear stress. The phase between *u* and *v* fluctuating velocities at $\eta \approx 1$ is zero for the most part i.e. in phase for wavenumbers ranging from 0.02 to 3, and for wavenumbers greater than 3, the signals are out of phase by $\pm 180°$. Therefore, the Reynolds shear stress is generated mostly by large coherent structures of wave numbers less than 3.

4.2 Two-Point measurement conclusion

Two-point measurements taken at two stream wise stations, with the fixed probe positioned at the center of the wake, were analyzed to reveal the structure of the turbulence. Three-dimensional space-time correlation functions were calculated from the measurements.

The normal correlations for zero time delay for the plane wake show that they are fairly symmetrical about Δy and Δz. The normal components R_{vv} and R_{ww} have a preferred orientation in the vertical and spanwise directions respectively. While the longitudinal correlations of both wakes are similar in form, in general difference in form and orientation prevailed over all indicating the difference in the turbulent structure of both wakes. Linear stochastic estimation reveals the presence of spanwise and double-roller eddy structures in the plane wake and for the ring wake only spanwise eddies were detected.

The current study is an advance in turbulence research concerning laterally curved wakes that are more prevalent in aircraft and submarine vehicles. Although a lot of research has been accomplished regarding plane wakes, there has been little research done on laterally curved wakes. Single and Two-point measurements reveal the statistical and structure of laterally curved turbulent wakes in contrast with plane turbulent wakes. The investigation of such canonical free-shear flows would help to test the development of turbulence models. Future broad band noise analysis of curved wakes would be of aeroacoustic interest complimenting previous research and to further obtain a fuller understanding of such wakes.