THE APPLICATION OF STATISTICAL QUALITY CONTROL

TO THE

CENTRIFUGAL CASTING OF IRON PIPE

BY

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PREFACE

In spite of the sensational success of statistical quality control in a wide variety of industries, its use is still not yet generally accepted. The foundry industry is one in which this valuable instrument has not yet received whole-hearted acclaim.

It is hoped, that in some small way, this paper will assist in emphasizing the possible advantages that the use of statistical quality control offers to foundry practice.

The particular application discussed on the following pages deals with only one phase of foundry production - the centrifugal casting of iron pipe. The reader, as he follows the statistical methods contained herein, perhaps, can visualize other applications of these same methods that would also be of benefit in foundry practice.

It will be noted that statistical devices other than quality control have been used in this investigation. These additional statistical tools used in conjunction with quality control form a highly desirable combination, further defining the statistical nature of a process and hastening the establishment of statistical quality control.

During the course of this study, valuable assistance has been received from several members of the faculty of Virginia Polytechnic Institute, Blacksburg, Virginia, and from the employees of the Lynchburg Foundry Company, Radford, Virginia.

Prof. W. G. Ireson, Associate Professor of Industrial Engineering, Virginia Polytechnic Institute, has given liberally of his time and advice throughout the preparation of this thesis. Prof. Paul T. Norton, Jr., Head of the Department of Industrial Engineering, Virginia Polytechnic Institute, has also been instrumental in the completion of this paper through his helpful cooperation and encouragement.

The counsel of Dr. Raj Candra Bose, Head of the Department of Statistics, University of Calcutta, Calcutta, India, and Dr. Boyd Harshbarger, Professor of Statistics, Virginia Polytechnic Institute, has been influential in designing the statistical tests to analyze process variance.

Mr. Samuel X. Cohen, Supervisor of Standards, under whose direction production data was made available, Mr. Thurman Coleman, Production Manager, Mr. Arlie Weeks, and Mr. Elmer Musselman, deLavaud Foremen, all of the Lynchburg Foundry Company, have been very cooperative during this investigation.

The author wishes to express his appreciation and gratitude for the valuable assistance rendered by these men and others who have contributed so generously of time and effort to the preparation of this thesis.

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THE APPLICATION OF STATISTICAL QUALITY CONTROL TO THE CENTRIFUGAL CASTING OF IRON PIPE

In 1922, a revolutionary improvement in foundry and metallurgical methods was introduced to the United States. The production of deLavaud Centrifugally Cast pipe answered a need for a pipe that would withstand rough handling in shipping and installation and that would provide additional properties that modern usage required. The particular properties that characterize deLavaud pipe result from the distinctive method of production. The superior metal structure of a finer and closer grain makes possible an improved tensile strength, greater ductility and unusually high impact strength. The design of the deLavaud machine allows absolute concentricity and uniformity of both wall thickness and density. Subsequent improvements in the mentioned properties led to what is now known as Super-deLavaud pipe.

This is an example of technical research having been applied to raise the standard of casting properties. Many such technological changes have been forthcoming in the last few decades. This scientific betterment has not only resulted in the improvement of production and services but has also necessitated similar advancement in the administration of such production and services. An illustration of this condition is the present emphasis being placed on constancy of product quality. The development of statistical quality control has made it possible to increase the efficiency of modern production methods by controlling the quality between economical limitations. Heretofore, if the efficiency of the method of production was to be increased, efforts were concentrated upon improved design or methods based on past experience or experimentation. "The fact is that the primary need in the foundry industry today is not so much new metallurgical discoveries, as the application of existing technical knowledge to control operations and casting properties within narrow limits."[1]

The Lynchburg Foundry Company in Radford, Virginia, has been for several years active in the successful production of Super-deLavaud pipe. Throughout the years of production, various innovations have been applied to the production of deLavaud pipe so that, at the present time, the physical properties of the pipe are unquestionable. However, though it is rare that a pipe fails to meet the tests imposed upon it, the variation in the weight of pipe does not seem to be controlled as closely as possible. Even though quality is restricted by minimum specifications, it cannot be allowed to vary uncontrolled above that minimum if production is to be economical. It is therefore of value to investigate the possibilities of more closely controlling the weight of pipe. In order to attack this problem logically, the present quality level must be determined, the nature of the present variation disclosed, an optimum quality level set forth, and the method of approaching that desired level of quality must be outlined. To obtain the necessary information and achieve a logical solution, the methods of statistical quality control will be used.

Statistical quality control is one of the more recent tools of management that has allowed administrational advances to keep pace with technological change. Although quality control was introduced back in the early 1920's by Walter E. Shewhart and his associates at Bell Telephone

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1 Harding, E. W., "Quality Control in the Foundry," Canadian Metals and Metallurgical Industries, Toronto, Westman Publications Ltd., Vol. 8, No. 6, pp. 39-46, (August 1944).

Laboratories, it has only recently acquired popular industrial acclaim. During World War II, quality control gained limited but enthusiastic acceptance. The government was instrumental in disseminating the nature and know-how of quality control by conducting short courses at various universities throughout the country. These courses gave some 1900 attendants the bare essentials of quality control.

The interest in and general acceptance of statistical quality control has been due to the spectacular success that has accompanied almost every application. There have been some notable failures. These failures, however, have been due to the improper administration of the system. Either management has failed to introduce quality control at the level of the worker or the system has been conducted by those who lack a sufficient working knowledge of quality control.

Fortunately, the success of quality control in the practical application does not depend upon an extensive knowledge of its complex mathematical foundation. This characteristic is not strange nor illogical when one recalls development of the electrical field made in ignorance of the electron or the "why" of electrical phenomena. It is, however, important to keep in mind that the essentials of a working knowledge of statistical quality control are mathematically justifiable.

Savings resulting from the use of statistical quality control have been most noticeable in those industries mass producing finished products and raw materials. However, many business organizations active in job lot production are numbered among those realizing profits from quality control.

In the operation of a foundry there is some mass production regardless of the size of the foundry. If the foundry is small and of the job lot type, they are at least interested in the production of molten metal in quantity. On a larger scale not only the raw material but also the finished product is mass produced. Most of the statistical work done by foundries has been to control the quality of molten metal. The articles that have appeared on this subject offer valuable suggestions that would be of assistance in the control of metallurgical properties. Aside from this basic information, the authors have expressed the opinion that further statistical applications in the foundry would be of great benefit. Mr. E. W. Harding of the International Meehanite Metal Co. Ltd. of Canada states[2] that statistical methods "can eventually be applied to other factors in casting production, such as defectives control, sand control, etc., as experience is gained with the system." Mr. E. M. Schrock expresses the profitableness of statistical analysis.[3] "We are convinced that through the expanding use of (statistical) programs of this type, metallurgical progress will be greatly expedited and thousands, yes, millions of dollars saved in operating costs." Although these men point the way for further applications of statistical control in the foundry, little material has been published dealing with specific statistical control of the finished foundry product.

If a foundry is not engaged in repetitive production of like castings, quality control can be of greatest assistance in controlling the quality

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[2] Harding, E. W., "Quality Control in the Foundry," Canadian Metals and Metallurgical Industries, Toronto, Westman Publications Ltd., Vol. 6, No. 8, pp. 39-46, (August 1944).

[3] Shrock, E. M., "Statistical Analysis of Metallurgical Problems," Metal Progress, August 1940.

of the desired material blends. This is not to say that the use of statistics would not be of benefit in controlling the production of job lot casting. The use of statistical methods other than quality control would be of assistance in determining optimum foundry and metallurgical conditions. Some foundries that produce like castings, but only in small lots, would be able to standardize production by the intermittent use of quality control.

It has been noted that the control of metallurgical properties in foundries has been more than satisfactory. The purpose of this paper is to indicate, in general, that the success can be extended to other foundry operations and specifically, that statistical quality control can be instrumental in controlling the weight of Super-deLavaud pipe and will serve as a guide to solve other foundry problems to which statistical methods are applicable.

The solution of the problem of controlling the weight of pipe by statistical methods will have several possible economies. An investigation of the statistical nature of the process will disclose those features of the process that largely contribute to excess variation. The knowledge of the relative variances will be of value in concentrating future improvement efforts where they will do the most good. This eliminates costly experimentation and the reliance upon past experience which often times, in this connection, leads to erroneous conclusions. When process control is gained, the pipe may be produced closer to the minimum weight, thus providing a saving to the consumer and to the producer. The presence of a quality control program also has the desirous effect of creating greater customer confidence in the product when the proof of constant quality is evident.

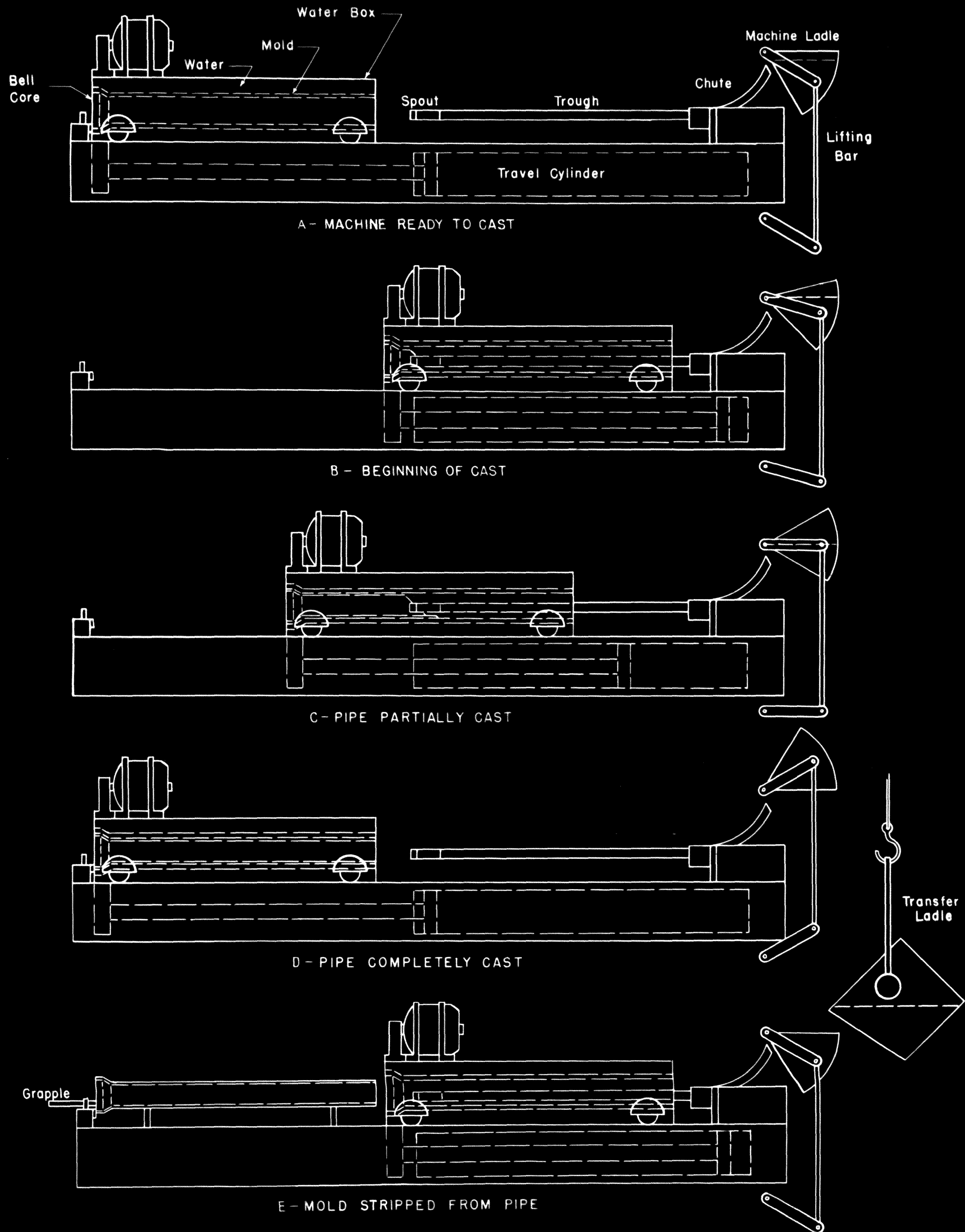
DESCRIPTION OF THE CASTING
OF DELAVAUD CENTRIFUGAL PIPE

Statistical quality control aids the prediction of future occurrences through the use of past data. This past data must necessarily be chosen with care if the results are to be satisfactory. It has been stated that the problem under consideration is one of weight reduction. The data chosen must reflect accurately the statistical nature of the weight variation. To select intelligently the pertinent data and to disregard that which is impertinent, an understanding of the method of production will be of utmost value. In following the method of production special notice should be given to features which are likely to cause process variation. This knowledge not only allows a logical selection of data, but also is essential in the interpretation of such data.

The functioning of the deLavaud machine has been described ably by Mr. Frank G. Carrington, Consultant, in the 1944 Spring issue of The Iron Worker, a quarterly publication of The Lynchburg Foundry Company. Mr. Carrington has had considerable experience with the centrifugal casting of iron pipe, both as a production man and as a designer in this country and in France.

Description of the Functioning of deLavaud Machine

"To one familiar with the older sand cast shops, the striking features of the deLavaud Process are the centrifugal casting machines in action and the extent to which scientific control is carried. The machine is made up of the bed or track, with the ladle and trough fixed to it, and a water box carrying the mold, which travels back and forth on the



OPERATION OF DELAVAUD PIPE CASTING MACHINE

FIG. 1

track. This lengthwise movement is given by the travel cylinder. The mold is a steel tube bored to the outside shape of the pipe, which is spun on rollers in the water box by the motor mounted on top. The water box forms the water jacket for cooling the outside of the mold. The ladle is tilted by the mechanically driven lifting bar to give an accurately controlled rate of flow into the chute and trough.

As the name implies, centrifugal force is set up in the molten metal when it is spun in the mold. This not only throws out impurities and prevents blow holes and shrink cavities but also shapes the metal into a tube. In this way no core is required except a small one to close the bell end of the mold and to shape the socket.

The casting operation is begun with the machine ladle full of iron, the water box moved forward to uncover the trough, and the bell core fixed in place (Figure 1A). The water box is then moved back until the trough will discharge at the bell. As soon as pouring has begun and the space between the mold and the bell core filled (Figure 1B), the water box and mold begin moving forward again so that the stream of metal winds like a ribbon around the inside of the mold (Figure 1C). This continues until the trough spout has come out of the mold, when the pipe is completely cast (Figure 1D). About 30 seconds are required to convert the molten metal in the ladle into a pipe.

By the time the spinning mold can be stopped, the pipe will have solidified. The bell core fastener can then be removed, a puller or grapple is fastened in the pipe, and by moving the water box toward the ladle, the mold will be stripped off. As the pipe is uncovered, skids are shoved under to support it (Figure 1E). When completely out of the mold, it is rolled to one side, so that the water box can be moved forward for the next bell core. Then, with the machine ladle again filled, the casting of another pipe can be begun. Thirty to forty of the smaller sizes will be cast per hour."

Description of Possible Causes of Process Variation

From Mr. Carrington's description of the deLavaud machine, it may be seen that the production may be afflicted with several sources of variance. It is important in the establishment of quality control that recognition be made of all possible sources of process variation. The following variables are offered as possible sources of assignable causes that are contributing or may contribute to the lack of quality control. The effective magnitude of assignable causes on process variance is discussed on succeeding pages in the analysis of variance.

Possible causes of variation will be discussed in the sequence in which they are likely to occur from the raw material to the finished product.

The chemical analysis of the molten metal is being controlled by the standard practice of weighing each constituent to insure correct composition of the final blend. The Lynchburg Foundry goes still further by controlling the moisture present in the cupola by air conditioning before the air enters the blowers. This last refinement was installed as a result of noted property variation due to the changing atmospheric moisture. The control of conditions in the cupola serves two functions. The correct constituents insure that the final quality of the cast pipe will meet certain physical requirements. The correct percentage of constituents and the temperature of the molten iron also influence the fluidity of the iron. In centrifugal casting, the fluidity of the molten iron must not vary significantly if the weight of each pipe is to remain constant under constant rates of machine operation. As the cast pipe produced seldom fail in their physical properties, the question of fluidity demands the most attention. The degree of fluidity may be caused to vary by, (1) changing the content of the carbon equivalent, carbon, or phosphorus, and by (2) changing the temperature of the molten metal, or (3) a combination of both.

When the molten metal leaves the cupola, it is transported in a transfer ladle to the deLavaud machines by means of an industrial truck. At the casting machines, the transfer ladle must wait until it is ready to be used. There is necessarily a variation in the time that the ladle must remain idle. This variation in time consequently causes a variation in the temperature at which the iron is poured into the machine ladle. Previous experience indicates that the temperature may vary as much as 40^{o}F during the use of a single transfer ladle. The optimum pouring temperature is considered to be about 2360^{o}F. If the temperature is excessively above the optimum, some scrap may be added to the transfer ladle to cool it down.

The iron, being within the accepted temperature range, is then poured from the transfer ladle into the machine ladle. The capacity of each machine ladle is slightly in excess of that required to pour one pipe of any given size. The machine ladle is tilted at a certain rate to insure even distribution of metal throughout the entire length of pipe. The tilting of the machine ladle is regulated by means of a rheostat. If, in the opinion of the foreman, the iron has higher than optimum fluidity, on the basis of the temperature, the tilting is slowed down. If, on the other hand, the iron is considerably below that of the optimum fluidity, the tilting of the ladle is speeded up.

Here we are confronted by the first of three variations that may be introduced by machine regulation. These are possible machine regulations and all of them may or may not be used at one time to correct for improper fluidity.

There are two other machine controls that may be exercised by the foreman, both having to do with the movement of the mold. The speed of rotation of the mold may be regulated by a rheostat similar to that controlling the tilt of the machine ladle. If the iron is judged to be quite fluid, the speed of rotation is increased, and if the iron is not fluid enough, the speed of rotation is decreased. In addition to the control of the speed of revolution, the travel of the mold may also be regulated by a rheostat. If the iron has high fluidity, the travel is increased; if the fluidity is low, the travel is decreased. The control of the machine ladle, speed of rotation of the mold, and travel of the mold are the only mechanical regulations available to the foreman to vary the weight of pipe that the machine will produce.

There is another possible cause of irregularity that would ordinarily escape notice. The mold travel of the three machines is powered by one hydraulic system. Due to the fact that there is only one system, variations in pressure, depending upon the relative movement of the machines, are likely to cause similar variations in the rate of travel of the mold.

Aside from the above mechanical cause of variation, there are several human factors that affect the weight of the pipe produced. Human error may be introduced in the person of the head coreman. The head coreman watches the pouring of the bell end of the pipe. When the molten iron has filled the space between the mold and bell core and starts to protrude beyond the core, the head coreman signals the machine operator to begin the mold travel. The judgment of the head coreman is, of course, subject to wide variation, even though he is constantly informed by an inspector as to the wall thickness of the last pipe made. The machine operator does not have regulatory control over the variation of machine movement but he is only able to start and stop the machine. Nevertheless, his judgment may be responsible for the variation in weight of both the bell and spigot ends of the pipe. If he does not react immediately to the signal of the head coreman, the bell will be heavy. If he stops the machine too soon when the spigot is being formed, a heavy spigot will result.

After leaving the machine and before entering the annealing oven, the pipe is transported to the weighing scales by means of an overhead crane. Possible source of error is equally present in the weight inspection as it is in any method of inspection. It is imperative if quality control is to be successful, that the means of inspecting be checked periodically to assure correctness of readings.

These various features all conceivably influence the final weight of pipe either directly or indirectly. Certainly, it is rare that all of the sources of variance affect the weight of pipe adversely at the same time. It may also be true that the causes of variability do not all act independently, but may interact to cause weight variation. It is undoubtedly true that all variances are different in their effective magnitude. When the statistical data has been collected and evaluated, the characteristics of some of the causes of variation may reveal themselves.

ESTABLISHMENT OF STATISTICAL QUALITY CONTROL

Definition of the Process Control Limits

"A phenomenon will said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomena will fall within the given limits."* The best prediction of future weight variance may be afforded by the compilation of sufficient past data; the nature of which will be representative of all pipe that will be produced, providing there is no basic change in the process. If this prediction denotes lack of statistical process control, the data may also indicate a method of establishing desired control. All data contained in this paper is taken from the past production records of 6 inch, bell and spigot, Class 150, Super-deLavaud pipe, (6" B&S 150), which represents the largest portion of this company's pipe production. The data of other pipe sizes and classes is not included because of the limitation of time and space, and because the method of calculation would be repetitious. The general conclusions drawn from the statistical study of 6" B&S 150 may well be applied to the production of pipe of other sizes and classes. It is not to be inferred

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* Shewhart, W. A., "Economic Control of Quality of Manufactured Product," D. Van Nordstrand Company, Inc., New York, 1931, p. 6.

that the statistical investigation of the production of remaining types of deLavaud pipe would not be beneficial. Such investigation perhaps might disclose characteristics peculiar to a certain type of pipe.

Fortunately, the Lynchburg Foundry Company has recorded the weight of each pipe, the thickness dimensions of each bell and spigot, the hour of the day when the pipe was made, the machine on which the pipe was produced, the number of defects and the reason of each defect. Because this discussion is concerned with the control of weight, only that data pertaining to the weight of pipe shall be used in the primary analysis. Table I shows the individual weights in the samples taken from the production of 6" B&S 150 pipe on machine No. 1 for the months of September and October, 1946.

To determine the present state of weight control by use of the available data, the mid point or average weight of pipe around which production is centered must be found. It is not enough to know only this central tendency; the extent of variation from the average must also be learned. The use of the average or $\bar{X}$ quality control chart will give this information. The average weight of all 6" B&S 150 pipe production or the average of the population ($\bar{\bar{X}}$) is estimated by the average of the sample or subgroup averages ($\bar{X}$). Because the only variation shown by the $\bar{X}$ chart is between subgroup averages, it must be supplemented by another control chart that indicates the variation occurring within subgroups. Either the range (R) chart or the standard deviation (σ) chart will give this information. The choice between the use of the range chart and the standard deviation chart depends upon the intended size of the subgroup. When using subgroups containing from one to ten specimens, the accuracy of both charts is comparable. Above the subgroup size of ten, the added accuracy of the standard deviation chart warrants the additional necessary calculations. As will be discussed below, the subgroup size to be used is five. Therefore, the range (R) chart may be used with reasonable assured accuracy. The R chart has one additional advantage; it is more meaningful to the worker.

The determination of the size and method of selection of subgroups must be initially established subject to future change. A subgroup is defined as[4] "a group of data, within which variations are believed to be due to nonassignable chance causes only. One of the essential features of the control chart method is to break up the inspection data into rational subgroups, that is, to classify the observed values into subgroups, within which variations may for engineering reasons be considered to be due to non assignable chance causes only, but between which there may be differences due to assignable causes whose presence is considered possible." The choice of subgroup selection to establish initial control charts oftentimes is more or less arbitrary because of the lack of information surrounding the conditions of production at the time that any individual or group of pipe was produced. As stated above, the primary consideration in the selection of subgroups should be an attempt to have minimum variation within subgroups and maximum variation between subgroups. With this in mind the subgroups are selected at the beginning of every hour, as there is no indication that any other method of selection would give a greater variation between subgroups or a lesser variation within subgroups.

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4 Control Chart Method of Controlling Quality During Production, American War Standard, Z1.3-1942, p. 38.

The size of the subgroup must be chosen with consideration for the rate of production and the possible interval of time within which an assignable cause might be introduced to the process. It has been found that when subgrouping is based on the order of production, the use of frequent small subgroups is most efficient for detecting trouble. For these reasons and ease of calculation the subgroup size of five is used.

Before the control limits are established, it is well to be aware of the government imposed specifications on centrifugally cast iron pipe. Federal Specification of Pipe, Water, Cast Iron (Bell and Spigot), No. WW-P-421 specifies that the weight of no single full-length pipe shall be less than the nominal tabulated weight by more than 5 per cent and that the total weight of any contract be not more than 2 per cent under the total nominal weight. The nominal weight for 6" B&S 150 pipe is 460 pounds. This means that no single full-length pipe shall be less than 437 pounds and that the average weight of pipe in any shipment will not be less than 451 pounds.

Table I presents the data in subgroups of five taken at the beginning of each hour of production from machine No. 1 which serves as a basis for the calculation of the control limits on the $\bar{X}$ and R charts. Columns 3, 4 and 5 of Table I have been calculated as follows:

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{n}$$

where n equals the size of the subgroup.

$$R = X_{max} - X_{min}$$

$$\sigma = \sqrt{\frac{X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2}{n} - \bar{X}^2}$$

The calculation of control limits is based upon the first 50 subgroups. These subgroups comprise what is termed the grand lot. It is desirable to have at least 25 subgroups in the grand lot. Because there is sufficient data to constitute 50 subgroups and because the reliability of calculations may be improved, the control limits are established on the basis of the first 50 subgroups, each of which must represent the process in a state of control.

Calculation of control limits:

Trial No. 1

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{N}$$

where N is the number of subgroups in the grand lot.

$$= \frac{23117.6}{50}$$

$$= 462.35$$

$$\bar{\bar{X}} = 462.35$$

$$\bar{R} = \frac{\Sigma R}{N}$$

$$= \frac{1105}{50}$$

$$= 22.10$$

Limits for $\bar{X}$:

$$\text{Upper Control Limit, (UCL)} = \bar{\bar{X}} + A_2\bar{R}$$

$$= 462.35 + .577(22.10)$$

$$= 475.10$$

Lower Control Limit, (LCL) $= \overline{\overline{X}} - A_2\overline{R}$

$= 462.35 - .577(22.10)$

$= 449.60$

For factor A_2 see Table B Appendix.

Limits for R:

(UCL) $= D_4\overline{R}$

$= 2.114(22.10)$

$= 46.72$

(LCL) $= D_3\overline{R}$

$= 0(22.10)$

$= 0$

For factors D_3 and D_4 see Table B Appendix.

A comparison of the values of $\overline{X}$ with which the control limits were calculated and the control limits themselves indicates that the $\overline{X}$ of subgroups 1, 3, 4, 6, 9, 10, 15, 19, 24, 25, 34, 42, 47, 48 and 50 fell either above or below the control limits of Trial No. 1. If the control limits are to be meaningful they must be established by subgroups that represent control. The out of control subgroups, therefore, are discarded and subgroups 51 to 65 added. The control limits are then recalculated.

Trial No. 2

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{N}$$

$$= \frac{23077.2}{50}$$

$$= 461.54$$

$$\bar{R} = \frac{\Sigma R}{N}$$

$$= \frac{1102}{50}$$

$$= 22.04$$

Limits for $\bar{X}$:

$$(UCL) = \bar{\bar{X}} + A_2\bar{R}$$

$$= 461.54 + .577(22.04)$$

$$= 474.26$$

$$(LCL) = \bar{\bar{X}} - A_2\bar{R}$$

$$= 461.54 - .577(22.04)$$

$$= 448.83$$

Limits for R:

$$(UCL) = D_4\bar{R}$$

$$= 2.114(22.04)$$

$$= 46.59$$

$$(LCL) = D_3\bar{R}$$

$$= 0(22.04)$$

$$= 0$$

In Trial No. 2, subgroups 51 and 61 are out of control. They are discarded and subgroups 67 and 69 are added to replace them. Subgroups 66 and 68 are not considered because they would be obviously out of control on either the $\bar{X}$ or R chart.

Trial No. 3:

$$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{N}$$

$$= \frac{23096.4}{50}$$

$$= 461.93$$

$$\bar{R} = \frac{\Sigma R}{N}$$

$$= \frac{1146}{50}$$

$$= 22.92$$

Limits for $\bar{X}$:

$$(UCL) = \bar{\bar{X}} + A_2\bar{R}$$

$$= 461.93 + .577(22.92)$$

$$= 475.15$$

$$(LCL) = \bar{\bar{X}} - A_2\bar{R}$$

$$= 461.93 - .577(22.92)$$

$$= 448.69$$

Limits for R:

$$(UCL) = D_4\bar{R}$$

$$= 2.114(22.92)$$

$$= 48.45$$

$$(LCL) = D_3\bar{R}$$

$$= 0(22.92)$$

$$= 0$$

TABLE I. --- Weights of 6" B&S 150 Super-deLavaud Pipe

Comprising Grand Lot

(Weight given in pounds)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R	Standard Deviation σ
1	500	500	494	496	494	496.8	6	-----
2	450	460	456	450	440	451.2	20	6.76
3	436	420	438	460	462	443.2	40	-----
4	470	442	442	440	440	446.8	30	-----
5	468	462	472	476	454	466.4	22	7.73
6	470	500	482	486	484	484.4	30	-----
7	480	462	458	460	468	465.6	22	7.94
8	450	460	450	450	460	454.0	10	4.90
9	452	450	448	448	446	448.8	6	-----
10	450	450	452	442	442	447.2	10	-----
11	470	470	468	468	464	467.2	10	3.71
12	462	456	462	458	450	457.6	12	4.45
13	470	470	468	460	464	466.4	10	3.88
14	472	460	462	460	472	465.2	12	5.60
15	472	474	480	500	489	483.0	28	-----
16	472	460	440	446	452	454.0	32	11.17
17	452	452	456	478	452	458.0	26	10.12
18	458	462	488	454	462	464.8	34	11.97

TABLE I. --- Weights of 6" B&S 150 Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R	Standard Deviation σ
19	500	492	486	458	458	478.8	42	-----
20	460	462	452	442	450	453.2	20	7.22
21	456	472	486	460	470	468.8	30	10.48
22	454	468	452	462	456	458.4	16	5.85
23	458	470	474	458	464	464.8	12	6.40
24	476	476	500	498	470	484.0	30	-----
25	482	486	480	512	492	490.4	32	-----
26	458	452	478	470	470	465.6	18	9.33
27	460	478	460	452	460	462.0	26	8.58
28	466	476	442	454	448	457.2	34	12.30
29	460	452	454	462	480	461.6	24	9.92
30	460	450	452	468	452	456.4	18	6.74
31	460	482	470	478	474	472.8	22	7.55
32	440	464	464	454	464	457.2	24	9.43
33	460	480	456	480	468	468.8	24	9.93
34	456	470	460	508	490	476.8	52	-----
35	470	472	478	470	470	472.0	8	3.10
36	470	470	478	452	460	466.0	26	9.03
37	468	462	468	470	452	464.0	18	6.57
38	458	460	458	456	460	458.4	4	1.50

TABLE I. --- Weights of 6" B&S 150 Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R	Standard Deviation σ
39	450	458	452	462	450	454.4	12	4.80
40	460	460	470	460	468	463.6	10	4.45
41	460	448	472	460	472	462.4	24	8.98
42	450	468	460	446	420	448.8	48	-----
43	478	472	454	460	450	462.8	28	10.63
44	464	466	440	450	458	455.6	26	9.58
45	486	460	452	454	462	462.8	34	12.17
46	448	470	470	460	452	460.0	22	9.01
47	450	450	448	450	440	447.6	10	-----
48	448	456	444	440	442	446.0	16	-----
49	460	458	464	462	458	460.4	6	2.33
50	450	430	440	436	421	435.4	29	-----
51	440	430	440	442	430	436.4	12	-----
52	464	450	478	452	456	460.0	28	10.20
53	484	446	462	468	474	466.8	38	12.69
54	458	458	460	462	466	460.8	8	2.99
55	500	488	458	454	470	474.0	46	17.57
56	450	470	458	456	470	460.8	20	7.96
57	468	434	450	450	452	450.8	34	10.78
58	450	462	460	460	454	457.2	12	4.49

TABLE I. --- Weights of 6" B&S Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R	Standard Deviation σ
59	440	458	472	442	458	454.0	32	11.80
60	450	484	468	476	490	473.6	40	13.94
61	472	476	474	472	480	474.8	8	-----
62	462	480	468	450	442	460.4	38	13.35
63	476	452	474	478	464	468.8	26	9.68
64	440	438	436	480	460	450.8	44	16.95
65	460	458	478	472	474	468.4	20	7.94
66	482	484	452	434	440	458.4	50	-----
67	446	462	482	460	476	465.2	36	12.69
68	476	490	492	506	522	497.2	46	-----
69	480	472	460	452	462	465.2	28	9.77

The calculation of the trial control limits based upon 50 subgroups, all of which are in control, indicates that the average of all 6" B&S 150 pipe produced on machine No. 1 is about 461.93 pounds. This is the best estimate that can be made of the production average without further sampling. As stated earlier, the R chart is here used in preference to the standard deviation (σ) chart because the relative accuracy of the two is believed to not be significantly different. However, to assure this contention, comparison is made of the two estimates of σ' (standard deviation of the population) by the use of $\bar{R}$ and by the use of $\bar{\sigma}$.

Estimation of σ' by use of $\bar{R}$:

$$\sigma' = \frac{\bar{R}}{d^2}$$

$$\sigma' = \frac{22.92}{2.326} = 9.84$$

Estimation of o by use of $\bar{o}$:

$$\bar{\sigma} = \frac{\Sigma\sigma}{N}$$

for subgroup values of o see Table I.

$$\bar{\sigma} = \frac{426.90}{50} = 8.538$$

$$\sigma' = \frac{\bar{\sigma}}{c^2}$$

$$= \frac{8.538}{.8407} = 10.16$$

It is seen that the two estimates of σ' are not greatly different. The effective difference of the two values may be better appreciated by comparing the calculations of the $\bar{X}$ control limits based on $\bar{R}$ and on $\bar{\sigma}$.

TABLE II. --- Weights of 6" B&S 150 Super-deLavaud Pipe

(Weight given in pounds)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R
51	454	450	454	450	450	451.6	4
52	476	486	470	482	462	475.2	24
53	452	452	432	428	420	436.8	32
54	458	444	458	456	468	456.8	24
55	440	450	458	456	460	452.8	20
56	460	478	482	476	482	475.6	22
57	474	472	476	476	476	474.8	4
58	468	482	470	482	486	477.6	18
59	446	442	440	442	444	442.8	6
60	440	436	440	438	454	441.6	18
61	428	442	424	410	428	426.4	32
62	472	460	456	458	470	463.2	16
63	458	448	444	456	444	450.0	14
64	454	464	458	466	468	462.0	14
65	450	462	480	448	446	457.2	34
66	472	474	470	488	474	475.6	18
67	432	450	492	510	464	469.6	78
68	482	484	478	460	460	472.8	22
69	450	450	460	460	450	454.0	10
70	458	448	460	450	458	454.8	12

TABLE II. --- Weights of 6" B&S 150 Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R
71	460	462	458	450	450	456.0	12
72	450	442	458	450	454	450.8	16
73	480	466	468	478	472	472.8	14
74	454	460	460	450	458	456.4	10
75	452	470	472	454	458	461.2	20
76	472	460	440	456	442	454.0	32
77	488	470	472	460	448	467.6	40
78	470	474	470	452	450	463.2	24
79	454	456	446	440	438	446.8	18
80	478	450	440	448	452	453.6	38
81	474	472	480	462	458	469.2	22
82	432	440	458	470	460	452.0	38
83	448	450	444	440	448	446.0	10
84	450	442	460	468	458	455.6	26
85	440	442	444	460	458	448.8	20
86	460	448	470	480	470	465.6	32
87	450	442	450	448	448	447.6	8
88	470	450	460	448	448	455.2	22
89	500	474	470	476	450	474.0	50
90	464	474	490	464	450	468.4	40

TABLE II. --- Weights of 6" B&S 150 Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R
91	468	470	454	456	440	457.6	30
92	460	454	468	460	460	460.4	14
93	470	450	480	460	462	464.4	30
94	510	466	444	440	438	459.6	72
95	464	452	442	440	472	454.0	32
96	454	452	464	464	440	454.8	24
97	450	470	452	470	450	458.4	20
98	418	408	540	462	450	455.6	132
99	440	440	450	440	446	443.2	10
100	470	470	424	454	450	453.6	46
101	450	432	450	450	448	446.0	18
102	452	450	458	456	458	454.8	8
103	440	480	470	468	442	460.0	40
104	462	472	460	460	452	461.2	20
105	440	448	448	470	460	453.2	30
106	474	458	442	432	436	448.4	42
107	428	404	440	442	442	431.2	38
108	450	458	452	430	448	447.6	28
109	442	458	460	470	460	458.0	28
110	470	470	472	450	470	466.4	22

TABLE II. --- Weights of 6" B&S 150 Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R
111	462	450	452	454	444	452.4	18
112	460	440	446	446	444	447.2	20
113	454	450	440	434	464	448.4	30
114	478	460	456	468	480	468.4	24
115	480	470	478	472	464	472.8	16
116	478	472	464	476	470	472.0	14
117	446	462	458	472	470	461.6	26
118	548	540	540	500	492	524.0	56
119	446	454	460	462	462	456.8	16
120	460	462	450	460	446	455.6	16
121	420	404	408	396	416	408.8	24
122	430	458	444	470	458	452.0	40
123	470	472	464	450	440	459.2	32
124	446	444	444	434	452	444.0	18
125	432	442	450	464	518	461.2	86
126	446	444	420	440	454	440.8	34
127	474	470	466	508	496	482.8	42
128	436	442	462	466	440	449.2	30
129	462	470	462	460	458	462.4	12
130	492	478	456	452	460	467.6	40

TABLE II. --- Weights of 6" B&S 150 Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R
131	470	470	452	454	450	459.2	20
132	452	450	448	452	454	451.2	6
133	466	468	490	480	484	477.6	24
134	474	442	470	472	460	463.6	32
135	462	460	452	454	452	456.0	10
136	462	470	464	458	462	463.2	12
137	464	468	454	468	470	464.8	16
138	480	462	462	466	448	463.6	32
139	458	442	432	430	442	440.8	28
140	442	464	448	454	442	450.0	22
141	448	454	450	450	448	450.0	6
142	440	450	460	460	452	452.4	20
143	470	444	460	474	462	462.0	30
144	456	452	448	448	460	452.8	12
145	450	454	406	460	456	445.2	54
146	468	460	462	458	476	464.8	18
147	438	454	442	446	458	447.6	20
148	450	440	446	440	458	446.8	18
149	454	440	458	450	470	454.4	30
150	440	442	460	466	464	454.4	26

TABLE II. --- Weights of 6" B&S 150 Super-deLavaud Pipe (continued)

Sub-Group No.	Measurement on each of 5 pipe per subgroup					Average $\bar{X}$	Range R
151	444	440	442	450	450	445.2	10
152	460	460	452	462	470	460.8	18
153	462	460	470	462	472	465.2	12
154	440	472	480	460	448	460.0	40
155	464	470	454	460	460	461.6	16
156	478	472	452	458	454	462.8	26
157	474	450	464	460	470	463.6	24
158	516	500	494	490	480	496.0	36
159	456	458	460	456	472	460.4	16
160	450	440	470	462	454	455.2	30
161	440	470	460	478	472	464.0	38
162	422	432	440	442	434	434.0	20
163	450	466	460	469	468	462.6	19
164	472	470	460	470	474	469.2	14
165	460	484	480	468	470	472.4	24
166	468	480	462	460	460	466.0	20
167	448	452	448	460	464	454.4	16
168	440	442	444	440	470	447.2	30
169	460	464	448	450	410	446.4	50
170	432	462	404	400	470	433.6	66

Control limits of $\overline{X}$ by use of $\overline{R}$. (Previously calculated on page 24.)

$$UCL = 475.15$$

$$LCL = 448.69$$

Control limits of $\overline{X}$ by use of $\overline{\sigma}$:

$$\begin{aligned} UCL &= \overline{\overline{X}} + A_1\overline{\sigma} \\ &= 461.93 + 1.596(8.538) \\ &= 475.56 \\ LCL &= \overline{\overline{X}} - A_1\overline{\sigma} \\ &= 461.93 - 1.596(8.538) \\ &= 448.30 \end{aligned}$$

The above comparison proves that the use of $\overline{R}$ in the calculation of control limits in this case is warranted. Therefore, the $\overline{X}$ and R charts are used as originally stated.

The graphical presentation of the control limits and the location of the subgroup averages and ranges is found on the control charts in Figure 2. Although the information given by the control charts is a repetition of the data in Table I and the results of calculation, it is an overall picture lending speed and accuracy to the interpretation of the data. Those subgroups which were out of control when the trial control limits were calculated do not appear on the chart and those subgroups that were the final basis of calculation have been numbered consecutively, 1-50. The control limits have been extended and subgroups 51-170 have been plotted in the order of production. The data for subgroups 51-170 appear in Table II. The plotting of these additional subgroups affords a comprehensive picture of the actual variation obtaining in the process.

Frequency Distributions of Production

Because the assurance with which control limits may be used depends upon the degree of congruency with which the distribution of the population that is being investigated approximates a normal distribution, it follows that it is important to plot the frequency distribution of the population in question.

Figures 3A, 3B, 3C, and 3D are histograms of frequency distributions taken under four different operating conditions. Figure 3E is a consolidation of the first four distributions representing the frequency distribution of the entire production. If Figure 3E had been the only distribution plotted, it would not necessarily follow that the minor distributions in Figures 3A, 3B, 3C, and 3D would approximate the normal curve. Therein lies the reason for plotting the first four distributions. Figures 3A, 3B, 3C, and 3D each represent the individual weights of 730 pipe taken from a different interval of production than were the pipe used in the calculation of the control limits. Table III shows the actual distribution represented in the five figures.

TABLE III - Frequency Distributions

	Shift 1		Shift 2		Total
Cells	Mach. 1	Mach. 2	Mach. 1	Mach. 2	
515-525	3	0	1	1	5
505-515	5	3	1	0	9
495-505	12	5	12	3	32
485-495	17	17	22	16	72
475-485	100	55	99	50	304
465-475	175	140	155	157	627
455-465	213	244	215	230	902
445-455	134	199	151	175	659
435-445	59	62	66	71	258
425-435	10	3	5	25	43
415-425	1	1	2	1	5
405-415	1	1	0	1	3
395-405	0	0	1	0	1
Total	730	730	730	730	2920

Part Name 6" B&S 150

Machine No. 1

Specifications Min 437 Avg 451

Inspected for Weight (lbs)

Data from sheets (See Table I & II)

Sample size 5

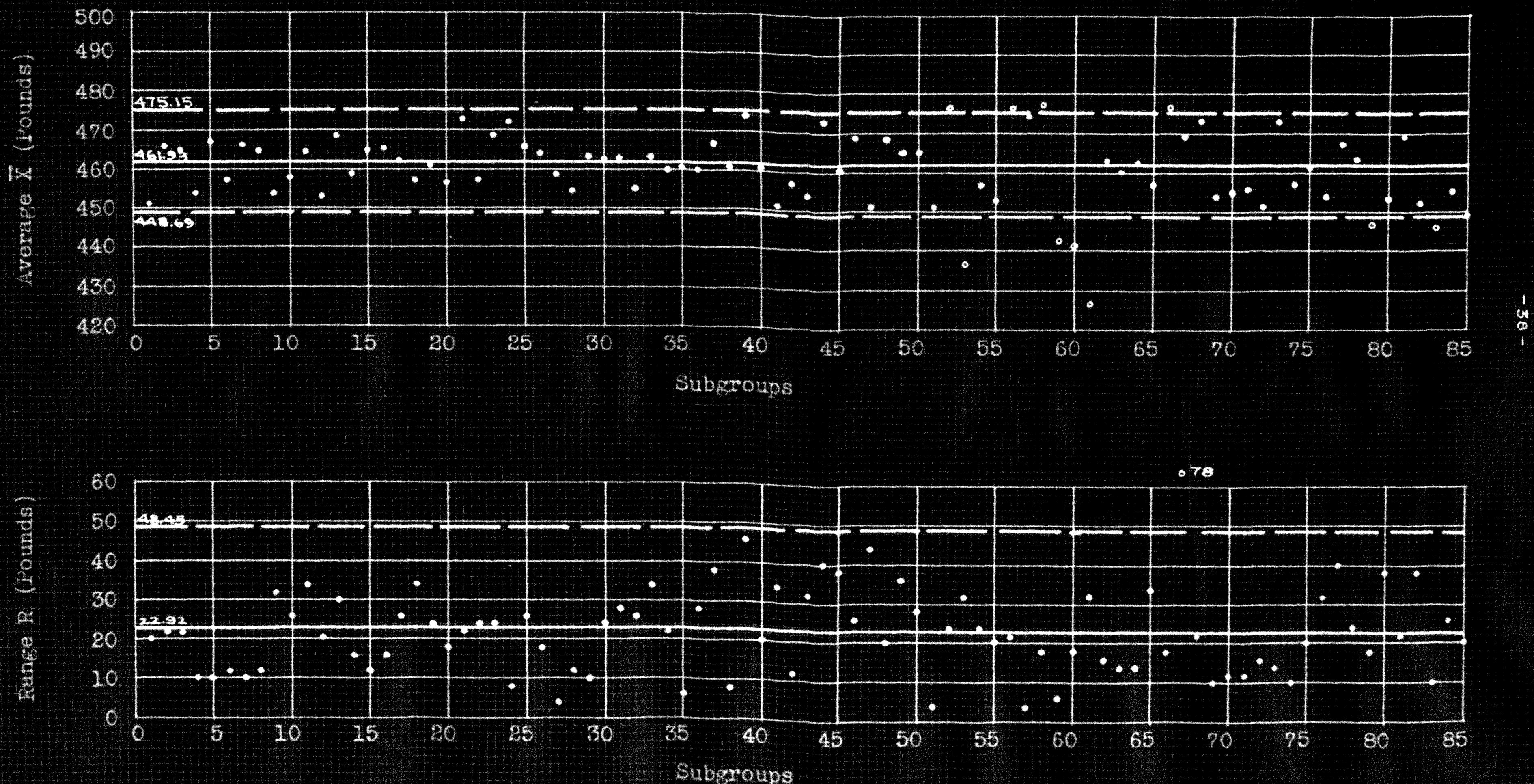

Fig. 2 AVERAGE AND RANGE CHARTS OF PAST DATA

Part Name 6" B&S 150

Inspected for Weight (lbs)

Machine No. 1

Data from sheets (See Table II)

Specifications Min 437 Avg 451

Sample size 5

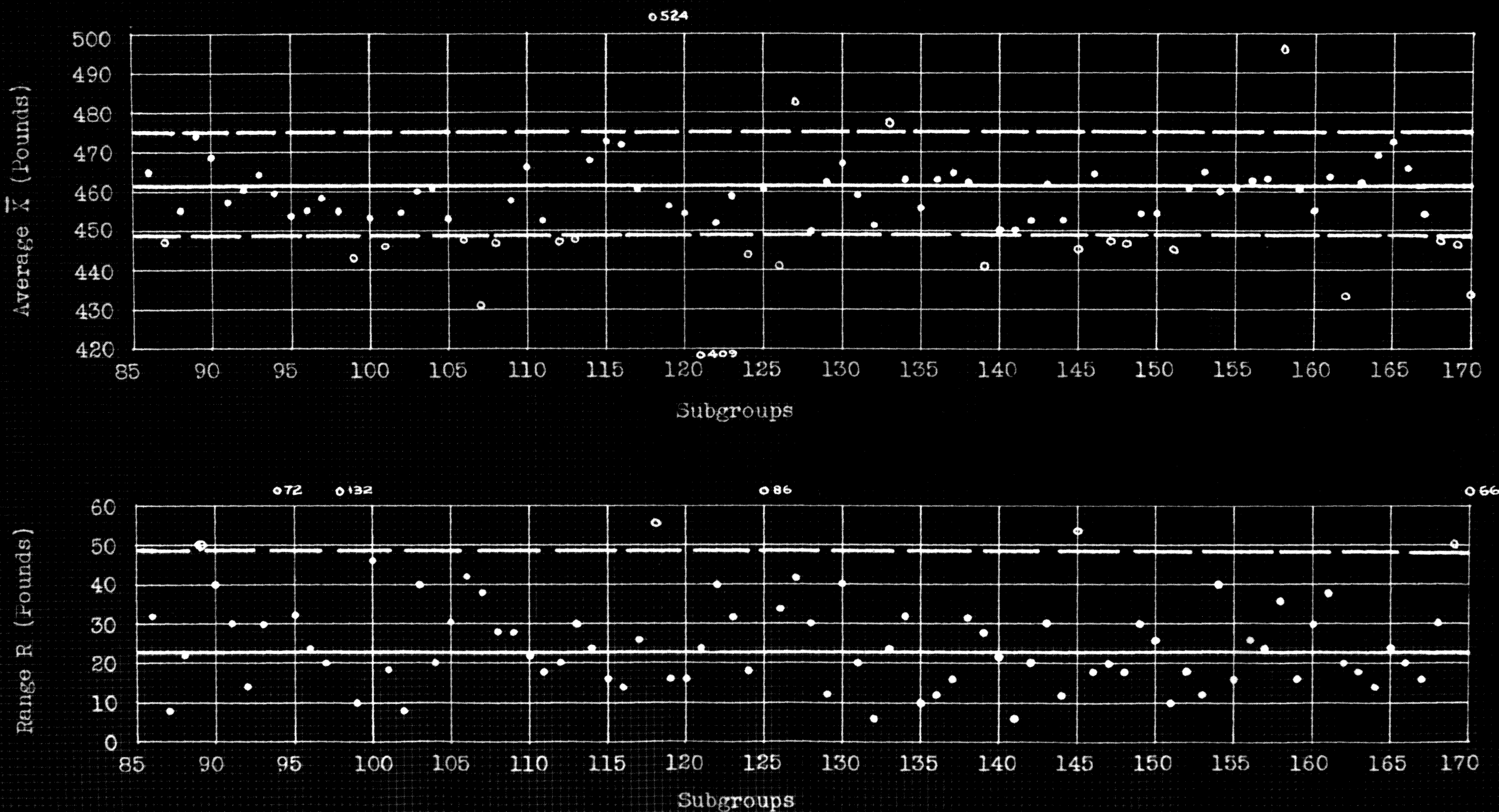

Fig. 2 AVERAGE AND RANGE CHARTS OF PAST DATA (Continued)

A normal curve appears on each figure representing the frequency distribution that would occur when plotting an infinite number of individual pipe taken from an infinite number of subgroups, 99.7 per cent of whose averages would fall within the $\pm 3\sigma$ control limits shown in Figure 2, providing that the data were continuous. To plot the normal curve, a transition is made of the normal equation

$$dy = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-\frac{1}{2}x^2}{\sigma^2}}\, dx$$

where y is the probability of x lying within the range x to x — dx, where x is the deviation from the mean $(X - \bar{X})$. The transition is accomplished by changing the actual units into standard deviation units. If Z represents the number of standard deviation units from the average, then $Z = \frac{x}{\sigma}$ and $\sigma = 1$. The normal equation then becomes

$$dy = \frac{1}{2\pi}\, e^{-\frac{Z^2}{2}}\, dx$$

The integration of this equation between the limits of 0 and Z gives the probability of a value of Z lying within those limits. Table C in the appendix gives the probability of a value of Z lying within the limits 0 to Z for different values of Z. If the theoretical frequencies can be calculated for each cell of the histogram of observed frequencies, it shall then be possible to draw the normal curve through those points. For a sample size of n, the theoretical class frequency (f_c) for any class whose mid-point is a distance $X - \bar{X}/\sigma$ from the observed average ($\bar{X}$) is

$$f_c = \frac{nC_i}{\sigma} y$$ where C_i is the class interval and y is the

value in Table C in the appendix. The use of this equation is illustrated in the following table:

TABLE IV: Determination of Normal Curve

$$\frac{nC_i}{\sigma} = \frac{730(10)}{10.16} = 71.85$$

Class Mid Point	$X - \bar{X}$	$\frac{X - \bar{X}}{\sigma}$	y	f_c
400	-61.9	-6.09	.0000	0
410	-51.9	-5.11	.0000	0
420	-41.9	-4.12	.0009	2
430	-31.9	-3.14	.0029	28
440	-21.9	-2.16	.0387	145
450	-11.9	-1.17	.2012	282
460	- 1.9	- .19	.3918	208
470	8.1	.80	.2897	59
480	18.1	1.78	.0818	6
490	28.1	2.77	.0086	0
500	38.1	3.75	.0004	0
510	48.1	4.73	.0000	0
520	58.1	5.72	.0000	0

The values f_c are then plotted to form the normal curve.

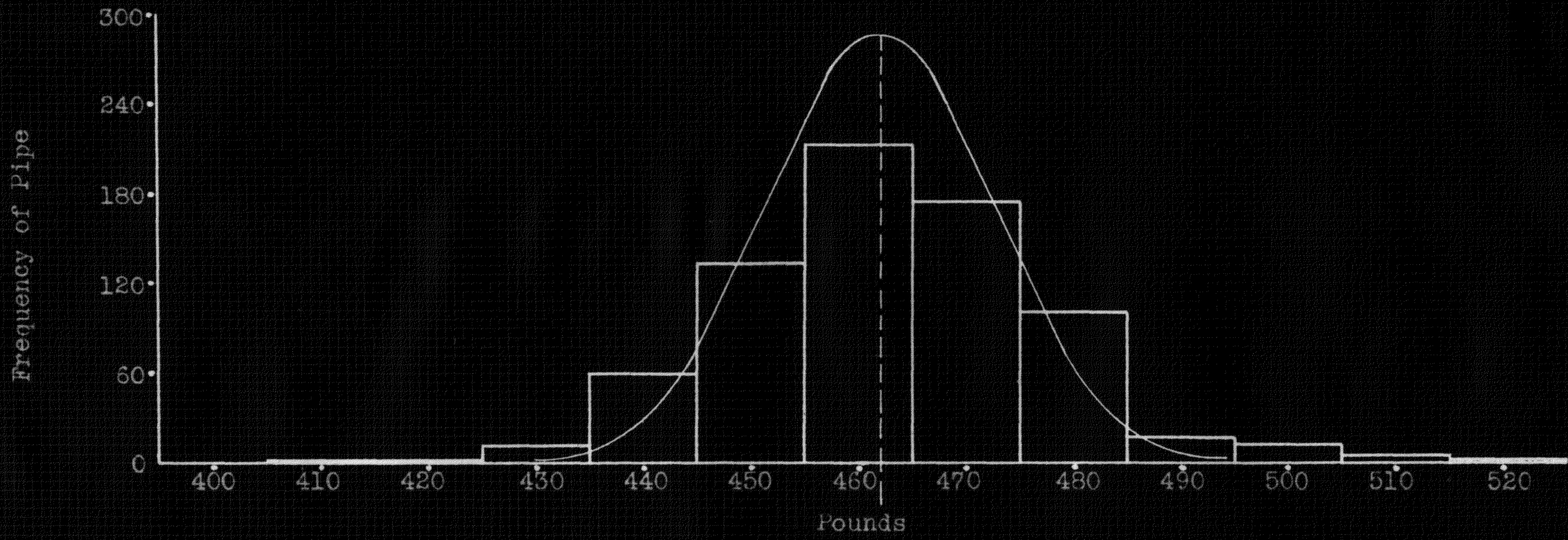

Fig. 3A Frequency Distribution of 6" B&S 150
Produced by Shift No. 1 - Machine No. 1,
As Compared With Normal Distribution

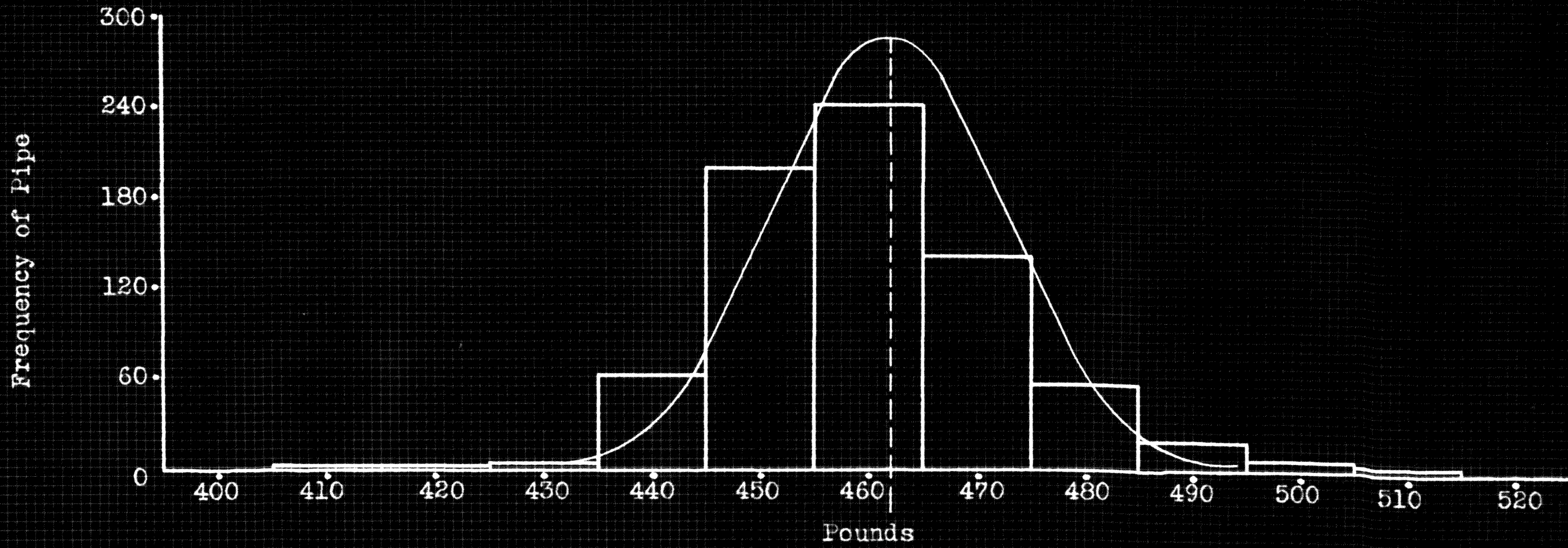

Fig. 3B Frequency Distribution of 6" B&S 150
Produced by Shift No. 1 - Machine No. 2,
As Compared With Normal Distribution

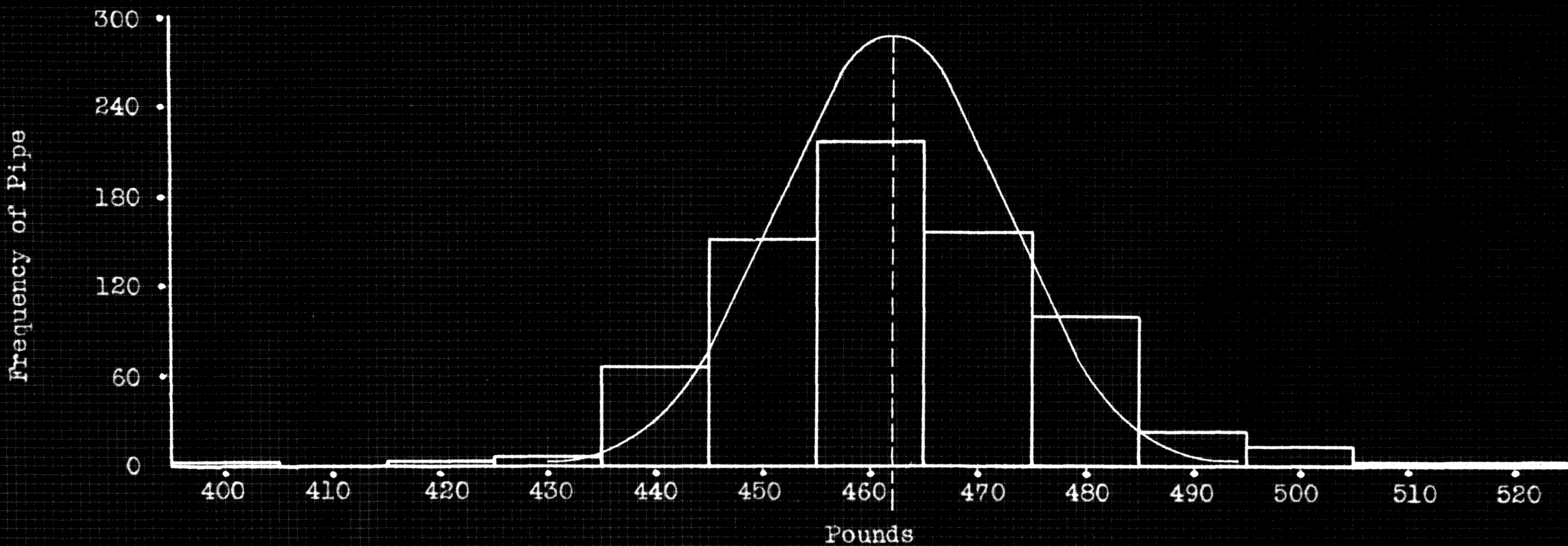

Fig. 3C Frequency Distribution of 6" B&S 150
Produced by Shift No. 2 - Machine No. 1,
As Compared With Normal Distribution

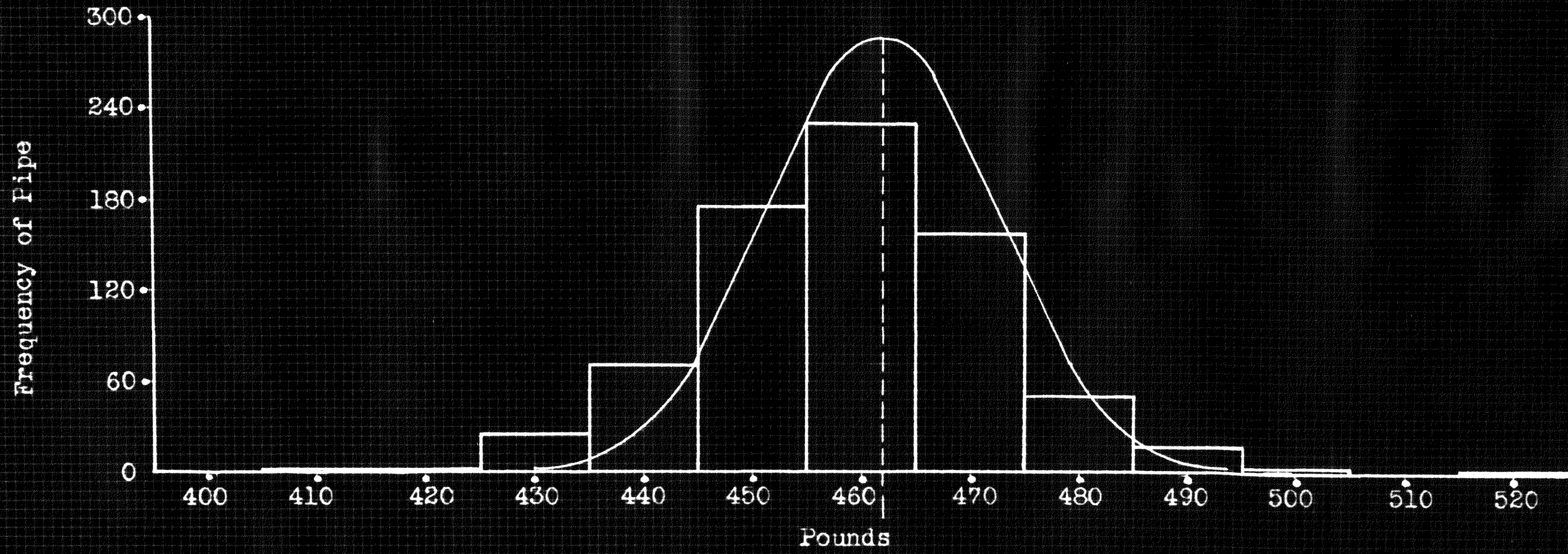

Fig. 3D Frequency Distribution of 6" B&S 150
Produced by Shift No. 2 - Machine No. 2,
As Compared with Normal Distribution

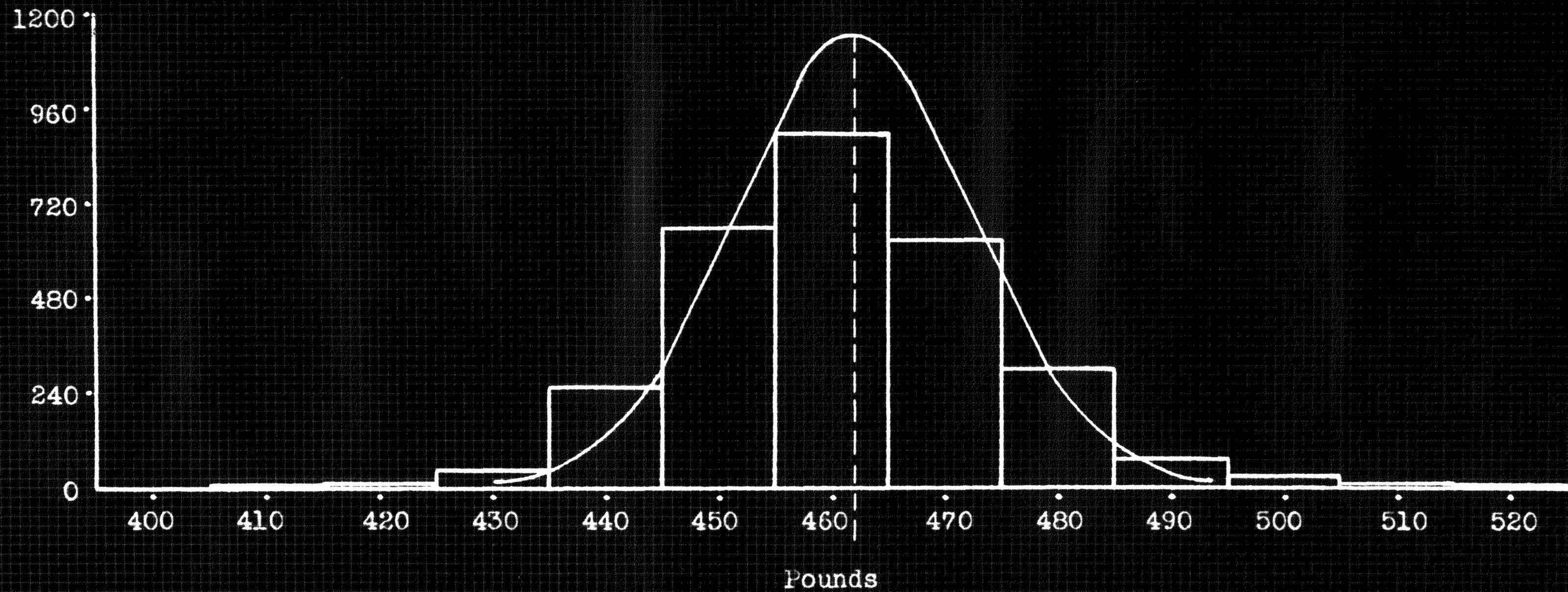

Fig. 3E Frequency Distribution of 6" B&S 150 Super-deLavaud Pipe

(Composite of Figs. 3A, B, C, and D)

Interpretation of Control Charts and Frequency Distributions

The control charts and the data of Table I and II are taken from only one of the two machines (machine No. 1) that is used in the making of 6" B&S 150 pipe, this being the only information available at that time. This might seem to restrict the information obtained regarding 6" B&S 150 pipe production in general. In anticipation of this possible discrepancy samples of production were taken at later dates from both machines and both shifts and the statistics, $\bar{\bar{X}}$ and $\bar{R}$, of those samples calculated. Each of the statistics checked did not practically differ from the $\bar{\bar{X}}$ and $\bar{R}$ used to determine the control limits in Figure 2. Therefore, any conclusions drawn from the charting of the production of machine No. 1, based on $\bar{\bar{X}}$ and $\bar{R}$, may be said to be true of 6" B&S 150 pipe production on machine No. 2.

The general information derived from the control charts and frequency distributions may be summed up as follows:

1. The process is out of control.

2. The $\bar{\bar{X}}$ of the process remains approximately at 462 pounds.

3. The distribution approaches the normal curve. That is, the distribution is unimodal; the mode coincides with the arithmetic mean; there is no appreciable skewness and the curve is monotonic.

When the control limits of the $\bar{X}$ and R charts are extended and subsequent data plotted, several averages and ranges of subgroups are seen to fall both above and below the control limits. The frequency distributions also make light of the fact that production is extending beyond the normal distribution. This apparent lack of control unfortunately cannot be assigned to any particular cause of variation on the basis of the evidence shown. The location of the subgroup averages do not conform to any pattern of occurrence that can be associated with any particular cause of variation. Rather, their location seems to be quite randomized. Even if there were evidence of trends in variation, their cause would be undeterminable because the operating conditions prevailing at that time would not be known.

Although, the indication of lack of control is present, subgroups 51 to 170 have a central tendency corresponding to the $\bar{\bar{X}}$ calculated from the first 50 subgroups of the grand lot. This tendency is substantiated by the frequency distributions of all operating conditions.

It seems highly probable that the production can be brought under control without great expense. It would be purely guesswork to describe at this point what should be done in order that control can be achieved. Only after the relative effects of the causes of variation have been evaluated will it be possible to say that this or that source of variation should be reduced or eliminated to bring the production into control. The analysis of these causes of variations shall be considered later.

However, at this point it is advisable to assume that the production can be controlled and to now consider at what level quality should be controlled and what further restrictions, if any, should be imposed on permissable variation. Federal Specifications cited earlier dictate that the $\bar{\bar{X}}$ of production cannot be below 451 pounds. The Specifications also stated that the weight of any one 6" B&S 150 pipe could not be less than 437 pounds. This does not mean that a production level must be maintained that allows no individual pipe to fall below 437 pounds, as shall be seen presently. Under prevailing conditions a certain number of pipe are falling below 437 pounds. The pipe of less than minimum weight cannot be considered as rejects because light weight pipe then belong to Class 100 and still retain their sales value. The question then arises, is it more advantageous to establish control at a level that will produce only Class 150 pipe or would it be better to allow a certain percentage of Class 100 pipe to be made in the lower range of Class 150 production? To answer this question, the percentage of Class 100 that results in the lower range of Class 150 pipe must be determined. This percentage can be calculated from the past data used to establish the control limits.

The lower control limit of the $\bar{X}$ chart is 448.69 pounds. This is the lower limit for averages or $\bar{\bar{X}} - 3\sigma_{\bar{X}}$. To find the percentage of Class 100 pipe produced the lower limit for individuals is $\bar{\bar{X}} - 3\sigma'$.

$$\sigma' = \frac{\bar{R}}{d_2} = 9.84$$

$$\begin{aligned} \text{Lower Control Limit} &= \bar{\bar{X}} - 3\sigma' \\ &= 461.93 - 3(9.84) \\ &= 432.41 \end{aligned}$$

To find the percentage of pipe that are produced between 437 and 432.41 pounds use is made of the Camp-Meidell inequality. The use of this inequality is justified by the nature of the frequency distribution of the production. It is obvious that the normal equation cannot be used because the observed frequency does not conform even approximately to the normal curve. The Camp-Meidell inequality is

$$P = 1 - \frac{1}{2.25\, t^2}$$

where P is the probability of a point falling within the limits $\pm$ t standard deviation units. The number of standard deviation units between 462 pounds ($\bar{\bar{X}}$) and 437 pounds is

$$t = \frac{X - \bar{\bar{X}}}{\sigma'} = \frac{437 - 462}{9.84}$$

$$t = 2.54$$

$$P = 1 - \frac{1}{2.25(2.54)^2}$$

$$= .9311$$

This percentage represents the number of pipe whose weight will lie between the limits of $\bar{\bar{X}} \pm (462 - 437)$. The percentage of pipe whose weight is less than 437 pounds is therefore $\frac{1 - .9311}{2}$ or 3.45%. This calculation or theoretical percentage is slightly larger than the actual percentage as revealed by production records. Consequently, the Class 100 pipe production shall be considered to be 3% of the Class 150 pipe production. The demand for 6" B&S 100 pipe is at present about equal to the number of Class 100 pipe that are produced in the lower range of Class 150 pipe. Assuming that the demand will remain constant, the control of quality at any level should not produce more than 3% of pipe falling below 437 pounds.

The restrictions that must necessarily be placed on the level of quality indicate three possible distributions of production.

Distribution A: Let it be assumed that production is controlled within the three sigma limits shown in Figure 2 and that 3% of the production will be allowed to fall below 437 pounds. To what extent will $\bar{\bar{X}}$ then be lowered? The number of standard deviation units between X and 437 pounds would be

$\frac{X-\bar{\bar{X}}}{\sigma'}$ and the area under the normal curve below $\frac{X-\bar{\bar{X}}}{\sigma'}$ would be 3%.

$$\frac{X-\bar{\bar{X}}}{\sigma'} = -1.88$$

The figure − 1.88 is obtained from a table for areas under the normal curve.

$$X = 437$$

$$\sigma' = 10.16$$

$$\bar{\bar{X}} = 1.88(10.16) + 437$$

$$\bar{\bar{X}} = 456.10$$

This is an effective reduction of the production weight average of approximately 5.8 pounds per pipe. On the basis of present costs and average monthly production, this lowering of the average weight would make possible the following monthly saving:

Cost of iron = \$50.00 per ton including the cost of material, melting, and shipping.

Monthly production = 8000 − 6" B&S 150

$\frac{8000(5.8)}{2000}$ = 23.2 tons saved per month

23.2 (\$50.00) = \$1160 saved per month

This saving can be realized by simply bringing the present process into control and lowering the average weight to 456.10 pounds without basically changing the method of production. The normal frequency distribution resulting from control at this level is shown graphically in Figure 4.

<u>Distribution B</u>: If it were practical, 451 pounds would be the optimum arithmetic mean of production. Although 451 pounds is the minimum shipping weight, it would not be practical to set $\bar{\bar{X}}$ at 451 pounds because if the

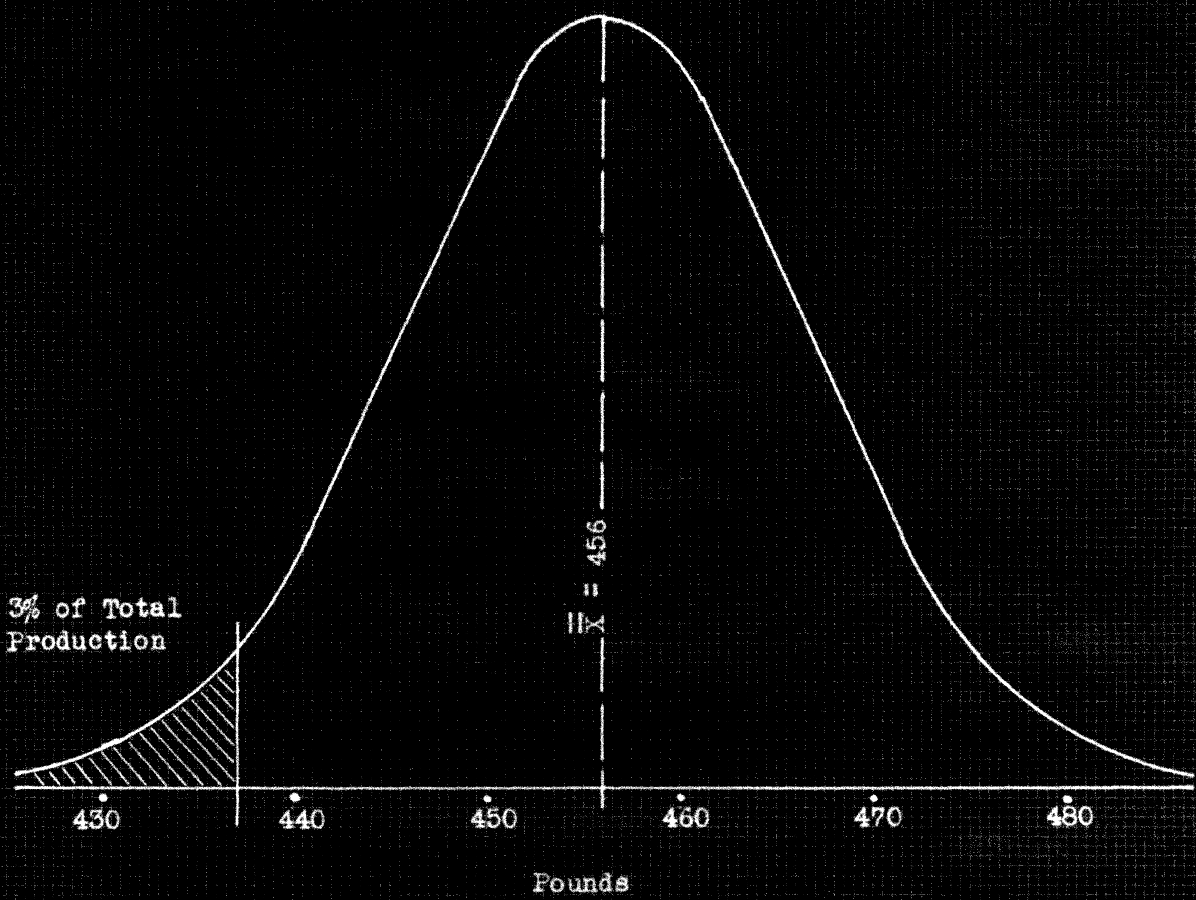

Fig. 4 Distribution "A": Normal Frequency Distribution with $\bar{\bar{X}}$ at 456, Including 3% of Class 100 Production.

production average were to vary slightly downward, the minimum specification would not be met. Therefore, if $\bar{\bar{X}}$ were placed at 453 pounds and the production controlled, continual meeting of the minimum specification would be assured. Locating $\bar{\bar{X}}$ at 453 pounds and still allowing 3% of production to fall below 437 pounds would necessarily mean a reduction in the standard deviation and a consequent narrowing of control limits.

$$\frac{X - \bar{\bar{X}}}{\sigma'} = -1.88$$

$$\frac{437 - 453}{\sigma'} = -1.88$$

$$\sigma' = \frac{16}{1.88} = 8.51$$

Reducing the standard deviation from 10.16 to 8.51 and lowering the average from 462 to 453 pounds would result in the following monthly saving:

$$462 - 453 = 9 \text{ pounds saved per pipe}$$

$$\frac{8000(9)}{2000} = 36 \text{ tons saved per month}$$

$$36(\$50.00) = \$1800 \text{ saved per month}$$

Proper re-engineering of the production method should be able to effect a reduction in σ' from 10.16 to 8.51. This distribution is shown graphically in Figure 5.

Distribution C: In this case it is assumed that the production of Class 100 pipe in the lower range Class 150 pipe production is undesirable, but $\bar{\bar{X}}$ is to remain at 453 pounds. This condition represents a further reduction in the standard deviation than is necessary in Distribution B.

$$\bar{\bar{X}} - X = 3\sigma'$$

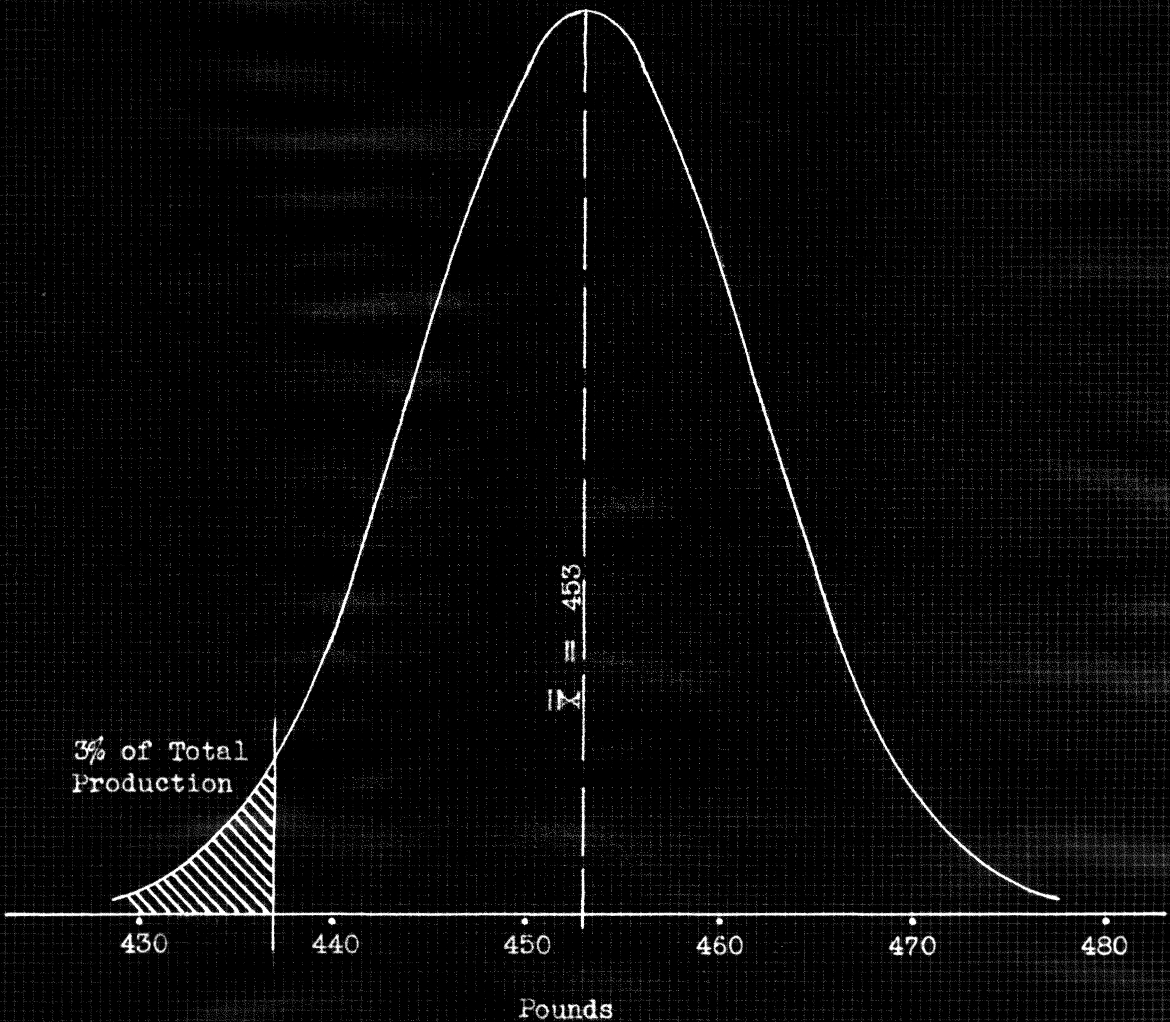

Fig. 5 Distribution "B": Normal Frequency Distribution with $\bar{X}$ at 453, Including 3% of Class 100 Production.

$$\sigma' = \frac{453 - 437}{3}$$

$$\sigma' = 5.33$$

There is no additional saving in this case than there was in Distribution "B" because both quality levels are the same with $\bar{\bar{X}}$ at 453 pounds. Distribution "C" is shown graphically in Figure 6.

The three distributions given represent three degrees of production improvement. Distribution "A" involves discovering and reducing the causes of excess variation to the extent that all subgroup averages will lie within the control limits shown in Figure 2. After control had been accomplished, $\bar{\bar{X}}$ would then be lowered to 456.10 pounds. An improvement of this type would require the least expense of the three distributions. Distribution "B" represents, in this writer's opinion, the most economical condition of production. The only possible feature that might meet with opposition is the fact that if 3% Class 100 pipe were to be produced it would be necessary to store the pipe in anticipation of future demand. However, it is felt that the expense of storage would be considerably less than the expense necessary to control the distribution as closely as is indicated by Distribution "C". It is questionable whether or not it would be possible to control all of the variables that enter into the production of centrifugal cast iron pipe so that the distribution would conform to Distribution "C". It is certainly true that if the control of these variables is possible the expense of re-engineering to achieve such control would be high. The logical sequence to acquire optimum control would be to first control production as described by Distribution "A" and then attempt the reduction of the standard deviation so that production would conform to Distribution "B".

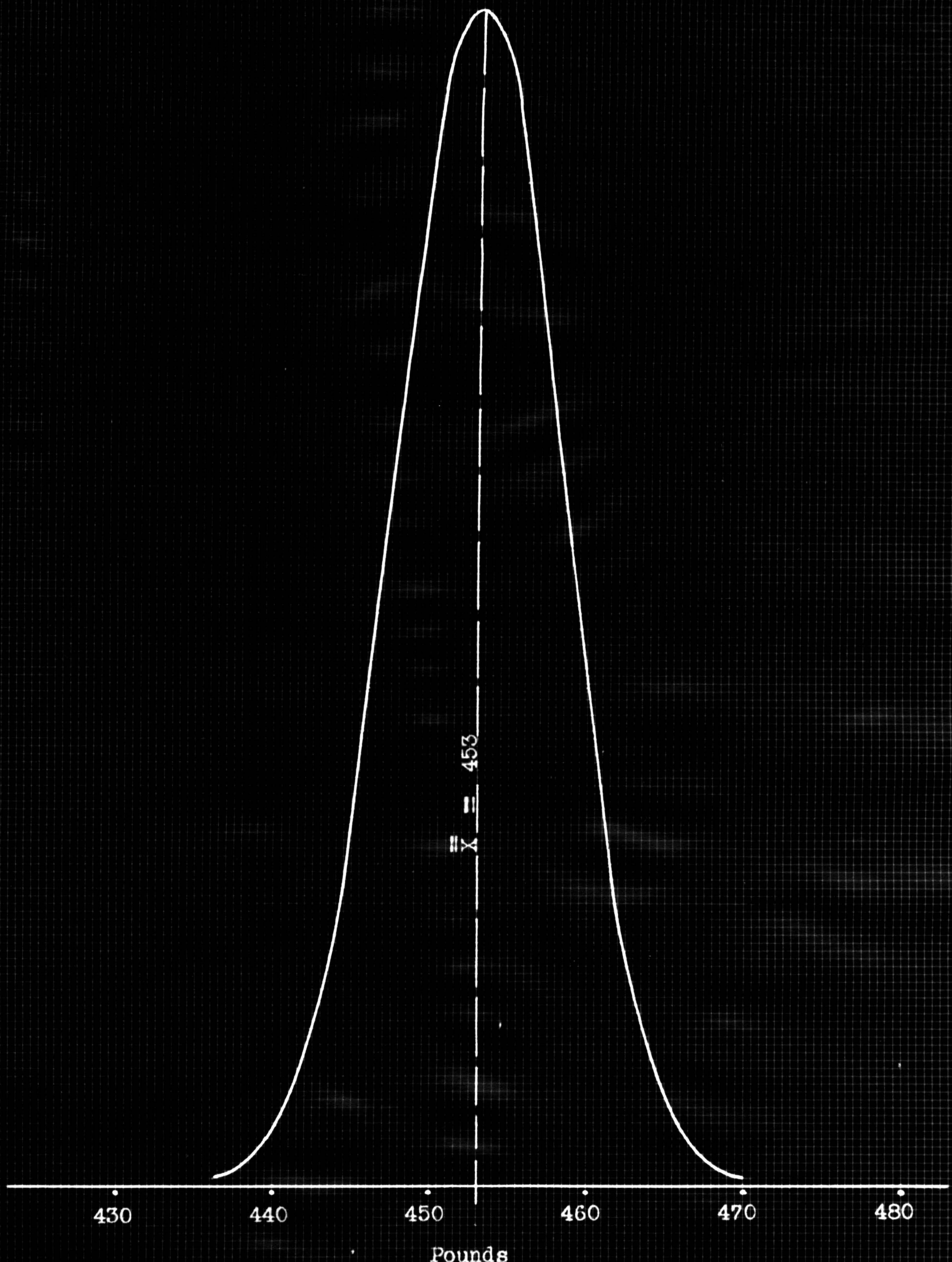

Fig. 6 Distribution "C": Normal Frequency Distribution with $\bar{\bar{X}}$ at 453, No Class 100 Production.

The interpretation of the data used to establish the initial control limits and the observed frequency distributions has been the basis for determining the desired production variation and level of quality. The problem of how to achieve this desired control still remains.

ANALYSIS OF VARIANCE

The information gained from the interpretation of the control charts indicates that the production of 6" B&S 150 pipe lacks control. The factors that may contribute to that lack of control have been discussed. The information of the control charts also indicate that the average weight should be revised downward and, if possible, the standard deviation should be reduced.

The primary consideration should be an effort to bring production into statistical control. After control has been achieved, manipulation of production average and standard deviation should follow.

It has been seen that location of subgroup averages and ranges do not disclose any significant regularity which evidences the characteristic of any particular cause of process variation. Production improvement by the reduction of variation may be approached in several ways. Improvements may be inaugurated by attacking that particular source of variation that seems to be the least costly to control. Improvements may be established because in the light of past experience it is "felt" that a certain variation is largely responsible for total process variation. Both of these approaches are non-scientific which in this particular case is likely to lead to incorrect conclusions and in the long run may prove to be uneconomical. At most, such an approach is not positive. Because statistical devices are available that should with a certain degree of assurity indicate the relative effect of various sources of process variation, several of such tests are here used to disclose the source of major variation.

It seems advisable at this point to review the possible sources of variation. These factors are as follows:

1. Shifts
2. Machines
3. Head-coremen
4. Temperatures
5. Hydraulic Pressures

Each of the above factors may differ in either means or variability. For example, Shift I and Shift II might differ significantly in both mean and variability, or they might have identical means and still differ in variability or they might be identical with respect to means and variability. To determine which factor contributes largely to production variability, difference in mean and variability should both be tested, with emphasis being placed on relative variabilities.

A sample of 20 pipe has been selected at random from each of the two shifts to test the significance between Shift I and Shift II.

<u>Tests of Significance Between Shifts</u>

Shift I	Shift II
460	470
452	470
454	472
462	478
480	470
460	470
450	470
452	478
468	452
452	460
460	468
482	462
470	468
478	470
474	452

440	458
464	460
464	458
454	496
464	460

$\Sigma X_1 = 1240$ $\qquad\qquad$ $\Sigma X_2 = 1302$

$\bar{X}_1 = 62.0$ $\qquad\qquad$ $\bar{X}_2 = 65.1$

$\Sigma X_1^2 = 79184$ $\qquad\qquad$ $\Sigma X_2^2 = 85916$

$$\Sigma(X_2 - \bar{X}_2)^2 = \Sigma X_1 - \left(\frac{\Sigma X}{n}\right)^2 \qquad \Sigma(X_2 - \bar{X}_2)^2 = \Sigma X_2 - \left(\frac{\Sigma X_2}{n}\right)^2$$

$$= 2304 \qquad\qquad = 1155.8$$

As all the weights of all pipe are between 400 and 499, 400 is subtracted from each individual weight to ease calculation.

Test for significant difference in variability:

$$F = \frac{\Sigma(X_1 - \bar{X}_1)^2}{\Sigma(X_2 - \bar{X}_2)^2} = \frac{2304}{1155.8}$$

$$= 1.99$$

F Table value for | P = .05, 19 degrees of freedom, 19 degrees of freedom | is 2.16. (For F Tables see any text on statistical methods. See Appendix II.) As the calculated value of 1.99 is less than 2.16, it may be concluded that Shifts I and II do not differ significantly in variability. This means the probability is less than one in twenty that the two samples representing the production of two shifts came from populations having different standard deviations.

Test for significant difference in means.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

$$= \frac{62.0 - 65.1}{\sqrt{\frac{2304 + 1155.8}{20 + 20 - 2}\left(\frac{1}{20} + \frac{1}{20}\right)}}$$

$$= 1.027$$

Because the two samples do not differ significantly in variability, $n_1 + n_2 - 2$ degrees of freedom may be used in obtaining the value to be compared from the t Table. There are no values appearing in the t Table for degrees of freedom 31 to ∞. The t Table value for $\left| \begin{matrix} P = .05 \\ \infty \text{ degrees of freedom} \end{matrix} \right|$ is 1.960. As the value calculated above is less than the table value, Shifts I and II do not differ significantly in means.

The weights of the individual pipe selected for tests of significance of the factors, machines, head-coremen, temperature and pressure were not chosen at random. They were selected under all possible combinations of the several factors. The tabulation is shown below.

		Machine 1			Machine 2		
		A	B	C	A	B	C
TEMPERATURE	HIGH	450	460	452	460	440	480
		466	452	446	460	444	466
	LOW	432	462	454	442	460	456
		478	462	448	474	454	448

Temperatures are divided into two ranges, "High" and "Low." The "High" temperatures represent the first pipe poured from each of twelve transfer ladles. The "Low" temperatures represent the last pipe poured from each of twelve transfer ladles. Different hydraulic pressures are designated as "A," "B," and "C."

Pressure		Conditions
A	- occurs when	both machines are moving down
B	- occurs when	one machine is moving down and the other machine is moving up.
C	- occurs when	one machine is moving down and the other is stationary.

It is noticed that the factor "head-coremen" has been left out. The effect of "head-coremen" and "machines" are both found in the factor "machines," since each regular head-coreman works on only one machine.

Tests for Significant Difference Between Machines

Machine 1	Machine 2
450	460
466	452
432	462
478	462
452	460
446	460
454	442
448	474
440	480
444	466
460	456
454	448
$\Sigma X_1 = 624$	$\Sigma X_2 = 722$

$$\bar{X}_1 = 52.00 \qquad \bar{X}_2 = 60.17$$

$$\Sigma X_1^2 = 34056 \qquad \Sigma X_2^2 = 44628$$

$$\Sigma(X_1 - \bar{X}_2)^2 = \Sigma X_1^2 - \frac{(\Sigma X_1)^2}{n} \qquad \Sigma(X_2 - \bar{X}_2)^2 = \Sigma X_2^2 - \frac{(\Sigma X_2)^2}{n}$$

$$= 1608 \qquad = 827.7$$

Test for significant difference in variability:

$$F = \frac{\Sigma(X_1 - \bar{X}_1)^2}{\Sigma(X_2 - \bar{X}_2)^2} = \frac{1608}{827.7}$$

$$= 1.94$$

The F Table value for | P = .05, 11 degrees of freedom, 11 degrees of freedom | is 2.82. As the calculated value is less than the value from the table, there is no significant difference between the variability of the machines.

Test for significant difference in means:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = 1.825$$

The t Table value for | P = .05, 22 degrees of freedom | is 2.074. The means of machines are not significantly different.

Tests for Significant Difference Between Temperatures

High Temperature	Low Temperature
450	432
466	478

452	454
446	448
440	460
444	454
460	462
452	462
460	442
460	474
480	456
460	448

$$\Sigma X_H = 676 \qquad \Sigma X_L = 670$$

$$\bar{X}_H = 56.33 \qquad \bar{X}_L = 55.83$$

$$\Sigma X_H^2 = 39472 \qquad \Sigma X_L^2 = 39212$$

$$\Sigma(X_H - \bar{X}_H)^2 = \Sigma X_H^2 - \frac{(\Sigma X_H)^2}{n} \qquad \Sigma(X_L - \bar{X}_L)^2 = \Sigma X_L^2 - \frac{(\Sigma X_L)^2}{n}$$

$$= 1390.7 \qquad = 1803.7$$

Test for significant difference in variability:

$$F = \frac{\Sigma(X_L - \bar{X}_L)^2}{\Sigma(X_H - \bar{X}_H)^2} = \frac{1803.7}{1390.7}$$

$$= 1.297$$

The F Table value for | P = .05, 11 degrees of freedom, 11 degrees of freedom | is 2.82. As the calculated value is less than the value from the table, there is no significant difference between the variability of temperatures.

Test for significant difference in means:

$$t = \frac{\bar{X}_H - \bar{X}_L}{\sqrt{\frac{\Sigma(X_H - \bar{X}_H)^2 + \Sigma(X_L - \bar{X}_L)^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = .1016$$

The t Table value for $\left| \begin{array}{l} P = .05 \\ 22 \text{ degrees of freedom} \end{array} \right|$ is 2.074. As the calculated value is less than the value from the table, there is no significant difference between the means of temperatures.

Tests for Significant Difference Among Pressures

Pressures

A	B	C
450	452	440
466	446	444
432	454	460
478	448	454
460	460	480
452	460	466
462	442	456
462	474	448
$\Sigma X_A = 462$	$\Sigma X_B = 436$	$\Sigma X_C = 448$
$\overline{X}_A = 57.75$	$\overline{X}_B = 54.50$	$\overline{X}_C = 56.00$
$\Sigma X_A^2 = 27956$	$\Sigma X_B^2 = 24480$	$\Sigma X_C^2 = 26248$
$\Sigma(X_A - \overline{X}_A)^2 = 1275.5$	$\Sigma(X_B - \overline{X}_B)^2 = 718$	$\Sigma(X_C - \overline{X}_C)^2 = 1160$

Because in this case there are three samples instead of two as in the previous cases, testing the significance can better be handled by the analysis of variance.

Within groups sum of squares:

$$\Sigma(X_A - \overline{X}_A)^2 + \Sigma(X_B - \overline{X}_B)^2 + \Sigma(X_C - \overline{X}_C)^2 = 3153.50$$

Between groups sum of squares:

$$\overline{X} = \frac{\overline{X}_A + \overline{X}_B + \overline{X}_C}{3} = 56.08$$

$$n_A(\overline{X}_A - \overline{X})^2 + n_B(\overline{X}_B - \overline{X})^2 + n_C(\overline{X}_C - \overline{\overline{X}})^2 = 43.33$$

Test for significant difference in variability:

$$F = \frac{\text{Between groups sum of squares} \;/\; \text{No. of groups} - 1}{\text{Within groups sum of squares} \;/\; n_A + n_B + n_C - \text{No. of groups}}$$

$$= \frac{42.33 \;/\; 2}{3153.5 \;/\; 21}$$

$$= .1409$$

The F Table value for | P = .05, 2 degrees of freedom, 21 degrees of freedom | is 19.44. The variability of the three groups does not differ significantly.

Test for significant difference in means:

$$t_{AB} = \frac{\bar{X}_A - \bar{X}_B}{\sqrt{\frac{\Sigma(X_A - \bar{X}_A)^2 + \Sigma(X_B - \bar{X}_B)^2}{n_A + n_B - 2}\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

$$= .544$$

The t Table value for | P = .05, 14 degrees of freedom | is 2.145. Pressures A and B do not differ significantly in means. It is not necessary to compare Pressure A and C or Pressure B and C because it is evident from the data that they too will not differ significantly in means.

Interpretation of Tests of Significance

None of the sources of variation differs significantly in either means or variability. In all cases the factors causing production variation were tested at the 5% (P = .05) level. Therefore, it may be said that the probability in each case is less than 1 in 20 that the samples representing any one of the factors came from populations having different means and variability. This still does not answer the question as to which factor is the largest contributor to total production variation. If one of the factors had evidenced significant difference, that factor would have been the predominant influence in total production variation.

It is not surprising that none of the factors examined exhibits significant difference. Attention is called to the control chart which showed lack of control but that lack of control was not great. Those subgroup averages which fell outside of the upper and lower control limits may well have been the additive effect of several sources of variation.

To determine which of the sources of variation has the greatest effect, use is made of the analysis of variance.

Sum of squares:

Pressures:

$$\frac{\Sigma X_A^2}{n_A} + \frac{\Sigma X_B^2}{n_B} + \frac{\Sigma X_C^2}{n_C} - \frac{(\Sigma X)^2}{n} =$$

$$\frac{462^2 - 436^2 - 448^2}{8} - \frac{1346^2}{24} = \quad 42.3$$

Machines:

$$\frac{\Sigma X_1^2}{n_1} + \frac{\Sigma X_2^2}{n_2} - \frac{(\Sigma X)^2}{n} =$$

$$\frac{624^2 + 722^2}{12} - \frac{1346^2}{24} = \quad 400.1$$

Temperatures:

$$\frac{\Sigma X_H^2}{n_H} + \frac{\Sigma X_L^2}{n_L} - \frac{(\Sigma X)^2}{n} =$$

$$\frac{676^2 + 670^2}{12} - \frac{1346^2}{24} = 1.4$$

Total

$$\Sigma X_{11}^2 + \Sigma X_{21}^2 + \Sigma X_{31}^2 \ldots\ldots\ldots + \Sigma X_{46}^2 - \frac{X^2}{n} =$$

$$78684 - \frac{1346^2}{24} = 3195.8$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Among Pressures	42.3	2	21.2
Between Machines	400.1	1	400.1
Between Temperatures	1.4	1	1.4
Error	2752.0	19	144.8
Total	3195.8	23	-----

By this means of comparing variance, it is seen that the source of variation which has the largest mean square is "Machines." This indicates that the variation in the production process is caused for the most part by the source of variation that has been designated as "Machines." It is interesting to note that the variation caused by temperature change is responsible for only a very small part of the total variation.

"Machines," however, include two possible sources of variation, machines and head-coremen. By further use of the t and F tests, it may be possible to determine which of these factors predominates.

A sample of 30 pipe is chosen from each of the two machines. The pipe from each of the two machines was made under the direction of the relief head-coreman, so that any difference noted between the two samples shall be due to machine difference alone.

Machine 1		Machine 2	
440	464	464	440
432	472	464	458
460	448	462	454
448	446	442	462
436	472	460	460
468	476	460	470
454	430	458	474
470	464	460	470
448	460	434	470
450	464	446	458
464	456	458	450
476	454	438	432
482	454	446	454
478	464	462	444
476	454	442	430

$\Sigma X_1 = 1760$ $\quad\quad$ $\Sigma X_2 = 1622$

$\bar{X}_1 = 58.67$ $\quad\quad$ $\bar{X}_2 = 54.07$

$\Sigma X_1^2 = 108856$ $\quad\quad$ $\Sigma X_2^2 = 91908$

$\Sigma(X_1 - \bar{X}_1)^2 = 5602.7$ $\quad\quad$ $\Sigma(X_2 - \bar{X}_2)^2 = 42119$

Test of significant difference between variabilities:

$$F = \frac{\Sigma(X_1 - \bar{X}_1)^2}{\Sigma(X_2 - \bar{X}_2)^2} = \frac{5602.7}{4211.9}$$

$$= 1.33$$

The F Table value for | P = .05, 29 degrees of freedom, 29 degrees of freedom | is 1.84. As the calculated value is less than the table value, there is no significant difference between the variabilities of the two machines.

Test for significant difference between means:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$t = 1.369$

The t Table value for | P = .05, ∞ degrees of freedom | is 1.960. There is no significant difference between the means of the two machines.

In the following tests, the effect of any machine difference has been eliminated by taking two samples of 30 pipe from one machine with each of the two samples having been made by one of two head-coremen.

Head-coreman 1		Head-coreman 2	
440	464	446	440
432	472	470	438
460	448	472	440
448	446	440	410
436	472	454	424
468	476	446	426
454	430	442	434
470	464	460	436
448	460	442	440
450	464	458	432
464	456	470	442
476	454	452	436
482	454	464	470
478	464	460	442
476	454	448	424

$\Sigma X_1 = 1760$ | $\Sigma X_2 = 1358$

$\bar{X}_1 = 58.67$ | $\bar{X}_2 = 45.27$

$\Sigma X_1^2 = 108856$ | $\Sigma X_2^2 = 68300$

$\Sigma(X_1 - \bar{X}_1)^2 = 5602.7$ | $\Sigma(X_2 - \bar{X}_2)^2 = 6827.9$

Test for significant difference between variabilities:

$$F = \frac{\Sigma(X_2 - \bar{X}_2)^2}{\Sigma(X_1 - \bar{X}_1)^2} = \frac{6827.9}{5602.7}$$

$$= 1.219$$

The F Table value for | P = .05, 29 degrees of freedom, 29 degrees of freedom | is 1.86. The head-coremen do not differ significantly with respect to variability.

Test for significant difference between means:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= 3.545$$

The t Table value for | P = .05, ∞ degrees of freedom | is 1.960. The head-coremen do differ significantly in means.

The two preceding tests have given added meaning to the previous conclusion that machines and head-coremen were causing the major portion of total variation. It may now be stated that the samples taken from the various combinations of production features causing variation, indicate that the head-coremen are responsible to a greater degree for the variation in the final weight of 6" B&S 150 pipe than is any other single factor.

The negative information derived from the analysis of variance is of equal importance. Attention is again called to the slightness of effect that variation in the temperature of the molten metal has on the final weight of the pipe.

The tests used in arriving at the above conclusions are valid for the samples considered. However, due to the nature of production it is difficult during production to achieve adequate data of such a positive nature as to alleviate all suspicion of the final conclusions. To assure complete validity, the data should be expanded and each reading should be taken so that the conditions under which it occurred can be accurately described.

FINAL SUMMARY

The statistical investigation of the production of 6" B&S 150 pipe has disclosed the necessity for corrective action in order to exercise control over the quality of pipe production. The investigation has also indicated the logical sequence that any such corrective action should follow to control the quality at a desired level. The outline of this sequence is given below.

1. Establish statistical quality control: To bring production into a state of control at the present average of 462 pounds, the total variation must be reduced. The analysis of variance gave the relative magnitudes of the sources of variation that are listed below in descending effect.

   1. Head-coremen
   2. Machines
   3. Hydraulic pressures
   4. Temperatures

As the source of variation, head-coremen, is the predominant contributor to the total variation, it is logical to presume that control of this one source of variation will greatly reduce total production variation. During the course of this study, such a correction has been made. The amount of metal poured into a pipe bell is regulated by a timing device which indicates to the operator when he should begin the travel of the mold. This eliminates the effect of variation introduced by the head-coreman. Unfortunately, the timing device is not an integral part of the machine, but may be used at the discretion of the operator. It has been observed that this new control is disregarded more than it is used, with the result that the variation between head-coremen again becomes present. The value of the timing device will not be known until statistical analysis indicates that the improvement has effectively reduced total variability. This unproven control is certainly a step in the direction toward achieving statistical quality control. It is possible that the incorporation of this simple device will result in the control of quality within the present $\pm$ 3 sigma limits and even substantially reduce the standard deviation.

2. <u>Choose an optimum production average</u>: Only after a state of quality control has been reached should an attempt be made to lower the weight average. Distribution B discussed on page 51 and shown graphically in Figure 5, page 54, seems to describe at least approximately the optimum or aimed-at population of 6" B&S 150 pipe production. To gain statistical control under the conditions of Distribution B, the average weight ($\bar{\bar{X}}$) must be lowered from its present location of 462 pounds to 453 pounds and the standard deviation must be reduced from 10.16 to 8.51. It should not be difficult to lower $\bar{\bar{X}}$ by resetting the rate of machine ladle tilt, mold travel or mold rotation. Effecting a reduction in standard deviation may be more difficult. The timing device referred to above should reduce the standard deviation a certain degree. This reduction may not, however, be sufficient. If this is the case, then effort should be directed to reduce the variation caused by the machines, providing, of course, that the variation caused by the head-coremen has been reduced to a practical minimum.

Distribution B admittedly produces a certain percentage of pipe below the individual minimum weight of 437 pounds for 6" B&S 150 pipe. The light weight pipe then become 6" B&S 100 pipe. According to past production records the demand for 6" B&S 100 pipe was only 3 per cent of the demand for 6" B&S 150 pipe. The production of 6" B&S 100 pipe as a portion of 6" B&S 150 pipe results in producing in anticipation of demand and is likely to present a problem of storage. The production of 6" B&S 100 pipe in this manner would have several advantages. It does allow $\bar{\bar{X}}$ to be lowered to 453 pounds without appreciably reducing the standard deviation. (Distribution C describes the conditions under which $\bar{\bar{X}}$ would be at 453 pounds without producing any Class 100 pipe.) If 6" B&S 100 pipe were produced as the lower portion of the 6" B&S 150 pipe production, the setup expense for a short run to specifically supply a small lot of 6" B&S 100 would be eliminated. For these reasons, the author believes that the expense of storage would be considerably less than the savings to be realized by the adoption of Distribution B as the quality goal for the production of 6" B&S 150 pipe.

3. Maintain the control charts: The effect of any improvement or new assignable cause of variation can only be evaluated by noting its influence on the production average and standard deviation. This is one advantage of the continuous use of the control charts. Also when the time interval between actual production and the plotting of points on the control charts is small, it is very likely that variations on the control charts may definitely be correlated with actual conditions causing such variations.

It should be remembered that the control charts constructed in the first part of this paper to establish initial control limits do not necessarily represent the best practice of selecting subgroups during production. For example, a chart would probably be needed for each machine and in order to secure maximum variation between subgroups and minimum variation within subgroups some other sampling scheme besides the order of production might be applicable.

The advantages of statistical quality control illustrated in this paper will be greatly increased when the charts are used in conjunction with production.

This particular application of statistical quality control has been concerned exclusively with 6" B&S 150 pipe. The possibility of improving quality by weight reduction and the savings resulting from such weight reduction have been demonstrated. The methods of this investigation may be applied to other sizes and classes of pipe production thus duplicating the advantages set forth herein, and it is hoped that this initial study may serve as an introduction for the use of statistical quality control in foundry operations other than the production of Super-deLavaud Centrifugally Cast Iron Pipe.

A P P E N D I C E S

APPENDIX I

Glossary of Symbols

A_1 = a multiplier of $\bar{\sigma}$ to determine the distance from the central line to 3-sigma control limits on an $\bar{X}$ chart.

A_2 = a multiplier of $\bar{R}$ to determine the distance from the central line to 3-sigma control limits on an $\bar{X}$ chart.

D_3 = a multiplier of $\bar{R}$ to determine the 3-sigma lower control limit on a chart for R.

D_4 = a multiplier of $\bar{R}$ to determine the 3-sigma upper control limit on a chart for R.

LCL = lower control limit on a control chart.

n = number of pieces or observed values in any given sample or subgroup.

N = number of subgroups in a lot or sub-lot.

R = the range, the difference between the largest value and the smallest value.

$\bar{R}$ = the average of a set of ranges.

UCL = upper control limit on a control chart.

X = a number representing a value of some variable; in statistical quality control, X is usually the value of some quality characteristic.

$\bar{X}$ = (bar X), the average (arithmetic mean) of two or more X values.

$\bar{\bar{X}}$ = (double bar X), the average of $\bar{X}$ values, an estimate of $\bar{X}$.

$\bar{X}'$ = (bar X prime), the true universe average.

σ = (sigma), the standard deviation of a set of numbers, the root mean square deviation about the average.

$\bar{\sigma}$ = (bar sigma), the average of a set of σ values.

σ' = (sigma prime), the known or estimated value of universe standard deviation.

APPENDIX II

Bibliography

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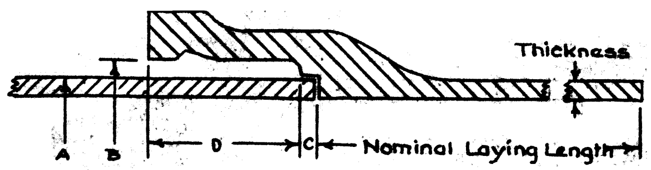
APPENDIX III

TABLE A --- Dimensions and Weights of Standard Bell and Spigot Super-deLavaud Pipe
Standard Water Bell



Nominal Diameter Inches	Class or Maximum Working Pressure	Dimensions in Inches				Average Thickness	Nominal Laying Length			
							12-Foot Length		18-Foot Length	
		A	B	C	D		Weight of Pipe Pounds	Weight Per Foot Including Bell	Weight of Pipe Pounds	Weight Per Foot Including Bell
3	150	3.80	4.60	.30	3.00	.33	140	11.8		
3	250	3.80	4.60	.30	3.00	.36	155	12.8		
4	150	4.80	5.60	.30	3.00	.34	195	16.4	285	15.9
4	250	4.80	5.60	.30	3.00	.38	220	18.4	325	17.9
6	150	6.90	7.70	.375	3.50	.37	315	26.3	460	25.5
6	250	6.90	7.70	.375	3.50	.43	350	29.3	515	28.5
8	150	9.05	9.85	.375	4.00	.42	475	39.4	690	38.3
8	200	9.05	9.85	.375	4.00	.46	510	42.4	745	41.3
8	250	9.05	9.85	.375	4.00	.50	545	45.5	800	44.3

TABLE A --- Dimensions and Weights of Standard Bell and Spigot Super-deLavaud Pipe
Standard Water Bell (Continued)

Nominal Diameter Inches	Class or Maximum Working Pressure	Dimensions in Inches				Average Thickness	Nominal Laying Length			
							12-Foot Length		18-Foot Length	
		A	B	C	D		Weight of Pipe Pounds	Weight Per Foot Including Bell	Weight of Pipe Pounds	Weight Per Foot Including Bell
10	150	11.10	11.90	.375	4.00	.47	640	53.3	935	51.8
10	200	11.10	11.90	.375	4.00	.52	700	58.3	1025	56.8
10	250	11.10	11.90	.375	4.00	.57	760	63.3	1115	61.9
12	150	13.20	14.00	.375	4.00	.50	810	67.4	1180	65.6
12	200	13.20	14.00	.375	4.00	.57	905	75.4	1325	73.7
12	250	13.20	14.00	.375	4.00	.62	990	82.5	1450	80.7

APPENDIX III (Continued)

TABLE B --- Factors for Computing Control Chart Lines, Small Samples*

Number of Observations in Sample, n	Chart for Averages		Chart for Standard Deviations	Chart for Ranges		
	Factors for Control Limits		Factor for Central Line	Factor for Central Line	Factors for Control Limits	
	A_1	A_2	c_2	d_2	D_3	D_4
2	3.759	1.880	0.5642	1.128	0	3.268
3	2.394	1.023	0.7236	1.693	0	2.574
4	1.880	0.729	0.7979	2.059	0	2.282
5	1.596	0.577	0.8407	2.326	0	2.114
6	1.410	0.483	0.8686	2.534	0	2.004
7	1.277	0.419	0.8882	2.704	0.076	1.924
8	1.175	0.373	0.9027	2.847	0.136	1.864
9	1.094	0.337	0.9139	2.970	0.184	1.816
10	1.028	0.308	0.9228	3.078	0.223	1.777
11	0.973	0.285	0.9300	3.173	0.259	1.744
12	0.925	0.266	0.9359	3.258	0.284	1.717
13	0.884	0.249	0.9410	3.336	0.308	1.692
14	0.848	0.235	0.9453	3.407	0.329	1.671
15	0.817	0.223	0.9490	3.472	0.348	1.652

* For complete table of factors for computing control chart lines see,

American War Standards Z1.1-1941 and Z1.2-1941, "Guide for Quality Control and Control Chart Method of Analyzing Data," American Standards Association, New York, 1941, p. 50.

APPENDIX III (Continued)

TABLE C ––– Ordinates of the Normal Distribution Curve

Values of $y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2}$ for Various Values of Z where $Z = \frac{X - \bar{X}}{\sigma}$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.398942	.398922	.398863	.398763	.398623	.398444	.398225	.397966	.397668	.397330
0.1	.396953	.396536	.396080	.395585	.395052	.394479	.393868	.393219	.392531	.391806
0.2	.391043	.390242	.389404	.388529	.387617	.386668	.385683	.384663	.383606	.382515
0.3	.381388	.380226	.379031	.377801	.376537	.375240	.373911	.372548	.371154	.369728
0.4	.368270	.366782	.365263	.363714	.362135	.360527	.358890	.357225	.355533	.353812
0.5	.352065	.350292	.348493	.346668	.344818	.342944	.341046	.339124	.337180	.335213
0.6	.333225	.331215	.329184	.327133	.325062	.322972	.320864	.318737	.316593	.314432
0.7	.312254	.310060	.307851	.305627	.303389	.301137	.298872	.296595	.294305	.292004
0.8	.289692	.287369	.285036	.282694	.280344	.277985	.275618	.273244	.270864	.268477
0.9	.266085	.263688	.261286	.258881	.256471	.254059	.251644	.249228	.246809	.244390
1.0	.241971	.239551	.237132	.234714	.232297	.229882	.227470	.225060	.222653	.220251
1.1	.217852	.215458	.213069	.210686	.208308	.205936	.203571	.201214	.198863	.196520
1.2	.194186	.191860	.189543	.187235	.184937	.182649	.180371	.178104	.175847	.173602
1.3	.171369	.169147	.166937	.164740	.162555	.160383	.158225	.156080	.153948	.151831
1.4	.149727	.147639	.145564	.143505	.141460	.139431	.137417	.135418	.133435	.131468
1.5	.129518	.127583	.125665	.123763	.121878	.120009	.118157	.116323	.114505	.112704
1.6	.110921	.109155	.107406	.105675	.103961	.102265	.100586	.098925	.097282	.095657
1.7	.094049	.092459	.090887	.089333	.087796	.086277	.084776	.083293	.081828	.080380
1.8	.078950	.077538	.076143	.074766	.073407	.072065	.070740	.069433	.068144	.066871
1.9	.065616	.064378	.063157	.061952	.060765	.059595	.058441	.057304	.056183	.055079
2.0	.053991	.052919	.051864	.050824	.049800	.048792	.047800	.046823	.045861	.044915
2.1	.043984	.043067	.042166	.041280	.040408	.039550	.038707	.037878	.037063	.036262
2.2	.035475	.034701	.033941	.033194	.032460	.031740	.031032	.030337	.029655	.028985

TABLE C —— Ordinates of the Normal Distribution Curve (Continued)

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.3	.028327	.027682	.027048	.026426	.025817	.025218	.024631	.024056	.023491	.022937
2.4	.022395	.021862	.021341	.020829	.020328	.019837	.019356	.018885	.018423	.017971
2.5	.017528	.017095	.016670	.016254	.015848	.015449	.015060	.014678	.014305	.013940
2.6	.013583	.013234	.012892	.012558	.012232	.011912	.011600	.011295	.010997	.010706
2.7	.010421	.010143	.009871	.009606	.009347	.009094	.008847	.008605	.008370	.008140
2.8	.007915	.007697	.007483	.007274	.007071	.006873	.006679	.006491	.006307	.006127
2.9	.005953	.005782	.005616	.005454	.005296	.005143	.004993	.004847	.004705	.004567
3.0	.004432	.004301	.004173	.004049	.003928	.003810	.003695	.003584	.003475	.003370
3.1	.003267	.003167	.003070	.002975	.002884	.002794	.002707	.002623	.002541	.002462
3.2	.002384	.002309	.002236	.002165	.002096	.002029	.001964	.001901	.001840	.001780
3.3	.001723	.001667	.001612	.001560	.001508	.001459	.001411	.001364	.001319	.001275
3.4	.001232	.001191	.001151	.001112	.001075	.001038	.001003	.000969	.000936	.000904
3.5	.000873	.000843	.000814	.000785	.000758	.000732	.000706	.000681	.000657	.000634
3.6	.000612	.000590	.000569	.000549	.000529	.000510	.000492	.000474	.000457	.000441
3.7	.000425	.000409	.000394	.000380	.000366	.000353	.000340	.000327	.000315	.000303
3.8	.000292	.000281	.000271	.000260	.000251	.000241	.000232	.000223	.000215	.000207
3.9	.000199	.000191	.000184	.000177	.000170	.000163	.000157	.000151	.000144	.000139
4.0	.000134	.000129	.000124	.000119	.000144	.000109	.000105	.000101	.000097	.000093
4.1	.000089	.000086	.000082	.000079	.000076	.000073	.000070	.000067	.000064	.000062
4.2	.000059	.000057	.000054	.000052	.000050	.000048	.000046	.000044	.000042	.000040
4.3	.000039	.000037	.000035	.000034	.000032	.000031	.000030	.000028	.000027	.000026
4.4	.000025	.000024	.000023	.000022	.000021	.000020	.000019	.000018	.000018	.000017
4.5	.000016	.000015	.000015	.000014	.000013	.000013	.000012	.000012	.000011	.000011
4.6	.000010	.000010	.000009	.000009	.000008	.000008	.000008	.000007	.000007	.000007
4.7	.000006	.000006	.000006	.000006	.000005	.000005	.000005	.000005	.000004	.000004
4.8	.000004	.000004	.000004	.000003	.000003	.000003	.000003	.000003	.000003	.000003
4.9	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
5.0	.000001	.000001	.000001	.000001	.000001	.000001	.000001	.000001	.000001	.000001
5.1	.000001	.000001	.000001	.000001	.000001	.000001	.000001	.000001	.000001	.000001
5.2	.000001	.000001								