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    FORCED CONVECTION HEAT TRANSFER FROM
    EXTENDED SURFACES WITH FINNED PINS
        by
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## I. INTRODUCTION

After the establishment of the fact from the works of Hsieh ${ }^{(1)}$ and Vatsaraj ${ }^{(2)}$ that extended surfaces in the form of finned pins can improve the rate of heat transfer by as much as 85 percent with an increase in effectiveness of 61 percent under free convection, it was decided to investigate the possibility of further increasing the heat transfer rate by increasing the film coefficient, since the film coefficient is the main controlling factor in the heat transfer performance of any heat transfer surface. This film coefficient can be increased by means of forced convection.

The object of this investigation consisted in finding the relation between the overall heat transfer coefficient (which depends on both conduction and convection processes through the finned pin) and the local individual film coefficient (which depends on the fluid, its velocity and other factors), optimizing the variables of the finned pin extended surfaces and determining the improvement in the rate of heat transfer.

The general procedure involved was as follows: a. Designing ductwork to get a uniform velocity profile at the test section and designing a device for changing the amount of air quantities so as to obtain the flow at different Reynolds numbers,
b. Optimizing the parameters of the finned pin for forced convection, and
c. Deriving an empirical eruation for the film coefficient in Nusselt's form for the finned pin.

## II. LITERATURE REVIEW

Ever since there was a need of a heat transfer surface or a heat exchanger in industry, heat transfer engineers have been trying to increase the rate of heat transfer with compact surfaces. For doing so they employed various types of fin geometries and extended surfaces. The growing demand for air-cooled heat exchangers gave an impetus to investigate analytically such surfaces which would require a minimum quantity of metal with compactness, be easy to manufacture, transfer more heat and have maximum efficiency. It was realized that all these qualities could not be combined in an assembly because one requirement would conflict with the others. Hence while designing such surfaces the most desirable requirements were selected over less preferable ones and the optimum qualities were used. The study led into extensive experimental and analytical work and various types of arrangements were designed. Some arrangements are:
(1) plate fin surfaces; (2) finned tube surfaces with circular and flattened cross section employing various shapes of fins;
(3) tube banks with circular and flattened cross sections having inside and outside flow; and (1) screen and sphere matrix surfaces. Xays and Ioncon ${ }^{(8)}$ have given the design and the flow friction data for 88 different arrangements.

Recently Professor Hsu, with his graduate students, Hsieh and Vatsaraj ${ }^{(22)}$, introduced a completely new type of extended
surfaces with finned pins. In this arrangement a few annular fins of rectangular cross section are installed equidistant on pins of small length. If banks of these finned pins are arranged and staggered on the heat transfer surface, a large increase in the rate of dissipation could be obtained with high effectiveness. In brief, the purpose is to increase the area of extendod surface without increasing the area of that part of the primary surface covered by the pins.

Jakob ${ }^{(11)}$ gave a good history of the derivation of the heat transfer equations for various fin configurations, which could be treated analytically. In all these treatments the heat transfer coefficient is assumed to be uniform or an average value about the extended surfaces is used although a great variation in the film coefficient is observed. Various other assumptions are also made in solving the differential equations. Harper and Brown ${ }^{(12)}$ first introduced the concent of effectiveness of extended surfaces which added another important point to be considered along with other considerations in the selection of any arrangement.

Nroith ${ }^{(13)}$ gave the mathematical equation for the heat transfer from a pin neglecting convection from the free end. Scheider ${ }^{(7)}$ gave a simplified equation for the heat transfer from a fin of rectangular cross section considering convection at the free end and used a correction length to take care of the end effect. Harper and prown (12) gave the equations for the efficiency and the effectiveness of the fins. Fsieh ${ }^{(1)}$
found that the combined finned pin would have characteristics related to pins as well as the fins and gave a mathematical equation in the combined form.

Avrami and Iittle ${ }^{(15)}$ found efficiencies of annular fins for different ratios of inner to outer radii of fins. They applied a two-dimensional analysis to a flat rectangular fin and found that a small height to thickness ratio and larger inner diameter of the fins would be more effective. Hsu (3) has narrated descriptively the derivation of various empirical equations for the forced convection inside small tubes and outside submerged bodies. Starner and MoManus (16) found the average heat transfer coefficient for four fin arrays, positioning their base at different angles, and dissipating the heat to the surrounding air. They found that similarly spaced parallel plates would give more heat transfer than vertical arrays or the plates at any other angle. Whey also found that the film coefficient would be reduced considerably by reducing three-dimensional flow into two-dimensional flow. Ward and Young ${ }^{(6)}$ studied the heat transfer and pressure drop of air flow across triangular pitch banks of finned tubing and gave the coefficients for different diameters of tubes, with different numbers of rows and columns.

Milding and Coogan (19) measured the leat transfer and pressure loss characteristics of internally finned tubes at different flow rates. Gardner and carnavos ${ }^{(18)}$ studied the
thermal contact resistance in finned tubing and found that contact pressure between the fin and the tube affects considerably the heat removal capability of finned tubing. They also gave a theoretical method for predicting the gap resistance between the tube and fin in terms of the fin and tube diameters.

Deissler and Loeffler ${ }^{(17)}$ observed the heat transfer and the flow friction characteristics of fluids flowing over surfaces at temperatures up to $2500^{\circ}$ and velocities up to a Mach number of 8. They employed Von Karman's expression for the eddy diffusivity in the outer portion of the boundary layer, with variable properties in the sublayer, to obtain the friction and heat transfer correlations for flow over a plate.

Theoclitus and Eckrich (20) gave an experimental technigue for determining the effectiveness of extended surfaces.

Appl and Hung ${ }^{(21)}$ utilized fins of restricted lencth with individually optimun profiles and prescribed minimum clearance between the fins. They gave equations for the number of fins to be used to provide the least veditht of fins for a given surface heat dissipation.
III. GHEORETICAL INVESTIGATION

## Nomenclature

$A=$ Area of pin, sq. ft. $=(\pi / 4) d^{2}$
$E=A$ parameter
$b=$ Thickness of fin, ft.

```
c
    d = Diameter of the pin
    D = Outside diameter of fin, ft.
De = Equivalent diameter of bundle of finned pins
\overline{a}}=\mathrm{ Density of air at ta
\mp@subsup{\lambda}{h}{}}=\mathrm{ lanometer reading in inches of water
d
    g = Acceloration of gravity
    H = Another parameter
    h = Average convective heat transfer coefficient,
        Btu/hr ft ' 
```

$I_{0}(x), I_{1}(x)=$ Zero and first order modified Ressel Function
of 'x', lst kind
$K_{o}(x), K_{1}(x)=$ Zero and first order nosifier fessel Function
of ' $x$ ', 2nd kind
$k=$ Thermal conductivity of finned pin metal, Dtu/hr ft ${ }^{\circ} \mathrm{F}$
$k_{a}=$ Thermal conductivity of air at $t_{a}$, Btu/hr ft $O_{F}$
$L=$ Length of pin, ft
$m=A$ variable $=\sqrt{4 h / k d}$
$N=A n o t h e r$ variable $=\sqrt{2 h / k b}$
$N_{N u}=$ Nusselt number $=h D_{e} / k_{a}$
$N_{P_{r}}=$ Prandtl number $=C_{p} \mu / k_{a}$
$N_{\text {Re }}=$ Reynolds number $=u a_{a} D_{e} / \mu$
$Q=$ Heat Transfer rate, Btu/ft ${ }^{2}$
$Q_{1}, Q_{2}, \Omega_{3}, \ldots, Q_{n}=$ Rate of heat transfer at the root of the pin with 1, 2, 3,....n fins spaced equidistant from each other.
$q=$ Rate of air flow, cu. ft. per sec.
$r_{1}=$ Radius of pin, ft. $=a / 2$
$r_{2}=$ Outside radius of fin, ft. $=\mathrm{D} / 2$
$S_{L}=$ Center to center longitudinal distance between consecutive pins in the direction of flow, ft.
$S_{T}=$ Center to center transverse distance between consecutive pins perpendicular to the flow, ft.
$s=$ Spacing of fins, ft.
$t_{a}=$ Temperature of atmosphere, $O_{F}$
$u=$ Velocity of air, ft/sec
$w=$ Density of water at $t_{a}$, $1 \mathrm{bm} / \mathrm{cu}$. ft.
$\theta_{0}=$ Temperature excess at the root of the fin, $\sigma_{r}$ $=t_{0}-t_{a}$
$\mu=$ Dynamic viscosity of air at $t_{a}, ~ l b m / h r, f t$.
Mathematical Equations for the Eeat Transfer From a Finned Pin
The forced convection equations for the heat transfer from extended surfaces, in the form of finned pins, can be derived from the free convection heat transfer equations for
the same configuration, given by Hsieh ${ }^{(1)}$.
The following assumptions are made in deriving these equations:
a. Heat concuction in the pin is one dimensional; i.e., temperature over any cross-section of the pin is uniform.
b. The temperature of the ambient air is constant.
c. The themal conductivity of the extended surface material is constant.
d. The annular fin is considered very thin so that heat loss through the onter edge to the ambient fluid is neglecter.
e. The heat transfer coefficient cver the surface of the firmed pin is considered constant.

Whe last assumption is a very rough approvimation in both cases, that is, in free as woll as in forcos convertion. But, in the absence of a rigorous theoretical procerure, a correction factor may be used with the onmirioal equations.

Let a single pin, having two fins on it, be held perpendicular to the direction of air flow, with an averaye film coefficient: "h' between the surface and the air. Por analytical purposes such an assembly may be divided into seven parts and numbered as shown in Figure 1. The suoscrinted quantities in this section are denoted such that the first subscript indicates the part being considered and the secone one indicates the value of ' $X$ ' as measured from the left end of that part. The fins are soldered or brazed on the pin.

The heat flux $Q_{7-0}$ from pin 7 as given by Kreith (13) is:


Figure 1 A Pin with Two Fins

$$
\begin{align*}
Q_{7-0} & =k A^{2} \theta_{7-0} \frac{\tanh m c+\mathrm{H}_{1}}{1+\mathrm{H}_{1} \tanh m \mathrm{~m}}  \tag{3-2-1}\\
& =k \operatorname{Am}_{7-0} \mathrm{~B}_{2}
\end{align*}
$$

where:

$$
\begin{align*}
\theta_{7-0} & =t_{7-0}-t_{a} \\
t_{7-0} & =\text { root temperature of pin } 7 . \\
m & =\sqrt{\frac{4 h}{k d}} \\
H_{1} & =\frac{h}{m k} \\
B_{2} & =\frac{\tanh m c+H_{1}}{I+H_{1} \tanh m c} \tag{3-2-1n}
\end{align*}
$$

The heat flux $0_{6-0}$ from fin 6 , given by schneider ${ }^{(7)}$ is:

$$
\begin{equation*}
Q_{6-0}=2 \pi r_{1} k \operatorname{tt} \theta_{6-0} \frac{\mathrm{~K}_{1}(\mathrm{Nr} 1) I_{1}\left(N r_{2}\right)-I_{1}\left(N r_{1}\right) \mathrm{K}_{1}\left(N r_{2}\right)}{\mathrm{K}_{0}\left(N r_{1}\right) I_{1}(\mathbb{N r})+I_{0}\left(N r_{1}\right) \mathrm{K}_{1}\left(N r_{2}\right)} \tag{3-2-2}
\end{equation*}
$$

$$
=B_{1} \theta_{6-0}
$$

where:

$$
\begin{aligned}
{ }^{\theta} 6-0 & =t_{6-0}-t_{a} \\
t_{6-0} & =\text { average root temperature of fin } 6 \\
N & =\sqrt{\frac{2 h}{\mathrm{kt}}} \\
I_{0} & =\text { Zero order modified Bessel function, first kind } \\
I_{1} & =\text { First order modified Dessel function, first kind } \\
K_{0} & =\text { Zero order modified Bessel function, second kind }
\end{aligned}
$$

$K_{1}=$ First order modified Bessel function, second kind

$$
\begin{equation*}
B_{1}=2 \pi r_{1} k N t \frac{K_{1}\left(N r_{1}\right) I_{1}\left(N r_{2}\right)-I_{1}\left(N r_{1}\right) K_{1}\left(N r_{2}\right)}{K_{0}\left(N r_{1}\right) I_{1}\left(N r_{2}\right)+I_{0}\left(N r_{1}\right) K_{1}\left(N r_{2}\right)} \tag{3-2-2A}
\end{equation*}
$$

$r_{1}$ and $r_{2}$ are the inner and the outer radii of the fin. The heat flux through pin 5 , if it were surrounded by a fluid, with film coefficient $h_{i}$, could be obtained by equating the heat flux through the end of pin 5 with the heat flux through the root of pin 7, thus

$$
Q_{5-t}=\Omega_{7-0}
$$

giving also

$$
\theta_{5-t}=\theta_{7-0}, \text { at } x=t
$$

and

$$
\begin{equation*}
\theta=\theta_{5-0} \text { at } x=0 \tag{3-2-3}
\end{equation*}
$$

The value of $h_{i}$ can be found from

$$
\begin{equation*}
Q_{6-0}=h_{i}(\pi d t) \theta_{6-0} \tag{3-2-4}
\end{equation*}
$$

or

$$
\begin{equation*}
h_{i}=\frac{B_{1}}{\pi d t} \tag{3-2-5}
\end{equation*}
$$

From the energy balance, the heat flux equation of pin 5 is given in the differential form

$$
\begin{equation*}
\frac{d^{2} \theta}{d x^{2}}-m_{i}^{2} \theta=0 \tag{3-2-6}
\end{equation*}
$$

where

$$
m_{i}=\sqrt{\frac{4 h_{i}}{k d}}
$$

Solving equation (3-2-6) with the help of the boundary conditions obtained in (3-2-3), gives

$$
\begin{equation*}
\frac{\theta}{\theta_{5-0}}=\frac{\cosh m_{i}(t-x)+H_{2} \sinh m_{i}(t-x)}{\cosh m_{i} t+n_{2} \sinh m_{i} t} \tag{3-2-7}
\end{equation*}
$$

where:

$$
\begin{equation*}
\mathrm{H}_{2}=\frac{\mathrm{m}}{\mathrm{~m}_{\mathrm{i}}} \mathrm{~B}_{2} \tag{3-2-8}
\end{equation*}
$$

In equation (3-2-7), substituting the first boundary condition from equation (3-2-3) i.e.,

$$
\text { at } x=t, \quad \theta=\theta_{5-t}=\theta_{7-0}
$$

the result is:

$$
\begin{aligned}
\theta_{7-0} & =\frac{\theta_{5-0}}{\cosh m_{i} t+H_{2} \sinh m_{i} t} \\
& =B_{3} \theta_{5-0}
\end{aligned}
$$

where:

$$
B_{3}=\frac{1}{\cosh m_{i} t+H_{2} \sinh m_{i} t}
$$

By differentiating equation (3-2-7) and putting $x=0$, we get:

$$
\left.\frac{d \theta}{d x}\right|_{x=0}=-m_{i} \theta_{5-0} \frac{\sinh m_{i} t+H_{2} \cosh m_{i} t}{\cosh m_{i} t+H_{2} \sinh m_{i} t}
$$

The heat flux through pin 5 can be written as:

$$
\begin{align*}
a_{5-0} & =-\left.k A \frac{d \theta}{d x}\right|_{x}=0 \\
& =k A m_{i} \theta_{5-0} \frac{\tanh m_{i} t+H_{2}}{1+H_{2} \tanh m_{i} t} \\
& =k A m_{i} \theta_{5-0} 0^{8} 4 \tag{3-2-9}
\end{align*}
$$

where:

$$
B_{4}=\frac{\tanh m_{i} t+H_{2}}{1+H_{2} \tanh m_{i} t}
$$

The heat flux at the end of pin $4, a_{4-b}$, at $x=b$ is found by equating it with the heat flux at root of pin $5, \Omega_{5-0}$
and the boundary conditions can be fixed

$$
\Omega_{4-b}=\Omega_{5-0}
$$

and

$$
\begin{equation*}
\theta_{4-b}=\theta_{5-0} \tag{3-2-10}
\end{equation*}
$$

where:
$\theta_{4-b}=$ The temperature difference at the end of pin 4 at $x=b$.

The differential equation for pin 4 is:

$$
\begin{equation*}
\Omega_{4-b}=-\left.k A \frac{d \theta}{d x}\right|_{x=b} \tag{3-2-11}
\end{equation*}
$$

Equating eq. (3-2-10) and eq. (3-2-11) and after rearranging, the result is:

$$
\begin{equation*}
\frac{d^{\theta} 4-b}{d x}=-m_{i} B_{4} \theta_{5-0} \tag{3-2-12}
\end{equation*}
$$

Acain from the energy halance, the dtrontial equation
for a differential element of a pin is.

$$
\frac{\mathrm{d}^{2} \theta}{\mathrm{dx}}-\mathrm{m}^{2} \theta=0
$$

The boundary conditions for pin 4 are:

$$
\begin{align*}
\text { at } x & =0 & \theta & =\theta_{4-0} \\
x & =b & \theta & =\theta_{4-b} \tag{3-2-13}
\end{align*}
$$

By using Eq. (3-2-10), the following solution of Eq. (3-2-12) is obtained:

$$
\begin{equation*}
\frac{\theta}{\theta_{4-0}}=\frac{\cosh m(b-x)+H_{3} \sinh m(b-x)}{\cosh m b+H_{3} \sinh m b} \tag{3-2-14}
\end{equation*}
$$

where:

$$
\mathrm{H}_{3}=\frac{\mathrm{m}_{i}}{m} \mathrm{~B}_{4}
$$

And the heat flux through the root of pin 4, $Q_{4-0}$, is:

$$
Q_{4-0}=\left.k A \frac{d \theta}{d x}\right|_{x=0}
$$

But $\left.\frac{d \theta}{d x}\right|_{x=0}$ is obtained by differentiating equation (3-2-14)
and putting $\mathrm{x}=0$, hence

$$
\begin{equation*}
\mathrm{Q}_{4-0}=\mathrm{kAm} \mathrm{\theta}_{4-0} \frac{\tanh m b+\mathrm{H}_{3}}{1+\mathrm{H}_{3} \tanh m b} \tag{3-2-15}
\end{equation*}
$$

The heat flux of pin $2,2_{2}-0$ is obtained in a similar manner thus:

$$
\begin{equation*}
\frac{\theta}{\theta_{2-0}}=\frac{\cosh m_{i}(t-x)+H_{4} \sinh m_{i}(t-x)}{\cosh m_{i} t+H_{4} \sinh m_{i} t} \tag{3-2-16}
\end{equation*}
$$

where:

$$
H_{4}=\frac{m}{m_{i}} B_{6}
$$

and

$$
B_{6}=\frac{\tanh m b+H_{3}}{1+H_{3} \tanh m b}
$$

Hence,

$$
\begin{equation*}
Q_{2-0}=\operatorname{kAm}_{i}{ }^{P} 8^{\theta_{2-0}} \tag{3-2-17}
\end{equation*}
$$

where:

$$
\mathrm{B}_{8}=\frac{\tanh m_{i} t+H_{4}}{1+H_{4} \tanh m_{i} t}
$$

The heat flux of pin $1, Q_{1-0}$, is analogous to equation (3-2-15) and can be written as:

$$
\begin{align*}
\Omega_{1-0} & =k A m \theta_{1-0} \frac{\tanh \operatorname{ma}+\mathrm{H}_{5}}{1+\mathrm{H}_{5} \tanh \operatorname{ma}} \\
& =k \operatorname{Am}_{1-0}{ }^{\mathrm{B}} 10 \tag{3-2-18}
\end{align*}
$$

where:

$$
\mathrm{B}_{10}=\frac{\tanh m a+H_{5}}{1+\mathrm{H}_{5} \tanh m a}
$$

and

$$
\mathrm{H}_{5}=\frac{\mathrm{m}_{i}}{\mathrm{~m}} \mathrm{~B}_{8}
$$

The equation of heat transfer for the finned pin with any number of fins can be found by generalizing the above equations. For doing so, it is assumed that all fins are equally spaced i.e., $a=b=c=s$. If $I$ is the length of pin and $n$
is the number of pins of $t$ thickness, then

$$
\begin{equation*}
s=\frac{L-n t}{n+1} \tag{3-2-19}
\end{equation*}
$$

Let

$$
\begin{equation*}
T=\tanh \mathrm{ms} \tag{3-2-20}
\end{equation*}
$$

and

$$
\begin{equation*}
T_{i}=\tanh m_{i} t \tag{3-2-21}
\end{equation*}
$$

where $m$ and $m_{i}$ are given by equations,

$$
\begin{align*}
m & =\sqrt{\frac{4 h}{k d}} q  \tag{3-2-22}\\
m_{i} & =\sqrt{\frac{4 h_{i}}{k d}} \tag{3-2-23}
\end{align*}
$$

and where

$$
\begin{equation*}
h_{i}=\frac{B_{1}}{\pi d t} \tag{3-2-24}
\end{equation*}
$$

$B_{1}$ and $B_{2}$ are given by equations $(3-2-2 A)$ and $(3-2-1 A)$ respectively.

In general, the heat transfer equation for the finned pin with $n$ number of fins can be written as:

$$
\begin{equation*}
Q_{\mathrm{n}}=k A m \theta_{0} \mathrm{P}_{\mathrm{p}} \tag{3-2-25}
\end{equation*}
$$

where $B_{p}$ is a coefficient parameter found for the finned pin having $n$ number of fins on it. In equation (3-2-18) $\mathrm{B}_{\mathrm{p}}$ was found to be $B_{10}$ for $n=2$.

Now the following equations with variable subscripts within the parentheses indicate the general equations with which parameter $D_{p}$ is found. The equations (3-2-26) to $(3-2-29)$ if repeated in sequence, from $n=1$ to $n=n$, will
give $B_{p}$ as $B^{B}(4 n+2)$. These equations are:

$$
\begin{align*}
&{ }^{H}(2 n)=\frac{m_{1}^{m}}{m_{i}}{ }^{B}(4 n-2)  \tag{3-2-26}\\
&{ }^{B}(4 n) \frac{T_{i}+H^{H}(2 n)}{1+T_{i}{ }^{H}(2 n)}  \tag{3-2-27}\\
&{ }^{H}(2 n+1)=\frac{m_{i}}{m}{ }^{B}(4 n)  \tag{3-2-28}\\
&{ }^{B}(3-2-27)  \tag{3-2-29}\\
&{ }^{B}(4 n+2)=\frac{T+H^{H}(2 n+1)}{1+T H^{H}(2 n+1)}
\end{align*}
$$

Thus, for example, for $n=8, B_{p}=B_{34}$ and will be found when values of the parameters $H$ and $B$ are found in sequence, starting from $n=1$ to $n=8$ in equations (3-2-26) to (3-2-29).

Application of the Mathematical Equation of Heat Transfer From a Finned Pin, to Free and Forced Convection Conditions.

In the section on Mathematical Equations for the Heat Transfer From a Finned Pin, the general equation for the heat transfer from a finned pin is given. This equation is applicable to natural convection if the value of the film coefficient $h$ is found for natural convection conditions. McAdams ${ }^{(9)}$ gave the form of the equations for the film coefficient $h$ for horizontal pipes in air at room temperature and at atmospheric temperatures for different ranges of Grashof
numbers $\mathrm{N}_{\mathrm{Gr}}$; for $\mathrm{N}_{\mathrm{Gr}}$ between $10^{3}$ and $10^{9}$.

$$
\begin{equation*}
h=0.27\left(\frac{\theta_{0}}{\mathrm{~d}}\right)^{1 / 4}, \text { Etu/hr } \mathrm{ft}^{2} \circ_{\mathrm{F}} \tag{3-3-1}
\end{equation*}
$$

and for $\mathrm{N}_{\mathrm{Gr}}$ between $10^{9}$ and $10^{12}$,

$$
\begin{equation*}
h=0.18\left(\theta_{0}\right)^{1 / 3} \mathrm{Btu} / \mathrm{hr} \mathrm{ft} \mathrm{o}_{\mathrm{F}} \tag{3-3-2}
\end{equation*}
$$

Where $\mathrm{N}_{\mathrm{Gr}}$ is given by

$$
\begin{aligned}
N_{G r} & =\frac{d^{3} \rho^{2} B G \theta_{O}}{2} \\
d & =\text { The diameter of pin (ft) } \\
\mu & =\text { The dynamic viscosity of fluid (lbm/hr ft) } \\
g & =\text { The gravitational acceleration (ft/hr }{ }^{2} \text { ) } \\
\rho & =\text { The mass density }\left(1 b_{m} / f^{3}\right) \\
\beta & =\text { The coefficient of volumetric expansion }
\end{aligned}
$$

and
$\theta_{0}=$ The temperature difference $\left({ }^{\circ} \mathrm{F}\right)$
Hsieh ${ }^{(1)}$ calculated the value of $h$ from equation (3-3-1) as $2.03 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2} \mathrm{O}_{\mathrm{F}}$ for his finned pin and used this value in calculating the heat transfer rates of the finned pin. The same formula for $h$ was used by Vatsaraj ${ }^{(2)}$, with various combinations of the dimensions of fins and pins, in calculating the heat transfer rates of the finned pin and getting the optimum dimensions. He used copper as the conducting metal and a fixed value of the temperature excess, $\theta_{0}$, of $100^{\circ} \mathrm{F}$ In order to determine, theoretically, the effect of the film coefficient $h$ on the final value of $Q$ for a finned pin,
values of $h$ are varied in the equations given in the section, Mathematical Equations for the Heat Transfer From a Einned Pin, keeping fixed values of $\theta_{0}, k, d, D$ and $t$. A graph of such calculated values is shown in Figure 2. From the curve it can be readily seen that at smaller values of the film coefficient, such as at $h=20 \mathrm{Btu} / \mathrm{ft}^{2} \mathrm{hr} \mathrm{O}_{\mathrm{F}}$, the increment in the heat transfer rate for a given increment in the film coefficient $h$ is more than the increments at higher values of $h$, such as at 150. Of course, the value of $\Omega$ goes on increasing with $h$, but the decreasing slope of the curve at higher values of $h$ shows that the effectiveness of extended surfaces drops very rapidly. For example, when the value of film coefficient $h$ is increased to ten times its natural convection value, the increase in the heat transfer rate is approximately four times; but if this value of $h$ were increased 50 times, the increase in the heat transfer is hardly ten times that obtained in free convection. Hence, for a small increase in $h$, substantial increase in the heat transfer can be obtained without appreciable loss of effectiveness. Before making any conclusions from this section it is necessary to determine the limitations of such a proposition and to optimize various parameters involved in the heat transfer from a finned pin. Optimization of the Parameters Involved in the Heat Transfer From a Finned Pin

In the equations for the heat transfer from a finned pin the following variables were involved:


Figure 2 Variation of Heat Transfer Rate with Film Coefficient. Computed at $\theta_{0}=1000 \mathrm{~F}$
h - film coefficient
d - diameter of pin
D - diameter of fin
$k$ - thermal conductivity of metal
$t$ - thickness of fin
1 - length of pin
n - number of fins
s - spacing of fins
The last variable can be eliminated by using the three variables mentioned imediately before it, $i . e ., t, 1$, and $n$. Hence there are seven variables which play a vital part in the heat transfer equations. It is very difficult to eliminate five variables in order to get an optimum value from the remaining two variables without ignoring the effect of any eliminated variables on the ultimate result.

A Fortran IV program was run on the IBM 7040 computer to compute the heat transfer with seven variables. Some of the most practicable values of these variable parameters were used to determine their effects on the heat transfer of a finned pin. The program is listed in Appendix A. From the results obtained, the heat transfer rates were plotted against the number of fins, for given lengths and diameters of the pins, and for given thicknesses and diameters of fins.
a. These rates are calculated for two different metals, copper and aluminum, having values of
thermal conductivity of 220 and $132 \mathrm{Btu} / \mathrm{hr}, \mathrm{ft}, \mathrm{O}_{\mathrm{F}}$ respectively. Figures $3(a)$ and $3(b)$ show the effects of thermal conductivities on the heat transfer for different ranges of the film coefficient $h$. It is noted that the heat transfer through copper is more than the heat transfer through aluminum, as was expected. The difference in the heat transfer rates is considerably higher at higher values of $h$, such as $50 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2} \mathrm{O}_{\mathrm{F}}$. Therefore, if forced convection with higher values of film coefficient is employed the material with higher thermal conductivity, that is, copper, should be preferred to obtain the most desirable effects. For the same reason, in this investigation the value of thermal conductivity used is $220 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}{ }^{{ }^{\circ} \mathrm{F} \text {. }}$
b. On Figures $3(a)$ and $3(b)$, different sets of graphs are drawn - one plotted for 0.01 inch thickness of the fin and the other for 0.020 inch. It is noted that by increasing the thickness of the fin, the heat transfer rate decreases, hence, as thin a fin as possible should be employed.

The following deductions are based on figures $4(a)$, $4(b), 4(c), 5(a), 5(b), 6(a)$, and $6(b)$.
c. With an increase in the length of pin there is always an increase in the rate of heat transfer.


Fig. 3(a) Optimization of thickness and thermal conductivity of fin metal $t=1 / 100^{\prime \prime}$ (Continuous lines showing $k=220 \mathrm{Btu} / \mathrm{ft} \mathrm{hr} \mathrm{oF}^{\mathrm{F}}$, discontinuous lines $k=132 \mathrm{Btu} / \mathrm{ft} \mathrm{hr}{ }^{\circ} \mathrm{F}$ )


Fig. 3(a) Optimization of thickness and thermal conductivity of fin metal $t=1 / 100^{\prime \prime}$
(Continuous lines showing $k=220 \mathrm{Btu} / \mathrm{ft} \mathrm{hr}$ of, discontinuous lines $k=132 \mathrm{Btu} / \mathrm{ft} \mathrm{hr}{ }^{\circ} \mathrm{F}$ )


Fig. 4(a) Optimization of fin diameters and lengths of pin of diameter $1 / 8^{\prime \prime}$
(Fin diameter $D=1 / 4^{\prime \prime}$ )


Fig. 4 (b) Optimization of fin diameters and lengths of pin of diameters 1/8"
(Fin diameter $D=1 / 2^{\prime \prime}$ )


Fig. 4(c) Optimization of fin diameters and lengths of pin of diameter 1/8"
(Fin diameter $D=3 / 4^{\prime \prime}$ )


Fig. 5(a) Optimization of fin diameters and lengths of pin of diameter 1/4"
(Fin diameter $D=1 / 2^{\prime \prime}$ )


Fig. 5(b) Optimization of fin diameters and lengths of pin of diameter 1/4"
(Fin diameter $D=3 / 4^{\prime \prime}$ )


Fig. 6(a) Optimization of fin diameters and lengths of pin of diameter 3/8"
(Fin diameter $D=1 / 2^{\prime \prime}$ )


Fig. 6(b) Optimization of fin diameters and lengths of pin of diameter 3/8"
(Fin diameter $D=3 / 4 "$ )

However, for a given number of fins, the increment in the rate of heat transfer with increase in the length diminishes gradually as the value of the film coefficient increases. Beyond a certain value of the film coefficient, increasing the length of pin reduces the rate of heat transfer.
d. With an increase in the diameter of the pin there is an increase in the rate of heat transfer of the finned pin.
e. With an increase in the outer diameter of the fin an increase in the rate of heat transfer of the finned pin is observed. However, when a fin of both larger outer and inner diameters is used, the increase in the heat transfer is not noticeable, especially for small values of $h$. At higher values of $h$, however, the increase in the rate of heat transfer is pronounced.
f. With an increase in the number of fins there is an increase in the rate of heat transfer of the finned pin. It is noticed that for a given value of the film coefficient $h$, increasing the number of fins, for a given length of pin, increases the rate of heat transfer of the finned pin. The increment in the heat transfer rate with an increase in number of fins reduces as the number increases.
g. The cumulative effect of increasing the length of the pin and the number of fins is that the rate of heat transfer of the finned pin will be increased. According to paragraph $c$, beyond a certain value of the film coefficient $h$, the rate of heat transfer decreases with an increase in the length of pin. The combined effect will be that the rate of heat transfer will decrease beyond a limiting value of the film coefficient $h$. The larger the number of fins the smaller will be the limiting value of film coefficient $h$. Hence, for a certain combination of the length of pin and the number of fins, the rate of heat transfer increases with the film coefficient $h$ only within the limiting value of film coefficient $h$.

From the last paragraph it can be seen that for maximum effectiveness, an optimum rate of heat transfer should be chosen within the limiting value of the film coefficient for the given extended surface.

We realized that by increasing the diameter of the pin, the weight of material used is increased in proportion to the square of the diameter, hence, the diameter of the pin should not be increased to a great degree.

In view of these factors, the dimensions of the extended surfaces for the experimental investigation were chosen as,
$d=3 / 8$ in.; $D=7 / 8 \mathrm{in}. ; \mathrm{n}=8$; and the optimum rate of heat transfer as $55 \mathrm{Btu} / \mathrm{hr}$ at the film coefficient $\mathrm{h}=10 \mathrm{Btu} / \mathrm{hr}$ $f t^{2} o_{F}$.

Dimensional Analysis for Convective Heat
Transfer Coefficient in Forced Convection
The derivation for the form of the equation for the film coefficient $h$ for forced convection over a finned pin is based on the dimensional analysis suggested by Hsu ${ }^{(3)}$ for forced convection inside tubes. In the present investigation, forced convection over extended surfaces in the finned pin enclosed in wind tunnel may be considered similar to forced convection in the small diameter tubes because of the absence of a free stream velocity and a free stream temperature due to the great obstruction of the pins. Hence the form of the empirical equation for the film coefficient will be similar to Nusselt's equation.

The functional relationship between the film coefficient $h$ and other factors can be expressed in the following form:

$$
\begin{equation*}
h=c u^{a} D^{b} \mu_{k} a_{p} m_{p}^{n} \tag{3-5-1}
\end{equation*}
$$

where:

```
    u = The velocity of fluid
    D = Diameter of pipe or the length of plate
    k = Thermal conductivity of the fluid
    c
    \mu= Dynamic viscosity of fluid
    p = Mass density of fluid
    C = A constant
```

The above quantities are expressed in the fundamental dimensions, i.e., L for length, $T$ for time, $M$ for mass, $\theta$ for temperature, and $H$ for energy, which can be replaced by (force) $x$ (distance) or

$$
\begin{equation*}
H=\left(M \frac{L}{T^{2}}\right) x(L)=M L^{2} T^{-2} \tag{3-5-2}
\end{equation*}
$$

Other quantities can be expressed as follows:
$u=L T^{-1}$
$D=L$
$\rho=M L^{-3}$
$c_{p}=H^{-1} \theta^{-1}=L^{2} T^{-2} \theta^{-1}$
$\mu=M L^{-1}-1$
$h=H L^{-2} T^{-1} \theta^{-1}=M T^{-3} \theta^{-1}$
$\mathrm{k}=\mathrm{HL}^{-1} \mathrm{~T}^{-1} \theta^{-1}=\mathrm{MLT}^{-3} \theta^{-1}$
By substituting these quantities in equation (3-5-3), the result is

$$
\begin{align*}
M T^{-3} \theta^{-1}= & \left(L^{a_{T}-a}\right)\left(L^{b}\right)\left(M^{c} L^{-c_{T}} T^{-c}\right)\left(M_{L} L_{T} d^{-3 d_{\theta}-d}\right) x \\
& \left(M^{m} L^{-3 m}\right)\left(L^{2 n_{T}} T^{-2 n_{\theta}}-n\right. \tag{3-5-4}
\end{align*}
$$

In order to balance equation (3-5-1) dimensionally, equation (3-5-4) should yield the following values of $a, b$, $c$ and $d$ in terms of $m$ and $n$.
$a=m$
$b=m-1$
$c=-m+n$
$\mathrm{d}=-\mathrm{n}+1$

With the above values of the exponents equation (3-5-1) becomes:

$$
\begin{equation*}
h=C u^{m} D^{m-1} \mu^{-m+n_{l}}{ }^{-n+1} \rho_{\rho}^{m} c_{\rho}^{n} \tag{3-5-6}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(\frac{h D}{k}\right)=c\left(\frac{u \rho D}{\mu}\right)^{m}\left(\frac{\left.c_{p \mu}\right)^{n}}{k}\right. \tag{3-5-7}
\end{equation*}
$$

or

$$
\begin{equation*}
N_{N_{u}}=C\left(N_{R e}\right)^{m}\left(N_{p_{r}}\right)^{n} \tag{3-5-8}
\end{equation*}
$$

If throughout the experiment $N_{P r}$ is kept constant, then:

$$
\begin{equation*}
N_{N u}=C_{1}\left(N_{R e}\right)^{n} \tag{3-5-9}
\end{equation*}
$$

For the finned pin bundles enclosed in a wind tunnel, equation (3-5-9) will be derived more appropriately by assuming a relationship of active factors involvec in a form suggested by ward and Young ${ }^{(6)}$ for finned tube arrays. These factors are:

$$
\begin{aligned}
& \text { D - Outer diameter of the fin } \\
& \text { d - Diameter of the pin } \\
& \text { s - Spacing of the fins } \\
& t \text { - Thickness of the fin } \\
& N_{\text {Pr }} \text { - Prandtl number }
\end{aligned}
$$

The relationtip
where $b, c, a_{1}$, and are arbitrary exponents and $A$ is a
constant. Values of these constants given by ward and young (6) for a seven array tube bundle, or more, are:

$$
\begin{align*}
A & =0.364 \\
b & =0.68 \\
c & =0.45 \\
d_{1} & =0.30 \\
e & =0.30 \tag{3-5-11}
\end{align*}
$$

If a correction factor $K_{1}$ along with constants given in equation (3-5-11) is used in equation (3-5-10), an empirical equation can be derived which would represent an equation for the finned pin arrays.

By using the value 0.74 for $N_{P_{r}}$ and actual values of $D, d, s$, and $t$ from the experiment, equation $(3-5-10)$ can be reduced to:

$$
\begin{equation*}
N_{N u}=K_{1}(.120)\left(N_{R e}\right) \cdot 684 \tag{3-5-12}
\end{equation*}
$$

Characteristic Length and Reynolds Number
for the Flow Across the Finned Pin Banks.
In calculating the rate of heat transfer for turbulent flow across the finned pin bundles, the concept of EQUIVALENT DIAMETER, or the characteristic length given by Hsu ${ }^{(3)}$, is used. While deriving an empirical equation in the previous section for forced convection over an extended surface in the finned pin form (Equation (3-5-9)) the diameter of the pipe, or the characteristic length was not defined. But the final equation did involve the Reynolds number, which
depended on the characteristic length or the equivalent diameter. The relationship for the Reynolds number is given by

$$
\begin{equation*}
N_{R e}=\frac{u_{m} D^{\rho}}{\mu} \tag{3-6-1}
\end{equation*}
$$

where:

$$
\begin{align*}
\rho= & \text { The mass density of the fluid } \\
\mu= & \text { The dynamic viscosity of the fluid } \\
u_{m}= & \text { The mean velocity, based on the cross-sectional } \\
& \text { area available for the flow of fluid in the bundles. } \\
D_{e}= & \text { The equivalent diameter of the finned pin bundle } \\
= & \frac{4 \text { (free cross-sectional area) }}{\text { wetted perimeter }} \tag{3-6-2}
\end{align*}
$$

The free cross-sectional area is given as:

$$
S_{L} S_{T}-\frac{\pi}{4} d^{2}
$$

Hence

$$
\begin{equation*}
D_{e}=\frac{4\left(S_{L} S_{T}-\frac{\pi}{4} a^{2}\right)}{\pi} \tag{3-6-3}
\end{equation*}
$$

where:
$S_{L}=$ Longitudinal center to center distance of pins.
$S_{T}=$ Transverse center to center distance of pins.
If $A_{f}$ is the free area available in $s q$. $f t .$, and $q$ is the quantity of fluid flowing in cu. ft./sec. then:

$$
\begin{equation*}
u_{m}=\frac{q}{A_{f}} \mathrm{ft} \cdot / \mathrm{sec} \tag{3-6-4}
\end{equation*}
$$

With these values known, the Reynolds number can be calculated by Equation (3-6-1).

## IV. EXPERIMENTAL TNVESTIGATION

## object of the Experiment

The object of this investigation was to find the heat removal capability of the finned pin at different flow rates of air.

After building the apparatus, the first step in the experiment was to find the rates of heat transfer from the finned pins, and from the plain pins at various Reynolds numbers. Next, it is desired to compare the heat transfer rates from finned pins with that obtained from the finned pins under natural convection conditions and with the heat transfer rates of the plain pins at different flow rates. Finally, it is desired to write an empirical equation for the film coefficient in Nusselt's form.

Apparatus and Test Eguipment
Design of the wind Tunnel and Measuring Equipment. The apparatus in this investigation consisted of a heat exchanger in the form of a heat box in which heat flow is restricted through two surfaces. The edges of the heat box were insulated by means of asbestos sheets. The extended surfaces consisted of finned pins, staggered in banks. The optimization of dimensions and the spacings of the fins and pins for this particular case was done by vatsaraj ${ }^{(2)}$.

Testing Fquipment. The test equipment for investigating the effect of forced convection on heat transfer through
extended surfaces with finned pins comprised of a wind tunnel designed to produce a uniform velocity profile immediately prior to the test section. Since the transfer of heat was from both sides of the heat box, it was considered necessary to provide separate flow sections for both surfaces, which was done by partitioning the sheet metal duct. An overall view of the equipment is given in Figure 7.

The entrance section of the duct was designed essentially in the shape of a diffuser. The optimization of the dimensions of the diffuser was according to empirically derived relations for optimum recovery suggested by Kline, Abbott, and Fox (3) for a straight-walled diffuser.

After correlating two-dimensional, conical, and annular geometries, Kline, Abbott, and Fox gave the following conditions of optima for the diffuser:
a. Minimum loss of total pressure for a given pressure rise.
b. Maximum pressure recovery for a given area ratio, regardless of length in the flow region.
c. Optimum recovery is to be ohtained for a given length in the direction of flow.
d. For the given inlet conditions, optimum recovery for any possible geometry is desired.

For this investigation the optimum recovery for a given length (see $c$ above) was considered since the length cannot be


Fig. $7 \underset{\text { velocities }}{\text { Experimental }}$ set-up for measurement of heat transfer at different air
increased beyond limits. The maximum actual pressure recovery, as a function of twice the diffuser-divergence angle will occur only when the rate of increase of losses becomes large, that is, after the average amount of stall in the diffuser begins to increase rapidly. Thus optimum recovery is obtained as angle is increased at constant ratio of length and entrance throat width. By keeping above ratio at about 5.5 and 20 as 11.4 , head loss due to dissipation is hardly 11 percent, actual pressure recovery is as high as 75 percent, and the pulsations are minimum, with the flow regime kept in the region of no appreciable stall. Thus the dimensions of the diffuser designed were 30 inches long, 3-3/8 inches by 5-1/2 inches at one end and $6-3 / 8$ inches by 5-1/2 inches at the other end.

Straighteners are provided in the form of egg crates. Width to length ratio for egg crates was kept $1: 6$ for most effective results. Two sets of screens were provided to finally ensure breaking away of any large eddies into uniform flow.

The quantity of air flow was measured at the suction side of the blower by means of a 3 inch standard nozzle. A micromanometer was used to measure the pressure differential. The length of straight pipe for suction was 10 d or 30 inches and pressure taps were located according to ASME specifications (Article 26, Flow Measurement, Part 5, April 1959).

The finned pin heat box assembly used was the same which
was made and used by vatsaraj ${ }^{(3)}$ for his experiment on optimizing the dimensions of finned pins for free convection. One-half-inch-thick asbestos sheets were used around the edges of the heater for insulation.

In order to control the quantity of flow a special multivane-type butterfly valve was fabricated in the Mechanical Engineering Department of VPI. Final check of velocity profile was done by means of a hot wire anemometer. As expected, the velocity profile was fairly uniform in both sections. Finally the temperatures at different sections of the heat exchanger were measured by means of copper constantan thermocouples and a strip chart recorder which read and recorded thermogenerated voltage in millivolts.

Construction of the Thermocouples. Forty-five thermocouples were installed at different locations of the testing section. Of these, 12 were located at the roots of pins, 18 at the roots of the fins, 10 at the end of pins, 4 at the exit section immediately after the heat box, and 1 at the fin for measuring the steady state temperatures. The arrangement and locations are shown in Figures 8, 9, 10, and 11.

Three connectors with 16 pins each were used to connect 16 thermocouples to the recorder, which facilitated the selectivity of measuring the temperatures of particular locations on the extended surfaces.

An errangement of numbering locations of which temperatures heve been noted in Aprendix B
(Figures in parentheses indicute thermocouples on the other side of the plate)


Fig. 8 Thermocouples on end of pins
(direction of flow being from right to left)

Fig. 9 Thermocouples on root of fins (direction of flow is perpendicular to the plene of paper)


Fig. 10 Thermocouples at the root of pins noted in Apendix $B$ (Direction of flow being from left to right. Figures in parentheses indiaste thermocouples on the other side of the plate.)


Fig. 11 Thermocouples measuring temreretures of outgoing air, noted in Appendix B
(Direction of $f$ low being per endicular to the plene of paper.)

The thermocouples at the roots of the pins were installed by silver solder. But the thermocouples at the roots of fins could not be soldered because the surface temperature of the pin could not be increased sufficiently due to excessive heat transfer. Hence, they were attached by drilling a small size hole, putting the thermocouple junction into it, and holding it firm by tapping around the hole with a punch. The thermocouple on the fin was attached by soft solder. For measuring the average temperatures at the roots of the pins both in finned pins and plain pins, the thermocouples were connected in parallel.

Calibration of the Instruments. The millivolt recorder was calibrated by means of a standard potentiometer. Voltages in $1,2,3,4$, and 5 millivolts were fed to each of the 16 junctions of the recorder by means of the standard potentiometer; and readings were recorded. If any of the 16 points showed deviation on the chart, correction adjustment was made.

A 3-inch standard nozzle with the standard ASME calibration curves was used as the diffuser.

Circuit Diagram for the Experiment. The circuit diagram for the test of the heater is shown in Figure 12. A current transformer was used to double the range of the wattmeter. A voltmeter and an ammeter were connected in parallel and in series, respectively, between the wattmeter and the heater.


This arrangemont gave the power input in watts free of losses in the watmeter. The watmeter facilitated in getting leat imputs inco desixed stens and in cautioning against orowloadig of tho hoatex.

## Test Procedure

The heat box was placed inside the wind tunnel and the power circuit was switched on. The blower which was also started, forced the air into the wind tunnel and through the finned pins on the heat box. For each reading several hours were required to reach the steady state conditions at the roots of the pins. The pin-root temperature noted on Table I of Appendix $B$ was determined by taking the average of the values of E.M.F. of the individual thermocouples numbered 34 to 45 shown in Figure 10 , while the pin-root temperature noted in Tables II and III of Appendix $B$ was found from the values of the E.M.F. of the averaging thermocouples. The rate of air flow was measured by the pressure differential indicated on the micromanometer.

First, the relationship of the Reynolds number and the root excess temperatures was determined at constant heat supply by varying the air flow by means of the baffle valve. Five to six readings were taken for each heat supply rate. The temperature readings of locations Nos. 1 to 45 , shown in Figures 8 to 11, are given in Table $I$ of Appendix B. The curves showing the relationship of the Reynolds numbers versus temperature excess are shown in Figure 13. Next, the root excess temperatures at different heat supply rates were measured by keeping the flow rate constant. A curve showing the heat transfer rates versus temperature excess at the


Figure 13. Curves Showing $\mathrm{N}_{\mathrm{Re}}$ vs. $\theta_{\mathrm{o}}$ Measured at Constant Heat Supply Rates
root of the pin is shown in Figure 14. The measuxed data are recorded in Table II of Appendix B.

## List of Apparatus

The following apparatus obtained from the Mechanical Engineering Department of Virginia Polytechnic Institute was used.

Ammeter: A.C. 0 to 10 Amps. Weston Instruments Model 1554177289.

Wattmeter: 0 to 1500 Watts: Maximum volts 400 , resistance ohms 22476, maximum amps. - 7.5; rated amps. - 5; by Daystrom Inc. Weston Instrument Div. Newark.

Voltmeter: Model no. 912 Weston. Range 3 to 300 volts and 3 to 1000 volts A.C. type.

Variac: Type W 20 MT 3 Variac Transformer manufactured by General Radio Co., Concord, Mass. Line voltage 120 50-50 cycles per second. Load 0 to 140 v. 20 amps.

Current Transformer: Weston current transformer Model 461 \#22278, type 5; Capacity 15 VA. Frequency 50 to 500 cps . Line voltage 120 .

Potentiometer Recorder: Model Mo. $153 \times 62$ - VI6- II-III - 23. Instrument no. 922805, Serial no. 5451; Range 0 to 5 MV; manufactured by Minneapolis-Honeywell Reg. Co. Brown Instrument Division, Philadelphia, Pa.

Hot Wire Anemometer: Range 0 to $20,000 \mathrm{fpm}$. Hastings Precision Airmeter. Model B-16A, Serial no. 135. Manufactured


Figure 14 Graph Showing $\theta$ o vs. Heat Transfer Rate of the Finned Pin at Constant ${ }^{\circ}$ Reynolds Numbers
by Hastings-Raydist Inc., Hampton, Va.
Micromanometer: Trimount Instrument Co., Chicago Ill. 8 - 6360 Serial. Reading $1 / 10,000$ of an inch precision. Nozzle: 3-inch Standard Aluminum Nozzle according to A.S.M.E. specifications.

## Motors and Blowers:

1. General Electric Co. A.C. Motors Model 5 KH 43 $A B$ 1060AX type $K H$, single phase, H.P. $-1,60 \mathrm{cps}$ Amps.-1.0; Volts - 115 V ; R.P.M. - 1725. No. TB 2465 with Blower manufactured by American Blower Co., Detroit, Michigan.
2. A.C. Motor Service B. Winding SPL-PH. Frame-151s H.P. 1/20; R.P.M. - 1725; Volts - 115; Amps. - 9; 60 cps.; Single phase; Serial no. 45031148, manufacturea by Johnson Fan \& Blower Corp. Chicago, Illinois.

## List of Materials

During this investigation the following materials were used:

Heat Transfer Box with Extended Surfaces. Made by Vatsaraj ${ }^{(2)}$, for which he used material as follows. Copper rods of $3 / 8$ inch diameter were used for making the pins of the finned pins. Copper foil of 0.01 inch thickness was used for making the fins. Copper plate of $1 / 4$ inch thickness was used for making the front and side plates of the heater
box. 1/8" thick asbestos sheet was used as insulation for supporting the heater wire. Nichrome wire with .652 ohms/ft. was used as heater resistance.

Insulation for the Sides of the Feater Box. $81 / 2^{\prime \prime} \mathrm{x}$ $11 / 2^{\prime \prime} \times 1 / 2^{\prime \prime}$ thick asbestos and $41 / 2^{\prime \prime} \times 11 / 2^{\prime \prime} \times 1 / 2^{\prime \prime}$ thick asbestos, two pieces of each were used.

Sheet Metal. \#20 gauge galvanized sheets were used for wind tunnel duct work.

Steel Pipe. $3^{\prime \prime}$ dia. $\times 3^{\prime}$ long steel pipe was used for the plenum chamber of the nozzle.

Thermocouple Wire. Matched 30 gauge copper constantan wires with plastic insulation and fiber glass coating were used.

## V. DISCUSSION

Discussion and Experimental Accuracy
The temperature data obtained for the roots of the pins showed variation from one row to the other and there was a variation from one column to the other. This temperature variation was highest when the temperature difference was greater and when the flow rate was low. The temperature of the first column was less than that of the last column. Similarly the temperature of the upper row was lower than the temperature of the bottom row.

The temperatures at the roots of the fins in one section of the wind tunnel differed from the temperatures at the similar roots of fins in the other section. This was due to unequal gap-resistances of pins in two different sections. Maximum variation observed was about five percent.

The data obtained from the power supply apparatus, the temperature measuring instrument and the flow measuring instruments employed in this investigation were reproducible.

The temperature near the testing equipment was found to be slightly higher than the room temperature. Maximum variation was observed when the heat input was at the maximum and the air flow at the minimum, showing that there was loss of heat through the insulation and the duct. Maximum rise in temperature of the surrounding area was observed at the last portion of the testing section and at the bottom side of it. The average heat loss, under the extreme conditions mentioned
above, was estimated to be $63.16 \mathrm{Btu} / \mathrm{hr}$ or 1.86 percent of the total input of one kilowatt.

Comparing Heat Transfers Through Extended Surfaces in the Form of Finned Pins by Forced Convection and

With Heat Pransfer by Pree Convection
The experiments for the relationship of the heat transfer rate and the root excess temperatures for the finned pins were conducted at four different flow rates, shown in Figure 14. However, the same relationship for any flow rates could be obtained for lower heat output ranges from Figure 13. Similar experiments for the plain pins were conducted and the relationship of the heat transfer rate and the root excess temperature was determined, which is shown in Pigure 15. The optimization of the variables of the plain pin was done by votsaraj ${ }^{(2)}$. The spacing of the pins was designed for free convection. Other variables such as length and diameter of the pin were determined for optimum heat transfer. Ferice the geometry of the plain pin optimized for free convection was used for forced convection. The primary surfaces of the plain pins ard the finned pins were equal. Therefore the same optinized plain pin was used for comparison of the heat transfer of the finned pin in forced convection.

The curves shown in Pigures 14 and 15 are again plotted in Figure 16 for comparison with the curves of similar relationship obtained by Vatsaraj ${ }^{(2)}$ for the plate, the plain pins, and the finned pins under free convection. Naturally, as

$\begin{aligned} & \text { Figure } 15 \text { Graph Showing } \theta \text { vs. Heat Transfer Rate of the Plain } \\ & \text { Pin at Constant Reynolds Number }\end{aligned}$


Figure 16 Comparison of the Heat Transfer Characteristics
expected the heat transfer rate by forced convection was much higher than that obtained by free convection for all root excess temperatures. The higher the Reynolds numbers the higher is the increase in heat transfer rate per unit temperature difference. The heat transfer rate for very high root excess temperatures under forced convection could not be obtained because the heat box was not designed for high heat input.

## Sample Calculations for the Reynolds Number and the Film Coefficient

For calculating the Reynolds number and the film coefficient, equations given in the section on Characteristic Length and Reynolds Number for the Flow Across the Finned Pin Banks are used. In equation (3-6-1), the product of the velocity and the density is replaced by the MASS VELOCITY $G_{\max }$, which is based on the minimum area.

$$
\begin{equation*}
G_{\max }=u_{m} \tag{5-3-1}
\end{equation*}
$$

Finned Pin Assembly. The measured dimensions for the finned pin banks are as follows:

$$
\begin{aligned}
& S_{\mathrm{L}}=3 / 8 \text { inch } \\
& S_{\mathrm{T}}=1-1 / 2 \text { inches }
\end{aligned}
$$

The center-to-center distance between the pins in adjacent transverse rows is $S_{L}$ ', which is related to s and $S_{T}$ by:

$$
S_{L}^{\prime}=\sqrt{S_{L}{ }^{2}+\frac{\left(S_{T}{ }^{2}\right)}{2}}
$$

or:
$S_{L}^{\prime}=0.83^{\circ}$
For calculating the mass velocity, the minimum-m cross-section as defined on page 324 by Knudsen and Katz ${ }^{(10)}$ is used.

Since $S_{T}$ is greater than $S_{L}$ ' hence, the width of minimum crossm-section of flow is ( $\left.S_{X_{s}}{ }^{\prime}-d\right)$
or

$$
\begin{aligned}
S_{L}^{\prime}-d & =0.838^{\prime \prime}-0.375^{\prime \prime} \\
& =0.463^{\prime \prime}
\end{aligned}
$$

The length of the minimum cross-section is $3-15 / 16^{*}$ and from Figure 9 it can be seen that there are five minimum cross-sections in the staggerec arrangement.

Hence, the minimum area

$$
\begin{aligned}
& =5 \times 0.463 \times 315 / 16 \times 1 / 144 \\
& =0.0632 \mathrm{sq} . \mathrm{ft} .
\end{aligned}
$$

Whe total rinimura area in both sections of the finned pin bundle is

$$
\begin{align*}
A_{f} & =2 \times 0.0632 \\
& =0.1264 \mathrm{sq} . \mathrm{ft} . \tag{5-3-2}
\end{align*}
$$

The equivalent aiameter $D e$ is calculated by the equation (3-6-3).

$$
\begin{align*}
D_{e} & =4 \times \frac{(3 / 8 \times 3 / 2)-\pi / 4(3 / 8)^{2}}{\pi \times 3 / 8 \times 12} \\
& =0.1275 \mathrm{ft} . \tag{5-3-3}
\end{align*}
$$

The reading of the micromanometer in inches of water $=0.2392^{\circ}$

The pressure differential $p$ at the nozzle due to the air flow is

$$
\begin{aligned}
& =(0.2392 \div 12) \times 62.2 \\
& =1.24 \mathrm{lb} / \mathrm{sq} . \mathrm{ft}
\end{aligned}
$$

The following properties of air are noted at $80^{\circ} \mathrm{F}$ :

$$
\begin{aligned}
& \rho=0.07394 \mathrm{Ib} / \mathrm{cu} / \mathrm{ft} \\
& \mathrm{k}=0.01499 \mathrm{Btu} / \mathrm{ft} \mathrm{hr} \mathrm{O}_{\mathrm{F}} \\
& \mu=1.2497 \times 10^{-5} \mathrm{Ib} / \mathrm{ft} / \mathrm{sec}
\end{aligned}
$$

The quantity of flow through $3^{\prime \prime}$ nozzle is given by

$$
\begin{aligned}
q & =-\frac{\pi}{4} \times(d)^{2} \frac{\sqrt{2 g \rho} \rho}{\rho} \\
& =\frac{\pi}{4} \times(0.25)^{2} \sqrt{\frac{2 \times 32.17 \times 1.24}{0.07394}} \\
& =1.61 \mathrm{cu} . \mathrm{ft} . / \mathrm{sec} .
\end{aligned}
$$

The area of cross section of the nozzle
$=\frac{\pi}{4} \times(3 / 12)^{2}$
$=\frac{\pi}{64} \mathrm{sq}$. ft.
hence, the velocity of air through the nozzle

$$
\begin{aligned}
& =\frac{\pi}{64} \div 1.610 \\
& =32.75 \mathrm{ft} / \mathrm{sec}
\end{aligned}
$$

The Reynolds number of air at the throat of the nozzle
$=\frac{32.75 \times 0.07394 \times 3}{1.2497 \times(10)^{-5} \times 12}$
$=48700$
The coefficient of discharge of air at a Reynolds number of 48700 is found from Reference (4) as 0.99.

Hence, the flow rate becomes

$$
\begin{aligned}
\mathrm{q} & =1.610 \times 0.99 \\
& =1.592 \mathrm{cu} . \mathrm{ft} . / \mathrm{sec}
\end{aligned}
$$

The mean velocity of flow in the wind tunnel across the finned pin

$$
\begin{align*}
u_{\mathrm{m}} & =1.592 \div 0.1264 \\
& =12.6 \mathrm{ft} / \mathrm{sec} \tag{5-3-4}
\end{align*}
$$

Hence, the mass velocity

$$
G_{\max }=12.6 \times 0.7394
$$

The Reynolds number $\mathrm{N}_{\text {pe }}$ is then given by

$$
\begin{align*}
\mathrm{N}_{R e} & =\frac{12.6 \times 0.07394 \times 0.1275}{1.2497 \times 10^{-5}} \\
& =9550 \tag{5-3--5}
\end{align*}
$$

and the Nusselt's number is calculated as

$$
\begin{aligned}
N_{N u} & =0.12 \times(9550)^{0.684} \\
& =64.8
\end{aligned}
$$

Therefore, the film coefficient is

$$
\begin{align*}
h & =\frac{64.8 \times 0.01499}{0.1275} \\
& =7.6 \mathrm{Btu} / \mathrm{ft}^{2} \mathrm{hr} \circ_{\mathrm{F}} \tag{5-3-6}
\end{align*}
$$

Plain Pin Assembly. The measured distances for the plain pin banks are as follows:

$$
S_{L}=3 / 8^{\prime \prime}
$$

$$
S_{T}=3 / 4^{\prime \prime}
$$

$$
S_{L}^{\prime}=0.531^{\prime \prime}
$$

Since $S_{T}$ is greater than $S_{L}$ ' the width of minimum crosssection of flow is ( $S_{L}$ ' $-d$ ).
or

$$
\begin{aligned}
S_{L^{\prime}}^{\prime}-d & =0.531^{\prime \prime}-0.1875^{\prime \prime} \\
& =0.3435^{\prime \prime}
\end{aligned}
$$

The length of the minimum cross-section is $3-15 / 16^{\prime \prime}$, and there are 11 minimum cross-sections in the staggered form.

Hence, the minimum area

$$
\begin{aligned}
& =11 \times 0.3435 \times 315 / 15 \times 1 / 144 \\
& =0.103 \mathrm{sq} . \mathrm{ft}
\end{aligned}
$$

The total minimum area in both sections of the plain pin bundle

$$
\begin{aligned}
& =2 \times 0.103 \\
& =0.206 \mathrm{sq} \cdot \mathrm{ft}
\end{aligned}
$$

The equivalent diameter $D_{e}$, is

$$
\begin{aligned}
& =\frac{\frac{3}{8} \times \frac{3}{4}-\frac{\pi}{4}\left(\frac{3}{16}\right)^{2}}{\times \frac{3}{16} \times 12} \\
& =0.1434 \mathrm{ft}
\end{aligned}
$$

The reading of the micromanometer in inches of water

$$
=0.2215^{\prime \prime}
$$

Hence, the pressure differential $D_{p}$ at the nozzle due to the air flow is

$$
\begin{aligned}
& =(0.2215 \div 12) \times 62.2 \\
& =1.148 \mathrm{lb} / \mathrm{sq} . \mathrm{ft} .
\end{aligned}
$$

The following properties of air are noted at $89.3^{\circ} \mathrm{F}$ :

$$
\begin{aligned}
& \rho=0.0726 \mathrm{lb} / \mathrm{cu} . \mathrm{ft} . \\
& \mathrm{k}=0.0152 \mathrm{Btu} / \mathrm{ft} \cdot \mathrm{hr} \cdot \mathrm{o}_{\mathrm{F}} \\
& \mu=1.266 \times 10^{-5} \mathrm{lb} / \mathrm{ft} . \mathrm{sec} .
\end{aligned}
$$

The quantity of air through the $3^{\prime \prime}$ nozzle is given by

$$
\begin{aligned}
q & =\frac{\pi}{4} \times(0.25)^{2} \times \sqrt{\frac{2 \times 32.17 \times 1.148}{0.0726}} \\
& =1.562 \mathrm{cu} . \mathrm{ft} . / \mathrm{sec}
\end{aligned}
$$

Since the area of cross-section of the nozzle $\frac{\pi}{64}$ sq. ft., the velocity of the air flow through the nozzle is

$$
\begin{aligned}
& =1.562 \div \frac{\pi}{64} \\
& =31.8 \mathrm{ft} / \mathrm{sec}
\end{aligned}
$$

The Reynolds number of air through the nozzle

$$
\begin{aligned}
& =\frac{31.8 \times 0.0726 \times 3}{1.266 \times 10^{-5} \times 12} \\
& =45600
\end{aligned}
$$

The coefficient of discharge for this flow rate is found as 0.99

Hence, the quantity of the air flow is

$$
\begin{aligned}
q & =1.562 \times 0.99 \\
& =1.548 \text { cu.ft.sec. }
\end{aligned}
$$

The mean velocity of air in the wind tunnel across the plain pin bundle

$$
\begin{aligned}
u_{\mathrm{m}} & =1.548 \div 0.206 \\
& =7.52 \mathrm{ft} / \mathrm{sec}
\end{aligned}
$$

Hence, the mass velocity

$$
G_{\max }=7.52 \times 0.0726
$$

and the Reynolds number is then given by

$$
\begin{aligned}
N_{R e} & =\frac{7.52 \times 0.0726 \times 0.1434}{1.266 \times 10^{-5}} \\
& =6190
\end{aligned}
$$

## Calculation of the coefficient in the

Empirical Equation for the Film Coefficient
In deriving the empirical equation for the forced convection film coefficient for the banks of the finned pins, it is assumed that there is no heat transfer due to radiation. The heat transfer rate for the Reynolds number 9550 from Figure 14 for $\theta_{0}=40^{\circ} \mathrm{F}$ is found to be $1495 \mathrm{ptu} / \mathrm{hr}$. The total number of the finned pins on the plate was 126. If " $y$ " is the heat transfer from one finned pin per degree Fahrenheit root temperature excess, then:

$$
126 x^{*} y \text { " } \times 40=1495
$$

or

$$
" y^{n}=0.296 \mathrm{Btu} / \mathrm{hr} O_{\mathrm{F}}
$$

Hence, for $100^{\circ} \mathrm{F}$ root excess temperature, the heat transfer rate is $29.6 \mathrm{Btu} / \mathrm{hr}$. From Figure 2, the value of the film coefficient corresponding to the heat transfer rate of 29.6 Btu/hr was found to be $4.39 \mathrm{Btu} / \mathrm{ft}^{2} \mathrm{hr} \mathrm{O}_{\mathrm{F}}$. But from equation $(5-3-6)$ the value of $h$ is $7.6 \mathrm{Btu} / \mathrm{ft}^{2} \mathrm{hr} \mathrm{O}_{\mathrm{F}}$. Therefore, the value of $K_{1}$ used in the equation $(3-5-12)$ is:

$$
\begin{aligned}
K_{1} & =4.39 / 7.6 \\
& =0.58
\end{aligned}
$$

Figure 17 shows two curves of Revnolds numbers versus the Nusselt numbers - one using the value of $K_{1}$ in the equation (3-5-12) as unity, and the other as 0.58.

## Effectiveness

The effectiveness of the forced convection heat transfer for the finned pins would be different for different Reynolds numbers. The effectiveness can again be defined in two ways, i.e., (a) with respect to the finned pin assembly for free convection and (b) with respect to the primary surface for free convection.

If $Q_{R}$ is the heat transfer for the finned pin assembly at any Reynolds numbers, $Q_{f}$ is the heat transfer of the finned pin assembly for free convection, and $Q_{p}$ is the heat transfer of the primary surface, i.e., of the plate for free convection, then the percentage effectiveness e,

$$
\begin{align*}
& e_{a}=\frac{Q_{R}-Q_{f}}{Q_{f}} \times 100  \tag{5-5-1}\\
& e_{b}=\frac{Q_{R}-Q_{p}}{Q_{p}} \times 100 \tag{5-5-2}
\end{align*}
$$



Figure 17 Reynolds Number versus Nusselt Number of the Finned Pin Banks.

The graph of the effectiveness for different Reynolds numbers is shown in Figure 18 where the effectiveness is plotted against the excess root temperature.


Figure 18 Effectiveness of the Finned Pin

## VI. CONCLUSION

It is seen that under similar flow conditions, the finned pin behaves better than the plain pin. As far as the heat removal capability with respect to the primary surface is concerned, at an average flow rate as used in the experiment, the finned pin is about 28 times more effective.
VII. SUMMARY

It has been shown that by increasing the film coefficient " $h$ " by means of forced convection the heat transfer rate of a finned pin can be considerably improved. For an average flow rate of air over the extended surfaces, throughout the investigation the increase in the effectiveness was as much as 4.60 times over the free convection process. However the effectiveness decreases with the increasing root excess temperatures. The heat loss through the insulation was about 1.86 percent, which will not impair the accuracy for engineering applications.

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## IX. EIBLIOGPAPIY

1. Hsieh, J.C., "Eleat Transfer from a Finned Pin to the Ambient Air: Thesis in Wechanical Ingineexing, Virginia Polytechnic Institute (1963).
2. Vatsaraj. P.C., "Optimization of the Desion of an Extenced Surface with Finned Pins", Thesis in Mechanical Fngineering, Virginia Polytechnic Institute (1964).
3. Hsu, S.T., "Incineering Heat Transfer, D. Van Mostrand Co. Inc., Princeton, N.J. (1963).
4. Supplement to A.S.M.F. Power Test Codes, Chapter 4, Flow Measurement part 5, heasurenent of Quantity of Materials PTC 19.5; 4-1959.
5. Kline, S.J.: Abbott, D.E.; Fox, R.W., "Optimum Design of Straight-walled Diffusers", Faper No. $58-\lambda-137$, A.S.M.F. Transactions, Journal of Basic Fingineering. V 81, Ser. D n3, Sert. 195\%, page 321.
6. Ward, D.J. and Young, E. K., "Yeat Franser anc Pressure Drop of Air in Forced Convection Across Triangular Pitch Eanks of Finnec Tubes", Chenical Ingineering Progress Symposium, Series no. 20. 1959, Vol. 55, page 37.
7. Schneider, D.S., "Conduction Heat Transfer", Second Printing, Adaison-Wesley PubIishing Co., Inc. (1957).
8. Kays, W.M. ard London, A.L., "Compact Heat Exchangers", MoGraw-Hill Book Co., Inc. (1958).
9. McAdams, Wi.I., "Heat Transmission", Third Edition, Mccraw Eill Bool Co., Inc. (1954).
10. Knudsen, J.G. and Katz, D. L. . "Fluid Nynamics and Heat Transfer , McGraw-Hill Book Co., Inc. (1958).
11. Jakob, M., Heat Mransfer", Voi. 1, John Wiley and Sons, Inc., New York, (1949).
12. Harper, W.B. and Brown, D.R., "Mathematical Equations for Heat Conduction in the Fins of Air Cooled Engines", NACA Report 158 (1922).
13. Krieth, F., "Principles of Heat Transfer", International Text Book Co., (1960).
14. Keller, H.H. and Somers, E.V., "Heat Transfer From An Annular fin of Constant Thickness", A.S.M.E. Transactions, Journal of Heat Transfer, May 1959, page 151.
15. Avrami, H. and Little, J.B., "Diffusion of Heat Through a Rectangular Bar and the Cooling and Insulating Effects of Fins", Journal of Applied Physics, Vol. 13, 1942, page 255.
16. Starner, K.E. and McManus, H.M., "An Experimental Investigation of Free-Convection Heat Transfer From Rectangular Fin Arrays", A.S.M.E. Transactions, Journal of Heat Transfer, Aug. 1963, page 273.
17. Deissler, R.G. and Loeffler, A.L. "Heat Transfer and Friction for Fluid Flowing Over Surfaces at High Temperatures and High Velocities", A.S.M.E. Transactions, Journal of Leat Transfer, Feb. 1959, page 89.
18. Gardner, K.A. and Carnavos, T.C., "Thermal-Contact Resistance in Finned Tubes", A.S.M.E. Transactions, Journal of Heat Transfer, Nov. 1960, page 279.
19. Hilding, W.E. and Coogan, C.H., "Heat Transfer and Pressure Loss in Internally Finned Tubes", Symposium on Air Cooled Heat Exchangers, A.S.M.E., August 1964, page 57.
20. Theoclitus, G., and Eckrich, T.L., "An Experimental Technique for Determining the Effectiveness of Extended Surfaces", Symposium on Air Cooled Heat Exchangers, A.S.M.E., August 1964, page 21.
21. Appl, F.C. and Hung, H.M., "Finned Surfaces With Optimum Fins of Restricted Length". Symposium on Air Cooled Heat Exchangers, A.S.M.E., August 1964, page 37.
22. Hsu, S.T., Hsieh, J.C., and Vatsaraj, B.C., A.S.M.F. Paper 65 - HT - 55, Eighth National Heat Transfer Conference, Los Angeles, August 8-11, 1965

## X. VITA

The author was born on August 1, 1938, in Sagar, India. After graduating from high school in 1953, he attended the Government College in Rewa, India. Then, joining Jabalpur University in the College of Engineering, he earned a B.S. degree in mechanical engineering with honors in June 1961. Before coming to the United States he worked for two years with the State Electricity Board as Assistant Engineer, managing a diesel and thermal power plant. In January 1964 he entered Virginia Polytechnic Institute, Department of Mechanical Engineering, to pursue a graduate program for the degree of Master of Science in mechanical engineering. He is a member of the American Society of Mechanical Engineers.
A. 5 Bhargave

## APPENDIX A

## FORTRAN PROGRAM FOR COMPUTING THE HEAT TRANSFER RATE

```
$IBFTC Dl
    2 READ (5,2000) DL,D,AL,TL,AK,HL
    2000 FORMAT (6E13.5)
    WRITE (6,2000)DL,D,AL,TL,AK,HL
    DIMENSION H(20),B(40)
    AM=SQRT (4.*HL/(AK*DL))
    CN=3QRT (2.*HL/ (AK*TLL))
    ANRl=CN*DL/2.
    ANR2=CN*D/2.
    H(1)=HL / (AM*AK)
    \=0.
    CALL BESSEL(ANRI,M,BJ,BY,BIO,BKO)
    M=1.
    CALL BESSEL (ANRI, M,BJ,BY,BII,BKI)
    CALL BESSEL (ANR2,M,BJ,BY,BI2,BK2)
2002 FORMAT(3E25.10,110
    B(1)=3.1415927*CN*AK*TL*DL*(BK1* B12-Bl1*BK2)
        (BKO*B12+B1O*BK2)
    BHI=B(1)/(3.1415927*DL*TL)
    BMI=SQRT (4.*BHI/(AK*DL))
    CH=BMI*TL
    TI=TANH (CH)
    DO 1 I=1,8
    AN=I
    S=(AL-AN*TL)/(AN+1.)
    T=TANH (AM*S)
    B(2)=(T+H(1))/(1.+T*H(1))
    H(2*I)=AM/BMI*B(4*I-2)
    B(4*I)=(TI+H(2*I))/(1.+TI*H(2*I))
    H(2*I+1)=BMI/AM*B(4*I)
    B(4*I+2)=(T+H(2*I+1))/(1.T*H(2*I+I))
    Q=AK*3.1415927/4.*DL**2*AM*100.*B(4*I+2)
    WRITE(6,>001) I,Q
2001 FORMAT(100X,12,E15.5)
    1 CONTINUE
    OOTO?
```

Measured Heat Transfer Data of the Finned Pins
at Constant Heat Supply Rate and Varying Air Flow
(Numbering of Locations Referred to Figure Numbers $8,9,10 \% 11$ )


| Sam. No. | n V | $O_{p}$ | Roots of the Pins (at the Entrance of Plow) |  |  |  |  |  |  | $\bigcirc$ | $\mathrm{my}^{8}$ | $0_{r}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | My | $4 \mathrm{O}_{\mathrm{F}}$ | My 5 | $\bigcirc_{\mathrm{p}}$ | vy | $\mathrm{O}_{\mathrm{F}}$ | My |  |  |  |
| 1 | 1.075 | 80.8 | 1.075 | 80.8 | 1.080 | 81.0 | 1.090 | 81.3 | 1.00 | 81.3 | 1.005 | 83.6 |
| 2 | 1.080 | 81.0 | 1.080 | 81.0 | 1.100 | 81.9 | 1.110 | 82.3 | 1.11 | 82.3 | 1.120 | 82.8 |
| 3 | 1.1 .30 | 83.2 | 1.130 | 83.2 | 1.135 | 83.5 | 1.140 | 83.6 | 1.15 | 84.1 | 1.150 | 94.1 |
| 4 | 1.170 | 85.0 | 1.170 | 85.0 | 1.180 | 85.0 | 1.185 | 85.2 | 1.10 | 85.4 | 1.200 | 86.4 |
| 5 | 1.210 | 86.7 | 1.210 | 86.7 | 1.230 | 87.6 | 1. 240 | 38.0 | 1.25 | 38.4 | 1.251 | 88.4 |
| 6 | 1.340 | 92.3 | 1.340 | 92.3 | 1.370 | 93.7 | 1.380 | 94.1 | 1.39 | 94.5 | 1.395 | 94.7 |
| 7 | 1.750 | 110.0 | 1.750 | 110.0 | 1.800 | 111.9 | 1.820 | 113.0 | 1.33 | 113.3 | 1.800 | 113.8 |
| 8 | 1.240 | 88.0 | 1.240 | 88.0 | 1.270 | 89.3 | 1.280 | 89.9 | 1.33 | 30.9 | 1.300 | 90.7 |
| 9 | 1.280 | 89.9 | 1.300 | 90.9 | 1.310 | 91.0 | 1.320 | 91.4 | 1.33 | 91.9 | 1.350 | 92.8 |
| 10 | 1.320 | 91.4 | 1.320 | 91.4 | 1.350 | 92.8 | 1.360 | 93.5 | 1.37 | 93.7 | 1.380 | 74.1 |
| 11 | 1.350 | 92.8 | 1.360 | 93.5 | 1.400 | 95.0 | 1.410 | 95.4 | 1.42 | 95.9 | 1.430 | 95.2 |
| 12 | 1.600 | 103.6 | 1.600 | 103.6 | 1.660 | 105.9 | 1.680 | 107.0 | 1.70 | 107.9 | 1.710 | 108.2 |
| 13 | 2.410 | 137.7 | 2.420 | 138.1 | 2.520 | 142.1 | 2.550 | 143.5 | 2.57 | 144.2 | 2.580 | 144.6 |
| 14 | 1.330 | 91.9 | 1.350 | 92.8 | 1.360 | 93.5 | 1.380 | 94.1 | 1.40 | 95.0 | 1.410 | 95.4 |
| 5 | 1.390 | 94.5 | 1.390 | 94.5 | 1.450 | 97.1 | 1.460 | 97.6 | 1.18 | 98.4 | 1.500 | 99.8 |
| 16 | 1.420 | 95.9 | 1.430 | 96.2 | 1.490 | 98.8 | 1.560 | 99.2 | 1.49 | 98.8 | 1.570 | 99.7 |
| 17 | 1.500 | 99.8 | 1.530 | 100.6 | 1.570 | 101.7 | 1.600 | 103.7 | 1.61 | 104.0 | 1.620 | 104.4 |
| 18 | 1.810 | 112.4 | 1.820 | 113.0 | 1.900 | 116.5 | 1.920 | 117.1 | 1.93 | 117.6 | 1.970 | 119.3 |
| 19 | 2.900 | 157.2 | 2.960 | 159.0 | 3.080 | 164.9 | 3.140 | 167.3 | 3.14 | 167.3 | 3.200 | 169.7 |
| 20 | 1.380 | 94.1 | 1.420 | 96.9 | 1.460 | 97.6 | 1.480 | 98.4 | 1.50 | 90.8 | 1.520 | 100.1 |
| 21 | 1.430 | 96.2 | 1.450 | 97.1 | 1.490 | 98.9 | 1.500 | 99.2 | 1.52 | 100.1 | 1.550 | 101.4 |
| 22 | 1.470 | 98.0 | 1.480 | 98.3 | 1.520 | 100.1 | 1.550 | 101.4 | 1.56 | 101.9 | 1.570 | 102.3 |
| 23 | 1.520 | 100.1 | 1.540 | 101.0 | 1.570 | 102.3 | 1.600 | 103.7 | 1.61 | 104.0 | 1.620 | 104.4 |
| 24 | 1.610 | 104.0 | 1.620 | 104.4 | 1.700 | 107.9 | 1.730 | 109.1 | 1.75 | 110.0 | 1.770 | 110.7 |
| 25 | 2.070 | 123.5 | 2.170 | 127.7 | 2.240 | 130.6 | 2.260 | 131.4 | 2.28 | 132.2 | 2.300 | 133.1 |
| 26 | 3.000 | 1.61 .7 | 3.150 | 167.7 | 3.160 | 168.1 | 3.209 | 169.3 | 3.24 | 171.3 | 3.300 | 173.7 |


| Sam. No. | 9 |  | Roots of the Fins (At the Entrance of Flow) |  |  |  |  |  |  | 3 | 14 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MV | ${ }^{\circ} \mathrm{F}$ | MV | ${ }^{\circ} \mathrm{F}$ | MV | ${ }^{\circ} \mathrm{F}$ | MV | $O_{F}$ | MV |  | MV | $4 \mathrm{O}_{\mathrm{F}}$ |
| 1 | 1.04 | 79.8 | 1.04 | 79.8 | 1.050 | 79.6 | 1.055 | 79.9 | 1.055 | 79.9 | 1.055 | 79.9 |
| 2 | 1.07 | 80.6 | 1.07 | 80.6 | 1.075 | 80.8 | 1.080 | 81.0 | 1.090 | 81.4 | 1.095 | 81.6 |
| 3 | 1.11 | 82.3 | 1.11 | 82.3 | 1.120 | 82.8 | 1.120 | 82.8 | 1.130 | 83.2 | 1.135 | 83.5 |
| 4 | 1.17 | 85.0 | 1.17 | 85.0 | 1.170 | 85.0 | 1.175 | 85.2 | 1.180 | 85.5 | 1.185 | 85.6 |
| 5 | 1.21 | 86.7 | 1.21 | 86.7 | 1.220 | 87.1 | 1.220 | 87.1 | 1.230 | 87.6 | 1.230 | 87.6 |
| 6 | 1.33 | 91.9 | 1.33 | 91.9 | 1.335 | 92.1 | 1.335 | 92.1 | 1.340 | 92.3 | 1.345 | 92.2 |
| 7 | 1.73 | 109.1 | 1.73 | 109.1 | 1.740 | 109.5 | 1.750 | 110.0 | 1.750 | 110.0 | 1.750 | 110.0 |
| 8 | 1.20 | 86.3 | 1.21 | 86.7 | 1.220 | 89.1 | 1.220 | 87.1 | 1.230 | 87.6 | 1.270 | 89.3 |
| 9 | 1.25 | 88.4 | 1.25 | 88.4 | 1.260 | 88.9 | 1.270 | 85.3 | 1.280 | 89.8 | 1.300 | 90.9 |
| 10 | 1.27 | 89.3 | 1.28 | 89.8 | 1.290 | 90.2 | 1.300 | 90.7 | 1.310 | 91.0 | 1.320 | 91.4 |
| 11 | 1.30 | 90.9 | 1.31 | 91.0 | 1.320 | 91.4 | 1.330 | 91.9 | 1.340 | 92.3 | 1.350 | 92.8 |
| 12 | 1.55 | 101.4 | 1.56 | 101.9 | 1.570 | 102.3 | 1.580 | 102.7 | 1.590 | 103.1 | 1.600 | 103.4 |
| 13 | 2.28 | 132.2 | 2.29 | 132.7 | 2.300 | 133.0 | 2.350 | 135.2 | 2.360 | 135.6 | 1.450 | 139.3 |
| 14 | 1.30 | 90.9 | 1.31 | 91.0 | 1.320 | 91.4 | 1.330 | 91.9 | 1.350 | 92.8 | 1.360 | 93.5 |
| 15 | 1.35 | 92.8 | 1.36 | 93.5 | 1. 360 | 93.5 | 1.370 | 93.8 | 1.420 | 95.8 | 1.450 | 97.1 |
| 16 | 1.37 | 93.8 | 1.38 | 94.0 | 1.390 | 94.5 | 1.400 | 95.0 | 1.420 | 95.8 | 1.460 | 97.4 |
| 17 | 1.42 | 95.8 | 1.44 | 96.0 | 1.450 | 97.1 | 1.460 | 97.4 | 1.490 | 98.8 | 1.500 | 99.2 |
| 18 | 1.69 | 107.4 | 1.72 | 108.7 | 1.730 | 105.1 | 1.750 | 110.0 | 1.770 | 110.9 | 1.780 | 111.3 |
| 19 | 2.76 | 152.0 | 1.79 | 153.2 | 2.800 | 153.7 | 2.830 | 154.4 | 2.860 | 136.0 | 2.890 | 157.6 |
| 20 | 1.35 | 92.8 | 1.36 | 93.2 | 1.370 | 93.7 | 1.440 | 95.0 | 1.410 | 95.4 | 1.450 | 97.1 |
| 21 | 1.40 | 95.0 | 1.41 | 95.4 | 1.400 | 95.0 | 1.400 | 96.3 | 1.450 | 97.0 | 1.480 | 98.3 |
| 22 | 1.43 | 96.2 | 1.44 | 96.3 | 1.450 | 97.1 | 1.450 | 97.1 | 1.480 | 98.3 | 1.520 | 100.1 |
| 23 | 1.47 | 98.0 | 1.48 | 98.3 | 1.480 | 98.3 | 1.490 | 98.4 | 1.520 | 100.1 | 1.540 | 101.0 |
| 24 | 1.45 | 101.0 | 1.55 | 101.4 | 1.580 | 101.9 | 1.580 | 102.7 | 1.590 | 103.1 | 1.680 | 107.0 |
| 25 | 1.95 | 118.4 | 1.96 | 118.9 | 1.970 | 119.3 | 2.020 | 121.4 | 2.080 | 123.9 | 2.100 | 124.7 |
| 26 | 2.68 | 148.7 | 2.72 | 150.4 | 2.780 | 152.8 | 2.950 | 159.7 | 2.980 | 160.9 | 3.100 | 165.7 |

```
TABLE I (Continued)
```

| $\begin{aligned} & \text { Sam. } \\ & \text { No. } \end{aligned}$ | Roots of the Fins <br> (At the Entrance of Flow) |  |  |  | Poots of the pins (at the Fxit) |  |  |  | Surface of the Fin |  | Outgoing Air |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.060 | 80.1 | 1.07 | 80.6 | 1.18 | 85.4 | 1.18 | 85.4 | ]. 05 | 79.7 | 1.090 | 81.4 |
| 2 | 1.100 | 81.9 | 1.11. | 82.3 | 1.20 | 86.3 | 1.20 | 96.3 | 1.06 | 80.1 | 1.100 | 81.9 |
| 3 | 1.140 | 83.6 | 1.15 | 84.1 | 1.25 | 88.4 | 1.25 | 88.4 | 1.10 | 81.9 | 1.140 | 83.6 |
| 4 | 1.186 | 85.8 | 1.19 | 85.9 | 1. 32 | 91.4 | 1.30 | 90.6 | 1.15 | 84.0 | 1.190 | 85.9 |
| 5 | 1.240 | 88.0 | 1.25 | 89.4 | 1.39 | 94.5 | 1.38 | 94.0 | 1.19 | 85.9 | 1.280 | 89.8 |
| 6 | 1.350 | 92.8 | 1.35 | 92.8 | 1. 62 | 105.9 | 1.59 | 103.1 | 1.30 | 90.6 | 1.470 | 58.9 |
| 7 | 1.760 | 110.3 | 1.78 | 111.3 | 2.20 | 129.0 | 2.10 | 124.7 | 1. 70 | 107.9 | 2.030 | 121.8 |
| 8 | 1.280 | 89.5 | 1.30 | 90.9 | 1.47 | 98.0 | 1.45 | 97.1 | 1. 21 | 86.7 | 1.300 | 90.6 |
| 9 | 1.310 | 91.0 | 1.32 | 91.4 | 1.56 | 101.9 | 1.53 | 100.7 | 1.26 | 88.9 | 1.370 | 93.8 |
| 10 | 1.330 | 91.9 | 1.34 | 92.3 | 1.62 | 1.04 .4 | 1.59 | 103.1 | 1.29 | 90.2 | 1.420 | 95.8 |
| 11 | 1.380 | 94.0 | 1.40 | 95.0 | 1.72 | 108.6 | 1.68 | 107.0 | 1.33 | 91.9 | 1.560 | 101.9 |
| 12 | 1.610 | 104.0 | 1. 65 | 105.7 | 2.13 | 126.0 | 2.08 | 123.9 | 1.59 | 102.3 | 1.910 | 116.9 |
| 13 | 2.460 | 139.7 | 2.49 | 141.0 | 3.35 | 175.7 | 3.22 | 170.5 | 2.32 | 134.0 | 2.780 | 152.8 |
| 14 | 1.380 | 94.1 | 1.40 | 95.0 | 1.67 | 106.7 | 1. 64 | 105.3 | 2.29 | 89.8 | 1.425 | 96.0 |
| 15 | 1.480 | 98.9 | 1.50 | 99.3 | 1. 80 | 112.1 | 1.76 | 110.3 | 7. 35 | 92.8 | 1.500 | 99.2 |
| 16 | 1.490 | 98.8 | 1. 51 | 99.8 | 1.82 | 113.0 | 1.78 | 111.2 | 1.35 | 92.8 | 1.550 | 101.4 |
| 17 | 1. 550 | 101.4 | 1.60 | 103.7 | 2.01 | 121.0 | 1.96 | 118.0 | 1. 4.5 | 97.1 | 1.700 | 107.9 |
| 18 | 1.800 | 112.0 | 1.90 | 116.3 | 2.59 | 145.0 | 2.51 | 141.8 | 1.78 | 110.3 | 2.300 | 133.0 |
| 19 | 3.000 | 161.7 | 3.10 | 165.8 | 4.29 | 212.6 | 4.18 | 208.2 | 2.86 | 156.0 | 3.370 | 176.5 |
| 20 | 1.460 | 57.6 | 1.50 | 99.8 | 1.83 | 113.4 | 1.79 | 111.7 | 1. 32 | 91.4 | 1. 520 | 100.1 |
| 21 | 1.500 | 99.3 | 1.53 | 100.6 | 1.90 | 116.5 | 1.97 | 115.0 | 1.36 | 93.5 | 1.580 | 102.7 |
| 22 | 1.530 | 100.6 | 1.55 | 101.4 | 1.96 | 118.9 | 1.92 | 117.1 | 1.37 | 93.8 | 1.610 | 104.0 |
| 23 | 1.590 | 103.1. | 1.60 | 103.7 | 2.00 | 120.6 | 1.98 | 119.7 | 1.77 | 110.7 | 1.770 | 110.7 |
| 24 | 1.700 | 107.8 | 1.72 | 108.3 | 2.21 | 129.3 | 2.18 | 128.0 | 1.94 | 118.0 | 1.950 | 118.4 |
| 25 | 2.180 | 123.0 | 2.20 | 12.8 .0 | 3.01 | 162.1 | 2.99 | 161.3 | 2.54 | 143.0 | 2.650 | 147.6 |
| 26 | 3.150 | 167.7 | 3.20 | 169.3 | 4.30 | 212.9 | 4.28 | 212.5 | 3.76 | 191.9 | 3.650 | 187.6 |


| Sam. No. | MV ${ }^{21}$ | $O_{F}$ | Outgoing $\text { My } 22$ | $\begin{gathered} \text { Air } \\ O_{F} \\ \hline \end{gathered}$ | MV 23 | ${ }^{\circ} \mathrm{F}$ | M ${ }^{24}$ | ${ }^{\circ}$ | Fnds of $M^{25}$ | f the ${ }^{\circ}{ }_{F}$ | $\begin{aligned} & \text { Pins } \\ & 26 \end{aligned}$ | ${ }^{\circ} \mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.09 | 81.4 | 1.09 | 81.4 | 1.090 | 81.4 | 1.05 | 79.7 | 1.05 | 79.7 | 1.060 | 80.1 |
| 2 | 1.57 | 102.3 | 1.10 | 81.9 | 1.570 | 102.3 | 1.07 | 80.5 | 1.07 | 80.6 | 1.080 | 81.0 |
| 3 | 1.12 | 83.8 | 1.14 | 83.6 | 1.120 | 82.8 | 1.12 | 82.8 | 1.12 | 82.8 | 1.125 | 83.0 |
| 4 | 1.17 | 85.0 | 1.18 | 85.4 | 1.160 | 84.4 | 1.15 | 84.0 | 1.15 | 84.0 | 1.170 | 85.0 |
| 5 | 1.24 | 88.0 | 1.24 | 88.0 | 1.220 | 87.1 | 1.19 | 85.9 | 1.19 | 85.9 | 1.210 | 86.7 |
| 6 | 1.40 | 95.0 | 1.40 | 95.0 | 1.370 | 93.8 | 1.30 | 90.6 | 1.30 | 90.6 | 1.350 | 92.8 |
| 7 | 1.70 | 107.9 | 1.74 | 109.5 | 1.770 | 110.7 | 1.57 | 102.3 | 1.58 | 102.7 | 1.650 | 105.7 |
| 8 | 1.28 | 89.8 | 1.28 | 89.8 | 1.310 | 91.0 | 1.21 | 86.7 | 1.21 | 86.7 | 1.230 | 87.6 |
| 9 | 1.33 | 91.9 | 1.33 | 91.9 | 1.370 | 93.8 | 1.25 | 88.4 | 1.25 | 88.4 | 1.280 | 89.8 |
| 10 | 1.37 | 93.8 | 1.37 | 93.8 | 1.420 | 95.8 | 1.28 | 89.8 | 1.28 | 84.8 | 1.320 | 91.5 |
| 11 | 1.44 | 96.6 | 1.44 | 96.6 | 1.450 | 97.1 | 1.32 | 91.4 | 1.32 | 91.4 | 1.370 | 93.8 |
| 12 | 1.73 | 109.1 | 1.86 | 114.6 | 1.780 | 111.2 | 1.54 | 101.0 | 1.55 | 101.4 | 1.620 | 104.4 |
| 13 | 2.55 | 143.5 | 2.37 | 136.0 | 2.580 | 144.6 | 2.07 | 123.5 | 2.12 | 125.5 | 2.250 | 131.0 |
| 14 | 1.37 | 93.8 | 1.37 | 93.8 | 1.425 | 96.0 | 1.28 | 84.8 | 1.28 | 89.8 | 1.320 | 91.4 |
| 15 | 1.47 | 98.0 | 1.47 | 98.0 | 1.510 | 99.8 | 1.34 | 92.3 | 1.35 | 92.8 | 1.390 | 94.5 |
| 16 | 1.48 | 98.4 | 1.48 | 98.4 | 1.550 | 101.4 | 1.34 | 92.3 | 1.35 | 92.8 | 1.400 | 95.0 |
| 17 | 1.64 | 105.3 | 1.61 | 104.0 | 1.700 | 107.9 | 1.44 | 96.6 | 1.45 | 97.1 | 1.520 | 100.1 |
| 18 | 2.17 | 127.6 | 2.01 | 121.0 | 2.140 | 126.4 | 1.72 | 108.6 | 1.75 | 110.0 | 1.850 | 114.2 |
| 19 | 3.20 | 169.3 | 3.12 | 166.5 | 3.350 | 175.7 | 2.78 | 152.8 | 2.92 | 158.5 | 2.010 | 162.1 |
| 20 | 1.45 | 97.1 | 1.46 | 97.4 | 1.520 | 100.1 | 1.33 | 91.9 | 1.34 | 92.3 | 1.380 | 94.0 |
| 21 | 1.52 | 100.1 | 1.52 | 100.1 | 1.580 | 102.7 | 1.37 | 93.8 | 1.38 | 94.0 | 1.430 | 96.2 |
| 22 | 1.53 | 100.7 | 1.53 | 100.7 | 1.610 | 104.0 | 1.38 | 94.0 | 1.39 | 96.5 | 1.440 | 96.6 |
| 23 | 1.68 | 107.0 | 1.62 | 104.4 | 1.690 | 107.4 | 1.43 | 96.2 | 1.48 | 98.4 | 1.680 | 107.0 |
| 24 | 1.85 | 114.2 | 1.80 | 112.1 | 1.920 | 117.1 | 1.57 | 102.3 | 1.63 | 104.9 | 1.810 | 112.4 |
| 25 | 2.50 | 141.4 | 2.56 | 143.9 | 2.750 | 151.6 | 2.00 | 120.6 | 2.11 | 125.1 | 2.330 | 134.3 |
| 26 | 3.45 | 179.7 | 3.27 | 172.5 | 3.570 | 182.1 | 2.57 | 148.3 | 2.88 | 156.8 | 2.885 | 157.0 |

TABLE I (Continued)


| Sam. No. | MV | 330 | MV 3 | $O_{F}$ | Poots of the Pins |  |  |  | MV 3 | $\mathrm{O}_{\mathrm{F}}$ | MV $38 \quad 0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | MV | $O^{2}$ |  |  |  |  |
| 1 | 1.19 | 85.9 | 1.08 | 81.0 | 1.1 .8 | 85.4 | 1.19 | 85.2 | 1.18 | 85.4 | 1. 13 | 83.2 |
| 2 | 1.20 | 86.3 | 1.14 | 83.6 | 1.18 | 85.0 | 1.20 | 86.3 | 1.18 | 85.4 | 1.14 | 83.6 |
| 3 | 1.25 | 89.4 | 1.18 | 85.0 | 1.22 | 87.1 | 1.25 | 88.4 | 1.23 | 87.6 | 1.19 | 85.2 |
| 4 | 1.31 | 91.0 | 1.25 | 88.4 | 1.29 | 90.2 | 1. 32 | 91.5 | 1.30 | 90.6 | 1.25 | 88.4 |
| 5 | 1.39 | 94.5 | 1.30 | 90.7 | 1.35 | 92.8 | 1.40 | 95.0 | 1.36 | 92.8 | 1. 30 | 90.7 |
| 6 | 1.60 | 104.7 | 1.47 | 98.0 | 1.58 | 102.7 | 1.62 | 104.4 | 1.57 | 102.3 | 1. 47 | 98.0 |
| 7 | 2.18 | 128.0 | 2.00 | 120.6 | 2.18 | 128.1 | 2.20 | 128.9 | 2.13 | 126.0 | 1.96 | 118.9 |
| 8 | 1.45 | 97.1 | 1.36 | 93.5 | 1.58 | 102.7 | 1.48 | 98.4 | 1.46 | 97.4 | 1. 38 | 94.0 |
| 9 | 1.54 | 101.0 | 1.42 | 95.8 | 1.56 | 101.9 | 1.56 | 101.9 | 1.54 | 101.0 | 1.46 | 97.4 |
| 10 | 1.59 | 103.1 | 1.47 | 98.0 | 1.62 | 104.4 | 1.62 | 104.4 | 1.58 | 102.7 | 1.50 | 99.2 |
| 11 | 1.69 | 107.9 | 1.53 | 100.7 | 1.70 | 107.9 | 1.70 | 107.9 | 1.67 | 106.7 | 1.56 | 101.9 |
| 12 | 2.09 | 124.3 | 1.85 | 114.2 | 2.10 | 124.7 | 2.12 | 125.5 | 2.07 | 123.5 | 1.87 | 115.0 |
| 13 | 3.23 | 172.9 | 2.94 | 159.3 | 3.32 | 174.5 | 3.34 | 175.3 | 3.24 | 171.3 | 2.90 | 157.7 |
| 14 | 1.63 | 104.9 | 1.50 | 99.0 | 1.69 | 107.4 | 1.68 | 107.0 | 1.66 | 105.9 | 1.55 | 101.4 |
| 15 | 1.76 | 110.3 | 1.60 | 103.7 | 1.81 | 112.4 | 1.80 | 112.1 | 1.78 | 111.2 | 1.65 | 106.7 |
| 16 | 1.79 | 111.7 | 1.62 | 104.4 | 1.84 | 113.8 | 1.82 | 113.0 | 1.81 | 112.4 | 1. 67 | 105.7 |
| 17 | 1.96 | 118.4 | 1.76 | 110.3 | 2.01 | 121.0 | 2.00 | 120.6 | 1.98 | 119.7 | 1.82 | 113.0 |
| 18 | 2.51 | 141.8 | 2.20 | 129.0 | 2.62 | 146.2 | 2.57 | 144.2 | 2.50 | 141.4 | 2.24 | 130.6 |
| 19 | 4.06 | 203.6 | 3.75 | 191.5 | 4.30 | 212.9 | 4.30 | 212.9 | 4.15 | 207.1 | 3.65 | 187.5 |
| 20 | 1.78 | 111.2 | 1.62 | 104.4 | 1.85 | 114.2 | 1.82 | 113.0 | 1.82 | 113.0 | 1.50 | 99.2 |
| 21 | 1.88 | 115.5 | 1.67 | 106.7 | 1.88 | 115.5 | 1.89 | 115.9 | 1.90 | 116.5 | 1.66 | 105.9 |
| 22 | 1.92 | 117.1 | 1.72 | 108.6 | 1.90 | 116.5 | 1.95 | 118.4 | 1.95 | 118.4 | 1.70 | 107.9 |
| 23 | 2.06 | 123.0 | 1.78 | 111.2 | 2.07 | 123.5 | 1.95 | 118.4 | 2.10 | 124.7 | 2.03 | 121.8 |
| 24 | 2.28 | 132.2 | 2.05 | 122.6 | 2.27 | 132.1 | 2.15 | 126.2 | 2. 2.5 | 131.0 | 2.20 | 129.0 |
| 25 | 3.11 | 166.1 | 2.75 | 151.6 | 3.08 | 164.0 | 2.91 | 158.1 | 3.20 | 169.7 | 3.00 | 161.7 |
| 26 | 4.27 | 211.9 | 4.20 | 209.0 | 4.50 | 220.6 | 3.99 | 200.5 | 4.50 | 220.6 | 4.32 | 213.7 |


| Sam. Mo. | Roots of the Pins |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | My 39 | $\mathrm{O}_{\mathrm{F}}$ | MV | $O_{F}$ |  | $O_{p}$ | My ${ }^{4}$ | $O_{F}$ | My 43 | ${ }^{\circ} \mathrm{F}$ | my 44 | $0^{\circ}$ |
| 1 | 1.13 | 83.2 | 1. 12 | 82.8 | 1.17 | 85.0 | 1.18 | 85.4 | 1.13 | 83.2 | 1.10 | 81.9 |
| 2 | 1.16 | 84.5 | 1.14 | 83.6 | 1.18 | 85.4 | 1.20 | 26.3 | 1.16 | 86.5 | 1.13 | 83.2 |
| 3 | 1.20 | 86.3 | 1.18 | 85.4 | 1.23 | 87.6 | 1.25 | 88.4 | 1.20 | 86.3 | 1.17 | 85.0 |
| 4 | 1.25 | 88.4 | 1.23 | 87.6 | 1.28 | 89.8 | 1.30 | 90.7 | 1.25 | 88.4 | 1.22 | 87.1 |
| 5 | 1.30 | 90.7 | 1.27 | 89.3 | 1.36 | 93.5 | 1.37 | 93.8 | 1.33 | 91.9 | 1.27 | 89.3 |
| 6 | 1.46 | 97.4 | 1.44 | 96.6 | 1.55 | 101.6 | 1.57 | 102.3 | 1.50 | 99.2 | 1.42 | 95.8 |
| 7 | 1.92 | 117.2 | 1.80 | 112.1 | 1.97 | 119.3 | 2.00 | 120.6 | 1.91 | 116.9 | 1.78 | 111.2 |
| 8 | 1.37 | 93.8 | 1.35 | 92.8 | 1.45 | 97.1 | 1.45 | 97.1 | 1.39 | 96.5 | 1.33 | 91.9 |
| 9 | 1.42 | 95.8 | 1.40 | 95.0 | 1.51 | 99.8 | 1.51 | 99.8 | 1.45 | 97.1 | 1.37 | 93.8 |
| 10 | 1.48 | 98.4 | 1.53 | 96.2 | 1.59 | 101.9 | 1.57 | 102.3 | 1.50 | 99.2 | 1.42 | 95.8 |
| 11 | 1.54 | 101.0 | 1.50 | 99.2 | 1.65 | 106.7 | 1.65 | 106.7 | 1.60 | 103.7 | 1.48 | 98.4 |
| 12 | 1.87 | 115.0 | 1.80 | 112.1 | 2.02 | 121.4 | 2.03 | 121.8 | 1.95 | 118.6 | 1.77 | 110.7 |
| 13 | 2.78 | 152.8 | 2.52 | 142.1 | 2.88 | 156.8 | 2.93 | 158.9 | 2.76 | 152.0 | 2.48 | 140.5 |
| 14 | 1.51 | 99.8 | 1.47 | 98.0 | 1.63 | 104.9 | 1.61 | 104.0 | 1.55 | 101.4 | 1.45 | 97.4 |
| 15 | 1.62 | 104.4 | 1.55 | 101.4 | 1.74 | 109.5 | 1.73 | 109.1 | 1.65 | 105.7 | 1.5? | 100.7 |
| 16 | 1.63 | 104.9 | 1.56 | 101.9 | 1.75 | 110.0 | 1.75 | 110.0 | 1.68 | 107.0 | 1.55 | 101.4 |
| 17 | 1.78 | 111.2 | 1.70 | 107.9 | 1.92 | 117.1 | 1.92 | 117.1 | 1.85 | 114.2 | 1.69 | 107.4 |
| 18 | 2.23 | 130.2 | 2.10 | 124.7 | 2.42 | 138.1 | 8.45 | 139.3 | 2.36 | 134.8 | 2.08 | 123.9 |
| 19 | 3.77 | 192.7 | 3.56 | 184.0 | 4.03 | 202.5 | 4.07 | 206.0 | 3.83 | 196.4 | 3.45 | 189.1 |
| 20 | 1.64 | 105.3 | 1.58 | 102.7 | 1.78 | 111.2 | 1.77 | 110.7 | 1.70 | 107.9 | 1.57 | 102.3 |
| 21 | 1.69 | 107.4 | 1.63 | 106.9 | 1.85 | 115.2 | 1.83 | 113.4 | 1.76 | 110.3 | 1.62 | 104.4 |
| 22 | 1.73 | 109.1 | 1.64 | 105.3 | 1.88 | 115.5 | 1.87 | 115.0 | 1.78 | 111.2 | 1.63 | 104.9 |
| 23 | 1.75 | 110.0 | 1.80 | 112.1 | 1.97 | 119.3 | 2.10 | 124.7 | 2.04 | 122.4 | 1.87 | 115.0 |
| 24 | 1.90 | 116.5 | 1.96 | 118.9 | 2.16 | 127.2 | 2.30 | 133.0 | 2.25 | 131.0 | 2.07 | 123.5 |
| 25 | 2.60 | 145.5 | 2.65 | 147.6 | 2.90 | 157.7 | 3.20 | 169.7 | 3.07 | 164.5 | 2.77 | 152.4 |
| 26 | 3.61 | 186.0 | 3.70 | 189.6 | 3.92 | 198.2 | 4.50 | 220.6 | 4.40 | 216.7 | 3.72 | 190.3 |

## TABLP I (Continued)

| Sample <br> Number | MV | $\mathrm{O}_{\mathrm{F}}$ | ```Average pin Root temperature in Op``` | ```memperature Difference in O``` |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1.12 | 82.8 | 83.7 | 7.70 |
| 2 | 1. 15 | 84.9 | 84.72 | 8.72 |
| 3 | 1.18 | 85.4 | 86.47 | 0.17 |
| 4 | 1.24 | 88.0 | 89.09 | 11.09 |
| 5 | 1.31 | 91.0 | 91.70 | 12.70 |
| 6 | 1.47 | 98.0 | 99.67 | 20.67 |
| 7 | 1.87 | 115.0 | 119.57 | 40.57 |
| 8 | 1.38 | 94.0 | 95.60 | 16.60 |
| 9 | 1.42 | 95.8 | 07.90 | 18.90 |
| 10 | 1.46 | 97.4 | 99.99 | 20.80 |
| 1.1 | 1.53 | 100.7 | 103.29 | 23.79 |
| 12 | 1.37 | 115.0 | 118.09 | 37.09 |
| 13 | 2.69 | 249.3 | 157.50 | 74.50 |
| 14 | 1.50 | 09.2 | 102.13 | 24.13 |
| 15 | 1.60 | 103.7 | 106.63 | 28.63 |
| 15 | 1.62 | 104.4 | 197.50 | 29.50 |
| 17 | 1.76 | 110.3 | 114.15 | 35.15 |
| 18 | 2.24 | 130.6 | 136.10 | 56.10 |
| 19 | 3.87 | 196.2 | 197.15 | 113.16 |
| 20 | 1.62 | 104.4 | 107.36 | 29.36 |
| 21 | 1.68 | 107.0 | 110.16 | 32.16 |
| 22 | 1.70 | 107.9 | 111.56 | 34.56 |
| 23 | 1.78 | 111.2 | 117.90 | 39.90 |
| 24 | ]. 93 | 117.6 | 125.70 | 45.70 |
| 25 | 2.57 | 144.2 | 157.30 | 76.30 |
| 2 F | 3.92 | 158.2 | 205.78 | 120.78 |

Measured Heat Transfer Data from the Finned Pins at Constant Flow Rate and Varying Heat Inputs

| $\begin{gathered} \text { Sam. } \\ \text { No. } \end{gathered}$ | ```Volt me- ter read. in Volts``` | Seat Supply Data |  | Air Flow Data |  |  |  | Data of the Avg. Thermocouples at the Roots of Pins |  | Temp Diff. $\theta$ in OF |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ```Am. me- ter read. in Amps``` | ```Watt me- ter read. in watts``` | ```Heat Input ``` | Pr. Diff. <br> in inches <br> of <br> Water | Room Temp in $\mathrm{O}_{\mathrm{F}}$ | Reynolds Numbers |  |  |  |
| 1 | 36.5 | 2.90 | 100.0 | 362.0 | 0.0438 | 90.0 | 4120.0 | 1.87 | 115.10 | 25.1 |
| 2 | 53.0 | 4.00 | 245.0 | 724.0 | 0.0438 | 90.0 | 4120.0 | 2.42 | 138.00 | 48.0 |
| 3 | 69.0 | 5.10 | 400.0 | 1202.0 | 0.0438 | 90.0 | 4120.0 | 3.17 | 168.70 | 72.7 |
| 4 | 82.0 | 5.90 | 550.0 | 1652.0 | 0.0438 | 30.0 | 4120.0 | 3.70 | 189.60 | 99.6 |
| 5 | 94.0 | 6.30 | 594.0 | 2025.0 | 0.0438 | 90.0 | 4120.0 | 4.20 | 209.00 | 119.0 |
| 6 | 102.5 | 6.80 | 696.0 | 2375.0 | 0.0438 | 90.0 | 4120.0 | 4.70 | 228.20 | 138.2 |
| 7 | 35.0 | 2.80 | 100.0 | 335.0 | 0.0973 | 89.0 | 6150.0 | 1.85 | 104.30 | 15.3 |
| 8 | 53.0 | 3.45 | 200.0 | 695.0 | 0.0973 | 89.0 | 6150.0 | 1.96 | 118.70 | 29.7 |
| 9 | 68.0 | 5.00 | 400.0 | 1161.0 | 0.0973 | 89.0 | 6150.0 | 2.31 | 133.50 | 4.4 .5 |
| 10 | 82.0 | 5.95 | 560.0 | 1668.0 | 0.0973 | 89.0 | 6150.0 | 2.75 | 151.50 | 62.5 |
| 11 | 92.0 | 6.60 | 700.0 | 2075.0 | 0.0973 | 89.0 | 6150.0 | 3.05 | 163.80 | 74.8 |
| 12 | 102.0 | 7.30 | 840.0 | 2540.0 | 0.0973 | 89.0 | 6150.0 | 3.54 | 193.20 | 94.2 |
| 13 | 34.0 | 2.75 | 90.0 | 319.0 | 0.2055 | 88.5 | 8560.0 | 1.47 | 98.10 | 9.6 |
| 14 | 53.0 | 4.05 | 250.0 | 732.0 | 0.2055 | 88.5 | 8660.0 | 1.74 | 109.50 | 21.0 |
| 1.5 | 68.5 | 5.00 | 400.0 | 1170.0 | 0.2055 | 88.5 | 8660.0 | 2.02 | 121.20 | 32.7 |
| 16 | 82.0 | 5.90 | 560.0 | 1650.0 | 0.2055 | 88.5 | 8660.0 | 2.30 | 133.20 | 44.7 |
| 17 | 93.0 | 6.60 | 705.0 | 2095.0 | 0.2055 | 88.5 | 9660.0 | 2.57 | 144.20 | 55.7 |
| 18 | 102.0 | 7.25 | 855.0 | 2520.0 | 0.2055 | 88.5 | 8660.0 | 2.96 | 160.20 | 71.5 |
| 19 | 111.0 | 7.90 | 1000.0 | 2990.0 | 0.2055 | 88.5 | 8560.0 | 3.29 | 173.40 | 84.9 |
| 20 | 41.0 | 2.60 | 100.0 | 364.0 | 0.2392 | 80.0 | 9550.0 | 1.28 | 89.70 | 9.7 |
| 21 | 57.0 | 3.50 | 200.? | 680.0 | 0.2392 | 80.5 | 9550.0 | 1.50 | 99.20 | 18.7 |
| 22 | 68.1 | 4.30 | 300.0 | 1000.0 | 0.2392 | 81.0 | 9550.0 | 1.71 | 108.10 | 27.1 |
| 23 | 78.5 | 4.90 | 395.0 | 1312.0 | 0.2392 | 82.0 | 9550.0 | 1.92 | 117.20 | 35.2 |
| 24 | 88.0 | 5.50 | 500.0 | 1668.0 | 0.2392 | 81.0 | 9550.0 | 2.12 | 125.40 | 44.4 |
| 25 | 97.0 | 6.10 | 600.0 | 2020.0 | 0.2392 | 80.0 | 9550.0 | 2.29 | 132.70 | 52.7 |
| 26 | 105.5 | 6.70 | 725.0 | 2410.0 | 0.2392 | 80.0 | 9550.0 | 2.55 | 143.40 | 63.5 |
| 27 | 118.0 | 7.40 | 875.0 | 2990.0 | 0.2392 | 78.0 | 9550.0 | 2.79 | 153.20 | 75.2 |
| 28 | 121.5 | 7.75 | 950.0 | 3210.0 | 0.2392 | 79.0 | 9550.0 | 2.99 | 161.50 | 82.5 |
| 29 | 124.0 | 8.00 | 1000.0 | 3395.0 | 0.2392 | 80.0 | 9550.0 | 3.13 | 166.80 | 86.8 |

Measured Heat Transfer Data from the Dlain Pin
at Constant Plow Pate and Varying Heat Inputs


The original idea of Professor Usu on extended surfaces with finned pins has been proved by Usieh and Vatsaraj that the finned pins can improve the rate of heat transfer by as much as 85 percent with an increase in effectiveness of 61 percent. The objective of conducting this investigation was to find the improvement in the rate of heat transfer by increasing the air-side film coefficient through forced convection. In this thesis, the variables of the finned pin were optimized for operation under a condition of forced convection. An experimental investigation was conducted to verify the superiority of the finned pin over other extended surfaces under forced convection.

1. Theoretical investigation consisted of
(a) Determination of the relationship between the overall heat transfer coefficient and the individual air-side film coefficient.
(b) Optimization of the variables of the finned pin, and
(c) Derivation of the empirical fromula for the film coefficient in the Musselt's cquation form.
2. Pxperimental investigation comnrised of:
(a) Fuilding an apoaratus to test the heat removal canability of the extended surfaces at various Peynolds numbers, and
(h) Testing for comparison of heat flow rates for
the finned pin and the plain oin.
3. Conclusions were based on: Comparisons of the heat transfer rates of the finned pin and the plain pin for different flow rates. At equal flow rates, the finned pin was shown to transfer more heat than the plain pin. For a reasonahle increase in the film coefficient under forced convection, the finned pin was found to improve in effectiveness by four and one-half times over that when under free convection, and about 28 times over the primary surface under free convection.
